A Model of the monetary system of Medieval Europe

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"any study of the money supply [of medieval Europe] needs to take account not only of the total face value of the currency, but also of the metals and denominations of which it is composed." (Mayhew (2004);82)

The history of commodity money systems is replete with denominational problems. In many instances these are complaints of an absence of small denomination coins (a 'scarcity of small change') but there were also occasions when there complaints of an absence of large denomination media of exchange. The complaints about the absence of small or large denomination coins reflect the very limited types of coins that circulated in commodity money systems. In medieval Europe mints typically produced only one type of coin, a silver penny stamped on both sides, weighing about 1.7 grams and being about 18 mm in diameter. Over the ensuing centuries pennies continued to be minted, but their silver content and fineness declined. Then in the early 13th century, the Italian city states began adding a second coin type, a 'grosso' that was very similar to Charlemagne's penny - of high fineness, but slightly heavier, around 2 gms. Most other European mints followed suit in the mid 13th century, issuing even larger silver coins even as they continued to mint the penny. In the mid-13th century, many mints also began issuing gold coins adding further choices to the coin types.

In this paper we build a model of a commodity money system with a limited number of types of coins and show how the choices of coin type influences economic welfare through the distribution of wealth and output. We also use the model to show the consequences of changes in the stock of the monetary commodity or the extent of trade on the economy. In a previous paper Redish and Weber (2010) we built a random matching monetary model with two indivisible coins and illustrated that a small change shortage could exist in the sense that adding small coins to the economy with large coins was welfare improving. However, in that paper we assumed a fixed quantity of coins of each type and compared the characteristics of equilibria with different stocks of, and sizes of, coins. The commodity value of the coins was imposed by assuming that each coin paid a dividend (essentially was a Lucas tree). In this paper we endogenize the quantity of money. We assume that there is a fixed stock of each monetary metal and allow agents to choose to mint coins (at a cost), or to melt the coins into jewelry. Thus the model combines a more fundamental way of generating valued commodity money, and an endogenous quantity of money (though not quantity of monetary metal) providing a much richer framework to discuss the shortage of small coins.

Our model is closely related to that of Velde and Weber (2000) and Lee, Wallace, and Zhu (2005). **?** model a commodity money system allowing agents to hold monetary metal as coins or in a form that yields direct utility - jewelery. The demand for money in their model is driven by a cash-in-advance constraint. Lee, Wallace, and Zhu (2005) build a random matching model in which agents can hold multiple units of various types of fiat money. Agents engage in pair-wise trade alternately with periods where they can rebalance their portfolio subject only to a wealth constraint. They prove the existence of a monetary steady state in which agents trade off the benefits of small denomination notes for transacting against their higher handling costs. As in Lee, Wallace, and Zhu (2005) we motivate the demand for money with a random matching model however, our agents use a commodity money and thus there is an additional margin - the marginal utility of jewelry - that bears on how many coins an agent wants to hold.

The paper proceeds as follows: in the next section we describe the types of coinage

Table 1: The penny coinages of the late 12th century

	Year	Weight	Fineness	Fine weight
Cologne	1160	1.4 gms	.925	1.30 gms
England	1160	1.46 gms	.925	1.35 gms
Paris	1160	1.28-.85 gms	$< .5$	$< .60$ gms
Melgueil	1125	1.1 gms	.42	0.46 gms
Lucca	1160	0.6 gms	0.6	0.35 gms
Barcelona	1160	0.66 gms	0.2	0.13 gms
Venice	1160			0.09 gms

Note: a U.S. dime weighs 2.268 gms.
Source: Spufford (1988); 102-3. Lane and Mueller (1985); 527.

produced in the mints of Venice and of England between 800 and 1400. These two locations are used in part because they are relatively well documented and in part because they represent two different examples of coinage structure. Section 2 then presents a model of a monetary system which enables us to evaluate the consequences of different denomination structures. We then use numerical examples to describe the equilibrium behavior of the economy and how the optimal denomination structure varied depending on the stock of monetary metal and the expansion in markets. Section 4 then uses the results to argue that the changes in denomination structure in medieval Venice and in England are consistent with those that would have been optimal responses to the changing economic environment.

1 The monetary system

From the time of Charlemagne until the end of the 12th century, the denier (or penny) was essentially the only denomination minted in Europe.[1] Charlemagne standardized the coinage of the many mints in the Holy Roman Empire with a penny of high fineness a diameter of about 18 mm. and a weight of about 1.7 grams. The English, not part of the Empire, also minted silver coins that were similar in fineness and weight to the denier. Not coincidentally 240 pennies weighed approximately one pound. Although pennies were the sole coin minted, accounts often used the collective noun shilling (or sol) to refer to a dozen pennies, and pound (or lire) to refer to a score of dozens (240 pence).

The uniformity that Charlemagne imposed did not outlive him. Within the empire, counts began operating their mints on their own account and throughout Europe, minting rights were assumed by bishops and seigneurs (the origin of the term seignorage) who became the local monetary authority. Even England, the country with the most centralized regime, had over 70 mints by the end of the 10th century. Throughout Europe only gradually did minting revert to the central (royal) authority. Each monetary authority chose the weight

[1]Some German states issued very different coins - 'bracteates' that were thin, ahd a large diameter and were only stamped on one face. A few states that issued deniers also minted obols - half deniers - however the extent of this coinage is hard to know. We follow Favier (1998; 127) who speaks of 'the single denomination the denier and nothing but the denier'.)

Table 2: The introduction of new coins, 13th and 14th centuries

	Coin	Metal	Fine weight	Value in unit of account	Silver equivalent
Venice					
1172	Denarius	Silver	.10 gms	1d	
1201	Grossus	Silver	2.1 gms	24d	
1284	Grossus	Silver	2.1 gms	32d	
1284	Ducat	Gold	3.5 gms	576d	50gms
France					
1200	Denier	Silver	0.36 gms	1d	
1266	Gros t	Silver	4.2 gms	12d	
1290	Royale	Gold	6.97gms	300d	100 gms
England					
1200	Penny	Silver	1.4 gms	1d	
1279	Farthing	Silver	.34 gms	.25d	
1344	Noble	Gold	8.8 gms	80d	103 gms
1351	Groat	Silver	4.68 gms	4d	

Sources: Lane and Mueller (1985), de Wailly (1857), Challis (1992)

and fineness of the penny that it minted and all diminished both but to very different extents. By 1200 the characteristics of the penny ranged widely (see Table 1) from the English penny, weighing 1.46 gms and 92.5% fine, to the many French coins weighing about 1 gram and about 35% fine, to the Italian coins that were about one quarter of a gram and less than 30% fine (Spufford (1988) p.102-3).

The use of a single type of coin in an economy made payments of both large and small amounts difficult. In the mid-13th century for example, English agricultural labourers earned about 1.6 pence per day and a penny bought for loaves of bread. Throughout Europe the options for paying large values were also limited. There were some Byzantine gold coins, particularly in the South of Europe. Silver ingots, stamped with their fineness and often weighing an integer number of marks (a mark weighed about 220 grams), were used on occasion but even for large payments pennies were used, frequently in sealed bags certified for their contents. Spufford (1988) p.210 cites the example of the papal collector for Northern Europe who collected over 70,000 marks in pence (close to 1 million pence!), as well as over 1,000 marks in ingots for remittance to the Pope.

The practice of minting only a single type coin ended in Italy in the last decade of the 12th century. In 1190 the Milanese began minting a grosso that was of high fineness and weighed over 2 gms. This coin only circulated locally, but a few years later the Venetians began minting a similar coin which circulated widely. The Venetian grosso contained roughly 24 times as much silver as the debased denari that were in circulation. Other mint authorities followed suit (see Table 2): the French introduced a 'gros' in 1266 and the English, a groat in 1351.[2] In England in contrast, the new denominations that were minted beginning in 1279 fractions of a penny: the halfpenny and farthing (quarter penny).

[2]The mint ordinance of 1279 allowed the minting of groats however numismatists argue that essentially

In the late 13th/ early 14th century the denomination structure became yet richer with the introduction of gold coins, but this still left a large gap in the denomination structure. Because gold is roughly twice as dense as silver (19.3gms/cm^3 compared to 10.5gms/cm^3) if gold were worth 10 times as much as silver by weight a gold coin of the same size as a 2 gm silver coin would be worth 20 times as much. While coining technology was restricted to hammer and shears the mints were not able to issue very heavy coins. It was not until screw presses and rolling mills were introduced into mints in the 16th century, that coins of up to 30 gms were produced; until then the largest silver coins weighed less than 5 gms.[3]

The sparse number of coin types led to frequent complaints about the coinage - some concerning a lack of small change and others of a lack of high value coins and underlies the quotation at the head of the paper.[4]

2 The Model

2.1 Environment

We build a random matching models in which agents are permitted to hold multiple numbers of each of two types of coins. The model has discrete time and an infinite number of periods. There is a nonstorable, perfectly divisible good and two metals (durable commodities) — silver and gold — in the economy. There are m_s ounces of silver and m_g ounces of gold in existence.

The way in which we model commodity money follows that used by Velde and Weber (2000). We assume that silver can be held in either of two forms, which we will refer to as *coins* and *jewelry*. Silver held in the form of jewelry yields utility directly to the holder, whereas silver held in the form of coins yields no utility to the holder. Because of difficulties in determining the weight and fineness of jewelry, we assume that only coins can be traded. There is a utility cost γ of holding a coin. All gold is held as jewelry.[5]

The technological limitations on minting in the medieval period placed restrictions on what the monetary authority could do with coinage. For one, although metals could be divided, they could be divided only imperfectly. This meant that coins had to be indivisible. Further, coins could not be too small, because of the possibility of loss, and they could not be too heavy or they would be difficult to carry. The role of the monetary authority in this environment was to choose how many ounces of metal to put into a coin of that metal subject to these limitations.

In our model, we allow for the minting of two types of silver coin distinguished by the weight of silver in each coin. We let b_s^1 be the ounces of silver that it puts in the first silver coin, and $b_s^2 = \eta b_s$ be the amount of silver in the second coin, where η is an integer greater

none were minted between 1279 and 1344; see Section 4.4 below.

[3] Cooper (1988) for the history of coining technology and the limitations it imposed; see **?** for discussion of the coin sizes).

[4] The literature on the scarcity of small change is plagued by confusion between the problem of a 'scarcity of small change' and a 'want of money'. Here we focus on the denomination question.

[5] Our reason for including gold is that we intend to consider the possible use of gold coins in a later version of the paper.

than one. As was the case with coins throughout most of the time during which commodity monies were used, these coins do not have denominations. They are simply an amount of silver that has been turned into coins with some type of standardized markings that allow one type of coin to be easily differentiated from a different type of coin.

Each period in the model is divided into two subperiods. In the first subperiod agents can trade in bilateral matches. At the beginning of this subperiod, an agent has a probability $\frac{1}{2}$ of being a consumer but not a producer and the same probability of being a producer but not a consumer. This assumption rules out double coincidence matches, and therefore gives rise to the essentiality of a medium of exchange.

After agents' types (consumer or producer) are revealed, a fraction $\theta \in (0, 1]$ of agents are matched bilaterally. In a match, the portfolios of both agents are known. However, past trading histories are private information and agents are anonymous. These assumptions rule out gift-giving equilibria and the use of credit. Thus, trading can only occur through the use of media of exchange, which is the role that the gold and silver coins can play.

In the second subperiod, agents can alter their mix of the two types of silver coins and jewelry by minting or melting. That is, in the second subperiod agents can change the form in which they hold the stock of silver, but they cannot change the total quantity in the economy.

2.2 Consumer choices

There is a $[0, 1]$ continuum of infinitely lived agents in the model that maximize expected discounted lifetime utility. The discount factor is β. Let c be the quantity of the good consumed; q, the quantity of the good produced; and j^s, the holdings of silver jewelry in terms of units of the small coin. The agent's period preferences are

$$u(c) - q + \mu(b_s^1 j^s, m_g)$$

with $u(0) = 0$, $u' > 0$, $u'' < 0$, and $u'(0) = \infty$. The disutility of good production is assumed to be linear without loss of generality. We assume that an agent's utility from holding silver and gold jewelry is $\mu(b_s j^s, m_g)$ with $\mu_i > 0$, $\mu_{ii} \leq 0$.

Let s_1 and s_2 be an agent's holdings of small and large silver coins, respectively, and . The agent's portfolio of metal holdings is

$$y = \{(s_1, s_2, j^s, m_g) : s_1, s_2, j^s, m_g \in \mathbb{N}\}.$$

We assume that in a single coincidence pairwise meeting, the potential consumer gets to make a take-it-or-leave-it (TIOLI) offer to the potential seller. This offer will be the triple (q, p_1, p_2), where $q \in \Re_+$ is the quantity of production demanded, $p_i \in \mathbb{Z}$ is the quantity of silver coins of type i offered. Offers with $p_1 < 0$ can be thought of as the seller being asked to make change.

Let $v(y) : \mathbb{N} \bigotimes \mathbb{N} \to \Re_+$ be the expected value of an agent's portfolio at the beginning of the second subperiod. The set of feasible TIOLI offers is a combination of special good output and coins that is a feasible coin offer and satisfies the condition that the seller be no worse off than not trading. Denoting this set by $\Gamma(y, \tilde{y})$,

$$\begin{aligned}\Gamma(y, \tilde{y}) &= \{(q, p_1, p_2) : q \in \Re_+, -\tilde{s_1} \leq p_1 \leq s_1, -\tilde{s_2} \leq p_2 \leq s_2, \\ &\quad -q + v(\tilde{s_1} + p_1, \tilde{s_2} + p_2, \tilde{j}^s, \tilde{m}^g) \geq v(\tilde{y})\},\end{aligned}$$

where $\tilde{y}$ denotes the seller's portfolio. In the second subperiod, the agent can mint or melt. Define $z_i \in \mathbb{Z}$ to be the quantity of silver coins of type i minted (> 0) or melted (< 0). Agents cannot melt more coins than they have, and agents cannot mint in terms of metal more coins that they have. Note that this does not prevent them from melting coins of one type and minting coins of the other from the metal gained by melting. Denote the set of minting and melting choices available to an agent by $\Omega(y)$

$$\Omega(y) = \{(z^s) : z_i \geq -s_i, z_1 + \eta z_2 \leq j^s\}$$

Agents can melt coins without incurring a cost. However, if agents mint, they incur a seigniorage cost $S(z_1, z_2; j^s, m_g)$. To keep the analysis tractable, we will assume that no metal is lost by an agent when minting. Instead, we assume that the seigniorage an agent has to pay is in terms of utility, but does depend how many coins are minted.

2.3 Equilibrium

The three components needed are the value functions (Bellman equations), the asset transition equations, and the market clearing conditions. We proceed to describe each in turn.

Value functions

Let $w_t(y_t)$ and $v_t(y_t)$ be the expected values of holding y_t at the beginning of the first and second subperiods of period t, respectively. Then the Bellman equation at the beginning of the second subperiod is

$$v_t(y_t) = \max_{(z_{1t}, z_{2t}) \in \Omega(y)} \{\beta w_{t+1}(s_{1t} + z_{1t}, s_{2t} + z_{2t}, j_t^s - z_{1t} - \eta z_{2t}) - S(z_{1t}, z_{2t}; j_t^s, m_g)\}$$

The Bellman equation at the beginning of the first subperiod is

$$\begin{aligned}w_t(y_t) &= \frac{\theta}{2} \sum_{\tilde{y}_t} \pi_t(\tilde{y}_t) \max_{(q_t, p_{1t}, p_{2t}) \in \Gamma(y, \tilde{y})} [u(q_t) + v_t(s_{1t} - p_{1t}, s_{2t} - p_{2t}, j_t^s, m_g)] \\ &\quad + (1 - \frac{\theta}{2}) v_t(y_t) + \mu(b_s^1 j_t^s, m_g) - \gamma(s_{1t} + s_{2t})\end{aligned}$$

where $\pi_t(y_t)$ is the fraction of agents with y_t at the beginning of the first subperiod. The first term on the right-hand side is the expected payoff from being a buyer in a single coincidence meeting, which occurs with probability $\frac{\theta}{2}$. The second term is the expected payoff either from being the seller in a single coincidence meeting or from not have a meeting. The third term is the utility from holding silver jewelry and gold, and the final term is the utility cost of carrying silver coins.

Asset holdings

Define $\lambda^b(k,k';y_t,\tilde{y}_t)$ to be the probability that a buyer with y_t meeting a seller with $\tilde{y}_t$ leaves with $s_1 = k, s_2 = k'$. That is,

$$\lambda^b(k,k';y,\tilde{y}) = \begin{cases} 1 & \text{if } k = s_1 - p_1(y,\tilde{y}) \text{ and } k' = s_2 - p_2(y,\tilde{y}) \\ \\ 0 & \text{otherwise.} \end{cases}$$

and define $\lambda^s(k,k';y_t,\tilde{y}_t)$ similarly for the seller.

Then the post-trade (pre-mint/melt) asset distribution is

$$\begin{aligned} \tilde{\pi}_t(k,k',j^s) &= \frac{\theta}{2}\sum_{y_t,\tilde{y}_t} \pi_t(y_t)\pi_t(\tilde{y}_t)[\lambda^b(k,k';y_t,\tilde{y}_t) + \lambda^s(k,k';y_t,\tilde{y}_t)] \\ & \quad +(1-\theta)\pi_t(k,k',j^s) \end{aligned}$$

The first term on the right-hand side is the fraction of single coincidence meetings in which the buyer leaves with k small silver coins and k' large silver coins. The second term is the fraction of such meetings in which the seller leaves with k small silver coins and k' large silver coins. The final term is the probability that no meeting occurs, in which case no coins change hands.

Next define $\delta(k,k',h,y_t)$ to be the probability an agent with y_t after trade has $s_1 = k$, $s_2 = k'$, and $j^s = h$ after minting or melting. Then the post-mint/melt (pre-trade next period) asset distribution is

$$\pi_{t+1}(k,k',h) = \sum_{y_t} \tilde{\pi}_t(y_t)\delta(k,k',h,y_t)$$

Of course, asset holdings must also satisfy $\sum_y \pi(y) = \sum_y \tilde{\pi}(y) = 1$.

Market clearing

The market clearing condition is that the stock of silver metal must be held. That is,

$$\sum_y b_s^1(s_1 + \eta s_2 + j^s)\pi(y) = m_s$$

Definition 1 ***Steady state equilibrium:*** *A steady state equilibrium is value functions w, v; asset holdings π and $\tilde{\pi}$; and quantities p_1, p_2, z_1, z_2, q that satisfy the value functions, the asset transition equations, and market clearing.*

This section includes discussion of how fixing the coin size fixes the price level; it also discusses the tradeoff that determines the optimal size of a coin - that is, on the one hand small coins deliver finer offers and therefore more trade but large coins economize on handling costs. If there were no handling costs the optimal single coin size would be very small.

3 Results – Single Silver Coin

Because of the long period in which there was only a single silver coin in medieval Europe and to get some intuition about how the model works, we begin by studying our economy when there is only a single silver coin.

We are unable to prove the existence of steady state equilibria or to obtain analytic results for even this simple case of our model. Therefore, we rely on computed equilibria for numerical examples to obtain our results. Specifically, we assume $u(q) = q^{1/4}, \mu(b_s^1 j_s, m_g) = 0.05(b_s^1 j_s)^{1/2} + 0.1581(m_g)^{1/2}, \beta = 0.9, \theta = \frac{2}{3}, m_s = 0.1, m_g = 0.01$, and $\gamma = 0.001$. Because we are interested in small change, we will consider various values for b_s^i. Although we have not attempted to calibrate the parameters in the numerical analysis, the relative weights of silver and gold in the jewelry utility function are chosen to approximate the relative market ratio of 1:10 for these metals during the early medieval period.

To avoid problems in keeping track of the quantity of silver in the economy, we define seigniorage in terms of foregone utility from holding jewelry. Specifically, we assume

$$S(z_{1t}; j_t^s, m_g) = \max[\mu(b_s j_t^s, m_g) - \mu(b_s j_t^s - b_s \sigma_s z_{1t}, m_g), 0]$$

where σ_s is the seigniorage rate. The term $\mu(b_s j_t^s, m_g)$ is the utility from holding jewelry before doing any minting. The term $\mu(b_s j_t^s - b_s \sigma_s z_{1t}, m_g)$ is the utility from holding that amount of jewelry less the amount of jewelry given up to pay the seigniorage tax on minting coins from jewelry. The max with 0 is because seigniorage is only paid on minting. We use a seigniorage rate of $\sigma_s = 0.04$ which was approximately the average seigniorage rate on silver during this period.

In the steady state, the welfare function is concave; that is, the welfare of individuals is increasing (at a decreasing rate) in the number of units of coins and the number of units of silver jewelery that they hold. Further, the distribution of coin and jewelry holdings is nondegenerate but does not have full support. This is shown in Figure 1, where we show the fraction of agents holding each pair of coin and jewelry. In this figure, the size of the silver coin is $b_s^1 = 0.009$, so that the available stock of silver is divided into 11.1 units in terms of coins. The average holding of coins is 2.23, while the average holding of jewelery is 8.87 units of silver in terms of coins. The most frequent holding (9.3% of the agents) is 10 jewels and 2 coins; 0.7 percent of agents hold neither coins or jewelry. The figure also shows that holdings of jewelry and coins increase with an agent's silver wealth and the each wealth level is associated with a unique coin-jewelry pair.

3.1 Varying coin sizes

Ex ante welfare in this economy depends upon the size of the single silver coin. This is shown in Figure 2.[6] The optimal size coin ($b_s^1 = 0.009$ in this case) reflects the trade off between

[6]Throughout the analysis, our welfare criterion is ex ante welfare, computed as $\bar{w} = \sum_y \pi(y)w(y)$. For comparison, we note that ex ante welfare in an economy in which a planner could instruct agents to give gifts of the optimal quantity of output $q* = 0.15$ is $\bar{w} = 1.89$. Welfare in the planner economy is higher both because agents could use all the silver for utility-yielding jewelry and because the level of output produced/exchanged would not depend on the coin-holding of each party.

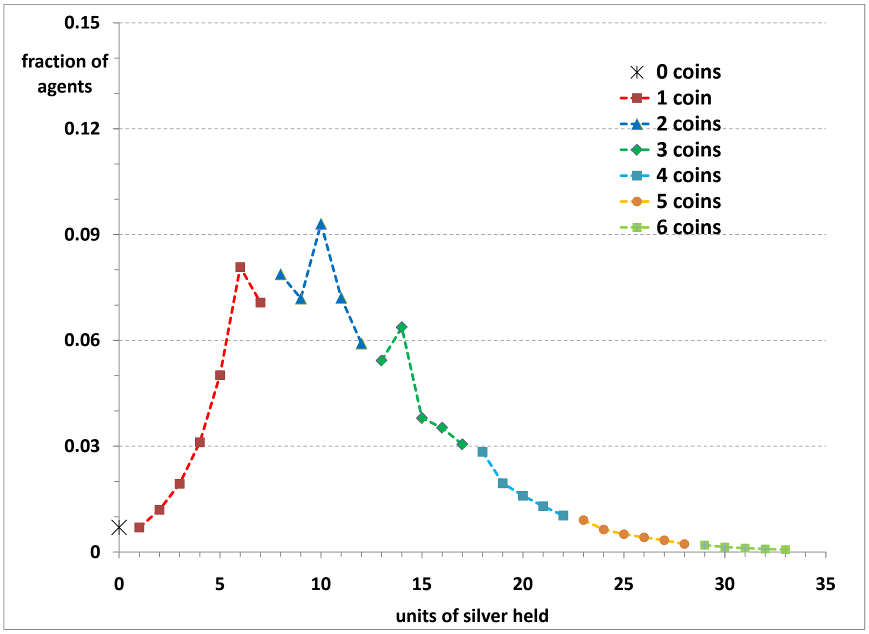

Figure 1: Distribution of coin and jewelry holdings, $b_s^1 = 0.009$

the benefit of having a less valuable coin (a coin with less silver in it) and therefore being able to make more finely calibrated offers, and the cost of having more handling more coins to get the same weight of silver in terms of coins. Having a single, large size coin in the economy saves on handling costs, but it means that potential buyers with coins will only be willing to trade if they encounter a potential seller who is willing to produce a large quantity of output. Otherwise, they are better off melting the coin into jewelry. Such meetings will not occur all that frequently, because the only potential sellers who are willing to produce a large quantity of output are those with small holdings of silver wealth. In contrast, a small size coin increases handling costs, but means that potential buyers will be willing to trade when they encounter potential sellers who are willing to produce a smaller quantity of output. Such meetings will occur more frequently, because there is a larger fraction of such potential sellers. Note that handling costs are critical in the determination of optimal coin size. For example, if γ were equal to zero, then it would be optimal to make the coin infinitely small.

Of course, because the types of trades that occur will differ depending on the size of the coin for the reasons just stated, the distribution of coin and jewelry holdings will depend on the size of the coin. This is illustrated by comparing Figure 1 above which shows coin and jewelry holdings when the coin size is $b_s = 0.009$ with Figure 3 which shows coin and jewelry holdings when the coin size is $b_s = 0.015$. The figures show that for the larger size coin there are more agents who have neither coins nor jewelry. Additionally, for any level of silver holdings in terms of units of coins, agents hold more silver wealth in the form of jewelry and less in the form of coins. The reason is the fewer trading opportunities available

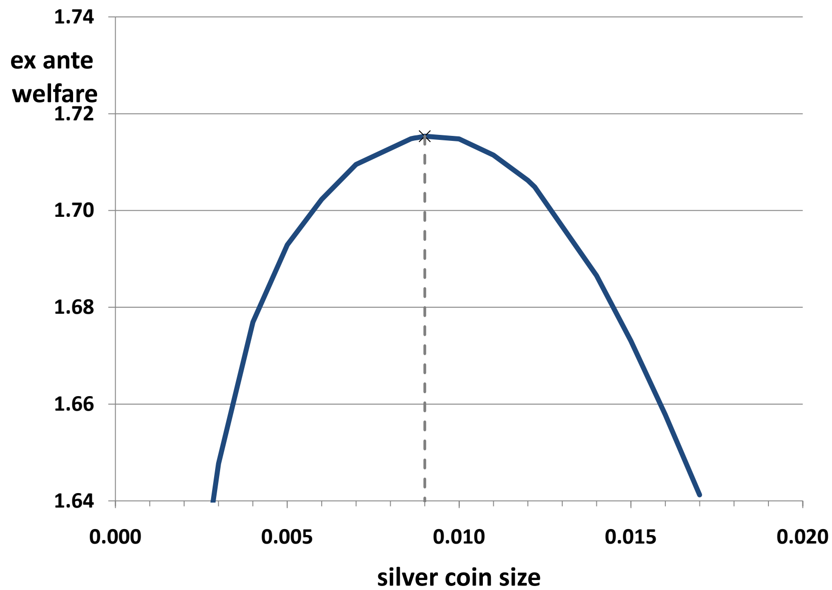

Figure 2: Ex ante welfare for various sizes of coins

with a larger coin.

Because there are more agents holding small numbers of coins and jewelry, it could be expected that the reason ex ante welfare is higher with the small size coin is that there are fewer poor in that economy. This is borne out in Figure 4, where we plot the cumulative distribution of agents by their silver wealth. It can be seen that there are far fewer agents with small amounts of silver wealth in the economy with the smaller coin than in the economy with the larger silver coin.

3.2 More frequent trading opportunities

We next compare economies with different matching probabilities (different θ). We find that the optimal coin size varies with the frequency of trading opportunities. Figure 5 shows that the optimal coin size initially increases with the frequency of trading opportunities, then decreases.

When trade is very infrequent (small θ), agents do not want to hold much silver in the form of coins and would prefer to keep their wealth in utility yielding jewelery. Small sized coins permit them to do this while still affording them the opportunity to trade when they happen to be the buyer in a single coincidence match. As trade becomes more frequent, agents are willing to hold more wealth in the form of coins and the cost of using small coins (γ) starts to matter more, leading them to want larger coins. After some point, here $\theta = 0.1$, however, the increasing amount of trade means that the flexibility in the offers more than offsets the handling cost of more coins, and so agents would prefer a smaller coin (on

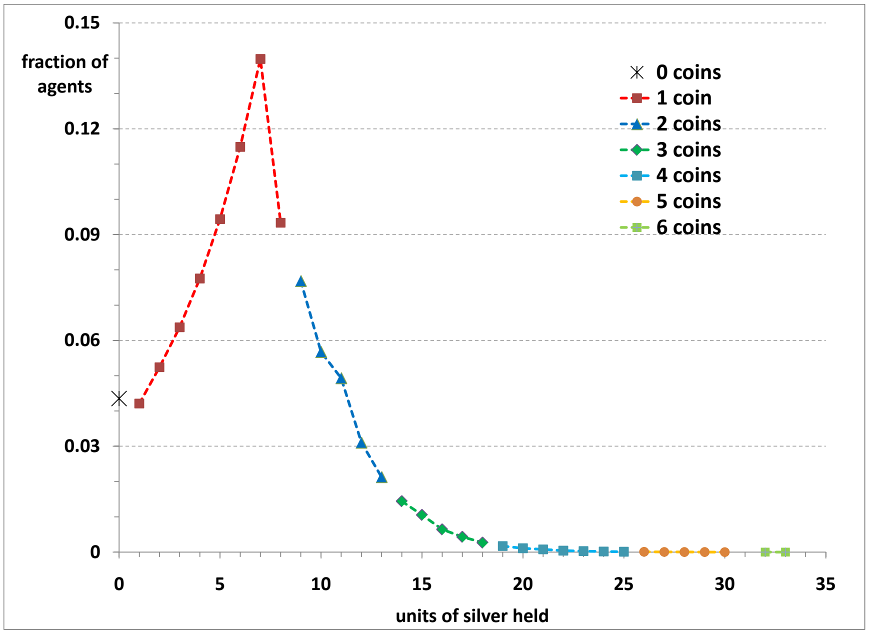

Figure 3: Distribution of coin and jewelry holdings, $b_s^1 = 0.015$

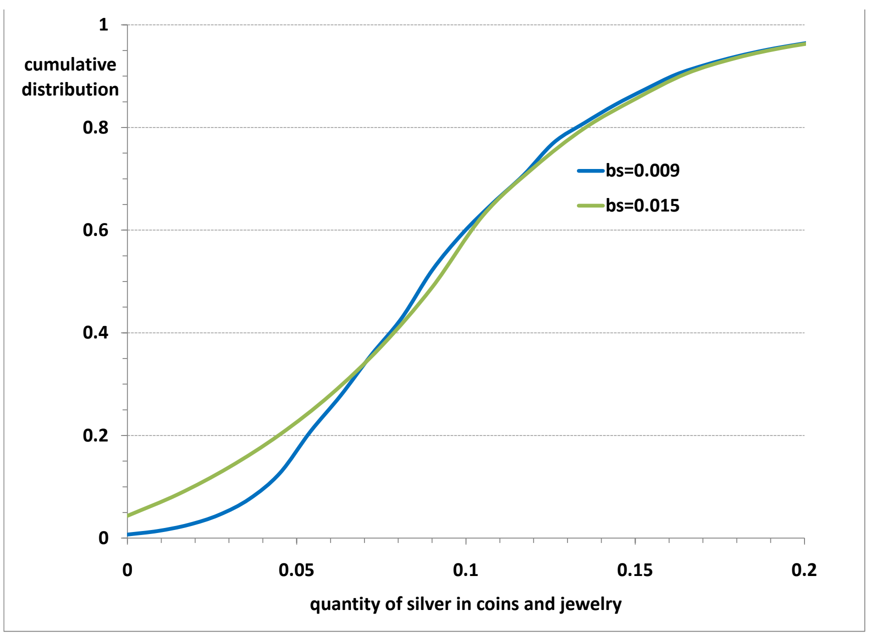

Figure 4: Cumulative distribution of silver holdings

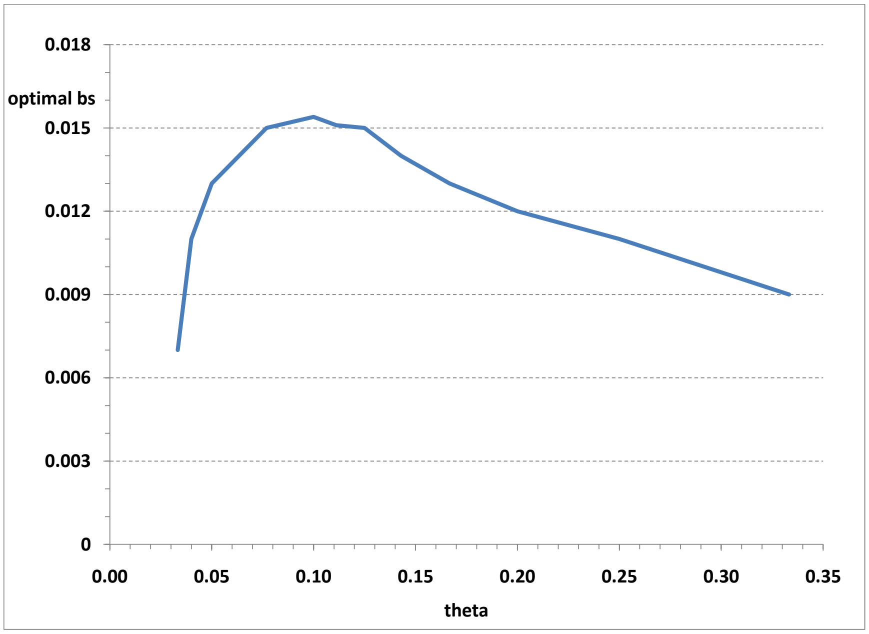

Figure 5: Optimal coin size for different amounts of trading

average).

3.3 Larger stock of silver

Finally, we compare single silver coin economies with different amounts of silver (m_s). Figure 6 shows that holding coin size fixed, ex ante welfare is higher, in general, in economies with larger quantities of silver. This is not surprising since agents are overall wealthier when there is more silver because they can potentially hold larger quantities of utility-yielding silver jewelry without reducing their holdings of coins. This is not true in economies with very small coins, however. When coins are quite small, the cost of holding coins is a substantial fraction of the size of the coins. In these case, a smaller quantity of silver imposes a smaller upper bound on potential coin holdings, which is welfare improving because it decreases the costs of holding coins.

Figure 6 also shows that from an ex ante perspective agents prefer that an economy with a larger stock of silver also have a larger size coin. When there is more silver, agents are wealthier on average, and they would choose to hold both more silver in the form of coins and more in the form of jewelry. A larger size coin allows them to do this with lower handling costs.[7]

Table 3 illustrates the effect of increasing the quantity of metal on average prices and output. The average price level is a weighted average of the amount of silver paid per unit

[7] Figure 6 makes it appear that optimal coin size increases proportionately with m_s. This is not true in general but is the result of the particular parameters choices we have made for the numerical results.

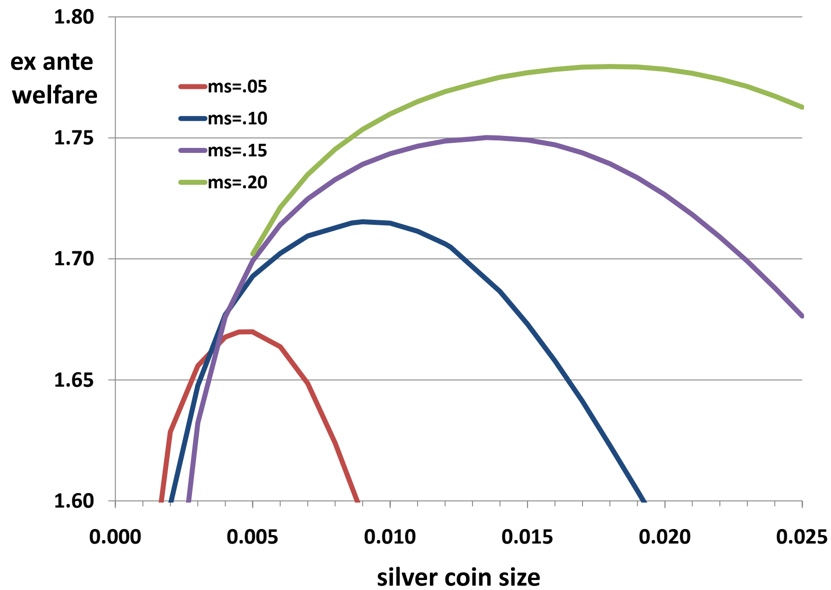

Figure 6: Ex ante welfare as the money stock increases

of the good traded in each match in which trade occurs, weighted by the probability of each match. Specifically, it is given by $\overline{p}$, where

$$\overline{p} = b_s^1 \sum_{y,\tilde{y}} \pi(y)\pi(\tilde{y}) \frac{p_1(y,\tilde{y})}{q(y,\tilde{y})}$$

.

As a point of comparison, consider the implications of a simple quantity theory of money: a doubling of the monetary metal would double the price of the good and leave quantities unchanged. Comparing rows 1 and 2 of the table shows that if the coin size is unchanged an increase in the quantity of metal leads to a fall in output and a more than doubling of the price level. Output falls in part because of the wealth effect of the increased metal (which reduces the incentive to work) and in part because the higher prices imply higher handling costs which act as a disincentive to production and trade.

When the coin size is doubled in addition to the doubling of the metal stock, the results are much closer to the Quantity theory predictions (compare rows 1 and 3). Output falls slightly and prices nearly double. In terms of the quantity theory, the fall in output and prices would be interpreted as a fall in velocity or increase in monetization.

3.4 Two coins

We turn now to the case where the monetary authorities permit the minting of coins of two different sizes. The agents who carry silver jewelery to the mint can opt for large or small

Quantity of metal	Size of coin (b_s^1)	Total output	Average price ($\overline{p}$	Average coin holdings
0.1	.009	0.105	0.246	2.04
0.2	.009	0.087	0.548	4.39
0.2	.018	0.103	0.458	2.07

Table 3: The effect of doubling the quantity of metal

silver coins, and equally can melt large or small coins into jewelery. In Figure 7 the curves depict ex ante welfare in an economy with 2 coin types where the second coin's weight is η times that of the original coin. The figure shows a number of features of the model.

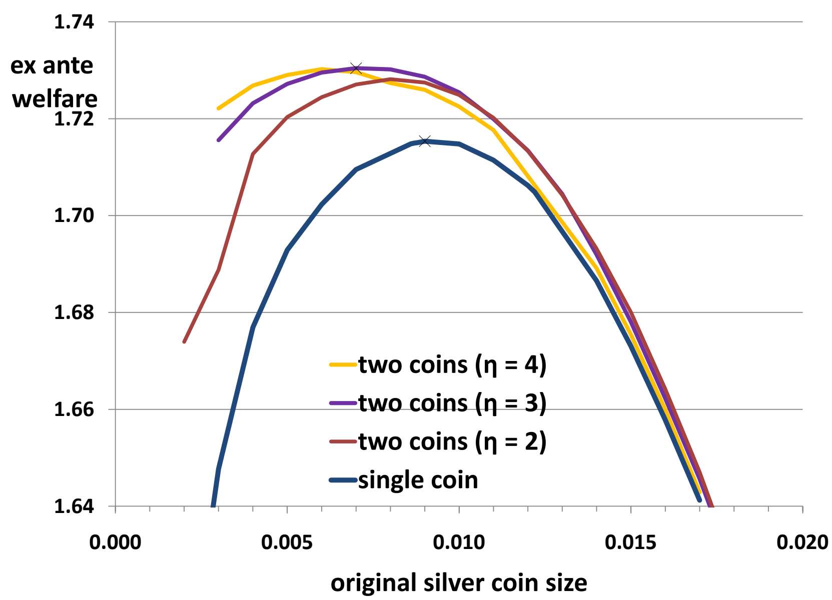

Figure 7: Optimal coin sizes if there are two coins

1. If the monetary authority picks coin sizes optimally (conditional on η) then ex ante welfare is higher if there are two coin types than if there is just one. A second, large coin potentially reduces the carrying cost of any given amount of coin wealth that an agent wants to hold.

2. The optimal coin sizes in two coin economies will span the size of the optimal coin in a single coin economy. For example, if there are two coins and the larger weighs three times as much as the original coin, the ex ante welfare maximizing single coin is of size $b_s = 0.009$ and the two coins have sizes $b_s^1 = 0.007$ and $b_s^2 = 0.021$.

3. Ex ante welfare in an economy with two coins is not always higher than ex ante welfare in all economies with a single coin. For example, ex ante welfare in an economy with a single coin of size $b_s^1 = 0.009$ is higher than the ex ante welfare in an economy with two coins of sizes ($b_s^1 = 0.004$ and $b_s^2 = 0.008$). If the two coins are both small, then agents will hold a large number of them and incur large handling costs. An economy with a single coin slightly larger than the larger of the two coins allows virtually the same trading opportunities and saves on handling costs.

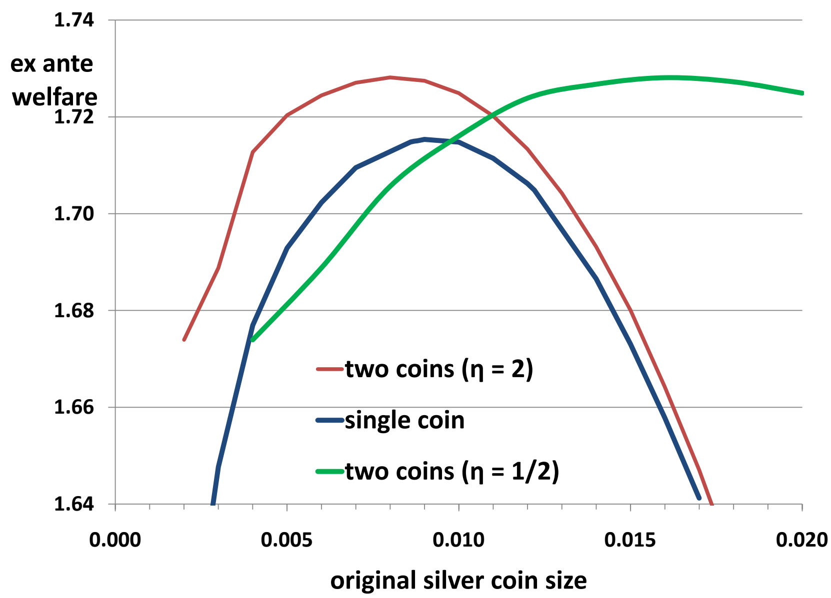

Figure 8: Optimal coin sizes if there are two coins

In Figure 8, which is a slight reworking of Figure 7, we compare a single coin economy with a given size coin to a two coin economy in which one of the two coins is of the same size. The figure shows that when the single coin is large (approximately $b_s^1 \geq 0.011$) in the figure, ex ante welfare will be higher in the two coin economy only if this similar coin is the larger of the two coins; that is, if the difference in the two economies is that the two coin economy has a coin smaller than the single coin economy. Otherwise, welfare would be lower if the two coin economy has a coin larger than the single coin economy. The opposite holds if the size of the coin in the single coin economy is small.

4 Historical application of the model

Two of the main features of economic development in the medieval period were the increase in urbanization and the increase in the stock of (mined) silver. Discussing the supply of silver first, the broad picture is one of a relatively static quantity of silver until the mid-12th century. In approximately 1160 the discovery and exploitation of silver mines at Freiberg produced a significant increase in the quantity of silver.[8] Indeed, Spufford (1988) titled the relevant chapter of his seminal monetary history of medieval Europe "New silver, c.1160-1320". The new silver found its way first to the Italian cities the most commercialized and trade-oriented part of the continent, and particularly to Venice the nearest city to the mines (Lane and Mueller (1985) 138). By the early 14th century, the silver had made its way throughout Europe. Spufford (2002); 12) estimates that in 1319 there were 800 tons of silver coin circulating in England, "a twenty-fourfold increase since the mid-twelth century".

Economic historians have described the "renaissance of markets" between the 10th and 13th centuries in Europe, as a process that spread intermittently beginning in Northern Italy and gradually spreading to the Low Countries, France and England (e.g. Epstein (2009) ch.4). Venice benefitted from its Maritime position and profited from acting as an entrepot for trade with the Levant and as the city grew richer it attracted a greater population. The acceleration of commercial activity and urbanization in England came perhaps a century later in England reflecting and stimulating the growth of the wool trade. In this section, we examine the impact of these changes through the lens of our monetary model.

4.1 Gradual debasement of the Italian coinage

Between 800 and 1150, the silver content of the denarius minted in the various Italian city states fell from about 1.7 gms to roughly .05 grams. This debasement is often blamed on the greed of monetary authorities. However, historians of Venice in this period have viewed the debasement as a reasonable response to economic expansion that exceeded the growth of monetary metal. For example, Cipolla (1963) argued that the dramatic growth of the Italian population between the mid-9th and mid-13th centuries and simultaneous increase int he division of labor significantly increased the demand for money. He argued (p.417) that the "inelastic supply of precious metals which did not expand proportionately to the increase in the demand for money [created circumstances such that] if prolonged deflationary pressure and a dangerous downward movement of prices were to be avoided" debasement of the denarius was necessary. Notice that the 'dangerous' deflation is a fall in prices measured in denari not in ounces of silver. He accepts that debasement would not avert a fall in the prices of goods in silver. Also note that while Cipolla argues that the supply of precious metal was inelastic, he discusses the availability of metal through trade rather than the other margin which was closer to home, the undoubted availability of silver in the form of plate.

In our model, the increase in trading is best captured by an increase in θ, and the model implies that in an economy with more trade there are benefits from a coin with less silver (e.g. from a debasement of the penny) in order that there are more coins in circulation and that each coin's value has decreased. As shown in Figure 5 (above some threshold level of economic activity) the optimal coin size decreases with increasing trade: producing more coins from a given quantity of metal by making smaller coins does bring an increase in handling costs, but the benefit of being able to make finer offers outweighs these costs.

[8] As the numismatists and historians note while rough measures of the quantity of mined can be estimated, the initial stock is not known. The conclusion as to the significance of the increase in part rests on the anecdotal discussion of contemporaries and on the change in volume of activity at mints.

This mechanism is analagous to the argument that Lane and Mueller (1985) makes for the debasement of the penny. They suggest (p.25) that the 'growing need for coin that arose from the increased use of markets and the general expansion of trade' implied that debasement was 'in the public interest'. In practice, the amount of silver in a coin could be reduced by making a lighter coin or a lower fineness coin.

4.2 Venice introduces the grosso

The era of the penny ended in the last decade of the 12th century when the Venetians introduced the grosso, a silver coin reminiscent of the Carolingian penny, weighing just over 2 gms and being over 95% fine. The Venetian denarius by that time contained about a twelth of a gram of pure silver and was about 25% fine, so that the grosso was worth about 24d.

The traditional explanation for the introduction of the grosso attributes it to the need for a convenient coin to facilitate buying materials and paying wages for outfitting a fleet for the Fourth Crusade in 1200 AD.[9] More recently, Lane and Mueller (1985) and Stahl (2000) have provided evidence that the coin was actually introduced in the early 1190s, that is, before the needs of the Crusade. Lane and Mueller (1985) (p.114) note, however, that the *success* of the grosso was assisted by the massive inflow of silver provided for outfitting the fleet, enough to mint 4 million grossi. The increasing production of German silver mines (and the importance of Venice as an entrepot for trade with the Levant) provided a further influx of silver.

As noted in Section 3.4 for our baseline economy, welfare is always higher if there are two coins in the economy relative to only having the smaller of the two coins. That said, the model does highlight the importance of the amount of available silver in motivating the provision of a large coin. Figure 9 compares the welfare gains from adding a second coin in economies with different amounts of monetary metal. In both cases there are gains from the second coin. However, consider a monetary authority that wished to add a second coin three times as large to a single coin with $b_s^1 = 0.009$. When the metal stock is 0.1 the welfare gains are modest, but when the metal stock is larger, the second coin adds much greater benefits.

4.3 England's introduction of farthings and halfpence

In contrast to the debasement of the penny in the Italian city states, the weight and fineness of the English penny declined very little from 800 to 1150. (See Table 1). By 1200 when relatively advanced Venice introduced the grosso as a large silver coin, the only coin being minted in England was still the penny, worth about three-quarters of a grosso. The inconvenience of the penny for day-to-day life is apparent from the prices of bread and ale, the two most common household purchases. A penny would buy 9 or 10 gallons of ale or

[9] Grierson (1979) makes this case and it is repeated, for example, in Spufford (1988). While Lane and Mueller (1985) argue that the introduction of the grosso preceded the Crusaders needs they also suggest that the grosso would have been a convenient coin for paying the workmen involved: weekly wages of 144d would be more easily counted out as 5 grossi and 14 denari. In his later history of medieval coinage Grierson (1991; 105) notes that " [the grossi] simplified and speeded up commercial transactions by reducing the number of coins involved in payments".

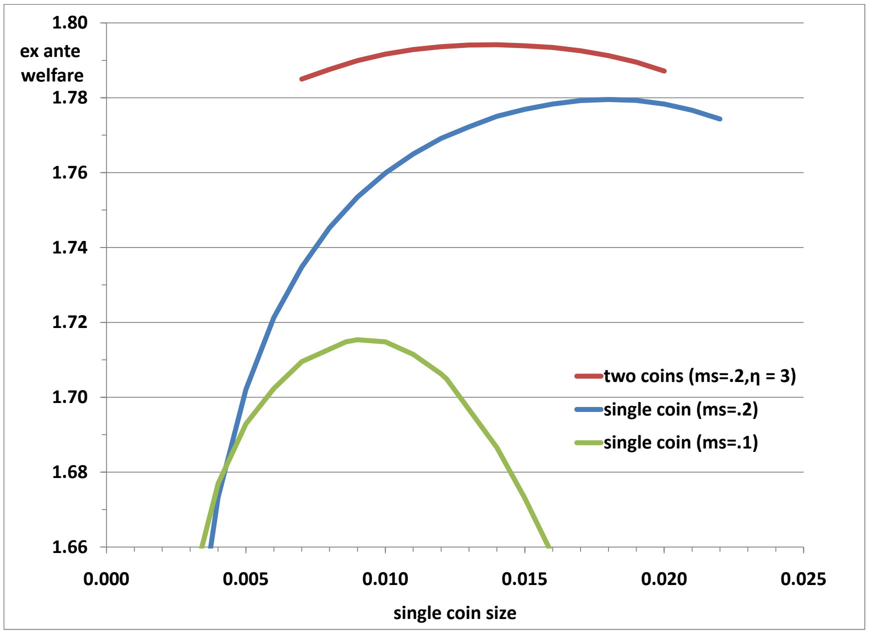

Figure 9: Welfare gains from a second coin

four 1 pound loaves of bread. The daily wage for a labourer in 1250 was roughly one penny (Britnell (2004) 24). On the other hand, for those amassing large amounts (Britnell cites the income of Canterbury Cathedral - £14,000 (3.4 million pennies) and King John's tax £57,000 (13 million pennies)) the penny coinage implied significant problems of handling and counting.

Through the 13th century economic activity in England expanded; the population increased and the growing wool trade and urbanization accompanied an expansion of commercial activity. One feature of this was the growth in the number of market towns between 1200 and 1349. Britnell (1981) shows that over the period 1200 to 1350 the most rapid expansion (in number and activity) occurred in the 25 years between 1250 and 1274.

Our model shows that an increase in trading frequency implies gains from reducing coin size and the introduction of smaller coins did indeed occur after this increase in the amount of trade. In 1279, Edward I introduced quarter pennies (farthings), and halfpennies in addition to the penny.[10]

Yet provision for the minting of halfpence and farthings did not eliminate concern amongst the English population over the issue of small change. There were two lingering concerns: insufficient coins were issued, and even a farthing was a relatively large denomination. The halfpence and farthings were minted immediately, (see Figure 10) but complaints of a shortage of these small coins continued through the 14th and 15th centuries. Ruding's history of the coinage (1840; I) documents petitions of the Commons and their complaints

[10]He also allowed the minting of groats but as described below there is no evidence (from mint records, hoards or anecdotes) than any groats were issued.

of ‘a want of halfpennies’ (1380); ‘the great mischief amongst the poor people for want of halfpennies’ (1404); ‘that little or nothing of small coins was struck... to the great harm of the people’ (1422); and the ‘hurt of the noble realm for default of halfpennies’ (1444).[11]

Allen (2007) states that the complaints led to frequent orders for the coinage of specific quantities of small change, but analyzing the evidence of die use and hoards he finds that the orders were not always followed.

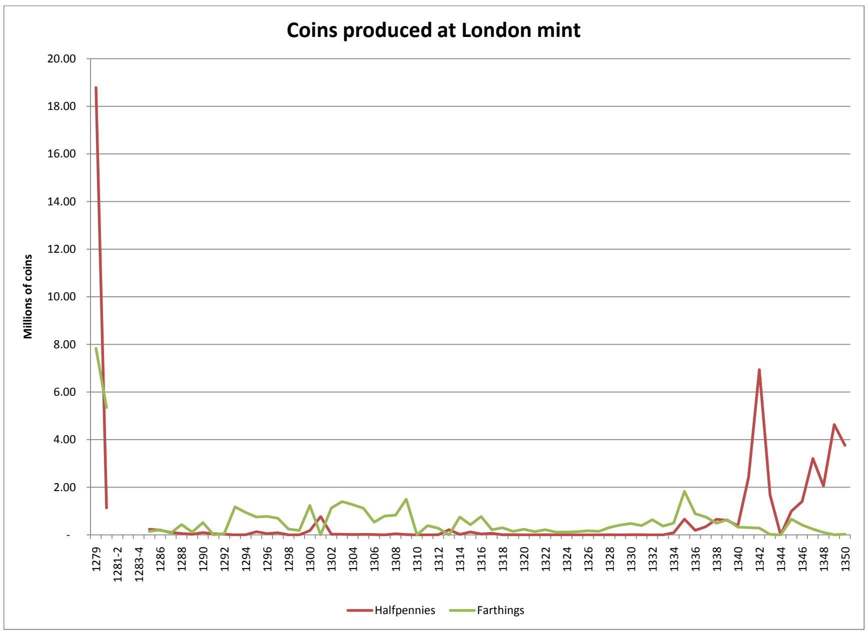

Figure 10: Output of farthings and halfpence, England

The farthing weighed 0.34 gms - one-sixth of the weight of a US dime - which came close to a lower bound on the feasible weight of a coin. Most European mints chose to issue billon coins (coins less than 50% fine) to supplement their silver coinage in order to have coins with lower value than a farthing. For example, the Venetians in the late 13th century minted a penny coin that was 25% fine and had one fifth the value of a farthing. The English issued no billon coin and no coin worth less than a farthing.

Christine Desan (2010) has argued that the English refusal to issue low denomination coins had widespread social implications that could not be overcome by the use of credit or adjustment of the price level. Our model provides some justification for the argument that the availability of different denominations affects social stratification. Figures 1, 3 and 4 show that the distribution of wealth in the economy depends on the size of the coin in circulation. If there are no small denomination coins, then the poorest 20% of agents hold less silver - are poorer - than if there are smaller coins.

[11] See Sargent and Velde (2002) and Redish (2000) for further discussion.

4.4 England's groat issues

Seventy years later, in 1351, Edward III ordered the coining of groats - a coin containing as much silver as four pence (as well as the existing silver denominations and a half groat).[12] The 1279 legislation had permitted the issue of groats but none had been minted, however, after 1351 a large numbers of groats were issued (Allen (2007)).

The failure of the groat in 1279 and its subsequent popularity after 1351 prompted Peter Spufford (1988; 234) to pose the question "What conditions determined the readiness of an area for the use of coins of a larger denomination than the penny?" He concludes that the key factor was the number and pay rate of urban wage-earners and soldiers. Between 1280 and 1350 the urban population of England increased, as did the wage rate: wages for a building laborer in the 1280s were about 9d. weekly, and in 1351 had risen to about 18d weekly. For Spufford, "In 18d weekly pay, a groat was marginally acceptable".

In the context of our model, the urban pay explanation seems incomplete. Workers were typically paid daily and wages of $1\frac{1}{2}$d/day don't seem to make groats an ideal medium. Studies of the 'scarcity of small change' such as Sargent and Velde (2002) cite frequent complaints about a lack of small change, particularly to buy bread and beer - the staple diet of the building-laborers.[13]

Spufford rejects the importance of an increase in the amount of monetary metal over the period, arguing that there there was plenty of money in 1279 to issue groats.[14] Our model emphasizes the importance of quantities of metal for the optimal coin size and recent (i.e.subsequent to Spufford's book) data on the stock of coins in England show that between 1279 and 1350 there was a doubling of the quantity of silver (per capita) in circulation in England. Figure 11 uses the estimates of Martin Allen (2001) for the amount of coin in circulation and of Clark (2007) for the English population. The figure shows a tripling of coin stocks per capita between 1279 and 1310. By 1351 the silver stocks have declined - in part perhaps because gold coins began to be minted in 1344. Silver coin stocks remain at over double their level in 1279.

As in the Venetian case, the move towards introducing a large silver coin was only successful after silver stocks had increased.

5 Conclusion

Monetary economists frequently imagine commodity money systems to be smoothly operating regimes where the fixed supply (or costs to produce) of the commodity provided a nominal anchor for the economy. However, while the commodity could anchor the value of the unit of account, commodity money systems had more difficulty in performing the medium of exchange function of money. Jevons famously pointed out that to provide a medium of exchange a commodity must be durable, portable and divisible, and for much of the last

[12] In 1344 the mint began issuing the noble, a gold coin worth 80d, as well as its halves and quarters.

[13] For example, Allen (2007) p.193 lists three petitions and a Parliamentary enquiry into the shortage of small change, between 1380 and 1402.

[14] "The sheer quantity of money in circulation does not seem to have been the efficient cause of change [between 1279 and 1351]" Spufford (1988); 234.

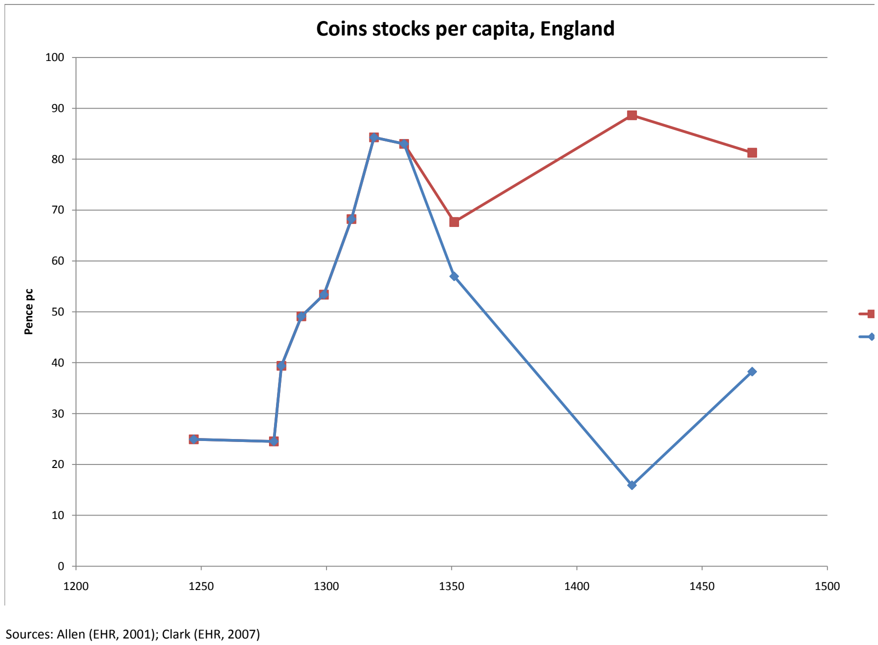

Figure 11: Value of coins stocks per capita, England

millennium gold and silver were adopted as monetary commodities because they had such qualities. Identifiability (for example, of the purity of the metal in a coin) and uniformity (permitting payments to be made by tale rather than by weight) were further desirable characteristics of money. The desire for these attributes promoted the use of coined metals and the monopolization of the right to mint coins. In turn, the monopoly production of coins gave the monetary authority (typically the Crown) two instruments of monetary policy: the rate of seignorage and the size of coins - the denomination structure.

Monetary historians have documented the difficulties created by these monetary systems and argued that denominational structures are an important contributor to those difficulties. In this paper we construct a model of a monetary economy in which a commodity can be used to produce an indivisible coin that can be used for transactions.

We use the model to analyze the implications of alternative monetary policy choices. In particular, we examine the impact of the denomination structure on welfare, and find that there is an optimal denomination structure. We show that as the trading opportunities rise the optimal size of the silver coins shrinks, and that as the stock of the monetary commodity rises the optimal size of the coin increases. We then document some examples of changing denomination structure in medieval Europe and use the model to show how changes in the economic environment would have influenced the economically efficient coin types.

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