Structural Identification of Production Functions

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Abstract

This paper examines some of the recent literature on the empirical identification of production functions. We focus on structural techniques suggested in two recent papers, Olley and Pakes (1996), and Levinsohn and Petrin (2003). While there are some solid and intuitive indentification ideas in these papers, we argue that the techniques, particularly those of Levinsohn and Petrin, suffer from collinearity problems which we believe cast doubt on the methodology. We then suggest alternative methodologies which make use of the ideas in these papers, but do not suffer from these collinearity problems.

1 Introduction

Production functions are a fundamental component of all economics. As such, estimation of production functions has a long history in applied economics, starting in the early 1800's. Unfortunately, this history cannot be deemed an unqualified success, as many of the econometric problems that hampered early estimation are still an issue today.

A production function relates productive inputs (e.g. capital, labor, materials) to outputs. The major econometric issue regarding estimation of

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production functions is the possibility that some of these inputs are unobserved. If this is the case, and if the observed inputs are chosen as a function of these unobserved inputs (as will typically be the case for a profit-maximizing or cost-minimizing firm), then OLS estimates of the marginal products of the observed inputs will be biased.

Much of the literature in the past half century has been devoted to solving this endogeneity problem. Two of the earliest solutions to the problem are instrumental variables (IV) and fixed-effects estimation (Mundlak (1961)). IV estimation requires finding variables that are correlated with observed input choices, but uncorrelated with the unobserved inputs. Fixed-effects estimation requires the assumption that the unobserved input or productivity is constant across time. While these techniques have been very successful in some instances, in others they have not.

The past five years has seen the introduction of a number of new techniques for identifiction of production function. One set of techniques follows the dynamic panel data literature, e.g. Chamberlain (1982), Arellano and Bover (1995), Blundell and Bond (1999). This paper focuses on a second set of techniques, advocated by Olley and Pakes (1996) and Levinsohn and Petrin (2003), that are more structural in nature. These technigues have been applied in a large number of recent empirical papers, including Pavcnik(2003), Sokoloff (2003), Sivadasan, J. (2004), Fernandes, Ozler and Yilmaz (2001), Criscuolo and Martin (2003), Kasahara and Rodrique (2004), and Topalova (2003). We argue in this paper that there may be significant problems with these estimation technologies - in particular that the methodologies have collinearity problems that can be problematic for interpretation of results. We illustrate this point, and also illustrate the (sometimes strong) assumptions that are required for the Olley and Pakes and Levinsohn and Petrin methodologies to remain identified in the wake of this collinearity problem. We then suggest some alternative estimation approaches. These approaches build upon the ideas in Olley and Pakes and Levinsohn and Petrin, e.g. using investment or intermediate inputs to "proxy" for productivity shocks, but do not suffer from these collinearity problems.

2 Review of Olley/Pakes and Levinsohn/Petrin

We start with a brief review of the techniques of Olley/Pakes (henceforth OP) and Levinsohn/Petrin (henceforth LP). Consider the following production function.

$$y_{it} = \beta_1 k_{it} + \beta_2 l_{it} + \omega_{it} + \epsilon_{it} \quad (1)$$

y_{it} is the log of output, k_{it} is the log of capital input, and l_{it} is the log of labor input. ω_{it} is firm i's "productivity shock" in time period t, observed to the firm (at time t) but unobserved to the econometrician. ϵ_{it} is assumed to be i.i.d. noise, e.g. measurement error in output. Assume that the productivity shock ω_{it} evolves exogenously following a first-order markov process, i.e.

$$p(\omega_{it}|\omega_{it-1}, ..., \omega_{i0}) = p(\omega_{it}|\omega_{it-1})$$

The classic endogeneity problem estimating equation (1) is that the firm's optimal choice of inputs k_{it} and l_{it} will generally be correlated with the unobserved productivity shock ω_{it}. This renders OLS estimates of the β's biased and inconsistent. As mentioned in the introduction, perhaps the two most commonly used solutions to this endogeneity problem are fixed effects (Mundlak (1961)) and instrumental variables estimation techniques. In this context, fixed-effects estimation requires the additional assumption that $\omega_{it} = \omega_{it-1} \forall t$. This is a strong assumption and in some cases can result in worse estimates than OLS (Griliches and Hausman (1986)). IV estimation requires instruments that are correlated with input choices k_{it} and l_{it} and uncorrelated with ω_{it}. Standard instrumental variables would be input prices, assuming the firm is in a competitive market for inputs. While in any particular application, such instruments may be available, they are often not. In addition, there is often an issue regarding the quality of these instruments. OP and LP take a more structural approach to identification of production functions.

2.1 Olley and Pakes

OP addres the endogeneity problem as follows. First, they assume that capital is a fixed input subject to an investment process. In other words, period t capital depends on period $t-1$ capital and an investment amount (i_{t-1}) that is decided in period $t-1$. Intuitively, one can see how this assumption regarding timing helps solve the endogeneity problem with respect to capital. Since k_{it} is actually decided upon at $t-1$, it is by construction uncorrelated with the innovation in ω_{it}, i.e. $\omega_{it} - E[\omega_{it}|\omega_{it-1}]$. This orthogonality can be used to form a moment to identify β_1.[1] We explicitly show how this is done in a moment.

More challenging is solving the endogeneity problem with respect to the assumed variable input, l_{it}. To accomplish this, OP make use of the investment variable i_{it}. Considering a firm's dynamic investment decision, OP state conditions under which a firm's optimal investment level i_{it} is a *strictly increasing* function of their current productivity ω_{it}, i.e.

$$i_{it} = f_t(\omega_{it}, k_{it}) \tag{2}$$

Note that this investment function must contain all current state variables for the optimizing firm, e.g. its current level of capital. The reason f is indexed by t is that variables such as input prices, demand, etc. also may be part of the state space. As these are typically unobserved (if they were observed, one could potentially use more standard solutions to the endogeneity problem, i.e. IV) OP treat these as part of f_t. The assumption here is that these variables are allowed to vary across time, but not across firms (i.e. firms face the same input markets).

Given that this investment function is strictly monotonic, it can be inverted to obtain

$$\omega_{it} = f_t^{-1}(i_{it}, k_{it}) \tag{3}$$

The essence of OP is to use this function to control for ω_{it} in the production function. Since f (and thus f^{-1}) would typically be very complicated to fully specify (as it is the policy function of a dynamic programming problem), OP treat it non-parametrically. Substituting this into the production function, we get:

$$y_{it} = \beta_1 k_{it} + \beta_2 l_{it} + f_t^{-1}(i_{it}, k_{it}) + \epsilon_{it} \tag{4}$$

The first stage of OP is to estimate this equation. Obviously, direct estimation of (4) does not identify β_1, since k_{it} is collinear with the non-parametric function. However, one does obtain an estimate of β_2, $\widehat{\beta}_2$ at this stage. One also obtains an estimate of the composite term $\beta_1 k_{it} + f_t^{-1}(i_{it}, k_{it})$, which we denote $\widehat{\Phi}_{it}$.

[1] In the special case where ω_{it} is a random walk, i.e. $\omega_{it} = \omega_{it-1} + \eta_{it}$, one can easily see how this can be done - if we first-difference the production function $(k_{it} - k_{it-1})$ is uncorrelated with the resulting unobserved term.

Given $\widehat{\beta}_2$ and $\widehat{\Phi}_{it}$, one can proceed to estimate β_1 following the discussion above. Note that we can write:

$$\omega_{it} = E[\omega_{it}|\omega_{it-1}] + \xi_{it}$$

where again, by the timing assumption regarding capital ξ_{it} is orthogonal to k_{it}. This provides the following moment for identification:

$$E[\xi_{it}|k_{it}] = 0$$

which can be operationalized with the construction

$$\xi_{it}(\beta_1) = \omega_{it} - E[\omega_{it}|\omega_{it-1}] = (\widehat{\Phi}_{it} - \beta_1 k_{it}) - \widehat{\Psi}(\beta_1)$$

where $\widehat{\Psi}(\beta_1)$ are predicted values from a non-parametric regression of $(\widehat{\Phi}_{it} - \beta_1 k_{it})$ on $(\widehat{\Phi}_{it-1} - \beta_1 k_{it-1})$. This moment identifies the capital coefficient β_1. Recapping the intuition behind identification here, β_2 is identified by using the information in firms' investment decisions i_{it} to control for the productivity shock that is correlated with l_{it}. β_1 is identified by the timing assumption that k_{it} is decided before the full realization of ω_{it}.

2.2 Levinsohn and Petrin

LP take a slightly different approach to the problem. Rather than using an investment equation to "invert" out ω_{it}, they use an intermediate input demand function. The motivation for this alternative equation is very reasonable. For the OP procedure to work, one needs the investment function to be *strictly* monotonic in ω_{it}. However, in actual data, investment is often very lumpy, and one often sees zeros. This casts doubt on this strict monotonicity assumption regarding investment. LP avoid this assumption by considering the following production function:

$$y_{it} = \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 m_{it} + \omega_{it} + \epsilon_{it}$$

where m_{it} is an intermediate input such as electricity, fuel, or materials. LP's basic idea is that since intermediate input demands are typically much less lumpy (and prone to zeros) than investment, the strict monotonicity condition is more likely to hold and these may be superior "proxies" to invert out the unobserved ω_{it}. LP consider the following intermediate input demand function:

$$m_{it} = f_t(\omega_{it}, k_{it}) \qquad (5)$$

Again, f is indexed by t, implicitly allowing input prices (and demand conditions) to vary across time. Note the timing assumptions implicit in this formulation. First, the intermediate input at t is chosen as a function of ω_{it}. This implies that the intermediate input is essentially chosen at the time production takes place. We describe this as a "perfectly variable" input. Secondly, note that l_{it} does not enter (5). This implies that labor is also a "perfectly variable" input. If l_{it} was chosen at some point in time before m_{it}, then l_{it} would impact the optimal choice of m_{it}.

Given this specification, LP proceed similarly to OP. Under the assumption that intermediate input demand (5) is monotonic in ω_{it}[2], we can invert:

$$\omega_{it} = f_t^{-1}(m_{it}, k_{it}) \tag{6}$$

Substituting this into the production function gives

$$y_{it} = \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 m_{it} + f_t^{-1}(m_{it}, k_{it}) + \epsilon_{it}$$

The first step of the LP estimation procedure is to estimate the above equation, as in OP treating f_t^{-1} non-parametrically. Again, only β_2 is identified as k_{it} and m_{it} are collinear with the non-parametric term. One also obtains an estimate of the composite term, in this case $\beta_1 k_{it} + \beta_3 m_{it} + f_t^{-1}(m_{it}, k_{it})$, which we again denote $\widehat{\Phi}_{it}$.

The second stage of the LP procedure again proceeds as OP, the only difference being that there is one more parameter to estimate, β_3. LP use essentially the same moment as OP to identify the capital coefficient, i.e.

$$E[\xi_{it}(\beta_1, \beta_3)|k_{it}] = 0$$

where

$$\xi_{it}(\beta_1, \beta_3) = \omega_{it} - E[\omega_{it}|\omega_{it-1}] = (\widehat{\Phi}_{it} - \beta_1 k_{it} - \beta_3 m_{it}) - \widehat{\Psi}(\beta_1, \beta_3)$$

where $\widehat{\Psi}(\beta_1, \beta_3)$ are predicted values from a non-parametric regression of $(\widehat{\Phi}_{it} - \beta_1 k_{it} - \beta_3 m_{it})$ on $(\widehat{\Phi}_{it-1} - \beta_1 k_{it-1} - \beta_3 m_{it-1})$. They also add an additional moment to identify β_3. This is

$$E[\xi_{it}(\beta_1, \beta_3)|m_{it-1}] = 0$$

[2] LP provide conditions on primitives such that this is the case.

Note that the innovation in ω_{it}, ξ_{it} is clearly *not* orthogonal to m_{it}. This is because ω_{it} is observed at the time that m_{it} is chosen. On the other hand, according to the model, ξ_{it} should be uncorrelated with m_{it-1}. Note that this moment is similar to the standard "lagged levels instrumenting for differences" used in the dynamic panel literature (e.g. Arellano and Bover).

Note that both the OP and LP procedures rely on a number of key structural assumptions. While these assumptions are described in these papers (see also Griliches and Mairesse (1998)), we summarize them here. A first key assumption is the strict monotonicity assumption - for OP investment must be strictly monotonic in ω_{it}, while for LP intermediate input demand must be strictly monotonic in ω_{it}. Monotonicity is required for the non-parametric inversion because otherwise, one cannot perfectly invert out ω_{it} and completely remove the endogeneity problem in (4).

A second key assumption is that ω_{it} is the *only* unobservable entering the functions for investment or the intermediate input. This rules out, e.g. measurement error or optimization error in these variables, or a model in which exogenous productivity is more than single dimensional. Again, the reason for this assumption is that if either of these were the case, one would not be able to perfectly invert out ω_{it}.

A third key set of assumptions of the models regard the timing of input choices. By this, we refer to the point in the ω process at which inputs are chosen. First, k_{it} is assumed to have been decided exactly at (OP) or exactly at/prior to (LP) time period $t-1$. Any later than this would violate the moment condition, as k_{it} would not be orthogonal to at least a component of the innovation in ω_{it}. For OP, any earlier than this would make the inversion imperfect. Regarding labor, first, this input must have no dynamic implications. Otherwise, l_{it} would enter the respective control functions and prevent identification of the labor coefficient in the first stage. Second, for LP, l_{it} is assumed to be a perfectly variable input (in other words, chosen at the time ω_{it} is observed). If labor was chosen earlier, then firms' choice of materials m_{it} would depend on labor and l_{it} would enter the control function, again preventing identification of the labor coefficient in the first stage.

3 Collinearity Issues

This paper argues that even if the above assumptions hold, there are potentially serious identification issues with these approaches (*). Intuitively, the problem can be described as follows. The above methods require reasonably strict assumptions on the unobservables process behind the data, in particular that this unobservable process can be summarized by a scalar. The problem with a scalar unobservable process driving much of the variation in the data is that it can cause significant collinearity problems. In other words, choice variables move together, making any attempt to separate their effects meaningless.

More formally, consider the first stage regressions of the two approaches, respectively:

$$y_{it} = \beta_1 k_{it} + \beta_2 l_{it} + f_t^{-1}(i_{it}, k_{it}) + \epsilon_{it} \tag{7}$$

and

$$y_{it} = \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 m_{it} + f_t^{-1}(m_{it}, k_{it}) + \epsilon_{it} \tag{8}$$

Recall that in the first stage, the main goal in both methods is to identify β_2, the coefficient on the labor input. What we now focus on is the question of whether β_2 can be identified from these regressions. Since ϵ_{it} is purely measurement error in output, the only real identification question here is whether l_{it} is collinear with the other terms in the respective regressions.

3.1 Levinsohn and Petrin

First, consider the LP technique. Phrased slightly differently, our question is whether there is anything that moves l_{it} around independently of a non-parametric function of m_{it} and k_{it}. We consider three separate cases - in the first, l_{it} and m_{it} are decided simultaneously, in the second, l_{it} is decided upon before m_{it}; in the third, l_{it} is decided upon after m_{it}.

First assume that l_{it} is decided at the same point in time as m_{it}. What determines l_{it}? Just like there is an input demand function for m_{it}, there is an input demand function for l_{it}. This should clearly be a function of the current state, which includes ω_{it} and k_{it}, i.e.

$$l_{it} = f_{2t}(\omega_{it}, k_{it})$$

Substituting in (6), we get:

$$l_{it} = f_{2t}(f_t^{-1}(m_{it}, k_{it}), k_{it}) = g_t(m_{it}, k_{it})$$

i.e. l_{it} is some function of m_{it} and k_{it}. If this is the case, (8) will not be identified - l_{it} will be collinear with the non-parametric function. Note that observing other variables, e.g. input prices p_l and p_m does not break the collinearity problem. These input prices will enter *both* input demands for m_{it} and l_{it} as they are both decided simultaneously. Generally, if l_{it} and m_{it} are determined simultaneously, they wil be functions of the same variables, and thus β_2 will not be identified - again, l_{it} will be collinear with the non-parametric function.

One theoretical possibility for identification in this case is if there is some random optimization error in firms' choices of l_{it}. Such optimization error will move l_{it} independently of the non-parametric function, identifying β_2.[3] However, note the strong assumptions that must accompany this assumption. First, the extent of identification is purely a function of the extent of this optimization error. Second, while one needs to assume there is enough optimization error in l_{it} to identify β_2, one simultaneously needs to assume exactly no optimization error in m_{it}. If there were optimization error in m_{it}, the inversion would not be valid.

Note that a related story, one where l_{it} is measured with error (certainly a believable story), does not solve the colinearity problem in LP. The problem here is the typical measurement error problem. In fact, one would expect the problem to be quite severe in this case, because all the "signal" in l_{it} is colinear with the non-parametric function, the only thing serviing to identify β_2 is the measurement error, and the estimate of β_2 will converge in probability to zero.

Second, suppose that l_{it} is decided upon before m_{it}. Regardless of what ω is at this point, or what expectations of future input prices are, in this case, firms' choices of the intermediate input m_{it} will now generally depend on l_{it}, i.e.:

$$m_{it} = f_t(\omega_{it}, k_{it}, l_{it})$$

[3] Such optimization error was considered prevalent by some early writers. As Marschak and Andrews (1944) note: "In addition, not all entrepreneurs may have the same urge, or ability, or luck to choose the most profitable combinations of production factors." (p. 156)

In this case, (8) becomes:

$$y_{it} = \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 m_{it} + f_{it}^{-1}(m_{it}, k_{it}, l_{it}) + \epsilon_{it} \tag{9}$$

and again, β_2 is not identified, as a non-parametric function of l_{it} is clearly collinear with a linear function of l_{it}.

Third suppose l_{it} is decided upon after m_{it}. Critical here is what happens to ω and other variables such as input prices between these two points of time. If neither changes between these points, it is as if l_{it} and m_{it} are chosen simultaneously - again, l_{it} and m_{it} will be functions of the same things and l_{it} will be collinear with the non-parametric term.

If ω evolves in some way between the choice of m_{it} and the choice of l_{it}, the collinearity problem disappears, as l_{it} is a function of the innovation of ω between the two points in time and thus variaes independently of the non-parametric function. However, this story is not consistent with the basic assumptions of LP. Since ω evolves *after* m_{it} is chosen, then the inversion of m_{it} will not pick up the correct ω to control for endogeneity in the production function.[4]

Still considering the situation where material inputs m_{it} are chosen *prior* to labor inputs l_{it}, this leaves us with the case where ω_{it} is constant between the two decisions, but other variables, such as the price of labor (or, e.g., demand shifters) vary. Intuitively, one might expect such variation to move l_{it} around independently of the non-parametric function. However, if this input price variation is constant across firms (varying across time), it is not helpful, as the non-parametric f_t^{-1} varies across time. Thus, anything that is constant across firms will not break the collinearity problem. What is needed is firm specific shocks to the price of l_{it} that occur between the m and l decisions. Such shocks would move l_{it} independently of the non-parametric function. Note, however, that these price shocks not only need to vary across firms, but also must have no persistence, otherwise, next period's m_{it+1} choice would depend on them. (*) Recall also that other input prices (as well as the expectation of the price of labor at the time m_{it} is chosen (since this enters ths m_{it} function)) one needs to assume *constant* across firms.

[4] Note that the part of ω_{it} that m_{it} does not pick up will be highly correlated with the variation in l_{it} independent of the non-parametric term (because it is the innovation in ω_{it} that is generating the independent variation). Thus, one would expect serious endogeneity issues in such a regression and one shouldn't be tempted to suggest that in this story, using ω_{it} as an imperfect proxy might eliminate "most" of the endogeneity problem.

To summarize, there appears to be only two potential stories that save the LP procedure from collinearity problems. The first is that there is a significant amount of optimization error in l_{it}, yet no optimization error in m_{it}. The second is that l_{it} is decided on after m_{it}, and that in between these two points in time, ω_{it} is constant but there is an i.i.d., firm-specific shock to the price of labor. Again, this story seems particularly unattractive since all other input prices are assumed to be constant across firms. Note that, in practice, one probably would not observe this collinearity problem. It is very likely that estimation of (**??**) would produce estimates of β_2. Our point is that unless one believes one of the above two stories, the extent to which the β_2 coefficient is identified is the extent to which the LP model is misspecified.[5]

3.2 Olley and Pakes

Now consider the OP model. Given the above results regarding LP, a reasonable question is whether the OP model also suffers from a collinearity problem. While there are also potential collinearity issues with OP, we argue that this collinearity can be broken under much more reasonable assumptions than in LP.

In OP, the question is whether l_{it} is collinear with $f_t^{-1}(i_{it}, k_{it})$. Again, if there is nothing else in the model, l_{it} is just a function of ω_{it} and k_{it}, so the collinearity problem is again present. Thus to get identification, one needs a story where something moves l_{it} around. Three potential stories with strong assumptions are similar to those for LP. First and second, the stories of optimization error or measurement error in the choice of l_{it} are again a possibility, but the strong assumptions around this are again required. Third, if one assumes that i_{it} is chosen before l_{it}, (and that ω is constant between these points in time), movement in the price of labor is again a possibility. For example, a firm-specific shock to the price of labor (or demand shock) between the two points in time would do the trick, though again this would need to be non-persistent. Again, this would need to occur despite the fact that other input prices, as well as the expectation of the price of labor at the point i_{it} is chosen have to be assumed constant across firms.

[5] Analagously, one might regress l_{it} on k_{it} and m_{it} and not find a perfect fit. In the context, our point would be that according to the LP assumptions, there is no really believable story why one wouldn't get a perfect fit in such a regession.

A better story in the OP context is one where l_{it} is actually not a perfectly variable input, and is chosen at some point in time between periods $t-1$ and t. Denote this point in time as $t-b$, where $0 < b < 1$. Suppose that ω evolves between the subperiods $t-1$, $t-b$, and t according to a first-order markov process, i.e.

$$p(\omega_{it}|I_{it-b}) = p(\omega_{it}|\omega_{it-b})$$

and

$$p(\omega_{it-b}|I_{it-1}) = p(\omega_{it-b}|\omega_{it-1})$$

In this case, labor input is not a function of ω_{it}, but of ω_{it-b}, i.e.

$$l_{it} = f_t\left(\omega_{it-b}, k_{it}\right)$$

Since ω_{it-b} cannot generally be written as a function of k_{it}, and i_{it}, l_{it} will *not* generally be collinear with the non-parametric term in (4), allowing the equation to be identified. Note the intution behind this - the fact that labor is set before production means that labor is determined by ω_{it-b}. The movement of ω between $t-b$ and t is what breaks the collinearity problem between l_{it} and the non-parametric function. (*) The maintained assumption that is required is that although l_{it} is not perfectly variable, it does not have dynamic implications. That is, labor is chosen at time $t-b$, and cannot be adjusted between time $t-b$ and t, but its choice at time t does not affect its choice (or anything else) beyond time t. Specifically, its choice has no implications for time $t+1$ or $t+1-b$.

Recall also why this more reasonable story *does not work* in the context of the LP model. In the LP model, if l_{it} is chosen before m_{it}, then m_{it} will directly depend on l_{it}, making the first stage unidentified. In OP, even if l_{it} is chosen before i_{it}, i_{it} does not depend on l_{it} (as long as one maintains the assumption that labor is a static choice variable). This is because i_{it}, unlike m_{it}, is not directly linked to period t outcomes, and thus l_{it} will not generally affect optimal choice of i_{it}. The fact that this type of story works for OP but does not work for LP is the reason that we describe the collinearity problem as being worse in the LP methodology.

4 Parametric Approaches to Levinsohn/Petrin?

The collinearity problem in LP is that in the first stage equation,

$$y_{it} = \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 m_{it} + f_t^{-1}\left(m_{it}, k_{it}\right) + \epsilon_{it} \quad (10)$$

the non-parametric function $f_t^{-1}(m_{it}, k_{it})$ will generally be collinear with l_{it} under the assumptions of the model. One approach to solving this collinearity problem might be to treat $f_t^{-1}(m_{it}, k_{it})$ parametrically. Note that even though l_{it} might again just be a function of m_{it} and k_{it}, if it is a different function of m_{it} and k_{it} than f_t^{-1} is, this parametric version is potentially identified. While using a parametric version makes more assumptions than the non-parametric approach, one might be willing to make such assumptions with relatively uncomplicated inputs such as materials.

Unfortunately, this parametric approach does not work in general for some popular production functions. In the case of Cobb-Douglas, the first order condition for m_{it} (conditional on k_{it}, l_{it}, and ω_{it}) is:

$$\beta_3 K_{it}^{\beta_1} L_{it}^{\beta_2} M_{it}^{\beta_3 - 1} e^{\omega_{it}} = \frac{p_m}{p_y}$$

assuming firms are price takers in both input and output markets. Recall that capital letters represent levels (rather than logs) of the inputs. Inverting this out for ω_{it} gives:

$$\begin{aligned} e^{\omega_{it}} &= \frac{1}{\beta_3}\frac{p_m}{p_y} K_{it}^{-\beta_1} L_{it}^{-\beta_2} M_{it}^{1-\beta_3} \\ \omega_{it} &= \ln(\frac{1}{\beta_3}) + \ln(\frac{p_m}{p_y}) - \beta_1 k_{it} - \beta_2 l_{it} + (1-\beta_3) m_{it} \end{aligned}$$

and plugging this inversion into the production function results in:

$$y_{it} = \ln(\frac{1}{\beta_3}) + \ln(\frac{p_m}{p_y}) + m_{it} + \epsilon_{it} \tag{11}$$

The important thing here is that β_2 has dropped out of the estimating equation, making a moment condition in ϵ_{it} worthless for identifying β_2. As such, with a Cobb-Douglas production function, a parametric approach cannot generally be used as a first stage to identify β_2.

One gets a similar result with a production function that is Leontief in the material inputs. Consider, for example:

$$Y_{it} = \min\left[\gamma_0 + \gamma_1 M_{it}, K_{it}^{\beta_1} L_{it}^{\beta_2} e^{\omega_{it}}\right] + \epsilon_{it}$$

With this production function, the first order condition for M_{it} satisfies

$$\gamma_0 + \gamma_1 M_{it} = K_{it}^{\beta_1} L_{it}^{\beta_2} e^{\omega_{it}}$$

as long as $\gamma_1 p_y > p_m$. At this optimum, note that:

$$y_{it} = \beta_1 k_{it} + \beta_2 l_{it} + \omega_{it} + \epsilon_{it} \tag{12}$$

which could form an estimating equation if not for endogeneity problems. Inverting out ω_{it} results in:

$$\begin{aligned} e^{\omega_{it}} &= \frac{\gamma_0 + \gamma_1 M_{it}}{K_{it}^{\beta_1} L_{it}^{\beta_2}} \\ \omega_{it} &= \ln(\gamma_0 + \gamma_1 M_{it}) - \beta_1 k_{it} - \beta_2 l_{it} \end{aligned}$$

and substituting into (12) results in

$$y_{it} = \ln(\gamma_0 + \gamma_1 M_{it}) + \epsilon_{it} \tag{13}$$

so again, as β_2 has dropped out of the estimating equation, making this procedure worthless for identifying β_2.

In summary, even with parametric assumptions, there is an identification problem in the LP technique using intermediate inputs to control for unobserved factors of production. However, it is possible that as one moves away from Cobb-Douglas production functions (or Hicks neutral unobservables), a parametric approach might be identified (see Van Biesebroeck (2003) for a related example).

5 Alternative Procedure 1

This section outlines an alternative estimation procedure that avoids the collinearity problems discussed above. This procedure draws on aspects of both the OP and LP procedures and is able to use either the 'investment as proxy' idea of OP, *or* the 'intermediate input as proxy' idea of LP. The main difference between this new approach and OP and LP is that in the new approach, no coefficients will be estimated in the first stage of estimation. In contrast, all the input coefficients are estimated in the second stage. However, as we shall see, the first stage will still be important to net out the untransmitted error ϵ_{it} from the production function. As they are straightforward extensions, we then show how our approach can treat labor as an input with dynamic implications. We lastly present a general framework under which one can think about identifying various input coefficient under various assumptions.

5.1 Non-Dynamic Labor

First we consider the case where labor has no dynamic implications. We consider the following value added production function (net of, e.g., materials),

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + \omega_{it} + \epsilon_{it} \tag{14}$$

but are techniques are equally applicable to the gross output production functions (e.g. $y_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \omega_{it} + \epsilon_{it}$) examined above.

Again following LP, we use the following form of the intermediate input function:

$$m_{it} = f_t\left(\omega_{it}, k_{it}\right) \tag{15}$$

which is again assumed to be monotonic so that (**??**) can be inverted to get:

$$\omega_{it} = f_t^{-1}(m_{it}, k_{it}) \tag{16}$$

Following the collinearity discussion of the previous section, we note that l will typically also be a function of the same state variables, and so we get, as before:

$$l_{it} = g_t(\omega_{it}, k_{it}) = g_t(f_t^{-1}(m_{it}, k_{it}), k_{it}) = h_t(m_{it}, k_{it}) \tag{17}$$

As previously noted, this resulted in a collinearity problem that could not be easily resolved, particularly within the LP framework. However, as will be clear shortly, the collinearity problem is not invincible in this case. Substituting in for ω_{it} in equation (14) and given the form of l_{it} in (17), we get the following expression for the production function:

$$y_{it} = \phi_t(m_{it}, k_{it}) + \epsilon_{it} \tag{18}$$

In this expression, ϕ_t combines all the production function terms, including the labor one. While β_1 is not identifiable from this equation, the ϕ function can be non-parametrically estimated in the same way that it is estimated in the first stage of either the OP or the LP procedures. As we shall see, the reason for doing this is to identify and remove the noise term, ϵ_{it}, from the dependent variable. However, with no coefficients obtained in the first stage, we still need to identify β_l and β_k. This now requires two independent moment conditions for identification in the second stage (in contrast, both

OP and LP (in the value added case) only require one second stage moment condition).

As in OP and LP, we assume that the productivity term follows a first-order Markov process

$$\omega_{it} = E[\omega_{it}|\omega_{it-1}] + \xi_{it}$$

Given the OP/LP timing assumption on k_{it}, this leads to the single moment condition used by both OP and LP, namely that:

$$E[\xi_{it}|k_{it}] = 0 \tag{19}$$

Constructing the empirical analogue of this moment again now uses the expression:

$$\xi_{it}(\beta_k, \beta_l) = \omega_{it}(\beta_k, \beta_l) - E[\omega_{it}|\omega_{it-1}; \beta_k, \beta_l] = (\widehat{\phi_{it}} - \beta_k k_{it} - \beta_l l_{it}) - \widehat{\psi}(\beta_k, \beta_l, \widehat{\phi_{it-1}}, k_{it-1}, l_{it-1})$$

As in OP/LP, values of $\widehat{\phi}_{it}$ are obtained in the first stage of estimation (which necessitated our first stage to net out ϵ_{it}). However, now constructing both $\omega_{it}(\beta_k, \beta_l)$ and $E[\omega_{it}|\omega_{it-1}; \beta_k, \beta_l]$ requires subtracting both $\beta_k k_{it}$ and $\beta_l l_{it}$ from $\widehat{\phi_{it}}$. Once again, the $\widehat{\psi}$ values are the predicted values from the nonparametric regression of of $\widehat{\omega_{it}}$ on $\widehat{\omega_{it-1}}$, which in this case means the regression of $(\widehat{\phi_{it}} - \beta_k k_{it} - \beta_l l_{it})$ on $(\widehat{\phi_{it-1}} - \beta_k k_{it-1} - \beta_l l_{it-1})$.

For our second needed moment condition we utilize the fact that under the assumptions of the model, *lagged* labor should be uncorrelated with the surprise in productivity, i.e.

$$E[\xi_{it}(\beta_k, \beta_l)|l_{it-1}] = 0 \tag{20}$$

This is because l_{it-1} is in the firm's information set at $t-1$ and thus must be uncorrelated with ξ_{it}. In summary, equations (19) and (20) are the two moment conditions used to identify the two coefficients, β_k and β_l in our model.

A few notes are in order here. First, note that there is nothing essential about using the intermediate input as the proxy in the above procedure. Using a monotonicity assumption on investment, instead of intermediate input use, and replacing each occurrence of an m in the above procedure with an i for investment would also work.[6,7] Second, the moment condition we use to

[6] As before, if investment is used, then we would need to assume that k_{it} is determined exactly at time $t-1$ (so that it proxies ω_{it-1} exactly), whereas with the intermediate input proxy, we can assume that k_{it} is determined at time $t-1$ or earlier.

[7] As noted above, this approach also works using gross output rather than value-added

identify the labor coefficient is very similar to the one used to identify the intermediate input coefficient in LP's gross output production function, i.e. $E[\xi_{it}|m_{it-1}] = 0$. In addition, a moment condition exactly of the form (20) is also used by both LP and OP as an overidentifying restriction of the model for the second stage procedure. However, there is a fundamental difference between what we are doing and what OP/LP do - in OP/LP, the labor coefficient is estimated in the first stage *without* using any of the information from (20). In our procedure we only use the information in (20). Given the possible collinearity problems with first stage identification described above, we prefer our method of identification.

Third, note that (18) is derived under the assumption that l_{it} is a function of the same state variables as is materials (i.e. labor is generated according to (17)). While in the above, we argue that it is hard to believe that there is enough optimization error in l_{it} to generate independent variation, it is true that any optimization error in l_{it} would invalidate this assumption. A more flexible model allowing a linear labor term in (18), i.e.

$$y_{it} = \theta l_{it} + \phi_t(m_{it}, k_{it}) + \epsilon_{it} \tag{21}$$

allows for less than perfect collinearity. In this situation, we would generally prefer, especially when using m_{it} as a proxy, not to interpret the estimated parameter θ as anything meaningful. This is identical to the first stage in LP and thus subject to the criticisms made above. We suspect that this is more likely to be specification error in ϕ than optimization error in l or i.i.d. price shocks.

5.2 Dynamic Labor

The above formulation, as does OP/LP, treats labor as a static variable. In other words, firms' choices of labor in period t only affect period t profits and

as the output measure (as in the LP procedure). In this case, all three coefficients (β_k, β_l, and β_m) need to be estimated in the second stage. Appropriate moments in this case would be $E[\xi_{it}(\beta_k, \beta_l, \beta_m)|k_{it}, l_{it-1}, m_{it-1}] = 0$. We do, however, need to verify that these moment conditions are not linearly dependent. Importantly, while the first part of this paper suggests that the variables $(k_{it}, l_{it-1}, m_{it-1})$ may be written as strict functions of each other, these functions will generally not be linear. As such, the above moment conditions will generally not be linearly dependent and provide sufficient identification to identify all three coefficients.

not the future. Our procedure can easily allow one to relax this assumption. Suppose labor has such dynamic implications. This could arise from, e.g., hiring or firing costs. Continue to make the LP assumption that labor and materials are perfectly variable inputs that are chosen at production time. Given labor is dynamic, it is now part of the state space. Specifically, firms choices of current labor and materials will depend on last period's labor choice, i.e.

$$m_{it} = f_t(\omega_{it}, k_{it}, l_{it-1}) \tag{22}$$

$$l_{it} = g_t(\omega_{it}, k_{it}, l_{it-1}) \tag{23}$$

Following the above, this simply generates a first stage of the form

$$y_{it} = \phi_t(m_{it}, k_{it}, l_{it-1}) + \epsilon_{it}$$

with the additional l_{it-1} variable. As in the above, no coefficients are identified in this first stage. Coefficients can be identified in the second stage by the same moments as in the last section, i.e.

$$E[\xi_{it}(\beta_k, \beta_l) | l_{it-1}, k_{it}] = 0 \tag{24}$$

Allowing labor to have dynamic effects in the OP framework is very similar, although it might be more natural in this case allow the proxy variable, investment, to depend on l_{it} rather than l_{it-1}, i.e.

$$i_{it} = f_t(\omega_{it}, k_{it}, l_{it}) \tag{25}$$

Substituting in generates a first stage of

$$y_{it} = \phi_t(i_{it}, k_{it}, l_{it}) + \epsilon_{it} \tag{26}$$

and estimation can proceed using the same second stage as above. Note that as long as $l_{it} = g_t(\omega_{it}, k_{it}, l_{it-1})$, this should be identical to allowing the investment decision to depend on $(\omega_{it}, k_{it}, l_{it-1})$.

One important point is that in both these cases, the dimension of our first stage non-parametric function has increased by one. As such, one needs to be even more careful about the caveats of non-parametric estimation, e.g. overfitting.

5.3 Non-Variable Labor

Recall that the standard LP procedure assumes that labor and materials are perfectly variable inputs and determined simultaneously (in contrast, note that OP does not rely on this assumption). While our procedure cannot relax the assumption that m_{it} is perfectly variable (because then m_{it} will typically not "invert out" ω_{it}), we can relax this assumption on labor. Suppose that at some point $t-b$ between $t-1$ and t, labor is chosen, possibly as a function of some intermediate value of ω, i.e.

$$l_{it} = g_t\left(\omega_{it-b}, k_{it}\right)$$

Then at t, the perfectly variable m_{it} is chosen. This is likely to depend on what was just chosen for l_{it}, i.e.

$$m_{it} = f_t\left(\omega_{it}, k_{it}, l_{it}\right) \tag{27}$$

While the first stage of the standard LP procedure will clearly not work in this case, our alternative procedure does. Our first stage is

$$y_{it} = \phi_t(m_{it}, k_{it}, l_{it}) + \epsilon_{it}$$

and identification of the coefficients in the second stage follow from the moments

$$E[\xi_{it}(\beta_k, \beta_l)|l_{it-1}, k_{it}] = 0 \tag{28}$$

Note that this approach is also consistent with labor having dynamic implications. In that case, labor will generally be set according to $l_{it} = g_t\left(\omega_{it-b}, k_{it}, l_{it-1}\right)$, but given that material input choice only affect current profits, equation (27) still holds and one ends up with the same first and second stages. This also increases the dimension of the non-parametric function relative to the basic model.

5.4 Relation to Dynamic Panel Models

6 Alternative Procedure 2

This section examines an alternative procedure to estimate production function coefficients. While it does break the potential collinearity problems of

OP/LP, it does rely on some additional assumptions, specifically an additional monotonicity assumption and some independence assumptions on the innovations in ω_{it}. It is also a bit more complicated than the procedure we suggest above. On the other hand, this procedure does allow one to learn something about the timing of input choice. In particular, it can help us understand the actual timing of input choice, e.g. whether labor is more of a flexible or

The intuition behind identification in this second approach follows directly from the intuition of identification of the coefficient on capital in OP (and LP). We make heavy use of the fact that if an input is determined prior to production, the *innovation* in productivity *between* the time of the input choice and the time of production should be orthogonal to that input choice. Moreformally, if ω_i is the productivity level of the firm at the time input i is chosen, and ω_p is the productivity level at the time of production, then:

$$(\omega_p - E[\omega_p|\omega_i]) \perp \quad i$$

This type of moment identifies the capital coefficient in OP and LP. Our approach simply extends this intuition to identification of parameters on labor inputs, combining this with non-parametrics to "invert out" values of the productivity shock at various decision times.

Consider a production model with 3 inputs, capital, labor, and an intermediate input, e.g. materials. We make the following timing assumptions regarding when k, l, and m are chosen. Suppose between periods $t-1$ and t, the following occurs, where $0 < b < 1$:

Time	Action
$t-1$	ω_{it-1} is observed, m_{it-1} is chosen, k_{it} is chosen, period $t-1$ production occurs
$t-b$	ω_{it-b} is observed, l_{it} is chosen
t	ω_{it} is observed, m_{it} is chosen, k_{it+1} is chosen, period t production occurs

Like OP/LP, we assume that k_{it} is determined at time $t-1$. Actually, like LP (but not OP), we only really need to assume that k_{it} is determined at either $t-1$ or earlier. For the more variable inputs, we assume that l_{it} is chosen at some time between $t-1$ and t, and that m_{it} is perfectly flexible and chosen at time t.

Note that we assume ω evolves between $t-1$, $t-t_b$, and t. As in our "story" behind OP, this movement is needed to alleviate possible collinearity problems between labor and other inputs. We assume that ω evolves as a first-order markov process between these stages, i.e.:

$$\begin{aligned} \omega_{it-b} &= g_1(\omega_{it-1}, \eta_{it}^b) \\ \omega_{it} &= g_2(\omega_{it-1}, \eta_{it}) \end{aligned} \tag{29}$$

where the η's are independent of the ω's (as well as all other variables that are chosen before their realizations). Note that this is a slightly stronger assumption than that of OP and LP, which assume only a first-order markov process. On the other hand, the fact that the g's are arbitrary functions allows some forms of heteroskedasticity. While our "staggered" input choice process might initially seem somewhat *ad-hoc*, we feel that it does capture some interesting aspects of reality.[8]

Given the above timing assumptions and assuming that labor is a static input, a firm's choice of labor will be a function of ω_{it-b}, i.e.

$$l_{it} = f_{1t}(\omega_{it-b}, k_{it}) \tag{30}$$

Since the firm's choice of labor in a given period is made before its choice of materials, the labor term must be taken into account when choosing the level of materials, i.e.

$$m_{it} = f_{2t}\left(\omega_{it}, k_{it}, l_{it}\right) \tag{31}$$

Once again, we will assume monotonicity of this equation in ω_{it}, allowing us to invert this function and obtain:

$$\omega_{it} = f_{2t}^{-1}(m_{it}, k_{it}, l_{it})$$

This term can be substituted into the production function from (14) to get:

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + f_{2t}^{-1}(m_{it}, k_{it}, l_{it}) + \epsilon_{it}$$

and collecting terms results in the first stage equation:

$$y_{it} = \phi(m_{it}, k_{it}, l_{it}) + \epsilon_{it} \tag{32}$$

[8] Though this is clearly a stylized model of what is likely a more continuous decision process.

This is exactly the same first stage as section 5.3, and the ϕ function can be estimated in the usual way. Similarly, we can construct the same moment condition for capital:

$$E[\xi_{it}(\beta_k, \beta_l)|k_{it}] = 0 \tag{33}$$

where $\xi_{it} = \omega_{it} - E[\omega_{it}|\omega_{it-1}]$, and $\xi_{it}(\beta_k, \beta_l)$ can be contructed in the usual way, i.e. by non-parametrically regressing $(\omega_{it}(\beta_k, \beta_l) = \widehat{\phi}(m_{it}, k_{it}, l_{it}) - \beta_l l_{it} - \beta_k k_{it})$ on $(\omega_{it-1}(\beta_k, \beta_l) = \widehat{\phi}(m_{it-1}, k_{it-1}, l_{it-1}) - \beta_l l_{it-1} - \beta_k k_{it-1})$.

What differs between this and the above procedures is the moment condition intended to identify the labor coefficient. Define ξ_{it}^b as the unexpected innovation in ω between time $t-b$ and t, i.e.

$$\xi_{it}^b = \omega_{it} - E[\omega_{it}|\omega_{it-b}]$$

Given that labor is chosen at $t-b$, it should be orthogonal to this innovation

$$E[\xi_{it}^b|l_{it}] = 0 \tag{34}$$

This is the moment condition we will use - what remains to be shown is how we can construct this moment given a value of the parameter vector. To do this, first note that the first stage estimates of (32) allow us to compute, conditional on the parameters, ω_{it} for all t. Call these terms $\omega_{it}(\beta_k, \beta_l)$. Now consider the firm's labor demand function (30). Substituting in (29) results in

$$\begin{aligned} l_{it} &= f_{1t}(g_1(\omega_{it-1}, \eta_{it}^b), k_{it}) \\ &= \widetilde{f}_{1t}(\omega_{it-1}(\beta_k, \beta_l), \eta_{it}^b, k_{it}) \end{aligned}$$

Note that conditional on (β_k, β_l), the only unobservable in this equation is η_{it}^b. Thus, assuming that the equation is strictly monotonic in η_{it}^b, one can use the methods of Matzkin (1999) to non-parametrically invert out η_{it}^b up to a normalization. Call this function $\tau(\eta_{it}^b; \beta_k, \beta_l)$. Again, the dependence on β_k and β_l comes from the fact that the ω_{it} are determined conditional on β_k and β_l. This non-parametric inversion relies on the assumption that η_{it}^b is independent of ω_{it-1} and k_{it}. The basic intuition is that for a given ω_{it-1} and k_{it}, one can form a distribution of l_{it}. $\tau(\eta_{it}^b; \beta_k, \beta_l)$ for a given i is simply the quantile of l_{it} in that distribution.

Next, note that since ω_{it} is a function of η^b_{it} and ω_{it-1}, we can also write it as a function of $\tau(\eta^b_{it};\beta_k,\beta_l)$ and ω_{it-1}, i.e.

$$\begin{aligned}\omega_{it-b}(\beta_k,\beta_l) &= g_1(\omega_{it-1}(\beta_k,\beta_l),\eta^b_{it}) \\ &= \widetilde{g}_1(\omega_{it-1}(\beta_k,\beta_l),\tau(\eta^b_{it};\beta_1,\beta_2))\end{aligned}$$

As a result, to construct $\xi^b_{it} = \omega_{it} - E[\omega_{it}|\omega_{it-b}]$, we can form the necessary conditional expectation by non-parametrically regressing $\omega_{it}(\beta_k,\beta_l)$ on $\omega_{it-1}(\beta_k,\beta_l)$ and $\tau(\eta^b_{it};\beta_k,\beta_l)$ (as an alternative to non-parametrically regressing $\omega_{it}(\beta_k,\beta_l)$ on $\omega_{it-b}(\beta_k,\beta_l)$). Denoting the residual from this regression by $\xi^b_{it}(\beta_k,\beta_l)$, we can form the moment

$$E[\xi^b_{it}(\beta_k,\beta_l)|l_{it}] = 0 \qquad (35)$$

to be used for estimation. Note that this procedure can easily be adjusted to allow for labor to have dynamic implications. One simply needs to include l_{it-1} in both the material and labor demand functions.

7 Empirical Example

See attached tables for some preliminary results.

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Apr-16-2005 **Summary - Value-Added Production Function**

Industry 381

	Constant	Capital Coeff	Unskilled Labor	Skilled Labor	# of Obs	Sum of Coefs
OLS - Full Sample	4.52	0.20	0.55	0.60	1569	1.35
Our Method - KE	4.97	0.27	0.26	0.51		1.04
Our Method - KM	4.81	0.29	0.29	0.48		1.06
Our Method - KEL	4.91	0.18	0.47	0.62		1.26
Our Method - KML	4.94	0.17	0.47	0.64		1.28
LP - Materials	4.52	0.20	0.39	0.40		1.00
LP - Electricity	3.99	0.33	0.43	0.48		1.24
OLS - Sample Dropping Investment Zeros	5.42	0.20	0.33	0.66	432	1.19
Our Method - KI	5.70	0.34	0.11	0.32		0.77
Our Method - KIL	6.13	0.15	0.23	0.70		1.08
OP - Investment	5.78	0.20	0.26	0.61		1.06

Industry 331

	Constant	Capital Coeff	Unskilled Labor	Skilled Labor	# of Obs	Sum of Coefs
OLS - Full Sample	3.93	0.23	0.56	0.56	1291	1.35
Our Method - KE	4.45	0.26	0.33	0.48		1.07
Our Method - KM	4.36	0.30	0.18	0.64		1.12
Our Method - KEL	4.27	0.19	0.50	0.66		1.36
Our Method - KML	4.43	0.16	0.51	0.70		1.37
LP - Materials	3.93	0.23	0.41	0.28		0.92
LP - Electricity	3.97	0.30	0.43	0.39		1.12
OLS - Sample Dropping Investment Zeros	3.94	0.29	0.49	0.51	249	1.29
Our Method - KI	3.84	0.47	0.16	0.28		0.90
Our Method - KIL	4.56	0.25	0.33	0.66		1.24
						0.00
OP - Investment	3.82	0.29	0.58	0.41		1.27

Industry 321

	Constant	Capital Coeff	Unskilled Labor	Skilled Labor	# of Obs	Sum of Coefs
OLS - Full Sample	4.59	0.25	0.42	0.53	1566	1.19
Our Method - KE	4.46	0.34	0.40	0.20		0.93
Our Method - KM	4.25	0.36	0.39	0.19		0.95
Our Method - KEL	4.75	0.21	0.48	0.49		1.18
Our Method - KML	4.87	0.20	0.48	0.49		1.17
LP - Materials	4.59	0.25	0.31	0.43		0.99
LP - Electricity	4.57	0.27	0.37	0.50		1.15
OLS - Sample Dropping Investment Zeros	5.39	0.21	0.32	0.53	395	1.07
Our Method - KI	4.34	0.52	0.14	0.03		0.69
Our Method - KIL	5.80	0.16	0.42	0.43		1.01
OP - Investment	5.45	0.22	0.26	0.55		1.04

Industry 311

	Constant	Capital Coeff	Unskilled Labor	Skilled Labor	# of Obs	Sum of Coefs
OLS - Full Sample	3.62	0.36	0.46	0.50	5372	1.31
Our Method - KE	4.32	0.36	0.05	0.72		1.13
Our Method - KM	4.18	0.32	0.34	0.52		1.18
Our Method - KEL	4.54	0.25	0.29	0.78		1.32
Our Method - KML	4.51	0.24	0.33	0.80		1.36
LP - Materials	3.62	0.36	0.28	0.24		0.88
LP - Electricity	2.54	0.56	0.37	0.23		1.16
OLS - Sample Dropping Investment Zeros	3.47	0.46	0.26	0.45	1333	1.18
Our Method - KI	3.41	0.55	0.13	0.25		0.93
Our Method - KIL	3.73	0.45	0.20	0.44		1.10
OP - Investment	3.32	0.51	0.21	0.40		1.12