Competition in Two-Sided Markets*

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August 2002: revised February 2004

Abstract

There are many examples of markets involving two groups of participants who need to interact via intermediaries or “platforms”, and where the benefit to joining a platform depends on the number of agents from the other side of the market who join the same platform. This paper presents theoretical models for three variants of such markets: a monopoly platform; a model of competing platforms where each agent must choose to join a single platform; and a model of “competing bottlenecks”, where one side of the market wishes to join all platforms. The main questions addressed are (i) what determines the structure of relative prices offered to the two groups, (ii) is the resulting outcome socially efficient, and (iii) how do the details of how platform charges are defined—for instance, on a lump-sum basis or on a per-transaction basis—affect the equilibrium outcome?

1 Introduction

There are many examples of markets or institutions involving two or more groups of participants who interact via intermediaries or “platforms”. Surplus is created—or destroyed in the case of negative externalities—when the groups interact. In most interesting cases, two-sided network effects are present, and the surplus enjoyed by a member of one group depends on the number of the other group who join the same platform (or depends on some other measure of the platform’s performance on the other side of the market). One class of examples involves buyers and retailers as the two groups, with the platforms being newspapers or

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*An early version of this paper was presented at the ESEM meeting in Venice, August 2002. I am grateful to Simon Anderson, Ted Bergstrom, Steve Coate, Jacques Crémer, Volker Nocke, Martin Peitz, Hyun Shin, Daniel Spulber, Jean Tirole, Xavier Vives, Myrna Wooders and most especially Julian Wright for discussion, correction and information.
yellow pages directories (which are used by retailers as a conduit for their advertising), or shopping malls (which act to bring the two groups physically together). See Evans (2003a, 2003b) and Rochet and Tirole (2003) for further examples of such markets.

Of course there are many examples of competing platforms that bring together two groups of agents in a surplus-enhancing manner but where cross-group network effects are absent. For instance, a firm needs to compete for labour at the same time as it competes for consumers of its output. However, such cases are different from those considered in this paper in that agents from one group generally do not care how well the firm performs in the market for the other group, but only about their own terms for dealing with the platform. A worker generally cares about the wage but not how many units of output are sold, and a consumer generally cares about the price, but not how many workers the firm employs.\footnote{One exception might be in a retailing context, where consumers care about the “service” they receive, and quality of service is likely to be improved when the retailer employs more staff. See section 2 for further examples where a firm competing for inputs and outputs can be considered as operating in a two-sided market with externalities.}

In addition, there are examples of cross-group externalities which are not mediated by a platform at all. Obvious examples come from economic geography, where one group is more likely to locate in an area where another, complementary group has located (and \textit{vice versa}).

The main questions addressed in the paper are (i) what determines the structure of relative prices offered to the two groups, (ii) when is the resulting allocation socially efficient, and (iii) how do the details of how platform charges are defined—for instance, on a lump-sum basis or on a per-transaction basis—affect the equilibrium outcome? No single, transparent model can hope to encompass all the features of markets with two-sided network effects. Therefore, following a selective literature review, in the following sections we present three variants of two-sided markets that cover a variety of plausible situations.

First is the benchmark case of a single platform (section 3). As one would expect, the principal market failure in this case is that the prices offered to both groups are too high. The second model is one of competing platforms, but in the special case where both sides of the market join a single platform (or “single-home”)—see section 4. To the extent that the markets are competitive, platforms will not make large profits overall, and any profit made on one side of the market is largely used to attract agents from the other side. The analysis will show that one group will be targeted more aggressively if that group is more competitive than the other, or if it causes relatively large external benefits to the other group. For instance, if group 1 gains little from the interaction with group 2, then it is plausible that group 1 will need an extra incentive to join a platform. The third model assumes that one side of the market joins a single platform (single-homes) while the other group joins each platform to gain access to all agents on the other side (or “multi-homes”). This model, which is termed the “competitive bottleneck” model, is analyzed in section 5. The analysis will show that competition is particularly intense on the single-homing side and essentially non-existent on the multi-homing side. For instance, if people tend to read just a single newspaper, perhaps because of time constraints, then the newspaper has a monopoly position over delivering

\footnote{One exception might be in a retailing context, where consumers care about the “service” they receive, and quality of service is likely to be improved when the retailer employs more staff. See section 2 for further examples where a firm competing for inputs and outputs can be considered as operating in a two-sided market with externalities.}
adverts to their readers and can therefore extract rents from advertisers. These rents are then used to attract readers. This outcome is inefficient since the price for advertising is set at too high a level.

2 The Literature

*Competition for inputs and outputs:* A firm typically has to compete for inputs (such as labour) and also to compete to sell its output to consumers. Stahl (1988) and Yanelle (1989) model firms as competing (either sequentially or simultaneously) both for inputs and for outputs. A natural strategy to investigate is for a firm to try to compete hard for inputs (by pricing low in that side of the market), in order to disadvantage its rival in the supply side of the market. When there is a fixed relationship between inputs and outputs (one unit of input is transformed into one unit of output), rationing plays an important role. Because of this, there are inter-group externalities present that affect the competitive outcome.2

Gehrig (1993) provides an interesting related analysis in the context of a monopoly platform. In his model there is a homogeneous product, and a group of buyers and group of sellers of this product. Buyers differ in their reservation price for a unit of the product, as do sellers. Without intermediation, buyers and sellers must *search* for a suitable trading partner, which entails delay and an uncertain eventual price. Suppose that a single platform is introduced which can *broadcast* a buying and selling price to all agents. Agents can choose whether to go to the platform or to remain in the search market. If the platform attracts fewer sellers than buyers, then buyers must be rationed according to some rule (as in Stahl and Yanelle), and similarly if there are more sellers than buyers. In this sense there are two-sided network externalities: for a given price, sellers are better off the more buyers there are on the platform (and they are worse off with more other sellers), since this reduces the chance that they will be rationed.

Finally, if wages, say, are determined by a bargaining process of some kind (perhaps as in the model of Stole and Zwiebel (1996)), then workers could care about how well the firm does in the product market since that affects the surplus over which they bargain.

*CREDIT CARDS:* Perhaps the main stimulus to the current work on two-sided markets has been the burgeoning literature on credit cards and other payment systems. Most of the work on payment systems has assumed a monopoly (non-cash) payment system—see Baxter (1983) for the pioneering contribution, followed more recently by (among others) Rochet and Tirole (2002), Schmalensee (2002), Wright (2003b) and Wright (2003a). More recently, Rochet and Tirole (2003) and Guthrie and Wright (2003) have extended the analysis to allow for competing payment systems.

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2See Gehrig (1996) and Yanelle (1997) for analyses of the banking industry, where the two sides of the market are “savings” and “loans”, that build on these earlier papers.
Consider Rochet and Tirole (2003) in more detail. One side of the credit card market consists of the retailers and the other side is the consumers. Retailers are assumed to be monopolies. Much of their analysis involves a model where each agent obtains a payoff from joining a platform which is proportional to the number of transactions carried out on the platform. In particular, platforms levy charges on a per-transaction basis, and there are no lump-sum joining fees. (This assumption implies the simplifying feature that an agent’s choice of card does not depend on the number of agents on the other side who use the same card.) The two credit card companies offer charges for using their payment system: a per-transaction charge to consumers and a per-transaction charge to retailers. Essentially, given these charges, retailers first decide which cards to accept (either one card, both cards, or no cards). Then consumers (who wish to buy a single unit from each monopoly retailer) decide for each retailer whether to use the retailer’s chosen card (if that shop accepts a single card) or, if both cards are accepted, which card to use.

A retailer deciding between taking one or both cards faces a trade-off. If it accepts a single card (the card offering a high intrinsic benefit or a low charge) then its consumers have a stark choice between paying by this card or not using a card at all. Alternatively, if it accepts both cards then (i) more consumers will choose to pay by some card but (ii) fewer consumers will choose to use the retailer’s favoured card. If one card reduces its charge to retailers, this will have two effects: first, it will cause some retailers which previously did not use either card to join its network, and second, it will cause some retailers who previously accepted both cards to accept only the low-charge card. The share of the charges that are borne by the retailers then depends (in part) on how closely consumers view the two cards as being substitutes. If consumers do not switch cards much in response to a price cut, then consumers should pay a large share of the total transaction charge; if consumers view the cards are close substitutes, then the retailers will bear most of the charges in equilibrium.

Rochet and Tirole also consider the case where there are fixed fees as well as per-transaction fees, under the assumption that consumers will choose a single card. In this case, whether a consumer decides to use a given card depends upon the number of retailers who accept the card. I analyze a very similar model (where the principal application is to advertising on media platforms) in section 5 below.

Call termination in telecommunications: Section 3.1 of Armstrong (2002) and Wright (2002) propose a model of competition between mobile telecommunications networks. Subscribers wish to join at most one mobile network (i.e., they single-home). People on the fixed telephony networks wish to call mobile subscribers. For a specified charge, someone can call any given mobile network, and in this sense the people who call mobile networks “multi-home”. A subscriber will choose a mobile network with the tariff that leaves him with the most

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3For maximum simplicity the analysis assumed that the people who called mobile subscribers were not themselves mobile subscribers. (But see section VI(ii) of Wright (2002) for discussion of the realistic case where there are calls between mobile networks.)
surplus. A network’s tariff has two ingredients: the charges for subscription and outbound calls that affect the subscriber’s welfare directly, and the charges the network makes to others for delivering calls to the subscriber (the so-called call termination charges). Unless the subscriber cares about the welfare of people who might call him, the latter charges affect the subscriber’s welfare only insofar as they affect the number of calls he receives. (High termination charges will typically act to reduce the number of calls made to mobile networks, and this is detrimental to a subscriber’s welfare if he obtains benefits from receiving calls.)

The tariffs that mobile networks set in equilibrium have low charges for subscription and outbound calls and high charges for call termination. In particular, the model predicts that high profits made on call termination are (partially) passed on to subscribers, perhaps in the form of subsidized handsets and the like. More precisely, the equilibrium call termination charge is chosen to maximise the welfare of mobile subscribers and mobile networks combined, and the interests of people who call mobile networks are ignored. This feature—that the single-homing side is treated well and the multi-homing side’s interests are ignored in equilibrium—is also a characteristic of the models presented in section 5 below. Although the market for subscribers might be highly competitive, so that mobile networks have low equilibrium profits, there is no competition for providing communication services to these subscribers.

Advertising in media markets: Gabszewicz, Lauussel, and Sonnac (2001) discuss the interaction of advertising, circulation and political position in the newspaper industry. They assume that readers do not care (positively or negatively) about the level of advertising in a newspaper. They model competition for the two sides of the market sequentially: once the newspapers’ political stances are chosen, prices are chosen (which determines the circulations for the two newspapers), and finally the newspapers choose their advertising charges. For the same reason as with call termination just discussed, there is essentially no competition between newspapers for advertising. (There is no welfare analysis in the model since the precise rationale for advertising is not modelled.) Interestingly, the authors find that the presence of advertising often acts to moderate the newspapers’ political stances, and the equilibrium in the first stage is for both newspapers to supply the same centrist stance.\footnote{Ferrando, Gabszewicz, Lauussel, and Sonnac (2003) provide a related analysis, but with the alternative assumption that advertisers can place an advert in only one outlet. This “single-homing” assumption for advertisers implies that media outlets do compete for advertising, but it is perhaps less natural than the assumption that advertisers may place adverts in several outlets.}

Anderson and Coate (2001) present a model where TV channels compete for viewers, and advertisers wish to gain access to these viewers. The cases where viewers pay a price to view and where viewing is free are considered. Viewers view adverts as a ‘nuisance’. For most of the paper it is assumed that viewers watch only a single channel over the relevant time horizon, and so a channel has a monopoly over providing advertising access to its viewers in a similar manner to the previous discussion of call termination in telecommunications. In this paper the role of advertising is explicitly modelled, and it is to inform viewers of a new
(monopoly) product. Welfare analysis is therefore possible, and one result in the
paper is that, when viewers pay for viewing there is too little advertising in equi-

Dukes (2003) and Dukes and Gal-Or (2003) discuss an interesting class of
models of advertising when advertisers compete amongst themselves for consumers. The latter paper
discusses whether a media platform would wish to restrict competition on its platform in
return for higher advertising fees. (A related model is presented in section 5.3 below.)
Baye and Morgan (2000) and Baye and Morgan (2001) present a model of advertising
on a monopoly media platform, showing among other results how advertising revenues are
likely to be greater than charges made to consumers to join the platform.
Rysman (2002) is a rare structural empirical investigation into markets with two-sided
network externalities. This paper estimates the importance of cross-group network effects
(on both sides of the market) in the market for yellow pages. He estimates that externalities
are significant on both sides of the market: consumers are more likely to use a directory
containing more adverts, while an advertiser will pay more to place an advert in a directory
that is consulted by more consumers.

Competing Matchmakers: A class of examples of two-sided markets is that of competing
matchmakers, such as dating agencies, real estate agents, and internet “business-to-business”
websites. The focus of these examples is naturally on the quality of a given match, and
heterogeneity of agents plays a crucial role. As a result, there is a rich set of contracting pos-
sibilities in these markets: for instance, one might have subscription charge in combination
with a transactions charge (i.e., a charge in the event of a successful match). For instance, a
new entrant might find it useful to charge its clients only in the event of a successful match,
since such a strategy means that potential customers are not deterred from joining the new
firm by the possibility that they will pay an up-front fee and yet too few people from the
other side have joined. A focus of this work is the possibility of asymmetric outcomes where
one platform corners both sides of the market. (These issues are ignored in the following
analysis.) Relevant work in this area is van Raalte and Webers (1998), Caillaud and Jullien

Competing Marketplaces: Gehrig (1998) presents a model with two marketplaces, in each of
which there are located a variety of differentiated firms. Consumers live on a line between
these two markets and decide which market to visit on the basis of a comparison of the
prices and varieties offered at the two ends. There is a separate tax authority for each
market which aims to maximize revenue, which it generates from taxing the transactions
within the market. The model assumes is that the fiscal authorities make their choices after
firms have made their location decisions, and so there is no scope for using fiscal policy to
attract firms to a market.

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5See Rosse (1970) for a very early empirical paper on advertising in the newspaper industry,
Ellison, Fudenberg, and Möbius (2003) investigates competing (standard) double-auction markets and, in particular, whether two auction markets can coexist even if there is no intrinsic product differentiation between them.

Pashigan and Gould (1998) and Pashigan, Gould, and Prendergast (2002) discuss the theory and empirics of pricing retailing space in shopping malls. In particular, they discuss how highly attractive shops (“anchor stores”) are given subsidized charges in order to attract consumers to the mall, and how other stores are thereby willing to pay higher charges in order to have access to these consumers.

Pricing complementary software products: Parker and Van Alstyne (2000) present an analysis of two-sided competition in software markets such as acrobat, and the analysis is done both for monopoly and for competition. They show how a monopolist might wish to distribute a complementary piece of software for free in order to stimulate demand for the fully functioning version. They then use this analysis to suggest why a monopolist in one segment (say, operating systems) might wish to enter a complementary market (say, web browsers) even if it ends up making no money in the competitive market.\footnote{See also Csorba and Hahn (2003).}

The present paper is most closely related to, and complementary with Rochet and Tirole (2003), which has already been discussed above in the context of credit cards. The differences between the papers are mainly in the specification of charges, costs and external benefits. The present paper, for the most part, assumes that platform charges take a lump-sum form and that costs take a per-consumer form. Rochet and Tirole focus on the case where both charges and costs are incurred per transaction. In particular, the charge an agent pays the platform is a function of the number of agents on the other side who join the platform. This has the feature that an agent’s choice of platform does not depend on her expectations about the number of agents on the other side who will join the same platform. Rochet and Tirole also focus very much on the issue of the endogeneity of when one group single-homes or multi-homes. The current paper, by contrast, takes a simplified approach of assuming that this preference is somehow exogenous. I return to this question in the paper’s conclusion.

3 A Monopoly Platform

Here we present the analysis for the case of a monopoly platform. While this framework does not apply to most of the examples of two-sided markets that come to mind, historically yellow pages directories were effectively a monopoly of the incumbent telephone company, sometimes shopping malls or nightclubs are far enough away from others that the monopoly paradigm might be appropriate, and sometimes there is only one newspaper or magazine in the relevant market.
Suppose there are two groups of agents, denoted \( l = 1, 2 \). If the “utility” (to be defined) offered to a member of group \( l \) is \( u_l \), suppose that the number of group \( l \) who participate is

\[ n_l = \phi_l(u_l) \]

for some increasing function \( \phi_l \). Next we describe how the utilities \( u_l \) are determined. The crucial ingredient is that a member of group \( l \) cares about the number of members of the other group \( m \) who go to the platform. (For simplicity, we ignore the possibility that a member of group \( l \) cares also about the number of members of group \( l \) who go to the same platform.) Therefore, utilities are determined in the following way: if the platform attracts \( n_m \) group \( m \) participants, the utility of a group \( l \) member at the platform is

\[ u_l = \alpha_l n_m - p_l , \]

where \( p_l \) is the platform’s charge to a group \( i \) participant. The parameter \( \alpha_l \) measures the inter-group externality for group \( l \) participants.

Turning to the cost side, suppose that the platform incurs a fixed cost of serving a group \( l \) participant of \( f_l \). If we consider the platform to be offering utilities \( \{ u_l \} \) rather than prices \( \{ p_l \} \), then the implicit price for group \( l \) is \( p_l = \alpha_l n_m - u_l = \alpha_l \phi_m(u_m) - u_l \). Therefore, expressed in terms of the offered utilities, the platform’s profit is

\[ \pi = [\alpha_1 \phi_2(u_2) - u_1 - f_1] \phi_1(u_1) + [\alpha_2 \phi_1(u_1) - u_2 - f_2] \phi_2(u_2) . \]

(1)

Let the aggregate consumer surplus of group \( l \) be \( v_l(u_l) \), where \( v_l'(u_l) \equiv \phi_l(u_l) \). Then total welfare, as measured by the unweighted sum of profit and consumer surplus, is

\[ w = \pi + v_1(u_1) + v_2(u_2) . \]

Notice that both the profit-maximizing and welfare-maximizing outcomes depend only on the sum of the externality parameters, \( \alpha = \alpha_1 + \alpha_2 \). It is easily verified that the first-best welfare maximizing outcome involves utilities satisfying:

\[ u_l = \alpha \phi_m(u_m) - f_l . \]

Or, since implicit prices are \( p_l = \alpha_l \phi_m(u_m) - u_l \), we see that the socially optimal prices are

\[ p_l = f_l - \alpha_l \phi_m(u_m) . \]

(2)

As one would expect, prices should ideally be set below cost (if \( \alpha_m > 0 \)) to take account of the externality enjoyed by the other side of the market. From expression (1), however, the profit-maximizing prices satisfy

\[ p_l = f_l - \alpha_m \phi_m(u_m) + \frac{\phi_l(u_l)}{\phi_l'(u_l)} . \]

(3)
Thus, the profit-maximizing prices are equal to the cost of providing service \((f_l)\), adjusted downwards by the externality effect \((\alpha_m\phi_m)\), and adjusted upwards by a factor related to the elasticity of the group’s participation. More precisely, if we write

\[
\eta_l(p_l \mid n_m) = \frac{p_l \phi_l(\alpha_l n_m - p_l)}{\phi_l(\alpha_l n_m - p_l)} > 0
\]

for the price elasticity of demand for group \(l\) for a given level of participation by the other group \(m\), then expression (3) can be written as a ‘Lerner’ formula:

\[
\frac{p_l - [f_l - \alpha_m n_m]}{p_l} = \frac{1}{\eta_l(p_l \mid n_m)}.
\]

Expression (4) states that the profit-maximizing prices are set proportionally above the welfare-maximizing prices by the factor equal to the inverse of the elasticity of participation. It is possible that the profit-maximizing outcome might involve group \(l\) being offered a subsidised service, i.e., \(p_l < f_l\). From (4), this happens if the elasticity of participation \(\eta_l\) is large and/or if the external benefit \(\alpha_m\) enjoyed by the other group is large. Indeed, the subsidy might be so large that the resulting price is negative (or zero, if negative prices are not feasible). This analysis applies, in a stylized way, to a market with a monopoly yellow page directory. Such directories typically are given to telephone subscribers for free, and profits are made from charges to advertisers. This analysis suggests that this outcome can be rationalised if readers of directories are in elastic supply and/or advertisers obtain a greater benefit from an additional reader than vice versa.

The analysis might sometimes apply to software markets where one type of software is required to create files in a certain format and another type of software is required to read such files. (The cases of “Acrobat”, or formats for audio and visual files come to mind.) For the analysis to apply accurately there needs to be two disjoint groups of agents: those who wish to read files and those that wish to create files. It does not readily apply when most people wish to perform both tasks.\(^7\)

This analysis is closely related to section 2 of Rochet and Tirole (2003). The two main differences with their analysis are the following. First, in the present model the level of participation is determined by the total surplus \(u_l = \alpha_l n_m - p_l\), whereas in Rochet and Tirole (2003) participation is determined by the surplus per transaction. This implies that participation decisions on one side do not depend on the number of participants on the other side. Second, Rochet and Tirole (2003) assume that costs are incurred on a per-transaction basis—they are proportional to \(n_1 n_2\)—whereas I assume they are incurred on a per-participant basis. For instance, they obtain the result (Proposition 1) that the monopolist sets charges proportional to a group’s elasticity of demand, whereas (3) expresses charges as being inversely proportional to elasticities (once account has been taken of externalities).

\(^7\)See Parker and Van Alstyne (2000) and Csorba and Hahn (2003) for analyses of this case.
4 Two-Sided Single-Homing

The second model involves competing platforms, but assumes that, for exogenous reasons, each participant (from either group) chooses to join a single platform.

4.1 A Simple Model

The model is made up of the following ingredients, which extend the previous monopoly model in a natural way. There are two groups of participants, 1 and 2. (These will be denoted l or m as before.) There are two platforms, A and B, which enable the participants to interact. (These will be denoted i or j.) Suppose that platform i offers a group m participant a utility level $u^i_m$. If group m is offered the choice of utilities $u^A_m$ and $u^B_m$ from the two platforms, the number who go to platform i is given by the familiar Hotelling specification:

$$n^i_m = \frac{1}{2} + \frac{u^i_m - u^i_j}{2t_m}.$$  \hspace{1cm} (5)

Here, $t_m$ is the differentiation parameter for group m (which might differ for the two groups). The utilities $u^i_m$ are determined in a similar manner to that described in section 3: if platform i attracts $n^i_m$ group m customers, the utility of a group l member at this platform is

$$u^i_l = \alpha_l n^i_m - p^i_l,$$  \hspace{1cm} (6)

where $p^i_l$ is the price the platform charges a group l member for its services. Putting (5) together with (6), and using the fact that $n^j_m = 1 - n^i_m$, gives the following implicit expressions for market shares:

$$n^i_1 = \frac{1}{2} + \frac{\alpha_1(2n^j_2 - 1) - (p^1_1 - p^j_1)}{2t_1}; \quad n^i_2 = \frac{1}{2} + \frac{\alpha_2(2n^j_1 - 1) - (p^2_1 - p^j_2)}{2t_2}.$$  \hspace{1cm} (7)

Expressions (7) show that, keeping prices for group 2 fixed, an extra group 1 customer for platform i attracts a further $\frac{\alpha_1}{t_1}$ group 2 customers onto its platform.

We suppose that the network externality parameters $\alpha_l$ are small compared to the differentiation parameters $t_l$ so that we can focus on symmetric equilibria where each platform captures half the market for both groups of participants. If this were not the case, the network externality effects outweigh brand preferences, and there would be equilibria where one platform corners both sides of the market. It turns out that the necessary and sufficient condition for a symmetric equilibrium to exist is the following condition on the consumer taste parameters:

$$4t_1 t_2 > (\alpha_1 + \alpha_2)^2$$  \hspace{1cm} (8)

and this inequality is assumed to hold in the following analysis.
Suppose that platforms $A$ and $B$ offer the two pairs of prices $(p^A_1, p^A_2)$ and $(p^B_1, p^B_2)$. Then solving the simultaneous equations (7) implies that markets shares are given explicitly by

\[ n^i_1 = \frac{1}{2} + \frac{1}{2} \frac{\alpha_1(p^i_2 - p^i_1) + t_2(p^i_1 - p^i_2)}{t_1t_2 - \alpha_1 \alpha_2}; \quad n^i_2 = \frac{1}{2} + \frac{1}{2} \frac{\alpha_2(p^i_1 - p^i_2) + t_1(p^i_1 - p^i_2)}{t_1t_2 - \alpha_1 \alpha_2}. \quad (9) \]

(Assumption (8) implies that $t_1t_2 - \alpha_1 \alpha_2 > 0$ in the above.)

As with the monopoly model, suppose that a platform has a fixed cost $f_i$ for serving a member of group $l$. Therefore, profits for platform $i$ are $(p^i_1 - f_i)n^i_1 + (p^i_2 - f_2)n^i_2$, or

\[
\pi^i = (p^i_1 - f_i) \left[ \frac{1}{2} + \frac{1}{2} \frac{\alpha_1(p^i_2 - p^i_1) + t_2(p^i_1 - p^i_2)}{t_1t_2 - \alpha_1 \alpha_2} \right] \\
+ (p^i_2 - f_2) \left[ \frac{1}{2} + \frac{1}{2} \frac{\alpha_2(p^i_1 - p^i_2) + t_1(p^i_1 - p^i_2)}{t_1t_2 - \alpha_1 \alpha_2} \right].
\]

This expression is quadratic in platform $i$’s prices, and it is concave in these prices if and only if assumption (8) holds. Therefore, platform $i$’s best response to $j$’s prices is determined by the first-order conditions. Given assumption (8), one can check there are no asymmetric equilibria. For the case of a symmetric equilibrium, the first-order conditions for equilibrium prices are

\[
p_1 = f_1 + t_1 - \frac{\alpha_2}{t_2}(\alpha_1 + p_2 - f_2); \quad p_2 = f_2 + t_2 - \frac{\alpha_1}{t_1}(\alpha_2 + p_1 - f_1). \quad (10)
\]

These expressions can be interpreted in the following manner. First, note that in a Hotelling model without network effects, the equilibrium price for group 1 would be $p_1 = f_1 + t_1$. In this setting the price is adjusted downwards by the factor $\frac{\alpha_2}{t_2}(\alpha_1 + p_2 - f_2)$. This adjustment factor can be decomposed into two parts. We claim that the term $(\alpha_1 + p_2 - f_2)$ represents the “external” benefit to a platform of having an additional group 2 customer. First, it makes extra profit $(p_2 - f_2)$ with an extra group 2 customer. Second, $\alpha_1$ measures the extra total revenue the platform can extract from its group 1 customers (without losing market share) when it has an extra group 2 customer. Thus $(\alpha_1 + p_2 - f_2)$ indeed represents the “external” benefit to a platform of having an additional group 2 customer. Finally, as discussed after expression (7), the term $\frac{\alpha_2}{t_2}$ measures how many extra group 2 customers a platform attracts when it attracts an extra group 1 customer. In sum, the adjustment factor

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8. This assumption is the requirement that the matrix of second derivatives of the profit function has a positive determinant.

9. An extra group 2 customer means that the utility of each of a platform’s group 1 customers increases by $\alpha_1$, and the utility of each of the rival platform’s group 1 customers falls by $\alpha_1$. Therefore, the relative utility for group 1 customers being on the platform increases by $2\alpha_1$ and each of the customers can bear a price increasing equal to this. Since a platform has half the group 1 customers, the total extra revenue it can extract from these customers is just $\alpha_1$. 

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\( \frac{\alpha_2}{t_2}(\alpha_1 + p_2 - f_2) \) measures the “external” benefit to the platform from attracting an extra group 1 customer; in other words, it measures the opportunity cost of raising the group 1 price enough to cause one group 1 customer to leave. We can summarize this discussion with an annotated version of formula (10):

\[
p_1 = \frac{f_1}{\text{cost of service}} + \frac{t_1}{\text{market power factor}} - \frac{\alpha_2}{t_2} \times (\alpha_1 + p_2 - f_2)
\]

Solving the simultaneous equations in (10) yields the following explicit expressions for equilibrium prices:

\[
p_1 = f_1 + t_1 - \alpha_2 ; p_2 = f_2 + t_2 - \alpha_1 . \tag{11}
\]

(Whilst these expressions are certainly “simple”, they are hardly intuitive, and this is why we focussed the discussion on the formulas (10) above.) Therefore, the price-cost margin for a particular group is equal to the differentiation parameter for that group (which represents how competitive that side of the market is), minus the externality that joining the platform has on the other group on the platform. Thus, a platform will target one group more aggressively than the other if that group is (i) on the more competitive side of the market and/or (ii) causes larger benefits to the other group than vice versa.\(^{10}\)

Each platform makes profit

\[
\pi = \frac{t_1 + t_2 - \alpha_1 - \alpha_2}{2} . \tag{12}
\]

Assumption (8) guarantees that this profit is positive. The cross-group network effects act to reduce profits compared to the case where \( \alpha_1 = \alpha_2 = 0 \), since firms have an additional reason to compete hard for market share. We discuss a model where platforms can choose tariffs that reduce, eliminate or even reverse these cross-group network effects by the use of more complex tariffs in section 4.2 below.

There is little scope for meaningful welfare analysis with this model since prices are just transfers between agents: any (symmetric) pair of prices offered by the two platforms will yield the same level of total surplus. One issue that can be discussed, however, is whether it is socially desirable to have two platforms rather than one. In the current context, have two platforms has the advantage that transport costs are small but that network effects are not fully exploited. To be precise, with two symmetric platforms, the average transport cost of a group \( i \) customer is \( \frac{1}{3}t_1 \) and the network benefit for a group \( i \) customer is \( \frac{1}{2} \alpha_i \). Total welfare (up to a constant) is therefore

\[
W_2 = \frac{\alpha_1 + \alpha_2}{2} - \frac{t_1 + t_2}{4} .
\]

\(^{10}\)It is quite possible given our assumptions that one price in the above expression is negative, and this happens if that side of the market involves a low cost, is competitive, or causes a large external benefit to the other side. In many cases it is not realistic to suppose that negative prices are feasible, in which case the analysis needs to be adapted explicitly to incorporate the non-negativity constraints.
With a single platform, located at the end of the Hotelling line, however, the average group
$i$ customer must incur the greater transport cost $\frac{t_1}{2}t_1$ but he enjoys higher network benefits
of $\alpha_1$, so that
\[ W_1 = \alpha_1 + \alpha_2 - \frac{t_1 + t_2}{2}. \]
Welfare with a single platform is higher than with two whenever $\alpha_1 + \alpha_2 > \frac{1}{2}(t_1 + t_2)$. This
inequality is perfectly compatible with our maintained assumption (8), and so it is possible
that there are too many platforms in equilibrium.

4.2 More Ornate Tariffs
The analysis so far has assumed that all consumers are charged a lump-sum “subscription fee”
to join a platform. There are several other kinds of competition that could be envisaged.
For instance, platforms could commit to supply consumer with fixed utilities instead of
charging a fixed price. Implicit in such a commitment would be to reduce the charge that
group 1 consumers pay if it turns out that the number of group 2 consumers is fewer than
expected. A more general formulation is for platforms to offer a kind of “two-part tariff”, in
which consumers pay a fixed charge $p$ together with an incremental charge, say $\gamma$, for each
consumer on the other side who goes to the same platform. That is to say, platform $i$’s
charge to group $l$ customers takes the form
\[ T_l^i = p_l^i + \gamma_l^i n_m^l, \]
where $p_l^i$ is the fixed part of the tariff and $\gamma_l^i$ is the extra charge per group $m$ customer at the
same platform.\textsuperscript{11} Special cases of this family of tariffs include: (i) $\gamma_l^i = 0$, where platforms
compete in fixed prices as in section 4.1, and (ii) $\gamma_l^i = \alpha_l$, where customers pay exactly the
benefit they enjoy from interacting with the other group. Thus, case (ii) corresponds to the
case where platforms commit to deliver a constant utility to customers, irrespective of their
success on the other side of the market. More generally, the $\gamma$ charge represent a kind of
“transactions” charge, akin to the transactions charges used in credit card markets.

Each platform now has four degrees of freedom in its strategy choice. Similarly to
expression (6), if a group 1 customer goes to platform $i$ she will obtain utility
\[ u_1^i = (\alpha_1 - \gamma_1^i) n_2^i - p_1^i, \]
and so from (5) the number of group 1 customers who join platform $i$ is
\[ n_1^i = \frac{1}{2} + \frac{(\alpha_1 - \gamma_1^i) n_2^i - (\alpha_1 - \gamma_1^i)(1 - n_2^i) - (p_1^i - p_1^i)}{2t_1} \]
(and similarly for group 2). One can then obtain explicit, but lengthy, formulas for $n_1^i$ and
$n_2^i$ in terms of the eight price parameters in the same way that expressions (9) were obtained.

\textsuperscript{11}Section 6 of Rochet and Tirole (2003) considers the same family of tariffs.
Platform $i$’s profits are
\[ \pi^i = (\gamma_1^i n_2^i + p_1^i - f_1) n_1^i + (\gamma_2^i n_1^i + p_2^i - f_2) n_2^i . \]

One can show that, when the two platforms choose the same pair of per-user charges $(\gamma_1, \gamma_2)$, the equilibrium fixed charges are given by
\[ p_1 = f_1 + t_1 - \alpha_2 + \frac{1}{2}(\gamma_2 - \gamma_1) \quad ; \quad p_2 = f_2 + t_2 - \alpha_1 + \frac{1}{2}(\gamma_1 - \gamma_2) . \] (13)

This expression generalizes (11) above. After some highly intricate analysis, one can then show any pair of per-user charges $(\gamma_1, \gamma_2)$ make up a symmetric equilibrium, provided that the corresponding fixed charges $p_i$ are as given in (13).\(^{12}\) The profit of each platform in such an equilibrium is given by
\[ \pi = \frac{t_1 + t_2 - \alpha_1 - \alpha_2}{2} + \frac{\gamma_1 + \gamma_2}{4} . \]

This profit is increasing in the per-user charges $\gamma_1$ and $\gamma_2$. The reason that high charges $\gamma_i$ yield high profits is that they reduce, or overturn, the cross-group network effects that make the market so competitive and destroy profits.

Thus, we see that when platforms consider the use of more complicated tariffs that depend on the platform’s success on the other side of the market, then a continuum of symmetric equilibria exist, which are ranked by the profit the generate.\(^{13}\) The question arises as to which of these equilibria is selected. One suggestion is that platforms coordinate on an equilibrium that generates high profits, i.e., on a pair of tariffs with large $\gamma_i$. An alternative viewpoint is that the “pure subscription” tariffs analyzed in section 4.1 are robust: if the rival platform offers a pure subscription tariff, then a platform would not wish to deviate by offering a tariff that depends on how well it performs on the other side of the market. But more generally, this analysis suggests that, while it is straightforward to analyze the case of pure subscription tariffs (as in section 4.1 of this paper) or the case of pure transaction tariffs (as emphasized in Rochet and Tirole (2003)), blending the two tariffs present major problems for the predictive power of the model (at least without some natural equilibrium selection device).\(^{14}\)

\(^{12}\) Details of this analysis are available from the author upon request.

\(^{13}\) There are also asymmetric equilibria. In technical terms, the source of the multiple equilibria is closely related to the multiple equilibria that exist in a (deterministic) supply function framework—see section 3 of Klemperer and Meyer (1989). The issue in the two settings is that, for a given choice of tariff by its rival, a firm has a continuum of best responses.

\(^{14}\) In their different model, Caillaud and Jullien (2003) find asymmetric equilibria when these kinds of two-part tariffs are used. Specifically, in a multi-homing model they show that one platform would choose to set a lower transaction charge than the other to deter entry (and this platform makes higher profits). See their Proposition 4.
5 Competitive Bottlenecks

In this section we describe the third and final class of models, which might be termed models of “competitive bottlenecks”. We modify the model of section 4.1 and suppose that, while group 1 continues to deal with a single platform (to single-home), group 2 wishes to deal with each platform (to multi-home). Implicitly in this model there is the idea that group 2 puts more weight on the network benefits of being in contact with the widest population of group 1 consumers than it does on the costs of dealing with more than one platform. The crucial difference between this model and that discussed in section 4.1 is that here group 2 does not make an “either-or” decision to join a platform. Rather, keeping the market shares for group 1 constant, a group 2 agent makes a decision to join one platform independently from its decision to join the other. In this sense, there is no competition between platforms to attract group 2 custom.

There are several examples of markets where this framework seems an appropriate stylized representation. For instance, as discussed in detail in section 2, people typically subscribe to a single mobile telephone network, but people from all other networks need to call them. People might read a single newspaper (perhaps due to time constraints) but producers might place adverts in all relevant newspapers. Consumers might choose to visit a single shopping mall (perhaps because of transport costs) but the same retailer might choose to locate a shop in several malls.

5.1 A Model of Consumers and Retailers

Suppose there are two platforms that facilitate interaction between a group of consumers (“group 1”) and a group of retailers (“group 2”). Consumers are assumed to join a single platform. There are a large number of retailers, and suppose that each has a monopoly position in its product(s). (See section 5.3 for a discussion of the case where retailers might compete with each other.) Retailers do not have any preferences for one platform over the other, except for the different numbers of consumers they might have. That is to say, if the two platforms have an equal number of consumers, retailers regard access to each platform as equally valuable.

Suppose that retailers are heterogeneous: if there are $n_i^1$ consumers on platform $i$, the number of retailers who are prepared to pay the lump-sum fee $p_2^i$ to join the platform is given by

$$n_2^i = \phi(n_1^1, p_2^i),$$

where the function $\phi$ is decreasing in price $p_2^i$ and increasing in $n_1^1$. The formulation in (14) is fairly general: it encompasses cases where retailers differ in their fixed costs, in their marginal costs, in the surplus they can extract from consumers, and so on. A retailer’s decision to join one platform does not depend on whether it chooses to join the rival platform.\(^{15}\) Let

\(^{15}\)At least, this is true if retailers are “atomistic”. If this were not the case then if a retailer which has
\( R(n_1, n_2) \) denote a platform’s revenue from retailers when it has \( n_1 \) group 1 consumers and sets its group 2 price such that \( n_2 \) retailers choose to join the platform. Formally, \( R \) is defined by the relation

\[
R(n_1, \phi(n_1, p_2)) \equiv p_2 \phi(n_1, p_2) .
\]

If a consumer’s utility is \( u^i_1 \) with platform \( i \), then suppose that the platform will attract

\[
n^i_1 = \Phi(u^i_1, u^i_2) \tag{15}
\]

consumers. The function \( \Phi \) is increasing in the first argument and decreasing in the second. Consumer utility \( u^i_1 \) is given by

\[
u^i_1 = U(n^i_2) - p^i_1 \tag{16}
\]

if the platform charges \( p^i_1 \) to its group 1 consumers and there are \( n^i_2 \) retailers on the platform. Here \( U \) is a function that measures the benefit a consumer has from greater retailer participation on the platform. (This function would be decreasing in cases where consumers found adverts to be a *nuisance*, say in a newspaper.) A platform’s total cost of serving the two sides is \( C(n^i_1, n^i_2) \). The profit of platform \( i \) is

\[
\pi^i = n^i_1 p^i_1 + R(n^i_1, n^i_2) - C(n^i_1, n^i_2) .
\]

Next we derive the number of retailers on each platform in equilibrium, as a function of the equilibrium market shares for consumers. Suppose that in equilibrium platform \( i \) offers utility \( \hat{u}^i_1 \) to its group 1 consumers and attracts the number \( \hat{n}^i_1 \) of such consumers (given by the function \( \Phi \)). Then the platform must be maximizing its profits given this group 1 utility \( \hat{u}^i_1 \). In other words, consider varying \( p^i_1 \) and \( n^i_2 \) so that \( \hat{u}^i_1 = U(n^i_2) - p^i_1 \) is constant. Writing

\[
p^i_1 = U(n^i_2) - \hat{u}^i_1 \text{ in (17) means that profit is}
\]

\[
\pi^i = \hat{n}^i_1 [U(n^i_2) - \hat{u}^i_1] + R(\hat{n}^i_1, n^i_2) - C(\hat{n}^i_1, n^i_2) .
\]

Given the equilibrium consumer numbers \( \hat{n}^i_1 \), platform \( i \) will choose to serve a number \( \hat{n}^i_2 \) of retailers, where \( \hat{n}^i_2 \) is chosen to maximize

\[
\hat{n}^i_1 U(\cdot) + R(\hat{n}^i_1, \cdot) - C(\hat{n}^i_1, \cdot) . \tag{18}
\]

The resulting equilibrium charge to group 2 is \( \hat{p}^i_2 \), where this charge satisfies

\[
\hat{n}^i_2 = \phi(\hat{n}^i_1, \hat{p}^i_2) . \tag{19}
\]

Notice that expression (18) measures the total welfare of platform \( i \) and its group 1 consumers as the number of retailers is varied. Therefore, the number of retailers is chosen already joined platform \( A \) decides to join platform \( B \), then this will draw consumers away from platform \( A \) and so cause a negative externality on profits from the retailer’s platform \( A \) business. Here, though, we ignore this possibility.
to maximize the interests of the platform and its consumers, and the interests of retailers are ignored. In particular, there is an unambiguous market failure present, in that there are too few retailers on each platform. (The aggregate welfare of retailers is an increasing function of the number of retailers.) How the resulting surplus is shared between platforms and consumers, i.e., how profitable the platforms are, depends in part on the strength of competition in the market for consumers (i.e., on the form of the function \( \Phi \)). Just as with the mobile telephony case discussed in section 2, even though platforms might compete vigorously for group 1 members, there is no competition for providing access by group 2 to these group 1 members, and this monopoly induces the usual excessive pricing for group 2. The market failure is not necessarily one of excessive profits—the platforms’ overall profits will be small if the market for consumers is competitive—but rather that there is an undesirable pattern of relative prices for the two groups of participants.

5.2 Application: Advertising on Media Platforms

Here we put more structure on the rather general model just discussed, with the aim of fully characterizing the competitive outcome. Specifically, consider situations where advertisers wish to make contact with potential consumers via platforms. Suppose there are two such platforms (which could be considered to be newspapers or yellow pages directories), \( A \) and \( B \). Adverts are placed on the platforms by monopoly retailers (“group 2”). A consumer will purchase a given product if she sees an advert for the product (and the price leaves her with non-negative surplus). As above, the crucial assumption is that consumers join one or other platform, but not both. The cost of producing and distributing an individual copy of a newspaper/directory is \( c(n_2) \) if there \( n_2 \) adverts on the platform. Thus the cost function \( C(n_1, n_2) \) in section 5.1 takes a multiplicative form:

\[
C(n_1, n_2) = n_1 c(n_2)
\]

where \( c \) is an increasing function. Retailers are differentiated by the parameter \( \sigma \): the type-\( \sigma \) retailer has a product that generates profit \( \sigma \) from each consumer who sees its advert. Thus an advert placed on a platform is worth \( \sigma \) for each consumer on the platform to the

---

\( ^{16} \) As discussed in the introduction, this section is closely related to Anderson and Coate (2001) and Rysman (2002), as well as section 5 of Rochet and Tirole (2003). One could analyze a slightly different application to shopping malls using similar techniques. There are perhaps two main differences. First, a platform’s cost function would plausibly take the additive form

\[
C(n_1, n_2) = f_1 n_1 + f_2 n_2
\]

instead of the multiplicative media platform case \( C(n_1, n_2) = n_1 c(n_2) \) considered in this section. Second, it is plausible to model retailers to have a fixed cost of joining a platform (the fixed cost associated with starting a shop) in addition to the platform’s charge. But the fundamental conclusion—that there will be too few retailers in equilibrium—will continue to hold in this alternative setting.
Let $F(\sigma)$ be the cumulative distribution function for $\sigma$ in the population of retailers.

If the lump-sum charge for placing an advert on platform $i$ is $p^i_2$, then a type-$\sigma$ retailer will be prepared to join the platform if $\sigma n^i_1 \geq p^i_2$. That is to say, the function $\phi$ in (14) is in this context given by

$$\phi(n^i_1, p^i_2) = 1 - F(p^i_2/n^i_1).$$

With an advertising demand function of this form, the revenue function $R$ is proportional to the consumer base:

$$R(n_1, n_2) = n_1 r(n_2)$$

where $r$ is a platform’s revenue-per-consumer function.\(^{18}\) In this case the expression (18) is precisely proportional to $n^i_1$, and the equilibrium advertising volume $\hat{n}_2$ is chosen to maximize

$$U(n_2) + r(n_2) - c(n_2)$$

which does not depend on $n_1$. Thus a platform’s decision on its advertising volume can be made independently of the size of its consumer base.

Suppose we give a specific functional for the consumer market share function $\Phi$ given by

$$n^i_1 = \frac{1}{2} + \frac{u^i_1 - u^j_1}{2t}$$

where $t$ is the parameter that measures the competitiveness of the market for consumers. Therefore, the symmetric equilibrium (if it exists) involves the two platforms sharing consumers equally: $\hat{n}^i_1 = \hat{n}^j_1 = \frac{1}{2}$. Expression (19) then shows that the equilibrium charge made to advertisers, $\hat{p}_2$, is given by

$$\hat{n}_2 = 1 - F(2\hat{p}_2),$$

where $\hat{n}_2$ maximizes (21).

Consider next overall welfare. The aggregate gross profit of retailers, not taking into account their payments to the platforms, if $n_2$ retailers are on each platform is denoted $V(n_2)$. This differentiates to give

$$V'(n_2) \equiv \frac{r(n_2)}{n_2}.$$
(The right-hand side of this expression is simply the price paid per advert.) The welfare-maximizing choice of \( n_2 \) maximizes

\[
U(n_2) + V(n_2) - c(n_2) .
\]  

(24)

Since \([V(\cdot) - r(\cdot)]\) is an increasing function \( n_2 \), it follows from comparing expression (21) with (24) that the equilibrium number of retailers is fewer than the welfare-maximizing number of retailers. Therefore, the equilibrium involves too little advertising. Moreover, this market failure is not mitigated by making the market for consumers more competitive (i.e., by reducing \( t \)).

We turn next to the outcome on the consumer side of the market. Similarly to section 4.2 above, it turns out that the equilibrium depends on the precise way in which charges are levied. In this context, the equilibrium price for consumers depends on the form of the charges levied on the advertisers. We consider two natural principles for charging advertisers: (i) advertising charges are explicitly made on a per-consumer basis, and (ii) advertisers are charged a lump-sum fee for placing an advert. The reason that this makes a difference to the competitiveness of the market for consumers is that it affects the profitability of a platform’s deviation in the consumer price. With case (i), if a platform attracts more consumers the number of adverts does not change. With case (ii), by contrast, having more consumers acts to attract more advertisers (keeping the lump-sum advertising charge fixed), and this in turn acts to attract still more consumers. (Note that the charging basis for advertising does not affect the equilibrium quantity, price or welfare of advertisers, as calculated above.) These two cases are discussed in turn.

**Per-consumer advertising charges**: Suppose that platform \( i \) offers advertising space for a charge \( \gamma^i \) per consumer that joins the platform. Then a type-\( \sigma \) advertiser will choose to join the platform if and only if \( \sigma > \gamma^i \), i.e., participation does not depend upon the number of consumers on the platform.\(^{19} \) In this case the analysis is extremely simple. Given the consumer choice specification in (22), the profit of platform \( i \) if it charges consumers \( p^i_1 \) (while the rival charges consumers \( p^j_1 \)) is

\[
\pi^i = \left( \frac{1}{2} - \frac{p^j_1 - p^i_1}{2t} \right) \left( p^i_1 + r(\hat{n}_2) - c(\hat{n}_2) \right) .
\]

Therefore, the symmetric equilibrium price for consumers is given by

\[
p_1 = c(\hat{n}_2) + t - r(\hat{n}_2) .
\]  

(25)

\(^{19} \)This feature is commonly seen in the credit card literature, where credit card charges are often assumed to be levied on a per-transaction basis, and the benefit is also assumed to be proportional to the number of transactions. See for instance Rochet and Tirole (2003). Also, in the telephony context discussed at the start of section 5 it is the case that calls to mobile platforms are levied on a per-subscriber basis.
Thus, a platform’s revenue from advertising, $r$, is passed onto consumers in the form of a low group 1 charge $p_1$. Platform profits in this case are given by the product differentiation parameter $t$.

**Lump-sum advertising charges:** To analyze this more complex case we need to calculate the consumer numbers that a platform attracts when it undercuts its rival in the market for consumers. When a platform undercuts its rival it will clearly attract more consumers; in consequence the platform attracts more advertisers (given that its charge for advertising is unchanged), which thereby attracts further consumers. This “feedback loop” is quite absent when advertising charges are levied per consumer. To be precise, platform $i$’s consumer market share, $n_1^i$, satisfies the following equation:

$$n_1^i = \frac{1}{2} + \frac{U(n_2(n_1^i)) - p_1^i - [U(n_2(1 - n_1^i)) - p_1^i]}{2t}.$$  

Here we have written the advertising volume $n_2$ on a platform as a function of the consumer numbers on the platform. That is to say,  

$$n_2(n_1^i) = 1 - F(\hat{p}_2/n_1)$$

where the equilibrium lump-sum charge to advertisers is given in expression (23). One can then calculate that the consumer demand function satisfies

$$\left. \frac{\partial n_1^i}{\partial p_1^i} \right|_{p_1^i = \hat{p}_1} = -\frac{1}{2t - 2n_2^i(\frac{1}{2})U'(\hat{n}_2)}.$$  

In particular, a platform’s consumer demand function is more elastic with respect to its consumer price than was the case with per-consumer advertising charges (when $\partial n_1^i/\partial p_1^i = -1/2t$). The platform’s total profit is given by

$$\pi^i = n_1^i \left[ p_1^i - c(n_2(n_1^i)) \right] + \hat{p}_2 n_2(n_1^i).$$

Using the expression (26), it follows that the equilibrium consumer price $p_1$ satisfies

$$p_1 = c(\hat{n}_2) + t - n_2^i(\frac{1}{2})[U'(\hat{n}_2) + \hat{p}_2 - \frac{1}{2}c'(\hat{n}_2)].$$  

However, using the first-order condition for the fact that $\hat{n}_2$ maximizes expression (21), some manipulations show that (27) can be written in the form

$$p_1 = c(\hat{n}_2) + t - r(\hat{n}_2) - \frac{1}{2} n_2^i(\frac{1}{2})U'(\hat{n}_2).$$  

Thus, comparing this expression with that for the per-consumer charging case in (25) we see that when advertising charges are levied on a lump-sum basis, the equilibrium price for consumers is lower than when they are levied on a per-consumer basis. Platform profits are
correspondingly lower with lump-sum charging. These results are akin to those presented in section 4.2, where the use of tariffs that depend positively on the platform’s success on the other side of the market were seen to relax competition and boost profits.

With either basis for advertising charges, but especially when advertising charges are lump-sum, the equilibrium group 1 price $p_1$ can be negative. For instance, in the per-consumer charge case (25), this is true if the revenue that a user generates, $r$, exceeds the unit cost of the newspaper/directory, $c$, by more than the market power term $t$. If, as seems plausible, there is a non-negativity constraint on prices, the equilibrium will presumably involve group 1 being allowed onto the platform for free. This could be a rationale for why yellow pages directories and some advertising-laden newspapers are supplied to consumers/readers for free, and for why a shopping mall might not wish to charge consumers for entry (even if it were feasible for them to do so).

5.3 Intra-Group Competition

An interesting issue is the equilibrium extent of competition between retailers within platforms.$^{20}$ For instance, a TV channel might charge more for a car advert if it promised not to show a rival manufacturer’s advert in the same slot. Or a mall might charge a high rent to a retailer with the promise that it will not let a competing retailer into the same mall. We expect that greater retailing competition will mean less profit per consumer for a retailer but each consumer will obtain higher surplus per retailer (or per retailer type). Thus we would expect that if the platform allowed retailing competition it would make less money from the retailing side of the market but more from the consumer side (if it charged for entry). One hypothesis which could be investigated is: platforms would allow competition within the platform if consumers were charged for entry, but if consumers had free entry then platforms would restrict competition in order to drive up the revenues obtained from retailers.

This topic deserves a separate paper to itself. Here we merely describe an illustrative example to show the plausibility of the above hypothesis. Suppose there are two platforms, $A$ and $B$. Suppose that platforms can serve any number of consumers and retailers costlessly (so that $C(n_1,n_2)$ in section 5.1 is equal to zero). If consumers receive utility $u_i$ from platform $i$, the market share of platform $i$ is given by expression (22). There is a single, homogeneous product supplied by a group of identical retailers. If the retail price for this product is $p$, each consumer demands a quantity $q(p)$ of the product. Let $v(p)$ be consumer surplus associated with this demand function. Each retailer has a unit cost $c$ for supplying the product (exclusive of payments to platforms). The monopoly price for retailers is $p^*$, which maximizes $q(p)(p - c)$. Suppose there is no fixed cost associated with a retailer locating in a given platform (other than the platform charge for entry).

$^{20}$Gehrig (1998) analyzes this issue in a related model. See also Dukes (2003) and Dukes and Gal-Or (2003) for models of advertising when advertisers compete for consumers.
A platform’s decision is whether to have retailing competition or not (i.e., whether to have more than one retailer on the platform). Suppose first that platforms can charge consumers for entry to the platform. If \( p_1^i \) is the platform’s entry charge for consumers, then

\[
 u^i_1 = \begin{cases} 
 v(c) - p^i_1 & \text{with competition} \\
 v(p^*) - p^i_1 & \text{with monopoly} 
\end{cases}
\]

Suppose that platform \( i \) decides to offer utility \( u_i \) to its consumers. (We will calculate the equilibrium level of utility later.) What is the most profitable way to generate this level of utility for the platform? If it chooses the competitive option the platform can levy the consumer charge \( p^i_1 = v(c) - u^i_1 \), and if it chooses the monopoly option it can levy the (lower) consumer charge \( p^*_1 = v(p^*) - u^i_1 \). Therefore, since the platform can extract the monopoly profit from the retailer in its charge to the retailer, the platform’s profit per consumer is

\[
 \pi_i = \begin{cases} 
 v(c) - u^i_1 & \text{with competition} \\
 v(p^*) - u^i_1 + q(p^*)(p^* - c) & \text{with monopoly} 
\end{cases}
\]

It follows that when consumers can be charged for access to the platform, the competitive option is the most profitable way for a platform to generate a given level of consumer utility. The platform makes all of its profit from the consumer side, and it does not choose to restrict competition among retailers. This is a “dominant strategy” and does not depend on the particular choice of utility \( u_i \) that the mall offers its consumers. It is straightforward to show that the equilibrium charge to consumers is \( p_1 = t \).

By contrast, suppose that platforms cannot charge consumers for entry, and so they must make their profit from the retailing side of the market. The only way a platform can set a positive charge to a retailer in the present stark framework is if the retailer is a monopoly. Thus, in this case we expect platforms to restrict competition and consumers will pay \( p^* \) for the product.

## 6 Conclusion

This paper has presented three simple formal models for how platforms behave in two-sided markets with network externalities. In the monopoly model, the main result was that the platform set excessive prices for both groups, and that the prices reflected the external benefit given by one group to the other. In the second model with single-homing, we saw that one group was targeted relatively aggressively in equilibrium when that group was on the more competitive side of the market, and/or it caused a greater benefit to the other group than \( \textit{vice versa} \). In the third model, where one group single-homed and the other multi-homed, we saw that the former group was targeted aggressively while the latter group was exploited. A
theme running through the paper was how the details of platform charges made a difference to the outcome. Since cross-group network externalities act to strengthen competition and to reduce profits, platforms can boost their profits by choosing tariffs that act to reduce the network externalities. This is achieved by making a customer on one side pay an additional charge for each customer on the other side who joins the same platform.

There are several clear limits to this analysis. Perhaps the most prominent of these is the assumption that agents exogenously single-home or multi-home. It would be valuable to relax this assumption in future work. There are two aspects to this issue. First, one should explicit consideration to whether an agent would wish to join one or two platforms. The option for an agent to join two platforms was not permitted in the model of section 4, even though such an action would enable the agent to take advantage of greater network externalities. Second, we need to consider a platform’s incentive to contractually require that an agent on one side of the market deals with it exclusively.

References


