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Power-hungry Candidates, Policy Favors, and Pareto Improving Campaign Contribution Limits*

Abstract

This paper argues that limits on campaign contributions may well be Pareto improving even under the most optimistic assumptions concerning the role of campaign advertising and the rationality of voters. The argument assumes that candidates use campaign contributions to convey truthful information to voters about their qualifications for office and that voters update their beliefs rationally on the basis of the information they have seen. It also assumes that campaign contributions are provided by interest groups and that candidates can offer to provide policy favors for their interest groups to attract higher contributions. The argument is developed in the context of a simple model of political competition with campaign contributions and informative advertising.

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1 Introduction

This paper argues that limits on campaign contributions could well be Pareto improving even under the most optimistic assumptions concerning the role of campaign advertising and the rationality of voters. The argument assumes that candidates use campaign contributions to convey *truthful information* to voters about their qualifications for office and voters update their beliefs *rationally* on the basis of the information they have seen. It also assumes that campaign contributions are provided by interest groups and that candidates can offer to provide policy favors for their interest groups to attract higher contributions.

The argument is developed in a simple model of electoral competition. There are two political parties representing opposing ideologies. Parties put forward candidates who represent their ideologies, but may have difficulty finding qualified candidates. Thus each party's candidate may be qualified or unqualified. Voters know a candidate's party affiliation but not whether he is qualified. Advertising allows a candidate to provide voters with this information. Such advertising can be advantageous for a qualified candidate because it may attract swing voters. Resources for campaign advertising are obtained by candidates from interest groups consisting of citizens of opposing ideologies. If elected, candidates are able to implement policy favors for their interest groups and, before the election, they can offer to implement such favors to extract larger contributions.

The starting point for the argument is the observation that the potential social benefit of contributions lies in giving qualified candidates an electoral advantage over unqualified opponents. With no contributions, there would be no mechanism for qualified candidates to get out the word to voters. Giving qualified candidates an electoral advantage potentially benefits all citizens, as it results in better leaders.

In order for campaign contributions to have this benefit, campaign advertising must be *effective* in that learning that a candidate is qualified will induce a non-trivial fraction of swing voters to switch their votes from unadvertised candidates. If advertising induces no voters to switch their votes then qualified candidates obviously have no electoral advantage. However, *when campaign contributions are unrestricted and candidates are sufficiently power-hungry, campaign advertising must be close to ineffective.* For if campaign advertising were effective, power-hungry candidates would promise a large number of favors to their interest groups to extract more resources for campaigning. Voters would rationally become cynical about candidates they learn are qualified, anticipating that they will implement large amounts of favors when in office. This cynicism would negate the effectiveness of campaign advertising.

Accordingly, when campaign contributions are unrestricted and candidates are sufficiently power-hungry, resources will be spent on campaigning but qualified candidates will not have much of an electoral advantage over unqualified opponents. Moreover, if elected, qualified candidates will implement *some* favors for their interest groups. This must be the case for advertising to be close to ineffective. It follows that banning campaign contributions would only result in a negligible reduction in the likelihood that leaders would be qualified, while eliminating the favors they would implement. This means that all regular citizens benefit from a contribution ban. The only possible losers are interest group members who no longer receive favors. But their expected gains from favors are dissipated by the contributions they make, meaning they are also better off. Thus, *banning contributions creates a Pareto improvement when candidates are sufficiently power-hungry.*

When candidates are less power-hungry, campaign advertising will be effective even with unrestricted contributions and, accordingly, contributions will give qualified candidates an edge over unqualified opponents. In such circumstances, *banning* contributions will reduce the probability that qualified candidates defeat their unqualified opponents. However, *limiting* contributions need not necessarily reduce this probability. This is because a limit reduces the level of favors qualified candidates provide and this may raise the effectiveness of campaign advertising. This increase in the *effectiveness* of advertising can compensate for the reduction in the *level* of advertising. In such circumstances, contribution limits again have the potential to be Pareto improving. Finally, even when limits necessarily reduce the probability that qualified candidates defeat their unqualified opponents, they may be Pareto improving if the reduction in this probability is compensated for by a large enough reduction in favors.

The organization of the remainder of the paper is as follows. The next section discusses the relationship of the paper to previous work on the regulation of campaign advertising and the more general literature on campaign finance. Section 3 presents the model. Section 4 characterizes equilibrium with unrestricted contributions and shows that when candidates are sufficiently power-hungry, campaign advertising is close to ineffective. The impact of contribution limits is analyzed in Section 5. It is first shown that banning contributions will be Pareto improving when candidates are sufficiently power-hungry. It is then argued that, when candidates are less power-hungry, limits need not necessarily reduce the probability that qualified candidates are elected and that in such circumstances they will be Pareto improving. Section 6 concludes with a summary of the argument and some suggestions for further research.

2 Related Literature

Despite the manifest policy significance of the topic, there have been few papers studying the welfare economics of campaign finance regulation. Partly this reflects the difficulty of incorporating campaign contributions into theories of electoral competition in a tractable way. Most efforts simply assume that campaign advertising buys the votes of "noise" voters, implying that it has no social benefit (see, for example, Baron (1994), Besley and Coate (2000), and Grossman and Helpman (1996)). Such an assumption obviously precludes a serious analysis of the case for contribution limits.

Work in which campaign advertising has a social benefit falls into two categories. First, there are those papers that assume that campaign advertising is *directly* informative (Austen-Smith (1987), Coate (2001), Ortuno-Ortin and Schultz (2000), and Schultz (2001)). The idea is that candidates can use advertising to provide voters with hard information about their policy positions, ideologies, or qualifications for office, thus permitting more informed choices. Second, there are those who argue that campaign advertising may best be understood as providing information *indirectly* (Potters, Sloof, and Van Winden (1997), Prat (1999) and (2000)). The idea is that candidates have qualities that interest groups can observe more precisely than voters and the amount of campaign money a candidate collects signals these qualities to voters.

Coate (2001) addresses the desirability of contribution limits in a world of directly informative advertising. The model used in this paper builds on Coate (2001), but differs in two key ways. First, voters are uninformed about candidates' "qualification for office" which is a "valence" characteristic that all voters value. In Coate (2001) voters are uninformed about candidates' ideologies and candidates are chosen strategically by competing political parties. This makes Coate's analysis more intricate because candidate types are endogenous. With a valence characteristic, it is natural to presume that all parties would field a candidate with a high value of the characteristic if they could find one, so it seems reasonable to treat the probability that parties select qualified candidates as exogenous. The second key difference is that in this analysis, candidates can offer policy favors to attract more contributions from the interest groups that support them. In Coate (2001) interest groups only give to help elect candidates whose ideologies they favor. This feature is key to explaining the difference in policy conclusions concerning the desirability of contribution limits. While in this paper limits can be Pareto improving, in Coate (2001) limits redistributes welfare *from* moderate voters *to* interest group members. This is because limits reduce the likelihood that parties select moderate candidates.

Prat (1999) addresses the case for limiting contributions in a world of indirectly informative advertising. In his analysis, two office-seeking candidates, who may differ in competence, compete by staking out positions in a one dimensional policy space. A single interest group with non-median policy preferences offers contributions to candidates in exchange for them moving their platforms towards its preferred policy position. Candidates the interest group believes to be more competent are offered larger contributions because they are more likely to win. This is because voters observe a noisy signal of competence and hence, ceteris paribus, are more likely to vote for the more competent candidate. In equilibrium, therefore, the more a candidate advertises, the higher is his competence. Campaign contributions are good for voters in the sense that they provide information about competence, but bad in that they lead candidates to distort policy away from the median voter's ideal. Banning contributions can raise voters' aggregate welfare when the losses in terms of information about competence are smaller than the costs of policy distortion. This is different from our argument which stresses that there need be no such trade off - banning contributions need not significantly impact the probability that competent candidates are elected. While Prat does not consider the distributional consequences of banning contributions, it seems likely that in his model banning is either Pareto inefficient or redistributes from citizens on the side of the interest group to those on the other side of the political spectrum.

While the literature on the specific topic of the welfare economics of campaign finance regulation is sparse, the general topic of campaign contributions has attracted much more attention. A significant strand of the literature is devoted to assessing the empirical relationship between campaign spending and votes - how effective is campaign advertising in delivering votes? (see, for example, Abramowitz (1988), Green and Krasno (1988), Jacobson (1980), (1985), Levitt (1994), and Palfrey and Erikson (2000).) In the model of this paper the effectiveness of campaign advertising is derived endogenously as part of the equilibrium (as in Coate (2001)). Moreover, a major lesson of the paper is that rules governing elections may be expected to have implications for the effectiveness of campaign advertising. In particular, ceteris paribus, campaign advertising may be more effective when limits are tighter. This has interesting implications for future empirical studies.

A further theme in the literature is the distinction between *service* and *position-induced* contributions (Morton and Cameron (1992)). The latter are contributions that are given because the donor shares some of the candidate's policy positions and wants to enhance his/her chances of winning. The former are contributions that are given in the expectation that the candidate will provide services for the donor if elected to office. Prior theoretical work has assumed *either* that contributions are position induced *or* that they are service-induced. In the model of this paper, the degree to which contributions are service or position induced is determined endogenously and is again affected by the rules governing elections.

3 The Model

The population consists of three groups of citizens - leftists, rightists, and swing voters. These groups differ in their ideologies measured on a 0 to 1 scale. Leftists and rightists have ideologies 0 and 1 respectively. Swing voters have ideologies that are uniformly distributed on the interval $[\mu - \tau, \mu + \tau]$. Reflecting the fluid nature of these voters' attitudes, the ideology of the median swing voter is ex ante uncertain. Specifically, μ is the realization of a random variable uniformly distributed on $[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon]$, where $\varepsilon < \frac{1}{2} - \tau$.[1] Leftists and rightists constitute an equal fraction of the community, so that the median swing voter is the median voter for the population as a whole.

The community must elect a representative. Candidates are put forward by two political parties: Party L - the leftist party, and Party R - the rightist party. Candidates are citizens

[1] The assumptions that swing voters are uniformly distributed over $[\mu - \tau, \mu + \tau]$ and that the ideology of the median swing voter is uniformly distributed over $[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon]$ are not key to the argument. They are simply made to ensure that the probability of winning function derived below has a simple and tractable form.

and hence are characterized by their ideologies. Each party must select from the ranks of its membership, so that Party L always selects a leftist and Party R a rightist. However, candidates differ in their qualifications for office, denoted by q. They are either "qualified" ($q = 1$) or "unqualified" ($q = 0$). A qualified candidate, for example, may be one who has previously held elective office.[2] All citizens, including party members, prefer a qualified candidate. Thus parties will always select qualified candidates if they are available. The probability that each party can find a qualified candidate is σ.

A citizen with ideology i enjoys a payoff from having a leader of ideology i' and qualifications q given by $\delta q - \beta |i - i'|$ where $|i - i'|$ is the *distance* from i to i'. The parameter δ measures the gains from having a qualified candidate in office. It is assumed that leftists and rightists always prefer a candidate of their own ideology even if he is unqualified which implies that δ is less than β. Candidates have the same payoffs as citizens except that the winning candidate enjoys an *ego-rent* r. This measures how *power-hungry* candidates are.

Swing voters do not have perfect information about candidates, in the sense of not knowing whether each party's candidate is qualified. Such information could be acquired, but swing voters are not politically engaged and choose to remain "rationally ignorant". However, candidates can convey information concerning their qualifications via advertising. For example, they can inform voters about the prior elected offices they have held.[3] Swing voters cannot ignore such advertising because it is bundled with radio or television programming.

Campaign advertising is governed by the following rules. First, candidates can only advertise

[2] Broader interpretations of q are possible. It could measure any valence characteristic such as managerial competence, policy creativity, charisma, image, or looks. The significance of valence characteristics for candidate elections has been stressed by numerous authors. See Aragones and Palfrey (2000) and Groseclose (2001) for two interesting recent contributions.

[3] There is widespread evidence that higher campaign spending leads to greater candidate familiarity (see, for example, Jacobson (1997)) and some evidence that it leads to greater familiarity with candidates' policy positions (see, for example, Jamieson (2000)). I am not aware of any studies directly investigating the relationship between campaign spending and voter knowledge of candidates' records (i.e., elected offices previously held, past accomplishments, etc).

their own characteristics; i.e., whether they are qualified. This rules out negative advertising. Second, candidates can only advertise the truth. The idea is that candidates have records which reveal their qualifications and that candidates cannot lie about their records. These two assumptions imply that only qualified candidates can benefit from campaign advertising.[4] The advertising technology is such that if a candidate spends an amount C, his message reaches a fraction $\lambda(C) = C/(C + \alpha)$ of the population, where $\alpha > 0$.[5]

Candidates' advertising is financed by campaign contributions from interest groups. There are two such groups - a group of leftists that contributes to Party L's candidates and a group of rightists that contributes to Party R's. Each group constitutes a fraction γ of the population. Contributions are shared equally by group members and the interest groups behave so as to maximize the expected payoff of their representative members.

After he has been selected, each party's candidate, if qualified, requests a contribution from his interest group to get the word out to voters. To obtain a larger contribution, a candidate may offer to implement policy favors. When a candidate provides a level of favors f each interest group member enjoys a monetary benefit $b(f)$ at the expense of a uniform monetary cost of f to each citizen. The function b is increasing and strictly concave, satisfying the conditions that $b(0) = 0$ and $b'(\delta) > 1$. The interest group agrees to a candidate's request if and only if it benefits it to do so.

In terms of timing, it is assumed that candidates make their requests before they or their interest group knows the type of their opponent.[6] Needless to say, swing voters do not observe

[4] This conclusion arises because there is only one possible difference between candidates and negative advertising is not permitted. However, the general conclusion that candidates with characteristics swing voters value should benefit more from advertising seems a natural implication of the informational perspective. Consistent with this, Jacobson (1989) shows that qualified candidates - defined as those who had previously held elective office - had higher levels of campaign spending and were more likely to win in U.S. House elections.

[5] Again, this specific functional form for the advertising technology is not key to the results and just helps produce a tractable probability of winning function.

[6] This assumption is made to simplify the argument. If interest groups know the type of the opposing party's candidate, they will be willing to contribute more to a candidate running against an unqualified than a qualified one. This is because the benefit to them of electing their own party's candidate is higher in the former case. This

the interaction between candidates and interest groups and hence do not observe the favors a candidate has promised.

Parties choose the best candidate they can find. Qualified candidates approach their interest group and decide what contribution to request and how many favors to offer. Interest groups decide whether or not to accept candidates' offers. Leftists and rightists always vote for the candidate put forward by the party representing their ideology. Swing voters, having possibly observed one or both candidates' advertisements update their beliefs about candidates' qualifications and vote for the candidate who yields them the highest expected payoff.[7] All these behaviors are described in greater detail in the sequel.

Throughout the analysis, we maintain the following additional assumptions on the parameter values.

Assumption 1: *(i)* $\tau \geq \varepsilon + \frac{\delta}{2\beta}$ *and (ii)* $\frac{\delta}{2\beta} \leq \varepsilon$.

The role of these will become apparent below.

3.1 Behavior of swing voters

At the time of voting, each swing voter may have seen advertisements from both, one, or neither candidate. Let (I_L, I_R) denote a swing voter's information where $I_K = 1$ if he has seen an

difference in contribution levels means that seeing an advertisement for a candidate provides information to voters about the likely type of his opponent. After all, a voter is more likely to see an advertisement for a candidate when he is running against an unqualified opponent. While it is perfectly to develop the argument taking this effect into account, it is an additional wrinkle that significantly complicates an already intricate analysis. Accordingly, the effect is assumed away here.

[7] We are therefore assuming "sincere" or "naive" voting. It is by now well known that in an election with a finite number of voters with private information, such behavior may not be fully rational. In particular, rational voters may choose to ignore their own private information on account of the "swing voter's curse" (Fedderson and Pesendorfer (1996)). In our model, there are a continuum of voters and hence such considerations do not arise. However, it would, of course, be possible to assume a finite number of swing voters and carefully model the equilibrium of the voting game. I have not taken this approach for two main reasons. First, a key assumption of the model is that there is aggregate uncertainty concerning the distribution of voter preferences. Specifically, the location of the median swing voter is unknown. In environments with aggregate uncertainty, the forces that lead voters to rationally ignore their private information are muted (Fedderson and Pesendorfer (1997)). Thus, given significant aggregate uncertainty, sincere voting may be a reasonable approximation of equilibrium behavior. Second, taking this approach would substantially complicate the development of the argument. In particular, the relationship between election outcomes and campaign spending is likely to be too complex to permit clean analysis of the contribution game.

advertisement from Party K's candidate and $I_K = \emptyset$ if not. Let $\rho_K(I_L, I_R)$ denote his belief that Party K's candidate is qualified conditional on informational state (I_L, I_R). Since only qualified candidates advertise, both $\rho_L(1, I_R)$ and $\rho_R(I_L, 1)$ must equal 1. The beliefs $\rho_L(\emptyset, I_R)$ and $\rho_R(I_L, \emptyset)$ will be derived as part of the equilibrium.

Swing voters will also have beliefs about the amount of favors that each party's candidate, if qualified, will provide to the interest group. In equilibrium, the amount of favors that voters think that candidates will implement must equal the amount that they actually will. Accordingly, we will not employ a separate notation to distinguish voters' beliefs from the actual levels promised. We let f_K denote the amount of favors that Party K's candidate, if qualified, will provide to the interest group.

Using this notation, the expected payoff of a swing voter with ideology i from Party L's candidate being elected when the voter has information (I_L, I_R) is $\rho_L(I_L, I_R)(\delta - f_L) - \beta i$, while that from Party R's candidate is $\rho_R(I_L, I_R)(\delta - f_R) - \beta(1 - i)$. Letting $i^*(I_L, I_R)$ be the ideology of the voter with information (I_L, I_R) who is just indifferent between the two parties candidates, we have that

$$i^*(I_L, I_R) = \frac{1}{2} + \frac{\rho_L(I_L, I_R)(\delta - f_L) - \rho_R(I_L, I_R)(\delta - f_R)}{2\beta}. \tag{1}$$

Swing voters for whom i is less than $i^*(I_L, I_R)$ vote for Party L's candidate, while those for whom i exceeds $i^*(I_L, I_R)$ vote for Party R's. Thus, using standard terminology, $i^*(I_L, I_R)$ is the *cut-point* for swing voters with information (I_L, I_R).

The assumption that swing voters are uniformly distributed on $[\mu - \tau, \mu + \tau]$ implies that when the median swing voter has ideology μ, the fraction of swing voters in informational state (I_L, I_R) voting for Party L's candidate is $\frac{1}{2} + \frac{i^*(I_L, I_R) - \mu}{2\tau}$. Assumption 1(i) implies that this fraction lies between zero and one when the two parties candidates are expected to implement the same level of favors.

3.2 Election probabilities

Given this voting behavior, the probability that each party's candidate will win may be computed. Suppose first that the two candidates are qualified and that they receive contributions C_L and C_R. Then, when the median swing voter has ideology μ, the fraction of swing voters voting for Party L's candidate is

$$\begin{aligned}&(\frac{1}{2}+\frac{i^*(1,1)-\mu}{2\tau})\lambda(C_L)\lambda(C_R)+(\frac{1}{2}+\frac{i^*(1,\emptyset)-\mu}{2\tau})\lambda(C_L)(1-\lambda(C_R))\\&+(\frac{1}{2}+\frac{i^*(\emptyset,1)-\mu}{2\tau})(1-\lambda(C_L))\lambda(C_R)+(\frac{1}{2}+\frac{i^*(\emptyset,\emptyset)-\mu}{2\tau})(1-\lambda(C_L))(1-\lambda(C_R)).\quad (2)\end{aligned}$$

The first term is those who have seen both candidates' advertisements; the second those who have seen only the advertisement of Party L's candidate; etc.

Party L's candidate will win if he gets at least half the swing voters vote. From (2), this requires that μ is less than $\mu^*(C_L, C_R)$ where

$$\begin{aligned}\mu^*(C_L,C_R) &= i^*(1,1)\lambda(C_L)\lambda(C_R)+i^*(1,\emptyset)\lambda(C_L)(1-\lambda(C_R))\\&\quad+i^*(\emptyset,1)(1-\lambda(C_L))\lambda(C_R)+i^*(\emptyset,\emptyset)(1-\lambda(C_L))(1-\lambda(C_R)).\quad (3)\end{aligned}$$

Since μ is uniformly distributed on $[\frac{1}{2}-\varepsilon, \frac{1}{2}+\varepsilon]$, the probability that Party L's candidate wins is

$$\pi(C_L,C_R)=\begin{cases}0 & \text{if } \mu^*(C_L,C_R)<\frac{1}{2}-\varepsilon\\ \frac{\mu^*(C_L,C_R)+\varepsilon-1/2}{2\varepsilon} & \text{otherwise}\\ 1 & \text{if } \mu^*(C_L,C_R)>\frac{1}{2}+\varepsilon\end{cases}. \quad (4)$$

If only Party L's candidate is qualified, he wins with probability $\pi(C_L, 0)$. Similarly, if only Party R's candidate is qualified, the probability that Party L's candidate wins is $\pi(0, C_R)$. If both candidates are unqualified, then no contributions are given and Party L's candidate wins with probability $\pi(0, 0)$.

3.3 Campaign contributions

Each qualified candidate, not knowing his opponent's type, must decide the level of favors to offer its interest group and how much to ask it for. Each interest group, must decide whether to accept the request. If it does so, it hands over the contribution and the candidate, if elected, will implement the agreed level of favors. If it does not, then it makes no contribution. Neither candidates nor interest groups observe the type of the opposing party's candidate at the time of contributing.

Recalling that C_K denotes the contribution a qualified candidate of Party K receives from his interest group and f_K the amount of favors he promises, interest group L's expected payoff from accepting Party L's candidate's request is

$$\sigma[\pi(C_L, C_R)(\beta + b(f_L) - f_L + f_R) + \delta - f_R] + (1 - \sigma)[\pi(C_L, 0)(\beta + \delta + b(f_L) - f_L)] - \beta - \frac{C_L}{\gamma}. \quad (5)$$

If the interest group does not accept the request, it would make no contributions and obtain a payoff:

$$\sigma[\pi(0, C_R)(\beta + f_R) + \delta - f_R] + (1 - \sigma)[\pi(0, 0)(\beta + \delta)] - \beta. \quad (6)$$

Thus, in order for the interest group to accept the request, (5) must exceed (6). Similar remarks apply to interest group R.

When Party L's candidate's request is accepted, his expected payoff is:

$$\sigma[\pi(C_L, C_R)(r + \beta + f_R - f_L) + \delta - f_R] + (1 - \sigma)\pi(C_L, 0)(r + \beta + \delta - f_L) - \beta. \quad (7)$$

Party L's candidate's request (C_L, f_L) maximizes his expected payoff subject to the constraint that the interest group will agree to it. Thus, (C_L, f_L) maximizes (7) subject to the constraint that (5) exceeds (6). Similarly, for Party R's candidate.

3.4 Political equilibrium

A *political equilibrium* consists of (i) candidate requests $((C_L, f_L), (C_R, f_R))$; (ii) voter belief functions $(\rho_L(I_L, I_R), \rho_R(I_L, I_R))$ describing swing voters' beliefs concerning the likelihood that candidates are qualified; and (iii) cut-points for the swing voters $(i^*(I_L, I_R))$ describing their voting behavior as a function of the information they have received in the campaign. Candidate strategies must be mutual best responses given voter behavior and the constraint of interest group acceptance. Voter beliefs must be consistent with candidates' strategies and voter behavior must be consistent with their beliefs.

The analysis will focus on political equilibria that are *symmetric* in the sense that candidates make the same request to their interest groups (i.e., $(C_L, f_L) = (C_R, f_R) = (C, f)$). In such an equilibrium, if $C > 0$, *Bayes Rule* implies that voters beliefs about unadvertised candidates must satisfy:

$$\rho_L(\emptyset, \emptyset) = \rho_R(\emptyset, \emptyset) = \rho_L(\emptyset, 1) = \rho_R(1, \emptyset) = \frac{\sigma[1 - \lambda(C)]}{\sigma[1 - \lambda(C)] + (1 - \sigma)}. \quad (8)$$

Thus, the probability that voters assign to an unadvertised candidate being qualified is independent of both his party affiliation and the information they have recieved about his opponent. If $C = 0$ then *Bayes Rule* implies that $\rho_L(\emptyset, \emptyset)$ and $\rho_R(\emptyset, \emptyset)$ must equal σ but has no implications for $\rho_L(\emptyset, 1)$ and $\rho_R(1, \emptyset)$. This is because the event of observing any candidate's advertisement does not arise along the equilibrium path when $C = 0$. Since it seems unreasonable to suppose that $\rho_L(\emptyset, 1)$ and $\rho_R(1, \emptyset)$ are anything other than σ when candidates are not expected to advertise, we focus only on symmetric equilibria which have the property that $\rho_L(\emptyset, 1)$ and $\rho_R(1, \emptyset)$ are σ when $C = 0$.[8] This assumption implies that (8) holds even when $C = 0$. Voters beliefs may therefore be summarized by a single variable ρ interpreted simply as the probability that voters assign to an unadvertised candidate being qualified. This must satisfy (8) in equilibrium.

[8] Henceforth, when we refer to a symmetric political equilibrium we will mean one where the beliefs satisfy this property. Note also that equilibria in which $C = 0$ and $\rho_L(\emptyset, 1)$ and $\rho_R(1, \emptyset)$ are not equal to σ are not *sequential equilibria* (Kreps and Wilson (1982)).

Turning to voter behavior, (1) and (8) imply that in a symmetric equilibrium, the cut-point for symmetrically informed swing voters is just $\frac{1}{2}$ (i.e., $i^*(1,1) = i^*(\emptyset,\emptyset) = \frac{1}{2}$). For asymmetrically informed voters, the cut points are given by:

$$i^*(1,\emptyset) = 1 - i^*(\emptyset,1) = \frac{1}{2} + \frac{(1-\rho)(\delta - f)}{2\beta}. \quad (9)$$

Voting behavior may therefore be described by the single variable $\xi = i^*(1,0) - \frac{1}{2}$. This variable measures the size of the interval of swing voters who are induced to vote for a candidate by seeing him advertise and nothing from his opponent. It therefore measures the *effectiveness* of campaign advertising in inducing swing voters to switch from their natural allegiances. In particular, when ξ is zero, campaign advertising is completely ineffective.

Using this notation, equation (3) may be written

$$\mu^*(C_L, C_R) = \frac{1}{2} + \xi(\lambda(C_L) - \lambda(C_R)). \quad (10)$$

Since Assumption 1(ii) implies that $\mu^*(C_L, C_R)$ must always lie between $\frac{1}{2} - \varepsilon$ and $\frac{1}{2} + \varepsilon$, the probability of winning function is given by:

$$\pi(C_L, C_R) = \frac{1}{2} + \frac{\xi}{2\varepsilon}(\lambda(C_L) - \lambda(C_R)). \quad (11)$$

This simple and tractable form of the probability of winning function is a consequence of our assumptions concerning the distribution of swing voters' ideal points. The expression nicely illustrates how ξ determines the productivity of campaign spending. In the sequel, we recognize the critical role of ξ by writing the probability of winning function as $\pi(C_L, C_R; \xi)$.

It follows from the above discussion that a symmetric political equilibrium may be completely described by four variables (C, f, ξ, ρ); C is the contribution given by interest groups to qualified candidates; f is the level of favors these candidates promise to interest groups to get their contributions; ξ is the effectiveness of advertising; and ρ is the probability voters assign to unadvertised candidates being qualified.

4 Equilibrium with Unrestricted Contributions

This section discusses the (symmetric political) equilibrium that would arise with no restrictions on the amount interest groups could contribute to candidates. It first provides a general characterization of equilibrium. It then shows what happens in the limit as candidates become increasingly power-hungry.

4.1 Preliminaries

As the first step towards characterizing equilibrium, we study the offers that candidates will make to their interest groups, taking as given the effectiveness of campaign advertising ξ. Let $U(C_L, f_L, C, f; \xi)$ be the expected utility of Party L's candidate if he is qualified and offers his interest group (C_L, f_L) when his qualified opponent offers his group (C, f); that is,

$$U = \sigma[\pi(C_L, C; \xi)(r + \beta + f - f_L) + \delta - f] + (1 - \sigma)\pi(C_L, 0; \xi)(r + \beta + \delta - f_L) - \beta. \quad (12)$$

Note that this is decreasing in f_L and increasing in C_L when advertising is effective.

Now let $G(C_L, f_L, C, f; \xi)$ denote the gain (gross of the contribution) to the leftist interest group from accepting the offer of Party L's candidate; that is,

$$\begin{aligned} G &= \sigma(\pi(C_L, C; \xi) - \pi(0, C; \xi))(\beta + f) + (1 - \sigma)(\pi(C_L, 0; \xi) - \frac{1}{2})(\beta + \delta) \\ &\quad + (b(f_L) - f_L)(\sigma\pi(C_L, C; \xi) + (1 - \sigma)\pi(C_L, 0; \xi)). \quad (13) \end{aligned}$$

Provided that advertising is effective, this gain is positive even when the interest group is promised no favors. This reflects the interest group's pure policy preference for a qualified candidate who shares its ideology. The gain is increasing in favors as long as b' exceeds 1 and increasing in the size of the contribution when advertising is effective.

Party L's candidate will optimally demand a contribution from his interest group sufficient to exhaust its gain from contributing. The level of favors will balance the gains of the interest group to the candidate's personal policy cost. In equilibrium, (C, f) must solve the problem:

$$\max_{(C_L,f_L)\in\Re^2_+} U(C_L,f_L,C,f;\xi) \ s.t. \ G(C_L,f_L,C,f;\xi) \geq \frac{C_L}{\gamma}. \qquad (14)$$

Henceforth, we refer to this as *Problem P*. It will be studied in more detail below.

Turning to the effectiveness of campaign advertising, we know from (9) that, in equilibrium, ξ is given by:

$$\xi = \frac{(1-\rho)(\delta - f)}{2\beta}. \qquad (15)$$

Effectiveness depends negatively on the level of favors and voters' beliefs concerning the likelihood that an unadvertised candidate is qualified. Using (8) and the functional form for λ, these beliefs are given by:

$$\rho = \frac{\sigma\alpha}{\alpha + C(1-\sigma)}. \qquad (16)$$

Note that ρ is decreasing in C, reflecting the logic that when contributions are plentiful, not having observed a candidate advertise increases the likelihood that he is unqualified.

We may conclude that (C, f, ξ, ρ) is an equilibrium if and only if (i) (C, f) solves *Problem P* given ξ and (ii) ξ and ρ satisfy equations (15) and (16). We can substitute the expression for ρ from (16) into the expression for ξ in (15) to obtain

$$\xi = \frac{(1-\sigma)(\alpha + C)(\delta - f)}{2\beta(\alpha + C(1-\sigma))}. \qquad (17)$$

An equilibrium can then be defined more compactly as a triple (C, f, ξ) such that (C, f) solves *Problem P* given ξ and ξ satisfies equation (17). The associated equilibrium beliefs may then be recovered from (16). Intuitively, equilibrium requires first that the offers qualified candidates make to interest groups must be optimal for them given the effectiveness of campaign advertising, and second that the effectiveness of advertising must be consistent with the amount of contributions qualified candidates receive and the favors they promise.

4.2 Characterization of equilibrium

Further progress necessitates a more detailed study of *Problem P*. Figure 1 presents a diagrammatic treatment. The family of convex curves represents the candidate's indifference map. The candidate dislikes favors and likes contributions, so that moving in a north-westerly direction increases the candidate's utility. The convexity of the indifference curves follows from the fact that the function $U(.,.,C,f;\xi)$ is quasi-concave.

The concave curve is the set of (C_L, f_L) pairs with the property that the interest group's gain $G(C_L, f_L, C, f;\xi)$ exactly equals the per-capita contribution $\frac{C_L}{\gamma}$. The constraint set for *Problem P* is the set of pairs on or below this curve. As drawn, this is a convex set. This will necessarily be the case when ξ is small and will typically be true more generally.[9] In equilibrium, the optimal choice for the candidate will be $(C_L, f_L) = (C, f)$ as illustrated in Figure 1.

Assuming that f is positive, the optimal choice occurs at the tangency of the candidate's indifference curves and the constraint set and this fact may be used to characterize (C, f). Define the function:

$$\Psi(C_L, f_L, C, f;\xi) = \frac{-\partial U/\partial f_L}{\partial U/\partial C_L} - \frac{\partial G/\partial f_L}{\frac{1}{\gamma} - \partial G/\partial C_L}. \qquad (18)$$

This is simply the difference between the candidate's and interest group's marginal rate of substitution between contributions and favors. Accordingly, if f is positive, we know that (C, f) must satisfy the pair of equations:

$$\Psi(C, f, C, f;\xi) = 0 \qquad (19)$$

and

$$G(C, f, C, f;\xi) = \frac{C}{\gamma}. \qquad (20)$$

[9] The constraint set will be convex if the extra contributions that can be extracted from a given increase in favors decreases with the level of favors. There are two forces working in this direction. First, the marginal benefit of favors is decreasing. Second, the marginal impact of contributions on the probability of winning is decreasing in the level of contributions. Against this, we have that the marginal benefit of contributing is higher at a higher level of favors. In Appendix B we solve explicitly for the curve describing the boundary of the constraint set and find the conditions under which it is concave.

These are simply the first order conditions for *Problem P*.

It remains to impose an assumption that guarantees that equilibrium does involve candidates providing favors. The required assumption is:

Assumption 2: $\Psi(\widetilde{C}, 0, \widetilde{C}, 0; \xi') < 0$ *where* $\xi' = \frac{(1-\sigma)\delta}{2\beta}$ *and* $\widetilde{C} = \frac{\gamma(\beta+(1-\sigma)\delta)}{2}$.

Note from (17) that ξ' is the effectiveness of advertising when $(C, f) = (0, 0)$. Thus, if the equilibrium involves no favors, ξ' is a lower bound on the effectiveness of advertising. On the other hand, $\widetilde{C}$ is an upper bound on the amount of contributions that candidates can receive if they grant no favors. Thus, the assumption says that even at the lowest possible level of advertising effectiveness and the highest level of contributions, there is a gain to candidates from offering favors. Assumption 2 rules out the possibility illustrated in Figure 2, in which contributions are purely position-induced. It will necessarily be satisfied for sufficiently high r and, for given r, is more likely to be satisfied the larger the size of the interest groups and the greater is the marginal value of favors to interest group members.[10]

We now have:

Proposition 1: *Suppose that Assumptions* 1 *and* 2 *are satisfied. Then, in any equilibrium* (C, f, ξ)*, candidates offer to implement favors for their interest groups to extract larger contributions. The contributions they receive allow them to defeat unqualified opponents with a probability between* $\frac{1}{2}$ *and* 1 (i.e., $\pi(C, 0; \xi) \in (1/2, 1)$). *The level of favors promised is less than the gain from having a qualified candidate (i.e.,* $f < \delta$*) and* (C, f, ξ) *must satisfy equations* (17), (19), *and* (20).

Thus, with unrestricted contributions, qualified candidates will offer favors to extract more contributions from their supporters. The campaign advertising these contributions finance gives them an electoral advantage over their unqualified opponents. Campaign contributions therefore play

[10] It follows from Fact 1 in Appendix A that Assumption 3 is equivalent to the requirement that $\widetilde{C}$ is less than $\sqrt{\frac{\xi'\alpha\gamma}{2\varepsilon}\{(b'(0)-1)r + \beta b'(0) + (1-\sigma)\delta b'(0)\}} - \alpha$.

the social role of raising the likelihood of qualified leaders. However, the favors qualified candidates implement reduce the benefits to non-interest group members from electing them. Moreover, the favors granted do not ultimately benefit the interest groups, because interest group members pay for them up front through their contributions.

Proposition 1 gives us three equations that may be solved for the three unknowns (C, f, ξ) and enables the numerical computation of equilibrium. If (C, f, ξ) satisfies the three equations then it will be an equilibrium provided that equations (19) and (20) are sufficient to imply that (C, f) solves *Problem P*. Provided that the constraint set in Figure 1 is convex, they will be sufficient. As noted above, the constraint set will necessarily be convex when ξ is small and will typically be convex more generally. Thus, if (C, f, ξ) satisfies the three equations it will typically be an equilibrium. The issue of the existence of equilibrium therefore boils down to the existence of a triple (C, f, ξ) satisfying the three equations. Section 5.3 below identifies some sufficient conditions for the existence of such a solution.

4.3 Power-hungry candidates

The logic of the equilibrium is that the effectiveness of advertising determines the incentives of candidates to offer favors and the level of favors feeds back into the determination of the effectiveness of advertising. When candidates are power-hungry one might expect them to be desperate to obtain more contributions and thus willing to promise large amounts of favors. But the level of favors must be less than the benefits of being qualified if campaign advertising is to be effective. One would therefore expect that equilibrium must involve a low level of advertising effectiveness to dampen candidates' propensity to offer favors. Thus, as candidates become more and more power-hungry, the effectiveness of campaign advertising should become smaller and smaller. This logic is confirmed in:

Proposition 2: *Suppose that Assumption 1 is satisfied. For all r, let $(C(r), f(r), \xi(r))$ be the* *equilibrium (or an equilibrium) that would arise with no limits when ego-rents are* r*. Then,*

$$\lim_{r\to\infty}(C(r), f(r), \xi(r)) = (\frac{\gamma(b(\delta)-\delta)}{2}, \delta, 0).$$

The conclusion that the effectiveness of advertising must go to zero may be understood diagrammatically. An increase in r raises the candidate's marginal value of contributions, thereby flattening his indifference curves.[11] For given $\xi > 0$, the candidate's indifference curves become horizontal as r goes to infinity. On the other hand, a reduction in ξ reduces the candidate's marginal value of contributions, steepening his indifference curves. Indeed, for given r, the candidate's indifference curves become vertical as ξ goes to zero. As r increases, the candidate's indifference curves become flatter and he is prepared to offer more and more favors. Since the level of favors must be strictly less than the gains from qualifications (δ) in any equilibrium and the slope of the boundary of the constraint set is positive over this range, the only way that the tangency condition (19) may hold as r gets larger and larger is for ξ to get smaller and smaller.

For the effectiveness of advertising to be zero the level of favors that qualified candidates are expected to implement must just equal the gains from them being qualified. Thus, the level of favors converges to δ. Note also that the Proposition implies that the equilibrium probability that a qualified candidate defeats an unqualified one tends to 1/2 as candidates become more power-hungry (i.e., $\lim_{r\to\infty}\pi(C(r), 0; \xi(r)) = 1/2$). Accordingly, *while resources are expended on campaign advertising, these resources do not make qualified candidates more likely to be elected.* This observation has important implications for the desirability of contribution limits.

5 Contribution Limits

This section first characterizes equilibrium with contribution limits. It then shows that when candidates are sufficiently power-hungry, banning contributions is Pareto improving. Finally, it

[11] The slope of the candidate's indifference curves $\frac{-\partial U/\partial f_L}{\partial U/\partial G_L}$ is given by $\frac{\sigma\pi(C_L,C;\xi)+(1-\sigma)\pi(C_L,0;\xi)}{\frac{\xi}{2\varepsilon}\lambda'(C_L)[r+\beta-f_L+\sigma f+(1-\sigma)\delta]}$.

argues that a similar logic implies that limiting contributions can be Pareto improving even when candidates are only mildly power-hungry.

5.1 Equilibrium with contribution limits

Suppose that the laws governing elections limit the amount of money an interest group can contribute. Let the limit be denoted by l. Candidates are now constrained in what they can obtain from their interest groups. In equilibrium, (C, f) must solve the problem:

$$\max_{(C_L,f_L)\in[0,l]\times\Re_+} U(C_L, f_L, C, f; \xi) \ s.t. \ G(C_L, f_L, C, f; \xi) \geq \frac{C_L}{\gamma}. \quad (21)$$

We will refer to this as *Problem P′*. An equilibrium is then a triple (C, f, ξ) such that (C, f) solves *Problem P′* given ξ and ξ satisfies equation (17).

Figure 3 illustrates how the introduction of a limit changes a candidate's constraint set. We may follow the strategy of the previous section and use the first order conditions for *Problem P′* to characterize equilibrium. If (C, f) solves *Problem P′* and the constraint binds (i.e., $C = l$), then (l, f) must satisfy the pair of equations:

$$\Psi(l, f, l, f; \xi) \leq 0 \qquad (22)$$

and

$$G(l, f, l, f; \xi) \geq \frac{l}{\gamma} \ (= \textit{if } f > 0). \quad (23)$$

The constraint that interest groups cannot contribute more than the limit, prevents the level of favors from being driven to the level where the slope of the candidate's indifference curve equals the slope of the boundary of the constraint set. Effectively, when the limit is binding, the candidate's indifference curve at the optimal choice can be flatter than the boundary of the constraint set as illustrated in Figure 3. This explains equation (22). With limits, it is possible that no favors are offered and contributions are purely "position induced" even under Assumption 3. This arises when interest groups would obtain a net gain from contributing the maximal level of contributions when the effectiveness of advertising is that which would arise if $(C, f) = (l, 0)$. Diagrammatically, the situation is as illustrated in Figure 4. In such an equilibrium, interest groups may obtain some surplus because candidates are unable to extract more contributions from them or offer them fewer favors. This explains equation (23).

We therefore have:

Proposition 3: *Suppose that Assumptions* 1 *and* 2 *are satisfied. Then, if* (C, f, ξ) *is an equilibrium under contribution limit* l *such that the limit binds (i.e.,* $C = l$*), then* (l, f, ξ) *satisfies equations* (17)*,* (22)*, and* (23).

If (l, f, ξ) satisfies the three equations then it will be an equilibrium under contribution limit l provided that equations (22) and (23) are sufficient to imply that (l, f) solves *Problem* P'. Again, provided that the constraint set in Figure 1 is convex, they will be sufficient.

5.2 Pareto improving contribution limits with power-hungry candidates

To understand the welfare implications of limits, it is necessary to understand both how the equilibrium is impacted by limits and how changes in the equilibrium impact citizens' payoffs. By deriving expressions for the equilibrium payoffs of the various types of citizens, we may establish:

Lemma 1: *If imposing a limit moves the community from some status quo* (C^*, f^*, ξ^*) *to a new equilibrium* (C, f, ξ) *such that (i)* $\pi(C^*, 0; \xi^*) \approx \pi(C, 0; \xi)$ *and (ii)* $f < f^*$*, then it makes all types of citizens strictly better off.*

Thus if introducing a limit does not appreciably change the probability a qualified candidate defeats an unqualified one and reduces the level of favors, it will create a Pareto improvement. That these conditions imply that leftists, rightists, and swing voters are better off seems natural. That they imply that interest group members are better off is less obvious. The key is to note that the *equilibrium* payoff of interest group members is decreasing in f. Intuitively, this is because interest group members pay for their own favors up front with their contributions and must also share the burden of favors granted to the other interest group.

Combining Lemma 1 with Proposition 2 enables us to establish:

Proposition 4: *Suppose that Assumption 1 is satisfied. Then, if candidates are sufficiently power-hungry (i.e., r is sufficiently large), banning contributions (i.e., setting $l = 0$) will create a Pareto improvement.*

To understand this result, note that if contributions were banned entirely then no favors would be promised and the probability that a qualified candidate defeats an unqualified one is just 1/2. The result now follows from the fact that, with no limits, as candidates become more power-hungry the probability that a qualified candidate defeats an unqualified one approaches 1/2 while the level of favors remains strictly positive.

5.3 The general case

The logic of the above argument is that when candidates are sufficiently power-hungry, banning contributions will have a negligible impact on the probability that a qualified candidate defeats an unqualified one, while reducing favors. This implies a Pareto improvement. With less power-hungry candidates, it is clear that *banning* contributions could lead to a significant reduction in the probability that qualified candidates win and hence this argument will not imply. We now argue that *limiting* contributions, while reducing favors, need not appreciably reduce the probability that qualified candidates win, in which case the same logic implies that limits could be Pareto improving.

Establishing this requires us to understand more completely the impact of limits on the equilibrium variables. Our strategy will be to first develop a deeper understanding of the determination of equilibrium with unrestricted contributions. We then use this to assess the impact of limits.

From Proposition 1 we know that any equilibrium with unrestricted contributions must satisfy equations (17), (19), and (20). To understand the solution to these equations, we first study the contribution-favor pairs that satisfy equations (17) and (20) taking ξ as given. We then use equation (19) to tie down the equilibrium level of effectiveness.

To this end, first consider equation (17). Let $C_o(f;\xi)$ be the level of contributions that qualified candidates must receive to generate an effectiveness of advertising ξ when qualified candidates provide an amount of favors f. Clearly, $C_o(f;\xi)$ will not be defined for all pairs $(f;\xi)$ - for example, there will exist no amount of contributions that will generate a high level of effectiveness when the level of favors is very high. For given ξ, $C_o(f;\xi)$ is well-defined for f values between $\max\{0, \delta - \frac{2\beta\xi}{1-\sigma}\}$, and $\delta - 2\beta\xi$. On this interval, $C_o(\cdot;\xi)$ is increasing at an increasing rate, approaching infinity as the level of favors approaches the upper limit of the interval. Intuitively, as the level of favors qualified candidates provide increases, the amount of contributions necessary to generate a given level of effectiveness increases.[12] The function $C_o(\cdot;\xi)$ is depicted in Figure 5 under the assumption that δ exceeds $\frac{2\beta\xi}{1-\sigma}$. For a given level of favors, it takes a higher level of contributions to generate a higher level of effectiveness, so that an increase in ξ shifts this curve to the left.

Now consider equation (20). Let $C_i(f;\xi)$ be the level of contributions that would make interest groups indifferent between accepting candidates offers when the level of favors promised is f and the effectiveness of advertising is ξ. For all ξ, $C_i(\cdot;\xi)$ is an increasing function, reflecting the fact that interest groups value favors. It may or may not be positive at $f = 0$ depending on the strength of the position-induced incentive to give and the effectiveness of campaign advertising.[13] The function $C_i(\cdot;\xi)$ is depicted in Figure 5 under the assumption that $C_i(0;\xi) = 0$. For a given level of favors, the gain from contributing is higher the more effective is advertising, so that an

[12] As observed earlier, when contributions are plentiful, not having observed a candidate's advertisement increases the likelihood that he is unqualified and thus increases the effectiveness of advertising.

[13] There may be two non-negative solutions to equation (20) when $f = 0$. One solution is always $C = 0$, since the gain from giving no contributions in exchange for no favors is obviously zero. But there will be a positive solution if $dG(0,0,0,0;\xi)/dC > \gamma$. $C_i(0;\xi)$ is the positive solution when it exists.

increase in ξ shifts this curve to the left.

For given levels of effectiveness ξ, we are interested in whether there exists a level of favors f such that $C_i(f;\xi) = C_o(f;\xi)$. If so, then at this level of favors and the associated level of contributions, both equations (17) and (20) are satisfied. We make the following assumptions on the functions C_i and C_o which serve to simplify the structure of the set of solutions.

Assumption 3: *(i) For all* $\xi \in [0, \frac{\delta}{2\beta}]$, $C_i(\cdot;\xi)$ *is strictly concave.*

(ii) For all $\xi \in [0, \frac{\delta}{2\beta}]$ *and* $f \in (\max\{0, \delta - \frac{2\beta\xi}{1-\sigma}\}, \delta - 2\beta\xi)$, $\frac{\partial C_i(f;\xi)}{\partial \xi} < \frac{\partial C_o(f;\xi)}{\partial \xi}$.

Part (i) is self-explanatory and part (ii) requires that a marginal increase in advertising effectiveness necessitates a larger increase in C_o than it generates in the interest groups' contribution C_i. An increase in ξ generates an increase in C_i because contributions now translate into higher winning probabilities and an increase in C_o simply because a higher level of contributions is necessary to generate increased effectiveness. In Appendix B, we compute the relevant derivatives of the two functions to spell out precisely what Assumption 3 implies. Both parts appear rather mild requirements and therefore can be thought of as characterizing the "regular case".[14]

We will distinguish two main cases. The first arises when the following assumption is satisfied.

Assumption 4: *(i)* $C_i(0;\xi') = 0$ *and (ii)* $\frac{\partial C_i(0;\xi')}{\partial f} < \frac{\partial C_o(0;\xi')}{\partial f}$ *where* $\xi' = \frac{(1-\sigma)\delta}{2\beta}$.

The first part says that when the effectiveness of advertising is given by ξ', candidates will be unable to extract contributions from interest groups without promising them favors. The second part says that $C_o(\cdot;\xi')$ has a steeper slope than $C_i(\cdot;\xi')$ at zero favors. Both parts of this assumption must hold for α sufficiently large.[15]

Under Assumptions 3 and 4, the maximum level of effectiveness for which there exists a level of favors f such that $C_i(f;\xi) = C_o(f;\xi)$ is just ξ'. If ξ lies in the interval $(0, \xi']$ the situation is

[14] Unfortunately, the complexity of the expressions make it difficult to identify nice sufficient conditions for Assumption 4 to be satisfied. However, both parts will necessarily be satisfied for sufficiently large α.

[15] Appendix B develops the implications of this assumption in more detail.

as depicted in Figure 5. There is a unique level of favors f such that $C_i(f;\xi) = C_o(f;\xi)$, which we denote $f(\xi)$. At this solution, $C_o(\cdot;\xi)$ cuts $C_i(\cdot;\xi)$ from below so that the function $f(\cdot)$ is decreasing on the interval $(0,\xi']$. For all ξ in the interval $(0,\xi']$, let $C(\xi) = C_o(f(\xi);\xi)$. Since C_o is increasing in f and ξ the sign of C' is indeterminate. The functions $(C(\cdot), f(\cdot))$ are illustrated in Figure 6.

For all ξ in the interval $(0,\xi']$, let $\Psi(\xi) = \Psi(C(\xi), f(\xi), C(\xi), f(\xi);\xi)$. Clearly, (C, f, ξ) satisfies equations (17), (19), and (20) if and only if $(C, f) = (C(\xi), f(\xi))$ and $\Psi(\xi) = 0$. We know that the candidate's indifference curve is vertical when $\xi = 0$, so that $\Psi(\xi) > 0$ for ξ sufficiently small. In addition, Assumption 2 implies that the candidate's indifference curve must be flatter than the boundary of the constraint set when $\xi = \xi'$ so that $\Psi(\xi') < 0$. Thus, since $\Psi(\cdot)$ is continuous, under Assumptions 1 - 4, there must exist (C, f, ξ) which satisfies equations (17), (19), and (20). Moreover, if there exists a unique such solution (C^*, f^*, ξ^*) it must also be the case that Ψ is positive on $(0, \xi^*)$ and negative on the interval $(\xi^*, \xi']$.

We are now in a position to understand the impact of limits. Suppose that there is a unique equilibrium with unrestricted contributions (C^*, f^*, ξ^*) and consider a limit $l < C^*$. The situation is illustrated in Figure 6. First note that the limit must reduce the level of favors. Even though it might be the case that there exists $\widetilde{\xi} < \xi^*$ such that $C(\widetilde{\xi}) = l$, it cannot be the case that $(f, \xi) = (f(\widetilde{\xi}), \widetilde{\xi})$ under the limit. This is because $\Psi(\widetilde{\xi}) < 0$ and hence equation (22) could not be satisfied.[16] The next point to note is that the limit will increase the effectiveness of campaign advertising. Intuitively, swing voters are more likely to be responsive to learning a candidate is qualified because they know that qualified candidates will implement lower levels of favors. This offsets the fact that not seeing an advertisement is less likely to mean that a candidate is unqualified because there is less advertising. We summarize these conclusions in the following

[16] There is no guarantee that there is a unique equilibrium under a particular limit in this case. Because $C(\xi)$ is not necessarily monotonic on $(0, \xi']$, there may be more than one $\xi > \xi^*$ such that $C(\xi) = l$.

proposition:

Proposition 5: *Suppose that Assumptions 1 - 4 are satisfied and that there exists a unique equilibrium with unrestricted contributions. Then, limiting contributions reduces the level of favors and increases the effectiveness of advertising.*

It follows that the limit need not reduce the probability that a qualified candidate defeats an unqualified candidate because the increase in the responsiveness of swing voters could compensate for the smaller fraction reached as a result of reduced campaign spending. A binding limit that leaves unchanged the probability that a qualified candidate wins will create a Pareto improvement by Lemma 3. A sufficient condition for the existence of such a limit is that $\pi(C(\xi), 0; \xi)$ is increasing at $\xi = \xi^*$. If this is the case, there must exist a limit $l < C^*$ that will reduce favors and leave the probability that a qualified candidate defeats an unqualified one unchanged. The condition requires that at the equilibrium level of effectiveness ξ^*, $-C^{'}$ is smaller than $\partial\pi/\partial\xi/\partial\pi/\partial C_L = C(C+\alpha)/\xi^*\alpha$.

The second main case arises when, instead of Assumption 4, the following assumption is satisfied:

Assumption 5: *(i)* $C_i(0;\xi') > 0$ *and (ii)* $\frac{\partial C_i(0;\underline{\xi})}{\partial f} > \frac{\partial C_o(0;\underline{\xi})}{\partial f}$ *where* $\underline{\xi}$ *satisfies* $C_i(0;\underline{\xi}) = C_o(0;\underline{\xi})$.

The first part says that when the effectiveness of advertising is given by ξ', candidates will be able to obtain contributions from interest groups without promising them favors. The second part says that $C_i(\cdot;\underline{\xi})$ has a steeper slope than $C_o(\cdot;\underline{\xi})$ at zero favors, where $\underline{\xi}$ is the level of effectiveness at which C_i equals C_o when favors are zero. Note that $\underline{\xi}$ must exceed ξ' given Assumption 3.

In this case, the structure of the solutions to equations (17) and (20) is more complicated. Let $\overline{\xi}$ be the maximum level of effectiveness for which there exists a level of favors f such that $C_i(f;\xi) \geq C_o(f;\xi)$. If ξ lies in the interval $[\underline{\xi}, \overline{\xi}]$ the situation is as depicted in Figure 7 and there are two solutions to the equation $C_i(f;\xi) = C_o(f;\xi)$, which we denote $f_-(\xi)$ and $f_+(\xi)$

respectively. At the former solution, $C_o(\cdot;\xi)$ cuts $C_i(\cdot;\xi)$ from above, at the latter from below. In the case in which ξ exactly equals $\overline{\xi}$, we have that $f_-(\xi) = f_+(\xi)$ and $C_o(\cdot;\xi)$ is tangent to $C_i(\cdot;\xi)$. If ξ lies in the interval $(0, \underline{\xi})$ the situation is as depicted in Figure 8 and there is a unique solution, which we denote $f_+(\xi)$. At this solution, $C_o(\cdot;\xi)$ cuts $C_i(\cdot;\xi)$ from below. Under Assumption 3, the function $f_-(\cdot)$ is increasing on the interval $[\underline{\xi}, \overline{\xi}]$ and $f_+(\cdot)$ is decreasing on $(0, \overline{\xi}]$.

For all ξ in the interval $[0, \overline{\xi}]$, let $C_+(\xi) = C_o(f_+(\xi);\xi)$ and for all ξ in the interval $[\underline{\xi}, \overline{\xi}]$ let $C_-(\xi) = C_o(f_-(\xi);\xi)$. Since C_o is increasing in ξ, $C_-(\xi)$ is increasing on $[\underline{\xi}, \overline{\xi}]$. However, the sign of $C_+(\xi)$ is indeterminate. The functions (C_+, f_+) and (C_-, f_-) are illustrated in Figure 9.

For all ξ in the interval $[0, \overline{\xi}]$ let $\Psi_+(\xi) = \Psi(C_+(\xi), f_+(\xi), C_+(\xi), f_+(\xi);\xi)$ and for all ξ in the interval $[\underline{\xi}, \overline{\xi}]$ let $\Psi_-(\xi) = \Psi(C_-(\xi), f_-(\xi), C_-(\xi), f_-(\xi);\xi)$. If (C, f, ξ) satisfies the three equations of Proposition 1 then *either* the effectiveness of advertising must lie in the interval $[\underline{\xi}, \overline{\xi}]$, $(C, f) = (C_-(\xi), f_-(\xi))$, and $\Psi_-(\xi) = 0$, *or* the effectiveness of advertising lies in the interval $(0, \overline{\xi}]$, $(C, f) = (C_+(\xi), f_+(\xi))$, and $\Psi_+(\xi) = 0$. In the former case we say that the equilibrium is in *Case A*, in the latter it is in *Case B*. Since $\underline{\xi} > 0$, the equilibrium must be in Case B for r sufficiently large.

We know that $\Psi_+(0) > 0$ and that, under Assumption 2, $\Psi_-(\underline{\xi}) < 0$. We also know that $\Psi_+(\overline{\xi}) = \Psi_-(\overline{\xi})$ and that Ψ_+ and Ψ_- are continuous. Thus, under Assumptions 1 - 3 and 5 there must exist (C, f, ξ) which satisfies equations (17), (19), and (20). Assuming that there exists a unique equilibrium (C^*, f^*, ξ^*), it follows that if the equilibrium is in Case A it must be that Ψ_- is negative on the interval $[\underline{\xi}, \xi^*)$ and positive on the interval $(\xi^*, \overline{\xi}]$, while Ψ_+ is positive on its entire range. If the equilibrium is in Case B it must be that Ψ_- is negative on its entire range, while Ψ_+ is positive on $(0, \xi^*)$ and negative on the interval $(\xi^*, \overline{\xi}]$.

To understand the impact of limits, suppose first that the status quo equilibrium is in Case A and that $l \geq C_-(\underline{\xi})$. The situation is as illustrated in Figure 10. Observe that the limit *reduces* the effectiveness of advertising and reduces the level of favors. The effectiveness of advertising is reduced, despite the fact that the level of favors decreases, because not seeing an advertisement is less likely to mean that a candidate is unqualified because there is less advertising. Thus, while the benefit of electing a qualified candidate has increased, swing voters are less likely to switch their votes from unadvertised candidates because unadvertised is less likely to imply unqualified. It follows that the probability that a qualified candidate defeats an unqualified candidate must fall since *both* the level of contributions *and* the effectiveness of advertising falls. If $l < C_{-}(\underline{\xi})$, then the limit reduces the level of favors to 0 and contributions become purely position-induced.[17] Again, the probability that a qualified candidate defeats an unqualified candidate falls.

If the status quo equilibrium is in Case B, then the situation is analagous to that arising under Assumption 4. The limit must reduce the level of favors and will increase the effectiveness of campaign advertising assuming that it is not too stringent. Figure 11 illustrates the situation. We summarize these conclusions in the following proposition:

Proposition 6: *Suppose that Assumptions 1 - 3 and 5 are satisfied and that there exists a unique equilibrium with unrestricted contributions. Then, limiting contributions reduces the level of favors but may increase or decrease the effectiveness of advertising.*

When the status quo equilibrium is in Case B, it follows that there may exist a limit that leaves unchanged the probability that a qualified candidate wins and hence creates a Pareto improvement. A sufficient condition for the existence of such a limit is that at the equilibrium level of effectiveness ξ^*, $-C_{+}^{'}$ is smaller than $\partial\pi/\partial\xi/\partial\pi/\partial C_L = C_{+}(C_{+} + \alpha)/\xi^*\alpha$.

Unfortunately, the problem is sufficiently complex that it is difficult to get convert the above sufficient condition or its earlier cousin into simple conditions on the primitives (other than that r be large!). However, numerical examples suggest that there is nothing paradoxical about the possibility of Pareto improving contribution limits even when candidates are only mildly power-hungry. The next sub-section presents two such examples.

[17] The effectiveness of advertising is given by $\xi = \frac{(1-\sigma)\delta(\alpha+l)}{2\beta(\alpha+l(1-\sigma))}$.

5.4 Examples

We present two examples, one in which Assumption 3 is satisfied and the other in which Assumption 4 applies. In both candidates are only mildly power-hungry.

Example 1

Assume that $\tau = 0.2$, $\varepsilon = 0.1$, $\sigma = 0.5$, $\beta = 100$, $\delta = 20$, $r = 200$, $\gamma = 0.05$, $\alpha = 4$, and $b(f) = 10f - (0.05)f^2$. Note that the ego rent is only twice the purely ideological gain from having a leader of one's own ideology, which is $\beta = 100$. Note also that, since each interest group comprises one twentieth of the population, the benefits received by the interest group from favors are significantly less than the cost of providing these favors. Thus, the transfer mechanism entails deadweight loss. It is straightforward to verify that, under these assumptions, Assumption 5 is satisfied.

Under this specification, there is unique equilibrium with unrestricted contributions in which $(C, f, \xi) = (3.3538, 13.452, 0.021205)$. Even with only mildly power-hungry candidates, much of the benefits from qualified candidates are dissipated via favors and campaign advertising is rather ineffective. This is reflected in the fact that the equilibrium probability that a qualified candidate defeats an unqualified one is only 0.54835.

With a limit of 3, the equilibrium is $(C, f, \xi) = (3, 11.722, 0.02634)$. Note that campaign advertising is more effective and the level of favors is reduced. The equilibrium probability that a qualified candidate defeats an unqualified one is now 0.55644, slightly above the status quo level. With limit 2.25, the equilibrium is $(C, f, \xi) = (2.25, 8.3649, 0.035473)$ and the probability that a qualified candidate wins further increases to 0.56385. The equilibrium with limit 2 is $(C, f, \xi) = (2, 7.3242, 0.038027)$ and the probability that a qualified candidate defeats an unqualified one is 0.56338. Thus, advertising is more effective than with the higher limit, but the probability that a qualified candidate wins falls slightly. With a limit of 1, $(C, f, \xi) = (1, 3.4688, 0.04592)$ and the probability that a qualified candidate wins is 0.54592, which is below the status quo level. With a limit of 0.5, $(C, f, \xi) = (0.5, 1.6933, 0.048459)$ and the probability that a qualified candidate wins falls to only 0.52692.

The picture this suggests is that the effectiveness of advertising is increasing in the stringency of the limit. Indeed, this is a general result as should be clear from Figure 6. The probability that a qualified candidate wins first rises and then falls as the limit becomes more stringent. It follows that there must exist a limit that reduces favors and leaves unaffected the probability a qualified candidate wins. Thus, by Lemma 1, there must exist a Pareto improving contribution limit.

Example 2

Assume the same parameter values as in Example 1, except that $\alpha = 0.5$, so that it is much cheaper to contact voters. With this new cheaper advertising technology, Assumption 6 is satisfied.

Under this specification, there is unique equilibrium with unrestricted contributions in which $(C, f, \xi) = (3.2231, 10.43, 0.042184)$. The equilibrium probability that a qualified candidate defeats an unqualified one is 0.68259. With a limit of 3, the equilibrium is $(C, f, \xi) = (3, 9.219\,5, 0.047165)$. Note that campaign advertising is more effective and the level of favors is reduced. The equilibrium probability that a qualified candidate defeats an unqualified one is now 0.70214, so that the limit slightly raises the probability that a qualified candidate is elected. With limit 2, the equilibrium is $(C, f, \xi) = (2, 4.654, 0.063942)$ and the probability that a qualified candidate defeats an unqualified one increases substantially to 0.75577. With a limit of 1, the equilibrium is $(C, f, \xi) = (1, 1.2701, 0.070237)$ and the probability that a qualified candidate wins is 0.73412. Thus, while the effectiveness of advertising increases, the probability that a qualified candidate wins goes down. With a limit of 0.5, the equilibrium is $(C, f, \xi) = (0.5, 0.16695, 0.06611)$ and the probability that a qualified candidate defeats an unqualified one is 0.66528, which is below the status quo level. Note also that the effectiveness of advertising decreases.

The picture this suggests is that the effectiveness of advertising is first increasing and then decreasing in the stringency of the limit. Thus, the status quo equilibrium is in Case B. The probability that a qualified candidate wins first rises and then falls as the limit becomes more stringent. Again, it follows that there must exist a limit that reduces favors and leaves unaffected the probability a qualified candidate wins so that there exists a Pareto improving contribution limit.

5.5 A final remark

It is important to note that even when imposing a limit implies a reduction in the probability that qualified candidates are elected, it still maybe the case that the limit is Pareto improving. This is because the gains from reduced favors may offset the losses from inferior sorting. To illustrate let (C^*, f^*, ξ^*) be the status quo and suppose that a limit leads to a new equilibrium (C, f, ξ) such that $\pi(C, 0; \xi) < \pi(C^*, 0; \xi^*)$. Then, provided that

$$[\sigma^2 + 2\sigma(1-\sigma)\pi](\delta - f) > [\sigma^2 + 2\sigma(1-\sigma)\pi^*](\delta - f^*), \quad (24)$$

it makes all types of citizens strictly better off. This condition ensures that any reduction in the probability a qualified candidate defeats an unqualified one is compensated by a reduction in the favors that such a candidate will provide. That these conditions imply that leftists, rightists, and swing voters are better off, follows directly from the expressions for their payoffs derived in the proof of Lemma 1. That they imply that interest group members are better off follows from the fact that the *equilibrium* payoff of interest group members is decreasing in π as well as f.

This admits a simple sufficient condition for limits to be Pareto improving. Under a contribution ban, we know that that $\pi = 1/2$ and $f = 0$. We also know that $\pi^* \leq 1$. It follows that (24) must hold if the status quo level of favors exceeds $\frac{(1-\sigma)}{2-\sigma}\delta$.

6 Conclusion

The basic logic of the argument presented in this paper is easily summarized. When candidates use campaign contributions to finance advertising that conveys truthful information to voters about their qualifications for office, contributions have the potential social benefit of helping elect more qualified leaders. But for contributions to have this benefit, voters who are informed that a candidate is qualified through campaign advertising must be induced to switch their votes from unadvertised candidates. However, when contributions are unrestricted and candidates are power-hungry, voters will rationally be cynical about qualified candidates, anticipating that they will implement favors for their contributors when elected. This cynicism will reduce the likelihood of voters switching their votes and, despite the fact that resources are spent on advertising, qualified candidates will not have much of an electoral advantage over unqualified opponents.

When campaign contributions are limited, candidates' incentive to offer favors to extract more contributions is dampened. While less money is available for campaign advertising, voters now anticipate that advertised candidates will implement fewer favors than in the unrestricted case and this may increase the likelihood that they will vote for them. In this way, limits can actually raise the likelihood that qualified candidates get elected. Moreover, if elected such candidates will implement lower levels of favors than in the unrestricted case. Thus, all regular citizens can be better off when contributions are limited. The only possible losers are contributors who receive lower levels of favors. But their expected gains from favors will be dissipated by the contributions they make, meaning they may also be better off.

While the underlying logic seems quite general, the argument has been formally developed in an undeniably simple model. It would be well worth investigating the robustness of the argument to alternative or more general specifications. One obvious assumption to change is that the candidates present interest groups with "take it or leave it" offers that allow them to extract all their surplus.

One could alternatively follow Grossman and Helpman (1994) in assuming the opposite; i.e., that interest groups make "take it or leave it" offers. It seems likely that the conclusion that even interest group members would benefit from contribution limits might need modification. That said, even when interest group members obtain some surplus from the favors they are given, they must still bear their share of the collective cost of granting other groups favors.

It would also be interesting to allow for a richer set of candidate types. For example, one could introduce multiple levels of qualifications. Presumably, equilibrium would involve only candidates with qualifications above some critical level advertising. This critical level would depend upon the cost of advertising. For a result like Proposition 2 to hold, it would have to be the case that, in equilibrium, more qualified candidates promised more favors. Alternatively, one could assume that candidates differed in their willingness to take favors - some were less power-hungry than others. Under the latter assumption, the number of times a voter had seen a candidate's advertisement might have some significance. There might be a penalty for advertising too heavily, because voters would take it as a signal of a candidate being more power-hungry and hence having promised more favors. This might limit the incentive of power-hungry candidates to offer favors even when contributions are unrestricted.[18] This extension becomes all the more interesting if parties can observe how power-hungry potential candidates are and can decide what type of candidates to run.

More generally, from an empirical perspective, it would be extremely interesting to exploit the cross-state variation in U.S. campaign finance regulations, to see if there is indeed systematic differences in the effectiveness of campaign advertising as the argument suggests. It would also be interesting to consider whether the type of argument developed here has implications for the

[18] The idea is that a qualified candidate's spending would generate a probability distribution over the number of messages voters had received. A certain fraction would have seen no messages, some fraction just one, etc. More spending would shift the probability distribution to the right. If less power-hungry spending candidates are expected to raise less money, voters who observed a higher number of messages would believe that a candidate was more likely to be power-hungry. This may dampen the effectiveness of large scale advertising campaigns abd thereby reduce the incentives of power-hungry candidates to raise money.

case for public financing of campaigns. In some U.S. states, candidates for statewide offices are entitled to public financing if (i) they have raised some minimum level of contributions from private citizens or groups and (ii) they forego taking further private contributions. Such a scheme would seem to have the potential of reducing favors and increasing the effectiveness of advertising, while not reducing the level of advertising. However, the downside is that public contributions must be financed via tax hikes.

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7 Appendix A: Proofs

The proofs will make use of the following Fact, whose (mechanical) proof is delegated to Appendix B.

Fact 1: Suppose that $f \leq \delta$ and that $G(C, f, C, f; \xi) \leq \frac{C}{\gamma}$. Then, $\Psi(C, f, C, f; \xi) \geq 0$ if and only if

$$C \geq \sqrt{\frac{\xi\alpha\gamma}{2\varepsilon}\{(b'(f)-1)r + \beta b'(f) + b(f) + (1-\sigma)(\delta - f)b'(f)\}} - \alpha.$$

Proof of Proposition 1: Let (C, f, ξ) be an equilibrium. By definition, we know that (C, f) must solve the problem

$$\max_{(C_L, f_L)\in\Re^2_+} U(C_L, f_L, C, f; \xi) \; s.t. \; G(C_L, f_L, C, f; \xi) \geq \frac{C_L}{\gamma}$$

and that (C, f, ξ) satisfies (17). To prove the proposition, we need to establish that f lies in the interval $(0, \delta)$ and that (C, f, ξ) satisfies (19) and (20). It will then follow that $C > 0$ and that $\pi(C, 0; \xi) \in (1/2, 1)$.

Observe first that it must be the case that $\xi > 0$. If not, then $\xi = 0$ which implies that $(C, f) = (0, 0)$ and hence, from (17), that $0 = \frac{(1-\sigma)\delta}{2\beta}$ - a contradiction. It follows that $G(C, f, C, f; \xi) = \frac{C}{\gamma}$. If not, then the candidate could ask for a slightly larger contribution and make himself better off, since $\xi > 0$. This proves (20).

Since $\xi > 0$, we know from (17) that $f < \delta$. Thus, it remains to show that $f > 0$ and that (C, f, ξ) satisfies (19). Since $U(\cdot, C, f; \xi)$ and $G(\cdot, C, f; \xi)$ are differentiable at (C, f), there exists $\mu \geq 0$ such that

$$\begin{aligned} \frac{\partial U}{\partial C_L} - \mu\left(\frac{1}{\gamma} - \frac{\partial G}{\partial C_L}\right) &\leq 0 \; (= \text{ if } C > 0) \; \text{(A.1)} \\ \text{and} \quad \frac{\partial U}{\partial f_L} + \mu\frac{\partial G}{\partial f_L} &\leq 0 \; (= \text{ if } f > 0) \; \text{(A.2)} \end{aligned}$$

We can now show that $f > 0$. Suppose, to the contrary, that $f = 0$. Equation (A.2) implies that $\mu \leq \frac{-\partial U/\partial f_L}{\partial G/\partial f_L}$. Equation (A.1) implies that $\mu(\frac{1}{\gamma} - \frac{\partial G}{\partial C_L}) \geq \frac{\partial U}{\partial C_L}$. Since $\xi > 0$, $\frac{\partial U}{\partial C_L} > 0$ and hence

$\frac{1}{\gamma} - \frac{\partial G}{\partial C_L} > 0$. Thus, this equation implies that

$$\mu \geq \frac{\partial U/\partial C_L}{\frac{1}{\gamma} - \frac{\partial G}{\partial C_L}}.$$

It follows that

$$\frac{-\partial U/\partial f_L}{\partial G/\partial f_L} \geq \frac{\partial U/\partial C_L}{\frac{1}{\gamma} - \frac{\partial G}{\partial C_L}}$$

or, equivalently, $\Psi(C, 0, C, 0; \xi) \geq 0$. By Fact 1, this means that

$$C \geq \sqrt{\frac{\xi\alpha\gamma}{2\varepsilon}\{(b'(0) - 1)r + \beta b'(0) + (1 - \sigma)\delta b'(0)\}} - \alpha.$$

We know that

$$\begin{aligned}
\frac{C}{\gamma} &= G(C, 0, C, 0; \xi) \\
&= (\pi(C, 0; \xi) - \frac{1}{2})(\beta + (1 - \sigma)\delta) \\
&< \frac{1}{2}(\beta + (1 - \sigma)\delta),
\end{aligned}$$

so that $C < \widetilde{C}$, as defined in Assumption 2. In addition, since $C > 0$,

$$\xi \geq \frac{(1 - \sigma)\delta}{2\beta} = \xi'.$$

It follows that

$$\widetilde{C} > \sqrt{\frac{\xi'\alpha\gamma}{2\varepsilon}\{(b'(0) - 1)r + \beta b'(0) + (1 - \sigma)\delta b'(0)\}} - \alpha,$$

which, by Fact 1, is inconsistent with Assumption 2. Thus, f cannot equal 0.

It only remains to show that (C, f, ξ) satisfies (19). Since $f > 0$, equation (A.2) implies that $\mu = \frac{-\partial U/\partial f_L}{\partial G/\partial f_L}$. Moreover, it follows from (20) that $f > 0$ implies that $C > 0$. This means that equation (A.1) implies that $\mu = \frac{\partial U/\partial C_L}{\frac{1}{\gamma} - \frac{\partial G}{\partial C_L}}$. It follows that

$$\frac{\partial U/\partial C_L}{\frac{1}{\gamma} - \frac{\partial G}{\partial C_L}} = \frac{-\partial U/\partial f_L}{\partial G/\partial f_L},$$

which implies that $\Psi(C, f, C, f; \xi) = 0$. QED

Proof of Proposition 2: We prove the result via a sequence of three claims.

Claim 1: *Let (C, f, ξ) be an equilibrium, then $C < \overline{C}$ where*

$$\overline{C} = \frac{\gamma}{2}(\beta + \delta) + \gamma(b(\delta) - \delta)(1 - \frac{\sigma}{2})$$

Proof: As shown in the proof of Proposition 1 it must be the case that $\xi > 0$ and that (C, f, ξ) satisfies equation (20). Since $\xi > 0$, we know that $f < \delta$. It follows that $b(f) - f < b(\delta) - \delta$, because (by assumption) $b'(\delta) > 1$ and b is concave. Using this and (20), we have that

$$\begin{aligned} \frac{C}{\gamma} &= G(C, f, C, f; \xi) \\ &= (\pi(C, 0; \xi) - \frac{1}{2})(\beta + \sigma f + (1 - \sigma)\delta) \\ &\quad + (b(f) - f)(\frac{\sigma}{2} + (1 - \sigma)\pi(C, 0; \xi)) \\ &\leq \frac{1}{2}(\beta + \delta) + (b(\delta) - \delta)(1 - \frac{\sigma}{2}). \end{aligned}$$

Multiplying through by γ yields the result. ∎

Claim 2: $\lim_{r \to \infty} \xi(r) = 0$.

Proof: We need to show that for all $\tau > 0$, there exists r_τ such that if $r \geq r_\tau$ it is the case that $\xi(r) \leq \tau$. Let τ be given. Let r_τ be any value of r satisfying both Assumption 2 and the inequality

$$\overline{C} < \sqrt{\frac{\tau\alpha\gamma}{2\varepsilon}\{(b'(\delta) - 1)r_\tau + \beta b'(\delta)\}} - \alpha.$$

Clearly, such an r_τ exists. Now let $r \geq r_\tau$. By Proposition 1, we know that

$$\Psi(C(r), f(r), C(r), f(r); \xi(r)) = 0,$$

which, by Fact 1, implies that

$$C(r) \geq \sqrt{\frac{\xi(r)\alpha\gamma}{2\varepsilon}\{(b'(f(r)) - 1)r + \beta b'(f(r)) + b(f(r)) + (1 - \sigma)(\delta - f(r))b'(f(r))\}} - \alpha.$$

Suppose that $\xi(r) > \tau$. Then because $b'' < 0$,

$$\begin{aligned} C(r) &\geq \sqrt{\frac{\xi(r)\alpha\gamma}{2\varepsilon}\{(b'(f(r)) - 1)r + \beta b'(f(r)) + b(f(r)) + (1-\sigma)(\delta - f(r))b'(f(r))\}} - \alpha \\ &> \sqrt{\frac{\tau\alpha\gamma}{2\varepsilon}\{(b'(\delta) - 1)r + \beta b'(\delta)\}} - \alpha > \overline{C}, \end{aligned}$$

which, by Claim 1, is a contradiction. Thus, it must be the case that $\xi(r) \leq \tau$. ■

Claim 3: *There exists $\widehat{\xi} > 0$ such that for all $\xi \in (0, \widehat{\xi})$ the pair of equations* (17) *and* (20) *have a unique solution $(C^*(\xi), f^*(\xi))$ in the domain $\Re_+ \times [0, \delta]$. Moreover, the functions $C^*(\cdot)$ and $f^*(\cdot)$ are continuous on $(0, \widehat{\xi})$ and*

$$\lim_{\xi \searrow 0}(C^*(\xi), f^*(\xi)) = (\frac{\gamma(b(\delta) - \delta)}{2}, \delta)$$

Proof: This claim may be established graphically by computing the loci of (C, f) combinations satisfying equations (17) and (20) for given ξ. Consider first equation (17). Let $C_o(f;\xi)$ be the level of contributions that qualified candidates must receive to generate an effectiveness of advertising ξ when qualified candidates provide an amount of favors f. When it is defined, C_o satisfies

$$\xi = \frac{(1-\sigma)(\alpha + C_o)(\delta - f)}{2\beta(\alpha + C_o(1-\sigma))}.$$

Solving this for C_o, we obtain

$$C_o(f;\xi) = \frac{\alpha[2\beta\xi - (1-\sigma)(\delta - f)]}{(1-\sigma)[\delta - f - 2\beta\xi]}$$

Thus, for given ξ, $C_o(f;\xi)$ is well-defined for f values between $\max\{0, \delta - \frac{2\beta\xi}{1-\sigma}\}$, and $\delta - 2\beta\xi$. On this interval, $C_o(\cdot;\xi)$ is increasing at an increasing rate, approaching infinity as the level of favors approaches the upper limit of the interval.

Now consider equation (20). Let $C_i(f;\xi)$ be the level of contributions that would make interest groups indifferent between accepting candidates offers when the level of favors promised is f and the effectiveness of advertising is ξ. Formally, C_i is implicitly defined by the equality:

$$G(C_i, f, C_i, f; \xi) = \frac{C_i}{\gamma}.$$

Note that there may be two non-negative solutions to this equation when $f = 0$. One solution is always $C = 0$, since the gain from giving no contributions in exchange for no favors is obviously zero. But there will be a positive solution if $\partial G(0, 0, 0, 0; \xi)/\partial C > 0$. We will let $C_i(0; \xi)$ be the positive solution when it exists.

It is possible to explicitly solve for $C_i(f; \xi)$. We have that

$$C_i(f; \xi) = \frac{a(f, \xi) + \sqrt{a(f, \xi)^2 + 16e(f, \xi)}}{8},$$

where:

$$\begin{aligned} a(f, \xi) &= 2\gamma\{\frac{\xi}{\varepsilon}(\beta + \sigma f + (1 - \sigma)\delta) + (1 + \frac{\xi}{\varepsilon}(1 - \sigma))(b(f) - f)\} - 4\alpha \\ e(f, \xi) &= 2\gamma\alpha(b(f) - f). \end{aligned}$$

Note that C_i is increasing in f and bounded above on $[0, \delta]$. Since

$$C_i(f; 0) = \frac{\gamma(b(f) - f)}{2},$$

it follows that $C_i(\cdot; \xi)$ is strictly concave on $[0, \delta]$ for sufficiently small ξ.

Given ξ, $(C, f) \in \Re_+ \times [0, \delta]$ is a solution of the pair of equations (17) and (20) if and only if $f \in [\max\{0, \delta - \frac{2\beta\xi}{1-\sigma}\}, \delta - 2\beta\xi)$, $C = C_o(f, 0)$ and $C_i(f, \xi) = C_o(f, \xi)$. We know that $C_o(f, \xi)$ must become larger than $C_i(f, \xi)$ as f approaches $\delta - 2\beta\xi$. Thus, by continuity, there exists a solution if $C_o(f, \xi)$ is smaller than $C_i(f, \xi)$ at $f = \max\{0, \delta - \frac{2\beta\xi}{1-\sigma}\}$. Moreover, if $C_i(\cdot, \xi)$ is strictly concave, then this solution must be unique.

We now claim that for ξ sufficiently small, $C_o(f, \xi)$ is indeed smaller than $C_i(f, \xi)$ at $f = \max\{0, \delta - \frac{2\beta\xi}{1-\sigma}\}$. For ξ sufficiently small, we have that $\max\{0, \delta - \frac{2\beta\xi}{1-\sigma}\} = \delta - \frac{2\beta\xi}{1-\sigma}$. Since $C_i(\delta - \frac{2\beta\xi}{1-\sigma}; \xi)$ is positive and $C_o(\delta - \frac{2\beta\xi}{1-\sigma}; \xi) = 0$, the claim follows. In addition, as noted above, for ξ sufficiently small, $C_i(\cdot;\xi)$ is strictly concave on $[0,\delta]$. It therefore follows that for sufficiently small ξ the pair of equations (17) and (20) have a unique solution $(C^*(\xi), f^*(\xi))$ in the domain $\Re_+ \times[0,\delta]$. The situation is illustrated in Figure A.1. That these solutions are continuous in ξ follows from the Implicit Function Theorem. Further, we know that $\delta - \frac{2\beta\xi}{1-\sigma} < f^*(\xi) < \delta - 2\beta\xi$, so that $\lim_{\xi\searrow 0} f^*(\xi) = \delta$. Finally, since $C^*(\xi) = C_i(f^*(\xi);\xi)$,

$$\begin{aligned}\lim_{\xi\searrow 0} C^*(\xi) &= C_i(\lim_{\xi\searrow 0} f^*(\xi), \lim_{\xi\searrow 0}\xi) \\ &= \frac{\gamma(b(\delta)-\delta)}{2}. \blacksquare\end{aligned}$$

It follows from Claims 2 and 3 that $\lim_{r\to\infty} C(r) = \lim_{\xi\searrow 0} C^*(\xi)$ and $\lim_{r\to\infty} f(r) = \lim_{\xi\searrow 0} f^*(\xi)$. From the proof of Claim 3 we know that

$$\lim_{\xi\searrow 0} C^*(\xi) = \frac{\gamma(b(\delta)-\delta)}{2}$$

and that

$$\lim_{\xi\searrow 0} f^*(\xi) = \delta.$$

The result now follows. QED

Proof of Proposition 3: If (l, f, ξ) is an equilibrium then it satisfies (17) by definition. Thus, we need only establish that (l, f, ξ) satisfies (22) and (23). We know that (l, f) must solve the problem

$$\max_{(C_L, f_L)\in[0,l]\times\Re_+} U(C_L, f_L, l, f;\xi) \ s.t.\ G(C_L, f_L, l, f;\xi) \geq \frac{C_L}{\gamma}.$$

Observe first that it must be the case that $G(l, f, l, f;\xi) = \frac{l}{\gamma}$ if $f > 0$. If not, then the candidate could reduce the amount of favors he promises. This yields (23).

We can now establish (22). If $l = 0$ then $f = 0$ and (22) follows from Assumption 2. Thus, suppose that $l > 0$. If $f = 0$, then we know that $l < \widetilde{C}$. In addition, since $l > 0$,

$$\xi > \frac{(1-\sigma)\delta}{2\beta} = \xi'.$$

Thus, using Assumption 2,

$$\begin{aligned} L &< \widetilde{C} < \sqrt{\frac{\xi'\alpha\gamma}{2\varepsilon}\{(b'(0)-1)r + \beta b'(0) + (1-\sigma)\delta b'(0)\}} - \alpha \\ &< \sqrt{\frac{\xi\alpha\gamma}{2\varepsilon}\{(b'(0)-1)r + \beta b'(0) + (1-\sigma)\delta b'(0)\}} - \alpha. \end{aligned}$$

This inequality implies (22).

If $f > 0$, then, since $U(\cdot, l, f; \xi)$ and $G(\cdot, l, f; \xi)$ are differentiable at (l, f), there exists $\mu \geq 0$ such that

$$\begin{aligned} \frac{\partial U}{\partial C_L} - \mu(\frac{1}{\gamma} - \frac{\partial G}{\partial C_L}) &\geq 0 \text{ (A.3)} \\ \textit{and} \quad \frac{\partial U}{\partial f_L} + \mu\frac{\partial G}{\partial f_L} &= 0 \text{ (A.4)} \end{aligned}$$

Equation (A.4) implies that $\mu = \frac{-\partial U/\partial f_L}{\partial G/\partial f_L}$. It follows from equation (A.3) that $\mu \leq \frac{\partial U/\partial C_L}{\frac{1}{\gamma} - \frac{\partial G}{\partial C_L}}$.Thus,

$$\frac{-\partial U/\partial f_L}{\partial G/\partial f_L} \leq \frac{\partial U/\partial C_L}{\frac{1}{\gamma} - \frac{\partial G}{\partial C_L}}.$$

Multiplying this expression through by $\frac{\partial G/\partial f_L}{\partial U/\partial C_L}$ yields (22). QED

Proof of Lemma 1: We begin by calculating the equilibrium payoffs of the various types of citizens. Assuming a symmetric equilibrium, we can divide the population into just three types: partisans (i.e., leftists and rightists), interest group members, and swing-voters. We deal with each in turn.

Consider a representative partisan. Given symmetry, the elected candidate is equally likely to be from either party. The expected payoff of the partisan is therefore $\delta - f - \beta/2$ if the elected candidate is qualified and $-\beta/2$ if not. Recall that both parties select a qualified candidate with probability σ^2 while only one party selects a qualified candidate with probability $2\sigma(1-\sigma)$. In the latter case, the qualified candidate wins with probability $\pi(C, 0; \xi)$ and hence the probability that a qualified candidate is elected is $\sigma^2 + 2\sigma(1-\sigma)\pi(C, 0; \xi)$. The expected payoff of the partisan is therefore

$$[\sigma^2 + 2\sigma(1-\sigma)\pi](\delta - f) - \frac{\beta}{2}. \qquad (A.5)$$

Interest group members provide campaign contributions to qualified candidates and also get policy favors enacted when their candidate wins. The expected payoff of a representative interest group member is therefore

$$[\sigma^2 + 2\sigma(1-\sigma)\pi](\delta - f + \frac{b(f)}{2}) - \frac{\beta}{2} - \frac{\sigma C}{\gamma}. \quad (A.6)$$

The fact that $b(f)$ is divided by two reflects the fact that the interest group only gets its favors implemented if the qualified candidate it is backing is elected.

The payoffs of swing voters are more complicated to compute because of the correlation between which party's candidate wins and the location of the median swing voter. Suppose first that both parties select unqualified candidates. Party L's candidate will win if the ideology of the median swing voter is less than $\mu^*(0, 0) = 1/2$. Thus, if μ is less than 1/2 then a swing voter with ideology i obtains a payoff $-\beta i$. The average swing voter's payoff is therefore $-\beta\mu$. If μ exceeds 1/2 then Party R's candidate wins and a swing voter with ideology i obtains a payoff $-\beta(1-i)$. In this case the average swing voter's payoff is $-\beta(1-\mu)$. Taking expectations over the realization of μ, the representative swing voter's expected payoff is

$$-\int_{\frac{1}{2}-\varepsilon}^{\frac{1}{2}} \beta\mu \frac{d\mu}{2\varepsilon} - \int_{\frac{1}{2}}^{\frac{1}{2}+\varepsilon} \beta(1-\mu)\frac{d\mu}{2\varepsilon} = -\beta(\frac{1-\varepsilon}{2})$$

The key point is that states in which the median swing-voter is left-leaning are states in which Party L's candidate will win.

Suppose now that both parties select qualified candidates. Party L's candidate will win if the ideology of the median swing voter is less than $\mu^*(C, C) = 1/2$. If μ is less than 1/2 then the majority of swing voters vote for Party L's candidate and a swing voter with ideology i obtains a payoff $\delta - f - \beta i$. The average swing voter's payoff is therefore $\delta - f - \beta\mu$. If μ exceeds 1/2 then Party R's candidate wins and a swing voter with ideology i obtains a payoff $\delta - f - \beta(1-i)$. In this case the average swing voter's payoff is $\delta - f - \beta(1-\mu)$ The representative swing voter's expected payoff is therefore

$$\delta - f - \int_{\frac{1}{2}-\varepsilon}^{\frac{1}{2}} \beta\mu \frac{d\mu}{2\varepsilon} - \int_{\frac{1}{2}}^{\frac{1}{2}+\varepsilon} \beta(1-\mu)\frac{d\mu}{2\varepsilon} = \delta - f - \beta(\frac{1-\varepsilon}{2})$$

Next consider the case in which just one party's candidate is qualified. For concreteness, assume that it is Party L's candidate. Party L's candidate will win if the ideology of the median swing voter is less than $\mu^*(C,0) = 1/2 + \xi\lambda(C)$. If μ is less than $1/2 + \xi\lambda(C)$ then the majority of swing voters vote for Party L's candidate and a swing voter with ideology i obtains a payoff $\delta - f - \beta i$. The average swing voter's payoff is therefore $\delta - f - \beta\mu$. If μ exceeds $1/2 + \xi\lambda(C)$ then Party R's candidate wins and a swing voter with ideology i obtains a payoff $-\beta(1-i)$. In this case the average swing voter's payoff is $-\beta(1-\mu)$. Taking expectations over the realization of μ and using the fact that $\xi\lambda(C) = \varepsilon(2\pi(C,0;\xi)-1)$ the representative swing voter's expected payoff is

$$\int_{\frac{1}{2}-\varepsilon}^{\frac{1}{2}+\varepsilon(2\pi-1)} [\delta - f - \beta\mu]\frac{d\mu}{2\varepsilon} - \int_{\frac{1}{2}+\varepsilon(2\pi-1)}^{\frac{1}{2}+\varepsilon} \beta(1-\mu)\frac{d\mu}{2\varepsilon} = (\delta - f)\pi - \beta(\frac{1}{2} - 2\varepsilon\pi(1-\pi)).$$

Aggregating over these three situations, the expected payoff of a swing voter is

$$[\sigma^2 + 2\sigma(1-\sigma)\pi](\delta - f) - \beta(\frac{1}{2} - \varepsilon\{\frac{\sigma^2 + (1-\sigma)^2}{2} + 2\sigma(1-\sigma)2\pi(1-\pi)\}). \quad (A.7)$$

We can now prove the Lemma. That partisans and swing voters will be strictly better off follows directly from (A.5) and (A.7). Thus, we need only deal with interest group members. Since (ii) implies that the status quo level of favors f^* is positive, we know that (20) holds at the status quo equilibrium. We can use this to express the status quo expected payoff of an interest group member as:

$$\delta[\sigma^2 + \frac{\sigma(1-\sigma)}{2} + (1-\sigma)\sigma\pi^*] - f^*\sigma\pi^* - \frac{\beta}{2}[1-\sigma+2\sigma\pi^*]. \ (A.8)$$

With the limit, it is clear that (23) holds and hence the expected payoff of an interest group member is at least

$$\delta[\sigma^2 + \frac{\sigma(1-\sigma)}{2} + (1-\sigma)\sigma\pi] - f\sigma\pi - \frac{\beta}{2}[1-\sigma+2\sigma\pi]. \ (A.9)$$

Since $\pi \simeq \pi^*$ and $f < f^*$, it is clear that $(A.9)$ exceeds $(A.8)$ and hence interest group members will be better off. QED

Proof of Proposition 4: For all r, let $(C(r), f(r), \xi(r))$ be the equilibrium (or an equilibrium) that would arise with no limits when ego-rents are r. Then, from Proposition 2 we know that $\lim_{r\to\infty} \pi(C(r), 0; \xi(r)) = 1/2$ and that $\lim_{r\to\infty} f(r) = \delta > 0$. Since banning contributions would eliminate favors and make the probability that a qualified candidate defeats an unqualified one equal 1/2, the result follows from Lemma 1. QED

8 Appendix B: Technical Notes

Convexity of the Constraint Set for Problem P: For all $f_L \in [0, \delta]$ let $\widetilde{C}_L(f_L; C, f; \xi)$ be the solution to $G(C_L, f_L, C, f; \xi) = \frac{C_L}{\gamma}$ (when $f_L = 0$ there might be two such solutions - let $\widetilde{C}_L$ be the largest). The curve $\widetilde{C}_L(\cdot; C, f; \xi)$ describes the boundary of the constraint set for *Problem P*. The constraint set will be convex if this curve is concave. It is possible to solve explicitly for this curve, so we may see when it will be concave.

Using (11) we have that

$$G = \frac{\xi}{2\varepsilon}\lambda(C_L)[\beta + \sigma f + (1-\sigma)\delta + b(f_L) - f_L] + (b(f_L) - f_L)[\frac{1}{2} - \frac{\xi}{2\varepsilon}\sigma\lambda(C)]$$

Thus, using the functional form for λ,

$$\frac{\xi}{2\varepsilon}\frac{\widetilde{C}_L}{(\widetilde{C}_L + \alpha)}[\beta + \sigma f + (1-\sigma)\delta + b(f_L) - f_L] + (b(f_L) - f_L)[\frac{1}{2} - \frac{\xi}{2\varepsilon}\sigma\lambda(C)] = \frac{\widetilde{C}_L}{\gamma}.$$

This implies that

$$4\widetilde{C}_L^2 - a(f_L, f, C, \xi)\widetilde{C}_L - e(f_L, f, C, \xi) = 0$$

where

$$\begin{aligned} a(f_L; f, C, \xi) &= 2\gamma\{\frac{\xi}{\varepsilon}(\beta + \sigma f + (1-\sigma)\delta + b(f_L) - f_L) + (b(f_L) - f_L)(1 - \frac{\xi}{\varepsilon}\sigma\lambda(C))\} - 4\alpha. \\ e(f_L; f, C, \xi) &= 2\gamma\alpha(b(f_L) - f_L)(1 - \frac{\xi}{\varepsilon}\sigma\lambda(C)). \end{aligned}$$

Thus,

$$\widetilde{C}_L(f_L; f, C, \xi) = \frac{a(f_L; f, C, \xi) + \sqrt{a(f_L; f, C, \xi)^2 + 16e(f_L; f, C, \xi)}}{8}.$$

Note first that when $\xi = 0$, we have that

$$\widetilde{C}_L(f_L; f, C, 0) = \frac{\gamma(b(f_L) - f_L)}{2},$$

which is strictly concave. Thus the constraint set will be concave for sufficiently small ξ. More generally, we have that:

$$\frac{\partial\widetilde{C}_L}{\partial f_L} = \frac{\frac{\partial a}{\partial f_L} + \frac{1}{2}(a^2 + 16e)^{-\frac{1}{2}}(2a\frac{\partial a}{\partial f_L} + 16\frac{\partial e}{\partial f_L})}{8}$$

and that

$$\frac{\partial^2 \widetilde{C}_L}{\partial f_L^2} = \frac{\frac{\partial^2 a}{\partial f_L^2} - \frac{1}{4}(a^2+16e)^{-\frac{3}{2}}(2a\frac{\partial a}{\partial f_L} + 16\frac{\partial e}{\partial f_L})^2 + \frac{1}{2}(a^2+16e)^{-\frac{1}{2}}(2(\frac{\partial a}{\partial f_L})^2 + 2a\frac{\partial^2 a}{\partial f_L^2} + 16\frac{\partial^2 e}{\partial f_L^2})}{8}.$$

Note that

$$\frac{\partial a}{\partial f_L} = 2\gamma(\frac{\xi}{\varepsilon} + (1 - \frac{\xi}{\varepsilon}\sigma\lambda(C)))(b'(f_L) - 1) > 0$$

$$\frac{\partial^2 a}{\partial f_L^2} = 2\gamma(\frac{\xi}{\varepsilon} + (1 - \frac{\xi}{\varepsilon}\sigma\lambda(C)))b''(f_L) < 0$$

$$\frac{\partial e}{\partial f_L} = 2\gamma\alpha(b'(f_L) - 1)(1 - \frac{\xi}{\varepsilon}\sigma\lambda(C)) > 0$$

and

$$\frac{\partial^2 e}{\partial f_L^2} = 2\gamma\alpha b''(f_L)(1 - \frac{\xi}{\varepsilon}\sigma\lambda(C)) < 0.$$

Thus, the first two terms in the expression for $\frac{\partial^2 \widetilde{C}_L}{\partial f_L^2}$ are negative. A sufficient condition for $\frac{\partial^2 C_i}{\partial f_L^2} < 0$ is therefore that the third term be negative, which requires that

$$(2(\frac{\partial a}{\partial f_L})^2 + 2a\frac{\partial^2 a}{\partial f_L^2} + 16\frac{\partial^2 e}{\partial f_L^2}) < 0.$$

While this is not a particularly tractable condition, it is clear that it will necessarily be satisfied for α sufficiently large.

Proof of Fact 1: By definition, $\Psi(C, f, C, f; \xi) \geq 0$ if and only if

$$\frac{-\partial U/\partial f_L}{\partial U/\partial C_L} \geq \frac{\partial G/\partial f_L}{\frac{1}{\gamma} - \partial G/\partial C_L}. \qquad (B.1)$$

From (12) we have that

$$\frac{\partial U(C, f, C, f; \xi)}{\partial f_L} = -\sigma\frac{1}{2} - (1-\sigma)\pi(C, 0; \xi)$$

and that

$$\frac{\partial U(C, f, C, f; \xi)}{\partial C_L} = \sigma\frac{\partial \pi(C, C; \xi)}{\partial C_L}(r+\beta) + (1-\sigma)\frac{\partial \pi(C, 0; \xi)}{\partial C_L}(r+\beta+\delta-f).$$

Thus,

$$\frac{-\partial U/\partial f_L}{\partial U/\partial C_L} = \frac{\sigma\frac{1}{2} + (1-\sigma)\pi(C,0;\xi)}{\sigma\frac{\partial\pi(C,C;\xi)}{\partial C_L}(r+\beta) + (1-\sigma)\frac{\partial\pi(C,0;\xi)}{\partial C_L}(r+\beta+\delta-f)}.$$

From (13) we have that

$$\frac{\partial G(C,f,C,f;\xi)}{\partial f_L} = (b'(f)-1)(\sigma\frac{1}{2} + (1-\sigma)\pi(C,0;\xi))$$

and that

$$\frac{\partial G(C,f,C,f;\xi)}{\partial C_L} = \sigma\frac{\partial\pi(C,C;\xi)}{\partial C_L}(\beta+b(f)) + (1-\sigma)\frac{\partial\pi(C,0;\xi)}{\partial C_L}(\beta+\delta+b(f)-f).$$

Thus,

$$\frac{\partial G/\partial f_L}{\frac{1}{\gamma} - \partial G/\partial C_L} = \frac{(b'(f)-1)(\sigma\frac{1}{2} + (1-\sigma)\pi(C,0;\xi))}{\frac{1}{\gamma} - \sigma\frac{\partial\pi(C,C;\xi)}{\partial C_L}(\beta+b(f)) + (1-\sigma)\frac{\partial\pi(C,0;\xi)}{\partial C_L}(\beta+\delta+b(f)-f)}.$$

It follows that $(B.1)$ holds if and only if

$$\begin{aligned} & \frac{1}{\sigma\frac{\partial\pi(C,C;\xi)}{\partial C_L}(r+\beta) + (1-\sigma)\frac{\partial\pi(C,0;\xi)}{\partial C_L}(r+\beta+\delta-f)} \\ \geq \quad & \frac{(b'(f)-1)}{\frac{1}{\gamma} - \sigma\frac{\partial\pi(C,C;\xi)}{\partial C_L}(\beta+b(f)) + (1-\sigma)\frac{\partial\pi(C,0;\xi)}{\partial C_L}(\beta+\delta+b(f)-f)}. \end{aligned}$$

Since $f \leq \delta$ and $G(C,f,C,f;\xi) \leq \frac{C}{\gamma}$ both the numerator and denominator of the right hand side are positive. Thus, $(B.1)$ holds if and only if

$$\begin{aligned} & \frac{1}{\gamma} - \sigma\frac{\partial\pi(C,C;\xi)}{\partial C_L}(\beta+b(f)) + (1-\sigma)(\frac{\partial\pi(C,0;\xi)}{\partial C_L})(\beta+\delta+b(f)-f) \\ \geq \quad & \{\sigma\frac{\partial\pi(C,C;\xi)}{\partial C_L}(r+\beta) + (1-\sigma)\frac{\partial\pi(C,0;\xi)}{\partial C_L}(r+\beta+\delta-f)\}(b'(f)-1) \end{aligned}$$

which is equivalent to

$$\begin{aligned} & \frac{1}{\gamma} \geq \sigma\frac{\partial\pi(C,C;\xi)}{\partial C_L}\{b'(f)(r+\beta) - r + b(f)\} \\ & +(1-\sigma)\frac{\partial\pi(C,0;\xi)}{\partial C_L}\{b'(f)(r+\beta+\delta-f) - r + b(f)\} \end{aligned}$$

Using (11), this can be rewritten as

$$\frac{1}{\gamma} \geq \frac{\xi\lambda'(C)}{2\varepsilon}\{b'(f)(r+\beta+(1-\sigma)(\delta-f)) - r + b(f)\}.$$

Using the functional form for $\lambda(C)$, this becomes

$$\frac{1}{\gamma} \geq \frac{\xi\alpha}{2\varepsilon(C+\alpha)^2}\{b'(f)(r+\beta+(1-\sigma)(\delta-f))-r+b(f)\},$$

which is equivalent to

$$C \geq \sqrt{\frac{\xi\alpha\gamma}{2\varepsilon}\{b'(f)(r+\beta+(1-\sigma)(\delta-f))-r+b(f)\}}-\alpha.$$

QED

Implications of Assumption 3: Part (i): From the proof of Proposition 2, we have that

$$C_i(f;\xi) = \frac{a(f,\xi)+(a(f,\xi)^2+16e(f,\xi))^{1/2}}{8},$$

where:

$$\begin{aligned}
a(f,\xi) &= 2\gamma\{\frac{\xi}{\varepsilon}(\beta+\sigma f+(1-\sigma)\delta)+(1+\frac{\xi}{\varepsilon}(1-\sigma))(b(f)-f)\}-4\alpha \\
e(f,\xi) &= 2\gamma\alpha(b(f)-f).
\end{aligned}$$

Thus, we have

$$\frac{\partial C_i(f;\xi)}{\partial f} = \frac{\frac{\partial a}{\partial f}+\frac{1}{2}(a^2+16e)^{-\frac{1}{2}}(2a\frac{\partial a}{\partial f}+16\frac{\partial e}{\partial f})}{8}$$

and that

$$\frac{\partial^2 C_i(f;\xi)}{\partial f^2} = \frac{\frac{\partial^2 a}{\partial f^2}-\frac{1}{4}(a^2+16e)^{-\frac{3}{2}}(2a\frac{\partial a}{\partial f}+16\frac{\partial e}{\partial f})^2+\frac{1}{2}(a^2+16e)^{-\frac{1}{2}}(2(\frac{\partial a}{\partial f})^2+2a\frac{\partial^2 a}{\partial f^2}+16\frac{\partial^2 e}{\partial f^2})}{8}.$$

Note that

$$\frac{\partial a(f,\xi)}{\partial f} = 2\gamma\{\frac{\xi}{\varepsilon}\sigma+(1+\frac{\xi}{\varepsilon}(1-\sigma))(b'(f)-1)\},$$

$$\frac{\partial^2 a(f,\xi)}{\partial f^2} = 2\gamma\frac{\xi}{\varepsilon}(1-\sigma)b''(f),$$

$$\frac{\partial e(f,\xi)}{\partial f} = 2\gamma\alpha(b'(f)-1),$$

and

$$\frac{\partial^2 e(f,\xi)}{\partial f^2} = 2\gamma\alpha b''(f).$$

Thus, the first two terms in the expression for $\frac{\partial^2 C_i}{\partial f^2}$ are negative. A sufficient condition for the third term to be negative is that

$$2(\frac{\partial a}{\partial f})^2 + 2a\frac{\partial^2 a}{\partial f^2} + 16\frac{\partial^2 e}{\partial f^2} < 0.$$

This will be satisfied for α sufficiently large.

Part (ii): From the proof of Proposition 2 we have that:

$$C_o(f;\xi) = \frac{\alpha[2\beta\xi - (1-\sigma)(\delta - f)]}{(1-\sigma)[\delta - f - 2\beta\xi]}$$

and hence

$$\frac{\partial C_o(f;\xi)}{\partial \xi} = \frac{\sigma 2\beta\alpha(\delta - f)}{(1-\sigma)[\delta - f - 2\beta\xi]^2}.$$

From the discussion of part (i) and the fact that $\partial e/\partial\xi = 0$, we have that:

$$\frac{\partial C_i(f;\xi)}{\partial \xi} = \frac{\frac{\partial a}{\partial \xi}\{1 + a(a^2 + 16e)^{-\frac{1}{2}}\}}{8}$$

where

$$\frac{\partial a(f,\xi)}{\partial \xi} = 2\gamma\{\frac{1}{\varepsilon}[\beta + \sigma f + (1-\sigma)\delta + (1-\sigma)(b(f) - f)]\}.$$

Thus, part (ii) will be satisfied if for all $f \in (\max\{0, \delta - \frac{2\beta\xi}{1-\sigma}\}, \delta - 2\beta\xi)$

$$\sigma 16\beta\alpha(\delta - f) > \frac{\partial a}{\partial \xi}\{1 + a(a^2 + 16e)^{-\frac{1}{2}}\}(1-\sigma)[\delta - f - 2\beta\xi]^2.$$

This will be satisfied for sufficiently large α.

Implications of Assumption 4: Using the expression for $C_i(f;\xi)$ derived in Proposition 2, we know that $C_i(0;\xi') = 0$ if and only if $a(0,\xi') \leq 0$ which implies that

$$2\gamma\frac{\xi'}{\varepsilon}(\beta + (1-\sigma)\delta) \leq 4\alpha.$$

Moreover, since $a(0,\xi') \leq 0$, we have that

$$\begin{aligned}\frac{\partial C_i(0;\xi')}{\partial f} &= \frac{\frac{\partial a}{\partial f} + (a\frac{\partial a}{\partial f} + 8(1-\gamma)\theta\alpha(b'(0)-1))/(-a)}{8} \\ &= \frac{2\gamma\alpha(b'(0)-1)}{-a(0,\xi')}.\end{aligned}$$

Using the expression for $C_o(f;\xi)$ derived in Proposition 2, we have that

$$\frac{\partial C_o(0;\xi')}{\partial f} = \frac{\sigma 2\beta\alpha\xi'}{(1-\sigma)[\delta - 2\beta\xi']^2}.$$

Thus, part (ii) of Assumption 4 requires that

$$\frac{2\gamma\alpha(b'(0)-1)}{-a(0,\xi')} < \frac{\sigma 2\beta\alpha\xi'}{(1-\sigma)[\delta - 2\beta\xi']^2}$$

or, equivalently, that

$$\sigma\delta 2\gamma(b'(0)-1) < \alpha[4\alpha - 2\gamma\{\frac{\xi'}{\varepsilon}(\beta + (1-\sigma)\delta)\}].$$

This must hold for sufficiently large α.

Implications of Assumption 5: From the discussion of Assumption 4, we know that $C_i(0;\xi') > 0$ if and only if $a(0,\xi') > 0$ which implies that

$$2\gamma\frac{\xi'}{\varepsilon}(\beta + (1-\sigma)\delta) > 4\alpha.$$

For part (ii) note first that $\underline{\xi}$ is well-defined given Assumption 3. On the interval $[\xi', \frac{\delta}{2\beta}]$ define the function $\phi(\xi) = C_i(0;\xi) - C_o(0;\xi)$. By Assumption 3 (ii) this function is decreasing. Moreover, $\phi(\xi) < 0$ for ξ sufficiently close to $\frac{\delta}{2\beta}$ and, by hypothesis, $\phi(\xi') > 0$. Thus, there is a unique $\underline{\xi}$ such that $\phi(\underline{\xi}) = 0$. Since $a(0,\underline{\xi}) > 0$, we have that:

$$\begin{aligned}\frac{\partial C_i(0;\underline{\xi})}{\partial f} &= \frac{\frac{\partial a}{\partial f} + (a\frac{\partial a}{\partial f} + 16\gamma\alpha(b'(0)-1))/a}{8} \\ &= \frac{\frac{\partial a}{\partial f}}{4} + \frac{2\gamma\alpha(b'(0)-1))}{a},\end{aligned}$$

where

$$a = 2\gamma\frac{\underline{\xi}}{\varepsilon}(\beta + (1-\sigma)\delta) - 4\alpha$$

and

$$\frac{\partial a}{\partial f} = 2\gamma\{\frac{\underline{\xi}}{\varepsilon}\sigma + (1 + \frac{\underline{\xi}}{\varepsilon}(1-\sigma))(b'(0) - 1)\},$$

Part (ii) will be satisfied when this derivative exceeds

$$\frac{\partial C_o(0;\underline{\xi})}{\partial f} = \frac{\sigma 2\beta\alpha\underline{\xi}}{(1-\sigma)[\delta - 2\beta\underline{\xi}]^2}.$$

This will be the case when $b'(0)$ is large enough or α small enough.

Figure 1

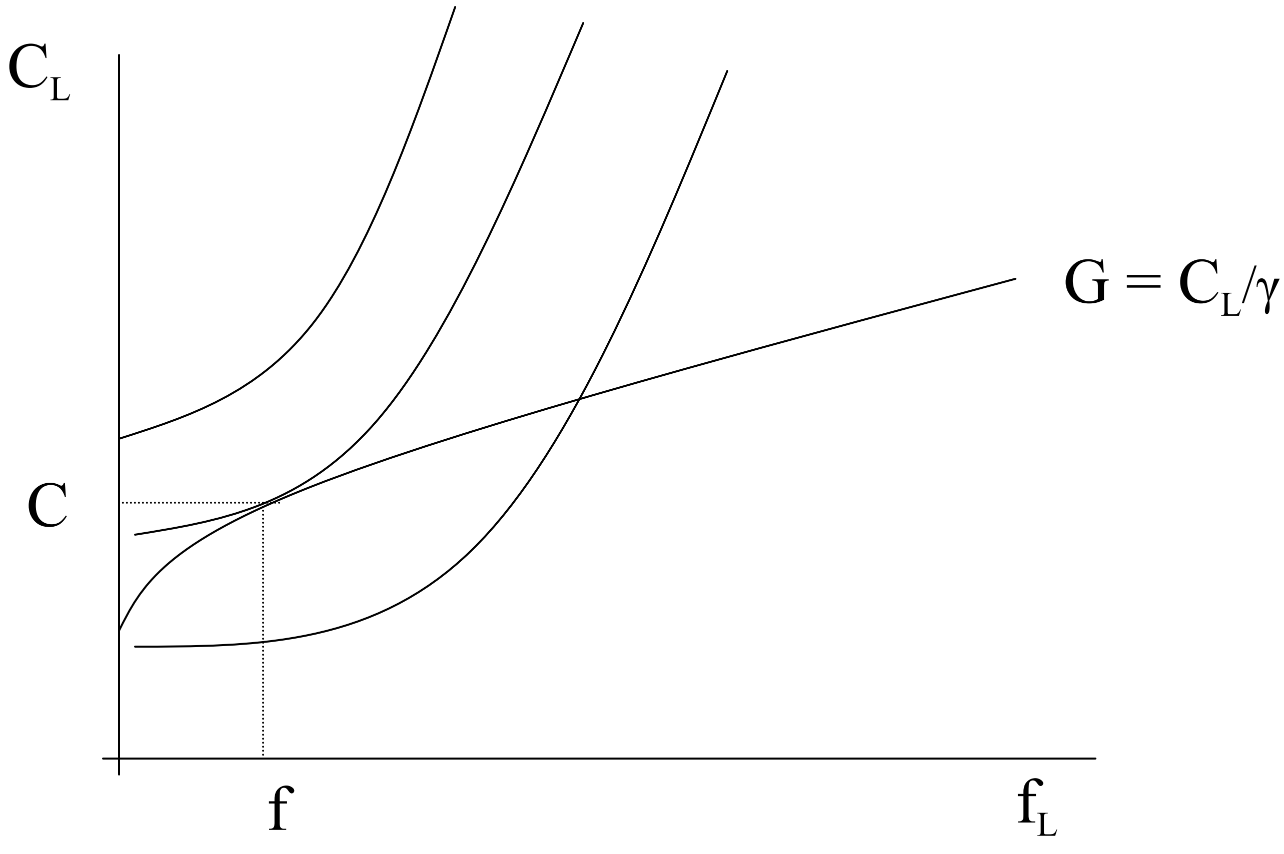


Figure 2

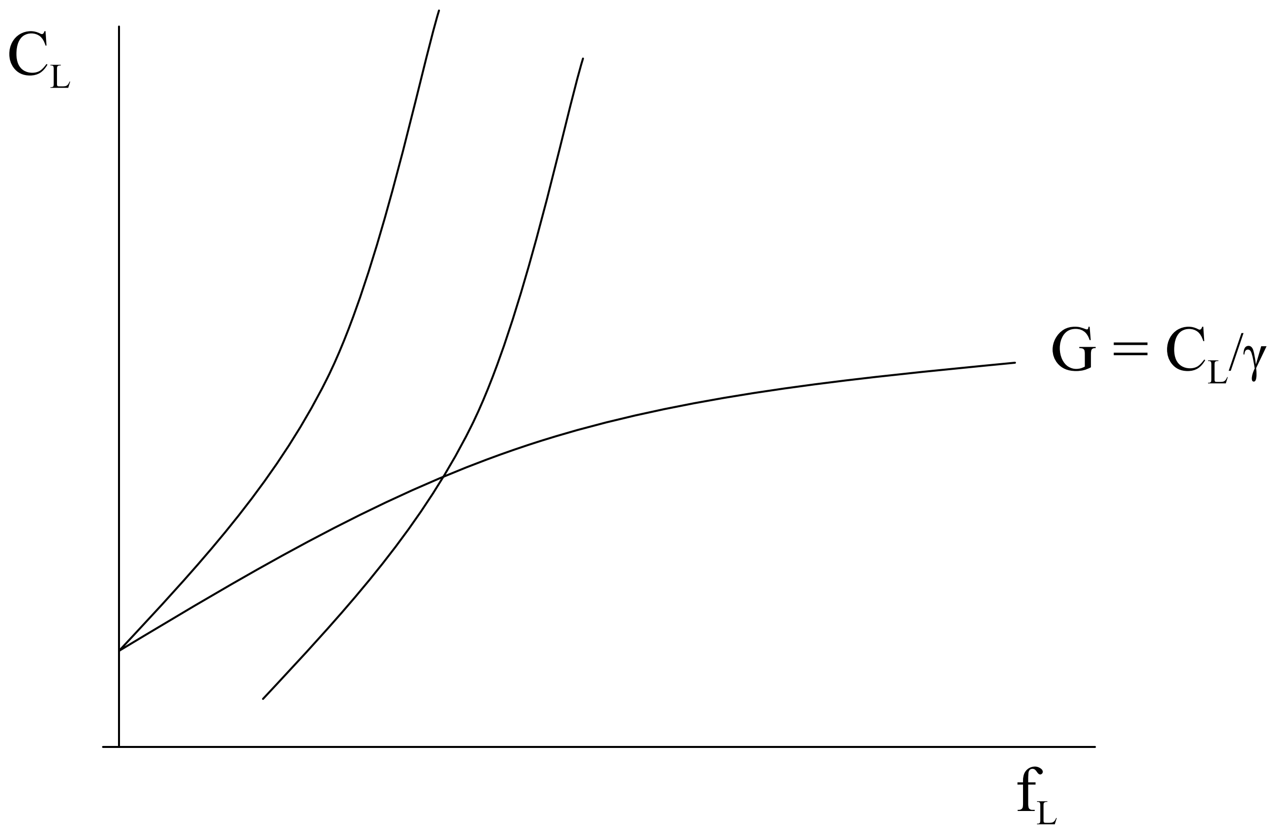


Figure 3

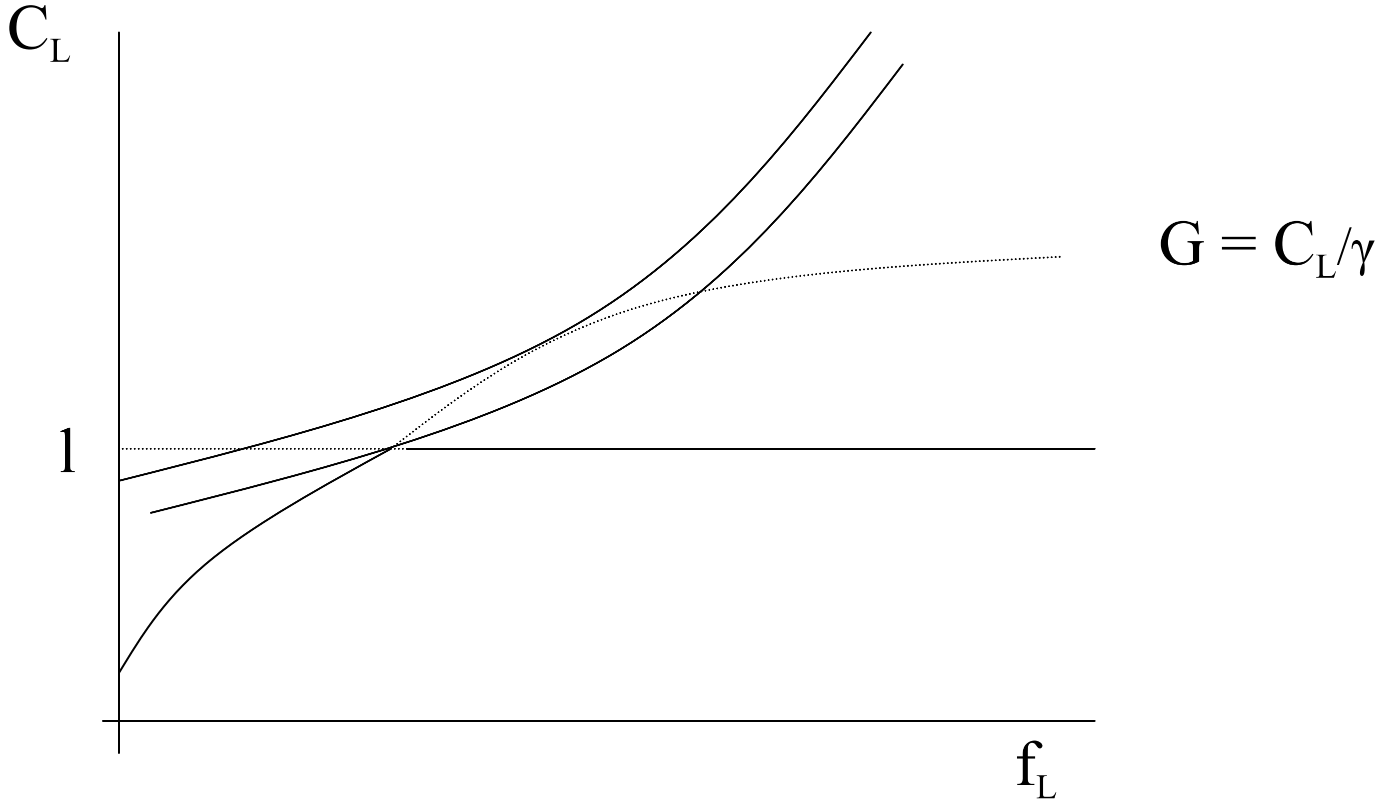
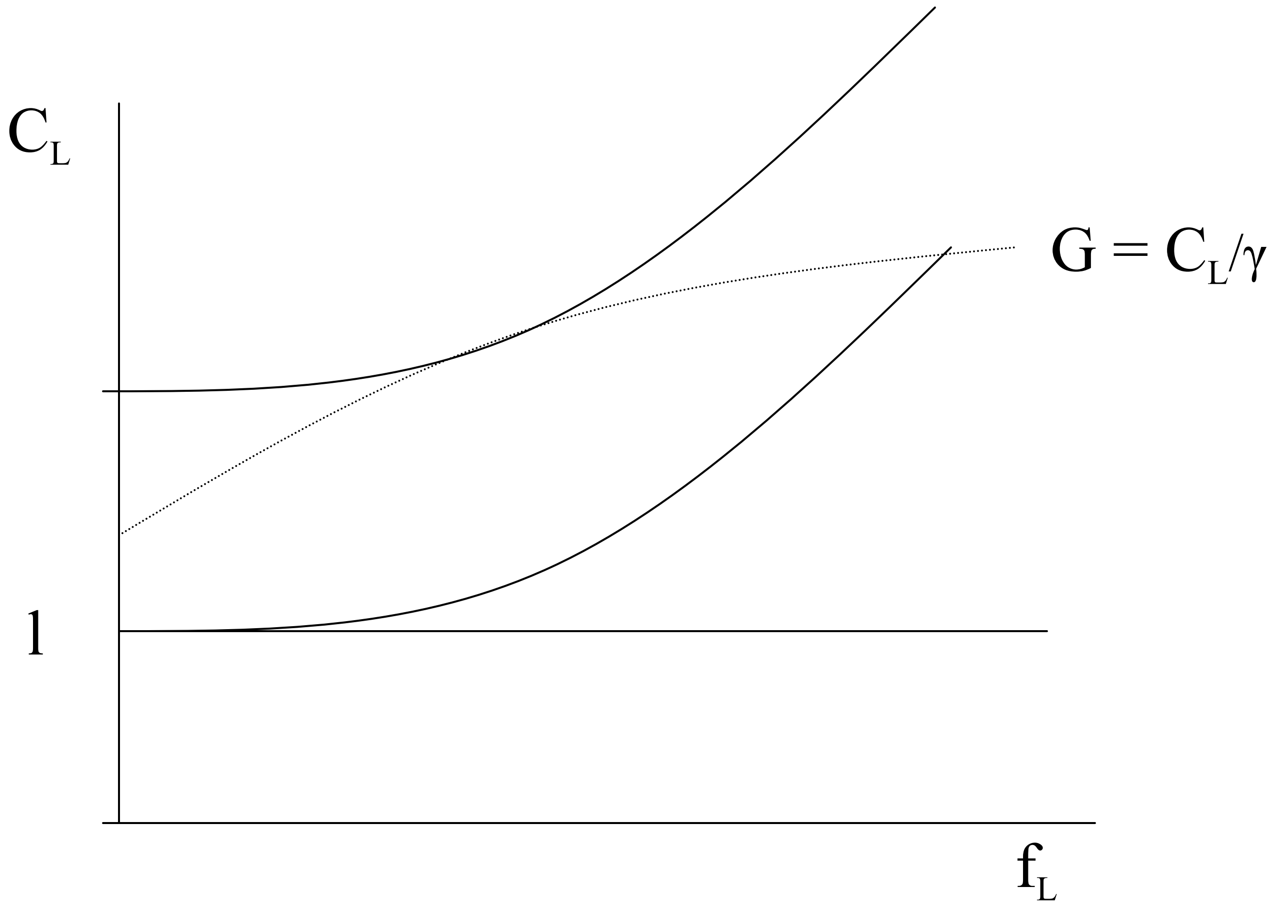


Figure 4



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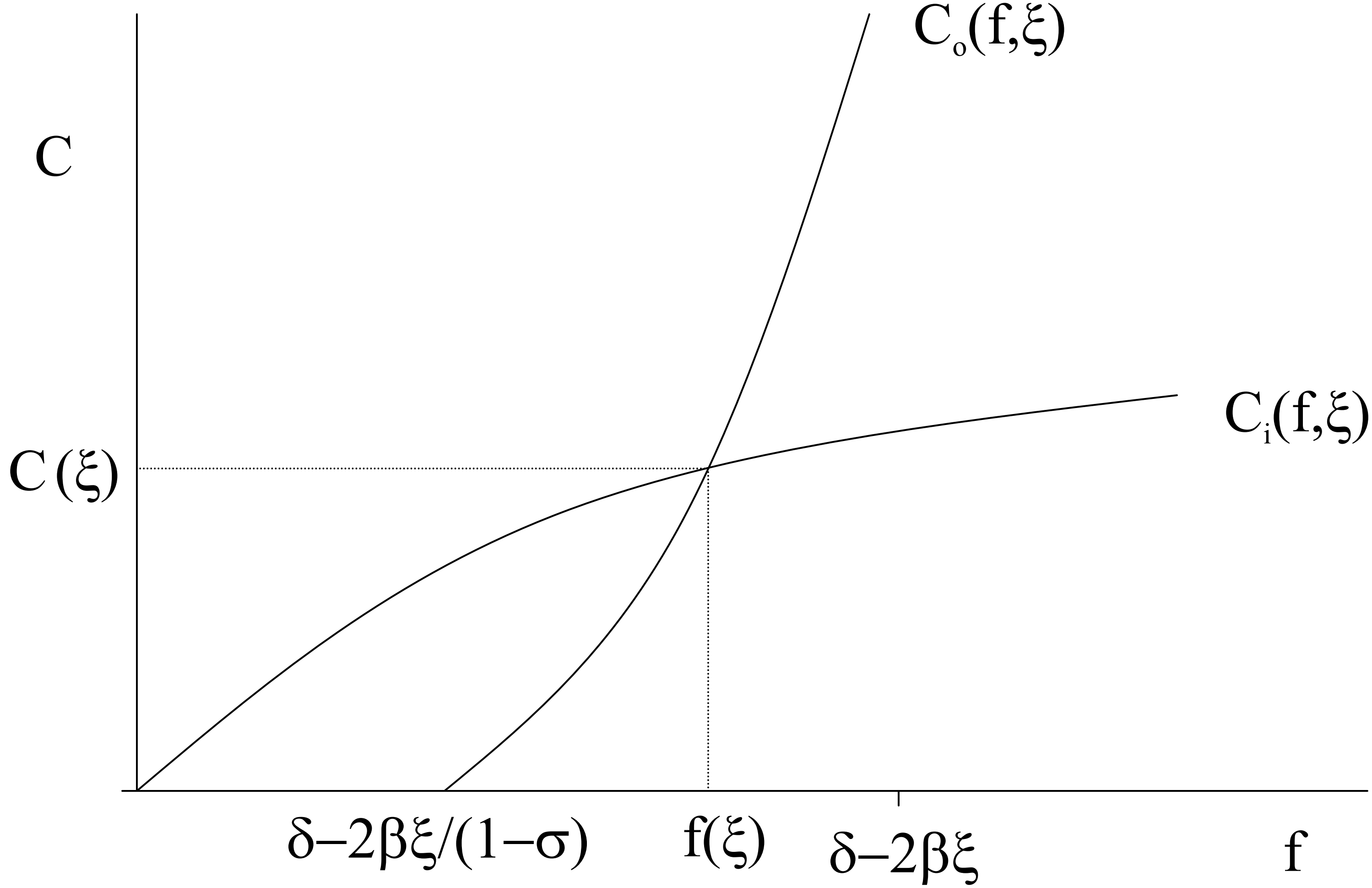


Figure 6

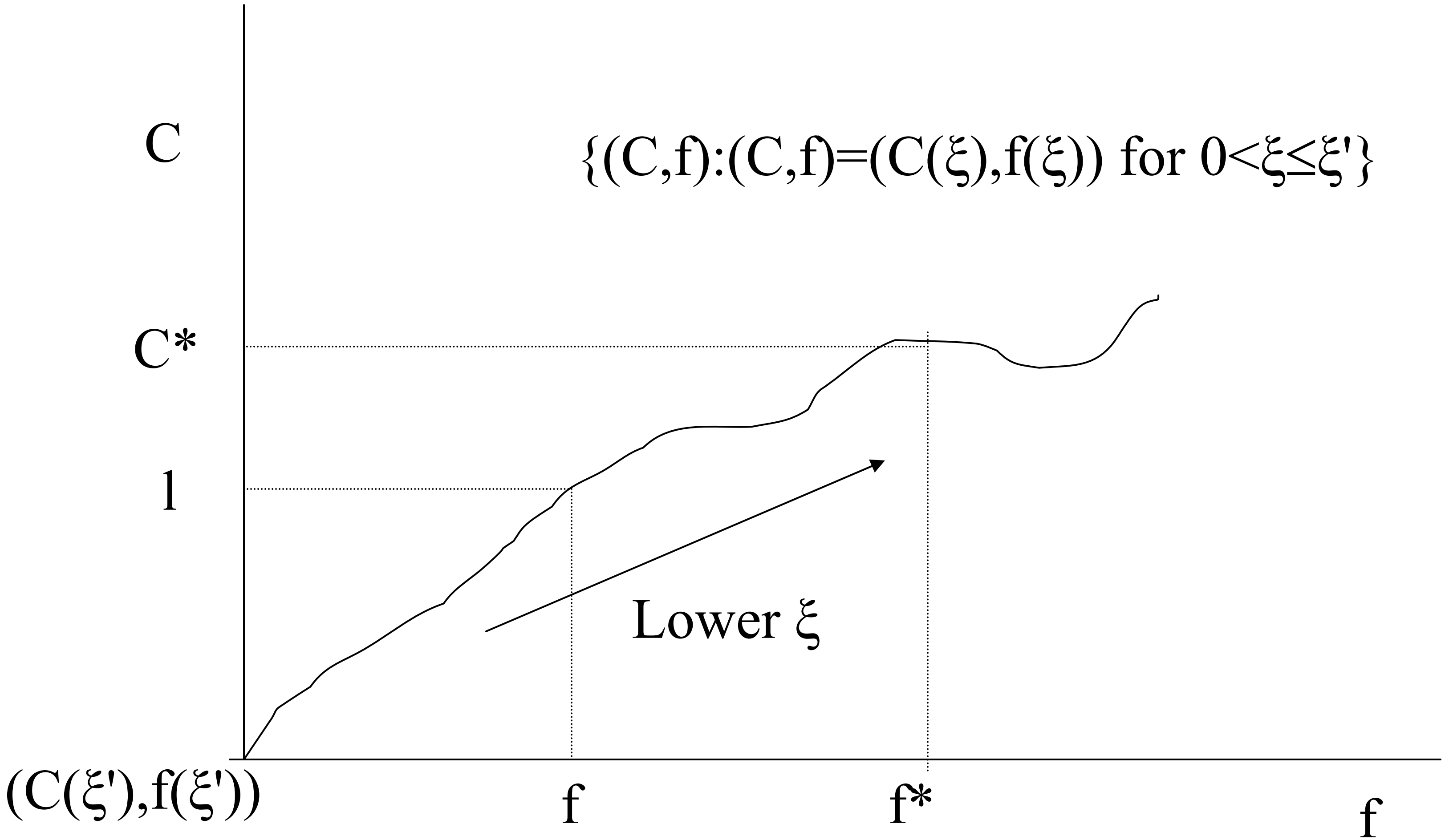


Figure 7

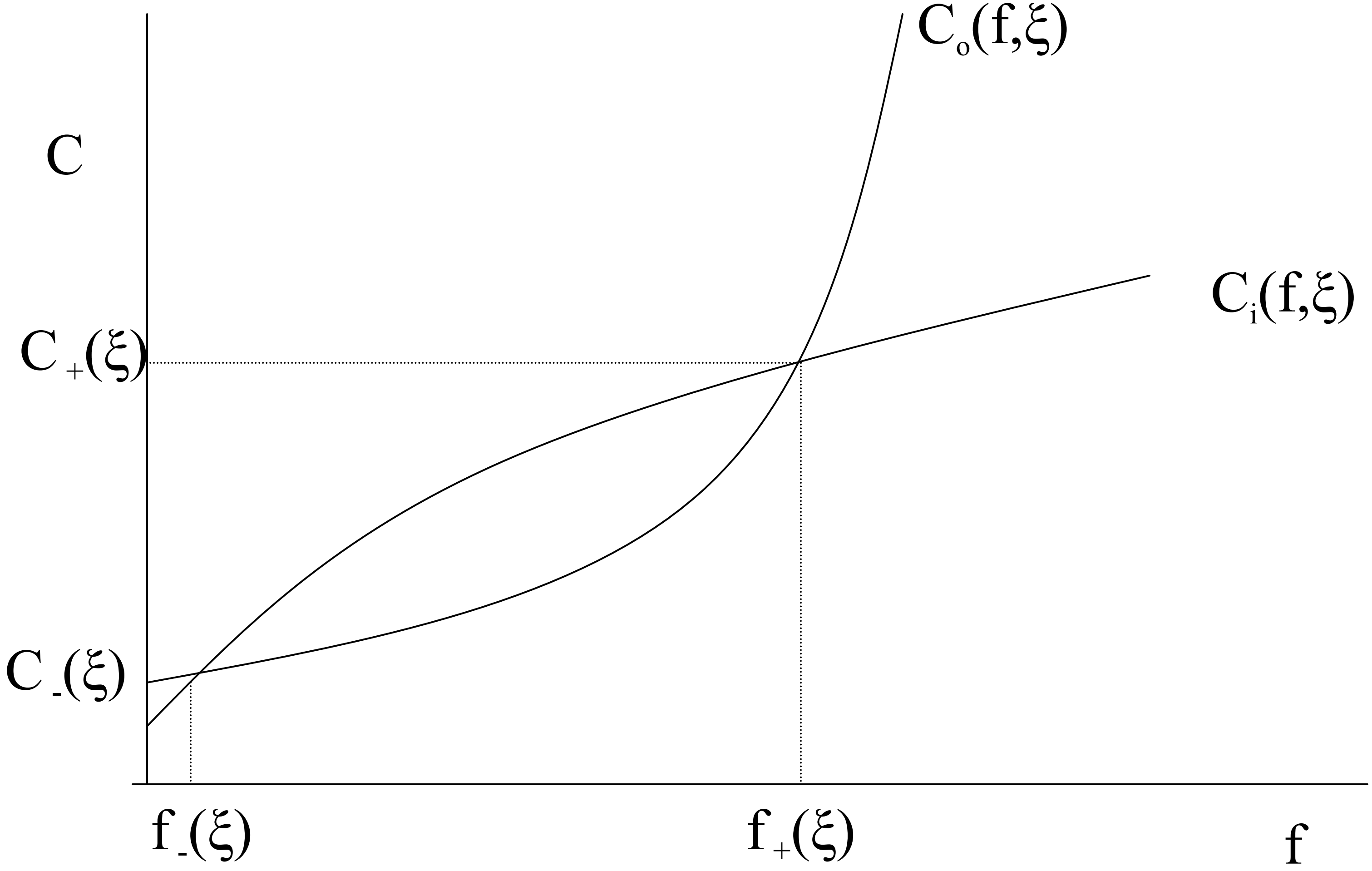


Figure 8

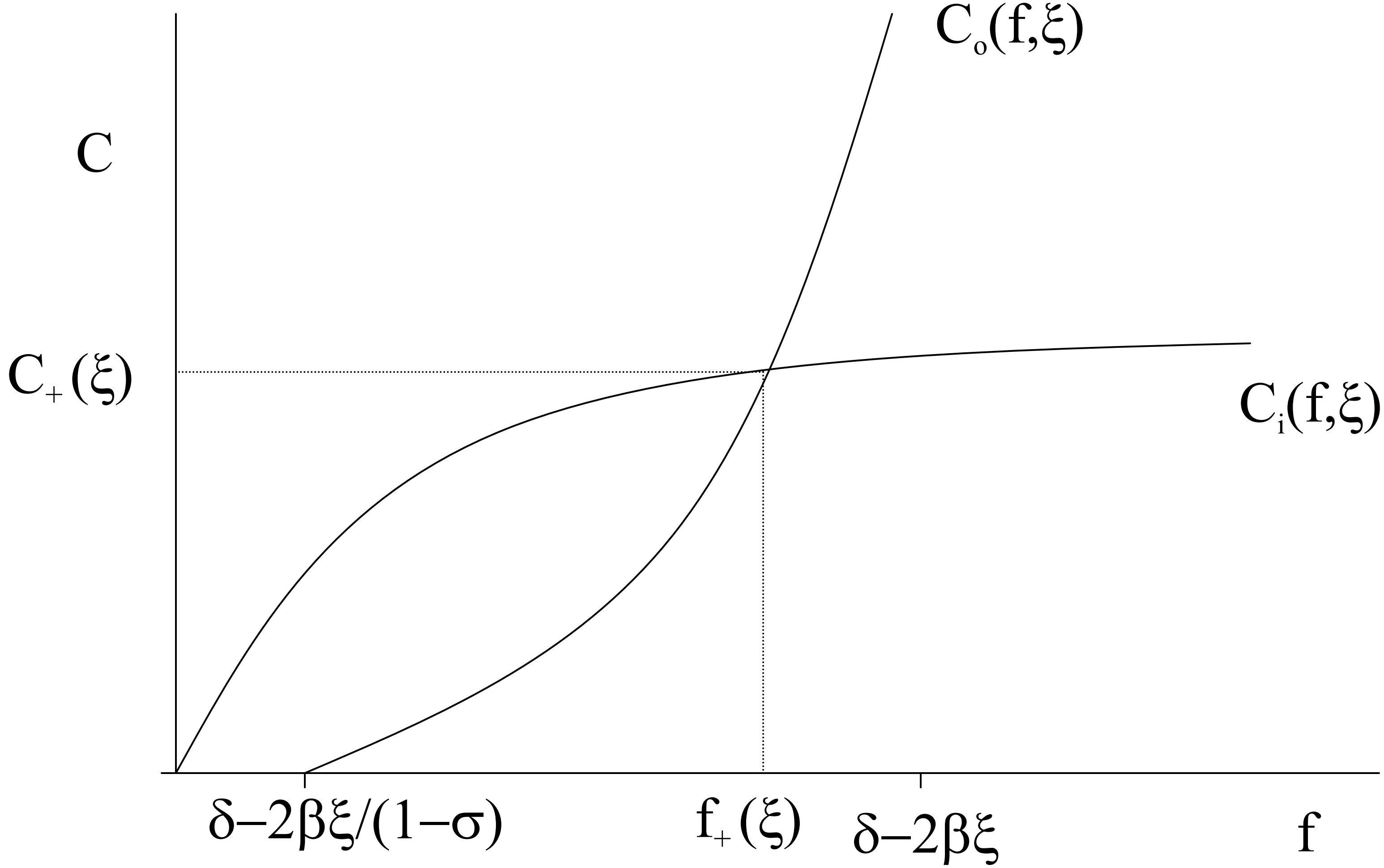
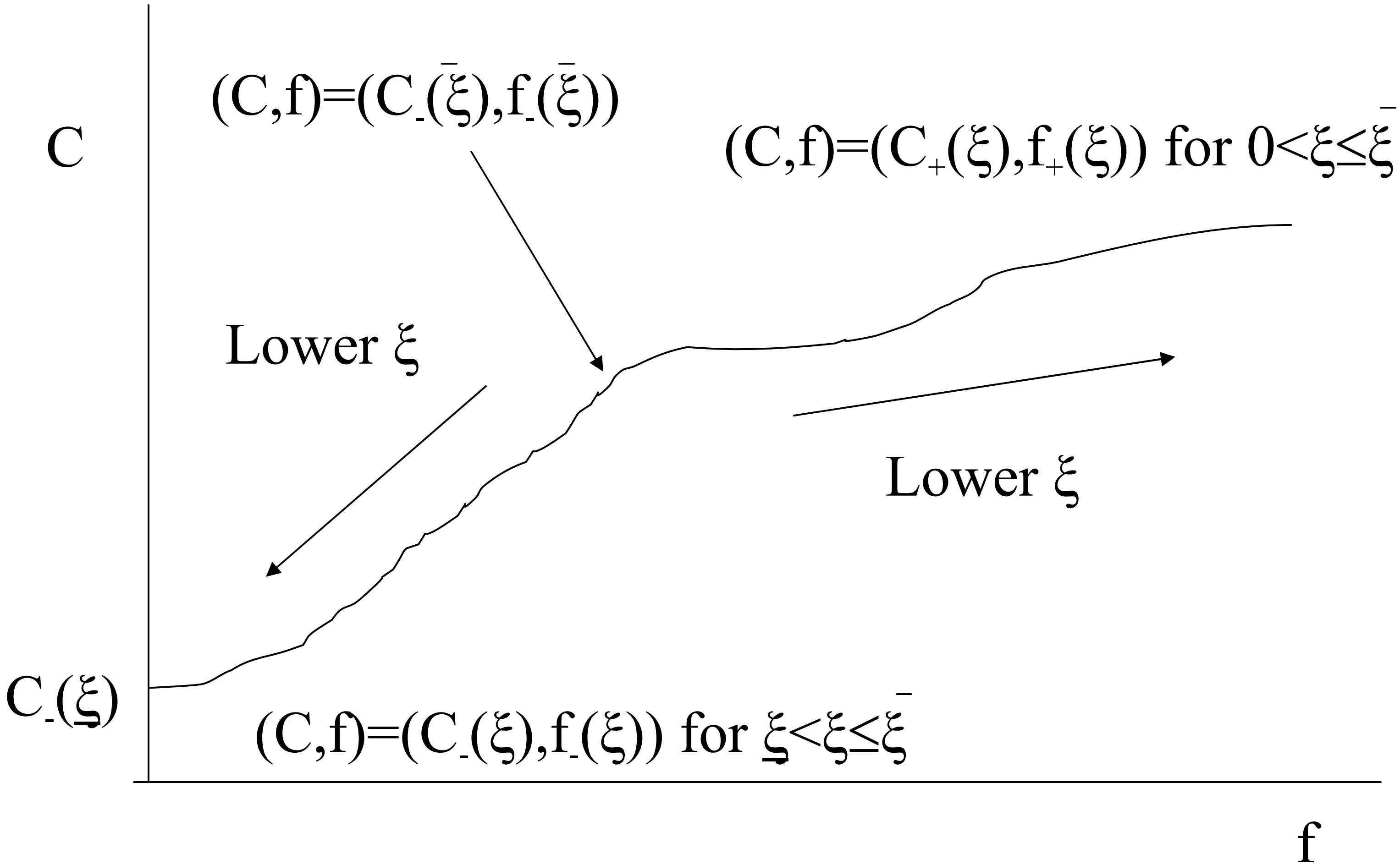


Figure 9



$(C,f)=(C_-(\bar{\xi}),f_-(\bar{\xi}))$
C
$(C,f)=(C_+(\xi),f_+(\xi))$ for $0<\xi\leq\bar{\xi}$
Lower ξ
Lower ξ
$C_-(\underline{\xi})$
$(C,f)=(C_-(\xi),f_-(\xi))$ for $\underline{\xi}<\xi\leq\bar{\xi}$
f

Figure 10

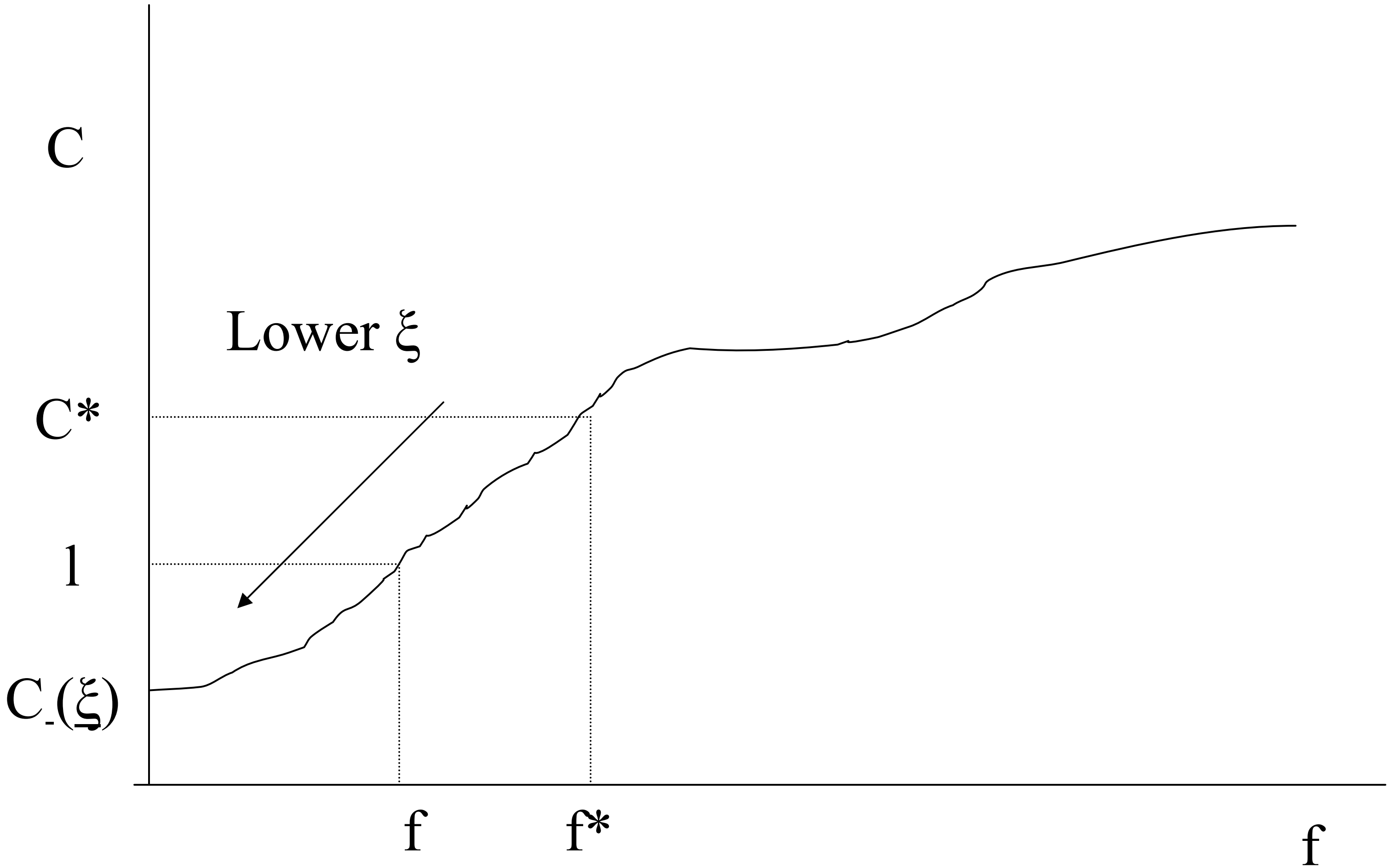


Figure 11

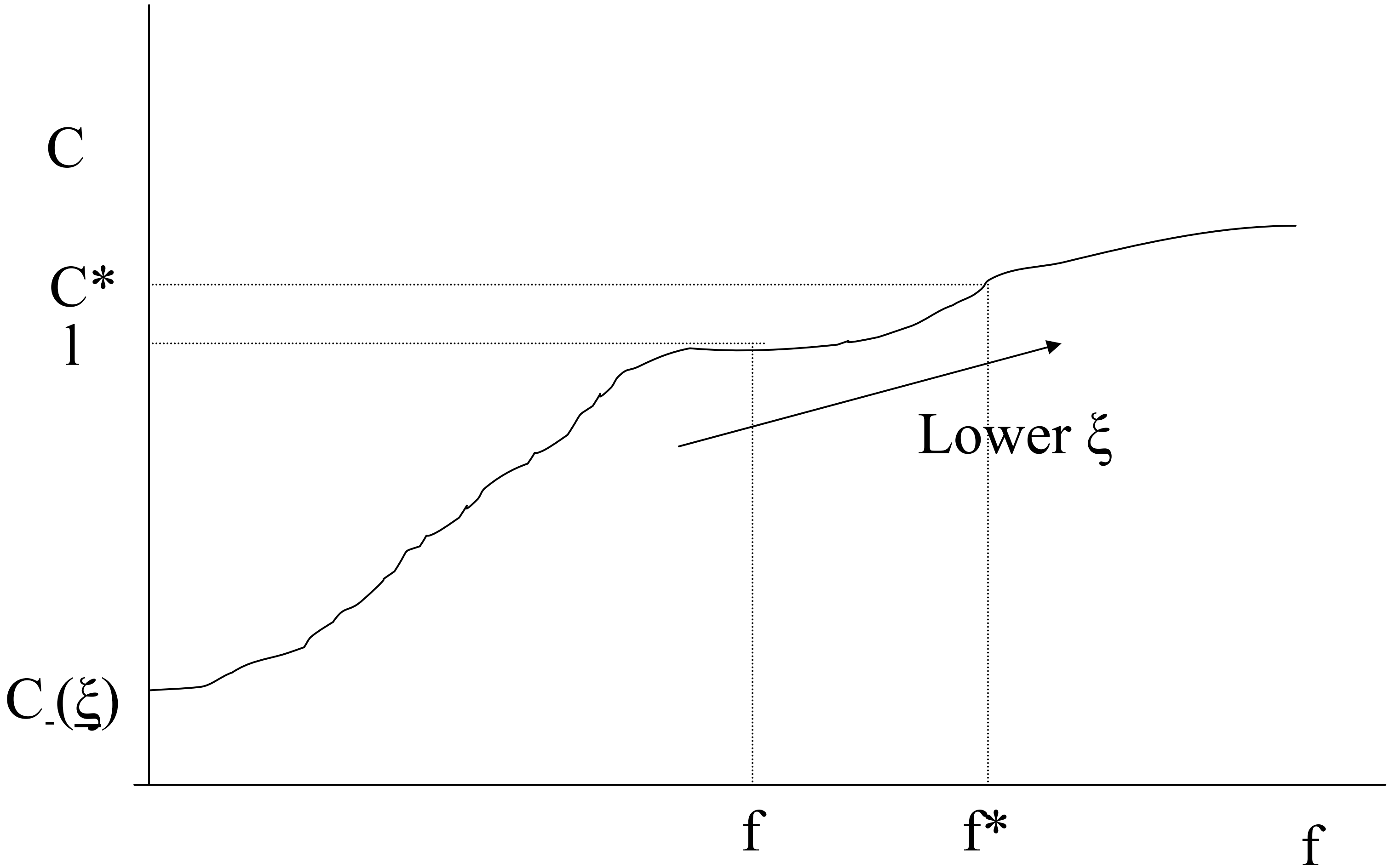


Figure A.1