

Bidding Asymmetries in Multi-Unit Auctions:  
Implications of Bid Function Equilibria in the  
British Spot Market for Electricity\*

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**Abstract**

The purpose of this paper is to test the implications of Bid Function Equilibria (Crespo (2000)) in the British spot market for electricity. Bid Function Equilibria (BFE) are Nash Equilibria of an oligopoly model of multi-unit auctions under complete information. Unlike existing theories of electricity supply, BFE predict *asymmetric* bidding by producers: a single firm (the “price-setter”) bids strategically while other firms (“non-price-setters”) bid cost. Using data on bid functions in the British spot market for electricity between 1993 and 1995, we find strong empirical support for the theory. Persistent asymmetries exist in bid functions consistent with one firm, National Power, the predominant price-setter in a BFE. Implications for the British electricity market remain to be considered.

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# 1 Introduction

The widespread deregulation of wholesale electricity markets is well underway. Competitive spot markets for electricity now exist in England and Wales, California, Australia, Spain, and Argentina. Other markets are being planned or implemented throughout the world, including much of the United States and Mexico. Despite the promise that spot markets generally yield competitive outcomes, concerns exist about this conclusion in electricity markets. Recent research by Wolak and Patrick (1998), Wolfram (1999b), and Borenstein, Bushnell, and Wolak (1999) have found that firms in these markets may exercise market power and offer electricity at prices considerably above marginal cost.

Existing economic theory does poorly at explaining these markups. Electricity markets are most accurately characterized as multi-unit procurement auctions. Despite having been applied to electricity, Supply Function Equilibria (hereafter SFE) introduced by Klemperer and Meyer (1989) map poorly to an institutional environment where generating units are discrete and have heterogeneous costs.<sup>1</sup> Models of Cournot interaction have similar difficulties.<sup>2</sup> Indeed, Wolfram (1999b) finds market power to be half or less of that predicted by these theories.

Recent advances in modeling multi-unit auctions, however, offer hope.<sup>3</sup> Ausubel and Cramton (1997), following a line of analysis dating to Wilson (1979), note that in markets involving the sale of multiple units of a homogenous good at a uniform price, buyers have an incentive to “reduce demand,” or understate their valuation for some of a good. If their (low) bid sets the market price, they earn a greater surplus on *all* units bought.<sup>4</sup> In electricity markets, this implies sellers bid greater than cost, trading off higher markups on inframarginal units against lost revenue on marginal units. Building on this literature, Crespo (2000) recently introduced Bid Function Equilibria to formalize these incentives and explicitly characterize the set of pure strategy Nash equilibria (PSNE) with unique outcomes. The basic model is one of oligopoly in a multi-unit auction under complete information, and extensions are derived to accommodate some important aspects of electricity markets.<sup>5</sup>

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<sup>1</sup>Calibration exercises by Green and Newbery (1992) and Green (1997) predict markups considerably larger than those found in the papers cited above.

<sup>2</sup>See Schmalensee and Golub (1985) and Borenstein and Bushnell (1999).

<sup>3</sup>For a more detailed review of recent multi-unit auctions literature, see Crespo (1999).

<sup>4</sup>In electricity markets where sellers bid in supply functions, this implies firms have an incentive to overstate their costs, earning a high price on all units supplied.

<sup>5</sup>In electricity markets, firms usually have fairly precise information regarding the marginal costs of other

The purpose of this paper is to test the implications of Bid Function Equilibria (hereafter BFE) in the England and Wales spot market for electricity. This extends recent work by Wolfram (1998) analyzing strategic bidding in the same market. As noted by Ausubel and Cramton (1997) and Borenstein, Bushnell, and Wolak (1999), strategic bidding introduces inefficiencies in market outcomes. Assessing the relevance of BFE permits quantifying the magnitude of these inefficiencies relative to alternative market structures and provides a mechanism for the design and governance of spot markets to mitigate them.

BFE differ in important ways from alternative theories of electricity supply. Unlike competitive markets characterized by marginal cost pricing, BFE permits strategic bidding by generators. Like SFE, producers are permitted to submit bid functions which are aggregated to obtain an industry supply curve. Unlike SFE, however, BFE acknowledges the discreteness inherent in electricity supply. Among theories applied to electricity markets, BFE is closest (and indeed extends) the general multi-unit auction theory cited above and tested in Wolfram (1998). Like this theory, generators have the incentive to inflate bids in an effort to increase the marginal price paid to all generating units. BFE differs from all these theories, however, in its predictions of *asymmetric* bidding behavior across producers. In particular, BFE predicts that for a given level of demand, a single firm (the “price-setter”, hereafter PS) chooses strategies aimed at setting the clearing price. The Nash Equilibrium response of all other firms (“non-price-setters”, hereafter NPS) is to bid their marginal costs. In essence, BFE finds that the intuition of the multi-unit auction theory is correct, *but only for the price-setter*.

The implications of BFE tested in this paper exploit this essential asymmetry. For instance, markups by the price-setter should increase in response to an increase in inframarginal capacity as the benefit of setting a higher equilibrium price increases. By contrast, bids by competitors should not be influenced by their inframarginal capacity. Furthermore, in the presence of uncertain shocks, bid markups for the price-setter near the margin should increase while those for non-price-setters should decrease. For the price-setter, this implies bid lumping, an empirical regularity in electricity markets.

Our results provide convincing support for the theory. There are strong and persistent asymmetries in bidding behavior across producers. While a single firm, National Power, appears as the predominant price-setter, we find evidence that all firms occasionally play this role. . Implications for the design and governance of the BSM remain to be considered.

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firms’ generating units.

The remainder of this paper is organized as follows. Section 2 described the England and Wales electricity market. Section 3 outlines the essential components of Bid Function Equilibria and Section 4 explores its implications for this market. Section 5 describes the available data and Section 6 presents the econometric specification and results. Section 7 will consider applications of these findings and a final section concludes.

## 2 The England and Wales Market for Electricity

### 2.1 Overview

Electricity markets have historically been served by monopolies subject to public regulation (Joskow (1997)). Over the past several decades, however, improvements in electric generation technology have caused regulators to reconsider whether the provision of electricity is most efficient when controlled as a natural monopoly. One proposed alternative has been the deregulation of wholesale generation. In place of tariffs based on cost of service, generators bid a series of prices for electricity that they are willing to supply into spot markets for electricity.<sup>6</sup> These spot markets have been designed to encourage competition in generation and ultimately yield lower prices for electricity.

The England and Wales (hereafter EW) market for electricity was among the first to implement such changes. In April, 1990, the UK began privatizing the markets for generation, transmission, and distribution of electricity. The generation assets of the extant monopoly were sold and an electricity “Pool” was established as the primary wholesale (spot) market for electricity. A separate transmission grid company, the National Grid Company (NGC), and 12 Regional Electricity Companies (RECs) were also created to transmit and distribute electricity to final customers. While the NGC and RECs remain subject to regulation, the spot market is unregulated and presumed to be competitive.

This section briefly describes salient features of the EW spot market for electricity.<sup>7</sup> Specifically, we characterize the market rules, the market structure, and the nature of generation

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<sup>6</sup>Throughout this paper, the word “bid(s)” refers to offer prices submitted by generators.

<sup>7</sup>Spot markets are often just one component of a deregulated generation market. Secondary markets are often used to complement the spot market and to provide hedging against price volatility. Bilateral contracts for power and their financial equivalent, “contracts for differences”, play a similar role. As the spot market can be viewed as the competitive engine of a deregulated generation market, we focus our efforts there. See, however, Wolak (1999) and Bushnell and Wolak (1999) for an analysis of the impact of such entities on strategic bidding in electricity markets.

technology in the market, and briefly survey analysis of the performance of the market to date. Interested readers should refer to Wolak and Patrick (1998) and Wolfram (1998) for additional details.

## 2.2 Market Rules

The rules for bidding into the spot market are as follows:

1. Demand is forecast by the NGC for each half-hour of the following day.<sup>8</sup>
2. Each generator submits bid prices for each generating unit they are willing to make available *for any and all half-hour markets* of the following day.
  - A generating unit is usually one turbine in a generating plant.<sup>9</sup>
3. The NGC uses these bid functions to obtain an aggregate supply curve. In each half-hour, it selects units to minimize cost subject to reliability constraints.
4. The intersection of the bid price of the marginal unit and forecast demand determines the System Marginal Price (SMP) paid to all units utilized in that half-hour.

In practice, bids submitted to the spot market have several components, including start-up costs, ramping constraints, and no-load variable costs.<sup>10</sup> Furthermore, the price paid to generators incorporates a capacity payment for providing reserves to the system in case of unexpectedly high demand.<sup>11</sup> In this paper, we abstract from these factors and focus on the SMP, the single price (per MWh) for electricity that accounts for the vast majority of revenue to firms during the sample period.

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<sup>8</sup>The forecasting algorithm is available to generators and they are capable of deriving the same forecasts as the NGC. This implies that demand is perfectly inelastic (in the short run) and known to all generators prior to bidding.

<sup>9</sup>A typical plant in Britain has between 2 and 11 turbines that range from under 200 to over 600 megawatts (Wolfram (1998)).

<sup>10</sup>These components are required given the production characteristics of electricity generation.

<sup>11</sup>This is called the Capacity Payment (CP). Together SMP plus the Capacity Payment equals the Pool Purchase Price (PPP).

### 2.3 Market Structure and Generation Technology

Since privatization, two companies – National Power (NP) and Power Gen (PG) have owned the greatest shares of generating capacity in the EW market. Due to market entry by independent power producers and reductions in their own capacity, the market shares of both generators have since declined. Between 1990 and 1998, the share of total market capacity has fallen from 47% to 30% for NP and from 27% to 25% for PG. (OFFER (1998)).<sup>12</sup>

Electricity generation is characterized by a diverse mix of technology. These can generally be described by their fuel type, implied marginal cost of production, and consequent pattern of use. Base load generation is utilized nearly all the time and includes large, low-cost generators. The cheapest are nuclear and sometimes hydroelectric plants. Intermediate-cost plants include mostly coal and some oil-burning turbines. High-cost plants typically burn natural gas, although combined cycle gas turbines (hereafter, CCGT's) are highly efficient gas burning plants that are usually considered to a part of base or intermediate load. Since privatization, there has been a significant increase in entry by these gas units along with the retirement of some less efficient coal units.

Generation portfolios are important because all generators are paid a single price set by the marginal plant required to meet market demand in the spot market. It is usually the fuel-burning plants, either coal or gas, that are marginal. Since NP and PG have generation portfolios consisting almost entirely of fuel-burning plants and no nuclear plants, they have played a disproportionately dominant role in setting the SMP. During the early years of the BSM, the two generators set the System Marginal Price (SMP) over 80% of the time.<sup>13</sup>

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<sup>12</sup>It is important to note that when the industry was privatized, the original plan was to award National Power with a much greater market share- about 70%. In turn, National Power agreed to own and operate all of the nuclear generation, which has more uncertain operating costs than other types of generation. However, just prior to the privatization, information regarding the potential costs of these plants became known and forced a break in the deal. At that point it was too late to make any reallocation of capacity between National Power and Power Gen, and the result has been a persistent although narrowing asymmetry in market shares.

<sup>13</sup>See Wolfram (1998).

## 2.4 Performance of the Spot Market

Performance of the spot market has been mixed. The Office of Electricity Regulation (OFFER) monitors the UK electricity market and has documented that average pool prices increased between 1990/91 and 1994/95 before reversing course and falling between 1994/95 and 1997/98 (OFFER (1998)).<sup>14</sup> Concerns about the exercise of market power by NP and PG prompted several investigations of bids into the spot market by the firms. The most recent of these, in February 1994, concluded pool prices exceeded avoidable costs. Rather than referring the firms to the UK Monopolies and Mergers Commission (MMC), OFFER and the firms reached agreements on price levels and the disposal (via divestiture or retirement) of 6 GW of coal- or oil-fired generation.<sup>15</sup> We consider the implications of these actions later in the paper.

Independent research of performance in the EW electricity market has largely substantiated OFFER's concerns. von der Fehr and Harbord (1993) estimate marginal costs and compare them to submitted bids in 1990 and 1991. They find significant differences over time in behavior with the latter bids greater than marginal costs. Following a similar approach, Wolfram (1999b) considers the period 1992 to 1994 and estimates price-cost margins on marginal units in the range of 0.20-0.27.

There is also evidence that suggests National Power has employed different bidding strategies than PowerGen (cf. Figures 1 and 2, taken from von der Fehr and Harbord (1993)). Indeed, we find that National Power sets the clearing price about 50% more often than Power Gen during most hours of the day. During the peak hours of the day when the highest markups are achieved, National Power is 75% more likely to be the price setter than Power Gen. This evidence, though preliminary, provides support for the empirical tests of BFE conducted in this paper.

## 3 Bid Function Equilibria

This section briefly summarizes results developed in Crespo (2000) and extends them in ways appropriate for modeling the England and Wales electricity market.

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<sup>14</sup>Volatility of prices has also increased significantly over time, particularly since 1994/95.

<sup>15</sup>In particular, NP and PG agreed to dispose of 4 GW and 2 GW, respectively, within two years. Furthermore, they agreed to bid such that prices would not exceed 24 Pounds/MWh time weighted and 25.5 Pounds/MWh demand weighted in October 1993 prices.

### 3.1 Overview

The basic model is that of a duopoly in a multi-unit auction with complete information. In this model, the auctioneer announces the number of units that are demanded in the auction and each firm responds by submitting a bid function. The bid functions are aggregated and the uniform clearing price is equal to the bid price of the unit where the inelastic demand intersects the aggregate bid function.

Demand is assumed *ex ante* known and perfectly inelastic. Marginal costs for each firm are step functions and are common knowledge. Prices and quantities take discrete values. Strategies map quantities into bids. In this setting, bidding marginal cost is *not* an equilibrium as firms have an incentive to “price out” units, where pricing out trades off higher revenue for inframarginal units against lost revenue for marginal units (von der Fehr and Harbord (1993), Ausubel and Cramton (1997)).

Bid Function Equilibria (BFE) are the unique Pure-Strategy Nash Equilibria (PSNE) in this setting. They are obtained in several steps. First, the price-setter (PS) is defined as the firm that owns the marginal unit in equilibrium. The other firm is called the non-price-setter (NPS). Next, we show that in equilibrium it is optimal for the NPS to bid marginal cost and for the PS to undercut the first extramarginal unit of the NPS. As such, as long as one firm finds it profitable to price out at least one unit, the outcome is asymmetric. This asymmetry describes the unique outcome found in every pure strategy Nash equilibrium of the model.

Given these strategies, the PS selects the optimal number of units to price out of the market. The solution to this problem is unique and pins down bids by the PS and NPS at the margin. Away from the margin, however, many bid functions yield equivalent equilibria; all that is required is that bids lie below certain upper bounds for both the PS and the NPS. Crespo (2000) describes these bounds in more detail.

Applying BFE to the EW market requires two significant extensions to the theory. The first allows for shocks to demand or availability, broadening the relevance of BFE and providing further implications about bids. The next extension considers the requirement that generator bid functions are fixed within the day. In practice, this muddies the implications of the theory for all but the peak demand period.

### 3.2 The Basic Model

The game,  $\Gamma_1 = \langle (1, 2), \beta, U, (\pi_i), M \rangle$  represents a duopoly where the players are denoted by  $i, j \in (1, 2)$ . The auction demand,  $M$ , is a positive integer. It is assumed that either firm is capable of supplying the full (inelastic) demand  $M$ , and each firm is required to submit separate prices for each of the  $M$  units.<sup>16</sup> The size of each unit is normalized to one. In this basic model, the cost functions of firms are symmetric. The cost of producing  $q$  units is  $C(q)$  with the marginal cost of producing the  $q^{th}$  unit given by  $c(q)$ . Strategies are expressed as bid functions  $\sigma_i : U \rightarrow \beta$ , where  $\sigma_i(q)$  is the price at which firm  $i$  is willing to supply its  $q^{th}$  unit. The domain of  $\sigma_i$  is  $U = \{1, 2, \dots, M\}$  and its range is the discrete interval,  $\beta = \{\lambda, 2\lambda, \dots, \bar{P}\}$ , where  $\bar{P}$  is an arbitrary price ceiling.  $\lambda$  here represents the bid increment, or smallest feasible change in bid.

Once bid functions are submitted, a scoring mechanism determines the number of units each firm sells into the market according to the  $M$  lowest bid prices in the aggregate of the pair  $(\sigma_1, \sigma_2) \equiv \sigma$ .<sup>17</sup> These quantities are denoted  $(q_1, q_2)$ . The compensation for each unit sold is the uniform clearing price,  $P \in \beta$ , equal to the highest accepted bid. An outcome is therefore  $O(\sigma) = \{q_1, q_2, P\}$ ,  $q_1 + q_2 = M$ , and  $P = \max_{i \in (1, 2)} \{\sigma_i(q_i)\}$ .<sup>18</sup> Profit is denoted  $\pi_i(\sigma_j, \sigma_i) = Pq_i - C_i(q_i)$ .

The remaining assumptions used in the basic model are given below.

- A1: The auction demand,  $M$  is known ex-ante.
- A2: The bid increment,  $\lambda$ , is small relative to the quantity increment, which is equal to one.
- A3: There is complete information regarding the other firm's payoff functions.

The restriction on the bid increment  $\lambda$  is provided so that the profitability of incremental undercutting will depend less on the size of the bid increment than on the price itself and

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<sup>16</sup>Extending the theory to  $n$  players requires only that demand may be supplied by any  $n - 1$  of them. Capacity constraints or transmission costs might violate this assumption. While of considerable policy interest (cf. Borenstein, Bushnell, and Stoft (1998), Joskow and Tirole (2000), Bushnell and Wolak (1999)), it is beyond the scope of this paper.

<sup>17</sup>In the event of a tie in bid prices for the last unit required to accommodate demand, we assume each firm will have one half of their respective unit taken in the auction.

<sup>18</sup>In most of the auction literature, outcomes are often expressed in revenues rather than prices. However, the latter alternative will conserve on notation and simplify forthcoming analysis.

the marginal cost of units near the margin. We consider the implications of relaxing the common knowledge assumption (A3) later in the paper.

The solution concept used in the basic model is Nash equilibrium. In this context, it will also be called a Bid Function Equilibrium.

**Definition 1** A Nash equilibrium in a duopoly model of a multi-unit auction with complete information is any pair of strategies  $\sigma$  such that:  $\pi_i(\sigma_j, \sigma_i) \geq \pi_i(\sigma_j, \tilde{\sigma}_i)$ , for all  $i \in (1, 2)$  where  $\sigma_i, \sigma_j$  and  $\tilde{\sigma}_i$  are mappings from  $U$  into  $\beta$ .

Thus, a pair of bid functions is a Nash equilibrium if for each firm there is no alternative bid function that would increase its profits given the bid function of the other firm and the auction demand  $M$ .

### 3.3 Properties and Existence of Pure Strategy Nash Equilibrium

#### 3.3.1 Bidding Behavior at the Margin

To begin the analysis, we first characterize the incentives facing firms when they choose the bid prices of units that are near the intersection of the aggregate bid function and the inelastic demand  $M$ . As the analysis will show, the incentives for firms to undercut the bid price of their counterpart is similar to the incentives found in Bertrand competition, with two important exceptions. First, if marginal costs are increasing and the output of firms is not symmetric, i.e.  $q_i \neq q_j$ , then the marginal cost of the two firms' marginal units will not be the same. Second, while the clearing price is determined by the bids for the marginal unit, undercutting a clearing price will lower the price paid to all inframarginal units. These differences introduce asymmetries in the benefits to undercutting a competitors price and drive the predictions of asymmetric bidding found in equilibrium.

We begin by defining a price-setter as a firm that sets the clearing price in equilibrium.

**Definition 2** Firm  $i$  is a Price-Setter for the outcome  $O(\sigma) = (q_i, q_j, P)$ , if  $\sigma_i(q_i) = P$ . A firm that is not a price-setter is called a Non-Price-Setter.

Suppose that  $\sigma = (\sigma_i, \sigma_j)$  is a pair of strategies that induces the outcome  $O(\sigma) = (q_i, q_j, P)$  such that firm  $i$  is the only price setter. We will first show that if  $P > c(q_j + 1)$  then firm  $j$  would prefer the strategy  $\hat{\sigma}_j$  which is equal to  $\sigma_j(q)$  for all  $q$  except for unit  $q_j + 1$  for which

$\sigma_j(q_j + 1) = P$ .<sup>19</sup> In other words, whenever price is above the marginal cost of firm  $j$ 's first extra-marginal unit, firm  $j$  prefers to tie the clearing price rather than price above it with unit  $q_j + 1$ . To verify this we can compare profits resulting from these two alternatives, and obtain the range of prices,  $P$  for which the profit from tying is greater than the profit from pricing above the clearing price.

$$Pq_j - \sum_{n=1}^{q_j} c(n) < P(q_j + \frac{1}{2}) - \sum_{n=1}^{q_j} c(n) - \frac{c(q_j + 1)}{2}, \text{ or}$$

$$0 < \frac{1}{2}P - \frac{c(q_j + 1)}{2}, \text{ or}$$

$$P > c(q_j + 1).$$

We can also show that if  $P > c(q_j + 1) + 2\lambda(q_j + 1)$  then firm  $j$  would prefer the strategy  $\hat{\sigma}_j$  which is equal to  $\sigma_j(q)$  for all  $q$  except for unit  $q_j + 1$  for which  $\sigma_j(q_j + 1) = P - \lambda$ . In other words, whenever price is above this level, firm  $j$  prefers to *undercut* the clearing price relative to *tying* it with unit  $q_j + 1$ . To verify this we can compare profits resulting from these two alternatives in a manner similar to that above, and obtain the range of prices,  $P$  for which the profit from undercutting is greater than the profit from tying.

### 3.3.2 Uniqueness of the Price Setter

The results of the previous subsection can be used to derive two propositions about the uniqueness of the price-setter.<sup>20</sup>

**Proposition 1** If  $\sigma$  is a PSNE with outcome  $O(\sigma) = (q_i, q_j, P)$  such that  $q_i = q_j = q$  then both firms are price setters. Furthermore,  $\sigma_i(q_i) = \sigma_j(q_j) = c(q + 1)$ .

**Proposition 2** If  $\sigma$  is a PSNE with outcome  $O(\sigma) = (q_i, q_j, P)$  such that  $q_i < q_j$  then only firm  $i$  is a price-setter.

Proposition 1 establishes that the only PSNE in which a tie occurs (and both firms are officially price-setters) is one in which both firms are bidding marginal costs. Crespo

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<sup>19</sup>We assume here a rationing rule that allocates one half the production to each firm in the event of a tie.

<sup>20</sup>Proofs for these propositions can be found in Crespo (2000).

(2000) demonstrates that this is only possible at very low levels of demand or with constant marginal costs.

Proposition 2 establishes that in all other situations, the price-setter is the one firm that is “pricing out” units. “Pricing out” involves bidding prices for one or more units that will result in their exclusion from the market (inducing  $q_i < q_j$ ) despite the clearing price being above their marginal costs. Because marginal costs are increasing for both firms, each unit that is priced out causes a more costly unit to be taken in the auction, increasing the clearing price and the markups over infra-marginal units. It is in this way that the price-setter trades off higher profits on inframarginal units against lost profits on marginal units.

The intuition for Propositions 1 and 2 can be obtained by relating bidding behavior at the margin in a BFE to Bertrand competition. In both models firms have incentives to undercut their counterparts bid price at the margin. In BFE, however, the price-setter is pricing out units. The marginal cost of his marginal unit is therefore less than the marginal cost of the non-price-setter’s marginal unit, even for ex ante identical firms. As such, he has an incentive to undercut the marginal cost of the other firms’ first extramarginal unit and be the unique price-setter in equilibrium.

### 3.3.3 The Non-Price Setter and Aggressive Bidding

The purpose of this section is to demonstrate that the best response of a non-price setter is to bid units at marginal cost. The proof utilizes the following definition of aggressive response strategies.

**Definition 3** The bid function  $\sigma_i : U \rightarrow \beta$  is an aggressive response to  $\sigma_j : U \rightarrow \beta$  if it induces an outcome  $O = (q_i, q_j, P)$  for which  $q_i = \max\{q : c(q) < P\}$ .

An aggressive response to  $\sigma_j$  is any strategy  $\sigma_i$  in which all of firm  $i$ ’s units with marginal cost below the clearing price are taken in the auction. By contrast, if firm  $i$  has a unit with marginal cost less than the clearing price and that unit is not taken in the auction, then firm  $i$  is not bidding as aggressively as it could without bidding units below marginal cost. A refinement of the definition above yields the definition of an aggressive strategy.

**Definition 4** The bid function  $\sigma_i : U \rightarrow \beta$  is an aggressive strategy if it induces an outcome  $O = (q_i, q_j, P)$  such that  $q_i = \max\{q : c(q) < P\}$  for any  $\sigma_j : U \rightarrow \beta$ .

An aggressive strategy  $\sigma_i$  is a bid function that is an aggressive response to any bid function  $\sigma_j : U \rightarrow \beta$ . Marginal cost bidding is an example of an aggressive strategy. These two forms of aggressive bidding are a common thread that links each of the results in this paper, beginning with Lemma 1.

**Lemma 1** Suppose  $\sigma$  is a Nash equilibrium of a multi-unit auction with complete information. If firm  $i$  is the Price Setter, then  $\sigma_j$  must be an aggressive response to  $\sigma_i$ .

Lemma 1 establishes that in any equilibrium, the firm that is not the price-setter must be bidding aggressively. The proof of the Lemma is an explanation for why any firm that is not the price setter gains from undercutting as long as there is at least one un-utilized unit with marginal cost less than the clearing price.

### 3.3.4 The Price Setter's Objective Function and Solution

Lemma 1 greatly simplifies the price-setter's decision problem. Even though the price setter is not established a priori, the definition of Nash equilibrium and Lemma 1 require that the price setter is playing a best response to an aggressive strategy.

To see how this line of reasoning enters into the price setter's choice of strategy, consider the example in which firm 1 chooses a price setting strategy. From Lemma 1, Firm 1 can assume that if he sets the price above the marginal cost of firm 2's first extra-marginal unit, then firm 2 would respond aggressively to undercut that clearing price. Thus, even if firm 1 does not necessarily interpret an aggressive response as an aggressive strategy, the solution to Nash equilibrium can be derived as if firm 1 internalizes a residual demand function given by the inelastic demand  $M$ , and a function involving the inverse of firm 2's cost function. This is formally stated in Corollary 1.

**Corollary 1** If firm  $i$  is the price setter in a multi-unit auction with complete information, then  $\sigma_j$  induces the residual demand  $D_i(P) = M - \max\{q : c(q) < P\}$ .

Along these lines, Nash equilibrium also requires that the price setter induces the highest price possible for whatever optimal level of output is chosen. That is, from Corollary 1 we know that if firm  $i$  is the price setter, then  $q_i = M - \max\{q : c(q) < P\}$ . Therefore, the highest price that firm  $i$  can induce in any outcome  $(q_i, q_j, P)$  is  $P = c(q_j + 1) - \lambda$ . This is formally stated in Corollary 2.

**Corollary 2** Suppose  $\sigma$  is a Nash Equilibrium of a multi-unit auction with complete information. If player  $i$  is the price setter, then  $P = c(q_j + 1) - \lambda$ .

Given Corollary 1 and Corollary 2, the price setter's best response to an aggressive strategy can now be simplified into a single objective function. Throughout this paper we will often express quantities in terms of their difference from the "null" quantities that firms would supply to the market if each firm chose an aggressive strategy. For example, if the demand  $M$  is an even integer and firms are symmetric, then the null quantity for both firms is  $q_i^o = q_j^o = \frac{M}{2}$ . Since we are only dealing with symmetric firms in this section, we will denote  $q^o = q_i^o = q_j^o$ .

**Definition 5** Let  $q_i^o$  be the realized output of firm  $i$  when  $\sigma$  is a pair of aggressive strategies.

In the following objective function, the price setter's quantity is expressed as the term  $(q^o - k_i)$  where  $k_i$  is the number of units firm  $i$  chooses to price out of the market in order to obtain a markup over his remaining infra-marginal units. In this case, we retain the subscript on  $k_i$  even when firms have symmetric cost functions because the subscript identifies the price setter. As stated in Corollary 2, the clearing price would be  $P = c(q^o + k_i + 1) - \lambda$ , where  $c(q^o + k_i + 1)$  is the marginal cost of firm  $j$ 's first extra-marginal unit. Combining these terms, the objective function is expressed in the following equation:

$$\max_{k_i \in \{0, 1, \dots, q^o\}} (q^o - k_i)[c(q^o + k_i + 1) - \lambda] - C(q^o - k_i) \quad (1)$$

Because we are dealing with discrete units and discrete bid prices, the optimization problem cannot be expressed in a differentiable first order condition. Nonetheless, we explain below how the solution can be obtained from an algorithm that utilizes an analogous difference equation.

$$\begin{aligned} k_i^* &= \max k_i \in \{0, 1, \dots, q^o\} \text{ s.t.} \\ (q^o - k_i)[\Delta c(q^o + k_i + 1)] - [c(q^o + k_i) - \lambda - c(q^o - k_i)] &\geq 0 \text{ or} \\ (q^o - k_i)[\Delta c(q^o + k_i + 1)] &\geq [c(q^o + k_i) - \lambda - c(q^o - k_i)] \end{aligned} \quad (2)$$

In equation (2), the term  $\Delta c(q^o + k_i + 1)$  is the incremental markup that can be achieved from pricing out the  $k_i^{\text{th}}$  unit when  $k_i - 1$  units have already been priced out. The term,

$(q^o - k_i)$  represents the number of firm  $i$ 's remaining units after the  $k_i^{th}$  unit is priced out. The product of these two yields the incremental revenue from pricing out the additional unit. The final term,  $[c(q^o + k_i) - \lambda - c(q^o - k_i)]$ , represents the foregone markup on the  $k_i^{th}$  unit. As in the standard monopolists pricing problem, the price-setter equates the marginal benefit of pricing out an additional unit against its marginal cost to determine the optimal number of units to price out.

This equation is strictly decreasing in  $k_i$ , ensuring the tractability of the price-setter's problem. For a given cost function it is possible to find an explicit value for  $k_i$ .<sup>21</sup> As long as the change in profit is non-negative a firm is willing to price out an additional unit.

Given  $k_i^*$  and firm  $j$ 's cost function, we can also define the optimal price  $P_i^*$ .

**Definition 6** Let  $P^* = c(q^o + k_i^* + 1) - \lambda$  be the optimal price chosen by firm  $i$  when firm  $j$  plays an aggressive strategy.

This notation is used in Lemma 2, which states the essential property of Nash equilibrium. That is, in every Nash equilibrium we know that the non-price setter is playing an aggressive response to a price setting strategy. Simultaneously, the price setter chooses a strategy that maximizes his own profits given that his counterpart is responding aggressively.

**Lemma 2** If  $\sigma$  is a Nash Equilibrium of a multi-unit auction with complete information and firm  $i$  is the price setter, then the outcome  $O(\sigma) = (q_i, q_j, P)$ , must be of the form:  $q_i = q^o - k_i^*$ ,  $q_j = q^o + k_i^*$ , and  $P = P_i^*$ . Such an equilibrium is called a **Bid Function Equilibrium**

It is important to note that while Lemma 2 provides a necessary criterion regarding a unique *outcome* (i.e.  $k^*$ ) in a Nash equilibrium, it does not establish the existence of unique PSNE *bid functions*. Because firms construct their residual demand curves from the inelastic demand and other firm's bid functions, it is the entire bid function of firms that must meet the sufficient criterion for Nash equilibrium.

It is possible, for example, to create bid functions that satisfy Lemma 2 at the margin, but are not an equilibrium. Suppose that two bid functions produce the unique outcome given in Lemma 2. If the bid price of one of the non-price-setter's extra-marginal units is too high

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<sup>21</sup>If the solution to equation 2 holds as an equality, then there are actually two values for  $k^*$  from the price-setter's perspective. However, the non-price setter is strictly better off when the price setter chooses the greater of these two values. We will assume that the greater value is chosen.

(above its marginal cost), then the price setter may wish to price out an additional unit in order to raise the clearing price to just below the bid price of that unit. A PSNE therefore requires an upper bound on the bid prices of the non-price-setter's extra-marginal units. Similarly, if the price of one of the price-setter's extra-marginal units is too high, then the non-price-setter may wish to begin pricing out units. Similar constraints can be derived for the bid prices of infra-marginal units.

Following Lemma 2 in Crespo (2000) there is a list of sufficient criterion for bid functions  $\sigma$  to satisfy the requirements of a pure strategy Nash equilibrium. These criterion provide an upper bound that specifies the highest equilibrium bid price on each unit. For example, if the bid prices of the non-price-setter's infra-marginal units are too high, (i.e., too close the clearing price) then the price setter may regret pricing out some or all of his  $k$  units. Instead, he could price some units back into the market without substantially lowering the clearing price. Similar conditions exist for both firms' extramarginal units.<sup>22</sup>

In Crespo (2000) it is also shown that bid functions that satisfy these criterion will always exist. This is stated in Theorem 1.

**Theorem 1** In any game  $\Gamma_1 = \langle (1, 2), \beta, U, (\pi_i), M, S \rangle$  representing a multi-unit auction with complete information, there exists a Nash equilibrium,  $\sigma$  that results in outcome  $O(\sigma) = (q_i, q_j, P)$ , where  $q_i = q^o - k_i^*$ ,  $q_j = q^o + k_i^*$ , and  $P = P_i^*$ .

While the intertemporal process explaining how firms arrive at this equilibrium cannot be obtained from the definition of Nash equilibrium *per se*, we can consider its implications in various contexts. For example, if we consider the "steady state" interpretation of Nash equilibrium, we might say that if both firms start out bidding at marginal cost, then either firm may have an incentive to deviate. This firm would then observe that the best response of the other firm is to continue bidding aggressively. If instead we consider the purely "deductive" interpretation of Nash equilibrium, then we might say that either firm who considers a deviation from an aggressive strategy knows that the best response of the other firm is an aggressive response.

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<sup>22</sup>The analogous criterion for the price setter's infra-marginal units is of lesser importance. For these bid prices to matter, the non-price-setter would have to consider "pricing in" some additional units, and from Lemma 1 we know that the non-price-setter has already bid all units with marginal cost less than the clearing price into the market.

### 3.4 Extensions to the Basic Model

Several extensions of the basic model are required to map it into the institutional environment of the England and Wales electricity market. These are considered below.

#### 3.4.1 Uncertainty and Bid Lumping

The basic model assumes that both firms know the other's profit function. While it is not implausible that firms in the EW electricity market know the cost characteristics of their rivals portfolio of plants, random shocks in the underlying economic environment may introduce some uncertainty into rivals' bid functions at the level of the day.

We therefore extend the basic model of Bid Function Equilibria above to admit a small probability of an exogenous and random shock to demand and/or availability. While forecast demand is unusually accurate in electricity markets (and in the England and Wales market is used to set the clearing price), there are times when the realized demand differs significantly from the forecasted demand. Similarly, shocks to availability due to maintenance and repair are common in electricity markets. Depending on the market rules for resolving the resulting imbalances of scheduled supply and realized demand, the presence of uncertain demand or availability can influence bidding behavior. In particular, the possibility of such shocks can dramatically tighten the bounds on bids away from the (perfect-information) bid price.

For example, in the England and Wales market an Uplift charge is used to compensate units that are brought on line when actual demand exceeds the forecasted demand. These units are paid their bid prices while units that are among those originally scheduled to run are paid the uniform clearing price. Because bid prices that are used to calculate the Uplift charge do not affect the uniform clearing price, there is an incentive for the price-setter to bid these units as competitively as possible without lowering the clearing price.<sup>23</sup> Specifically, it can be shown that for extra-marginal units belonging to the price-setter that are near the margin, the optimal bid price is  $\sigma_i(q) = \max(c(q), P)$  where  $P$  is the equilibrium clearing price absent demand shocks.

This strategy results in a bid function for the price setter that is characterized by *lumping* at the bid price. An illustration of this is given in Figure 3.

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<sup>23</sup>Recall that extra-marginal units for the price-setter may have marginal costs significantly below the clearing bid price (cf. equation (2)).

### 3.4.2 Multiple Demand Periods

The basic model assumes that there is a single bid function for each demand period. In practice, firms must submit a single bid function for each of the 48 half-hours in the day in the EW electricity market.

We therefore extend Theorem 1 to apply to auctions where each generator submits a bid function that will be applied to multiple values for  $M$ . In the simplest case, the demand periods are sufficiently dispersed that the implications of BFE apply to each demand period in the multi-period case.

Sometimes, however, these bid functions are problematic because the peak price for the price-setter can violate the upper bound on their extramarginal off-peak bid prices. When the price  $P_P$  is sufficiently high, this may induce the non-price-setter (in the peak) to price out off-peak units that would otherwise run during both the off-peak and peak period. By doing this, he now sets the price so that it is just below the peak price  $P_P$  in *both* demand periods.

This incentive can be eliminated by moving to a repeated game framework. While many equilibria are possible in such a setting, one plausible one would invoke a punishment phase if the non-price-setter were to violate these bounds.<sup>24</sup> Note that in dynamic extensions of the static game, there are equilibria where the same firm is the price-setter in both the peak and off-peak periods, or where one firm is the price setter during the peak period while the other is the price setter during the off-peak period. This finding is important given the variation in the identity of the actual price-setter observed in the data.

## 4 Implications of the Theory

This section considers the implications of Bid Function Equilibria in the England and Wales market for electricity.

The testable implications examined in this paper exploit the asymmetry in behavior predicted by BFE. Specifically, BFE predicts that in each demand period, one firm acts as a price-setter, bidding strategically, while the others act as non-price-setters, bidding their

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<sup>24</sup>This is particularly true if one recognizes the regulatory oversight present in the EW electricity market. If the non-price-setter were to price off-peak units as high as peak units, this might well attract regulatory intervention in the market.

marginal costs.

Before we begin, however, two remarks are in order. The first is a brief note on terminology. The difference between a unit's bid price and its marginal cost is called its *bid markup*. The difference between the (common) clearing price and each unit's marginal cost is called the *markup*. As we are interested in strategic bidding by generators, bid markups are the focus of our empirical tests. The second remark considers the impact to the predictions of BFE of the requirement that bid functions in the EW market be fixed within the day. As discussed above, this means that firms must incorporate the strategic impact of multiple demand periods into a single bid function, muddying the predictions of the theory for most demand periods.<sup>25</sup> We consider the impact of this rule after the introduction of each implication below.

**Implication 1:** Bid markups for the price-setter's marginal unit increase with inframarginal capacity, decrease with marginal capacity, and increase with the change in costs of non-price-setting firms (cf. equation (2)). Bid markups for the non-price-setter's marginal unit do not.

This implication follows from Lemma 1 and Lemma 2 describing the optimal behavior of the non-price-setter(s) and price-setter at the margin. The intuition is straightforward. The price-setter bids strategically, trading off additional profits from pricing out an additional unit (as a function of inframarginal capacity and the change in costs of non-price-setting firms) against the lost profits on the marginal unit (as a function of the bid markup and capacity of the marginal unit). By contrast, the non-price-setter bids marginal costs, implying no relationship between bid markups and these things.

In practice, that bid functions are fixed throughout the day considerably complicates the testing of this implication. While we expect the implications of BFE to be satisfied throughout bid functions, this presumes firms set bids for each unit *as if it were marginal*. Even so, that a firm may be a price-setter during some demand periods and a non-price-setter during others muddies the predictions of the theory, implying that the average effect on firms' bid markups of the variables above should lie between the extremes in Implication 1.

In *peak* demand periods, however, the multiplicity of demand periods has much less impact as there are no greater periods of demand to interfere with bounds on bids for extramarginal

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<sup>25</sup>In practice, demand follows a regular hump-shaped pattern within the day. There are generally one or two "peak" demand periods, multiple moderate demand periods ("shoulders"), and multiple low demand periods.

units. We therefore take pains in the empirical implementation to explore bidding behavior in general *and* in the peak period. We discuss this point in greater detail in the next section.

While the previous implication considered firm behavior at the margin, our second implication considers behavior of bid markups away from the margin:

**Implication 2:** Bid markups for the price-setter are greater for units that are close to the margin. Bid markups for non-price-setters are smaller for units that are closer to the margin.

Recall BFE requires that that only the bids of the price setter's *marginal* units are subject to strategic manipulation. These include both the price setting unit and the "priced out" units. As such, bid markups should be higher for these units. For the non-price-setter, the opposite effect occurs. Non-price-setting strategies are constrained by the bounds enforcing price-setting strategies. These bounds are tightest at the margin.<sup>26</sup> As such, bid markups should be smallest for these units.

In practice, the presence of multiple demand periods complicates testing this implication throughout the bid function. As such, this effect is tested exclusively during peak demand periods. So, too is our final implication:

**Implication 3:** The bid function of the price setter exhibits *lumping* at the clearing price. The bid function of the non-price-setter does not.

Our final implication considers the extension of BFE to allow for shocks to demand or availability. While BFE without such shocks permit a range of bids for extra-marginal units for the price-setter, the possibility they could become marginal in the presence of shocks requires their bid equals that of the marginal unit without shocks. This causes the bid function of the price-setter to flatten (i.e. lump) in the region of the clearing price. Following the discussion of Implication 2 above, bids for the non-price-setter are closest to marginal cost at the margin.

It is important to note the relationship of the implications above to those previously considered in the literature. Wolfram (1998) recognized that the BSM could be modeled as a multi-unit, first price auction and tested for strategic behavior in light of this result. Her primary testable implication was similar to Implication 1 above: generators have an incen-

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<sup>26</sup>Indeed at the margin, BFE predict non-price-setters bid exactly marginal cost. Furthermore, marginal changes in the non-price-setters bid yield marginal changes in the price-setter's bids. Away from the margin, marginal changes in the non-price-setter's bids require non-marginal changes in the price-setter's bids. As such, there is a range of possible equilibria supporting a range of non-price-setters' bids.

tive to bid up generating units in order to trade off higher margins for inframarginal units against lost revenue on marginal units.<sup>27</sup> For several measures of inframarginal capacity, her empirical results suggested that such units did indeed bid higher. For some measures, however, including a direct measure of inframarginal capacity, the evidence was weak.<sup>28</sup> Bid Function Equilibria imply that only the price-setting firm bids strategically while other firms bid at (or near) marginal cost. This suggests that the results in Wolfram (1998) were weakened by pooling the price-setting and non-price-setting firms. In the results that follow, we take care to allow for asymmetric effects across firms.

## 5 Data

In this section we describe the data that we use to test the implications of BFE. The key variables described in Section 4 relate bid markups to characteristics of firm's generating portfolios, marginal costs, and bid functions, where the bid markup over unit  $i$  for a given bid function is simply the bid price submitted for unit  $i$  minus the approximated marginal or fuel cost constructed for unit  $i$ . Generator portfolio information is available from the individual generators and a complete list is published by the Electricity Association. Marginal cost and market (bid) data are described below. Both of these data sets relate to the time period from January 1993 to December 1995.

### 5.1 Marginal Cost Data

Due to the nature of electricity generation and the elaborate exchanges of information between generators and regulators prior to deregulation, we are fortunate to have fairly precise estimates of marginal cost curves at the firm level. Following Wolfram (1999b) and Green and Newbery (1992), the short run marginal (energy) cost of producing electricity from a fossil fuel plant (oil, coal, and natural gas) depends on the type of fuel the plant burns, the price at which that fuel is purchased, and the efficiency rating of the plant. We are therefore able to construct an approximation of the marginal cost curves of the two largest generators, National Power and Power Gen, as well as an approximation of an aggregate marginal cost curve for the entire market. The details of this procedure are described below.

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<sup>27</sup>For instance, her estimating equation is an imperfect-information analog to equation (2) above.

<sup>28</sup>cf. Wolfram (1998), Table 4.

A list of all plants and their efficiency ratings was last made public in the 1987/1988 Central Electricity Generating Board (CEGB) Statistical Yearbook. Although efficiency ratings of plants are not expected to change substantially over a few years, there have been some important plant additions and plant retirements since the CEGB publication. A list of these additions and retirements is published by the Electricity Association. The plants that have been added during the 1990's include mostly combined cycle gas turbines (CCGT's). These plants convert the waste heat produced as a by product of burning natural gas into steam that can be used to power an additional turbine. Following the assumptions in Green and Newbery (1992) and Wolfram (1998) we assume that the thermal efficiencies of these plants are 45 percent. As pointed out in Wolfram (1999b), coal plants set the SMP over 80 percent of the time, so it is helpful to have precise estimates of the marginal costs associated with these plants. For this reason, we substitute more recent measures of efficiency ratings of coal plants published in Rainbow, Doyle, and Price (1993) where they differ significantly from those published in the CEGB Statistical yearbook.

At the time of privatization, regulators and industry representatives were concerned about the effects of competition in generation on the nations large but vulnerable coal industry. Due to this concern, generators were "locked in" to purchase prices for coal in contracts that started at 150 pence per gigajoule in March 1993 and were lowered periodically to 134 pence per gigajoule in 1995. For coal prices after this period and the prices of other fossil fuels, we use those published quarterly in the periodical Energy Trends. Energy Trends published the average of the actual fuel prices paid by generators. Because generators purchase much of their needed fuels at prices based on long term contracts and these can differ from the spot market prices, this is a more accurate figure than a spot market price.

For non-fossil fuel plants, we use approximations of marginal cost that are based on the same procedures used in Green and Newbery (1992) and Wolfram (1999b). Energy from pumped storage plants and energy imported from France and Scotland is assigned its minimum quarterly bid price, and for nuclear energy, we use approximations between 11 and 13 British pounds per megawatt-hour, depending on the technology used by the reactor.

## **5.2 Market Data**

Market data is made available by The Electricity Pool, who provides these services on behalf of the National Grid Company. We will use 51,278 observations on pool bids, prices, and quantities during nearly every half-hour of each day during six month periods from 1993

through 1995. This data consists of five sets of observations:

1. Day-ahead demand as forecasted by the National Grid Company.
2. Generator bid functions for each day during the months covered between 1993 and 1995.
3. The generation capacity declared available for each unit as provided daily by the generators
4. The system marginal price for each demand period.
5. The identity of the marginal generating unit. This data is available beginning in January of 1995

Table 1 presents some summary statistics for some of the key variables used in the paper.<sup>29</sup>

## 6 Testing the Implications of BFE: Specification and Results

This section presents our econometric specifications and the results of our tests of BFE. We first introduce our research strategy for identifying the price-setter in the England and Wales market for electricity. We then present results analyzing the impact of characteristics of generator portfolios and bid functions on bid markups.<sup>30</sup>

### 6.1 Identifying the Price-Setter

As discussed in Section 4, the predictions of BFE differ for the price-setter and non-price-setters. Identifying the price-setter is therefore a critical first step in testing the implications of BFE.

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<sup>29</sup>Relative to Wolfram (1998), the greatest difference in our datasets (due to our differences in the years considered) is in the level of bids and markups - ours average almost twice as much. In future revisions, we will assess the sensitivity of our results to including potentially irrelevant extramarginal bid information.

<sup>30</sup>In previous versions, we also explored the implications of BFE on “technological inefficiency” (TI) (i.e.  $k^*$ ). As the TI specification yielded qualitatively similar conclusions about the implications of BFE in the EW electricity market, we retain only the more informative bid markup specification. For completeness, however, we report the TI results in an Appendix.

We consider two approaches to this problem. The first relies on institutional information. As described earlier, National Power (NP) and PowerGen (PG) are the two largest generators of electricity in the EW market, accounting for between 74% and 54% of system capacity in our sample period. Moreover, they own an even greater percentage of marginal generation. As such, our prior beliefs are that one of them is more likely by far to be the price-setter in any demand period than any other producer. Casual inspection of the lumpiness of the representative bid functions in Figure 1 suggest National Power is the likely price-setter, at least for these bid functions. To allow for the possibility that either is a price-setter, however, in the specifications outlined below, we take care to allow for different effects for each of these firms relative to the other firms in the market. This allows the data to identify the price-setter(s) in the EW market.

A second approach would rely on the identity of the price-setter provided in the data. Several difficulties arise in using this information, however. The greatest is that the firm that actually sets price need not be the price-setter in a BFE. While BFE does predict that the firm owning the marginal unit will be the price-setter, uncertainty in demand or availability of the type described in Section 3.4.1 induce noise into the bidding process and can cause the non-price-setters to own the marginal unit.<sup>31</sup> Moreover, the identity of the price-setter is clearly endogenous: bid markups are functionally related to the price-setting firm. While generator fixed effects offset somewhat this concern by controlling for markups on individual units, that a given unit may be priced out in some days and not others remains a problem. Furthermore, appropriate instruments are difficult to find. As such, in the results that follow, we rely on the first approach outlined above. In subsequent revisions, we will assess the robustness of our results using the second approach.

## 6.2 Bid Markups and BFE

Our first specification uses the entire bid function of each generator to test the implications of BFE. Equation (2), repeated below, forms the foundation for the specification.

$$(q^o - k_i)[\Delta c(q^o + k_i + 1)] \geq [c(q^o + k_i) - \lambda - c(q^o - k_i)]$$

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<sup>31</sup>This can occur as the optimal bid price for the price-setter is just under the marginal cost of the first extramarginal unit of the non-price-setter. Noise in the price-setting process could cause this unit to “accidentally” set the price.

In the presence of bid lumping, it is more likely that negative demand shocks or positive availability shocks would cause the non-price-setter to set price as the price-setter lumps bids *to the right* of the marginal unit.

As units differ in size in the EW market, the tradeoff above should account for that difference. Let  $\Delta q_i \equiv q(k_i) - q(k_i - 1)$  be the size of the  $k_i^{th}$  unit. Incorporating this into the equation above yields:

$$(q^\circ - k_i)[\Delta c(q^\circ + k_i + 1)] \geq [c(q^\circ + k_i) - \lambda - c(q^\circ - k_i)]\Delta q_i \quad (3)$$

Equation (3) merely states that generators continue to price out units while the increase in profits on inframarginal units exceeds the loss in profits on marginal units. *At the margin*, there will therefore be a relationship between bid markups,  $P^* - c^* = c(q^\circ + k_i) - \lambda - c(q^\circ - k_i)$  and each of inframarginal capacity,  $(q^\circ - k_i)$ , marginal capacity,  $\Delta q_i$ , and the price change from pricing out the  $k_i^{th}$  unit,  $\Delta c(q^\circ + k_i + 1)$ . Taking logs and substituting an equality for the weak inequality above yields the following specification:

$$\begin{aligned} \ln(m_{it}) = & \ln(ACB_{it})\beta_1 + \ln(UC_{it})\beta_2 + \ln(\Delta c_{it})\beta_3 \\ & + X'_t\gamma_1 + u_i + \epsilon_{it} \end{aligned} \quad (4)$$

where the subscripts  $i$  and  $t$  denote generating unit  $i$  and the demand period  $t$ , and each of the variables included is described below.

The dependent variable, the (log) markup, denoted  $\ln(m_{it})$ , is defined as the bid price submitted for unit  $i$  minus the approximated marginal or fuel cost constructed for unit  $i$ . (log) Available Capacity Below,  $\ln(ACB_{it})$ , is defined as the amount of infra-marginal capacity with respect to unit  $i$  that a firm declares available on a given day. (log) Unit Capacity,  $\ln(UC_{it})$  is defined as the average number of megawatts declared available from a particular unit in a given day, and the (log) cost change,  $\ln(\Delta c_{it})$ , is defined as (one plus) the difference in marginal cost of unit  $i$ 's first and second extramarginal competitor's units.<sup>32</sup> Additional independent variables included to capture heterogeneity in cost and demand conditions over generating units and time include time dummies,  $X_t$ , and generating unit fixed effects,  $u_i$ .  $\epsilon_{it}$  is a random disturbance term that is assumed to be orthogonal to all the other variables.<sup>33</sup>

Note that Equation (4) is closely related to a similar specification in Wolfram (1998).<sup>34</sup> BFE, however, predicts that equation (4) applies *only* to the price-setter. Allowing for

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<sup>32</sup>We add one to the difference as in many cases competitors have adjacent units with equal marginal costs.

<sup>33</sup>Wolfram (1998) also includes the average daily load as an independent variable. We exclude this variable as average load only enters into the price setter's objective function as it affects inframarginal capacity, which is already included.

<sup>34</sup>In particular, we include the change in competitors costs,  $\Delta c_{it}$ , as a measure of the benefit of pricing

different firm effects along the lines described in subsection 6.1 above for each of the  $\beta$  above yields the following specification:

$$\begin{aligned} \ln(m_{it}) = & \ln(ACB_{it})D'\vec{\beta}_1 + \ln(UC_{it})D'\vec{\beta}_2 + \ln(\Delta c_{it})D'\vec{\beta}_3 \\ & + X'_t\gamma_1 + u_i + \epsilon_{it} \end{aligned} \tag{5}$$

where  $D \equiv (1, NP, PG)'$  is the  $3 \times 1$  vector consisting of a constant, a dummy variable for National Power (NP), and a dummy variable for PowerGen (PG),  $\vec{\beta}_s \equiv (\beta_s, \beta_s^{NP}, \beta_s^{PG})$  for  $s \in \{1, 2, 3\}$  is a corresponding  $3 \times 1$  vector of parameters measuring the effect of variable  $s$  on all firms, an incremental effect for NP, and an incremental effect for PG. These dummies will allow us to detect whether there are any consistent asymmetries in the bidding strategies of National Power, PowerGen, and other generators.

**Remarks** Several remarks are in order. First, note that identification of the model is based on the assumption that  $\epsilon_{it}$  is orthogonal to the included regressors. This assumption requires perhaps the greatest justification for Available Capacity Below (ACB).<sup>35</sup> We feel this is a reasonable assumption for several reasons. As generators decide simultaneously on both bids and availability, ACB is plausibly endogenous. Including fixed effects, however, specifically addresses this issue. Any persistent relationship between bidding and availability for a given unit will be captured in the unit fixed effect. Identification of the effect of ACB therefore arises from variation *over time* for a given unit. This happens for two primary reasons. First, generating units require regular maintenance and repair, a process which is plausibly exogenous to the firm's bidding decisions. As this happens, however, the ACB for a given unit will vary and so too the incentives to price out that unit. Second, new units are entering and being retired in this period. For now, we maintain the assumption that these decisions are exogenous to the bidding behavior of firms.<sup>36</sup> In subsequent revisions,

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out an additional unit and exclude a variable (called Bid Impact) indicating when the unit of a competitor that has marginal cost closest to unit  $i$  is unavailable. Since  $\Delta c_{it}$  is calculated daily, it will incorporate if a competitors unit is unavailable.

<sup>35</sup>The remaining regressors, unit capacity and the change in competitors' costs are plausibly exogenous to the firm.

<sup>36</sup>While an admittedly strong assumption, there is some justification for it in the literature. The NGC surveys plant disconnections and decommissionings for possibility of strategic manipulation. It has found no support for this view (NGC, 1999). Furthermore, Rainbow, Doyle, and Price (1993) cite numerous non-strategic factors for closing coal plants during this period (the vast majority of plant closures in our sample are coal plants). Furthermore, entry has been exclusively with the construction of CCGT plants whose operating characteristics (and consequent impact on bid functions) are largely outside the control of each firm.

we will test these identification assumptions.

Note further that basing the analysis of the entire bid function of each firm reduces concerns about mis-measurement in the analysis. For instance, included in  $\epsilon_{it}$  are elements assumed to be observed by firms but not the econometrician. These include shocks to bidding behavior (e.g. bidding heterogeneity), shocks to cost (e.g. fuel price variation), and shocks to expectations of other firms' availability (e.g. rival units' maintenance and repair).<sup>37</sup> This would be a problem if we were looking only at the marginal unit (as in conventional supply and demand analysis), where shocks could introduce correlation between the *observed* ACB and  $\epsilon$ . Along the entire bid function, however, ACB is fixed and cannot respond to shocks in  $\epsilon$ .<sup>38</sup>

Finally, while we are looking at the entire bid function, note that equation (5) above holds only for the *marginal unit* priced out by the price-setter. As such, it will accurately describe the price-setter's behavior only in instances where they behave as if each unit will be marginal at some point in the day. If the price-setter is pricing out multiple units, however, the relationship above may not apply for most of the units.<sup>39</sup> We return to this point later on in our discussion of the impact of shocks to the price-setter's bid function.

### 6.3 Results

The first two columns of Table 2 present the parameter estimates for specifications (4) and (5). From the first column, the signs are as expected for the impact of ACB and UC, but are opposite expectation for  $\Delta c$ . Furthermore, as equation (3) implies markups should increase one-for-one with percentage increases in ACB, the magnitude of this effect is also off.<sup>40</sup>

Relaxing the imposition of a common impact across firms, however, yields very different

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<sup>37</sup>Note that while unobserved demand shocks are not relevant to the firms' bidding (as SMP is set by the interesection of the day-ahead aggregate bid function and the day-ahead demand estimate), shocks to rivals' availability has the effect of shifting a firm's residual demand curve.

<sup>38</sup>Nonetheless, it is possible that cost shocks could induce a bias in the ACB coefficient. To the extent these represent unobserved changes in fuel prices, they are likely to be correlated across units for a given time period but uncorrelated over time. As such, we could use levels of  $ACB_{is}$  for  $s \neq t$  to instrument for  $ACB_{it}$ .  $s$  should be chosen to maximize the correlation between the two measures of availability while allowing for enough time for persistence in the cost shock to not be a problem.

<sup>39</sup>It still could if the optimal bid for an infra- or extra-marginal unit *assuming it were marginal* was below the bounds on the optimal bid function.

<sup>40</sup>Similar relationships should exist between markups and UC and  $\Delta c$ , though these are sufficiently discontinuous that such a test would be extremely sensitive to influential observations.

conclusions. While all generators appear to bid larger markups the greater their inframarginal capacity ( $\beta_1 > 0$ ), the effect is 74% larger for National Power and 35% larger for PowerGen.<sup>41</sup> The impact of unit capacity, while modest for other generators, is 7 times as great for National Power and more than 3 times as great for PowerGen. The effect of cost differences, while strongly negative for other generators, is less negative for PowerGen and positive for National Power.<sup>42</sup>

Two conclusions suggest themselves from these findings. First, there are significant asymmetries across firms in bidding strategies which cannot be justified by size differences across firms.<sup>43</sup> Second, *all* bidders bid positive markups, though the magnitude of the effects favor a “strategic ordering” of National Power followed by PowerGen followed by the remaining firms.

Further justification of BFE arise when we consider that bid functions are fixed throughout the day. Specifically, the extension of BFE to multiple demand periods does not exclude equilibria where firms share the role (over days or within a day) of acting as the price-setter. If this is the case, however, two things should be true. First, the *sum* of the coefficients on ACB across firms should equal 1. Second, the coefficient on ACB for each firm should correspond to the frequency within days that they are the price-setter. Why should these hold? Recall that BFE predicts markups should increase one-for-one with percentage increases in ACB for the price-setter and be unrelated to ACB for non-price-setters. If this is true and price-setting rotates among firms over time, (a) the coefficient on ACB will reflect the average incidence of price-setting in the sample and (b) the sum of these averages should equal 1. For our specification,  $\sum_k \beta_{1k} = 1.05$  with  $\frac{\beta_{1k}}{1.05} = 0.44, 0.31, 0.25$  for National Power, PowerGen, and others, respectively.<sup>44</sup> The corresponding share of times each firm sets the price for our sample is 0.50, 0.33, and 0.16.

The balance of the results abstract from the issue of multiple demand periods by focusing on the peak demand period (where bid functions are unaffected by possible supply in other periods). The third column of Table 2 generalizes the specification to allow for differing

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<sup>41</sup>Note: parameters labeled NP and PG measure *incremental* effects. For example, the aggregate effect of ACB for NP in the 2nd column is  $0.26 + 0.19 = 0.45$ .

<sup>42</sup>In all cases discussed here, we can reject the null hypothesis of equality of effects for each reported effect below at conventional confidence levels.

<sup>43</sup>ACB controls for differences in available capacity below and finds significantly different effects for firms of different sizes for the same inframarginal capacity.

<sup>44</sup>The  $\chi^2$  statistic = 27.49, well above reasonable critical values. The reported standard errors, however, have not been corrected for possible heteroscedasticity or autocorrelation.

effects on the level and impact of ACB on markups for all units within 5,000 MWh of the peak demand for the day.<sup>45</sup>

We turning first to the markup levels for units near the peak, reported in the third column of Table 2, to test the second implication of BFE, that bid markups should be higher for the price-setter and lower for non-price-setters at the peak. In fact, this is exactly what we find. Unit markups are significantly higher for National Power, slightly lower for other firms, and significantly lower for PowerGen near the peak load relative to non-peak loads.

The final implication, that of bid lumping for the price-setter, is also supported by the results in the third column. The bid function of one firm, National Power, deviates in ways predicted by the theory. Near the peak, markups jump ( $\beta_0^{\text{Peak},NP} > 0$ ) and flattens ( $\beta_1^{\text{Peak},NP} < 0$ ) for units near peak demand. By contrast the slope of the bid function for units owned by PowerGen and other firms actually increases near the peak. Figure 4 [Pending] provides a picture representative of these effects.<sup>46</sup> Both these findings are consistent with National Power being the predominant price-setter in the EW market for electricity.

The fourth column of Table 2 show the danger of estimating the specification emphasizing marginal conditions (as in the first two columns of the table) in the peak demand period. In periods of peak demand, the price-setter has the greatest incentive to price out units. As such, it is reasonable to think that  $k^*$  will be greatest in these periods.<sup>47</sup> In the presence of shocks to demand or availability, however, the price-setter has a strong incentive to ensure bids do not exceed the maximum of  $P^*$ , the clearing price, and  $c(q)$ , the marginal cost of rivals' extramarginal units. In this setting, the relationship tested by the marginal condition in equation (3) no longer holds: bid markups for a given unit are *not* related to the capacity of that unit and the incremental marginal cost of rivals' first and second extramarginal units. Instead, markups for the marginal unit are related to the capacity *of all  $k^*$  units* and the incremental cost of rivals' first through  $k^{*th}$  unit. Indeed, with bid lumping, one would expect a negative relationship between markups and rivals' cost changes, as markups decline over the (lumped) extramarginal units while cost changes dramatically increase (cf. Figure X).<sup>48</sup> In fact, this is what we find. If subsequent revisions, we intend to exploit the

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<sup>45</sup>We chose this magnitude as it is close to the median measured “technological inefficiency” in the market. See the Appendix for a description of this measure.

<sup>46</sup>Include Figure with example of bid functions without and with lumping.

<sup>47</sup>This is also suggested in Figure 1 from von der Fehr and Harbord (1993).

<sup>48</sup>At peak periods, generators are sometimes required to bring on very high-cost and rarely used units. This induces large changes in  $\Delta c$ .

non-marginal predictions of BFE for further implications of the theory.

## 7 Application: Measuring the Effects of Changes to Generation Portfolios

During the years since privatization there has considerable criticism by regulators and politicians regarding the magnitude of the sustained markups in generation. This criticism has motivated several actions taken by regulators, including forced divestiture of several plants that are thought to play a role in the exercise in market power aimed at manipulating prices away from their competitive level.

Within this framework, it is possible to ascertain the effects that forced divestiture and other changes to generation portfolios have had on markups. Table X [pending] presents the results of several approaches to this problem. The first column presents the average percentage change in markups predicted by the estimates in Table 2 associated with the observed reductions in plant availability across our time period. The second column presents the same information for the peak demand period during the day. The results ... [pending]

A fundamental difficulty with this exercise is the endogeneity of long-run plant reduction decisions. If these are voluntary reductions on the part of firms, they could be done to increase the profitability (and markups) of the remaining generators.<sup>49</sup> As the results presented so far rely on reduced form estimates of the relationship between markups and availability, they may not be stable under reductions in availability by firms. To address this possibility, the third column of Table X presents the average percentage change in peak markups *predicted by BFE* for the same observed changes. The results... [pending]

A final exercise demonstrates the usefulness of BFE as a policy tool for the design and governance of electricity markets. Rather than predict the change in markups associated with observed capacity reductions, we ask what would be the change associated with specific policies?

What types of policies would yield the greatest impact to markups? Recall the primary prediction of BFE is that bid markups reflect strategic price-setting by a single price-setter facing a residual demand curve. Bid markups are a function of the level and slope of this

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<sup>49</sup>See Wolak and Patrick (1998). This issue is simply the long-run analog to possibly endogenous plant availability decisions. We intend to address this issue in future revisions.

curve, itself given by industry demand less the portfolio of non-price-setters' generation capacity. Significant inframarginal capacity by the price-setter increases the level of the residual demand curve while a concentration of capacity at the margin decreases (in absolute value) its slope.<sup>50</sup> Either of these effects increase bid markups.

Columns 4-7 of Table X present the average percentage change in peak markups associated with policies of ... [pending]. These will be chosen to highlight both profit-maximizing and markup-minimizing strategies. These should prove useful to regulators guiding the long-run management of generation capacity in newly formed electricity markets by providing intuition for the types of changes in generation portfolios likely to increase or decrease market power among generators.

## 8 Conclusion and Extensions

The purpose of this paper is test the implications of Bid Function Equilibria in the British spot market for electricity. Using data on bid functions in this market between 1993 and 1995, we find strong empirical support for the theory. Strong and persistent asymmetries exist in bid functions consistent with a single firm, National Power, as the predominant price-setter in a BFE. Implications for the British market remain to be considered.

Having demonstrated that there is substantial empirical evidence to support the use of Bid Function Equilibria to model generators in spot markets for electricity, it is important to recognize the potential for extensions of this research. Perhaps the most promising extension is to apply Bid Function Equilibria in a structural model of short-run strategic availability decisions by generators. Furthermore, the wealth of data and repeat nature of play in electricity markets provides ample opportunity to study tacit collusion in the market. BFE is critical to this undertaking by characterizing expected outcomes in a static environment relative to those obtained through repeat play.

BFE has significant potential both in other electricity markets as well as analyzing multi-unit auctions in general (e.g. electromagnetic spectrum or some Internet auctions). For example example, the past year has seen an emergence of a literature examining market power issues in the newer California deregulated market for electricity.<sup>51</sup> Similar issues have

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<sup>50</sup> Also important to the slope are the cost characteristics of competing firms at the margin.

<sup>51</sup> See for example Borenstein and Bushnell (1999), Bushnell and Wolak (1999), Borenstein, Bushnell, and Wolak (1999), and Wolfram (1999a).

arisen in the New England (NEISO) and Pennsylvania, New Jersey, and Maryland (PJM) markets. Although we would have to generalize BFE to include idiosyncratic aspects of these markets, these additions would not undermine its basic features. While the analysis of multi-unit auctions outside electricity would require the important and non-trivial extension of BFE to the case of imperfect information about rivals' bid functions, it would likely provide important insights into economic performance in these markets.

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## A Technological Inefficiency: Evidence of BFE

To measure the technological inefficiency that is a by-product of BFE requires constructing an aggregate marginal cost curve and measuring the deviation from least-cost generation induced by generators' bid functions. We construct the aggregate marginal cost function (AMC) using actual market data on input prices and plant efficiencies. The AMC is used to rank each generating unit according to the position of the generating unit's first MW/h along this function. The position of a particular unit is calculated each day by summing the available capacity from each unit with a lower marginal cost.<sup>52</sup> For example, if there are 10 generating units, each capable of producing 20 Mw/h's then the position of the highest cost unit is 180. The natural log of the absolute value of the difference in MW/h's between the point where AMC intersects with the load forecast,  $L_t$  and the rank of the price setting unit,  $RPS_t$  forms the our dependent variable,  $\ln TECH_t = |RPS_t - L_t|$ . We use the absolute value because demand variation throughout the day along with possible supply or demand shocks make it likely that the actual price setting unit may sometimes belong to an aggressive bidder. As the price-setter bids greater than marginal cost for marginal units, if the price-setter owns the marginal unit, then  $(RPS_t - L_t) = -k_1^*$ . If, however, an aggressive bidder owns the marginal unit, it is a relatively high-cost unit and  $(RPS_t - L_t)$  should be about equal to  $+k_1^*$ . As defined,  $\ln TECH_t$  should therefore be unaffected by the identify of the owner of the marginal unit.

Our baseline model is then given by the following specification:

$$\begin{aligned} \ln TECH_t = & \beta_0 + \ln(ABL_t^{NP})\beta_1 + \ln(ABL_t^{PG})\beta_2 \\ & + \ln(ABL_{Peak}^{NP})\beta_3 + \ln(ABL_{Peak}^{PG})\beta_4 \\ & + X_t'\gamma + \epsilon_t \end{aligned} \quad (6)$$

where  $\ln TECH_t$  is defined above, the subscript  $t$  denotes a particular half hour demand period,  $ABL$  measures the Availability Below Load, defined as the amount of available capacity belonging to the generator that is inframarginal with respect to the load in period  $t$ , NP denotes National Power, PG denotes Power Gen, Peak denotes the peak demand period of the day,  $X_t$  denotes day and month dummies, included to capture heterogeneity in cost and demand conditions over time, and  $\epsilon_t$  is a random disturbance term that is assumed to be orthogonal to all the other variables.

The implication of BFE tested in this specification is that technological inefficiency should increase with the inframarginal capacity of the price-setter. As described above, we allow for asymmetric behavior for National Power and PowerGen, the two major generators in the BSM. We also allow separate (incremental) effects for the peak demand period as the

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<sup>52</sup>In contrast to our approach, Wolfram (1998) ranks units according to bid prices rather than marginal costs, and then uses instrumental variables to account for the endogeneity of this measure.

confounding effects of a common bid function for all 48 periods are absent during the peak demand period.

Table 3 presents the OLS estimates of the critical parameters,  $\beta_0 - \beta_4$ , in the technological inefficiency specification. First note the strong asymmetry in the impact of inframarginal capacity (ABL) for National Power ( $\beta_1$ ) versus PowerGen ( $\beta_2$ ): increases in ABL increase technological inefficiency for the former and decrease it for the latter. In both cases, we can reject the null hypothesis of no impact of ABL. This effect is even stronger during peak demand periods ( $\beta_3$  v.  $\beta_4$ ), though one cannot reject at very low confidence levels the null hypothesis of no incremental difference. Together, these findings are broadly consistent with a BFE with NP as a price-setter.

Interpreting the magnitude of the coefficients above requires care. Increases in ABL for either firm can occur for two reasons. First, demand might increase, causing ABL for both firms to rise.<sup>53</sup> In this case, a 10% increase in demand yields a  $(1.97\% - 1.70\% = )$  0.27% increase in technological inefficiency.<sup>54</sup> Alternatively, ABL may increase due to a reallocation of generation between firms, e.g. due to maintenance or outages. In this case a 10% increase in National Power's ABL corresponds to a 10% decrease in PowerGen's ABL, yielding a  $(1.97\% + 1.70\%) = 3.67\%$  increase in technological inefficiency. This effect is even greater when one considers the greater size of National Power.<sup>55</sup>

In sum, these results provide strong evidence in favor of BFE: there is an asymmetry in effects of inframarginal capacity on technological inefficiency across firms and the asymmetry suggests National Power is the (frequent) price-setter. As explained earlier, this systematic behavior of technological inefficiency is a direct implication of BFE and is not directly implied by other theories such as Cournot or SFE.

While useful, however, this specification is limiting. The dependent variable,  $\ln |RPS_t - L_t|$ , cannot directly report on the impact to markups of strategic bidding. Since markups are critical to the design and governance of electricity markets, we next consider an additional specification based on bid markups analagous to that used by Wolfram (1998).

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<sup>53</sup>Demand is simply equal to the sum of ABL for all generators.

<sup>54</sup>This exercise assumes each generators ABL increases by an amount proportional to the increase in demand. This will be the case if the share of generation provided by each firm across the demand shock equals their average share before the shock.

<sup>55</sup>So that a 10% increase in NP ABL corresponds to a greater than 10% decrease in PG ABL.

Table 1: Sample Statistics

Variable	Mean	SDev	Mean	SDev	Mean	SDev
Across Entire Bid Function						
Bids, Cost, and Markups:						
Bid Price	131.89	299.56	241.97	406.22	108.05	203.30
Marginal Cost	19.45	11.19	21.45	9.71	20.31	11.18
Markup	116.29	293.27	221.65	400.25	89.98	197.25
Log(Markup)	2.04	2.36	3.12	2.58	2.30	2.14
Regressors:						
Avail. Cap. Below	31580	16101	36458	13244	35281	13303
Log(ACB)	8.39	1.58	9.36	0.76	17.75	1.66
Unit Capacity	276	226	261	232	305	221
Log(UC)	5.04	1.28	4.88	1.35	5.12	1.36
$\Delta$ Cost	9.55	72.76	6.50	45.72	10.67	87.32
Log( $\Delta$ Cost)	0.61	0.96	0.47	0.87	0.65	0.98
Natl. Power	0.42	0.49	—	—	—	—
PowerGen	0.25	0.43	—	—	—	—
Near Peak Demand Periods						
Regressors:						
Near Peak Demand	0.16	0.37	0.20	0.40	0.24	0.43
ACB*Peak	6219	14265	7513	15076	9398	16962
Log(ACB)*Peak	1.43	3.27	1.88	3.72	2.11	3.78
Observations	192,168		80,691		47,339	

Table 2: Parameter Estimates  
Markup Specification

	Specifications:	Baseline		Firm Effects		Peak Demand Pds. Level & ACB		Peak Demand Pds. All Variables	
Param	Variable	Est	StdErr	Est	StdErr	Est	StdErr	Est	StdErr
Effects for all demand periods:									
$\beta_1$	$\ln(ACB_{it})$	0.36	0.00	0.26	0.01	0.26	0.01	0.26	0.01
	$\ln(ACB_{it}^{NP})$			0.19	0.01	0.23	0.01	0.20	0.01
	$\ln(ACB_{it}^{PG})$			0.09	0.01	0.07	0.01	0.07	0.01
$\beta_2$	$\ln(UC_{it})$	-0.36	0.01	-0.11	0.01	-0.11	0.01	-0.11	0.01
	$\ln(UC_{it}^{NP})$			-0.67	0.02	-0.66	0.02	-0.69	0.02
	$\ln(UC_{it}^{PG})$			-0.30	0.05	-0.27	0.05	-0.35	0.05
$\beta_3$	$\ln(\Delta c_{it})$	-0.03	0.00	-0.16	0.00	-0.15	0.00	-0.15	0.00
	$\ln(\Delta c_{it}^{NP})$			0.32	0.01	0.32	0.01	0.33	0.01
	$\ln(\Delta c_{it}^{PG})$			0.08	0.01	0.08	0.01	0.07	0.01
Incremental effects near peak demand periods:									
$\beta_0^{\text{Peak}}$	All Firms					-0.25	0.08	0.85	0.17
	NP					1.03	0.12	-0.76	0.21
	PG					-0.98	0.12	-3.72	0.22
$\beta_1^{\text{Peak}}$	$\ln(ACB_{it})$					0.05	0.01	0.08	0.01
	$\ln(ACB_{it}^{NP})$					-0.13	0.02	-0.11	0.02
	$\ln(ACB_{it}^{PG})$					0.10	0.02	0.11	0.02
$\beta_2^{\text{Peak}}$	$\ln(UC_{it})$							-0.23	0.03
	$\ln(UC_{it}^{NP})$							0.29	0.03
	$\ln(UC_{it}^{PG})$							0.45	0.03
$\beta_3^{\text{Peak}}$	$\ln(\Delta c_{it})$							-0.05	0.03
	$\ln(\Delta c_{it}^{NP})$							-0.46	0.04
	$\ln(\Delta c_{it}^{PG})$							0.05	0.03

Table 3: Parameter Estimates  
 Technological Inefficiency Specification<sup>a</sup>

Param	Variable	Estimate	Std. Error
$\beta_0$	Constant	8.125	0.202
$\beta_1$	$\ln(ABL_t)^{NP}$	0.197	0.020
$\beta_2$	$\ln(ABL_t)^{PG}$	-0.170	0.020
$\beta_3$	$\ln(ABL_{Peak})^{NP}$	0.255	0.148
$\beta_4$	$\ln(ABL_{Peak})^{PG}$	-0.245	0.157

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<sup>a</sup>See Appendix for discussion.