A Model of Collusion Timing*

Nicolas de Roos†

Department of Economics,

Yale University

November, 2000

Job Market Paper

Abstract

For any cartel, the entrance of a new competitor is a dangerous development. This paper develops a dynamic model that, in contrast to much of the collusion literature, affords a substantial role to entry. Heterogeneous firms make collusion, entry, exit, and investment decisions within an evolving environment. The model is calibrated using demand and cost estimates from the lysine market, a market in which collusion recently played a dramatic role. The collusive agreement adopted is based on a simple rule of thumb which is motivated by the details of collusion in the lysine market. The model provides one rationale for the emergence of a price war upon entry, and a means for examining the timing of the decision to reinstigate collusion. It is found that an entrant will wait until it has built up a market share comparable to its competitors before agreeing to collude. Further, allowing for collusive possibilities tends to give rise to a less concentrated industry with reduced consumer welfare. The model is able to characterise the experience with entry in the lysine market and could provide insights into any market with collusive possibilities, the potential for entry, and some uncertainty about the characteristics of potential entrants.

*I am grateful to my advisers, Steve Berry, Ariel Pakes, and David Pearce for invaluable guidance. I would like to thank Yianis Sarafidis, Chris Timmins, Pat Bayer, John Rust, Chris Avery, Dirk Bergemann, Martin Pesendorfer, Eugene Choo, Günter Hitsch, Katja Seim, Hugo Benitez-Silva, Ricky Lam, Jason Draho, Oleg Melnikov, Nikolay Moshkin, Rupa Athreya, Amil Dasgupta, and the seminar participants at Yale and Harvard for useful comments, support, and encouragement. I also thank John Connor for patient, detailed discussions of the lysine market, and Gautam Gowrisankaran for providing computer code for the Pakes-McGuire algorithm.

†Author's contact information: e-mail: nicolas.deroos@yale.edu; phone: (203) 432 3595; homepage: www.econ.yale.edu/ njd7

1 Introduction

The emergence of a new competitor poses a serious challenge for any collusive arrangement. Careful examination of the experience of collusion in the market for lysine reveals that entry played a pivotal role. However, most theoretical models of collusion ignore the role of entry. In fact, most theoretical models allow no role for any firm actions that alter the collusive environment. This paper develops a dynamic model of collusion in which firms must respond to, and can influence, the evolution of their environment. Firms are free to make decisions about entry, exit, investment, and whether to collude. The model provides one rationale for the emergence of a price war upon entry, and a means for examining the timing of the decision to reinstigate collusion. It also facilitates a comparison between a world with and without collusive possibilities along the dimensions of industry structure and consumer and producer welfare.

The model is applied to the market for lysine, where the entry of a competitor dramatically altered the experience with collusion. Demand and cost estimates for the lysine market are used as parameters in the firm profit functions. Knowledge of the operation of collusion in that market is used to shape key assumptions of the model. In particular, the nature of the collusive agreement in the model reflects the simple rule of thumb that operated in the lysine market. However, the model has a more general applicability. It could provide lessons for any market with collusive possibilities, the potential for entry, and some uncertainty about the characteristics of potential entrants.

We might expect the response of incumbent firms to the entry of a competitor to depend critically on how serious a threat the entrant is expected to be. Incumbent firms may seek to drive out a feeble entrant, or simply ignore it. In contrast, incumbents may need to completely recast their behaviour upon the entry of a more substantial competitor. The model is designed to examine such a situation. Agreement of the recent entrant is required before a collusive arrangement can be settled. It was just this kind of entry that unsettled the lysine market. Hence, the lysine market provides an interesting examination of the model's performance. It should be noted, however, that due to the rich nature of the experience with collusion in the lysine market, we cannot hope to explain all the nuances of firm behaviour in that market. Rather, I characterise a key ingredient in the collusive experience in the lysine market.

I follow Fershtman and Pakes (2000) in developing a model with heterogeneous firms operating in a changing environment. Firms engage in repeated quantity competition in a market for a homogeneous product, subject to capacity constraints.[1] These capacity constraints determine the profits enjoyed by firms. Each period, firms can choose to indulge in investment spending

[1] In applying this model to the lysine market, I interpret capacity as the stock of interested customers at a firm's fingertips, and not as the stock of physical production capacity.

aimed at increasing capacity, and thus allowing greater profit opportunities in the future. Firms are free to exit should their competitive environment become too hostile. Potential entrants join the fray if the competitive environment becomes conducive to them. Incumbent firms make decisions about whether to collude, taking these factors into account.

Every cartel must decide how much to produce and how to split the proceeds from collusion. Economic theory provides no clear guidance on this matter. The present model appeals to empirical observation by adopting the simple rule of thumb that operated in the market for lysine. The collusive agreement specifies that participating firms receive the market share they enjoyed at the time collusion was initiated and that these market shares are maintained throughout the agreement. This assumption appeals to the bounded rationality of firms. The rule of thumb is only (at best) approximately optimal, but may be an easy rule on which to coordinate. This is a departure from most existing models of collusion, which assume the existence of an optimal collusive mechanism, or at the very least a collusive mechanism involving a high degree of rationality on the part of firms. That is, participating firms are assumed to devise complex conditional punishment and reward regimes to create the incentives necessary to sustain an optimal degree of collusion.

An additional motivation for this rule of thumb relates to the inherent uncertainty of incumbent firms about the characteristics of a new entrant. Incumbent firms will have only incomplete information about crucial attributes of an entrant, such as its marginal cost or production capacity. These variables are key elements in the determination of an optimal collusive arrangement. We might expect an entrant to build up market share over time as it advertises, customers learn about its product, and it develops a distribution network, but exactly how much of the market should it receive in an immediate agreement? Unless the entrant can credibly convey its private information, it may not be possible to negotiate an optimal agreement.

In the absence of an optimal collusive mechanism, firms may gravitate toward an obvious arrangement that could serve as a focal point. In this context, a clear focal point is the existing market shares of the firms. Schelling (1960) presents experimental evidence suggesting that without communication, parties with coinciding or opposing interests will tend to coordinate on a focal point even if it yields an asymmetric or indeed "unfair" outcome. He argues that the status quo also provides a strong attraction in situations affording explicit communication. Schmalensee (1987, p.357) notes that if side payments are impossible and firm costs are asymmetric, colluding firms may wish to maintain market shares at their non-collusive levels, particularly if firms have imperfect information on their rivals' costs. In the next section, I briefly discuss the manifestations of this problem in the lysine market.

The crucial trade off for the entrant (or for a firm with a small capacity relative to its competitors) can then be summarised as follows. Should the entrant agree to collude today, it can enjoy collusive payoffs immediately, but will have a minor share in the collusive agreement for its working life. If the entrant waits, it obtains the reduced profits arising from the non-cooperative regime, but has the prospect of a potentially much larger share of the collusive regime in the future.

Several other assumptions play a central role in the model. First, there are convex adjustment costs to increasing output. Such costs could take the form of physical capacity constraints, advertising, customer search, or distribution costs. Second, entry is assumed to interrupt the collusive agreement as firms must negotiate a new agreement that accounts for the entrant. This assumption follows naturally from our consideration of entrants of substance rather than fringe entrants. Finally, firms are able to collude on output, but not investment.[2] This restriction implicitly assumes that firms are better able to coordinate on output than investment. This may arise because output is easier to observe and verify, and the returns to investment are uncertain. This is particularly so if we interpret investment more broadly than simply investment in productive capacity.

The results suggest that if we permit entry and investment in an industry with collusive possibilities, we might expect a much richer set of firm behaviour. In equilibrium, we observe periods of successful collusion, price wars due to entry and punishment, and entry deterrence. An entrant will tend to build up a market share comparable with its competitors before colluding, unless imminent entry is anticipated or entry deterrence might prove fruitful. The reason is that, by waiting and foregoing the collusive profits in the short-run, the entrant can obtain a larger slice of the payoffs to collusion in the future. Relative to a world without collusion, if we permit collusion, we tend to obtain a less concentrated industry as potential entrants seek to benefit from the increased payoffs associated with collusion. Despite this result, we tend to observe higher prices and reduced consumer surplus if we allow collusive possibilities.

The rest of the paper is organised as follows. Below, I briefly discuss some of the related literature on collusion. Section two describes the operation of collusion in the lysine market. This provides both a context for discussion and a real application for the model. Section three discusses the model. Section four sketches the computational algorithm used to solve the model. Results are presented in section five. The type of firm behaviour observed in equilibrium is described and compared to a model without the possibility of collusion. The model is simulated, generating industry and welfare statistics. I then examine the flavour of short term dynamics we might observe by simulating the model repeatedly, choosing as a starting point the industry structure prevailing in the lysine market at the time of a major entry into the market. Finally, I illustrate the implications of changing key parameters for the model's predictions. Avenues for refinement and further research are discussed in section six. Concluding remarks are contained

[2] Fershtman and Pakes (2000) adopt this assumption in a similar context.

in section seven.

1.1 Related Literature

The collusion literature has highlighted the tension between the profitability of a collusive agreement and the enforcement of the agreement. Stigler (1964) discusses the industry conditions conducive to the enforcement of a cartel. Friedman (1971) demonstrates that the incentives for collusion can be maintained through a regime of punishments and rewards. The patience of the participating firms is crucial for determining the viability of collusion. The models of Green and Porter (1984) and Abreu, Pearce, and Stacchetti (1986) reveal that, if firms cannot perfectly monitor the behaviour of their rivals, periodic price wars may be necessary to maintain the incentives for collusion.

An assumption common to these models is that firms are symmetric and operate in an unchanging environment. However, introducing dynamic considerations and firm heterogeneity can dramatically alter the nature of collusion. Rotemberg and Saloner (1986) demonstrate that allowing demand to vary with time alters the nature of the collusive agreement that can be sustained. A high level of demand results in an increased temptation to cheat on the collusive agreement, making collusion less effective in peak demand periods. Compte, Jenny and Rey (1997) consider the problem of optimal collusion when firms face asymmetric capacity constraints, finding that collusion is more difficult to sustain with asymmetric firm capacities when aggregate capacity is limited.

Fershtman and Pakes (2000) develop a dynamic model of collusion with heterogeneous firms. In their collusion decisions, firms explicitly consider the entry, exit, and investment decisions of incumbents and potential competitors. It is found that collusion is particularly hard to sustain if one of the firms is likely to exit in the near future. Moreover, allowing for the possibility of collusion can have a dramatic impact on industry structure. The present paper differs in several respects, two of which are crucial for the flavour of the model's predictions. First, the nature of the collusive agreement is markedly different. Fershtman and Pakes assume that the terms of the collusive agreement are negotiated each period through a static Nash bargaining game of perfect information. In the current paper, the collusive agreement is designed to capture some of the informational asymmetries, especially with respect to a new entrant. The collusive agreement specifies that, for the life of the agreement, firms receive shares in the cartel profits based on their market shares at the time of the agreement. This reflects the inability of competing firms, and a recent entrant in particular, to credibly convey key information that would determine its market share under collusion. The implication is that a prospective entrant must establish itself through a price war before it can receive favourable terms from a collusive agreement. In contrast, in the model of Fershtman and Pakes, an entrant will often anticipate entering a comparatively benign industry in which collusion is maintained despite entry.

Second, in the current paper the flavour of punishment is quite different. In the model of Fershtman and Pakes, as in most of the literature on collusion, punishment is intended to deter deviation from the collusive agreement by any firm attempting to skim additional profits in the short term before its competitors can detect its deviant behaviour and coordinate a response. By its nature, this kind of punishment will not be observed in equilibrium. In the present model, punishment is more characteristic of a general deterioration of the agreement. That is, a firm will willingly invoke the punishment regime if it no longer believes the collusive agreement is in its interests. As we shall see, this kind of punishment is observed in equilibrium.

Other authors have examined empirical applications of collusion models. Porter (1983) finds the existence of price wars in the Joint Executive Committee railroad cartel and concludes that the pattern of pricing behaviour is consistent with the Green and Porter model. Ellison (1994) reexamines this data set, contrasting the Green and Porter model with the model of Rotemberg and Saloner. Levinstein (1997) compares the price wars in the pre-World War I bromine industry with the predictions of the Green and Porter and Abreu, Pearce, and Stacchetti price war models. It is argued that the nature of the most severe price wars is more consistent with a bargaining and renegotiation process than with problems of imperfect monitoring and demand uncertainty. Based on an examination of the recent international lysine price-fixing conspiracy, de Roos (2000) finds that imperfect monitoring models of price wars provide only limited guidance for firm behaviour.

2 Collusion in the Market for Lysine

The market for lysine provides a unique opportunity to study the operation of an international cartel in a legal environment hostile to collusion. The original motivation for the current model arose from an examination of this market. In this section, I will provide a brief description of the operation of collusion in the lysine market.[3] This should serve the dual purposes of clarifying the nature of the model and providing a real application. I will first describe some essential features of the lysine market, and then describe briefly the history of collusion in the lysine market.

Lysine is an essential amino acid for the lean muscle development of hogs and poultry. Being a chemical compound, it is a homogeneous product. There is a great deal of heterogeneity

[3] For more detailed discussion of the operation of collusion in the market for lysine, see Connor(1998a,1998b) and de Roos(2000).

in the capacities, locations, and costs of firms in the lysine market. Prospective entrants face entry barriers arising from technological patents and the cost and length of time required to build a new lysine plant. Prices and volumes of lysine suppliers are contractual and not directly observable.

By the end of the 1980s, the lysine market was dominated by three Asian based firms; Ajinomoto and Kyowa Hakko of Japan, and Sewon of South Korea. Testimony by officials at Ajinomoto reveals that price fixing was a feature of this period (Connor, 2000). In 1989, Archer Daniels Midland (ADM) began construction of the world's largest lysine factory. ADM began production in February 1991, precipitating a severe price war. During the price war, Ajinomoto and Kyowa Hakko tried unsuccessfully to raise prices several times. Subsequently, ADM suggested the formation of a lysine trade association, with the first meeting taking place in June 1992. Lysine prices rose shortly afterwards. Thus, in the market for lysine, collusion was interrupted by the emergence of a large-scale entrant.

A cartel comprising the major firms operated with moderate success over the next year. However, no consensus was achieved on the operating mechanism of the cartel. Cooperation in the monitoring of sales and prices was scant. Firms were suspicious of rivals' costs and capacities. A second price war began in early 1993, and was resolved later that year. The character of the subsequent phase of collusion was considerably different. Uncertainty about costs and capacities was largely resolved. A centralised monitoring scheme was initiated. A system of global volume quotas was agreed to, based on the current market shares of firms. A compensation scheme involving intra-cartel sales operated for those firms not meeting their quotas.

The cartel operated successfully for a period of about two years before the FBI intervened in June 1995. The behaviour of prices following the breakup of the cartel is consistent with the existence of tacit collusion. Prices rose precipitously beginning in 1996. Late in 1999, another large firm, a joint venture between Cargill and Degussa, entered the market. In this case, prices declined before the entry of Cargill and Degussa.

An interesting puzzle in this brief history is why collusion broke down at all when ADM entered the market. Incumbent firms had advanced warning of the construction of ADM's plant. Would it not have been better to simply let ADM into the collusive agreement immediately and thus prevent a price war? It is improbable that a price war arose from attempts by incumbents to scare the upstart ADM out of the market. The sheer size of ADM's plant seems to signal ADM's intentions of remaining in the market. Furthermore, the incumbent firms were the first to attempt to raise prices.

Two alternative, interrelated explanations offer themselves. First, firms may have been signalling in advance of a collusive market share agreement. This explanation requires that firms are uncertain about their rivals' capacities or costs or some other strategic variables, and expect the terms of the collusive agreement to depend on these factors. Hence, firms signal their low cost or high capacity in a price war prior to collusion. Second, the entrant may have wished to gain market share in advance of a collusive agreement. This presumes that the terms of the collusive agreement depend on the market shares of the participants prior to collusion.

I contend that these two explanations represent two sides to the same coin. Because of the high degree of uncertainty in the lysine market about marginal costs and production capacities, there were no obvious terms to set for a collusive agreement. Firms were unable to credibly convey their private information.[4] Hence, the only obvious focal point for an agreement was the existing market shares of the firms. The actual collusive agreement specified that firms received market shares based on their actual market shares over the past year. These market shares were to be maintained over the life of the agreement. This kind of agreement naturally encourages any entrant to first build up market share before agreeing to collude. The fact that ADM appears to have been the principal stumbling block for collusion is consistent with this interpretation.

This history may be repeating itself with the recent entry of Cargill/Degussa. The model I develop below begins with the presumption that the collusive agreement specifies that firms maintain their market shares. From this starting point, I can make inferences about the timing decision of firms contemplating collusion.

3 The Model

The model adapts the framework set out in Ericson and Pakes (1995). In this framework, firms solve a discrete time, infinite horizon problem involving endogenous entry, exit, and investment decisions. Each period, firms engage in price, quantity, or quality competition subject to constraints imposed by a set of firm-specific state variables. This process determines profits each period. In addition, firms can influence the vector of state variables that determines profit opportunities through investment spending.

The solution concept is Markov-perfect Nash equilibrium (MPNE).[5] In an MPNE, firms have perceptions about the distributions of the state variables, conditional on their actions. Firms choose optimal actions based on these perceptions. The realised conditional distributions of the state variables depend on the actions of all the firms. In equilibrium, these realised distributions accord with firms' perceptions.

[4] In fact, ADM conducted tours of its plant in June 1992 for Sewon and in April 1993 for Ajinomoto.

[5] The solution concept departs from that of Maskin and Tirole (1988) in permitting firms to condition on information not relevant for payoffs.

Figure 1: Sequence of Events

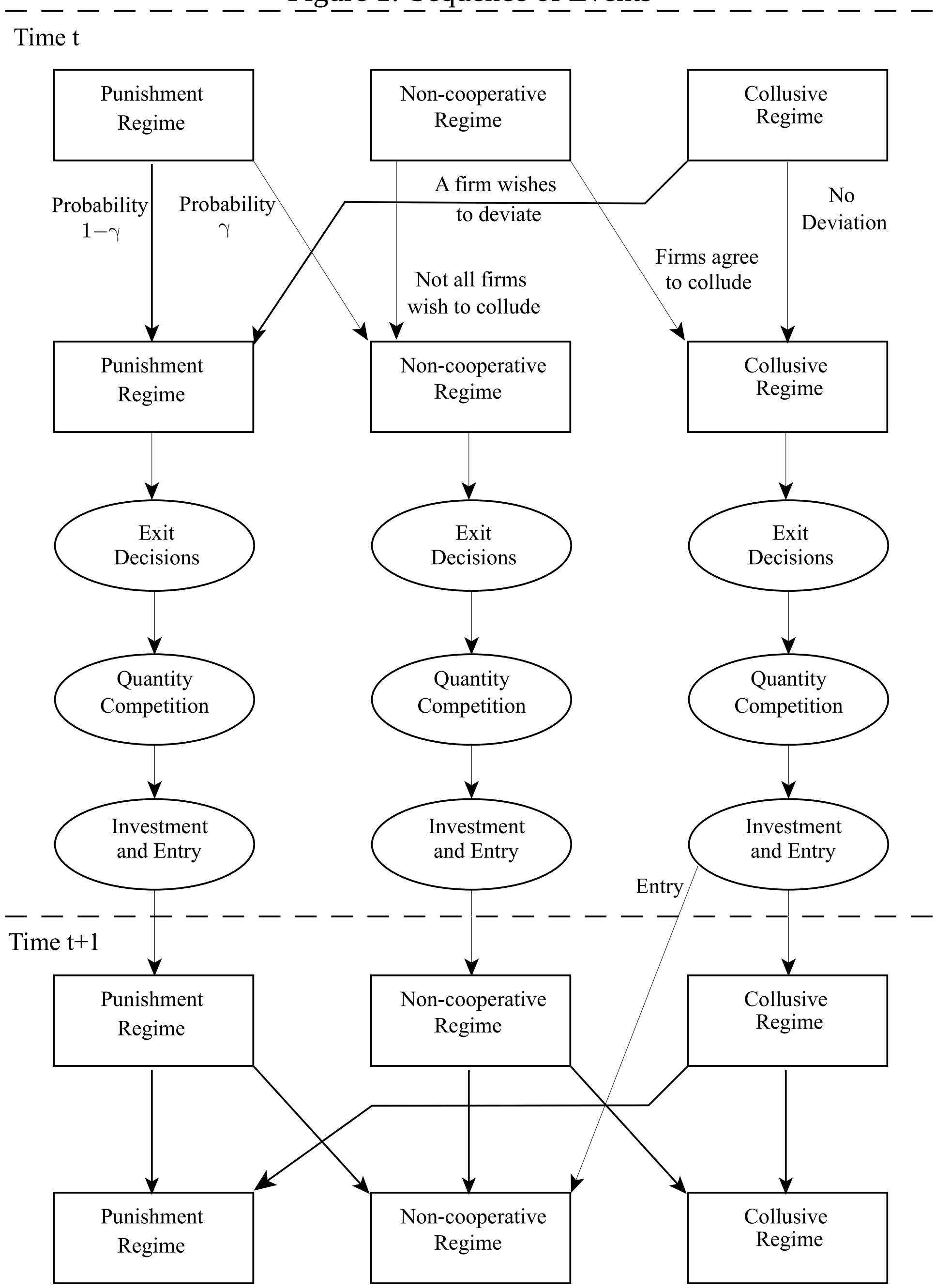


Figure 1 describes the sequence of events taking place each period. There are three competitive regimes which yield different profits each period: a non-cooperative regime, a collusive regime, and a punishment regime. Profits in the non-cooperative and punishment regimes are determined by the Nash equilibrium in quantities to a one-shot capacity-constrained game. Profits in the collusive regime are determined by joint profit maximisation subject to capacity constraints and a market share constraint. The market share constraint specifies that firms must produce output in the ratio given by their ratio of capacities at the time collusion was instigated.[6] Hence, firms have an incentive to build up capacity prior to a collusive agreement.

Play switches from the non-cooperative regime to the collusive regime if all firms wish to collude. It is assumed that any firm that is indifferent between colluding and not colluding will vote for collusion.[7] Should another firm enter the market, play reverts to the non-cooperative regime until all firms, including the entrant, again agree to collude. At any time, firms may choose to exit the collusive agreement even if entry does not occur. However, this will invoke a punishment regime. In this punishment regime, firms behave as in the non-cooperative regime, except that there is no possibility of collusion. With fixed probability, γ, firms negotiate their way back to the non-cooperative regime. Hence, the expected length of the punishment phase is $1/\gamma$ periods. Once firms have negotiated their way out of the punishment regime, they then must achieve consensus to begin colluding.

The sequence of play each period is as follows. At the beginning of the period, collusion decisions are made. That is, if we are in the non-cooperative regime, firms decide if they wish to collude. If all firms agree to collude, then play switches to the collusive regime. If we are in the collusive regime, firms decide whether they wish to break the collusive agreement. If any of the firms wishes to break collusion, play switches to the punishment regime. If we are in the punishment regime, with exogenous probability γ the firms renegotiate their way to the non-cooperative regime. Following the collusion decision, incumbent firms decide whether to exit. Firms then make output decisions which determine profits. Finally, incumbent firms decide on investment spending and potential entrants decide whether to enter. Entrants take one period to set up operations and begin production in the next period.[8]

[6]A more realistic market share constraint would specify production in the ratio of sales rather than capacity at the time of collusion. However, this would result in a considerably more computationally demanding problem without adding significantly to the flavour of the results.

[7]This assumption rules out the uninteresting equilibrium where all firms decide not to collude each period.

[8]This is unrealistically short for the lysine market.

3.1 The State Space

To make computation feasible, the state space is assumed to be discrete. The set of feasible capacities, Ω, can be mapped onto the set of positive integers. Several restrictions are made to the state space to make the problem more manageable. First, following Ericson and Pakes (1995), each firm's capacity can take on a finite set of values.[9] Second, payoffs are independent of the ordering of a firm's competitors. Hence, we do not have to consider different permutations of competitors' states. Finally, successive elements of Ω are assumed to increase exponentially rather than linearly as in Ericson and Pakes (1995).[10] That is, the set of feasible capacities is given by $\Omega \equiv \left\{\tau, \tau g, \tau g^2, \ldots, \tau g^{\bar{k}}\right\}$, where τ and $\bar{k}$ determine the minimum and maximum possible capacity, respectively. This assumption considerably restricts the state space because states that generate the same ratio of capacities amongst firms at the time of collusion are equivalent.[11]

In the non-cooperative and punishment regimes, the state space, S, is fully described by the vector of capacities of the incumbent firms, $\omega_t = \{\omega_{i,t}\}_{i=1}^{n_t}$, where n_t is the number of active firms in period t, and $\omega_{i,t} \in \Omega$.[12] However, in the collusive regime, a firm's share of the collusive profits depends on its capacity at the time the collusive agreement was negotiated. Hence, each firm's state is given by a tuple comprising its current capacity and its share in total capacity at the beginning of the collusive regime. That is, the state space, S^C, is given by $\{\omega_t, \mu_t\}$, where $\mu_t = \{\mu_{i,t}\}_{i=1}^{n_t}$ is the vector of capacities at the time the collusive agreement was struck, and $\mu_{i,t} \in \Omega$.[13]

3.2 Profit Functions

Profits each period are determined by quantity competition for a homogeneous product. The inverse demand function is given by $P(Q_t) = a - bQ_t$, where Q_t is market output, and a and b are demand parameters. Current production decisions have no impact on state probabilities. Hence, the production game can be treated in isolation from the investment, exit, entry, and collusion decisions.

In the non-cooperative regime, the profit vector, $\pi^N(\omega_t) = \left\{\pi_i^N(\omega_t)\right\}_{i=1}^{n_t}$, is determined by the unique solution to a one-shot capacity-constrained quantity game. It is calculated recursively as follows. Define $q_{i,t}$ to be firm i's output in period t. In the absence of capacity constraints,

[9] Ericson and Pakes show there is a maximum state level that can be reached in equilibrium.

[10] This assumption implies that there are some economies of scale in capacity generation.

[11] The state space could be further restricted dramatically by ruling out state vectors at which collusion would never be agreed to. However, this requires some educated guesswork about the equilibrium prior to computation.

[12] As the above discussion indicates, this notation considerably exaggerates the extent of the state space.

[13] Again, the notation exaggerates the extent of the state space.

$q_{i,t} = \tilde{q} \equiv \frac{a-mc}{(n_t+1)b}$, where mc is the constant marginal cost of production. This is the Cournot-Nash equilibrium output for the unconstrained game. We introduce capacity constraints as follows. Starting with the smallest firm, we check to see if $\omega_{i,t} \geq \tilde{q}$. If so, then $q_{i,t} = \tilde{q}, i = 1, \ldots, n_t$. If $\omega_{i,t} < \tilde{q}$, then $q_{i,t} = \omega_{i,t}$, and we redefine $\tilde{q} \equiv \frac{a-mc-b\omega_{i,t}}{n_t b}$ and verify whether for the next smallest firm, $\omega_{j,t} \geq \tilde{q}$, and so on.

To calculate the collusive profit vector, $\pi^C(\omega_t, \mu_t) = \left\{\pi_i^C(\omega_t, \mu_t)\right\}_{i=1}^{n_t}$, let s_t be the vector of collusive shares with $s_{i,t} \equiv \mu_{i,t} / \sum_{j=1}^{n_t} \mu_{j,t}$. Then, in the absence of capacity constraints, $q_{i,t} = s_{i,t}\frac{a-mc}{2b}$, firm i's share of a monopolist's optimal output. To incorporate capacity constraints, we allocate excess production over capacity to the remaining firms according to their shares.

3.3 Investment

Let $\eta_{i,t} \in \{1, g\}$ represent the outcome of the firm's investment process. The probability of successful investment is an increasing, concave function of investment spending. Let $\nu_t \in \{1, g\}$ be the outcome of some exogenous process capturing developments in the industry. If we take ω to describe physical capacities, then an obvious interpretation for ν is the stochastic (industry-wide) decay of capacity. If ω represents the stock of interested customers for each firm, we could think of ν as an industry-wide demand shock. Then, the transition of firm i's capacity is governed by

$$\omega_{i,t+1} = \omega_{i,t}\frac{\eta_{i,t+1}}{\nu_{t+1}} \tag{1}$$

$$\eta_{i,t} = \begin{cases} g & \text{with probability } \frac{\alpha x_{i,t}}{1+\alpha x_{i,t}}, \\ 1 & \text{with probability } \frac{1}{1+\alpha x_{i,t}} \end{cases} \tag{2}$$

$$\nu_t = \begin{cases} g & \text{with probability } \delta, \\ 1 & \text{with probability } 1-\delta. \end{cases} \tag{3}$$

where $\alpha > 0$ is a constant determining the effectiveness of investment, and $x_{i,t}$ is firm i's investment expenditure in period t.

3.4 Entry and Exit

Each period, before firms engage in quantity competition, they have the option of exiting the industry. A firm which exits receives a pay-off of ϕ and takes no further part in the game. A firm will therefore exit if the expected discounted value of remaining in the market is less than ϕ.

Entry decisions are made concurrent with investment decisions. A single potential entrant observes an entry cost draw, x_e, from a uniform distribution $U\left(x_e^{min}, x_e^{max}\right)$ and then decides whether to enter. Should the firm decide to enter, it receives capacity ω_e and begins production in the following period. The entrant will choose to enter if the expected discounted value of entry at capacity ω_e is greater than or equal to the observed entry cost.

3.5 The Bellman Equations

I define a separate Bellman equation for each of the three regimes.[14] The superscripts P,N, and C refer to the punishment, non-cooperative, and collusive regimes, respectively. A negative subscript denotes omission of a single element. Thus, $\omega_{-i} \equiv (\omega_1, \ldots, \omega_{i-1}, \omega_{i+1}, \ldots, \omega_{n_t})$. The Bellman equations below describe the value to firm j for each (ω, μ) in the state space, for each of the regimes. Collusion decisions are based on these values.

$$
\begin{aligned}
V^P\left(\omega_j;\omega_{-j}\right) &= \max\Big\{\phi, \pi^N\left(\omega_j;\omega_{-j}\right) + \max_{x\geq 0}\big[-x \\
&\quad +\beta\sum_{\omega'}\left(\gamma V^N\left(\omega'_j;\omega'_{-j}\right) + (1-\gamma)V^P\left(\omega'_j;\omega'_{-j}\right)\right) \\
&\quad p\left(\omega'_j|\omega_j,x\right)p^P\left(\omega'_{-j}|\omega\right)\big]\Big\} \qquad (4) \\
V^N\left(\omega_j;\omega_{-j}\right) &= \max\Big\{\phi, \pi^N\left(\omega_j;\omega_{-j}\right) + \max_{x\geq 0}\big[-x \\
&\quad +\beta\sum_{\omega'}\Big(I^C\left(\omega'_j;\omega'_{-j}\right)V^C\left(\omega'_j;\omega'_{-j},\omega'_j;\omega'_{-j}\right) \\
&\quad +\Big(1-I^C\left(\omega'_j;\omega'_{-j}\right)\Big)V^N\left(\omega'_j;\omega'_{-j}\right)\Big) \\
&\quad p\left(\omega'_j|\omega_j,x\right)p^N\left(\omega'_{-j}|\omega\right)\big]\Big\} \qquad (5) \\
V^C\left(\omega_j;\omega_{-j},\mu_j;\mu_{-j}\right) &= \max\Big\{\phi, \pi^C\left(\omega_j;\omega_{-j},\mu_j;\mu_{-j}\right) + \max_{x\geq 0}\big[-x \\
&\quad +\beta\sum_{\omega'}\Big(I^P\left(\omega'_j;\omega'_{-j},\mu_j;\mu_{-j}\right)V^P\left(\omega'_j;\omega'_{-j}\right) \\
&\quad +\left(1-I^P\left(\omega'_j;\omega'_{-j},\mu_j;\mu_{-j}\right)\right)V^C\left(\omega'_j;\omega'_{-j},\mu_j;\mu_{-j}\right)\Big) \\
&\quad p\left(\omega'_j|\omega_j,x\right)p^C\left(\omega'_{-j}|\omega,\mu\right)\Big]\Big\}, \qquad (6)
\end{aligned}
$$

where $\pi^C\left(\omega_j;\omega_{-j},\mu_j;\mu_{-j}\right) = \pi^C_j(\omega,\mu)$ and $\pi^N\left(\omega_j;\omega_{-j}\right) = \pi^N_j(\omega)$ are the profit functions for collusion and non-cooperation, respectively; $I^C\left(\omega_j;\omega_{-j}\right) \in \{0,1\}$ is an indicator function governing transition from the non-cooperative regime to the collusive regime, with

[14] Alternatively, the system could be represented by a single Bellman equation by incorporating an additional state variable indicating the current regime of play.

$$I^{C}\left(\omega_{j};\omega_{-j}\right)=1\Leftrightarrow V^{C}\left(\omega_{j};\omega_{-j},\omega_{j};\omega_{-j}\right)\geq V^{N}\left(\omega_{j};\omega_{-j}\right)\ \forall j; \tag{7}$$

and $I^{P}\left(\omega_{j};\omega_{-j},\mu_{j};\mu_{-j}\right)\in\{0,1\}$ is an indicator function governing transition from the collusive regime to the punishment regime, with

$$I^{P}\left(\omega_{j};\omega_{-j},\mu_{j};\mu_{-j}\right)=1\Leftrightarrow V^{P}\left(\omega_{j};\omega_{-j}\right)>V^{C}\left(\omega_{j};\omega_{-j},\mu_{j};\mu_{-j}\right)\text{ for any } j. \tag{8}$$

In each of the regimes, the firm can choose to exit and receive ϕ, or receive the continuation value. The continuation value comprises current profits plus the expected discounted value of future returns. The simplest case is the punishment regime. Here, profits are given by the non-cooperative profit function. The continuation value depends on which competitive regime we are in next period, the firm's own state next period, ω'_{j}, and the states of its competitors (including the entrant if entry occurs) next period, ω'_{-j}. The investment policy function, $x=x^{P}\left(\omega_{j};\omega_{-j}\right)$, depends on this continuation value. With probability γ, play switches to the non-cooperative regime. The distribution of the firm's own state in the next period, $p(\omega'_{j}|\omega_{j},x)$, is determined by its level of investment this period, x, while the distribution of its competitors' states next period, $p^{P}(\omega'_{-j}|\omega)$, depends on the investment, exit, and entry decisions made by its competitors this period.

In the non-cooperative regime, profits are given by the non-cooperative profit function. The continuation value again depends on next period's competitive regime, the firm's own state next period, and the states of its competitors next period. This determines the non-cooperative investment policy function, $x^{N}\left(\omega_{j};\omega_{-j}\right)$. The collusive policy function, $I^{C}(.)$, depends on the current vector of capacities. A value of one indicates that play switches to the collusive regime next period.

Finally, in the collusive regime, profits are given by the collusive profit function. Note that the collusive profit function (and therefore also the collusive value function) depends on both the vector of current capacities, and the vector of capacity shares at the time collusion took place. The punishment policy function, $I^{P}(.)$, and the investment policy function, $x^{C}\left(\omega_{j};\omega_{-j},\mu_{j};\mu_{-j}\right)$, depend on this expanded state vector. If $I^{P}(.)=1$, play switches to the punishment regime next period.

We are now in a position to discuss the entry decision. A single potential entrant observes an entry cost draw of x_{e} before deciding whether to enter. The entrant then spends the remainder of the period constructing a plant with capacity ω_{e}. It takes no part in the quantity game. In the following period, the entrant becomes an incumbent with capacity ω'_{e} with probability $p_{e}\left(\omega'_{e}\right)$. The stochastic nature of ω'_{e} arises from the uncertainty about industry wide developments during the period of plant construction. That is, $p_{e}\left(\omega_{e}\right)=1-\delta$ and $p_{e}\left(\omega_{e}/g\right)=\delta$. The entry decision will depend on the current competitive regime for two reasons. First, the value function differs by competitive regime. Second, rival firms' investment decisions, and hence the state transition probabilities, depend on the competitive regime. Equations (9)-(11) summarise the entry decisions for the punishment, non-cooperative, and collusive regimes, respectively.

$$\beta \sum_{\omega'} V^P\left(\omega'_e;\omega'_{-e}\right) p_e\left(\omega'_e\right) p^P\left(\omega'_{-e}|\omega\right) \quad > \quad x_e \tag{9}$$

$$\beta \sum_{\omega'} V^N\left(\omega'_e;\omega'_{-e}\right) p_e\left(\omega'_e\right) p^N\left(\omega'_{-e}|\omega\right) \quad > \quad x_e \tag{10}$$

$$\beta \sum_{\omega'} V^N\left(\omega'_e;\omega'_{-e}\right) p_e\left(\omega'_e\right) p^C\left(\omega'_{-e}|\omega,\mu\right) \quad > \quad x_e. \tag{11}$$

Let the resulting entry policies be given by $\chi_e^P\left(\omega_e;\omega_{-e}\right)$, $\chi_e^N\left(\omega_e;\omega_{-e}\right)$, and $\chi_e^C\left(\omega_e;\omega_{-e},\mu_{-e}\right)$ for the punishment, non-cooperative, and collusive regimes, respectively. Because the cost of entry, x_e, is random, the entry policy is a probability measure reflecting the perceptions of the incumbent firms of the probability of entry at the time investment decisions are made. In the collusive regime, the entry condition depends on the non-cooperative value function because entry breaks the collusive agreement. Notice, however, that the transition probabilities of the incumbent firms depend on the investment decisions made in the collusive regime.

4 Computational Algorithm

The reader can omit this section without loss of continuity. The computational algorithm used to solve for the equilibrium of the model is based on the method described in Pakes and McGuire (1994). The method involves iterative computation of the value and policy functions.[15] First, I calculate the non-cooperative profits, $\pi^N(.)$, for each $\omega \in S$ and the collusive profits, $\pi^C(.)$, for each $(\omega,\mu) \in S^C$. I begin with an initial value function and investment policy function for each of the competitive regimes, $\left(V^{H,0}(.), x^{H,0}(.)\right)$, $H \in \{N,P,C\}$. The superscripts refers to the competitive regime, and the iteration number, respectively. To complete one iteration, I cycle through all the elements of the state space of the non-cooperative regime, obtaining updated value and policy functions. I then do the same for the punishment and collusive regimes. In equilibrium, the value and policy functions will not change from one iteration to the next.

Consider first the calculations involved to obtain the iteration $k+1$ functions for the non-cooperative regime. I first determine the entry policy, $\chi_e^{N,k+1}(.)$ by testing equation (10) using $V^{N,k}(.)$ and $x^{N,k}(.)$. I next update the collusion policy function, $I^{C,k+1}(.)$, by verifying equation

[15] The computational algorithm is written in the C programming language. Further details are available on request.

(7) using $V^{C,k}(.)$ and $V^{N,k}(.)$. The entry and collusion policies are then used to update the exit and investment policies. Firm j will exit if $V_j^{N,k}(\omega) < \phi$. If it exits, we can update j's investment and value functions by setting $x_j^{N,k+1}(\omega) = 0$ and $V_j^{N,k+1}(\omega) = \phi$. If it doesn't decide to exit, firm j's investment policy in iteration $k+1$ is given by

$$\begin{aligned} x_j^{N,k+1}(\omega) &= argmax_{x\geq 0}\left[-x + \beta \sum_{\omega'} \left(I^{C,k+1}\left(\omega_j';\omega_{-j}'\right) V^{C,k}\left(\omega_j';\omega_{-j}',\omega_j';\omega_{-j}'\right)\right.\right. \\ &\quad + \left.\left(1 - I^{C,k+1}\left(\omega_j';\omega_{-j}'\right)\right) V^{N,k}\left(\omega_j';\omega_{-j}'\right)\right) \\ &\quad \left. p\left(\omega_j'|\omega_j, x\right) p^N\left(\omega_{-j}'|\omega_{-j}, x_{-j}^{N,k}, \chi_e^{N,k+1}\right)\right]. \end{aligned} \tag{12}$$

Notice that the distribution of states next period, $p^N(.|.)$, depends on the current iteration entry policies, $\chi_e^{N,k+1}(\omega)$, and the investment policies of competitors derived from the previous iteration, $x_{-j}^{N,k}(\omega)$. We can use $x_j^{N,k+1}(\omega)$ to update the value function for firm j for iteration $k+1$,

$$\begin{aligned} V_j^{N,k+1}(\omega) &= \pi_j^N(\omega) - x_j^{N,k+1} + \beta \sum_{\omega'} \left(I^{C,k+1}\left(\omega_j';\omega_{-j}'\right) V^{C,k}\left(\omega_j';\omega_{-j}',\omega_j';\omega_{-j}'\right)\right. \\ &\quad + \left.\left(1 - I^{C,k+1}\left(\omega_j';\omega_{-j}'\right)\right) V^{N,k}\left(\omega_j';\omega_{-j}'\right)\right) \\ &\quad p\left(\omega_j'|\omega_j, x_j^{N,k+1}\right) p^N\left(\omega_{-j}'|\omega_{-j}, x_{-j}^{N,k}, \chi_e^{N,k+1}\right). \end{aligned} \tag{13}$$

A similar set of calculations is then performed for the punishment regime. Entry policy, $\chi_e^{P,k+1}(.)$, is determined by equation (9). Firm j will exit if $V_j^{P,k}(\omega) < \phi$. If j does not exit, we calculate its iteration $k+1$ investment policy with

$$\begin{aligned} x_j^{P,k+1}(\omega) &= argmax_{x\geq 0}\left[-x + \beta \sum_{\omega'} \left(\gamma V^{N,k}\left(\omega_j';\omega_{-j}'\right) + (1-\gamma) V^{P,k}\left(\omega_j';\omega_{-j}'\right)\right)\right. \\ &\quad \left. p\left(\omega_j'|\omega_j, x\right) p^P\left(\omega_{-j}'|\omega_{-j}, x_{-j}^{P,k}, \chi_e^{P,k+1}\right)\right]. \end{aligned} \tag{14}$$

We then use $x_j^{P,k+1}(\omega)$ to update the value function for firm j for iteration $k+1$,

$$\begin{aligned} V_j^{P,k+1}(\omega) &= \pi_j^N(\omega) - x_j^{P,k+1} + \beta \sum_{\omega'} \left(\gamma V^{N,k}\left(\omega_j';\omega_{-j}'\right) + (1-\gamma) V^{P,k}\left(\omega_j';\omega_{-j}'\right)\right) \\ &\quad p\left(\omega_j'|\omega_j, x_j^{P,k+1}\right) p^P\left(\omega_{-j}'|\omega_{-j}, x_{-j}^{P,k}, \chi_e^{P,k+1}\right). \end{aligned} \tag{15}$$

Collusion entry policy, $\chi_e^{C,k+1}(.)$, is determined by equation (11). Firm j will exit if $V_j^{C,k}(\omega,\mu) < \phi$. If j does not exit, we calculate its iteration $k+1$ investment policy with

$$
\begin{aligned}
x_j^{C,k+1}(\omega,\mu) \quad = \quad & argmax_{x\geq 0}\Bigg[-x+\beta\sum_{\omega'}\Big(I^{P,k+1}\left(\omega_j';\omega_{-j}',\mu_j;\mu_{-j}\right)V^{P,k}\left(\omega_j';\omega_{-j}'\right) \\
& +\Big(1-I^{P,k+1}\left(\omega_j';\omega_{-j}',\mu_j;\mu_{-j}\right)\Big)V^{C,k}\left(\omega_j';\omega_{-j}',\mu_j;\mu_{-j}\right)\Big) \\
& p\left(\omega_j'|\omega_j,x\right)p^C\left(\omega_{-j}'|\omega_{-j},x_{-j}^{C,k},\chi_e^{C,k+1}\right)\Bigg]. \qquad (16)
\end{aligned}
$$

We then use $x_j^{C,k+1}(\omega,\mu)$ to update the value function for firm j for iteration $k+1$,

$$
\begin{aligned}
V_j^{C,k+1}(\omega,\mu) \quad = \quad & \pi_j^C(\omega,\mu)-x_j^{C,k+1}+\beta\sum_{\omega'}\Big(I^{P,k+1}\left(\omega_j';\omega_{-j}',\mu_j;\mu_{-j}\right)V^{P,k}\left(\omega_j';\omega_{-j}'\right) \\
& +\Big(1-I^{P,k+1}\left(\omega_j';\omega_{-j}',\mu_j;\mu_{-j}\right)\Big)V^{C,k}\left(\omega_j';\omega_{-j}',\mu_j;\mu_{-j}\right)\Big) \\
& p\left(\omega_j'|\omega_j,x_j^{C,k+1}\right)p^C\left(\omega_{-j}'|\omega_{-j},x_{-j}^{C,k},\chi_e^{C,k+1}\right). \qquad (17)
\end{aligned}
$$

Up to now, we have assumed that we know the extent of the state spaces, S, and S^C. However, the dimensions of the state spaces depend on the maximum allowable capacity of a firm, $\tau\bar{g}^k$, and the maximum number of firms, N. As in Pakes and McGuire (1994), the maximum allowable capacity is determined by the point at which the monopolist stops investing. In Pakes and McGuire, the maximum number of firms is obtained by calculating the equilibrium for a restricted number of firms and increasing the number of firms until a potential entrant would no longer enter at any element of the state space. Due to computational limitations, I impose an upper limit of three firms.[16] I am unable to verify whether a 4^{th} firm would wish to enter.

5 Results

The results below are broken into three sections. The first section characterises the nature of the equilibrium for a given set of parameters. Static parameters are based on an empirical examination of the lysine market. Here, I compare the results obtained in the complete model with the results obtained in a stripped-down model ignoring the possibility of collusion. I

[16] As an example, with 3 firms and a set of feasible capacities, Ω, of dimension 20, the state space comprises 1,771 elements for the non-cooperative and punishment regimes and 1,955,401 elements for the collusive regime. With 4 firms and the same set of feasible capacities, the state space expands to 10,626 and 282,340,296 elements for the non-cooperative and collusive regimes, respectively.

Table 1: Parameters Used for the Base Model

Parameter	Description	Value
a	Demand intercept	1.651
b	Demand slope	0.0857
mc	Marginal cost of production	0.527
α	Investment efficiency	1.2
ω_e	Entrant's starting capacity	$\tau g^2 = 2.88$
x_e^{min}	Minimum entry cost	13.0
x_e^{max}	Maximum entry cost	18.0
ϕ	Scrap value of firm	1.0
τ	Minimum capacity	2.0
g	Capacity growth factor	1.2
$\bar{k}$	Number of feasible capacity levels	16
β	Discount factor	0.98
δ	Capacity depreciation rate	0.4
γ	Renegotiation probability	0.05
N	Maximum number of firms	3

employ a variant of the punishment regime to this end. In the second section, I compare the model's results with the experience in the lysine market. This exercise also highlights the type of short-term dynamics the model can produce. In the third section, I examine the consequences of varying key parameters.

5.1 The Base Case

A list of the parameters used and their values for the base model is contained in Table 1. The static parameters I choose are based on the demand estimates and cost data obtained for the lysine market in de Roos (2000). The slope of the demand curve, b, is derived from the demand elasticity of the linear demand model. The intercept, a, is calculated using the values of explanatory variables at the time of ADM's entry into the market. The constant marginal cost, mc, is derived from a parameterisation of ADM's cost function allowing for learning by doing.

The dynamic parameters are chosen to produce a relatively concentrated industry. The high discount factor of 0.98 is chosen to reflect the monthly planning horizon that appeared to operate in the lysine market, and the monthly cost and demand data used for the estimation of the static parameters. The maximum number of firms was set at 3. This is not designed to mimic the market structure in the lysine market (where there were 5 major firms), but rather reflects the enormous computational burden of the state space.

5.1.1 A Model Without Collusion

The point of comparison is a model without the possibility of collusion. This is given by the punishment regime with $\gamma = 0$. Hence, there is no possibility of transition to either the non-cooperative or the collusive regimes. This model is essentially the dynamic model of Pakes and McGuire (1994). A general characterisation of this model for a different static profit function and set of parameters can be found there. Firms tend to invest more when they have a low capacity. Investment tends to decrease with the capacities of rivals. Entry is more likely to occur, the smaller and fewer the incumbent firms.

I simulated the industry for $1,000,000$ iterations and observed the distributions of several industry characteristics. The initial state vector used was (4,0,0). That is, we start the simulations with a single firm with capacity of τg^4. The results are presented in column 1 of Table 2. The industry contains the maximum number of firms (three) for about 12% of the periods, and is a duopoly for almost all the remaining periods. Firms charge a markup over marginal costs of about 71% on average. Entry and exit are highly correlated. Many firms are unable to establish themselves but, once entrenched, firms tend to enjoy a long operating life.

5.1.2 The Full Model

Once we allow collusive possibilities, we can generate a richer tapestry of firm behaviour. I will first describe some features of the model allowing for collusion, drawing comparisons with the model without collusion. Then, industry statistics for the two models are compared.

There are two kinds of environment that are conducive to collusion. First, firms tend to switch to the collusive regime when they have similar capacities. The smallest firms will generally present the greatest stumbling block to collusion because they obtain only a small share of the profits in a collusive agreement. Given the high discount factor assumed, smaller firms are prepared to suffer through a spell of non-cooperative profits while they invest heavily to later obtain a higher share of the collusive profits. Second, there are some elements in the state space where firms will agree to collude with asymmetric market shares. This occurs when there are only two active firms, and both are not overly large. There are two possible reasons for this. First, when the incumbent firms are relatively small, entry is very likely in the near future. By colluding, firms obtain the collusive profits immediately and, because entry breaks the collusive regime, also obtain a larger share of collusive profits some time in the future. This is less

Table 2: Industry Statistics for Model With and Without Collusion

Industry Feature	Without Collusion	With Collusion
Periods with 0 firms	0.0%	0.0%
Periods with 1 firm	0.06%	0.05%
Periods with 2 firms	88.13%	39.48%
Periods with 3 firms	11.81%	60.47%
Periods with entry	0.22%	0.51%
Periods with exit	0.21%	0.53%
Periods in collusive regime	–	70.46%
Periods in non-cooperative regime	–	6.45%
Periods in punishment regime	–	23.10%
Mean market price	0.904	1.032
	(0.050)	(0.093)
Mean investment by incumbents	1.297	1.574
	(0.658)	(0.738)
Mean firm production	4.116	2.772
	(0.499)	(0.843)
Mean firm capacity	6.712	6.875
	(2.779)	(4.130)
Mean one-firm concentration ratio	0.492	0.408
	(0.063)	(0.093)
Mean consumer surplus	3.270	2.283
	(0.417)	(0.727)
Mean producer surplus	1.932	1.904
	(0.972)	(1.318)
Mean total surplus	5.201	4.188
	(1.128)	(1.414)
Mean firm value	3.701	1.385
	(16.86)	(12.95)
Mean firm lifespan	984.0	492.8
	(1399.6)	(660.0)

Standard errors are in parentheses ().

likely to occur when incumbent firms are very large because the probability of entry is markedly reduced. Second, incumbent firms may seek to deter entry. For a given state vector, the probability of entry is lower in the punishment regime. Incumbents could reduce the probability of entry by colluding and subsequently entering the punishment regime.

Punishment tends to occur if firms have collusive shares that are misaligned with their current shares of total market capacity. Punishment will occur over a greater range of capacity vectors the more unequal the collusive shares of the incumbent firms. If firms have equal collusive shares, punishment will never occur if there are three firms in the market.[17] However, if there are only two firms in the market, punishment tends to occur if the firms are not overly large, even if the firms' collusive shares are equal. As discussed, this reflects attempts to deter entry. Entry is less likely for a given state vector in the punishment regime than either the collusive or non-cooperative regimes.

Figure 2 describes the elements of the state space in which entry occurs with a probability of at least 0.15 for the model without collusion, and for each competitive regime of the model with collusion.[18] The axes index the capacity levels of each incumbent firm, with a value of 0 indicating that a firm is inactive and a positive value k indicating a capacity of τg^k. We need only two axes because there are at most three firms in the market. Entry will always occur into a monopoly. Entry is more likely in the model with collusion, reflecting the option value of collusion. Within the model of collusion, entry is more likely in the non-cooperative regime than the punishment regime for the same reason. Because we switch from the collusive to the non-cooperative regime if entry occurs, entry behaviour is almost identical in the collusive and non-cooperative regimes. Differences in the entry behaviour of these two regimes reflect only the differences in investment spending in these regimes. For example, at the state vector (7,3,0), entry is more likely into the collusive regime because the largest firm invests less in the collusive regime than in the non-cooperative regime.

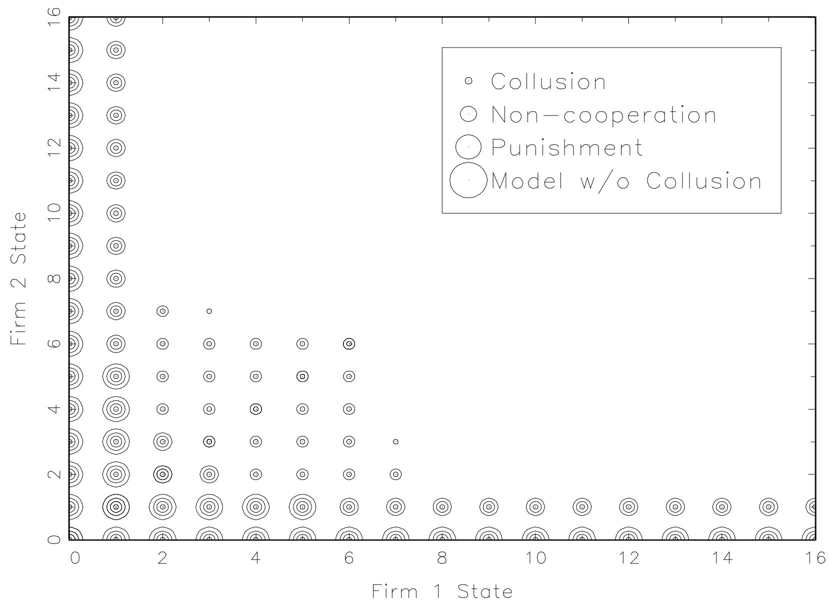
Investment behaviour is also different once we permit collusion. In the non-cooperative regime, investment is tailored to speed the onset of collusion. That is, in state tuples "near" collusion, the larger firms will tend to invest less, and the smaller firms invest more, relative to the punishment regime or the model without collusion. Similarly, firms that wish to maintain collusion will tend to bend their investment spending to reduce the likelihood of reaching states in which punishment occurs.

Column 2 of Table 2 presents industry statistics for the model allowing for collusion. Industry statistics are generated in the same manner as the model without collusion. That is,

[17] This is not a general result, but holds for the current set of parameters.

[18] For the collusive regime, I examine only elements of the state space in which firms have equal shares of the cartel profits.

Figure 2: Entry Region by Model and Regime



beginning in the non-cooperative regime with a state vector of (4,0,0), the industry is simulated for 1,000,000 iterations, and industry characteristics are observed. There are typically a greater number of firms in the model with collusion, but each firm tends to produce less. This difference in industry structure is a reflection of the greater attractiveness of entry in the model with collusion. There is also a greater incidence of entry and exit in the model incorporating collusion.

Firms charge higher prices in the collusion model, and the industry with collusion is characterised by greater investment, and lower consumer and total surplus.[19] Interestingly, producer surplus is also lower on average in the industry with collusion, reflecting a less concentrated industry structure and greater industry-wide investment. Firms are longer lived and earn greater lifetime profits in the industry without collusion. This result is a culmination of several factors. First, with a less concentrated industry, producer surplus is spread among a greater number of firms. Second, firms tend to be shorter lived, another reflection of the greater incidence of entry

[19]This contrasts with the results of Fershtman and Pakes (2000). In their model, consumer surplus is higher if we allow collusion because the collusive industry generates a greater variety of products.

and exit. Finally, because entry is more likely for a given state and entry costs are stochastic, entrants will on average incur greater costs of entry in the model with collusion.

Table 3 presents some additional statistics on collusion and punishment. The top panel describes the length of time it takes firms to collude after we enter the non-cooperative regime (after either the end of punishment, or the entry of a new firm while firms are colluding), the length of the collusive regime, and the behaviour of prices when we switch competitive regimes. With the base parameters, on average collusion occurs relatively quickly, but there is a large variance to collusion times. Collusion is rapid when there are only two firms in the market for two reasons. First, entry may be anticipated relatively quickly. Therefore, firms will be willing to collude even with asymmetric market shares, anticipating imminent entry. Second, firms may make efforts to deter entry. They can attempt this by colluding quickly and then entering the punishment regime. When there are three firms in the market, the smallest firm will wish to build up market share prior to collusion. Collusion will then occur much later if the smallest firm experiences a bad sequence of investment outcomes, but is able to remain in the market.

Collusion is long-lasting on average, but there is a great deal of variety in the success of collusion, reflecting the variety of states in which collusion is instigated. Collusion will end quickly if there are two small firms in the market when collusion begins. For the reasons described above, either entry or punishment is therefore likely in the near future. Collusion appears to last longer when it is broken by entry rather than punishment. This is largely because, for the parameter values chosen, entry deterrence will be fruitless when the incumbent firms are particularly small. Therefore, collusion will tend to be broken by entry when there are two small active firms. We are more likely to arrive at this situation when collusion occurs initially with three small firms. Transition from three small firms to two small firms will often occur comparatively quickly.

As one might expect, prices jump substantially when we enter the collusive regime, and fall when collusion breaks down. The fall in prices when collusion is broken by entry is more dramatic because of the combined effect of a change in competitive regime and a less concentrated industry. The bottom panel describes the conditions prevailing at the time of collusion and punishment. It can be seen that we only ever enter the punishment regime with two firms in the market. This is because, with three firms, collusion only occurs when firms have equal market shares, and punishment will never occur while there are three firms in the market enjoying equal collusive shares. At the onset of collusion, there are typically two firms operating in the market. However, as we will see below, collusion is usually characterised by three firms. This apparent discrepancy arises because, with two firms in the market, a brief spell of collusion will often occur as a prelude to entry deterrence.

Table 4 presents a comparison of industry characteristics for each of the regimes of the

Table 3: Characteristics of Collusion and Punishment

	Mean	Standard Deviation
Periods taken to collude		
With 2 firms	2.20	4.92
With 3 firms	9.97	8.67
All cases	3.99	6.83
Length of collusive regime		
Before entry	26.52	89.37
Before punishment	55.91	124.2
All cases	52.18	120.7
Price change at collusion		
	22.10%	9.14
Price change on breaking collusion		
Due to punishment	-13.12%	7.15
Due to entry	-17.90%	10.22
	At Collusion	At Punishment
Percentage of Periods with n firms active		
$n = 2$	76.94%	100%
$n = 3$	23.06%	0%

collusion model. As might be expected, the collusive regime yields the highest industry prices and producer surplus. There are typically more firms operating in the collusive regime, but each firm tends to produce less. Consumer surplus is substantially lower than in the other regimes. Interestingly, the lowest prices and the highest consumer surplus tend to be observed in the non-cooperative regime. The punishment and non-cooperative regimes share a common quantity competition stage game. We observe lower prices in the non-cooperative regime on average because there tends to be a greater number of firms in this regime. Firms tend to have a larger capacity in the punishment regime because we always enter the punishment regime with two firms. A greater incidence of entry keeps the average capacity per firm in the non-cooperative regime down.

Table 4: Industry Statistics for Different Regimes

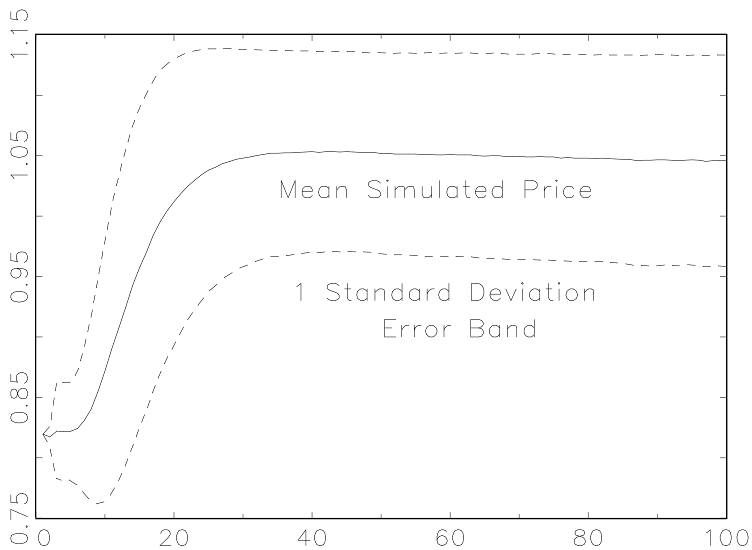
Industry Feature	Collusive	Non-cooperative	Punishment
Periods with 0 firms	0.0%	0.0%	0.0%
Periods with 1 firm	0.02%	0.09%	0.13%
Periods with 2 firms	23.05%	43.02%	88.61%
Periods with 3 firms	76.93%	56.89%	11.26%
Periods with entry	0.24%	1.65%	1.10%
Periods with exit	0.16%	0.15%	1.67%
Mean investment by incumbents	1.566	1.452	1.475
	(0.739)	(1.059)	(0.750)
Mean firm production	2.368	3.597	4.109
	(0.421)	(0.591)	(0.545)
Mean firm capacity	6.800	6.481	7.309
	(4.232)	(4.008)	(3.705)
Mean market price	1.089	0.859	0.907
	(0.005)	(0.052)	(0.053)
Mean one-firm concentration	0.378	0.417	0.497
	(0.083)	(0.083)	(0.064)
Mean consumer surplus	1.842	3.672	3.242
	(0.028)	(0.458)	(0.421)
Mean producer surplus	2.087	1.075	1.578
	(1.030)	(1.978)	(1.671)
Mean total surplus	3.929	4.747	4.820
	(1.042)	(2.055)	(1.863)

Standard errors are in parentheses ().

5.2 The Lysine Market

The results of the previous section suggest the model is capable of explaining the type of collusion experienced in the lysine market. In particular, successful phases of collusion, reversion to punishment, and price wars following entry were all observed in equilibrium. This section serves two purposes. First, I wish to examine more closely the predictions of the model following an entry similar to the entry of ADM into the lysine market. The goal is not to replicate every nuance of firm behaviour in the lysine market, but rather to examine whether the flavour

Figure 3: Mean Simulated Prices



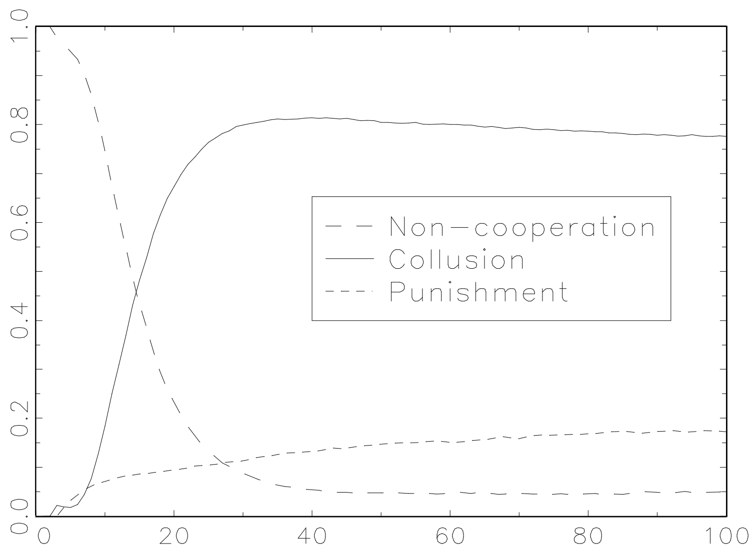
of events in the lysine market can be captured by the model.[20] Second, I wish to illustrate the type of short term dynamics the model can generate.

The initial capacities of the incumbent firms are chosen to reflect the state of affairs after ADM entered. That is, I choose a starting state vector of $(7,7,2)$, meaning the incumbent firms have capacities of τg^7 and the entrant has capacity τg^2. The model is then simulated 10,000 times for 100 periods. The time horizon is assumed to be monthly, reflecting the frequency with which lysine discussions were held, and the frequency of the data used for demand and cost estimates. Figures 3 and 4 present the results of these simulations.

Figure 3 depicts the market price in each period, averaged across the simulations. One standard deviation error bands on the simulated price are also shown. A great deal of variety in the sample paths is masked. In the majority of simulations, there is a sustained price war before collusion is negotiated. Prices drop immediately after entry, and then rise when the collusive regime begins. The average market price begins to rise after only two periods. This is because in some simulations, exit occurs quickly, immediately followed by collusion by the remaining

[20] It should be noted that the dynamic parameters are not estimated to fit the lysine market, but are only guided by the experience in that market. In fact, parameter choices are constrained by computational considerations which limit the size of the state space that can be handled.

Figure 4: Fraction of Simulations in Each Regime



incumbents.[21]

Figure 4 shows for each period the fraction of simulations in each of the competitive regimes. It can be seen that we tend to enter the collusive regime in a larger fraction of periods over time, with the distribution settling down after about 30 periods. The fraction of simulations in the punishment regime also begins to rise after about 4 periods. Punishment will occur very quickly only if the entrant has a sequence of adverse investment outcomes and exits the market, and the remaining incumbents collude quickly at asymmetric market shares, anticipating imminent entry. If an entrant does not oblige quickly, the incumbents may wish to break out of collusion for one of two reasons. If the incumbent with the smaller collusive share has a sequence of fortunate investment outcomes relative to its rival, it may wish to break the agreement. Alterna-

[21]To control for this phenomenon, a subset of the simulations was chosen in which exit did not occur within the 100 periods of the simulation. The severity of the price war is then more apparent. The average price across the simulations falls for the first 4 months, and no collusion occurs for at least 5 months. The fraction of simulations with collusion then rises rapidly over time. However, the reader should notice the distortions induced by this selection exercise. An implication of this sample selection is that firms' investment activities were systematically more successful than anticipated and/or the decay of capacity was systematically overpredicted. However, the exercise does mimic ADM's successful entry into the market.

tively, if both firms are relatively small, they may wish to enter the punishment regime to try to deter entry.

A comparison with the discussion of section 2 suggests that in principle the model can provide useful insights for the lysine market. However, there are some features that would be difficult to explain in the current model. First, the gradual nature of price movements in the lysine market associated with the onset of price wars and the beginning of collusion cannot be captured by the model. The smoothness of price movements in the lysine market reflects the existence of contracts and attempts to avoid arousing the suspicion of the anti-trust authorities, features not incorporated in the model. Second, the model assumes perfect information. This does not allow price wars that arise due to the existence of demand uncertainty and imperfect monitoring. de Roos (2000) notes that these factors may have contributed to the second price war in the lysine market. Other desirable features not incorporated in the present model are discussed in the extensions section.

5.3 Comparative Dynamics

In this section, I examine the consequences of varying some key parameters. Tables 5 and 6 summarise characteristics of industry equilibrium for three experiments. Table 5 presents general statistics on the industry equilibrium, and table 6 presents information about collusion and punishment for the three experiments. The results should be compared with the base model, which is included in the first column. In the base case, the cost of entry, x_e, was drawn from the uniform distribution $U(13,18)$. In the second column, the cost of entry is increased so that $x_e \sim U(15,20)$. As we might expect, the frequency of entry and exit is reduced and a more concentrated industry results on average. Consequently, we tend to have higher prices. Firms produce more on average, but industry production is typically lower because fewer firms are active. Consumer surplus is reduced, but producer surplus actually rises on average. This is because of the reduced industry investment required to sustain a more concentrated industry. There are also implications for collusion. Collusion prevails for a greater fraction of periods. The reason is that the average length of collusion is greater with higher entry costs. However, notice that the average length of collusion is actually slightly smaller when collusion is broken by entry. The apparent increase in stability of collusion with higher entry costs arises for a more subtle reason. Because entry is more costly, colluding firms feel the need to engage in entry deterring behaviour over a smaller subset of the state space. Consequently, it appears that punishment is less prevalent in the industry with greater entry costs.

In column 3, the discount factor is reduced from 0.98 to 0.96. This has a dramatic impact on industry structure. A more concentrated industry tends to result because firms have a

Table 5: Industry Statistics for Different Parameter Values

Industry Feature	Base Case	Increased Entry Costs	Lower Discount Factor	Smaller Demand
Periods with 0 firms	0.0%	0.0%	0.01%	0.0%
Periods with 1 firm	0.05%	0.04%	0.77%	0.38%
Periods with 2 firms	39.48%	73.38%	98.60%	99.03%
Periods with 3 firms	60.47%	26.58%	0.61%	0.58%
Periods with entry	0.51%	0.24%	0.68%	0.40%
Periods with exit	0.53%	0.23%	0.66%	0.39%
Periods in collusive regime	70.46%	76.70%	77.32%	94.77%
Periods in non-cooperative regime	6.45%	5.21%	7.06%	2.60%
Periods in punishment regime	23.10%	18.09%	15.63%	2.63%
Mean market price	1.032	1.049	1.069	1.009
	(0.093)	(0.078)	(0.071)	(0.032)
Mean investment by incumbents	1.574	1.392	1.244	1.223
	(0.738)	(0.717)	(0.644)	(0.607)
Mean firm production	2.772	3.100	3.397	2.862
	(0.843)	(0.741)	(0.495)	(0.214)
Mean firm capacity	6.875	6.867	4.771	5.597
	(4.130)	(2.984)	1.490)	(2.218)
Mean one-firm concentration ratio	0.408	0.461	0.520	0.504
	(0.093)	(0.079)	(0.057)	(0.037)
Mean consumer surplus	2.283	2.149	2.003	1.413
	(0.727)	(0.604)	(0.497)	(0.199)
Mean producer surplus	1.904	2.168	2.283	1.469
	(1.318)	(1.151)	(1.429)	(1.138)
Mean total surplus	4.188	4.317	4.287	2.881
	(1.414)	(1.167)	(1.632)	(1.192)
Mean firm value	1.385	2.797	2.806	6.712
	(12.95)	(15.39)	(10.51)	(14.55)
Mean firm lifespan	492.8	942.5	295.7	504.0
	(660.0)	(1283.6)	(355.2)	(604.5)

Standard errors are in parentheses ().

Table 6: Collusion and Punishment Characteristics for Different Parameters

	Base Case	Increased Entry Costs	Lower Discount Factor	Smaller Demand
Mean periods taken to collude				
With 2 firms	2.20	3.89	5.50	6.28
	(4.92)	(5.86)	(7.32)	(7.73)
With 3 firms	9.97	8.78	1.23	3.32
	(8.67)	(8.11)	(3.29)	(11.12)
All cases	3.99	4.53	5.44	6.22
	(6.83)	(6.42)	(7.30)	(7.82)
Mean length of collusive regime				
Before entry	26.52	17.56	53.59	252.4
	(89.37)	(92.94)	(66.26)	(296.6)
Before punishment	55.91	83.44	71.98	205.8
	(124.2)	(126.0)	(71.04)	(266.8)
All cases	52.18	79.60	66.02	237.4
	(120.7)	(125.2)	(70.06)	(288.2)
At collusion percentage of Periods with *n* firms active				
$n = 2$	76.94%	86.91%	98.59%	98.09%
$n = 3$	23.06%	13.09%	1.41%	1.91%
At punishment percentage of Periods with *n* firms active				
$n = 2$	100%	100%	99.73%	97.35%
$n = 3$	0%	0%	0.27%	2.65%

Standard errors are in parentheses ().

reduced incentive to invest. This is because they care less about future profitability. Notice that average firm capacity is considerably lower in this experiment, reflecting the reduced level of investment. We might expect entry to be less attractive with a lower discount factor as the cost of entry is imposed initially and the benefits to industry participation accrue in the future. However, entry actually occurs more often because active firms are on average smaller and less numerous, making the conditions for entry more favourable. Interestingly, in this example, collusion is more prevalent when firms are less patient. This is due to two factors. First, relative to the base case, collusion is less often broken by punishment. Hence, firms need endure the punishment regime less frequently. Second, the average length of collusion is greater than the base case. The main reason for this is the absence of entry deterrence. Because entry is very unlikely when there are two firms in the market, there is no incentive to engage in entry deterring behaviour.

In the final column, the demand intercept is reduced from 1.651 to 1.5. Notice that this is an equivalent experiment to increasing the constant marginal cost parameter, except that the market price will differ by a constant. As we might expect, a more concentrated industry with a lower average market price results. Average firm capacity is again lower, reflecting reduced average investment. The most dramatic effect is the increased sustainability of collusion. This arises largely because entry and entry deterrence are both much less likely when there are multiple firms in the market. Entry almost always occurs into a monopoly. Hence, collusion will tend to prevail until incumbents have a sequence of extremely unlucky investment outcomes and the market is reduced to a single active firm.

6 Extensions

There are many possible avenues for extension of the current model. Policy experiments could be conducted through some fairly minor changes to the model. A simple way to incorporate the influence of the anti-trust authorities would be to include in the collusive regime a probability of detection, leading to pecuniary punishment. The detection probability could potentially be conditional on the level of prices or the rate of change of prices, although this will add to the computational complexity. There has been a recent surge in the number of successful international cartel prosecutions, with a contributing factor being a change in the amnesty program. An amnesty policy could be incorporated into the model by allowing any firm to blow the whistle on the collusive regime and avoid any punishment by the authorities.

Some more substantive enhancements to the model are also desirable. I will point out some features that appear particularly important based on consideration of the lysine market. First, a central prediction of the model is the tendency for firms to build up a market share comparable with their competitors before colluding. This prediction can only carry force if firms are on an equal footing. In the current model, firms differ only in their capacity constraints. A more realistic setting would allow for other asymmetries. Cost asymmetries are especially important. We might expect an entrant with a relative cost disadvantage to pursue something less than a market share comparable to its competitors before caving in to the temptation of collusion. Second, an important element guiding firm behaviour in the lysine market was the expectation of future market growth. Altering the discount factor could represent a rough approximation for market growth. However, to account more precisely for growth necessitates an expansion of the state space.

Third, an important assumption of the model is that the entrant must be a party to any collusive agreement. We could relax this assumption by calculating a separate value function in which incumbent firms collude without the participation of a recent entrant. Incumbent firms could then compare this value function with the collusive value function when making collusion decisions. More generally, we could consider collusion by any subset of the incumbent firms. This is a particularly thorny problem, even in the absence of dynamic considerations.[22] Finally, a central element to many price war models is the asymmetry of information. Incorporating this feature is a serious challenge, but would considerably enrich the descriptive power of the model.

7 Conclusions

This paper has developed a dynamic model of collusion that focuses on the role of entry. The model is sufficiently rich to allow a wide variety of behaviour in equilibrium. Successful collusion, price wars due to entry or punishment, and entry deterrence are all potential elements of equilibrium. Key assumptions of the model are motivated by recent empirical observations of collusion. The model could provide guidance for any market with collusive possibilities, the potential for entry, and some uncertainty about the characteristics of potential entrants.

A central prediction of the model is that a firm entering a market characterised by collusion will tend to build up a market share comparable with its competitors before agreeing to collude. This was an important feature of the market for lysine, where collusion has recently played a dramatic role. However, firms may collude quickly with asymmetric market shares if they anticipate entry, or if they wish to deter entry. The results highlight the intimate connection between the success of collusion and the prospects for entry. In market structures where entry is unlikely and recourse to entry deterrence unnecessary, collusion is more likely to be sustained.

[22] See, for example, Bernheim, Peleg and Whinston (1987).

The results also reinforce the conclusion of Fershtman and Pakes (2000) that anti-trust analysis must consider the impact of collusion on market structure.

References

[1] Abreu, D., D. Pearce, and E. Stacchetti (1986), "Optimal Cartel Equilibria with Imperfect Monitoring", *Journal of Economic Theory*, 39, pp. 251-269.

[2] Compte, O., F. Jenny, and P. Rey (1997), "Capacity Constraints, Mergers and Collusion", *mimeo*, University of Tolouse.

[3] Connor, John (1998a), "Archer Daniels Midland: Price-Fixer to the World", 3rd Edition, Purdue University Department of Agricultural Economics Staff Paper 98-10.

[4] Connor, John (1998b), "Lysine Production, Trade, and the Effects of International Price Fixing", Purdue University Department of Agricultural Economics Staff Paper 98-18.

[5] Connor, John (2000), "'Our Customers are our Enemies': the Lysine Cartel of 1992-1995", *Review of Industrial Organisation*, forthcoming.

[6] de Roos, Nicolas (2000), "Examining Models of Collusion: the Market for Lysine", Manuscript, Yale University.

[7] Ellison, G. (1994), "Theories of Cartel Stability and the Joint Executive Committee", *RAND Journal of Economics*, 25, pp. 37-57.

[8] Ericson, R. and A. Pakes (1995), "Markov-Perfect Industry Dynamics: A Framework for Empirical Work", *Review of Economic Studies*, 62, pp. 53-82.

[9] Fershtman, C. and A. Pakes (2000), "A Dynamic Game with Collusion and Price Wars", *RAND Journal of Economics*, 31 (2), pp. 207-236.

[10] Friedman, J. (1971), "A Non-cooperative Equilibrium for Supergames", *Review of Economic Studies*, 28, pp. 1-12.

[11] Green, E. J. and R. H. Porter (1984), "Non-cooperative Collusion Under Imperfect Price Information", *Econometrica*, pp. 87-100.

[12] Klepper, Steven (1996), "Entry, Exit, Growth and Innovation Over the Product Life Cycle", *American Economic Review*, 86(3), pp. 562-583.

[13] Levinstein, M. C. (1997), "Price Wars and the Stability of Collusion: A Study of the Pre-World War I Bromine Industry", *Journal of Industrial Economics*, 45, pp. 117-147.

[14] Maskin, E. and J. Tirole (1988), "A Theory of Dynamic Oligopoly: I and II", *Econometrica*, 56, pp. 549-600.

[15] Pakes, A. and P. McGuire (1994), "Computing Markov-Perfect Nash Equilibrium: Numerical Implications of a Dynamic Differentiated Product Model", *RAND Journal of Economics*, 25, pp. 555-589.

[16] Porter, R. H. (1983), "A Study of Cartel Stability: The Joint Executive Committee, 1880-1886", *Bell Journal of Economics*, 14, pp. 301-314.

[17] Rotemberg, J. J. and G. Saloner (1986), "A Supergame-Theoretic Model of Price Wars during Booms", *American Economic Review*, 76, pp. 390-407.

[18] Schelling, Thomas (1960), *The Strategy of Conflict*, Harvard University Press.

[19] Schmalensee, Richard (1987), "Competitive Advantage and Collusive Optima", *International Journal of Industrial Organization*, 5, pp. 351-367.

[20] Stigler, G. J. (1964), "A Theory of Oligopoly", *Journal of Political Economy*, 72, pp. 44-61.