Bilateral Bargaining with Externalities*

by

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This paper provides an analysis of a non-cooperative but bilateral bargaining game between agents in a network. We establish that there exists an equilibrium that generates a cooperative bargaining division of the reduced surplus that arises as a result of non-pecuniary externalities between agents. That is, we provide a non-cooperative justification for a cooperative division of a non-cooperative surplus. This also represents a non-cooperative foundation for the Myerson-Shapley value as well as a new bargaining outcome with properties that are particularly useful and tractable in applications. We demonstrate this by examining firm-worker negotiations and buyer-seller networks. *Journal of Economic Literature* Classification Number: C78.

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1. Introduction

There are many areas of economics where market outcomes are best described by an on-going sequence of interrelated negotiations. When firms negotiate over employment conditions with individual workers, patent-holders negotiate with several potential licensors, and when competing firms negotiate with their suppliers over procurement contracts, a network of more or less bilateral relationships determines the allocation of resources. To date, however, most theoretical developments in bargaining have either focused on the outcomes of independent bilateral negotiations or on multilateral exchanges with a single key agent.

The goal of this paper is to consider the general problem of the outcomes that might be realised when many agents bargain bilaterally with one another and where negotiation outcomes are interrelated and generate external effects. This is an environment where (1) surplus is not maximised because of the existence of those external effects and the lack of a multilateral mechanism to control them; and (2) distribution depends upon which agents negotiate with each other. While cooperative game theory has developed to take into account (2) by considering payoff functions that depend on the precise position of agents in the graph of network relationships, it almost axiomatically rules out (1). In contrast, non-cooperative game theory embraces (1) but restricts the environment considered – symmetry, two players, small players, etc. – to avoid (2).

Here we consider the general problem of a set of agents who negotiate in pairs. All agents may be linked, or certain links may not be possible for other reasons (e.g., antitrust laws preventing horizontal arrangements among firms). Our environment is such that pairs of agents negotiate over variables that are jointly observable. This might be a joint action – such as whether trade takes place – or an individual action undertaken by one agent but observed by the other (e.g., effort or an investment). We specify a non-cooperative game whereby each pair of agents in a network bargains bilaterally in a pre-specified sequence (although the order does not matter for the equilibrium we focus on). Pairwise negotiations utilise an alternating offer approach where offers and acceptances are made in anticipation of deals reached later in the sequence. Moreover, those negotiations take place with full knowledge of the network structure and the ability to make terms contingent upon that structure should it change. Specifically, the network may become 'smaller' should other pairs of agents fail to reach an agreement.

We consider a situation in which the precise agreement terms cannot be directly observed outside a pair, and we focus on an attractive equilibrium outcome of the incomplete information game. That outcome involves agents negotiating actions that maximise their joint surplus (as in Nash bargaining) taking all other actions as given. Hence, with externalities, outcomes are what might be termed "bilaterally efficient" rather than socially efficient.

The equilibrium set of transfers also gives rise to a precise structure; namely, a payoff that depends upon the weighted sum of values to particular coalitions of agents. This has a cooperative bargaining structure but with several important differences. First, the presence of externalities means that coalitions do not maximise their surplus, as equilibrium actions are bilaterally efficient rather than socially efficient. Second, coalitions may impose externalities on other coalitions; thus, the partition of the whole space is relevant. Thus, the equilibrium outcome is a Shapley allocation generalized to partition function spaces (as in Myerson 1977b) and networks in those partition spaces, but over a surplus that is characterised by bilateral rather than social efficiency.[1] Third, the restricted communication space may give rise to further inefficiencies, if certain agents are missing links between them and cannot negotiate, but instead choose individually optimal actions. Jackson and Wolinsky (1996) generalise Shapley/Myerson values to environments where a coalition's payoff depends on the link structure; the equilibrium outcomes and payoffs in this model likewise depend on the link structure.

In sum, we have a non-cooperative foundation for a generalised Shapley division of a non-cooperative surplus; which is easy to use in applied settings. To our knowledge, no similar simple characterisation exists in the literature for a multi-agent bargaining environment with externalities.

The usefulness of this solution to applied research seems clear. The seminal paper in the theory of the firm, Hart and Moore (1990), and myriad subsequent papers assume that agents receive the Shapley value in negotiations, as it captures the impact of substitutability but without the extreme solutions of other concepts such as the core. However, there is an inherent discomfort to applying Shapley values in non-cooperative settings. Because Shapley values do not take into account externalities, the theory of the firm has been limited in terms of which types of strategic interactions can be studied. In contrast, the analysis presented here allows us to study a fully non-cooperative game of non-contractible and contractible investments and actions between agents (see de Fontenay and Gans 2005 for an example). Moreover, it allows us to contribute to the modeling of buyer-seller networks. Up until now, the papers addressing this issue have

[1] In the absence of externalities, it reduces to the Myerson value, and if further the network is complete, it reduces to the Shapley value.

needed to restrict their attention to environments with a restrictive network structure, such as common agency, or to an environment with no externalities (Cremer and Riordan, 1987; Kranton and Minehart, 2001; Inderst and Wey, 2003; Prat and Rustichini, 2003; Bjornerstedt and Stennek, 2002). Our solution combines the intuitiveness and computability of Shapley values, with a serious treatment of the consequences of externalities for payoffs and efficiency. As such, it is capable of general application in these environments.[2]

The paper proceeds as follows. In the next section, we review the current literature on non-cooperative foundations of the Shapley and Myerson values. Section 3 then introduces our action space which is the principal environmental restriction in this paper. Our extensive form game is introduced in Section 4. The equilibrium outcomes of that game are characterised in Sections 4 and 5; first with the equilibrium outcomes as they pertain to actions and then to distribution. Section 6 then considers particular economic applications including wage bargaining with competing employers and buyer seller networks. A final section concludes.

2. Literature Review

Winter (2002, p.2045) argues that "[o]f all the solution concepts in cooperative game theory, the Shapley value is arguably the most 'cooperative,' undoubtedly more so than such concepts as the core and the bargaining set whose definitions include strategic interpretations." Despite this, the Shapley value has emerged as an outcome in a number

[2] There is also a literature on inefficiencies that arise in non-cooperative games with externalities (see, for example, Jeheil and Moldovanu (1995). The structure of our non-cooperative game is of a form that eliminates these and we focus, in particular, on equilibria without such inefficiencies. As such, that literature can be seen as complementary to the model here.

of non-cooperative settings. Harsanyi (1985) noted the emergence of the Shapley value in games that divide surplus based on unanimity rules. However, recent attempts to provide a non-cooperative foundation for the Shapley value have focused, for the most part, on the outcomes of a series of bilateral negotiations.

Gul (1989) proposed a game where a single agent can generate utility from consuming resources that are initially dispersed. His trading game has individual agents meeting randomly to conduct bilateral trades. Each bilateral negotiation involves one agent being selected at random to make a take-it-or-leave-it purchase offer to the other agent. Successful trades result in the seller leaving the game with their earnings. Essentially, a trade is equivalent to a seller agreeing to join the buyer's coalition. Eventually, sufficient trades occur that the grand coalition is formed with, for sufficiently patient players, the unique stationary subgame perfect outcome (with no delay) with each agent receiving (in expectation) their Shapley value. The economic environment is quite specialised here, however, as it essentially amounts to a sequence of discrete trades with no other actions being feasible.

Stole and Zwiebel (1996) examined an environment where a firm bargains bilaterally with a given set of workers. While their treatment is for the most part axiomatic, focusing on a natural notion of stable agreements, they do posit an extensive form game for their environment. In this extensive form game, there is a fixed order in which each worker bargains with the firm. Any given negotiation has the worker and firm taking turns in making offers to the other party that can be accepted or rejected. Rejected offers bring with them an infinitesimal probability of an irreversible breakdown where the worker leaves employment forever. Otherwise, a counter-offer is possible. If the worker and firm agree to a wage (in exchange for a unit of labour), the negotiations move on to the next worker. The twist is that agreements are not binding in the sense that, if there is a breakdown in any bilateral negotiation, this automatically triggers a replaying of the sequence of negotiations between the firm and each remaining worker. This new subgame takes place as if no previous wage agreements had been made (reflecting a key assumption in Stole and Zwiebel's axiomatic treatment that wage agreements are not binding and can be renegotiated by any party at any time).

Stole and Zwiebel (1996, Theorem 2) claim that this extensive form game gives rise to the Shapley value as the *unique* subgame perfect equilibrium outcome (something they also derive in their axiomatic treatment). However, we demonstrate below that the informational structure between different bilateral negotiations must be more precisely specified (Stole and Zwiebel implicitly assume that the precise wage that is paid to a worker is not observed by other workers), and certain specific 'out of equilibrium' beliefs specified, for their result to hold. As will be apparent below, our extensive form bargaining game – consisting of a sequence of bilateral negotiations based on the Binmore, Rubinstein and Wolinsky outcome – is a natural extension of theirs to more general economic environments.

Finally, we note the influential contribution of Hart and Mas-Colell (1996) to this literature. They do not model an extensive form game based on bilateral offers and negotiations but instead consider rounds where players have opportunities to make offers to all 'active' players (i.e., players who have not had a proposal rejected). If this is accepted by all 'active' players, the game ends. If it is not accepted by one player there is a chance that the proposer will be excluded from the game. Hart and Mas-Colell (1996) demonstrate that there is a unique subgame perfect equilibrium of this game that results in each active player receiving its Shapley value.[3] As Winter (2002) surveys, this game has given rise to a variety of extensions but in general the institutional environment requires the ability of proposers to make offers to all, for single rejections to nullify agreements and for a commitment to cause proposers to risk exit following rejection.

In summary, prior extensive form games that generate Shapley value outcomes as equilibrium outcomes, while significant, have been based in somewhat restrictive economic environments. Either the set of choices is restricted to decisions to join coalitions or not – as in Gul (1989) and Stole and Zwiebel (1996) – or alternatively, the institutional environment involves communication structures and commitment not present in many important economic environments.

3. Observability of Actions

Because we consider an environment where all negotiations are bilateral, we similarly restrict the observability of actions to no more than two agents. We assume below that individual actions (such as effort expended or an investment) may be observable, and hence negotiable, with at most one other agent. Similarly, a joint action (such as exchange of goods, services or assets) may be observed and negotiated by the two agents concerned. However, in each case, agents outside of the agreement cannot observe the action taken. Importantly, what this means is that agents cannot negotiate agreements contingent upon negotiations that one or neither of them is a party to. To

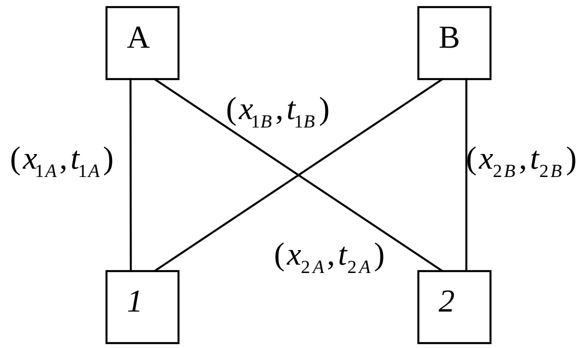
[3] A related multilateral mechanism based on bids for the surplus is provided by Perez-Castrillo and Wettstein (2001) with the property that the Shapley value is obtained in every equilibrium outcome of the game and not just in expectation.

assume otherwise would be inconsistent with our restriction to bilateral bargaining and would suggest instead that a multilateral bargaining protocol might be more appropriate.

As an example, consider an environment where there are 2 sellers (*A* and *B*) and two buyers (1 and 2) of a product, where 1 and 2 compete against each other in a downstream market. Each buyer and seller can negotiate over the quantity of the product traded between them; e.g., 1 and *A* negotiate over x_{1A} and so on. If upstream products are homogenous, the buyers' values are functions, $b_1(x_{1A}+x_{1B}, x_{2A}+x_{2B})$ and $b_2(x_{2A}+x_{2B}, x_{1A}+x_{1B})$, respectively (where the partial derivative of the second term is negative). Assume that the sellers have no costs. In exchange for the product, buyers pay the sellers a transfer; for example, 1 pays *A*, t_{1A}. Each pair trades a quantity and pays a transfer between them.

The network of bilateral negotiations is as depicted in Figure 1. Notice that the two buyers and the two sellers are assumed here not to negotiate with one another (say for antitrust reasons). Our observability requirements will also presume that 2 will not be able to observe (x_{1A}, t_{1A}) or (x_{1B}, t_{1B}). This means that when 2 negotiates with *A*, agreements cannot be made contingent upon (x_{1A}, t_{1A}), (x_{1B}, t_{1B}) and (x_{2B}, t_{2B}).

Figure 1: Buyer-Seller Network



To formalise this, consider a set of agents, $N = \{1, 2, ..., n\}$. There are three types of actions:

1. Individually observable actions taken by *i*: let $\mathbf{a}_i$ be the vector of such actions.
2. Jointly observable actions by *i* and *j* (*i* < *j*): let $\mathbf{x}_{ij}$ be the $m_{ij} \times 1$ vector of such actions.
3. Transfers between *i* and *j*: there is a single payment from *i* to *j*, t_{ij}, that may be positive or negative or zero. That is, $t_{ij} = -t_{ji}$

As noted earlier, it is clear that $\mathbf{a}_i$, $\mathbf{x}_{ij}$ and t_{ij} are observed by *i*. (A1) formalizes our unobservability assumption. Let $A = \left(\mathbf{a}_1, \mathbf{a}_2, ...\mathbf{a}_n\right)$, $X = \left(\mathbf{x}_{12}, \mathbf{x}_{13}, ...\mathbf{x}_{1n}, \mathbf{x}_{23}, \mathbf{x}_{24}, ...\mathbf{x}_{2n}, ...\mathbf{x}_{n-1,n}\right)$ and $T = \left(t_{12}, t_{13}, ...t_{1n}, t_{23}, t_{24}, ...t_{2n}, ...t_{n-1,n}\right)$ be the vectors of realized actions.

(A1) (Unobservable Actions) *During negotiations, agent i cannot observe* $A / \mathbf{a}_i$, $X / \{\mathbf{x}_{ij}\}_{j \in N}$ *and* $T / \{t_{ij}\}_{j \in N}$.

In particular, this means that even if it is negotiating with *i*, *j* cannot directly communicate to *i* the outcomes of a previous negotiation with, say, *k*. Instead, *i* must form beliefs over those actions it cannot observe and expectations about outcomes in the future. We let *i*'s beliefs over a particular action be superscripted with *i* and marked with a tilde. That is, *i*'s beliefs regarding $\mathbf{x}_{kl}$ would be $\tilde{\mathbf{x}}_{kl}^{i}$.

To simplify notation, throughout most of the paper we focus on the simple case where **there are no individual actions, and all actions are joint actions** – requiring, say, *i* and *j* to agree for $\mathbf{x}_{ij}$ to be anything other than 0. For instance, in our example, if *A* and 1 cannot agree, *A* supplies no inputs to 1 and 1 does not pay *A*. At the end of Section 6, we extend the results to a more general action space.

4. Bargaining Game

We begin by stating some additional notation, before defining our extensive form bargaining game.

Set-up and notation

The most natural way to describe the set of bilateral negotiations is by a graph (*N*, *L*) which has the set of agents as its vertices each connected by a set of edges or links, $L \subseteq L^N = \left\{\{i, j\} \middle| \{i, j\} \subseteq N, i \neq j\right\}$. Thus, the potential number of links in a *complete* graph (N, L^N) is $n(n$-1)/2. An individual link between *i* and *j* will be denoted $ij \equiv \{i, j\}$ (and therefore $ij = ji$). *L* describes the state space of potential bilateral agreements. If $ij \in L$, then agents *i* and *j* can still come to a bilateral agreement. If $ij \notin L$, then agents *i* and *j* cannot negotiate by assumption, or they have reached a disagreement state. If a pair $ij \in L$ were to disagree (i.e., their negotiation breaks down), the new state is denoted: $L - ij \equiv L \setminus \{ij\}$. Define *L*/*S*, the graph *L* restricted to the set of agents, *S*: $L / S = \left\{ ij \middle| ij \in L, i \in S, j \in S \right\}$. Finally, for any network, *L*, let $S(L) \equiv \left\{ i \middle| \exists j \text{ s.t. } ij \in L \right\}$.

L describes a network of bilateral relationships. That network connects sub-groups of agents or perhaps all agents. More precisely,

Definition (Connectedness). *Agents i and j are connected in network L if there exists a sequence of agents* $(i_1, i_2, ..., i_t)$ *such that* $i_1 = i$ *and* $i_t = j$ *and* $\{i_l, i_{l+1}\} \in L$ *for all* $l \in \{1, 2, ..., t-1\}$. *i is directly connected to j if* $ij \in L$.

Definition (Component). *A set of agents* $h \subset N$ *is a component of N in L if (i) all* $i \in h, j \in h, i \neq j$ *are connected in* (*N*,*L*)*; and (ii) for any* $i \in h, j \notin h$, *i and j are not connected. The set of all components of* (*N*,*L*) *is C*(*L*).

Finally, for some analysis that follows it will be convenient to partition the set of agents. $P = \{P_1, ..., P_p\}$ is a partition of the set *N* if and only if (i) $\bigcup_{i=1}^{p} P_i = N$; (ii) $P_i \neq \varnothing$; and (iii) for all $j \neq k$, $P_j \cap P_k = \varnothing$. We define *p* as the cardinality of *P*. The set of all partitions of *N* is P^N. For a given network (*N*, *K*), we can now define a graph, (*N*, K^P), imputed from a partition, *P*. That is, $K^P = \{ jk \in K \,|\, \exists i \text{ s.t. } j, k \in P_i \}$. In other words, (*N*, K^P) is a graph partitioned by *P*.

Starting with a network (*N*, *L*), agents *i* and *j* negotiate bilaterally over choices, $x_{ij}(K) \in \mathbf{R}$ and payments $t_{ij}(K) \in \mathbf{R}$ for each $K \subseteq L$ with $ij \in K$. For a given $1 \times m$ vector $X = \left(\mathbf{x}_{12}, \mathbf{x}_{13}, ...\mathbf{x}_{1n}, \mathbf{x}_{23}, \mathbf{x}_{24}, ...\mathbf{x}_{2n}, ...\mathbf{x}_{n-1,n} \right)$ of all players' actions, and a set of transfers t_{ij}, an agent's payoff is $u_i(X) - \sum_{j \in N} t_{ij}$. Thus, we are assuming a transferable utility environment where total surplus generated is not affected by transfers. We assume that $u_i(X)$ is *strictly concave* in $\mathbf{x}_{ij}$ for any $X \setminus \{\mathbf{x}_{ij}\}_{j \in N}$, and that $\sum_{i \in N} u_i(X)$ is globally concave.

If there is no link (or a broken link) between *i* and *j*, we assume that the relevant choice variables $\mathbf{x}_{ij}$ and t_{ij} are 0 (this may be a normalization, for $\mathbf{x}_{ij}$).

This notation also allows us to define what we mean by a (constrained) efficient set of agreements.

Definition (Constrained Efficiency): *For a given graph K, a vector of actions* X^{K*} *is constrained efficient if*

$$X^{K*} = \arg\max_X \sum_{i \in N} u_i(X) \text{ subject to } \mathbf{x}_{ij} = 0 \text{ if } ij \notin K .$$

X^{K*} *is unique because the problem is globally concave.* Let $v(N,K) = \sum_{i \in N} u_i(X^{K*})$.

Thus, for a given network, (N,K), an agreement is constrained efficient if the choices agreed upon maximise the sum of utilities of all agents, whether they are party to an agreement or not. Similarly, for a coalition $S \subseteq N$ and set of links *K*, define $v(S,K)$, the constrained efficient actions by agents in *S* alone, when other agents take no actions, that maximize the utilities of agents in *S*.

Next, we define bilateral efficiency, as distinct from efficiency and constrained efficiency:

Definition (Bilateral Efficiency). *For a given graph, K, a vector of actions,* $\hat{X}^K = \left(\hat{\mathbf{x}}_{12}(K), \hat{\mathbf{x}}_{13}(K), ..., \hat{\mathbf{x}}_{1n}(K), \hat{\mathbf{x}}_{23}(K), ... \hat{\mathbf{x}}_{n-1,n}(K)\right)$ *satisfies bilateral efficiency if and only if:*

$$\hat{\mathbf{x}}_{ij}(K) = \begin{cases} \arg\max_{\mathbf{x}_{ij}} u_i\left(\mathbf{x}_{ij}, \hat{X}^K \setminus \hat{\mathbf{x}}_{ij}(K)\right) + u_j\left(\mathbf{x}_{ij}, \hat{X}^K \setminus \hat{\mathbf{x}}_{ij}(K)\right) & \text{if } ij \in K \\ 0 & \text{if } ij \notin K \end{cases}.$$

Consistent with this definition, we define: $\hat{v}(N,K) \equiv \sum_{i \in N} u_i(\hat{X}^K)$ where $\hat{X}^K$ is bilaterally efficient. For any coalition, $S \subseteq N$, $\hat{v}(S,K)$. is defined analogously. Note that the values $\hat{v}(.)$ are unique given our concavity assumptions on $u_i(.)$.

All of these concepts can be illustrated by returning to our buyer-seller network example. In this situation, Figure 1 depicts the set of links, $L = \{1A, 1B, 2A, 2B\}$ and we have assumed that $u_1(X^L) = b_1(x_{1A} + x_{1B}, x_{2A} + x_{2B})$, $u_2(X^L) = b_2(x_{2A} + x_{2B}, x_{1A} + x_{1B})$ and $u_A(X^L) = u_B(X^L) = 0$. An efficient outcome would involve $v(N,L) = \max_{x_{1A}, x_{1B}, x_{2A}, x_{2B}} b_1(x_{1A} + x_{1B}, x_{2A} + x_{2B}) + b_2(x_{2A} + x_{2B}, x_{1A} + x_{1B})$. If, however, 1 and A could no longer negotiate or trade with one another, the network would become $K = \{1B, 2A, 2B\}$ and (x_{1A}, t_{1A}) would be set equal to $(0,0)$ with $v(N,K) = \max_{x_{1B}, x_{2A}, x_{2B}} b_1(x_{1B}, x_{2A} + x_{2B}) + b_2(x_{2A} + x_{2B}, x_{1B})$. Conversely, bilateral efficient outcomes in the same (N,L) network would involve $\hat{x}_{1A}(N,L) = \arg\max_{x_{1A}} b(x_{1A} + \hat{x}_{1B} \mid \hat{x}_{2A} + \hat{x}_{2B})$ and so on. Finally, if we were to partition the set of agents into $P = \{\{1, A\}, \{2, B\}\}$ then, $L^P = \{1A, 2B\}$ and $K^P = \{2B\}$.

In terms of what the parties negotiate over, recall that these are agreements contingent on the state of the network. So 1 and A could negotiate, say, a quantity $x_{1A}(L) = 3$ and transfer $t_{1A}(L) = 2$ as well as $x_{1A}(1A, 1B, 2B) = 4$ and $t_{1A}(1A, 1B, 2B) = 5$ and so on. That is, they consider all possible networks that could emerge and they can negotiate different quantities and transfers that would be payable upon the final realisation of any particular network. In principle, the transfers and quantities paid under each network contingency could be the same and the contract could be a full commitment. However, in the equilibrium we focus on below, this will not be the case.

Information regarding the bargaining network

In what follows, a *breakdown* in bargaining between i and j is a situation where the network changes from (N, L) where $ij \in L$ to $(N, L - ij)$. It will also be considered irreversible as the link between i and j can never subsequently be restored. Thus, as breakdowns are possible, the network will potentially move from one with many links to ones that are subsets of the original network. For convenience, we will sometimes describe networks in terms of states with a current state and potential future states.

A key assumption here is:

(A2) (Knowledge of the Bargaining Network) *The current state of the network is common knowledge.*

This assumption is necessary in order for agents to negotiate contracts that are contingent upon the state of the network. As we will see below, this assumption would also be necessary if, rather than writing contracts contingent upon networks that may arise, agents negotiated contracts based only on the current state of the network and renegotiated them in the event a new network arose (following a breakdown).

Extensive form

We are now in a position to define the full extensive form game. Given (N, L), fix an order of the directly connected pairs, $\{ij\}_{ij \in L}$. The precise order will not matter to the solution that follows. Bargaining proceeds as follows. Each pair negotiates in turn. A bilateral negotiation takes the following form: randomly select i or j. That agent, say i, makes an offer of a set of contingent actions and transfers $\left\{\mathbf{x}_{ij}(K), t_{ij}(K)\right\}_{K \subseteq L, ij \in K}$ to *j*, for all networks $K \subseteq L$ such that $ij \in K$. Notice that offers are *contingent* upon the potential agreement state (*K*). *j* either accepts the offer or rejects it. If *j* accepts it, the offers $\left\{\mathbf{x}_{ij}(K), t_{ij}(K)\right\}_{K \subseteq L, ij \in K}$ are all fixed and we proceed to the next pair. If *j* rejects the offer, with probability 1-σ negotiations end and the bargaining game with the remaining pairs continues over a new network $(N, L - ij)$ on the original order. Otherwise, negotiations between *i* and *j* continue with another randomization: either *i* or *j* is picked with 50% probability to make the next offer, over *the same network K.*

This specification of an individual bilateral negotiation is very similar to that of Binmore, Rubinstein and Wolinsky (1986) for stand-alone bilateral negotiations. Here, however, bilateral negotiations are not isolated and are embedded within a sequence of negotiating pairs; also, they cover multiple contingencies simultaneously.[4]

Belief structure

Given that our proposed game involves incomplete information, to demonstrate the existence of certain equilibrium outcomes in the game, we will need to impose some structure on 'out of equilibrium' beliefs. This is an issue that has drawn considerable

[4] Inderst and Wey (2003) also consider negotiations over contingencies. They model multilateral negotiations as occurring simultaneously; an agent involved in more than one negotiation delegates one agent to bargain on their behalf in each negotiation.. This alternative specification may be appropriate for situations where negotiations take place between firms. Agents could not observe the outcomes of negotiations they were not a party to. This would avoid the need to specify beliefs precisely in any equilibrium. As our model applies more generally than just between firms, we chose not to rely on a similar specification here. It is clear, however, that a 'delegated agent' specification (where applicable) would generate a unique equilibrium outcome.

Note that Inderst and Wey's treatment of individual negotiations is axiomatic rather than a full extensive-form, in the sense that they merely posit that agents split the surplus from negotiations in each different contingency.

attention in the contracting with externalities literature (McAfee and Schwartz, 1994; Segal, 1999; Rey and Vergé, 2004).

It is not our intention to revisit that literature here. Suffice it to say that the most common assumption made about what players believe about actions that they do not observe, or that have not yet happened is the simple notion of "passive" beliefs. We will utilise it through this paper. To define it, let $\{(\hat{\mathbf{x}}_{ij}(K), \hat{t}_{ij}(K))_{K \subseteq L}\}_{ij \in L}$ be a set of equilibrium agreements between all negotiating pairs.

Definition (Passive Beliefs). *When i receives an offer from j of* $\mathbf{x}_{ij}(K) \neq \hat{\mathbf{x}}_{ij}(K)$ *or* $t_{ij}(K) \neq \hat{t}_{ij}(K)$, *i does not revise its beliefs regarding any other action in the game.*

At one level, this is a natural belief structure that mimics Nash equilibrium reasoning.[5] That is, if *i*'s beliefs are consistent with equilibrium outcomes – as they would be in a perfect Bayesian equilibrium – then under passive beliefs, it holds those beliefs constant off the equilibrium path as well. At another level, this is precisely why passive beliefs are not appealing from a game-theoretic standpoint. Specifically, if *i* receives an unexpected offer from an agent it knows to be rational, a restriction of passive beliefs is tantamount to assuming that *i* makes no inference from the unexpected action. Nonetheless, as we demonstrate here, passive beliefs play an important role in generating tractable and interpretable results from our extensive form bargaining game; simplifying the interactions between different bilateral negotiations.

[5] McAfee and Schwartz (1995, p.252) noted that: "one justification for passive beliefs is that each firm interprets a deviation by the supplier as a tremble and assumes trembles to be uncorrelated (say, because the supplier appoints a different agent to deal with each firm)." Similarly, the passive beliefs equilibrium in this paper is trembling hand perfect in the agent perfect form.

Externalities

Most of our important results are in environments with non-pecuniary externalities; here we define the absence of such externalities:

Definition (No Non-Pecuniary Externalities). *$u_i(X^L)$ is independent of $\mathbf{x}_{jk}$ for all* $\{jk \mid i \neq j, i \neq k\}$.

That is, *i*'s utility is only affected by observable actions made by agents it is directly connected to. Notice that, in a non-cooperative game, pecuniary externalities can still exist through the transfers that are agreed upon in other bilateral negotiations that themselves impact on the value of an agreement between a particular pair.

Feasibility

Below we will focus on an equilibrium outcome of our non-cooperative bargaining game whose convenient characterisation will at times rely upon agreements being reached in all bilateral negotiations in a network (N, L). However, in general, an equilibrium with this property may not exist. For instance, as Maskin (2003) demonstrates, when an agent may be able to free ride upon the contributions and choices of other agents, that agent may have an incentive to force breakdowns in all their negotiations so as to avoid their own contribution. Maskin demonstrates that this is the case for situations where there are positive externalities between groups of agents (as in the case of public goods).

The idea that an agent or group of agents may not wish to participate in a larger coalition is something that was an issue for Stole and Zwiebel (1996). Specifically, agents will not want to unilaterally or bilaterally trigger breakdowns in negotiations if the payoffs they expect to receive are feasible.

Definition (Feasible Payoffs). *A set of payoffs* $\Upsilon_i(N,L)$ *for each i is* ***feasible*** *if and only if, for any sub-network* $L' \subseteq L$ *and any link* $ij \in L'$, $\Upsilon_i(N,L') \geq \Upsilon_i(N,L'-ij)$.

As did Stole and Zwiebel (1996), we assume that the primitives of the environment are such that feasibility is assured.

(A3) (Feasibility) *In equilibrium, all payoffs are feasible*.

Notice that our buyer-seller network example the payoffs are feasible as $b_1(.)$ and $b_2(.)$ are independent of the purchases of the other buyer and hence, $\hat{v}(N,K) \geq \hat{v}(N,K-ij)$ for all $ij \in K$ and $K \subseteq L$. However, if these buyers were competitors in some other market, then it is possible that their purchases could enter into the utilities of each other. In this case, an externality would be present and we cannot take it for granted that outcomes were feasible.

In general, feasibility is something that would have to be confirmed for each environment. If it did not hold, then our bargaining game will not have an equilibrium outcome that involved all negotiations at the initial network state being concluded with an agreement. Some would break down.

However, as our goal here, is to consider when many inter-related bilateral negotiations might give rise to computationally straightforward results, it is useful to state a result where for one environment with externalities feasibility holds. This occurs when there is component superadditivity.

Definition (Component Superadditivity). *For a given set of agents, h, and graph K, suppose that* $v(h,K) \geq v(h,K-ij)$. *If this condition holds, for each component* $h \in C(K)$ *and each graph,* $K \subseteq L$, *then* $v(N,L)$ *satisfies component superadditivity.*

This leads to a first result:

***Lemma 1**: Suppose that $v(N,L)$ satisfies component superadditivity, then the equilibrium payoffs will be feasible.*

All proofs are in the appendix.

5. Equilibrium Outcomes: Actions

In exploring the outcomes of this non-cooperative bargaining game, it is useful to focus first on the equilibrium actions that emerge before turning to the transfers and ultimate payoffs. Of course, the equilibrium described is one in which actions and transfers are jointly determined. It is for expositional reasons that we focus on each in turn.

***Theorem 1.** Suppose that all agents hold passive beliefs regarding the outcomes of negotiations they are not a party to. Given (N,L), as $\sigma \to 1$, there exists a perfect Bayesian equilibrium in which equilibrium actions $\hat{X}^L$ are bilaterally efficient.*

This result says that actions are chosen to maximise joint utility holding those of others as given. It is easy to see that, in general, the outcome will not be efficient.

The intuition behind the result is subtle. Consider a pair, *i* and *j*, negotiating in an environment where all other pairs have agreed to the equilibrium choices in any past negotiation, there is one more additional negotiation still to come and that this negotiation involves *j* and another agent, *k*. Given the agreements already fixed in past negotiations, the final negotiation between *j* and *k* is simply a bilateral Binmore, Rubinstein, Wolinsky bargaining game. That game would ordinarily yield the Nash bargaining solution if *j* and *k* had symmetric information regarding the impact of their choices on their joint utility, $u_j(x_{ij}, x_{jk}, .) + u_k(x_{ij}, x_{jk}, .)$. This will be the case if *i* and *j* agree to the equilibrium $\hat{x}_{ij}$.

However, if *i* and *j* agree to $x'_{ij} \neq \hat{x}_{ij}$, *j* and *k* will have different information. Specifically, while, under passive beliefs, *k* will continue to base its offers and acceptance decisions on an assumption that $\hat{x}_{ij}$ has occurred, *j*'s offers and acceptances will be based on x'_{ij}. That is, *j* will make an offer, $(t'_{jk}, x'_{jk}(x'_{ij}))$, that maximises $u_j(x'_{ij}, x_{jk}, .) - t_{jk}$ rather than $u_j(\hat{x}_{ij}, x_{jk}, .) - t_{jk}$ subject to *k* accepting that offer. Moreover, we demonstrate that *j* will reject offers made to it by *k*.

In this case, the question becomes: will *i* and *j* agree to some $x'_{ij} \neq \hat{x}_{ij}$? If they do, this will alter the equilibrium in subsequent negotiations. *j* will anticipate this, however, the assumption of passive beliefs means that *i* will not. That is, even if they agreed to $x'_{ij} \neq \hat{x}_{ij}$, *i* would continue to believe that $\hat{x}_{jk}$ will occur. For this reason, *i* will continue to make offers consistent with the proposed equilibrium. On the other hand, *j* will make an offer, (t'_{ij}, x'_{ij}), that maximises $u_j(x_{ij}, x'_{jk}(x_{ij}), .) + t'_{ij}(x_{ij}) - t'_{jk}(x'_{jk}(x_{ij}))$ rather than $u_j(x_{ij}, \hat{x}_{jk}, .) - t_{ij} - \hat{t}_{jk}$ subject to *i* accepting that offer. We demonstrate that this is equivalent to *j* choosing:

$$x'_{ij} \in \arg\max_{x_{ij}} u_j(x_{ij}, x'_{jk}(x_{ij}), .) + u_i(x_{ij}, \hat{x}_{jk}, .) + u_k(\hat{x}_{ij}, x'_{jk}(x_{ij}), .)$$

which, by the envelope theorem applied to x'_{jk}, has $x'_{ij} = \hat{x}_{ij}$, the bilaterally efficient action. By a similar argument, *j* does not find it worthwhile to deviate in a series of several negotiations.

Finally, it is useful to state a case where the perfect Bayesian equilibrium outcome under passive beliefs is efficient:

Corollary 1. *Assume the conditions of Theorem 1 and that there are no non-pecuniary externalities for all i. Then given* (N, L)*, as* $\sigma \to 1$*, there exists a perfect Bayesian equilibrium in which actions are constrained efficient.*

In effect, Corollary 1 can be viewed as a generalisation of the results of Segal (1999, Proposition 3) that when there are no externalities, contracts are efficient.

6. Equilibrium Outcomes: Transfers and Payoffs

We are now in a position to consider the equilibrium transfers and payoffs. As was determined above, when there are externalities present, sequential bilateral bargaining does not lead to a maximised surplus. Instead, under passive beliefs, it yields a Nash equilibrium where actions are taken ignoring externalities on other agents. In this sense, the outcome is very different from what might emerge from cooperative bargaining.

However, we demonstrate here that while surplus is determined in a non-cooperative manner, under the same passive beliefs assumption, surplus division takes on a form attractively similar to cooperative bargaining outcomes. In particular, depending upon the nature of externalities and the network of bilateral negotiations, the division of whatever surplus is created gives agents a generalization of their Myerson value on that reduced surplus. As such, the division of surplus between players has an appealing coalitional structure even if the surplus is non-cooperatively determined.

Some definitions

It is useful at this point to state some additional definitions from cooperative bargaining theory using our notation.

A characteristic function $V(S)$ assigns a payoff to a coalition S of players, who are cooperating with each other but not with anyone else in their coalition. If the possibility of cooperation between agents is represented as a link in a graph, when players form a coalition, they have severed their links to anyone outside their coalition. Cooperative game theory assumes that coalition S cooperates internally so as to maximize its total payoff. However, beginning with Myerson (1977a), the cooperative literature allows for the possibility that cooperation may be restricted initially to an (exogenously given) graph (N, L), even before any coalitions S have broken links with other players. Thus players in a coalition S who are not connected within S are assumed to be unable to cooperate with each other; there may be several components in S, and cooperation is restricted to components. Jackson and Wolinsky (1996) further extend the restrictions imposed by graphs by allowing the structure of the graph within a component to affect the payoff of the component; we adopt that extension here. Therefore, let a characteristic function $V(h,L)$ assign a payoff to a component h of a graph L. A coalition S may be composed of several components, thus we define $V(S,L)=\sum_{h\in C(L),h\subseteq S}V(h,L)$. (Note that $V(S, L)$ is not defined if any $i\in S$ is connected to any $j\in N\backslash S$.)

Most of cooperative game theory is concerned with environments in which there are no externalities between players in different components. *If there are no component externalities,* $V(h,K)=V(h,K') \ \ \forall K,K' \ \ s.t. \ \text{ if } \ i\in h \Rightarrow i\in K\cap K'$.

Definition (Myerson Value). *Given a characteristic function $V(S,L/S)$, and an environment in which there are no component externalities between players in different components, the Myerson value of agent i, $\Phi_i(L)$ is:*

$$\Phi_i(N,L)=\sum_{S\subseteq N:i\notin S}\frac{|S|!\big(|N|-1-|S|\big)!}{|N|!}\Big(V\big(S\cup i,L/(S\cup i)\big)-V(S,L/S)\Big).$$

Notice that for a complete, graph, L^N, $\Phi_i(N, L^N)$ is the Shapley value.

In the spirit of cooperative game theory, the characteristic function should represent the total payoff to a coalition when members take the actions (among those possible) that maximize the coalition's payoff; in other words, the constrained efficient actions. Therefore Jackson and Wolinsky (1996) would define the characteristic function as $V^{coop}(S,K) = v(S,K) \equiv \max_{X^{K/S}} \sum_{i \in S} u_i(X^{K/S})$. Notice, however, that for **any** characteristic function, a Myerson value can be calculated.

The Myerson value is somewhat restrictive in that it is not defined in situations where different groups of agents impose externalities upon one another. Myerson (1977b) generalised the Shapley value to consider externalities by defining it for games in partition function form. Here we define a further generalization of the Myerson value to allow for a partition function space as well as a graph of potential communications (as in Navarro 2003).[6] The characteristic function $V(h,L)$ in such an environment depends on the structure of the entire graph, both within component h and in other components.

Definition (Generalized Myerson Value). *Given a characteristic function $V(S,L)$, the Generalized Myerson value of agent i in graph (N,L), $\Upsilon_i(N,L)$, is:*

$$\Upsilon_i(N,L) = \sum_{P \in P^N} \sum_{S \in P} (-1)^{|P|-1}(|P|-1)! \left[\frac{1}{|N|} - \sum_{\substack{i \notin S' \in P \\ S' \neq S}} \frac{1}{(|P|-1)(|N|-|S'|)} \right] V(S, L^P).$$

It is easy to demonstrate that when there are no externalities between different components (i.e., $u_i(X^L)$ is independent of x_{kl} for any k and l not in the same component

[6] Navarro (2003) includes a definition of the Generalized Myerson Value or Myerson Value in Partition Function Form, and similar properties to those derived in the appendix.

as *i*), this value is equivalent to the Myerson value and, in addition, if it is defined over a complete graph, it is equivalent to the Shapley value.[7]

Some Issues: An Illustrative Example

Before turning to consider these results, it is useful to highlight some important technical issues by way of an illustrative example. Consider a situation in which there are three agents (1, 2 and 3), each of whom can negotiate bilaterally with one another; that is, our starting point is a complete graph. We will denote this initial network by 123. If there is a breakdown in negotiations between one pair that will result in a network of 1-2-3, 1-3-2 or 2-1-3 respectively; with the middle agent the agent who has not had a breakdown with any of the other two agents. If there are two breakdowns in negotiations, the networks may become 12, 13 or 23. Finally, if all three negotiations breakdown, the state becomes 0.

We suppose also that there are only joint actions and, using the result in Theorem 1, those actions will lead to a payoff to agent *i* of $u_i(K)$; for example, if network 1-2-3 occurs, the expected negotiated actions are such that $u_1(1-2-3)$ is generated to agent 1.

To see how payoffs and transfers are determined in equilibrium, note that, as the probability of a breakdown anywhere, σ, goes to 0, we can treat negotiations over

[7] How would cooperative game theorists choose $V(S,L)$? They would expect agents to maximize the payoff to their component, but how would they act towards other components? There are relatively few applications to guide us. Myerson (1977c) considers an environment in which each coalition takes the action that maximizes the coalition's payoff (because, in graphical terms, all members are directly linked), while taking as given the actions of other coalition. In other words, the coalitions are playing a Nash equilibrium strategy against the other coalitions. (Assumptions must be made to ensure the uniqueness of the Nash equilibrium.) Note once again that for any characteristic function $V(S,L)$, the generalized Myerson value can be calculated.

transfers in each state as separate bilateral negotiations between each negotiating pair. If this is 12, then, then our bargaining game results in the Nash bargaining solution:

$$\begin{aligned} &u_1(12)-t_{12}-u_1(0)=u_2(12)+t_{12}-u_2(0) \\ &\Rightarrow t_{12}(12)=\tfrac{1}{2}\left(u_1(12)-u_2(12)+u_2(0)-u_1(0)\right) \end{aligned} \tag{1}$$

With 13 and 23 defined similarly. For 1-2-3 these are:

$$\begin{aligned} &u_1(1-2-3)-t_{12}(1-2-3)-u_1(23) \\ &=u_2(1-2-3)+t_{12}(1-2-3)-t_{23}(1-2-3)-\left(u_2(23)-t_{23}(23)\right) \end{aligned} \tag{2}$$

$$\begin{aligned} &u_2(1-2-3)+t_{12}(1-2-3)-t_{23}(1-2-3)-\left(u_2(12)+t_{12}(12)\right) \\ &=u_3(1-2-3)-t_{23}(1-2-3)-u_3(12) \end{aligned} \tag{3}$$

(with 1-3-2 and 2-1-3 defined similarly). And for 123, these are:

$$\begin{aligned} &u_1(123)-t_{12}(123)-t_{13}(123)-\left(u_1(1-3-2)-t_{13}(1-3-2)\right) \\ &=u_2(123)+t_{12}(123)-t_{23}(123)-\left(u_2(1-3-2)-t_{23}(1-3-2)\right) \end{aligned} \tag{4}$$

$$\begin{aligned} &u_1(123)-t_{12}(123)-t_{13}(123)-\left(u_1(1-2-3)-t_{12}(1-2-3)\right) \\ &=u_3(123)+t_{13}(123)+t_{23}(123)-\left(u_3(1-2-3)+t_{23}(1-2-3)\right) \end{aligned} \tag{5}$$

$$\begin{aligned} &u_2(123)+t_{12}(123)-t_{23}(123)-\left(u_2(2-1-3)-t_{12}(2-1-3)\right) \\ &=u_3(123)+t_{13}(123)+t_{23}(123)-\left(u_3(2-1-3)+t_{13}(2-1-3)\right) \end{aligned} \tag{6}$$

with the total number of transfer prices over all contingent negotiations being 12. While solving for transfers would appear to be possible with 12 bargaining equations and 12 unknowns, equations (4), (5) and (6) are linearly dependent. For there are many consistent transfer prices – $t_{12}(123)$, $t_{13}(123)$ and $t_{23}(123)$ – that will satisfy those equations. In other cases, the transfer prices are uniquely determined. It is for this reason that we refer in theorems to equilibrium payoffs rather than equilibrium transfers themselves. Nonetheless, even though particular transfer prices are not uniquely determined in some networks, payoffs are uniquely determined.

The recursive structure of the equations provides a simple algorithm for finding the solution. Thus, in this game, it is straightforward to demonstrate that in equilibrium, agents receive:

$$\begin{aligned}\Phi_1(123) = {} & \tfrac{1}{3}\left(u_1(123)+u_2(123)+u_3(123)\right)+\tfrac{1}{3}\left(2u_1(23)-u_2(23)-u_3(23)\right)\\ & +\tfrac{1}{6}\left(u_1(12)+u_2(12)-2u_3(12)\right)+\tfrac{1}{6}\left(u_1(13)-2u_2(13)+u_3(13)\right)\\ & +\tfrac{1}{6}\left(-2u_1(0)+u_2(0)+u_3(0)\right)\end{aligned}$$

$$\begin{aligned}\Phi_2(123) = {} & \tfrac{1}{3}\left(u_1(123)+u_2(123)+u_3(123)\right)+\tfrac{1}{6}\left(-2u_1(23)+u_2(23)+u_3(23)\right)\\ & +\tfrac{1}{6}\left(u_1(12)+u_2(12)-2u_3(12)\right)+\tfrac{1}{3}\left(-u_1(13)+2u_2(13)-u_3(13)\right)\\ & +\tfrac{1}{6}\left(u_1(0)-2u_2(0)+u_3(0)\right)\end{aligned}$$

$$\begin{aligned}\Phi_3(123) = {} & \tfrac{1}{3}\left(u_1(123)+u_2(123)+u_3(123)\right)+\tfrac{1}{6}\left(-2u_1(23)+u_2(23)+u_3(23)\right)\\ & +\tfrac{1}{3}\left(-u_1(12)-u_2(12)+2u_3(12)\right)+\tfrac{1}{6}\left(u_1(13)-2u_2(13)+u_3(13)\right)\\ & +\tfrac{1}{6}\left(u_1(0)+u_2(0)-2u_3(0)\right)\end{aligned}$$

These outcomes are, in fact, each agent's Generalized Myerson values for the complete graph L^N. We demonstrate below that this is a general outcome in environments with externalities.

Notice that these payoffs do *not* depend on network states where there are two bilateral negotiations despite that fact that $\sum_i u_i(123)$ does not equal $\sum_i u_i(1-2-3)$ as it does in Myerson (1977b). Payoffs here only depend on payoffs to agents under graphs created by partitioning the initial graph. Jackson and Wolinsky (1996) demonstrate a similar outcome for the Myerson value. Here, the outcome arises for essentially the same reason: that each pair of Nash bargaining equations represents a condition of balanced contributions. This is a property that makes these bargaining outcomes particularly useful in applications, as we do not need to solve for non-cooperative action outcomes in any other network besides the initial network, under all possible partitions; a substantial reduction in the number of cases.[8]

General Result

We are now in a position to state our main result.

Theorem 3. *Given* (N,L)*, as* $\sigma \to 1$*, there exists a perfect Bayesian outcome of our extensive form bargaining game with bilaterally efficient actions* $\hat{X}^L$*, and with each agent i receiving:*

$$\hat{\Upsilon}_i(N,L) = \sum_{P \in P^N} \sum_{S \in P} (-1)^{|P|-1} (|P|-1)! \left[\frac{1}{n} - \sum_{\substack{i \notin S' \in P \\ S' \neq S}} \frac{1}{(|P|-1)(n-|S'|)} \right] \hat{v}(S, L^P) .$$

In other words, each agent receives their generalized Myerson value associated with characteristic function $V(S,L) = \hat{v}(S,L)$. *Agents take their bilaterally efficient actions, rather than those which maximize the payoff to their component*. Thus, in equilibrium, we have a generalized Myerson value type division of a reduced surplus. That surplus is generated by a bilaterally efficient outcome in which each bilateral negotiation maximises the negotiator's own sum of utilities while ignoring the external impact of their choices on other negotiations (as in Theorem 1).

As in Theorem 1, the proof relies upon the agents holding passive beliefs in equilibrium. For this reason, Theorem 3 is an existence proof. Without passive beliefs, the equilibrium outcomes are more complex and do not reduce to this simple structure. That simplicity is, of course, the important outcome here. What we have is a bargaining solution that marries the simple linear structure of cooperative bargaining outcomes with

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[8] There are $n(n$-1)/2 possible links, each of which can take a value of 0 or 1; thus there are potentially $2^{n(n-1)/2}$ possible networks to consider. However there are at most $B(n(n$-1)/2) possible partitions of the initial network (where $B(s)$ is the Bell number for s objects).

easily determined actions based on bilateral efficiency. As we demonstrate below, that allows it to be of practical value in applied work.

To that end, directly following on from Theorem 3, are the following corollaries:

Corollary 2. *Suppose that for all* $i \in N$, $u_i(\mathbf{X}^{N,L})$ $u_i(X^L)$ *is independent of* x_{kl} *for any k and l not in the same component as i. Given* (N,L), *as* $\sigma \to 1$, *there exists a perfect Bayesian outcome of our extensive form bargaining game with each agent receiving:*

$$\hat{\Phi}_i(L) = \sum_{S \subseteq N : i \notin S} \frac{|S|!\left(|N|-1-|S|\right)!}{|N|!}\left(\hat{v}\left(S \cup i, L/(S \cup i)\right) - \hat{v}(S, L/S)\right).$$

Thus, if *i*'s utility is not affected by actions of agents that it is not connected to, we obtain the Myerson (or Shapley value) type division of a reduced surplus based on bilateral efficiency. On the other hand, with a stronger condition, we have a non-cooperative foundation for the Myerson-Shapley value in that context:

Corollary 3. *Suppose that for all* $i \in N$, $u_i(X^L)$ *satisfies no non-pecuniary externalities. Given* (N,L), *as* $\sigma \to 1$, *there exists a perfect Bayesian outcome of our extensive form bargaining game with each agent receiving their Myerson value.*

Non-Binding Agreements

It is possible, however, that, in some environments, agents will not be able to make agreements that are contingent upon the state *K*. This is a central assumption in, for example, Stole and Zwiebel (1996) who assume that labour supply contracts are non-binding and so can be unilaterally broken if there is a change in a publicly observed state.

To explore this, suppose that, given *K*, a sequence of pairs $\{ij\}_{ij \in K}$ is fixed and agent pairs make alternating offers to one another regarding a single choice and payment pair. If they agree, for example, to $\left(x_{ij}(K), t_{ij}(K)\right)$, the next pair in the sequence negotiates. However, if a breakdown occurs, then the state changes to $K - ij$ and a new subgame occurs in which a sequence of pairs in $K - ij$ is fixed and bilateral negotiations take place in sequence. On the other hand, if there is no breakdown in a sequence then the agreements $\{x_{ij}(K), t_{ij}(K)\}_{ij \in K}$ stand and each agent's payoff is determined.

This case involves *non-binding* agreements. An interpretation of this is that while each pair might arrive at an agreement, if there is a change in circumstance – that is, the state of agreements, *K* – then any individual agent can re-open negotiations with any other agent it is still linked to in *K*. This is precisely the generalisation of the Stole-Zwiebel bargaining game to our more general environment.

It is straightforward to demonstrate that the proofs of all results – in particular, Theorems 1 and 3 – are unchanged by this. The reason is that in those proofs we focus on an equilibrium where agreements contingent upon a state maximise the joint payoffs of the parties concerned. This is precisely what would happen if, in fact, the parties were to re-negotiate contract terms *following* the observation of a state (*K*) rather than prior to it. Indeed, this simplifies the belief structure considerably as they are the subgame perfect outcomes following a breakdown whereas in our contingent contract case they are the expectation of agreements signed by others.

More General Action Spaces

For simplicity, we have considered a fairly restrictive action space, composed exclusively of joint actions. In this section, we extend our results to the fully general action space. For a given graph, *K*, a vector of actions (A^K, X^K) is made up of individually and jointly observable actions. Let $\mathbf{a}_i$ be *i*'s actions that are observable to *i* alone, $\mathbf{a}_j$ similarly defined for j and $\mathbf{x}_{ij}(K) = \left(\mathbf{x}_{ij}^i(K), \mathbf{x}_{ij}^j(K), \mathbf{x}_{ij}^{ij}(K)\right)$ be made up of $\mathbf{x}_{ij}^i$, the vector of actions taken by *i*, $\mathbf{x}_{ij}^{j}$ be the vector of actions taken by *j*, and $\mathbf{x}_{ij}^{ij}$ be the vector of joint actions (requiring cooperation between *i* and *j* to be other than 0).

Definition (Bilateral Efficiency with individual and joint actions). *For a given graph, K, a vector of actions,* $(\hat{A}^K, \hat{X}^K)$ *satisfies bilateral efficiency if and only if:*

$$\hat{\mathbf{x}}_{ij}(K) = \arg\max_{\mathbf{x}_{ij}} u_i\left(\mathbf{x}_{ij}, \hat{A}^K, \hat{X}^K \setminus \hat{\mathbf{x}}_{ij}(K)\right) + u_j\left(\mathbf{x}_{ij}, \hat{A}^K, \hat{X}^K \setminus \hat{\mathbf{x}}_{ij}(K)\right) \text{ for all } ij \in K$$

$$\hat{\mathbf{a}}_i(K) = \arg\max_{\mathbf{a}_i} u_i\left(\mathbf{a}_i, \hat{A}^K \setminus \hat{\mathbf{a}}_i(K), \hat{X}^K\right) \text{ for all } i$$

Consistent with this definition, we now define: $\hat{v}(N,K) \equiv \sum_{i \in N} u_i(\hat{A}^K, \hat{X}^K)$.

Corollary 4. *Theorem 1, Theorem 3, and Corollary 2 hold for* $\hat{v}(N,K) \equiv \sum_{i \in N} u_i(\hat{A}^K, \hat{X}^K)$ *defined for individual and joint actions.*

Notice that the proofs of Theorem 1 and 3 are proven in terms of $\hat{v}(N,K)$, therefore the extension to the expanded action space is immediate. Note that if the level of $\mathbf{x}_{ij}^{i}$ has no effect on u_j when *i* and *j* are not directly connected, for any *i* and *j*, then Corollaries 1 and 3 hold. Otherwise, when *i* and *j* are not linked, the actions $\hat{\mathbf{x}}_{ij}^{i}$ are not constrained efficient, as they maximise *i*'s payoff but not the joint payoff of *i* and *j*, and therefore not total payoff.

7. Applications

We now consider how our basic theorems apply in a number of specific contexts where cooperative game theoretic outcomes have played an important role.

Stole and Zwiebel's Wage Bargaining Game

Stole and Zwiebel (1996) develop a model of wage bargaining between a number of workers and a single firm. The workers cannot negotiate with one another or as a group. Thus, the relevant network has an underlying ‘star’ graph with links between the firm and each individual worker. A key feature of Stole and Zwiebel’s model is that bargaining over wages is non-binding; that is, following the departure of any given worker (that is, a breakdown), either the firm or an individual worker can elect to renegotiate wage payments. As noted earlier, while Stole and Zwiebel posit an extensive form bargaining game as a foundation for their axiomatic treatment of bargaining, the equilibria in this game are not properly characterised. Nonetheless, Theorem 3 now provides that characterization; confirming their Shapley value outcome.

Theorem 3 now demonstrates that an assumption that wage contracts are non-binding is not necessary to motivate the Stole-Zwiebel wage bargaining outcome. Instead, wage contracts could be made contingent upon the number of workers employed by the firm. Thus, the economic driving force behind Stole and Zwiebel’s labour market results is an environment that gives individual workers some bargaining power in ex post wage negotiations rather than the non-binding nature of wage contracts.[9]

Nonetheless, what is significant here is that, when a firm cannot easily expand the set of workers it can employ ex post, there will be a wage bargaining outcome with workers and the firm receiving their Myerson values (as in Corollary 2). This happens if workers are not identical, differ in their outside employment wages, and have variable work hours. Moreover, if there were many firms, each of whom could bargain with any available worker ex post, each firm and each worker would receive their Myerson value over the broader network. As such, our results demonstrate that a Myerson value outcome

[9] This is also true of the results of Wolinsky (2000) who uses an axiomatic argument to justify a Shapley value wage bargaining outcome. de Fontenay and Gans (2003) examine a situation where a breakdown in negotiations causes a link with one worker (the insider) to be severed and a link to be established, if possible, with a new worker. The above results do not apply to breakdowns that create links as well as remove them.

can be employed in significantly more general environments than those considered by Stole and Zwiebel.

It is instructive to expand on this latter point as it represents a significant generalisation of the Stole and Zwiebel model and yields important insights into the nature of wage determination in labour markets. Suppose that there are two identical firms, 1 and 2, each of whom can employ workers from a common pool with a total size of n. All workers are identical with reservation wages normalized here to 0 and if, say, firm 1 employs n_1 of them, it produces profits of $F(n_1)$; where $F(.)$ is non-decreasing and concave. The firms only compete in the labour and not the product market.[10]

The maximum industry value satisfies:

$$\max_{n_1,n_2} F(n_1) + F(n_2) \text{ subject to } n_1 + n_2 = n \tag{7}$$

Denote this maximised value by $v(n)$.. In this case, as there are no non-pecuniary externalities, Corollary 3 applies and each firm receives:[11]

$$\tilde{\pi}(n) = \frac{1}{(n+1)(n+2)}\left(\sum_{i=1}^{n}(i+1)\big(v(i) - 2F(i)\big) + (n+2)\sum_{i=1}^{n}F(i)\right) \tag{8}$$

Each worker receives a wage $\tilde{w}(n)$ where:

$$\tilde{w}(n) = \frac{1}{n}v(n) + \frac{2}{n(n+1)(n+2)}\sum_{i=0}^{n}\big((2i-n)F(i) - (i+1)v(i)\big) \tag{9}$$

It is relatively straightforward to demonstrate that $\tilde{w}(n)$ is decreasing in n as in the single firm Stole and Zwiebel model.

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[10] It should be readily apparent that our model here will allow for competing, non-identical firms as well as a heterogeneous workforce.

[11] The complete derivation of these values can be provided by the authors on request.

What is interesting is to examine the effect of firm competition in the labour market by considering wage outcomes when the two firms above merge. In this case, the bargaining wage, $\tilde{w}_M(n)$, becomes:

$$\tilde{w}_M(n) = \frac{1}{n}v(n) - \frac{1}{n(n+1)}\sum_{i=0}^{n} v(i) \qquad (10)$$

One would normally expect that $\tilde{w}_M(n) < \tilde{w}(n)$ as there is a reduction in competition for workers with a merged firm; pushing wages down. However, this is only the case if:

$$\frac{1}{n(n+1)(n+2)}\sum_{i=0}^{n}(n-2i)\big(v(i)-2F(i)\big) > 0 \qquad (11)$$

does not always hold. For example, suppose that $F(i) = \sqrt{i}$, and suppose that workers can work part time for each firm, then $v(i) = 2F(\tfrac{1}{2}i)$. In this case, the LHS of (11) becomes $\frac{2}{n(n+1)(n+2)}\sum_{i=0}^{n}(n-2i)\big(F(\tfrac{1}{2}i)-F(i)\big)$. The terms within the summation move from negative to positive and so if $F(i)-F(\tfrac{1}{2}i)$ is decreasing in *i*, then the entire expression will be negative so that $\tilde{w}_M(n) > \tilde{w}(n)$. Thus, the simple intuition may not hold.

The model here reveals why workers may be able to appropriate more surplus facing a merged firm than two competing ones. While it is true that in negotiations with an individual worker, a merged firm's bargaining position is improved as workers must leave the industry if negotiations breakdown, it does not have the advantage a competing firm does in being able to draw in a replacement worker from another firm.[12] Interestingly, this suggests that competing firms may have two reasons not to expand the

[12] This result is related to de Fontenay and Gans (2003) who consider replacement workers in a single firm Stole-Zwiebel environment and demonstrate that replacement workers are more effective than internal workers in keeping bargained wages down.

pool of available workers as Stole and Zwiebel argue a single firm might. First, expanding that pool provides a positive externality to the other firm and so there is a free-rider effect if that expansion (say through training) is costly. But, second, the presence of a second firm may result in a lower bargained wage, thereby, alleviating incentives to inefficiently expand production to keep them down. Indeed, (11) is less likely to hold as n expands.

General Buyer-Seller Networks

Perhaps the most important application of the model presented here is to the analysis of buyer-seller networks. These are networks where buyers purchase goods from sellers and engage in a series of bilateral transactions; the joint actions between them being the total volume of trade. Significantly, it is often assumed – for practical and antitrust reasons – that the buyers and sellers do not negotiate with others on the same side of the market. Hence, the analysis takes place on a graph with restricted communication and negotiation options.

In this literature, models essentially fall into two types. The first assumes that there are externalities between buyers (as might happen if they are firms competing in the same market) but that there is only a single seller (e.g., McAfee and Schwartz, 1994; Segal, 1999; de Fontenay and Gans, 2004) while the second assumes that there are no externalities between buyers but there are multiple buyers and sellers (Cremer and Riordan, 1987; Kranton and Minehart, 2001; Inderst and Wey, 2003; Prat and Rustichini, 2003; Bjornerstedt and Stennek, 2002).[13] In each case, however, the underlying

[13] Horn and Wolinsky (1988) permit externalities between buyers but sellers are constrained to deal with a single buyer.

bargaining or market game differs from the model here ranging from a series of take it or leave it offers (McAfee and Schwartz, 1994) to auction-like mechanisms (Kranton and Minehart, 2001) to a simultaneous determination of bilateral negotiations (Inderst and Wey, 2003).

Nonetheless, regardless of the type of model, this literature is predominantly focused upon whether bilateral transactions between buyers and sellers can yield efficient outcomes. The broad conclusion is that where there are externalities between buyers, the joint payoff of buyers and sellers is only maximised when those externalities are not present.

Our environment here encompasses both of these model types – permitting externalities between buyers (and indeed sellers) as well as not restricting the numbers or set of links between either side of the market. In so doing, we have demonstrated that when there are no non-pecuniary externalities – i.e., the only externalities for variables that are bilaterally contractible between agents occur through prices – then industry profits are maximised (Corollary 1). Thus, it provides a general statement of the broad conclusion of the buyer-seller network literature. Similarly, we have a fairly precise characterization of outcomes when there are externalities: firms will produce Cournot quantities, in the sense that the contracts of upstream firm A with downstream firm 1 will take the quantities sold by A to downstream firms $2,\ldots m$ as given; and the quantities sold by B to downstream firms $1,\ldots m$ as given.

Ultimately, the framework here allows one to characterize fully the equilibrium outcome in a buyer-seller network where buyers compete with one another in downstream market. The key advantage is that the cooperative structure of individual firm payoffs makes their computability relatively straightforward. For example, consider a situation with *m* identical downstream firms each of who can negotiate with two (possibly heterogeneous) suppliers, *A* and *B*. In this situation, applying Theorem 3, *A*'s payoff is:

$$\begin{aligned}\Upsilon_A &= \sum_{x=0}^{m}\binom{m}{x}\left(\sum_{i=0}^{x}\frac{(-1)^{x-i}}{m-i+2}\binom{x}{i}\right)\hat{v}_{A,B}(m-x)\\ &+\sum_{x_A=0}^{m}\sum_{x_B=0}^{m-x_A}\binom{m}{x_A}\binom{m-x_A}{x_B}\left(\begin{array}{l}\left(-\sum_{i=0}^{m-x_A-x_B}\frac{(-2)^{m-x_A-x_B-i}}{m-i+2}\binom{m-x_A-x_B}{i}\right)\left(\hat{v}_A\left(x_A \mid x_B\right)+\hat{v}_B\left(x_B \mid x_A\right)\right)\\ +\frac{(-1)^{m-x_A-x_B}}{m-x_B+1}\hat{v}_A\left(x_A \mid x_B\right)\end{array}\right)\end{aligned} \qquad (12)$$

where $\hat{v}_{A,B}(m-x)$ is the bilaterally efficient (i.e., Cournot) surplus that can be achieved when both suppliers can both supply *m*–*x* downstream firms and $\hat{v}_A(x_A|x_B)$ is the bilaterally efficient surplus generated by *A* and x_A downstream firms when those x_A downstream firms can only be supplied by *A* and there are x_B downstream firms that can only be supplied by *B* (with no downstream firms able to be supplied by both). Thus, with knowledge of $\hat{v}_{A,B}(m-s)$, $\hat{v}_A(x_A|x_B)$ and $\hat{v}_B(x_A|x_B)$, using demand and cost assumptions to calculate Cournot outcomes, it is a relatively straightforward matter to compute each firms' payoff.

Significantly, this solution can be used to analyse the effects of changes in the network structure of a market. The linear structure makes comparisons relatively simple. For example, Kranton and Minehart (2001) explore the formation of links between buyers and sellers while de Fontenay and Gans (2005) explore changes in those links as a result of changes in the ownership of assets. The cooperative game structure of payoffs – in particular its linear structure – makes the analysis of changes relatively straightforward. It is also convenient for analyzing the effect of non-contractible investments.[14]

Other environments with externalities

The framework is well-suited to a number of environments with more specific externalities. For instance, a patent holder, or several patent holders with competing innovations, may bargain sequentially with potential licensees. One very interesting area to explore is upstream externalities, which have received little attention: for instance, firms negotiating over oil contracts on adjoining tracts of land, when the amount pumped from one property negatively affects the reserves or the cost of pumping on adjoining properties.

8. Conclusion and Future Directions

This paper has analysed a non-cooperative, bilateral bargaining game that involves agreements that may impose externalities on others. In so doing, we have demonstrated that the generation of overall surplus is likely to be inefficient as a result of these externalities but surplus division results in payoffs involving the weighted sums of surplus generated by different coalitions. As such, there exists an equilibrium bargaining outcome that involves a cooperative division of a non-cooperative surplus; a generalisation of both the Shapley and Myerson values in cooperative bargaining. This is both an intuitive outcome but also one that provides a tractable foundation for applied work involving interrelated bilateral exchanges.

[14] See Inderst and Wey (2003) for an analysis of investments in a related framework.

Appendix

Proof of Lemma 1

The strategy of the proof of this is to demonstrate first that we can map our game and its payoffs with externalities to one that is equivalent but is a coalitional game without externalities with Shapley values corresponding to the payoffs in the original game. We can then use that equivalence, to apply the results of Myerson (1977a) and show that no agent has an incentive to cause a breakdown in bilateral negotiations.

Owen (1986) shows that every coalitional game, Γ, that assigns a payoff $\gamma(T)$ to each subset T of N can be represented as a **unique** linear combination of unanimity games. A unanimity game u^S assigns the following payoffs:

$$u^S(T) = \begin{cases} 1 & \text{if } \quad S \subseteq T \\ 0 & \quad \text{otherwise} \end{cases} \tag{13}$$

Given that there are 2^N possible subsets, this implies that there are 2^N parameters that define any particular game, Γ.

Shapley values satisfy three properties: linearity, symmetry, and no payments to "carriers," that is, players who add nothing to value. Those last two properties imply that the Shapley value payoff to the players of a unanimity game must be $1/s$ for each of the s members of S, and 0 for everyone else. So the payoff to players in a unanimity game is pinned down uniquely by the properties.

Linearity means the following: Suppose game Γ is a linear combination of two games, Γ_1 and Γ_2. Then the Shapley value of each player in game Γ, θ_i^Γ, has the same linear relation to the Shapley values of games Γ_1 and Γ_2: If, for any set T, $\gamma(T) = a\gamma_1(T) + b\gamma_2(T)$, then $\theta_i^\Gamma = a\theta_i^{\Gamma_1} + b\theta_i^{\Gamma_2}$. Linearity implies that if each game Γ is a unique linear combination of unanimity games, then its Shapley value is uniquely defined: If, for any T, $\gamma(T) = \sum_{S\subseteq N} a_S^\Gamma u^s(T)$ then $\theta_i^\Gamma = \sum_{S\subseteq N} a_S^\Gamma \theta_i^{u^S}$, for all i. So the Shapley value of agents in game Γ are uniquely defined by 2^N coefficients.

The next step is to take a given network state and determine whether the payoffs are feasible. If this is done for any arbitrary network state, the proof will carry over to all possible states.

Consider the set of players h (including i and j) who form a component in the current form of the graph, g (where $ij \in g$); the players in h are not necessarily a component in the graph $g - ij$.

Now let's imagine two games with no externalities and no graph structure, Γ and Γ', which are defined **only** for those k players in h. The Shapley values for game Γ are called θ_i^{Γ}, and for game Γ' are called $\theta_i^{\Gamma'}$ (for each $i \in h$). Let v_{Γ} and $v_{\Gamma'}$ be the characteristic functions of the coalitional games Γ and Γ', respectively. No restrictions are imposed on the games except the following:

$v_{\Gamma} = v(h, g)$ (1 equation)

$v_{\Gamma'} = v(h, g - ij)$ (1 equation)

For every *i*, $\theta_i^{\Gamma} = \Phi_i(g)$ (*N* equations)

For every *i*, $\theta_i^{\Gamma'} = \Phi_i(g - ij)$ (*N* equations)

Where $\Phi_i(g)$ is *i*'s payoff in the original game.

Therefore there are *N*+1 constraints imposed on the values of each game and of its Shapley values. Any game Γ and its Shapley value are uniquely defined by 2^N parameters, the coefficients a_S^{Γ} that define the relationship to unanimity games. Therefore, so long as $N + 1 \leq 2^N$, which always holds, there are enough degrees of freedom to write down games Γ and Γ' that satisfy these conditions.

Component superadditivity of the game with externalities (which holds by assumption) implies that $v(h, g) \geq v(h, g - ij)$, and, therefore, that the same holds for the game without externalities. Without specifying additional, notation, this condition also has to hold for an augmented game including possible sub-graphs. This requires another 2^N equations or inequalities to satisfy. We still have enough degrees of freedom so long as $2N + 2 + 2^N \leq 2(2^N)$, which holds whenever $N \geq 3$. (When $N < 3$, the original game has no externalities be definition and so feasibility is not an issue.)

Finally, following the proof in Myerson (1977a), we define a final game $\Gamma'' = \Gamma - \Gamma'$. Since, by construction, Γ'' has value of 0 for any coalition that does not include *i* and *j*, and weakly positive value for any set that includes *i* and *j*, the Shapley value of *i* for game Γ'' is positive, by the representation of the Shapley value as an expected marginal contribution. Thus, $\theta_i^{\Gamma''} \geq 0$, for all *i*, and thus, by the linearity of Shapley values, $\theta_i^{\Gamma} \geq \theta_i^{\Gamma'}$, and therefore $\Phi_i(g) \geq \Phi_i(g - ij)$.

Proof of Theorem 1

We will prove this proposition first for an alternative game – the *nonbinding contracts game* – whereby pairs do not negotiate contingent agreements based on the network state but instead agreements are non-binding and can, following a breakdown (i.e., change in the network state) be renegotiated. Those negotiations are pairwise and take place in the original order between all remaining pairs. We then demonstrate that the equilibrium outcome of the nonbinding contracts game is also an equilibrium outcome of our baseline game where binding contingent contracts are negotiated by each pair.

As we are solving for a Nash equilibrium, we need only consider the incentives for one player, *i*, to deviate. Without loss in generality, let the current state of the network be *L*, and let $(\hat{X}^L, \hat{T}^L) = (\{\hat{\mathbf{x}}_{ij}(L)\}_{ij \in L}, \{\hat{t}_{ij}(L)\}_{ij \in L})$ be the equilibrium outcome and also agents' beliefs regarding unobserved actions. Let us assume for simplicity that *i* always gets to make the first offer, noting that if this were not the case, as σ approached 1, player *i* would simply reject any offer that differed from the offer that they would have made.

Suppose *i* is involved in *s* negotiations, and re-name the agents that *i* negotiates with as "1 to *s*". When *i* comes to negotiate with player *s*, in their final round, if *i* has deviated in previous negotiations, *i* can offer a deviation that *s* will accept in this round.

Without loss in generality, suppose that *i* has deviated in only a single past negotiation, agreeing to $\left(\mathbf{x}'_{ij}(L), t'_{ij}(L)\right)$ rather than $\hat{\mathbf{x}}_{ij}(L)$ and $\hat{t}_{ij}(L)$. In making the first offer to *s*, *i* solves the following problem:

$$\max_{\mathbf{x}_{is}, t_{is}} u_i(\mathbf{x}_{is}, \mathbf{x}'_{ij}, \hat{X}^L \setminus \{\hat{\mathbf{x}}_{is}(L), \hat{\mathbf{x}}_{ij}(L)\}) - t_{is} - t_{ij}' - \sum_{k \in N \setminus \{i,j,s\}} \hat{t}_{ik}$$

$$\text{subject to } u_s(\mathbf{x}_{is}, \hat{X}^L \setminus \{\hat{\mathbf{x}}_{is}(L)\}) + t_{is} + \sum_{k \in N \setminus \{i,s\}} \hat{t}_{ks} \geq \sigma V_s + (1-\sigma)\Omega_{si}$$

where V_s is *s*'s expectation of their payoff if it makes a counter-offer, and Ω_{si} is *s*'s payoff if there is a breakdown in negotiations between *i* and *s* and a renegotiation subgame is triggered. By subgame perfection, neither of these values is affected by the current offer: the game after a breakdown is an independent subgame, because breakdown is irrevocable. And because any player can unilaterally renegotiate after a breakdown of any player, no enforceable agreements can be written on that subgame. The transfer payment provides a degree of freedom that allows *i* to make the constraint bind; therefore:

$$t_{is} = \sigma V_s + (1-\sigma)\Omega_{si} - \sum_{k \in N \setminus \{i,s\}} \hat{t}_{ks} - u_s(\mathbf{x}_{is}, \hat{X}^L / \{\hat{\mathbf{x}}_{is}(L)\})$$

and *i* solves:

$$\begin{aligned} \max_{\mathbf{x}_{is}} \; & u_i(\mathbf{x}_{is}, \mathbf{x}'_{ij}, \hat{X}^L / \{\hat{\mathbf{x}}_{is}(L), \hat{\mathbf{x}}_{ij}(L)\}) \\ & + u_s(\mathbf{x}_{is}, \hat{X}^L / \{\hat{\mathbf{x}}_{is}(L)\}) \\ & - \sum_{j=1}^{s-1} \hat{t}_{ij} - \sigma V_s - (1-\sigma)\Omega_{si} \end{aligned}$$

where the last three terms of the expression do not depend on $\mathbf{x}_{is}$. Nonetheless, a past deviation may cause a deviation in future negotiations. Let us call this new value $\mathbf{x}'_{is}(\mathbf{x}_{ij})$.

The issue becomes, anticipating this, will that past deviation actually occur? Consider *i*'s negotiation with *j*. Without loss of generality, we will assume that *j* is *i*'s (1-*s*)$^{\text{th}}$ negotiation. Under passive beliefs, *j*'s offers will not change even following an alternative offer from *i*; as it does not use this information to revise $\tilde{\mathbf{x}}^j_{is} = \hat{\mathbf{x}}^j_{is}$. *i* does anticipate this and when making an offer to *j*, solves:

$$\max_{\mathbf{x}_{ij}, t_{ij}} u_i(\mathbf{x}'_{is}(\mathbf{x}_{ij}), \mathbf{x}_{ij}, \hat{X}^L / \{\hat{\mathbf{x}}_{is}(L), \hat{\mathbf{x}}_{ij}(L)\}) - t_{ij} - t'_{is}(\mathbf{x}_{ij}) - \sum_{j=1}^{s-2} \hat{t}_{ij}$$

$$\text{subject to } u_j(\mathbf{x}_{ij}, \hat{X}^L / \{\hat{\mathbf{x}}_{ij}(L)\}) + t_{ij} \geq \sigma V_j + (1-\sigma)\Omega_{ji}$$

Substituting in the constraint and *i*'s expected $t'_{is}(\mathbf{x}_{ij})$, we have:

$$\begin{aligned} \max_{\mathbf{x}_{ij}} \ & u_i(\mathbf{x}'_{is}(\mathbf{x}_{ij}), \mathbf{x}_{ij}, \hat{X}^L / \{\hat{\mathbf{x}}_{is}(L), \hat{\mathbf{x}}_{ij}(L)\}) \\ & + u_j(\mathbf{x}_{ij}, \hat{X}^L / \{\hat{\mathbf{x}}_{ij}(L)\}) \\ & + u_s(\mathbf{x}'_{is}(\mathbf{x}_{ij}), \hat{X}^L / \{\hat{\mathbf{x}}_{is}(L)\}) \\ & - \sigma V_j - (1-\sigma)\Omega_{ji} - \sigma V_s - (1-\sigma)\Omega_{si} - \sum_{j=1}^{s-2} \hat{t}_{ij} \end{aligned}$$

where again the terms in the last line do not depend on $\mathbf{x}_{ij}$. Note that, if the equilibrium quantities $(\hat{X}^L, \hat{T}^L) = (\{\hat{\mathbf{x}}_{ij}(L)\}_{ij \in L}, \{\hat{t}_{ij}(L)\}_{ij \in L})$ *are bilaterally efficient*, implying $\frac{\partial u_i(X^L)}{\partial \hat{\mathbf{x}}_{is}(L)} + \frac{\partial u_s(X^L)}{\partial \hat{\mathbf{x}}_{is}(L)} = 0$ for all *i* and *s*, then by the envelope theorem $\mathbf{x}'_{is}(\hat{\mathbf{x}}_{ij}(L)) = \hat{\mathbf{x}}_{is}(L)$. Consequently, this maximisation problem gives the same solution as:

$$\max_{\mathbf{x}_{ij}} u_i(\mathbf{x}_{ij}, \hat{X}^L / \{\hat{\mathbf{x}}_{is}(L)\}) + u_j(\mathbf{x}_{ij}, \hat{X}^L / \{\hat{\mathbf{x}}_{is}(L)\}) .$$

Thus, there is no deviation from the equilibrium negotiations with *j* under bilaterally efficient quantities and hence, no deviation in subsequent negotiations. No other equilibrium exists, because a profitable deviation exists if the quantities negotiated are not bilaterally efficient.

The completion of the proof for the contingent contract case can be easily done by noting that the transferable utility in each contingency can be used to support an equilibrium whereby each contingent negotiation is treated as a separate negotiation (see proof of Theorem 3 for more detail).

Proof of Theorem 3

The proof of this theorem has two parts. First, we need to establish the set of conditions that characterise the unique cooperative game allocation in a partition function environment when the communication structure is restricted to a graph. Second, we will demonstrate that an equilibrium of our non-cooperative bargaining game satisfies these conditions.

Part 1: Conditions Characterising the Generalised Myerson Value

Myerson (1977a) examines a communication structure restricted to a graph – something that is extended by Jackson and Wolinsky (1996) – and demonstrates that the Myerson value is the unique allocation of the surplus under a fair allocation condition and a component balance condition. Myerson (1977b) defines a cooperative value for a game in partition function space but does not examine this on a restricted communication structure nor does he provide a characterisation of that outcome based on conditions such as fair allocation and component balance. Given our general environment here, we first fill these gaps.

Let $v(S, K^P)$ be the underlying value function of a game in partition function form with total number of agents (S) and graph of communication (K). Here are some definitions important for the results that follow.

Definition (Allocation Rule). *An allocation rule is a function that assigns a payoff vector,* $\mathbf{Y}(N,v,L)\in \mathrm{R}^N$ *, for a given (N, v, L).*

Definition (Component Balance). *An allocation rule,* $\mathbf{Y}$ *, is component balanced if* $\sum_{i\in h}\Upsilon_i(N,v,L)=v(h,L)$ *for every* $h\in C(L)$ *, where* $v(h,L)=\sum_{i\in h}u_i$ *.*

Definition (Fair Allocation). *An allocation rule,* $\mathbf{Y}$ *, is fair if* $\Upsilon_i(N,v,L)-\Upsilon_i(N,v,L-ij)=\Upsilon_j(N,v,L)-\Upsilon_j(N,v,L-ij)$ *for every* $ij\in L$ *.*

The final two conditions are amendments of similar conditions imposed in Myerson (1977a) and Jackson and Wolinsky (1996) but for the notation in this paper.

The method of proof will be the following. First, Lemma 1 establishes that under component balance and fair allocation, there is a unique allocation rule. Second, we show that the generalized Myerson value satisfies fair allocation and component balance. Thus, using Lemma 1, this implies that the generalized Myerson value is the unique allocation rule for this type of cooperative game.

First, we note the following result from Navarro (2003):

Lemma 2 (Navarro 2003). *For a given cooperative game (N, v, L), under component balance and fair allocation, there exists a unique allocation rule.*

Next we demonstrate that the generalized Myerson value satisfies fair allocation. Let i and j be linked together by a graph L, where payoffs to groups are described by a component additive payoff function $v(.|L)$. Suppose that each agent i receives their generalized Myerson value from the game (N, v, L) in partition function form:

$$\Upsilon_i(N,L)=\sum_{P\in P^N}\sum_{S\in P}(-1)^{p-1}(p-1)!\left[\frac{1}{|N|}-\sum_{\substack{i\notin S'\in P\\ S'\neq S}}\frac{1}{(p-1)(|N|-|S'|)}\right]v(S,L^P).$$

We aim to show that $\left(\Upsilon_i(L,v)-\Upsilon_i(L-ij,v)\right)-\left(\Upsilon_j(L,v)-\Upsilon_j(L-ij,v)\right)=0$.

$$\left(\Upsilon_i(L,v)-\Upsilon_i(L-ij,v)\right)-\left(\Upsilon_j(L,v)-\Upsilon_j(L-ij,v)\right)$$
$$=\sum_{P\in P^N}\sum_{S\in P}(-1)^{p-1}(p-1)!\left(\begin{array}{l}-\left(\sum_{\substack{i\notin S'\in P\\ S'\neq S}}\frac{1}{(p-1)(|N|-|S'|)}\right)\left(v(S,L^P)\ -v(S,(L-ij)^P)\right)\\ +\left(\sum_{\substack{j\notin S'\in P\\ S'\neq S}}\frac{1}{(p-1)(|N|-|S'|)}\right)\left(v(S,L^P)\ -v(S,(L-ij)^P)\right)\end{array}\right)$$

Consider any partition *P*, and any set S' of that partition. If *i* and *j* are members of S', it does not appear in the summation. If neither *i* nor *j* are members of S', it appears in the top and the bottom line of the parenthesis, and cancels out. Thus, the only relevant case is when *i* is a member of S' and *j* is not, or vice versa; but if *i* and *j* are not members of the same set of the partition, then $L^P=(L-ij)^P$, and therefore $v(S,L^P)=v(S,(L-ij)^P)$, and the term disappears. □

Third, we demonstrate that the generalized Myerson value satisfies component balance. Let *i* and *j* be linked together by a graph *L*, where payoffs to groups are described by a component additive payoff function *v*(.|*L*). Suppose that each agent *i* receives their generalized Myerson value from the game (*N*, *v*, *L*) in partition function form. We will show that for every component, $h\in C(L)$, $\sum_{i\in h}\Upsilon_i(N,L)=\nu(h,L)$.

To do this, we first show that component balance is implied by two of the properties that Myerson (1977b) proved for the extension of Shapley values to games in partition function form: Value Axiom 2 (that carriers get all the value) and Value Axiom 3 (that adding two partition function games gives an addition of their values). Let $\mathbf{Y}$ be the allocation under the game (*N*, *v*, *L*). For a given component, *h*, let $\mathbf{Y}^1$ be the allocation so that for all $i\in h$, $\Upsilon_i^1=\Upsilon_i(N,L)$ and for all $i\notin h$, $\Upsilon_i^1=0$. Similarly, let $\mathbf{Y}^2$ be the allocation so that for all $i\in h$, $\Upsilon_i^2=0$ and for all $i\notin h$, $\Upsilon_i^2=\Upsilon_i(N,L)$. By Axiom 2, the set of agents in *h* gets all the value in allocation 1, and *N**h* gets all the value in allocation 2. By Axiom 3, the vector of payoffs in 1 and 2 sum up to $\mathbf{Y}$.

Given the same *h*, consider a partition of *N* into *h* and *N**h*. Then define two games in partition function form with characteristic functions, (a) $\nu(h,L^{\{h,N\setminus h\}})$ and (b) $\nu(N\setminus h,L^{\{h,N\setminus h\}})$. Let Υ_i^a and Υ_i^b be the Myerson values (in partition function space) for an agent associated with the first and second games respectively. By the carrier axiom,

$$\sum_{i\in h}\Upsilon_i^a(h,L^{\{h,N\setminus h\}})=\nu(h,L)$$
$$\sum_{i\in N\setminus h}\Upsilon_i^b(h,L^{\{h,N\setminus h\}})=\nu(N\setminus h,L)$$

Now we add these two games (a) and (b) together, obtaining the original game in partition function form. By Axiom 3, the payoff to each agent is the sum of their payoffs under (a) and (b). But agents in *h* only have a non-zero payoff in game (a), therefore:

$$\sum_{i\in h}\Upsilon_i(N,L)=\sum_{i\in h}\Upsilon_i^a(h,L^{\{h,N\setminus h\}})+\sum_{i\in h}\Upsilon_i^b(h,L^{\{h,N\setminus h\}})=\sum_{i\in h}\Upsilon_i^a(h,L^{\{h,N\setminus h\}})=v(h,L)\,.$$

Part 2: The non-cooperative bargaining game satisfies these conditions

We want to show that the non-cooperative bargaining game satisfies fair allocation and component balance over a cooperative game with value function $\hat{v}(N,L)$ as determined by bilateral efficiency. Note that Theorem 1 demonstrates that the unique equilibrium of the bargaining game under passive beliefs involves achieving bilateral efficiency. This defines an imputed value function. We now want to show that for this equilibrium the two conditions are satisfied for the game with this value function. *Again we do this first for the bargaining game with non-binding contracts before turning to the binding contingent contract case.*

When *i* and *j* negotiate, the current state of the network is *L*. When *i* and *j* bargain together, let t_{ij}^i be the transfer that *i* offers, which would give a payoff $\hat{v}_i^i$ and $\hat{v}_j^i$ to *i* and *j* respectively; *j*'s offer t_{ij}^j would, if accepted, lead to payoffs $\hat{v}_i^j$ and $\hat{v}_j^j$ respectively. Given that the transfers are chosen to make the incentive constraint bind, the offers satisfy:

$$\begin{aligned}\hat{v}_i^j&=\sigma\left(\tfrac{1}{2}\hat{v}_i^j+\tfrac{1}{2}\hat{v}_i^i\right)+(1-\sigma)\Upsilon_i(N,L\setminus ij)\\ \hat{v}_j^i&=\sigma\left(\tfrac{1}{2}\hat{v}_j^j+\tfrac{1}{2}\hat{v}_j^i\right)+(1-\sigma)\Upsilon_j(N,L\setminus ij)\end{aligned}\tag{14}$$

where $\Upsilon_i(N,L-ij)$ is the payoff to *i* after a breakdown with *j*. (Recall that if an offer is rejected, the order of offers is randomized again; so either *i* or *j* may make the next offer, with 0.5 probability each)

The payoff of a player, $\hat{v}_i$, is simply their utility from the actions taken plus equilibrium transfers $\hat{t}_{ki}$ from other players (which may be negative):

$$\hat{v}_i^i=\hat{u}_i^i-\hat{t}_{ij}^i+\sum_{k\in N\setminus j}\hat{t}_{ki}\qquad\text{and}\qquad\hat{v}_i^j=\hat{u}_i^j-\hat{t}_{ij}^j+\sum_{k\in N\setminus j}\hat{t}_{ki}$$

(where transfer t_{ki} is zero if *i* and *k* do not have a bargaining link). Also, the total amount that *i* and *j* have to divide is given by the other bargaining relationships: if $\hat{t}_{ki}$ is the equilibrium transfer from *k* to *i*:

$$\hat{v}_i^i+\hat{v}_j^i=\hat{v}_i^j+\hat{v}_j^j=\hat{u}_i+\sum_{k\in N\setminus\{i,j\}}\hat{t}_{ki}+\hat{u}_j+\sum_{k\in N\setminus\{i,j\}}\hat{t}_{kj}\tag{15}$$

Using (14) to substitute out $\hat{v}_i^j$ and $\hat{v}_j^i$ in the first part of (15):

$$\hat{v}_i^i + \frac{\sigma}{2-\sigma}\hat{v}_j^j + \left(\frac{2-2\sigma}{2-\sigma}\right)\Upsilon_j(N, L \setminus ij) = \frac{\sigma}{2-\sigma}\hat{v}_i^i + \left(\frac{2-2\sigma}{2-\sigma}\right)\Upsilon_i(N, L \setminus ij) + \hat{v}_j^j$$
$$\Rightarrow \left(\frac{2-2\sigma}{2-\sigma}\right)\hat{v}_i^i + \left(\frac{2-2\sigma}{2-\sigma}\right)\Upsilon_j(N, L \setminus ij) = \left(\frac{2-2\sigma}{2-\sigma}\right)\hat{v}_j^j + \left(\frac{2-2\sigma}{2-\sigma}\right)\Upsilon_i(N, L \setminus ij)$$
$$\Rightarrow \quad \hat{v}_i^i + \Upsilon_j(N, L \setminus ij) = \hat{v}_j^j + \Upsilon_i(N, L \setminus ij)$$

Note from (14) that in the limit, as σ tends towards zero, payoffs $\hat{v}_i^i$ and $\hat{v}_i^j$ become the same payoff $\hat{v}_i$, and therefore:

$$\hat{v}_i + \Upsilon_j(N, L - ij) = \hat{v}_j + \Upsilon_i(N, L - ij)$$

which is the balanced contributions condition.

Now consider condition (15) and its analogue for every bargaining link in the component that includes *i* and *j*. In the limit, as σ tends towards zero, the condition becomes:

$$\hat{v}_i = u_i + \sum_{k \in N \setminus i} \hat{t}_{ki}$$

for each *i*, where transfer t_{ij} is zero if *i* and *j* do not have a bargaining link. Therefore, for a given component, *h*:

$$\sum_{i \in h} \hat{v}_i = \sum_{i \in h}\left(u_i + \sum_{k \in N \setminus i} \hat{t}_{ki}\right) = \sum_{i \in h}\left(u_i + \sum_{k \in h \setminus i} \hat{t}_{ki}\right) = \sum_{i \in h} u_i$$

because there are no transfers to agents that you do not bargain with. The non-zero transfers in this summation term are all between agents in *h*, and, therefore, the summation includes both $\hat{t}_{ij}$ and (-$\hat{t}_{ij}$), which cancel out. This demonstrates component balance.

Binding Contingent Contracts

We now complete the proofs of Theorems 1 and 3 demonstrating that they apply for the game with binding contingent contracts. Let an arbitrary order of negotiations be fixed, and suppose the order of negotiations is known to all players. In the negotiation between *i* and *j*, *i* and *j* negotiate over all possible contingencies that may still occur.

The proof will show that the equilibrium actions and transfers – consistent with Theorem 3 – for the nonbinding contracts bargaining game also form an equilibrium of the contingent contract bargaining game.

Suppose that when any *i* makes an offer to any *j*, their equilibrium offer is composed of:

- an offer of the bilaterally efficient actions $\hat{\mathbf{x}}_{ij}(K)$ for each contingency *K* in which *i* and *j* are still linked;

- an offer of the transfers $\hat{t}^i_{ij}(K)$ that satisfy (14) for each contingency *K* in which *i* and *j* are still linked.

Suppose that *i* and *j* are the first pair to negotiate, in network *L*. They expect all other pairs to negotiate the agreements described above. Therefore, actions $\hat{\mathbf{x}}_{ij}(L)$ and $\hat{t}^i_{ij}(L)$ satisfy (14), and hence, are the outcome of bilateral bargaining between *i* and *j*.

There is a zero probability of the other contingencies, therefore *i* and *j* are indifferent as to the actions and transfers negotiated in other contingencies. Notice, however, that if *i* and *j* assign any positive probability to any contingency other than *L* (or a number of other contingencies), these contingent offers automatically satisfy our pairwise bargaining conditions (14) and (15). Suppose for instance that they assign probability μ to some other contingency *K*: To satisfy the above conditions, *i*'s offer must satisfy

$$\max_{\substack{\mathbf{x}_{ij}(K),t_{ij}(K)\\ \mathbf{x}_{ij}(L),t_{ij}(L)}} \mu\left(u_i(\mathbf{x}_{ij}(K),\hat{X}^K\setminus\{\hat{\mathbf{x}}_{ij}(K)\})-\sum_{k\in N\setminus\{i,j\}}\hat{t}_{ik}(K)-t_{ij}(K)\right)$$
$$+(1-\mu)\left(u_i(\mathbf{x}_{ij}(L),\hat{X}^L\setminus\{\hat{\mathbf{x}}_{ij}(L)\})-\sum_{k\in N\setminus\{i,j\}}\hat{t}_{ik}(L)-t_{ij}(K)\right)$$

subject to

$$\begin{array}{l}\mu\left(u_j(\mathbf{x}_{ij}(K),\hat{X}^K\setminus\{\hat{\mathbf{x}}_{ij}(K)\})+t_{ij}(K)+\sum_{k\in N\setminus\{i,j\}}\hat{t}_{jk}(K)\right)\\ +(1-\mu)\left(u_j(\mathbf{x}_{ij}(L),\hat{X}^L\setminus\{\hat{\mathbf{x}}_{ij}(L)\})+t_{ij}(L)+\sum_{k\in N\setminus\{i,j\}}\hat{t}_{jk}(L)\right)\end{array} \geq \begin{array}{l}\mu\left(\sigma\hat{v}^j_j(K)+(1-\sigma)\Upsilon_j(K\setminus ij)\right)\\ +(1-\mu)\left(\sigma\hat{v}^j_j(L)+(1-\sigma)\Upsilon_j(K\setminus ij)\right)\end{array}$$

Clearly the equilibrium offers from the nonbinding game satisfy these conditions.

Would *i* and *j* have an incentive to deviate and negotiate different contingent contracts, in order to influence the other negotiations? For example, would *i* and *j* want to negotiate a contract that is very favourable to *i* in the event of a breakdown between *i* and *k*, in order to improve *i*'s bargaining power in negotiations with *k*? Let us name that contingency {*L**ik*}, and that contract $\hat{\mathbf{x}}_{ij}(L\setminus ik)$ and $\tilde{t}_{ij}(L\setminus ik) << \hat{t}_{ij}(L\setminus ik)$. Given that we are assuming passive beliefs by all players, a deviation by these two would not change *k*'s equilibrium beliefs, and, therefore, would not improve *i*'s bargaining position with *k*:

$$\max_{\mathbf{x}_{ik}(L),t_{ik}(L)}\left(u_i(\mathbf{x}_{ik}(L),\hat{X}^L\setminus\{\hat{\mathbf{x}}_{ik}(L)\})-\sum_{k\in N\setminus\{i,j,k\}}\hat{t}_{ik}(L)-t_{ij}(K)-\tilde{t}_{ik}(K)\right)$$

$$\text{subject to } u_j(\mathbf{x}_{ij}(L),\hat{X}^L\setminus\{\hat{\mathbf{x}}_{ij}(L)\})+t_{ij}(L)+\sum_{k\in N\setminus\{i,j\}}\hat{t}_{jk}(L) \;\geq\; \sigma\hat{v}^j_j(L)+(1-\sigma)\Upsilon_j(K\setminus ij)$$

Notice that the transfer does not enter *k*'s constraint at all and, therefore, does not affect the outcome of the negotiation.

Now let us consider a negotiation that is further down in the line of negotiations. Suppose that *i* and *j* negotiate after players *a* and *b*. Then if *a* and *b* have not had a breakdown in negotiations, *i* and *j* do not negotiate over the contingencies in which *a* and *b* have a breakdown, for instance, as that will clearly not occur. However, if *a* and *b* have indeed had a breakdown, they negotiate over those contingencies, and not over any contingencies in which *a* and *b* are still linked. From *a* and *b*'s point of view, therefore, *i* and *j* behave in the same way as in the nonbinding contract game: they negotiate a contract in whatever contingency they find themselves in, not constrained by any earlier agreement. Therefore they expect them to reach the agreements described in the nonbinding contract game.

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