Abstract

In this paper, we test if firms competing in an electricity auction submit bids that approximate a benchmark for optimal behavior. First, we derive an equilibrium model of bidding into uniform-price auction spot markets for electricity generators maximizing static profits. Under assumptions of the structure of bidding as a function of private information, firms bid supply functions that maximize ex post unilateral profits for each possible realization of residual demand. Given data on marginal costs of generation, this provides a convenient and computationally straightforward method to construct a theoretical “equilibrium” benchmark against which we can compare the actual strategies of bidders.

Next, we use these results to analyze the evolution of competition in the newly deregulated electricity market in Texas. We use detailed data on demand and firm-level bids and marginal costs to compare actual bids to the theoretical benchmark ex post optimal bids. Using several metrics of performance, we find that the largest seller offered bids that were close to ex post optimal. However, the other sellers deviated from optimal bidding in important ways and we explore various explanations for the observed deviation. Sellers with larger stakes in the market generally were closer to theoretical benchmark optimal behavior. We also find some evidence of learning over the first year of the market’s operation.
1 Introduction

Recent econometric analysis of auction markets, along with much of the “new empirical industrial organization” (NEIO) literature have relied on behavioral assumptions to make inference about unobserved valuations and/or marginal costs. For example, data on individual bids are combined with a model of equilibrium bidding to recover estimates of the underlying valuation of the good. Such inferences are an important input into subsequent questions regarding the efficiency and pattern of surplus allocation achieved by the current market mechanism, and to prospective questions regarding market design. In most instances, the “testing” of the underlying strategic behavior has been left to laboratory experiments, where the researcher “assigns” valuations or marginal costs to subjects, and compares subject behavior against behavior predicted by equilibrium models of competition. Outside of the laboratory, it is difficult to test equilibrium models because data usually are not available on bidder valuations. However, such data are available in electricity markets.

In this paper, we analyze the Texas electricity balancing market, where we have the unique advantage of having access to both very detailed data on firm strategies, and on electricity generation costs. An analysis of this market has a number of distinct advantages over other field settings. In many other product markets, the economist usually has an incomplete view of firms’ decision problems and the way the market clears. Firms simultaneously choose a variety of instruments in the short-run, including prices, output, advertising, and inventory. The precise means by which price is determined and output is rationed must often be assumed. However, the operation of electricity auctions is more transparent. Firms sell a homogenous good into a market with a mechanism of price determination that is well-defined to both the firm and the researcher. We can observe an accurate picture of the problem an electricity generator faces when bidding into the auction, and model generators’ optimal bidding behavior. Given reliable data on the marginal generation costs of the firm, we can thus attempt to “test” theories of strategic pricing, by comparing actual vs. predicted pricing behavior.

The daily electricity auction market in Texas opened in August 2001 when Texas deregulated its electricity market. The vast majority of power in the Texas market is traded bilaterally; however, we have very detailed data on the prices and quantities of trades on the balancing market, where a “spot price” is determined to eliminate excess supply or demand. We model competition on the balancing market as a uniform price auction game, as an application of Wilson’s (1979) share auction model. This modelling approach can be seen as a generalization of Klemperer and Meyer’s (1989) supply-function equilibrium (SFE) model, which has been an influential benchmark in the modelling of deregulated electricity markets since Green and Newberry (1990).¹ In our model, firms compete by submitting entire supply schedules, as they do in the

¹The supply function equilibrium approach has also been utilized in Green (1992), Rudkevich (1999), Baldick, Grant and Kahn (2001), Crawford, Crespo and Tauchen (2002).
balancing market. In contrast to the Klemperer and Meyer’s SFE model, however, our model allows generators to possess private information, aside from allowing for aggregate uncertainty in market demand. With the addition of private information, we are led to seek Bayesian Nash equilibria of the uniform price share auction game, as in Wilson (1979).

The source and nature of private information in the balancing market is an important modelling component that needs to specified clearly. In almost all deregulated electricity markets, it is reasonable to assume that firms have a good idea of their opponents’ marginal cost schedules – a lot of engineering information is available about the type of technology employed by different generators, and firms have a good idea about the spot price of fuel. In this respect, the assumption of SFE models that marginal cost is observable is not very far-fetched. However, even if firms have a good idea of each others’ marginal cost schedules, they seldom have information about competitors’ contract obligations. As first pointed out by Wolak (2003), these contract obligations may significantly affect bidders’ incentives to exercise market power, hence this constitutes a very important source of private information for firms competing in the balancing market. Specifically, this source of private information incorporates a “private values” component to the uniform price auction game.

Given this modelling framework, intuition suggests that the exact specification of the informational environment (the distribution of private information across bidders, and distribution of aggregate uncertainty) may play a very important role in strategic behavior. In particular, the construction of a “theoretical” benchmark for bidder strategies may depend critically on assumptions regarding bidders’ knowledge or expectations about the distribution of their opponents’ private information. However, our analysis in section 3, shows that although this intuition may be true in general, if we impose the restriction that bidders’ private information regarding their contract positions enter additively separably and linearly into their strategic supply functions, such restricted strategies will lead to an ex-post optimal equilibrium of the game, in that a bidder following the prescribed strategy would not change his decision even after seeing any ex-post realization of his rivals’ bids. This leads to the somewhat surprising conclusion that neither the distribution of private information across bidders nor the distribution of aggregate uncertainty play a role in shaping a bidder’s decision.

Intuitively, this “additive separability and linearity of private information” restriction is analogous to the condition that aggregate demand uncertainty enters additively into the market demand in Klemperer and Meyer’s SFE model, which also enables for the existence ex-post optimal equilibria in that model. Hence, many of the basic economic intuitions from the SFE model follow through to our model. Namely, under this additivity restriction, the “residual demand

\[\text{2} \text{Although contract positions maybe correlated across firms, it is difficult to think of a reason why one firm would change its assessment of its marginal costs upon learning another firm’s contract position; hence we do not believe that private information introduces a “common value” element to this market.} \]
function" faced by a firm will shift in a parallel fashion in response to realizations of rival bidders’ private information and aggregate uncertainty. This allows one to conveniently “trace out” a set of ex-post optimal best response points corresponding to different parallel shifts of the residual demand curve. An ex-post optimal equilibrium is one in which these “traced-out” points form an equilibrium of the game.

Although the imposition of such an a priori restriction on equilibrium strategies may seem ad hoc in a general empirical setting, in the context of the extant theoretical literature on uniform price share auctions or supply-function equilibria, the restriction is standard. In particular, prior theoretical literature has focused on obtaining closed-form solutions for equilibrium strategies in uniform price auction settings, and, to our knowledge, every analytical example that has been constructed in the literature satisfies the “additivity in private information” restriction that we impose.\(^3\) We provide an explicit characterization of such equilibrium strategies for an analytically tractable example in the Appendix of our paper.

Perhaps more importantly for our empirical application, the ex-post optimality restriction allows for a computationally convenient way to make an empirical assessment of how “effective” observed bidding strategies are. That is, under the assumption that residual demand shifts are parallel, we can construct the ex-post optimal bid function for a given bidder in our data set – provided that one has data on bidders’ marginal costs.\(^4\) Fortunately, for ERCOT, as in many other electricity markets, economic researchers have access to detailed information on generation technologies and fuel costs (the same information available to rival firms). Using this information, we are able to construct a detailed data set on marginal costs schedules of the firms operating in the balancing market. Given our estimates of the (hourly) marginal cost schedules of the firms in the balancing market, we are able to compare the “ex-post optimal” supply functions with actual bid schedules in the Texas balancing market.

The results from this exercise are quite instructive. We find that the firm with the largest stake on the balancing market, Reliant Energy Corp., submits bid schedules that mimic the “ex-post optimal” bid schedule remarkably well. To see an example, see Figure 1 on the next page. This figure depicts Reliant’s optimization problem on February 8, 2002 in the auction period from 6:00-6:15pm. Reliant’s residual demand is downward-sloping implying that it has some potential to exercise market power and bid something different from marginal cost. The marginal cost for supplying positive or negative balancing power is shown with the solid line. Reliant has the capacity to increase production by almost 1900 MW at a cost increasing up to about $28/MWh, and decrease production by up to 2300 MW at a cost savings of about $14/MWh.

\(^3\)See Wilson (1979), Wang and Zender (2000) for analytical characterizations of equilibria in uniform price auctions which satisfy this restriction.

\(^4\)Observe the considerable computational convenience afforded by the ex-post optimality assumption: in particular, we do not have to explicitly solve for a Bayesian Nash equilibrium of a game in which the strategy space is comprised of entire supply functions.
Figure 1

Example of Actual and Optimal Bidding

Reliant on February 8, 2002  6:00-6:15pm

- Residual demand
- Bid curve
- Ex-post optimal bid
- MC curve
Given the actual realization of residual demand and the marginal cost function, the optimal \((p, q)\) and actual bid are shown. Reliant chose a bid very close to the ex post optimal level. We can also compare the other points on Reliant’s bid function to ex post optimal points. If other realizations of residual demand are effectively parallel shifts (e.g. due to weather shocks shifting out total demand), we can calculate the ex post optimal bids for other possible demand realizations. The starred (*) line shows the set of profit-maximizing bids for other realizations of residual demand. It is optimal to bid above marginal cost for positive levels of balancing demand and below marginal cost for negative demand. Apparently, Reliant’s actual bids are fairly close to the optimal bid function for other residual demand realizations as well. This suggests that not only is there potential to exercise market power, but Reliant appears to exercise much of that market power.

Observe that the closeness of Reliant’s actual bid schedule and the “ex-post optimal” benchmark we computed is quite remarkable, in that Reliant neither observes the ex-post realization of the residual demand curve, nor should necessarily have the belief that residual demand will shift in a parallel fashion, as we have assumed. Of course, this is also quite remarkable from the point of view that computing equilibrium strategies in this game is a difficult computational task. Nevertheless, as we demonstrate in section 5, Reliant appears to perform well in several different performance metrics we construct utilizing ex-post information.

We also find that the observed behavior of several other firms in the industry display interesting and economically meaningful deviations from the “ex-post optimal” benchmark we have constructed. We first document the rather puzzling fact that firms bidding into the balancing market do not appear to make full use of the strategy space available to them: each firm is allowed to submit up to 40 price-quantity points on its bid schedule, however, firms appear to submit much fewer price-quantity points. Since ex-post realizations of the residual demand function is quite continuous, one may expect that the ex-post optimal best response will also be as continuous as possible; however, firms appear to prefer “coarser-grained” strategies than is available to them. It is noteworthy that the major players in the market utilize more bid points over time which could indicate increased sophistication.

We then document an interesting pattern we observe in the bid data that suggests that, especially in the beginning of the market operation, some firms were quite reluctant to supply or take away power from the balancing market – even if doing so was profitable for them. This they ensured by submitting a large vertical step on their supply schedule corresponding to the point where their balancing supply is equal to zero. However, we also document that the tendency to “avoid” the balancing market disappears as time progresses.

Our results suggest the following conclusions: first, the assumption of ex-post optimality of observed bid functions, although a priori quite strong, appears to be a good description of the behavior of the firm with the highest stakes in the balancing market. We believe that this is a confirmation that strategic equilibrium models such as ours or the SFE are accurate descriptors of strategic
agents.

Second, there do appear to be important deviations from “benchmark” optimal behavior for firms with somewhat smaller stakes in this market. Firms with larger stakes tend to realize a higher fraction of potential profits. This suggests there may be fixed costs of participating in the market. However, there are a few noteworthy exceptions – several firms perform quite well despite having less money at stake. Therefore, it may be the case that the participation costs are more perceived than real, and there may be a sticky market for managerial efficiency in this new market.

Finally, we find some evidence of learning behavior by firms over our sample period. The game being played in this environment is complex, and ex-post optimality requires good knowledge of the “shape” of possible realization of the residual demand curve – hence playing the game over and over can contribute greatly to a strategic agent’s understanding of the game, and the contingencies he or she may face.

The outline of the remaining sections is as follows: in section 2, we describe the institutional setting of the Texas electricity balancing market, describe the construction of our data set, and present summary evidence on market prices and deviations from marginal cost pricing. In section 3, we model strategic bidding in this market as a uniform price share auction. We discuss the empirical implications of our model. In section 4, we compare the theoretical “benchmark” ex-post optimal bids with the actual bids in the data. Section 5 discusses these results and explores possible explanations for deviations from ex-post optimal bidding.

2 How Bidding Occurs in ERCOT’s Balancing Energy Market

Restructured electricity markets around the world differ in how energy is procured. Some markets settle the vast majority of transactions through a single uniform-price auction or pool. The Texas market follows a different model. The vast majority of transactions are negotiated bilaterally. Rather than serving as a market maker, the system operator takes the more passive role of collecting schedules of bilateral trades and only running a small residual market for balancing supply and demand in real-time. In a bilaterals market, it is difficult to access the overall competitiveness of the market because bilateral trades are not public information and do not provide transparent information for empirical analyses. Therefore, this paper does not attempt to measure the level of competition of the entire market but rather focuses on the balancing market.

The ERCOT system is the largest control area in the United States with a 2002 installed capacity of 77,000 MW. The system is largely separated from other control areas in the country so there are virtually no imports into ERCOT. Approximately 2-5% of energy is traded in the Balancing Energy Services auction. Generation (“supply”) and load (“demand”) are bid into this auction through an agent called a Qualified Scheduling Entity (QSE). Many of the generation owners (PGCs) act as their own QSE, but some PGCs use another QSE
to submit their bids.

On the day before and up to about one hour before production, ERCOT accepts schedules of quantities of electricity to inject and withdraw at specific locations on the grid. In the first year of market operation, all QSEs were required to submit “balanced” schedules meaning that the amount of generation exactly covers forecast load. Firms did not always obey this balanced schedule requirement and occasionally left small fractions of contract obligations to be procured on the balancing market. During the period of balanced schedules, the balancing market was intended to include transactions for actual demand differing from the forecast (e.g. an unpredictably hot day) or generation differing from the day-ahead schedule (e.g. an outage at a plant). Also, the balancing market served as a source of adjusting for day-ahead schedules that were not technically feasible due to interzonal transmission congestion. Beginning in November 2002, QSEs were permitted to submit “unbalanced” schedules with some restrictions on the deviations. This rule change expanded the balancing market to more resemble a exchange for trading spot energy.

As the time of production and consumption nears, ERCOT estimates how much balancing energy is demanded. Because there are (virtually) no sources of demand that can respond to prices in real-time, balancing demand is effectively perfectly inelastic. The balancing demand can be positive or negative. In the early months of operation, the market was overscheduled and demand was often negative.

Bidders can offer to either increase (“inc”) or decrease (“dec”) the amount of power supplied. QSEs submit hourly inc and dec bid schedules. The bids must be increasing monotonic functions and can have up to 20 “elbow” points per bid stack per zone (i.e. 20 inc and 20 dec bids per congestion zone). Those bids apply to each of the four 15-minute intervals of the hour. Figure 2 displays a sample bidder’s interface for one participant’s daily bids to “dec”. The bidder can either type or paste in up to 20 elbow points for each hour. The bids may be changed for any hour up until one hour prior to the operating hour. The bidder also has access to the up-to-the-hour history of calls for its generating units. In addition, the bidder observes real-time information on its units’ generation, the load it is obligated to serve, and its net short or long position in the balancing market. A sample of this operations interface is shown in Figure 3.

Four times every hour, ERCOT clears the balancing market by intersecting the hourly aggregate bid function with the 15-minute perfectly inelastic demand function. ERCOT instructs the generators on the amount of power to supply ten minutes before the beginning of each interval. The procurement process is a uniform-price auction so each unit is paid (or pays) the market clearing price.\footnote{The actual deployment is based on other factors as well. ERCOT takes the QSE portfolio deployments and allocates the incremental and decremental generation among the generating units based upon a specific algorithm which considers factors such as the rate at which units can ramp up generation and the location of units within a zone. In the event that units are allocated to solve intrazonal congestion, the generators also receive unit-specific premia payments.}

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Bidders appear to have a great deal of information on the competitive environment when they choose their bid functions. Our conversations with several market participants suggest that some traders have good information on their rivals’ marginal costs. The generating units have very similar production technologies, and there are publicly available data on the fuel efficiency of each unit. Traders appear to know fairly accurately the major units that are on and off-line at any point in time. Also, the traders have interfaces with the ERCOT system that provide access to real-time charts of the grid’s electrical frequency. If a major unit were to go off-line, the frequency would briefly dip and indicate that an outage likely had occurred. In addition, market players can purchase real-time data on the generation of large plants. An energy information company named Genscape has developed a technology that measures real-time megawatt output with remote, wireless sensors installed near the transmission lines out of a plant. Generators and traders can purchase subscription service to access the output data in real-time. This service also sends alerts to traders within a minute or two after a large plant significantly ramps up generation or trips offline. Such information can be useful strategic information not only when initial bids are submitted but also if the trader wants to update bids up to an hour before the market clears. Figure 4 shows a sample interface with the ability to choose particular plants and retrieve real-time generation data.

This may provide the trader with strategically important information on residual demand. A trader equipped with knowledge of the marginal costs of rivals and an equilibrium mapping of costs to bids, will be able to calculate the aggregate bids of rival firms. Because residual demand is the perfectly inelastic total demand minus bids by all other firms, the bidder can calculate the residual demand function up to parallel shifts left and right in total demand (e.g. weather shocks).\footnote{The actual calculation of rival bids is a bit more nuanced if the rivals possess private information on their bilaterally contracted sales. Below we show that the presence of this private information shifts residual demand in a parallel fashion just as total demand shocks do.}

Even if she does not know the operation status of every rival unit, every trader has access through ERCOT to the aggregate bid stack with a two to three day lag. By knowing the recent aggregate bid stack as well as her own recent bids, the trader can infer the recent aggregate bids by all rivals which is sufficient to calculate residual demand. To the extent that the rivals’ bids several days before are similar to current rival bids, the trader can infer the shape of residual demand before placing their bids.

ERCOT is geographically divided into a small number of zones. When transmission lines between the zones are not congested, the balancing market can be thought of as an integrated market with a single market clearing price. If however, we restrict our sample to periods when these factors are unlikely to substantially alter the deployment that would be implied by the bids. Below we compare our simulated prices to the actual prices as a test of the validity of this assumption.
transmission lines between zones are congested, ERCOT can have separate market clearing prices for each zone. Beginning in 2002, firms could buy rights to collect revenues based on the price differences between zones and these transmission congestion rights could be used to hedge interzonal price risk. Transmission congestion poses two complications that lead us to restrict our sample to uncongested time periods. First, we do not have data on transmission rights and hence cannot measure all sources of revenue. Second, transmission capacity limits affect whether the observed bids are technically feasible. In hours with no interzonal transmission congestion, a firm’s residual demand over the entire ERCOT control area is the total ERCOT balancing demand minus the bids by rivals in all zones. Therefore, this analysis uses only bid hours with no congestion which represents slightly over half of the hours during the sample period. This avoids the complication posed by congestion unless firms’ uncertainty as to whether there will be congestion affects their bidding strategy, a possibility that we will try to investigate in section 5.

During the first month of the market in August 2001, the rules regarding the assignment of congestion costs created incentives for a bidding game that makes the market difficult to analyze. The prevalence of this other bidding game which we will not discuss decreased after the opening month. Therefore, we begin our analysis with September 2001. We analyze weekdays from September 2001 through the end of the first year of operation in July 2002. Although we can analyze any 15 minute interval, we focus on 6:00-6:15pm when there is no congestion during the entire 6:00-7:00pm bid hour. The average number of MW traded (both positive and negative) during this time interval is 904 MW. To compare this with other intervals, the average MW transacted across all intervals varies only slightly over the day with an average volume of 852 MW. Figure 5 shows the probability density of the (perfectly inelastic) demand for balancing energy. Demand for balancing power is both negative and positive in many hours although on average demand is negative, especially during the early months of the sample. Demand can vary substantially during the 15-minute intervals within a bidding hour. The average difference between the minimum and maximum balancing demand is 625 MW for the 6:00-7:00pm bidding hour. Because firms can submit only one bid function for each 15-minute interval, optimal bids must cover a wide range of possible demand quantities.

For 6:00-6:15pm, prices averaged $28.82 during intervals of positive balancing demand and $17.62 during intervals of negative demand. In contrast, total ERCOT loads were not substantially different – load averaged 38,210 MW during the positive demand intervals and 33,792 MW during the negative demand intervals. Given the technology mix of the ERCOT system, these price differences appear larger than what could plausibly be the difference in marginal system costs for only 4,418 MW more GW of production. These prices could be consistent with market power. If generators exercise market power, we expect price to be higher than marginal cost during intervals of positive demand and lower than marginal cost during hours of negative demand.\(^7\) Below we measure

\(^7\)An important qualifier is that this holds if contract positions going into the
Table 1: Generation Ownership

<table>
<thead>
<tr>
<th>Owner</th>
<th>% of Installed Capacity</th>
</tr>
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<tbody>
<tr>
<td>TXU</td>
<td>24</td>
</tr>
<tr>
<td>Reliant Energy</td>
<td>18</td>
</tr>
<tr>
<td>City of San Antonio Public Service</td>
<td>8</td>
</tr>
<tr>
<td>Central Power &amp; Light</td>
<td>7</td>
</tr>
<tr>
<td>City of Austin</td>
<td>6</td>
</tr>
<tr>
<td>Calpine</td>
<td>5</td>
</tr>
<tr>
<td>Lower Colorado River Authority</td>
<td>4</td>
</tr>
<tr>
<td>Lamar Power Partners</td>
<td>4</td>
</tr>
<tr>
<td>Guadalupe Power Partners</td>
<td>2</td>
</tr>
<tr>
<td>West Texas Utilities</td>
<td>2</td>
</tr>
<tr>
<td>Midlothian Energy</td>
<td>2</td>
</tr>
<tr>
<td>Dow Chemical</td>
<td>1</td>
</tr>
<tr>
<td>Brazos Electric Power Coop</td>
<td>1</td>
</tr>
<tr>
<td>Others</td>
<td>16</td>
</tr>
</tbody>
</table>

whether prices are above or below marginal costs.

Many generators sell power into both the bilateral market and balancing market in ERCOT. Table 1 breaks down the generation capacity by owner. The two largest incumbent utilities, Reliant and TXU, own the largest shares. Although the utilities were not required to divest plants, they are required to sell to some existing customers at a fixed price. Also, each was required to sell 15% of generation through a virtual divestiture auction beginning in 2002. Many of the other generation owners were regulated utilities before restructuring. The largest merchant generator is Calpine which owns natural gas fired plants comprising 5% of capacity. Generation technology is a mix of natural gas, coal, nuclear, hydroelectric, and wind generating units.

We analyze firm-level bidding behavior into the balancing market. As discussed above, bids are submitted by QSEs and some QSEs submit bids for more than one firm. For example, in 2001 the same QSE submitted bids for both Reliant and City Public Service of San Antonio (CPS) in the South zone. This poses complications for our analysis. In these instances, it is impossible to separate out the bids for each firm. We use all QSE bids in our analysis but can only interpret the bids as “firm-level behavior” if the QSE bids primarily for a single balancing market are roughly zero. We allow for contracts in our estimation below.
Table 2: Balancing Sales for Firms with Identifiable Bids

<table>
<thead>
<tr>
<th>Firm with Identifiable Bids</th>
<th>Average Sales (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliant Energy</td>
<td>473</td>
</tr>
<tr>
<td>TXU</td>
<td>156</td>
</tr>
<tr>
<td>Calpine</td>
<td>78</td>
</tr>
<tr>
<td>City of Austin</td>
<td>40</td>
</tr>
<tr>
<td>Brownsville Public Utility Board Tenaska</td>
<td>39</td>
</tr>
<tr>
<td>Cogen Lyondell</td>
<td>37</td>
</tr>
<tr>
<td>Central Power &amp; Light</td>
<td>28</td>
</tr>
<tr>
<td>Bryan Texas Utilities</td>
<td>25</td>
</tr>
<tr>
<td>Lamar Power Partners</td>
<td>23</td>
</tr>
<tr>
<td>Tractebel Power</td>
<td>18</td>
</tr>
<tr>
<td>Rio Nogales Power Project</td>
<td>15</td>
</tr>
<tr>
<td>BP Energy</td>
<td>12</td>
</tr>
<tr>
<td>West Texas Utilities</td>
<td>10</td>
</tr>
</tbody>
</table>

Note: Sales implied by bidstack and total balancing demand.

firm in a particular zone. We interpret the bidding as firm-level behavior only if a single generation owner supplies at least 90% of the total generation (day-ahead and balancing) supplied through the QSE for which we have bid data. This limits our ability to analyze behavior over parts of our sample. The largest supplier of balancing power, Reliant Energy, cannot be separated from CPS in 2001 so we do not measure Reliant’s individual behavior in 2001. For 2002, we can separate the vast majority of Reliant’s production from CPS because a new zone was created (Houston) that contains most of Reliant’s generating units. Another major player is TXU which was the incumbent utility for much of the northern part of the state. TXU’s bids can be separated in the North in both years, the South in 2001, but not the West in either year. However, the vast majority of TXU’s generation capacity is located in zones for which we can separately identify TXU’s bids. This is not a problem for Calpine, the third largest seller, because Calpine acts as its own QSE.

Given the bids we can separately identify, Table 2 show the average sales for each 6:00-6:15pm time interval by firm. Reliant is by far the largest participant in the market selling an average of 473 MWh. The other large incumbent utility, TXU, averages substantially less relative to its capacity at 156 MWh. We suggest below that this relatively low quantity of balancing sales results from an apparent reluctance of TXU to supply negative “dec” output to the balancing market, even when it appears profitable to do so. The third largest seller is Calpine, which is the merchant generator owning several combined cycle gas-fired plants.
2.1 Is There Evidence of Any Market Power?

A precondition for testing strategic behavior is evidence of non-price-taking behavior. In this market, strategic behavior can yield prices either above or below marginal cost depending upon whether balancing demand is positive or negative. Interestingly, market power can lead to prices that are too high or too low.

Consider a firm that has submitted day-ahead schedules to generate exactly the same quantity as its contract obligations and therefore has no residual contract obligations upon entering the balancing market. If firms bid perfectly competitively into the balancing market, they will bid along the marginal cost curve to increase output and bid negative supply down the marginal cost curve to decrease output. However if they are not price takers, the bid function will not correspond to the MC function and will depend on whether demand for balancing energy is positive or negative.

If demand is positive, a firm that sells more than one unit has incentives to markup the bid price above marginal cost. Although it reduces profits by sacrificing an additional unit or two of profitable sales, the firm raises the price earned on all inframarginal units. This is just the standard oligopoly result that a firm acts as a monopolist on residual demand.

However, firms will mark down bids below marginal costs if total balancing demand is negative. Suppose our generator has load obligations to serve its customers 100 MW and has submitted a day-ahead schedule to generate 100 MW. Additionally, suppose that total electricity demand (but not demand from its customers) is lower. The grid operator demands negative generation and accepts bids for willingness to pay to reduce production. The grid operator accepts offers from the highest bids to reduce demand and works down the aggregate bid curve. If our generator were to produce 10 MW less, it would still have to supply 100 MW to meet its load obligation. It would be net short 10 MW and must pay the balancing price for the 10 MW it is short. A price-taker bids (down) its marginal cost curve because it is willing to reduce it costs by its MC and purchase the additional MWs of load obligation at the market price if the price is less than marginal cost. However, a firm exercising market power will bid below marginal cost to sell itself into a “short” position but lower the price at which it must buy back its short position. Or, put differently, a firm bids above marginal cost to exercise “monopoly power” if it is a net seller, and bids below marginal cost to exercise “monopsony power” if it is a net buyer. Therefore, positive price-cost margins in hours with positive balancing demand and negative margins in hours with negative demand are consistent with the exercise of some form of market power.

We measure price-cost margins for positive demand “inc” intervals and negative demand “dec” intervals. In the inc intervals, we measure the difference between the balancing zonal price and the marginal cost of the lowest cost unit that is operating and has excess capacity. In dec intervals, we measure the difference between the zonal price and marginal cost of the highest cost unit that
is operating at any positive level of output and can sell negative output.  

Table 3 shows price-cost margins for the largest sellers and all other sellers during periods of positive and negative balancing demand. The largest sellers with strong incentives to bid strategically have the largest positive margins during inc intervals and the largest negative margins (in absolute value) during the dec intervals. This is consistent with some form of strategic behavior. In the remainder of the paper, we will model this strategic behavior, and test the predictions of this model using observed bidding and marginal cost data. Interestingly, the smaller sellers also exhibit margins consistent with the exercise of market power. Below we analyze the bids of these small firms and find that they do not have sufficiently large inframarginal sales to justify bidding prices that differ substantially from marginal costs. Rather they appear to bid inelastically to avoid participating in the balancing market. Below we offer possible explanations for the observed behavior.

Table 3: Average Price-Cost Margins ($/MWh) 6:00-6:15pm

<table>
<thead>
<tr>
<th>Size of Seller</th>
<th>Demand</th>
<th>Big</th>
<th>Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inc</td>
<td>$10.61</td>
<td>$9.28</td>
<td></td>
</tr>
<tr>
<td>Dec</td>
<td>-$17.08</td>
<td>-$9.39</td>
<td></td>
</tr>
</tbody>
</table>

Big sellers are Reliant, TXU, and Calpine. Small are all others.

3 An Equilibrium Model of Bidding in the ERCOT Balancing Market

As discussed in the previous section, the vast majority of power in the Texas market is traded bilaterally; only a small fraction of residual or balancing demand is traded through a uniform-price auction. Data are only available on the prices and quantities of trades in this balancing market, so we analyze this market conditional on the bilateral sales. In reality, the analysis of the two-stage game with endogenous contract choices adds a further strategic dimension. However, the analysis here will assume that these contracts have been written a long enough time ago that they might be taken as “sunk” decisions from the purposes of a bidder making real time decisions on the balancing market.

8We assume that nuclear, wind, and hydro units have no short-run operational flexibility and cannot inc or dec on short notice.
We follow the uniform price share auction setup of Wilson (1979) to model competition in the spot market. There are $N$ firms, with costs of generation (at time $t$) given by $C_i(t)$, which is random from the perspective of bidders. We construct a static game in which the total demand $ar{D}_t$ is taken as given (but maybe uncertain from the perspective of the firms), and model the competition to satisfy that demand. Some firms may have signed fixed price contracts for certain quantities of power each hour, given by $QC_{it}$.

In each time period $t$, each firm submits a supply schedule, $S_{it}(p)$. In the balancing market setup of ERCOT, $S_{it}(p)$ can be thought of as being the sum of a “day ahead schedule” component, and a balancing schedule component. (Notice that the lumping of these two temporally separated components does not affect the strategic nature of the game if bidders are not provided any information about each others actions until the market clears.) Given the supply schedules of each firm, ERCOT computes the market clearing price, $p_{c_t}$, which satisfies the below market clearing condition:

$$\sum_{i=1}^{N} S_{it}(p_{c_t}, QC_{it}) = \bar{D}_t(p_{c_t})$$

Each firm gets paid $S_{it}(p_{c_t})p_{c_t}$ due to the uniform pricing rule. Hence, firm $i$’s “ex-post” profit, upon the realization of market clearing price $p_{c_t}$ is:

$$\pi_{it} = S_{it}(p_{c_t})p_{c_t} - C_{it}(S_{it}(p_{c_t})) - (p_{c_t} - PC_{it})QC_{it}$$

The firm’s payoff from its contract position in the balancing market is $-(p_{c_t} - PC_{it})QC_{it}$, since it has to refund its customers any differential between the contract and market prices for the contracted sales. Wolak (2003) has shown this is identical to a contract for differences.

Clearly, the most important source of uncertainty in the profit equation above is $p_{c_t}$, the market clearing price at time $t$. In a strategic equilibrium, the uncertainty in $p_{c_t}$, from the perspective of firm $i$, will be due to two factors:

1. Uncertainty in market demand, $ar{D}_t$
2. The unobserved component(s) of $i$’s competitors’ profit maximization problems, which can consist of the following:
   
   (a) Contract positions by each firm, $QC_{jt}$
   (b) Contract prices obtained by each firm: $PC_{jt}$

A firm bidding into the market is assumed to know its rivals’ total cost functions. (The assumption is that, if we, as econometricians, have been able to gather this
information from public sources, the firms competing in this market will also have gained this information). This knowledge would be sufficient to calculate equilibrium bids by rival firms except the firm does not know its’ rivals contract positions. Those contract positions determine the quantities for which the rivals will bid above and below marginal cost. This private information affects the rivals’ bidding incentives and, therefore, generates uncertainty of firm i’s residual demand. In addition, observe that, among the above sources of informational asymmetry between firms, component (b) contract prices $PC_{it}$ do not enter into the profit maximization problem.

Thus specified, this problem is a “share auction” problem first analyzed by Wilson (1979). We will also argue that this is a private value share auction, since observing other firms’ contract positions is unlikely to change a given firm’s own evaluation of its generation costs, as would be in a common value model.

Note that our model setup differs slightly from the “supply function equilibrium” (SFE) model of Klemperer and Meyer (1989), which has been used frequently in the analysis of competition in electricity spot markets since Green and Newbery (1990). In the SFE model, firms also have perfect information about each others’ marginal costs. However, the only source of uncertainty is aggregate demand; there is no private information. Thus the SFE model can be seen as a special case of a share auction model. In the share auction model, the strategies of the firms are also supply functions, but these are also functions of their private information, which is their contract position. That is, we are looking for a Bayesian Nash equilibrium characterization of the auction, in which firms’ strategies are of the form $S_{jt}(p, QC_{jt})$.

To characterize this Bayesian Nash equilibrium, we follow the method of Wilson (1979). Let us define a probability measure over the realizations of the market clearing price, from the perspective of firm i, conditional on submitting a “hypothetical” supply schedule, $\hat{S}_{it}(p)$, while his competitors are submitting their Bayesian Nash equilibrium supply schedules:

$$H_{it}(p, \hat{S}_{it}(p)) = \Pr(p_{C} \leq p | \hat{S}_{it}(p))$$

$$= \Pr\left(\sum_{j \neq i} S_{jt}(p, QC_{jt}) + \hat{S}_{it}(p) \geq \tilde{D}_{t}(p) | \hat{S}_{it}(p)\right)$$

where the second line follows from the fact that the event “$p_{C} \leq p$” is equivalent to there being excess supply at price $p$.

Now, let’s rewrite the firm’s profit maximization problem from an ex-ante standpoint:

$$\max_{\hat{S}_{it}(p)} \int_{p}^{\hat{p}} \left\{ \hat{S}_{it}(p)p - C_{it}(\hat{S}_{it}(p)) - (p - PC_{it})QC_{it} \right\} dH_{it}(p, \hat{S}_{it}(p))$$
where the expectation is taken over all possible realizations of the market clearing price (from \( p \) to \( \bar{p} \)), weighted by the probability density, \( dH_{it}(p, S_{it}(p)) \).

Using first an integration by parts, we can write the Euler-Lagrange necessary condition for the (pointwise) optimality of the supply schedule \( S_{it}^*(p) \) to be:

\[
p - C'_{it}(S_{it}^*(p)) = (S_{it}^*(p) - QC_{it}) \frac{H_S(p, S_{it}^*(p))}{H_p(p, S_{it}^*(p))}
\]

where

\[
H_p(p, S_{it}^*(p)) = \frac{\partial}{\partial p} \Pr(p^c_t \leq p | S_{it}^*(p))
\]

\[
H_S(p, S_{it}^*(p)) = \frac{\partial}{\partial S} \Pr(p^c_t \leq p | S_{it}^*(p))
\]

i.e. \( H_p(p, S_{it}^*(p)) \) is the “density” of market clearing price when \( S_{it}^*(p) \). \( H_S(p, S_{it}^*(p)) \) can be interpreted is the “shift” in the probability distribution of the market clearing price, due to a change in \( S_{it}^*(p) \), i.e. this is the term that captures the “market power” of firm \( i \). Notice that this derivative is always positive (assuming all bidders submit increasing supply functions), since an increase in supply lowers the market clearing price, making the probability that the market clearing price is lower than a given price \( p \) higher.

Observe that the above first-order condition can be seen as a “markup” expression, where the markup in price above the marginal cost depends on how much “market power” firm \( i \) can exercise by shifting the distribution of the market clearing price through its own supply function \( S_{it}^*(p) \). As an intuitive consequence, observe that if \( H_S(p, S_{it}^*(p)) \rightarrow 0 \), i.e. there is no market power, hence price equals marginal cost. Also observe that where \( S_{it}^*(p) - QC_{it} = 0 \), \( p = C'_{it}(S_{it}^*(p)) \). This has an interesting consequence for the empirical exercise to follow:

**Proposition 1** If \( C'_{it}(S_{it}^*(p)) \) is observed (which we do in our data), along with supply functions, one can calculate the contract position \( QC_{it} \), by finding where the strategic supply function of the firm intersects its marginal cost function.

Observe also that:

**Proposition 2** Marginal schedules \( C''_{it}(S_{it}^*(p)) \) are nonparametrically identified through knowledge of \( H_{it}(p, S_{it}^*(p)) \) (hence \( H_p(p, S_{it}^*(p)) \) and \( H_S(p, S_{it}^*(p)) \)) and contract quantities, \( QC_{it} \).
Notice, however, that the empirical implementation of the above identification lemma is complicated by the need to econometrically estimate \( H_{it}(p, S^*_it(p)) \), which reflects the equilibrium ex-ante expectations of each bidder regarding the distribution of the market clearing price conditional on their bidding strategies.\(^9\) Notice also that the form of the markup relation suggests that the equilibrium bid strategies are \textit{ex-ante optimal} in the sense that if bidders were given an opportunity to see each others’ supply functions, they might have an incentive to modify their bids.

Going in the reverse direction, the computation of equilibrium strategies, \( S_{it}(p, QC_{it}) \) for a given specification of the primitives of the competitive setup (number of firms who are participating, \( N \), the cost curves \( C_{it}(q) \), \( i = 1, \ldots, N \), the distribution of contract quantities, the distribution of the uncertain demand component) is also complicated by the fact that \( H_{it}(p, S^*_it(p)) \) will depend in a complex way on the distribution of private information and the distribution of demand noise. Specifically, \( H(p, S^*_it(p)) \) is determined endogenously through the market-clearing condition (1), depends on the distribution of contract positions, \( QC_{jt} \), across competing firms, and the distribution of demand noise.

However, we now show that the form of the markup relation (2) is greatly simplified when the functional form of the supply function strategies, \( S_i(p, QC_i) \) is restricted to a class of strategies that are \textit{additively separable and linear in the private information} (AS-LPI) possessed by bidders:

**Proposition 3** If supply function strategies \( S_i(p, QC_i) \) are restricted to the class of AS-LPI strategies: \( S_i(p, QC_i) = \alpha_i(p) + \beta_i QC_i \), and demand is additively separable in the stochastic component, i.e. \( \hat{D}(p) = D(p) + \varepsilon \), where \( D(p) \) is deterministic and \( \varepsilon \) is random, the markup relation (2) is given by

\[
p - C^*_i(S_i(p, QC_i)) = \frac{S_i(p, QC_i) - QC_i}{-\frac{\partial D(p)}{\partial QC_i} - \alpha_i(p)}
\]

or, the more familiar “inverse-elasticity” markup rule:

\[
p - C^*_i(S_i(p, QC_i)) = \frac{S_i(p, QC_i) - QC_i}{-\hat{D}'_i(p)}
\]

where \( \hat{D}'_i(p) \) is the price derivative of the ex-post realization of the residual demand curve faced by bidder \( i \).

**Proof.** Start with the most general form of strategies, which can be written as \( S_i(p, QC_i) = \alpha_i(p) + \beta_i(p, QC_i) \), i.e. an additively separable term and a non-separable term. Now use the market clearing condition (1) above to represent the event \( \{p^*_i \leq p|S_i\} \), i.e. there is excess supply at \( p \), conditional on bidder \( i \) bidding \( S_i \):

\[
\sum_{j \neq i} \alpha_j(p) + \beta_j(p, QC_j) \geq \hat{D}(p) - S_i
\]

\[
\sum_{j \neq i} \beta_j(p, QC_j) \geq \hat{D}_i(p) - S_i - \sum_{j \neq i} \alpha_j(p)
\]

\(\footnote{Which is done, in a discriminatory (pay-as-bid) share auction context, by Hortacsu (2002).} \)
Observe that when \( S_i(p, QC_i) = \alpha_i(p) + \beta_i QC_i \), and demand is also additively separable into \( \tilde{D}(p) = D(p) + \varepsilon \), the last expression becomes:

\[
\sum_{j \neq i} \beta_j QC_j - \varepsilon \geq D(p) - S_i - \sum_{j \neq i} \alpha_j(p)
\]

where the left hand side is a (bidder-specific) random variable, \( \theta_i \), that does not depend on \( p \), and the right hand side is a deterministic function of price.

Now, let \( \Gamma_i(.) \) denote the cdf of \( \theta_i \), and \( \gamma_i(.) \) denote the pdf. Given this:

\[
H_p(p, S_i) = \frac{\partial}{\partial p} \Pr(p^*_i \leq p|S_i)
\]

\[
= \frac{\partial}{\partial p} \Pr(\theta_i \geq D(p) - S_i - \sum_{j \neq i} \alpha_j(p))
\]

\[
= \frac{\partial}{\partial p} [1 - \Gamma(D(p) - S_i - \sum_{j \neq i} \alpha_j(p))]
\]

\[
= -\gamma(D(p) - S_i - \sum_{j \neq i} \alpha_j(p)) \frac{\partial}{\partial p}(D(p) - S_i - \sum_{j \neq i} \alpha_j(p))
\]

and

\[
H_S(p, S_i) = \frac{\partial}{\partial S_i} \Pr(p^*_i \leq p|S_i)
\]

\[
= \frac{\partial}{\partial S_i} \Pr(\theta_i \geq D(p) - S_i - \sum_{j \neq i} \alpha_j(p))
\]

\[
= \frac{\partial}{\partial S_i} [1 - \Gamma(D(p) - S_i - \sum_{j \neq i} \alpha_j(p))]
\]

\[
= -\gamma(D(p) - S_i - \sum_{j \neq i} \alpha_j(p)) \frac{\partial}{\partial S_i}(D(p) - S_i - \sum_{j \neq i} \alpha_j(p))
\]

Evaluating the derivatives gives \( \frac{H_p(p, S_i)}{H_S(p, S_i)} = -[D'(p) - \sum_{j \neq i} \alpha'_j(p)] \). Now, observe that with the above restrictions, the residual demand function faced by firm \( i \) (for a given realization of the random variables \( \{\varepsilon, QC_i, i = 1,..N\} \), is given by:

\[
RD_i(p, \varepsilon, QC_{-i}) = D(p) + \varepsilon - \sum_{j \neq i} \alpha_j(p) - \sum_{j \neq i} \beta_j QC_j
\]

with derivative:

\[
RD'_i(p) = D'(p) - \sum_{j \neq i} \alpha'_j(p)
\]
which yields the result in the proposition.

Several notes are in order regarding this result. First, note that the linearity-in-private-information restriction is an a priori restriction on bidding strategies, and it is not necessarily true that every specification of marginal cost functions, \( C_0^i(q) \) and joint distribution of contract quantities \( QC_i \), will lead to equilibrium strategies of this form. The Appendix provides an example in which firms possess linear marginal cost curves, under which equilibrium strategies are analytically solvable, and satisfy this functional form restriction.

Second, note the intuitive content of the AS-LPI restriction, which says that, in equilibrium, the residual demand function faced by bidders is additively separable and linear in its random component – i.e. all uncertainty is due to shifts in the residual demand curve and no rotations of the residual demand curve are allowed due to realizations of its random (from the perspective of bidder \( i \)) elements. Given this, it is easily seen that (subject to the concavity of the profit function), that the bid function \( S_i(p) \) provides a pointwise best-response to every possible realization of the residual demand curve. This also means that the class of AS-LPI equilibrium strategies, when they exist, are ex-post optimal, in the sense that seeing other bidders’ supply functions would not change bidder \( i \)'s choice of supply function.

Third, observe that the restrictions imposed are crucial, since, without them:

\[
RD_i'(p, \varepsilon, QC_{-i}) \neq D'(p) - \sum_{j \neq i} \alpha_j'(p)
\]

Also, note that without the linearity restriction, we can not collapse the stochastic terms (from the perspective of bidder \( i \)) into a single scalar random variable, \( \theta_i \), which does not depend on \( p \). \(^{10}\)

Fourth, observe that this result has two immediate empirical applications:

**Proposition 4** If supply function strategies \( S_i(p, QC_i) \) are restricted to the class of AS-LPI strategies: \( S_i(p, QC_i) = \alpha_i(p) + \beta_i QC_i \), and demand is additively separable in the stochastic component, i.e. \( \tilde{D}(p) = D(p) + \varepsilon \), one can:

1. compute the entire marginal cost curve rationalizing a supply curve \( S_i^c(p) \) observed in the data, using a single realization of the residual demand curve

2. equivalently, given information about marginal costs, one can compute the ex-post optimal supply curve \( S_i^{xp}(p) \), which is the ex-post best response to the observed realization of the residual demand curve.

\(^{10}\)Note that in the one other case where we can do this, we do not obtain the ex-post optimality result, since when \( S_i(p, QC_i) = \alpha_i(p) + \beta_i p QC_i \) and \( \tilde{D}(p) = D(p) + \varepsilon \), i.e. the stochastic elements of the game only rotate the residual demand curve, the market clearing condition becomes \( \theta_i = \frac{1}{p} \sum_{j \neq i} \beta_j QC_j - \varepsilon \geq \frac{1}{p} D(p) - S_i - \sum_{j \neq i} \alpha_j(p) \), but \( RD_i(p, \varepsilon, QC_{-i}) \neq RD_i(p, \varepsilon, QC_{-i}) \).
We start with part (2). Observe that under the above restriction, a single realization of the residual demand curve, $RD_i(p, \varepsilon, QC_{-i})$ is enough to compute $\frac{d}{dp}RD_i(p, \varepsilon, QC_{-i}) = RD'_i(p)$ for all realizations. Then, for a range of prices, $p \in [\bar{p}, \bar{p}]$, one can solve the equation for $S$, in terms of $p$ and $QC_i$

$$ p - MC_i(S) = \frac{S - QC_i}{-RD'_i(p)} $$

(4)

to trace out $S^{eq}(p, QC_i)$, which constitutes an ex-post best-response to all possible realizations of residual demand.

Observe that part (1) follows exactly using the same logic. This time we know (or assume) $S^{o}(p) = S^{eq}(p, QC_i)$, thus for each $p$, we solve the equation:

$$ p - MC_i = \frac{S^{eq}(p, QC_i) - QC_i}{-RD'_i(p)} $$

(5)

for $MC_i$, in terms of $p$, $S^{eq}(p, QC_i)$ and, $QC_i$. Incidentally, this gives us $MC_i$ as a function of $S^{eq}(p, QC_i)$, which exactly traces out the marginal cost function for all points on the support of $S^{eq}(p, QC_i)$.

Hence, the theoretical discussion above suggests the following sets of empirical applications:

1. “Pointwise” testing of ex-post optimal bidding using marginal cost data, by looking at whether the markup relation (4) holds for the actual market-clearing price and quantity supplied by each bidder on a particular day.
2. Testing of entire ex-post optimal equilibrium/best response schedules using marginal cost data and residual demand information from a single auction
3. “Pointwise” identification of marginal cost schedules using bid data, by evaluating the markup relation (4) at the actual market-clearing price and quantity supplied by each bidder on a particular day – this can be used to trace out marginal cost schedules by utilizing data across auctions
4. Identification of entire marginal cost schedules using data from a single auction, through (5).

Empirical applications similar to (1) and (3) have previously been conducted by Sweeting (2002) and Wolak (2003). We should note that the analyses of both Sweeting (2002) and Wolak (2003) rely on the assumption that the market clearing point on a bidder’s supply function is an ex-post “best-response” to the realized residual demand curve, and hence an “equilibrium rationale” for their respective empirical exercises depends on similar restrictions on the theoretical structure of the game. Note, however, the distinction that they exploit the “pointwise” ex-post optimality of the supply functions, whereas we will impose the condition that the entire supply function is an ex-post optimal.
Our novel contributions in the empirical application domain are items (2) and (4) above: that the observed bid schedules convey more information about a bidder’s strategic thinking, than single points on these schedules. More concretely, Part (1) of Proposition 4 above suggests that, armed with knowledge of marginal cost schedules, $MC_i(q)$, an economic researcher can test whether an observed supply schedule $S^o_i(p)$, for a bidder with contract position, $QC_i$, corresponds to ex-post best response (or, under the maintained set of restrictions, equilibrium) behavior $S^{BP}(p, QC_i)$. No assumptions need to be made about the “expectation” of bidders regarding the set of residual demand functions they are going to face to trace out entire ex-post optimal bid schedules – EXCEPT for the assumption that bidders are playing an equilibrium of the game where the “independence of price derivative” condition holds. This is in contrast to “pointwise” tests of best-response behavior – where the “best-response” price that could have been charged by a bidder is compared with the actual price.

Of course, the finding of a deviation from equilibrium behavior can call into question many things, one of which includes the key theoretical restriction we have used above, the “independence of price derivative of residual demand”. Unfortunately, given only one ex-post realization of the residual demand curve per auction, this assumption can only be tested by imposing statistical restrictions on the distribution of residual demand curves across auctions. We do not doubt that there are reasonable ways of imposing such restrictions, but once again one has to face the problem that the econometrician’s assumptions/beliefs about the nature of the stochastic components of the model correspond with the statistical model in bidders’ minds.

Similarly, in terms of estimating marginal costs our method can be contrasted with the “best-response estimation” method of Wolak (2003), in that his method is used to estimate only a single point on a bidder’s marginal cost schedule from a given realization of the residual demand function. To use Wolak’s (2003) procedure to estimate entire marginal cost schedules rather than single points, one has to make statistical assumptions regarding the set of possible residual demand realizations that a bidder “expects” to face. However, the assumption of ex-post optimality allows one to eschew such statistical assumptions.

3.0.1 An important practical consideration

The main practical consideration with all of these applications is that, in real-life data sets, supply functions and marginal cost schedules alike are not represented as continuous functions, but as step functions comprising of discrete (price, quantity) pairs.

This complicates the calculation of derivatives, especially of the all-important “residual demand derivative”. We will therefore follow a parametric approach to surmount this problem, by using the idea of Wolak (2003) to “smooth” step functions in the data using kernel functions. We can thus use the smoothing parameter, $h$, as the focus of robustness analyses of our results.

In particular, let the collection of price-quantity pairs $\{(p_1, q_1), ..., (p_K, q_K)\}$
represent the residual demand curve seen in the data. The smoothed version of this function will be:

\[ RD(p) = \sum_{k=1}^{K} q_k K\left( \frac{p - p_k}{h} \right) \]

where \( K(.) \) is a kernel function. With this representation, the derivative of residual demand will be given by:

\[ RD'(p) = \sum_{k=1}^{K} q_k \frac{1}{h} K'(\frac{p - p_k}{h}) \]

We will use these “smoothed” versions of the residual demand and residual demand derivatives to calculate the “ex-post optimal” bid schedules, as outlined in the proof of Proposition 3.

4 Comparison of Actual Bid Schedules to Ex-post Optimal Bid Schedules

We construct ex-post optimal bid schedules and compare these bids to actual bids for September 2001 to July 2002. We use several metrics of the distance between optimal and actual bids. The first method is a “pointwise” test of optimal bidding behavior, in that it looks at the realized intersection of residual demand with the bidder’s supply schedule, and quantifies how far away this point is from being static profit maximization. The second method uses the technique described in the previous section to derive the entire ex-post optimal bid curve, and calculates the “foregone” profit of the bidder by not utilizing this ex-post optimal bid curve. We focus on the largest sellers of balancing power.

We briefly summarize how we calculate ex post optimal bids. We apply the model of the previous section to the balancing market. In ERCOT, many transactions are scheduled day-ahead. We can easily modify the model to fit ERCOT’s balancing market. Demand \( D_t \) is now defined as total ERCOT demand minus the total quantity scheduled day-ahead. \( MC_{it} \) is now the marginal cost of changing output from the firm’s day-ahead schedule. And \( QC_{it} \) is now the amount that the firm is long or short on power after the day-ahead schedule and upon entering the balancing market.

For a given firm in period \( t \), the residual demand is given by the perfectly inelastic total balancing demand minus the supply bids by the rivals. We use the market realization of balancing demand and observed bids by all rivals to calculate the ex post residual demand: \( RD_{it}(p) = D_t - \sum_{j \neq i} S_{jt}(p_{jt}, QC_{jt}) \).
In the ERCOT balancing market, this function is well-approximated by a step function. The slope of residual demand (which is simply negative the slope of rival supply) is therefore either zero or infinity at every point. However, the firm cannot know at which points the steps occur, so we model the firm’s prior of the residual demand as being a smooth function fitted through the step function. Following Wolak, we use a normal kernel to smooth the residual demand function and use the slope of that function to measure the firm’s perceived slope of residual demand.

We require a measure of the firm’s marginal cost function of supplying power to the balancing market. We have data on the generating units operating on a given hour and the declared capacity of each unit in the particular hour. Also, we know how much generation has been scheduled day-ahead by the firm, so we can calculate the capacity available to increase and decrease generation in the balancing market. Given a financial obligation to supply a certain quantity of MW, it is reasonable to assume the generators will produce that output in a least cost manner. Thus the “residual marginal cost” going into the balancing market can be characterized as the total marginal cost function with the origin re-centered at the day-ahead schedule. We construct the marginal cost function of all the firm’s generating units and then subtract the supply already scheduled day-ahead to develop our measure of “residual marginal cost”. Total marginal cost is the marginal production cost of units that are reported by ERCOT to be operating and available in period $t$. Some units do not have the ability to adjust production from the planned day-ahead level on the short notice required of the balancing market. Therefore, we exclude nuclear, wind, and hydroelectric generating units from the marginal cost portfolio for providing balancing energy. Our marginal cost represents the variable fuel, operating and maintenance, and SO2 permit costs of coal and natural gas fired units. Gas units, and to a lesser extent coal units, can be adjusted on relatively short notice to other production levels. A large literature has developed on measuring the marginal cost of electricity production (for example, see Wolfram[1999], Borenstein, Bushnell and Wolak [2003], Mansur [2002], Puller [2002], Joskow and Kahn [2002], Bushnell and Saravia [2002]) and we use the same approach. Details of the data and additional institutional considerations are discussed in the data appendix. Our “residual marginal cost” is the costs of providing more power (INCing) or the cost savings of reducing production (DECing) from the day-ahead scheduled quantity.

We use firm (portfolio) bid schedules for each firm for the 6:00-7:00pm hour of each non-congested weekday from September 2001 to July 2002. We need to know the amount sold to the balancing market $q_{it}$ for a given realization of demand. We intersect the actual bid with the realization of residual demand to determine sales. We assume a step function which is a good approximation to the actual dispatch algorithm used. This simulation of the actual auction predicts prices with only a 5% error.
4.1 Qualitative assessment of bidding

Figure 6 displays somewhat representative actual and ex post optimal bid functions for the three largest suppliers – Reliant, TXU, and Calpine – and for one small seller, Guadalupe. In each panel, competitive bidding is offering the “MC curve” and optimal bidding is offering the “Ex-post optimal bid”. Reliant’s bids are much “closer” to the optimal bids than to the marginal cost function. In addition, Reliant’s performance for incremental supply (Balancing MW > 0) is relatively similar to its performance for decremental supply.

TXU is somewhat “close” to the ex-post optimal bid function on the INC side, but bids significantly below optimal prices on the DEC side. We see this tendency for TXU to offer DECs at only very low prices throughout our sample period.

Calpine offers some DEC bids but does not offer to INC supply. Although Calpine does offer INC bids in some periods, much of Calpine’s bids are to DEC supply. Those DEC offers are often are at prices substantially below ex post optimal bids.

Guadalupe submits bids that are much steeper than optimal. Because it is a small seller, Guadalupe’s residual demand function is relatively flat as compared to the residual demand of the larger players. Nevertheless, small sellers have some potential to exercise market power. However, the actual bid functions are significantly above the optimal bids in INC periods and below optimal bids in DEC periods. This suggests that bids significantly different from marginal cost are not intended as a means to exercise market power, but rather to avoid being called upon to change production from day-ahead schedules. Many small sellers show similar bidding patterns.

Finally, note that the firms use coarse-grained strategies that use a small number of bidpoints. All three bidders use far fewer bidpoints than the maximum permitted, and it appears that using more bidpoints would allow the firms to better fit the relatively smooth optimal bid function. These figures are suggestive of the level of bid optimality, but more formal metrics are required.

4.2 “Pointwise” tests of optimality using ex-post realizations of residual demand

We parameterize the difference between the observed and optimal bid function by writing the first-order condition as:

$$ p - C'_{it}(S_{it}^{obs}(p, QC_{it}) + q_{it}^{DA}) = \theta_{it} \frac{RD_{it}(p) - QC_{it}}{RD_{it}(p)} $$

(6)

where $\theta_{it}$ is a multiplicative error term.
There are multiple interpretations of the error term. $\theta_{it}$ can be interpreted as a reduced form “conduct” parameter, in that $\theta_{it} = 0$ implies “perfectly competitive” behavior, and $\theta_{it} = 1$ implies rational “Bayes-Nash” behavior – though we would like to make it very clear that when we use the term “conduct” parameter, we are not referring to any equilibrium concept that incorporates $\theta_{it}$’s in a logically consistent manner. One could interpret this “conduct” parameter as a firm who does not properly internalize the effect of the marginal unit sold on the revenue from inframarginal sales. Perhaps in the early months of the market, the firms’ transition from a regulated regime to a market involves substantial learning.

Another interpretation is that our ex post measure of the slope of residual demand (and therefore the marginal revenue function) differs from the firm’s ex ante estimate of the slope prior to bidding. We measure the (smoothed) actual realization of residual demand whereas a firm only has a prior on the slope when it bids. Because the slope enters the first-order condition multiplicatively, the error should also be multiplicative. Unfortunately, we do not know how firms smoothed residual demand in their estimation, so this multiplicative error includes both “optimization error” and differences between our smoothing and the firms’ smoothing algorithm. Therefore, our $\theta$ measures of performance can be quite noisy.

Finally, we consider the possibility of an additive error to the first order condition. To the extent that we mismeasure the marginal costs, the error should be modeled as a non-zero mean additive error. If we mismeasure heat rates or fuel spot prices, marginal costs and hence price-cost margins are mismeasured. However, we believe these measurement errors to be small. There is an well-established literature on measuring the marginal costs of electricity production. We measure the variable costs of units that are operating with certain declared capacity in the relevant time period, and we believe these measures to be fairly precise. Therefore, we believe the best way to model the error is multiplicative.

Observe that the multiplicative error term allows us to use various points on the bid, marginal cost, and demand schedules to test the model. Consider the points labeled in Figure 7. This is a figure we can construct for each time period so we suppress the $t$ subscript.

1. Point A: where the “day-ahead shifted” marginal cost schedule intersects the supply schedule. This is used to calculate $QC_{it}$. Under our maintained assumption of the $\theta$-parameterized first order condition, a firm’s bid function intersects the marginal cost function at the contract position entering the balancing market $QC_{it}$. Intuitively, if it is maximizing profit and there is no error (i.e. $\theta = 1$), a firm will bid above marginal cost when it is a net seller and below marginal cost when it is a net buyer, and just equal to marginal cost when it has no net position. No net position corresponds to the contract quantity. We intersect the actual bid with our

\[11\] We should also note that a mismeasured marginal cost can have small or large effects on our estimate of $QC_{it}$ depending on how flat the bid function is near its intersection with marginal cost (see below).
measure of marginal cost to measure $QC_{it}$ (Note that we also can identify $QC_{it}$ under any multiplicative error $\theta_{it} \neq 0$ because the bid function for different $\theta$'s is just the optimal bid function pivoted around Point A. In other words, we can identify the contract position as long as the firms are sophisticated enough to know that they should bid above marginal cost when they are long and below marginal cost when they are short, even if the firms err in the size of the markup. Our conversations with several market participants leads us to believe that traders recognize the rationale for mark-upping up bids for quantities greater than the contract position (and vice versa), but have different heuristics for choosing the size of the markup. So even if firms do not satisfy the theorized first-order condition, we may be able to identify the contract position.)

2. Point B: where the marginal cost schedule intersects the marginal revenue curve implied by the ex-post “realized” residual demand curve, $RD_{it}(p)$. The point in the residual demand curve corresponding to this quantity is the rational Bayes-Nash behavior and corresponds to $\theta_{it} = 1$.

3. Point C: where the supply schedule of bidder $i$, $S_{it}(p, QC_{it})$, intersects the residual demand curve, $RD_{it}(p)$. If point C is not equal to point B, the firm is not playing the (ex post) profit-maximizing equilibrium. Here the error term $\theta \neq 1$ represents a price-cost margin that does not satisfy the first order condition – it is too low given the elasticity of residual demand. The difference in Point B and Point C in practice allows us to measure the magnitude of the deviation from strategic equilibrium behavior in terms of $\theta_{it}$. Below we also measure the difference in foregone profits.

4. Points D and E: Others points on the supply schedule provide information on the firms bidding behavior that are relevant for other possible realizations of residual demand. We can measure the optimality of bidding for demand realizations that did not occur if uncertainty manifests itself as parallel shifts. These shifts can result from private information (in our equilibrium bidding model), shocks to the perfectly inelastic total balancing demand (e.g. a weather shock), or the fact that an hour’s bids are used to clear demand four times during each hour. We add a reasonable amount of uniformly distributed noise to each period’s total balancing demand, and evaluate the optimality of bidding using other points on the supply schedule.

Results: Results are displayed in Table 4 to test if the first order condition where $\theta = 1$ is satisfied in our data. The times series of $\theta$s is quite noisy due to the multiplicative nature of the error. For example, if the actual slope of residual demand is steeper than predicted, the $\theta$ represents the scale of the error. Because the measure is quite noisy, we use the median as our measure of central

---

12Data for April 2002 is currently incomplete but will be included in future versions of this paper.
Table 4: Median $\theta$ Pointwise Measure of Performance

<table>
<thead>
<tr>
<th>Actual Realizations of Residual Demand</th>
<th>Interval</th>
<th>Reliant</th>
<th>TXU</th>
<th>Calpine</th>
</tr>
</thead>
<tbody>
<tr>
<td>INC</td>
<td>1.78</td>
<td>0.79</td>
<td>15.90</td>
<td></td>
</tr>
<tr>
<td>DEC</td>
<td>1.23</td>
<td>2.85</td>
<td>9.64</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>20 Simulated Residual Demands Each Period</th>
<th>Interval</th>
<th>Reliant</th>
<th>TXU</th>
<th>Calpine</th>
</tr>
</thead>
<tbody>
<tr>
<td>INC</td>
<td>1.40</td>
<td>0.77</td>
<td>12.54*</td>
<td></td>
</tr>
<tr>
<td>DEC</td>
<td>1.37</td>
<td>2.99</td>
<td>7.68*</td>
<td></td>
</tr>
</tbody>
</table>

Note: Uses twenty simulated residual demands created by adding uniformly distributed noise to actual demand. Cannot calculate theta if $RD(p) = QC$.

* 95% confidence interval does not include $\theta = 1$.

tendency to make inferences about the behavior. In incremental intervals when balancing demand is positive, both TXU and Reliant fail to achieve optimal prices but they are closer to optimal prices ($\theta = 1$) than to non-strategic pricing ($\theta = 0$). However, in decremental intervals, Reliant charges prices much closer to optimal than TXU. TXU’s prices are substantially below ex post optimal levels in DEC intervals (i.e. it exercises “too much” market power than is individually profitable when demand is negative). This finding for TXU is consistent with the figure in the previous section. Calpine bids significantly higher than optimal in INC intervals and significantly lower than optimal in DEC intervals, as we saw in the figures in the previous section.

When we examine the times series of median $\theta$s (not reported), we see interesting patterns. TXU’s bidding is relatively close to optimal in 2001. Even in the first few months of the market, TXU displayed sophistication in its bidding strategy. However, starting in 2002, TXU displays large deviations from optimal bidding. However, these monthly measures of deviation are largely driven by the frequency of INC and DEC intervals. TXU performs poorly during DEC intervals because it bids much lower than optimal. Reliant’s monthly average $\theta$s display less variability, but the patterns of TXU and Reliant variability appear correlated.

There is a great deal of noise in these measures of optimal bidding. Some of this variability is to be expected given the multiplicative error. However, there is also reason to believe that other (unmodeled) factors have important effects on bidding. We explore possible explanations for these deviations below.


4.3 Testing ex-post optimality of bid schedules

Although the multiplicative error term characterization has the advantage of being interpreted as a reduced form “conduct” parameter, there are other ways of quantifying the observed deviations from equilibrium bidding strategies. Perhaps a more relevant metric to evaluate our bidders’ performance is to see how much profit they have foregone ex-post by deviating from the ex-post optimal bidding schedule.

However, to construct the “counterfactual” profit of a bidder if it were employing the ex-post optimal bidding schedule, we first have to find the ex-post optimal bidding schedule. Following the first part of Proposition 4, we find the pointwise solution to the optimal $S_i(p)$ that solves equation (4) for each firm and each time period. Recall that this set of ex post optimal bids also characterizes the Bayesian Nash equilibrium in which uncertainty leads to parallel shifts in residual demand. This allows us to calculate $RD_{i,t}(p)$ for other possible realizations of residual demand. This assumption is not unreasonable because shifts in (perfectly inelastic) total balancing demand shift residual demand to the left and right in a parallel fashion. In addition, firms bid hourly supply functions that are used to clear demand every 15 minutes. The average difference between the highest and lowest 15 minute demand in the 6-7pm bidding hour is over 600 MW, and this difference has the same effect as additional demand uncertainty.

To calculate the profit deviation, we calculate the difference of the producer surplus obtained at the actual submitted price/quantity point (point C in figure 7), and the surplus obtained at the ex-post optimal point (point B in figure 7). We calculate this difference each firm-period for 20 simulations of residual demand which we construct by adding uniformly distributed noise to the actual demand. These simulations allow us to evaluate the optimality of several points on the bid function that would be the market clearing price under possible realizations of residual demand. The results are generally robust to the scale of the noise added.

We also calculate the producer surplus achieved relative to a benchmark of “suboptimal” behavior to compare how much distance is closed between the benchmark of suboptimal pricing and optimal pricing. One possible benchmark is behaving non-strategically and bidding marginal cost. However, it appears that the “default” behavior is to bid to avoid being called to supply balancing power. As shown in the sample figures, smaller firms choose to bid only small quantities relative to both competitive and optimal bidding. Therefore, we measure performance as the fraction of (dollar) distance between “no bidding” and ex post optimal bidding that is realized by the actual bids. Producer surplus is:

\[\text{Producer surplus} = \frac{\text{Actual surplus} - \text{Suboptimal surplus}}{\text{Optimal surplus} - \text{Suboptimal surplus}}\]

We cannot use a measure such as the fraction of possible profits achieved because some firms tend to be short on their contract positions entering the balancing market, and we would have to make an assumption about the contract price. The measure we construct differences out the contract price and avoids this complication.
Table 5: Difference between Ex Post Optimal and Actual Producer Surplus

<table>
<thead>
<tr>
<th>Firm</th>
<th>Foregone Profits ($/hour-day)</th>
<th>Pct Achieved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliant</td>
<td>1,295</td>
<td>83%</td>
</tr>
<tr>
<td>TXU</td>
<td>2,240</td>
<td>45%</td>
</tr>
<tr>
<td>City of Austin</td>
<td>1,111</td>
<td>24%</td>
</tr>
<tr>
<td>Calpine</td>
<td>1,573</td>
<td>22%</td>
</tr>
<tr>
<td>Central Power and Light (CPL)</td>
<td>2,027</td>
<td>15%</td>
</tr>
<tr>
<td>West Texas Utilities (WTU)</td>
<td>1,593</td>
<td>15%</td>
</tr>
<tr>
<td>Lamar</td>
<td>1,198</td>
<td>8%</td>
</tr>
</tbody>
</table>

Pct Achieved is the percent of possible gains in producer surplus achieved relative to not bidding into the balancing market.

\[
\pi = P^{BAL}(Q_i^{BAL}) - TC(Q_i^{BAL}) - (P^{BAL} - PC)QC
\]

We calculate \( \pi \) for three scenarios: (1) ex post optimal bidding, (2) actual bidding, and (3) bidding to avoid the balancing market (i.e. not bidding at all and buying/selling at the market clearing price any net short or long position on contracts). The measure of performance is:

\[
\text{Percent Achieved} = \frac{\pi^{Actual} - \pi^{Avoid}}{\pi^{EPO} - \pi^{Avoid}}
\]

**Results:** We focus on the larger suppliers of balancing power. The average dollar difference between actual and ex post optimal bidding ranges from about $1,000-$2,000/hour-day for this bidding interval.\(^{14}\) The foregone profits by firm are shown in table 5. The larger firms tend to have higher foregone profits and this is driven to some degree by the amount money at stake for each firm. TXU and Reliant have more money at stake than other firms, but Reliant leaves less money on the table than TXU. Smaller firms (e.g. Lamar Power Partners) have smaller foregone profits but, as we shall see below, do not necessarily reap a large fraction of the potential gains of optimal bidding.

\(^{14}\text{Note that this “money left on the table” is for one particular hour of one day. (Technically, this is the foregone producer surplus for one fifteen minute interval if those prices and quantities prevailed for all four 15 minute intervals.) We would need to analyze other bidding hours to determine if foregone profits are similar in other periods of the day.}\)
Our second measure of performance, PercentAchieved, suggests interesting patterns across firms. Reliant achieves 83% of the potential gain in producer surplus from choosing ex post optimal bids versus not bidding. TXU achieves around half that fraction of possible gains, and this is driven in part by the very low bidding to DEC. Smaller players in the market have even weaker performance measures. This should not be surprising given the previous figures showing very little willingness to bid into the market by small firms. These overall differences in performance are positively correlated with the stakes in the market. Figure 8 shows the percent achieved as a function of the average volume of sales under optimal bidding. Firms that would sell more into the market (if they bid optimally) tend to achieve a larger fraction of possible gains. This suggests that firms with more incentive to participate in the market tend to bid more effectively. If there are costs to participating in the market and strategically calculating bids, larger players may be more likely to bid optimally. However, there are noteworthy exceptions to this trend in the City of Bryan, BP Energy, and Mirant who achieve large fractions of potential gains for relatively small players. Perhaps, there is a sticky market for managerial talent and the larger players later will hire more skilled bidders.

There is some evidence of a time trend in improved performance that provides evidence of “learning”. Figure 9 shows the monthly measure of PercentAchieved for the largest firms under ex post optimal bidding. We do not see strong evidence of learning such as a convergence to ex post optimal bidding. However, a time trend fit through the month-firm means does show a statistically significant increase in performance of 2.4% per month.

We explore other possible explanations for the deviations from optimal bidding in the next section.

5 Explaining Deviations from Ex-Post Optimal Bidding: Discussion

What causes the observed deviations from ex-post optimal bidding? A closer examination of our data sheds some interesting insights into what may be driving the observed deviations. We will group these possible causes under the following headings:

1. Constraints on optimizing behavior. There may be two constraints that bidders face when forming their optimal strategies. The first is limitations on the strategy space imposed by the rules of the game. The second are limitations imposed by bounded rationality because the game is complex and bidders may not have fully grasped the strategic context to take full advantage of their strategic options. We will discuss some evidence regarding bidders’ observed lack of use of sophisticated strategies that points to this possibility.
2. Unmeasured cost components, in particular adjustment costs. It is possible that firms face fixed costs of changing their units' generation from the day-ahead schedule. This is possible if the marginal unit is a coal unit or there are costs to moving units away from certain “sweet spots” in the production function. This is a possible explanation for the large vertical steps in the bid function for TXU at q=0. We find that the marginal generation unit is very often a gas unit which is more flexible than coal units. Also, we find evidence suggesting that if there were large adjustment costs, firms found ways to reduce these costs over time.

3. Dynamic pricing. One might be concerned that tacit collusion could arise in a daily repeated auction between firms with a great deal of information about one another. In fact, INC bids above (unilateral) ex post optimal prices or DEC bids below ex post optimal prices are consistent with collusion. However, as we argue below, we do not believe collusion explains the observed bidding behavior because the most likely “conspirators” would be the small firms.

4. Transactions costs of participation for small firms. As seen in Figure 6 for Guadalupe Power Partners, small producers bid INC prices that are substantially higher and DEC prices that are substantially lower than optimal. Some of the small firms bid in the minimum amount required which is the ERCOT requirement to bid at least 15% of the day-ahead schedule to DEC. Therefore, it appears that some small firms may not believe there is enough money at stake in the balancing market to warrant participation.

5. Transmission constraints. Firms strategic behavior may differ in hours when transmission lines are congested and uncongested across zones. If firms cannot perfectly predict congestion, the strategic effects of congested hours may spillover into the uncongested hours we analyze. We test for this possibility.

5.1 Constraints on Optimal Bidding Behavior?

Although the model presented in section 3 presents a complete equilibrium model of the competitive environment in the balancing market, one may have many concerns about whether this model presents a realistic characterization of actual bidder behavior. In particular, since the strategic supply function submitted by the bidders is an “ex-post rational” response to any equilibrium realization of residual demand curves, one may expect bidders to submit a large number of points on their supply curves to optimize their profits in a pointwise manner. Ex post profit-maximization can only be achieved if the firm submits a bid function that connects the profit-maximizing price-quantity pair for each possible realization of residual demand. This may not occur either because bidding rules do not allow such bid functions or because firms are not sufficiently sophisticated.
We can evaluate whether the bidding rules significantly constrain the ability to maximize ex post profits. Bidding rules impose two important constraints on the shape of bid functions. First, bid functions into the INC and DEC balancing markets must be monotonic. It is possible that the set of ex post optimal price-quantity points is a cloud of points that cannot be connected by a monotonic (step) function. We have yet to assess whether the set of realized optimal \((p, q)\) points on “similar days” are more of an “increasing function” or a “cloud of points”, and that is a future research priority.

The second major constraint of the bidding rules is the number of allowable price-quantity points in each bid function. It is not possible to submit (almost) continuous supply functions that connect every possible realization of residual demand. The bid rules for the ERCOT balancing market only allow each bidder 40 price-quantity points in each geographic zone (20 points for each zone’s inc and dec bid stacks). This affords generators a large degree of flexibility in bidding, especially those firms that own generation in more than one zone. During our sample period September 2001-July 2002 for intervals 6:00-6:15pm, only one firm uses the maximum number of bid points. Guadalupe Power Partners uses 80 bidpoints across its two zones in one hour and uses an average of 25 bid points per hour. Interestingly, this firm sells relatively small quantities of power to the balancing market but uses on average the largest number of bidpoints. The second most prolific user of bid points is Reliant Energy, the largest seller into the balancing market. However, Reliant uses a maximum of 24 bidpoints out of the 80 bidpoints allowed across its two zones. Therefore, we do not believe that the limit on the number of bidpoints significantly constrains the bidding behavior.

We use the number of bidpoints each hour as one measure of bid sophistication. If the support of residual demand is relatively large and a firm is profit-maximizing, it will submit a bid function with a sufficiently large number of elbow points to connect the profit-maximizing points for the various possible realizations of residual demand. In contrast, a firm that bids in marginal costs might be expected to bid in one bid point for each generating unit owned because each unit has approximately constant marginal cost. We count the number of bidpoints for each period and analyze trends. Simple inspection of the data reveal that the vast majority of firms changed the number of bidpoints used from day to day and week to week. However the number of points used by each bidder is not necessarily increasing with time.

We analyze summary statistics of the number of bid points. Table 6 displays the average number of bidpoints used over the entire sample and for each month. Across all bidders, there is a slight upwards trend in the number of bidpoints. We expect large sellers to utilize more bid point for several reasons. Non-strategic bidders who bid marginal cost require approximately one bid point per unit. On the other hand, any firm selling more than one unit has some incentive to bid strategically and raise price on inframarginal sales. Presumably, bidding strategically has some labor costs of analyzing the market rules, considering the strategic behavior of rivals, estimating residual demand and choosing how to bid on that residual demand. Smaller players have less money at stake and
less incentive to incur the costs of optimal bidding. Clearly, large sellers have greater incentive to use multiple bid points to trace out the ex post optimal \((p, q)\) points.\(^{15}\)

For the entire sample, large sellers average 9.3 points compared to the 8.1 points used by smaller sellers. For big sellers, the number of utilized bidpoints increases continually over the sample from 7.4 in September 2001 to 10.7 in July 2002. For small sellers, the number rises for about the first half of the sample and then declines.

Table 6: Average Bidpoints Used Per Bid Period

<table>
<thead>
<tr>
<th></th>
<th>All Bidders</th>
<th>Small</th>
<th>Big</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sep-01</td>
<td>7.3</td>
<td>7.3</td>
<td>7.4</td>
</tr>
<tr>
<td>Oct-01</td>
<td>8.2</td>
<td>8.2</td>
<td>8.2</td>
</tr>
<tr>
<td>Nov-01</td>
<td>8.0</td>
<td>7.7</td>
<td>8.4</td>
</tr>
<tr>
<td>Dec-01</td>
<td>8.6</td>
<td>8.4</td>
<td>8.8</td>
</tr>
<tr>
<td>Jan-02</td>
<td>8.7</td>
<td>8.3</td>
<td>9.1</td>
</tr>
<tr>
<td>Feb-02</td>
<td>10.0</td>
<td>9.8</td>
<td>10.1</td>
</tr>
<tr>
<td>Mar-02</td>
<td>9.9</td>
<td>10.3</td>
<td>9.6</td>
</tr>
<tr>
<td>Apr-02</td>
<td>8.3</td>
<td>7.3</td>
<td>10.0</td>
</tr>
<tr>
<td>May-02</td>
<td>9.3</td>
<td>8.8</td>
<td>10.0</td>
</tr>
<tr>
<td>Jun-02</td>
<td>9.4</td>
<td>8.2</td>
<td>10.7</td>
</tr>
<tr>
<td>Jul-02</td>
<td>8.8</td>
<td>7.6</td>
<td>10.7</td>
</tr>
</tbody>
</table>

| Full Sample | 8.6 | 8.1 | 9.3 |

Big (small) bidders are those that average more (less) than 20MW sales per interval.

The largest three sellers of power into the balancing market have distinct trends in the average number of bidpoints used. Figure 10 displays the average points used per period by month for the largest sellers. Reliant is the largest seller averaging 473 MW per interval. Recall that data are only available for 2002 because Reliant’s 2001 bids were aggregated with the City of San Antonio Public Service bids and separation of the two bid functions is not possible. The number of utilized bidpoints is increasing during 2002. TXU has two distinct

\(^{15}\)Of course any unilateral profit-maximizing firm albeit large or small has incentives to trace out all the ex post optimal points. Ignoring the (labor) costs of bidding strategically, all firms should utilize the maximum number of bid points.
upward trends interrupted by a discrete decrease in January 2002. This decrease is coincident with TXU consolidating all its bids under a single QSE at the beginning of 2002. It is not clear why the number should decrease when bids are consolidated, but if we treat the consolidation as a regime shift, the trends are upward throughout the sample period. Finally, the third largest seller, Calpine, displays a continual upward trend in bidpoints.

One is tempted to interpret the time trend of increased bidpoints as an increase in sophistication. However, other explanations could confound this interpretation. For example, if firms have more capacity available to bid into the balancing market, then firms must bid functions with more bidpoints, holding constant the level of sophistication. Controlling for the amount of capacity available for sale into the balancing market, the time trend in bidpoints is still positive. Also, firms may require more points to bid optimally if the support of the probability distribution of market clearing prices grows larger over time. In future analysis, we will explore the relationship more formally between the number of bidpoints other proxies for sophistication.

5.2 Adjustment costs?

Above we find that TXU’s performance compared to the theoretical benchmark is noticeably worse than Reliant’s. This is somewhat puzzling given that TXU owns the largest amount of capacity and is the second largest player in the balancing market, and this may suggest that they have sufficient incentives to act strategically.

Although TXU does bid a substantial amount of its residual capacity into the balancing market, the bid function has a large range of prices at which quantity offered is zero. For example, TXU might not offer to increase production until price reaches $25 and not decrease production until price is $5. Because prices are often in this interval, TXU is not called to supply balancing energy, especially during DEC intervals. We call this difference of prices a “bid-ask spread”. Note that this spread between INC and DEC prices for just 1MW of production is not consistent with market power. A firm exercising market power will indeed bid above marginal cost to INC and below to DEC. But the spread should be a smooth function of the amount sold because it is that inframarginal quantity that provides incentives to bid above or below costs.

This bid-ask spread is consistent with there being a fixed cost to participating in the market. It is not clear what these costs could be. One possibility is that supplying balancing energy requires TXU to adjust its generation from its original schedule. Plant operators need to be on hand to respond to a balancing call within a few minutes. The operator then must adjust the production of one or more units to meet the new generation level. Perhaps the mere “hassle” of adjusting production on short notice is a (fixed) cost. It is difficult to assess whether these costs are substantial enough to discourage participation. (A $10 margin on 100MWh summed across several hours of the day would appear to be substantial).
TXU’s tendency to “avoid the market” is much stronger on the DEC side
than the INC side. TXU is much closer to ex post optimal bidding when INCing,
but TXU does not offer to DEC until price is substantially lower than optimal.
One possible explanation for this apparent large “cost” to DECing is that the
marginal unit that would decrease output cannot respond (at all) to balancing
calls on short notice. This could be the case if systematically the marginal unit
were a coal or lignite unit rather than a more flexible natural gas unit. We
identify the technology of the marginal production unit each interval. TXU has
a natural gas unit as the marginal unit or lower in the marginal cost function in
68% of the intervals in our sample. In the remaining 32% of intervals, TXU only
has coal or lignite units to DEC. However, the periods when no gas units are
available to DEC do not appear to be associated with larger bid-ask spreads.
Also, such an adjustment cost explanation would not account for the closer to
optimal bids on the INC side because the same unit used to DEC would also
be used to INC. Therefore, we do not believe there are engineering adjustment
cost explanations for this behavior.

However, this bid-ask spread decreases with time suggesting that TXU (and
other firms) either found ways to reduce the fixed cost or realized the market
was sufficiently lucrative to warrant participation. Figure 11 shows the monthly
average bid-ask spread for each bid. TXU averages a $40 spread during the
first two months of the market, but the spread decreases over time and is just
below $5 by July 2002. TXU’s decreasing bid-ask spread despite using the same
generating units is evidence against the adjustment cost explanation. However,
the decrease is not uniform over time which suggests that it is not strictly
“learning” that drives the spread. The other large sellers also demonstrated
generally downward trends in the bid-ask spread over time. Calpine started the
market with a large bid-ask spread around $40 before decreasing to under $10
for all months in 2002 except April. Reliant (which we only begin to observe in
January 2002) has the lowest spread which averages under $4.

The evolution of TXU’s behavior is more consistent with learning than trans-
actions costs. First, transactions costs do not appear to be prohibitively large.
TXU had substantially more money on the table than many other participants.
And as noted above, TXU’s “bid-ask” spread decreases over time suggesting
that the apparent fixed costs of participation were more perceived than real.
Second, transactions costs to participation in the balancing market would affect
both INC and DEC bidding, but TXU is not far from optimal bidding on the
INC side.

The suboptimal DEC pricing is difficult to understand but it may disappear
with time. Market analysts have suggested that TXU was known for having a
strong tendency to rely on its own inefficient units to serve demand even when
cheaper power was available on the spot market. TXU appeared to be biased
toward supplying its large retail load obligation with its own generating units
rather than purchasing cheaper generation from other firms. TXU company
officials claim that in early 2003 (after our sample period), TXU began to replace
its own inefficient generation with purchases of cheaper power from bilateral
transactions or the balancing market.

35
5.3 Dynamic Pricing

We consider the possibility that the observed bids exceeding ex post optimal prices (in absolute value) result from collusion. In order to identify potential collusion, we must identify coalitions of players that priced above ex post optimal prices in a consistent pattern. Potential candidates for dynamic pricing are those that bid above unilateral ex post optimal prices for quantities above their contract position and below optimal prices for quantities below their contract position. In general, the firms that fit this description are smaller firms. Reliant, the largest seller, bids very close to ex post optimal on average, and TXU bid close to optimally for incremental sales. Although we have not performed any systematic analysis of potential coalitions, we believe it unlikely that a collusive coalition would form that included the small but not large players. It is more likely that the small players exhibited such steep bid functions due to real or perceived costs of participating in the balancing market. In future versions of the paper, we will carefully search for any potential coalitions of small players displaying behavior consistent with tacit collusion.

5.4 Transactions Costs of Participating in the Balancing Market

Smaller firms tend to submit bids that are substantially steeper than ex-post optimal bids. This apparent exercise of “too much market power” is probably more accurately interpreted as attempt to avoid being called to supply balancing power. One might expect that firms that did not want to bother with the costs of “thinking strategically” might simply bid their marginal cost. However, if there are operational or labor costs to adjusting production on short notice, the firms may opt to bid so as to avoid balancing calls. The transaction costs of participating in the market may be too high to justify the low stakes available for small participants. These transactions costs generate cost inefficiencies when the small players (who often own cheap combined-cycle generators) are not called to supply incremental balancing power and more expensive units owned by larger generators are called instead.

Anecdotal evidence from market participants and regulators also offers some insights as to why we see deviation from profit-maximizing behavior for smaller firms. In contrast to large vertically integrated utilities with a presence in other markets, the small municipal utilities (munis) do not have experience in other markets. Some municipal utilities may view their role as serving the customers’ load and avoiding risks of volatile prices. We have heard some munis described as “scared” of the market because they do not have good forecasts of prices. Munis with extra capacity claim they do not bid that capacity into the market but rather “save” the capacity to protect against the possibility of an outage at another unit. Although a trader or an economist would argue it is still optimal to bid that capacity into the market, it appears that some firms are not willing to do so.
5.5 Transmission Constraints?

We analyze ERCOT-wide bidding by restricting our sample to intervals without transmission congestion. Transmission congestion cannot be perfectly forecasted so firms must submit bids without knowing if congestion will occur. The possibility of congestion can alter the optimal bidding strategy. Congestion alters the residual demand function and may alter the best-response prices. For example, an importing zone cannot accommodate additional imports once the line capacity has been reached and the only bids that are technically feasible are other bids from that zone. This introduces kinks and convexities into the residual demand function that change the profit-maximization decision. In addition, ERCOT sold rights to the rents on congested transmission lines beginning in February 2002. For this reason, we have only analyzed intervals when there was no transmission congestion during that bid hour (so we exclude any 6:00-6:15pm intervals when there was congestion during the 6:00-7:00pm bid hour). We believe intrazonal or local congestion is also likely to be rare during intervals with no interzonal congestion.

However, if firms cannot perfectly forecast congestion, their bids during intervals that ex post are not congested may reflect the bid strategy if there were congestion. As a result, the strategic effect of congestion may spillover to uncongested hours. To test for possible strategic spillover, we estimate whether the profitability of actual bids is systematically related to the frequency of congestion in other intervals during the same month. We regress the monthly average of each firm’s measure of profitability, PercentAchieved, on the average sales volume under optimal bidding, the percent of 6:00-7:00pm intervals congested during that month, and the percent of 6:00-7:00pm intervals with DEC demand during that month. We include firm fixed effects which explain a large fraction of the variation in profitability. Because figure 8 indicates that large firms may differ in profitability, we report results using only the seven largest firms and for all firms. Results are reported in table 7. If the bidding strategy under congestion spilled over to uncongested hours, the effect would reduce profitability because bidding would reflect a strategy which is (presumably) profit enhancing under a state of the world that was not realized. After controlling for firm effects, we find that the high prevalence of congestion has the expected effect on bidding profitability (negative), but it is not statistically significant. These results do not rule out the possibility that congestion plays a role in determining profitability in uncongested hours, but we fail to find strong support for this explanation.

6 Conclusions and future research

Our test of a model of equilibrium bidding reveals interesting patterns in the optimality of bidding by various firms. Our analysis of the ERCOT balancing
Table 7: Model of Profitability

<table>
<thead>
<tr>
<th>Dependent Variable: Monthly Fraction of Profits Improved from No Bidding to Ex Post Optimal Bids</th>
<th>Largest 7</th>
<th>All Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume Optimal Output (GW)</td>
<td>0.00</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>(0.31)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>Pct Intervals Congested</td>
<td>-0.20</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>Monthly Fraction DEC</td>
<td>-0.40</td>
<td>-0.15</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.77</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Model includes firm fixed effects.
Robust standard errors in parentheses.

Market yields the rather comforting— at least for economic theory— conclusion that, even in the presence of considerable uncertainty and private information, and high requirements for strategic sophistication, the firm with the highest stake in this market appears to behave in a manner that is close to the prediction of a rather stylized economic model of strategic behavior (we should emphasize the word “stylized” here, since we believe that the condition we imposed for the ex-post optimality of equilibrium strategies is quite strong). The other large firms achieve a smaller fraction of potential profits, suggesting that larger stakes provide better incentives to bid optimally. We measure a slight upward trend in performance which suggests there may be “learning” over our sample period.

Interestingly, small firms with less incentive to bid strategically do not bid their marginal costs, but rather bid so as not to be called upon to supply balancing power. The bidding “to avoid the market” by small firms generates productive inefficiencies when those firms that would be called upon to increase or decrease output under efficient dispatch, are passed up because they did not bid substantial quantity into the balancing market. In future work, we plan to quantify this inefficiency. The cost of participating in the balancing market may generate such inefficiencies. However, a few small firms do perform quite well, which suggests that the fixed costs of “thinking strategically” may not be prohibitively large for small firms. If there is stickiness in the market for managerial efficiency in this relatively new market, it may take time for large
players to exhibit optimal bidding.

A confirmation of basic predictions of the uniform price share auction model of the balancing market is important in the sense that one could use this model to forecast bidder behavior in the future, and in balancing markets being setup in different parts of the world. A basic confirmation of the fact that bidders strategize in a manner predicted by the theory also opens up a whole host of interesting “counterfactual” exercises. For example, as the theoretical work of Ausubel and Cramton (2002) shows, strategic behavior in multi-unit uniform price auctions can cause inefficiencies in production – whereas the implementation of a multi-unit Vickrey auction can (at least theoretically) achieve efficiency. However, the efficient allocation implemented by the Vickrey auction may come at the cost of higher prices in the market. Hence, in the future versions of this paper, we will include simulations of the Vickrey auction mechanism, and compare the efficiency/price performance of this alternative mechanism with the uniform price auction being employed in Texas. This calculation will also enable us to calculate the “distributional” consequences of the inefficiencies caused by the uniform price auction mechanism – i.e. we will be able to assess whether strategic behavior by large bidders such as Reliant ends up being costly for smaller generators; compared to the efficient benchmark.
APPENDIX ON BID AND COST DATA

Bids into the balancing market are available for each hour of the sample. The bids are not unit-specific but rather are the aggregate bids for all units for which the QSE bids. These bids represent all bids to supply balancing power (i.e. there are not “imports” being ignored). As discussed in the main text, QSEs also submit ramp rates and unit-specific bid premia to be used in some cases of intrazonal congestion. We restrict our sample to periods when these other bid components are unlikely to lead to a dispatch substantially different from the one implied by the bid stack. (We analyze hours without interzonal congestion for a non-shoulder period). The bids implied by the bid stack alone predict the actual prices with only 5% error, so we believe the bid stack is a good model of actual deployment, or at the very least the bidder intent.

QSEs occasionally bid for more than one “owner” (PGC) of generating units. For example, in the South zone in 2001, the QSE named Reliant bid for both the Reliant PGC and the City of San Antonio Public Service Company. We match the bid functions to all units that the QSE bid for. So for all units owned by both Reliant the PGC and City of San Antonio in the South in 2001, we match the bid function to the generation data. This allows us to plot the bid function on top of the marginal cost of supplying into the balancing market. However, interpretation of the results becomes problematic when an observed bid function represents the bids by more than one owner. Because the results are some combination of two firms’ behavior, we will not interpret results in such situations. We only interpret our results for bids where we know that at least 90% of all electricity generated by owners using that QSE can be attributed to a single owner. We make one exception to this 90% rule. TXU Generation comprises 87% of the generation for TXU the QSE in North 2002. Because this is such a major player, we nevertheless refer to this as TXU the generation company.

We measure the variable costs of output using data on each unit’s fuel costs and the rate at which the unit converts the fuel to electricity. Only certain units are sufficiently flexible to supply power on short notice. We assume that nuclear, wind, and hydroelectric units cannot or do not supply to the balancing market. Clearly, this is a critical assumption. If the observed bids for each firm (which do not differentiate by unit) incorporate the intent to adjust hydro, nuclear, or wind units, we have mismeasure the marginal cost corresponding to that bid function. In addition, we assume that only units that are currently operating can supply into the balancing market. Data on the availability of generating units are from ERCOT. For each 15-minute interval, we know the generating unit, whether it is operating, and its hourly available generating capacity. For each unit, we know the ownership at each point in time during our sample. Almost no units changed ownership, although about 15% of incumbents’ capacity was virtually divested to new owners in 2002.

We measure the marginal cost of units that burn natural gas and coal. Data on fuel efficiency (i.e. average heat rates) are from Henwood Energy Services. Each unit is assumed to have constant marginal cost up to it’s hourly operating
capacity. This assumption, which is common in the literature, assumes that average heat rates are good approximations of incremental heat rates for the relatively small changes in output implied by balancing sales. This assumption is most problematic for combined-cycle gas turbines at lower levels of utilization.

Daily gas spot prices measure the opportunity of fuel for natural gas units. We use prices at the Agua Dulce, Katy, Waha, and Carthage hubs for units in the South, Houston, West, and North zones, respectively. We assume a gas distribution charge of $0.10/mmBtu. Coal prices are monthly weighted average spot price of purchases of bituminous, sub-bituminous, and lignite in Texas, reported in Form FERC-423 by the Energy Information Administration.

Coal-fired plants in Texas are required to possess federal emission permits under the Clean Air Act Amendments for each ton of SO2 emissions. Therefore, we require a measure of the emission permit cost for each MWh of electricity production. In order to measure average emission rates, we merge hourly net metered generation data from ERCOT with hourly emission data from EPA’s Continuous Emission Monitoring System to calculate each unit’s average pounds of SO2 emissions per net MWh of electricity output. The emissions each hour are priced at the average monthly EPA permit price reported on the EPA website (this price is the average of Cantor-Fitzgerald and Fieldston prices). The EPA does not (yet) report prices for 2002, so we use the March 2002 EPA spot auction clearing price as the emission permit price for all months in 2002.

We do not account for the sales of operating reserves. In electricity markets, the grid operator procures the right to purchase power from generating units that are to be operating with excess capacity available to provide power on short notice. We do not incorporate the possibility that some of the available capacity to INC in our data may be sold as reserves. The amount of operating reserves procured are small as a fraction of total demand but could represent a larger fraction of a given firm’s output depending upon which firms sell the operating reserves.

We measure the marginal cost of INCing or DECing from the day-ahead schedule of output. We account for the fact that units cannot DEC down to zero output without incurring costs of startup and facing constraints on minimum downtime. It is unlikely that revenue from the balancing market would be sufficiently lucrative to compensate a unit for shutting down. Rather, firms are likely to supply balancing energy services along the operational levels of currently operating units. Therefore, we assume that each operating unit cannot DEC to a level below 20% of its maximum generating capacity. Therefore, each firm’s marginal cost function is essentially the portfolio’s marginal cost for the upper 80% of its units’ maximum generating capacity.
APPENDIX ON EXISTENCE OF AN EQUILIBRIUM

An Analytic Example for Ex-Post Optimal Equilibrium in Linear Supply Functions:

Assume a symmetric setup with \( N > 2 \) firms in the market (the \( N > 2 \) restriction will become apparent in the solution).

Let the total cost function be quadratic of the form: \( TC(Q) = k + aQ + \frac{1}{2}bQ^2 \).

This gives the marginal cost function: \( MC(Q) = a + bQ \), or the "true" supply function: \( S^{true}(p) = \frac{1}{b}p - \frac{a}{b} \).

Let the contract position of firm \( i \) be \( QC_i \), where \( QC_i \) are iid random variables with distribution \( F(QC) \).

We are going to look for symmetric Bayesian-Nash equilibria of the game in linear supply functions, of the form:

\[
S_i(p) = S(p, QC_i) = \alpha + \beta p + \gamma QC_i
\]

Given such supply functions, the probability that the market clearing price is below a given price \( p \), conditional firm \( i \)'s strategy, \( S_i(p) \), is defined by the condition:

\[
S_i(p) + \sum_{j \neq i} S_j(p) \geq Q^{tot}
\]

ie. there is "excess supply at \( p \)." Equivalently:

\[
Q^{tot} - \gamma \sum_{j \neq i} QC_j \leq (N - 1)\alpha + (N - 1)\beta p + S_i(p)
\]

Notice that the left-hand side of this equation collects together all of the random elements of this problem, from the perspective of bidder \( i \).

That is,

\[
H(p, S_i(p)) = \Pr\{Q^{tot} - \gamma \sum_{j \neq i} QC_j \leq (N - 1)\alpha + (N - 1)\beta p + S_i(p)\}
\]

where \( H(p, S_i(p)) \) depends on the distribution of the random variable \( Z_i = Q^{tot} - \gamma \sum_{j \neq i} QC_j \).

Recall, however, the first-order condition for optimality derived in the paper:
\[
p - MC(S_i(p)) = (S_i(p) - QC_i) \frac{\partial H(p, S_i(p))}{\partial S_i} \frac{\partial S_i}{\partial p}
\]

A look at the expression for \( H(p, S_i(p)) \) reveals:

\[
\frac{\partial H(p, S_i(p))}{\partial S_i} \frac{\partial S_i}{\partial p} = \frac{1}{(N-1)\beta}
\]

The really surprising thing about this is that this "simplification" does not depend on the distribution of \( QC_i \)'s. It holds for any (non-atomic) distribution with full support over the range of prices.

How does this help? Plugging in for \( MC(Q) = a + bQ \):

\[
p - (a + bS_i(p)) = \frac{1}{(N-1)\beta} (S_i(p) - QC_i)
\]

\[
\beta(N-1)p - \beta(N-1)a + QC_i = S_i(p)[1 + \beta(N-1)b]
\]

Now, use the expression: \( S_i(p) = \alpha + \beta p + \gamma QC_i \)

\[
\beta(N-1)p - \beta(N-1)a + QC_i = (\alpha + \beta p + \gamma QC_i)[1 + \beta(N-1)b]
\]

For this equality to hold, we need to have the conditions (equating coefficients on like terms):

\[
\begin{align*}
\frac{\beta(N-1)}{1 + \beta(N-1)b} & = \beta \\
-\frac{\beta(N-1)a}{1 + \beta(N-1)b} & = \alpha \\
\frac{1}{1 + \beta(N-1)b} & = \gamma
\end{align*}
\]

The first equation has two solutions, \( \beta = 0 \) and \( \beta = \frac{N-2}{(N-1)b} \). We focus on the second solution, since \( \beta = 0 \) yields the bidding strategy, \( S_i(p) = QC_i \), which can actually be equilibrium bidding strategies, since the residual demand curve seen by every bidder is vertical.

Using the second solution, we get:

\[
\begin{align*}
\beta & = \frac{N-2}{(N-1)b} \\
\alpha & = -\frac{a}{b} \frac{N-2}{N-1} \\
\gamma & = \frac{1}{N-1}
\end{align*}
\]
Hence, the equilibrium strategies are:

\[ S_i(p) = S_i(p, QC_i) = \frac{N-2}{(N-1)p}p - \frac{a}{b} \frac{N-2}{N-1} + \frac{1}{N-1} QC_i \]

Contrasting this with the "true" supply functions given by the marginal cost functions, \( S_i^{true}(p) = \frac{1}{b}p - \frac{a}{b} \), yields that equilibrium strategies are "above" the true supply function (intercept term \( -\frac{a}{b} \frac{N-2}{N-1} \) is higher, and steeper \( \frac{N-2}{(N-1)p} < \frac{1}{b} \), but we are looking at slope when quantity is on the horizontal axis).

Notes:

1. When \( N = 2 \), the only equilibrium candidate that survives is \( S_i(p) = QC_i \).

2. If the market demand function, \( Q^{tot}(p) \) is not perfectly inelastic, then the "trivial" equilibrium \( S_i(p) = QC_i \) does not survive, since the residual demand function faced by each firm is no longer vertical.

3. What if firms were asymmetric, i.e. \( MC_i(Q) = a_i + b_i Q \)? In this case, we look for asymmetric equilibria of the form: \( S_i(p, QC_i) = \alpha_i + \beta_i p + \gamma_i QC_i \), and get a system of \( 3N \) equations to solve for \( \{\alpha_i, \beta_i, \gamma_i \} \), since:

\[
\frac{\partial H_i(p, S_i(p))}{\partial S_i} \frac{\partial S_i}{\partial p} = \frac{1}{\sum_{j\neq i} \beta_j}
\]

yielding, for each \( i = 1, ..., N \)

\[
\frac{\sum_{j\neq i} \beta_j}{1 + b_i \sum_{j\neq i} \beta_j} = \beta_i
\]
\[
\frac{-a_i \sum_{j\neq i} \beta_j}{1 + b_i \sum_{j\neq i} \beta_j} = \alpha_i
\]
\[
\frac{1}{1 + b_i \sum_{j\neq i} \beta_j} = \gamma_i
\]

Note that the first set of \( N \) equations, \( \frac{p \sum_{j\neq i} \beta_j}{1 + b_i \sum_{j\neq i} \beta_j} = \beta_i \) can be solved numerically for \( \beta_i \) as a function of \( b_1, ..., b_N \) (we have not been able to find an analytic solution). Given \( \beta_i \), one can then solve for \( \alpha_i \) and \( \gamma_i \).
References


Figure 1

Example of Actual and Optimal Bidding

Reliant on February 8, 2002 6:00-6:15pm

- Residual demand
- Bid curve
- Ex-post optimal bid
- MC curve
Figure 2
Sample Bidding Interface
Figure 3
Sample Bidder’s Operations Interface

![Sample Bidder’s Operations Interface Image]
Figure 4
Sample Genscape Interface
Figure 5

Density of Balancing Demand

Mean = -257
Stdev = 1035
Min = -3700
25\(^{th}\) Pctile = -964
75\(^{th}\) Pctile = 390
Max = 2713

Sample is uncongested weekdays 6:00-6:15pm from September 2001-July 2002.
Figure 6: Examples of Actual and Optimal Bid Functions

Reliant on February 8, 2002 6:00-6:15pm

TXU on March 6, 2002 6:00-6:15pm
Figure 6 (contd)

Calpine

Calpine on March 19, 2002 6:00-6:15pm

Guadalupe Power Partners

Guadalupe Power Partners on May 3, 2002 6:00-6:15pm
Figure 7: Identification Strategy

\[
\theta_i (RD_i(p) - QC_i) \overline{\text{MC}_i(q)}
\]
Percent of Possible Gains Achieved Relative to Not Bidding as a function of the (absolute value of) quantity sold under optimal bidding.

Percent Achieved = \( \frac{\pi^{Actual} - \pi^{Avoid}}{\pi^{EPO} - \pi^{Avoid}} \)
Figure 9

Percent of Possible Gains Achieved Over Sample Period

Percent of Possible Gains Achieved Relative to Not Bidding as a function of the (absolute value of) quantity sold under optimal bidding.
Figure 10

Average Bid Points Used Per Period
For Three Largest Sellers

Avg. Number of Bid Points

- Reliant
- Calpine
- TXU
Figure 11

"Bid-Ask" Spread For Largest Sellers

Difference between INC and DEC bid prices at q=0, excluding hockey stick hours.