Gaussian Noise, Consumer Confusion, and Asymptotic Markups

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Abstract

For a host of issues, it is important to know whether competition lowers markups. Progress on this question has been hampered by analytical intractability. Only special cases have been amenable to analysis. To shed light on the general case, we use results from extreme value theory and characterize markups in the leading model of competition with random utility, the Perloff-Salop (1985) model. We show that markups are asymptotically proportional to $\left(nF'\left[F^{-1}\left(1-n^{-1}\right)\right]\right)^{-1}$, where n is the number of competing firms, and F is the distribution function for preference heterogeneity. This formula implies that for most distributions of noise, the markup decreases very slowly with the number of firms. For the Gaussian case asymptotic markups are proportional to $1/\sqrt{\ln n}$, implying that mark-ups converge slowly in n. Increasing competition in an environment with Gaussian noise will only produce weak pressure on prices. We show the economic origin for this formula: the asymptotic markup is proportional to the expected gap between the highest draw and second highest draw in a sample of n draws.

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1 Introduction

In the Perloff-Salop (1985) model of markups n identical firms pick prices. Then consumers with i.i.d. taste shocks buy from the firm that offers the highest perceived net surplus.[1] Perloff and Salop analyze the symmetric equilibrium and express the equilibrium markup as a function of the number of competing firms, n, and the density function of consumer noise.

In this paper we use the Perloff-Salop framework to characterize the effect of competition on markups. We take the Perloff-Salop model as our starting point because their framework incorporates noise. We believe that noise is a key feature of any market in which some consumers do not know the true value of the products they are buying and therefore make errors when they evaluate those products.

It is already well-known that markups may or may not fall as the number of competing firms rises. For example, when noise is exponentially distributed, markups do *not* depend on n (Perloff and Salop 1985). Similarly, when noise follows the related logit (i.e., Gumbel) distribution, markups asymptote to a non-zero constant (Anderson et al 1992). However, for other cases markups have a familiar negative relationship to n. For example, when noise is uniformly distributed, markups are proportional to $1/n$ (Perloff and Salop 1985).

All three of these illustrative distributions — exponential, logit, and uniform — are appealing for their analytic tractability rather than their realism. The uniform case has no tails while the exponential and logit cases have improbably fat tails. In this paper we apply extreme value theory to develop an asymptotic approximation that can be used to analytically characterize general noise distributions.

We show that markups are asymptotically proportional to $1/\left(nF'\left[F^{-1}\left(1-n^{-1}\right)\right]\right)$, where F is the distribution function for noise. Moreover, we show that this is actually a precise limit pricing result. For most distributions that we study, the Perloff-Salop markup is asymptotically equal to the expected gap between the highest draw and second highest draw in a sample of n draws.

We pay particular attention to the Gaussian case because it is believed to characterize a wide range of real-world distributions. For the Gaussian case we show that asymptotic markups are

[1] The Perloff-Salop framework does not take a position on whether this noise reflects true variation in consumer surplus or just consumer errors in product evaluations. The Perloff-Salop model builds on the random utility framework developed by Luce (1959) and McFadden (1981).

proportional to $1/\sqrt{\ln n}$. This formula implies that mark-ups converge extremely slowly to zero as n rises. Hence, the Gaussian case turns out to behave much more like the exponential and logit cases than like the uniform case. Our analysis implies that rising competition in an environment with a Gaussian noise distribution will only produce weak downward pressure on prices.

The tools that we develop also enable us to characterize markups for several other distributions including two fat-tailed cases — log-normal and power-law — in which mark-ups *increase* as the number of competing firms increase.

Finally, we show that our results do not depend on the unbounded tails of the distributions that we study. Our results are preserved when we truncate these distributions, as long as the truncation point is large.

The rest of this paper formalizes these claims. Section 2 presents the Perloff-Salop model and our extreme value results. Section 3 applies these results to derive markups for seven noise distributions. Section 4 discusses extensions including truncation and a formal statement of our limit pricing result. Section 5 concludes.

2 The main result

In the Perloff-Salop (1985) model identical firms pick prices and consumers with i.i.d. taste shocks choose among the firms. To set up the notation, assume that firm i picks price p_i. Assume that a particular consumer receives net utility $\sigma\varepsilon_i - p_i$ by purchasing the good of firm i, where ε_i is i.i.d. across firms and consumers. Without loss of generality, ε_i has zero mean and unit standard deviation.

In a symmetric-price equilibrium,[2] the demand function of firm i is the probability that the consumer's surplus at firm i, $\sigma\varepsilon_i - p_i$, exceeds the consumer's surplus at all other firms, which charge $p_j = p$,

$$\mathcal{D}(p_i, p) = P\left(\sigma\varepsilon_i - p_i > \max_{j\neq i} \sigma\varepsilon_j - p\right) = P\left(p - p_i > \max_{j\neq i} \sigma\varepsilon_j - \sigma\varepsilon_i\right) \equiv D(p - p_i).$$

[2] If the logarithm of the density of ε is concave, the existence of the equilibrium is ensured by Caplin and Nalebuff (1991), Theorem 2 and Proposition 7. The question of the equilibrium when the density is not concave is an open one. The Technical Appendix to this paper, available on the authors' web page, discusses the existence of the equilibrium for the distributions used is in this paper that are not log-concave, the lognormal distribution and the unbounded power law distributions.

The simplified demand function, D, takes as its argument the average surplus x of firm i relative to its competitors. Firms maximize profit, π_i, by setting their price equal to $\arg\max_{p_i} \pi_i \equiv (p_i - c)\,\mathcal{D}(p_i, p)$, where c is the marginal cost of production. Perloff and Salop (1985) show that this (normed) equilibrium markup is

$$p - c = \mu_n \sigma \tag{1}$$

$$\mu_n = \frac{1}{n(n-1)\int F(x)^{n-2} f(x)^2\, dx}, \tag{2}$$

where n is the number of firms.

To interpret the Perloff-Salop markup equation, call M_{n-1} the largest of $n-1$ noise realizations: $M_{n-1} \equiv \max_{j\in\{1,\dots,n\}, j\neq i} \varepsilon_i$. Then, $D(x) = P\left(\varepsilon_i > \frac{-x+\sigma M_{n-1}}{\sigma}\right)$, so

$$D(x) = E\left[\overline{F}\left(\frac{-x}{\sigma} + M_{n-1}\right)\right], \tag{3}$$

where $\overline{F}(x) = \int_x^\infty f(y)\, dy$ is the countercumulative distribution function. This formulation emphasizes that the demand for good i is driven by the properties of the right-hand tail of the countercumulative distribution function, $\overline{F}$. We can also confirm that the Perloff-Salop markup is $p - c = D(0)/D'(0)$.

The properties of the symmetric equilibrium can be derived from the behavior of $D(x)$ at $x = 0$. Specifically, (3) gives:

Lemma 1 *In a symmetric Bertrand equilibrium,*

$$p - c = \frac{\sigma}{nE[f(M_{n-1})]} \tag{4}$$

where M_{n-1} is a random variable with cumulative density function $P(M_{n-1} \le x) = F(x)^{n-1}$. This is a rewriting of Perloff-Salop (1985)'s formula (1).

Before proceeding with a formal asymptotic approximation to this markup, we first explain the intuition for the asymptotic result. We begin by characterizing the right-hand tail of the noise distribution. Recall that M_{n-1} is the maximum value of $n-1$ draws. First, we observe that $E\left[\overline{F}(M_{n-1})\right] = 1/n$. On average there is a $1/n$ chance of drawing a noise realization that dominates the largest element in a random set of $n-1$ noise realizations. This suggest that if we define[3]

$$A_n \equiv \overline{F}^{-1}(1/n), \tag{5}$$

then M_{n-1} will be close to A_n.

Call S_{n-1} the second-highest draw. $E\left[\overline{F}(S_{n-1})\right] = 2/n$, so it likely that $S_{n-1} \simeq \overline{F}^{-1}(2/n)$. Intuitively, to set its optimum price, a firm conditions on its getting the largest draw, then evaluates the likely draw of the second highest firm, and engages in limit pricing, where it charges a markup equal to the difference between its draw and the next highest draw. This heuristic reasoning (which will be justified later) suggests:

$$\begin{aligned} p-c &\simeq M_{n-1} - S_{n-1} \simeq \overline{F}^{-1}(1/n) - \overline{F}^{-1}(2/n) = \overline{F}^{-1}(1/n) - \overline{F}^{-1}(1/n + 1/n) \\ &\simeq \frac{d\overline{F}^{-1}(x)}{dx}_{|x=1/n} \cdot \frac{1}{n} \text{ by Taylor expansion} \\ &= \frac{1}{nf(A_n)} \end{aligned}$$

So we conjecture $p - c \sim 1/[nf(A_n)]$. Our two main Propositions make this reasoning rigorous. Proposition 2 gives an asymptotic characterization of the Perloff-Salop markup. Proposition 5 gives the rigorous counterpart of the above heuristic reasoning.

The following Proposition shows that this heuristic argument generates the right approximation for the Gaussian, logit (Gumbel), exponential, and lognormal distributions. The Proposition also shows that the approximation remains accurate up to a corrective constant $\Gamma(2+\xi)$ in other cases.

Proposition 2 *In a symmetric Bertrand equilibrium:*

$$p - c \sim \frac{\sigma}{nf(A_n)\Gamma(2+\xi)}, \tag{6}$$

where $A_n = F^{-1}(1-1/n)$, Γ *is the Gamma function, and* $\xi = \lim_{x \to F^{-1}(1)} \left(\overline{F}/f\right)'(x)$ *is the characteristic index of the distribution. Table 1 presents values of* A_n *and* ξ *for many distributions.*

Proof. See Appendix B. ■

[3] We use the usual convention (see Resnick 1987) that $F^{-1}(t) = \inf\{x : F(x) \geq t\}$.

This final proposition yields useful formulae, since the key mathematical objects, A_n, $f(A_n)$, and ξ are easy to calculate for most distributions of interest. Finally, to apply this formula, it is useful to remember $\Gamma(1) = \Gamma(2) = 1$.

3 Noise Distributions and Markups

To analyze the impact of competition on markups, we examine the equilibrium markup for various noise distributions. It is useful to consider seven well-studied analytically tractable distributions.

First, we consider the case in which ε is uniformly distributed between -1 and 1,

$$f_{\text{Uniform}}(\varepsilon) = \frac{1}{2} 1_{|\varepsilon|<1}. \tag{7}$$

This generalizes to a density in $[0, -1]$ that has a power law distribution near 0^-

$$f_{\text{Bounded power law}}(\varepsilon) = \alpha(-\varepsilon)^{\alpha-1}, \tag{8}$$

with $\alpha > 1$. Another paradigmatic example is the Weibull distribution, defined in $(-\infty, 0)$

$$f_{\text{Weibull}}(\varepsilon) = \alpha(-\varepsilon)^{\alpha-1} e^{(-\varepsilon)^{\alpha}}. \tag{9}$$

We also consider the Gaussian density,

$$f_{\text{Gaussian}}(\varepsilon) = \frac{1}{\sqrt{2\pi}} e^{-\varepsilon^2/2}, \tag{10}$$

the Gumbel density (where $\theta \simeq 0.577216$ is Euler's constant), which is also known as the logit density,

$$f_{\text{Gumbel}}(\varepsilon) = \exp\left(-e^{-\varepsilon-\theta} - \varepsilon - \theta\right), \tag{11}$$

the exponential density,

$$f_{\text{Exponential}}(\varepsilon) = e^{-(\varepsilon+1)} 1_{\varepsilon>-1}, \tag{12}$$

the log-normal density,

$$f_{\text{Lognormal}}(\varepsilon) = \frac{1}{(\varepsilon + \sqrt{e})\sqrt{2\pi}} e^{-\ln\left(\varepsilon+\sqrt{e}\right)^2/2} 1_{\varepsilon > -\sqrt{e}}, \tag{13}$$

and the power law[4] density on $[1, \infty)$

$$f_{\text{Power law } \zeta}(\varepsilon) \sim \zeta \varepsilon^{-\zeta-1}. \tag{14}$$

Another type of power law distribution is the Fréchet distribution, defined on $[0, \infty)$

$$f_{\text{Fréchet}}(\varepsilon) = \zeta \varepsilon^{-\zeta-1} e^{-\varepsilon^{-\zeta}}. \tag{15}$$

The shift factors θ, 1, and $\sqrt{e}$ ensure that the mean of ε is 0 in each distribution. The densities are ranked from thinnest to fattest tails.[5]

We calculate the Bertrand outcome for the seven distributions discussed above. Some of our calculations are asymptotic expansions, which hold for large n and small positive t. Table 1 reports values for the key ingredients in our calculations.[6] In this table, f is the density, $\overline{F}(x) \equiv \int_x^\infty f(y)\,dy$ is the countercumulative function, $A_n \equiv \overline{F}^{-1}(1/n)$, $h(t) \equiv f\left(\overline{F}^{-1}(t)\right)$, and ξ is the characteristic index of F (i.e., an index of the fatness of the distribution, see Appendix A). For application of Proposition 2, note that $f(A_n) = h(1/n)$.

Table 1: Distributions and Associated Functions.

[4]From an empirical perspective, we do not know whether the fat-tailed case is relevant. We speculate that it might apply in markets with fat tailed distribution of sales – for instance, the book market. See Chevalier and Goolsbee (2004) and Sornette et al. (2003). Movies (De Vany 2004) also have power law distributions. Power laws generally arise in markets where word of mouth creates snowballing effects (Simon 1955, Gabaix 1999, and the survey in Gabaix and Ioannides 2003).

[5]A density g has weakly fatter tails than a density f if there is a positive constant D such that for all x above a certain threshold $f(x) \leq Dg(x)$.

[6]The proof is a consequence of e.g. Embrechts et al. (1997, p.155-7) and simple calculations.

	$A_n \equiv \overline{F}^{-1}(1/n)$	$h(t) \equiv f\left(\overline{F}^{-1}(t)\right)$	ξ
Uniform	$1-2/n$	$1/2$	-1
Bounded power law	$1-kn^{-1/\alpha}+o\left(n^{-1/\alpha}\right)$	$\sim \alpha k^{-1}t^{1-1/\alpha}$	$-1/\alpha$
Gaussian	$\sim \sqrt{2\ln n}$	$\sim t\sqrt{2\ln \frac{1}{t}}$	0
Logit (Gumbel)	$\sim \ln n$	$\sim t$	0
Exponential	$\ln n - 1$	t	0
Lognormal	$\sim e^{\sqrt{2\ln n}}$	$\sim te^{-\sqrt{2\ln\frac{1}{t}+\frac{1}{2}\ln\left(2\ln\frac{1}{t}\right)}}$	0
Power law	$\sim kn^{1/\zeta}$	$\sim \zeta k^{-1}t^{1+1/\zeta}$	$1/\zeta$

We now show how markups change as competition intensifies. Proposition 3 provides closed form expressions for the markups in different distributional cases for fixed σ and a fixed number of competitors, n.

Proposition 3 *The Bertrand equilibrium generates the following markups. For uniform noise (7),*

$$p-c=\frac{2}{n}\sigma. \tag{16}$$

For bounded power law noise (8) with $\alpha \geq 1$,

$$p-c=\frac{\Gamma(1-1/\alpha+n)}{\alpha\Gamma(2-1/\alpha)\Gamma(1+n)}\sigma \sim \frac{1}{\alpha\Gamma(2-1/\alpha)}n^{-1/\alpha}\sigma. \tag{17}$$

For Weibull noise (9) with $\alpha \geq 1$,

$$p-c=\frac{1}{\alpha\Gamma(2-1/\alpha)}\frac{n^{1-1/\alpha}}{n-1}\sigma \sim \frac{1}{\alpha\Gamma(2-1/\alpha)}n^{-1/\alpha}\sigma \tag{18}$$

For Gaussian noise (10),

$$p-c\sim\frac{1}{\sqrt{2\ln n}}\sigma. \tag{19}$$

For Gumbel noise (11),

$$p-c=\frac{n}{n-1}\sigma. \tag{20}$$

For exponential noise (12),

$$p - c = \sigma. \tag{21}$$

For log-normal noise (13),

$$p - c \sim e^{\sqrt{2 \ln n} - \frac{1}{2} \ln(2 \ln n)} \sigma. \tag{22}$$

For power-law noise (14) with exponent $\zeta > 1$,

$$p - c = \frac{\Gamma\left(1 + 1/\zeta + n\right)}{\zeta \Gamma\left(2 + 1/\zeta\right) \Gamma\left(1 + n\right)} \sigma \sim \frac{k}{\zeta \Gamma\left(2 + 1/\zeta\right)} n^{1/\zeta} \sigma. \tag{23}$$

For Fréchet noise (15) with exponent $\zeta > 1$,

$$p - c = \frac{1}{\zeta \Gamma\left(2 + 1/\zeta\right)} \frac{n^{1+1/\zeta}}{n - 1} \sigma \sim \frac{1}{\zeta \Gamma\left(2 + 1/\zeta\right)} n^{1/\zeta} \sigma \tag{24}$$

The distributions in Proposition 3 are presented in increasing order of fatness of the tails. For the uniform distribution, which has the thinnest tails, the markup is proportional to $1/n$. This is the same equilibrium markup generated by the Cournot model. However the uniform/Cournot case is unrepresentative of the general picture. Proposition 3 implies that markups scale with $n^{-1/\alpha}$. For the distributions reported in Proposition 3 ξ is bounded above by one, so the uniform case is an extreme case.

For the distributions with the fattest tails, the markups paradoxically[7] *rise* as the number of competitors *increases*. Markups rise since the price elasticity *falls* as n gets large. Intuitively, for fat-tailed noise, as n increases, the difference between the best draw and the second best draw, which is proportional to $1/\left[nf\left(A_n\right)\right]$, increases with n. However, even though markups rise with n, profits per firm go to zero since firm prices scale with $n^{1/\zeta}$ but sales per firm are proportional to $1/n$.

Thin-tailed distributions (e.g., uniform) and fat-tailed distributions (e.g., power-laws) are the extreme cases in Proposition 3. Most of the distributional cases imply that competition typically has remarkably *little* impact on markups. For instance with Gaussian noise, the markup, $p - c$, is proportional to $1/\sqrt{\ln n}$, and the elasticity of the markup with respect to n is $-1/\ln n$. So $p - c$ converges to 0, but this convergence proceeds at a glacial pace. Indeed, the elasticity of the markup

[7] See Bénabou and Gertner (1993), Rosenthal (1980), Spector (2002) for perverse competitive effects generated by different microfoundations.

with respect to n converges to zero.

To illustrate the slow convergence, we normalize the markup at $n = 10$ to be 1 and calculate the markup as the number of competitors expands by factors of 10. Table 2 shows that a highly competitive industry with $n = 1,000,000$ firms will retain a third of the markup of a highly concentrated industry with only $n = 10$ competitors. We also compare markups in the Perloff-Salop model to those in the Cournot model, which features markups proportional to $1/n$ and a markup elasticity w.r.t. n of -1.

Table 2: Mark-ups as a function of the number of competitors, n: cases of Gaussian noise and uniform noise (Cournot competition).

n	Markup with Gaussian noise	Markup with Uniform noise
10	1.00	1.00
100	0.61	0.1
$1,000$	0.47	0.01
$10,000$	0.40	0.001
$100,000$	0.35	0.0001
$1,000,000$	0.32	0.00001

We normalize the markup for $n = 10$. We integrate numerically Eq. (4). The asymptotic result (19) provides a good approximation for these exact results.

In cases with moderate fatness, such as the Gumbel (i.e., logit), exponential, and log-normal densities, the markup again shows little (or no) response to changes in n. Finally, the case of *bounded* power law noise (17) shows that an infinite support is not necessary for our results. In this case the markup is proportional to $1/n^{1/\alpha}$ and markup decay is slow for large α. In section 4 we show that *all* of our results can be reformulated for truncated distributions.

In practical terms, these results imply that in markets with noise we should not necessarily expect increased competition to dramatically reduce markups. The mutual fund industry may exemplify such stickiness. Currently 10,000 mutual funds are available in the U.S. and many of these funds offer similar portfolios. Even in a narrow class of homogenous products, such as medium capitalization value stocks or S&P 500 index funds it is normal to find 100 or more competing funds (Hortacsu and Syverson 2004). Despite the large number of competitors in such sub-markets, mutual funds still charge high annual fees, often more than 1% of assets under management. Most interestingly, these fees have not fallen as the number of homogeneous competing funds has increased by a factor of 10 over the past several decades.

The general pattern above is that the markup μ_n behaves as $\mu_n \simeq kn^{\xi}$. To following Proposition makes this precise. We interpret n as a continuous variable in the expression of the markup, Eq. 2.

Proposition 4 *If F is a regular distribution with characteristic index $\xi = \lim_{x \to F^{-1}(1)} \left(\overline{F}/f\right)'(x)$, the asymptotic elasticity of the markup with respect to the number of firms is:*

$$\lim_{n\to\infty} \frac{n}{\mu_n} \frac{d\mu_n}{dn} = \xi. \tag{25}$$

The proof is in Appendix B.

There is a interesting consequence. We can call "distributions with declining right tail" distributions such that $f'(x) \leq 0$ for each large enough, i.e. such that the density of the right tail is weakly declining. As $\left(\overline{F}/f\right)'(x) = -1 - \frac{\overline{F}}{f^2} f'(x) \geq 0$, this implies that if the index exists, then $\xi = \lim_{x \to F^{-1}(1)} \left(\overline{F}/f\right)'(x) \geq -1$. Hence Proposition 4 implies that for distributions with declining right tail, the mark-up μ_n decreases more slowly than $1/n$. This is a sense in which the uniform density case is an extreme case: for distributions with declining right tail, the markup declines (weakly) slowly than for the uniform density.

4 Discussion and Extensions

4.1 The limit pricing interpretation of our asymptotic approximation

Let M_n and S_n represent the highest and second highest draws of n i.i.d. signals. We represent the expected difference between the two as $\delta_n \equiv E\left[M_n - S_n\right]$. The following Proposition characterizes the expected value of this gap and shows that it is equal to the normed markup in the Perloff-Salop model for distributions with $\xi = 0$ (i.e., the Gaussian, Gumbel, exponential, and log-normal distributions).

Proposition 5 *Call $\delta_n \equiv E[M_n - S_n]$ the expected value of the difference between the largest value and the second value of n i.i.d. draws. Then, for large n,*

$$\delta_n \sim \frac{\Gamma(1-\xi)}{nf(A_n)}. \tag{26}$$

In particular, for the Gaussian, Gumbel (i.e., logit), exponential, and log-normal distributions $\delta_n \sim \mu_n$.

This result has an interpretation in auction theory. Consider a second-price auction with n buyers who have independent valuations ε. The winner of the auction has valuation M_n and pays the second price S_n. So his profit is $M_n - S_n$. Hence δ_n is the expected profit of the winner in a second price auction[8].

So the economics of the Perloff-Salop model with Gaussian, Gumbel, exponential, or log-normal distributions asymptotically matches the economics of the second-price auction model. Asymptotically, the two models – Perloff Salop and limit pricing/second price auction — yield isomorphic results.

4.1.1 Two case studies

We now apply the preceding analysis to two examples: the exponential and Gaussian distributions. For the exponential distribution, $\delta_n = 1$ for any $n > 1$ (Reiss 1989, p.36–37). Hence, $p - c = \sigma$, independently of n.

For the Gaussian distribution, we report four statistics: the expected value of the highest draw in a sample of n draws, the expected value of the difference between the highest and second highest draws, our asymptotic approximation to the markup, and the exact Perloff-Salop markup.

[8] Additionally, the proof of Proposition 5 allows us easily to calculate the expected revenue of the seller, $s_n = E[S_n]$, in an auction with n buyers with independent valuation. One finds $s_n \sim \Gamma(2-\xi) A_n$ for $\xi \geq 0$, and $s_n \to F^{-1}(0)$ for $\xi < 0$.

Table 4: The economic foundations of equilibrium markups, in the Gaussian case.

n	$E[M_n]$	$E[M_n - S_n]$	$1/\sqrt{2 \ln n}$	Perloff-Salop Markup
10	1.54	0.54	0.47	0.65
100	2.51	0.36	0.33	0.40
$1,000$	3.24	0.29	0.27	0.31
$10,000$	3.85	0.25	0.23	0.26
$100,000$	4.38	0.22	0.21	0.23
$1,000,000$	4.86	0.20	0.19	0.21

M_n (resp. S_n) is the largest (resp. second largest) value of n i.i.d. draws from a standard Gaussian distribution. We integrate numerically to calculate the statistics in columns 1, 2, and 4 (cf. Eq. 4).

For large n the equilibrium markup approximates the expected gap between the highest and second highest draws. Moreover, even for $n = 1,000,000$, the highest draw remains far from the second highest draw. Hence, as we expect, the markup remains robustly large even in highly competitive markets.

4.2 Truncated distributions

The assumption of unbounded support is *not* necessary for the property that the elasticity of the markup with respect to n may be small. We have already analyzed one case — bounded power law noise (17) — that illustrates this point.

We extend the analysis of truncated distributions in the current section. We develop two additional examples — one numerical and one analytic — that also illustrate weak competitive effects with truncated distributions. Moreover, these examples suggest that the markup approximations for untruncated distributions can also be applied to the case of truncated distributions.

Intuitively, truncation need not matter since our markup calculations in Proposition 3 depend only on the properties of the density in a neighborhood of $A_n \equiv \overline{F}^{-1}(1/n)$. We would expect that the same equilibrium markups will apply to truncated noise distributions as long as n is large *and* the truncation point X is chosen such that $\overline{F}(X) \ll 1/n$.[9]

We first present numerical simulations for markups using the Perloff-Salop model with Gaussian noise and Gaussian noise truncated at $X = 6$ (i.e., truncated six standard deviations into the tails). These calculations confirm the conjecture that truncation does not matter for $n \ll 1/\overline{F}(X)$. This cutoff is approximately $10^9 = 1/\overline{F}(X)$ for $X = 6$.

Table 3: Mark-ups as a function of the number of competitors, n: cases of Gaussian noise and Gaussian noise truncated at $X = 6$.

n	Gaussian noise	Gaussian noise truncated at 6
10	1.00	1.00
100	0.61	0.61
$1,000$	0.47	0.47
$10,000$	0.40	0.40
$100,000$	0.35	0.35
$1,000,000$	0.32	0.32

We integrate numerically Eq. (4).

We complement the numerical analysis for truncated Gaussian noise with analytic results for truncated exponential noise: $f(\varepsilon) = e^{-\varepsilon} 1_{\varepsilon \in [0,X]} / \left(1 - e^{-X}\right)$.[10] Let μ_n represent the normed markup, so $\mu_n \equiv \frac{p-c}{\sigma}$. Explicit calculations show that $\mu_n = \left[1 - e^{-X}\right] / \left[1 + (n-1)\, e^{-X}\right]$. As expected, $\lim_{X \to \infty} \mu_n = 1$, which is the same value as the normed markup for the untruncated exponential distribution. Also, $\mu_n \simeq 1$ for $1 \gg 1/n \gg e^{-X} = \overline{F}(X)$, as expected.

[9] Research papers in physics make frequent use of such "intermediate asymptotics" (e.g., Barenblatt 1996). However, the rigorous mathematics of intermediate asymptotics is underdeveloped.

[10] Recentering the distribution to keep a zero mean does not change the results — the equilibrium markup is preserved under a mean shift of the distribution.

4.3 Implications for consumer surplus

Sometimes the random utility framework is criticized as generating a too high consumer surplus. Indeed, if the distribution is unbounded, the total surplus goes to ∞ as the number of firms increases. Our analytical results allow us to examine this criticism.

Expected surplus is $\sigma E[M_n] = \sigma m_n$, where M_n is the highest of n draws. Intuitively, it is reasonable to expect this to be approximately equal to $A_n \equiv \overline{F}^{-1}(1/n)$. Indeed, the proof of Proposition 5 shows that $m_n = E[M_n] \sim \Gamma(1-\xi) A_n$ for $\xi \geq 0$. The value of A_n for a truncated distribution is bounded above by the value of A_n for the analogous non-truncated distribution. So we study the latter case for simplicity.

For all the distributions that we study except the unbounded power law case, A_n rises only slowly with n. Hence, even for unbounded distributions, and large numbers of producers, consumer surplus is quite small.

For example, for the case of Gaussian noise when consumer preferences have a standard deviation of \$1, $A_n \sim \sqrt{2\ln n}$. So with a million toothpaste producers consumer surplus averages only \$5.25 per tube. Hence, our framework — even with unbounded distributions — does not generate counterfactual predictions about consumer surplus or bizarre predictions about the prices that cartels would hypothetically set.

5 Conclusion

Consumers have noisy product evaluations. We study the effect of competition in such a market.

Using extreme value theory, we characterize markups in the Perloff-Salop (1985) model. We derive an asymptotic approximation for the Perloff-Salop markup and show that for realistic distributions the markup has a natural limit pricing property; the asymptotic Perloff-Salop markup is equal to the expected gap between the highest draw and second highest draw in a sample of n draws, where n is the number of competing firms.

For the Gaussian case asymptotic markups are proportional to $1/\sqrt{\ln n}$, implying that mark-ups converge slowly in n. Increasing competition in an environment with Gaussian noise (even truncated Gaussian noise) will only produce weak pressure on prices.

6 Appendix A: Elements of Extreme Value Theory

Coefficients of Regular Variation We recommend Embrechts *et al.* (1997) and Resnick (1987) for excellent expositions of extreme value theory. The following concept will be important in the proofs.

Definition 6 *A function g defined in a right neighborhood of 0 has regular variation with exponent ρ if*

$$\forall u > 0, \lim_{t \to 0^+} g(ut)/g(t) = u^\rho. \tag{27}$$

This means that, for small $t > 0$, $g(t)$ behaves like t^ρ, perhaps up to a constant or slowly varying function. For instance, (27) holds if $g(t) = t^\rho$ and $g(t) = t^\rho [\ln(1/t)]^\alpha$ for some α. If g is differentiable, then $\rho = \lim_{t \to 0^+} t g'(t)/g(t)$ (Resnick 1987, p.21).

Three Types of Distributions In extreme value theory, there are three classes of distributions. They are ordered by increased fatness of the right tail. A useful indicator of their fatness is the characteristic index $\xi = \lim_{x \to F^{-1}(1)} \left(\overline{F}/f\right)'(x)$. As Table 1 indicates, ξ is an index of the fatness of the distribution. Distributions with fatter tails have a weakly larger ξ.

Type 1 is the "domain of attraction of the Fréchet." It comprises power law tail distributions of the type (14). Their support is unbounded on the right, and $\xi = 1/\zeta > 0$.

Type 2 is the "domain of attraction of the Weibull". It comprises very "thin tailed" distributions of the type (8), such as the uniform distribution. Their support has an upper bound, and $\xi = -1/\alpha < 0$

Type 3 is the "domain of attraction of the Gumbel". It comprises distributions of medium thinness, such as the Gaussian, Gumbel, Exponential, and Gamma distributions. Their support may or may not be unbounded on the right, and $\xi = 0$.

7 Appendix B: Proofs

7.1 The main approximation result

We will prove a more general Proposition than Proposition 2. We characterize the large n behavior of $E[J(M_{n-1})]$, where J is a function, and $M_{n-1} = \max_{i=1\ldots n-1} \varepsilon_i$ is the maximum of $n-1$ random variables with CDF F. The result allows us replace the complicated integral $E\left[J\left(M_{n-1}\right)\right]$ by the much simpler deterministic expression $J\left(A_n\right)\Gamma\left(1+\rho\right)$.

We make the following hypotheses on $j(t) := J\left(\overline{F}^{-1}(t)\right)$, defined for $t \in (0,1)$.

1. j has regular variations with exponent $\rho > 0$.

2. There is a $t_1 > 0$, such that j is non-decreasing, differentiable and positive in $(0, t_1]$.

3. There is a $R \geq 0$ s.t. $\sup_{t\in(0,1)} j(t)(1-t)^R < \infty$.

Hyp. 3 means that if j goes to ∞ at $t \to \infty$, then it does not go faster than some power law.

Proposition 7 *Suppose that J satisfies the three hypotheses above, and M_{n-1} is the maximum of $n-1$ i.i.d. random variables with CDF F. Set $A_n = \overline{F}^{-1}(1/n)$, where $\overline{F}(x) \equiv 1 - F(x)$. Then:*

$$\lim_{n\to\infty} \frac{E\left[J\left(M_{n-1}\right)\right]}{J\left(A_n\right)\Gamma\left(1+\rho\right)} = 1 \tag{28}$$

Proof. We call $m_{n-1} = \min_{i=1\ldots n-1} y_i$ the minimum of $n-1$ i.i.d. uniform $[0,1]$ variables y_i. Its cumulative is:

$$P\left(m_{n-1} > t\right) = P\left(\forall i = 1\ldots n-1, y_i > t\right) = \prod_{i=1}^{n-1} P\left(y_i > t\right) = (1-t)^{n-1} \tag{29}$$

M_{n-1} has the distribution: $P\left(M_{n-1} < X\right) = F(x)^{n-1}$. So M_{n-1} has the same distribution as $\overline{F}^{-1}\left(m_{n-1}\right)$. Indeed:

$$\begin{aligned} P\left(\overline{F}^{-1}\left(m_{n-1}\right) < X\right) &= P\left(m_{n-1} > \overline{F}(x)\right) = \left(1 - \overline{F}(x)\right)^{n-1} \text{ by (29)} \\ &= F(x)^{n-1} = P\left(M_{n-1} < X\right). \end{aligned}$$

Defining $j(t) = J\left(\overline{F}^{-1}(t)\right)$, this yields the representation

$$\begin{aligned} I_n &\equiv E\left[J\left(M_{n-1}\right)\right] = E\left[J\left(\overline{F}^{-1}\left(m_{n-1}\right)\right)\right] = E\left[j\left(m_{n-1}\right)\right] = \int_0^1 j(x)(n-1)(1-x)^{n-2}\,dx \\ &= \int_0^n j\left(\frac{u}{n}\right)\frac{n-1}{n}\left(1-\frac{u}{n}\right)^{n-2} du. \end{aligned} \tag{30}$$

Before proceeding, we give the following *heuristic* argument for why the result is true. As $\left(1-\frac{u}{n}\right)^{n-1} \to e^{-u}$, we can expect to hold the following series of "heuristic asymptotic equalities", signalled by $\simeq$

$$\begin{aligned} I_n &\simeq \int_0^n j\left(\frac{u}{n}\right) e^{-u} du = j\left(\frac{1}{n}\right) \int_0^n \frac{j\left(u/n\right)}{j\left(1/n\right)} e^{-u} du \\ &\simeq j\left(\frac{1}{n}\right) \int_0^n u^\rho e^{-u} du \simeq j\left(\frac{1}{n}\right) \int_0^\infty u^\rho e^{-u} du = j\left(\frac{1}{n}\right) \Gamma\left(1+\rho\right) = J\left(A_n\right) \Gamma\left(1+\rho\right). \end{aligned}$$

We now proceed to a rigorous proof. Let $\eta > 0$, define $K_n = \frac{n}{n-1} \frac{I_n}{j(1/n)}$, and will show that for n large enough, $\left|K_n - \Gamma\left(1+\rho\right)\right| < 4\eta$. This will prove the Proposition. To do this, we give ourselves $0 < \varepsilon < L$, and we start from (30) use the following decomposition:

$$\begin{aligned} K_n &= \frac{n}{n-1} \frac{I_n}{j\left(1/n\right)} = a_n + b_n + c_n + d_n \\ a_n &= \int_0^\varepsilon g_n\left(u\right) du, b_n = \int_\varepsilon^L g_n\left(u\right) du, c_n = \int_L^{\varepsilon n} g_n\left(u\right) du, d_n = \int_{\varepsilon n}^n g_n\left(u\right) du \\ g_n\left(u\right) &= \frac{j\left(u/n\right)}{j\left(1/n\right)} \left(1-\frac{u}{n}\right)^{n-2}. \end{aligned} \tag{31}$$

We study each term in turn. We first observe that, by choosing ε low enough,there is an n_0 s.t. if $n > n_0$, a_n, b_n and c_n are non-negative, in virtue of the positivity part of Hyp.2. We now start by analyzing a_n.

$$\begin{aligned} a_n &= \int_0^\varepsilon \frac{j\left(u/n\right)}{j\left(1/n\right)} \left(1-\frac{u}{n}\right)^{n-2} du \le \int_0^\varepsilon \frac{j\left(\varepsilon/n\right)}{j\left(1/n\right)} du \text{ by Hyp. 2} \\ &= \frac{j\left(\varepsilon/n\right)}{j\left(1/n\right)} \varepsilon \sim \varepsilon^{\rho+1} \end{aligned}$$

If we take ε is small enough so that $\varepsilon^{\rho+1} < \eta/2$, there is an n_1 such that for $n > n_1$,

$$0 \le a_n \le \eta \tag{32}$$

We now turn to b_n .

$$b_n = \int_\varepsilon^L \frac{j\left(u/n\right) u^{-\rho}}{j\left(1/n\right)} u^\rho \left(1-\frac{u}{n}\right)^{n-2} du$$

By Resnick (1987, p.17), Hyp. 1 implies that $\frac{j(u\lambda)}{j(u)} u^{-\rho} \to 1$ locally uniformly for $u \in (0,\infty)$. This means that, for a given η', there is a $\lambda_1 > 0$ such that $\forall \lambda \in (0, \lambda_1), \forall u \in [\varepsilon, L], \left|\frac{j(u\lambda)}{j(\lambda)} u^{-\rho} - 1\right| \leq \eta'$. This implies, for $n > n_2 = 1/\lambda_1$,

$$1 - \eta' < \frac{b_n}{\int_\varepsilon^L u^\rho \left(1 - \frac{u}{n}\right)^{n-2} du} < 1 + \eta'$$

Because $\left(1 - \frac{u}{n}\right)^{n-2} \to e^{-u}$ uniformly in $[\varepsilon, L]$ (Dieudonné, p.127), $\int_\varepsilon^L u^\rho \left(1 - \frac{u}{n}\right)^{n-2} du \to \int_\varepsilon^L u^\rho e^{-u} du$, which implies that for n greater than some n_3,

$$1 - \eta' < \frac{\int_\varepsilon^L u^\rho \left(1 - \frac{u}{n}\right)^{n-2} du}{\int_\varepsilon^L u^\rho e^{-u} du} < 1 + \eta'$$

Also, as $\Gamma(1+\rho) = \int_0^\infty u^\rho e^{-u} du$, if we choose ε small enough and L large enough,

$$1 - \eta' < \frac{\int_\varepsilon^L u^\rho e^{-u} du}{\Gamma(1+\rho)} < 1 + \eta'$$

The last 3 displayed Eqs. imply

$$\left(1 - \eta'\right)^3 < \frac{b_n}{\Gamma(1+\rho)} < \left(1 + \eta'\right)^3$$

If we choose η' small enough, we ensure

$$|b_n - \Gamma(1+\rho)| < \eta \tag{33}$$

We now study c_n. We pick some $\rho' > \rho$ and study $\phi(x) = \ln\left[j(x) x^{-\rho'}\right]$. $x\phi'(x) = xj'(x)/j(x) - \rho'$, and Hyp. 1 ensures that it tends to $\rho - \rho' < 0$ for $x \to 0$. So if we take ε small enough, then for all $x \in (0, \varepsilon]$, $\phi'(x) < 0$. This implies that for $1/n < u/n \leq \varepsilon$, $j(1/n)(1/n)^{-\rho'} > j(u/n)(u/n)^{-\rho'}$, and, for $L > 1$

$$c_n = \int_L^{\varepsilon n} \frac{j(u/n)}{j(1/n)} \left(1 - \frac{u}{n}\right)^{n-2} du \leq \int_L^{\varepsilon n} u^{\rho'} \left(1 - \frac{u}{n}\right)^{n-2} du$$

Also, inequality $1 - x \leq e^{-x}$ implies $\left(1 - \frac{u}{n}\right)^{n-2} \leq e^{-\frac{n-2}{n}u} \leq e^{-u/3}$ for $n \geq n_4 = 3$, which implies

$$c_n \leq \int_L^{\varepsilon n} u^{\rho'} e^{-u/3} du \leq \int_L^\infty u^{\rho'} e^{-u/3} du$$

The right hand side of the last equation goes to 0 as $L \to \infty$. So if L is large enough, it is less than η. We conclude that for $n \geq n_4$,

$$0 \leq c_n < \eta. \tag{34}$$

We finally study d_n. By Hyp. 3, call F s.t.$\left|j(t)(1-t)^R\right| < F$ for $t \in (\varepsilon, 1)$. By Resnick (1987, Proposition 0.8.ii), Hyp. 1 implies that $j(1/n) > (1/n)^{\rho'}$ for some $\rho' > \rho$ and n large enough, so

$$\begin{aligned} |d_n| &= \left|\int_{\varepsilon n}^{n} \frac{j(u/n)}{j(1/n)}\left(1-\frac{u}{n}\right)^{n-2} du\right| \leq \left|\int_{\varepsilon n}^{n} \frac{F}{j(1/n)}\left(1-\frac{u}{n}\right)^{n-2-R} du\right| \leq \frac{F}{j(1/n)}\int_{\varepsilon n}^{n}(1-\varepsilon)^{n-2-R} du \\ &= \frac{F}{j(1/n)} n(1-\varepsilon)^{n-1-R} du \leq F n^{1+\rho'}(1-\varepsilon)^{n-1-R} du \end{aligned}$$

The last expression goes to 0 when $n \to \infty$. So for all n above a certain n_5,

$$|d_n| < \eta. \tag{35}$$

Combining (31)-(35), we conclude that, for $n > \max(n_0, ..., n_5)$,

$$|K_n - \Gamma(1+\rho)| < 4\eta$$

which proves the Proposition. ■

7.2 Proof of Proposition 2

First, we observe that for our regular distributions with characteristic index ξ, the coefficient of regular variation of $f\left(\overline{F}^{-1}(t)\right)$ is $1+\xi$.

Proof. If F is in the domain of attraction of the Fréchet (as in Eq. 14), we first prove the proposition in the case of pure power laws with $k=1$. Write $f(x) = \zeta x^{-\zeta-1}$, and $\overline{F} = x^{-\zeta}$. Then $\overline{F}^{-1}(t) = t^{-1/\zeta}$, and $f\left(\overline{F}^{-1}\right)(t) = \zeta t^{1+1/\zeta}$. Because in this case $\xi = 1/\zeta$, this means $\rho = 1+\xi$. In the general case where F is in the domain of attraction of the Fréchet, we use Proposition 0.8 of Resnick (1987, p.22-23).

If F is in the domain of attraction of the Weibull (as in Eq. 8), the same proof works. We say that the upper bound is 1, and have $f(x) = \alpha(1-x)^{\alpha-1}$, and $\overline{F} = (1-x)^{\alpha}$. Then $\overline{F}^{-1}(t) = 1 - t^{1/\alpha}$, and $f\left(\overline{F}^{-1}\right)(t) = \alpha t^{1-1/\alpha}$. Because in this case $\xi = -1/a$, this means $\rho = 1+\xi$. In the general case where F is in the domain of attraction of the Weibull case, we use Proposition 0.8 of Resnick (1987, p.22-23).

If F is in the domain of attraction of the Gumbel, Resnick (1987, p. 66) shows, with $x = \overline{F}^{-1}(t)$: $t\frac{d}{dt}\ln\left[f\left(\overline{F}^{-1}(t)\right)\right] = -\overline{F}(x) f'(x)/f(x)^2 \to 1$, which implies $\rho = 1$. ■

So applying Proposition 7 to $J(x) = f(x)$ gives: $E[f(M_{n-1})] \sim f(A_n)\Gamma(2+\xi)$. Substituting this into (4) we get (6).

7.3 Proof of Proposition 3

For approximate results we use Eq. (6) in Proposition 2. We also exploit the distributional statistics in Table 1, and $\Gamma(n+k)/\Gamma(n) \sim n^k$ for fixed k and $n \to \infty$, which comes from Stirling's formula.

To obtain exact results we use Eq. (4) in Lemma 1, and identity $\int_0^1 t^{a-1}(1-t)^{b-1}dt = \Gamma(a)\Gamma(b)/\Gamma(a+b)$. For the Weibull and Fréchet distributions, Proposition 1, and the fact that $M_n =^d n^{\xi}\varepsilon$ (Embrechts et al. 1997, p.124) offers a nice way to simplify the calculations.

7.4 Proof of Proposition 4

Differentiating $\ln \mu_n^{-1}$ yields:

$$-\frac{d\ln\mu_n}{d\ln n} = 1+\frac{n}{n-1}+\frac{(n-1)n\int F(x)^{n-2} f(x)^2 \ln F(x)\,dx}{(n-1)\int F(x)^{n-2} f(x)^2\,dx} = \frac{2n-1}{n-1}+\frac{nE[f(M_{n-1})\ln F(M_{n-1})]}{E[f(M_{n-1})]}.$$

The proof of Proposition 2 established that the tail exponent of $f\left(\overline{F}^{-1}(t)\right)$ is $1+\xi$. The same proof shows that the tail exponent of $f\left(\overline{F}^{-1}(t)\right)\ln F\left(\overline{F}^{-1}(t)\right)$ is $2+\xi$. Using Proposition 7, we get:

$$\frac{nE[f(M_{n-1})\ln F(M_{n-1})]}{E[f(M_{n-1})]} \sim \frac{n\Gamma(3+\xi) f(A_n)\ln F(A_n)}{\Gamma(2+\xi) f(A_n)} = \frac{n\Gamma(2+\xi)(2+\xi)\ln(1-1/n)}{\Gamma(2+\xi)} \sim -(2+\xi)$$

so we conclude that $\lim_{n\to\infty} -\frac{d\ln\mu_n}{d\ln n} = 2-(2+\xi) = -\xi$.

7.5 Proof of Proposition 5

We use the asymptotic results from Reiss (1989, p.160-161), which details how the largest and second largest order statistics, appropriately normalized, converge to universal distributions $G_{i,\beta,1}$ and $G_{i,\beta,2}$. Type $i = 1,2,3$ correspond respectively to the domain of attraction of the Fréchet, Weibull and Gumbel, i.e. $\xi > 0$, $\xi < 0$ and $\xi = 0$. As reported by Reiss (1989, p.160-161), with $\beta = \zeta$ in the Fréchet case, and $\beta = -\alpha$ in the Weibull case, and $\beta = \infty$ in the Gumbel case,

$$\begin{aligned} P\left(b_n^{-1}\left(M_n - a_n\right) \leq x\right) &\rightarrow G_{i,\beta,1}(x) \\ P\left(b_n^{-1}\left(S_n - a_n\right) \leq x\right) &\rightarrow G_{i,\beta,2}(x) \end{aligned}$$

the cumulative distribution functions $G_{i,\beta,k}$ are given by: $G_{1,\beta,k}(x) = \exp\left(-x^{-\beta}\right)\sum_{j=0}^{k-1}\frac{x^{-j\beta}}{j!}, x > 0$; $G_{2,\beta,k}(x) = \exp\left(-(-x)^{-\beta}\right)\sum_{j=0}^{k-1}\frac{(-x)^{-j\beta}}{j!}, x < 0$; $G_{3,\infty,k}(x) = \exp\left(-e^{-x}\right)\sum_{j=0}^{k-1}\frac{e^{-jx}}{j!}, -\infty < x < \infty$. The centering constants are: For the Fréchet, $a_n = 0$, $b_n = A_n \sim kn^{\xi}$, for Weibull: $a_n = \inf\{x, F(x) = 1\}, b_n = a_n - \overline{F}^{-1}(1/n) \sim kn^{\xi}$, and for the Gumbel: $a_n = A_n, b_n = 1/(nf(A_n))$. Using the natural limits for the 3 types, $(U_i, V_i) = (0,\infty), (-\infty, 0), (-\infty,\infty)$ respectively, this implies:

$$\begin{aligned} b_n^{-1}\delta_n &= E\left[\left(b_n^{-1}\left(M_n - a_n\right)\right) - b_n^{-1}\left(S_n - a_n\right)\right] \\ &\rightarrow \int_{U_i}^{V_i} x\left(G'_{i,\beta,1} - G'_{i,\beta,2}\right)dx = \left[x\left(G_{i,\beta,1} - G_{i,\beta,2}\right)\right]_{U_i}^{V_i} - \int_{U_i}^{V_i}\left(G_{i,\beta,1} - G_{i,\beta,2}\right)dx \\ &= -\int_{U_i}^{V_i}\left(G_{i,\beta,1} - G_{i,\beta,2}\right)dx = I_i \end{aligned}$$

where

$$\begin{aligned} I_1 &= \int_0^{\infty}\exp\left(-x^{-\beta}\right)x^{-\beta}dx = \frac{1}{\beta}\Gamma\left(1 - \frac{1}{\beta}\right) \text{ by change of variable } y = x^{-\beta} \\ I_2 &= \frac{-1}{\beta}\Gamma\left(1 - \frac{1}{\beta}\right) \text{ likewise} \\ I_3 &= \int_{-\infty}^{\infty}\exp\left(-e^{-x}\right)e^{-x}dx = \left[\exp\left(-e^{-x}\right)\right]_{-\infty}^{\infty} = 1. \end{aligned}$$

□

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