Do new competitors, new customers, new suppliers,... sustain, destroy or create competitive advantage?

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PRELIMINARY
1 Introduction

Many of the subjects of interest in strategy and industrial organization involve additions to the players in a game. Some examples: (i) a firm entering an industry with a new substitute or complementary product, or a new technology, or simply more capacity; (ii) a firm developing the capability to imitate an incumbent’s activities; (iii) a new customer or segment changing demand for some product; (iv) an entrepreneurial venture altering the game incumbents are playing; (v) a spin-off or divestiture; (vi) a new source of supply for inputs; (vii) the transition from short run to long run in a competitive market; (viii) a patent expiring and allowing other to produce, etc.

This paper explores how increasing the number of players in a game affects the equilibrium payoffs of existing players. Specifically, we study a general coalitional game (within which any of the examples just mentioned can be described), and ask how adding a player changes an existing player’s minimum equilibrium payoff. We focus primarily on whether that minimum payoff is zero, or more generally, equal to the player’s next best alternative to being in the game. There is nothing about our methodology that requires this focus on the minimum and whether it is zero; indeed, the same sort of reasoning can be applied to whether the minimum takes on some other value, or whether the maximum does so, etc. Instead, as we argued in earlier work (MacDonald and Ryall, 2004b), whether the minimum is positive or is a question of special interest since it describes whether the forces competition alone suffice to guarantee supra-normal profit, an outcome we equate with the familiar term competitive advantage.

Our results focus on a comparison of a firm’s (generally, any player’s) pre- and post-entry competitive advantage. In our earlier work (MacDonald and Ryall (2004a) and (2004b)), we provided a complete characterization of a player’s having a competitive advantage in a general coalitional game, i.e., a result of the form: a firm’s minimum equilibrium payoff is positive if and only if condition $X$ is satisfied. This result, which we review below, necessarily forms the basis for all the results in this paper since it applies to both the pre- and post-
entry games. Thus, our new results are of the form: a firm has competitive advantage pre-entry, but not post entry, if and only if condition $X$ is satisfied in the pre-entry game, but not in the post-entry game. The result therefore provides a complete description of the features of the pre-and post-entry games that lead to entry destroying competitive advantage. The features of the game that lead to the firm not having competitive advantage pre-entry, but having competitive advantage post-entry, can be described in an analogous fashion, i.e., entry creates competitive advantage. Entry sustaining competitive advantage is defined and characterized similarly. Altogether, our first result, Proposition 1, provides a complete description of the features of the pre- and post-entry games that result in entry destroying, creating, or sustaining competitive advantage.

(The remaining case, in which the firm lacks competitive advantage pre- and post-entry, is described by “none of the above conditions”.)

The basic result from our earlier work, Theorem 1, below, shows that there are two fundamental, opposing, forces that shape how value must be distributed in equilibrium: (i) there is only so much value to distribute; and (ii) players have alternatives they can act on, and in equilibrium, they must prefer the equilibrium outcome to any available alternative. The more value there is to distribute, the more feasible ways there are to distribute it in a way that would dominate players’ alternatives. That is, more value to distribute widens the range of payoffs a player can earn in equilibrium, including lowering the minimum. On the other hand, any improvement in players’ alternatives further constrains how value can be distributed while dominating players’ alternatives. This narrows the range of payoffs a player can earn in equilibrium.\footnote{In what follows we will refer to the impact of agents’ alternatives as “competition”. That is, a player’s having alternatives means there are competing activities in which he can engage.}

The application of Theorem 1 to entry’s impact on competitive advantage, Proposition 1, describes how these forces operate to create, destroy, or sustain competitive advantage when entry, in particular, changes the game. That is, the entrant generally brings additional value to the economic activities in which players ultimately engage. This tends to widen the range of possible equilibrium
payoffs for players, including lowering the minimum payoff. But the entrant also creates new alternatives in which players might engage, which has the opposite effect, in particular, increasing the minimum payoff. Whether entry creates, destroys or sustains competitive advantage depends on the interplay of these effects. Corollaries 1-4 provide further perspective on entry’s impact on competitive advantage by stating the conditions in terms of the entrant’s “added value.”

Entry’s impact on players’ alternatives comes in three varieties. First, pre-entry, the aggregate value that all players could create without the entrant is exactly the value that can be distributed in the pre-entry game. Post-entry, obtaining this value is simply one of the alternatives that the players other than the entrant might act on. Second, entry opens up a specific alternative for each player, i.e., to create value by interacting with just the entrant. Third, entry creates new opportunities for groups of players, i.e., to create value by interacting with the entrant. Each of these three new opportunities has different effects. Our second set of results, Propositions ??-4, provides further insight into how entry impacts competitive advantage by exploring the operation of these three effects one at a time.

Imitation is a specific form of entry, one much discussed in strategy. Indeed, if there is one idea that is not controversial, it is that to enjoy a sustained performance advantage, a firm must possess value-producing resources that are difficult to imitate. For example, Saloner et. al. (2001, page 49) say, “If a firm’s competitive advantage is based on its capabilities, a sustainable advantage requires either that imitation is difficult or that the firm can improve its capabilities (learn) before its rivals catch up.” The intuition is as follows. Imitation permits a competitor to provide new, equally attractive alternatives to customers. In order to persuade those customers not to act on those alternatives, the incumbent firm must allow the customers to keep a greater share of the gains from trading with it than they had previously enjoyed. As a result, its payoff is diminished and possibly eliminated altogether, i.e., entry destroyed competitive advantage. In MacDonald and Ryall (2004b), we formalized and
explored two versions of this idea, and found that the impact of imitation varies greatly with what one has in mind by “ability to imitate.” One version is what we called capability imitation: the imitator can do anything the incumbent can do, viz., a clone of the incumbent. More precisely, the entrant is a capability imitator if in every economic interaction involving the firm, replacing the firm with the imitator results in the same value created. We showed that (i) capability imitation might or might not eliminate competitive advantage, and (ii) entry forces both the firm and the entrant to earn the same range of payoffs (not necessarily zero or the same payoff). Our second version of imitation, unlimited product imitation describes the situation in which the firm and its imitator can supply an identical product at the same constant marginal cost, i.e., no capacity constraints, no diminishing returns to scale, no limited managerial talent,... Unlimited product imitation invariably both eliminates competitive advantage and forces both the firm and entrant to earn zero payoff. Even in the specific case of imitation, the impact of an entrant on an incumbent firm depends greatly on the way entry influences the total value that player’s might appropriate, and the alternatives available to players.

The use of coalitional game theory to study issues in strategy is growing rapidly. The coalitional approach to strategy was first suggested by Brandenburger and Stuart (1996) who, in particular, propose the usefulness of the “value-added” concept in analyzing business strategy; also see Brandenburger and Stuart (2004) and MacDonald and Ryall (2004b). Lippman and Rumelt (2003) discuss the benefits of coalitional game theory for strategy research and compare various solution concepts traditionally used to solve coalitional games. Gans, MacDonald and Ryall (2005) demonstrate how the coalitional framework can be used in practice and discuss the interpretation of the mathematical primitives of the model. Adner and Zemsky (2006) use value-added concepts to analyze the sustainability of competitive advantage. De Fontenay and Gans (2004) use coalitional game theory to analyze the strategic implications of outsourcing.

We begin with examples illustrating the basics of the methodology and a some of our results, then develop the formal notation, review the basic charac-
terization result, and discuss the new results.

2 Examples

We begin with examples that illustrate some of the mechanics we employ, and give the flavor of our general results.

The pre-entry game includes a firm, \( f \), and two buyers. The firm has one unit of production capacity, and can costlessly produce either one of two goods, which we will think of as components of a system of some sort. For example, the goods might be a video device and a computer. Buyer 1 has some use for one of the goods, valuing it at $10, but also values the whole system, at $30. Buyer 2 has no use for either good on its own, but also values the system at $30.

The conditions describing competitive appropriations, \( \pi_f, \pi_1 \) and \( \pi_2 \) (\( \pi_i \) is buyer \( i \)'s appropriation) are

\[
\pi_f \geq 0, \pi_1 \geq 0, \pi_2 \geq 0,
\]

\[
\pi_f + \pi_1 \geq 10, \pi_f + \pi_2 \geq 0, \pi_1 + \pi_2 \geq 0,
\]

and

\[
\pi_f + \pi_1 + \pi_2 = 10.
\]

(Since there is just one unit of capacity, the $30 value of the system cannot be achieved.) Observe that buyer 2’s added value (i.e., the aggregate value the whole group can generate, less the value \( f \) and buyer 1 can generate on their own) is zero. Thus, since a player’s appropriation is bounded above by his/her added value, \( \pi_2 = 0 \). The conditions describing competitive appropriations simplify to:

\[
\pi_f \geq 0, \pi_1 \geq 0,
\]

\[
\pi_f + \pi_1 \geq 10.
\]
and

$$\pi_f + \pi_1 = 10,$$

from which it follows immediately that $$0 \leq \pi_f \leq 10$$ and $$\pi_1 = 10 - \pi_f$$. In particular, $$f$$ does not have competitive advantage since $$\pi_f = 0$$ is possible. The examples that follow thus illustrate whether entry creates competitive advantage.

### 2.1 Example of Proposition 2

Now assume an entrant, $$e$$, joins the game, and that $$e$$ has precisely the same capabilities as $$f$$ in the sense that buyers do not care whether they purchase from $$f$$ or $$e$$. However, in this example, we assume that compatibility issues mean that a system must be purchased from one firm, i.e., a buyer could purchase one unit from each firm, but buyer 1 would only achieve value of $10 by doing so, and buyer 2 would continue achieve $0. The conditions describing competitive appropriations, $$\pi_f, \pi_e, \pi_1$$ and $$\pi_2$$ are

$$\pi_f \geq 0, \pi_e \geq 0, \pi_1 \geq 0, \pi_2 \geq 0,$$

$$\pi_e + \pi_f \geq 0, \pi_f + \pi_1 \geq 10, \pi_f + \pi_2 \geq 0, \pi_e + \pi_1 \geq 10, \pi_e + \pi_2 \geq 0, \pi_1 + \pi_2 \geq 0,$$

$$\pi_f + \pi_e + \pi_1 \geq 10, \pi_f + \pi_e + \pi_2 \geq 0, \pi_f + \pi_1 + \pi_2 \geq 10, \pi_e + \pi_1 + \pi_2 \geq 10,$$

and

$$\pi_f + \pi_e + \pi_1 + \pi_2 = 10.$$

In this example, the incompatibility of firms’ products means buyer 2 has a zero added value. However, the fact that buyer 1 now has two equally good sources of one unit means that both the firm and entrant also have a zero added value. Thus, only buyer 1 can have strictly positive appropriation, i.e., $$\pi_2 = 10$$.

In this example entry created no extra value, since entry brought more capacity, but, as a result of the incompatibility, not in a way that allowed any
more value to be achieved. Moreover, entry did not create any value alternative for the \( f \) and \( e \) pair alone, since buyer 1 is needed to create value. And further, as a result of the incompatibility, entry did not open up any new alternatives for groups of players including \( f \), e.g. by including \( e \), the group of and either buyer can create the same value the could without \( e \). The only new alternatives created by \( e \)'s entry involve groups not including \( f \), i.e., buyer 1, or buyer 1 and buyer 2. Consistent with Proposition 2, entry under these circumstances can never create competitive advantage for \( f \), and, indeed, forces \( f \)'s appropriation to zero.

### 2.2 Example of Proposition 3

In this example we continue to assume \( e \)'s product is not compatible with \( f \)'s, but in a less extreme manner. That is, buyer two continues to value the incompatible products at $0, but buyer 1 can achieve $11 in value from both goods, and $10 from just one. We also suppose that the system, but neither good individually, has some alternative use outside the game, valued at $2. The conditions describing competitive appropriations become

\[
\begin{align*}
\pi_f &\geq 0, \pi_e \geq 0, \pi_1 \geq 0, \pi_2 \geq 0, \\
\pi_e + \pi_f &\geq 2, \pi_f + \pi_1 \geq 10, \pi_f + \pi_2 \geq 0, \pi_e + \pi_1 \geq 10, \pi_e + \pi_2 \geq 0, \pi_1 + \pi_2 \geq 0, \\
\pi_f + \pi_e + \pi_1 \geq 11, \pi_f + \pi_e + \pi_2 \geq 0^2, \pi_f + \pi_1 + \pi_2 \geq 10, \pi_e + \pi_1 + \pi_2 \geq 10,
\end{align*}
\]

and

\[
\pi_f + \pi_e + \pi_1 + \pi_2 = 11.
\]

Buyer 2's added value continues to be zero, and thus \( \pi_2 = 0 \). Suppose \( \pi_f = 0 \), i.e., entry does not create competitive advantage. Then, if the outside alternative available to \( f \) and \( e \) is to be unattractive, we must have \( \pi_e \geq 2 \). But this leaves at most $9 for buyer 1, making the option of simply buying one unit from
$f$ quite attractive. In fact, in this example, the unique competitive appropriations are $\pi_f = \pi_e = 1$, $\pi_1 = 9$ and $\pi_2 = 0$. That is, $f$ and $e$ must appropriate at least $\$2$ between them, leaving at most $\$9$ for buyer 1. But each of $f$ and buyer 1, and $f$ and buyer 2, can appropriate $\$10$ between them. $f$ and $e$ receiving exactly $\$1$ is the only way to divide $\$11$ in the required fashion.

In this example, entry allowed more value to be achieved, but also opened up a new opportunity for $e$ and $f$, i.e., acting on the outside option. This created competitive advantage where none existed pre-entry.

2.3 Example of Proposition 4

Finally, we assume that entry does not yield a new outside use for the system, but that the good are fully compatible. The conditions describing competitive appropriations are

\[
\pi_f \geq 0, \pi_e \geq 0, \pi_1 \geq 0, \pi_2 \geq 0,
\]

\[
\pi_e + \pi_f \geq 0, \pi_f + \pi_1 \geq 10, \pi_f + \pi_2 \geq 0, \pi_e + \pi_1 \geq 10, \pi_e + \pi_2 \geq 0, \pi_1 + \pi_2 \geq 0,
\]

\[
\pi_f + \pi_e + \pi_1 \geq 30, \pi_f + \pi_e + \pi_2 \geq 30, \pi_f + \pi_1 + \pi_2 \geq 10, \pi_e + \pi_1 + \pi_2 \geq 10,
\]

and

\[
\pi_f + \pi_e + \pi_1 + \pi_2 = 30.
\]

Post-entry, since either buyer values the whole system, both have zero added value, and so $\pi_1 = \pi_2 = 0$. Conditions describing competitive appropriations simplify to

\[
\pi_f \geq 10, \pi_e \geq 10,
\]

\[
\pi_f + \pi_e \geq 30,
\]
\[ \pi_f + \pi_e = 30, \]
i.e., \( 10 \leq \pi_f \leq 20 \) and \( \pi_e = 30 - \pi_f \).

Pre-entry, \( f \)'s appropriation lies between 0 and $10$; post-entry, \( f \)'s appropriation lies between $10$ and $20$, i.e., entry created competitive advantage. The intuition is as follows. Pre-entry, there is not enough capacity to produce anything buyer 2 values, so \( f \) and buyer 1 are effectively in a pure bargaining situation. Thus, whatever \( f \) appropriates is not a consequence of competition, and \( f \) does not have competitive advantage. Post entry, there is sufficient capacity to produce the system, which both buyers value at $30$. Specifically, \( f \) and \( e \) each produce to capacity, and one buyer purchases a system. Buyers have become highly effective competitors – in particular, each having zero added value – and so the $30$ must be shared between \( f \) and \( e \). The sole constraint is that this sharing is such that neither \( f \) nor \( e \) could improve by selling one good to buyer 1 instead of being part of formation of a system.

In this example, as is true in general, an entrant typically both allows more value to be created (i.e., $30$ versus $10$) and opens up new alternatives \( (f \) and \( e \) together have capacity to create $30$ total with \( e \)ther buyer). The former tends to lower \( f \)'s minimum equilibrium payoff, whereas the latter tends to increase it. In this example, the latter dominates: entry creates competitive advantage.

### 3 Preliminaries

#### 3.1 Notation and assumptions

Our goal is to study the value appropriation possibilities for an agent, for example, an incumbent firm, before and after the entry of another agent. The entering agent could be an imitating firm, but there are lots of other possibilities – a new supplier, a new customer, a firm offering a complementary product,... The coalitional game framework we employ is general enough to accommodate any kind
of entry. The simplest and most transparent way to carry out the analysis involves specifying the post-entry game in detail, and then comparing pre-and post-entry appropriation possibilities for the agent of interest by treating the pre-entry case as the special one in which (in a way that will be made precise shortly) the entering agent can be “ignored”. To that end, we assume the game has \(n+1\) agents, indexed by \(i; i = 1, ..., n + 1 < \infty\). Given the players, the balance of the specification of any coalitional game involves a description of the value that is expected ultimately to be created by the agents – the aggregate value that will be appropriated by the agents in the game – as well as the value of the alternatives that are available to subgroups of the agents. The idea is that the value any group of agents will ultimately appropriate must exceed or equal what that same group could obtain by acting on alternatives available to it. Otherwise, why would that group not act on its alternative instead? We begin by describing all the possible groups of agents in a way that facilitates analysis of the pre- and post-entry games. Then, we specify the value that will ultimately be distributed and the value subgroups of agents can obtain from available alternatives.

We employ \((n + 1)\)-length vectors of zeros and ones to describe the groups, where a vector with a 1 in the \(i^{th}\) position means the group includes agent \(i\); a “generic” vector will be labelled \(g\), and entities associated with \(g\) will have a \(g\)-subscript. It will be clear from the context whether \(g\) is a row or a column. The agent whose appropriation we will focus on will be the first in any such vector, and the entrant the last. For concreteness, we will call the first agent the “incumbent firm,” \(f\), and the last agent the “entrant,” \(e\).

The analysis of the pre- and post-entry games is facilitated by organizing all the possible groups of agents in a particular way. The group of all agents is

\[ I \equiv (1, 1, ..., 1). \]

\(^3\)The basics of this framework are explained at length in MacDonald and Ryall (2004); also see Brandenburger and Stuart (2003).
The group including only $f$ is

$$I_f \equiv (1, 0, ..., 0),$$

whereas the group of all agents other than $f$ is

$$I_{-f} \equiv (0, 1, ..., 1).$$

Analogously, the group including only $e$ is

$$I_e \equiv (0, ..., 0, 1),$$

and the group of all agents other than $e$ is

$$I_{-e} \equiv (1, ..., 1, 0).$$

The group including only $f$ and $e$ is

$$I_{fe} \equiv (1, 0, ..., 0, 1),$$

and the group including all agents other than $f$ and $e$ is

$$I_{-fe} \equiv (0, 1, ..., 1, 0).$$

This leaves $2^{n+1} - 8$ groups that include at least one, but not every, agent other than $f$ and $e$, and possibly one or both of $f$ and $e$. These groups can be divided into four subsets each consisting of $2^{n-1} - 2$ groups. The first subset is the collection of groups that include $f$ but not $e$. This collection can be described by the $(2^{n-1} - 2) \times (n + 1)$ matrix

$$[1G0],$$

where $1$ ($0$) is a column vector of ones (zeros) of length $(2^{n-1} - 2)$ and $G$ is the $(2^{n-1} - 2) \times (n - 1)$ matrix in which no two rows are the same and no row is either all zeros or all ones. Similarly, the collections of groups that include neither $f$ nor $e$ is described by

$$[0G0];$$
those including $e$ but not $f$ are

$[0G1]$; \hspace{1cm} (3)

and those including both $e$ and $f$ are

$[1G1]$. \hspace{1cm} (4)

Any of the groups can produce value “on its own” in the sense that agents in the group can engage in whatever transactions are technically and institutionally feasible. We denote the value that can be generated by group $g$ as $v_g$. Hence, the aggregate value anticipated in the post-entry industry is $v_I$; the value available to agent $i$ acting alone is $v_{I_i}$; and so on. We assume, without loss of generality, that $v_{I_i} = 0$ (a normalization).

There are $2^{n+1} - 1$ distinct and nonempty groups of players that can be formed from $n + 1$ players. Let $v$ be the $(2^{n+1} - 1)$-vector of nonnegative numbers in which each component corresponds to a nonempty group; i.e., $v$ is a vector including the $v_g$ values, for all $g$. Consistent with the group-identification scheme introduced above, $v_f$ is the value that can be produced by the firm on its own, $v_{-f}$ the value that can be produced by all the agents without the firm, and so on. Similarly $v_{10}$ is the $(2^{n-1} - 2)$-length vector that includes the components of $v$ corresponding to the rows of (1); $v_{00}$ is the vector that includes the components of $v$ corresponding to the rows of (2); etc.

Note that $v$ includes all information on the value creation opportunities available to the agents in both the pre- and post-entry games. That is, $v_I$ is the aggregate value that will be produced in the industry post-entry. The other components of $v$ (including $v_{1_{-e}}$) are the values that any subgroup $g$ could obtain on its own in the post-entry game. Likewise $v_{1_{-e}}$ is the aggregate value that will be produced in the industry pre-entry; i.e., $e$ is not “active” in the pre-entry game. Other components of $v$, corresponding to groups $g$ that do not include $e$, are the values that these groups could obtain on their own in the pre-entry game.

Finally, to avoid trivial cases, we assume that any group $g$ is no less produc-
tive when either $f$ or $e$ is included: for all groups $g$ not including $f$

\[ v_{g+f} \geq v_g, \]  

(5)

and for all groups $g$ not including $e$

\[ v_{g+e} \geq v_g. \]  

(6)

### 3.2 Competitive distributions and appropriation

Whatever economic activity ultimately transpires – either pre- or post-entry – the resulting value will be appropriated by the participating agents. Let $\pi$ be an $(n + 1)$-vector describing each agent’s appropriation; $\pi$ is called a distribution of value. When required, we will refer to $f$’s ($e$’s) appropriation as $\pi_f$ ($\pi_e$). A distribution of value, $\pi$, is competitive in the pre-entry game if:

\[ \mathbf{I} \cdot \pi \leq v_{1-e}, \]  

(7)

and,

\[ \mathbf{I}_f \cdot \pi \geq 0, \]  

(8)

\[ \mathbf{I}_{-e} \cdot \pi \geq v_{1-e}, \]  

(9)

\[ \mathbf{I}_{-fe} \cdot \pi \geq v_{1-fe}, \]  

(10)

\[ [100] \pi \geq v_{10}, \]  

(11)

\[ [000] \pi \geq v_{00}. \]  

(12)

Condition (7) requires that only the value agents can produce without the entrant, $v_{1-e}$, be distributed among the other agents. The other conditions, (8) – (12), are necessary if, given $\pi$, agents are assumed to participate voluntarily in producing $v_{1-e}$. The logic behind this is that, if any of (8) – (12) do not hold, there is some group, $g$, not including $e$, for which $g \cdot \pi < v_g$. That is, the agents in $g$ could all be made strictly better off by not participating in the contemplated transactions (i.e., whatever produces value $v_{1-e}$) and, instead, producing $v_g$ via some other activity. (8) – (12) is a complete description of the competitive opportunities and alternatives players have that do not involve $e$. 

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For what follows, it is useful to recall a some basics about competitive distributions in coalitional games. One is that for any agent, say \( f \), the levels of appropriation for that agent that are consistent with competition (i.e., \( \pi \) is a competitive distribution) form a closed interval, i.e., a lowest possible value, a highest possible value, and everything between. We denote this interval by \([\pi_f^{\min}, \pi_f^{\max}]\). Generally, \( \pi_f^{\min} < \pi_f^{\max} \), i.e., competition does not normally determine the firm’s appropriation uniquely. To determine the firm’s appropriation when \( \pi_f^{\min} < \pi_f^{\max} \), additional assumptions are required regarding the resolution of super-competitive factors.\(^4\) Second, if there is more value to distribute – e.g., \( v_{L-e} \) is larger – the interval of appropriation for \( f \) consistent with competition cannot shrink, and generally grows. Third, if the alternatives available to agents improve – e.g., any of the components of \( v_{L-e}, v_{L-f_e}, v_{10} \) or \( v_{00} \) increase – the interval of appropriation for \( f \) consistent with competition cannot grow, and generally shrinks. Finally, and more specific to the pre- and post-entry setup studied here, (7) and (9) imply that exactly \( v_{L-e} \) is distributed among the agents in the pre-entry industry, and that \( \pi_e = 0 \). (If \( \pi_e > 0 \), (7) requires that strictly less than \( v_{L-e} \) be distributed among the agents in the pre-entry game, violating (9).)\(^5\)

A distribution of value, \( \pi \), is competitive in the post-entry game if (7) is replaced by

\[
\mathbf{I} \cdot \pi \leq v_1,
\]

\(\text{Equation } 13\)

\(^4\)See Brandenburger and Stuart (2004) for a very intuitive and applicable approach. Since we are concerned only with the effects of competition on the value appropriated by industry participants, we leave open the question of how extra-competitive issues are resolved.

\(^5\)The last conclusion is straightforward, but important in the sense that it would be hard to interpret (7)-(12) as describing a competitive situation where the entrant plays no role if \( \pi_e > 0 \) were possible.
and, in addition to (8) – (12), \( \pi \) satisfies

\[
\begin{align*}
I_e \cdot \pi & \geq 0, \tag{14} \\
I \cdot \pi & \geq v_I, \tag{15} \\
I_{-f} \cdot \pi & \geq v_{I_{-f}}, \tag{16} \\
I_{f_e} \cdot \pi & \geq v_{I_{f_e}}, \tag{17} \\
[1G1] \pi & \geq v_{11}, \tag{18} \\
[0G1] \pi & \geq v_{01}. \tag{19}
\end{align*}
\]

Condition (13) replacing (7) accounts for the fact that post-entry, \( v_I \) is available for distribution instead of \( v_{I_{-e}} \). Including (14) – (19) takes account of the fact that the entrant opens up new alternatives for players, while removing none, and that if a distribution, \( \pi \), is to survive competition, these new alternatives must not offer any group the prospect of improvement over what \( \pi \) offers.\(^6\)

Let \( [\hat{\pi}^\text{min}_f, \hat{\pi}^\text{max}_f] \) be the range of appropriation for \( f \) that is consistent with competition post entry.

That (13) replaces (7), and (14)-(19) are in addition to (8) – (12), foreshadows the two general effects that an entrant has on an incumbent’s appropriation possibilities. First, since (6) implies \( v_I \geq v_{I_{-e}} \), there is no less, and generally more, value to distribute post entry. This, by itself, tends to \textit{expand} the range of appropriation for \( f \) that is consistent with competition. On the other hand, post-entry, agents have many alternatives – i.e., those involving \( e \) – that were unavailable pre-entry; this, by itself, has the same sort of impact as improving agents’ alternatives pre-entry, i.e., it \textit{narrow}s the range of appropriation for \( f \) that is consistent with competition. The net effect of \( e \)’s presence on \( f \) reflects the relative strength of these two forces; our results explain how this relative strength is determined.

\(^6\)Notice that (13) and (15) imply \( I \cdot \pi = v_I \). We assume that there is at least one competitive distribution \( \pi \) in each of the pre- and post-entry industries.
4 Competitive advantage

If every competitive distribution has \( f \) appropriating more than its next-best alternative (here normalized to 0), then \( f \) is guaranteed an economic profit, i.e., \( \pi^\text{min}_f > 0 \). In this case the forces of competition alone guarantee \( f \) economic profit no matter how the extra-competitive forces mentioned above operate. Whenever \( \pi^\text{min}_f > 0 \), we will say that the incumbent has a competitive advantage, where we emphasize that \( f \)'s value appropriation is guaranteed by competition.

When \( f \) does not have a competitive advantage, defined in this way, \( f \) might still appropriate, provided \( \pi^\text{max}_f > 0 \); however, any value \( f \) appropriates in this case will be a consequence of some supra-competitive activity like bargaining or luck, and not the consequence of competition.

It helps to have some notation to keep track of whether and when \( f \) has a competitive advantage. So \( CA^{pre} \) means \( \pi^\text{min}_f > 0 \), and \( CA^{post} \) means \( \pi^\text{min}_f > 0 \). In earlier work (MacDonald and Ryall (2004)), we provided a complete characterization of when a player in an arbitrary coalitional game has a competitive advantage as just defined. Since this basic result applies to \( f \) and to both the pre- and post-entry games, it will be helpful briefly to review that result since it necessarily plays a key role in what follows.

4.1 Characterization (MacDonald & Ryall, (2004))

Agent \( f \) failing to have a competitive advantage in the pre-entry game is equivalent to the existence of a distribution, \( \pi \), satisfying (7)-(12), and for which \( \pi_f = 0 \). This observation allows us to characterize \( CA^{pre} \); a characterization of \( CA^{post} \) follows analogously.

To see how the characterization works, assume \( \pi_f = 0 \). Then (8) is always satisfied; using (5), if (9) is satisfied then (10) is too; and, again employing (5), if (11) is satisfied then (12) is too. Combining (7) and (9), the conditions whose satisfaction is equivalent to \( f \) not having a competitive advantage in the pre-entry game are

\[
L_{-e} \cdot \pi = v_{L_{-e}}
\]
and
\[ [0G0] \pi \geq v_{10}, \]  
(20)
where the latter is (11), taking account of the fact that when \( \pi_f = 0, [1G0] \pi = [0G0] \pi \).

Inequalities (20) require that \( \pi \), with \( \pi_f = 0 \), distributes value so that no group of agents (where \( e \) is not a member of any such group, since we are considering the pre-entry game) could appropriate more value by acting on some alternative including \( f \). These are the most attractive alternatives since \( f \) would be no worse off by being included, and every such alternative is at least as valuable when \( f \) is included (i.e., (5)). Now notice that if there is sufficient value to distribute, i.e., \( v_{1-e} \) is large enough, then (20) can always be satisfied, i.e., \( CA_{pre} \) does not hold. Thus, \( CA_{pre} \) holds if and only if \( v_{1-e} \) is not too large.

More precisely, let
\[ m_{v_{pre}} \equiv \min_{\pi \in \mathbb{R}_{++}^{n+1}} \{ I \cdot \pi \mid [0G0] \pi \geq v_{10} \} . \]  
(21)
\( m_{v_{pre}} \) is called \( f \)'s minimum value in the pre-entry game, and is the least value that can be distributed to players other than \( f \) and \( e \), while simultaneously making none of the alternatives described by (20) attractive.\(^7\) If \( m_{v_{pre}} \) is larger than the value available for distribution, \( v_{1-e} \), there is no way value can be distributed, with \( f \) receiving nothing, without making some alternative attractive to some group including \( f \); thus, \( f \) must appropriate. Conversely, if \( f \) appropriates in every competitive distribution, then the available resources must be too meagre to distribute without making some alternative attractive to some group including \( f \) if \( \pi_f = 0 \).

**Theorem 1** (MacDonald and Ryall) \( CA_{pre} \) if and only if \( m_{v_{pre}} > v_{1-e} \).

Analogous reasoning can be applied to \( f \) failing to have a competitive advantage in the post-entry game. If this is the case, there is a distribution, \( \pi \),

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\(^7\) As stated, the minimization problem does not require \( \pi_f = \pi_e = 0 \); however, any minimizing \( \pi \) will have this feature.
satisfying (8)-(19) and that has \( \pi_f = 0 \). To develop the equivalent to Theorem 1, assume, again, that \( \pi_f = 0 \). Then, as above, (8) is satisfied. Also, (10) is satisfied if (9) is, which, since \( \mathbf{I}_{-e} \cdot \pi = \mathbf{I}_{-e} \cdot \pi \) when \( \pi_f = 0 \), we will write

\[
\mathbf{I}_{-e} \cdot \pi \geq v_{1_{-e}}. \tag{22}
\]

Similarly, (11) and (12) can be combined, exactly as above, to yield (20). Next, (16) is satisfied if (15) is, and (13) and (15) together imply that, post entry, exactly \( v_I \) will be distributed:

\[
\mathbf{I} \cdot \pi = v_I. \tag{23}
\]

Also, (14) is satisfied if (17) is, which, using \( \mathbf{I}_{fe} \cdot \pi = \mathbf{I}_{e} \cdot \pi \), we write as

\[
\mathbf{I}_{e} \cdot \pi \geq v_{1_{e}}. \tag{24}
\]

Finally, using (5), (19) is met if (18) is:

\[
[0G1] \pi \geq v_{11}, \tag{25}
\]

where, as above, we note the fact that when \( \pi_f = 0 \), \([1G1] \pi = [0G1] \pi \). Thus, the question of whether \( f \) has a competitive advantage in the post-entry game can be answered by determining whether the value that might be distributed, as described by (23) are sufficient to allow (20), (22), (24) and (25) to be satisfied.

Define

\[
mv_{\text{post}} \equiv \min_{\pi \in \mathbb{R}^{n+1}_+} \{ \mathbf{I} \cdot \pi \mid \mathbf{I}_{-e} \cdot \pi \geq v_{1_{-e}}, \mathbf{I}_{e} \cdot \pi \geq v_{1_{e}}, [0G1] \pi \geq v_{11} \text{ and } [0G0] \pi \geq v_{10} \}. \tag{26}
\]

**Theorem 2** (MacDonald and Ryall) \( CA_{\text{post}} \) if and only if \( mv_{\text{post}} > v_I \).

Note that the minimization problems (21) and (26) have the same objective, and that the constraints in the former are a subset of those in the latter; thus \( mv_{\text{pre}} \leq mv_{\text{post}} \). Moreover, in (26) the aggregate value distributed to agents other than \( f \) or \( e \) must be at least \( v_{1_{-e}} \); thus \( mv_{\text{post}} \geq v_{1_{-e}} \).

**Lemma 1** \( mv_{\text{post}} \geq \max\{mv_{\text{pre}}, v_{1_{-e}}\} \).
Finally, observe that (6) implies \( u_I \geq u_{I-e} \). Define \( e \)’s added value by
\[
mv_e \equiv v_I - v_{I-e};
\]

\( mv_e \geq 0 \).

5 The impact of entry on competitive advantage

5.1 Basic result

Given Theorems 1 and 2, Lemma 1, and the fact that \( v_I \geq v_{I-e} \), we can be specific about the impact of entry on \( f \)’s competitive advantage. There are exactly four entities that, together, determine this effect: \( mv_{pre}, mv_{post}, v_{I-e}, \) and \( v_I \).

**Proposition 1** Given the pre- and post-entry games, entry

1. destroys competitive advantage – \( CA^{pre} \) and not \( CA^{post} \) – if and only if both \( mv^{pre} > v_{I-e} \) and \( mv^{post} \leq v_I \);

2. creates competitive advantage – not \( CA^{pre} \) and \( CA^{post} \) – if and only if \( mv^{pre} \leq v_{I-e} \) and \( mv^{post} > v_I \); and

3. sustains competitive advantage – \( CA^{pre} \) and \( CA^{post} \) – if and only if \( mv^{pre} > v_{I-e} \) and \( mv^{post} > v_I \).

**Remark 1** Given the pre- and post-entry games, entry does not sustain, create, or destroy competitive advantage – neither \( CA^{pre} \) nor \( CA^{post} \) – if and only if \( mv^{pre} \leq v_{I-e} \) and \( mv^{post} \leq v_I \).

Given Proposition 1, there are two sources of insights about how entry impacts \( f \)’s competitive advantage. One is further exploration of Proposition 1 itself.\(^8\) The other is exploration of the sources of the different patterns of values and minimum values that lead to the different cases in Proposition 1. We examine these in turn.

\(^8\)Corollaries (1)-(4), being corollaries, follow immediately from Proposition 1. However, in each case a small calculation is required. These calculations are included as footnotes.
5.1.1 Corollaries of Proposition 1

**Corollary 1** Given the pre- and post-entry games, entry *destroys* competitive advantage if and only if\(^9\)

\[ av_e \geq mp_{post} - vl_{-e} \geq mp_{pre} - vl_{-e} > 0. \]

Two observations follow immediately. First, an entrant whose added value is sufficiently large always destroys \(f\)'s competitive advantage. The intuition is straightforward. Entry provides players both new alternatives — which, as discussed earlier, tends to increase \(\pi^f_{min} \) — along with more value, which has the opposite effect. If \(av_e\) is large enough, the latter effect always outweighs the former. Second, if the entrant is to destroy competitive advantage, a strictly positive \(av_e\) is necessary (the theoretically smallest possible value for \(av_e\) is zero).

The argument is easy. If \(f\) has a competitive advantage in the pre-entry game, the reason is that, assuming \(f\) does not appropriate, the alternatives available to players are too valuable to be dominated with the available resources. Entry produces even more alternatives. So if the entrant brings no new value to the game, i.e., \(av_e = 0\), the same forces that gave \(f\) competitive advantage pre-entry are augmented by entry. Since the resources available to oppose these forces are no greater, \(f\) must continue to appropriate post-entry.

The necessity of \(av_e > 0\) for entry to destroy \(f\)'s competitive advantage provides an interesting perspective on a familiar situation in which entry is thought to eliminate competitive advantage. Consider a monopolist, \(f\), with a constant returns production technology, and a collection of customers, at least some of whom value the monopolist’s product at more than its marginal cost. Now assume another firm enters, offering the same product, and having the same constant returns technology. Given identical costs and constant returns, the entrant has \(av_e = 0\), and so, according to Corollary 1, *cannot* destroy competi-

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\(^9\)Since Lemma 1 implies \(mp_{post} \geq mp_{pre}\), the inequalities in part 1 of 1 can be combined to yield

\[ vl \geq mp_{post} \geq mp_{pre} > vl_{-e}. \]

Subtracting \(vl_{-e}\) throughout, then applying the definition of \(mp_e\), completes the argument.
tive advantage. But given the identical products and costs, it is immediate that 
\[ \pi_f^{\min} = \pi_f^{\max} = 0, \] i.e., \( f \) has no competitive advantage post-entry. These conclusions are mutually consistent only if \( f \) had no pre-entry competitive advantage. That is, the forces of competition in the pre-entry game did not guarantee the monopolist could appropriate. This is exactly correct. Given constant marginal cost, the monopolist is effectively in a pure bargaining game with each consumer, where the bargaining is over the surplus (value less production cost) from that one transaction. Thus \( \pi_f^{\min} = 0 \) and \( \pi_f^{\max} = S \), where \( S \) is the aggregate surplus from consumption of the good by all consumers who value it at marginal cost or more. In this situation any profit the monopolist extracts from consumers pre-entry is purely the result of bargaining/negotiation, and not a consequence of the competitive alternatives available to it or to consumers.\(^{10}\) The forces of competition are very weak pre-entry, and, in particular, \( f \) has no competitive advantage. Entry does nothing to make competition to transact with \( f \) more attractive, and creates an alternative for consumers that makes bargaining with \( f \) unnecessary, and, in fact, forces \( \pi_f = \pi_e = 0 \). Entry did not destroy the ability of alternatives to guarantee appropriation for \( f \). Instead, it eliminated the possibility of \( f \) appropriating via bargaining.

**Corollary 2** Given the pre- and post-entry games, entry *creates* competitive advantage if and only if\(^{11}\)

\[
mv^{post} - v_{1-e} > av_e \geq 0 \geq mv^{pre} - v_{1-e}.
\]

\(^{10}\) Specifically, the textbook monopoly model assumes the firm has all the bargaining power. Given constant returns, the monopolist’s threatening, for example, to exclude one customer and sell to another is not credible, and there is no obvious justification for the bargaining power assumption. With increasing marginal cost or capacity constraints, this assumption has greater justification. However, with increasing marginal cost, entry generally does not remove competitive advantage.

\(^{11}\) Since (6) implies \( v_1 \geq v_{1-e} \), the inequalities in part 2 of 1 can be combined to yield

\[
mv^{post} > v_1 \geq v_{1-e} \geq mv^{pre}.
\]

Subtracting \( v_{1-e} \) throughout, then applying the definition of \( mp_e \), completes the argument.
Using Lemma 1 and the fact that \( f \) has no pre-entry competitive advantage, the theoretically smallest value \( m_{v_{\text{post}}} \) can take on is \( v_{1_{-e}} \), so \( m_{v_{\text{post}}} - v_{1_{-e}} \) is \( m_{v_{\text{post}}} \) measured relative to its smallest possible value. Two observations follow from Corollary 2. First, if entry generates new alternatives for groups including \( f \) that are sufficiently valuable, in the sense that \( m_{v_{\text{post}}} - v_{1_{-e}} \) is large, then \( f \) always has competitive advantage post-entry. In this case, although the additional resources entry brings make it easier for given alternatives involving to \( f \) to be dominated, entry also offers new alternatives. And if these alternatives are valuable enough, \( f \) must appropriate post-entry even if failing to do so was possible pre-entry. Second, for entry to create competitive advantage, it is necessary that \( m_{v_{\text{post}}} \) be strictly greater than its smallest possible value, \( v_{1_{-e}} \). If \( m_{v_{\text{post}}} = v_{1_{-e}} \), then the resources available to render unattractive the new alternatives entry brings, i.e., \( v_{I} \), are ample and \( f \) need not appropriate.

Return to the example of monopoly with entry. In that example, \( m_{v_{\text{post}}} = S (= v_{1_{-e}} = v_{I}) \). So, consistent with what we found before, entry has no prospect of creating competitive advantage for \( f \). Altogether, in the monopoly with entry example, entry neither creates nor destroys competitive advantage.

**Corollary 3** Given the pre- and post-entry games, entry *sustains* competitive advantage if and only if\(^{12}\)

\[
m_{v_{\text{post}}} - v_{1_{-e}} > av_{e} \geq 0 > v_{1_{-e}} - m_{v_{\text{pre}}}
\]

**Corollary 4** Given the pre- and post-entry games, *if entry affects competitive* advantage if and only if

\[
m_{v_{\text{post}}} - v_{1_{-e}} > m_{v_{\text{pre}}} \geq 0 > v_{1_{-e}} - m_{v_{\text{pre}}}
\]

\(^{12}\)Since (6) implies \( v_{I} \geq v_{1_{-e}} \), the second inequality in part 3 of 1 gives

\[
m_{v_{\text{post}}} > v_{I} \geq v_{1_{-e}}.
\]

Subtracting \( v_{1_{-e}} \) throughout, then applying the definition of \( m_{p_{e}} \), gives

\[
m_{v_{\text{post}}} - v_{1_{-e}} > m_{p_{e}} \geq 0.
\]

The first inequality in part 3 of 1 is equivalent to

\[
v_{1_{-e}} - m_{v_{\text{pre}}} < 0.
\]

This, with the previous inequalities, yields the result.
advantage, it creates it if and only if
\[ mv_{\text{post}} - mv_{\text{pre}} > av_e, \]
and destroys it if and only if
\[ mv_{\text{post}} - mv_{\text{pre}} < av_e. \]

5.1.2 Impact of new the new alternatives entry brings

Recall the minimization problems (21) and (26). Both minimizations determine the least value that can be distributed only to players other than \( f \) (i.e., \( f \) receives zero) without making some alternative including \( f \) preferable for some group. Compared to the pre-entry game, entry introduces three new kinds of alternatives, described by: (22), (24) and (25).

The first new alternative, (22), is the one that involves all players other than \( e \), including \( f \), simply acting on their own and sharing \( v_{1-e} \). Whereas the agents other than \( e \) ultimately had to share \( v_{1-e} \) pre-entry, post-entry doing this is simply one of the available alternatives. Does this new choice, by itself, have any particular effect on \( f \)'s competitive advantage? To answer this, assume both \( v_{1-e} \) and \( v_{11} = v_{10} \). That is, the alternatives available to to any group including \( f \) (other than the group of all players except \( e \)) are not improved by including \( e \). Under these assumptions, (26) becomes

\[
mv_{1}^{\text{post}} \equiv \min_{\pi \in \mathbb{R}^{n+1}} \left\{ I \cdot \pi \mid I_{-fe} \cdot \pi \geq v_{1-e} \right\} \quad (27)
\]

13 If entry affects competitive advantage, it either creates it or destroys it. In the former, both
\[ mv_{\text{pre}} \leq v_{1-e} \text{ and } mv_{\text{post}} > v_{1}, \]
or
\[ mv_{\text{post}} - mv_{\text{pre}} > mp_e. \]

In the latter both
\[ mv_{\text{pre}} > v_{1-e} \text{ and } mv_{\text{post}} \leq v_{1}, \]
or
\[ mv_{\text{post}} - mv_{\text{pre}} < mp_e. \]

14 To see this, observe that when \( v_{11} = v_{10} \), any \( \pi \) satisfying \( [0G0] \pi \geq v_{10} \) also satisfies

24
i.e., comparing (27) to (21), $I_{-fe} \cdot \pi \geq v_{1e}$ is the sole additional constraint that has any bearing on the magnitude of $mv_{1}^{post}$, and thus on the $mv_{1}^{post}$ versus $v_{I}$ comparison that determines whether $f$ has competitive advantage post-entry.

**Proposition 2** Given the pre- and post-entry games, if $v_{1fe} = 0$ and $v_{11} = v_{10}$, entry

1. Does not create competitive advantage;

2. Destroys competitive advantage if and only if $v_{1e} < mv_{pre} \leq v_{I} - e$;

3. Sustains competitive advantage if and only if $v_{1} < mv_{pre}$.

The argument is as follows. For part 1, suppose $f$ does not have competitive advantage pre-entry, i.e., $mv_{pre} \leq v_{1e}$. Then there is at least one distribution $\pi$ that satisfies $[O(0G)] \pi \geq v_{10}$ and $I_{-fe} \cdot \pi \leq v_{1e}$. Thus the minimum value in (27) can be achieved by some distribution satisfying $[O(0G)] \pi \geq v_{10}$ and also $I_{-fe} \cdot \pi = v_{1e}$, i.e., $mv_{1}^{post} = v_{1e}$. Since $v_{1e} \leq v_{1}$, $f$ has no competitive advantage post-entry. For part 2, suppose $f$ has competitive advantage pre-entry, i.e., $mv_{pre} > v_{1e}$. It follows that any minimizing distribution in (27) also has $I_{-fe} \cdot \pi > v_{1e}$, in which case (21) and (27) are the same problem, i.e., $mv_{1}^{post} = mv_{pre}$. Thus, whether $f$ also has competitive advantage post-entry hinges on a comparison of $v_{1}$ and $mv_{pre}$. This also settles whether entry sustains competitive advantage, i.e., part 3.

The intuition is as follows. When the entrant does not make any group including $f$ more valuable, it does nothing to make the alternatives involving $f$ any harder to dominate, which leaves $\pi_{f}^{min}$ at best, unchanged. So if $f$ lacks competitive advantage, i.e., $\pi_{f}^{min} = 0$, $e$’s participation cannot create $\tilde{\pi}_{f}^{min} > 0$. On the other hand, if $\pi_{f}^{min} > 0$, the fact that the entrant generally has a positive added value means there are more resources available, which, for the reasons set out earlier, tends to lower $\pi_{f}^{min}$. If $v_{1} \geq mv_{pre}$, these additional resources are sufficient to remove $f$’s competitive advantage, i.e., $\tilde{\pi}_{f}^{min} = 0$.

$[O(0G)] \pi \geq v_{11}$, but not the converse. Thus the latter set of inequalities can be eliminated from (26).
Next, what happens when including $e$ in a group including $f$ more valuable?\textsuperscript{15} First, consider the group including only $f$. Pre-entry, this group’s alternative is just $v_f = 0$, whereas post-entry $f$ and $e$ can share $v_{ife}$. Once again assuming $v_{11} = v_{10}$, but allowing $v_{ife} > 0$,\textsuperscript{3}

$$mv_{2}^{\text{post}} \equiv \min_{\pi \in \mathbb{R}_{+}^{n+1}} \{ I \cdot \pi | I_{ife} \cdot \pi \geq v_{1e}, I_{e} \cdot \pi \geq v_{ife} \text{ and } [0G0] \pi \geq v_{10} \}.$$\textsuperscript{(28)}

**Proposition 3** Given the pre- and post-entry games, if $v_{11} = v_{10}$, entry

1. Creates competitive advantage if and only if

$$mv_{pre} - v_{1e} \leq 0 \leq av_e < v_{ife} ;$$

2. Destroys competitive advantage if and only if

$$0 < mv_{pre} - v_{1e} \leq v_{ife} + mv_{pre} - v_{1e} \leq av_e ;$$

and

3. Sustains competitive advantage if and only if

$$v_{1} - mv_{pre} < av_e < v_{ife} + mv_{pre} - v_{1e} .$$

For the first part, observe that when $f$ does not have a competitive advantage pre-entry,

$$mv_{2}^{\text{post}} = \min_{\pi \in \mathbb{R}_{+}^{n+1}} \{ I \cdot \pi | I_{ife} \cdot \pi \geq v_{1e} \text{ and } [0G0] \pi \geq v_{10} \} + v_{ife} = v_{1e} + v_{ife} .$$

\textsuperscript{15}To determine the impact on $f$’s competitive advantage of the option of all players other than $e$, including $f$, simply acting on their own and sharing $v_{1e}$, we set $v_{ife} = 0$ and $v_{11} = v_{10}$, effectively prohibiting $v_{ife}$ and $v_{11}$ from having any impact. To examine their impact alternative by alternative, it is tempting to do so while prohibiting the constraint $I_{ife} \cdot \pi \geq v_{1e}$ from having any impact, say by setting $v_{1e} = 0$. This would be a mistake. Assuming $v_{ife} = 0$ and $v_{11} = v_{10}$ simply structures how the pre- and post-entry games differ. Assuming $v_{1e} = 0$ fundamentally changes the pre-entry game. Thus, the next two Propositions make no new assumption about $v_{1e}$.\textsuperscript{26}
The first equality follows from the fact that \( \pi_e \) enters the minimization only via the constraint \( I_e \cdot \pi \geq v_{1e} \), and so \( \pi_e = v_{1e} \) in any solution. The second equality follows from observing that given any distribution of value \((0, \pi_{-e}, 0)\) that solves (21), if any component of \( \pi_{-e} \) is increased so that \( I_{-e} \cdot \pi \geq v_{1e} \) is satisfied, and \( \pi_e = v_{1e} \), (29) is also solved, with the minimized value being \( mv^2_{\text{post}} = v_{1e} + v_{1e} \). It follows that conditions describing entry creating competitive advantage when it did not exist pre-entry, i.e.,

\[
mv^{\text{pre}} \leq v_{1e} \leq v_1 < mv^2_{\text{post}},
\]
can be written

\[
mv^{\text{pre}} \leq v_{1e} \leq v_1 < v_{1e} + v_{1e}.
\]

Subtracting \( v_{1e} \) yields the result.

For part 2, \( f \)'s having a competitive advantage pre-entry requires \( mv^{\text{pre}} > v_{1e} \). And competitive advantage is destroyed by entry if and only if \( mv^2_{\text{post}} \leq v_1 \). That \( f \) has competitive advantage pre-entry implies \( I_{-e} \cdot \pi > v_{1e} \) in the minimization defining \( mv^2_{\text{post}} \). Therefore

\[
mv^2_{\text{post}} \equiv \min_{\pi \in \mathbb{R}^{n+1}} \{ I \cdot \pi \mid I_e \cdot \pi \geq v_{1e}, \text{ and } [0G0] \pi \geq v_{10} \}.
\]

It follows that for any \( \pi = (0, \pi_{-e}, 0) \) yielding \( mv^{\text{pre}} \), \((0, \pi_{-e}, v_{fe})\) yields \( mv^2_{\text{post}} \) and that \( mv^2_{\text{post}} = mv^{\text{pre}} + v_{fe} \). Thus, \( mv^2_{\text{post}} \leq v_1 \) is equivalent to \( mv^{\text{pre}} + v_{fe} \leq v_{1e} \), or \( v_{fe} + mv^{\text{pre}} - v_{1e} \leq av_e \). Since \( v_{1e} \geq 0 \), we have that competitive advantage exists pre-entry, and is destroyed by entry, if and only if

\[
0 < mv^{\text{pre}} - v_{1e} \leq v_{fe} + mv^{\text{pre}} - v_{1e} \leq av_e.
\]

For part 3, \( f \)'s having a competitive advantage pre-entry requires \( mv^{\text{pre}} > v_{1e} \), or \( v_1 - mv^{\text{pre}} < av_e \). Also, as in part 2, that \( f \) has competitive advantage pre-entry implies \( mv^2_{\text{post}} = mv^{\text{pre}} + v_{fe} \). Thus \( f \) continuing to have competitive advantage requires \( mv^{\text{pre}} + v_{fe} > v_1 \), or \( mv^{\text{pre}} + v_{fe} - v_{1e} > av_e \). It follows that

\[
v_1 - mv^{\text{pre}} < av_e < mv^{\text{pre}} + v_{fe} - v_{1e}.
\]
To proceed, consider including $e$ in groups including $f$ and at least one other player. Assuming $v_{1e} = 0$, but allowing $v_{11} \geq v_{10}$, but allowing $v_{1e} > 0$,

$$mv^\text{post}_3 \equiv \min_{\pi \in \mathbb{R}^{n+1}_{+}} \{ \mathbf{I} \cdot \pi \mid \mathbf{I} - f_e \cdot \pi \geq v_{1-}, [\mathbf{0G1}] \pi \geq v_{11} \text{ and } [\mathbf{0G0}] \pi \geq v_{10} \}.$$  

(29)

**Proposition 4** Given the pre- and post-entry games, if $v_{1e} = 0$, entry

1. Creates competitive advantage if and only if
   $$mv^{pre} - v_{1-} \leq 0 < av_e < mv^\text{post}_3 - v_{1-}.$$  

2. Destroys competitive advantage if and only if
   $$0 < mv^{pre} - v_{1-} \leq mv^\text{post}_3 - v_{1-} \leq av_e;$$  

and

3. Sustains competitive advantage if and only if
   $$v_{1} - mv^{pre} < av_e < mv^\text{post}_3 - v_{1-}.$$  

For the first part, recall that conditions describing entry creating competitive advantage when it did not exist pre-entry are

$$mv^{pre} \leq v_{1-} \leq v_{1} < mv^\text{post}_3.$$  

For part 2, conditions describing entry removing competitive advantage are

$$v_{1-} < mv^{pre} \leq mv^\text{post}_3 < v_{1}.$$  

Subtracting $v_{1-}$ yields the result.

For part 3, $f$’s having a competitive advantage pre-entry again requires $mv^{pre} > v_{1-}$, or $v_{1} - mv^{pre} < av_e$. And the condition describing $f$’s also having competitive advantage post-entry is $mv^\text{post}_3 < v_{1}$. Subtracting $v_{1-}$ from this inequality, and combining it with the previous one, yields the result.
References


