# Identification of Equilibrium Models of Local Jurisdictions*

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**Abstract**

Research over the past several years has led to development of models characterizing equilibrium in a system of local jurisdictions. More recently, there have been a number of studies which have tried to estimate these models. The evidence suggests that simple parametric models can explain the observed sorting of individuals by income among local jurisdictions reasonably well. One drawback of the approach adopted in most empirical studies is that identification of important parameters of the model may be primarily due to functional form assumptions which are hard to evaluate. In this paper, we provide a discussion of identification and estimation of hierarchical equilibrium models in a semi-parametric framework. We show that a broad class of models is identified without imposing strong parametric restriction on the distribution of unobserved tastes for local public goods. We then extend the framework and consider a broad class of non-hierarchical model.

JEL classification: C51, H31, R12

## 1 Introduction

Research over the past several years has led to development of models characterizing equilibrium in a system of local jurisdictions. More recently, there have been a number of studies which have tried to estimate these models. The evidence suggests that simple parametric models can explain the observed sorting of individuals by income among local jurisdictions reasonably well. One drawback of the approach adopted in most empirical studies is that identification of important parameters of the model may be primarily due to functional form assumptions which are hard to evaluate. In this paper, we provide a discussion of identification and estimation of equilibrium models of local jurisdictions in a semi-parametric framework. We provide different sets of regularity conditions which imply that a generic hierarchical locational equilibrium model is identified without imposing parametric assumptions on the joint distribution of income and tastes. In the first case we assume that tastes and income are independently distributed. We show that the marginal distribution of tastes is non-parametrically identified. The second case considers a model in which preferences and income are not independently distributed. Point identification is still possible if we observe the equilibrium in the metropolitan area at two distinct points of time.

In applications it is reasonable to assume that public good provision is measured with error. We show in this paper that the same set of assumptions on the distribution of measurement error which were used in the parametric analysis of Epple and Sieg (1999) and Epple, Romer, and Sieg (2001) are also sufficient to identify and estimate the semi-parametric model presented in this paper. Based on these results, we develop a new semi-parametric estimation algorithm. Our new estimation algorithm has a number of advantages over algorithms which are currently used. In particular, it is computationally simpler than the parametric approach.

After we have established identification of hierarchical locational equilibrium models, we turn our attention to non-hierarchical models. Non-hierarchical models arise if we introduce horizontal differentiation in tastes for local public goods and amenities among households. The main problem encountered in non-hierarchical models is that the ascending bundles property does not hold in these models. While it is still possible to order communities by increasing housing prices, households may differ in their orderings of public good provision. This failure to agree on a single ordering of communities increases the complexity of the model significantly.

We use discrete distributions to model horizontal taste differentiation for local public goods. Discrete distributions have the advantage that they allow us to limit the admissible number of ordering of communities to a small tractable number. Moreover, the resulting sorting model is hierarchical once we condition on the discrete household type. The resulting non-hierarchical model can, therefore, be viewed as a mixture of hierarchical models of the type estimated in Epple and Sieg (1999). We show that it is exceedingly hard to get non-parametric point identification of the distribution of tastes if the discrete types are unobserved. Point identification for model with observed types can be established using a similar argument as in the hierarchical case.

The rest of the paper is organized as follows. Section 2 provides a brief overview of hierarchical locational equilibrium models. Section 3 discusses identification. Section 4 focuses on semiparametric estimation. In Section 5 we develop a new class of non-hierarchical sorting models and discusses identification of those models. Section 6 offers some conclusions.

# 2 Hierarchical Locational Equilibrium Models

## 2.1 Theory

The model considers the problem of public good provision and residential decisions in a system of multiple jurisdictions.[1] The economy consists of a continuum of households living in a metropolitan area. The homogeneous land in the metropolitan area is divided among a number of communities, each of which has fixed boundaries. Jurisdictions may differ in

[1]This literature was inspired by Tiebout (1956). See, for example, Epple, Filimon, and Romer (1984), Goodspeed (1989), Epple and Romer (1991), Nechyba (1997a, 1997b) and Fernandez and Rogerson (1996, 1998).

the amount of land contained within their boundaries. Individuals differ in their endowed income, $y$, and in a taste parameter, $\alpha$, which reflects the household's valuation of the public good. The continuum of households is implicitly described by the joint distribution of $y$ and $\alpha$, $F(\alpha, y)$.

A household with taste parameter, $\alpha$, and income, $y$, is referred to as $(\alpha, y)$. A household living in a community has preferences defined over a local public good, $g$, a local housing good, $h$, and a composite private good, $b$. Denote with $p$ the gross-of-tax price of a unit of housing services in the community. Households pay taxes that are levied on the value of housing services. Let $t$ be an *ad valorem* tax on housing. The preferences of a household are represented by a utility function, $U(\alpha, g, h, b)$ that is twice differentiable in its arguments and strictly quasi-concave in $g$, $h$ and $b$. Households maximize their utility with respect to the budget constraint:

$$\begin{aligned} &\max_{(h,b)} \; U(\alpha, g, h, b) \\ &s.t. \;\; (1+t)\, p^h \, h \;=\; y \;-\; b \end{aligned} \tag{1}$$

It is convenient to represent the preferences of a household living in community $j$ using the indirect utility function, $V(\alpha, y, g_j, p_j)$, derived by solving the optimization problem given in equation (1). Consider the slope of an "indirect indifference curve" in the $(g_j, p_j)$-plane:

$$M(\alpha, y, g_j, p_j) \;=\; \frac{dp_j}{dg_j}\Big|_{V=\bar{V}} \tag{2}$$

If $M(\cdot)$ is monotonic in $y$, then, for given $\alpha$, indifference curves in the $(g_j, p_j)$-plane satisfy the "single-crossing" property. Likewise, the monotonicity of $M(\cdot)$ in $\alpha$ provides single-crossing for given $y$.

There is a finite number $J$ of local communities. Let $C_j$ denote the population living in community $j$. We also assume that the budget of community $j$ must be balanced.[2] This

[2] We impose this assumption for simplicity to close the model. The analysis can be easily extended to incorporate lump sum transfers, for example, from the state government to the local governments. The estimator developed in Section 4 exploits first-order conditions implied by optimal household demands for

implies that:

$$t_j \; p_j^h \int_{C_j} h(p_j, \alpha, y) \;\; f(\alpha, y) \; dy \; d\alpha \Big/ P(C_j) \;=\; c(g_j) \tag{3}$$

where $c(g)$ is the cost per household of providing $g$, $h(\cdot)$ denotes housing demand, and

$$P(C_j) \quad = \quad \int_{C_j} f(\alpha, y) \; dy \; d\alpha \tag{4}$$

is the size of community $j$.

We assume that the pair $(t, g)$ in each community is chosen by majority rule. In each community, voters take the $(t, g)$ pairs in all other communities as given when making their decisions. One can make a variety of assumptions about voter sophistication regarding anticipation of the way changes in the community's own $(t, g)$ pair affect the community's housing prices and migration into or out of the community. For example, utility-taking voters base their voting decisions on the housing price and migration effects that would occur if the utility in the next best alternative community is given. The community budget constraint, housing market clearing, and perceived migration effects define the function $p(g)$ that determines the government-services possibility frontier, i.e. GPF $= \{g(t), p(t) \; | t \in R^+\}$. For given tax and expenditure policies in other communities, a point on the GPF that cannot be beaten in a majority vote is a majority equilibrium.

Consider a point $(g_j^*, p_j^*)$ on community j's GPF, and let $\tilde{\alpha}_j(y)$ define a set of voters who weakly prefer $(g_j^*, p_j^*)$ to any other $(g_j, p_j)$ on the GPF. It follows that $(g_j^*, p_j^*)$ is a majority voting equilibrium for the given GPF if

$$\int_0^\infty \int_{\alpha_j(y)}^{\tilde{\alpha}_j(y)} \;\; f(\alpha, y) \; d\alpha \; dy \;=\; \frac{1}{2} \int_0^\infty \int_{\alpha_j(y)}^{\alpha_{j+1}(y)} \;\; f(\alpha, y) \; d\alpha \; dy \tag{5}$$

where $\tilde{\alpha}_j(y)$ defines a locus of pivotal voters.

Mobility among communities is costless, and in equilibrium every household lives in his

public goods, which are unaffected by lump sum transfers.

or her preferred community. To close the model we assume that the housing stock in each community is owned by absentee landlords. As a consequence, we can characterize housing supply in each community by a simple housing supply function, $H^s(p^h)$. Having specified all components of a (generic) equilibrium model, we define an intercommunity equilibrium as follows:

An **intercommunity equilibrium** consists of a set of communities and a partition of households across communities such that every community has a positive population; a vector of prices and taxes; an allocation of public goods; and an allocation for every household, such that:

1. Every household living in community $j$ maximizes its utility subject to the budget constraint.

2. Each household lives in one community and no household wants to move to a different community.

3. The housing market clears in every community.

4. The budget of every community is balanced.

5. There is a voting equilibrium in each community.

If household preferences satisfy single-crossing properties, the existence of an intercommunity equilibrium has been shown in somewhat simpler versions of this model. If an equilibrium exists, necessary conditions for equilibrium in this model impose a number of restrictions on the equilibrium allocation that apply quite broadly. Let $(g_i, p_i)$ and $(g_j, p_j)$ be the level of public good provision and gross-of-tax housing price in community $i$ and $j$, respectively, and suppose that some individuals prefer $(g_j, p_j)$ and others prefer $(g_i, p_i)$. Then locational choices in equilibrium will satisfy three properties: boundary indifference, stratification and ascending bundles.[3]

[3]These models are thus similar to vertical differentiation models studied in the IO literature. See, for example, Shaked and Sutton (1982) and Breshnahan (1987).

We consider in the application that follows a preference function that is sufficiently general to subsume specifications that have generally been adopted in applied equilibrium models of local jurisdictions. We assume additive separability of the indirect utility function in the subutility function for the public good and the subutility for the private goods bundle:

$$V(\alpha, y, g, p) \;=\; \alpha\, V^g(g) \;+\; V^b(y, p) \tag{6}$$

Our results below apply for any well-behaved utility function $V^b(y, p)$ for the private goods bundle that preserves the single-crossing property in equation (2). We adopt a constant elasticity of substitution formulation to capture the trade-offs between the public good and the private goods components. While this specification offers a reasonable degree of generality and subsumes functional forms used to date in computational analysis, the approach we develop could be used to investigate identification and semi-parametric estimation with other functional forms. For concreteness, we adopt a from for $V^b(y, p)$ that implies a constant price and income elasticities for housing. Thus we assume that:

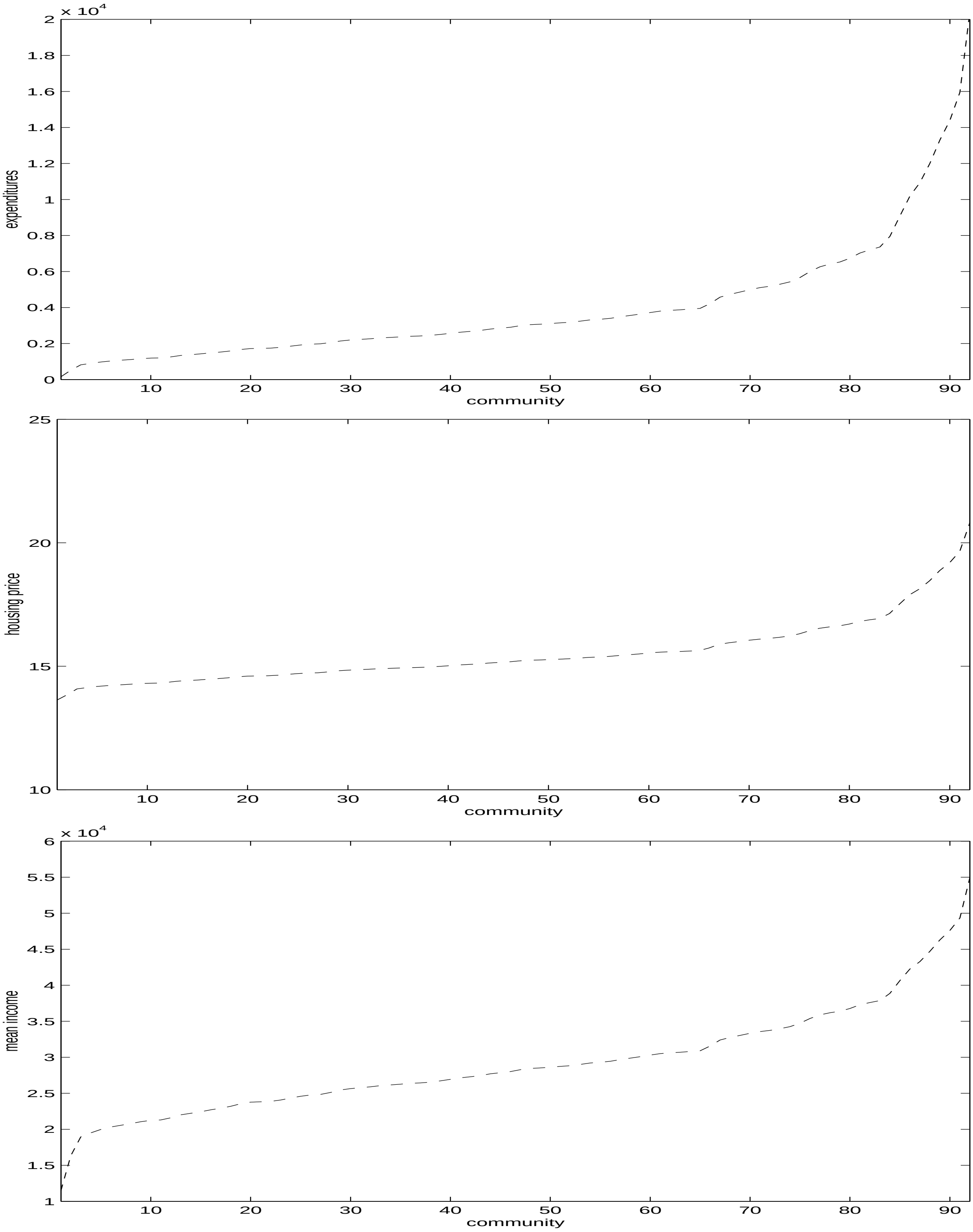
$$V(\alpha, y, g_j, p_j) \;=\; \left\{\alpha\, g_j^{\rho} \;+\; \left[e^{\frac{y^{1-\nu}-1}{1-\nu}}\; e^{-\frac{B p_j^{\eta+1}-1}{1+\eta}}\right]^{\rho}\right\}^{\frac{1}{\rho}} \tag{7}$$

where $\eta$ is the price elasticity of housing, $\nu$ is the income elasticity of housing, and $B$ is the intercept in the housing demand equation.

## 2.2 A Computational Example

To illustrate some of the important results of this paper, we use the following computational example which is a rough approximation of the 92 communities in the Boston metropolitan area. The indirect utility function is given by equation (7). The joint distribution of income and tastes is assumed to be bivariate log-normal. The parameters of the utility function and the distribution of tastes are as follows $\mu_y = 9.79; \mu_\alpha = -3.118; \sigma_y = 0.755; \sigma_\alpha = 0.815; \lambda = 0.0; \nu = 0.938; \eta = -0.3; B = 0.34; \rho = -0.24$. The housing supply in each community is fixed. The expenditure levels, housing prices and mean income levels in equilibrium are

Figure 1: Equilibrium



plotted in Figure 1.

# 3 Identification

## 3.1 Identification of the Joint Distribution of Tastes and Income

In the spirit of semi-parametric estimation, we assume that the utility function is known up to finite vector of parameters and given by equation (7).[4] To simplify the analysis, let us initially assume that the parameters of the indirect utility function are known. Hence we focus on the identification of $f(\alpha, y)$.[5] The joint distribution of tastes and income can be factored as follows:

$$f(\alpha, y) \quad = \quad f_y(\alpha)\, f(y) \tag{8}$$

where $F_y(\alpha)$ denotes the conditional density of $\alpha$ given $y$ and $F(y)$ is the unconditional distribution of $y$. We do not impose any restrictions on the conditional distribution of tastes given income, since tastes for public goods are inherently unobserved.

Given the parametric specification of the indirect utility function, boundary indifference implies that the set of individuals who are indifferent between adjacent communities is implicitly characterized by the following equation:

$$\alpha_j(y) \quad = \quad \left[ e^{\frac{y^{1-\nu}-1}{1-\nu}} \right]^{\rho} \; \frac{Q(p_j) - Q(p_{j-1})}{g_{j-1}^{\rho} - g_j^{\rho}} \quad j = 2, ..., J \tag{9}$$

where $Q(p_i) = e^{-\,\rho\, \frac{B p_i^{\eta+1}-1}{1+\eta}}$. By construction we also have $\alpha_1(y) = 0$ and $\alpha_{J+1} = \infty$.

Stratification implies that a measure of the number of households with income $y$ living

[4]Our analysis is thus similar in spirit to the recent work by Bajari and Benkard (2002) on identification of hedonic models. In contrast Ekeland, Heckman, and Nesheim (2002) and Heckman, Matzkin, and Nesheim (2002) study identification of hedonic models without imposing parametric restrictions on preferences.

[5]Identification of the parameters of $V(\cdot)$ is discussed below in Section 3.2.

in community $j$ is given by:

$$P_j(y) = F_y(\alpha_{j+1}(y)) - F_y(\alpha_j(y)) \tag{10}$$

We observe households' residential decisions by income.[6] Hence, we observe $P_j(y)$ which is the measure on the left-hand side of equation (10). We also observe the level of public good provision, $g_j$, and housing prices, $p_j$ without error.[7] This implies that $\alpha_j(y)$ is observed as well. Equation (10) therefore implies that we can identify $J-1$ points of the conditional distribution function of $\alpha$ given $y$. These points correspond to the values of $\alpha_j(y)$.

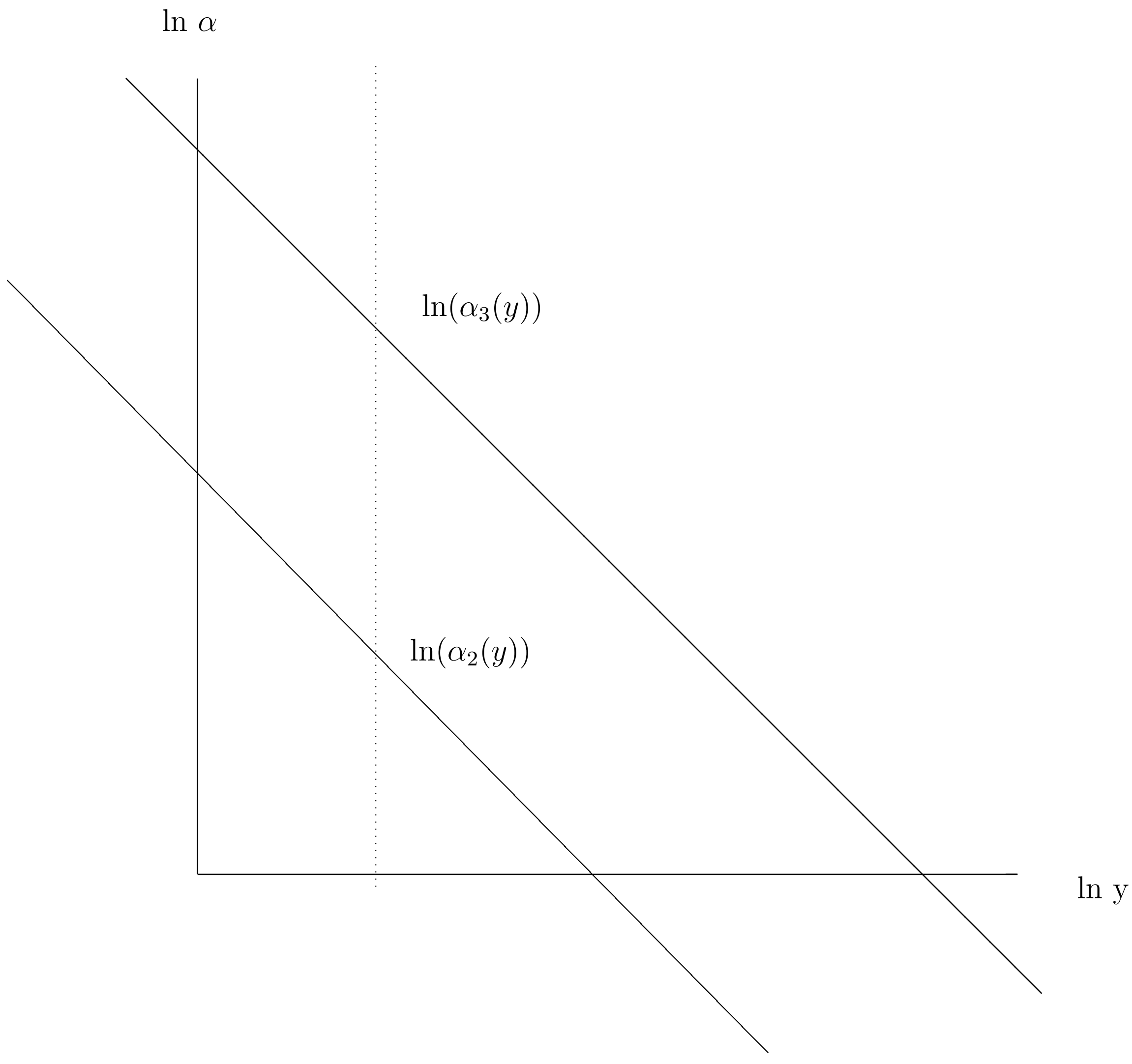
This result is illustrated in Figure 2, which considers an example with three communities. Given our choice for the indirect utility function, it is easier to consider sorting in the space of $(\ln(y), \ln(\alpha))$. Given the structure of the model we can arbitrarily transform the distribution of $\ln(\alpha)$ given $\ln(y)$ on the intervals $(-\infty, \ln(\alpha_2(y)))$, $(\ln(\alpha_2(y)), \ln(\alpha_3(y)))$, and $(\ln(\alpha_3(y)), \infty)$ without affecting the sorting of individuals among communities in equilibrium, as long as the transformed distribution has the correct mass points at the boundaries. As a consequence the conditional distribution of $\ln(\alpha)$ given $\ln(y)$ is not identified in the interior of the three intervals.

This point can also be illustrated using the computational example introduced in Section 2.2. The discussion above suggests that we can approximate the conditional density of $\alpha$ given $y$ using a histogram. This histogram is constructed from the J-1 pairs $(\alpha_j(y), F_y(\alpha_j(y))$ which are identified from the observed sorting by income among communities. Figure 3 shows the true conditional distribution and the histogram which is nonparametrically identified. The results indicate that the histogram approximates the true density for most parts the distribution. Failure of identification of $f_y(\alpha)$ for low values of $\alpha$ is due to the fact that the second community in our example (Boston) is relatively large. Almost 22 percent of the population in the metro area lives in that community. This implies that the

[6] Thus $F(y)$ is nonparametrically identified.

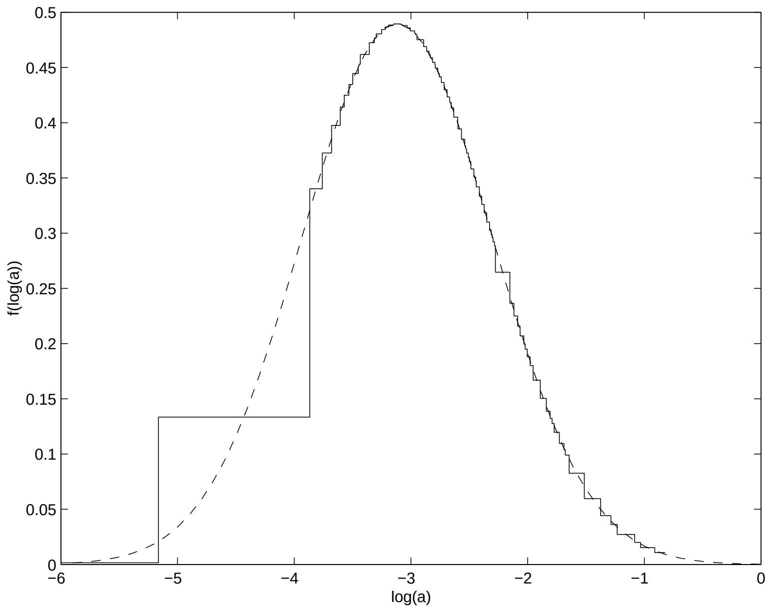
[7] In practice, housing prices are not observed and need to be estimated as discussed in Sieg, Smith, Banzhaf, and Walsh (2002). Thus we implicitly assume that we can consistently estimate housing prices,prior to estimating locational equilibrium models. Moreover, public good provision may only be observed with error as discussed in detail in Section 4.

Figure 2: Identification of $F_y(\alpha)$



difference between $\alpha_2(y)$ and $\alpha_3(y)$ is large in our example. All other communities in the example are small. This implies that the density function can be closely approximated by the histogram for most values of $\alpha$.

Figure 3: Conditional Distribution of Tastes




If there are many communities, we would expect that the difference between $\alpha_j(y)$ and $\alpha_{j+1}(y)$ will be small for most community pairs. Moreover we expect that $[\alpha_2(y), \alpha_J(y)]$ covers large parts of the support of the distribution of $\alpha$. We thus conjecture that failure of identification at points in the interior of $[\alpha_j(y), \alpha_{j+1}(y)]$, will not be problematic in most practical application. In particular, the conditional distribution of $\alpha$ given $y$ for $\alpha \in (\alpha_j(y), \alpha_{j+1}(y))$ can be approximated using linear interpolation in analyzing policies that shift the boundaries.

Suppose, we add communities to our model such that the size of all communities gets small as the number of communities increases. More specifically, assume that

$$\lim_{J \to \infty} [\alpha_{j+1}(y) - \alpha_j(y)] = 0 \quad \forall j \tag{11}$$

In that case, it is not hard to show that we get identification of the complete conditional distribution function $F_y(\alpha)$.

We thus conclude that in an economy with a finite number of communities, we can identify $J-1$ points of the conditional distribution of $F_y(\alpha)$. The observed sorting of households by income does not impose any restrictions on the the conditional distribution at any other points, except the points on the boundaries of adjacent communities. As the number of communities in the economy increases, the histogram which is identified of the sorting by income will closely approximate the conditional distribution of tastes.

As we have seen above, we endogenize public good provision by specifying a voting mechanism. These voting models impose additional restrictions on the joint distribution $F(\alpha, y)$. The intuition is the following: each voting model generates a locus for the set of decisive voters in each community. This locus is implicitly defined by the majority voting condition that 50 percent of households are on either side of this locus (equation 5). This locus thus effectively partitions each community into two areas with equal mass and restricts the distribution of $F(\alpha, y)$ for values which are not on the boundary of two adjacent communities. We thus conclude that voting models impose J additional restrictions on the joint distribution of $F(\alpha, y)$. These restrictions allow us to constrain the conditional distribution of $F_y(\alpha)$ in the interior of $[\alpha_j(y), \alpha_{j+1}(y)]$.

### 3.2 Identification of the Parameters of the Indirect Utility Function

Let $\theta_0 = (B_0, \eta_0, \nu_0, \rho_0)$ denote the true parameter values under which the data were generated. So far we have assumed that $\theta_0$ is known to the econometrician. This is a strong assumption and needs to be relaxed. Given the functional form assumption used for the indirect utility function identification of $\nu_0$, $\eta_0$, and $B_0$ is straightforward if we observe housing consumption. Roy's identity implies that housing consumption is given by

$$h = B\, p^{\eta}\, y^{\nu}. \tag{12}$$

We thus conclude that the three parameters of the housing demand equation are identified from the observed joint distribution of housing and income. Moreover, it is easy to allow for more flexible functional forms in characterizing the demand for housing. The main problem encountered in generalizing the indirect utility function is that we need to maintain the single-crossing properties as we consider broader classes of indirect utility functions. We thus restrict our analysis to the simpler utility function given in equation (7).

Identification of $\rho_0$ is more problematic. If $\rho_0$ is unknown to the econometrician, the analysis above should be viewed as a conditional analysis in the sense that $\alpha_j(y)$ is really a function of $\rho$:

$$\alpha_j(y) \quad \equiv \quad \alpha_j(y|\rho) \tag{13}$$

Ascending bundles implies that for any $i > j$ we have that

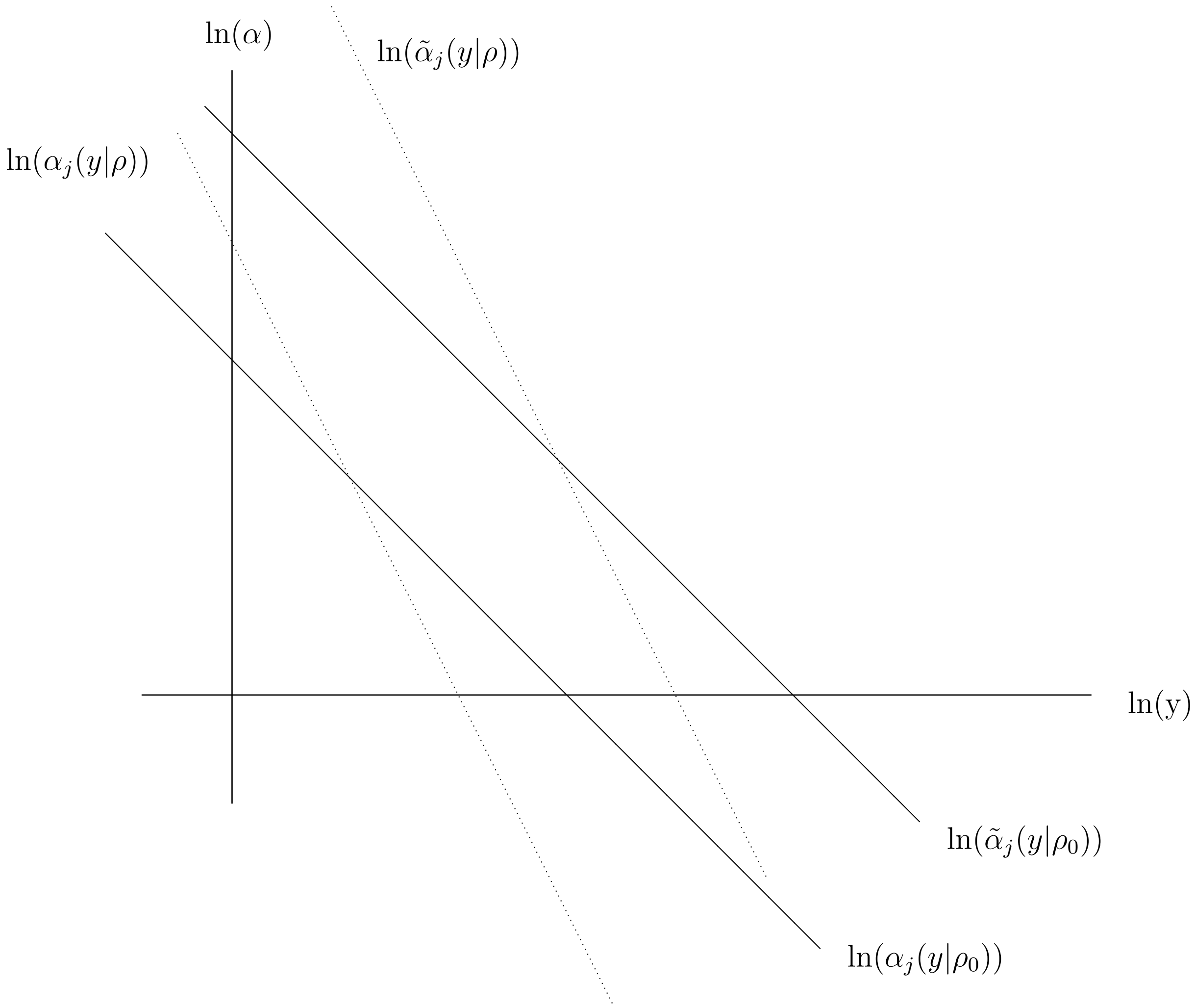
$$\alpha_i(y|\rho) \quad > \quad \alpha_j(y|\rho) \tag{14}$$

Unfortunately, single crossing implies that this condition will always hold no matter what negative value of $\rho$ we chose.

This problem is illustrated in Figure 4. In this example, we consider a single boundary indifference locus. Changing $\rho$ from $\rho_1$ to $\rho_2$ implies that we are rotating the boundary indifference locus. Holding the distribution of $(\alpha, y)$ constant such a shift implies a change in the populations of both communities. However, the key insight of this analysis is that we can always transform the conditional distribution $F_y(\alpha)$, such that the marginal distributions of income in each community are unchanged. For example, at the income level indicated by the dotted line, we just need to contract the conditional distribution of tastes to leave the two marginal income distributions unchanged. More formally, for each value of $\rho$ we can define a new distribution of $\alpha$ as follows:

$$G_y(\alpha_j(y|\rho)) \quad \equiv \quad F_y(\alpha_j(y|\rho_0)) \tag{15}$$

Figure 4: Identification of $\rho$



Then by construction, the observed equilibrium and sorting of households by income among communities for $(\rho, G_y(\alpha_j(y|\rho))$ is observationally equivalent to the one given by $(\rho_0, F_y(\alpha_j(y|\rho_0))$. The observed sorting of households by income, therefore, does not impose any restrictions on $\rho$ in a single cross-section of communities.

This lack of identification of $\rho_0$ is also illustrated in Figure 5. In this figure we plot the implied histograms which are identified of the observed sorting by income for the correct value $\rho_0$ as well as $0.5 * \rho_0$ and $2 * \rho_0$. We find that histograms evaluated at values different from $\rho_0$ are well defined. By construction, these histograms are consistent with the observed sorting by income and housing across communities.[8]

Moreover, imposing voting equilibrium conditions does not alter this conclusion. Stratification implies that $\alpha_j(y|\rho_0) < \tilde{\alpha}_j(y|\rho_0) < \alpha_j(y|\rho_0)$. The demand for public goods for individuals at the upper boundary must be larger than the demand for public goods of the decisive voter. The demand at the lower boundary must be less than the demand of the decisive voter. While it may appear that these equations impose some restrictions on the admissible set of $\rho's$, this is, unfortunately, not the case. The intuition is the following. A change of $\rho$ away from $\rho_0$ implies that we are rotating the boundary indifference loci as well the the loci of the decisive voters as illustrated in Figure 4. Since the slope of the boundary indifference locus equals the slope of the locus of the decisive voters, we have for any value of $\rho$:

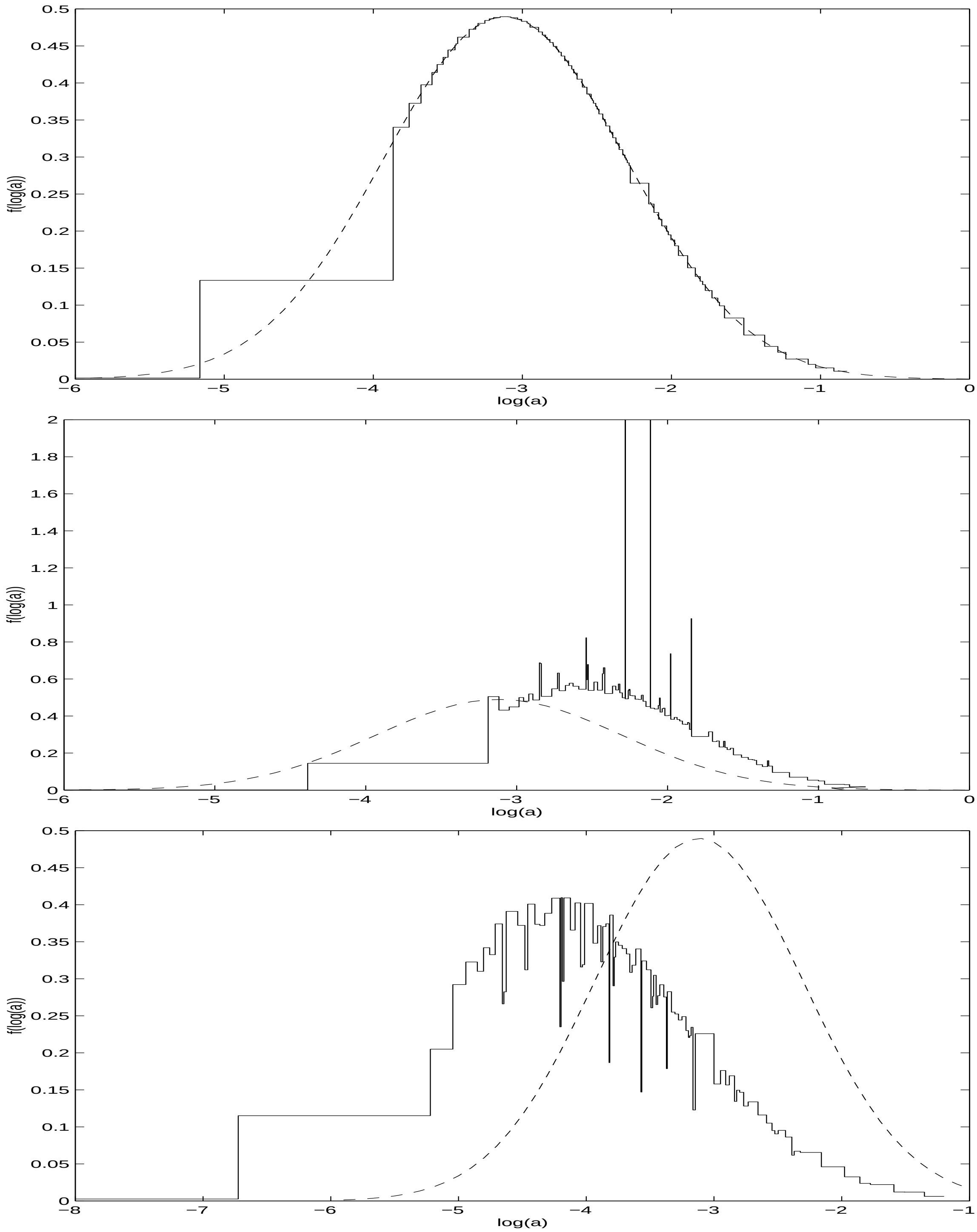
$$\alpha_{j-1}(y|\rho) \; < \; \tilde{\alpha}_j(y|\rho) \; < \; \alpha_j(y|\rho) \qquad j = 1, ..., J \tag{16}$$

Using a similar argument as above, we can always construct a distribution $G_y(\alpha|\rho)$ such that the new locus $\tilde{\alpha}_j(y|\rho)$ satisfies equation (5) and thus is a majority voting equilibrium. We thus conclude that imposing voting equilibrium restrictions does not help us with the identification of $\rho_0$, either.[9]

[8] We can also extend the analysis such that the histograms are also consistent with public good provision through majority rule. We thus conclude that $\rho_0$ is not identified under the set of assumptions imposed so far.

[9] Point identification of $\rho_0$ is possible if the econometrician observes the joint distribution of income and

Figure 5: Histograms as a Function of $\rho$



In order to get identification of $\rho_0$, we need to impose more structure on the problem. There are a number of approaches that one could take in order to reduce the admissible set of $\rho$ or obtain point identification of $\rho_0$. In the remaining parts of this section, we provide an informal analysis of these approaches. One approach imposes additional restrictions on the conditional distribution of $\alpha$. If tastes and income are independent, equation (10) simplifies to the following equation:

$$P_j(y) \quad = \quad F(\alpha_{j+1}(y)) - F(\alpha_j(y)) \tag{17}$$

The intuition for identification of $\rho_0$ is the following. We can still identify the conditional density functions of $\alpha$ as a function of $\rho$. If we evaluate, these conditional density functions at the correct value $\rho_0$, we should find that the conditional density function are all the same and equal the unconditional density function. If, however, we evaluate these density functions at values different from $\rho_0$, we will get a violation of independence.

More formally speaking, recall that $G_y(\alpha_j(y|\rho))$ is defined as:

$$G_y(\alpha_j(y|\rho)) \quad \equiv \quad F(\alpha_j(y|\rho_0)) \tag{18}$$

Fix the value of $\rho$ at $\hat{\rho}$ and consider two levels of income $y_1 < y_2$ and two communities $k < l$ such that:

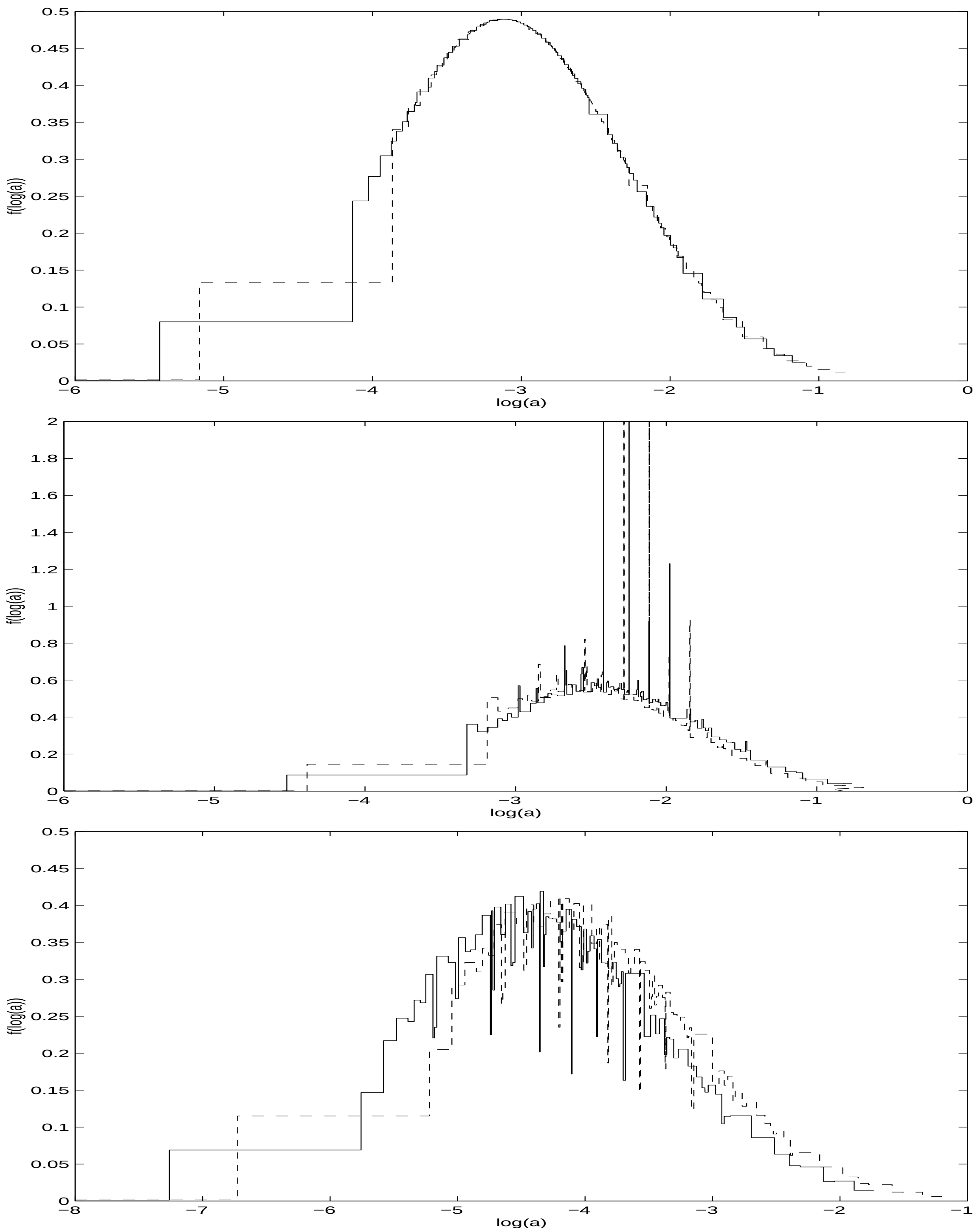
$$\alpha_k(y_1|\hat{\rho}) \quad = \quad \alpha_l(y_2|\hat{\rho}) \tag{19}$$

Equation (9) implies that $\alpha_j(y|\rho)$ is a nonlinear function in $\rho$. Thus we have:

$$\alpha_k(y_1|\rho_0) \quad \neq \quad \alpha_l(y_2|\rho_0) \tag{20}$$

(marginal) willingness to pay for a change in public good provision instead of voting outcomes. Similarly, we can construct bounds for $\rho_0$ if we can bound (marginal) willingness to pay. Sieg, Smith, Banzhaf, and Walsh (2001) provided a detailed discussion of how to conduct a welfare analysis based on these types of models.

Figure 6: Identification through Independence



which implies that

$$G_y(\alpha_k(y_1|\hat{\rho})) \quad \neq \quad G_y(\alpha_l(y_2|\hat{\rho})) \tag{21}$$

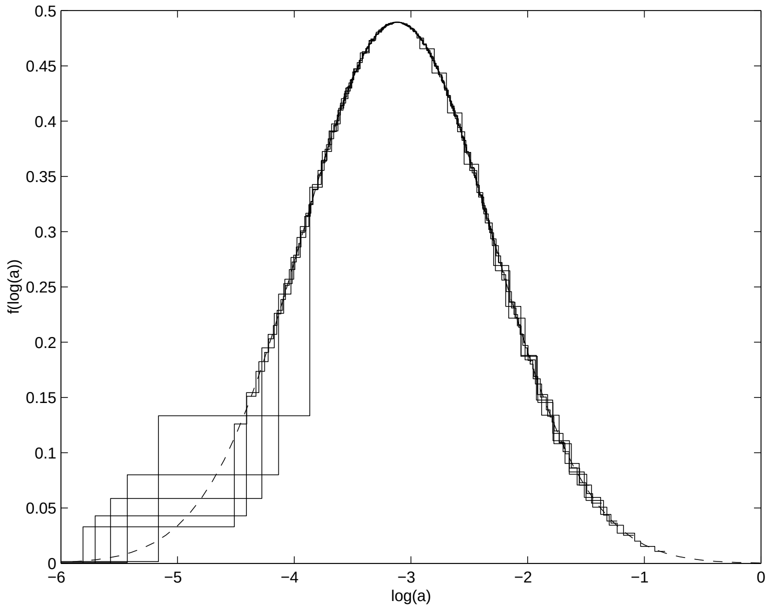
which is a violation of the independence assumption.

This result is illustrated in Figure 6 using our computational example. In each panel of this figure we plot the implied conditional distributions of $\alpha$ for $y = 23675$ and $y = 43675$. The three panels correspond to $\rho_0$, $0.5 * \rho_0$ and $2 * \rho_0$. The upper panel shows that the histograms of the two conditional distributions are virtually identical. The main difference between the two histograms is that they are evaluated at different points of $\alpha$. But they are estimates of the same distribution. In contrast, the two lower panels show the estimates at values of $\rho$ which are different from $\rho_0$. In that case, we get some significant differences between the two conditional density functions and hence a violation of independence.

Independence also allows us to identify the distribution of $\alpha$ at more than $J - 1$ points. We can combine histograms for different values of income. If tastes and income are independently distributed, these histograms are all estimators of the same unconditional density function. Moreover, they are evaluated at different points for different levels of income. By combining these histograms we can identify the distribution of $\alpha$ at a large number of points. To illustrate this point, we plot histograms for five levels of income ranging from 23675 to $100,000$. The results are illustrated in Figure 7.

Instead of assuming that income and tastes are independently distributed, one could also impose restrictions on the shape of the conditional distribution of $F_y(\alpha)$. For example, one could assume that the conditional distribution of tastes given income is unimodal. As seen in Figures 5 and 6, the conditional histograms evaluated at levels of $\rho$ different from $\rho_0$ are inherently very spiky. If one is willing to rule out these types of distributions, we conjecture that we should be able to get relatively tight bounds for $\rho_0$. We can also get point identification of $F_y(\alpha)$ if we use a parametric distribution of $(\alpha, y)$. As discussed in

Figure 7: Unconditional Distribution of Tastes



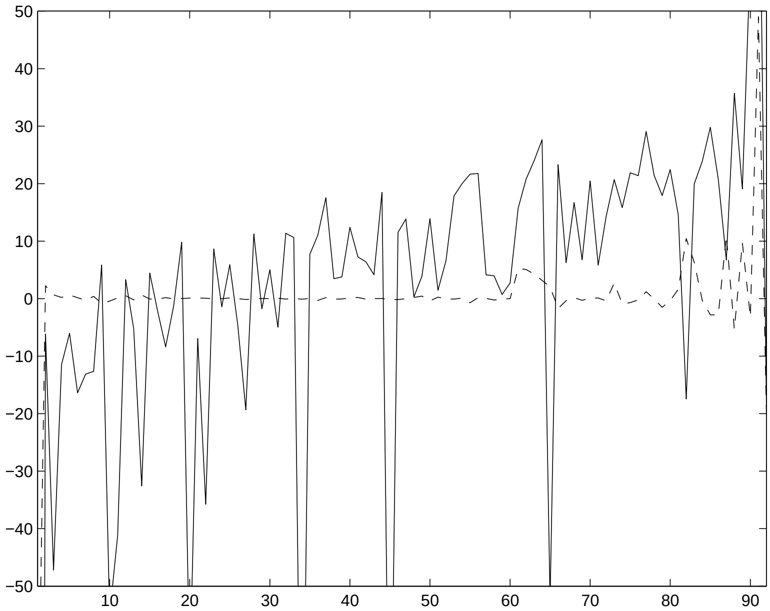

detail in Epple and Sieg (1999), $\rho_0$ is identified if $F(\ln(\alpha), \ln(y)) = N_2(\mu_y, \mu_\alpha, \lambda, \sigma_y^2, \sigma_\alpha^2)$.[10]

Independence of tastes and income is a strong assumption which is likely to be violated in many applications. We can achieve identification of $\rho_0$ without imposing independence if we observe locational equilibria at two successive points of time. If we assume that preferences remain constant, but income changes across time periods, then we obtain point identification of $\rho_0$. The intuition is the following. Conditional on $\rho$, the model is identified using the equilibrium allocation observed in the first period. We can thus predict the equilibrium in the second period as a function of $\rho$. Since the distribution of income changes between the two periods, the equilibrium in period 2 will be different from the equilibrium in period 1. Thus the observed sorting by income will be distinctly different in both periods. We will only be able to to predict the equilibrium sorting in period 2 at the correct value of $\rho_0$.

To illustrate, this point we compute a second equilibrium of the model which is identical

[10]The fact that the parametric model estimated in Epple and Sieg (1999) fits the observed income distributions fairly well, suggests that the bivariate normal model is, at minimum, a good benchmark for a more general semi-parametric analysis.

Figure 8: Prediction Error




to the one discussed in Section 2.2, except that we assume that mean income is 10 percent higher than in the baseline economy. We then predict the fraction of households with income at the median of the baseline model in each of the 92 communities using the conditional distribution of $\alpha$ evaluated at $\rho_0$ and the implied distribution at $2 * \rho_0$. In Figure 8 we plot the relative prediction error based on the approximation of $G_y(\alpha|\rho_0)$ (dashed line) and $G_y(\alpha|2*\rho_0)$ (solid line) . As we can see, we make substantial prediction errors if we predict the equilibrium using the incorrect value $2*\rho_0$. If we use the correct value $\rho_0$, the prediction error is small and only due to the fact that the conditional distribution of $\alpha$ is only identified in the baseline equilibrium at $J-1$ points. Without this discreteness problem, there would be no prediction error at $\rho_0$.

## 4 Semiparametric Estimation

So far, we have assumed that public good provision is observed without error. In practice it is likely that we will observe some measures of public good provision. However, our observed measures may not be perfect, but are subject to measurement error. Moreover, observed measures of public good provision are unlikely to satisfy the ascending bundles property due to measurement errors. In that case, identification of the parameters of the model is more complicated. To formalize these ideas, suppose, we observe, $\tilde{g}_j$, which is given by:

$$\tilde{g}_j = g_j + \epsilon_j \tag{22}$$

where $\epsilon_j$ denotes measurement error. In this case, identification of $F_y(\alpha)$ is only possible if we impose additional restrictions on the distribution of measurement error. Of course, it is not surprising that some such conditions are required in order to proceed when there is measurement error.

Ascending bundles implies that the housing prices and local public good provision are monotonic functions of the rank of the community, i.e. $p_i > p_j$ if and only if $g_i > g_j$. Thus the level of public good provision is monotonically increasing in the (price) rank of a community. Let us denote the rank of community $j$ by $r_j$. Hence ascending bundles implies that in equilibrium the following equation holds

$$g_j = g(r_j) \tag{23}$$

for some unknown function $g(\cdot)$. Substituting equation (23) into equation (22), we obtain

$$\tilde{g}_j = g(r_j) + \epsilon_j \tag{24}$$

Furthermore suppose that $E[\epsilon_j|r_j] = 0$, i.e. the measurement error in $g$ is uncorrelated with the rank of a community. In that case $g(r_j)$ is nonparametrically identified and can be consistently estimated using the techniques discussed in Matzkin (1994). Once we have nonparametrically identified $g(r_j)$ we are for all practical purposes again in the case in which we observe public good provision without error. We thus conclude that nonparametric identification of $g(r_j)$ implies nonparametric identification of $F_y(\alpha)$.

Suppose we observe the locational equilibrium in a metropolitan area at two distinct points of time. The following procedure can be used to estimate the model semiparametrically using a micro sample:

1. We estimate interjurisdictional housing prices for each community in both time periods using price index models as discussed in detail in Sieg et al. (2002). We then rank the communities in each time period by increasing housing prices.

2. We estimate the parameters $B$, $\nu$ and $\eta$ (equation (12)) by regressing housing expenditures on housing prices and incomes (assuming measurement error in housing).

3. We estimate $g_j$ in each time period by regressing observed measures of public good provision on a function of the community rank (equation (24)) using a series estimator which imposes the monotonicity restrictions implied by the ascending bundles property. We compute the predicted levels of public good provision for both time periods and characterize boundary indifference conditions as a function of $\rho$.

4. We then specify the joint distribution of income and tastes using a semi-nonparametric approach suggested by Gallant and Nychka (1987) and estimate the remaining parameters of the model by matching the observed income distributions of the communities in both periods with their predicted counterparts using EMM (Gallant and Tauchen, 1996) or simulated method of moments while imposing the restriction that the conditional distribution of tastes is the same in both periods.

This estimation procedure has a number of advantages over the estimation procedure developed in Epple and Sieg (1999). First, by implementing this estimation algorithm, we obtain a better fit of the observed income distributions. If we knew $\rho_0$, the distribution of $F_y(\alpha)$ is estimated such that we perfectly replicate the observed income distributions among communities. If we need to estimate $\rho$ and the model is correctly specified, we should be able to get close approximations of the income distributions in both time periods. If we cannot get good fits of the income distributions in both periods, then our model is likely to be misspecified. Second, the estimation procedure is computationally simpler. In particular, it does not require implementation of the computationally intensive inversion algorithm which constrains the community specific intercepts developed in Epple and Sieg (1999). Third, there is no need to implement the estimation procedure sequentially. Forth, we can also impose voting restrictions in step 4 of the estimation algorithm and estimate along the lines discussed in Epple et al. (2001) different voting models.

Finally, the estimation procedure can be extended to account for multiple public goods as long as there is no taste heterogeneity for these different components. Consider the following example. Suppose we also observe a vector of community specific amenities $x_j$. Suppose these amenities are measured without error. Furthermore assume that household preferences only depend on the linear index $g_j + x_j' \gamma$. In that case, we would replace equation (24) with the following equation

$$\tilde{g}_j \;=\; g(r_j) \;-\; x_j' \gamma \;+\; \epsilon_j \tag{25}$$

which can be estimated, for example, using techniques proposed by Robinson (1988).[11]

# 5 Non-Hierarchical Models

## 5.1 Horizontal Differentiation in Tastes for Public Goods

In the previous section, we have considered a strictly hierarchical economy with essentially one public good. These economies satisfy the ascending bundles property. All households agree on the ordering of communities. The price rank of a community must therefore equal its public good rank. In a more general model, we have multiple public goods. In those models, it is plausible that households have heterogeneous tastes for the different types

[11] This approach is also consistent with the interpretation that $\epsilon_j$ is unobserved community specific amenity.

of the public goods. For example, some households may have stronger tastes for public education, others are more concerned with protection from crime or environmental quality. Heterogeneity in tastes for public goods implies heterogeneity in preferences over community orderings which leads to violation of the ascending bundles property. In this section, we discuss how to allow for some heterogeneity in perceived community orderings.[12]

In principle, it is not hard to introduce heterogeneity for public goods into the analysis. In the previous section, we assumed that public good provision is one-dimensional. Suppose now we have two public goods $x_1$ and $x_2$. Suppose that $\gamma$ measures the relative weight that households of type $\gamma$ assign to the two public goods. One way to to allow for horizontal differentiation in preferences, is to assume that the level of public good provision as perceived by household of type $\gamma$ is given by:

$$g(\gamma) \quad = \quad g(x_1, x_2|\gamma) \tag{26}$$

Following Berry and Pakes (2002), we could treat $\gamma$ as a random variable with continuous joint density. In that case, household types are characterized by the tuple $(y, \alpha, \gamma)$. The main problem with this approach is that it is exceedingly hard to characterize household sorting in that case. In a model with $J$ communities, there are in principle $J!$ orderings of communities. Defining preferences over characteristics does not significantly reduce the complexity of the problem. For example, consider the case in which communities are characterized by a $K$ dimensional vector of characteristics and assume that preferences over characteristics are linear. If we do not impose any restriction on the set of preferences except linearity, the number of potential orderings of products is bounded by $J^{2(K-1)}$. Thus characterizing the composition of communities can be exceedingly cumbersome.[13]

A promising way to reduce the number of possible orderings of communities is to impose

[12] Some of the issues encountered in these models are discussed in the earlier literature on characteristics models due to Gorman (1980) and Lancaster (1966). See also the discussion in Heckman and Scheinkman (1987).

[13] Berry and Pakes (2002) provide a detailed analysis of the parametric model used in Berry, Levinsohn, and Pakes (1995) without the idiosyncratic errors. See also Bajari and Benkard (2002).

additional restrictions on the support of the distribution of $\gamma$. One way to do this is to use discrete distributions to model taste heterogeneity for public goods. Following Heckman and Singer (1984), we consider a model in which there are a finite number of discrete types of households, $i = 1, .., I$, which have preferences for public goods denoted by $\gamma_i$. The probability of each type is denoted by $P_i$. Under these assumptions public good provision is now type specific and can be written as:

$$g_{ij} = g(x_{1j}, x_{2j} | \gamma_i) \qquad i = 1, ..., I \tag{27}$$

Hence the number of possible orderings of communities in this specification of the model is at most $I$.

Introducing discrete taste heterogeneity for community specific amenities allows us to extend the framework developed in the previous section while at the same time maintaining a number of important features of the model. In particular, we will show that the extended model is a mixture of simple hierarchical models.

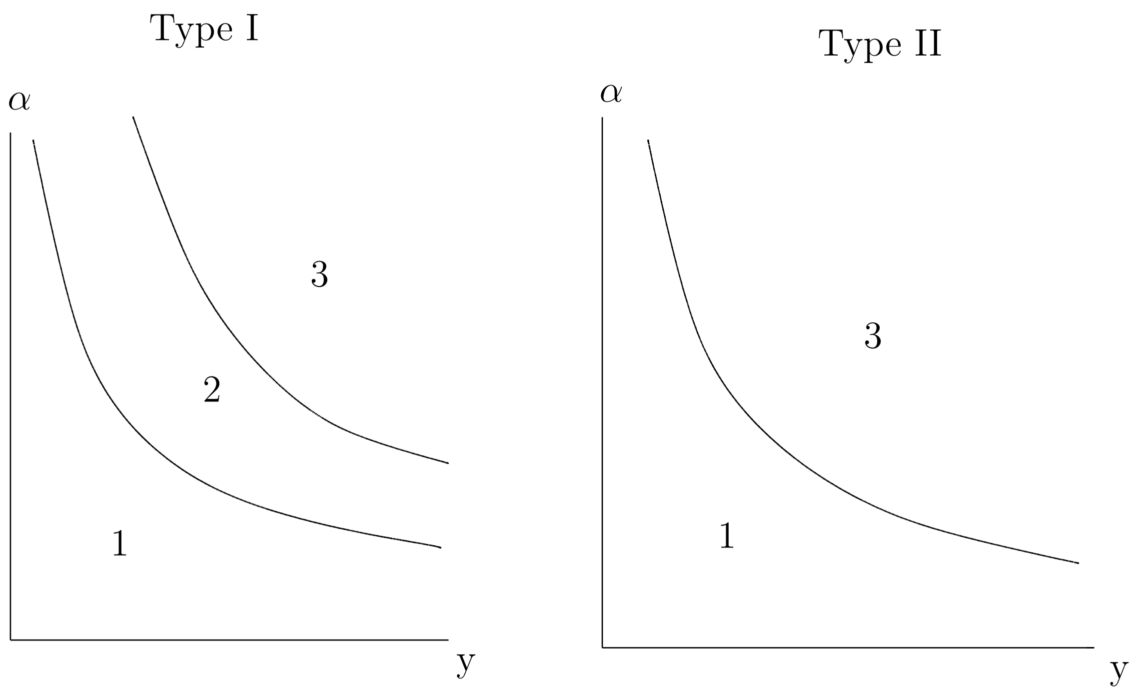
To characterize household sorting in this class of non-hierarchical models, we start with the observation that we can still order communities by ascending prices. Of course, the price rank of a community does not necessarily have to equal to the public good rank of household type $i$. Instead there may now be communities, for some type, which are strictly dominated by other communities. To illustrate this point consider two communities $j$ and $k$ such that $p_j < p_k$. Let us consider the case in which $g_{ij} < g_{ik}$. In this case, the pairwise boundary indifference condition is still well defined. Some households will prefer community $j$ others community $k$ depending on tastes and income. However, this is no longer the case if $g_{ij} > g_{ik}$. From the perspective of household type $i$ community $j$ strictly dominates community $k$: it has higher levels of public goods and lower housing prices. Thus our model predicts that no household of type $i$ would choose community $k$.

The argument above suggests that we need to characterize the effective choice set for each household type, before we can analyze household sorting. The effective choice set, denoted $S_i$, consists of all communities that are not dominated by other communities for type $i$:

$$S_i \quad = \quad \{ j : \nexists\, k \;\; \text{s.t.} \quad V(\alpha, y, g_{ik}, p_k) > V(\alpha, y, g_{ij}, p_j) \;\; \forall \, (\alpha, y) \, \} \tag{28}$$

In particular, a community $j$ is not in the effective choice set of type $i$, if there exists a $k$ such that $p_k < p_j$ and $g_{ik} > g_{ij}$. Furthermore, communities in the effective choice set of type $i$ can be ranked in hierarchical fashion. Sorting among communities in the effective choice set thus satisfies boundary indifference, stratification, and ascending bundles properties. We can therefore characterize sorting of households of type $i$ as discussed in detail in the hierarchical case, if we restrict our attention to communities in the effective choice set.

Figure 9: The Distribution of Households Types across Communities




This figure illustrates the distribution of households across communities.

To illustrate basic ideas consider the example shown in Figure 9. We assume that there are three communities ranked by increasing housing prices: $p_1 < p_2 < p_3$. There are two types of households. Type 1 has preferences such that $g_{11} < g_{12} < g_{13}$. Type 2's preferences imply that $g_{22} < g_{21} < g_{23}$. Consequently the effective choice sets are $S_1 = \{1, 2, 3\}$ and

$S_2 = \{1, 3\}$. Type 2 will not reside in equilibrium in community 2. The size of community 1 is given by the sum of the two lower triangles in Figure 9. In contrast the size of community 2 is just the integral between the two boundary indifference curves in the left panel of Figure 9.

In summary, there are two types of communities. The first community type is in the effective choice set and has a strictly positive population of type $i$ households. The other type of community is strictly dominated by a community in the effective choice set which has lower housing prices. These communities are not chosen in equilibrium by household type $i$.

To characterize choices of a household type $i$, we need to define the adjacent communities for each community in the effective choice set. Let $l(j)$ and $u(j)$ denote the adjacent lower and upper communities of community $j$.[14] In our example in Figure 9, we have for household type 1 the following mapping for the upper adjacent communities: $u(1) = 2$, $u(2) = 3$ , $u(3) = 4$. For household type 2, the mapping is $u(1) = 3$, and $u(3) = 4$. Similarly, we have $l(1) = 0$, $l(2) = 1$ , $l(3) = 2$ for type 1 and $l(1) = 0$, and $l(3) = 1$ for type 2.

One we have characterized adjacent communities in the effective choice set, we are in a position to characterize admission spaces for all communities. If community $j$ is in the effective choice set of type $i$, the set of households of type $i$ living in community $j$ is given by:

$$C_{ij} \;=\; \left\{(\alpha, y) \,\middle|\, K_{il(j)} \;+\; \ln(Q_j - Q_{l(j)}) \;\leq\; \ln(\alpha) \;-\; \rho \left(\frac{y^{1-\nu}-1}{1-\nu}\right) \;\leq\; K_{iu(j)} + (Q_{u(j)} - Q_j)\right\}$$

where

$$\begin{aligned} K_{il(1)} &= -\infty \\ K_{il(j)} &= -\ln(g^{\rho}_{il(j)} \;-\; g^{\rho}_{ij}) \\ K_{iu(j)} &= -\ln(g^{\rho}_{ij} \;-\; g^{\rho}_{iu(j)}) \end{aligned} \tag{29}$$

[14]For notational convenience, the lower adjacent community of the lowest community is denoted by 0 and the upper adjacent community of the highest community by $J + 1$.

$$K_{iu(J)} = \infty$$

If community $j$ is not in the effective choice set of type $i$, i.e. if $j \notin S_i$, we have:

$$C_{ij} = \emptyset \tag{30}$$

Suppose community $j$ is in the effective choice set of household type $i$. Let $\alpha_{i,l(j)}(y)$ characterize the boundary to the lower adjacent community in the effective choice set of household type $i$. Similarly, let $\alpha_{i,u(j)}(y)$ denote the boundary between community $j$ and the next higher community. Let $1_{j \in S_i}$ denote an indicator function which is equal to 1 if community $j$ is in the effective choice set of household type $i$ and 0 otherwise.

Then the size of the population of households of type $i$ with income $y$ living in community $j$ is given by:

$$\begin{aligned} P_{ij}(y) &= 1_{j \in S_i} \int_{\alpha_{i,l(j)}(y)}^{\alpha_{i,u(j)}} f_y^i(\alpha) \; d\alpha \\ &= 1_{j \in S_i} \left[ F_y^i(\alpha_{i,u(j)}(y)) \; - \; F_y^i(\alpha_{i,l(j)}(y)) \right] \end{aligned} \tag{31}$$

where $f_y^i(\alpha|y)$ is the density of $\alpha$ conditional on income and household type. Summing over all discrete types yields the total population of community $j$:

$$P_j(y) = \sum_{i=1}^{I} 1_{j \in S_i} \left[ F_y^i(\alpha_{i,u(j)}(y)) \; - \; F_y^i(\alpha_{i,l(j)}(y)) \right] P_i \tag{32}$$

Equation (32) thus suggests that the observed sorting of households by income can be characterized by a mixture of hierarchical sorting models.

## 5.2 Observed Types

Suppose that household types are observed. This will be the case if the household type is characterized by observed demographics such as race or age. In this case, we often observe the conditional distribution of income by type in each community. Hence we observe

$P_{ij}(y)$. The analysis of identification in this case is similar to the one discussed in Section 3. Equation (31) implies that we can identify the conditional distribution of $F_y^i(\alpha)$ at a finite number of points $J_i - 1$ where $J_i$ is the number of communities in the effective choice set of household type $i$.

The main difference to the analysis in Section 3 is that there is an additional parameter $\gamma_i$ which needs to be estimated. Identification of $\gamma_i$ comes from two sources. First, $\gamma_i$ determines the effective choice set of households which is observed by the econometrician if the household type is observed. Second, the boundary indifference loci $\alpha_{i,l(j)}(y)$ and $\alpha_{i,l(j)}(y)$ are functions of $\gamma_i$. Thus identification of $\gamma_i$ is closely related to identification of $\rho$. The same arguments that we used to establish identification of $\rho$ are sufficient to establish identification of $\gamma_i$.

### 5.3 Unobserved Types

If types are unobserved, the econometrician has less information that can be used to establish identification. In particular, the econometrician does not observe the right hand side of equation (31), but only the right hand side of (32). Suppose for simplicity the econometrician knows the parameters of the indirect utility function as well as the values of $\gamma_i$ for $i = 1, ...I$. Furthermore assume that we observe prices and the two measures of public good provision without error. In that case, we can compute the effective choice sets of all household types and characterize the boundaries of adjacent communities. However, even in this case we do not get point identification of the distribution of the unobservables $F_y^i(\alpha)$ and $P_i$. Each value $P_j(y)$ is a linear combination of up to $I$ terms of the form $[F_y^i(\alpha_{i,u(j)}(y)) \; - \; F_y^i(\alpha_{i,l(j)}(y))] \; P_i$. We thus conclude that in general we cannot identify the $F_y^i(\alpha)$ and $P_i$ separately from the observed sorting of households by income among communities. We thus conclude that the general non-hierarchical model with unobserved types is non-parametrically not identified.

One way to achieve identification is to impose stronger assumptions on the joint distribution of tastes and income. If income and tastes are independently distributed, equation

(32) simplifies to the following equation:

$$P_j(y) \;=\; \sum_{i=1}^{I} 1_{j \in S_i} \, [F^i(\alpha_{i,u(j)}(y)) \;-\; F^i(\alpha_{i,l(j)}(y))] \, P_i \tag{33}$$

We are trying to infer I conditional distributions of $\alpha$ and $I$ parameters characterizing the distribution of $\gamma$ from J observed income distributions. We conjecture that if $J > I$ nonparametric identification may be possible. But do not have a proof for this conjecture.

# 6 Conclusions

In the first part of this paper we have discussed identification of hierarchical models of locational equilibrium in a semiparametric framework. We have shown that the distribution of tastes conditional on income is identified at J-1 points if (1) the parameters of the indirect utility function of households are known to the econometrician; (2) the parameters of utility are not known, but we observe equilibria in two periods; (3) the parameters of the indirect utility function are not known, but tastes and income are independently distributed; or (4) the parameters of the indirect utility function are not known, but we also observe the joint distribution of income and (marginal) willingness to pay for a change in local public good provision. In many applications observed measures of public good provision include measurement error. Nonparametric identification of the distribution of tastes conditional on income requires additional assumptions on the conditional distribution of the measurement error. The same moment restrictions which imply parametric identification in Epple and Sieg (1999) also imply nonparametric identification. We have provided a simple algorithm which can be used to estimate the parameters of the model.

Hierarchical locational equilibrium models are closely related to hedonic models pioneered by Rosen (1974). Recently, Ekeland et al. (2002) and Heckman et al. (2002) have developed a new approach for identification and estimation of hedonic models.[15] In hedonic

[15] Related issues are also discussed in Bartik (1987) and Epple (1987). Nesheim (2001) proposes a parametric approach for identification and estimating hedonic models which is based on a full-solution algorithm.

models, the choice space is continuous and preferences are smooth. Hence optimal choices can be characterized by standard first and second order conditions. The key insight of the two papers above is that the inherent nonlinearities of hedonic price functions are sufficient to identify certain classes of hedonic models based on the first order conditions of optimal behavior. In contrast to hedonic models, locational equilibrium models are based on the assumption, that there is only a finite number of communities. Thus optimal community choices are not characterized by tangency conditions, but by a sequence of inequality constraints. We have shown that this discreteness property of locational equilibrium models limits our ability to identify the conditional distribution of unobserved tastes. However, we can accurately approximate the conditional distribution of tastes, if the number of communities in the sample is reasonably large.

In the second part of the paper, we have introduced a broad class of non-hierarchical locational equilibrium models in which horizontal taste heterogeneity for public goods is introduced through discrete distributions characterizing household types. Non-hierarchical models raise issues that require additional theoretical development. In particular, when more than one good is publicly provided at the local level, existence of equilibrium under majority rule is problematic with simultaneous voting on the provision of all goods. Since differences in preferences are surely present in reality, a modeling strategy is needed that captures the way in which such differences are reconciled by local collective choice processes. A relatively natural approach is to view issues as being voted separately, with the outcome on each being taken as given when voting on the other.[16] The two most important locally provided public services, education and public safety, are typically provided by two distinct local governments, a school district and a municipality. Thus, expenditures on education and police are accurately characterized as being determined via voting in distinct elections. Other key factors affecting household choice of location, environmental (air) quality and proximity to places of employment, are exogenous to local governments and thus do not raise collective choice issues. These considerations lead us to the view that no intractable

[16]See Mackay and Weaver (1983) p. 619 for discussion of this and related modeling approaches. Existence with voting one issue at a time also depends on voter expectations if the outcome on one issue is expected by voters to affect the outcome of votes on other issues (Denzau and Mackay, 1981).

issues arise in characterizing equilibrium in non-hierarchical models.

We have shown that sorting of households in these models can be characterized by a mixture of hierarchical models. Our results indicate that identification is problematic if household types are unobserved. The observed distribution of households by income among the set of communities is not sufficient to achieve nonparametric point identification of the distribution of unobserved tastes. This result suggests that identification may have to rely on functional form assumptions. If the households differ by observed demographics such as race, age and household size, identification is possible. These models will allow us to study sorting of households within system of jurisdictions along many new dimensions. Estimation of these models will require access to restricted use data provided by the U.S. Census. Implementation of these model is an important area of future research.

The methods discussed above can also be used if households differ, for example, by unobserved tastes for housing consumption. The distribution of tastes for housing will be identified of the observed distribution of housing consumption and sorting of households among communities with different housing prices. More generally, we have not fully exploited the observed sorting of individuals by housing in the analysis. One drawback of the utility specification used in this paper is that housing is assumed to be separable of public goods. This simplifies the analysis of sorting by income in the hierarchical model. However, it also limits our ability to identify the distributions of unobserved tastes from the the observed to distribution of housing in non-hierarchical economies. It may be desirable to relax this assumption, especially in models with multiple public goods and horizontal taste differentiation. If housing consumption is only separable with respect to one of the two measures of public goods, it may be possible to achieve identification of both taste distributions from the joint distribution of income and housing across communities.

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