# Bargaining with Arrival of New Traders

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**Abstract**

A seller meets a potential buyer who has private information about her valuation of the asset. They bargain dynamically over the transaction price. The bargaining is affected by the possibility that some event occurs that would end the bargaining with some division of the surplus. The arrivals of events are driven by exogenous Poisson processes. We characterize the unique stationary equilibrium of this game and in particular the dynamics of trade and prices in the limit as the time between offers goes to zero. As a general result we provide conditions for when the Coase conjecture does not hold. The main relevant factor is that the seller's expected payoff conditional on arrival of an event is sensitive to the buyer's value. We show that the expected time to trade is a non-monotonic function of the arrival rate. Applying the model to arrival of a second trader (buyer or seller) with common value, we show how market tightness and thickness affect the division of surplus and the time to trade. When buyer valuations fall, average transaction prices drop and the time on the market gets longer.

## 1 Introduction

We analyze a model of a thin market in which a seller bargains over a price of a non-divisible asset with a privately informed buyer. As bargaining continues, an external event can happen that influences the competitive positions of the players: a second buyer can arrive, a second seller can arrive or public information may get released. For example, suppose you have put your house on the market. So far only one buyer has expressed interest. He informs you that your original price is too high and asks you to reduce it. What do you consider before responding? Out of many factors that you may take into account two important ones are: 1) How likely it is that other serious buyers will show up in the short run. 2) How likely it is that if you wait to reduce the price, the current buyer will find another house and "disappear".

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Arrivals of new traders is a common feature of many markets (housing, labor, financial markets to name a few). A key characteristic of such markets is that trade/bargaining over price takes time and the bargaining dynamics are heavily influenced by the market conditions. For example, the asking price of a house takes time to drop, and how long it takes may depend on whether it is a "sellers' market" or a "buyers' market." In this paper we try to connect bargaining theory and demand-supply theory.

One of our main findings is that delay is a very natural consequence of bargaining in the context of a thin market: the possibility of arrival of new buyers (and subsequent competition between them) leads to delay in reaching compromise. In equilibrium the seller decreases prices over time, but does it slowly. This creates a non-trivial amount of delay even as bargaining frictions disappear and even though it is common knowledge that immediate trade is efficient. In contrast, the seminal papers by Gul, Sonnenschein and Wilson (1986) (GSW) and Fudenberg, Levine and Tirole (1986) (FLT) have shown that if the bargaining is insulated from external events then the equilibrium outcome exhibits Coasian dynamics (Coase 1972). That is, as bargaining frictions disappear, so that the seller is allowed to make offers very frequently, trade takes place with no delay.

That is not to say that there are no bargaining models that exhibit delay. For example, delay occurs in a model with two sided private information about fundamentals and overlap in values (e.g. Cramton 1984 or Cho 1990), with irrational players (Abreu and Gul 2000), with higher order beliefs (Feinberg and Skrzypacz 2005) with disagreement about continuation play (Yildiz 2004), with correlation between seller's cost and buyer's value (Evans 1989, Vincent 1989 and Deneckere and Liang 2006) or with the possibility that players can commit to delay (Admati and Perry 1987). The novelty is that even in the simplest FLT/GSW framework adding the possibility of arrival of the second buyer leads to delay. Such arrivals are a natural possibility of real-life transactions and hence can be a common reason for delay.

We characterize the equilibrium and analyze the determinants of delay and division of surplus. Interestingly, although the delay part of the Coase conjecture does not hold, the prediction about seller payoffs does hold: as bargaining frictions disappear the seller payoffs converge just to his outside option (of waiting for the arrival of the event). Moreover, we show that delay and surplus division are tied to how the seller's outside option changes with the buyer's valuation. The more sensitive the seller's (post arrival) payoff is to the current buyer's type, the longer, and hence more inefficient, the negotiations will be. For example, the less diverse are tastes of potential buyers, the longer the bargaining takes (see Section 4.2).

Delay seems to be cyclical in many markets. For example, in the housing market it is perceived that when prices are lower (a down market) houses stay longer on the market. In the labor market, a down market is usually correlated with longer unemployment spells. Standard demand-supply analysis does not have predictions for how prices should correlate with time on the market. Yet, such patterns seem to be consistent with our model. In particular, as we show in Section 4 a weaker distribution of the buyer values leads to lower prices and longer bargaining (even keeping the relative arrival rates of buyers and sellers constant).

Finally, we show how relative market conditions influence the division of surplus. In the full information bargaining models, such as Rubinstein (1982), the division of surplus is highly dependent on the relative impatience of players. In our model the potential arrival of outside events acts similar to discounting, introducing 'impatience' that depends on the market conditions (the seller may be impatient because he is afraid the buyer will find another house, the buyer may be impatient because he is afraid that another buyer will compete against him). The relative market conditions intuitively affect the division of surplus, creating a natural connection between bargaining and demand-supply analysis: prices are lower in buyer's markets (with relatively more sellers) and higher in seller's markets (with relatively more buyers).

The main intuition why there is delay in equilibrium is closely related to the bargaining models with interdependent values, as presented by Evans (1989), Vincent (1989) and Deneckere and Liang (2006). In these models the seller does not know the cost of supplying the asset and the buyer has private information that determines both the value and the cost. If the lowest possible value is below the average cost, delay must occur. The reasoning is that otherwise the individual rationality constraint of at least one of the agents would be violated: to satisfy buyer's IR with no delay implies prices not higher than the lowest buyer's value, but then the seller would lose money on average. In our model the seller knows his physical cost of delivering the asset, but the buyer has information about the (endogenous) opportunity cost: by trading today the seller forgoes the option to trade after an event arrives. As long as the post-arrival seller profits depend on the buyer type (which is the case whenever a competing buyer arrives), the delay is necessary. The main difference between our model and the previous work on bargaining with interdependent values is that the interdependence is created by market conditions and hence we can obtain interesting insights about thin markets. On the methodological side, we present most of the analysis in the continuous time limit, which greatly simplifies the analysis. We believe that this approach is promising for many other applications.

There are previous bargaining papers that allow for arrival of new traders (in particular buyers) without obtaining equilibrium delay. This difference in results is caused by different assumptions about post-arrival competition, mainly that the post-arrival profits do not depend on the current buyer type. For example, Inderst (2003) only allows the seller to choose whether to keep the original buyer or switch to the new one but if he does switch, then the value of the original value is irrelevant for his continuation value.[1] As a result, in his model the Coase conjecture continues to hold.

Our work is also related to other models of dynamic trade with asymmetric information and with no frictions, for example Noldeke and Van Damme (1990), Swinkels (1999) and Kremer and Skrzypacz (2006). The main difference between these papers and the setup we propose to analyze is that they

[1] The same happens in Alberto Trejos and Randall Wright (1995) where the newly arrived traders simply displace the old ones.

have monopoly only on one side of the market and competition on the other side, while we propose to study a temporary bilateral monopoly. Similarly to Kremer and Skrzypacz (2006) the outside events lead to delay in trade, but the equilibrium dynamics are very different (for example, in the current model we obtain perfect separation of types through type, while with competition on one side of the market such separation is not possible).

The paper is organized as follows: Section 2 presents the general model. Section 3 characterizes the equilibrium of the game in the continuous time limit. Section 4 studies how different properties of the market (tightness, thickness, etc) impact the bargaining dynamics. Extensions are discussed in Section 5 and Section 6 concludes. Most proofs are in the Appendix.

# 2 The Model

To describe the model, we start with an abstract description of a general bargaining game with arrival of new events. In Section 4 we analyze in detail the case when the events stand for the arrival of new traders.

## 2.1 General Bargaining

There is a seller and a buyer. The seller has an indivisible good (or asset) to sell. The buyer has a privately known type $v \in [0,1]$ that represents his value of the asset and that is distributed according to a *c.d.f.* $F(v)$ that has an atomless distribution and full support.

Time is discrete and periods have length $\Delta$. The timing within periods is as follows. In the beginning of the period an event $\omega \in \Omega$ arrives with probability $\Delta\lambda(\omega)$ that ends the game. There is a finite set of possible events, $\Omega$, and their (Poisson) arrival rates are exogenous.[2] For now, we treat these events as a reduced form of some continuation play. If no event arrives, the seller makes a price offer $p$. Then the buyer decides whether to accept this price or to reject it. If he accepts, the game ends. If he rejects, the game moves to the next period. A strategy of the seller is a mapping from the histories of rejected prices to current period price offers. A strategy of the buyer is a mapping from the history of rejected prices to an acceptance strategy (which specifies the set of prices that the buyer accepts in the current period).

The payoffs are as follows. If the game ends with the buyer accepting price $p$ at time $t$, then the seller's payoff is $e^{-rt}p$ and the buyer's payoff is $e^{-rt}(v-p)$, where $r$ is a common discount rate.[3] If

[2] In particular, we assume that the arrival rates $\lambda(\omega)$ are independent of $v$ and of the history of the game. Only one event can arrive in the game.

[3] We focus on the case $\Delta \to 0$, i.e. no bargaining frictions, so it is more convenient to count time in absolute terms rather than in periods. Period $n$ corresponds to real time $t = n\Delta$.

the game ends with event $\omega$ arriving at time $t$, then the payoffs are:

$$e^{-rt}W_{\omega}(v) \text{ for the buyer,}$$
$$e^{-rt}\Pi_{\omega}(v) \text{ for the seller.}$$

For notational convenience let $\Lambda = \sum \lambda(\omega)$ denote the total arrival rate of events. Also, let $W(v) = \sum_{\omega} \frac{\lambda(\omega)}{\Lambda} W_{\omega}(v)$ and $\Pi(v) = \sum_{\omega} \frac{\lambda(\omega)}{\Lambda} \Pi_{\omega}(v)$ denote the expected payoffs conditional on an arrival and the buyer's value being $v$. Finally, define $\overline{\Pi}(k) = \int_0^k \Pi(v) \frac{f(v)}{F(k)} dv$ which is the seller's expected payoff conditional on some event arriving immediately and types being distributed according to a truncated $F(v)$ over $v \in [0, k]$.

To justify the reduced-form payoffs consider the following example. Let the arrival represent a second buyer arriving and suppose the seller runs an English auction upon arrival. If values are independent, then $\Pi(v) = \int_0^1 \min\{x, v\} \, dF(x)$ and $W(v) = \int_0^v F(x) \, dx$. If the values are common, then $\Pi(v) = v$ and $W(v) = 0$. We provide additional examples later.

We assume:

**Assumption 1**

i) $\frac{e^{-\Delta r}\Delta\Lambda}{(1-e^{-\Delta r}(1-\Delta\Lambda))}(\Pi(v) + W(v)) < v$, so that immediate trade is efficient.[4]

ii) $W(v)$ is continuous and increasing, with $v - W(v)$ strictly increasing.

iii) $\Pi(v)$ is continuous, strictly increasing and differentiable.

iv) $\Pi(0) = W(0) = 0$, so that no player expects negative payoff regardless of the realized $v$.

These assumptions are not too restrictive and are satisfied in many environments (including the two examples above).[5]

**Stationary Equilibrium**

We focus on stationary equilibria that are characterized by the following strategies:

1. Buyer's acceptance rule $\kappa(p)$ that specifies the lowest type that accepts offer $p$. Since $v - W(v)$ is strictly increasing it can be easily shown that in any stationary equilibrium a *skimming property* holds so the buyer strategy can be described by the simple rule. (The skimming property means that if a type $v$ accepts offer $p$ then it is a strict best response for all types $v' > v$ to accept it as well. The stationarity requires that the cutoff be independent of the history of the game.)

2. Seller's pricing rule $P_{\Delta}(k)$ that specifies the price he offers given he believes remaining types are distributed according to a truncated $F(v)$ over $v \in [0, k]$.

These strategies imply the equilibrium path of prices and cutoff types, for example the first price

[4] A sufficient condition is $\Pi(v) + W(v) \leq v$.

[5] For comparison, as we discussed in the Introduction, Inderst's (2003) model violates (iii) because the outside option of the seller is not increasing in the current buyer's valuation in his environment.

is $P_\Delta(1)$, the next cutoff is $\kappa(P_\Delta(1))$ and so on. For clarity, let $k_{+\Delta} = \kappa(P_\Delta(k))$ denote next period cutoff given current cutoff $k$ and the strategies. Let $V_\Delta(k)$ be the expected continuation payoff of the seller given a cutoff $k$.

**Definition 1** *For a given $\Delta$, the continuous increasing function $V_\Delta(k)$ and the increasing, left-continuous functions $P_\Delta(k)$ and $\kappa(p)$ (the three functions depend implicitly on $\Delta$) are called a consistent collection if for all $k$ :*

*(i)*

$$V_\Delta(k) = \Delta\Lambda\overline{\Pi}(k) + (1-\Delta\Lambda)\left[P_\Delta(k)\frac{F(k)-F(k_{+\Delta})}{F(k)} + \frac{F(k_{+\Delta})}{F(k)}e^{-\Delta r}V_\Delta(k_{+\Delta})\right]$$

*where* $k_{+\Delta} = \kappa(P_\Delta(k))$.

*(ii)* $P_\Delta(k) \in \arg\max_p \left[p\frac{F(k)-F(\kappa(p))}{F(k)} + \frac{F(\kappa(p))}{F(k)}e^{-\Delta r}V_\Delta(\kappa(p))\right]$.

*(iii)* $\underbrace{k_{+\Delta} - P_\Delta(k)}_{\text{trade now}} = e^{-\Delta r}(\Delta\Lambda\underbrace{W(k_{+\Delta})}_{\text{arrival}} + (1-\Delta\Lambda)\underbrace{(k_{+\Delta} - P_\Delta(k_{+\Delta}))}_{\text{trade tomorrow}})$ *where* $k_{+\Delta} = \kappa(P_\Delta(k))$.

Condition (i) states that the expected continuation payoff given a cutoff $k$ is calculated using the equilibrium strategies $P_\Delta(k)$ and $\kappa(p)$. Condition (ii) captures the seller's lack of commitment: in every period he chooses price to maximize his payoff (instead of committing to a whole sequence of prices at time 0). Condition (iii) is the buyer's incentive compatibility constraint: the new cutoff type $k_{+\Delta}$ has to be indifferent between accepting $P_\Delta(k)$ today or trading next period at $P_\Delta(k_{+\Delta})$ (while facing the risk of arrival and getting $W(k_{+\Delta})$ instead). In other words, conditions (i) and (ii) guarantee that the seller plays a best response to $\kappa(p)$ and condition (iii) guarantees that the buyer plays a best response given the expected path of prices.

**Definition 2** *For a given $\Delta$, a stationary equilibrium is a consistent collection $V_\Delta(k)$, $P_\Delta(k)$ and $\kappa(p)$ and a probability distribution over first-period prices $H(p_0)$ such that*

*(i) if $p_0$ is in the support of prices offered in the initial period, then $p = p_0$ maximizes*

$$p(1-F(k_{+\Delta})) + F(k_{+\Delta})e^{-\Delta r}V_\Delta(k_{+\Delta})$$

*where* $k_{+\Delta} = \kappa(p)$

*(ii) At any $t > 0$, if $p^l$ is the minimum price offered for all times $\tau \in \{0, \Delta, 2\Delta ... t-\Delta\}$ (potentially off the equilibrium path) then at $t$ the seller offers price $p = P_\Delta(\kappa(p^l))$*[6]

*(iii) At any $t$ the buyer of type $k$ accepts price $p$ if $\kappa(p) \leq k$*

[6]Additionally, if the seller deviates to a price $p'$ such that $k' = \kappa(p')$ and yet $p' \neq \max\{p|\kappa(p) = k'\}$ (which can happen only if $\kappa(p)$ is constant over a range and is never a seller best response since he can increase the price without changing the probability of trade) then the seller randomizes between prices $p_1$ and $p_2$ to rationalize the acceptance of $p'$ by type $k'$. The prices $p_1$ and $p_2$ are the maximum and minimum elements of the seller maximization problem (ii) in Definition 1 given the cutoff $k'$.

The equilibrium strategies $P_\Delta(k)$ and $\kappa(p)$ implicitly define a monotonically decreasing step function $K_\Delta(t)$ which specifies the highest remaining type in equilibrium as a function of time. Similarly we can define the monotonically decreasing step function $T_\Delta(v)$ with $T_\Delta(1) = 0$ which specifies the time at which each type $v$ trades conditional on no arrival.

Note that the definition of equilibrium only allows the seller to mix in the initial period. This is without loss of generality as shown by Ausubel and Deneckere (1989) (AD henceforth) (AD Proposition 4.3). Conditions (ii) and (iii) specify that on path strategies follow the $P_\Delta(k)$ and $\kappa(p)$, and off-path are induced by returning to the path after any deviation.

**Remark 1** *The above definition of a stationary equilibrium follows Gul (2001). AD call these equilibria weak-Markov (and strong-Markov when $\kappa(p)$ is strictly increasing). The existence of these equilibria is proven in Fudenberg, Levine and Tirole (1985) and in AD for the game without arrival of events and in Deneckere and Liang (2006) in bargaining with interdependent values. These proofs can be extended to the present setup. Furthermore, since we are in the no-gap case these equilibria may not be unique.*[7]

The equilibrium strategies in discrete time are known to be in general analytically intractable (other than in special cases, for example for uniform distribution of values, see Stokey (1981)). In the Appendix we analyze the equilibria in discrete time and show that they all (even if they are not unique) converge to the same equilibrium path as $\Delta \to 0$.[8] In contrast, this continuous-time limit turns out to be relatively easy to characterize so that in the rest of the paper we focus on this limit which we call *the limit-equilibrium.*

As we show later in Theorem 2, the limit-equilibrium can be described by three functions $V(k)$, $P(k)$ and $K(t)$ that satisfy the continuous-time limits of the consistency requirements of the definitions above. Note that in discrete time we have used the $\kappa(p)$ function and now we use the $K(t)$ function. In discrete time $\kappa(p)$ combined with $P_\Delta(k)$ defines uniquely the time path of cutoffs $K_\Delta(t)$. However, as $\Delta \to 0$ the $\kappa^{-1}$ and $P_\Delta$ functions converge and $k_{+\Delta} \to k$. In the limit-equilibrium $P(k)$ plays a dual role: it is at the same time the price the seller asks given a cutoff $k$ and the reservation price of type $k$. Hence, $P(k)$ is not sufficient to determine the time at which type $k$ trades.

**Definition 3** *The strictly increasing and continuous functions $V(k)$ and $P(k)$ and a strictly decreasing, continuous function $K(t)$ are called a consistent limit-collection if for all $t$ such that $K(t) = k$ with $k \in [0,1]$ :*

*(i)* $rV(k) = \Lambda\left(\overline{\Pi}(k) - V(k)\right) + (P(k) - V(k))\frac{f(k)}{F(k)}\left(-\dot{K}\right) + V'(k)\dot{K}$

[7] Additionally, there exist non-stationary equilibria as shown by AD. Finally, if we assumed a gap case (i.e. $v > 0$), then we conjecture that like in the other papers one can prove that there exists a unique sequential equilibrium; that this equilibrium is stationary and as the gap converges to zero, this equilibrium converges to a stationary equilibrium of the game we analyze. We focus on the no gap case because the limit equilibrium is simpler in that case.

[8] The general intuition behind the proof is related to the uniform convergence of equilibria shown in AD.

*(ii)* $\dot{K}$ *solves*

$$\max_{\dot{K}\in(-\infty,0]} \Lambda\left(\overline{\Pi}(k)-V(k)\right)+(P(k)-V(k))\frac{f(k)}{F(k)}\left(-\dot{K}\right)+V'(k)\dot{K}$$

*and* $V(k) \geq \left[P\left(\tilde{k}\right)\frac{F(k)-F(\tilde{k})}{F(k)}+\frac{F(\tilde{k})}{F(k)}V\left(\tilde{k}\right)\right]$ *for all* $\tilde{k}\in[0,k]$
*(iii)* $(r+\Lambda)(k-P(k))=\Lambda W(k)-P'(k)\dot{K}$
*where* $\dot{K}$ *represents the right derivative of* $K(t)$ *and* $P'(k)$ *the left derivative of* $P(k)$*.*

Analogously to the discrete time conditions presented above, condition (i) states that the expected flow payoff given a cutoff $k$ is calculated using the limit-equilibrium strategies. Condition (ii) captures the seller's lack of commitment: in every instant he chooses the rate at which he trades with different types to maximize his payoff (instead of committing to a whole schedule at time at 0). Condition (iii) is the buyer's incentive compatibility constraint in the limit expressed in flow terms: the cutoff type $k$ has to be indifferent between accepting $P(k)$ today or trading at the next instant. The left hand side of (iii) captures the cost of delaying trade while the right hand side captures the benefits.

## 3 Characterization of the Limit-Equilibrium

In this Section we characterize the consistent limit-collection $V(k)$, $P(k)$ and $K(t)$ that satisfy Definition 3. At some points of the analysis it is convenient to use the continuous and strictly decreasing function $T(v)=K^{-1}(v)$ which specifies the (equilibrium path) time at which a buyer of type $v$ trades.

Formally, we work with conditions in Definition 3. Informally we refer to them as best responses in the limit as $\Delta\to 0$ and we call the limit-collection the limit-equilibrium. We show later in Theorem 2 that this informal approach is valid because not only the equilibrium conditions in Definition 1 converge to those in Definition 3 but also the all discrete-time equilibria converge to the limit-equilibrium.

**Seller's problem.** Recall from Definition 3 that the seller's problem in a limit-equilibrium is given by:

$$rV(k)=\max_{\dot{K}\in(-\infty,0]} \Lambda\left(\overline{\Pi}(k)-V(k)\right)+(P(k)-V(k))\frac{f(k)}{F(k)}\left(-\dot{K}\right)+V'(k)\dot{K} \tag{1}$$

This condition has a clear interpretation. The left-hand side is the expected equilibrium payoff expressed in flow terms. The right hand side represents the possible sources of the flow: upon arrival (which happens with a probability flow $\Lambda$) the game ends with the seller earning in expectation $\overline{\Pi}(k)$ (and as the game ends he forgoes $V(k)$). With a flow probability $\frac{f(k)}{F(k)}\left(-\dot{K}\right)$ the buyer accepts current offer, $P(k)$, which also ends the game. Finally, if the game does not end immediately, the continuation payoff drops, as the seller becomes more pessimistic about $v$, as captured by $V'(k)\dot{K}$.

Note that (1) is linear in $\dot{K}$, so that a continuous and strictly decreasing $T(v)$ can be consistent with equilibrium only if the seller is indifferent over all possible $\dot{K}$. This linearity is the source of Coasian dynamics when $\Lambda = 0$. In that case, for any strictly increasing $P(k)$ the seller wants to run down the demand function as fast as possible. Therefore the equilibrium $P(k)$ in the limit becomes flat at 0. The outside option in our model provides a counterbalance for the seller's temptation to run down the demand curve, leading to a strictly downward-sloping $P(k)$. But we still require that the coefficients on $\dot{K}$ add up to 0:

$$
\begin{aligned}
(P(k) - V(k)) \frac{f(k)}{F(k)} &= V'(k) \\
&\Updownarrow \\
f(k) P(k) &= \frac{\partial}{\partial k} [V(k) F(k)]
\end{aligned}
$$

Furthermore, if indeed the seller is indifferent over all $\dot{K}$ it is easy to calculate $V(k)$ by evaluating a strategy of $\dot{K} = 0$, that is of asking high enough prices to avoid trade until the arrival:

$$V(k) = \frac{\Lambda}{\Lambda + r} \overline{\Pi}(k) \tag{2}$$

That implies that the equilibrium prices have to satisfy:

$$P(k) = \frac{\Lambda}{\Lambda + r} \Pi(k) \tag{3}$$

That pins down the unique candidate for the limit-equilibrium $P(k)$ and $V(k)$. Note that $P(k)$ represents both the prices asked by the seller given a threshold $k$ and the reservation prices the buyer with value $k$ (in discrete time these are a bit different, but in the limit these two coincide) and that since $\Pi(v)$ is strictly increasing so is $P(k)$.

**Buyer's problem.** We now turn to the buyer's best response problem and show how condition (iii) of the consistent limit-collection definition is derived. Then we use this condition to determine the time at which each type trades in the limit-equilibrium, a unique candidate $K(t)$ (and the corresponding $T(v)$).

Start by denoting by $B(v)$ the expected payoff of buyer with value $v$ (at the beginning of the game). Looking at the direct-revelation representation of the buyer's strategy, he plays a best response if and only if:

$$B(v) = \max_{v'} e^{-(r+\Lambda)T(v')} \left(v - P(v')\right) + \int_0^{T(v')} \Lambda W(v) e^{-(\Lambda + r)s} ds \tag{4}$$

and $v' = v$ is a solution to this problem. In words, the buyer can mimic another type $v'$ to trade at a different price, $P(v')$, and at the corresponding time $T(v')$. The first part on the RHS reflects the surplus from trading before the arrival of an event and the second part stands for the possibility that the arrival happens before $T(v')$.[9] Taking the first order condition of this expression we obtain the condition (iii) of the consistent limit-collection:[10]

$$(r+\Lambda)(k-P(k)) = \Lambda W(k) - P'(k)\dot{K} \tag{5}$$

As discussed before, the LHS is the cost of delaying trade (due to discounting and possibility of arrival) and the RHS is the benefit of waiting consisting of the reduction of price and the payoffs from a possible arrival.

Since we have pinned down $P(k)$ from the seller's problem, we use the buyer's indifference condition to characterize $K(t)$. Substituting the candidate $P(k)$, (3) in (5) yields:

$$-\dot{K} = (\Lambda+r)\frac{(r+\Lambda)K(t) - \Lambda(\Pi(K(t)) + W(K(t)))}{\Lambda\Pi'(K(t))} \tag{6}$$

which together with a boundary condition $K(0)=1$ pins down uniquely a candidate $K(t)$. Since we have assumed $\frac{\Lambda}{\Lambda+r}(\Pi(v)+W(v)) < v$, the numerator is strictly positive for all $K(t)>0$ and the since $\Pi'(v)>0$, so is the denominator. Therefore this differential equation yields a strictly decreasing continuous $K(t)$.

**Summarizing:**

**Theorem 1** *There exists a unique consistent limit-collection* $V(k)$, $P(k)$, $K(t)$. *It is characterized by* (2), (3), (6) *and the boundary condition* $K(0)=1$.

Additionally the buyer's expected payoff (at time 0) can be found using either directly $\{T(v), P(v)\}$, or applying the envelope theorem to problem (4):

$$B'(v) = e^{-(r+\Lambda)T(v)} + \frac{\Lambda}{\Lambda+r}\left(1-e^{-(r+\Lambda)T(v)}\right)W'(v)$$

and using the boundary condition $B(0)=0$.

Although we find the limit-equilibrium conditions intuitive in their own right, in the Appendix we show that the equilibrium values and paths we found can be also formally derived as limits of equilibrium values and strategies in discrete time:

**Theorem 2** *For any sequence of stationary equilibria as* $\Delta \to 0$, *the consistent collections that form these equilibria converge to the consistent limit-collection described in Theorem 1. That is, as* $\Delta \to$

[9] This expression looks at the problem at time 0, but if it is satisfied, due to the stationarity of the equilibrium (and the environment), the off-equilibrium incentives are satisfied as well.

[10] The local incentive compatibility conditions are sufficient because when $t(v)$ is strictly decreasing then the buyer's objective function (4) is supermodular in $v'$ and $v$.

0, $V_\Delta(k) \to V(k)$, $P_\Delta(k) \to P(k)$ *and the time at which each type trades* $T_\Delta(v) \to T(v)$ *(all convergences are point-wise).*

This result shows that focusing the analysis on the limit-equilibria not only is convenient in terms of analytical tractability but that it also is consistent with the discrete time equilibria since all discrete time equilibria converge to a unique limit-equilibrium.

## 3.1 Properties of the Limit-Equilibrium.

We now present some general properties of the limit-equilibrium.

In the preavious section we characterized $T(v)$, the time at which type $v$ trades condition on no arrival. We can further define the expected time at which type $v$ trades $\tau(v)$ which is given by taking into account the possibility that arrival takes place before $T(v)$.

$$\begin{aligned}\tau(v) &= \underbrace{\left(\int_0^{T(v)} \Lambda e^{-\Lambda s} ds\right)}_{\Pr\ arrival\ before\ T(v)} \times \underbrace{\left(\int_0^{T(v)} s \frac{\Lambda e^{-\Lambda s}}{1-e^{-\Lambda T(v)}} ds\right)}_{E[arrival\ time\ |<T(v)]} \\ &\quad + \underbrace{\left(1-\int_0^{T(v)} \Lambda e^{-\Lambda s} ds\right)}_{\Pr\ no\ arrival\ before\ T(v)} \underbrace{T(v)}_{\substack{\text{time to trade conditional} \\ \text{on no arrival}}}\end{aligned}$$

Finally, we can define the (unconditional) expected time to trade as $\int_0^1 \tau(v)\, dF(v)$.

**Proposition 1**

*(i) (Delay): For all* $0 < \Lambda < \infty$ *the expected time to trade is strictly positive.*

*(ii) (Coase conjecture): as* $\Lambda \to 0$, *the expected time to trade and transaction prices converge to* 0 *for all types (i.e.* $T(v) \to 0$ *and* $P(k) \to 0$*).*

*(iii) (Competition Effect): as* $\Lambda \to \infty$ *the expected time to trade converges to zero.*

Part (i) shows that when the bargaining is subject to external influences, delay is to be expected, which is our first main result. It follows directly from our characterization, but the key force behind delay is that $\Pi(v) > 0$. The intuition is as follows: suppose that there is no delay. Then the transaction prices for all types have to be close to zero. But then the seller's payoff would be close to zero as well, less than $\frac{\Lambda}{\Lambda+r}\overline{\Pi}(k)$. But that leads to a contradiction since the seller can guarantee himself that by just waiting for the arrival of the event.

Part (ii) shows that our limit-equilibrium converges to the equilibria in GSW and FLT: as we take the probability of arrivals to zero (convergence of the model) trade takes place immediately and the buyer captures all the surplus (convergence of equilibrium). Finally, part (iii) states that if the events arrive with a very high frequency, the game ends immediately due to the exogenous arrival.

Combining the three parts shows that the expected time on the market is non-monotonic in the arrival of events. When there are very few traders or very many traders the expected time to trade is very small, but in the intermediate case of thin markets the delay can be large.

The next proposition characterizes how the time on the market and the ex-ante expected payoffs depend on the distribution of values:

**Proposition 2** *Suppose* $\Pi(v)$ *and* $W(v)$ *are independent of the distribution of values,* $F(v)$ *.*
*(i) The limit-equilibrium* $P(k)$ *and* $K(t)$ *are independent of the distribution of values,* $F(v)$*.*
*(ii)(Weak markets and time on the market) Consider two distributions of buyer's values* $F$ *and* $H$ *such that* $F$ *first order stochastically dominates* $H$*. The expected time to trade is longer if the distribution of values is* $H$ *(and average prices are lower).*
*(iii) (Dispersion of values and efficiency of trade) Consider two distributions of values* $F$ *and* $H$ *such that* $F$ *second order stochastically dominates* $H$*. Then the ex-ante expected sum of payoffs is higher under distribution of values* $H$*.*

To illustrate this surprising result (that the equilibrium $P(k)$ and $K(t)$ are independent of the distribution of values) consider the following example. Suppose that the event represents an arrival of one more buyer who has the same valuation as the original buyer. Upon arrival the seller runs an English auction.[11] As a result, $\Pi(v) = v$ and $W(v) = 0$ independently of the distribution. In that case, the proposition states that the equilibrium path of prices $P(k)$ and the times at each type trades, $T(v)$ are independent of $F(v)$![12]

The intuition behind this result is as follows. First, since the inability to commit and the associated requirement of time-consistency drives the expected payoff of the seller down to his outside option for any $k$ ($V(k) = \frac{\Lambda}{\Lambda+r}\overline{\Pi}(k)$) it has to be that prices satisfy $P(k) = \frac{\Lambda}{\Lambda+r}\Pi(k)$. They depend only on the current cutoff and not the whole distribution (unless $\Pi(v)$ does). Second, $K(t)$ is pinned down by the indifference condition of the buyers. Since the current marginal buyer's incentives do not depend on the distribution (unless $W(v)$ does) the limit-equilibrium is independent of $F(v)$. Clearly low valuation buyers would like the seller not to spend time sorting through high types. The problem is that they have no credible way in which to signal to the seller that they have a low value.

Arrival of new traders or outside options is necessary for delay but another important ingredient is that the seller's outside value depends on the buyer's type. In particular, we can establish the following general comparative statics:

[11] Alternatively, the event can represent an arrival of information that reveals the value of the buyer allowing the seller to capture all the surplus.

[12] If instead the second buyer's value were independent but distributed identically to the first buyer's value then: $W(v) = F(v)(v - E[V_2|V_2 \leq v])$ and $\Pi(v) = F(v)E[V_2|V_2 \leq v] + (1 - F(v))v$. And therefore the equilibrium depends on $F(v)$.

**Proposition 3**

*(i) Consider two environments, one with* $\Pi_1(v)$ *and the other with* $\Pi_2(v)$ *and either* $W_1(v) = W_2(v)$ *or* $\Pi_1(v) + W_1(v) = \Pi_2(v) + W_2(v)$. *Then if* $\Pi_1'(v) \geq \Pi_2'(v)$ $\forall v$, *the expected time to trade is shorter in the environment with* $\Pi_2(v)$.

*(ii) In the limit as* $\Pi'(v) \to 0$ $\forall v$, *expected time to trade converges to zero and the buyer asymptotically captures all the surplus.*

The second part of this Proposition shows that the Coase conjecture holds in the limit as $\Pi'(v) \to 0$ $\forall v$. Given our assumption $\Pi(0) = 0$, $\Pi'(v) \to 0$ $\forall v$ implies that $\Pi(v) \to 0$ $\forall v$. To separate slope versus level effects consider the case where $\Pi'(v) = 0$ *but* $\Pi(v) = c > 0$ $\forall v$. In this case, in equilibrium the seller offers price $p = \frac{\Lambda}{\Lambda+r}c$ and either trade happens immediately or there is no trade until arrival. For there to be trade with delay it is necessary that $\Pi'(v) > 0$. Intuitively, the seller makes a first offer $p = \frac{\Lambda}{\Lambda+r}\Pi(1)$. Since this offer is accepted by the highest types of the buyer, the seller's outside option decreases a bit and next period he is willing to make lower offers. In this way he slowly skims through all buyer types.

But why does it happen slowly? Why don't we get almost immediately to $p = 0$, like in the Coase conjecture? The reason is that if the seller ran 'the clock' too fast then some buyer types would have an incentive to wait for a lower price - their reservation prices would decrease. But then the seller would prefer to stop trading, since he would get a higher expected payoff from just waiting for an arrival than from trading at these low prices. On the other hand, the seller cannot run too slowly through the demand either, since then the reservation prices would be so high, that the seller would prefer to collect the whole area below the demand *before* the arrival. Therefore the speed at which price decreases has to be such that the reservation prices of the buyer keep the balance between the incentive to speed up and slow down the trade.

Following this logic, if $\Pi_1'(v) \geq \Pi_2'(v)$ $\forall v$, then under $\Pi_1$ the seller's outside option drops faster as his belief of the current buyer cutoff type falls. This makes him offer lower prices at $k' = v - \varepsilon$, that is prices as a function of $k$ decrease at a faster rate for the steeper $\Pi(v)$. Hence, if the seller ran the clock (with respect to $K(t)$) at the same speed, prices would drop faster in time under $\Pi_1$. But then the buyers would have an incentive to wait for lower prices, leading to a contradiction that the $k$ changes through time. To keep the current cutoff types willing to trade at the current prices the seller has to go through the types slower under $\Pi_1$, as claimed in the first part of the last proposition.

This result allows us to compare our dynamics to existing literature. For example, in Inderst (2003) (and other papers that have the new buyers replace the existing buyer), $\Pi'(v) = 0$ and there is no delay.[13] Taking the limit $\Pi'(v) \to 0$ in our model leads to the same limiting outcome. It is essential for there to be delay that the outside value of the seller depends on the buyer's type. The more sensitive the outside value of the seller to the buyer's private information, the greater the delay/inefficiency. As

[13] Inderst (2003) assumes that upon the second buyer arriving, the seller can only choose to keep on bargaining with the current buyer or switch to bilateral bargaining with the new one, which implies $\Pi'(v) = 0$ in his model.

we explained in the Introduction, the correlation of the seller's outside option with the buyer value endogenously creates a bargaining environment with interdependent values, as studied by Evans (1989), Vincent (1989) and Deneckere and Liang (2006), and hence the main economic intuition behind the delay is similar to that in those papers.

# 4 Arrival of New Traders

In this section we return to the original motivation of studying thin markets. In particular, we apply the general model to analyze bargaining dynamics when new traders can arrive to the market. We are mostly interested in the determinants of delay/time on the market and the division of surplus.

## 4.1 Common Value

Suppose that there are two events that can happen: either a second seller with an identical good arrives or a second buyer with identical valuation arrives (we call it *the common value* case). The arrival rates are $\lambda(\omega_s) = \lambda_s$ and $\lambda(\omega_b) = \lambda_b$. If the second buyer arrives, the game ends with the seller capturing all the surplus. If the second seller arrives, the sellers compete away all surplus, selling the asset at 0. In this context we can refer to $\Lambda = \lambda_b + \lambda_s$ as the market thickness: how frequently new traders are found. The larger $\Lambda$, the thicker the market. We can also use $\frac{\lambda_b}{\lambda_s}$ as a measure of relative market tightness. It is a buyers' market if $\frac{\lambda_b}{\lambda_s}$ is low, so that sellers are abundant relative to buyers.

The expected payoffs conditional on arrival in this case are:

$$W(v) = \frac{\lambda_s}{\Lambda} v \ , \ \Pi(v) = \frac{\lambda_b}{\Lambda} v \ .$$

and clearly they satisfy Assumption 1 of the general model.

Using the equilibrium conditions (3) and (6) we can calculate the limit-equilibrium $P(k)$ and $T(v) = K^{-1}(v)$ in a closed form:

$$P(v) = \frac{\lambda_b}{\Lambda + r} v \ , \qquad T(v) = -\frac{\lambda_b}{r(\Lambda + r)} \ln v \ . \tag{7}$$

The corresponding value functions are:

$$\begin{aligned} V(k) &= \frac{\lambda_b}{\Lambda + r} \int_0^k v \frac{f(v)}{F(k)} dv \ , \qquad (8) \\ B(v) &= \frac{\lambda_s v + r v^{\frac{\lambda_b + r}{r}}}{\Lambda + r} . \end{aligned}$$

(Recall that $B(v)$ is the expected payoff of the buyer of type $v$ at the beginning of the game while

$V(k)$ is the expected payoff of the seller given current cutoff $k$).

Using the general characterization of the limit-equilibrium we obtain the following corollary that describes the effects of arrival rates on the outcomes:

**Corollary 1** *In the limit-equilibrium of the model with entry on both sides of the market and common value:*
*i) (Market Tightness) Keeping* $\Lambda = \lambda_b + \lambda_s$ *constant (the sum of arrival rates of the second buyer and seller), a decrease in the ratio* $\frac{\lambda_b}{\lambda_s}$*, implies a shorter equilibrium time on the market, a lower seller's expected payoff and a higher buyer's payoff. In the limit, as* $\frac{\lambda_b}{\lambda_s} \to 0$ *we get immediate trade with the buyer capturing all the surplus.*
*ii) (Market Thickness) Keeping the ratio* $\frac{\lambda_b}{\lambda_s}$ *fixed, delay is non-monotonic in the sum* $\lambda_b + \lambda_s$*. It converges to zero as* $\lambda_b + \lambda_s \to \infty$ *and also as* $\lambda_b + \lambda_s \to 0,$ *while it is greater than zero for intermediate values.*

These results can be useful for understanding the impact of market makers on the market equilibrium.
The first result shows that trade is more efficient when it is a buyers' market. This is because the higher the likelihood of arrival of the second seller, the more impatient the current seller gets, which makes him offer lower prices. In the limit, if only new sellers can arrive then trade takes place immediately and the buyers capture all the surplus as in FLT or GSW.

The second result shows that the efficiency is not monotonic in the liquidity of the market. In the limit as we approach perfect competition ($\lambda_b + \lambda_s \to \infty$) trade takes place immediately. Trade is also immediate when there is a bilateral monopoly with no possibility of arrival. But when we have a thin market there is some delay in trade.

Notice that in line with Proposition 2 the equilibrium $P(k)$ and $K(t)$ in (7) are independent of the distribution $F(v)$. Does it mean that the distribution of values has no impact on the expected trade dynamics? No. In fact, as a corollary to Proposition 2 we get the following results about the impact of the distribution on efficiency and time to trade:

**Corollary 2** *In the limit-equilibrium of the model with entry on both sides of the market and common value:*
*i) (Weak markets and time on the market) Consider two distributions of buyer's values* $F$ *and* $H$ *such that* $F$ *first order stochastically dominates* $H$*. The expected time to trade is longer if the distribution of values is* $H$ *(and average prices are lower).*
*ii) (Dispersion of values and efficiency of trade) Consider two distributions of values* $F$ *and* $H$ *such that* $F$ *second order stochastically dominates* $H$*. Then the ex-ante expected sum of payoffs is higher under distribution of values* $H$ *but the expected time to trade is lower under* $F$*.*

These results can be derived directly from the expressions (7) and (8) by noting that $T(v)$ is decreasing and convex, the $V(1)$ depends only on the average $v$ and $B(v)$ is convex in $v$.[14]

This proposition points to an interesting finding that trade takes longer in markets with weaker distributions of valuations. This could help explain some of the cyclical patterns in real estate markets and in labor markets. In a downturn prices in both these markets respond sluggishly but both time to sale and exit out of unemployment tend to respond much more significantly:

> "One important point to keep in mind, however, is that while declines in nominal house prices are relatively rare, the volume of housing market transactions tends to be more responsive to a slowing economy. A flattening out of an observed price series may in fact mask a buildup of inventory of unsold houses." FRBSF Economic Letter 2002-13; May 3, 2002.

More research (theoretical and empirical) is needed to determine if our model provides relevant insights about the causes of these cyclical patterns.

## 4.2 Taste Diversity and Time on the Market.

So far we have assumed that the two buyers have the same valuation. In many markets, however, it is natural to think that there are different groups of potential buyers of the asset, and that even though valuations within a group can be very similar, they would differ across groups quite a bit. For example, families with school age children could be one group with similar valuations for a given house. The group of retirees, on the other hand, could value the same house differently. The first group would put more weight on the quality of the school district while the latter care more about the quality of the walking paths. Similarly, if a firm is being sold, there are different groups of potential buyers such as competing firms and private equity funds that have different motives for purchasing the target.

To illustrate the effects of diverse taste groups of potential buyers on the bargaining dynamics, we parameterize the problem as follows. Assume there are $n$ different groups of buyers. All members of a given group share the same valuation but valuations across groups are *i.i.d.* according to $F(v)$. Now, when the second buyer arrives, with probability $\gamma = \frac{1}{n}$ he belongs to the same group (and has the same valuation) as the current buyer (and this is common knowledge). Otherwise, with probability $(1-\gamma)$, he belongs to a different group and his value is independent of the first buyer value. Therefore, a larger $\gamma$ stands for a less diverse market place. In either case an English auction is used to allocate the good. For simplicity assume $\lambda_s = 0$.

[14] The only new claim in this corollary is that the expected time to trade is longer when the distribution of values is more dispersed and it follows directly from $t(v)$ being convex (see (7)).

In this case the expected payoffs conditional on arrival are:

$$\begin{aligned} W(v_1) &= (1-\gamma) F(v_1)(v_1 - E[v_2 | v_2 \le v_1]) \\ \Pi(v_1) &= \gamma v_1 + (1-\gamma)(F(v_1) E[v_2 | v_2 \le v_1] + (1 - F(v_1)) v_1) \end{aligned}$$

Applying the general analysis above, we can establish the following comparative statics with respect to the taste diversity:

**Proposition 4** *The limit-equilibrium, has the following comparative statics with respect to an increase in the number of groups,* $n \uparrow (\downarrow \gamma)$
*(i) The expected time to trade decreases.*
*(ii) The payoff of the seller falls.*
*(iii) For any* $t$ *the price offered is lower.*

Part $(i)$ of the Proposition follows from noting that $\frac{\partial \Pi'(v_1)}{\partial \gamma} = F(v_1) > 0$ and using the result from Proposition 3. $(ii)$ and $(iii)$ follow from noting that $\Pi(v_1)$ is decreasing in $n$ since the second term of $\Pi(v_1)$ is smaller than $v_1$ and using equations (2) and (3) which respectively characterize the seller's value and prices.

This result suggests that sellers would benefit more from specializing in a narrow market. Intensively targeting a given group of potential buyers rather than casting a very wide net. This benefit of specialization must be balanced against the potential drop in the contact frequency, $\lambda_b$.

# 5 Extensions of the Model

Before we conclude, we discuss two additional applications of the model: arrival of information and the impact of auction format .

## 5.1 Arrival of Information

So far we have looked at different events which involved the arrival of new traders. Instead we could consider events which are characterized by the arrival of information. A possibility is to assume that public information can arrive that reveals the value of the buyer. Additionally, the bargaining situation (power) can change depending on the state of nature $\omega \in \Omega$. That is, the players split the surplus according to shares $\alpha_\omega, (1-\alpha_\omega)$ . In this case the payoffs are:

$$\begin{aligned} W(v) &= v(1-\bar{\alpha}), \\ \Pi(v) &= v\bar{\alpha} . \end{aligned}$$

where $\bar{\alpha} = \sum_{\omega} \frac{\lambda(\omega)}{\Lambda} \alpha_{\omega}$ is the expected bargaining power of the seller.

The offered prices and the time to trade are given by:

$$P(k) = \frac{\Lambda}{\Lambda + r} \bar{\alpha} k \ , \qquad T(v) = -\frac{\bar{\alpha}\Lambda}{r(\Lambda + r)} \ln v$$

Using Proposition (3), or directly from the equations above, we can show that as the expected bargaining power of the seller decreases:
i) The expected time to trade decreases.
ii) The surplus for the seller falls.
iii) For any $t$ the price offered is lower.

Finally, a case we believe to be interesting but which we leave for future work is when the buyer's valuation is not fully revealed by a unique signal. Instead, there is a gradual revelation of the buyer's type as time goes by and more and more signals arrive.

### 5.2 The Auction Format

In the applications we have analyzed so far we have assumed that the seller would run an English auction with no reserve price. McAfee and Vincent (1997) have shown that without commitment and symmetric bidders there is revenue equivalence between first price and second price auctions. Furthermore, when the time between auctions goes to zero the seller's expected revenues converge to those of a second price auction with no reserve. In our setup since first and second buyers are not symmetric at the beginning of the auction[15], different auction formats will yield different revenues.

Optimal auctions usually treat weaker bidders more favorably. With i.i.d. ex-ante distributions of the two buyers, the first buyer is going to have a weaker (truncated) distribution. However, treating the first bidder more favorably in the auction can make him more stubborn during the bargaining phase. Can that lead to time-inconsistency of the optimal auction choice of the seller (i.e. that he would like to choose one format ex-ante and another one ex-post)?

Using the analysis above we can show that in fact no such time-inconsistency would arise. The intuition is that since the lack of commitment to prices drives the seller's payoff down to his outside option, maximizing ex-post revenues, maximizes ex-ante payoffs as well. [16]

[15] The asymmetry arises endogenously even with i.i.d. valuations in our setup because of the updating that takes place during the previous bargaining with the first buyer.

[16] Modeling the impact of different auction formats is somewhat delicate because in general optimal bids depend on the beliefs the new buyer has about the value of the first buyer, and these in turn depend on how much of prior bargaining he can observe. The analysis is tractable when we assume that he observes the whole history.

# 6 Conclusions

When bargaining takes place in the context of a thin market, in which other traders might show up, trade will no longer take place immediately with the informed party capturing all the rents. Although many other explanations have been proposed for the observed delay in bargaining, we believe this to be a very natural one. Furthermore it shows that delay is to be expected outside the extreme cases of perfect competition or bilateral monopolies. Nonetheless, part of the Coase conjecture continues to hold. The seller's value is driven down to his outside option of waiting for an arrival. This is what connects the characteristics of the market to the bargaining dynamics. For example, a higher ratio of buyers in the market leads to higher prices and longer times to trade. This, in turn, could affect the decision of traders to enter the market in the first place. The present model does not allow us to capture this general equilibrium effect since the arrival rates are exogenous in the model. We plan to address this issue in future work.

# 7 Appendix

We present first a series of Lemmas that will be used in proving our main Propositions.

**Lemma 1 (No Quiet Period)** *For all $\Delta > 0$, all stationary equilibria must have trade with positive probability in every period.*

**Proof.** Suppose that there exists an equilibrium in which after a cutoff type $k^*$ is reached, there is a period in which the probability of trade is zero. That implies that next period cutoff type is also $k^*$ and hence (by definition of equilibrium) the price the seller sets in this and all future periods is simply $P(k^*)$ and there is no trade till the end of the game.

The seller's expected continuation payoff is simply the expected payoff upon arrival discounted by the expected time of arrival:

$$V_\Delta(k^*) = \frac{\Delta\Lambda}{1-(1-\Delta\Lambda)\,e^{-\Delta r}}\overline{\Pi}(k^*)$$

Suppose that the seller deviates to a price $p' = \frac{\Delta\Lambda}{1-(1-\Delta\Lambda)e^{-\Delta r}}\overline{\Pi}(k^*) + \varepsilon$. If this price is accepted by some types (i.e. $\kappa(p') < k^*$) then for all $\varepsilon > 0$ we have a contradiction, since the seller payoff would be greater than $V_\Delta(k^*)$ (no matter what the cutoff type $k' = \kappa(p')$ the seller must obtain from the remaining types at least $\frac{\Delta\Lambda}{1-(1-\Delta\Lambda)e^{-\Delta r}}\overline{\Pi}(k')$, as in the proposed equilibrium, but obtains a strictly higher payoff from types $(k', k^*)$).

Suppose that this price is rejected for sure. That implies that in the continuation game $k = k^*$ and hence the seller returns to $P(k^*)$ forever. As a result, the buyer expected continuation payoff is $\frac{\Delta\Lambda}{1-(1-\Delta\Lambda)e^{-\Delta r}}W(k)$. But since $\frac{\Delta\Lambda}{1-(1-\Delta\Lambda)e^{-\Delta r}}(W(v) + \Pi(v)) < v$, there exists an $\varepsilon > 0$ such that types close to $k^*$ would be strictly better off accepting $p'$, a contradiction. ■

**Lemma 2 (No Atoms)** *For all $\varepsilon > 0$ there exists a $\Delta > 0$ such that in all stationary equilibria, the probability of trade after the initial period is $< \varepsilon$. In other words, as $\Delta \to 0$, on the equilibrium path $k - k_{+\Delta} \to 0$ and $F(k) - F(k_{+\Delta}) \to 0$.*

**Proof.** First note that as $\Delta \to 0$ prices cannot jump i.e. $P_\Delta(k) - P_\Delta(k_{+\Delta}) \underset{\Delta \to 0}{\to} 0$. Otherwise, the buyer of type $k$ would wait an instant to trade at the lower price.

Now consider a sequence of equilibria such that for all $\Delta > 0$ there is a mass of at least $\varepsilon$, of buyers that trade at some time $\tau_\Delta > 0$ for all $\Delta$, i.e. $\varepsilon \leq F(k) - F(k_{+\Delta})$. This implies that for all $\Delta$ there exists a buyer with value $\bar{k} = \frac{k + k_{+\Delta}}{2}$ that trades at the same time as type $k_{+\Delta}$ and $\varepsilon_1 > 0$ such that $k - k_{+\Delta} > \varepsilon_1$.

If type $k$ is the lowest type willing to trade at price $P_\Delta(k_{-\Delta})$ and type $k_{+\Delta}$ is the lowest type willing to trade at price $P_\Delta(k)$, they buyer's optimality requires that:

$$\begin{aligned} k - P_\Delta(k_{-\Delta}) &= e^{-\Delta r}(\Delta\Lambda W(k) + (1 - \Delta\Lambda)(k - P_\Delta(k))) \\ k_{+\Delta} - P_\Delta(k) &= e^{-\Delta r}(\Delta\Lambda W(k_{+\Delta}) + (1 - \Delta\Lambda)(k_{+\Delta} - P_\Delta(k_{+\Delta}))) \end{aligned}$$

Note that

$$\begin{aligned} & v - e^{-\Delta r}(\Delta\Lambda W(v) + v(1 - \Delta\Lambda)) \\ = & v\left(1 - e^{-\Delta r}\right) + e^{-\Delta r}\Delta\Lambda(v - W(v)) \end{aligned}$$

is strictly increasing in $v$ (because $v - W(v)$ is strictly increasing). Therefore, $\varepsilon_1 > 0$ implies that there exists $\varepsilon_2 > 0$ such that

$$\begin{aligned} \bar{k} - P_\Delta(k) &> e^{-\Delta r}(\Delta\Lambda W(\bar{k}) + (1 - \Delta\Lambda)(\bar{k} - P_\Delta(k_{+\Delta}))) + \varepsilon_2\left(1 - e^{-\Delta r}(1 - \Delta\Lambda)\right) \\ \bar{k} - P_\Delta(k_{-\Delta}) &< e^{-\Delta r}(\Delta\Lambda W(\bar{k}) + (1 - \Delta\Lambda)(\bar{k} - P_\Delta(k))) - \varepsilon_2\left(1 - e^{-\Delta r}(1 - \Delta\Lambda)\right) \end{aligned}$$

Rearanging:

$$\begin{aligned} & \frac{e^{-\Delta r}\Delta\Lambda}{(1 - e^{-\Delta r}(1 - \Delta\Lambda))}\left(W(\bar{k}) + (P_\Delta(k) - P_\Delta(k_{+\Delta}))\right) + P_\Delta(k_{+\Delta}) + \varepsilon_2 \\ < & \ \bar{k} < \frac{e^{-\Delta r}\Delta\Lambda}{(1 - e^{-\Delta r}(1 - \Delta\Lambda))}\left(W(\bar{k}) + (P_\Delta(k_{-\Delta}) - P_\Delta(k))\right) + P_\Delta(k) - \varepsilon_2 \end{aligned}$$

Now in the limit as $\Delta \to 0$ using $P_\Delta(k) - P_\Delta(k_{-\Delta}) \to 0$ and $\lim_{\Delta \to 0^+} \frac{\Delta\Lambda}{(e^{\Delta r} - (1 - \Delta\Lambda))} = \frac{\Lambda}{r + \Lambda}$ we get:

$$\frac{\Lambda}{r + \Lambda} W(\bar{k}) + P_\Delta(k_{+\Delta}) + \varepsilon_2 \leq \bar{k} \leq \frac{\Lambda}{r + \Lambda}\left(W(\bar{k})\right) + P_\Delta(k) - \varepsilon_2$$

Which implies there cannot exist such a $\bar{k}$ and therefore there cannot be a mass of buyers trading after the initial period. ■

**Lemma 3** *For all stationary equilibria as* $\Delta \to 0$ $\frac{P_\Delta(k)-P_\Delta(k_{+\Delta})}{\Delta} \to O\,(const)$*.*

**Proof.** The buyer's optimality condition for all $\Delta$ is:

$$\underbrace{k_{+\Delta} - P_\Delta(k)}_{\text{trade now}} = e^{-\Delta r}(\Delta\Lambda \underbrace{W(k_{+\Delta})}_{\text{arrival}} + (1-\Delta\Lambda)\underbrace{(k_{+\Delta} - P_\Delta(k_{+\Delta}))}_{\text{trade tomorrow}})$$

We now re-group the terms:

$$k_{+\Delta}\left(1 - e^{-\Delta r}(1-\Delta\Lambda)\right) = e^{-\Delta r}\Delta\Lambda W(k_{+\Delta}) + (P_\Delta(k) - P_\Delta(k_{+\Delta})) + 1 - e^{-\Delta r}(1-\Delta\Lambda) P_\Delta(k_{+\Delta})$$

divide by $\Delta$ :

$$k_{+\Delta}\frac{\left(1 - e^{-\Delta r}(1-\Delta\Lambda)\right)}{\Delta} = e^{-\Delta r}\Lambda W(k_{+\Delta}) + \frac{(P_\Delta(k) - P_\Delta(k_{+\Delta}))}{\Delta} + \frac{1 - e^{-\Delta r}(1-\Delta\Lambda)}{\Delta} P_\Delta(k_{+\Delta})$$

and take the limit:

$$\lim_{\Delta\to 0}\frac{\left(1 - e^{-\Delta r}(1-\Delta\Lambda)\right)}{\Delta} = \Lambda + r$$

$$\frac{(P_\Delta(k) - P_\Delta(k_{+\Delta}))}{\Delta} \to \Lambda W(k) + (\Lambda + r)(P_\Delta(k) - k) = O\,(const)$$

■

**Lemma 4** *For all stationary equilibria, the seller's value when the highest remaining type is* $k$ *,*$V_\Delta(k)$ *converges to* $V(k) = \frac{\Lambda}{r+\Lambda}\overline{\Pi}(k)$ *as* $\Delta \to 0$*.*

**Proof.** First, we can bound the seller's payoff from below by considering a deviation to (completely) slow down the trade. Since the seller can always choose to wait for the arrival of an event, his value must at least be equal to the expected discounted payoff upon arrival. That is, in all stationary equilibria and for all $\Delta > 0$ the seller's value $V_\Delta(k)$ must satisfy:

$$V_\Delta(k) \geq \frac{\Delta\Lambda}{1-(1-\Delta\Lambda)e^{-\Delta r}}\overline{\Pi}(k)$$

As $\Delta \to 0$ the RHS converges to $\frac{\Lambda}{r+\Lambda}\overline{\Pi}(k)$.

Second, we can bound the seller's payoff from above by considering a deviation to speed up trade. In particular, suppose that the highest remaining type is $k$ and suppose that the seller deviates and instead of asking for $P_\Delta(k)$ he asks for $P_\Delta(k_{+\Delta})$. For this not to be a profitable deviation, in all stationary equilibria and for all $\Delta > 0$ the seller's payoff must satisfy:

$$P_{\Delta}(k)\left[F(k)-F(k_{+\Delta})\right]+e^{-\Delta r}U(k_{+\Delta})\geq P_{\Delta}(k_{+\Delta})\left[F(k)-F(k_{+2\Delta})\right]+e^{-\Delta r}U(k_{+2\Delta}) \qquad (9)$$

where to simplify notation we used $U(k)\equiv F(k)V_{\Delta}(k)$ and $k_{+2\Delta}\equiv\kappa(P_{\Delta}(k_{+\Delta}))$

By definition of $V_{\Delta}(k)$ we can write,

$$\begin{aligned} U(k_{+\Delta}) &= \Delta\Lambda\overline{\Pi}(k_{+\Delta})F(k_{+\Delta})+ \\ &\quad (1-\Delta\Lambda)\left[P_{\Delta}(k_{+\Delta})\left(F(k_{+\Delta})-F(k_{+2\Delta})\right)+e^{-\Delta r}U(k_{+2\Delta})\right] \end{aligned}$$

Substituting it to (9) and rearranging terms we get:

$$\begin{aligned} &\left[P_{\Delta}(k)-P_{\Delta}(k_{+\Delta})\right]\left[F(k)-F(k_{+\Delta})\right]-P_{\Delta}(k_{+\Delta})\left[F(k_{+\Delta})-F(k_{+2\Delta})\right]\left(1-e^{-\Delta r}(1-\Delta\Lambda)\right) \\ \geq\; &-e^{-\Delta r}\Delta\Lambda\overline{\Pi}(k_{+\Delta})F(k_{+\Delta})+e^{-\Delta r}\left(1-(1-\Delta\Lambda)e^{-\Delta r}\right)U(k_{+2\Delta}) \end{aligned}$$

Divide by $\Delta$

$$\begin{aligned} &\frac{P_{\Delta}(k)-P_{\Delta}(k_{+\Delta})}{\Delta}\left[F(k)-F(k_{+\Delta})\right]-P_{\Delta}(k_{+\Delta})\left[F(k_{+\Delta})-F(k_{+2\Delta})\right]\frac{1-e^{-\Delta r}(1-\Delta\Lambda)}{\Delta} \\ \geq\; &-e^{-\Delta r}\Lambda\overline{\Pi}(k_{+\Delta})F(k_{+\Delta})+e^{-\Delta r}\frac{1-e^{-\Delta r}(1-\Delta\Lambda)}{\Delta}U(k_{+2\Delta}) \end{aligned}$$

Now, recall from Lemma 3 that $\frac{P_{\Delta}(k)-P_{\Delta}(k_{+\Delta})}{\Delta}\to O(const)$ and from Lemma 2 that $F(k)-F(k_{+\Delta})\to 0$. Using again

$$\lim_{\Delta\to 0+}\frac{1-e^{-\Delta r}(1-\Delta\Lambda)}{\Delta}=r+\Lambda$$

we get that in the limit

$$\Lambda\overline{\Pi}(k)F(k)\geq(r+\Lambda)U(k)$$

This implies the lower bound

$$\lim_{\Delta\to 0}V_{\Delta}(k)\leq\frac{\Lambda}{r+\Lambda}\overline{\Pi}(k)$$

Combining it with the opposite bound (that we obtained in the first step) yields the result:

$$V_{\Delta}(k)\underset{\Delta\to 0}{\to}V(k)=\frac{\Lambda}{r+\Lambda}\overline{\Pi}(k)$$

■

**Lemma 5** *The path of prices for all discrete time stationary equilibria* $P_{\Delta}(k)$ *converge to* $P(k)=\frac{\Lambda}{r+\Lambda}\Pi(k)$ *as* $\Delta\to 0$.

**Proof.** XXXTo be completedXXX

First recall that, since there cannot be any price jumps in the limit, all sequences of equilibrium pricing rules $P_{\Delta}(k)$ must converge to a continuous function . Hence, if there is a sequence of equilibrium pricing rules $P_{\Delta}(k)$ converging to something different than $P(k)$, they must differ from $P(k)$ in an open interval. So suppose that there exists a sequence of equilibrium pricing rules $P_{\Delta}(k)$ such that, as $\Delta \to 0,\ P_{\Delta}(k) \to \tilde{P}(k) \neq P(k)$ for $k \in \left(\underline{k}, \overline{k}\right)$.

Consider first the case $\tilde{P}(k) > P(k)$ for $k \in \left(\underline{k}, \overline{k}\right)$. This sequence could not be part of an equilibrium since the seller could deviate by speeding up trade. Consider for example the deviation to a pricing rule that goes twice as fast through types when $k \in \left(\underline{k}, \overline{k}\right)$ :

$$P_{2\Delta}(k) = \begin{cases} P_{\Delta}(k) \ \ for\ k \notin \left(\underline{k}, \overline{k}\right) \\ P_{\Delta}(\kappa(P_{\Delta}(k))) \quad for\ k \in \left(\underline{k}, \overline{k}\right) \end{cases}$$

The value to the seller at the first cutoff lower than $\overline{k}$ from following $P_{\Delta}(k)$ is:

$$\begin{aligned} V_{\Delta}(k_0) &= \sum_{n=0}^{N-1} e^{-rn\Delta} \left( \Delta\Lambda (1-\Delta\Lambda)^n \frac{F(k_n)}{F(k_0)} \overline{\Pi}(k_n) + (1-\Delta\Lambda)^{n+1} P_{\Delta}(k_n) \frac{F(k_n) - F(k_{n+1})}{F(k_0)} \right) \\ &\quad + \frac{F(k_N)}{F(k_0)} e^{-rN\Delta} (1-\Delta\Lambda)^N V_{\Delta}(k_N) \end{aligned}$$

where $N$ is the number of periods for which $k \in \left(\underline{k}, \overline{k}\right)$ and $\{k_n\}$ are the sequence of equilibrium cutoff types. $k_0$ is the first cutoff type $\leq \overline{k}$ and $k_N$ is the last cutoff type $\geq \underline{k}$. We will focus on the subsequence along which $N$ is odd for notational convenience.

The value to the seller at $\overline{k}$ from following instead $P_{2\Delta}(k)$ is:

$$\begin{aligned} V_{2\Delta}(k_0) &= \sum_{n=0}^{\frac{N-1}{2}} e^{-rn\Delta} \left( \Delta\Lambda (1-\Delta\Lambda)^n \frac{F(k_{2n})}{F(k_0)} \overline{\Pi}(k_{2n}) + (1-\Delta\Lambda)^{n+1} P_{\Delta}(k_{2n}) \frac{F(k_{2n}) - F\left(k_{2(n+1)}\right)}{F(k_0)} \right) \\ &\quad + \frac{F(k_N)}{F(k_0)} e^{-r\frac{N}{2}\Delta} (1-\Delta\Lambda)^{\frac{N}{2}} V_{\Delta}(k_N) \end{aligned}$$

From the expressions above it can be shown that there exists a $\Delta^* > 0$ such that for all $\Delta < \Delta^*$ $V_{2\Delta}\left(\overline{k}\right) > V_{\Delta}\left(\overline{k}\right)$. Hence there cannot be a sequence of equilibrium pricing rules $P_{\Delta}(k)$ such that, as $\Delta \to 0,\ P_{\Delta}(k) \to \tilde{P}(k) > P(k)$ for $k \in \left(\underline{k}, \overline{k}\right)$. Intuitively, since in the limit $P(k) > \frac{\Lambda}{\Lambda+r}\Pi(k)$ implies that the term multiplying $\dot{K}$ in the seller's problem:

$$rV(k) = \max_{\dot{K} \in (-\infty, 0]} \Lambda\left(\overline{\Pi}(k) - V(k)\right) + \left(V'(k) - (P(k) - V(k)) \frac{f(k)}{F(k)}\right) \dot{K}$$

is smaller than zero. Noting that the $\dot{K}$ resulting from the proposed deviation is twice as large as the original $\dot{K}$ (and recall both are negative) it leads to a higher value to the seller.

Next suppose there exists a sequence of equilibrium pricing rules $P_\Delta(k)$ such that, as $\Delta \to 0$, $P_\Delta(k) \to \tilde{P}(k) < P(k)$ for $k \in (\underline{k}, \overline{k})$. These pricing rules cannot be part of an equilibrium sequence since the seller can improve with the following deviation. Let:

$$\widehat{P}_\Delta(k) = \begin{cases} P_\Delta(k) \;\; for\ k \notin (\underline{k}, \overline{k}) \\ \frac{\Delta\Lambda}{1-(1-\Delta\Lambda)e^{-\Delta r}}\Pi(k) \quad for\ k \in (\underline{k}, \overline{k}) \end{cases}$$

Since $\frac{\Delta\Lambda}{1-(1-\Delta\Lambda)e^{-\Delta r}}\Pi(k) \to P(k)$ as $\Delta \to 0$ it must be that there exists a $\Delta^* > 0$ such that for all $\Delta < \Delta^*$ $\widehat{P}_\Delta(k) > P_\Delta(k)$ $for\ k \in (\underline{k}, \overline{k})$. By deviating to these higher prices the seller is better off since if there is trade he is trading at a higher price and if there is no trade the seller is better off since in the limit for any $v$ if the seller instead of trading for $\tilde{P}(v) < \frac{\Lambda}{\Lambda+r}\Pi(v)$ simply waits for an arrival he gets an expected present value of $\frac{\Lambda}{\Lambda+r}\Pi(v)$. Therefore, the seller would improve by deviating to $\widehat{P}_\Delta(k)$. This implies, there cannot exist a sequence of equilibrium pricing rules $P_\Delta(k)$ that don't converge to $P(k)$. ■

**Lemma 6** *In the limit, as $\Delta \to 0$, in any sequential equilibrium there cannot be an atom of trade at $t = 0$.*

**Proof.** Suppose there exists some $\bar{k} < 1$ such that all types $v \geq \bar{k}$ trade at $t = 0$. Then the seller can improve by raising $\bar{k}$ to 1 as we proceed to show.

$$\begin{aligned}
\left(1 - F(\bar{k})\right) P(\bar{k}) + F(\bar{k}) V(\bar{k}) &< F(1) V(1) \\
\left(1 - F(\bar{k})\right) \frac{\Lambda}{\Lambda + r}\Pi(\bar{k}) + F(\bar{k}) \frac{\Lambda}{\Lambda + r}\bar{\Pi}(\bar{k}) &< \frac{\Lambda}{\Lambda + r}\bar{\Pi}(1) \\
\left(1 - F(\bar{k})\right) \Pi(\bar{k}) + F(\bar{k}) \int_0^{\bar{k}} \frac{\Pi(v) f(v)}{F(\bar{k})} dv &< \left(1 - F(\bar{k})\right) \int_{\bar{k}}^1 \frac{\Pi(v) f(v)}{1 - F(\bar{k})} dv + F(\bar{k}) \int_0^{\bar{k}} \frac{\Pi(v) f(v)}{F(\bar{k})} dv
\end{aligned}$$

■

**Lemma 7** *Consider a sequence of stationary equilibria as $\Delta \to 0$. Let the equilibrium path of types be defined by $k_1 = \kappa(p_1)$ and $k_{t+1} = \kappa(P_\Delta(k_t))$. Then as $\Delta \to 0$, $\frac{k_{t+1}-k_t}{\Delta} \to \dot{K}(t)$.*

**Proof.** Recall the buyer optimality condition in discrete time: $\underbrace{k_{+\Delta} - P_\Delta(k)}_{\text{trade now}} = e^{-\Delta r}(\Delta\Lambda \underbrace{W(k_{+\Delta})}_{\text{arrival}} + (1 - \Delta\Lambda) \underbrace{(k_{+\Delta} - P_\Delta(k_{+\Delta})))}_{\text{trade tomorrow}}$ where $k_{+\Delta} = \kappa(P_\Delta(k))$

Subtracting $e^{-\Delta r}(1 - \Lambda\Delta)(k_{+\Delta} - P_\Delta(k))$ from both sides, dividing by $\Delta$ and taking $\Delta \to 0$ we get (using that $P_\Delta(k)$ converges to $P(k)$ and that there are no atoms in the limit) the following limit of the indifference condition:

$$(r + \Lambda)(k - P(k)) = \Lambda W(k) - P'(k)\dot{K}$$

which is the optimality condition for the limit-equilibrium. ■

**Proof of Theorem** 1

*The fact that equations* (3) *and* (6) *together with the boundary condition* $K(0) = 1$ *characterize an equilibrium is discussed in detail in Section 3. Uniqueness follows from noting that only necessary conditions were used to characterize this equilibrium.*

**Proof of Theorem** 2

*Lemmas* 4 and 5 *show that in the limit as* $\Delta \to 0$ *all discrete time equilibria deliver the same value to the seller and the same transaction prices given a current cutoff type. Lemmas 6 and 7 then show that how the types change through time also converges to* $K(t)$ (the first lemma shows convergence of the initial condition while the second shows that the continuation path converges).

**Proof of Proposition** (1)

Parts (i) and (ii) *follow directly from equations* (3) *and* (6). Part (iii) follows from the fact that as $\Lambda \to \infty$ the expected time for an arrival $\frac{1}{\Lambda} \to 0$.

**Proof of Proposition** (2).

*From equations* (3) and (6) *we can see that if* $\Pi(v)$ *and* $W(v)$ *are independent of* $F(v)$ *then* $P(k)$ *and* $\dot{K}$ *are independent of* $F(v)$ *and therefore the equilibrium is independent of* $F(v)$.■

**Proof of Proposition** (3).

*(1) From equation (6) we can see that keeping* $\Pi_1(v) + W_1(v) = \Pi_2(v) + W_2(v)$ *or simply* $W_1(v) = W_2(v)$ *leads to* $-\dot{K}_2 > -\dot{K}_1$ *which implies that buyers with the same valuation will trade faster in the environment with* $\Pi_2(v)$. *This follows because the boundary conditions are the same and with* $\Pi_2(v)$ *we go through types faster since* $-\dot{K}_2 > -\dot{K}_1$.

*(2) As* $\Pi'(v) \to 0$ $\Pi(v) \to 0$ *this implies the seller's value:* $V(k) = \frac{\Lambda}{\Lambda+r}\overline{\Pi}(k) \to 0$ *and prices are also converging to zero* $P(k) = \frac{\Lambda}{\Lambda+r}\Pi(k) \to 0$. *Trade on the other hand is taking place faster since* $-\dot{K} \to \infty$ *therefore there will be no delay in trade and the buyer will capture all the surplus.*■

**Proof of Corollary** (1)

*The first part follows directly from the equations for* $V(k)$, $B(v)$ and $t^*(v)$. *The second part follows directly from part (iii) of Proposition* (1).■

**Proof of Corollary**(2)

*The first part follows from* $t^*(v)$ *decreasing in* $v$ *and being invariant to the distribution of types. Hence the expectation of* $t^*(v)$ *is lower under* $F$. *(Since we are keeping* $\Lambda$ *fixed, we can focus on trade only conditional on acceptance and disregard trade after arrival).*

*For part (ii) note that the total achievable surplus is constant, equal to the expected value of* $v$. *The expected payoff of the seller is the same under* $F$ *and* $H$ *since it is simply* $\frac{\lambda_b}{\Lambda+r}$ *share of the expected value. Finally, the payoff of the buyer* $B(v)$ *is convex in* $v$, *so that the average buyer payoff is higher under* $H$, *so that the total equilibrium expected surplus is higher.*■

**Proof of Proposition** (4) .

$\Pi(v_1)$ *can be re-written as:*

$$\Pi(v_1) = \gamma v_1 + (1-\gamma)\left(\int_0^{v_1} xf(x)\,d(x) + (1-F(v_1))\,v_1\right)$$

*Hence,*

$$\begin{aligned}\Pi'(v_1) &= \gamma + (1-\gamma)(v_1 f(v_1) + (1-F(v_1)) - f(v_1)v_1) \\ &= \gamma + (1-\gamma)(1-F(v_1)) \\ &= 1 - F(v_1) + F(v_1)\gamma\end{aligned}$$

*Therefore:*

$$\frac{\partial \Pi'(v_1)}{\partial \gamma} = F(v_1) > 0$$

*Therefore, the larger* $\gamma$ *the larger* $\Pi'(v)$ $\forall v$ *and from Proposition* (3) *this implies that delay is decreasing in the number of different buyer classes.* (*ii*) and (*iii*) follow from noting that $\Pi(v_1)$ is decreasing in $n$ since the second term of $\Pi(v_1)$ is smaller than $v_1$ and using equations (2) and (3) which respectively characterize the seller's value and prices.■

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