Turnover of Used Durables in a Stationary Equilibrium: Are Older Goods Traded More?*

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Abstract

Akerlof (1970) observed that there may be an inverse relationship between the used good's quality and its volume of trade: "cherries" are traded less than "lemons". This paper develops a model that addresses a related and more general question: what is the relationship between the good's age (or physical condition) and its trade volume on the used market? The key result is that this relationship is non-monotonic. Trade volume is relatively low in the very beginning and in the middle of a good's life. This result helps explain the observed variations of resale rate across vintages for the US market of used cars.

JEL classification: D58, L62

1 Introduction

Durable goods are an important part of consumption and investment. The distinguishing characteristic of a durable is its potential for resale. There are many markets with active resale and significant volume of trade in used goods. For example, 52% of automobiles owned by households in 1995 were purchased used; more than half of Boeing 707 aircraft ever manufactured were resold to other operators at some point in their life (Goolsbee (1998)); more than 68% of all machine tools sold in the US in 1960 were used (Waterson (1964)).

Perhaps, the best known research on used goods is Akerlof's (1970) famous "lemons" model. He showed that there may be an inverse relationship between the used good's

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quality and its volume of trade: "cherries" are traded less than "lemons". One interpretation of Akerlof's result is that older goods must be traded more frequently than newer ones.

This paper presents a model that addresses a related and more general question: what is the relationship between the good's age (or physical condition) and its trade volume on the used market? The question is not only of theoretical importance: there are some interesting regularities in the data that need to be explained.

1.1 Evidence

Few durable goods have a secondary market as large and active as automobiles do, making it a natural choice for evidence. Several relationships between depreciation and trade volume have been established by Porter and Sattler (1999) who collected ownership histories on 250,000 vehicles. They document the following stylized facts:

- More reliable vehicle makes and more expensive vehicles are traded less frequently;
- "The rate of decline of a used car model's prices is negatively and significantly correlated with the length of ownership tenure" (p. 3). More generally, vehicles with more convex used price profiles are traded more frequently.

There is also direct evidence on the probability of trade (resale rate) as a function of the car's age that comes from the Nationwide Personal Transportation (NPTS) Survey.[1] These data are discussed in detail in Section 4.1 and add the following facts to the list:

- Young (0-1 year old) cars have very low resale rates;
- The cars of middle age (6-10 years old) have significantly lower resale rates than either younger (2-5 years old) or older (10-15 years old) cars;
- More reliable makes are traded, on average, at older ages.

To bring together and interpret this evidence, the next subsection proposes and defends a dynamic equilibrium modeling approach.

1.2 Modeling approach

What kind of model does one need to explain the evidence presented? A representative agent approach is of limited help in analyzing the problem, because consumer's buying and selling decisions critically depend on prices, and they are exogenous. A

[1] http://www-cta.ornl.gov/npts/1995/Doc/index.shtml

static model is also of little use, because in a static setting the volume of trade will heavily depend on the distribution of initial endowment of goods. It is clear, therefore, that one must have a dynamic framework that describes individual buying and selling decisions as well as equilibrium interactions in the aftermarkets.

Durable goods may be traded for different reasons. Akerlof (1970) argued that asymmetric information about quality induces transactions, because the seller values the "lemon" less than the buyer. Another reason for trade is quality deterioration that is common knowledge. Consumer resells her depreciated good in order to replace it with a higher quality one. Hendel and Lizzeri (1999) present a dynamic framework with two period lived goods that allows direct comparison between the adverse selection and the physical depreciation effects on used good prices and trade volume. They find that "... physical depreciation is more important than adverse selection in determining the observed price profiles" (p. 1099). Bond (1982) tested the adverse selection hypothesis empirically using maintenance data for pick-up trucks and found no evidence that the "lemons" problem is important. The above arguments suggest that one can hope to explain resale of durable goods in a framework where physical depreciation is common knowledge.

Physical depreciation gives consumers motive to eventually replace their current good. The old good is then sold to another consumer or scrapped. The timing of replacement depends on prices of all vintages. These prices, in turn, depend on how other consumers behave. Solving for this dynamic equilibrium is a formidable problem. In its general version, the state variable for the consumer is the entire distribution of durable goods holdings, since the price for used goods is a function of this distribution. Several simplified versions of the problem have been considered in the literature, but none have focussed on explaining variations in resale rates across vintages.

1.3 Related literature

Different strands of literature have made progress in understanding resale of durable goods in a dynamic context. The first strand is the equilibrium models of secondary markets (Bond (1983), Jovanovic (1998), Kursten (1991), Rust (1985)) that assume no market frictions. The absence of frictions makes the replacement problem very simple: it is always optimal for the consumer to trade every period. The implication is that in equilibrium there cannot be any variation in resale rate across vintages: the resale rate is always 100%. One way to change this implication is to induce the agents to make infrequent purchases by assuming some sort of microeconomic frictions.

This is the common theme of the second strand of literature on durable goods. When replacement is costly, the agent's decision on the stock of a durable usually follows an (s, S) rule. Because (s, S) behavior is very difficult to aggregate (see, for example, Caplin and Leahy (1999) for an excellent literature review and discussion of what the difficulties are) most of the (s, S) models would be impossibly complex if they incorporated secondary markets.

There are also a few papers that simplify the general problem by making very special assumptions. Holmes and Schmitz (1990) consider a stationary equilibrium in the market of durable capital goods (businesses) with transfer costs. In their model the size of the secondary market as well as the range and distribution of qualities traded are determined endogenously. However, because of a particular type of agent heterogeneity that the authors consider, goods over age 1 are traded if and only if the owner exits the economy. In other words, resale rates may be high only during the first period of a good's life and are low in all subsequent periods.

Porter and Sattler (1999) make another set of special assumptions: they consider two period lived goods and two period lived consumers. This dramatically reduces the set of feasible trades for the consumer and allows the authors to obtain a closed form solution. In their model, different consumer types specialize in different holding patterns. High-end consumers buy new cars and sell them next period, middle range consumers buy new cars and hold them till they fall apart, and low-end consumers buy used cars. This model produces comparative statics consistent with the stylized facts that the authors present. However, since there is just one vintage of used goods, the structure is not rich enough to analyze the variation in resale rates. More importantly, the simplifying assumption that consumers live for just two periods is also a limitation of the model: a significant number of resale transactions must occur because consumers exit the economy.

It is clearly desirable to have a model that is tractable and yet rich enough to produce a non-trivial resale pattern.

1.4 Results

This paper develops a dynamic full market model with transaction costs where individual optimal replacement cycles are embedded into an equilibrium framework. Transaction costs are central to the model, because they make the agents hold goods for multiple periods. Trade arises in the model because goods depreciate with age and because consumers have heterogenous preferences. Deterioration of good's quality is common knowledge. Trade allows both buyer and seller to adjust to their optimal vintage. Prices and quantities are determined simultaneously in a stationary equilibrium.

The key result is that the relationship between the age of the good and its resale rate is *non-monotonic*. Resale rates are typically low in the very beginning and in the middle of the good's life. It is not surprising that very young used goods are not traded a lot. Consumers faced with positive transaction costs hold their goods for more than one period and resell them later. A more subtle effect is that mid-life resale rates are relatively low as well. This is so because the optimal holding time for the goods is the shortest at the extremes of consumer type distribution and the longest in the middle. The middle consumers are the ones who buy mid-life used goods and, because of the long holding time, demand fewer of them per period. In contrast, consumers at the extremes of the type distribution replace their goods relatively quickly, making resale rates for young and old used goods relatively high.

The model is applied to the data from the US market for used automobiles. The analysis provides new insights into the relationship between the good's characteristics and its resale pattern. It is shown that less reliable automobiles must have more convex used price profiles. Also, less reliable automobiles are traded more frequently and earlier in life. The effect of warranty expiration on resale rate of automobiles is investigated as well. If expected repair costs are large enough, there is a surge of resale at the expiration date. The expectation of future repair costs lowers the prices for used cars that are still on warranty, contributing to the initial super-fast depreciation of young vehicles. The evidence supports these findings.

The plan of the paper is as follows: Section 2 introduces the model; Section 3 describes the steady state equilibrium; Section 4 presents the evidence and describes the numerical results. Section 5 concludes.

2 The model

2.1 Basic Framework, Assumptions and Notation

Goods: Time is discrete. Durable goods provide useful service for the first T periods of their life, where T is exogenous. If the good has age $t = 0, ..., T-1$ at the beginning of the period, it provides the flow of service equal to x_t during this period. Older goods provide less service:

$$0 < x_{t+1} < x_t,\ t = 0, ..., T-2.$$

The good falls apart and becomes useless when it reaches age T:

$$x_t = 0,\ \ t \geq T.$$

Without loss of generality, the flow of service from the brand-new good, x_0, is normalized to 1.

Consumers: Consumers derive utility from the durable good and the numeraire commodity. Each consumer is infinitely lived and can use only one durable good at a time. Consumers differ in their marginal utility of service denoted $h \in [0, h_{\max}]$, distributed according to an atomless density $n(h)$. Utility is linear in the numeraire commodity c. That is, the current period utility to consumer of type h from having a good x and c units of the numeraire is

$$u_h(x, c) = xh + c. \tag{1}$$

This particular form of utility function is very common in the industrial organization literature.

Consumers have the same, rational expectations about future prices, and maximize their lifetime utility of owning an infinite sequence of durable goods. For each good in this sequence, they choose what vintage to buy and how long to hold it.

New goods: The focus of the model is on the consumer side of the market. It is therefore assumed that any quantity of new goods can be supplied at an exogenously given price p_0.

Resale: Goods can be bought and sold at the end of every period. The price the buyer pays for the good of age $t \geq 0$ is denoted p_t. Each resale involves a stochastic transaction cost Λ paid by the seller. Individual sellers make independent draws of Λ from the following distribution:

$$\Lambda = \begin{cases} 0, & \text{with prob. } \alpha \\ \lambda_t p_t, & \text{with prob. } 1-\alpha. \end{cases}$$

Here $\lambda_t \leq 1$ is a fraction of market price that is paid as transaction cost when a good of age t is sold.

Since an infinite number of consumers make independent draws from the transaction cost distribution, there is no aggregate uncertainty in the model.

Free disposal: Any good can be scrapped, and this does not involve a transaction cost. Assume that the good's scrap value is zero. Anyone can pick up a useless free good from the scrap heap and secure a utility of zero. Therefore, consumers whose present value of participating in the market is negative choose not to participate.

2.2 Discussion of Assumptions

A single new good: This assumption makes sense if there is little substitutability between different types of new goods. Consumers buy used goods not just because they cost less, but because comparably priced new goods do not have features they want. For example, although some new cars are available for under $ 10,000, a family of 7 may still better off buying a used minivan for the same price.

Constant useful life: In the worst case scenario, the physical lifetime constraint is binding, and goods are not scrapped until age T. In a steady state where all goods are scrapped at age T, demand for new goods must equal $\frac{1}{T}$ per consumer. However, this does not imply that aggregate demand for new goods is always constant. The *number* of consumers who participate in the market changes in response to the parameters of the model, and so does demand for new goods.

Quasi-linear utility: this is an essential simplifying assumption, which allows to obtain important monotonicity properties that make numerical computation of equilibrium feasible for reasonably large values of T. When utility is quasi-linear, it is possible to prove that consumer's optimal value function satisfies a particular single-crossing property *irrespective of prices.* This implies that the optimal decision rules are monotonic functions of consumer type no matter what the prices are. The importance of this monotonicity property for computing the steady state equilibrium is discussed in Section 3.3.

The assumption of quasi-linear utility has another important benefit. Neither the sequence $\{x_t\}_{t=0}^{T}$ nor transaction costs $\{\lambda_t\}_{t=1}^{T}$ need to be convex to for the decision rules to be monotonic. This allows to compute the steady state equilibrium for possibly non-convex depreciation paths. An example of such analysis and its importance for interpreting the evidence on resale rates are discussed in Section 4.2.4.

Transaction costs: Transaction costs capture, in reduced form, various market frictions that may arise when a durable is resold. The most common frictions are information costs and search costs. For example, many used car dealers offer warranties, and buyers pay a premium over the trade-in value to receive a certified vehicle. If the sale is not through the dealer, the seller must pay for advertising and spend time showing the good to prospective buyers.

Making more general assumptions about the distribution of transaction costs will have the same effect as adding another dimension to consumer type: consumers' decision rules will depend not only on their type, but also on the current draw from the transaction cost distribution. While there is no conceptual difficulty with relaxing this assumption, the monotonicity result may go away. In this case computation will become extremely taxing, because the numerical method that uses monotonicity of decision rules will no longer apply.

Assuming that transaction cost sometimes equals zero is the simplest way to get a model consistent with the observation (presented in Section 4.1) that *all* vintages have non-zero trade volume. When $\alpha > 0$, all vintages are traded with probability of at least α. Besides, it is actually easier to find the equilibrium with $\alpha > 0$. If transaction costs are always bounded away from zero, some vintages may not be traded in equilibrium. Such equilibria are usually supported by multiple prices. This makes computation problematic, because if equilibrium price is non-unique, it cannot be found as a fixed point of some operator. However, the equilibrium with $\alpha = 0$ can be computed as a limit of equilibria with positive α.

Assuming that transaction cost does not exceed the market price ($\lambda_t \leq 1$) guarantees that resale is weakly preferred to scrappage and that goods whose prices are positive are not scrapped. This assumption imposes more structure on the problem as it significantly narrows down the kinds of equilibria that can arise. As long as transaction cost never exceeds equilibrium price, choosing alternative functional forms for the sequence $\{\lambda_t\}_{t=1}^{T}$ does not change the analysis.

3 Steady state equilibrium

For the model economy to be in *steady state,* we require that all markets clear every period and that prices and quantities traded stay constant over time. Prices and

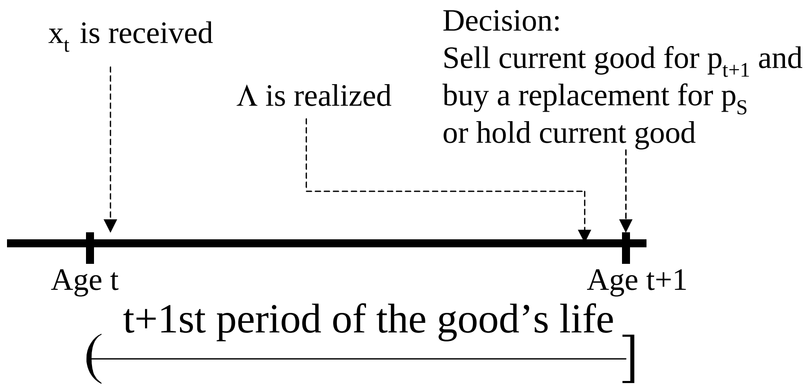

Figure 1: The timeline

quantities that give rise to the steady state equilibrium must be determined simultaneously. The analysis proceeds as follows. First, the optimization problem of any consumer h is solved conditional on prices of all vintages. Then the optimal decision rules are aggregated to determine the steady state distribution of durable goods holdings, i.e. the state of the economy that replicates itself indefinitely when all consumers behave according to their decision rules. The steady state holdings distribution generates constant resales and constant purchases for every vintage. These resales and purchases make up steady state supply and demand, conditional on vintage prices. Finally, supply is equated to demand for each vintage in order to determine the equilibrium price vector.

3.1 Optimal replacement problem

We will now describe the consumer's decision problem in a stationary environment where the distribution of durable goods holdings is the same every period. In this environment, used goods prices are constant over time and depend only on the age of the good. Consumers take current and future prices as given and make their decisions according to the timeline shown on Figure 1.

Consumer h arrives at the beginning of the current period with a good of age $t \in \{0, ..., T-1\}$. The age of her current good is the state variable for the consumer. In the current period, this good provides the flow of service equal to x_t, which she values at $x_t h$. At the end of the current period, the consumer makes an independent draw from the transaction cost distribution. After the consumer sees the realization of transaction cost, she decides whether to sell the good and purchase a replacement or keep the good for another period.[2]

In equilibrium, all consumers will follow the stationary decision rule: they will sell

[2]Since it is assumed that $\lambda \leq 1$, consumer will never striclty prefer scrappage to resale.

their goods either when they draw a zero transaction cost or when the good reaches a threshold age, whichever happens first. After resale, each consumer type will update to their most preferred vintage, and the holding cycle will start again. Let $V(h;t)$ be the discounted present value of consumer h having the good of age t at the beginning of the current period and let $\beta < 1$ be the discount factor.[3] Technically speaking, the value function V and the optimal decision rules will depend on the price vector as well, but this dependence is suppressed in the notation for convenience. In this condensed notation, the Bellman equation describing the optimal replacement problem reads:

$$V(h;t) = x_t h + \alpha\beta \max\left\{\max_S \left(V(h;S) - p_S\right) + p_{t+1}, V(h;t+1)\right\} +$$

$$+ (1-\alpha)\,\beta \max\left\{\max_S \left(V(h;S) - p_S\right) + p_{t+1}(1-\lambda_{t+1}), V(h;t+1)\right\}. \tag{2}$$

The first term is the value of service provided by the good during the current period, the second term is the expected value of the resale opportunity with zero transaction cost, and the third term is the expected value of the resale opportunity with positive transaction cost. Let

$$S_h = \arg\max_S \left(V(h;S) - p_S\right) \tag{3}$$

be the optimal vintage for consumer h. Since the problem is stationary, this expression implies that consumer h will purchase the good of age S_h every time she replaces her durable. We will call S_h the *buying point* for consumer h. Since for every t

$$\max_S \left(V(h;S) - p_S\right) \geq V(h;t+1) - p_{t+1},$$

any consumer who draws a zero transaction cost resells her good immediately and buys her optimal vintage. Consumer who draws a positive transaction cost replaces her good when the gain from resale exceeds the transaction cost:

$$\max_S \left(V(h;S) - p_S\right) - \left(V(h;t+1) - p_{t+1}\right) > \lambda_{t+1} p_{t+1}.$$

Let

$$\tau_h = \min\left\{t : V(h;S_h) - p_{S_h} - \left(V(h;S_h+t) - p_{S_h+t}\right) > \lambda_{S_h+t} p_{S_h+t}\right\}. \tag{4}$$

The above expression[4] says that consumer h will sell her good at the age $S_h + \tau_h$ even if she draws a positive transaction cost. We will use the term *selling point* for $S_h + \tau_h$

[3] Since utility is linear in the numeraire commodity, utility and wealth are equivalent, so

$$\beta = \frac{1}{1+r},$$

where r is the interest rate.

[4] Since scrapping the good does not involve any transaction cost ($\lambda_T = 0$), any good will be replaced before age T:

$$S_h + \tau_h \leq T$$

and the term *holding time*[5] for τ_h. The interval $[S_h, S_h+\tau_h-1]$ will be called the *holding interval.*

Suppose that consumer h follows a decision rule with the buying point S and the holding time τ. That is, she always buys vintage S and holds it for at most τ periods. We can compute her lifetime utility by substituting these decision rules into the Bellman equation (2). This yields the following expressions the value function on the holding interval:

$$V(h;S+i-1)=x_{S+i-1}h+\alpha\beta\left(V(h;S)-p_S+p_{S+i}\right)+(1-\alpha)\,\beta V(h;S+i),\; i=1,...,\tau-1;$$

$$\begin{aligned}V(h;S+\tau-1) &= x_{S+\tau-1}h+\alpha\beta\left(V(h;S)-p_S+p_{S+\tau}\right)+\\ &\quad +(1-\alpha)\,\beta\left(V(h;S)-p_S+p_{S+\tau}-\lambda_{S+\tau}p_{S+\tau}\right).\end{aligned}$$

Using the notation

$$\gamma=(1-\alpha)\beta,\;\alpha\beta=\beta-\gamma$$

and making recursive substitution, we can find the expression for the optimal value function at the buying point:

$$V(h;S)-p_S=$$

$$=\max_{\tau}\left(\frac{h\sum_{i=1}^{\tau}x_{S+i-1}\gamma^{i-1}-p_S+(\beta-\gamma)\sum_{i=1}^{\tau}p_{S+i}\gamma^{i-1}+\gamma^{\tau}p_{S+\tau}\left(1-\lambda_{S+\tau}\right)}{(1-\beta)\frac{1-\gamma^{\tau}}{1-\gamma}}\right). \tag{5}$$

The right hand side of this expression equals consumer's expected lifetime utility measured at the buying point. Let $U(h;S,\tau)$ be the expected utility to consumer h of having the buying point S and the holding time τ:

$$U(h;S,\tau)=\frac{h\sum_{i=1}^{\tau}x_{S+i-1}\gamma^{i-1}-p_S+(\beta-\gamma)\sum_{i=1}^{\tau}p_{S+i}\gamma^{i-1}+\gamma^{\tau}p_{S+\tau}\left(1-\lambda_{S+\tau}\right)}{(1-\beta)\frac{1-\gamma^{\tau}}{1-\gamma}}. \tag{6}$$

The optimal decision rule is the one that gives the consumer maximum expected utility:

$$\begin{aligned}(S_h,\tau_h) &= \arg\max U(h;S,\tau), \\ \text{s.t. } 0 &\le S\le T-1 \\ 1 &\le \tau\le T-S.\end{aligned} \tag{7}$$

If initially the consumer has a good of age $t_0\in[S_h,S_h+\tau_h-1]$, then by construction she will find it optimal to follow the decision rule (S_h,τ_h). However, since

[5] Strictly speaking, τ_h is the *maximum* holding time for durable goods purchased by consumer h. The actual holding time is a random variable distributed on $\{1,...,\tau_h\}$.

we have not determined the optimal value function outside the holding interval, we do not know how consumer will behave if her initial state is not in $[S_h, S_h + \tau_h - 1]$. Neither can we claim that the optimal policy takes the form of an (S, s) rule. This is because the value function may not be concave.[6] Nevertheless, if the decision rules satisfy (7), it is sufficient to guarantee that all consumers will abide by the equilibrium behavior, since in equilibrium the ages of goods owned by consumers will *always* belong to their respective holding intervals.

We will now establish an important monotonicity property for the decision rules. It turns out that no matter what the prices are, higher types prefer to buy younger goods and resell them earlier. Formally, the buying point S_h and the selling point $S_h + \tau_h$ are step-functions that map $[0, h_{\max}]$ into $\{0, .., T\}$. The following proposition says that these step-functions are monotonic in consumer type.

Proposition 1: (monotonicity of decision rules). Let (S_h, τ_h) be the solution to the optimal replacement problem. Then S_h and $S_h + \tau_h$ are non-increasing functions of h for almost all prices.[7]

Proof: See Appendix.

The proposition says that decision rules are monotonic regardless of prices. This significantly narrows down the class of decision rules to consider and makes computation of equilibrium much more efficient. Proposition 1 holds regardless of prices, because utility function is assumed to be quasi-linear in money. With a more general form of utility function, decision rules will be monotonic only if prices and transaction costs satisfy certain convexity restrictions. Since it is not clear whether equilibrium prices will satisfy the same restrictions, monotonicity cannot be used for computation under weaker assumptions on utility function.

We will now turn to characterizing the distribution of durable goods across consumer types that gives rise to the steady state equilibrium.

3.2 Steady state holdings distribution

In steady state, the distribution of durable goods across consumer types must stay the same every period and replicate itself indefinitely. This holdings distribution will generate constant purchases and constant resales for every vintage, which will make up steady state supply and demand.

[6] If, for example, a consumer is given the good whose age is greater than her selling point, she may not want to return to the buying point immediately, but may instead choose to keep holding the good.

[7] Except for price vectors for which all three functions S_h, $S_h + \tau_h$ and τ_h have a discontinuity at the same point $\hat{h} \in [h_{\min}, h_{\max}]$. However, the subset of such price vectors has measure zero.

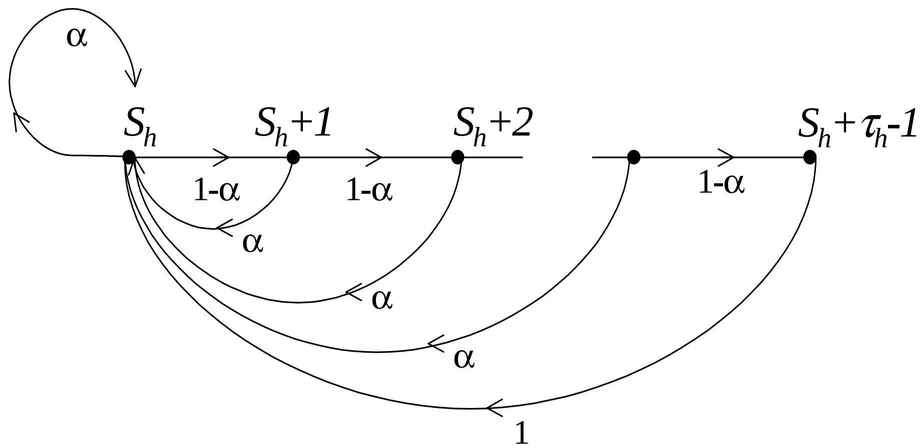

Figure 2: States and transition probabilities on the holding interval

Let $f(h,t)$ be the number[8] of consumers of type $h \in [0, h_{\max}]$ who hold the goods of age $t = 0, ..., T-1$ at the beginning of the current period.[9] In steady state, $f(h,t)$ must be the same every period. The optimal decision rules (S_h, τ_h) impose a certain law of motion on the holdings distribution $f(h,t)$. In particular, consumers do not own goods whose ages are outside of their holding interval:

$$f(h,t) = 0, \;\; t < S_h \text{ or } t > S_h + \tau_h - 1. \tag{8}$$

Inside the holding interval, states and transition probabilities for any consumer h are depicted on Figure 2. In every state, consumer draws a zero transaction cost with probability α and returns to her buying point S_h next period. With probability $1-\alpha$, she draws a positive transaction cost and transits to the state where her good is one period older. In addition, *all* consumers who are one period away from their selling point return to the buying point next period. This law of motion can be expressed as

$$f(h,t) = (1-\alpha)\, f(h,t-1), \; S_h + 1 \leq t \leq S_h + \tau_h - 1. \tag{9}$$

The transitions of consumer h across states inside her holding interval can be sum-

[8]Since market participation is endogenous, $f(h,t)$ is not a density:

$$\sum_{t=0}^{T-1} \int_0^{h_{\max}} f(h,t) dh \leq 1.$$

[9]Since resale, purchase or scrappage happen at the *end* of the period, no one holds goods of age T at the *beginning* of the period.

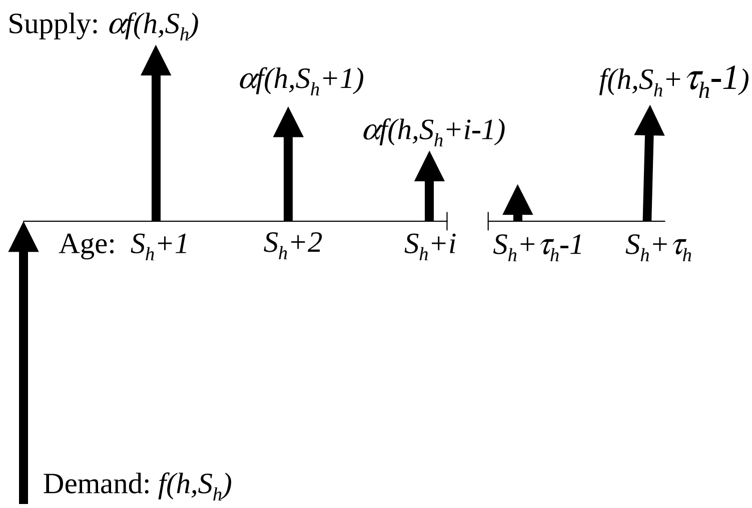

Figure 3: Supply and demand by vintage for consumer type h.

marized by the following $\tau_h \times \tau_h$ matrix:

$$\Pi_h = \begin{pmatrix} \alpha & 1-\alpha & 0 & ... & 0 \\ \alpha & 0 & 1-\alpha & ... & 0 \\ ... & ... & 0 & ... & ... \\ \alpha & 0 & ... & ... & 1-\alpha \\ 1 & 0 & ... & 0 & 0 \end{pmatrix}$$

The number of consumers in each state must be constant over time. Therefore, for each consumer type the steady state holdings distribution can be found as the stationary distribution for the transition matrix Π_h. Solving for this stationary distribution yields

$$f(h,t) = \begin{cases} 0, & t < S_h \text{ or } t > S_h + \tau_h - 1 \\ \frac{\alpha(1-\alpha)^{t-S_h}}{1-(1-\alpha)^{\tau_h}} n(h), & S_h \leq t \leq S_h + \tau_h - 1. \end{cases} \tag{10}$$

Having found the steady state holdings distribution, we can compute steady state flows of goods as functions of consumer type[10] and age of the good. Adding the law of motion equations in (9), we obtain the material balance equation

$$f(h, S_h) = \sum_{i=1}^{\tau_h - 1} \alpha f(h, S_h + i - 1) + f(h, S_h + \tau_h - 1). \tag{11}$$

Its left hand side is the total number of new goods purchased per period by consumers of type h, and the right hand side is the total number of goods sold per period. Figure

[10]More precisely, consumer type and (S_h, τ_h) which are themselves functions of h.

3 illustrates the material balance equation. It shows the inflow of goods at age S_h which equals the sum of outflows at all other ages in the holding interval.

Each outflow term on the right-hand side of the material balance equation equals consumer h's contribution to supply of goods of the corresponding age, as shown on Figure 3. Accordingly, the supply function for consumers of type h whose decision rule is (S_h, τ_h) can be written as:

$$q_s(h,t) = \begin{cases} 0, & t < S_h + 1 \text{ or } t > S_h + \tau_h \\ \alpha f(h, t-1), & S_h + 1 \le t \le S_h + \tau_h - 1 \\ f(h, t-1), & t = S_h + \tau_h. \end{cases} \tag{12}$$

Note that supply depends on the vintage prices through S_h and τ_h. Similarly, the inflow term of (11) equals consumer h's contribution to demand for goods of age S_h. Therefore, demand function for consumer h whose decision rule is (S_h, τ_h) can be expressed as:

$$q_d(h,t) = \begin{cases} 0, & t \neq S_h \\ f(h, S_h), & t = S_h. \end{cases} \tag{13}$$

Using the expressions (10), (12) and (13) and integrating over consumer types with the same decision rule, we can determine the steady state supply and demand for every vintage. The steady state supply and demand depend on the price vector $\mathbf{p} = (p_t)_{t=0}^{T}$ through the decision rules (S_h, τ_h). All the elements are now in place to define the steady state equilibrium.

3.3 Equilibrium

Definition: Steady state equilibrium consists of the price vector $\mathbf{p} = (p_0, p_1, ..p_{T-1}, 0)$, the steady state holdings distribution $f(h,t)$, the marginal consumer $h_{\min}$ and the optimal decision rule (S_h, τ_h), $h \in [h_{\min}, h_{\max}]$ such that:

1. The steady state holdings distribution $f(h,t)$ is given by (10) for every participating consumer $h \in [h_{\min}, h_{\max}]$ and $f(h,t) = 0$ for non-participating consumers $h \in [0, h_{\min})$;

2. Prices for all useful goods are positive

$$\begin{aligned} p_t &> 0, \; t = 0, .., T-1 \\ p_T &= 0, \end{aligned}$$

and supply equals demand for any used good that is not scrapped:

$$Q_s(t) = \int_{h_{\min}}^{h_{\max}} q_s(h,t)dh = \int_{h_{\min}}^{h_{\max}} q_d(h,t)dh = Q_d(t), \;\; t = 1, .., T-1;$$

3. Consumers choose the decision rule that maximizes their lifetime utility

$$(S_h, \tau_h) = \arg\max_{(S,\tau)} U(h, S, \tau);$$

4. The marginal consumer[11] is indifferent between buying a used good and taking a scrapped good for free

$$h_{\min} = \frac{p_{T-1}}{x_{T-1}}.$$

As follows from the definition, we restrict attention to those equilibria where used goods prices are positive and all useful vintages are traded.[12] This implies that goods are not scrapped before age T. If new goods are cheap, some consumers will find it worthwhile to scrap them before the end of their physical life. For example, plastic cups are durable, and yet because they are so cheap, consumers throw them away after the first use. Considering equilibria with positive prices makes sense when the cost of producing new goods is sufficiently high. All else being equal, if the new goods are expensive enough, vintages that are still useful are not scrapped, and there exists an equilibrium with positive prices. This is because more expensive new goods make more expensive used goods, and as production cost rises, equilibrium prices of all useful vintages must eventually rise above zero.

There is a practical way to find parameter values for which the equilibrium price vector is positive. In general, the higher are the transaction costs, the higher are the equilibrium prices. This result is intuitive. As transaction costs increase, supply of used goods of all ages is reduced, because sellers hold their goods for a longer time. This implies that prices must rise to bring the market in equilibrium. Therefore, if equilibrium prices are positive when transaction costs are zero, they will necessarily be positive in any equilibrium with non-zero transaction costs.

Computing the steady state equilibrium does not boil down to solving a system of equations, except for the case when transaction costs are zero. The reason is that S_h and τ_h cannot be explicitly expressed through prices. Instead, the method of computation is to set up a mapping from the price vector into itself, with equilibrium price vector as a fixed point. Monotonicity of decision rules established in Proposition 1 is crucial for the method to work, since monotonicity guarantees that the fixed point mapping is not a correspondence. Unfortunately, an existence proof cannot be constructed based on this algorithm, because this mapping can take some positive prices into negative, and Brouwer's fixed point theorem does not apply. Uniqueness of the steady state is even harder to prove, because there is no analytical expression for the fixed point operator.

[11]Non-negativity of utility $U(h, S_h, \tau_h) \geq 0$ is equivalent to $h \geq \frac{p_{T-1}}{x_{T-1}}$. The proof is in the Appendix.

[12]For arbitrary values of p_0 and $\{x_t\}$, T-period life may not be a binding constraint. It is not necessary that all goods are used for the entire duration of their physical life. In particular, if market price $p_{T_e} = 0$ for some age $T_e < T$, all goods of ages greater than T_e are scrapped.

Nevertheless, it is possible to establish the following important property of the steady state equilibrium:

Proposition 2 In any steady state equilibrium with positive prices, the holding time τ_h is (weakly) increasing in h around $h_{\min}$ and (weakly) decreasing in h around $h_{\max}$.

Proof: The proof will use two properties of the steady state equilibrium: that equilibrium decision rules are monotonic functions of consumer type (Proposition 1) and that all goods are scrapped at age T. The latter statement follows from the fact that prices are positive and that $\lambda_t \leq 1$ for every t. Since every consumer prefers resale to scrappage, scrapping the good before age T cannot be anyone's optimal decision rule.

In steady state, the total number of goods held in the economy at a moment in time equals the number of participating consumers, $1 - h_{\min}$. Because all goods are retired at age T, $\frac{1-N(h_{\min})}{T}$ consumers buy new goods each period, and an equal number of consumers retire their goods. Therefore, in equilibrium, there must be an interval of types whose buying point is 0 (new goods) and an interval of types whose selling point is T. Take two consumers h_1 and $h_2 < h_1$ whose buying point is 0. Proposition 1 implies that higher type h_1 must hold the good for a shorter time: $\tau(h_1) \leq \tau(h_2)$. Monotonicity of decision rules implies that consumers whose buying point is 0 are at the top of the type distribution. Therefore, holding time $\tau(h)$ will be *decreasing* in h for some interval at the top of the type distribution. Similarly, take two consumers h_1' and $h_2' < h_1'$ whose selling point is T. They must be at the bottom of the type distribution and the higher type h_1' must buy a younger good: $S(h_1') \leq S(h_2')$. As a result, holding time $\tau(h)$ will be *increasing* in h near the bottom of the type distribution. ■

The proof did not rely on the fact that the economic lifetime of the goods, T, is exogenous. Proposition 2 will hold as long as in equilibrium all the goods have the same, perhaps endogenous, lifetime. This property will play an important role in explaining the observed holding patterns for cars. Before turning to the numerical results, we will present the evidence from the US market for used automobiles.

4 Application: used automobiles

4.1 Evidence on resale of automobiles

The evidence on resale of automobiles in the US comes from the 1995 Nationwide Personal Transportation (NPTS) Survey.[13] This dataset has more than 75,000 observations. It is convenient for analyzing resale patterns, because ages of vehicles are not top-coded or grouped into intervals.

[13] This survey is conducted by the U.S. Department of Transportation. http://www-cta.ornl.gov/npts/1995/Doc/index.shtml

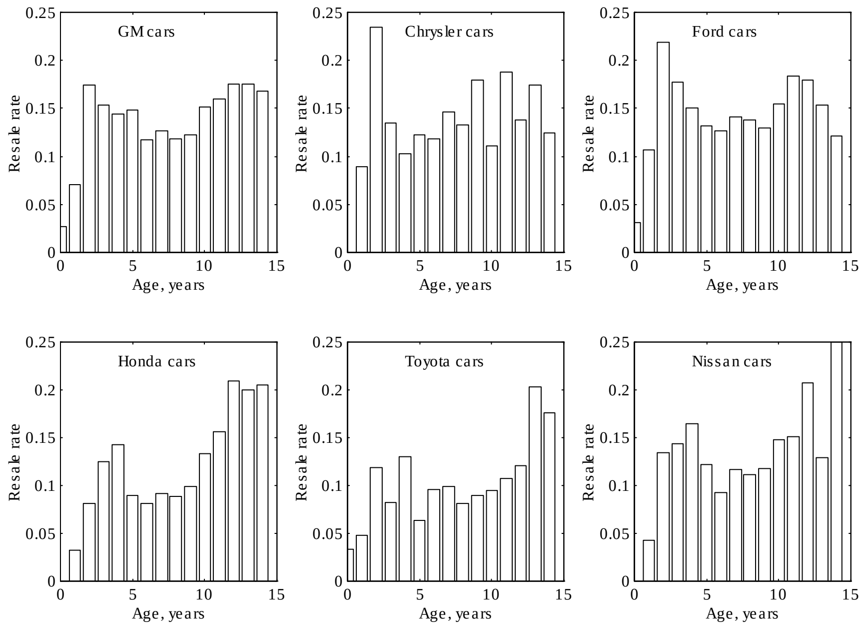

Figure 4: Resale rate as a function of age for different car makes

The data were collected for 6 major automobile manufacturers in the model year range 1982-1996, which covers more than 75% of all cars in the sample. Figure 4 shows resale rates as a function of model year, which is taken to be a proxy for vehicle's age. The vertical axis of each plot shows the observed fraction of vehicles of a particular age purchased *in used condition* in 1995. The horizontal axis shows the vehicle's age. Ages run 15 years back, with model year 1996 at age 0 through model year 1982 at age 14.

There are several factors that can make resale rate vary with age. Since resale rate is a *fraction*, the echo effects from high sales in the past are controlled for: the variations in resale rate across vintages are not due to different *numbers* of vehicles across vintages. Resale rates may also vary because some vintages of cars are very popular. For example, Ford Taurus was just introduced in 1986, and the observed resale rate for 1986 Taurus is high. The "good vintage" effects are controlled for by pooling together all car models by the same manufacturer. Because introduction of new models is usually staggered, only a fraction of observations for a particular model year can conceivably be from the good vintage. Besides, the effect of good vintage on quantity traded is ambiguous. A good vintage is not only something that consumers want to *buy* (increased demand), but also something that other consumers want to *keep* (reduced supply).

All categories exhibit similar regularities in resale patterns: resale rate is very low for 1-year old cars, it peaks when vehicles are 2-4 years old, then resale rate stays

Manufacturer (Nobs)	**First significant drop in resale rate (P-value)**	**First subsequent rise in resale rate (P-value)**
GM (15,538)	6-th year (0.012)	10-th year (0.013)
Chrysler (4,318)	3-d year (0.0003)	9-th year (0.028)
Ford (8,731)	3-d year (0.017)	10-th year (0.1)
Honda (3,145)	5-th year (0.022)	12-th year (0.1)
Toyota (3,180)	5-th year (0.003)	13-th year (0.07)
Nissan (1,998)	Not significant at 10%	14-th year (0.06)

Table 1: Changes in resale rate and their significance

relatively low for several years, and then goes up again when the vehicle is about 10 years old.

The patterns on Figure 4 are confirmed by tests summarized in Table 1. If resale rate drops significantly within one year, and then rises significantly within some other year, it cannot be a monotonic function of age. The hypothesis that resale rate for cars that are more than 2 years old is monotonic in vehicle's age can be rejected for most makes. The test based on the difference in resale rates for two consecutive years is not very tight, because it does not use the information that resale rate may be increasing or decreasing for several years in a row. For example, resale rate for Toyotas starts increasing during the 9th year, but the first *significant* increase occurs only during the 13th year. Unfortunately, testing a complex monotonicity hypothesis is impractical in most cases, because it is very hard to derive the distribution of the test statistic.[14]

The observations record only purchases, but not the vehicle ownership histories. Ideally, one would like to exclude fleet sales by rental car companies, because they are done by agents who hold multiple vehicles at a time and thus face a different decision problem. However, since fleet sales usually involve 1 year old cars, excluding them from the sample would likely lower the resale rate at 1 year and make the non-monotonic pattern on Figure 4 even more pronounced.

The evidence also shows that frequency of trade varies by vehicle make. On average, Hondas and Toyotas are traded significantly less than other makes considered. Hendel and Lizzeri (1999) have a similar finding: they use the data from 1991 Consumer Expenditure Survey to conclude that Fords are traded more often than Hondas. The next section explains why resale rates are relatively low in the middle of a car's life and why some automobiles are traded more frequently than others.

[14] The difficulty is that the form of the test statistic depends on the sign of the constraints tested. The number of regions in the parameter space where the test statistic takes a different form grows exponentially in the number of constraints. See, for example, Gourieroux, Holly and Monfort (1982).

4.2 Numerical results: prices and resale rates

4.2.1 Choice of parameter values

To evaluate the model numerically, we must first choose suitable parameter values. The distribution of consumer types is assumed to be uniform on $[0, h_{\max}]$, which seems appropriate given the paper's focus on the peaks of resale. Under this assumption the variation in resale rates cannot be due to high density of types that prefer a particular vintage. It should also be noted that any density $n(h)$ can be transformed into the uniform by appropriately choosing the units of h and redefining (1) accordingly.[15]

The physical lifetime of a car is taken to be $T = 15$ years. Depreciation is assumed to be exponential, with a constant rate δ:

$$x_t = x_0(1-\delta)^t,\ t = 1, ..., T-1.$$

We measure the transaction cost of selling a used vehicle with the difference between its market price and the trade-in value. In the model, the ratio of this difference to market price is equal to λ. It turns out that the ratio of transaction cost to price rises with vehicle's age and roughly doubles[16] over a vehicle's lifetime. Accordingly, we set

$$\lambda_t = \lambda_1 2^{\frac{t-1}{T-2}},\ t = 2, ..., T-1. \tag{14}$$

In the model, goods of all ages are traded with probability of at least α. Typically, resale rates are minimal (i.e. equal to α) for young vintages . We can therefore choose the value of α to match the resale rate for 1-2 year old vehicles in the data.[17] This implies $\alpha = 0.1$.

The values of p_0, $h_{\max}$, λ_1 and δ are chosen by fitting the equilibrium price predicted by the model to the normalized price series for used automobiles reported by Porter and Sattler (1999, Table 4). The resulting benchmark parameter values are $p_0 = 3.6$, $h_{\max} = 1.6$, $\lambda_1 = 0.075$, $\delta = 0.063$. The unit of measurement is the stream of service from the brand-new good, $x_0 = 1$. To be more specific, the price of a new car approximately equals to the present value of 5 years of its service to the median consumer. The real interest rate is set to 0.04, which implies $\beta = 0.96$.

4.2.2 Numerical results

The simulations for prices and quantities traded are presented on Figure 5. The right plot shows the equilibrium price (the solid line) against the price series data (*). The model fits the price data quite well: the discrepancy between predicted used goods

[15] If $h \sim N[0, h_{\max}]$, then $h' = N(h)$ is uniform on $[0, 1]$. Since this transformation is monotonic, it is the same as redefining the units of h.

[16] This calculation was performed using trade-in and market values repotrted by Edmund's: http://www.edmunds.com/used/

[17] Average resale rate for used 1 year old vehicles is 0.058, and for used 2 year old vehicles it is 0.144. Source: NPTS 1995.

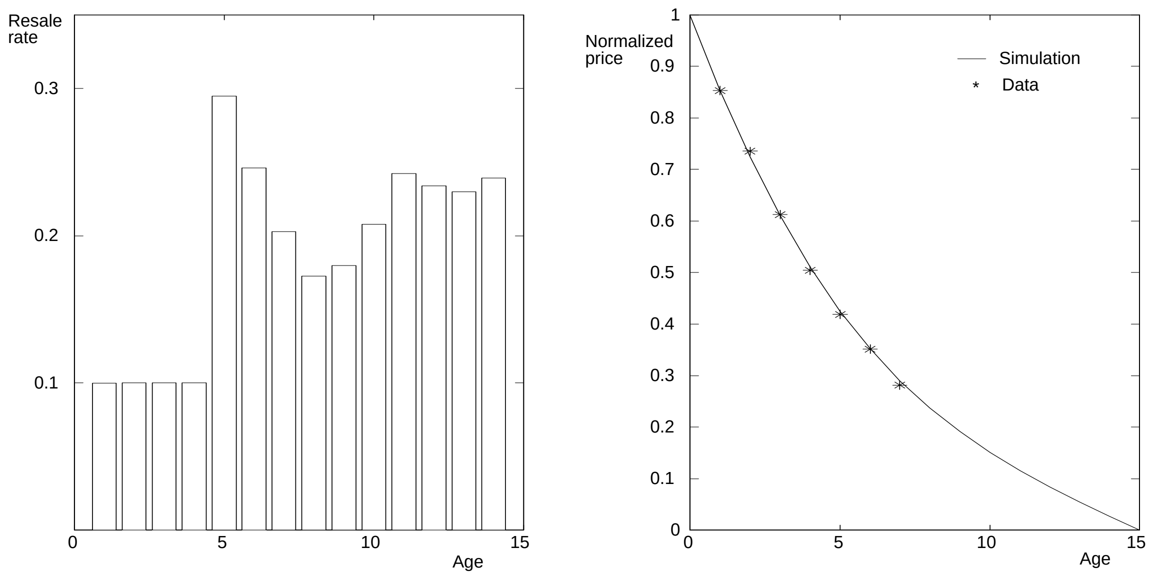

Figure 5: Equilibrium resale rate and prices, benchmark case

prices and their values in the data is 3% or less. The trade intensity plot shows the simulated resale rate as a function of age. Resale rate is minimal during the first 4 years, because consumers who draw positive transaction costs do not sell their goods until the age of 5. Resale rate has a peak at 5 years, is relatively low for 6-10 year old cars and becomes relatively high for 11-15 year old cars.

To understand how this pattern arises, we need to look at the equilibrium decision rules summarized in Table 2. Each row of the table corresponds to a different decision rule, starting with the highest type. The first column shows the interval of consumer types who have a particular decision rule. For example, the top 7.6% of consumer types buy goods of age zero and sell them at age 5, the next 7.3% buy goods of age zero and sell them at age 6, etc., and finally, the bottom 15% of consumer types do not participate in the market.

We will demonstrate using Table 2 that relatively high resale rates for 11-15 year old cars are due to relatively short holding times for these cars. According to (13), demand for goods of age t equals the number of consumers whose buying point is t and who are currently at this buying point. The longer is the holding time for the good, the less is the steady state number of consumers at the buying point.[18] Take, for example, 8 year old cars. Consumers who buy these cars hold them for either

[18] According to (10), the number of consumers at the buying point

$$f(h, S_h) = \frac{\alpha n(h)}{1-(1-\alpha)^{\tau_h}}.$$

This expression is monotonically decreasing in holding time τ_h.

Percentile of type distribution	Buying point, S	Selling point, $S+\tau$	Holding time, τ
100-92.4	0	5	5
92.4-85.1	0	6	6
85.1-78.7	0	7	7
78.7-73.3	0	8	8
73.3-73.0	0	9	9
73.0-69.7	1	9	8
69.7-67.7	2	9	7
67.7-66.7	2	10	8
66.7-63.7	3	10	7
63.7-61.4	4	10	6
61.4-61.0	4	11	7
61.0-54.3	5	11	6
54.3-53.0	5	12	7
53.0-47.4	6	12	6
47.4-46.4	6	13	7
46.4-41.0	7	13	6
41.0-40.8	8	13	5
40.8-36.5	8	14	6
36.5-34.4	9	14	5
34.4-32.0	9	15	6
32.0-27.2	10	15	5
27.2-22.5	11	15	4
22.5-18.9	12	15	3
18.9-16.4	13	15	2
16.4-15.0	14	15	1
15.0-0.0	Do	not	participate

Table 2: Decision rules by consumer type, benchmark case

5 or 6 years (Table 2). Therefore, it takes them up to[19] 5-6 years to return to the buying point. Compare them, say, to the consumers who buy 14 year old cars. Their holding time is just 1 year, and they turn over much faster: next period *all of them* return to the buying point. As a result, demand for 14 year old cars is higher, and so is the resale rate. As shown in the last column of Table 2, the holding times at the extremes of the type distribution are shorter than in the middle. Because consumers in the middle of the type distribution turn over slowly, resale rates for the goods they buy are relatively low.

Roughly speaking, the number of consumers who buy a particular vintage is in the numerator of the resale rate, and their holding time is in the denominator.[20] Holding time is not the only factor that affects demand for a particular vintage. Demand for goods of age t is proportional to the *total number* of consumers whose buying point is t. For example, demand for 5 year old goods is high, because a total of 8% of the population buys 5 year old goods. In contrast, demand for 8 year old goods is lower, because only 4.5% of consumers buy them.

It is also true that the large quantity of consumers at a particular buying point can counteract the effect of short holding time, in which case the resale rate need not have a trough at middle ages. This is usually the case when large transaction costs almost prevent resale and holding times do not vary a lot across consumers. When holding time is almost the same for everybody, the quantity of consumers at the buying point becomes the decisive factor in determining the resale rate.

We will now turn to comparative statics exercises which help explain the relationship between a car's reliability and frequency and timing of its resale.

4.2.3 Depreciation effects

The effects of faster depreciation on used goods prices and resale patterns are investigated by increasing the value of δ from 0.06 to 0.08 and comparing the resulting equilibrium with the benchmark case. The results are shown on Figure 6. Cars that depreciate faster have more convex prices and shorter holding intervals. Hence less reliable cars are traded more frequently and earlier in life.

The evidence shows strong support for these predictions. Porter and Sattler (1999) (Tables 6, 7 and 8) report that unreliable vehicles are traded more frequently. They also find (p. 3) that "the rate of decline of a used car model's prices is negatively and significantly correlated with the length of ownership tenure". In the model, more convex prices imply shorter holding intervals.

[19] If a consumer draws a zero transaction cost, she returns to the buying point next period. That is why some consumers do not reach their selling point.

[20] Observe that the number of consumers at the buying point, $f(h, S_h)$, approximately equals to

$$\lim_{\alpha \to 0} f(h, S_h) = \frac{n(h)}{\tau_h}.$$

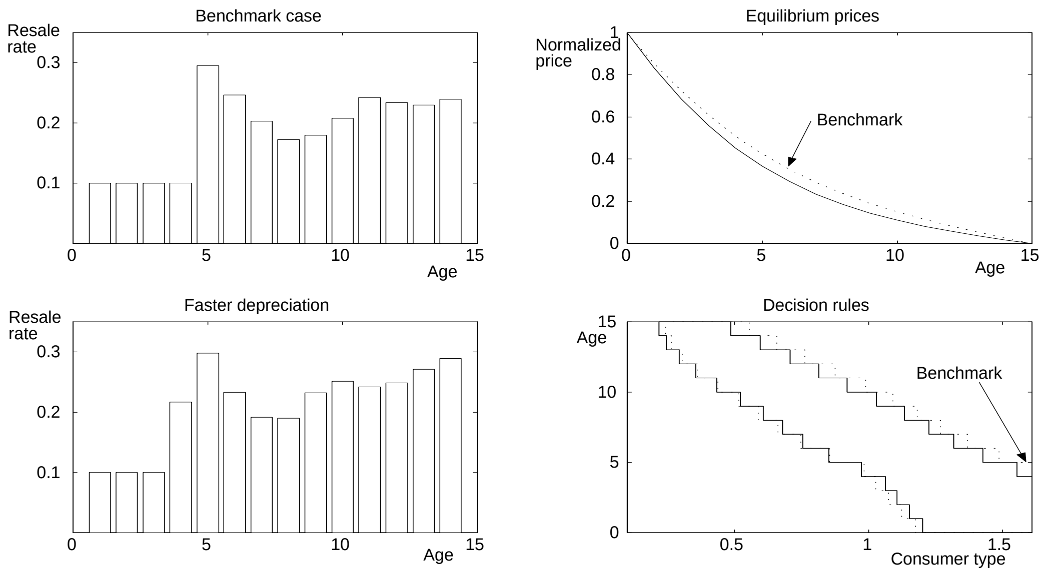

Figure 6: Effects of faster depreciation

There is also evidence that reliable vehicles are traded later in life. According to Porter and Sattler (1999, Table 3), two makes with the highest reliability are Honda and Toyota. The median[21] selling age for a used Honda or Toyota is 7.1 years. In contrast, the median selling age for a Pontiac or GMC, two of the less reliable makes, is 6.1 years.

Not shown on the figure is the effect of raising the price of the new good, which works in the opposite direction. All other things being equal, more expensive cars have less convex prices. Therefore, they are traded less frequently and later in life. Porter and Sattler (1999) also note this fact and explain it by the positive correlation between initial price and quality. The analysis here implies that the effect may have less to do with quality than was originally thought. According to the model, when a car is more expensive, a higher fraction of its original price is passed onto the next owners.[22] This makes the price sequence less convex and resale less frequent.

4.2.4 Warranty effects

Expiration of warranty must have an important impact on resale patterns, because owners anticipate repair costs which affect their utility from the good. In the model, x measures the value of service net of repair expenses, so the appropriate way to

[21] The median is computed with respect to the distribtuion of resale rates by age. Source: NPTS 1995.

[22] This result is *not* due to the assumption of constant lifetime. If lifetime were endogenous, more expensive cars would live longer, making their prices even less convex.

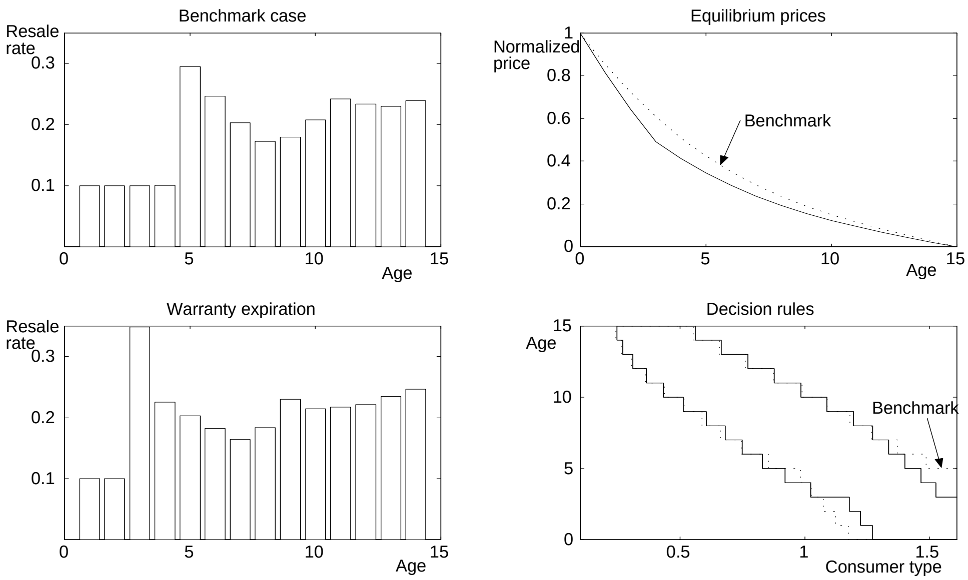

Figure 7: Effects of warranty expiration

model warranty expiration would be to subtract repair costs from x beginning from the 4th year of vehicle's life.

If repair costs are large enough,[23] warranty expiration can produce a peak of resale at the expiration date, as shown on Figure 7. High-end consumers who have large marginal utility of service start to resell their goods at 3 years (note the dramatically narrower holding intervals for these consumers). The equilibrium price becomes more convex, with a kink at 3 years. The expectation of future repair costs lowers the prices for used cars that are still on warranty, making the initial depreciation of young vehicles even faster. The intuition for this result is the following. The buyers of 1-3 year old cars will optimally hold them for a relatively long time *after* the warranty expires. Hence they expect to be repeatedly hit with repair costs and their willingness to pay decreases.

Warranty expiration is another reason why unreliable car models may be traded earlier in life. If repair costs are significant, resale becomes concentrated at the date of warranty expiration.

4.2.5 Transaction cost effects

Transaction costs are another important determinant of resale rates. If transaction costs are zero, each good is traded every period, and resale rate equals 1 for every useful vintage. When transaction costs are infinite, the good is traded if an only if the consumer draws a zero transaction cost. This implies that resale rates for all

[23] To get a distinctly differnt holding pattern, it was assumed that repair costs reduce x by 15%.

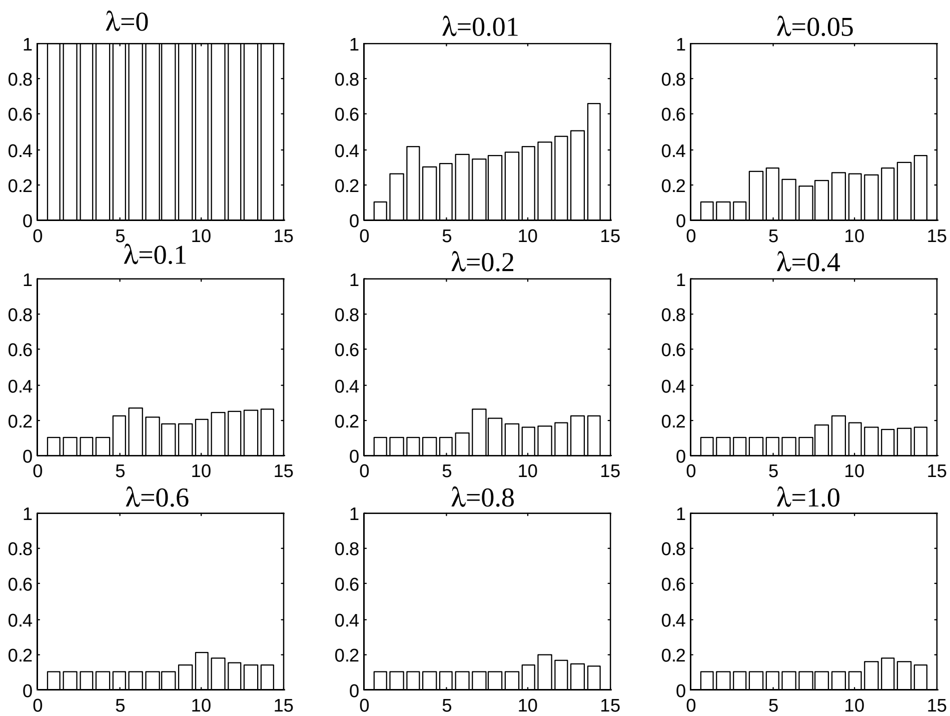

Figure 8: Resale rates for different transaction cost levels.

useful vintages are equal to α.

Figure shows how trade intensity depends on transaction cost level. Each plot corresponds to a different value of transaction cost level λ.[24] For simplicity, we depart from the specification in (14) and assume that $\lambda_t = const$ for all t.

As transaction costs increase, resale rates go down, peaks of resale occur later, and the number of such peaks decreases. Also, the trough in resale rate disappears at higher levels of transaction costs, because there is little variation in holding time across consumer types. The greater are the transaction costs, the longer are the holding intervals. As transaction costs increase, supply of used goods is reduced, because goods are held for a longer time. This makes buyer's prices for all used goods go up. Therefore, as transaction costs increase, the price sequence becomes less and less convex.

5 Conclusion

This paper developed a dynamic full equilibrium model of durable goods markets. The unique feature of the model is that it allows multiple holding periods for durables and at the same time considers equilibrium interactions in the aftermarkets. Prices and quantities traded for every vintage are determined endogenously in a stationary equilibrium. The model can account for the variations in resale rate for the used automobiles in the US. One point of this paper is that these variations can be explained without informational asymmetries. Variable resale rates arise in the situation where deterioration of good's quality is common knowledge, due to preference heterogeneity and physical depreciation alone. Another important contribution is that the model provides an explicit aggregation of microeconomic consumption patterns for durable goods.

This framework has applications beyond explaining the trade volume for used autos. For example, one can use a related model in an environment where workers are heterogenous and firms face hiring costs. Human capital is a durable asset whose services can be rented to firms. Firms use different technologies, and workers possess different skills. Human capital may depreciate due to technological progress or it may appreciate due to learning. Either way, this will induce labor market transactions. Slow learners will eventually fall behind their firm's technology and will be rented by technological laggards. Fast learners will "outlearn" the firms they are currently with and leave to work for technological leaders. Such a model can generate variations of labor turnover by skill level as well as explain how skill premium depends on the rate of technological progress and the learning response of workers.

Another potential application has to do with the normative economics of marriage.

[24] Equilibrium with $\lambda = 1$ is not the same as equilibrium with infinite transaction costs. When $\lambda = 1$, consumers are indifferent between scrapping and selling their goods, so resale rates are above α for some vintages. In contrast, when λ is infinite, all consumers strictly prefer scrappage to resale.

If men strictly prefer younger wives and yet the costs of remarriage are significant, then the model can predict the efficient pattern of turnover of spouses of different ages. This may give a better interpretation of divorce data and may lead to new policy advice on minimizing the social costs of divorce. Hence, the model, appropriately modified, has a variety of other interesting applications.

References

[1] Akerlof, G. A. [1970], "The Market for "Lemons": Quality Uncertainty and the Market Mechanism", *Quarterly Journal of Economics*, 84, 488-500.

[2] Bond, E. [1983], "Trade in Used Equipment with Heterogenous Firms", *Journal of Political Economy* 91, 688-705.

[3] Bond, E. [1982], "A Direct Test of the "Lemons" Model: The Market for Used Pickup Trucks", *The American Economic Review* 72, 836-839.

[4] Caplin, A. and John Leahy [1999], "Durable Goods Cycles", NBER Working Paper No 6987

[5] Eberly, J. [1994], "Adjustment of Consumers' Durables Stocks: Evidence from Automobile Purchases", *Journal of Political Economy,* 102, 403-436.

[6] Gooslbee, A. [1998], "Factor Prices and the Retirement of Capital Goods", *Review of Economic Dynamics,* 1, 474-496.

[7] Gourieroux C., Alberto Holly and Alain Monfort [1982], "Likelihood Ratio Test, Wald Test, and Kuhn-Tucker Test in Linear Models with Inequality Constraints on the Regression Parameters", *Econometrica,* Vol. 50, No. 1. (Jan., 1982), pp. 63-80.

[8] Holmes, T. and James A. Schmitz [1990], "A Theory of Entrepreneurship and its Application to the Study of Business Transfers", *Journal of Political Economy*, 1990, vol. 98, 265-294.

[9] Hendel I. and A. Lizzeri [1999], "Adverse Selection in Durable Goods Markets", *The American Economic Review* 89, 1097-1115.

[10] Jovanovic, B. [1998], "Vintage Capital and Inequality", *Review of Economic Dynamics,* 1, 497-530.

[11] Kursten, W. [1991], "A Theory of Second-hand Markets: The Rapid Depreciation of Consumer Durables and Product Differentiation Effects," *Journal of Institutional and Theoretical Economics*, 147, 459-476.

[12] Porter, R. and Peter Sattler, [1999], "Patterns of Trade in the Market for Used Durables: Theory and Evidence", NBER Working paper No 7149.

[13] Rust, J. [1985], "Stationary Equilibrium in a Market for Durable Assets," *Econometrica*, 53, 783-805.

[14] Waterson, A. [1964], "Good Enough for Developing Countries?", *Finance and Development*, 1, 89-96.

Appendix

Proof of Proposition 1 (monotonicity of decision rules) Before we prove the proposition, let us prove the following lemma:

Lemma Suppose that $h > h'$. Then

$$U(h,S,\tau)-U(h,S+1,\tau-1) > U(h',S,\tau)-U(h',S+1,\tau-1),\ \text{for any } S \geq 0, \tau > 1$$

$$U(h,S,\tau)-U(h,S,\tau+1) > U(h',S,\tau)-U(h',S,\tau+1),\ \text{for any } S \geq 0, \tau \geq 1$$

$$U(h,S,\tau)-U(h,S+1,\tau) > U(h',S,\tau)-U(h',S+1,\tau),\ \text{for any } S \geq 0, \tau \geq 1$$

Proof: Rearranging the terms in (6), consumer's lifetime utility can be expressed as

$$U(h,S,\tau) = h\frac{X(S,\tau)}{R(\tau)} + \sum_{t=0}^{T} b_t(S,\tau)p_t - \lambda_{S+\tau}p_{S+\tau}b_{S+\tau}(S,\tau),$$

where

$$X(S,\tau) = \sum_{i=1}^{\tau} x_{S+i-1}\gamma^{i-1}$$

is the discounted stream of service from the good,

$$R(\tau) = 1-(\beta-\gamma)\sum_{i=1}^{\tau}\gamma^{i-1} - \gamma^{\tau} = (1-\gamma^{\tau})\frac{1-\beta}{1-\gamma}$$

is the discount factor for a τ-period replacement cycle, and

$$b_t(S,\tau) = \begin{cases} 0, & t \leq S \text{ or } t > S+\tau \\ -1/R(\tau), & t = S \\ (\beta-\gamma)\gamma^{t-1-S}/R(\tau), & S+1 \leq t \leq S+\tau-1 \\ \beta\gamma^{\tau-1}/R(\tau), & t = S+\tau \end{cases}$$

is the discount factor for prices. Using this notation

$$U(h,S,\tau) - U(h',S,\tau) = \frac{X(S,\tau)}{R(\tau)}(h-h'),$$

$$U(h,S+1,\tau-1) - U(h',S+1,\tau-1) = \frac{X(S+1,\tau-1)}{R(\tau-1)}(h-h').$$

To prove the first inequality of the lemma we must establish that

$$\frac{X(S,\tau)}{X(S+1,\tau-1)} > \frac{R(\tau)}{R(\tau-1)} = \frac{1-\gamma^{\tau}}{1-\gamma^{\tau-1}}.$$

This follows directly from the fact that $x_S > x_{S+i}$:

$$\frac{X(S,\tau)}{X(S+1,\tau-1)} = \frac{x_S + \gamma X(S+1,\tau-1)}{X(S+1,\tau-1)} = \frac{x_S}{\sum_{i=1}^{\tau-1} x_{S+i}\gamma^{i-1}} + \gamma >$$

$$> \frac{1-\gamma}{1-\gamma^{\tau-1}} + \gamma = \frac{1-\gamma^{\tau}}{1-\gamma^{\tau-1}}.$$

Similarly, to prove the second inequality of the lemma we must establish that

$$\frac{R(\tau+1)}{R(\tau)} = \frac{1-\gamma^{\tau+1}}{1-\gamma^{\tau}} > \frac{X(S,\tau+1)}{X(S,\tau)}.$$

Rewriting the right hand side and using the monotonicity of x_t, we obtain:

$$\frac{X(S,\tau+1)}{X(S,\tau)} = \frac{X(S,\tau) + \gamma^{\tau} x_{S+\tau}}{X(S,\tau)} < 1 + \frac{\gamma^{\tau}(1-\gamma)}{1-\gamma^{\tau}} = \frac{1-\gamma^{\tau+1}}{1-\gamma^{\tau}}.$$

To prove the third inequality, simply observe that because x_t is decreasing,

$$X(S,\tau) > X(S+1,\tau),$$

which implies that

$$U(h,S,\tau) - U(h',S,\tau) = \frac{X(S,\tau)}{R(\tau)}(h-h') >$$

$$> \frac{X(S+1,\tau)}{R(\tau)}(h-h') = U(h,S+1,\tau) - U(h',S+1,\tau).$$

■

Corollary: For any $t > 0$

$$U(h,S,\tau) - U(h,S,\tau+t) > U(h',S,\tau) - U(h',S,\tau+t),$$

$$U(h,S,\tau) - U(h,S+t,\tau-t) > U(h',S,\tau) - U(h',S+t,\tau-t),$$

$$U(h,S,\tau) - U(h,S+t,\tau) > U(h',S,\tau) - U(h',S+t,\tau).$$

Proof: Observe that

$$U(h,S,\tau) - U(h,S,\tau+t) =$$

$$= (U(h,S,\tau) - U(h,S,\tau+1)) + (U(h,S,\tau+1) - U(h,S,\tau+2)) + ...$$

$$... + (U(h,S,\tau+t-1) - U(h,S,\tau+t)).$$

and according to the Lemma, the desired inequality holds for each of the terms. ■

Proof of the Proposition: We will prove the following: over any interval in h where S_h is constant, $S_h + \tau_h$ is non-increasing in h. Over any interval where $S_h + \tau_h$ is constant, the buying point S_h is non-increasing in h. Over any interval where τ_h is constant, the S_h is non-increasing in h.

The proof will consist of three parts. First, if two consumers $h > h'$ have the same buying point, $S_h = S_{h'}$, then the higher type sells her good earlier: $\tau_h \leq \tau_{h'}$.

We will show that if $(S_{h'}, \tau_{h'})$ is the solution to the optimal replacement problem for h', then any higher type $h > h'$ will be strictly better off holding her good for $\tau_{h'}$ periods rather than for $\tau > \tau_{h'}$ periods. That is, it must be the case that $\tau_h \leq \tau_{h'}$.

This follows from the corollary to the lemma:

$$U(h, S_{h'}, \tau_{h'}) - U(h, S_{h'}, \tau_{h'} + t) > U(h', S_{h'}, \tau_{h'}) - U(h', S_{h'}, \tau_{h'} + t) > 0, \forall t > 0.$$

Second, if selling points for the good are the same, $S_h + \tau_h = S_{h'} + \tau_{h'}$, then the higher type buys a younger good: $S_h \leq S_{h'}$. We will prove that if $(S_{h'}, \tau_{h'})$ is the solution to the optimal replacement problem for h', then any higher type $h > h'$ will be strictly better off buying the good of age $S_{h'}$ rather than any good of age $S > S_{h'}$.

Again, using the corollary to the lemma, we obtain:

$$U(h, S_{h'}, \tau_{h'}) - U(h, S_{h'} + t, \tau_{h'} - t) > U(h', S_{h'}, \tau_{h'}) - U(h', S_{h'} + t, \tau_{h'} - t) > 0, \forall t > 0.$$

Similarly, if holding times for two consumers h' and $h > h'$ are the same, the higher type is better off with $S_{h'}$ than any buying point $S > S_{h'}$:

$$U(h, S_{h'}, \tau_{h'}) - U(h, S_{h'} + t, \tau_{h'}) > U(h', S_{h'}, \tau_{h'}) - U(h', S_{h'} + t, \tau_{h'}) > 0, \forall t > 0.$$

Three statements just proven would imply monotonicity if not for one special case. Suppose that for some price vector there exists a point $\hat{h}$ such that S_h, $S_h + \tau_h$ and τ_h are all discontinuous at the same point $\hat{h}$. Then the proposition does not tell anything about the relationship between $S_{\hat{h}+0}$ and $S_{\hat{h}-0}$. Although it is possible to choose such a special price vector, the set of these vectors will be a measure zero subset of the price space. ■

Claim: In any steady state equilibrium market participation constraint reads

$$h_{\min} = \frac{p_{T-1}}{x_{T-1}}.$$

Proof: The consumer who participates in the market must be at least as well off as from taking the best available free good ($x_T = 0$) every period. Then the marginal consumer $h_{\min}$ who is indifferent between participating and not participating must get zero utility.

We must now establish that consumers whose utility is positive belong to an interval $[h_{\min}, h_{\max}]$. This is immediate, since

$$U(h, S_h, \tau_h) > U(h, S_{h_{\min}}, \tau_{h_{\min}}) > U(h_{\min}, S_{h_{\min}}, \tau_{h_{\min}}) = 0$$

Next, we must show that either $S_{h_{\min}} = T-1$ and $\tau_{h_{\min}} = 1$ or $p_{T-1} = 0$ and $h_{\min} = 0$. First consider the case when $p_{T-1} > 0$. This implies that supply must be equal to demand

$$Q_s(T-1) = Q_d(T-1).$$

In equilibrium, the lowest type $h_{\min}$ must buy a good of age $T-1$ and retire it at age T. Suppose this is not true, that is, there is $S' < T-1$ and $\tau' > 0$ such that

$$U(h_{\min}, S', \tau') > U(h_{\min}, T-1, 1).$$

Buying a good of age $T-1$ every period yields the smallest possible present value of service. That is, for every (S', τ'),

$$\frac{X(S', \tau')}{R(\tau')} > \frac{X(T-1, 1)}{R(1)}.$$

Therefore, for every h

$$U(h, S', \tau') > U(h_{\min}, S', \tau').$$

This says that if the decision rule $(T-1, 1)$ is not optimal for the lowest type, it cannot be optimal for anybody else. Therefore, demand for goods of age $T-1$ equals zero, which means there is excess supply at $T-1$.

Now suppose that $p_{T-1} = 0$, which implies that all consumers participate in the market, that is, $h_{\min} = 0$. ■