Antitrust in Innovative Industries*

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First Version: July 2003
This Version: December 29, 2004

Abstract

We study the effects of antitrust policy in industries with continual innovation. A more protective antitrust policy may have conflicting effects on innovation incentives, raising the profits of new entrants, but lowering those of continuing incumbents. We show that the direction of the net effect can be determined by analyzing shifts in innovation benefit and supply holding the innovation rate fixed. We apply this framework to analyze several specific antitrust policies. We show that in some cases, holding the innovation rate fixed, as suggested by our comparative statics results, the tension does not arise and a more protective policy necessarily raises the rate of innovation.

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*We thank Shane Greenstein, Ken Hendricks, Volker Nocke, Richard Schmalensee, Scott Stern, and participants in seminars at the FTC, Harvard, Northwestern, Penn, UT-Austin, the 2003 NBER Summer Institute meeting on Innovation Policy, the 2003 International Industrial Organization Conference, and the 2004 Duke-Northwestern-Texas IO Theory Conference for helpful discussions and comments. We also thank the NSF (SES-0318438) and the Searle Foundation for financial support.

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1 Introduction

This paper is concerned with the effects of antitrust policy in markets in which innovation is important. Traditionally, antitrust analyses have largely ignored issues of innovation, focusing instead on the price/output effects of contested practices.[1] Yet, over the last two decades, intellectual property and innovation have become central to competition in a large share of the economy. In the wake of these changes, and sparked by the recent *Microsoft* case, a number of commentators have expressed concerns that traditional antitrust analysis might be poorly suited to maximizing welfare in such industries. As Evans and Schmalensee [2002] put it, in these industries

> "...firms engage in dynamic competition *for the market* — usually through research-and development (R&D) to develop the 'killer' product, service, or feature that will confer market leadership and thus diminish or eliminate actual or potential rivals. Static price/output competition on the margin *in the market* is less important."

The effects of antitrust policy on innovation are poorly understood. In the *Microsoft* case, for example, the most significant issue arising from Microsoft's allegedly exclusionary practices was almost surely their effect on innovation. Microsoft argued that while a technological leader like Microsoft may possess a good deal of static market power, this is merely the fuel for stimulating dynamic R&D competition, a process that it argued works well in the software industry. Antitrust intervention in this process would run the risk of reducing the rate of innovation and welfare. The government, in contrast, argued that Microsoft's practices prevented entry of new firms and products, and therefore would both raise prices and retard innovation.[2] How to reconcile these two views, however, was never clear in the discussion surrounding the case.

On closer inspection, these two conflicting views reveal a fundamental tension in the effects of antitrust policy on innovation. Policies that protect new entrants from incumbents raise a successful innovator's initial profits and may thereby encourage innovation, as the government argued. But new entrants hope to become the next Microsoft, and will want to engage in the same sorts of entry-disadvantaging behaviors should they succeed. Thus,

[1] Issues of innovation have been considered when discussing "innovation markets" in some horizontal merger cases in which there was a concern that a merger might reduce R&D competition. See, e.g., Gilbert and Sunshine [1995].

[2] For further discussion, see e.g., Evans and Schmalensee [2002], Fisher and Rubinfeld [2000], Gilbert and Katz [2001], and Whinston [2001].

by lowering the profits of incumbency, protective policies may actually retard innovation, as Microsoft alleged. Disentangling these two effects is the central focus of this paper.

We study the effects of antitrust policy in innovative industries using models in which innovation is a continual process, with new innovators replacing current incumbents, and holding dominant market positions until they are themselves replaced. Although a great deal of formal modeling of R&D races has occurred in the industrial organization literature (beginning with the work of Loury [1979] and Lee and Wilde [1980]; see Reinganum [1989] for a survey), this work has typically analyzed a single, or at most a finite sequence, of innovative races.[3] Instead, our models are closer to those that have received attention in the recent literature on growth (e.g., Grossman and Helpman [1991], Aghion and Howitt [1992], Aghion et. al [2001]). The primary distinction between our analysis and the analysis in this growth literature lies in our explicit focus on how antitrust policies affect equilibrium in such industries.[4]

The paper is organized as follows. In Sections 2 and 3, we introduce and analyze a simple stylized model of antitrust in an innovative industry. Our aim is to develop a model that yields some general insights into the effect of antitrust policies on the rate of innovation, and that we can apply to a number of different antitrust policies in the remainder of the paper. In Section 2, we study the simplest version of this stylized model, in which in each period a single potential entrant conducts R&D. The model captures antitrust policy in a reduced form way, by assuming that it alters the profit flows that an incumbent and a new entrant can earn in competition with each other, as well as the profits of an uncontested incumbent. In the model, a more protective antitrust policy — one that protects entrants at the expense of incumbents — increases a new entrant's profits, but

[3] Three exceptions are O'Donoghue et al. [1998] and Hunt [2004], who use continuing innovation models to examine optimal patent, and Fudenberg and Tirole [2000] who study dynamic limit pricing in markets with network externalities using a model of continuing innovation.

[4] The growth literature often considers how changes in various parameters will affect the rate of innovation, sometimes even calling such parameters measures of the degree of "antitrust policy" (e.g., Aghion et al. [2001] refer to the elasticity of substitution as such a measure). Here we are much more explicit than is the growth literature about what antitrust policies toward specific practices do. This is not a minor difference, as our results differ substantially from those that might be inferred from the parameter changes considered in the growth literature. As one example, one would get exactly the wrong conclusion if one extrapolated results showing that more inelastic demand functions lead to more R&D (e.g., Aghion and Howitt [1992]) to mean that allowing an incumbent to enhance its market power through long-term contracts leads to more R&D.

also affects the profitability of continuing incumbents. Since successful entrants become continuing incumbents, both of these effects matter for the incentive to innovate.

Using this simple stylized model, we develop some general insights into the effect of antitrust policies on the rate of innovation. We do so by characterizing equilibrium in terms of "innovation benefit" and "innovation supply" functions, which provides a very simple approach to comparative statics. This approach to comparative statics tells us that a policy change will increase innovation when a certain weighted sum of the profit changes for a new entrant and a continuing incumbent increase, holding fixed the level of innovation. This weighted sum corresponds to the change in the present discounted value of successful entry, which we call the "innovation prize," holding fixed the rate of innovation. Using this condition, we show that a more protective antitrust policy "front-loads" an innovative new entrant's profit stream, and that this feature tends to increase the level of innovative activity. We also note here how the degree of market growth can alter the effects of antitrust policy.

In Section 3, we extend our comparative statics approach to substantially more general innovation benefit and supply settings. The extension to innovation supply, for example, allows us to consider supply settings with constant returns to scale R&D technologies, with N potential entrants, with free entry, or with "free entry with a limited idea" as in work by Fudenberg and Tirole [2000] and O'Donoghue et al. [1998]. We show using our innovation benefit and supply approach that in each case, or in any other setting satisfying several basic properties, the condition characterizing comparative statics is the same.

With the comparative statics results of Section 3 in hand, in Sections 4 and 5 we develop applications to specific antitrust policies. In Section 4, we focus on incumbent behaviors designed to reduce the level of entrant R&D. We begin with a surprising observation: *in many circumstances involving R&D-deterring activities, the apparent tension between effects on entrant and continuing incumbent profits does not arise at all.* In these cases, holding the rate of innovation fixed, limits on R&D-deterring activities raise the profits of *both* new entrants *and* incumbents, and so our characterization result tells us that the rate of innovation necessarily increases. We then analyze two models of antitrust policies toward specific R&D-deterring activities. First, we study a model of long-term (exclusive) contracts and show that a more protective antitrust policy necessarily stimulates innovation, and also raises both aggregate and consumer welfare. This model falls into the class of cases in which the tension indeed does not arise. Then, we consider a model of compatibility choice in an industry characterized by network externalities. Here we identify cases in which innovation necessarily increases when incumbents are forced to make their products more compatible with those of future entrants (which, again, are cases in which the tension does not arise), as well as cases in which innovation may decline. (Appendix C also contains an extension of our long-term contracting model to the case of uncertain innovation size when there is a cost of rapidly implementing new innovations. The key new feature in this model is that antitrust policy has a "selection effect," altering the set of innovations that enter the market. We show that in this situation, limitations on long-term contracting may retard innovation.)

The framework that we develop is not limited to exclusionary behaviors. In Section 5, we consider the effects of voluntary deals, looking specifically at (nonexclusive) licensing by incumbents of entrants' innovations and at price collusion. We show using our comparative statics results that restrictions on these behaviors necessarily lower the rate of innovation. Moreover, restrictions on licensing necessarily lower welfare, while restrictions on collusion may either raise or lower welfare depending on whether innovation is initially socially excessive or insufficient (standard deadweight loss distortions are assumed away in this model).

The analysis of Sections 2-5 makes the strong assumption that only potential entrants do R&D. While useful for gaining understanding, this assumption is rarely descriptive of reality. In Section 6, we turn our attention to models in which both incumbents and potential entrants conduct R&D. Introducing incumbent investment has the potential to substantially complicate our analysis by making equilibrium behavior depend on the level of the incumbent's lead over other firms. We study two models in which we can avoid this state dependence. In one model, we assume that the previous leading technology enters the public domain whenever the incumbent innovates. In this model, the incumbent does R&D solely to avoid displacement by a rival. In our second model, we assume that the profit improvement from a larger lead is linear in the size of the lead and that potential entrants win all "ties," which again leads the incumbent's optimal R&D level to be stationary. In this model, the incumbent does R&D to improve its profit flows until the time that it is displaced by a rival. Interestingly, in both models there are a wide range of circumstances in which a more protective policy can increase the innovation incentives of *both* the incumbent and potential entrants.

In the policies considered in Sections 2-6 antitrust policy affects entrant and incumbent profit flows, which shift only the innovation benefit. In Section 7, we consider antitrust policies that have other effects. We first consider predatory activities, which alter an entrant's probability of survival. As when policy affects profit flows, limitations on predatory activities affect only innovation benefit. We then consider policies that instead involve shifts in innovation supply. These include limitations on behaviors that raise rivals' R&D costs, such as buying up needed R&D inputs or engaging in costly litigation to challenge entrants' patents.

Section 8 concludes and briefly discusses the relation of our analysis to issues in intellectual property protection, where some similar issues arise.

2 A Stylized Model of Antitrust in Innovative Industries

We begin by developing a stylized model of continuing innovation. Our aim is to develop a model that yields some general insights into the effect of antitrust policies on the rate of innovation, and that we can apply to a number of different antitrust policies in the remainder of the paper. In this section, we develop the simplest possible version of this model, which we substantially generalize in the next section.

The model has discrete time and an infinite horizon. There are two firms who discount future profits at rate $\delta \in (0,1)$. In each period, one of the firms is the "incumbent" I and the other is the "potential entrant" E. In the beginning of each period, the potential entrant chooses its R&D rate, $\phi \in [0,1]$; the cost of R&D is given by a strictly convex function $c(\phi)$.[5] The R&D of the potential entrant yields an innovation — which we interpret to be a particular improvement in the quality of the product — with probability ϕ. If the potential entrant innovates, it receives a patent, enters, competes with the incumbent in the present period, and then becomes the incumbent in the next period, while the previous incumbent then becomes the potential entrant. In this sense, this is a model of "winner-take-all" competition. While the patent provides perfect protection (forever) to the innovation itself, the other firm may overtake the patent holder by developing subsequent innovations.

Antitrust policies can impact incentives for innovation in various ways. Through most of the paper we will be interested in the effects of an antitrust policy α that affects the incumbent's competition with an entrant who has just received a patent. Many antitrust policies are of this type and we will analyze some of these in detail in Sections 4 and 5. (In Section 7 we discuss policies that have other effects.) To this end, we denote the incumbent's profit in competition with a new entrant by $\pi_I(\alpha)$, and the profit of the entrant by $\pi_E(\alpha)$, which we assume are differentiable functions of α. We let $\pi'_E(\alpha) > 0$, so that a higher α represents a policy that is more "protective" of the entrant in the sense that it raises the profit of the entrant in the period of entry. Less clear, however, is the overall

[5] Note that $c(\cdot)$ must be convex if the entrant can randomize over its R&D strategy. We assume *strict* convexity to simplify exposition in the simplest version of the model.

effect of an increase in α on the incentive to innovate, since an increase in α will alter as well the value of becoming a continuing incumbent. We also denote by the differentiable function $\pi_m(\alpha)$ the profit of an incumbent who faces no competition in a period. (In Sections 4 and 5, when we consider specific applications, we show how these values can be derived from an underlying model of the product market.)

We examine stationary Markov perfect equilibria of the infinite-horizon game using the dynamic programming approach. Let V_I denote the expected present discounted profits of an incumbent, and V_E those of a potential entrant (both evaluated at the beginning of a period). Then, since innovation occurs with probability ϕ, these values should satisfy

$$V_I = \pi_m(\alpha) + \delta V_I + \phi\left[\pi_I(\alpha) - \pi_m(\alpha) + \delta(V_E - V_I)\right], \tag{VI}$$

$$V_E = \delta V_E + \phi\left[\pi_E(\alpha) + \delta(V_I - V_E)\right] - c(\phi). \tag{VE}$$

Also, a potential entrant's choice of ϕ should maximize its expected discounted value. Letting $w \equiv \pi_E(\alpha) + \delta(V_I - V_E)$ denote the expected discounted benefit from becoming a successful innovator—what we shall call the *innovation prize*—the optimal innovation level is

$$\Phi(w) = \arg\max_{\phi\in[0,1]} \{\phi w - c(\phi)\}.$$

Note that the maximizer is unique since the objective function is strictly concave. Note also that $\Phi(\cdot)$ is continuous (by Berge's Theorem of the Maximum) and nondecreasing (by the Monotone Selection Theorem of Milgrom and Shannon [1994]). Function $\Phi(\cdot)$ gives the innovation decision of the entrant as a function of the innovation prize w, and its graph gives us an "innovation supply curve", which we label IS in Figure 1.

Consider now the determinants of the innovation prize w. Subtracting (1) from (1), solving for $(V_I - V_E)$, and substituting its value into $w \equiv \pi_E + \delta(V_I - V_E)$, allows us to express the innovation prize as $w = W(\phi, \alpha)$, where

$$\begin{aligned} W(\phi,\alpha) &= \pi_E(\alpha) + \delta\left[\frac{\phi\pi_I(\alpha) + (1-\phi)\pi_m(\alpha) - \phi\pi_E(\alpha) + c(\phi)}{1-\delta+2\delta\phi}\right] \\ &= \frac{[1-\delta(1-\phi)]\,\pi_E(\alpha) + \delta\left[\phi\pi_I(\alpha) + (1-\phi)\pi_m(\alpha) + c(\phi)\right]}{1-\delta+2\delta\phi}. \end{aligned} \tag{IB}$$

This equation defines an "innovation benefit curve" — the value of the innovation prize as a function of the innovation rate ϕ. In Figure 2 we graph it (labeled IB) along with the IS curve. Points where the two curves intersect represent equilibrium values of (ϕ, w).

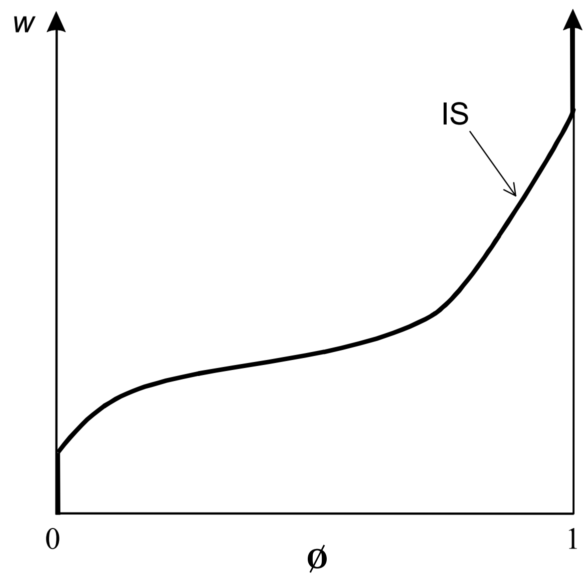

Figure 1: The Innovation Supply Curve

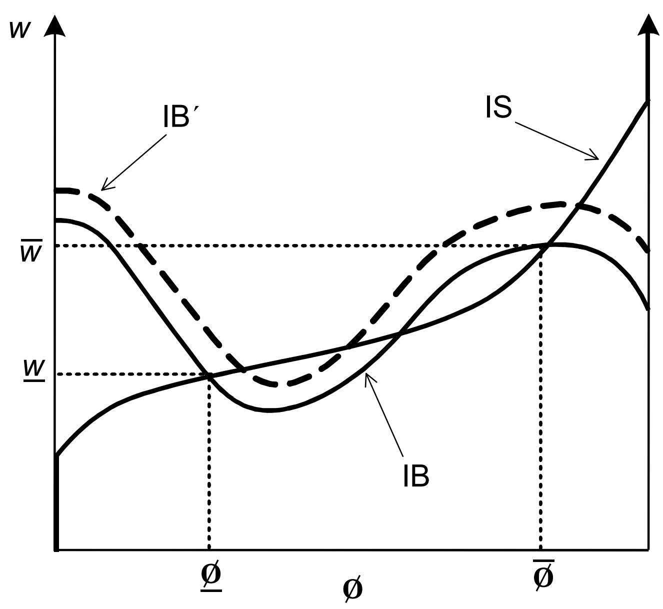

Figure 2: Equilibrium Comparative Statics

Note that the IS curve does not depend on α at all. As seen in Figure 2, if α shifts the IB curve up (down) at all values of ϕ, then it increases (decreases) the equilibrium innovation rate in the "largest" and "smallest" equilibria (denoted by $\underline{\phi}$ and $\overline{\phi}$ respectively in Figure 2). This can be established formally using comparative statics results of Milgrom and Roberts [1994], which we do in the next section. When there is a unique equilibrium, this result implies determinate comparative statics. Also, as evident in Figure 2, the same local comparative statics must hold (using the Implicit Function Theorem) at any "stable" equilibrium (at which the IB curve must cut the IS curve from above) if the IB function is shifted up or down in a neighborhood of the equilibrium. In what follows, we will say that a change in policy "increases (decreases) innovation" whenever these comparative statics properties hold. Differentiating $W(\phi, \alpha)$ with respect to α, we see that the protectiveness of antitrust policy increases (decreases) innovation if

$$\pi_E'(\alpha) + \delta \left[\frac{(1-\phi)\pi_m'(\alpha) + \phi \pi_I'(\alpha)}{1-\delta(1-\phi)} \right] \geq (\leq)\ 0 \tag{1}$$

for all $\phi \in [0, 1]$.

Condition (1) indicates how to sort through the potentially conflicting effects of antitrust policy on innovation incentives that arise from the policy's dual effects on a successful entrant's initial profits and on its returns from achieving incumbency. It shows that a change in policy encourages (discourages) innovation precisely when it raises (reduces) the incremental expected discounted profits over an innovation's lifetime: The first term on the left side of (1) is the change in an entrant's profit in the period of entry due to the policy change, while the second term is equal to the change in the value of a continuing incumbent (the numerator is the derivative of the flow of expected profits in each period of incumbency conditional on still being an incumbent; the denominator captures the "effective" discount rate, which includes the probability of displacement), and thus of the entrant's value once it is itself established as the incumbent.

In interpreting condition (1), it is helpful to think about the case in which the monopoly profit π_m is independent of the antitrust policy α, so that $\pi_m'(\alpha) = 0$. In this case, condition (1) tells us that innovation increases (decreases) if

$$\pi_E'(\alpha) + \left[\frac{\delta\phi}{1-\delta(1-\phi)} \right] \pi_I'(\alpha) \geq (\leq)\ 0. \tag{2}$$

Thus, innovation increases if a weighted sum of $\pi_E'(\alpha)$ and $\pi_I'(\alpha)$ increases, where the weight on $\pi_E'(\alpha)$ exceeds the weight on $\pi_I'(\alpha)$ due to discounting ($\delta < 1$). This implies that a more protective antitrust policy raises innovation whenever $\pi_I'(\alpha) + \pi_E'(\alpha) \geq 0$; that is, provided that an increase in α does not lower the joint profit of the entrant and the incumbent in the period of entry. Intuitively, observe that a successful innovator earns $\pi_E(\alpha)$ when he enters, and earns $\pi_I(\alpha)$ when he is displaced. A more protective antitrust policy that raises π_E and lowers π_I has a *front loading effect*, effectively shifting profits forward in time. Since the later profits π_I are discounted, this front loading of profits necessarily increases the innovation prize provided that the joint profit $\pi_I + \pi_E$ does not decrease.

Observe also that the weight on $\pi_I'(\alpha)$ is increasing in ϕ and in δ. Thus, the larger is δ or ϕ, the more likely is a more protective policy to reduce innovation. For ϕ, this is so because larger ϕ moves forward the expected date when the entrant will itself be displaced. For δ, this is so because with larger δ the discounted value of the profits in the period in which the entrant is displaced are greater. In the limit, as $\delta \to 1$, the amount by which the joint profit $\pi_E + \pi_I$ can be dissipated while still encouraging innovation converges to zero: the cost of a one dollar reduction in the value π_I that the entrant will receive when he is ultimately displaced is exactly equal to the gain from receiving a dollar more in the period in which he enters.

Up to this point (and in the remainder of the paper) we have focused on a stationary setting. However, the front-loading feature of protective antitrust policies suggests that the rate of market growth may alter the impact of antitrust policy on innovation. As an illustration, imagine that profits in period 1 are instead $\beta\pi_I(\alpha), \beta\pi_E(\alpha)$, and $\beta\pi_m(\alpha)$ for $\beta \leq 1$, and are $\pi_I(\alpha), \pi_E(\alpha)$, and $\pi_m(\alpha)$ from period 2 on. Following a similar derivation to that above, a change in α will now increase (decrease) the period 1 innovation rate ϕ_1 if

$$\beta\pi_E'(\alpha) + \delta\left[\frac{(1-\phi)\pi_m'(\alpha) + \phi\pi_I'(\alpha)}{1-\delta(1-\phi)}\right] \geq (\leq)\ 0.$$

Thus, the greater is market growth (the lower is β), the less likely is it that period 1 innovation will increase. For example, when $\beta < 1$ and $\pi_m'(\alpha) = 0$, an increase in α may reduce innovation even when joint profits upon entry increase $(\pi_I'(\alpha) + \pi_E'(\alpha) \geq 0)$.

3 Comparative Statics for More General Innovation Supply and Benefit

The fact that the comparative statics argument above did not depend on the particular shapes of the innovation supply and innovation benefit curves suggests that we can substantially generalize it. For one thing, we can generalize the innovation benefit curve by allowing all three of the profits $\pi_I, \pi_E,$ and π_m to be affected by the rate of innovation ϕ. (This may happen because the price at which consumers purchase a durable good or accept a long-term contract may depend on their expectation of the innovation rate, as in the examples studied in Section 4.1 and Appendix C.) Denoting these profits by $\pi_I(\alpha, \phi)$, $\pi_E(\alpha, \phi)$, and $\pi_m(\alpha, \phi)$, we see that the argument of Section 3 continues to hold if we reinterpret the derivatives in (1) as being partial derivatives with respect to α holding ϕ fixed.

We can also allow for alternative models of innovation supply, such as having more than one potential entrant engage in R&D, or even allowing free entry (i.e., infinitely many potential entrants). (We still do not allow the incumbent to do R&D; we discuss relaxing this assumption in Section 6.) To do so, we define the industry's "innovation rate" ϕ as the probability that the incumbent technology is displaced with an innovation. For a symmetric industry, ϕ also determines the potential entrants' *individual* R&D investments [and, hence, their R&D cost $c(\phi)$] and the probability $u(\phi)$ that a *given* potential entrant becomes a new incumbent (i.e., moves "up"). The expected present discounted profits of an incumbent (V_I) a potential entrant (V_E) can then be described by

$$\begin{aligned} V_I &= \pi_m(\alpha) + \delta V_I + \phi\left[\pi_I(\alpha) - \pi_m(\alpha) + \delta(V_E - V_I)\right], && \text{(VI*)} \\ V_E &= \delta V_E + u(\phi)\left[\pi_E(\alpha) + \delta(V_I - V_E)\right] - c(\phi). && \text{(VE*)} \end{aligned}$$

Subtracting, we can express the innovation prize $w = \pi_E + \delta(V_I - V_E)$ with the following function:

$$W(\phi, \alpha) = \frac{\left[1 - \delta(1 - \phi)\right]\pi_E(\alpha, \phi) + \delta\left[\phi\pi_I(\alpha, \phi) + (1 - \phi)\pi_m(\alpha, \phi) + c(\phi)\right]}{1 - \delta + \delta(\phi + u(\phi))}. \qquad \text{(IB*)}$$

As for the innovation supply curve, which describes the entrants' R&D response to a given innovation prize, our comparative statics results hold as long as the curve is described by a correspondence $\Phi(w)$ satisfying the following three properties:

(IS1): $\Phi(\cdot)$ is nonempty- and convex-valued;

(IS2) $\Phi(\cdot)$ has a closed graph;

(IS3) Any selection from $\Phi(\cdot)$ is nondecreasing (i.e., if $w' > w$, $\phi' \in \Phi(w')$, and $\phi \in \Phi(w)$, then $\phi' \geq \phi$).

For example, if $\Phi(\cdot)$ is a function (i.e., single-valued), (IS1) is vacuous, (IS2) means that the function is continuous, and (IS3) means that it is nondecreasing. Other correspondences satisfying (IS1)-(IS3) are obtained by taking a nondecreasing function and "filling in" its jumps, as illustrated in Figure 3.

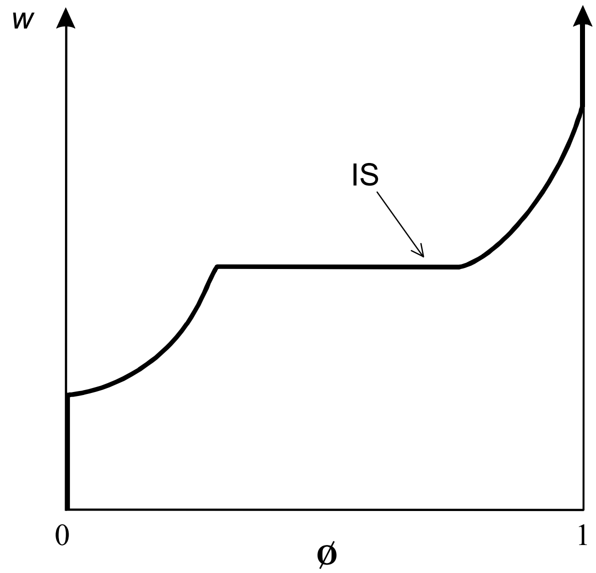

Figure 3: An IS curve satisfying (IS1)-(IS3)

Properties (IS1)-(IS3) ensure that antitrust policy affects the largest and smallest equilibrium innovation rates in the same direction in which it shifts the innovation benefit curve, which we can determine by partially differentiating $W(\phi, \alpha)$:

Proposition 1 *If the innovation supply correspondence $\Phi(\cdot)$ satisfies (IS1)-(IS3) and the innovation benefit function $W(\phi, \alpha)$ is continuous in ϕ, then the largest and smallest equilibrium innovation rates exist, and both these rates are nondecreasing (nonincreasing) in α, the protectiveness of antitrust policy, if*

$$\pi_E'(\alpha, \phi) + \delta \left[\frac{(1-\phi)\pi_m'(\alpha, \phi) + \phi \pi_I'(\alpha, \phi)}{1 - \delta(1-\phi)} \right] \geq (\leq)\ 0 \tag{3}$$

for all $\phi \in [0, 1]$.

Proof. The equilibrium innovation rates are the fixed points of the composite correspondence $\gamma(\alpha, \cdot) \equiv \Phi(W(\alpha, \cdot))$. (IS1)-(IS3) and the continuity of $W(\alpha, \cdot)$ imply that $\gamma(\alpha, \phi) = [\gamma_L(\alpha, \phi), \gamma_H(\alpha, \phi)] \neq \emptyset$, and that the correspondence $\gamma(\alpha, \cdot)$ is continuous but for upward jumps (Milgrom and Roberts [1994]). Furthermore, if $W(\phi, \alpha)$ is nondecreasing (nonincreasing) in α, then so are $\gamma_L(\alpha, \phi)$ and $\gamma_H(\alpha, \phi)$. Corollary 2 of Milgrom and

Roberts [1994] then establishes that the largest and smallest fixed points of $\gamma(\alpha,\cdot)$ exist and are nondecreasing (nonincreasing) in α. ■

If there are multiple equilibria, this result does allow some equilibria to move in the direction opposite to that predicted by Proposition 1. However, the same local comparative statics holds for any locally unique equilibrium $\phi(\alpha)$ at which we have *crossing from above*, i.e., for some interval $\left[\underline{\phi},\overline{\phi}\right]$, $\Phi(W(\alpha,\phi)) \subset [\phi,1]$ for $\phi \in \left[\underline{\phi},\phi(\alpha)\right]$ and $\Phi(W(\alpha,\phi)) \subset [0,\phi]$ for $\phi \in \left[\phi(\alpha),\overline{\phi}\right]$, which is a necessary condition for (Lyapunov) stability.[6]

Proposition 2 *Suppose that the innovation supply correspondence $\Phi(\cdot)$ satisfies (IS1)-(IS3) and the innovation benefit function $W(\phi,\alpha)$ is continuous in ϕ. Suppose, in addition, that for any $\alpha \in [\underline{\alpha},\overline{\alpha}]$ there is a unique equilibrium innovation rate $\phi(\alpha)$ on an interval $\left[\underline{\phi},\overline{\phi}\right]$ and that the IB curve crosses the IS curve from above on this interval. Then $\phi(\alpha)$ is nondecreasing (nonincreasing) if condition (3) holds for all $\alpha \in [\underline{\alpha},\overline{\alpha}]$ and $\phi \in \left[\underline{\phi},\overline{\phi}\right]$.*

Proof. Let

$$\chi(\phi) = \begin{cases} \underline{\phi} & \text{for} \quad \phi \in \left[0,\underline{\phi}\right), \\ \phi & \text{for} \quad \phi \in \left[\underline{\phi},\overline{\phi}\right], \\ \overline{\phi} & \text{for} \quad \phi \in \left(\overline{\phi},1\right]. \end{cases}$$

Crossing from above implies that $\phi(\alpha)$ is also the unique fixed point of the composite correspondence $\gamma(\alpha,\cdot) \equiv \chi(\Phi(W(\alpha,\cdot)))$ on the interval $\left[\underline{\phi},\overline{\phi}\right]$. Since this correspondence satisfies (IS1)-(IS3), the proof of Proposition 1 implies the result. ■

We now provide three examples in which (IS1)-(IS3) hold and so Propositions 1 and 2 determine the comparative statics effect of antitrust policy on the rate of innovation.

3.1 Example: One entrant with constant returns R&D technology

Suppose that we still have one entrant but the R&D cost function is $c(\phi) = c \cdot \phi$ for $c > 0$. In this case, the innovation supply is no longer a function: $\Phi(w) = 0$ if $w < c$, $[0,\infty]$ if $w = c$, and 1 if $w > c$. However, since $\Phi(\cdot)$ satisfies (IS1)-(IS3), Propositions 1 and 2 tell us that the same comparative statics apply.

[6] Specifically, suppose that the industry in period t adaptively expects the future innovation rate to be ϕ_{t-1}, and so chooses innovation rate $\phi_t \in \Phi\left(W\left(\alpha,\phi_{t-1}\right)\right)$. If we have "crossing from below," then the dynamics starting at any disequilibrium $\phi \in \left[\underline{\phi},\overline{\phi}\right]$ must leave the interval $\left[\underline{\phi},\overline{\phi}\right]$, so that the equilibrium is not (Lyapunov) stable.

3.2 Example: More than one potential entrant

Suppose that there are $N > 1$ potential entrants in any period (in addition to the single incumbent). In the beginning of each period, each potential entrant i independently chooses its R&D rate $\psi_i \in [0, 1]$; the cost of R&D is given by a convex function $\gamma(\psi_i)$. The R&D of a given potential entrant i yields a discovery with probability ψ_i (we assume that the discoveries are independently realized). We shall focus on symmetric equilibria, in which all potential entrants choose the same equilibrium level of R&D, denoted by ψ. In this case, the likelihood that at least one of the N potential entrants makes a discovery is given by $\phi = \left[1 - (1 - \psi)^N\right]$. Thus, in a symmetric equilibrium with aggregate innovation rate ϕ, the potential entrants' individual R&D choices are $\psi_N(\phi) = 1 - (1 - \phi)^{1/N}$.

Among the potential entrants who make a discovery, only one may receive the patent for the discovery. Denote by $r_N(\psi)$ the probability that a given potential entrant receives a patent, conditional on it making a discovery, when all other potential entrants are doing R&D at level ψ. We assume that $r_N(\cdot)$ is a strictly decreasing function.[7] A potential entrant who is successful at receiving a patent enters and competes with the incumbent in the present period, and then becomes the incumbent in the next period, while the previous incumbent then becomes a potential entrant.

Note that in the symmetric equilibrium, the probability that a given entrant becomes the incumbent is $u(\phi) = \psi_N(\phi) r_N(\psi_N(\phi))$, and the entrant's R&D cost is $c(\phi) = \gamma(\psi_N(\phi))$. The innovation benefit curve is then given by substituting these expressions in (IB*) above.

As for the innovation supply, the equilibrium individual innovation rate solves the following equilibrium condition for a given value of w

$$\psi = \arg\max_{\psi' \in [0,1]} \left\{\psi' r_N(\psi) w - \gamma(\psi')\right\}. \tag{4}$$

As shown in Lemma A.1 in Appendix A, this describes a unique equilibrium level of ψ, which is a continuous and nondecreasing function of w. These properties are preserved for the *aggregate* equilibrium innovation rate ϕ, and so the aggregate innovation supply $\Phi(\cdot)$ satisfies (IS1)-(IS3). Therefore, Propositions 1 and 2 apply to this model.

[7] When the patent is awarded randomly to one of the successful innovators,

$$r(\psi) = \sum_{k=0}^{N-1} \left(\frac{1}{k+1}\right) \begin{pmatrix} N-1 \\ k \end{pmatrix} \psi^k (1-\psi)^{N-1-k} = \frac{1-(1-\psi)^N}{\psi N}.$$

3.3 Example: Free entry

In some circumstances it may be more appropriate to assume that there is free entry into R&D competition.[8] This assumption can be interpreted as a limiting case of a very large number N of potential entrants each of whom engages in an infinitesimal amount of R&D while the aggregate innovation rate is positive. An innovator's conditional probability of getting a patent can then be written using the expression in footnote 7 as

$$\bar{r}(\phi) = \lim_{N\to\infty} r_N(\psi_N(\phi)) = \lim_{N\to\infty} \frac{\phi}{\left(1-(1-\phi)^{1/N}\right)N} = -\frac{\phi}{\ln(1-\phi)},$$

which is a continuous decreasing function of $\phi \in [0,1]$. From (4), the first-order condition for each potential entrant to choose a positive infinitesimal R&D is $w\bar{r}(\phi) = \gamma'(0)$, which determines the innovation supply function $\Phi(w) = \bar{r}^{-1}(\gamma'(0)/w)$ for $w > \gamma'(0)$, and $\Phi(w) = 0$ for $w \leq \gamma'(0)$. Since this is a continuous and nondecreasing function, the comparative statics is again described by Propositions 1 and 2.

3.4 Example: Free entry with a limited idea

In the models of Fudenberg and Tirole [2000] and O'Donoghue et al. [1998], there are infinitely many potential entrants, but in each period only one of them is randomly drawn to receive an "idea" that enables him to invest in R&D. Suppose that this potential entrant then observes a randomly drawn implementation cost γ and chooses whether to invest in implementing the innovation. If he does, he is certain to become the next incumbent.

Observe first that the optimal strategy of potential entrants takes the form of choosing a cost threshold $\bar{\gamma}$ below which to implement their idea. This is equivalent to choosing the probability of innovation $\phi = \Pr\{\gamma \leq \bar{\gamma}\} \equiv F(\bar{\gamma})$, and the associated expected innovation cost $\int_0^{\bar{\gamma}} \gamma dF(\gamma)$. In other words, the innovation supply is equivalent to having a single potential entrant who chooses an innovation probability ϕ at an expected cost of $\int_0^{F^{-1}(\phi)} \gamma dF(\gamma)$, which is a convex function of ϕ. Thus, the innovation supply curve is determined just as in the single-firm model, and satisfies (IS1)-(IS3). For the innovation benefit curve, on the other hand, in (IB*) we take $c(\phi) = u(\phi) = 0$ since each potential entrant in expectation does not incur any cost and has zero chance of innovating. Once again, the comparative statics are described by Propositions 1 and 2.

[8] The fixed N model is the appropriate model when there are a limited number of firms with the capability of doing R&D in an industry (perhaps because of complementary assets they possess due to participation in related industries).

4 Applications: R&D-Deterring Activities

In this section, we use the results of Section 3 to analyze two models in which an incumbent engages in activities designed to deter the R&D of potential entrants. The models we consider are all versions of the "quality ladder" models introduced in the recent literature on economic growth (e.g., Aghion and Howitt [1992]; Grossman and Helpman [1991]). Our results in this section will apply for any continuous nondecreasing innovation supply function.

Before turning to these applications, we first make a surprising observation: In many circumstances involving R&D-deterring activities, the tension between a more protective policy's effect on a new entrant and its effect on an incumbent will not arise, at least once we hold the rate of innovation fixed, as Proposition 1 suggests we do.

To see why, consider the following stylized model: Imagine that at the end of each period the firm with the leading technology can commit to some behavior $d \in \mathbb{R}$ that affects both its profits and an entrant's profits in the following period. Let these profits be given by functions $\widehat{\pi}_m(d)$, $\widehat{\pi}_I(d)$, and $\widehat{\pi}_E(d)$, where $\widehat{\pi}'_E(d) < 0$ (for simplicity we assume that these functions do not depend on ϕ, although this is inessential). Consider a stationary Markov perfect equilibrium in which potential entrant R&D is a decreasing function of the state variable d (chosen in the previous period) given by the function $\phi^*(d)$ with $\phi^{*\prime}(d) < 0$. Then, the equilibrium level of d absent any antitrust constraint, d^*, maximizes

$$\phi^*(d)[\widehat{\pi}_I(d) + \delta V_E] + (1 - \phi^*(d))[\widehat{\pi}_m(d) + \delta V_I], \tag{5}$$

and satisfies the first-order condition

$$[\phi\widehat{\pi}'_I(d^*) + (1-\phi)\widehat{\pi}'_m(d^*)] + \phi^{*\prime}(d^*)\{[\widehat{\pi}_I(d^*) + \delta V_E] - [\widehat{\pi}_m(d^*) + \delta V_I]\} = 0.$$

Thus, $[\widehat{\pi}_I(d^*)+\delta V_E]-[\widehat{\pi}_m(d^*)+\delta V_I] \leq 0$ (which says that it is valuable to the incumbent to prevent entry) implies that $[\phi\widehat{\pi}'_I(d^*)+(1-\phi)\widehat{\pi}'_m(d^*)] \geq 0$: starting at d^*, a small reduction in d, *holding ϕ fixed*, actually *raises* the expected profit of a continuing incumbent (as shown in Figure 4). This tells us that if the antitrust policy α requires that $d \leq d^* - \alpha$, then for small enough α no tension arises: *both* terms in condition (3) are nonnegative, and so innovation necessarily increases when a slightly protective antitrust policy is introduced.[9]

More generally, suppose that the function $\phi\widehat{\pi}_I(d) + (1-\phi)\widehat{\pi}_m(d)$ is a *pseudo-concave* function of d, i.e., it is increasing below some peak d^{**} and decreasing above d^{**}, as in Figure 4. (Thus, d^{**} is the level of d that an incumbent unconcerned with preventing entry

[9]Formally, taking $\pi_j(\alpha) \equiv \widehat{\pi}_E(d^* - \alpha)$ for $j = m, I, E$, we have $\phi\pi'_I(\alpha) + (1-\phi)\pi'_m(\alpha) \geq 0$.

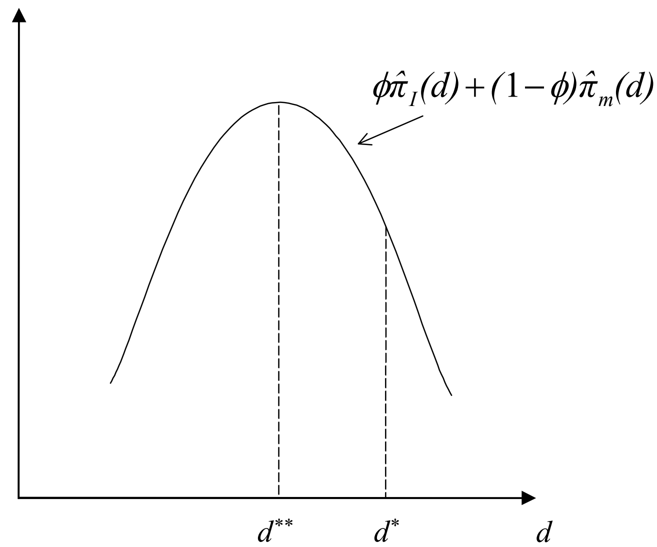

Figure 4: Antitrust and Expected Incumbent Profits

would choose.) Then, for equilibria in which the antitrust constraint is not so severe as to force d below d^{**}, both incumbent and entrant profits increase with an increase in α holding ϕ fixed, and hence so does the rate of innovation.

The key insight here comes because of Proposition 1's characterization, which instructs us to think about profit effects holding the rate of innovation ϕ fixed. This frequently eliminates the apparent tension between the effects of policy changes on entrant and incumbent profits: If (on the margin) an R&D-deterring activity involves a sacrifice in incumbent profit in return for a reduction in the probability of entry then, *holding ϕ fixed*, both entrant and incumbent profits are increased by a more protective policy, and so innovation increases.

We now turn to two more specific applications in which we derive the relevant profit functions from fundamentals, and in which there is a fully specified consumer side, so that welfare analysis is possible.

4.1 Long-term (exclusive) contracts

We first consider a model in which the incumbent can sign consumers to long-term contracts. There are at least 2 firms and a continuum of infinitely-lived consumers of measure 1 who may consume a nonstorable and nondurable good with production cost $k \geq 0$. R&D may improve the quality of this good and consumers value "generation j" of the good at $v_j = v + j \cdot \Delta$. At any time t, one firm — the current "incumbent" — possesses a perfectly effective and infinitely-lived patent on the latest generation product j_t. Likewise, at time t there is a patentholder for each of the previous generations of the product $(j_t - 1, j_t - 2, ...)$. We assume, as in Sections 2 and 3, that at time t only firms other than the incumbent in the leading technology — the potential entrants — can invest in developing the generation $j_t + 1$ product. One implication of this assumption is that in each period t the holder of the patent on generation $j_t - 1$ is a firm other than the current incumbent, who holds the patent on the current leading generation j_t.

Suppose that in each period t, the incumbent can offer long-term contracts to a share β_{t+1} of period $t+1$ consumers. The contracts specify a sale in period $t+1$ at a price q_{t+1} to be paid upon delivery. (In our simple model, this is equivalent to an exclusive contract that prevents the consumer from buying from the entrant, subject to some irrelevant issues with the timing of payments.) The antitrust policy restricts the proportion of consumers that can be offered long-term contracts: $\beta_{t+1} \leq 1 - \alpha$. We assume that the production cost k exceeds the quality increment Δ, so that an entrant cannot profitably make a sale to a consumer who is bound to a long-term contract.

The timing in period t is:

- **Stage** $t.1$**:** Each potential entrant i observes the share of captured consumers β_t and chooses its innovation rate ψ_{it}. Then innovation success is realized.

- **Stage** $t.2$**:** Firms name prices p_t^i to free period t consumers.

- **Stage** $t.3$**:** Free period t consumers accept/reject these offers.

- **Stage** $t.4$**:** The firm with the leading technology chooses to offer to a share $\beta_{t+1} \leq 1 - \alpha$ of period $t+1$ consumers a period $t+1$ sales contract at price q_{t+1} to be paid upon delivery.

- **Stage** $t.5$**:** Period $t+1$ consumers accept/reject these contract offers (they assume that they have no effect on the likelihood of future entry).[10]

Observe that when $\alpha = 0$ so that no long-term contracts can be written, we have a simple model in which Bertrand competition between the leading firm and firms further down the ladder occurs each period. In Appendix B, we discuss this benchmark quality ladder model, and the distortions that arise in the rate of innovation relative to the first-best level.

[10] We assume throughout that consumers all accept if accepting is a continuation equilibrium (we do not allow consumers to coordinate). The leading firm could achieve this by, for example, committing to auction off the desired number of long-term contracts.

In general, innovation may be either insufficient or excessive because of "Schumpeterian" and "business stealing" effects.

Here we focus on Markov perfect equilibria. In particular, we study equilibria in which potential entrants in stage $t.1$ condition their innovation choices only on the current share of captive customers β_t, and in which the choices at all other stages are stationary. (Note that since period t contracts expire at the end of that period, there is no relevant state variable affecting the contracting choice of the leading firm at stage $t.4$.)

It is immediate that in any such equilibrium, the prices offered to free customers in any period t are $k+\Delta$ by the firm with the leading technology j_t, who wins the sale, and k by the firm with technology $j_t - 1$. Now consider a consumer's decision of whether to accept a long-term contract. If in period t the expected probability of entry in period $t+1$ is ϕ_{t+1}, a period $t+1$ consumer who rejects the leading firm's long-term contract offer anticipates getting the period t surplus level $v + (j_t - 1)\,\Delta - k$ plus an expected gain in surplus of $\phi_{t+1}\Delta$ due to the possibility of technological advancement in period $t+1$. Thus, he will accept the contract if and only if the price q_{t+1} satisfies $v + j_t\Delta - q_{t+1} \geq v + \left(j_t - 1 + \phi_{t+1}\right)\Delta - k$. Hence, the maximum price the incumbent can receive in a long-term contract is $q_{t+1} = k + (1 - \phi_{t+1})\Delta$, which leaves the consumer indifferent about signing.

How many consumers will the leading firm sign up in period t? Observe first that if the probability of entry ϕ_{t+1} were independent of β_{t+1}, then the leading firm would be indifferent about signing up an extra consumer: its period t expectation of the profit from a free consumer in period $t+1$ is $(1 - \phi_{t+1})\Delta$, which exactly equals its maximal expected profit from a long-term contract. However, ϕ is nonincreasing in β_{t+1}, because an increase in the share of captive customers reduces the profits a successful entrant can collect in period $t+1$.[11] Therefore, the incumbent optimally signs up as many long-term customers as the antitrust constraint allows, i.e., $\beta_{t+1} = 1 - \alpha$ in every period. We can therefore fit this model into our basic model by taking

$$\begin{aligned} \pi_m(\alpha,\phi) &= \alpha\Delta + (1-\alpha)(1-\phi)\Delta \\ \pi_I(\alpha,\phi) &= (1-\alpha)(1-\phi)\Delta \\ \pi_E(\alpha,\phi) &= \alpha\Delta. \end{aligned} \tag{6}$$

Observe, first, that in this model an increase in α does indeed raise π_E. More signifi-

[11] Formally, the innovation rate of a potential entrant in period $t+1$ when a share β_{t+1} of period $t+1$ consumers have signed long-term contracts is $\Phi((1-\beta_{t+1})\Delta + \delta(V_I^{t+2} - V_E^{t+2}))$, where V_I^{t+2} and V_E^{t+2} are the continuation values at the start of period $t+2$, which are independent of β_{t+1}. By (IS3), this innovation rate is nonincreasing in β_{t+1}.

cantly,

$$[\phi\pi_I(\alpha,\phi)+(1-\phi)\pi_m(\alpha,\phi)]=(1-\phi)\alpha\Delta+(1-\alpha)(1-\phi)\Delta=(1-\phi)\Delta.$$

Thus, holding ϕ fixed, the expected profit of a continuing incumbent is *independent of* α. The reason is that, holding ϕ fixed, it is a matter of indifference to both the incumbent (and consumers) whether consumers accept a long-term contract. This is a case in which an R&D-deterring activity has no cost to the incumbent. Thus, we see immediately that condition (3) is satisfied, and so we have:

Proposition 3 *In our basic model of long-term (exclusive) contracts, restricting the use of long-term (exclusive) contracts encourages innovation.*

Consider now the welfare effects of a once-and-for-all increase in the policy α in some period τ. We assume that this intervention occurs just after stage $\tau.1$.[12] The payoff effects of such a change begin in period $\tau+1$. Note, first, that the increase raises consumer surplus: consumers are indifferent about signing exclusives when the innovation rate is held fixed, but an increase in the innovation rate delivers to them higher-quality goods at the same prices. What about the sum of consumer surplus and current incumbent (i.e., the firm with the leading technology just after stage $\tau.1$) profits? There is no direct effect of the policy change on either consumers or the current incumbent because, holding ϕ fixed, both are indifferent about whether long-term contracts are signed. What about the indirect effect caused by the increase in ϕ? Intuitively, an innovation in period $\tau+1$ reallocates surplus $\alpha\Delta$ from the incumbent to period $\tau+1$ consumers. However, in subsequent periods the innovation confers an expected benefit Δ to consumers but at an expected cost to the incumbent that is less than Δ as long as the probability of future displacement is positive (i.e., $\phi>0$).

Now consider the effects on potential entrants. Observe that in each of the examples we considered in Sections 3.1-3.4, $u(\Phi(w))w-c(\Phi(w))$ is nondecreasing in w, i.e., each potential entrant is better off if the innovation prize increases. We refer to this as the *Value Monotonicity Property.* Since we are moving upward along the upward-sloping IS curve

[12] We make this assumption to simplify the analysis. By doing so, the equilibrium innovation rate transitions immediately to its new steady state value and all payoff changes begin in period $\tau+1$. If, instead, the intervention occurs at the start of the period, there would be a one period transitory effect on the innovation rate because in period τ the share of captive customers facing an entrant would be the level from before the policy change while the continuation values V_I and V_E starting in period $\tau+1$ would be the levels in the new steady state.

when α increases, the increase in ϕ caused by the increase in α must be associated with an increase in w. When the Value Monotonicity Property is satisfied, this implies that each potential entrant becomes (weakly) better off. Finally, what about the current incumbent? This turns out to be ambiguous: on one hand, the increase in ϕ speeds the incumbent's displacement. On the other hand, the value V_E that the incumbent receives when he is displaced may increase. This reasoning leads to the following result:

Proposition 4 *A once-and-for-all restriction on the share of long-term (exclusive) contracts that raises innovation raises consumer surplus, aggregate welfare, and (when the Value Monotonicity Property is satisfied) the values of potential entrants. The effect on the current incumbent's value is ambiguous.*

Proof. In Appendix A.

It is perhaps surprising that the welfare effect of an increase in α is necessarily positive, given that the equilibrium innovation rate may be above the first-best level due to business stealing (see Appendix B). Note, however, that long-term contracts involve an inefficiency since when entry occurs the incumbent makes sales of an old technology to captive consumers. Thus, even if an increase in the share of captive consumers brings a socially excessive innovation rate closer to the first-best level, the waste effect dominates and aggregate welfare is reduced.[13]

In the model above, holding ϕ fixed, incumbent and entrant profits both increased with a more protective policy, and so innovation necessarily increased. In Appendix C, we consider an extension of this long-term contracting model in which the innovation sizes (Δ) are uncertain *ex ante* and there is a cost of rapidly implementing new innovations. The key new feature in this model is that antitrust policy has a "selection effect": by altering the profitability of entry, antitrust policy alters the set of innovations that rapidly enter the market. We show in Appendix C that in this situation, limitations on long-term contracting may retard innovation.

4.2 Compatibility in a network industry

We next consider a model of compatibility choices by a leading firm in an industry with network externalities. The model is patterned after Fudenberg and Tirole [2000] who

[13]Indeed, observe that potential entrants, who directly suffer from the business-stealing effect, are necessarily better off when α increases.

studied limit pricing in a dynamic model.[14] Overlapping generations of consumers live for two periods, and make purchases only when young. Each generation is of unit mass. The value of consumption in a period t is $v + j_t\Delta + v(N)$ if the consumer consumes the leading quality good j_t and this good has a "network size" of N. We follow the convention in the network externalities literature and assume that consumers in each generation coordinate their purchases, acting as a single agent and purchasing from a single firm. We also assume that $\Delta > v(2) - v(1)$. This implies that the firm with the leading quality will also have the leading effective quality once we include network benefits, regardless of the network sizes of the various goods.

There are many ways in which compatibility is determined in actual markets. Here we focus on one that leads to a relatively simple model that fits our framework. We assume that each firm that offers its product to consumers chooses a price p and also a compatibility level $\beta \in [0, 1]$ of this product with higher quality products. Network benefits are then determined as follows: the network size enjoyed by generation g of consumers who have bought good j is 2 if all consumers in a period consume it, is 1 if the other existing generation of consumers consumes a higher quality good, and is $1 + \beta_l$ if the other existing generation of consumers consumes a lower quality product l which has compatibility level β_l with higher quality products. That is, while consumers of high quality products can benefit from the existence of consumers who consume a lower quality product (to the extent that this good's producer allows), the reverse is not true.[15,16] The cost of producing a product with compatibility level β is $k(\beta)$. We define $\underline{k} \equiv \min k(\beta)$.

In each period t the game proceeds as follows:

- **Stage** $t.1$**:** Each potential entrant i (firms other than the producer of the leading quality product) observes the purchase choice and compatibility level β_{t-1} of the current old generation of consumers. Potential entrants then make their R&D choices

[14] There are several key differences: we have only one type of consumer (so limit pricing is not possible), firms make compatibility choices, and patent protection lasts forever.

[15] As an example, consider 386 and 486 chips: software designed for 386 machines also worked on 486 machines, but not the reverse.

[16] In essence, we assume that the higher quality product can costlessly achieve as much backward compatibility as the lower quality firm allows. Were we to allow the higher quality firm a choice of whether it wants backward compatibility (at no cost), it would always choose the maximal possible level. Regarding the lower quality product, our assumptions allow its producer to commit to a compatibility level. This may be thought of as a product design choice. For example, a patented interface may prevent a new entrant from achieving backward compatibility.

and innovation success is realized.

- **Stage** $t.2$**:** Firms choose compatibility levels β_t^i and name prices p_t^i to young consumers.

- **Stage** $t.3$**:** Young consumers make their purchase decisions.

Our policy parameter $\alpha \in [0,1]$ will put a lower bound on the compatibility level that the firm with the leading technology can choose.[17] We focus here on Markov perfect equilibria in which the leading firm (whether a continuing incumbent or a new entrant) always wins the sales to young consumers, and in which the probability of entry in period $t+1$ when period t's leading quality firm sold today's old consumers a product with compatibility level β_t, $\phi^*(\beta_t)$, is increasing in β_t.[18] We let $V_I(\beta)$ denote the value of an incumbent seller who has the highest quality and yesterday sold products of quality β to today's old consumers. It will also be useful to define

$$B(\phi) \equiv v(2) + \delta[(1-\phi)v(2) + \phi v(1)].$$

This is the discounted expected network benefit that a young consumer anticipates if he buys the product of a continuing incumbent seller (who has previously sold to today's old consumers) when the probability of entry tomorrow is ϕ.

Consider, first, the period t pricing, compatibility choice, and profit of a continuing incumbent who has the highest quality in a period without entry. Its relevant competitor is the firm with the next-highest quality product, the previous incumbent. The previous incumbent can offer the young consumers a surplus of[19]

$$[v + (j_t - 1)\Delta] + (1+\delta)v(1) - \underline{k}.$$

The continuing incumbent therefore sets its compatibility level equal to

$$\beta^*(\alpha) \in \arg\max_{\widetilde{\beta} \geq \alpha} \Delta + \{[B(\phi^*(\widetilde{\beta})) - k(\widetilde{\beta})] - [(1+\delta)v(1) - \underline{k}]\} + \delta V_I(\widetilde{\beta})$$

[17] We remark below on the effects of having all firms subject to this constraint.

[18] Off the equilibrium path, when a firm other than the leading quality firm has won the sales to yesterday's young consumers, the probability of entry will be $\phi^*(1)$ since the leading quality firm will have no existing network, leaving entrants in the same strategic situation as when a leading quality firm makes sales but chooses to be fully compatible.

[19] Recall that, in this equilibrium, if today's young consumers buy from the previous incumbent, then tomorrow's young consumers are certain to buy from a different firm.

and earns $\{\Delta + [B(\phi^*(\beta^*(\alpha))) - k(\beta^*(\alpha))] - [v(1)(1+\delta) - \underline{k}]\}$ today from sales to current young consumers and a continuation value of $V_I(\beta^*(\alpha))$.

Now consider the period t pricing, compatibility choice, and profit of an entrant against a continuing incumbent that prior to this entry was the highest quality firm and yesterday sold a product with compatibility level β_t to the current old consumers. Its relevant competitor is the continuing incumbent. The continuing incumbent can offer the young consumers a surplus of[20]

$$[v + (j_t - 1)\Delta] + [v(2) + \delta v(1) - \underline{k}].$$

The entrant will therefore set its compatibility level equal to

$$\beta^*(\alpha) \in \arg\max_{\widetilde{\beta} \geq \alpha} \Delta + \{[B(\phi^*(\widetilde{\beta})) + v(1+\beta_{t-1}) - v(2) - k(\widetilde{\beta})] - [v(2) + \delta v(1) - \underline{k}]\} + \delta V_I(\widetilde{\beta})$$

and will earn $\{\Delta + [B(\phi^*(\beta^*(\alpha)) + v(1+\beta_{t-1}) - v(2) - k(\beta^*(\alpha))] - [v(2) + \delta v(1) - \underline{k}]\}$ from current consumers and a continuation value of $V_I(\beta^*(\alpha))$.[21] Of course, in equilibrium, we will have $\beta_t = \beta^*(\alpha)$.

Now consider equilibria in which the antitrust constraint binds, i.e., in which $\beta^*(\alpha) = \alpha$. For these equilibria we have:

$$\begin{aligned}
\pi_m(\alpha, \phi) &= \Delta + \{[B(\phi) - k(\alpha)] - [v(1)(1+\delta) - \underline{k}]\} \\
\pi_I(\alpha, \phi) &= 0 \\
\pi_E(\alpha, \phi) &= \Delta + \{[B(\phi) + v(1+\alpha) - v(2) - k(\alpha)] - \\
&\quad [v(2) + \delta v(1) - \underline{k}]\}.
\end{aligned} \tag{7}$$

To see the effect of a more protective antitrust policy on innovation, consider first the case in which $k(\cdot)$ is decreasing so that it is costly to block compatibility by a higher quality entrant. In this case, a firm choosing to be incompatible incurs increased production costs today to deter R&D and reduce tomorrow's likelihood of entry. As is evident from (7), in this case π_m and π_E both increase when α increases, while π_I remains unchanged. Again, no tension arises, and Propositions 1 and 2 tell us that with this type of R&D-deterring behavior, innovation increases with a more protective policy.

When $k(\cdot)$ is not everywhere decreasing in β, however, increased protectiveness may instead lower innovation. For example, suppose that $k(\cdot)$ is convex with its minimum at

[20] Once again, in this equilibrium, if today's young consumers buy from the previous incumbent, then tomorrow's young consumers are certain to buy from a different firm.

[21] From this one can verify that the innovation rate will be nonincreasing in β_{t-1} in any model satisfying (IS1)-(IS3).

$\underline{\beta} \in (0,1)$. Examining (7), we see that increasing α above $\underline{\beta}$ may reduce innovation: it certainly lowers π_m and may even lower π_E [if $v(1+\alpha) - k(\alpha)$ falls].[22] In this case, forcing compatibility above the level that minimizes costs may reduce R&D. At such compatibility levels, we are no longer in a region where the incumbent is trading off reduced profits today for reduced R&D tomorrow.

A full investigation of the welfare effects of a more protective policy in this model is beyond our scope here. Nevertheless, it is worth noting that in this model innovation can be excessive even with only a single potential entrant (in contrast to our benchmark model in Appendix B). The reason is that an externality exists between the two generations of consumers in each period: when the young purchase an entrant's product they leave old consumers with lower network benefits. Indeed, while young consumers have a benefit of $[\Delta - v(2) + v(1 + \beta^*(\alpha))]$ in the first period that an entrant is in the market, the old consumers lose $[v(2) - v(1)]$. Thus, when $\frac{\Delta}{1-\delta} < [2v(2) - v(1) - v(1 + \beta^*(\alpha))]$, an innovation lowers aggregate welfare, even ignoring R&D costs. Thus, it would not be surprising if a more protective policy that raises innovation could lower aggregate welfare here.

5 Applications: Voluntary Deals

Although we have motivated our analysis by discussing examples of exclusionary behaviors, our framework is not limited to such applications. Another sort of behavior to which we can apply our model is a voluntary deal between an incumbent and a new entrant. With the innovation rate held fixed, voluntary deals — by definition — raise both parties' payoffs.[23] By Proposition 1, such deals should therefore increase innovation. Here we briefly consider two examples of such deals.

5.1 Licensing of the entrant's technology

Imagine that in our long-term contracting model the incumbent can license a new entrant's technology for serving his captive consumers. Specifically, assume that the incumbent is then able to make a take-it-or-leave-it offer to these captive buyers, offering to give them instead the entrant's better product for an additional payment of Δ. The incumbent and

[22] If the policy instead applied to all firms, then we would have $\underline{k} = k(\alpha)$; in this case π_m and π_E would still be increasing in α, so innovation would increase with α.

[23] This conclusion depends on the fact that we have only two parties negotiating. With more than two active firms, profits for some or all parties may fall when voluntary deals are allowed (see Segal [1999]).

the entrant split the gain from the agreement. We now denote the lower bound on the share of free consumers by α^* and let our policy parameter of interest $\alpha \in [0,1]$ be the probability that such a deal is allowed. Under these assumptions, $\pi_m(\alpha,\phi) = \alpha^*\Delta + (1-\alpha^*)(1-\phi)\Delta$, $\pi_I(\alpha,\phi) = (1-\alpha^*)[(1-\phi)\Delta + \alpha(\Delta/2)]$, and $\pi_E(\alpha,\phi) = \alpha^*\Delta + (1-\alpha^*)\alpha(\Delta/2)]$, and so both π_I and π_E will increase, while π_m will be unaffected when α increases. As such, Propositions 1 and 2 tell us that the rate of innovation will increase if such voluntary licensing deals are allowed. Moreover, using similar reasoning to that in Proposition 4, we can show that allowing such deals also increases aggregate welfare.[24]

5.2 Price collusion

As another example of a voluntary deal, set the share of long-term contracts to be zero in our long-term contracting model (so we are in the benchmark model of Appendix B), and suppose that an incumbent and an entrant may be able to collude in their pricing to consumers in the period in which the entrant enters. Let $\alpha = 1$ if collusion is allowed, and $\alpha = 0$ if it is not allowed. Assume also that a technology enters the public domain in the period after it is superseded, so that the only opportunity for collusion is in the period of entry.[25] Hence, in periods without entry the continuing incumbent earns Δ as before. In periods with entry, the entrant will make the sale to consumers at a price of $k + (1+\alpha)\Delta$, and split equally the joint gain from collusion $\alpha\Delta$ with the incumbent by means of a side payment. Hence, $\pi_E(\alpha) = (1+\alpha/2)\Delta$, $\pi_I(\alpha) = \alpha\Delta/2$, and $\pi_m(\alpha) = \Delta$. The rate of innovation is therefore increasing in the degree of allowed collusion α. In this case, however, the welfare effects are not clear. We cannot use the same type of argument as in Proposition 4 to show that welfare increases, because the direct effect of the change on the current incumbent plus consumers is negative. Indeed, observe that there is no direct efficiency effect arising from this collusive arrangement; it is merely a transfer from consumers to the firms. Thus, the sign of the effect on aggregate welfare is determined simply by whether we were initially in a situation of over- or under-investment in R&D relative to the socially optimal symmetric rate of innovation.[26]

[24] As in the long-term contracting model, both the direct effect of the policy change and the indirect effect of the induced increase in innovation on the current incumbent plus consumers is positive. Since potential entrants are again be better off if the rate of innovation increases (assuming that the Value Monotonicity Property holds), this implies that aggregate welfare increases.

[25] The purpose of this assumption is just to limit the size of the gain from collusion.

[26] In a model with more general demand functions there would be an additional efficiency loss from the collusive deal because of increased pricing distortions.

6 Incumbent Innovation

The analysis above imposed the strong restriction that only potential entrants engaged in R&D. Although useful for gaining insight, this assumption is clearly not representative of most settings of interest. In this section, we explore how our conclusions are affected when incumbent firms may also engage in R&D.

Allowing incumbent firms to engage in R&D has the potential to considerably complicate the analysis. In particular, once we allow for incumbent investment, we need to introduce a state space to keep track of the incumbent's current lead over the potential entrants. In general, the rates of R&D investment by the incumbent and its challengers may be state dependent (see, for example, Aghion et. al. [2001]).

Here we focus on two special cases in which R&D strategies are nonetheless stationary. Although clearly restrictive, these two models do have the virtue of capturing two distinct motives for incumbent R&D: (i) preventing displacement by an entrant, and (ii) increasing the flow of profits until displacement by increasing the lead over the previous incumbent.

6.1 R&D to prevent displacement

Suppose the incumbent can do R&D denoted by ϕ_I, while the potential entrants' aggregate R&D is now denoted by ϕ_E. Similarly, the incumbent's and entrants' respective cost functions are denoted by $c_I(\phi_I)$ and $c_E(\phi_E)$ (we allow for the fact that the cost of achieving a discovery may differ between the incumbent and the potential entrants). In this first model, we assume that if the leading quality level in period t is j_t, then quality level $j_t - 1$ is freely available to all potential producers. That is, it enters the public domain. Thus, the incumbent never has a lead greater than one step on the ladder. If so, the only reason for an incumbent to do R&D is to try to get the patent on the next innovation in cases in which at least one potential entrant has made a discovery – that is, to prevent its displacement.[27]

[27] In the usual sort of (Poisson) continuous-time model considered in the R&D literature (see, e.g., Lee and Wilde [1980], Reinganum [1989], and Grossman and Helpman [1991]), the probability of ties is zero, and so one might worry that our formulation here is dependent on a merely technical feature of the discrete-time set-up. Indeed, in such a model, the incumbent would do no R&D here. However, the usual continuous-time model relies on the implicit assumption that following an innovation, all firms reorient their R&D activity instantaneously to the next technology level. If we were to instead use a continuous-time model in which there is a fixed time period after a rival's success before which R&D for the next technology level cannot be successful, then we would get effects that parallel those in our discrete-time model (where the discount factor δ reflects how quickly R&D activity can be reoriented to the next technology level.)

With these assumptions, we need not keep track of any states, and there is a stationary equilibrium. We restrict ourselves here to making two main points: (i) there are some simple cases in which the incumbent chooses zero innovation in equilibrium and so all our previous results go through, and (ii) more protective antitrust policy may have positive direct effects on *both* the incumbent's and the entrants' innovation rates.

To proceed, let $r_I(\phi_E)$ denote the probability that the incumbent who innovates preserves the incumbency, which allows us to express the probability that the incumbent is displaced by an entrant as $d(\phi_I, \phi_E) \equiv (1-\phi_I)\phi_E + \phi_I(1-r_I(\phi_E))$. Also, let $u(\phi_I, \phi_E)$ denote the probability that a given entrant innovates and then becomes the next incumbent.

The values V_I and V_E of the incumbent and a potential entrant must then satisfy

$$V_I = \pi_m(\alpha,\phi) + \delta V_I + d(\phi)\{\pi_I(\alpha,\phi) - \pi_m(\alpha,\phi) - \delta(V_I - V_E)\} - c_I(\phi_I) \tag{VI**}$$

$$V_E = \delta V_E + u(\phi)[\pi_E(\alpha,\phi) + \delta(V_I - V_E)] - c_E(\phi_E), \tag{VE**}$$

where $\phi = (\phi_I, \phi_E)$. The incumbent will choose R&D to solve

$$\begin{aligned} \phi_I &\in \arg\max_{\psi_I \in [0,1]} \psi_I [r_I(\phi_E) - (1-\phi_E)] w_I - c_I(\psi_I), \text{ where} \\ w_I &\equiv \pi_m(\alpha,\phi) - \pi_I(\alpha,\phi) + \delta(V_I - V_E) \end{aligned} \tag{8}$$

is the incumbent's expected gain from getting a patent conditional on one of the entrants making a discovery. As for the potential entrants, they are facing innovation prize

$$w_E = \pi_E(\alpha,\phi) + \delta(V_I - V_E), \tag{9}$$

and will choose $\phi_E \in \Phi(w_E, \phi_I)$, where $\Phi(\cdot, \phi_I)$ is the entrants' innovation supply correspondence.

The following example shows some cases in which the incumbent does no R&D in equilibrium, and so our previous results apply without modification:

Example 1 *Suppose that there is a single entrant and that when the entrant and incumbent both innovate they each have a 1/2 probability of receiving the patent. Then* $r_i(\phi_{-i}) = (1-\phi_{-i}/2)$ *for* $i = I, E$, *and the entrant's R&D level is* $\phi_E \in \arg\max_{\psi_E \in [0,1]} \psi_E r_E(\phi_I) w_E - c_E(\psi_E)$. *Suppose also that the entrant and incumbent share the same R&D cost function. Comparing with (8), we see two countervailing asymmetries between the entrant and incumbent:*

Thus, our discrete-time formulation captures an arguably realistic feature of the economics of R&D.

- *When $\pi_m \geq \pi_I + \pi_E$, which captures the "efficiency effect" (that a monopoly maximizes industry profits), the incumbent's innovation prize w_I is at least as large as the entrants' innovation prize w_E.*

- *The incumbent's innovation prize is multiplied by $r_I(\phi_E) - (1 - \phi_E)$, while the entrant's prize is multiplied by $r_E(\phi_I)$. This difference captures the "replacement effect": the incumbent's innovation only results in him in "replacing" himself and so has no value if the entrant does not innovate, while an entrant's innovation is valuable when the incumbent does not innovate.*

Now consider an extension of our long-term (exclusive) contracting model to allow for incumbent investment. We now assume that a long-term contract is a commitment by the incumbent to deliver his best current product in the next period. In this case, consumers gain Δ in surplus when the incumbent gets a new patent regardless of whether they have signed a long-term contract since the previous leading product enters the public domain. The price of a long-term contract will therefore be $q_{t+1} = k + (1 - d(\phi))\Delta$, and the profit functions can be written as

$$\begin{aligned} \pi_m(\alpha, \phi) &= \alpha\Delta + (1-\alpha)(1 - d(\phi))\Delta \\ \pi_I(\alpha, \phi) &= (1-\alpha)(1 - d(\phi))\Delta \\ \pi_E(\alpha, \phi) &= \alpha\Delta. \end{aligned} \tag{10}$$

Thus, here the efficiency effect is zero: $\pi_m = \pi_I + \pi_E$. Then, due to the replacement effect, in equilibrium we must have $\phi_E \geq \phi_I$. If, in addition, the common R&D cost function has constant returns to scale so that $c_E(\phi) = c_I(\phi) = c \cdot \phi$, then we have $\phi_I = 0$.[28]

To examine the effects of antitrust policy in the general case in which both the incumbent and potential entrants do R&D , solve (VI**) and (VE**) for $(V_I - V_E)$ and substitute it in (8, 9). This yields (suppressing the arguments of functions)

$$w_I = \frac{1}{1 - \delta + \delta(d + u)} \left\{ \pi_m - (1-\delta)\pi_I + \delta u(\pi_m - \pi_E - \pi_I) + \delta(c_E - c_I) \right\}, \tag{11}$$

$$w_E = \frac{1}{1 - \delta + \delta(d + u)} \left\{ \delta\pi_m + (1-\delta)\pi_E - \delta d(\pi_m - \pi_E - \pi_I) + \delta(c_E - c_I) \right\}. \tag{12}$$

[28] The same conclusion obtains the free entry case with the same long-term contracting model. Indeed, in this case, $r_I(\phi_E) = \bar{r}(\phi_E)$. Also, if the incumbent does not invest in R&D, then each entrant makes an infinitesimal investment $\psi_E \in \arg\max_{\widetilde{\psi}_E \in [0,1]} \widetilde{\psi}_E \bar{r}(\phi_E) w_E - c_E(\widetilde{\psi}_E)$. Then (8) implies that, as long as the incumbent and entrants share the same cost function, the incumbent will indeed choose a zero investment.

Consider first the *direct* effects of a change in the policy α. For the incumbent, the former captures the change in its R&D incentives holding fixed the R&D of potential entrants ϕ_E, and has the same sign as the change in w_I caused by the change in α holding (ϕ_I, ϕ_E) fixed. Similarly, the direct effect for the potential entrants has the same sign as the change in w_E caused by the change in α holding (ϕ_I, ϕ_E) fixed. The following proposition summarizes these direct effects:

Proposition 5 *In the model of incumbent R&D to prevent displacement, the direct effect of a more protective antitrust policy (an increase in α) on potential entrant R&D is positive (negative) if*

$$\pi_E'(\alpha, \phi) + \delta \left[\frac{(1 - d(\phi))\pi_m'(\alpha, \phi) + d\pi_I'(\alpha, \phi)}{1 - \delta(1 - d(\phi))} \right] \geq (\leq) 0 \tag{13}$$

for all α and all $\phi \in [0,1]^2$. The direct effect on the incumbent R&D is positive (negative) if

$$\pi_m'(\alpha, \phi) - (1 - \delta)\pi_I'(\alpha, \phi) + \delta u(\phi) \left[\pi_m'(\alpha, \phi) - \pi_I'(\alpha, \phi) - \pi_E'(\alpha, \phi)\right] \geq (\leq)\ 0 \tag{14}$$

for all α and all $\phi \in [0,1]^2$.

Observe first that the condition (13) describing the direct effect on entrant innovation of the policy change is the same as condition (3), but with the probability of incumbent displacement now being $d(\phi)$ instead of ϕ. Turning to the direct effect on the incumbent, note that the direct effects of a more protective antitrust policy on incumbent and potential entrant innovation can *both* be positive. For instance, this is always true in the exclusive contracts model described in the above example. Indeed, in this case, holding innovation fixed, a more protective antitrust policy increases the continuing monopoly profit π_m and reduces the contested incumbent's profit π_I, thus raising the incumbent's incentive to avoid displacement.

The direct effects are not determinative, however, of the overall change in equilibrium innovation rates, because there are interactions between the R&D levels of the incumbent and potential entrants since the level of ϕ_i affects the innovation value w_{-i} $(i = I, E)$.

6.2 R&D to increase profit flows

We next consider a model in which rivals do not get access to the second best technology when the incumbent innovates. Thus, the incumbent can increase its flow of profits by innovating, until the time when it is displaced. Specifically, let s denote the number of steps that the incumbent is ahead of its nearest rival (this is our state variable). The variable

s affects the incumbent's profit flow when entry does not occur, which we now denote by $\pi_m^s(\alpha, \phi_E)$ (it does not affect either π_I or π_E). We now make two assumptions that will imply that there is an equilibrium in which the R&D levels of the incumbent and potential entrants do not depend upon s. Specifically, we assume that $\pi_m^s(\alpha, \phi) = \mu(\alpha, \phi) + s\pi_m(\alpha, \phi)$ and that an entrant gets the patent whenever at least one entrant has made a discovery.

Example 2 *Consider again the extension of the long-term (exclusive) contracts model to incumbent innovation introduced in Example 1. A contract again is a promise to deliver the incumbent's leading technology product in the next period. To fit into our framework here, however, we change the timing of the payment in this contract, assuming instead that the payment is made when the contract is signed. Following similar reasoning as in Example 1 we see that*[29,30]

$$\begin{aligned}
\pi_m^s(\alpha, \phi) &= \alpha s\Delta + (1-\alpha)[(s+\phi_I)(1-\phi_E)\Delta - (1-\delta)k] \\
&= (1-\alpha)[\phi_I(1-\phi_E)\Delta - (1-\delta)k] + s\Delta[\alpha + (1-\alpha)(1-\phi_E)] \qquad (15) \\
\pi_I(\alpha, \phi) &= -(1-\alpha)k \\
\pi_E(\alpha, \phi) &= \alpha\Delta + (1-\alpha)\delta[k + (1+\phi_I)(1-\phi_E)\Delta]
\end{aligned}$$

It is clear that there is a solution in which the entrants' innovation ϕ_E and value V_E are stationary. To begin, we allow the incumbent's R&D and value functions to depend on s: ϕ_I^s and V_I^s. In this case, we can write the value equations as

$$\begin{aligned}
V_I^s &= \mu + s\pi_m(\phi_E, \phi_I^s) + \delta V_I^s + \phi_E\{[\pi_I(\phi_E, \phi_I^s) - (\mu + s\pi_m(\phi_E, \phi_I^s))] + \delta[V_E - V_I^s]\} \\
&\quad + \phi_I^s(1-\phi_E)\{\pi_m(\phi_E, \phi_I^s) + \delta(V_I^{s+1} - V_I^s)\} - c_I(\phi_I^s), \qquad (16)
\end{aligned}$$

for $s \geq 1$, and

$$V_E = \delta V_E + u(\phi_E)\left[\pi_E + \delta\left(V_I^1 - V_E\right)\right] - c_E(\phi_E). \qquad (17)$$

The incumbent's equilibrium innovation rate satisfies

$$\phi_I^s \in \arg\max_{\psi\in[0,1]} \psi(1-\phi_E)\{\pi_m(\phi_E, \phi_I^s) + \delta[V_I^{s+1} - V_I^s]\} - c_I(\psi), \qquad (18)$$

[29] Observe that the price of a long-term contract, which is paid when signed in period t, is $q_t = \delta[k + (s+\phi_I)(1-\phi_E)\Delta]$. A continuing uncontested incumbent in period t sells to free consumers (for a profit of $\alpha s\Delta$), delivers on contracts written in period $t-1$, and writes new contracts for period $t+1$ delivery. An incumbent who faces new entry in period t only delivers on period $t-1$ contracts. An entrant in period t sells to free consumers (at a profit of $\alpha\Delta$), and writes new contracts for period $t+1$ deliveries.

[30] Note that with this change in the timing of payments, an increase in α that leaves more free consumers can lower π_E as defined here.

These equations have a solution in which both the incumbent's innovation ϕ_I^s and the value difference $V_I^{s+1} - V_I^s$ are independent of s. Indeed, using (16) for s and $s+1$ and assuming that $\phi_I^s = \phi_I$ for all s, we obtain

$$V_I^{s+1} - V_I^s = \frac{\pi_m\left(\phi_E, \phi_I\right)\left(1-\phi_E\right)}{1-\delta+\delta\phi_E},$$

which is independent of s, and so (18) becomes

$$\phi_I \in \arg\max_{\psi\in[0,1]} \psi\pi_m\left(\phi_E, \phi_I\right)\left[\frac{1-\phi_E}{1-\delta+\delta\phi_E}\right] - c_I(\psi), \tag{19}$$

which implies that the incumbent's innovation can indeed be independent of s. Note that the direct effect of antitrust policy on incumbent innovation in this equilibrium is determined solely by $\pi_m'\left(\alpha, \phi\right)$.

We next solve for the entrants' innovation benefit function. Subtracting (17) from (16) for $s=1$ to solve for $(V_I^1 - V_E)$, we can write the entrants' innovation benefit function as (suppressing arguments of functions)

$$\begin{aligned} w_E &= \pi_E + \delta\left(V_I^1 - V_E\right) \qquad (20)\\ &= \left\{\frac{[1-\delta\left(1-\phi_E\right)]\pi_E + \delta(1-\phi_E)\mu + \delta\phi_E\pi_I + \delta(1-\phi_E)\left[1+\frac{\phi_I}{1-\delta+\delta\phi_E}\right]\pi_m - \delta(c_I - c_E)}{1-\delta+\delta(\phi_E + u\left(\phi_E\right))}\right\}. \end{aligned}$$

This parallels the case without incumbent investment except for the term multiplying π_m, which now accounts for the possibility of rising incumbent profits over time (the difference vanishes when $\phi_I = 0$). The direct effect of α on ϕ_E can be seen by differentiating (20) with respect to α. To summarize:

Proposition 6 *In the model of incumbent R&D to increase profit flows, the direct effect on incumbent R&D of a more protective antitrust policy (an increase in α) is positive (negative) if $\pi_m'(\alpha, \phi) \geq (\leq)\ 0$ for all α and all $\phi \in [0,1]^2$, while the direct effect on potential entrant R&D is positive (negative) if*

$$\pi_E'\left(\alpha,\phi\right) + \delta\left[\frac{\left(1-\phi_E\right)\left[\mu'\left(\alpha,\phi\right) + \left(1+\frac{\phi_I}{1-\delta+\delta\phi_E}\right)\pi_m'\left(\alpha,\phi\right)\right] + \phi_E\pi_I'\left(\alpha,\phi\right)}{1-\delta+\delta\phi_E}\right] \geq (\leq)\ 0$$

for all α and all $\phi \in [0,1]^2$.

For example, using (15) we see that in the long-term contracting model the direct effect on ϕ_I of an increase in α is always positive. On the other hand, the direct effect on ϕ_E

may now be either positive or negative. When ϕ_I is close to zero, the direct effect is the same as absent incumbent innovation, and so is necessarily positive. However, when ϕ_I is large, this conclusion can be reversed. Considering now the indirect effects in the long-term contracting model, we see that increases in ϕ_E necessarily lower incumbent innovation, while increases in ϕ_I raise entrant innovation. Thus, when both direct effects are positive, we can be sure that ϕ_E increases when α rises; when instead the direct effect on entrant innovation is negative we can be sure that ϕ_I increases when α rises.

7 Other Types of Antitrust Policies

In the analysis up to this point we have considered policies that alter the profits that incumbents and entrants earn in competition with one another. Some antitrust policies have other types of effects. In this section, we briefly consider two such examples.

7.1 Predatory activities

In some situations, antitrust may affect not only an entrant's profits in competition with the incumbent, but also the entrant's probability of survival. To focus solely on this effect, take π_I, π_E, and π_m as fixed and suppose that a new entrant's probability of survival following its entry is $\lambda(\alpha)$, where $\lambda(\cdot)$ is increasing in α.

Now the innovation prize is

$$w = [\pi^E + \delta\lambda(\alpha)(V_I - V_E)]. \tag{21}$$

If $(V_I - V_E)$ were fixed, an increase in α would necessarily increase innovation. Now,

$$\begin{aligned} V_I &= \pi_m + \delta V_I + \phi\left[\pi_I - \pi_m + \delta\lambda(\alpha)\left(V_E - V_I\right)\right], & \text{(VI}^{***}\text{)} \\ V_E &= \delta V_E + u(\phi)\left[\pi_E + \delta\lambda(\alpha)\left(V_I - V_E\right)\right] - c\left(\phi\right). & \text{(VE}^{***}\text{)} \end{aligned}$$

Subtracting (VE***) from (VI***), we can express the innovation prize with the following function:

$$W(\phi,\alpha) = \left\{\pi_E + \left(\frac{\delta\lambda(\alpha)}{1-\delta+\delta\lambda(\alpha)(\phi+u(\phi))}\right)\left[\phi\pi_I + (1-\phi)\,\pi_m - u(\phi)\pi_E + c(\phi)\right]\right\}. \tag{22}$$

The fraction $\delta\lambda(\alpha)/[1-\delta+\delta\lambda(\alpha)(\phi+u(\phi))]$ is increasing in α. Hence, provided that $(V_I - V_E)$ is positive, a more protective antitrust policy that raises the likelihood of entrant survival necessarily increases the innovation benefit.[31] Propositions 1 and 2 then tell us that innovation increases with this change in α.

We illustrate these effects with a simple model of predatory pricing:

7.1.1 Predatory Pricing

Consider a setting in which the entrant's probability of survival after its first production period is an increasing continuous function $\lambda(\pi_E)$ of its first-period profit. (This could be due to the entrant's financial constraints in an imperfect credit market, as in Bolton and Scharfstein [1990].) In this situation, the incumbent will be willing to price below the marginal cost k in the period following entry to increase the likelihood of forcing the entrant out of the market. To see this, consider first what the pricing equilibrium would be absent any antitrust constraint. In any such equilibrium, the entrant still wins, and consumers are indifferent between the two firms' products: the incumbent charges price p and the entrant charges price $p + \Delta$. This is an equilibrium if and only if p satisfies

$$p \leq k - [\lambda(p + \Delta - k) - \lambda(0)] (V_I - V_E) \leq p + \Delta.$$

The first inequality ensures that the incumbent prefers to lose at price p [rather than undercutting the price by ε, losing money on the sale, but increasing his chances of survival by $\lambda(p + \Delta - k) - \lambda(0)$]. The second inequality ensures that the entrant prefers to win at price $p + \Delta$. Assuming that $V_I - V_E > 0$, the middle expression is decreasing in p. Note also that both inequalities hold when $p = k - \Delta$. Thus, at the highest equilibrium price p^* the first inequality binds, i.e.

$$p^* = k - [\lambda(p^* + \Delta - k) - \lambda(0)] (V_I - V_E).$$

Note that $p^* \in (k - \Delta, k)$. We focus on the equilibrium in which the incumbent charges p^*, since this strategy for the incumbent weakly dominates charging any $p < p^*$.

Now consider an antitrust policy that imposes a price floor $\alpha < k$ on the incumbent. Suppose that the floor is binding, i.e., $p^* < \alpha$. In this case, $\pi_E(\alpha) = \alpha + \Delta - k$, $\pi_I(\alpha) = 0$, and $\pi_m(\alpha) = \Delta$: thus, a higher α raises $\pi_E(\alpha)$ upon entry, does not affect $\pi_I(\alpha)$ or $\pi_m(\alpha)$, and raises $\lambda(\alpha)$. If the policy only had an effect on π_E but not on λ, then by Propositions

[31] For example, this will always be true whenever $V_E = 0$ (say, because of a constant returns to scale R&D technology) and π_m and π_I are non-negative. Another sufficient condition is $\phi\pi_I + (1 - \phi)\pi_m \geq u(\phi)\pi_E$ for all ϕ.

1 and 2 it would stimulate innovation. However, the policy also increases the entrant's probability of survival λ, which also increases innovation.[32]

As in the model of long-term contracts, an increase in α holding ϕ fixed eliminates an inefficiency, here the inefficient loss of a new innovation. However, unlike the long-term contracting model, we cannot conclude that an increase in α necessarily raises aggregate welfare. To see one example in which welfare falls when α increases, suppose that the probability of survival $\lambda(\cdot)$ is constant at $\overline{\lambda}$ around $\alpha + \Delta - k$. Then a small increase in α will raise the price the entrant receives in his first period in the market (and lower consumers' payoffs in that period), have no effect on an entrant's survival probability, and will raise the level of R&D. Because the first effect is a pure transfer, the overall effect in welfare will be determined simply by whether we have too much or too little R&D given the survival rate $\overline{\lambda}$, which can go either way just as in Appendix B.[33] By way of contrast, if we instead have a perfectly inelastic innovation supply, α affects aggregate welfare only through an increased probability of the entrant's survival, which unambiguously raises welfare whenever $\lambda(\cdot)$ is strictly increasing.

7.2 Shifting innovation supply

In some cases incumbents may take actions that instead affect innovation supply. For example, an incumbent may buy up needed R&D inputs, thereby raising potential entrants' R&D costs. As another example, incumbents may engage in patent litigation claiming that an entrant's innovation infringes its own patent, raising the cost or lowering the probability of the entrant receiving a patent. Formally, we now denote the innovation supply correspondence by $\Phi(\cdot, \alpha)$. We say that an innovation supply correspondence satisfying (IS1)-(IS3) is nondecreasing (nonincreasing) in the policy parameter α if for $\alpha'' > \alpha'$ we have $\max \Phi(w, \alpha') \geq (\leq) \max \Phi(w, \alpha)$ and $\min \Phi(w, \alpha') \geq (\leq) \min \Phi(w, \alpha)$ for all w. As the following propositions establish, increases in innovation supply lead to increases in innovation in the same senses as before (we omit the proofs, which are similar to those earlier):

[32] In a more general model with differentiated products, predation would make both the entrant and incumbent lose money. Thus, increasing α would raise both firms' profits as well as the entrant's probability of survival, and so would again increase innovation.

[33] The reason we cannot use an argument like that leading to Proposition 4 is that the price increase has a direct negative effect on consumers plus the current incumbent; in contrast, in the long-term contracting model, all direct effects on the consumers plus the current incumbent were positive.

Proposition 7 *If the innovation supply correspondence $\Phi(\cdot,\alpha)$ satisfies (IS1)-(IS3) and the innovation benefit function $W(\phi,\alpha)$ is continuous in ϕ, then the largest and smallest equilibrium innovation rates exist, and both these rates are nondecreasing (nonincreasing) in α, the protectiveness of antitrust policy, if $\Phi(\cdot,\alpha)$ is nondecreasing (nonincreasing) in α for any $\phi \in [0,1]$.*

Proposition 8 *Suppose that the innovation supply correspondence $\Phi(\cdot,\alpha)$ satisfies (IS1)-(IS3) and the innovation benefit function $W(\phi,\alpha)$ is continuous in ϕ. Suppose, in addition, that for all $\alpha \in [\underline{\alpha},\overline{\alpha}]$ there is a unique equilibrium innovation rate $\phi(\alpha)$ on an interval $\left[\underline{\phi},\overline{\phi}\right]$ and that the IB curve crosses the IS curve from above on this interval. Then $\phi(\alpha)$ is nondecreasing (nonincreasing) if $\Phi(\cdot,\alpha)$ is nondecreasing (nonincreasing) in α for all $\alpha \in [\underline{\alpha},\overline{\alpha}]$ and $\phi \in \left[\underline{\phi},\overline{\phi}\right]$.*

Thus, rightward (leftward) shifts of the innovation supply correspondence cause the rate of innovation to increase (decrease) in every stable equilibrium. Returning to the two examples mentioned above, these incumbent behaviors shift both innovation benefit and supply. If an uncontested incumbent over-buys needed R&D inputs at the end of each period, this will both raise potential entrants' R&D costs and lower π_m. Both effects cause innovation to decrease. Patent litigation, on the other hand, will not only shift the IS curve leftward but also lower both π_I and π_E. Again, both effects lower the rate of innovation.

8 Conclusion

In this paper we have studied the effects of antitrust policies in industries in which innovation is central to competition using models of continuing innovation that are closely related to recent models in the growth theory literature. By using a stylized model with reduced form profit functions, and by characterizing the equilibrium innovation rate in terms of innovation benefit and innovation supply, we are able to develop comparative static results that apply to a wide range of market settings and antitrust policies.

In general, a tension arises in discerning the effects of antitrust policy on innovation in such settings. On the one hand, limiting incumbent behaviors that reduce the initial profit of entrants tends to increase the incentives for R&D. But these same limitations will affect a successful entrant once it in turn becomes the next incumbent, and so may reduce innovation incentives. Our results show how to disentangle these two effects, and we illustrate their implications for a number of antitrust policies. Interestingly, once one looks at the effects on entrant and incumbent profits holding the rate of innovation fixed – as our comparative statics results instruct us to do – limitations on R&D-deterring activities often involve no tension at all, as both entrant and incumbent profits increase holding the rate of innovation fixed.

Finally, the tension between effects on entrant and continuing incumbent profits that is our focus also arises in other policy settings, most notably intellectual property protection. While the traditional tension considered in patent policy is between the (presumed) welfare benefit of creating greater ex ante R&D incentives and the welfare cost of greater ex post monopoly distortions, the tension we focus on instead suggests that there are conflicting effects of intellectual property protection on the incentives to innovate: it reduces profits for new innovators (who may produce infringing innovations) but raises profits for previous innovators. This tension is at the heart of recent work by Hunt [2004] and Llobet et al. [2000] who focus on the effect of leading breadth (novelty requirement) rules. In Appendix C, we provide a brief discussion of the connection between antitrust policy and leading breadth rules in an extension of our long-term exclusive contracting model. While leading breadth rules are able to control the ease of entry without some of the inefficiencies associated with long-term exclusive contracts, they require courts to determine the degree of improvement brought by new innovations. In contrast, enforcement of antitrust policy, which requires no such determination, can achieve similar effects by indirectly determining how the speed or likelihood of entry depends on innovation size. The implications of the tension we identify for intellectual property protection more broadly (and possibly other policies) and the interactions between various policies that affect innovation both seem fruitful areas for further research.

Appendix A: Proofs

Lemma 1 *The model with $N > 1$ entrants has a unique symmetric innovation equilibrium, and the equilibrium innovation rate is a continuous nondecreasing function of w.*

Proof. The symmetric equilibrium R&D rates are fixed points of correspondence $\sigma(w, \cdot) \equiv B(wr(\cdot))$, where $B(a) = \arg\max_{\psi' \in [0,1]} [a\psi' - \gamma(\psi')]$. Note that $B(a)$ is a nonempty closed interval for each a (by convexity and continuity of $\gamma(\cdot)$), and any selection from $B(\cdot)$ is nondecreasing (by the Monotone Selection Theorem of Milgrom and Shannon [1994])). Therefore, $\sigma(w, \psi) = [\sigma_L(w, \psi), \sigma_H(w, \psi)] \neq \emptyset$, with both $\sigma_L(w, \psi)$ and $\sigma_H(w, \psi)$ nondecreasing in w.

If $\sigma(w, \cdot)$ has two fixed points ψ', ψ'' with $\psi' < \psi''$, then $r(\psi') > r(\psi'')$, and since any selection from $B(\cdot)$ is nondecreasing we must have $\psi' \geq \psi''$, a contradiction. Thus, $\sigma(w, \cdot)$

has a unique fixed point, which we denote by $\Psi(w)$.

Note that correspondence $\sigma(\cdot,\cdot)$ has a closed graph, since $B(\cdot)$ has a closed graph by the Maximum Theorem, and $r(\cdot)$ is a continuous function. This in turn implies that $\sigma(w,\cdot)$ is "continuous but for upward jumps" as defined by Milgrom and Roberts [1994] (σ_L can only jump downward and σ_H can only jump upward, no matter from which direction we take $\psi \to \overline{\psi}$). Then Corollary 2 of Milgrom-Roberts applies to show that $\Psi(w)$ is nondecreasing in w.

Finally, the graph of $\Psi(\cdot)$ can be obtained by intersecting the graph of $\sigma(\cdot,\cdot)$ in the (ψ', w, ψ) space with the set described by $\psi' = \psi$ and projecting the intersection on the first axis. Since closedness is preserved under intersection and projection, we see that the graph of $\Psi(\cdot)$ is closed, and so $\Psi(\cdot)$ is a continuous function. ■

Proof of Proposition 4. We consider in turn the change in the payoffs of entrants, the current incumbent, consumers, and the current incumbent plus consumers.

Potential Entrants: If ϕ increases then w must have increased by (IS3). Using (VE*), we see that

$$(1-\delta)V_E = u(\phi)w - c(\phi),$$

which implies that a potential entrant's value V_E has weakly increased if the Value Monotonicity Property holds.

Sum of Current Incumbent and Consumers: We first compute a lower bound on the value change of the current incumbent (the firm with the leading technology just after stage $\tau.1$). A policy change just after stage $\tau.1$ changes the current incumbent's profits only beginning in the next period. From equation (VI*) we see that we can write

$$\begin{aligned}(1-\delta+\delta\phi)V_I &= [(1-\phi)\pi_m + \phi\pi_I] + \delta\phi V_E \\ &= [(1-\alpha)(1-\phi)\Delta + (1-\phi)\alpha\Delta] + \delta d V_E \\ &= (1-\phi)\Delta + \delta\phi V_E.\end{aligned} \tag{23}$$

Again, since V_E has weakly increased, a lower bound on the change in the current incumbent's value V_I starting at time $\tau+1$ is the change in

$$\frac{\delta(1-\phi_0)(1-\phi)\Delta}{(1-\delta+\delta\phi)}. \tag{24}$$

Now consider the consumers. Consumer welfare does not change until period $\tau+1$ either. Since every consumer is always indifferent between signing an exclusive and being free, we can derive consumer welfare from period $\tau+1$ on by assuming that all consumers are free. Thus, consumer welfare starting in period $\tau+1$ is

$$\delta[(v_{j_\tau}+\phi_0\Delta-k-\Delta)+\phi\frac{\Delta}{1-\delta}]+\delta^2[(v_{j_\tau}+\phi_0\Delta-k-\Delta)+\phi\frac{\Delta}{1-\delta}]+... = \delta[(v_{j_\tau}-k-\Delta)+\phi\frac{\Delta}{(1-\delta)^2}], \tag{25}$$

where v_τ is the value of the quality of the leading good at the start of period τ. This establishes that consumers are better off, since ϕ increases. Now adding (24) and (25), a lower bound on the change in the sum of consumer plus current incumbent payoffs is given by the change in

$$\frac{\delta(1-\phi_0)(1-\phi)\Delta}{(1-\delta+\delta\phi)}+\delta\phi\frac{\Delta}{(1-\delta)^2},$$

which is increasing in ϕ.

Current Incumbent: Finally, consider the current incumbent. Consider the simplest model with two firms (one potential entrant). If $V_E=0$, which will be true if $c(\phi)=c\phi$ for some $c>0$, then (23) implies that the incumbent is worse off since ϕ increases. On the other hand, suppose that ϕ is fixed at some $\overline{\phi}$ (e.g., $c(\cdot)$ is finite only at $\overline{\phi}$), then (23) implies that the incumbent is better off since V_E increases. ■

Appendix B: Welfare Effects in a Benchmark Quality Ladder Model

Suppose that in each period t firms engage in Bertrand competition to make sales. Thus, $\pi_E=\pi_m=\Delta$ and $\pi_I=0$.[34] Specializing (IB*) to this case, the innovation prize is given by

$$\begin{aligned} W(\phi) &= \left[\frac{\Delta(1-\delta)+\delta[\phi\Delta+(1-\phi)\Delta+c(\phi)]}{1-\delta+\delta(\phi+u(\phi))}\right] \\ &= \left[\frac{\Delta+\delta c(\phi)}{1-\delta+\delta(\phi+u(\phi))}\right]. \end{aligned} \tag{26}$$

For now we need not be specific about the nature of innovation supply; we assume only that it is described by a continuous nondecreasing innovation supply function.

Since we now have a fully-specified consumer side (unlike in Sections 2 and 3), we can compare the equilibrium innovation rate to the symmetric innovation rate that maximizes

[34] We focus here on the undominated equilibrium in which the incumbent (who makes no sales) charges a price equal to cost and the entrant with technology j_t+1 charges a price of Δ.

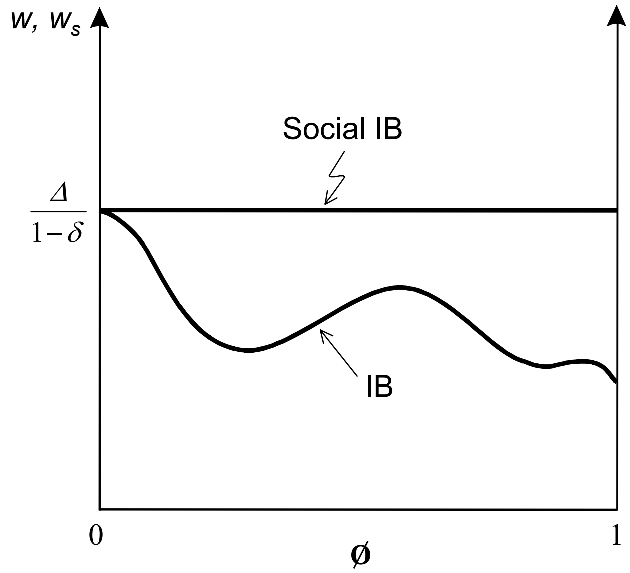

Figure 5: Private and Social Innovation Benefit Curves

aggregate welfare.[35] Let us begin by considering the social innovation benefit. To this end, observe that a technological advancement in period t raises gross consumer surplus in every subsequent period by Δ. The *social innovation prize* w_s is therefore equal to the present discounted value of this change, $w_s = \left(\frac{\Delta}{1-\delta}\right)$. From (VE*) we see that $V_E \geq 0$ implies $u(\phi)w - c(\phi) \geq 0$. Substituting from this inequality for $c(\phi)$ in (26) implies that

$$w \leq \frac{\Delta}{1-\delta+\delta\phi}.$$

Thus, $w \leq \frac{\Delta}{1-\delta} = w_s$, so that the (private) innovation benefit curve always lies weakly below the social innovation benefit curve, as in Figure 5. This difference is due to the "Schumpeterian effect" that arises because an innovator is eventually replaced even though its innovation raises surplus forever.

Given the social benefit w_s (which is independent of ϕ), we can determine the socially optimal symmetric innovation rate by constructing a social innovation supply curve

[35] In general, the socially optimal innovation plan may be asymmetric. We focus here on the best symmetric plan since our aim is to see how changes in the symmetric equilibrium innovation rate affect welfare.

$\Phi_s(\cdot)$ giving the socially optimal symmetric innovation rate for a given level of the social innovation prize. Given the relation between the social and private innovation benefit shown in Figure 5, it is immediate that if the social innovation supply curve coincides with the private one then the equilibrium rate of innovation must be below the socially optimal rate [note that (IS3) and the fact that w_s is independent of ϕ implies a unique socially optimal innovation rate]. This is true, for example, when there is a single potential entrant (and, hence, a single research lab) and when there is free entry with a limited idea.

In contrast, innovation may be excessive when there a fixed number $N > 1$ of potential entrants. For example, consider the case where the patent is awarded randomly among the firms who make a discovery and $c(\cdot)$ is differentiable. In this case, $r_N(\psi)$ takes the value in footnote 7. Given an innovation prize w, the socially efficient innovation rate would obtain by letting each firm internalize its contribution to social surplus by giving it the innovation prize only when it is the only successful innovator, leading to the equilibrium condition $\psi \in \arg\max_{\psi' \in (0,1)} (1-\psi)^{N-1} \psi' - c(\psi')$. Comparing with (4), we see that since $r_N(\psi) > (1-\psi)^{N-1}$, the private innovation supply exceeds the social: $\Phi_s(w) \leq \Phi(w)$, with strict inequality for all $w > c'(0)$. (Formally, the comparison follows from applying Milgrom and Roberts' [1994] Corollary 2 to compare the fixed points of the firms' private and social best-response correspondences.) This difference is due to the "business stealing effect" that arises because a potential entrant is sure to get a patent when all other firms have failed (in which case the innovation is socially useful), but also gets the patent in some cases when another firm has succeeded (in which case it is not).[36]

As is evident in Figure 5, these two differences make the comparison between the equilibrium and socially optimal rates of innovation ambiguous when there are a fixed number $N > 1$ of potential entrants. As $\delta \to 0$ the social and private innovation prizes both converge to Δ, and so only the latter innovation supply difference is present (provided that $N > 1$). As Figure 5 suggests (and Proposition 7 and 2 in Section 7 show formally), in this case any equilibrium innovation rate will exceed the socially optimal rate. At the other extreme, when $\delta \to 1$, we have $w_s \to \infty$ while $w \leq \Delta/\phi$. Thus, as long as $\lim_{\phi \to 1} \gamma'(\phi) = \infty$, the equilibrium innovation rate will be bounded below 1, while the socially optimal innovation rate will converge to 1.

[36] In Aghion and Howitt [1992], two additional distortions are present: an "appropriability effect" (an incumbent monopolist captures less than his full incremental contribution to social surplus in a period) and a "monopoly distortion" effect (an incumbent produces less than the socially optimal quantity in each period). These two distortions are absent here because of our assumption of homogeneous consumer valuations and Bertrand competition.

Appendix C: Uncertain Innovation Size and Selection Effects

In this section, we consider an extension of the long-term contracting model in which innovations are random and innovators must incur costs to bring them to market quickly. In such a setting, antitrust policy can affect not only the rate of discovery, but also the speeds with which different types of innovations make it to market. Thus, antitrust policy also involves "selection effects." Intuitively, some innovations may bring only small benefits to their innovators, but may create large costs for the incumbents they replace. This may lead to circumstances in which more protective antitrust polices retard innovation.

To explore this possibility, we consider an extension of the long-term contracting model in which a new innovator must pay $K > 0$ to enter the market immediately. If he does not incur this cost, he enters in the following period at no cost. We assume that the distribution of innovation sizes Δ is given by the cdf $F(\cdot)$ and for convenience we define $G(\Delta) \equiv 1 - F(\Delta)$.

To begin, observe that in this setting, if α is the share of free consumers, a new innovator will enter immediately if and only if his innovation size Δ_E satisfies $\alpha\Delta_E \geq K$, or equivalently, $\Delta_E \geq \widehat{\Delta}(K,\alpha) \equiv \frac{K}{\alpha}$.

Consider now a consumer's decision of whether to accept a contract from an incumbent whose product's value is v_I and whose innovation size was Δ_I, when the innovation rate is ϕ and the cut-off type for immediate entry is $\widehat{\Delta}$. If the consumer accepts he gets $v_I - q_{t+1}$, while if he rejects he gets $(v_I - \Delta_I - c) + \phi G(\widehat{\Delta})\Delta_I$. Hence, the incumbent will charge $q_{t+1} = c + [1 - \phi G(\widehat{\Delta})]\Delta_I$.

For given innovation sizes Δ_E and Δ_I we have the following profits for an entrant and incumbent respectively:

$$
\begin{aligned}
\pi_m(\alpha,\phi,\Delta_I) &= [\alpha\Delta_I + (1-\alpha)(1-\phi G(\frac{K}{\alpha}))\Delta_I] \\
&= [1-(1-\alpha)\phi(1-F(\frac{K}{\alpha}))]\Delta_I \\
\pi_I(\alpha,\phi,\Delta_I) &= [\alpha F(\frac{K}{\alpha})\Delta_I + (1-\alpha)(1-\phi G(\frac{K}{\alpha}))\Delta_I] \\
&= [\alpha F(\frac{K}{\alpha}) + (1-\alpha)(1-\phi G(\frac{K}{\alpha}))]\Delta_I \\
\pi_E(\alpha,\phi,\Delta_E) &= Max\{\alpha\Delta_E - K, 0\}
\end{aligned}
\tag{27}
$$

It is straightforward to see that condition (3) extends to the case of uncertain innovation, where now innovation increases (decreases) if

$$\overline{\pi}'_E(\alpha,\phi) + \delta\left[\frac{(1-\phi)\overline{\pi}'_m(\alpha,\phi) + \phi\overline{\pi}'_I(\alpha,\phi)}{1-\delta(1-\phi)}\right] \geq (\leq) 0, \tag{28}$$

and the $\overline{\pi}$ functions are the expected profit functions (the expectation is taken with respect to the innovation size Δ). Taking expectations of (27), we get expected profit functions of

$$\begin{aligned} \overline{\pi}_m(\alpha,\phi) &= [1-(1-\alpha)\phi(1-F(\frac{K}{\alpha}))]\overline{\Delta} \\ \overline{\pi}_I(\alpha,\phi) &= [\alpha F + (1-\alpha)(1-\phi G(\frac{K}{\alpha}))]\overline{\Delta} \\ \overline{\pi}_E(\alpha,\phi) &= \int_{\frac{K}{\alpha}}^{\infty} (\alpha\Delta_E - K) f(\Delta_E) d\Delta_E, \end{aligned} \tag{29}$$

where $\overline{\Delta} = \int \Delta dF$.

Examining (29), it useful to think of a change in α as involving two effects on the profit functions, a direct effect of the change in α holding the cut-off type $\widehat{\Delta} = k/\alpha$ fixed, and an indirect effect through a change in $\widehat{\Delta}$.

Consider the direct effect. Just as in the basic long-term (exclusive) contracting model of Section 4.1, the expected profit of a continuing incumbent, $\phi\overline{\pi}_I + (1-\phi)\overline{\pi}_m = [1-\phi(1-F(\widehat{\Delta}))]\overline{\Delta}$, is unaffected by a change in α holding $\widehat{\Delta}$ fixed. On the other hand, the entrant's expected profit upon successful innovation, $\overline{\pi}_E$, continues to be increasing in α, just as in the basic model, although here, this effect also approaches zero as $F(\widehat{\Delta}) \to 1$ since then nearly all entrants are in any case waiting for existing exclusive contracts to lapse before entering.

Now consider the effect of a decrease in the cut-off type $\widehat{\Delta}$. By the envelope theorem, this has no effect on the expected profit of a successful innovator, $\overline{\pi}_E$, since the marginal type $\widehat{\Delta}$ who is entering is earning zero, but it reduces the expected profit of a continuing incumbent, $[1-\phi(1-F(\widehat{\Delta}))]\overline{\Delta}$, by speeding his replacement. Thus, in this model, signing more customers to long-term contracts — an R&D deterring activity — raises an incumbent's profit holding ϕ fixed. So, in this model, the tension between the effects of antitrust policy on entrant and incumbent profits is present, even holding ϕ fixed.

Whether an increase in α increases or decreases innovation depends on whether the direct or indirect effect dominates. Formally:

Proposition 9 *In the long-term (exclusives) model with random innovation size and costs of rapid implementation, restricting the use of long-term contracts increases (decreases) the rate of innovation ϕ if*

$$\left(\frac{\alpha}{f(\widehat{\Delta})\widehat{\Delta}}\right)\left(\frac{\int_{\widehat{\Delta}}^{\infty}\Delta f(\Delta)\phi\Delta_E}{\overline{\Delta}}\right) \geq (\leq) \left(\frac{\delta s}{1-\delta+\delta s}\right). \tag{30}$$

For example, restricting the use of long-term contracts increases the rate of innovation ϕ if $f(\widehat{\Delta}) \approx 0$. In this case, the indirect effect of a change in the cut-off type is of negligible importance since there is almost no change in the likelihood of a successful innovator entering immediately. On the other hand, it lowers the rate of innovation if $\alpha > 0$, the support of Δ is bounded with $f > \underline{f} > 0$ on this support, and $F(\widehat{\Delta}) \approx 1$. When $F(\widehat{\Delta})$ is close to 1, the direct effect on $\overline{\pi}_E$ approaches 0 and the indirect effect dominates, and so the innovation rate falls.

Of course, even when an increase in α causes the rate of innovation ϕ to fall, successful innovations are more likely to come into the market quickly, since the cut-off type $\widehat{\Delta}$ decreases. Thus, the welfare effects of this change in innovation appear ambiguous. The following example illustrates the effect of changes in α on the rate of innovation and welfare.

Example 3 *Suppose there is only one potential entrant and let $c(\phi) = c\phi$. Then the innovation supply is $\Phi(w) = 0$ if $w < c$, $[0,1]$ if $w = c$, and 1 if $w > c$. Thus, for equilibria with interior innovation rates, we must have [using (IB*)]*

$$\left(\frac{\overline{\pi}_E(\alpha)(1-\delta+\delta\phi)+\delta\left[(1-\phi)\overline{\pi}_m(\alpha)+\phi\overline{\pi}_I(\alpha)\right]+\delta c\phi}{1-\delta+2\delta\phi}\right) = c,$$

Solving for c and substituting for the expected profit functions, we have[37]

$$\begin{aligned} c &= \overline{\pi}_E(\alpha) + \left(\frac{\delta}{1-\delta+\delta\phi}\right)\left[(1-\phi)\overline{\pi}_m(\alpha)+\phi\overline{\pi}_I(\alpha)\right] \\ &= \int_{\frac{K}{\alpha}}^{\infty}(\alpha\Delta_E - K)f(\Delta)d\Delta_E + \left(\frac{\delta}{1-\delta+\delta\phi}\right)\left[1-\phi(1-F(\frac{K}{\alpha}))\right]\overline{\Delta} \\ &= \int_{\frac{K}{\alpha}}^{\infty}(\alpha\Delta_E - K)f(\Delta)d\Delta_E + \delta\left\{\left(\frac{1}{1-\delta}\right)\overline{\Delta} - \left(\frac{1}{1-\delta+\delta\phi}\right)\phi[(1-F(\frac{K}{\alpha})) + \frac{\delta}{1-\delta}]\overline{\Delta}\right\}. \end{aligned}$$

This equation describes an interior equilibrium innovation rate ϕ. We now assume that $\Delta \sim U[0,1]$ and that $\alpha \in [K,1]$. We also let $\delta = .9$ (a "period" is two years) and $K = 0.3$. Letting $\phi^*(\alpha)$ denote the equilibrium value of ϕ given α, the solid lines in Figures 6-8 graph the values of $\phi^*(\alpha)$ for $c = 0.5$, $c = 1$, and $c = 3$.

[37] Observe that the expression in curly brackets in the last line makes sense: the continuation payoff of an entrant starting in the period after entry is exactly equal to the present discounted social value of the innovation less the present discounted social value of the first innovation to follow it.

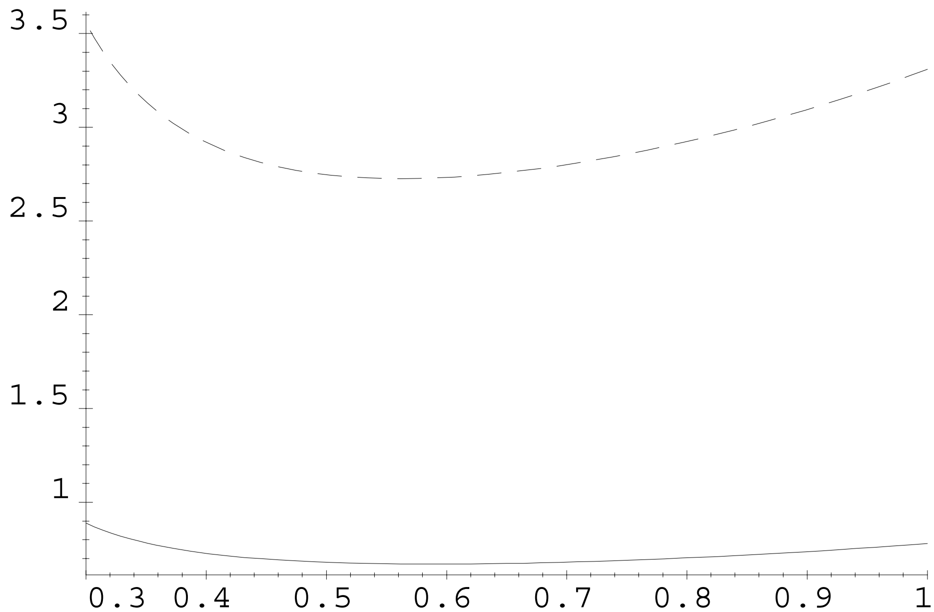

Figure 6: $c = 0.5$

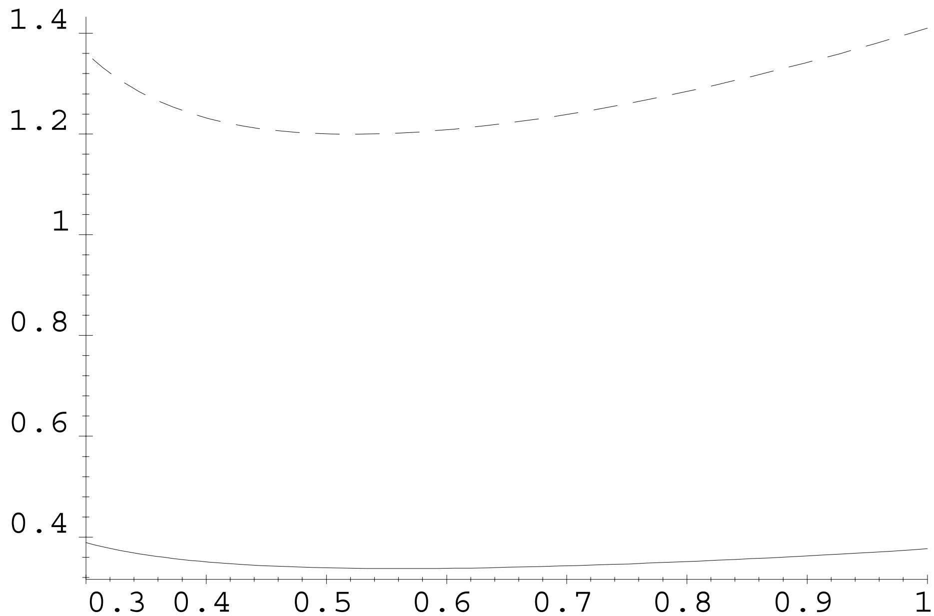

Figure 7: $c = 1$

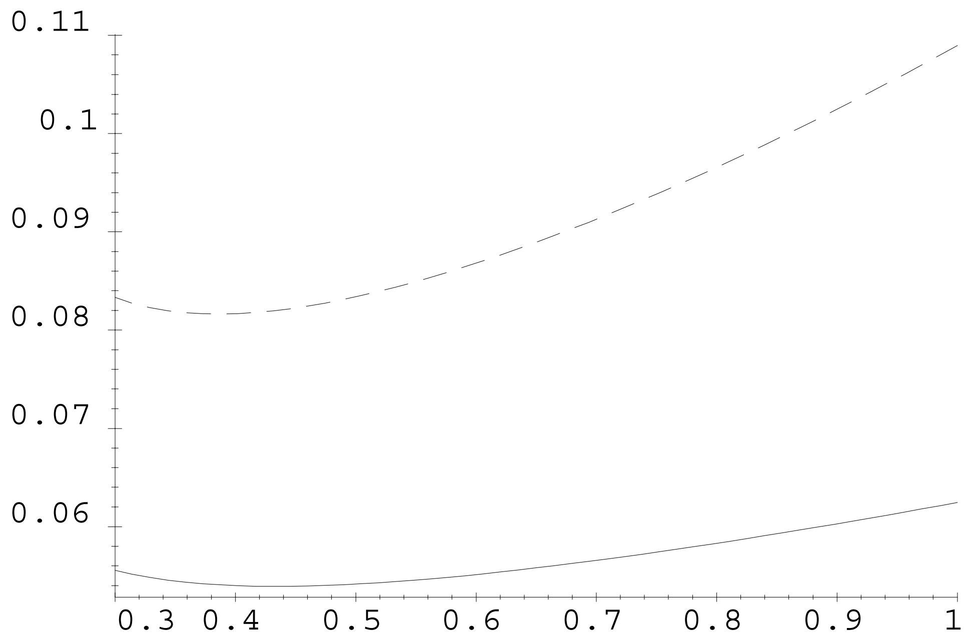

Figure 8: $c = 3$

To consider the welfare effects of these changes, we consider as before an intervention just after stage $\tau.1$. Once again, all of the payoff effects of this change begin in period $\tau+1$. Observe, first, that with a constant returns R&D technology, any interior equilibrium has $V_E = 0$ both before and after the change. So the only effects are those on consumers and the firm with the leading technology just after stage $\tau.1$ (the relevant current incumbent). When $V_E = 0$, (1) implies that

$$V_I = \left[\frac{1-\phi(1-F(\frac{K}{\alpha}))}{1-\delta+\delta\phi}\right]\Delta_I,$$

where Δ_I is the size of this leading firm's innovation. On the other hand, the continuation payoff of consumers starting in period $\tau+1$ is[38]

$$\left(\frac{v_I - c - \Delta_I}{1-\delta}\right) + \left[\frac{\phi(1-F)}{1-\delta+\delta\phi} + \left(\frac{1}{1-\delta}\right)\left(\frac{\delta\phi}{1-\delta+\delta\phi}\right)\right]\Delta_I$$
$$+ \left(\frac{1}{1-\delta}\right)\left(\frac{\delta\phi}{1-\delta+\delta\phi}\right)\phi[(1-F(\frac{K}{\alpha})) + \frac{\delta}{1-\delta}]\overline{\Delta},$$

[38]This can be calculated by observing that consumers start with a baseline net surplus of $(v_I - c - \Delta_I)$ in each period, gain Δ_I from the first subsequent innovation, and gain $\overline{\Delta}$ from each innovation thereafter.

where v_I is the value of the incumbent's product. Putting these together, discounted aggregate welfare starting in period $\tau + 1$ is

$$\left(\frac{v_I - c}{1-\delta}\right) + \left(\frac{1}{1-\delta}\right)\left(\frac{\delta\phi}{1-\delta+\delta\phi}\right)\phi[(1 - F(\frac{K}{\alpha})) + \frac{\delta}{1-\delta}]\overline{\Delta}.$$

The dashed lines in Figures 6-8 graph aggregate welfare as a function of α for the cases $c = 0.5$, $c = 1$, and $c = 1.5$. In each case, the optimal policy is either a ban on long-term contracts ($\alpha = 1$) or unrestricted contracting ($\alpha = 0.3 = K$). The optimum is unrestricted contracting when $c = 0.5$, and is no long-term contracting when $c = 1$ or $c = 3$. (When $c = 1$, unrestricted contracting maximizes innovation, but its first-period innovation-suppressing effect tips the welfare comparison towards no long-term contracting.)

Antitrust Policy and Intellectual Property Protection

The results for this model of random innovation size are related to those of Hunt [1994]'s study of patent policy in a model of continuing innovation (see also O'Donoghue et al [1998]). In particular, Hunt shows that "leading patent breadth," the requirement that an innovation be at least a certain minimal amount better than the current leading technology to get a patent, can increase the rate of innovation and aggregate welfare. Here, a greater share of contractually-bound consumers shifts upward the cut-off level of innovation that comes into the market rapidly, and so has an effect very much like a larger leading patent breadth.[39]

This similarity between the effects of antitrust policy and patent policy raises the question of how optimal antitrust policy should be affected by the ability to also optimally

[39] The structure of Hunt [2004]'s model is very similar to ours. Greater leading breadth can raise the rate of innovation because it can block innovations of small size that generate little profit for an entrant but destroy large profit levels for an incumbent.

In contrast, O'Donohue et al [1998]'s model is quite different. Their innovation process takes the form of "free entry with a limited idea" but innovation size varying instead of the cost of implementation. In their model (where the rate of ideas is exogenous), increasing leading patent breadth necessarily *reduces* the number of innovations that can enter the market without infringing an incumbent's patent. They assume, however, that infringing innovations can be licensed to the current incumbent, who then implements them. Since increased breadth increases the length of time until the incumbent is displaced by a noninfringing innovation, the incumbent is more willing to license infringing innovations the larger is leading breadth. In the limit, as leading breadth grows infinity large, the incumbent "owns the entire quality ladder" and implements exactly the first-best set of innovations.

set patent policy. While a full analysis is beyond our scope here, some insight can be gained by considering the introduction of a simple leading breadth policy into our model. Imagine, then, that we can also set directly a cut-off level Δ_C such that no innovation of size less than Δ_C can come into the market immediately. Suppose we start with an antitrust policy α that is less than 1 and an equilibrium cut-off level equal to $\widehat{\Delta}$. It is clear that nothing is changed if we set $\Delta_C = \widehat{\Delta}$. However, once we have done this, we change the effects of raising α. In particular, now an increase in α no longer has any effect on the set of innovations being immediately implemented; only the "direct effect" of an increase in α on profit levels remains, which we have seen causes innovation to increase. Moreover, this increase in innovation without any change in the set of innovations being immediately implemented necessarily raises welfare. Hence, the optimal antitrust policy when patent policy is available sets $\alpha = 1$. Intuitively, while allowing long-term contracts can be used to prevent small innovations from coming to market, it does this only at the cost of introducing an inefficiency. A leading breadth requirement in patent policy can achieve the same objective without this inefficiency. Thus, if innovation sizes are verifiable, it is better to use patent policy. However, antitrust policy does not require that innovation sizes be observable, and so may sometimes be the best way to achieve this end.[40]

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