Disruptive Technologies and the Emergence of Competition

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Abstract

We formalize the phenomenon of disruptive technologies that initially serve isolated market niches and, as they mature, expand to displace established technologies from mainstream segments. Using a novel model of horizontal and vertical differentiation with discrete customer segmentation, we show how the emergence of technology competition depends on numerous factors including the number of firms using each technology, the size of the different market segments, the trajectory of technological advance, and the extent to which consumers have decreasing marginal utility from product improvements. Our theory suggests a richer and more dynamic approach to market definition than the static analysis of substitution usually employed in antitrust deliberations.

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1. Introduction

New technologies are often commercialized in a specialized niche. Some stay in their niche, while others go on to penetrate mainstream segments and compete with incumbent technologies. Long studied by historians of technology (e.g., Basalla, 1988); economic historians (e.g., Rosenberg, 1976), and business and marketing strategists (e.g., Foster, 1986; Moore, 1994), the emergence of technology competition has received renewed attention through the work of Christensen (1997), which originated with an analysis of the competitive dynamics in the hard disk drive industry. Consider the following characterization:

In 1985, the hard disk drive market for personal computers was divided into two segments: desktop computer users who cared about capacity, and portable computer users who cared about both capacity and portability. A 5.25-inch disk technology, which offered higher capacity than the 3.5-inch alternative, was used in the desktop segment. The 3.5-inch hard drives, which were smaller and more energy efficient, served the emerging market for portable computers. Thus, the two technologies were initially isolated, each limited to serving consumers in a different market segment. With time, the performance of both technologies improved, but the 5.25-inch drives always offered significantly higher capacity than the 3.5-inch drives. By 1988, however, the 3.5-inch drives had expended from the portable segment to capture the low-end of the desktop segment.

Christensen terms technologies like the 3.5-inch drives **disruptive technologies**. Such technologies offer a novel mix of attributes compared to the established technology, but are inferior to the established technology according to the needs of consumers in the primary (mainstream) market segment. The disruptive technology is, therefore, initially purchased by consumers in a secondary (niche) market segment who place high value on the new technology's attribute mix. As the new technology matures, its performance improves, but its perceived quality in the primary segment remains inferior to that of the established technology. Despite this performance inferiority, the new technology enters the primary segment and captures the low-end from the established technology. Christensen (1997) documents similar dynamics in a variety of settings including laser and inkjet printers, minimill and integrated steel plants, and department store and discount store retail formats.

The idea of disruptive technologies has had a profound effect on the way in which both scholars and managers approach the management of technology. For scholars, disruptive technologies highlight the question of the boundaries of technology competition and how those boundaries change over time (Adner, 2001). For managers, disruptive technologies highlight the danger posed to incumbent firms from too quickly dismissing new technologies as inferior and irrelevant to their market positions.[1] The question of whether or not two technologies compete has not, however, been the focus of formal economic theorizing.

One can interpret the vast economics literature on product differentiation as being about technology competition. For example, different positions in a Hotelling model can be interpreted as arising from firms using different technologies. However, the received literature does not address the general issue of the emergence of competition between distinct technologies and, moreover, existing models are not well suited for analyzing the particular dynamics of disruption described above.

One limitation of the received literature is that consumer heterogeneity is usually assumed to be continuous, rather than segmented discretely. Further, existing models that combine horizontal and vertical differentiation are not very

[1] See for example the January 25, 1999 *Forbes* cover story "Danger: Stealth Attack," about Christensen and the challenge posed by disruptive technologies.

tractable (Neven and Thisse, 1990; Heeb, 2001). Given these two limitations, one cannot (easily) use existing models to study the expansion of a new technology from a discrete niche segment to the low-end of the mainstream segment.[2] In addition, because the literature assumes Bertrand price competition, there is only a single firm at each position.[3] Thus, these models do not make a clear distinction between firms and technologies and hence cannot elucidate the effect of competition among multiple firms using a given technology on the emergence of competition between technologies.

We develop a novel model of vertical and horizontal differentiation that is well suited to studying the emergence of competition between distinct technologies.[4] There are two product technologies—a new technology and an established technology—which differ in their marginal costs of production and in the attributes of their associated products. These attributes improve over time as the technologies mature. Improvement is along a fixed technology trajectory that determine the relative attribute levels. We focus on Cournot competition in which there are an arbitrary number of firms using each of the two technologies. We check the robustness of our main results by extending the analysis to a Bertrand duopoly where there is one firm using each technology.

Consumers belong to one of two discrete segments—a primary segment and a secondary segment.[5] A consumer's perceived quality of a product depends on

[2]Moreover, the common assumption in models of differentiation that the market is covered (i.e., all consumers buy some product) implies that some consumers are always choosing between the two technologies and hence that the two technologies are always in direct competition. Thus, the received literature assumes away the question of whether or not two technologies compete.

[3]With two or more firms at a given location and Bertrand competition, price is reduced to marginal cost.

[4]Our model builds on the the simulation model developed by Adner (2000).

[5]Examples of discrete consumer heterogeneity are numerous, including personal versus professional users; differences across national markets such as Europe versus US; differences across industry sector in business-to-business markets; and for the case of component technologies such as disk drives, differences in the architecture of the end products.

how they value its mix of attributes. The defining feature of a market segment is that all consumers in the segment have the same perceived quality for a given product, while quality perceptions may vary across segments. Thus, segments capture horizontal differentiation. Within segments, consumers vary in their willingness-to-pay for quality as in a standard model of vertical differentiation. The established technology is perceived as the higher quality alternative by the primary segment and the new technology is perceived as the higher quality alternative in the secondary segment. We assume that there is no price discrimination, based on either market segment or willingness-to-pay.[6]

Our main research question is whether or not competition emerges between the two technologies; that is, whether or not the new technology is disruptive. In the initial excitement surrounding Christensen's work, analysts and incumbents tended to see disruptive threats everywhere.[7] That many of these threats did not materialize highlights the importance of distinguishing between disruptive threats and technologies that will remain in an isolated segment.[8]

Our analytic approach is to focus on the incentives for firms using the new technology to enter and hence disrupt the primary segment. To do this, we identify an **isolated equilibrium** in which firms optimize as if the new technology was limited to the secondary segment. We then identify the factors that create an incentive for firms using the new technology to deviate from this equilibrium

[6]In the economics literature, our model is well motivated by Sutton's (1998, chapter 6) description of the flow meter industry where there are multiple technologies (e.g., electromagnetic and turbine flow meters) that differ along multiple attributes (e.g., ease of instalation and durability) and where customers can be segmented based on type of application (e.g., oil versus gas pipelines).

[7]For example, consider the following quote from *The Economist*: "Starting with this issue, *The Economist Technology Quarterly* will offer readers a foretaste of what new developments are threatening—no, guaranteeing—to disrupt the way business is done in the years ahead" (December 9, 2000).

[8]For example, Porter (2001) claims that incumbents were too quick to treat new internet-based business models as disruptive technologies.

and disrupt the primary segment.

Our model captures the basic dynamic in which new technologies first enter the secondary segment and then, as they mature, sometimes expand into the primary segment as well. We find that whether the new technology is disruptive depends on the degree of similarity in preferences across segments, the technology trajectories, and the extent of decreasing marginal utility from product improvement, as well as more traditional parameters such as the size of the two segments, the marginal costs of the technologies and the number of firms using each technology.

The intuition for our results rests on an understanding of the demand function faced by firms using the new technology. For sufficiently high prices (or equivalently low output), their product is only consumed by the secondary segment because that is where the new technology creates the most value. For sufficiently low price (or equivalently high output), the product based on the new technology is disruptive. Specifically, it is purchased by those consumers in the primary segment who are most willing to trade-off quality for a lower price (i.e., the low-end consumers). Thus, the decision by firms using the new technology to disrupt the primary market involves moving from a high price, low volume "niche" strategy to a low price, high volume "mass market" strategy.

Consider, for example, the effect of the marginal cost of each technology on disruption. The lower the marginal cost of the new technology, the more attractive is a high volume strategy and the more likely it is that the new technology is disruptive. On the other hand, the lower the marginal costs of the established technology, the greater the optimal output of firms using that technology and hence the lower the scope for firms using the new technology to increase their volumes by disrupting the primary market. Thus, lower costs of the established technology make disruption less likely.

By characterizing the boundaries of technology competition, this paper contributes to debates about market definition that are often central to antitrust cases. The standard approaches to market definition in antitrust deliberations focus on the degree of substitution between products as measured by both own-price and cross-price elasticities. This approach has been criticized as being inherently static, and hence unable to accommodate the possibility of future competition from alternative technologies (Teece and Coleman, 1998).[9] In contrast, our model explicitly incorporates dynamics by allowing product attributes to improve over time. Moreover, we go beyond the standard focus on consumers' willingness to substitute and identify multiple factors that determine the boundaries of competition. Thus, for example, we find that one of the key drivers of whether a new technology currently in a niche will later compete with an established technology is the extent to which consumer utility exhibits decreasing returns to product improvements. The greater such decreasing returns, the more consumers' quality assessments converge over time. As result, a smaller price cut is required to enter the mainstream segment and technology competition is more likely to occur.

The paper proceeds as follows. Section 2 defines the model and Section 3 characterizes the demand functions. Section 4 defines the benchmark isolated equilibrium. Section 5 establishes our approach to identifying the effect of the model parameters on the existence of that equilibrium. Sections 6 and 7 characterize how the boundaries of competition at a given point in time depend on the parameters of the model. Section 8 characterizes how the boundaries of competition change over time. Section 9 extends the analysis to Bertrand competition and to allow for an endogenous number of firms. The paper concludes with

[9] See also the antitrust case *Bourns, Inc v Raychem Corporation*, US District Court, Central District of California, Case No CV 98-1765 CM and the associated discussion in Pleatsikas and Teece (2001) for problems with using traditional approaches to market definition in settings characterized by multiple evolving technologies and multiple customer segments.

Section 10.

2. The Model

We divide the model specification into four parts, as follows.

2.1. Consumers

A consumer's payoff from consuming one unit of a product is $\theta v - p$, where v is the perceived quality of the product, θ is the consumer's willingness-to-pay for quality, and p is the price of the product. Consumers purchase one unit of the product that has the highest payoff, unless all payoffs are negative in which case they do not purchase.

Consumers belong to either a primary market segment or to a secondary market segment, which we index by $i = 1, 2$, respectively. The size of segment i is $S_i > 0$. The defining feature of a market segment is that all consumers in the segment have the same perception of product quality (v), while consumers in different segments may have different perceptions of product quality (details given below). Consumers within a segment vary in their willingness-to-pay for quality. In particular, we assume that $\theta \sim U[0, 1]$ in each segment.

2.2. Technologies

There is a new technology and an established technology, which we index by $\phi = N$ for the new technology and by $\phi = E$ for the established technology. The marginal cost of the new product (i.e., the product made with the new technology) is denoted $c_N \geq 0$. The marginal cost of the established product (i.e., the product made with the established technology) is denoted $c_E \geq 0$.

The perceived quality of the new product for consumers in the primary segment is denoted v_1, while the perceived quality of the new product for consumers in the secondary segment is denoted v_2. The perceived quality of the established product in the primary segment is denoted v_E, while its perceived quality in the secondary segment is assumed to be 0.[10] Consistent with Christensen's description of disruptive technologies as having lower quality in the primary segment, we assume that $v_E > v_1$ and because the new technology is well suited to the secondary segment we assume that $v_2 \geq v_1$.

2.3. Product Attributes and Perceived Quality

Following Lancaster (1971), we model the perceived quality of the new product as coming from its underlying attributes, which we assume to be improving over time. In particular, the new product has two attributes x and y which improve according to

$$\begin{aligned} x_t &= \gamma t \\ y_t &= (1-\gamma)t \end{aligned}$$

where $t \geq 0$ indexes the maturity of the technology and $\gamma \in [0,1]$ is the technology trajectory.

A consumer's perceived quality is a function of a product's attributes and a preference parameter α_i that varies across segments. We denote this function by $V(x, y; \alpha)$ and assume that it is strictly increasing in each of its first two arguments. The perceived quality of the new product by consumers in segment i is then $v_i = V(x, y; \alpha_i)$.

[10]This assumption allows us to focus on whether the new technology disrupts the mainstream segment. The analysis can easily be extended to the case where both technologies have the potential to serve both segments. See Section 9 for further discussion.

We consider four possible utility functions:

$$V(x,y;\alpha) = \left(x^{\alpha}y^{1-\alpha}\right)^{\beta} \qquad \text{(Cobb-Douglas)}$$

$$V(x,y;\alpha) = \alpha x^{\beta} + (1-\alpha)y^{\beta} \qquad \text{(Additive Exponential)}$$

$$V(x,y;\alpha) = \alpha \log x + (1-\alpha)\log y \qquad \text{(Log Linear)}$$

$$V(x,y;\alpha) = K - \frac{\alpha}{x} - \frac{1-\alpha}{y} \qquad \text{(Bounded)}$$

where $\beta \in (0,1]$ parameterizes the extent of decreasing returns to product improvements for the first two utility functions.[11] Note that higher values of α are associated with greater importance of attribute x.

At times we explicitly write $v_1(t)$ and $v_2(t)$ to reflect that perceived qualities are an (increasing) function of t. We restrict attention to $t \geq t_N$ where $v_2(t_N) = c_N$ because before this point no consumer values the new product above marginal cost.

We do not explicitly link the perceived quality of the established product to underlying attributes, but rather make some general assumptions about how its quality in the primary segment evolves with t. We assume that $v_E'(t) \geq 0$ and that $v_E(t_N) > c_E$, which assures that there is positive output of the established product when the new technology first becomes viable. For bounded utility, we additionally assume that $v_E(t) \leq K$ for all t.

2.4. Firms and Competition

There are $n_E \geq 1$ firms using the established technology and we refer to these as established-technology firms. There are $n_N \geq 1$ firms using the new technology and we refer to these as new-technology firms. We assume that at least one of

[11] For the case of bounded utility we assume that $K > c_N$ because otherwise the entrant technology will never have positive output.

the firms using the new technology does not use the established technology.[12]

We introduce the following notation for firm output. Let $q_{N,k}$ for $k = 1, \ldots n_N$ denote the output of new products by each of the new-technology firms and $q_{E,k}$ for $k = 1, \ldots n_E$ denote the output of the established product by each of the established-technology firms. Let $Q_\phi = \sum_{k=1}^{n_\phi} q_{\phi,k}$ for $\phi = N, E$ be the total output of each type of product.

Prices vary across new and established products. Let $P_N \geq 0$ and $P_E \geq 0$ denote the price of new and established products, respectively. Prices do not vary across firms using the same technology as their products are homogeneous. Nor do prices vary across consumers as there is no price discrimination based on either market segment or on willingness-to-pay. Let $P_N(Q_N, Q_E)$ and $P_E(Q_E, Q_N)$ be the inverse demand curves.

A Cournot equilibrium is a set of quantities for each firm which satisfy

$$q^*_{\phi,k} \in \arg\max_q q(P_\phi(Q^*_\phi - q^*_{\phi,k} + q, Q^*_{-\phi}) - c_\phi) \text{ for } \phi = N, E \text{ and } k = 1, \ldots n_\phi,$$

where $Q^*_\phi = \sum_{k=1}^{n_\phi} q^*_{\phi,k}$ for $\phi = N, E$. (Section 9.1 extends key results to the case of Bertrand competition.)

3. The Demand Functions

We start the analysis by characterizing consumer demand for each of the products. We first derive the demand curves and then the inverse demand curves that are needed to characterize Cournot equilibria. We then characterize the relationship

[12] In many settings, most firms use a single technology. Christensen (1997) reports that across four generations of hard disk technology, *none* of the leading firms from one generation had a significant position in the next generation. More generally, established firms are often found to have significant trouble exploiting new technologies (Utterback, 1996), which are then left to entrant firms which usually have their hands full commercializing a single technology.

between disruption and demand.

Consider the demand for the new product. Part of the demand comes from consumers in the secondary segment. For these consumers, the established product is not relevant as its perceived quality is 0. Thus, consumers are choosing between buying the new product and buying no product at all. All consumers for whom $\theta v_2 - P_N > 0$ or equivalently $\theta > P_N/v_2$ buy the new product. Given that $\theta \sim U[0,1]$ and that the measure of segment 2 consumers is S_2, demand for the new product from this segment (i.e., the measure of segment 2 consumers with $\theta > P_N/v_2$) is $\max\{S_2[1 - P_N/v_2], 0\}$.

Now consider whether there is demand for the new product coming from the primary segment. Consumers in this segment have two alternatives to the new product: buying nothing for a payoff of 0 and buying the established product for a payoff of $\theta v_E - P_E$. When there exists a $\theta \in [0,1]$ such that $\theta v_1 - P_N > \max\{\theta v_E - P_E, 0\}$ at least some consumers strictly prefer the new product. Since $\theta v_1 - P_N - \max\{\theta v_E - P_E, 0\}$ is increasing in θ up to P_E/v_E and decreasing thereafter, the new product has positive demand from the primary segment when $(P_E/v_E)v_1 - P_N > 0 \Leftrightarrow P_N < v_1 P_E/v_E$. Thus for P_N sufficiently low relative to P_E, some consumers in the primary segment purchase the new product.

Finally, consider the extent of demand in the primary segment, given that it exists. If the price of the new product is so low that $v_1 - P_N > v_E - P_E$, then even the consumer with the highest willingness-to-pay for quality ($\theta = 1$) prefers the new product to the established product. In this case, all consumers for whom $\theta v_1 - P_N > 0$ buy, which results in a demand of $S_1[1 - P_N/v_1]$. If, however, P_N is higher, so that $v_1 - P_N < v_E - P_E$, then consumers with a sufficiently high willingness-to-pay will still prefer the established product. The indifferent consumer has $\theta v_1 - P_N = \theta v_E - P_E$. Hence, only consumers with $P_N/v_1 < \theta < (P_E - P_N)/(v_E - v_1)$ buy the new product, which results in a

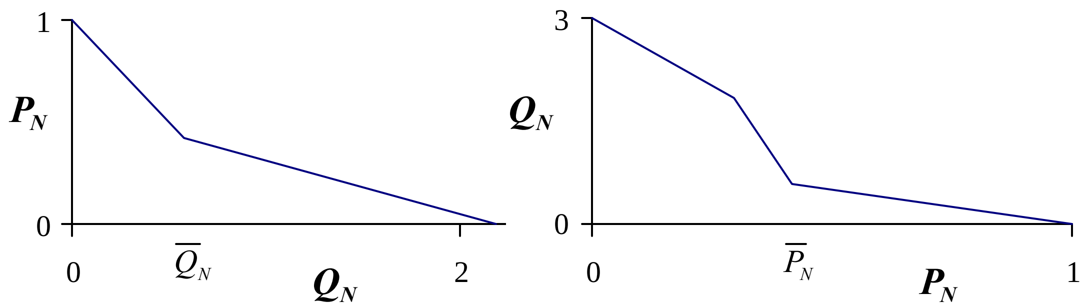

Figure 3.1: Demand and inverse demand curves for the new product with $v_2 = v_E = 1$, $v_1 = .8$, $S_1 = 2$, $S_2 = 1$, $P_E = .6$ and $Q_E = .8$

demand of $S_1\left[\frac{P_E - P_N}{v_E - v_1} - \frac{P_N}{v_1}\right]$. Putting these results together, and extending the analysis to the established product, we have the following result.

Lemma 3.1. *The demand function for the new product is*

$$Q_N(P_N, P_E) = \begin{cases} S_1(1 - \frac{P_N}{v_1}) + S_2(1 - \frac{P_N}{v_2}) & \text{if } P_N < P_E + v_1 - v_E \\ S_1(\frac{P_E - P_N}{v_E - v_1} - \frac{P_N}{v_1}) + S_2(1 - \frac{P_N}{v_2}) & \text{if } P_E + v_1 - v_I \le P_N < \bar{P}_N(P_E) \\ S_2(1 - \frac{P_N}{v_2}) & \text{if } \bar{P}_N(P_E) \le P_N < v_2 \\ 0 & \text{if } v_2 \le P_N. \end{cases}$$

and the demand for the established product is

$$Q_E(P_E, P_N) = \begin{cases} S_1(1 - \frac{P_E}{v_E}) & \text{if } P_E < \bar{P}_E(P_N) \\ S_1(1 - \frac{P_E - P_N}{v_E - v_1}) & \text{if } \bar{P}_E(P_N) \le P_E < P_N + v_E - v_1 \\ 0 & \text{if } P_N + v_E - v_1 \le P_E. \end{cases}$$

where $\bar{P}_N(P_E) = v_1 P_E / v_E$ *and* $\bar{P}_E(P_N) = v_E P_N / v_1$.

Proof The derivation of $Q_N(P_N, P_E)$ is given in the text. The derivation of $Q_E(P_E, P_N)$ is as follows. Demand can only come from segment 1, as the perceived quality of the established product in segment 2 is 0. For $P_E < v_E P_N / v_1$, no consumer in segment 1 buys the new product (see text above) and the established product is bought by those with $\theta v_E - P_E > 0$, which yields a demand of $S_1 [1 - P_E/v_E]$. If $P_N + v_E - v_1 \leq P_E$, then all consumers prefer the new product to the established product and demand is 0. For intermediate values of P_E, the indifferent consumer is the one for whom $\theta v_1 - P_N = \theta v_E - P_E$ and demand is $S_1(1 - \frac{P_E - P_N}{v_E - v_1})$. **QED**

To characterize Cournot equilibria we need inverse demand curves. These are found by taking as given the total output of each type of product, Q_N and Q_E, and solving for the market clearing prices for which $Q_N = Q_N(P_N, P_E)$ and $Q_E = Q_E(P_E, P_N)$ (unless output is too great in which case one or both prices are reduced to 0). Figure 3.1 illustrates the demand and inverse demand curves for the new product.

Lemma 3.2. *When $Q_E \leq S_1$, the inverse demand function for the new product is*

$$P_N(Q_N, Q_E) = \begin{cases} \frac{(S_2 - Q_N)v_2}{S_2} & \text{if } Q_N \leq \bar{Q}_N(Q_E) \\ \frac{(S_1 + S_2 - Q_E - Q_N)v_1 v_2}{v_1 S_2 + v_2 S_1} & \text{if } \bar{Q}_N(Q_E) < Q_N < S_1 + S_2 - Q_E \\ 0 & \text{if } S_1 + S_2 - Q_E \leq Q_N. \end{cases}$$

where $\bar{Q}_N(Q_E) = (v_2 - (1 - Q_E/S_1)v_1)S_2/v_2$. When $Q_E \geq S_1$, the inverse demand function is $P_N(Q_N, Q_E) = \max\{(S_2 - Q_N)\frac{v_2}{S_2}, 0\}$. The inverse demand function for the established product when $Q_N \leq S_2$ is

$$P_E(Q_E, Q_N) = \begin{cases} \frac{(S_1 + S_2 - Q_E - Q_N)v_1 v_2}{v_1 S_2 + v_2 S_1} + \frac{(S_1 - Q_E)(v_E - v_1)}{S_1} & \text{if } Q_E \leq \bar{Q}_E(Q_N) \\ \frac{(S_1 - Q_E)v_E}{S_1} & \text{if } \bar{Q}_E(Q_N) < Q_E < S_1 \\ 0 & \text{if } S_1 \leq Q_E. \end{cases}$$

where $\bar{Q}_E(Q_N) = (v_1 - (1 - Q_N/S_2)v_2)S_1/v_1$*, while for* $Q_N \geq S_2$ *it is*

$$P_E(Q_E, Q_N) = \begin{cases} \frac{(S_1+S_2-Q_E-Q_N)v_1v_2}{v_1S_2+v_2S_1} + \frac{(S_1-Q_E)(v_E-v_1)}{S_1} & \text{if } Q_E \leq S_1 + S_2 - Q_N \\ \frac{(S_1-Q_E)(v_E-v_1)}{S_1} & \text{if } S_1 + S_2 - Q_N < Q_E < S_1 \\ 0 & \text{if } S_1 \leq Q_E. \end{cases}$$

Proof See Appendix.

We now define when the new technology is disruptive and then characterize the relationship between disruption and demand.

Definition 3.3. *The new technology is* ***disruptive*** *if a positive measure of consumers in the primary segment purchase the new product.*

Proposition 3.4. *(i) With quantity competition and* $0 < Q_E < S_1$*, the new technology is disruptive for* Q_N *sufficiently large* $Q_N > \bar{Q}_N(Q_E) > 0$*; with price competition and* $0 < P_E < v_E$*, the new technology is disruptive for* P_N *sufficiently low* $P_N < \bar{P}_N(P_E)$*.*
(ii) When the new technology is disruptive, there exist $0 < \theta_1 < \theta_2 \leq 1$ *such that low-end consumers, those with* $\theta \in (\theta_1, \theta_2)$*, buy the new product and high-end consumers, those with* $\theta > \theta_2$*, buy the established product.*
(iii) The slopes of both the new and the established product inverse demand curves decrease with an expansion of Q_N *that leads to disruption:*

$$\left.\frac{\partial P_\phi}{\partial Q_\phi}\right|_{Q_N=Q_{N,1}} < \left.\frac{\partial P_\phi}{\partial Q_\phi}\right|_{Q_N=Q_{N,2}} < 0, \text{ for } \phi = N, E$$

where $0 \leq Q_{N,1} < \bar{Q}_N(Q_E) < Q_{N,2}$*.*

Proof Parts (i) and (ii) follow from Lemmas 3.1 and 3.2. For part (iii), suppose $Q_E < S_1$. Then one can find values $0 \leq Q_{N,1} < \bar{Q}_N(Q_E) < Q_{N,2}$. We have $\frac{\partial P_N(Q_{N,1},Q_E)}{\partial Q_N} = -v_2/S_2 < -v_2/S_2(\frac{v_1}{v_1+v_2S_1/S_2}) = \frac{\partial P_N(Q_{N,2},Q_E)}{\partial Q_N} < 0$. Further,

$\frac{\partial P_E(Q_E,Q_{N,1})}{\partial Q_E} = -v_E/S_1$, while $\frac{\partial P_E(Q_E,Q_{N,2})}{\partial Q_E} = -v_E/S_1 + v_1/S_1 < 0$ if $Q_{N,2} > S_2$ and $\frac{\partial P_E(Q_E,Q_{N,2})}{\partial Q_E} = -v_E/S_1 + v_1/S_1(v_1S_2/(v_1S_2 + v_2S_1)) < 0$ for $Q_{N,2} < S_2$, both of which are greater than $-v_E/S_1$. **QED**

Part (i) of the proposition establishes that disruption arises from new-technology firms pursuing a high output, and hence low price, strategy. Two factors create the price (or equivalently output) differential. First, the established product is present in the primary but not the secondary segment and hence selling the new product is harder there. In addition, new products have a higher perceived quality in the secondary segment and hence selling the new product is easier there. (Thus, part (i) would not hold if $Q_E = 0$ and $v_1 = v_2$.) Part (ii) establishes that our model is consistent with the observation that disruptive technologies take the low-end of the primary segment. This follows from the assumption that they have the lower perceived quality in that segment.

Finally, part (iii) implies that disruption makes demand for both types of product more elastic. For the established product, this is because demand is more responsive to price changes when marginal consumers are considering a competing product rather than not purchasing. For the new product, this is because demand is more responsive when there are marginal consumers in both, rather than just one, segment. The kink in the new product demand curve, which creates the possibility of multiple locally optimal outputs, is important for the analysis in the next section.

4. The Isolated Equilibrium

To identify disruptive technologies we define a benchmark equilibrium in which the new technology is isolated in the secondary segment. The benchmark is the equilibrium when $v_1 = 0$. In this case, the new technology is clearly not disruptive as it is unattractive to customers in the primary segment. Thus, the two segments form independent markets. The primary segment is served only by the n_E established-technology firms and they face the linear inverse demand curve $P_E(Q_E) = (S_1 - Q_E)\frac{v_E}{S_1}$. The secondary segment is served only by the n_N new-technology firms and they face the linear inverse demand curve $P_N(Q_N) = (S_2 - Q_N)\frac{v_2}{S_2}$. The equilibrium in each segment is then the standard n-firm Cournot outcome with symmetric costs and linear demand. Let Q^I_ϕ and P^I_ϕ for $\phi = N, E$ denote the total output and prices when the technologies are isolatd in this way.

Lemma 4.1. *For $v_1 = 0$, the equilibrium prices and total output are*

$$\begin{aligned} P^I_N &= \frac{v_2 + n_N c_N}{n_N + 1} \text{ and } Q^I_N = \frac{n_N}{n_N + 1}(1 - \frac{c_N}{v_2})S_2 \\ P^I_E &= \frac{v_E + n_E c_E}{n_E + 1} \text{ and } Q^I_E = \frac{n_E}{n_E + 1}(1 - \frac{c_E}{v_E})S_1 \end{aligned}$$

The equilibrium is symmetric in that each firm using technology ϕ has an output of the associated product given by $q^I_\phi = \frac{1}{n_\phi}Q^I_\phi$.

Proof Suppose $v_1 = 0$. With a linear inverse demand function and symmetric costs there is a unique Cournot equilibrium, which is symmetric. With the specific demand functions, equilibrium output is $q^*_{N,k} = \frac{S_2}{n_N+1}(1-\frac{c_N}{v_2})$ and $q^*_{E,k} = \frac{S_1}{n_E+1}(1-\frac{c_E}{v_1})$, which lead to the stated total output and prices. **QED**

Denote the profits of a new-technology firm when $v_1 = 0$ by $\pi^I_N = q^I_N(P^I_N - c_N)$. We now define the benchmark equilibrium.

Definition 4.2. *An* ***isolated equilibrium*** *is one in which $(Q^*_N, Q^*_E) = (Q^I_N, Q^I_E)$.*

Lemma 4.3. *(i) The new technology is not disruptive in an isolated equilibrium. (ii) If an isolated equilibrium does not exist, then the new technology is disruptive with positive probability.*

If the new technology were disruptive in the isolated equilibrium, then the slope of the demand curve would be flatter than $P_N(Q_N)$—see Proposition 3.1, part (iii)—and hence the optimal output would be greater than q_N^I, which contradicts (Q_N^I, Q_E^I) being an equilibrium. In addition to the isolated equilibrium, there are two other possible Nash equilibria. A pure strategy equilibrium involving disruption and a mixed strategy equilibrium in which the new technology is disruptive with positive probability.[13]

We now derive necessary and sufficient conditions for the existence of an isolated equilibrium. There are two ways in which the isolated equilibrium may breakdown. First, at the output levels (Q_N^I, Q_E^I) the new product could be bought by both segments, in which case q_E^N is not locally optimal due to the shift in the slope of the demand curve. Second, a new-technology firm might jump to a higher output level in order to penetrate the primary segment. Given the concavity of the new-product demand function, such a deviation might be globally optimal. The optimal deviation onto the disruptive part of the demand curve by a firm using only the new technology is

$$\hat{q} = \arg \max_{q \geq \bar{Q}_N(Q_N^I) - Q_{-k}^I} q(P_N(Q_{-k}^I + q, Q_E^I) - c_N) \tag{4.1}$$

where $Q_{-k}^I = Q_N^I - q_N^I$ is the total output of all but one new-technology firm in an isolated equilibrium.[14] Let $\hat{Q} = Q_{-k}^I + \hat{q}$ be the total output at the optimal disruptive deviation and let $\hat{P} = P_N(\hat{Q}, Q_E^I)$ and $\hat{\pi} = \hat{q}(\hat{P} - c_E)$ be the associated price and profit. Global optimality of the isolated equilibrium then requires that

[13] The mixed strategy arises because the best response to disruption by the established firm may be to increase output so much that disruption is no longer optimal for new-technology firms. Outputs can be strategic complements in this model because of the flattening of the inverse demand curve that comes with disruption.

[14] We consider deviations by firms using only the new technology because they have a greater incentive to disrupt. Disruption imposes a negative externality on firms using the established technology as it lowers the price of the established product.

$\pi_N^I \geq \hat{\pi}$. Part (i) of the following proposition shows that this condition is sufficient for local optimality as well.

Lemma 4.4. *(i) The isolated equilibrium exists* iff $\Delta_I = \pi_N^I - \hat{\pi} \geq 0$. *(ii) If* $\Delta_I = 0$ *then* $Q_N^I < \bar{Q}_N(Q_E^I) < \hat{Q}$.

Proof See Appendix

In what follows, we are interested in the boundaries of the set of parameters for which the isolated equilibrium exists, i.e., those parameters for which $\Delta_I = 0$. According to part (ii) of Lemma 4.4, at this boundary a firm using (only) the new technology is indifferent between an output q_N^I that does not lead to disruption and another $\hat{q}$ that does.

5. Linking Parameters to Equilibrium Existence

In order to make precise statements about how the existence of the isolated equilibrium depends on a given parameter of the model, we introduce the following definition.

Definition 5.1. *Let* $a = \{a_1, \ldots, a_n\}$ *denote the set of parameters of a model. An equilibrium of the model is* ***increasing*** *in the* j*th parameter if existence of the equilibrium for any vector* $\{a_1, \ldots, a_n\}$ *implies that the equilibrium exists for all feasible vectors* $\{a_1, \ldots, \hat{a}_j, \ldots a_n\}$ *such that* $\hat{a}_j > a_j$. *An equilibrium of the model is* ***decreasing*** *in the* j*th parameter if existence of the equilibrium for any vector* $\{a_1, \ldots, a_n\}$ *implies that the equilibrium exists for all feasible vectors* $\{a_1, \ldots, \hat{a}_j, \ldots a_n\}$ *such that* $\hat{a}_j < a_j$.

An attractive feature of these definitions is that they imply the existence of a critical value of a parameter above or below which the equilibrium exists.[15] See

[15] To make this statement precise, let a_{-j} denote values for all parameters other than jth and suppose that given these values the equilibrium may or may not exist depending on the value

Figure 6.1 and the associated discussion in the next section for more details. The following result is useful in proving that an equilibrium is increasing or decreasing in some parameter.

Lemma 5.2. *Let $a = \{a_1, \ldots, a_n\}$ denote the set of parameters of a model. Suppose there exists a continuous function $\Delta(a)$ such that an equilibrium exists* iff *$\Delta(a) \geq 0$. A sufficient condition for the equilibrium to be increasing in parameter a_j is $\frac{\partial \Delta(a)}{\partial a_j} \geq 0$ for all a such that $\Delta(a) = 0$. A sufficient condition for the equilibrium to be decreasing in parameter a_j is $\frac{\partial \Delta(a)}{\partial a_j} < 0$ for all a such that $\Delta(a) = 0$.*

Proof Suppose there exists a function $\Delta(a)$ such that an equilibrium exists *iff* $\Delta(a) \geq 0$ and that the equilibrium exists for some vector of parameter values $b = \{b_1, \ldots, b_n\}$. Hence, $\Delta(b) \geq 0$. If $\Delta(a)$ is continuous and $\frac{\partial \Delta(a)}{\partial a_j} \geq 0$, it must be that $\Delta(b_1, \ldots, \hat{b}_j, \ldots, b_n) \geq 0$ for all $\hat{b}_j > b_j$, i.e., the equilibrium is increasing in the jth parameter. A similar argument holds for the decreasing case. **QED**

In the following section, we use Lemma 5.2 in conjunction with Lemma 4.4 to characterize how the parameters of the model affect the existence of the isolated equilibrium. *Note that when the isolated equilibrium is increasing (decreasing) in a parameter, the likelihood of disruption is decreasing (increasing).*

6. The Effect of Size, Cost, Quality and Concentration

In this section, we consider the existence of an isolated equilibrium at a given point in time. Thus, we take as given the quality levels v_1, v_2 and v_E. Numerous parameters have an unambiguous effect on whether the new technology is isolated or disruptive.

of a_j. If the equilibrium is increasing (decreasing) in a_j, then there exists an $\bar{a}_j(a_{-j})$ such that the equilibrium exists *iff* $a_j \geq (\leq)\ \bar{a}_j(a_{-j})$.

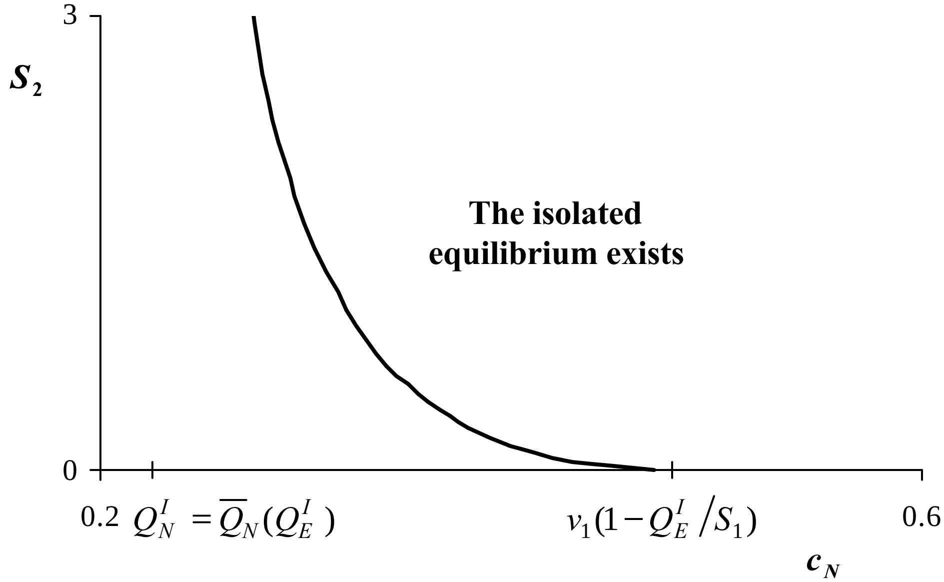

Figure 6.1: Values of c_N and S_2 for which the isolated equilibrium exists when $v_1 = 0.8$, $v_2 = 1$, $S_1 = 2$, $n_N = 2$ and $Q_E^I = 0.8$.

Proposition 6.1. *The isolated equilibrium is increasing in c_N, S_2, v_2, v_E, and n_E. It is decreasing in c_E, S_1, v_1, and n_N.*

Proof See Appendix

Before turning to the intuition for these results, we clarify what they mean. Figure 6.1 shows the values of c_N and S_2 for which the isolated equilibrium exists when the other parameters take on the indicated values. First note that existence may or may not depend on a given parameter depending on values of the other parameters. In the figure, the existence of the equilibrium only depends on S_2 for $c_E \in (.22, .48)$.[16] If there is dependence and the equilibrium is increasing or decreasing in the parameter, then there exists a critical value of the parameter.

[16]The upperbound occurs when $c_N = v_1(1-Q_E^I/S_1)$ at which point the entrant product does not appeal to any of the primary segment consumers unserved by the incumbents even when price is set to marginal cost. The lower bound occurs when c_N is such that $Q_N^I = \bar{Q}_N(Q_E^I)$ as then (Q_N^I, Q_E^I) leads to disruption directly.

For values of c_N in the interval $(.22, .48)$, the critical value of S_2 is the graphed line. As the isolated equilibrium is increasing in S_2, existence holds for values equal to or above the line. Moreover, these critical values are monotonic in the other parameters. Specifically, if both parameters are increasing or both are decreasing, then the critical value is monotonically decreasing; if one parameter is increasing and the other is decreasing, then the critical value is increasing. Applying this to the figure, as the isolated equilibrium is increasing in both c_N and S_2, and the critical value of S_2 is decreasing in c_N.

The intuition for the results in Proposition 6.1 come from the fact that disruption occurs when a new-technology firm finds it profitable to pursue a low price, high volume strategy (Proposition 3.4, part (i)). The effect of each parameter comes from its impact on the profitability of such a strategy relative to the profitability of the lower volume, higher price strategy of remaining in the secondary segment. Thus, for example, the isolated equilibrium is increasing in v_1 because a higher quality of the new product in the primary segment means that the new-product price does not fall as much with disruption, which makes a disruptive strategy more attractive. On the other hand, disruption is less attractive as c_N increases because higher costs reduce the margins that are earned on the increase in volume that comes with disruption. Thus, the isolated equilibrium is increasing in c_N.

The effect of several of the parameters comes from their impact of the amount of additional volume generated by disruption. The bigger the size of the primary segment, the more volume generated by disruption, and hence, the new technology is more likely to be disruptive for larger values of S_1. Relatedly, the more output produced by established-technology firms (Q_E^I), the less primary segment demand is available for the new product and hence the lower the volume generated by disruption. From Lemma 4.1, we have that Q_E^I is decreasing in c_E and hence the isolated equilibrium is decreasing in c_E as well (i.e, disruption is more likely). Conversely, Q_E^I is increasing in v_E and n_E and hence these variables make disruption less likely.

The effect of the remaining parameters comes from their impact on the attractiveness of staying in the secondary segment. The larger the size of the secondary segment, the more attractive it is to stay there. The higher the new product's perceived quality in the secondary segment, the greater the price premium for staying there. Finally, the more firms there are competing in the secondary segment, the lower the price and the volume for any individual firm and the less attractive it is to stay there. Thus, the isolated equilibrium is increasing in S_2 and v_2 and decreasing in n_N.

Many of these findings are consistent with prior empirical work on disruptive technologies. For example, our finding that the likelihood of disruption increases with the new technology's cost advantage (i.e., the isolated equilibrium is increasing in c_N and decreasing in c_E) and the relative segment sizes (i.e., the isolated equilibrium is increasing in S_2 and decreasing in S_1) is consistent with Christensen's claims (1997, pages 15 and 81).

7. The Effect of the Technology Trajectory

This section focuses on the how the technology trajectory γ, which determines the new product's mix of attributes, and thus its perceived quality in each segment, affects the likelihood of disruption. Recall that product attributes evolve according to $x_t = \gamma t$ and $y_t = (1-\gamma)t$ and that quality in segment i is $v_i = V(x_t, y_t; \alpha_i)$. We find that the likelihood of disruption depends on how γ compares to the segment preference parameters α_1 and α_2, which determine the weight placed on each attribute in consumer utility functions.

We define the trajectory that maximizes the new product's perceived quality in segment i by

$$\gamma_i^* = \arg\max_{\gamma} V(\gamma t, (1-\gamma)t; \alpha_i).$$

This γ_i^* is well defined for all four of the utility function that we consider, because $V(\gamma t, (1-\gamma)t; \alpha_i)$ is a concave function of γ with an optimal value that is independent of t. Moreover, the optimal trajectory always satisfies the following properties.

Lemma 7.1. *The quality maximizing trajectory for segment i is increasing in α_i, i.e., $\partial\gamma_i^*/\partial\alpha_i > 0$. For $\alpha_i > 1/2$, $\gamma_i^* \in [\alpha_i, 1]$ and for $\alpha_i \leq 1/2$, $\gamma_i^* \in [0, \alpha_i]$.*

Proof See Appendix.

Shifts in the technology trajectory away from one segment's optimal trajectory towards the other segments optimum have an unambiguous effect on disruption.

Proposition 7.2. *For $\gamma \in [\gamma_1^*, \gamma_2^*]$, the isolated equilibrium is increasing in γ if $\alpha_1 < \alpha_2$ and decreasing in γ if $\alpha_2 < \alpha_1$.*

Proof See text.

Consider the case where $\alpha_1 < \alpha_2$. From Lemma 7.1, we know that $\gamma_1^* < \gamma_2^*$.[17] Suppose $\gamma \in [\gamma_1^*, \gamma_2^*]$. Here, an increase in γ decreases v_1 and increases v_2. As the isolated equilibrium is decreasing in v_1 and increasing in v_2, the two effects work in the same direction and the isolated equilibrium is increasing in γ. For the case where $\alpha_2 < \alpha_1$, the effect of γ on the v_i's reverses and the isolated equilibrium is decreasing. Thus, we capture the intuitive idea that as the technology trajectory shifts away from the preferences of the secondary segment and towards the preferences of the primary segment, disruption becomes more likely.

[17] Further, for $\alpha_1 < 1/2 < \alpha_2$ we have that $\gamma_1^* \leq \alpha_1 < 1/2 < \alpha_2 \leq \gamma_2^*$.

The effect of changes in γ outside of $[\gamma_1^*, \gamma_2^*]$ is indeterminate for two reasons. The first reason is that both v_1 and v_2 move in the same direction, and hence the effect of γ depends in part on which of the two is changing faster, which itself depends on the curvature of the utility function. Second, scaling up or down both quality levels has its own effect on disruption, as identified in the following lemma.

Lemma 7.3. *For $\alpha_1 = \alpha_2$ and hence $\gamma_1^* = \gamma_2^*$, the isolated equilibrium is increasing in γ for $\gamma < \gamma_i^*$ and decreasing for $\gamma > \gamma_i^*$.*

When $\alpha_1 = \alpha_2$, then $v_1 = v_2$ and these values are increasing in γ for $\gamma < \gamma_i^*$ and decreasing for $\gamma > \gamma_i^*$. If, for example, the qualities are increasing, the price charged on any given output is increasing as well, and the resulting higher margins encourage an increase in output, which makes disruption more likely. This effect is also present when the α_i's are different and it may offset the effect from any differential movements in the v_i's. Thus, factors like $\gamma \notin [\gamma_1^*, \gamma_2^*]$ which affect the perceived quality in both segments in the same way, but at different rates, have ambiguous effects on the likelihood of disruption.

8. The Effect of Technological Advance

Disruptive technologies have an important dynamic element: they start in a new market segment and then move to the mainstream segment as they mature. In our model, technology maturity (t) causes attributes, and hence qualities, to improve over time. Recall that the there is positive output of the new product only after time t_N and that the initial sales are in the secondary market segment.

Proposition 8.1. *If $v_E(t_N)$ is sufficiently large relative to c_E, then the isolated equilibrium is decreasing in t and either the isolated equilibrium exists for all $t > t_N$ or there exists a $t_D > t_N$ such that it exist iff $t \leq t_D$.*

Proof See Appendix.

While the new technology starts out in the secondary segment, it may become disruptive at some later time and move into the primary segment. [18] While we have focused on the question of whether or not disruption occurs, Proposition 8.1 raises the additional issue of whether disruption occurs earlier or later. We find that the two questions are closely related.

Proposition 8.2. *Suppose the isolated equilibrium exists iff $t \leq t_D$. Then t_D is increasing in c_N, S_2, v_2, v_E, and n_E while it is decreasing in c_E, S_1, v_1, and n_N. For $\gamma \in [\gamma_1^*, \gamma_2^*]$, t_D is increasing (decreasing) in γ for $\alpha_1 < (>)\ \alpha_2$.*

Proof At $t = t_D$, $\Delta_I = 0$. From Proposition 6.1, $\partial\Delta_I/\partial\rho_H > 0$ for $\rho_H \in \{c_N, S_2, v_2, v_E, n_E\}$. Hence increasing the value of ρ_H implies that $\Delta_I > 0$ and disruption now occurs for a value of t greater than t_D. Conversely, $\partial\Delta_I/\partial\rho_L < 0$ for $\rho_L \in \{c_E, S_1, v_1, n_N\}$. Hence increasing the value of ρ_L implies that $\Delta_I < 0$ and disruption occurs for a value of t less than t_D. **QED**

The factors that we identify as deterring disruption, also serve to delay disruption. As discussed in Section 6.1, when there exists a critical threshold for a parameter such that the technology is disruptive above that threshold, then the critical threshold is increasing in those parameters for which the isolated equilibrium is decreasing and decreasing in those for which it is increasing. Proposition 8.2 simply applies that logic to the threshold t_D.

All that remains is to characterize whether the new technology is disruptive in the limit as t gets large. For all but the bounded utility function, the main effect of $t \to \infty$ is that the ratio of marginal costs to perceived quality goes to

[18] The condition that $v_E(t_N)$ is sufficiently large relative to c_E rules out the possibility that disruption might be reversed (i.e.,that the isolated equilibrium breaks down and later reemerges). This condition can be interpretted as a restriction that the incumbent technology is sufficiently mature by time t_N.

zero. In this limit, whether or not disruption occurs depends on the following closed form expression.[19]

Lemma 8.3. *In the limit as* $\max\{c_N, c_E\}/\min\{v_E, v_1, v_2\} \to 0$*, the condition for an isolated equilibrium to exist converges to*

$$\frac{v_1(t)}{v_2(t)} < \frac{4S_2}{\mathcal{N}(4S_2 + \mathcal{N}S_1)} \text{ where } \mathcal{N} = \left(\frac{n_N + 1}{n_E + 1}\right)^2. \tag{8.1}$$

Proof See Appendix

Inequality (8.1) is consistent with our prior results on the drivers of disruption: it depends on the relative size of the two segments, the relative number of firms using each technology and the relative perceived quality of the new product across segments. As long as $n_N \leq n_E$, the right-hand side of the condition is less than 1 and hence disruption occurs if consumers in the two segments are sufficiently similar in terms of their assessment of the new product's quality.[20]

There are several factors that determine the ratio $v_1(t)/v_2(t)$. The first is the extent of consumers' decreasing marginal utility from product improvements. These are greatest for the bounded and log linear utility functions, for which $\lim_{t\to\infty}(v_1(t)/v_2(t)) = 1$. With Cobb-Douglas and linear exponential utility, decreasing marginal utility is less pronounced and $v_1(t)/v_2(t)$ is a constant independent of t. In the case of Cobb-Douglas this constant is

$$\frac{v_1(t)}{v_2(t)} = \left(\frac{1-\gamma}{\gamma}\right)^{\beta(\alpha_2-\alpha_1)}.$$

[19] Although we model the effect of t as coming entirely through increases in attributes, marginal production costs also tend to fall as technologies improve. Hence, even in the case of bounded utility Lemma 8.3 would apply if cost reductions caused $\max\{c_N, c_E\}/K$ to go to zero with t.

[20] The factors from Proposition 6.1 that are missing in Lemma 8.3 are c_E, c_N and v_E. The costs drop out because what matters is cost relative to quality, which is going to zero by assumption. Incumbent quality v_E is absent because what matters for disruption is the output of the incumbents Q_E^I, which is independent of v_E when c_E/v_E goes to zero.

We thus have the following.

Proposition 8.4. *Suppose $n_E \leq n_N$ and t is arbitrarily large. (i) An isolated equilibrium does not exist if utility is log linear. (ii) An isolated equilibrium does not exist if utility is bounded and $c_N \leq c_E$. (iii) an isolated equilibrium does not exist if utility is Cobb-Douglas or linear exponential and any of the following hold:*

a) The technology trajectory is sufficiently central (i.e., $\gamma \to 1/2$);

b) Preference's are sufficiently homogeneous across segments (i.e., $\alpha_1 \to \alpha_2$);

c) There are sufficiently decreasing returns to product improvement (i.e., $\beta \to 0$).

Proof If utility is not bounded, we have that $\lim_{t\to\infty}(\max\{c_N, c_E\}/\min\{v_E(t), v_1(t), v_2(t)\}) = 0$ and an isolated equilibrium only exists when condition (8.1) is satisfied. For $n_E \leq n_N$, $4S_2/(\mathcal{N}(4S_2 + \mathcal{N}S_1)) < 1$. For log linear utility the condition is then satisfied for t large. For the case of additive exponential we have

$$\frac{v_1(t)}{v_2(t)} = \frac{\alpha_1\gamma^\beta + (1-\alpha_1)(1-\gamma)^\beta}{\alpha_2\gamma^\beta + (1-\alpha_2)(1-\gamma)^\beta}$$

Hence, for both additive exponential and Cobb-Douglas we have

$$\lim_{\gamma\to 1/2}\frac{v_1(t)}{v_2(t)} = \lim_{\alpha_1\to\alpha_2}\frac{v_1(t)}{v_2(t)} = \lim_{\beta\to 0}\frac{v_1(t)}{v_2(t)} = 1$$

and condition (8.1) is satisfied in each of the above limits for $n_E \leq n_N$. **QED**

As we have seen, differences in the number of firms using each technology can deter disruption. Thus, Proposition 8.4 restricts attention to $n_N \leq n_E$. For log utility, this restriction on rivalry is sufficient to assure disruption in the limit.

For bounded utility, the established technology might also be protected by a cost advantage.

For Cobb-Douglas and exponential utility, the limit behavior depends on the extent to which consumers in the primary segment have a greater appreciation for the new product (v_1/v_2). Valuations diverge to the extent that the technology trajectory is skewed towards the attribute favored by the secondary segment. To the extent that segment preferences are homogeneous (α_1 close to α_2) or that the trajectory is central (γ close to $1/2$), consumer perceptions converge and disruption occurs. Further, the stronger the decreasing marginal utility from product improvement (β small), the more muted are any differences.[21]

Our results are consistent with Christensen's observations regarding the dynamics of disruption and with his discussion of how disruption arises from "performance oversupply," which we formalize as decreasing marginal utility. Beyond consistency with existing observations, the value of our formalization is that it allows for new and more specific insights into the influence of numerous factors including rivalry among firms using each technology, technology trajectories and attribute weightings, on the emergence of technology competition. Many of these are novel independent variables that can inform future empirical work in this area.

9. Extensions

The model can be extended along several lines. In Section 9.1 we consider Bertrand competition and disruptive technologies that come not just "from below" (i.e., $v_1 < v_E$) but also "from above" (i.e., $v_1 > v_E$). In Section 9.2 we

[21] Returning to the disk drive example, consider how γ and a_2 affect the disruptive threat posed by 3.5-inch drives. The more technology advance improves capacity along with portability (i.e., the more central the trajectory) the greater the threat. In addition, the more weight the portable segment places on capacity, the greater the threat.

explore the effect of endogenous entry. In Section 9.3, we discuss the implications of our results for a setting in which there are two technologies that both have the possibility to disrupt the others home segment.

9.1. Bertrand Competition

With price competition, we restrict attention to $n_N = n_E = 1$ as otherwise firms price at marginal cost. A Bertrand equilibrium is then a pair of prices P_N^* and P_E^* that satisfy

$$P_\phi^* \in \arg\max_P Q_\phi(P, P_{-\phi}^*)(P - c_\phi) \text{ for } \phi = N, E.$$

We do not need to redefine the isolated equilibrium for the case of Bertrand competition. When $n_N = n_E = 1$ and $v_1 = 0$, each firm operates as a monopolist in its segment and the equilibrium does not depend on whether competition is Cournot or Bertrand.

We now show that the effect of the various parameters of the model on the existence of the isolated equilibrium does not depend on our assumption of Cournot competition.

Proposition 9.1. *With a Bertrand duopoly, and either $v_1 \leq v_E$ or $v_1 > v_E$, the isolated equilibrium is increasing in c_N, S_2, v_2, and v_E while it is decreasing in c_E, S_1, and v_1. For $\gamma \in [\gamma_1^*, \gamma_2^*]$, the isolated equilibrium is increasing (decreasing) in γ for $\alpha_1 > (<)\ \alpha_2$.*

Proof See Appendix.

Note that these results hold independently of the relative ranking of v_1 and v_E. Thus, in addition to addressing the case of disruption from below, they also characterize the case of technology competition from above, in which the perceived quality of the new technology is higher in both the secondary and the primary segment. Examples of the latter include jet versus propeller engines in the military and commercial aircraft market; radial versus bias ply in the European and American tire markets; and xerography versus carbon paper in the large and small enterprise markets.[22]

Next we turn to the effect of the form of competition.

Lemma 9.2. *If an isolated equilibrium exists with Bertrand competition, then it also exists in the Cournot model.*

Proof See Appendix.

Thus, technology competition emerges sooner with price competition than with quantity competition. With Cournot competition the price of the established product falls with disruption so that all output from the established-technology firms still sells. In contrast, with Bertrand competition, price is fixed and the output of the established-technology firms falls with disruption. As a result, the volume of the new product is enhanced more by disruption with price competition and hence disruption is more likely to occur.

One implication of Lemma 9.2 is that conditions which cause disruption as t gets large in the case of Cournot competition (see Proposition 8.4), also cause disruption in the case of Bertrand competition. In summary, our main results about the drivers of disruption are not dependent on an assumption of Cournot competition.[23]

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[22] While Cournot competition is useful for accomidating multiple firms using a given technology, the case of $v_1 > v_E$ is much less tractable.

[23] One difference is that while the condition for disruption for $t \to \infty$ is independent of $v_I(t)/v_1(t)$ in the case of Cournot, this ratio does enter into the limit condition for Bertrand.

9.2. Endogenous Number of Firms

For the Cournot model it is possible to endogenize the number of firms using each technology.[24] A full analysis of the effect of entry on disruption requires a characterization of profits when the entry causes the isolated equilibrium to breakdown, and hence is beyond the scope of this paper. Nonetheless, a limited exploration of the topic yields some interesting insights.

Assume that there is a fixed cost F_ϕ for firms using technology $\phi = N, E$. Let n_ϕ^I be the number of firms of type ϕ that can cover their fixed costs under the assumption that the new technology is isolated:

$$\pi_\phi^I(n_\phi^I) \geq F_\phi \text{ and } \pi_\phi^I(n_\phi^I + 1) < F_\phi.$$

Proposition 9.3. *Suppose $n_N = n_N^I$ and $n_E = n_E^I$. The isolated equilibrium is increasing in c_N, v_E and F_N and it is decreasing in c_E, v_1 and F_N. The effect of S_1, S_2 and v_2 are ambiguous.*

Proof We have that

$$\begin{aligned} \pi_N^I(n_N) &= \frac{v_2 S_2}{(n_N+1)^2}(1 - \frac{c_N}{v_2})^2, \\ \pi_E^I(n_E) &= \frac{v_E S_1}{(n_E+1)^2}(1 - \frac{c_E}{v_E})^2. \end{aligned}$$

Hence n_N^I is weakly increasing in v_2 and S_2 and it is weakly decreasing in c_N and F_N, while n_E^I is weakly increasing in v_E and S_1 and it is weakly decreasing in c_E

[24] The Bertrand model with fixed costs always results in one firm using each technology given that more than one firm of a given type results in marginal cost pricing.

and F_E. We again seek to sign Δ_I. The effect of the fixed costs is

$$\begin{aligned} \frac{\partial \Delta_I}{\partial F_N} &= \frac{\partial \Delta_I}{\partial n_N}\frac{\partial n_N^I}{\partial F_N} \geq 0, \\ \frac{\partial \Delta_I}{\partial F_E} &= \frac{\partial \Delta_I}{\partial n_E}\frac{\partial n_E^I}{\partial F_E} \leq 0. \end{aligned}$$

As v_1 affects neither n_E^I nor n_N^I, its effect is as before. The remaining parameters have both the effect identified in Proposition 6.1 and an additional indirect effect through n_E^I and n_N^I. For all parameters but S_1, S_2 and v_2 the two effects work in the same direction. **QED**

Thus, some of our earlier results depend on there being a fixed number of firms. Interestingly, the three parameters with ambiguous effects (S_1, S_2 and v_2) are the ones that do not matter in a perfectly competitive setting.

Proposition 9.4. *Suppose $n_\phi = n_\phi^I$ and $F_\phi = F$ for $\phi = N, E$. As $F \to 0$, the condition for the isolated equilibrium to exist converges to $v_1/c_N \leq v_E/c_E$.*

Proof We have $\lim_{F\to 0} n_\phi^I = \infty$ and hence $\lim_{F\to 0} P_\phi^I = c_\phi$, $\lim_{F\to 0} Q_N^I = S_2(1-c_N/v_2)$ and $\lim_{F\to 0} Q_E^I = S_1(1-c_E/v_E)$. An isolated equilibrium exists in the limit *iff* $\lim_{F\to 0} P_N(Q_N^I, Q_E^I) = c_N$ (as otherwise new-technology firms can profitably expand output). This requires that $\lim_{F\to 0}(Q_N^I - \bar{Q}_N(Q_E^I)) = v_1/c_N - v_E/c_E \leq 0$. **QED**

With perfect competition (so that $P_\phi = c_\phi$), market boundaries depend entirely on the relative price/performance ratio of two alternatives for customers in the primary segment. It is only when new-technology firms have market power that market boundaries depend on the relative size of the segments and the perceived quality in the secondary segment.

9.3. Convergence

Although we have focused on an asymmetric model where only the new technology had positive quality in both segments, our results can be easily extended to a more symmetric setup. Suppose that there are two technologies A and B and two market segments 1 and 2. Denote the quality of technology $\phi \in \{A, B\}$ in segment $i \in \{1, 2\}$ by $v_{\phi,i} > 0$. Suppose technology A is better suited for segment 1 ($v_{A,1} > v_{A,2}$) while technology B is better suited for segment 2 ($v_{B,2} > v_{B,1}$). One can again define an isolated equilibrium in which technology A is limited to segment 1 and technology B is limited to segment 2. In this context, our results can address two questions: How do the parameters effect the incentive of firms using each type of technology to deviate from the isolated equilibrium? Is there technology convergence in the long-run? Technology convergence (Yoffie, 1997) refers to a situation where technologies serving initially distinct market segments begin to compete.[25]

One can apply Proposition 6.1 to each of the technologies to characterize how different parameters affect the incentive of firms using a technology to enter the home segment of the other technology. Thus the incentive of firms using technology A to enter segment 2 are increasing in $v_{A,2}$, S_2, n_A, c_B and decreasing in $v_{A,1}$, $v_{B,2}$, S_1, n_B, c_A; while the incentives for firms using technology B to enter segment 1 are increasing in $v_{B,1}$, S_1, n_B, c_A and decreasing in $v_{B,2}$, $v_{A,1}$, S_2, n_A, c_B.

Suppose that each $v_{\phi,i} = V(\gamma_\phi t, (1 - \gamma_\phi)t; \alpha_i)$ so that each technology is improving along its own trajectory. One can apply Proposition 8.4 to address whether there is convergence in the long-run. That is, whether at least one firm

[25]Convergence is thus conceptualy very similar to disruption. The difference being that disruption is used when discussing the bluring of market boundaries between a new and established technology while convergence is used in discussing the bluring of market boundaries between two established technologies.

has the incentive to deviate from the isolated equilibrium in the long-run. Thus, for example, there is always convergence for log utility. For exponential utility there is convergence if there are sufficiently great decreasing returns ($\beta \to 1$), if preferences are sufficiently homogeneous ($\alpha_1 \to \alpha_2$) and if the trajectory is sufficiently central for the technology with more firms ($\gamma_\phi \to 1/2$ and $n_\phi \geq n_{-\phi}$).[26]

10. Conclusion

Market definition is usually taken as a purely empirical question. The standard approach for theoretical work in industrial organization is to take as given the set of competing firms and the set of relevant consumers. We take the boundaries of competition as our central research question. A better understanding of where competition emerges is important both for analyzing technology evolution and for defining markets in antitrust deliberations. Inspired by recent empirical observations of disruptive technologies, we propose a novel model of horizontal and vertical differentiation that is well suited to addressing the boundaries of technology competition. We show that market boundaries can be shaped by the profit maximizing behavior of firms.

To summarize our findings, we start with the observation that market boundaries depend on the degree of substitution, which itself depends on price-performance ratios (Scherer, 1980). In the context of our model, this means that the market boundary (i.e., whether the new technology enters the primary segment) depends on the price-performance ratio of the new product (P_N/v_1) and the established product (P_E/v_E), as viewed by consumers in the primary segment. Central to our theory is that these prices result from profit maximizing behavior by the firms using the technologies. Thus, for example, P_E depends on the marginal

[26]Note that as either $n_A \leq n_B$ or $n_B \leq n_A$, one can drop sometimes drop this condition in addressing convergence.

cost of established products (c_E) and the number of firms using the established technology (n_E) and hence these factors influence market boundaries. For the new technology, P_N not only depends on marginal costs (c_N) and the number of firms (n_N), but also on the extent to which *firms using the new technology view consumers in the primary segment as good substitutes for those in the secondary segment* in terms of volumes (S_1/S_2) and quality perceptions (v_1/v_2). If the secondary segment offfers sufficiently superior volumes and valuations, the firms will not choose to disrupt. Since, by definition, antitrust deliberations occur in markets with small numbers, it seems important to identify the ways in which the strategic behavior of firms affects these market boundaries. Having identified the critical role played by the perceived quality levels v_1, v_2, and v_E, we consider how these shift with consumer preferences (α_i), the new technology's trajectory (γ), the extent of consumers' decreasing marginal utility from product improvements (β) and the maturity of the new technology (t). This elaborated approach to market boundaries might be useful in antitrust settings where the analyst is trying to predict the future threat posed by substitute technologies.

The analysis of our model is not yet complete. Having addressed whether technology competition emerges, it remains to study the effects of such competition. We are interested in the effect of technology competition on the profits, market shares and concentration ratios of firms using each of the technologies. While we model technology progress as exogenous, one could study the incentives of firms in our model to engage in product and process innovation, as well as the optimal trajectory of technology advance. It would be interesting to know how these change, as well, with the emergence of technology competition. Put another way, the question remains: How disruptive are disruptive technologies?

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11. Appendix

Proof of Lemma 3.2 Note that $Q_E(0, P_N) = S_1$ for all $P_N \geq 0$ follows from $v_E > v_1$. If $Q_E > S_1$ the supply of the established product exceeds the maximum demand and $P_E = 0$, while for $Q_E \leq S_1$ a market clearing price $P_E \geq 0$ must exist. Further, when $Q_E \leq S_1$ the established product is purchased only by those consumers in segment 1 with a sufficiently high θ, in particular by all those with $\theta > \bar{\theta} = 1 - \frac{Q_E}{S_1}$.

We now solve for $P_N(Q_N, Q_E)$. Suppose $Q_E \geq S_1$ and hence $P_E = 0$. Since $Q_N(0,0) = S_2$, we have that for $Q_N > S_2$ output of the new product exceeds the maximum demand and $P_N = 0$. For $Q_N \leq S_2$ a market clearing price exists and is given by $Q_N = S_2(1 - P_N/v_2) \Leftrightarrow P_N = (S_2 - Q_N)\frac{v_2}{S_2}$. This establishes $P_N(Q_N, Q_E) = \max\{(S_2 - Q_N)\frac{v_2}{S_2}, 0\}$ when $Q_E > S_1$.

Now suppose $Q_E < S_1$. If all of the new product is bought by segment 2, we have the market clearing condition $Q_N = (1 - P_N/v_2)S_2$, which yields $P_N = (S_2 - Q_N)\frac{v_2}{S_2}$. The cusumer in segment 2 most tempted by the new product has $\theta = \bar{\theta}$ and this customer does not buy if $v_1\bar{\theta} \leq P_N$. Hence, none of the new product is bought by consumers in segment 2 if $v_1\bar{\theta} \leq (S_2 - Q_N)\frac{v_2}{S_2} \Leftrightarrow Q_N \leq \bar{Q}_N(Q_E) = (v_2 - (1 - Q_E/S_1)v_1)S_2/v_2$. If $\bar{Q}_N(Q_E) < Q_N < S_1 + S_2 - Q_E$, then market clearing requires that $S_1(\frac{P_E - P_N}{v_E - v_1} - \frac{P_N}{v_1}) + S_2(1 - \frac{P_N}{v_2}) = Q_N$ and $S_1(1 - \frac{P_E - P_N}{v_E - v_1}) = Q_N$ which yields $P_N = (S_1 + S_2 - Q_E - Q_N)v_1v_2/(v_1S_2 + v_2S_1)$. Since $Q_N(0,0) = S_1 + S_2 - Q_E$, a value of Q_N greater than this amount results in $P_N = 0$.

We now turn to $P_E(Q_E, Q_N)$. Suppose $Q_E < S_1$ as otherwise $P_E = 0$. If there exists a consumer in segment 2 indifferent between the established and new product, then the market clearing condition is $Q_E = S_1(1 - \frac{P_E - P_N}{v_E - v_1}) \Longleftrightarrow P_E = P_N + (S_1 - Q_E)\frac{v_E - v_1}{S_1}$. We proceed by considering two regions of Q_N.

Case 1: $Q_N \geq S_2$. Then there is always an indifferent consumer. $P_N = 0$ if $Q_E > \bar{Q}_E = S_1 + S_2 - Q_N$, while for $Q_E < \bar{Q}_E$ we have $P_N = (S_1 + S_2 - Q_E - Q_N)v_1v_2/(v_1S_2 + v_2S_1) > 0$. Case 2: $Q_N < S_2$. There is an indifferent consumer *iff* $Q_N > \bar{Q}_N(Q_E) \Longleftrightarrow Q_E < \bar{Q}_E(Q_N) = (v_1 - (1 - Q_N/S_2)v_2)S_1/v_1$. For $Q_E < \bar{Q}_E$, $P_N = (S_1 + S_2 - Q_E - Q_N)v_1v_2/(v_1S_2 + v_2S_1)$. For $Q_E \geq \bar{Q}_E$, the market clearing condition is $Q_E = S_1(1 - P_E/v_E) \Longleftrightarrow P_E = (S_1 - Q_E)v_E/S_1$.
QED

Proof of Lemma 4.4 We start with part (ii). If $Q_N^I > \bar{Q}_N(Q_E^I)$ then $\pi_N^I = q_N^I(P_N(Q_N^I) - c_N) < q_N^I(P_N(Q_N^I, Q_E^I) - c_N) \leq \hat{\pi}$ and hence $\Delta_I \neq 0$. If $Q_N^I = \bar{Q}_N(Q_E^I)$ then $q(P_N(\bar{Q}_N(Q_E^I) - Q_{-k}^I + q, Q_E^I) - c_N)$ is increasing in q for $q = \bar{Q}_N(Q_E^I) - Q_{-k}^I$ and hence $\pi_N^I < \hat{\pi}$ and again $\Delta_I \neq 0$. We conclude that $\Delta_I = 0$ implies that $Q_N^I < \bar{Q}_N(Q_E^I)$. When $\hat{Q} = \bar{Q}_N(Q_E^I)$, we have $\hat{\pi} < \pi_N^I$ and hence $\Delta_I \neq 0$. Since $\bar{Q}_N(Q_E^I) \leq \hat{Q}$ by assumption, $\Delta_I = 0$ implies that $\bar{Q}_N(Q_E^I) < \hat{Q}$.

Now consider part (i). First, note that we may have $\pi_N^I \neq \pi_N(Q_N^I, Q_E^I)$ as π_N^I was defined under the restriction $v_1 = 0$. When $v_1 > 0$, $P(Q_N^I, Q_E^I) \geq P_N^I$, which implies that $\pi_N(Q_N^I, Q_E^I) \geq \pi_N^I$. Suppose an isolated equilibrium exists. With constant marginal costs and linear demand, it must be symmetric. Further, it must be that $\pi_N(Q_N^I, Q_E^I) = \pi_N^I$ as otherwise $Q_N^I > \bar{Q}_N(Q_E^I)$ and hence q_N^I would not be locally optimal. Global optimality of q_N^I then implies that $\pi_N^I \geq \hat{\pi} \Rightarrow \Delta_I \geq 0$. Now suppose $\Delta_I \geq 0$. From the proof of (ii) we have that $Q_N^I < \bar{Q}_N(Q_E^I)$ and hence $\pi_N(Q_N^I, Q_E^I) = \pi_N^I$ and q_N^I is locally optimal. $\Delta_I \geq 0$ then assures global optimality as well and an isolated equilibrium exists. **QED**

Proof of Propostion 6.1 Let ρ denote an arbitrary parameter of the model. Note that Δ_I is continuous in the parameters of the model follows from the continuity of the underlying profit functions. Hence, from Lemma 5.2, we can determin the effect of ρ on the existence of the equilibrium by signing $\partial\Delta_I/\partial\rho$ for all values of the parameters such that $\Delta_I = 0$. Henceforth, assume that $\Delta_I = 0$, ie $\pi_N^I = \hat{\pi}$.

For $\Delta_I = 0$ we have that $Q_N^I < \bar{Q}_N(Q_E^I) < \hat{Q}$, which implies that $q_N^I < \hat{q}$. Further, $\Delta_I = 0 \Rightarrow q_N^I(P_N^I - c_N) = \hat{q}(\hat{P} - c_N)$, which when combined with $q_N^I < \hat{q}$ implies that $P_N^I > \hat{P} > c_N$ and $\hat{q}\hat{P} \geq q_N^I P_N^I$.

As both q_N^I and $\hat{q}$ are interior, we can use the envelope theorem to write

$$\begin{aligned}
\frac{\partial \pi_N^I}{\partial \rho} &= q_N^I \left[\frac{\partial P_N(Q_N^I)}{\partial \rho} + \frac{\partial P_N(Q_N^I)}{\partial Q_N}\frac{\partial Q_{-k}^I}{\partial \rho} - \frac{\partial c_N}{\partial \rho}\right] \\
\frac{\partial \hat{\pi}}{\partial \rho} &= \hat{q}\left[\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial \rho} + \frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial Q_N}\frac{\partial Q_{-k}^I}{\partial \rho} + \frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial Q_E}\frac{\partial Q_E^I}{\partial \rho} - \frac{\partial c_N}{\partial \rho}\right]
\end{aligned}$$

and from the first order conditions characterizing q_N^I and $\hat{q}$ we have that

$$q_N^I \frac{\partial P_N(Q_N^I)}{\partial Q_N} = P_N^I - c_N, \tag{11.1}$$

$$\hat{q}\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial Q_N} = \hat{P} - c_N. \tag{11.2}$$

Putting these last four equalities together we have

$$\begin{aligned}
\frac{\partial \Delta_I}{\partial \rho} = & -\frac{\partial Q_{-k}^I}{\partial \rho}(P_N^I - \hat{P}) + \frac{\partial c_N}{\partial \rho}(\hat{q} - q_N^I) + \\
& q_N^I \frac{\partial P_N(Q_N^I)}{\partial \rho} - \hat{q}\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial \rho} - \hat{q}\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial Q_E}\frac{\partial Q_E^I}{\partial \rho}.
\end{aligned} \tag{11.3}$$

We now turn to specific parameter values, starting with $\rho = c_N$:

$$\frac{\partial \Delta_I}{\partial c_N} = -\frac{\partial Q_{-k}^I}{\partial c_N}(P_N^I - \hat{P}) + (\hat{q} - q_N^I) > 0$$

since $\partial Q_{-k}^I / \partial c_N < 0$ from Lemma 4.1. Hence, the isolated equilibrium is increasing in c_N. For c_E we have

$$\frac{\partial \Delta_I}{\partial c_E} = -\hat{q}\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial Q_E}\frac{\partial Q_E^I}{\partial c_E} < 0$$

since $\partial P_N(\hat{Q}, Q_E^I)/\partial Q_E < 0$ and $\partial Q_E^I/\partial c_E < 0$. Hence the isolated equilibrium is decreasing in c_E.

Turning now to the valuation of the two products in segment 1 we find

$$\frac{\partial \Delta_I}{\partial v_E} = -\hat{q}\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial Q_E}\frac{\partial Q_E^I}{\partial v_E} > 0$$

since $\partial Q_E^I/\partial c_E > 0$ and

$$\frac{\partial \Delta_I}{\partial v_1} = -\hat{q}\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial v_1} < 0$$

since $\partial P_N(\hat{Q}, Q_E^I)/\partial v_1 > 0$. The effect of v_2 is given by

$$\frac{\partial \Delta_I}{\partial v_2} = -\frac{\partial Q_{-k}^I}{\partial v_2}(P_N^I - \hat{P}) + q_N^I\frac{\partial P_N(Q_N^I)}{\partial v_2} - \hat{q}\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial v_2}. \tag{11.4}$$

Using the following

$$\begin{aligned} \frac{\partial P_E(Q_N^I)}{\partial v_2} &= \frac{S_2}{v_2^2}P_N^I\frac{\partial P_N(Q_N^I)}{\partial Q_N}, \\ \frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial v_2} &= \frac{S_2}{v_2^2}\hat{P}\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial Q_N}, \\ \frac{\partial Q_{-k}^I}{\partial v_2} &= \left(\frac{n_N - 1}{n_N + 1}\right)\frac{S_2}{v_2^2}c_N, \end{aligned}$$

and equalities (11.1) and (11.2) one can rewrite (11.4) as

$$\frac{\partial \Delta_I}{\partial v_2} = \frac{S_2}{v_2^2}\left[(P_N^I - c_N)(P_N^I - \frac{n_N - 1}{n_N + 1}c_N) - (\hat{P} - c_N)(\hat{P} - \frac{n_N - 1}{n_N + 1}c_N)\right] > 0$$

since $P_N^I > \hat{P} > c_N$.

The effect of the number of firms is

$$\begin{aligned}\frac{\partial \Delta_I}{\partial n_N} &= -\frac{\partial Q_{-k}^I}{\partial n_N}(P_N^I - \hat{P}) < 0 \\ \frac{\partial \Delta_I}{\partial n_E} &= -\hat{q}\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial Q_E}\frac{\partial Q_E^I}{\partial n_E} > 0\end{aligned}$$

since $\partial Q_{-k}^I/\partial n_N > 0$ and $\partial Q_E^I/\partial n_E > 0$.

Finally, the effect of segment size is determined by

$$\frac{\partial \Delta_I}{\partial S_1} = -\hat{q}\left[\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial S_1} + \frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial Q_E}\frac{\partial Q_E^I}{\partial S_1}\right],$$

which is difficult to sign directly. We therefore develope a slightly different expression for the dependence of P_E on S_1. Recall from the proof of Lemma 3.2 that the willingness-to-pay of the consumer indifferent between N and E products is $\bar{\theta} = (1 - Q_E^I/S_1)$. Since $Q_E^I = S_1\frac{n_E}{n_E+1}(1 - c_E/v_E)$, $\bar{\theta}$ is independent of S_1 and demand for the new product depends on S_1 according to $P_N(\hat{Q}, Q_E^I) = (S_1\bar{\theta} + S_2 - \hat{Q})\frac{v_1 v_2}{v_1 S_2 + v_2 S_1}$ and then

$$\frac{\partial \Delta_I}{\partial S_1} = -\hat{q}\frac{\partial}{\partial S_1}\left[\frac{(S_1\bar{\theta} + S_2 - \hat{Q})v_1 v_2}{v_1 S_2 + v_2 S_1}\right] = -\hat{q}\left[v_1\bar{\theta} - \hat{P}\right] < 0$$

since $\hat{P} < v_1\bar{\theta}$ for the new product to be bought by consumers in segment 2. Signing $\partial\Delta_I/\partial S_2$ using (11.3) is very tedious. Instead, note that $\pi_N^I = S_2(1 - c_N/v_2)^2/(n_N + 1)^2$ is proportional to S_2 while $\hat{\pi}$ is proportional to $(S_1 + S_2)$ and hence less than proportional to S_2. Thus, for $\pi_N^I = \hat{\pi}$ we have $\partial\pi_N^I/\partial S_2 > \partial\hat{\pi}/\partial S_2$ and hence $\partial\Delta_I/\partial S_2 > 0$ for $\Delta_I = 0$. **QED**

Proof of Lemma 7.1 The proof is straight forward. We have that

$$\gamma_i^* = \frac{\alpha_i^r}{\alpha_i^r + (1-\alpha_i)^r}$$

where $r = 1$ for both log linear and Cobb-Douglas utiltity, $r = 1/2$ for bounded utility and $r = 1/(1-\beta)$ for additive exponential utility.

We only report the case of additive exponential utility. Then $v_i(\gamma) = V(\gamma k, (1-\gamma)k; \alpha_i) = \alpha_i \gamma^\beta + (1-\alpha_i)\gamma^\beta + k^\beta$. The function is concave since

$$\frac{\partial^2 v_i}{\partial \gamma^2} = -\beta(1-\beta)\left[\alpha\gamma^{\beta-2} + (1-\alpha)(1-\gamma)^{\beta-2}\right] < 0.$$

The first order condition for γ can be rewritten as

$$\left(\frac{\gamma^*}{1-\gamma^*}\right)^{1-\beta} = \frac{\alpha_i}{1-\alpha_i} \Leftrightarrow \gamma^* = \frac{\alpha_i^{\frac{1}{1-\beta}}}{\alpha_i^{\frac{1}{1-\beta}} + (1-\alpha_i)^{\frac{1}{1-\beta}}}.$$

That γ_i^* is increasing in α_i follows from $\partial^2 V/\partial\alpha_i\partial\gamma = \beta/\gamma^{1-\beta} + \beta/(1-\gamma)^{1-\beta} > 0$.

We now show that $\gamma_i^* \geq \alpha_i$ for $\alpha_i \geq 1/2$. If $\alpha_i = 1/2$ then $\gamma_i^* = 1/2$. Let $z_i = (1-\alpha_i)/\alpha_i$ so that we can write $\gamma_i^* = 1/(1 + z_i^{1/(1-\beta)})$. Note that $z_i < 1$ and hence $\ln z_i < 0$ for $\alpha_i > 1/2$. Then $\gamma_i^* > \alpha_i$ for $\alpha_i > 1/2$ follows from

$$\frac{\partial \gamma^*}{\partial \beta} = -\ln z \frac{z^{\frac{1}{1-\beta}}}{(1-\beta)^2\left(1 + z^{\frac{1}{1-\beta}}\right)^2} > 0$$

and $\gamma_i^* = \alpha_i$ for $\beta = 0$. An analogous argument yields $\gamma_i^* \leq \alpha_i$ for $\alpha_i \leq 1/2$. **QED**

Proof of Proposition 8.1

We start by proving that the isolated equilibrium is decreasing in t. Suppose $\Delta_I = 0$. We have

$$\frac{\partial \Delta_I}{\partial t} = \frac{\partial \Delta_I}{\partial v_1}\frac{\partial v_1}{\partial t} + \frac{\partial \Delta_I}{\partial v_2}\frac{\partial v_2}{\partial t} + \frac{\partial \Delta_I}{\partial v_E}\frac{\partial v_E}{\partial t}.$$

Since $\lim_{v_E/c_E \to \infty} \frac{\partial Q_E^I}{\partial v_E} = 0$, we can take $\frac{\partial \Delta_I}{\partial v_E} = \frac{\partial \hat{\pi}}{\partial Q_E}\frac{\partial Q_E^I}{\partial v_E} = 0$. For the case of Cobb-Douglas and additive exponential we have that $v_i(t) = t^\beta w_i$ where $w_i = v_i(1)$ is a constant independent of t. Note that

$$\begin{aligned} P_N(Q_N) &= t^\beta (S_2 - Q_N) w_2 \\ P_N(\hat{Q}, Q_E) &= t^\beta \frac{(S_1 + S_2 - Q_E - \hat{Q}) w_1 w_2}{w_1 S_2 + w_2 S_1} \end{aligned}$$

and hence

$$\begin{aligned} \frac{\partial \Delta_I}{\partial v_1}\frac{\partial v_1}{\partial t} + \frac{\partial \Delta_I}{\partial v_2}\frac{\partial v_2}{\partial t} &= q_N^I \frac{\partial P_N(Q_N^I)}{\partial t} - \hat{q}\frac{\partial P_N(\hat{Q}, Q_E^I)}{\partial t} \\ &= -\frac{\beta}{t}\left(\hat{q}\hat{P} - q_N^I P_N^I\right) - (P_N^I - \hat{P})\frac{\partial Q_{-k}^I}{\partial v_2}\frac{\partial v_2}{\partial t} \\ &< 0. \end{aligned}$$

We conclude that $\partial \Delta_I / \partial t < 0$ for $c_E / v_E(t_N)$ sufficiently small and Cobb-Douglas or additive exponential utility. The same result holds *a fortiori* for bounded or log linear since $\partial v_2 / \partial t$ is greater relative to $\partial v_1 / \partial t$ with these utility functions and hence $\partial \Delta_I / \partial t$ is still negative.

The isolated equilibrium decreasing in t implies that either an isolated equilibrium exists for all $t \geq t_N$ or there exists a t_D such that it only exists for $t \in [t_N, t_D]$. Finally, $v_1(t_N) < v_2(t_N) = c_N \Rightarrow \hat{\pi}(t_N) < 0 \Rightarrow \Delta_I(t_N) > 0 \Rightarrow t_D > t_E$. **QED**

Proof of Lemma 8.3 In the limit as $\max\{c_N, c_E\}/\min\{v_E, v_1, v_2\} \to 0$, we have $q_N^I = S_2/(n_N + 1)$, $Q_E^I = S_1 n_E/(n_E + 1)$, $\hat{q} = \frac{1}{2}(\frac{S_1}{n_E+1} + \frac{2S_2}{n_N+1})$, $\pi_N^I = \frac{v_2 S_2}{(n_N+1)^2}$, and $\hat{\pi} = \frac{1}{4}\frac{v_1 v_2}{v_1 S_2 + v_2 S_1}(\frac{S_1}{n_E+1} + \frac{2S_2}{n_N+1})^2$. One can then show that $\pi_N^I - \hat{\pi} < 0$ is equivalent to inequality (8.1). **QED**