Market Access Costs and Trade Dynamics*

Costas Arkolakis†

Yale University

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Abstract

I introduce trade dynamics into a static model of international trade with product differentiation, heterogeneous productivity firms, and increasing marginal market penetration costs. I interpret firms as ideas that materialize into production, where an idea is a way to produce a differentiated good with a given productivity. Adapting a stochastic process similar to Reed, the model endogenously generates a right tail cross-sectional Pareto distribution of firms' productivities based on two minimal assumptions: continuous entry of ideas at a certain rate and productivities of ideas that evolve according to a geometric Brownian motion. The cross-sectional predictions of the dynamic model for the distribution of domestic and exporting sales of firms are in line with firm-level data. In addition, the model delivers new predictions consistent with panel data observations on domestic and exporting firm-level sales. It predicts that many small firms enter and exit the market very frequently and that the growth rate as well as the variance of the growth rate of sales is higher for small firms.

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†Contact: Department of Economics, Yale University, 37 Hillhouse ave., New Haven, CT, 06511. Email: costas.arkolakis@yale.edu.

1 Introduction

Recent empirical research has established a series of facts on cross-sectional observations of sales of firms by exporting destination. This empirical work was followed by studies such as those by Eaton and Kortum (2002), Bernard, Eaton, Jensen, and Kortum (2003), Eaton, Kortum, and Kramarz (2005), and Arkolakis (2006), which have shown that models with firm productivity heterogeneity can closely predict key aspects of this empirical evidence. In addition, these models are kept highly parsimonious and are often used to perform policy research. In addition to the cross-sectional data, over the past few years new firm level data are becoming available. These newly available data allows us to explore the panel dimension on firms exporting sales (Bernard, Redding, and Schott (2006), and Eaton, Eslava, Kugler, and Tybout (2007)).

In this paper, I develop a model that studies firm-level trade dynamics. In particular, I introduce dynamics into the model developed by Arkolakis (2006), where firms have to pay an increasing marketing cost to reach additional consumers in each country. Following Kortum (1997) and Eaton and Kortum (2001), I assume that new ideas arrive at an exogenously given rate in each of the countries. Essentially, to each firm I associate an idea that allows it to produce a differentiated good and potentially earn profits. After an idea is "born", its productivity is expected to grow over time. Deviations from the expected growth rate follow a Brownian motion. The resulting stochastic process of productivities is the one introduced by Reed (2001) while the dynamics are introduced in a firm-level model following Luttmer (2006).

The new model endogenously generates a cross-sectional distribution of productivities with a right Pareto tail. Given this Pareto cross-sectional distribution of productivities and the rest of the setup of the model that is based on Arkolakis (2006), the model delivers cross-sectional predictions for the export sales of firms that are essentially identical to those outlined by Arkolakis (2006). In addition, the introduction of trade dynamics delivers three new predictions. First, the model predicts that many small firms enter and exit the market very frequently. This observation is consistent with facts presented by Eaton, Eslava, Kugler, and Tybout (2007) for Colombian exporters. Second, the growth of export sales is higher for small firms, which is consistent with the facts reported by Eaton, Eslava, Kugler, and Tybout (2007). This finding is also consistent with a series of studies on domestic sales data that report that Gibrat's law (the independence of firms' size and growth rates) does not hold for small firms. Such evidence can be found in Mansfield (1962), Hart and Oulton (1996), among others, and is reviewed by Sutton (1997). Third, the model predicts that the variance of growth rates of export sales is higher for firms with lower sales, when restricting attention to the sales of firms selling to a given destination. This is a testable hypothesis that is consistent with domestic firm-level sales data reviewed by Sutton (2002). However, the model also predicts the same pattern for firms' exports per destination, a hypothesis that has not been tested empirically.

Previous theoretical models with heterogeneous firms and firm dynamics include the one-country models of Jovanovic (1982), Klette and Kortum (2004), and Luttmer (2006) and the two-country model of Opromolla and Irarrazabal (2006). Jovanovic (1982) and Klette and Kortum (2004) develop a model that is consistent with panel data observations on firm dynamics such as the ones mentioned above. Luttmer develops a model that delivers a cross-sectional distribution of sales close to the one observed in the data but his model is not consistent with the facts on firm dynamics trade stated above. Opromolla and Irarrazabal (2006) extend Luttmer (2006) in a two-country context. The model presented in this paper contributes to this literature with an analytically tractable, multi-country model of trade that is consistent with the main cross-sectional and panel data observations on firms domestic and exporting sales.

2 Model

The model described in this section extends the static version of Arkolakis (2006). It introduces a stochastic process for productivities found in Reed (2001). It incorporates dynamics into a model with heterogeneous productivity firms following Luttmer (2006).

I assume that time is continuous and indexed by t. I will refer to the importing country with an index j and to the exporting country with i, where $i, j = 1, ..., N$. There is a continuum of consumers in each economy i of measure $H_{it} = H_i e^{g_\eta t}$ at each point of time. The consumer in country i, has preferences over a composite good C_{jt} from which she derives utility according to

$$\left(E \int_0^{+\infty} r e^{-rt} C_{jt}^{\frac{\gamma-1}{\gamma}} dt \right)^{\frac{\gamma}{\gamma-1}}$$

where $r > 0$ is the discount rate and $\gamma > 0$ is the intertemporal elasticity of substitution.

On the balanced growth path constructed below, the aggregate variables grow at a some rate g_κ (to be specified), implying that $C_{jt} = C_j e^{g_\kappa t}$. To ensure that the value of the aggregate endowment is finite, the discount rate must exceed the rate of growth of the aggregate variables and thus,

Assumption 1

$$r + \gamma g_\kappa > g_\kappa + g_\eta$$

with $g_\eta \geq 0$.

The composite good is made of a continuum of differentiated commodities

$$C_{jt} = \left(\sum_{i=1}^{N} \int_{\omega \in \Omega_{it}^h} q_{ijt} (\omega)^\rho \, d\omega \right)^{\frac{1}{\rho}}$$

where $q_{ijt}(\omega)$ is the demand for a good ω from a consumer from country j and $\sigma = 1/(1-\rho)$ is the elasticity of substitution among different varieties of goods. At a given point of time t, a consumer has access to a set of goods Ω_{it}^h from country $i = 1, ..., N$. Goods are produced by firms with potentially different productivities. The productivities of firms selling to country j are potentially drawn from $[0, +\infty)$. We will consider a symmetric equilibrium where all firms with the same productivity from the same country i choose to charge the same price in country j. Given the large number of consumers and firms and the symmetric CES Dixit-Stiglitz preferences, we can re-index variables at each point of time t as a function of productivities of the firms producing the goods, z, and their country of origin i. In this symmetric equilibrium, each consumer from country i has access to the same measure of goods of a given type $M_{ijt}\mu_{ijt}(z)\, n_{ijt}(z)$, at time t.[1] Here, M_{ijt} stands for the the measure of firms from country i selling in country j at time t, $n_{ijt}(z)$ for the fraction of goods of a given type that a consumer from country j has access to at time t, and $\mu_{ijt}(z)$ for the pdf of the distribution of productivities of firms from country i conditional on selling to country j at a given time t. The measure of consumers reached by a firm of type z from country i, in country j is $n_{ijt}(z)\, L_j$.

[1] See Arkolakis (2006) for the details of this argument.

Each household earns labor income for w_{jt} for selling his unit labor endowment on the labor market and profit flows π_{jt} from the ownership of domestic firms. Thus, the demand for good z from country i by a consumer from country j is

$$q_{ijt}(z) = \frac{p_{ijt}(z)^{-\sigma}}{P_{ijt}^{1-\sigma}} y_{jt}$$

where $y_{jt} = w_{jt} + \pi_{jt}$ and

$$P_{jt}^{1-\sigma} = \sum_{\upsilon=1}^{N} M_{\upsilon jt} \int_{0}^{+\infty} p_{\upsilon jt}(z)^{1-\sigma} n_{\upsilon jt}(z) \mu_{\upsilon jt}(z) \, dz \ . \tag{1}$$

In the equation above, $p_{\upsilon jt}(z)$ is the price that a firm with productivity z from source country υ charges in country j. Given the above assumptions, we have that $C_{jt}P_{jt} = y_{jt}$. Finally, firms from source country υ that have drawn z below the productivity threshold $z^*_{\upsilon jt}$ choose not to sell to country j. Given the above, total demand faced by a firm of type z from country i when selling to country j is

$$n_{ijt}(z) L_j \frac{p_{ijt}(z)^{-\sigma}}{P_{ijt}^{1-\sigma}} y_{jt}$$

2.1 Firm

Each firm is the unique monopolist over the technology used to produce a differentiated commodity. However, only firms that chose to produce positive amounts of their unique commodity appear as operating. Firms decide on the quantity of the good produced using a constant returns to scale production function $q(z_{\tau,a}) = z_{\tau,a} l$, where l is the amount of labor used in production and $z_{\tau,a}$ is the labor productivity of the firm of age a that was born in time τ, $\tau = t - a$. Productivities evolve independently across firms according to

$$z_{\tau,a} = \bar{z}_i \exp(g_E \tau + g_I a + \sigma_z W_{\tau,a}) \tag{2}$$

where $W_{\tau,a} \sim N(0, a)$ is a standard Brownian motion and $\bar{z}_i$ is an initial condition. New firms born with a productivity $\bar{z}_i \exp(g_E t)$ and the productivity increases with age at a rate g_I. The firms have to pay market penetration costs that are a function of the number of consumers reached at a given market. I model these market penetration costs as in Arkolakis (2006) and

I assume that they have to be incurred at each instant by the firm. While this assumption is clearly abstracting from reality, it comes closer to it compared to a fixed cost that is necessary to be paid at each period of time required by previous models (see for example Melitz (2003), Luttmer (2006)). A more detailed development of the dynamics of the market penetration costs induced by state dependence on previous market penetration is left for future research.

The labor requirement of a firm willing to reach a fraction of consumers n in a market of population size L is

$$f(n, L_j) = \frac{L_j^{\alpha}}{\psi} \frac{1-(1-n)^{-\beta+1}}{-\beta+1} .$$

where $\beta \in [0, +\infty)$ and $\alpha \in [0,1]$. Assuming, as in Arkolakis (2006), that firms incur these costs in both domestic and foreign wages yields the following total market penetration cost faced by a firm from country i selling to country j:

$$f(n, L_j) = w_j^{\gamma} w_i^{1-\gamma} \frac{L_j^{\alpha}}{\psi} \frac{1-(1-n)^{-\beta+1}}{-\beta+1} .$$

In addition to the marketing cost to reach consumers, the firm has to pay a variable trade cost modeled in the standard iceberg formulation. This implies that a firm operating in country i and selling to country j must ship $\tau_{ij} > 1$ units in order for one unit of the good to arrive at the export destination. For simplicity, I assume that $\tau_{ii} = 1$.

Given the constant returns to scale production technology and the separability of the marketing cost function across countries, the decision of the firm to sell to a given country is independent of the decision to sell to other countries. Thus, a firm with productivity z at time t from country i solves the following maximization problem for each country j:[2]

$$\begin{aligned} \pi_{ijt}(z) = \quad & \max_{n_{ijt}, p_{ijt}} \left\{ n_{ijt} L_{jt} y_{jt} \frac{p_{ijt}^{1-\sigma}}{P_{jt}^{1-\sigma}} - n_{ijt} L_{jt} y_{jt} \frac{\tau_{ijt} p_{ijt}^{-\sigma} w_{it}}{P_{jt}^{1-\sigma} z} - w_{jt}^{\gamma} w_{it}^{1-\gamma} \frac{L_{jt}^{\alpha}}{\psi} \frac{1-[1-n_{ijt}]^{-\beta+1}}{-\beta+1} \right\} \\ & \text{s.t. } n_{ijt} \in [0,1] \ \ \forall t . \end{aligned}$$

Total profits of a particular firm are the summation of the profits from exporting activities in all the $j = 1, ..., N$ countries (or a subset thereof). Notice that given the current set-up, the

[2]Slighly abusing the notation I denote the decision of the firm as a function of its productivity z, supressing time of birth and age information. Given that the optimization decision is static what is important is the current level of productivity, rather than the time path. I will keep the notation parsimonious throughout the text whenerer is possible.

decision of the firm is essentially static. Thus, at a given moment of time, the firm's problem is the same as in Arkolakis (2006).

For the case of $\beta \geq 0$, the optimal decisions of the firm in the multi-country model are:

$$p_{ijt}(z) = \tilde{\sigma} \frac{\tau_{ijt} w_{it}}{z} \ . \tag{3}$$

where

$$\tilde{\sigma} = \frac{\sigma}{\sigma - 1}$$

For $z \geq z_{ijt}^*$,

$$n_{ijt}(z) = 1 - \left[L_{jt}^{1-\alpha} y_{jt} w_{jt}^{-\gamma} z^{\sigma-1} \left(\tilde{\sigma} \tau_{ij} w_{it} \right)^{1-\sigma} \psi P_{jt}^{\sigma-1} / \left(w_{it}^{1-\gamma} \sigma \right) \right]^{-1/\beta} \ , \tag{4}$$

and $n_{ijt}(z) = 0$ for $z < z_{ijt}^*$, where z_{ijt}^* is given by

$$z_{ijt}^* = \sup_{z \geq b_{it}} \left\{ \pi_{ijt}(z) = 0 \right\} \ . \tag{5}$$

The above implies that firms from country i that choose to operate at period t in market j have

$$z_{ijt}^* \geq L_{jt}^{1-\alpha} y_{jt} w_{jt}^{-\gamma} z^{\sigma-1} \left(\tilde{\sigma} \tau_{ijt} w_{it} \right)^{1-\sigma} \psi P_{jt}^{\sigma-1} / \left(w_{it}^{1-\gamma} \sigma \right) \tag{6}$$

Solving for the first order conditions and substituting them out together with (6) in the expression for the sales per firm, we have that sales of a firm z from country i in country j are

$$r_{ijt}(z) = \begin{cases} L_{jt}^{\alpha} y_{jt}^{\gamma} y_{it}^{1-\gamma} \frac{1}{\tilde{\psi}} \left[\left(\frac{z}{z_{ijt}^*} \right)^{\sigma-1} - \left(\frac{z}{z_{ijt}^*} \right)^{(\sigma-1)/\tilde{\beta}} \right] & \text{if } z \geq z_{ijt}^* \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

where

$$\tilde{\beta} = \frac{\beta}{\beta - 1}, \ \tilde{\psi} = \frac{\psi}{\sigma(1-\eta)}$$

2.2 Entry and Exit

Following Kortum (1997), an idea is a way to produce a good ω with productivity z. Thus, ideas are essentially firms if they materialize into production. I assume that each country innovates at a constant rate and thus ideas flow at a rate $g_\eta (1-\alpha)$.[3] Each idea is exclusive and is owned by a firm that has the monopoly over the good related to that idea (monopolistic competition).[4] New goods are produced with an initial productivity, $\bar{z}_{it}$, that is given by equation (2) for $a = 0$. $\bar{z}_i$ is drawn from an initial distribution $G(\bar{z}_i)$.

The productivity of producing a good follows a geometric Brownian motion with a drift that is described by (2). Ideas cannot disappear, but if are not used in production, they remain idle while waiting for a chance to be used (if their productivity surpasses z^*_{ijt} at a given time t).[5] Notice that the productivity of incumbent ideas is generally improving at a rate g_I with deviation from this trend due to the Brownian shocks on the growth rate. The incumbent ideas also face competition from new ideas, and thus new firms, that arrive continuously.

2.3 Balanced Growth Path Equilibrium

For simplicity of exposition I consider a simple case regarding the entry of firms: at each moment of time, all the entry happens at one level of productivity, $\bar{z}_{it}$, but because of technological progress, this level increases over time at a given rate (in particular $\bar{z}_{it} = \bar{z}_i \exp(g_E t)$). Extending this simple case to one in which new entrants arrive with a productivity that is distributed according to a particular distribution is straightforward. To solve for the cross-sectional distri-

[3]This rate is the one necessary to solve for a balanced growth path. The assumption that this rate is a function of the growth rate of population is consistent with the context proposed here. Ideas could be related to population and thus the rate of arrival of these ideas could be ultimately thought of as a function of the population.

[4]The setup of monopolistic competition can be thought also in the context of Eaton, Kortum, and Kramarz (2005). In particular, as a limit case of a model where firms can choose to produce a single good out of a number of potential varieties and where this number tends to infinity.

[5]It is straightforward to add an assumption that ideas dissapear at an exogenous rate g_λ.

bution, I consider the stationary balanced growth path. I define the detrended variable[6]

$$\begin{aligned}\phi &= \bar{z}_i \exp\left(g_E\tau + g_I a + \sigma_z W_{\tau,a}\right) / \exp\left(g_E\left(\tau + a\right)\right) = \\ &= \bar{z}_i \exp\left(\left(g_I - g_E\right) a + \sigma_z W_{\tau,a}\right)\end{aligned}$$

The logarithm of this expression gives

$$\phi' = \ln \phi = \ln \bar{z}_i + \left(g_I - g_E\right) a + \sigma_z W_{\tau,a}$$

and thus the variable ϕ' follows a simple Brownian motion with an initial condition $\bar{z}_i$. Given the entry process, this differential equation generates the following Kolmogorov forward equation

$$0 = -\eta\left(1-\alpha\right) f\left(\phi'\right) - \mu f'\left(\phi'\right) + \frac{1}{2}\sigma_z^2 f''\left(\phi'\right) \ , \tag{8}$$

where f is a probability density function and $J_i e^{g_\eta(1-\alpha)t} f\left(\phi'\right)$ the density of firms with productivity ϕ'. The entry point is at $\bar{z}_i' = \ln \bar{z}_i$.

The process of productivities that was considered above has to satisfy a set of conditions. The requirement that $f\left(\phi'\right)$ is a probability density implies that

$$f\left(\phi'\right) \geq 0 \ \ \forall \phi' \in \left(-\infty, +\infty\right) \ , \tag{9}$$

and

$$\int_{-\infty}^{\bar{z}_i'} f\left(\phi'\right) d\phi' + \int_{\bar{z}_i'}^{+\infty} f\left(\phi'\right) d\phi' = 1 \ . \tag{10}$$

Also, $-\infty$ is an absorbing barrier and thus,

$$\lim_{\phi' \downarrow -\infty} f\left(\phi'\right) = 0 \ . \tag{11}$$

Finally, the requirement that the total flows into the distribution at point $\bar{z}_i'$ equal the entry

[6] Essentially, considering the detrended variable ϕ, I consider a no-growth version of the model and thus follow the notation of Arkolakis (2006).

rate η[7]

$$-\left(g_I-g_E\right)\left[f\left(\bar{z}_i^{\prime}-\right)-f\left(\bar{z}_i^{\prime}+\right)\right]+\frac{1}{2}\sigma_z^2\left[f^{\prime}\left(\bar{z}_i^{\prime}-\right)-f^{\prime}\left(\bar{z}_i^{\prime}+\right)\right]=\eta\ . \tag{12}$$

The solution of the above system is (see appendix):

$$f\left(\phi^{\prime}|\bar{z}_i^{\prime}\right)=\begin{cases}\frac{\theta_1\theta_2}{\theta_1-\theta_2}e^{\theta_1\left(\phi^{\prime}-\bar{z}_i^{\prime}\right)} & \text{if } \phi^{\prime}<\bar{z}_i^{\prime}\\ \frac{\theta_1\theta_2}{\theta_1-\theta_2}e^{-\theta_2\left(\phi^{\prime}-\bar{z}_i^{\prime}\right)} & \text{if } \phi^{\prime}\geq\bar{z}_i^{\prime}\end{cases}$$

where

$$\begin{aligned}\theta_1 &= \frac{g_I-g_E+\sqrt{\left(g_I-g_E\right)^2+2\sigma_z^2 g_\eta\left(1-\alpha\right)}}{\sigma_z^2}>0\\ \theta_2 &= -\frac{g_I-g_E-\sqrt{\left(g_I-g_E\right)^2+2\sigma^2 g_\eta\left(1-\alpha\right)}}{\sigma_z^2}>0\end{aligned}$$

where I require that:

Assumption 2

The growth rate of incumbent firms is positive, thus, $g_I-g_E>0$.

The resulting distribution of $\phi\in[0,+\infty]$ for the point of entry $\bar{z}_i$ is the so-called double Pareto distribution (Reed (2001)) with probability density function[8]

$$f\left(\phi|\bar{z}_i\right)=\begin{cases}\frac{\theta_1\theta_2}{\theta_1+\theta_2}\left(\frac{\phi}{\bar{z}_i}\right)^{\theta_1-1} & \text{if } \phi<\bar{z}_i\\ \frac{\theta_1\theta_2}{\theta_1+\theta_2}\left(\frac{\phi}{\bar{z}_i}\right)^{-\theta_2-1} & \text{if } \phi\geq\bar{z}_i\end{cases}$$

The requirements that the Pareto distribution of productivities has a finite mean as well as that the average sales are finite are necessary and are guaranteed by the following two assumptions:

[7]This last condition guarantees that the total inflow of firms at each point of time equals the entry rate. Similar conditions are commonly used in labor models to characterize the behavior of the distribution at a point of entry to or exit from a particular occupation (see Moscarini (2005)).

[8]This distribution is the result of a process first introduced by Reed (2001). In the appendix, I provide an alternative proof of that result. It can also be thought of as a limit case in the distribution of firms derived by Luttmer (2006) when the entry exit cutoff goes to $-\infty$. However, in the case of Luttmer (2006), this would imply that firms never exit and this is not consistent with the existence of indivisibilities in the production that are postulated there.

Assumption 3

$$g_\eta (1-\alpha) > g_I - g_E + \sigma_z^2/2$$

Assumption 4

$$\frac{g_\eta (1-\alpha)}{(\sigma - 1)} > g_I - g_E + \frac{\sigma_z^2}{2}(\sigma - 1)$$

which given assumption 2, also implies that $g_\eta, 1-\alpha > 0$.

The double Pareto distribution is illustrated in (1). Notice that in this distribution, at each moment of time, a constant fraction of ideas $\theta_1/(\theta_1+\theta_2)$ is above the threshold $\bar{z}_i$. Thus, for the shake of exposition, I assume that the parameters of the model are such that $z_{ijt}^*/\exp(g_E t) > \bar{z}_i$ for the rest of the paper. The (detrended) cross-sectional distribution is a Pareto at $[\bar{z}_i, +\infty)$ with parameter θ_2, and the model at each point of time collapses to the model of Arkolakis (2006) with the number of potential entrants being $\theta_1/(\theta_1+\theta_2) J_i e^{g_\eta (1-\alpha) t}$. In addition, the rate of growth of aggregate variables is $g_\kappa = g_E + g_\eta (1-\alpha)/(\sigma-1)$. It is straightforward to verify that there exists a balanced growth path that satisfies all the steady state equations appearing in Arkolakis (2006) and in this paper $\forall t$.[9]

Proposition 1 *Given assumptions 1-4, there exists a balanced growth path for the economy described above.*

Proof. *By assumption we have that $H_{it} = H_i e^{g_\eta t}$ and $J_{it} = J_i e^{g_\eta (1-\alpha) t}$, $\bar{z}_{it} = \bar{z}_i \exp(g_E t)$. Define $z_{ijt}^* = z_{ij}^* \exp(g_E t)$, $w_{it} = w_i e^{g_\kappa t}$, $P_{it} = P_i$, $C_{it} = C_i e^{g_\kappa t}$. Given these assumptions and definitions, the cross-sectional distribution of productivities of operating firms is Pareto. In addition, when we replace the variables considered by Arkolakis (2006) with their dynamic analoges all the steady state equations of the equilibrium presented in Arkolakis (2006) are satisfied $\forall\, t$. The values of z_{ij}^*, w_i, P_i, and C_i can be found by solving the system of equations when $t=0$.* ■

[9] It is also straightforward to show existence of the balanced growth path when the parameters are such that a double Pareto distribution emerges.

2.4 Firm Dynamics

The static framework in Arkolakis (2006) is successful in delivering the stylized cross-sectional firm-level predictions as documented by Eaton, Kortum, and Kramarz (2005). Given this success, the extension to a dynamic framework allows to focus on the new theoretical predictions related to firm dynamics. Generalizing the framework of this model in order to allow the cross-sectional distribution of productivities of operating firms to be double-Pareto rather than Pareto will give additional testable implications. The skewness of the distribution of productivities, and thus of sales, will be higher for the countries with higher total exports. This is consistent with the facts reported by Eaton, Kortum, and Kramarz (2005). Simultaneously, the Pareto tail of the distribution of productivities allows to match the Pareto-like distribution of sales for the largest exporters to each country.

The productivity process generated here, while following Reed (2001), is quite different from other processes proposed in the literature that generate a right tail Pareto cross-sectional distribution of productivities (see Gabaix (1999), Luttmer (2006)). In previous models a lower bound productivity was required to bound the size of the firm and prevent the distribution from becoming degenerate. In the setup proposed in this paper, entry is the only force that keeps the distribution from becoming degenerate while ideas have a productivity without a lower bound. The entry of new firms close to the threshold of operation, is the force that prevents the distribution from widening out and that creates the Pareto tails.

The first important observation is that, due to the stochastic nature of productivities, firms with a productivity close the threshold of entry z^*_{ijt} continuously enter and exit a particular market j. Given that the firms that just surpassed the threshold of entry z^*_{ijt} are tiny and that the distribution has Pareto tails a large part of the new entrants will have tiny sales. All these implications of the model, are very consistent with the trade data reported by Eaton, Eslava, Kugler, and Tybout (2007).

Before proceeding to the predictions about growth rates of sales, it will be useful to give some intuition for the main mechanisms at work in this model. The assumptions on the distribution of productivities imply that the expected growth rate of productivity is the same for all incumbent firms, independent of their size, since the mean of the Brownian motion is 0. Given the assumption on constant returns to scale and the Dixit-Stiglitz demand specification (constant price elasticity), this translates into identical expected growth rate of sales per consumer across all incumbent firms. However, the marketing cost function exhibits increasing cost elasticity for reaching additional consumers. Thus, the same expected growth rate in per-consumer sales for the incumbent firms translates into percentage increases in the number of consumers reached that are larger for initially smaller firms. The following proposition is a straightforward implication of mechanism mentioned above:

Proposition 2 *Given assumption 2, the expected growth rate of incumbent firms is larger the smaller their sales.*

Proof. *Using Ito's lemma, I apply the derivative of the natural logarithm of sales with respect to age. This gives the expected growth rate of the firm as a function of the current productivity and the sales of the firm. That is,*

$$\frac{\partial \ln r_{ij\tau,a}(z)}{\partial a} = g_\eta \alpha + g_\kappa + (\sigma - 1)(g_I - g_E)\, h(z)$$

where

$$h(z) = \frac{\left(\frac{z}{z^*_{ijt}}\right)^{\sigma-1} - \frac{1}{\tilde{\beta}}\left(\frac{z}{z^*_{ijt}}\right)^{(\sigma-1)/\tilde{\beta}}}{\left(\frac{z}{z^*_{ijt}}\right)^{\sigma-1} - \left(\frac{z}{z^*_{ijt}}\right)^{(\sigma-1)/\tilde{\beta}}} \xrightarrow{\beta \to 0} 1$$

and $h_z < 0$. ■

Notice that the growth rate of wage per capita and population imply that nominal GDP grows at a rate $g_\kappa + g_\eta$. Assumption 4 ensures that the expected growth rate of the largest incumbent firms is not larger than the growth rate of nominal GDP, thus guaranteeing the existence of a stationary equilibrium. Deviations from the independence of growth rates of sales from firms' size have been recognized as early as Mansfield (1962). They has been recently verified from a large literature reviewed by Sutton (1997) and verified for exporting sales data to given destinations by Eaton, Eslava, Kugler, and Tybout (2007). On the other hand, this deviation appears to be vanishing when considering samples of large firms as Hart and Oulton (1996) point out. All these predictions are valid in the model given that $h_z < 0$ and $h(z) \to 1$ as z becomes large. It is important to notice that the same mechanism at work is the one that implies the higher growth rates of smaller exporters in trade liberalization episodes, consistent with the theoretical and empirical analysis of Arkolakis (2006).

The final theoretical result of this paper refers to the variance of the growth rate of sales of firms and its relation to the size of firms. At this time, there is no study in the trade data reporting this fact. However, a large body of empirical literature, reviewed by Sutton (2002), uses domestic sales data to establish an inverse relationship between the sales of firms and the variance of their growth rates.

The following proposition identifies this fact in the model.

Proposition 3 *In each market, the variance of the growth rates of firms is bigger the smaller their sales there.*

Proof. *Applying Ito's lemma once more, this time with respect to the variance term, we get that the mean variance of the growth rate of sales is*

$$var\left(\ln r_{ij\tau,a}\left(z\right)\right)=\frac{\partial \ln r_{ij\tau,a}\left(z\right)}{\partial W_{\tau,a}}=\sigma_z\left(\sigma-1\right)h\left(z\right)$$

■

Given $\beta > 0$, the constant variance of the growth rate of productivity shocks translates into increased variance of the growth rates of sales to a destination for the firms with smaller sales to that destination.

3 Conclusion

The model presented in this paper has all the potential to be used in applied empirical work: it is highly tractable, closely matches the key cross-sectional observations on firms' domestic and exporting sales data, and is qualitatively consistent with some of the key observations related to panel data on firms domestic or exporting sales. Predictions of the new model could also be used as a compass for new empirical research. For example, the new model predicts that the variance of export sales of firms is larger for firms that sell small amounts, a fact already documented for domestic sales. Ongoing empirical research such as the one of Eaton, Eslava, Kugler, and Tybout

(2007) will help precisely quantify key aspects of the panel dimension of the firm-level data on trade and allow for the calibration of this model and for future quantitative policy analysis. The construction of a model of trade that is highly tractable and that matches important aspects of firm-level trade data could allow to take important steps toward understanding aspects of dynamic firm-level behavior.

4 Appendix

4.1 Deriving the Stationary Distribution of Productivities

A simple guess for the solution of the Kolmogorov equation (8) is $f(\phi') = A_1 e^{\theta_1 \phi'} + A_2 e^{-\theta_2 \phi'}$ where θ_1 and $-\theta_2$ are given by the two solutions of the quadratic equation $\frac{1}{2}\sigma_Z^2 \theta^2 - \mu\theta - \eta(1-\alpha)$. Using condition (11) set $A_2 = 0$ for $\phi' < \bar{z}_i'$ and using the requirement that $f(\phi')$ is a probability density set $A_1 = 0$ for $\phi' \geq \bar{z}_i'$.

Finally, from the characterization of the flows at the entry point (12), we pick A_1, A_2 such that

$$\begin{aligned} \frac{1}{2}\sigma_z^2 \left[f'(\bar{z}_i'-) - f'(\bar{z}_i'+)\right] &= \eta \implies \\ \frac{1}{2}\sigma_z^2 \left(A_1 \theta_1 e^{\theta_1 \bar{z}_i'} + A_2 \theta_2 e^{-\theta_2 \bar{z}_i'}\right) &= \eta \end{aligned}$$

which in combination with (10) that gives

$$\int_{-\infty}^{\bar{z}_i'} A_1 e^{\theta_1 \phi'} d\phi' + \int_{\bar{z}_i'}^{+\infty} A_2 e^{-\theta_2 \phi'} d\phi' = 1$$

imply that

$$\begin{aligned} A_1 &= \frac{\theta_1 \theta_2}{\theta_1 - \theta_2} e^{-\theta_1 \bar{z}_i'} \\ A_2 &= \frac{\theta_1 \theta_2}{\theta_1 - \theta_2} e^{\theta_2 \bar{z}_i'} \end{aligned}$$

Notice, that the solutions also satisfy the first term in the LHS of (12) since the above solutions imply that $f(\bar{z}_i'-) = f(\bar{z}_i'+)$. In other words the distribution is continuous, but the derivative has a kink at $\bar{z}_i'$.

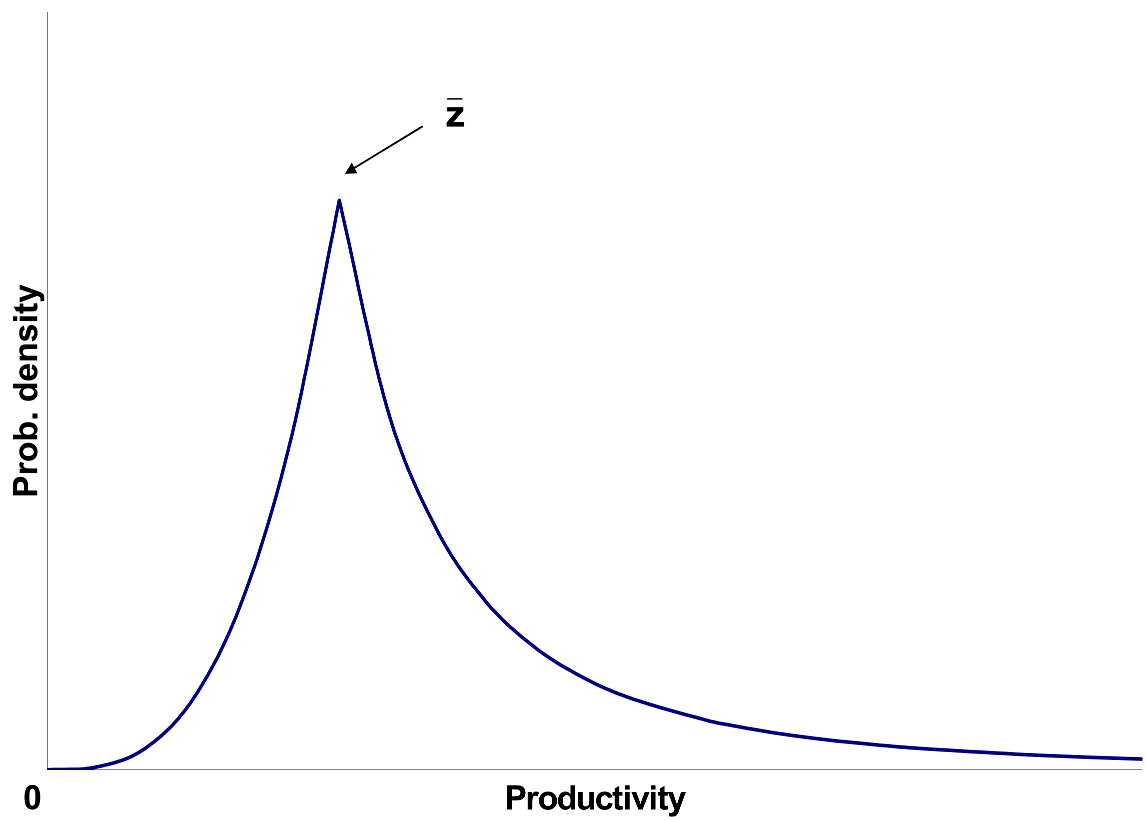

Figure 1: Double Pareto distribution

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