A Theory of Capital Controls as Dynamic Terms of Trade Manipulation*

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Abstract

This paper develops a theory of optimal capital controls in which the motive for capital controls is the manipulation of interest rates and other prices. We study a dynamic endowment economy where countries are free to impose capital controls, but are not allowed to use import and export taxes, because of existing trade agreements. In the one good case, we show that it is optimal to tax capital inflows (or subsidize capital outflows) in periods of positive growth and to tax capital outflows (or subsidize capital inflows) in periods of negative growth. In the long-run, if the endowments converge to a steady state, taxes on international capital flows converge to zero. With many goods, capital controls also influence terms of trade in any period. As a result, optimal capital controls depend both on the level and composition of growth across goods. We also examine how these forces play out in a non-cooperative equilibrium.

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1 Introduction

Since the end of World War II, bilateral and multilateral trade agreements have lead to dramatic tariff reductions around the world, contributing to a spectacular increase in world trade over the last sixty years; see Subramanian and Wei (2007) and Baier and Bergstrand (2007). Starting in the mid-1980*s*, the world has also experienced a dramatic increase in capital markets integration, with increased cross-border flows both across industrial countries and between industrial and developing countries; see Kose, Prasad, Rogoff, and Wei (2009). In other words, there has been a big increase both in intratemporal and intertemporal trade across countries.

The primary goal of multilateral trade policy coordination, promoted by the World Trade Organization, has been to eliminate relative price distortions in intratemporal trade. At the same time, international efforts toward increased capital openness have taken a different approach, focusing less on relative price distortions and more on the effects of capital controls on macroeconomic and financial stability. Consequently, the multilateral institutions that promote capital market integration, like the International Monetary Fund, have taken a different, more nuanced, approach, as exemplified in the following quote from Eichengreen and Mussa (1998) (italics added).[1]

> Article VIII of the IMF's Articles of Agreement, which defines current account convertibility as freedom from restrictions on the making of payments and transfers for current international transactions and makes this an explicit objective of IMF policy, *does not proscribe the imposition of such price-based restrictions as import tariffs and taxes on underlying transactions*. Correspondingly, capital account convertibility means the removal of foreign exchange and other controls, but not necessarily all tax-like instruments imposed on the underlying transactions, which need not be viewed as incompatible with the desirable goal of capital account liberalization.

Trade policy's focus on relative price distortions has its intellectual background in the international trade literature. Terms-of-trade manipulation always played a central role in the analysis of optimal tariffs, going back at least to Johnson (1951). Recent examples of the prominent role of terms-of-trade effects both in the theoretical and in the empirical literature are Bagwell and Staiger (1999) and Broda, Limao, and Weinstein (2008). On the other hand, the theory of capital controls has payed relatively less attention to relative price effects.

[1]See Ostry, Ghosh, Habermeier, Chamon, Qureshi, and Reinhardt (2010), for a recent example of the IMF recommendations on the appropriate use of capital controls

The objective of this paper is to take one step to bridge the gap between the trade policy approach and the macroeconomic approach to capital controls. In particular, we use a traditional trade policy approach to study the use of tax-like instruments in intertemporal trade, focusing on their effects on relative prices. Our objective is not to argue that the only motive for observed capital controls are relative price distortions or that the removal of these distortions should be the only goal of international policy coordination. Rather, we want to develop some basic tools to think about capital controls as a form of intertemporal trade policy. We develop these tools in the context of models where other effects of capital controls are muted: namely, there are no effects on output levels, nominal prices and nominal exchange rates, and no effects on financial stability. Hopefully, the tools developed here can be incorporated in models where these other channels are active, and our contribution will help to shed light on their interactions.

From an Arrow-Debreu point of view, there is really no difference between intertemporal trade and intratemporal trade. So, from this point of view, one only needs to relabel goods by time period and the same approach used to study static terms-of-trade manipulation can be used to analyze dynamic terms-of-trade manipulation. Our paper builds on this simple idea, exploiting the time-separable structure of preferences typically used in macro applications.

The only precedent that we know of our approach to intertemporal trade policy is the textbook treatment of a two-period one-good model in Obstfeld and Rogoff (1996). Expanding the analysis to many periods and many goods allows us to develop two broad ideas. The first is that optimal capital controls in a given period are not guided by the absolute desire to alter the intertemporal price of the goods produced in that period, but rather by the relative strength of this desire between two consecutive periods. If a country is a net seller of goods dated t and $t+1$ in equal amounts, and faces equal elasticities in both periods, there is no incentive for the country to distort the saving decisions of its consumers at date t. It is the time variation in the incentive to distort intertemporal prices that leads to non-zero capital controls.

We develop this first idea in the context of a two country, infinite horizon, endowment economy with two goods. In this model the only relative prices are real interest rates. We look at the optimal unilateral policy chosen by the home country, assuming that the foreign country is passive. In this economy, our principle yields sharp implications for the direction of optimal capital flow taxes. In particular, it is optimal for Home to tax capital inflows (or subsidize capital outflows) in periods in which the home endowment is growing faster than the rest of the world and to tax capital outflows (or subsidize capital inflows) in periods in which it is growing more slowly. Accordingly, if relative endowments converge to a steady state, then taxes on international capital flows converge to zero.

The intuition for this result is as follows. Consider Home's incentives to distort domestic consumption in each period. In periods of larger trade deficits, it has a stronger incentive, as a buyer, to distort prices downward by lowering domestic consumption. Similarly, in periods of larger trade surpluses, it has a stronger incentive, as a seller, to distort prices upward by raising domestic consumption. Since periods of faster growth at home tend to be associated with either lower future trade deficits or larger future trade surpluses, Home always has an incentive to raise future consumption relative to current consumption in such periods. This is exactly what taxes on capital inflows or subsidies on capital outflows accomplish through their effects on relative distortions across periods. Interestingly, although this mechanism emphasizes interest rate manipulation, the net financial position of Home is not the relevant object to look at to identify the periods in which capital controls are imposed.

The second idea that we emphasize is that the incentive to distort trade over time does not depend only on the overall growth of the country's output relative to the world, but also on its composition. In a multi good world in which countries have different preferences, a change in the time profile of consumption not only affects the interest rate but also the relative prices of consumption goods in each given period. This is an effect familiar from the literature on the transfer problem, which goes back to the debate between Keynes (1929) and Ohlin (1929). In our context this means that distorting its consumers' decision to allocate spending between different periods a country also affects its static terms of trade. Even if all static trade distortions are banned, say because of a trade agreement, intratemporal prices will not necessarily be at their undistorted levels if capital controls are allowed.

We develop this idea by extending our dynamic model to the case of many goods. We still assume that Home is choosing its policy unilaterally, but that it can only impose taxes/subsidies on capital flows and not good-specific taxes. However, the country takes into account static terms of trade effects in its optimization problem. In this model, we explore the role of compositional effects in driving the incentive to impose capital controls. We provide a general formula that identifies the role of the compositional effect and then focus on a simple example in which this effect can be derived analytically. In this situation, we show that our predictions on capital flow taxes in the one-good model survive if variations in the endowment growth are concentrated in the import-oriented sector. However, if these variations occur in the export-oriented sector and the intertemporal elasticity of substitution is low enough, the incentive to distort the real interest rate identified in the one-good model may be dominated by an opposing incentive to distort static terms of trade and the predictions can be reversed.

We conclude by considering the case of capital control wars in which both countries set capital controls optimally, taking as given the capital controls chosen by the other country. In equilibrium, far from cancelling each other out, the net distortion on capital flows is typically larger than in the unilateral case. In this situation, we show that results derived in the one-good case are affected in two ways. First, unlike in the case in which capital controls are set unilaterally, domestic consumption may decrease with domestic endowments, even though both countries would have liked domestic consumption to increase in the absence of policy response from the other country. Second, even if domestic consumption is increasing in domestic endowments, as in our earlier analysis, one can only relate periods of growth in one country to its taxes on capital flows relative to the other country, not their absolute level.

Our paper attacks an international macroeconomic question following a classical approach of the international trade literature and using tools from the dynamic public finance literature. In international macro, the closest literature is a growing theoretical literature demonstrating, among other things, how restrictions on international capital flows may be welfare-enhancing in the presence of various credit market imperfections; see e.g. Calvo and Mendoza (2000), Aoki, Benigno, and Kiyotaki (2010), Jeanne and Korinek (2010), and Martin and Taddei (2010). In addition to these second-best arguments, there also exists an older literature emphasizing the so-called "trilemma": one cannot have a fixed exchange rate, an independent monetary policy and free capital mobility; see e.g. McKinnon and Oates (1966), or more recently, Obstfeld, Shambaugh, and Taylor (2010). To the extent that a having fixed exchange and an independent monetary policy may be welfare-enhancing, such papers also offer a rationale for capital controls.

On the international trade side, the literature on optimal taxes in open economies is large and varied; see Dixit (1985) for an overview. However, the common starting point of most trade policy papers is that international trade is balanced. They therefore abstract from intertemporal considerations.[2] That being said, some situations which arise naturally in a dynamic setting, such as the convergence to a steady state, have no clear counterpart in a static setting. More importantly, the existing trade policy literature tends to focus on low-dimensional general equilibrium models, i.e., with only two goods. Thus there are no 'off-the-shelf' results that directly apply to the dynamic endowment econ-

[2] A notable exception is Bagwell and Staiger (1990), though their focus is on self-enforcing trade agreements. See Staiger (1995) for an overview of that literature.

omy considered in our paper.[3] Another benefit of our high-dimensional environment is that it allows us to study the interaction between intratemporal and intertemporal considerations and derive its implications for optimal capital controls.

Finally, in terms of methodology, we follow the public finance literature and use the primal approach to characterize first optimal wedges rather than explicit policy instruments, see e.g. Lucas and Stokey (1983). Since there are typically many ways to implement the optimal allocation in an intertemporal context, this approach will help us clarify the equivalence between capital controls and other policy instruments.

The rest of our paper is organized as follows. Section 2 describes a simple one-good economy. Section 3 characterizes the structure of optimal capital controls in this environment. Section 4 extends our results to the case of many goods. Section 5 considers the case of capital control wars. Section 6 offers some concluding remarks.

2 Basic Environment

2.1 A Dynamic Endowment Economy

There are two countries, Home and Foreign. Time is discrete and infinite, $t = 0, 1, ...$ and there is no uncertainty. The preferences of the representative home consumer are represented by the utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where c_t denotes consumption, u is an increasing, concave function, and $\beta \in (0, 1)$ is the discount factor. The preferences of the representative Foreign consumer have a similar form, with asterisks denoting foreign variables.

Both domestic and foreign consumers receive an endowment sequence denoted by $\{y_t\}$ and $\{y_t^*\}$, respectively. We make two simplifying assumptions: world endowments are fixed across periods, $y_t + y_t^* = Y$, and the home and foreign consumer have the same discount factor, $\beta = \beta^*$. Accordingly, in the absence of distortions, there should be perfect consumption smoothing across time in both countries.

We assume that both countries begin with zero assets at date 0. Let p_t be the price of a unit of consumption in period t on the world capital markets. In the absence of taxes, the

[3] With more than two goods, existing trade policy papers typically offer either: (i) partial equilibrium results under the assumption of quasi-linear preferences; (ii) sufficient conditions under which seemingly paradoxical results may arise (e.g. Feenstra, 1986; Itoh and Kiyono, 1987); or (iii) fairly weak restriction on the structure of optimal trade policy (Bond, 1990).

intertemporal budget constraint of the home consumer is

$$\sum_{t=0}^{\infty} p_t(c_t - y_t) \leq 0.$$

The budget constraint of the foreign consumer is the same expression with asterisks on c_t and y_t.

2.2 A Dynamic Monopolist

For most of the paper, we will focus on the case in which the home government sets capital flow taxes in order to maximize domestic welfare, assuming the foreign country is passive: it does not have any tax policy in place and does not respond to variations in Home's policy. We will look at the case where both countries set taxes strategically in Section 5.

In order to characterize Home's optimal policy, we follow the public finance literature and use the primal approach. That is, we approach the home government's optimal policy problem by studying a planning problem in which equilibrium quantities are chosen directly and address implementation issues later.

Formally, we assume that the home government's objective is to maximize the lifetime utility of the representative home consumer subject to (i) utility maximization by the foreign consumer at (undistorted) world prices p_t, and (ii) market clearing in each period. The foreign consumer first-order conditions are given by

$$\beta^t u^{*\prime}(c_t^*) = \lambda^* p_t, \tag{2}$$

$$\sum_{t=0}^{\infty} p_t(c_t^* - y_t^*) = 0, \tag{3}$$

where λ^* is the Lagrange multiplier on the foreign consumer's budget constraint. Moreover, goods market clearing requires

$$c_t + c_t^* = Y. \tag{4}$$

Combining equations (2)-(4), we can express Home's planning problem as

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{P}$$

subject to

$$\sum_{t=0}^{\infty} \beta^t u^{*\prime}(Y - c_t)(c_t - y_t) = 0. \tag{5}$$

Equation (5) is an implementability constraint, familiar from the optimal taxation literature. Note that given a sequence of domestic consumption, condition (5) is also sufficient to ensure the existence of a feasible, utility-maximizing consumption sequence for the Foreign country. The argument is constructive: given $\{c_t\}$, the proposed sequence $\{c_t^*\}$ is obtained from market clearing (4) and the sequence of prices is computed from (2), so that (3) is implied by (5), ensuring that the foreign consumer's sufficient conditions for optimality are met.

3 Optimal Capital Controls

3.1 Optimal Allocation

We first describe how home consumption $\{c_t\}$ fluctuates with home endowments $\{y_t\}$ along the optimal path. Next we will show how the optimal allocation can be implemented using taxes on international capital flows.

The first-order condition associated with Home's planning problem is given by

$$u'(c_t) = \mu \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t) \right], \tag{6}$$

where μ is the Lagrange multiplier on the implementability constraint. This condition immediately leads to our first observation. Although the entire sequence $\{y_t\}$ affects the level of current consumption through their effects on the Lagrange multiplier μ, we see that variations in current consumption c_t along the optimal path only depend on variations in the current value of y_t.

The next proposition shows that, whatever the shape of the utility functions u and u^*, there is a monotonic relationship between domestic consumption and domestic endowments along the optimal path.

Proposition 1 *(Procyclical consumption) For any two periods t and s, if the home endowment is larger in s, $y_s > y_t$, then home consumption is also higher, $c_s > c_t$.*

Figure 1 provides a graphical representation of Home's planning problem. On the x-axis we have domestic consumption c, which determines foreign consumption, $Y - c$, by market clearing. The decreasing blue curve labeled $u'(c)$ represents the marginal utility of

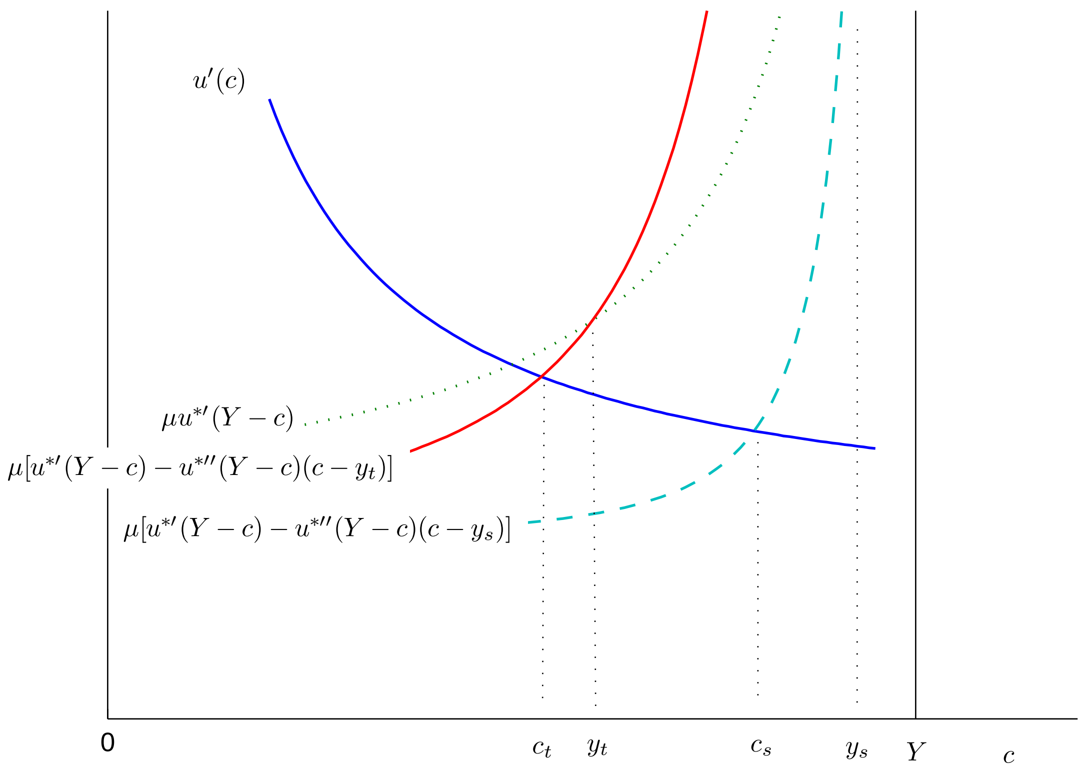

Figure 1: Consumption Allocation at the Optimal Home's Policy

the home consumer. The increasing red curve represents the marginal revenue associated to reducing home consumption by a small amount and thus (i) increasing net sales, and (ii) decreasing the intertemporal price of the t-dated good. The optimal consumption choice at t corresponds to the point where the two curves meet, that is, where marginal revenue is equalized with the marginal cost of reducing home consumption, which is just $u'(c)$. The same reasoning applies in period s, but with the marginal revenue represented by the light blue dashed curve.

Figure 1 gives an intuition for Proposition 1. As the endowment increases from y_t to y_s, the curve $u'(c)$ does not move. At the same time, the marginal revenue curve shifts down, as the price decrease associated to a reduction in c applies to a larger amount of inframarginal units sold. This induces Home to consume more, explaining why consumption is procyclical along the optimal path.

As a preliminary step in the analysis of optimal capital flow taxes, we conclude this section by describing how the "wedge" between the marginal utility of domestic and foreign consumption varies along the optimal path. Formally, define

$$\tau_t \equiv \frac{u'(c_t)}{\mu u^{*\prime}(c_t^*)} - 1. \tag{7}$$

By market clearing, we know that $c_t^* = Y - y_t$. Thus combining the definition of τ_t with the strict concavity of u and u^*, we obtain the following corollary to Proposition 1.

Corollary 1 *(Countercyclical wedges) For any two periods t and s, if the home endowment is larger in s, $y_s > y_t$, then the wedge is lower, $\tau_s < \tau_t$.*

The existing trade policy literature provides a useful way to interpret Corollary 1. By equations (6) and (7), we have

$$\tau_t = -\frac{u^{*\prime\prime}(Y - c_t)}{u^{*\prime}(Y - c_t)}(c_t - y_t). \tag{8}$$

Condition (8) is closely related to the well-known optimal tariff formula involving the elasticity of the foreign export supply curve in static trade models with two goods and/or quasi-linear preferences. This should not be too surprising since τ_t measures the difference between the marginal utility of domestic and foreign consumption. According to equation (8), the wedge τ_t is positive in periods of trade deficit and negative in periods of trade surplus. This captures the idea that if (time-varying) trade taxes were available, Home would like to tax imports if $c_t - y_t > 0$ and tax exports if $c_t - y_t < 0$. Corollary 1, however, goes beyond this simple observation by establishing a monotonic relationship between τ_t and y_t. This insight will play a key role in our analysis of optimal capital controls, to which we now turn.[4]

3.2 Optimal Taxes on International Capital Flows

It is well-known from the Ramsey taxation literature that there are typically many combinations of taxes that can implement the optimal allocation; see e.g. Chari and Kehoe (1999). Here, we focus on the tax instrument most directly related to world interest rate

[4]A natural question is whether the same logic implies a monotonic relationship between τ_t and net imports, $c_t - y_t$. Perhaps surprisingly, this is not necessarily the case. While the sign of the slopes of $u'(c)$ and $\mu u^{*\prime}(Y - c)$ is unambiguous (by concavity) the sign of the slope of the marginal return curve could in general be positive or negative. This opens up the possibility that a positive output shock may lead to an increase in import volumes, and so by Proposition 1, that an increase in import volumes may be accompanied by a decrease in τ_t along the optimal path.

manipulation: taxes on international capital flows.[5]

For expositional purposes, we assume that consumers can only trade one-period bonds on international capital markets, with Home imposing a proportional tax θ_t on the gross return on net asset position in international bond markets. Standard arguments show that any competitive equilibrium supported by intertemporal trading of consumption claims at date 0 can be supported by trading of one-period bonds. As we discuss later in Section 3.4, none of the results presented here depend on the assumption that one-period bonds are the only assets available.

With only one-period bonds, the per-period budget constraint of the Home consumer takes the form

$$q_t a_{t+1} + c_t = y_t + (1 - \theta_t)\, a_t - l_t, \tag{9}$$

where a_t denotes the current bond holdings, l_t is a lump sum tax, and $q_t \equiv p_{t+1}/p_t$ is the price of one-period bonds at date t. In addition, consumers are subject to a standard no-Ponzi condition, $\lim_{t\to\infty} p_t a_t \geq 0$. In this environment the Home consumer's Euler equation takes the form

$$u'(c_t) = \beta(1 - \theta_t)(1 + r_t)u'(c_{t+1}). \tag{10}$$

where $r_t \equiv 1/q_t - 1$ is the world interest rate. Given a solution $\{c_t\}$ to the Home's planning problem (P), the world interest rate is uniquely determined as

$$r_t = \frac{u^{*\prime}(Y - c_t)}{\beta u^{*\prime}(Y - c_{t+1})} - 1,$$

using equations (2) and (4). Thus, given $\{c_t\}$, we can use (10) to construct a unique sequence of taxes $\{\theta_t\}$. We can then set the sequence of assets positions and lump-sum transfers

$$\begin{aligned} a_t &= \sum_{s=t}^{\infty} (p_s/p_t)\,(c_s - y_s)\,, \\ l_t &= -\theta_t a_t, \end{aligned}$$

which ensure that the per-period budget constraint (9) and the no-Ponzi condition are satisfied. Since (9), (10), and the no-Ponzi condition are sufficient for optimality it follows that given prices and taxes $\{c_t\}$ is optimal for Home's representative consumer. This

[5]Other tax instruments that could be used to implement the optimal allocation include time-varying trade and consumption taxes (possibly accompanied by production taxes in more general environments). See Jeanne (2011) for a detailed discussion of the equivalence between capital controls and trade taxes.

establishes that any solution $\{c_t\}$ of (P) can be decentralized as a competitive equilibrium with taxes.

A positive θ_t can be interpreted as imposing simultaneously a tax θ_t on capital outflows and a subsidy θ_t to capital inflows. Obviously, since there is a representative consumer, only one of the two is active in equilibrium: the outflows tax if the country is a net lender, $a_t > 0$, and the inflows subsidy if it is a net borrower, $a_t < 0$. Similarly, a negative θ_t can be interpreted as a subsidy on capital outflows plus a tax on capital inflows. The bottom line is that $\theta_t > 0$ discourages domestic savings while $\theta_t < 0$ encourages them.

In the Foreign country, since there are no capital flow taxes, the Euler equation takes the form

$$u^{*\prime}(c_t^*) = \beta(1 + r_t)u'(c_{t+1}^*). \tag{11}$$

Combining the definition of the wedge (7) with the Home and the Foreign Euler equations (10) and (11), we obtain the following relationship between wedges and taxes on capital flows:

$$\theta_t = 1 - \frac{1 + \tau_t}{1 + \tau_{t+1}}. \tag{12}$$

The previous subsection has already established that variations in domestic consumption c_t along the optimal path are only a function of the current endowment y_t. Since τ_t is only a function of c_t, equation (12) implies that variations in θ_t are only a function of y_t and y_{t+1}. Combining equation (12) with Corollary 1, we then obtain the following result about the structure of optimal capital controls.

Proposition 2 *(Optimal capital flow taxes) Suppose that the optimal policy is implemented with capital flows taxes. Then the optimal tax schedule satisfies the following properties:*

1. *if $y_{t+1} > y_t$ tax capital inflows/subsidize capital outflows ($\theta_t < 0$);*
2. *if $y_{t+1} < y_t$ tax capital outflows/subsidize capital inflows ($\theta_t > 0$);*
3. *if $y_{t+1} = y_t$ do not distort capital flows ($\theta_t = 0$).*

Proposition 2 builds on the same logic as Proposition 1. Suppose, for instance, that Home is running a trade deficit in periods t and $t + 1$. In this case, the Home government wants to exercise its monopsony power by lowering domestic consumption in both periods. But if Home grows between these two periods, $y_{t+1} > y_t$, the number of units imported from abroad is lower in period $t + 1$. Thus the Home government has less incentive to lower consumption in that period. This explains why a tax on capital inflows is optimal in period t: it reduces borrowing in period t, thereby shifting consumption from period t to period $t+1$. The other results follow a similar logic. Note that, although the only motive for capital controls in our model is interest rate manipulation, the net financial position of Home in any particular period, debtor or creditor, is not the relevant variable to look at to sign the optimal direction of the tax. This is because the effect of a capital flow tax is to affect the relative distortion in consumption decisions between two consecutive periods. Therefore, what matters is whether the monopolistic incentives to restrict domestic consumption are stronger in period t or $t+1$. In our simple endowment economy, these incentives are purely captured by the growth rate of the endowment, but the general principle extends beyond our model.

Proposition 2 has a number of interesting implications. Consider first an economy that is catching up with the rest of the world in the sense that $y_{t+1} > y_t$ for all t and is thus borrowing. According to our analysis, it is optimal for this country to tax capital inflows and to subsidize capital outflows in order to encourage domestic savings. The basic intuition is that a growing country will export more tomorrow than today. Thus it has more incentive to increase export prices in the future, which it can achieve by raising future consumption through a subsidy on capital outflows. Put simply, for an economy catching up with the rest of the world, larger benefits from future terms-of-trade manipulation lead to foreign reserve accumulation.

Consider instead a country that at time t borrows from abroad in anticipation of a temporary boom. In particular, suppose that $y_{t+1} > y_t$ and $y_s = y_t$ for all $s > t+1$. In this situation, the logic of Proposition 2 implies that, at time t, at the onset of the boom, it is optimal to impose restrictions on short-term capital inflows, i.e., to tax bonds with one-period maturity and leave long-term capital inflows unrestricted.[6] This example provides a different perspective on why governments may try to alter the composition of capital flows in favor of longer maturity flows in practice (e.g. Magud, Reinhart, and Rogoff (2011). In our model, incentives to alter the composition of capital flows do not come from the fear of "hot money" but from larger benefits of terms-of-trade manipulation in the short run.

Finally, Proposition 2 has sharp implications for the structure of optimal capital controls in the long-run.

Corollary 2 *(No tax in steady state) In the long run, if endowments converge to a steady state,* $y_t \to y$*, then taxes on international capital flows converge to zero,* $\theta_t \to 0$*.*

Corollary 2 is reminiscent of the Chamley-Judd result (Judd, 1985; Chamley, 1986) of zero capital income tax in the long-run. Intuitively, Home would like to use its monopoly

[6]The tax on two period bonds is easily shown to be $(1-\theta_t)(1-\theta_t+1)-1$ and Proposition 1 implies that it's zero in our example.

power to influence intertemporal prices to favor the present value of its income. However, at a steady state all periods are symmetric, so it is not optimal to manipulate relative prices. Note that a steady state may be reached with trade balance, trade deficit or trade surplus. Which of these cases applies depends on the entire sequence $\{y_t\}$. Our analysis demonstrates that taxes on international capital flows are unaffected by these long-run relative wealth dynamics. For instance, even if Home, say, becomes heavily indebted, it is not optimal to lower long run interest rates. In our model, even away from a steady state, taxes on international capital flows are determined by the endowments at t and $t+1$ only.

3.3 An Example with CRRA Utility and Aggregate Fluctuations

Up to now we have focused on the case of a fixed world endowment. Thus we have looked at how optimal capital controls respond to a reallocation of resources between countries, keeping the total pie fixed. This provides a useful benchmark in which all fluctuations in consumption reflect Home's incentives to manipulate the world interest rate. Here we show that if domestic and foreign consumers have identical CRRA utility functions, then our results extend to economies with aggregate fluctuations. We also take advantage of this example for a simple exploration of the magnitudes involved in terms of quantities and welfare.

Our characterization of Home's optimal policy extends immediately to the case of a time-varying world endowment: just replace Y with Y_t in equation (6). Under the assumption of identical CRRA utility functions, $u(c) = u^*(c) = c^{1-\gamma}/\,(1-\gamma)$ with $\gamma \geq 0$, we then obtain a simple relationship between the home share of world endowments, y_t/Y_t and the home share of world consumption, c_t/Y_t:

$$\left(\frac{c_t/Y_t}{1-c_t/Y_t}\right)^{-\gamma} = \mu\left[1+\gamma\left(\frac{c_t/Y_t - y_t/Y_t}{1-c_t/Y_t}\right)\right].$$

The left-hand side is decreasing in c_t/Y_t, whereas the right-hand side is increasing in c_t/Y_t and decreasing in y_t/Y_t. Thus the implicit function theorem implies that, along the optimal path, the Home share of world consumption, c/Y, is strictly increasing in the Home share of world endowments, y/Y. Put simply, if utility functions are CRRA, Proposition 1 generalizes to environments with aggregate fluctuations.

Now consider the wedge τ_t between the marginal utility of domestic and foreign consumption in period t. Under the assumption of CRRA utility functions we have

$$\tau_t = \frac{1}{\mu}\left(\frac{c_t/Y_t}{1-c_t/Y_t}\right)^{-\gamma} - 1.$$

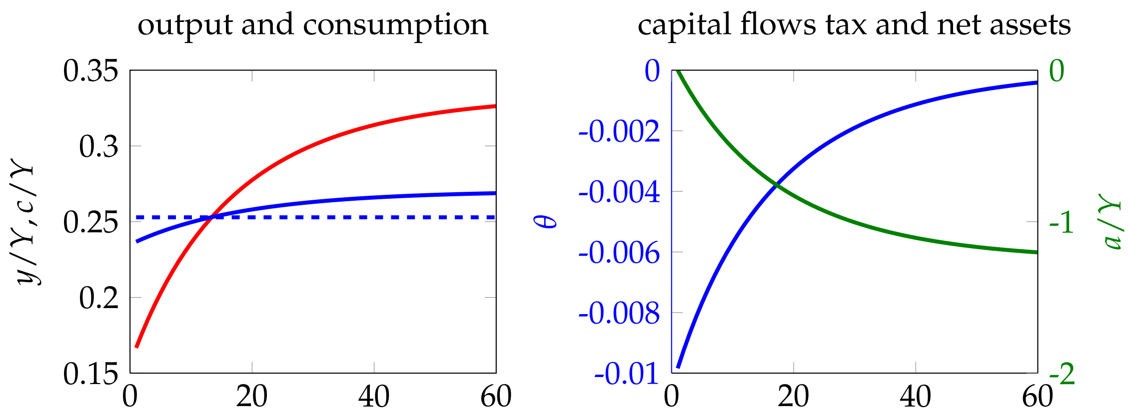

Figure 2: Optimal Allocation and Taxes for a Country Catching Up

Note: In the left panel, the red line is the exogenous path for the endowment, the blue line is consumption at the Home's optimal policy, and the dashed line is the efficient no-tax benchmark. In the right panel, the blue line is the capital flow tax and the green line the home assets-to-world-GDP ratio.

According to this expression, if c/Y is strictly increasing in y/Y along the optimal path, then τ is strictly decreasing. The same logic as in Section 3.2 therefore implies that optimal taxes on capital flows must be such that $\theta_t < 0$ if and only if $y_{t+1}/Y_{t+1} > y_t/Y_t$. In other words, if utility functions are CRRA, Proposition 2 also generalizes to environments with aggregate fluctuations.

As a quantitative illustration of our theory of optimal capital controls, suppose that foreign endowments $\{y_t^*\}$ are growing at the constant rate $g = 3\%$ per year and that Home is catching up with the rest of the world. To be more specific, suppose that Home's endowment is 20% of world endowments at date 0 and that it is converging towards being 50% in the long run, with the ratio y_t/y_t^* converging to its long run value at a constant speed $\eta = 0.05$.[7]

Figure 3.3 shows the path of Home's share world endowments and consumption, assuming a unit elasticity of intertemporal substitution, $\gamma = 1$. For comparison, we also plot the path for consumption and for the trade balance in the benchmark case with no capital controls. In this case, consistently with consumption smoothing, Home consumes a fixed fraction of the world endowment in all periods. Although optimal capital controls reduce consumption smoothing, intertemporal trade flows remain several times larger than domestic output. The optimal tax on capital inflows is less than 1% at date 0 and vanishing

[7]That is, we assume that

$$y_t/y_t^* - a = (y_0/y_0^* - a)\, e^{-\eta t},$$

for some $a = 0.5/0.5 = 1 > y_0/y_0^* = 0.2/0.8 = 0.25$.

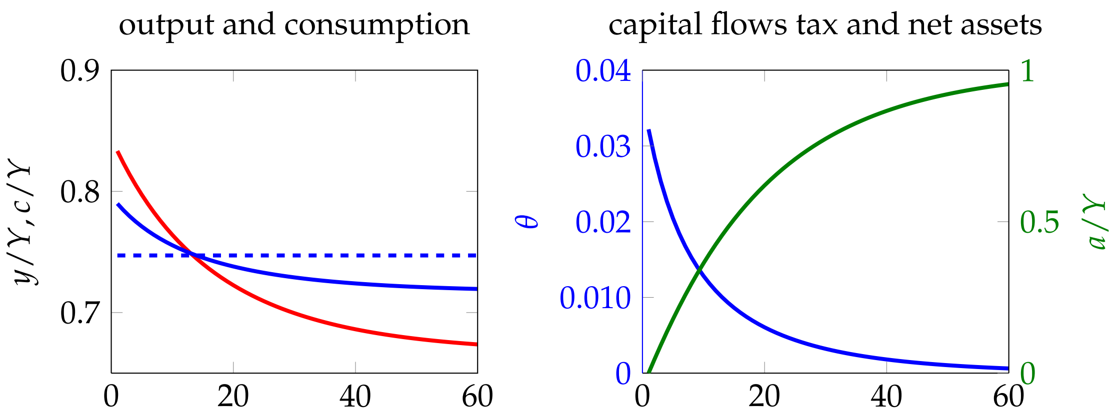

Figure 3: Optimal Allocation and Taxes for the Foreign

Note: In the left panel, the red line is the exogenous path for the endowment, the blue line is consumption at the Foreign's optimal policy, and the dashed line is the efficient no-tax benchmark. In the right panel, the blue line is the capital flow tax and the green line the foreign assets-to-world-GDP ratio.

in the long run, following the same logic as for Corollary 2. Perhaps surprisingly, the optimal tax on capital inflows decreases as the value of Home's debt increases. Compared to the benchmark with no capital controls, optimal taxes are associated with an increase in domestic consumption of 0.12% and a decrease in foreign consumption of 0.07%. Though the welfare impact of optimal capital controls is admittedly not large in this particular example, it is not much smaller than either the estimated gains of international trade or financial integration.[8]

Figure 3.3 considers the same exogenous path for endowments, but assumes that it is the foreign country, instead of the home country, that is imposing capital flow taxes optimally. Formally, Home's share of world endowments is now 80% at date 0 and is converging towards being 50% in the long-run. In this situation, we see that optimal taxes on international capital flows are about three times as large as in the previous case, reflecting the greater size of the rest of the world, and hence its greater ability to manipulate world interest rates. The impact on welfare is larger as well. In this second example, domestic consumption increases by 0.27% compared to the benchmark with no capital controls, whereas foreign consumption decreases by 1.27%. With higher values of γ, i.e. lower intertemporal elasticity of substitution, we obtain similar though slightly higher numbers.

[8] According to a fairly large class of trade models, the current welfare gains from international trade in the United States are between 0.7% and 1.4% of real GDP; see Arkolakis, Costinot, and Rodríguez-Clare (2009). Similarly, the welfare gain from switching from financial autarky to perfect capital mobility is roughly equivalent to a 1% permanent increase in consumption for the typical non-OECD country; see Gourinchas and Jeanne (2006).

For instance, for $\gamma = 2$, the increase in domestic consumption is equal to 0.48% and the decrease in foreign consumption to 2.5%.[9]

3.4 Initial Assets, Debt Maturity, and Time-Consistency

So far, we have focused on environments in which: (i) there are no initial assets at date 0 and (ii) one-period bonds are the only assets available. We now briefly discuss how relaxing both assumptions affects our results. We also show that if more debt instruments are available, the optimal allocation is time-consistent: a future government free to choose future consumption, but forced to fulfill previous debt obligations would not want to deviate from the consumption path chosen by its predecessors.

Let $a_{t,s}$ represents holdings at time t of bonds maturing at time s. Suppose the home consumer enters date 0 with initial asset holdings $\{a_{0,t}\}_{t=0}^{\infty}$. The asset holdings now enter the intertemporal budget constraints of the home and foreign consumer. In particular, the home budget constraint generalizes to

$$\sum_{t=0}^{\infty} p_t(c_t^* - y_t^* - a_{0,t}^*) = 0.$$

The other equilibrium conditions are unchanged, so Home's planning problem becomes

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{P_0}$$

subject to

$$\sum_{t=0}^{\infty} \beta^t u^{*\prime}(Y - c_t)(c_t - y_t + a_{0,t}^*) = 0. \tag{13}$$

Compared to the case without initial assets, the only difference is the new implementability constraint (13), which depends on $\{y_t - a_{0,t}^*\}$ rather than $\{y_t\}$. Accordingly, Proposition 1 and Corollary 1 simply generalize to environments with initial assets $\{a_{0,t}^*\}_{t=0}^{\infty}$ provided that they are restated in terms of changes in $y_t - a_{0,t}^*$ rather than changes in y_t.

Throughout our analysis, we assumed that Home can freely commit at date 0 to a consumption path $\{c_t\}$. Now that we have recognized the role of the initial asset positions, this assumption may seem uncomfortably restrictive. After all, along the optimal path, the debt obligations $\{a_{t,s}\}_{s=t}^{\infty}$ held at date t will typically be different from the obliga-

[9] In general, changing γ has non-monotonic effects on the welfare impact of optimal capital controls. Intuitively, since Home and Foreign have the same intertemporal elasticity of substitution, changing γ affects both the marginal revenue and the marginal cost of a dynamic monopolist, as described in Figure 1.

tions $\{a^*_{0,s}\}_{s=t}^{\infty}$ held at date 0. Accordingly, a government at later dates may benefit from deviating from the consumption chosen at date 0.

We now demonstrate that this is not the case if the government has access to bonds of arbitrary maturity. The basic idea builds on the original insight of Lucas and Stokey (1983). At any date t, the foreign consumer is indifferent between many future asset holdings $\{a^*_{t+1,s}\}_{s=t+1}^{\infty}$. Given a consumption sequence $\{c^*_t\}$ that maximizes her utility subject to her budget constraint, she is indifferent between any bond holdings satisfying

$$\sum_{s=t+1}^{\infty} p_s(c^*_s - y^*_s - a^*_{t+1,s}) = 0. \tag{14}$$

As we formally establish in the appendix, this degree of freedom is sufficient for the home government at date 0 to construct sequences of debt obligations $\{a^*_{t,s}\}_{s=t}^{\infty}$ for all $t \geq 1$ such that the solution of

$$\max_{\{c_s\}} \sum_{s=t}^{\infty} \beta^s u(c_s)$$

subject to

$$\sum_{s=t}^{\infty} \beta^s u^{*\prime}(Y - c_s)(c_s - y_s + a^*_{t,s}) = 0 \tag{15}$$

coincides with the solution of (P_0) at all dates $t \geq 0$. In short, if Home and Foreign can enter debt commitments at all maturities, the optimal allocation derived in Section 3.1 is time-consistent.

4 Intertemporal and Intratemporal Trade

How do the incentives to tax capital flows change in a world with many goods? In a one good economy, the only form of term of trade manipulation achieved by taxing capital flows is to manipulate the world interest rate. In a world with many goods, distorting the borrowing and lending decisions of domestic consumers also affects the relative prices of the different goods traded each period. In this section, we explore how these intratemporal relative price considerations change optimal capital flow distortions.

In order to maintain the focus of our analysis on optimal capital controls, we proceed under the assumption that Home is constrained by an international free-trade agreement, so good specific taxes/subsidies are not permitted. As in the previous section, Home is still allowed to impose taxes on capital flows that distort intertemporal decisions. This means that while the government cannot control the path of consumption of each specific good i, it can still control the path of aggregate consumption. As we shall see, in general, the path of aggregate consumption can affect relative prices at any point in time, thus creating additional room for terms of trade manipulation.

4.1 The Monopolist Problem Revisited

The basic environment is the same as in Section 2.1, except that there are $n > 1$ goods. Thus domestic consumption and output, c_t and y_t, are now vectors in R_+^n. We assume that the domestic consumer has homothetic preferences represented by

$$\sum_{t=0}^{\infty} \beta^t U(C_t), \tag{16}$$

where U is increasing and strictly concave, $C_t \equiv g(c_t)$ is aggregate domestic consumption at date t, and g is increasing, concave, and homogeneous of degree one. Analogous definitions apply to U^* and $C_t^* \equiv g^*(c_t^*)$.

In the absence of taxes, the intertemporal budget constraint of the home representative consumer is now given by

$$\sum_{t=0}^{\infty} p_t \cdot (c_t - y_t) \leq 0,$$

where $p_t \in R_+^n$ denotes the intertemporal price vector for period-t goods and $\cdot$ is the inner product. A similar budget constraint applies in Foreign.

As in Section 2.2, we use the primal approach to characterize Home's optimal policy. In this new environment, the home government's objective is to set consumption $\{c_t\}$ in order to maximize the lifetime utility of its representative consumer, given by (16), subject to (i) utility maximization by the foreign representative consumer at (undistorted) world prices p_t; (ii) market clearing in each period; and (iii) a free trade agreement that rules out good specific taxes or subsidies.

Constraint (i) can be dealt with as we did in the one-good case. In vector notation, the first-order conditions associated utility maximization by the foreign consumer generalize to

$$\beta^t U^{*\prime}(C_t^*)\, g_c^*(c_t^*) = \lambda^* p_t, \tag{17}$$

$$\sum_{t=0}^{\infty} p_t \cdot (c_t^* - y_t^*) = 0. \tag{18}$$

Next, note that if Home cannot impose good specific taxes or subsidies, the relative price of any two goods i and j in period t, p_{it}/p_{jt}, must be equal in the two countries and equal to the marginal rates of substitution $g_i(c_t)/g_j(c_t)$ and $g_i^*(c_t^*)/g_j^*(c_t^*)$. Accordingly, the consumption allocation (c_t, c_t^*) in period t is Pareto efficient and solves

$$C^* (C_t) = \max_{c,c^*} \{g^*(c^*) \qquad \text{subject to} \quad c + c^* = Y \text{ and } g(c) \geq C_t\} \tag{19}$$

for some C_t. Therefore, constraints (ii) and (iii), can be captured by letting Home choose an aggregate consumption level C_t, which identifies a point on the static Pareto frontier. The consumption vectors at time t are then given by the corresponding solutions to problem (19), which we denote by $c(C_t)$ and $c^*(C_t)$.

We can then state Home's planning problem in the case of many goods as

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{P'}$$

subject to

$$\sum_{t=0}^{\infty} \beta^t \rho(C_t) \cdot (c(C_t) - y_t) = 0, \tag{20}$$

where $\rho(C_t) \equiv U^{*\prime}(C^*(C_t))\, g_c^*(c^*(C_t))$ and equation (20) is the counterpart of the implementability constraint in Section 2.2.

4.2 Optimal Allocation

With many goods, the first-order condition associated with Home's planning problem generalizes to

$$U'(C_t) = \mu \left[\rho(C_t) \cdot \frac{\partial c(C_t)}{\partial C_t} + \frac{\partial \rho(C_t)}{\partial C_t} \cdot (c(C_t) - y_t) \right], \tag{21}$$

where μ still denotes the Lagrange multiplier on the implementability constraint. Armed with condition (21), we can now follow the same strategy as in the one-good case. First we will characterize how $\{C_t\}$ covaries with $\{y_t\}$ along the optimal path. Second we will derive the associated implications for the structure of optimal capital controls.

The next proposition describes the relationship between domestic consumption and domestic endowments along the optimal path.

Proposition 3 *Suppose that between periods t and $t+1$ there is a small change in Home endowments $dy_{t+1} = y_{t+1} - y_t$. Then domestic consumption is higher in period $t+1$, $C_{t+1} > C_t$, if and only if $\partial \rho(C_t)/\partial C_t \cdot dy_{t+1} > 0$.*

In the one good case, $\partial \rho(C_t)/\partial C_t$ simplifies to $-u^{*\prime\prime}(Y - c_t)$, which is positive by the concavity of u^*. Therefore, whether domestic consumption grows or not only depends on whether the level of domestic endowments is increasing or decreasing. In the multi-good case, by contrast, this also depends on the composition of domestic endowments and on how relative prices respond to changes in C_t.

In order to highlight the importance of these compositional effects, in an economy with many goods, consider the effect of a small change in domestic endowment that leaves its market value unchanged at period t prices. That is, suppose $\rho(C_t) \cdot dy_{t+1} = 0$. In the one good case this can only happen if the endowment level does not change, thereby leading to a zero capital flows tax. In the multi-good case this is no longer the case. According to Proposition 3, consumption would grow if and only if

$$\mathrm{Cov}\left(\frac{\rho_i'(C_t)}{\rho_i(C_t)}, \rho_i(C_t) dy_{it+1}\right) > 0.$$

Here, what matters is whether the composition of endowments tilts towards goods that are more or less price sensitive to changes in C_t. We come back to the role of this compositional effects in more detail in Section 4.4.

4.3 Optimal Taxes on International Capital Flows

In line with Section 3.2, we again assume that consumers can only trade one-period bonds on international capital markets. But compared to Section 3.2, there is now one bond for each good. Since Home cannot impose good specific taxes/subsidies, it must impose the same proportional tax θ_t on the gross return on net lending in all bond markets. So the per period budget constraint of the domestic consumer takes the form

$$p_{t+1} \cdot a_{t+1} + p_t \cdot c_t = p_t \cdot y_t + (1 - \theta_t)(p_t \cdot a_t) - l_t,$$

where $a_t \in R_+^n$ now denotes the vector of current asset positions and l_t is a lump sum tax. As before, the domestic consumer is subject to the no-Ponzi condition, $\lim_{t\to\infty} p_t \cdot a_t \geq 0$. The first-order conditions associated with utility maximization in Home are given by

$$U'(C_t)\, g_i(c_t) = \beta(1-\theta_t)(1+r_{it})U'(C_{t+1})g_i(c_{t+1}), \text{ for all } i = 1, ..., n.$$

where $r_{it} \equiv p_{it}/p_{it+1} - 1$ is a good-specific interest rate. Let $P_t \equiv \min_c \{p_t \cdot c : g(c) \geq 1\}$ denote Home's consumer price index at date t. Using this notation, the previous conditions can be rearranged in a more compact form as

$$U'(C_t) = \beta(1-\theta_t)(1+R_t)U'(C_{t+1}), \tag{22}$$

where $R_t \equiv P_t/P_{t+1} - 1$ is Home's real interest rate at date t.[10] Since there are no taxes abroad, the same logic implies

$$U^{*\prime}(C_t^*) = \beta(1+R_t^*)U^{*\prime}(C_{t+1}^*), \tag{23}$$

where $R_t^* \equiv P_t^*/P_{t+1}^* - 1$ is Foreign's real interest rate at date t. Equations (22) and (23) are the counterparts of the Euler equations (10) and (11) in the one-good case. Combining these two expressions we obtain

$$\theta_t = 1 - \frac{U'(C_t)}{U^{*\prime}(C_t^*)}\frac{U^{*\prime}(C_{t+1}^*)}{U'(C_{t+1})}\frac{(1+R_t^*)}{(1+R_t)}.$$

If we follow the same approach as in the one-good case and let $\tau_t \equiv U'(C_t)/\mu U^{*\prime}(C_t^*) - 1$ denote the wedge between the marginal utility of domestic and foreign consumption, we can rearrange Home's tax on international capital flows as

$$\theta_t = 1 - \left(\frac{1+\tau_t}{1+\tau_{t+1}}\right)\left(\frac{P_{t+1}/P_{t+1}^*}{P_t/P_t^*}\right).$$

With many goods, the sign of θ_t depends on (i) whether the wedge τ_t between the marginal utility of domestic and foreign consumption is increasing or decreasing and (ii) whether Home's real exchange rate, P_t/P_t^*, appreciates or depreciates between t and $t+1$. Like in the one-good case, one can check that the wedge is a decreasing function of Home aggregate consumption C_t. In the next proposition we further demonstrate that an increase in C_t is always associated with an appreciation of Home's real exchange rate. Combining these two observations with Proposition 3, we obtain the following result.

Proposition 4 *Suppose that the optimal policy is implemented with capital flows taxes and that between periods t and $t+1$ there is a small change in Home endowments $dy_{t+1} = y_{t+1} - y_t$. Then it is optimal:*

1. *to tax capital inflows/subsidize capital outflows ($\theta_t < 0$) if $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1} > 0$;*

2. *to tax capital outflows/subsidize capital inflows ($\theta_t > 0$) if $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1} < 0$;*

[10] To see this formally, let us denote $c_t(1) \equiv \arg\min_c \{p_t \cdot c : g(c) \geq 1\}$. The associated first-order conditions are given by (i) $p_i = \lambda g_i[c_t(1)]$ and (ii) $g[c_t(1)] = 1$. This implies

$$P_t = \sum_i p_i c_{it}(1) = \lambda \sum_i g_i[c_t(1)]\, c_{it}(1) = \lambda g[c_t(1)] = \lambda,$$

where the third equality uses the fact that g is homogeneous of degree one. Combining this equality with condition (i), we obtain $P_t = p_i/g_i[c_t(1)]$. Since g_i is homogeneous of degree zero, this further implies $P_t = p_i/g_i(c_t)$ for all $i = 1, ..., n$.

3. *not to distort capital flows ($\theta_t = 0$) if $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1} = 0$.*

In order to understand better how intertemporal and intratemporal considerations affect the structure of the optimal tax schedule, let us decompose the price vector in period t into an intertemporal price and an intratemporal vector of relative prices: $p_t = P_t^* \pi_t$, where $\pi_{it} \equiv p_{it}/P_t^*$ denotes the price of good i in terms of Foreign consumption at date t.

Using the previous decomposition, we see that the sign of the expression $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1}$ in Proposition 4 is the same as the sign of the following expression[11]

$$\frac{P^{*\prime}(C_t)}{P^*(C_t)} \sum_i \pi_i(C_t)\, dy_{it+1} + \sum_i \frac{\pi_i'(C_t)}{\pi_i(C_t)} \pi_i(C_t)\, dy_{it+1}. \tag{24}$$

The first term captures the intertemporal price channel and is proportional to the change in the value of output. It is possible to show that $P^{*\prime}(C_t) > 0$. Thus an increase in the value of home output—all else equal—pushes in the direction of a tax on capital inflows/subsidy to capital outflows. This follows the same logic as in the one-good case.

The new element is the second term in (24), which captures intratemporal terms-of-trade effects. The sign of this term depends on the elasticity of relative prices to changes in domestic consumption. To sign this term we need to know more about preferences. The simplest case is the case of symmetric preferences, in which g and g^* are the same. In that case, the Pareto set in the Edgeworth box is a straight line and relative prices are independent of the point we choose (i.e., of C_t). Not surprisingly, in this case the analysis boils down to the one-good case. Therefore, the interesting case is the case of asymmetric preferences, which we now turn to.

4.4 An Example with CRRA and Asymmetric Cobb-Douglas Utility

In this subsection we focus on a simple example in which the effects of intratemporal considerations can be captured analytically. There are two goods. The upper-level utility function is CRRA and the lower-level utility is Cobb-Douglas:

$$U(C) = \frac{1}{1-\gamma} C^{1-\gamma}, C = c_1^{\alpha} c_2^{1-\alpha}, \tag{25}$$

[11] Just notice that

$$\rho_t = \lambda^* \beta^{-t} p_t = \lambda^* \beta^{-t} P_t^* \pi_t,$$

from the optimality condition of Foreign's consumers and so

$$\frac{\rho_i'(C_t)}{\rho_i(C_t)} = \frac{P^{*\prime}(C_t)}{P^*(C_t)} + \frac{\pi_i'(C_t)}{\pi_i(C_t)}.$$

where $\gamma \geq 0$ and $\alpha > 1/2$. Utility functions in Foreign take the same form, but the roles of goods 1 and 2 are reversed

$$U^* \left(C^*\right) = \frac{1}{1-\gamma} \left(C^*\right)^{1-\gamma}, C^* = \left(c_2^*\right)^{\alpha} \left(c_1^*\right)^{1-\alpha}. \tag{26}$$

Since $\alpha > 1/2$, Home has a higher relative demand for good 1 in all periods. Without risk of confusion, we now refer to good 1 and good 2 as Home's "import-oriented" and "export-oriented" sectors, respectively. The next proposition highlights how this distinction plays a key role in linking intertemporal and intratemporal terms-of-trade motives.[12]

Proposition 5 *Suppose that equations (25)-(26) hold with $\gamma \geq 0$ and $\alpha > 1/2$ and that between periods t and $t+1$ there is a small change in Home endowments $dy_{t+1} = y_{t+1} - y_t$. If growth is import-oriented, $dy_{1t+1} > 0$ and $dy_{2t+1} = 0$, it is optimal to tax capital inflows/subsidize capital outflows ($\theta_t < 0$). Conversely, if growth is export-oriented, $dy_{1t} = 0$ and $dy_{2t+1} > 0$, it is optimal to tax capital inflows/subsidize capital outflows ($\theta_t < 0$) if and only if $\gamma > \left(\frac{2\alpha-1}{\alpha}\right)\left(\frac{P_t^* C_t^*}{P_t^* C_t^* + P_t C_t}\right)$.*

The idea behind the first part of Proposition 5 is closely related to Proposition 2. In periods in which Home controls a larger fraction of the world endowment of good 1, the incentive to subsidize consumption C increases. Here, however, the reason is twofold. First, a larger endowment of good 1 means that Home is running a smaller trade deficit (or a bigger trade surplus), which reduces the incentive to depress the intertemporal price P^*. Second, it means that within the period the country is selling more of good 1. Since Home's preferences are biased towards good 1, an increase in C drives up the price of good 1, which further increases the incentives to subsidize consumption.

By contrast, when endowment growth is export-oriented, intertemporal and intratemporal considerations are not aligned anymore. If the elasticity of intertemporal substitution, $1/\gamma$, is low enough, the intertemporal motive for terms-of-trade manipulation dominates and we get the same result as in the one good economy. If instead that elasticity is high enough, the result goes in the opposite direction. Namely, it is possible that when Home receives a larger endowment of good 2, it decides to subsidize aggregate consumption less, even though the increase in y_2 is reducing its trade deficit. Intuitively, Home now benefits from reducing its own consumption since this increases the price of

[12] Another simple example that can be solved analytically is the case of tradable and non-tradable goods. If there only is one tradable good, then Proposition 2 applies unchanged to changes in the endowment of the tradable good. The only difference between this case and the one-good case studied in Section 3 is that taxes on capital inflows/subsidies on capital outflows ($\theta_t < 0$) now are always accompanied by a real exchange rate appreciation, whereas taxes on capital outflows/subsidies on capital inflows ($\theta_t > 0$) now are always accompanied by a real exchange rate depreciation

good 2 due again to the fact that, relative to Foreign's, Home's preferences are biased towards good 1. Proposition 5 formally demonstrates that the intratemporal terms-of-trade motive is more likely to dominate the intertemporal one if demand differences between countries are large and/or Foreign accounts for a large share of world consumption.

In order to illustrate the quantitative importance of this effect, we return to the exercise presented in Section 3.3 in which Home is catching up with the rest of the world. For simplicity, the world endowments of both goods are assumed to be constant over time. In the first panel of Figure XXX, the intertemporal elasticity of substitution is set to unity, $\gamma = 1$, and demand differences are set such that $\alpha = 3/4$. The blue curve represents the optimal tax on capital inflows in the import-oriented scenario: Home's endowment of good 2 is fixed, but its endowment of good 1 is 20% of world endowments at date 0 and is converging towards being 50% in the long run, with the ratio y_{1t}/y^*_{1t} converging to its long run value at a constant speed $\eta = 0.05$. The green curve instead represents the optimal tax on capital inflows in the export-oriented scenario: Home's endowment of good 1 is fixed, but its endowment of good 2 is growing. In order to make the two scenarios comparable, the growth rate of good 2's endowments is chosen such that the share of Home's world income in all periods is the same as in the import-oriented scenario. In all periods we see that the optimal tax on capital inflows is lower in the export-oriented scenario. While taxes converge to zero under both scenarios, the tax on capital inflows at date 0 is four times larger in the import-oriented scenario than in the export-oriented one: 1.6% versus 0.4%. In the second panel of Figure XXX, we repeat the same experiments under the assumption that $\gamma = 0.33$. In this situation, the intratemporal terms-of-trade motives now dominate the intertemporal ones under the export-oriented scenario. When Home's endowments grow, but growth is concentrated in sector 2, Home finds it optimal to subsidize rather than tax capital inflows. At date 0, the optimal *subsidy* on capital inflows is now around 0.4%.

5 Capital Control Wars

In this section we go back to the one-good case, but consider the case in which both countries set capital controls optimally, taking as given the capital controls chosen by the other country. As before, we assume that consumers can only trade one-period bonds on international capital markets, but we now let both Home and Foreign impose proportional taxes θ_t and θ^*_t, respectively, on the gross return on net asset position in international bond markets. At date 0, we assume that the two countries simultaneously choose the sequences $\{\theta_t\}$ and $\{\theta^*_t\}$, and commit to them.

5.1 Nash Equilibrium

We look for a Nash equilibrium, so we look at each country's optimization problem taking the other country's tax sequence as given. Focusing on Home's government problem, the optimal taxes can be characterized in terms of a planner problem involving directly the quantities consumed, as in the case of unilateral policy. Given the sequence $\{\theta_t^*\}$ the foreign consumer's Euler equation can be written as

$$u^{*\prime}(c_t^*) = \beta(1-\theta_t^*)(1+r_t)u^{*\prime}(c_{t+1}^*).$$

Since $1 + r_t = p_t/p_{t+1}$, a standard iterative argument then implies

$$p_t = \beta^t \left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\left[p_0 u^{*\prime}(c_t^*)/u^{*\prime}(c_0^*)\right]. \tag{27}$$

Accordingly, Home now maximizes (1) subject to the implementability constraint

$$\sum_{t=0}^{\infty} \beta^t u^{*\prime}(Y-c_t)\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right](c_t - y_t) = 0.$$

This yields the optimality condition

$$u'(c_t) = \mu\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\left[u^{*\prime}(c_t^*) - u^{*\prime\prime}(c_t^*)(c_t - y_t)\right], \tag{28}$$

which further implies

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{1}{(1-\theta_t^*)}\frac{u^{*\prime}(c_t^*) - u^{*\prime\prime}(c_t^*)(c_t - y_t)}{u^{*\prime}(c_{t+1}^*) - u^{*\prime\prime}(c_{t+1}^*)(c_{t+1} - y_{t+1})}. \tag{29}$$

From the domestic consumer's Euler equation, we also know that

$$u'(c_t) = \beta(1-\theta_t)(1+r_t)u'(c_{t+1}). \tag{30}$$

Combining equations (29) and (30) with equation (27), we obtain after simplifications

$$1-\theta_t = \frac{1 - \frac{u^{*\prime\prime}(c_t^*)}{u^{*\prime}(c_t^*)}(c_t - y_t)}{1 - \frac{u^{*\prime\prime}(c_{t+1}^*)}{u^{*\prime}(c_{t+1}^*)}(c_{t+1} - y_{t+1})}. \tag{31}$$

The planning problem of Foreign's government is symmetric. So the same logic implies

$$1-\theta_t^* = \frac{1-\frac{u''(c_t)}{u'(c_t)}(c_t^*-y_t^*)}{1-\frac{u''(c_{t+1})}{u'(c_{t+1})}(c_{t+1}^*-y_{t+1}^*)}. \tag{32}$$

Substituting for the foreign tax on international capital flows in equation (29) and using the good market clearing condition (4), we obtain

$$\frac{u'(c_t)+u''(c_t)(c_t-y_t)}{u^{*\prime}(Y-c_t)-u^{*\prime\prime}(Y-c_t)(c_t-y_t)} = \frac{u'(c_{t+1})+u''(c_{t+1})(c_{t+1}-y_{t+1})}{u^{*\prime}(Y-c_{t+1})-u^{*\prime\prime}(Y-c_{t+1})(c_{t+1}-y_{t+1})},$$

which implies

$$\frac{u'(c_t)+u''(c_t)(c_t-y_t)}{u^{*\prime}(Y-c_t)-u^{*\prime\prime}(Y-c_t)(c_t-y_t)} = \alpha, \text{ for all } t \geq 0, \tag{33}$$

where $\alpha \equiv [u'(c_0)+u''(c_0)(c_0-y_0)]/[u^{*\prime}(Y-c_0)-u^{*\prime\prime}(Y-c_0)(c_0-y_0)] > 0$. This is the counterpart of equation (6) in Section 2. In particular, using equations (28) and (30) and their counterparts in Foreign, one can check that $\alpha = \lambda\mu^*/\lambda^*\mu$, where λ and λ^* are the Lagrange multipliers associated with the intertemporal budget constraints in both countries.

5.2 Main Results Revisited

With the analysis of Section 3 in mind, one might have expected that an increase in y would necessarily lead to an increase in c. Indeed, we have established that if Home were to impose taxes unilaterally, it would like to increase c in response to a positive shock in y. The same logic implies that if Foreign were to impose taxes unilaterally, it would like to decrease c^*—i.e., to increase c as well—in response to a positive shock in y. Thus both unilateral responses point towards an increasing relationship between c and y. Yet, as the next lemma demonstrates, the relationship between endowments and consumption along the Nash equilibrium is more subtle.

Lemma 1 *Along the Nash equilibrium, consumption c is increasing in y if and only if the following condition holds:*

$$\frac{(y_t-c_t)}{2}\frac{\partial\left[u''(c_t)+\alpha u^{*\prime\prime}(Y-c_t)\right]/\partial c_t}{\left[u''(c_t)+\alpha u^{*\prime\prime}(Y-c_t)\right]} < 1. \tag{34}$$

In addition to the effects that were present in Section 3, the change in Foreign's tax may now lead Home to operate in a region in which prices are more manipulable. Broadly speaking, Lemma 1 states that if there is a large number of inframarginal units and if changes in consumption have large effects on changes the price of these inframarginal units, then c may be decreasing in y along the Nash equilibrium. A sufficient condition to rule out this scenario is that net exports, $y - c$, is not too large in equilibrium. Another sufficient condition is that utility functions are quadratic, which implies $\partial \left[u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t)\right] / \partial c_t = 0$. In this case, changes in consumption have no effects on changes in prices.

Under the assumption that inequality (34) holds, we can use Lemma 1 together with the domestic and foreign consumer's Euler equations to characterize capital control wars the same way we characterized optimal capital controls in Section 3. Our main result about capital control wars can be stated as follows.

Proposition 6 *Suppose that inequality (34) holds. Then along the Nash equilibrium, domestic and foreign capital flows taxes are such that:*

1. *If $y_{t+1} > y_t$, then domestic interest rates are higher than foreign interest rates ($\theta_t < \theta_t^*$).*
2. *If $y_{t+1} < y_t$, then domestic interest rates are lower than foreign interest rates ($\theta_t > \theta_t^*$).*
3. *If $y_{t+1} = y_t$, then domestic and foreign interest rates are equal ($\theta_t = \theta_t^*$).*

If there are no intertemporal distortions abroad, $\theta_t^* = 0$, then like in Section 3, an increase in domestic endowments, $y_{t+1} > y_t$, leads to a tax on capital inflows or a subsidy to capital outflows, $\theta_t < 0$, which is associated with higher domestic interest rates, $(1 - \theta_t)(1 + r_t) > 1 + r_t$. In general, however, we cannot sign θ_t and θ_t^*. The intuition for this result is a combination of the intuition for the unilateral policy of Home and Foreign. Suppose, for instance, that Home is running a trade deficit in period t. An increase in Home's endowment reduces the trade deficit and reduces Home's incentives to repress domestic consumption. Foreign's incentives are symmetric, meaning that Foreign has less incentives to stimulate foreign consumption. The increase in domestic consumption and the reduction in foreign consumption between periods t and $t + 1$ can be achieved in two ways: by a tax on capital inflows at Home, $\theta_t < 0$, or by a tax on capital outflows in Foreign, $\theta_t^* > 0$. Because of the general equilibrium response of world prices, we do not necessarily need *both*. All we need is that $\theta_t < \theta_t^*$, i.e. that domestic interest rates are higher than foreign interest rates.

To conclude, we compare the equilibrium capital control tax to the unilateral optimal taxes for both countries, i.e. the best response to a zero tax, using again the parameterized example presented in Section 3.2. In Figure 4, we see that a capital control war leads to a larger interest rate differential between the two countries (as a percentage of the

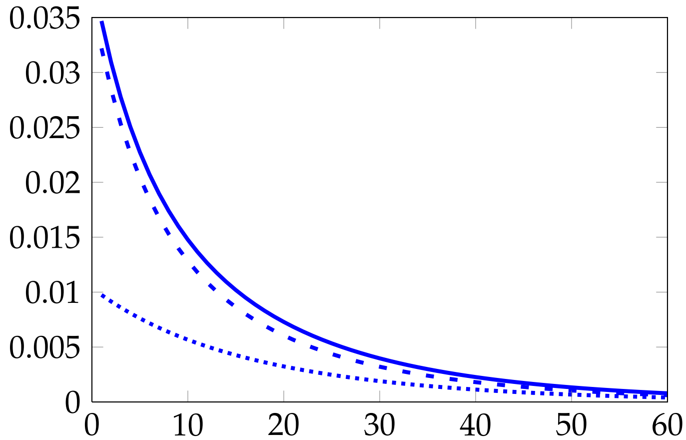

Figure 4: Taxes with capital control war versus unilateral optimal taxes. The solid line represents the equilibrium capital control war, while the dashed and dotted line show the two optimal unilateral distortions (from the perspective of the rest of the world).

world return to net lending) than either one of the two unilateral outcomes considered in Section 3.2. Far from cancelling each other out, the net distortion on capital flows is therefore larger when both countries set capital controls optimally. Compared to the benchmark with no capital controls, a capital control war here decreases consumption by 0.49% in the country catching-up and by 0.05% in the rest of the world. Interestingly, even though the interest rate differential is close to its value when the rest of the world sets capital controls unilaterally, both countries are worse off in the Nash equilibrium. In this particular example, neither country wins the capital control war.

6 Concluding Remarks

In this paper we have developed a simple theory of optimal capital controls in which the motive for capital controls is the manipulation of interest rates and other prices. We have studied a dynamic endowment economy where countries are free to impose capital controls, but prohibited by trade agreements from imposing import and export taxes. In the one good case, we have shown that it is optimal to tax capital inflows (or subsidize capital outflows) in periods of positive growth and to tax capital outflows (or subsidize capital inflows) in periods of negative growth. In the long-run, if the endowments converge to a steady state, taxes on international capital flows converge to zero. With many goods, capital controls also influence terms of trade in any period. As a result, optimal capital controls depend both on the level and composition of growth across goods. We have also examined how these forces play out in capital control wars.

In order to maintain the focus of our analysis on optimal capital controls, we have abstracted from environments with intratemporal and intertemporal trade in which countries can set good-and-time specific trade taxes. An interesting question that arises once these more general policy instruments are allowed is whether restricting static trade distortions, as we did in Section 4, may contribute to reduce the incentives to use capital controls. We leave this issue for future research.

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7 Appendix

Proof of Proposition 1. The first- and second-order conditions associated with Home's planning problem imply:

$$u'(c_t) - \mu \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t) \right] = 0, \tag{35}$$

$$\frac{\partial}{\partial c_t} \left\{ u'(c_t) - \mu \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t) \right] \right\} < 0. \tag{36}$$

Differentiating equation (35), we get after simple rearrangements

$$\frac{\partial c_t}{\partial y_t} = \frac{\mu u^{*\prime\prime}(Y - c_t)}{\frac{\partial}{\partial c_t} \left\{ u'(c_t) - \mu \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t) \right] \right\}} > 0, \tag{37}$$

where the inequality directly derives from inequality (36) and the strict concavity of u^*. Inequality (37) implies that for any pair of periods, t and t', such that $y_{t'} > y_t$, we must have $c_{t'} > c_t$. ■

Section 3.4. Let us focus on date 0 and date 1. Let $\{c_t\}_{t=1}^{\infty}$ and $\{c_t'\}_{t=1}^{\infty}$ denote the optimal consumption paths for all dates $t \geq 1$ from the point of view of Home's government at date 0 and date 1, respectively. We want show that one can construct $\{a_{1,s}^*\}_{s=1}^{\infty}$ satisfying equation (14) at $t = 0$ such that $c_t' = c_t$ for all $t \geq 1$. The first-order conditions associated with Home's planning problem at dates $t = 0$ and $t = 1$ imply

$$\begin{aligned} u'(c_t) &= \mu_0 \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t + a_{0,t}^*) \right], && (38) \\ u'(c_t') &= \mu_1 \left[u^{*\prime}(Y - c_t') - u^{*\prime\prime}(Y - c_t')(c_t' - y_t + a_{1,t}^*) \right]. && (39) \end{aligned}$$

For a given value of μ_1, let us construct $a^*_{1,t}(\mu_1)$ such that

$$\mu_0\left[u^{*\prime}(Y-c_t)-u^{*\prime\prime}(Y-c_t)(c_t-y_t+a^*_{0,t})\right] \\ =\mu_1\left[u^{*\prime}(Y-c_t)-u^{*\prime\prime}(Y-c_t)(c_t-y_t+a^*_{1,t}(\mu_1))\right],$$

which can be rearranged as

$$\mu_1 a^*_{1,t}(\mu_1)=\mu_0 a^*_{0,t}+(\mu_1-\mu_0)\left[\frac{u^{*\prime}(Y-c_t)}{u^{*\prime\prime}(Y-c_t)}-(c_t-y_t)\right].$$

By construction, if the previous condition holds, then, for any μ_1, equation (39) holds as well if $c'_t=c_t$ for all $t\geq 1$. Now let us choose μ_1 such that:

$$\sum_{s=1}^{\infty}u^{*\prime}(Y-c_s)\left[c_s-y_s+a^*_{1,s}(\mu_1)\right]=0.$$

Thus, equation (14) is satisfied at $t=0$. Since equation (14) evaluated at $t=0$ and equation (15) evaluated at $t=1$ are identical, we have constructed $\{a^*_{1,s}\}_{s=1}^{\infty}$ such that $c'_t=c_t$ for all $t\geq 1$. The argument for other dates is similar. ■

Proof of Proposition 3. The basic strategy is the same as in the proof of Proposition 1. The first- and second-order conditions associated with Home's planning problem imply:

$$U'(C_t)-\mu\left(\sum_i\rho_i(C_t)\frac{\partial c_i(C_t)}{\partial C_t}+\sum_i\frac{\partial\rho_i(C_t)}{\partial C_t}(c_i(C_t)-y_{it})\right)=0, \tag{40}$$

$$\frac{\partial}{\partial C_t}\left\{U'(C_t)-\mu\left(\sum_i\rho_i(C_t)\frac{\partial c_i(C_t)}{\partial C_t}-\sum_i\frac{\partial\rho_i(C_t)}{\partial C_t}(c_i(C_t)-y_{it})\right)\right\}<0. \tag{41}$$

Differentiating equation (40), we get after simple rearrangements

$$dC_t=-\frac{\mu\sum_i\frac{\partial\rho_i(C_t)}{\partial C_t}dy_{it}}{\frac{\partial}{\partial C_t}\left\{U'(C_t)-\mu\sum_i\rho_i(C_t)\frac{\partial c_i(C_t)}{\partial C_t}-\sum_i\frac{\partial\rho_i(C_t)/\partial C_t}{\rho_i(C_t)}\rho_i(C_t)(c_i(C_t)-y_{it})\right\}}.$$

By inequality (41), we therefore have $dC_t>0$ if and only if $\sum_i\frac{\partial\rho_i(C_t)}{\partial C_t}dy_{it}>0$. ■

Proof of Proposition 4. In the main text, we have already established that

$$\theta_t=1-\left(\frac{1+\tau_t}{1+\tau_{t+1}}\right)\left(\frac{P_{t+1}/P^*_{t+1}}{P_t/P^*_t}\right), \tag{42}$$

with the wedge τ_t such that

$$\tau_t = \frac{U'(C_t)}{\mu U^{*\prime}(C_t^*)} - 1 = \frac{U'(C_t)}{\mu U^{*\prime}(C^*(C_t))} - 1.$$

Since U and U^* are concave and C^* is decreasing in C_t along the Pareto frontier, we already know from Proposition 3 that

$$\tau_{t+1} < \tau_t \text{ if and only if } \sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it} > 0. \tag{43}$$

Now notice that by the envelope theorem, $C^{*\prime}(C_t)$ is equal to the opposite of the lagrange multiplier associated with the constraint in (19). Thus the first-order conditions associated with that program imply

$$g_i^*(c_t) = -C^{*\prime}(C_t)\, g_i(c_t) \tag{44}$$

In footnote 10, we have already formally established that $P_t = p_i / g_i(c_t)$. The same logic applied to Foreign implies $P_t^* = p_i / g_i^*(c_t)$. Combining the two previous observations with equation (44), we obtain

$$\frac{P_t}{P_t^*} = -C^{*\prime}(C_t).$$

Since g is concave and homogeneous of degree one, we also know that C^* is (weakly) concave in C_t. By Proposition 3, we therefore have

$$P_{t+1}/P_{t+1}^* > P_t/P_t^* \text{ if and only if } \sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it} > 0. \tag{45}$$

Combining equation (42) with conditions (43) and (45), we finally get $\theta_t < 0$ if and only if $\sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it} > 0$. ■

Proof of Proposition 5. Suppose that $dy_{it+1} > 0$ and $dy_{jt+1} = 0$. By Proposition 4, we know that $\theta_t < 0$ if and only if $\partial \rho(C_t)/\partial C_t \cdot dy_{t+1} > 0$, where $\rho(C_t) \equiv U^{*\prime}(C^*(C_t))\, g_c^*(c^*(C_t))$. Thus if $dy_{it+1} > 0$ and $dy_{jt+1} = 0$, $\theta_t < 0$ if and only if

$$\frac{U^{*\prime\prime}(C^*(C_t))}{U^{*\prime}(C^*(C_t))} \frac{\partial C^*(C_t)}{\partial C_t} + \left[\frac{g_{ii}^*(c^*(C_t))}{g_i^*(c^*(C_t))} \frac{\partial c_i^*(C_t)}{\partial C_t} + \frac{g_{ij}^*(c^*(C_t))}{g_i^*(c^*(C_t))} \frac{\partial c_j^*(C_t)}{\partial C_t} \right] > 0. \tag{46}$$

Let us first demonstrate that

$$\frac{U^{*\prime\prime}(C^*(C_t))}{U^{*\prime}(C^*(C_t))} \frac{\partial C^*(C_t)}{\partial C_t} = \frac{\gamma P_t}{P_t^* C_t^*}. \tag{47}$$

In the proof of Proposition 4, we have already established that

$$\frac{\partial C^*(C_t)}{\partial C_t} = -\frac{P_t}{P_t^*}.$$

Equation (47) directly derives from this expression and the fact that $U^*(C^*) = \frac{1}{1-\gamma}(C^*)^{1-\gamma}$. Let us now demonstrate that

$$\begin{aligned}
\frac{g_{11}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{12}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} &= \frac{\alpha}{(1-\alpha)C_t}\left(\frac{(2\alpha-1)\,c_2(C_t)}{(1-\alpha)\,c_2^*(C_t)+\alpha c_2(C_t)}\right), && (48)\\
\frac{g_{12}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{22}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} &= -\frac{1}{C_t}\left(\frac{(2\alpha-1)\,c_2(C_t)}{(1-\alpha)\,c_2^*(C_t)+\alpha c_2(C_t)}\right). && (49)
\end{aligned}$$

Since $g^*(c^*) = (c_2^*)^\alpha (c_1^*)^{1-\alpha}$, simple algebra implies

$$\begin{aligned}
\frac{g_{11}^*(c_t^*)}{g_1^*(c_t^*)} &= -\frac{\alpha}{c_{1t}^*}, \frac{g_{12}^*(c_t^*)}{g_1^*(c_t^*)} = \frac{\alpha}{c_{2t}^*},\\
\frac{g_{12}^*(c_t^*)}{g_2^*(c_t^*)} &= \frac{(1-\alpha)}{c_{1t}^*}, \frac{g_{22}^*(c_t^*)}{g_2^*(c_t^*)} = -\frac{(1-\alpha)}{c_{2t}^*}.
\end{aligned}$$

Thus

$$\begin{aligned}
\frac{g_{11}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{12}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} &= \alpha\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t},\\
\frac{g_{12}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{22}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} &= -(1-\alpha)\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t}.
\end{aligned}$$

Let us compute $\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t}$. By definition, $c(C_t)$ and $c^*(C_t)$ are solution of

$$\max_{c,c^*} \ (c_2^*)^\alpha (c_1^*)^{1-\alpha}$$

subject to

$$\begin{aligned}
c_1^\alpha c_2^{1-\alpha} &\geq C_t,\\
c_1 + c_1^* &\leq Y_1,\\
c_2 + c_2^* &\leq Y_2.
\end{aligned}$$

The associated first-order conditions imply

$$\frac{c_2^*\left(C_t\right)}{c_1^*\left(C_t\right)} = \beta \left(\frac{Y_2 - c_2^*\left(C_t\right)}{Y_1 - c_1^*\left(C_t\right)} \right), \beta \equiv \left[\alpha/\left(1-\alpha\right)\right]^2, \tag{50}$$

and

$$\left(\frac{Y_2 - c_2^*\left(C_t\right)}{Y_1 - c_1^*\left(C_t\right)} \right) = \left(\frac{C_t}{Y_2 - c_2^*\left(C_t\right)} \right)^{-\frac{1}{\alpha}}. \tag{51}$$

Combining the two previous expressions, we obtain

$$\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t} = -\frac{1}{\alpha}\left(\frac{1}{C_t} - \frac{\partial \ln\left[Y_2 - c_2^*(C_t)\right]}{\partial C_t}\right) = -\frac{1}{\alpha}\left(\frac{1}{C_t} - \frac{\partial \ln\left[c_2(C_t)\right]}{\partial C_t}\right). \tag{52}$$

Let us compute $\frac{\partial \ln[c_2(C_t)]}{\partial C_t}$. Using the resource constraint, we can express equation (50) as

$$c_1\left(C_t\right) = \frac{\beta c_2\left(C_t\right) Y_1}{\left[Y_2 - \left(1-\beta\right) c_2\left(C_t\right)\right]}.$$

Together with equation (51), using again the resource constraint, this implies

$$c_2\left(C_t\right) = C_t \left[\frac{Y_2 - \left(1-\beta\right) c_2\left(C_t\right)}{\beta Y_1}\right]^{\alpha}.$$

Taking the log and differentiating, we obtain after rearrangements

$$\frac{\partial \ln\left[c_2(C_t)\right]}{\partial C_t} = \frac{Y_2 - \left(1-\beta\right) c_2\left(C_t\right)}{C_t\left[Y_2 - \left(1-\alpha\right)\left(1-\beta\right) c_2\left(C_t\right)\right]}. \tag{53}$$

Equations (52) and (53) imply

$$\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t} = \frac{1}{C_t}\left(\frac{\left(\beta-1\right) c_2\left(C_t\right)}{Y_2 + \left(1-\alpha\right)\left(\beta-1\right) c_2\left(C_t\right)}\right).$$

Using the definition of $\beta \equiv \left[\alpha/\left(1-\alpha\right)\right]^2$, we can rearrange the previous expression as

$$\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t} = \frac{1}{(1-\alpha)C_t}\left(\frac{\left(2\alpha-1\right) c_2\left(C_t\right)}{\left(1-\alpha\right) c_2^*\left(C_t\right) + \alpha c_2\left(C_t\right)}\right). \tag{54}$$

To conclude the proof of Proposition 5, first note that equations (47), (48), and (54) imply

$$\frac{U^{*\prime\prime}\left(C^*(C_t)\right)}{U^{*\prime}\left(C^*(C_t)\right)}\frac{\partial C^*(C_t)}{\partial C_t}+\left[\frac{g_{11}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t}+\frac{g_{12}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t}\right]$$
$$=\frac{\gamma P_t}{P_t^* C_t^*}+\frac{\alpha}{(1-\alpha)C_t}\left(\frac{(2\alpha-1)\,c_2\left(C_t\right)}{(1-\alpha)\,c_2^*\left(C_t\right)+\alpha c_2\left(C_t\right)}\right)>0.$$

Thus if $dy_{1t+1}>0$ and $dy_{2t+1}=0$, then $\theta_t<0$. Second note that equations (47), (49), and (54) imply

$$\frac{U^{*\prime\prime}\left(C^*(C_t)\right)}{U^{*\prime}\left(C^*(C_t)\right)}\frac{\partial C^*(C_t)}{\partial C_t}+\left[\frac{g_{12}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t}+\frac{g_{22}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t}\right]$$
$$=\frac{\gamma P_t}{P_t^* C_t^*}-\frac{1}{C_t}\left(\frac{(2\alpha-1)\,c_2\left(C_t\right)}{(1-\alpha)\,c_2^*\left(C_t\right)+\alpha c_2\left(C_t\right)}\right).$$

According to this expression, $\theta_t<0$ if and only if

$$\gamma>\left(\frac{P_t^* C_t^*}{P_t C_t}\right)\left(\frac{(2\alpha-1)\,c_2\left(C_t\right)}{(1-\alpha)\,c_2^*\left(C_t\right)+\alpha c_2\left(C_t\right)}\right). \tag{55}$$

Since utility functions are Cobb-Douglas, $g(c)=c_1^{\alpha}c_2^{1-\alpha}$ and $g^*(c^*)=\left(c_2^*\right)^{\alpha}\left(c_1^*\right)^{1-\alpha}$, we know that

$$\begin{aligned}p_{2t}c_2\left(C_t\right)&=(1-\alpha)\,P_tC_t\\ p_{2t}c_2^*\left(C_t\right)&=\alpha P_t^* C_t^*\end{aligned}$$

Combining these two observations with inequality (55), we conclude that $\theta_t<0$ if and only if $\gamma>\left(\frac{2\alpha-1}{\alpha}\right)\left(\frac{P_t^*C_t^*}{P_t^*C_t^*+P_tC_t}\right)$. ■

Proof of Lemma 1. Let us rearrange equation (33) as

$$F(c_t,y_t)\equiv\left[u'(c_t)-u''(c_t)(y_t-c_t)\right]-\alpha\left[u^{*\prime}(Y-c_t)-u^{*\prime\prime}(Y-c_t)(c_t-y_t)\right]=0. \tag{56}$$

Note that $\partial F(c_t,y_t)/\partial y_t>0$ by strict concavity of u and u^*. Thus

$$dc_t/dy_t=-\,\partial F(c_t,y_t)/\partial y_t/\,\partial F(c_t,y_t)/\partial c_t>0$$

if and only if $\partial F(c_t,y_t)/\partial c_t<0$. Differentiating equation (56) we get

$$\frac{\partial F(c_t,y_t)}{\partial c_t}=\frac{\partial}{\partial c_t}\left[u'(c_t)-u''(c_t)(y_t-c_t)\right]-\alpha\frac{\partial}{\partial c_t}\left[u^{*\prime}(Y-c_t)-u^{*\prime\prime}(Y-c_t)(c_t-y_t)\right],$$

which can be rearranged as

$$\frac{\partial F(c_t, y_t)}{\partial c_t} = \left[u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t)\right] \left\{2 - (y_t - c_t) \frac{\partial \left[u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t)\right] / \partial c_t}{\left[u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t)\right]}\right\}.$$

Thus $\partial F(c_t, y_t)/\partial c_t < 0$ if and only if $\frac{(y_t - c_t)}{2} \frac{\partial [u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t)]/\partial c_t}{[u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t)]} < 1$. ∎

Proof of Proposition 6. The foreign and domestic consumers' Euler equations imply

$$\frac{1 - \theta_t}{1 - \theta_t^*} = \frac{u'(c_t)}{u^{*\prime}(c_t^*)} \frac{u^{*\prime}(c_{t+1}^*)}{u'(c_{t+1})}$$

Using the good market clearing condition (4), we can rearrange this expression as

$$\frac{1 - \theta_t}{1 - \theta_t^*} = \frac{u'(c_t)}{u^{*\prime}(Y - c_t)} \frac{u^{*\prime}(Y - c_{t+1})}{u'(c_{t+1})}.$$

By Lemma 1, we know that if inequality (34), then c_t is increasing in y_t. Since u and u^* are concave, the previous expression therefore implies

$$\frac{1 - \theta_t}{1 - \theta_t^*} < 1 \text{ if and only if } y_t > y_{t+1}.$$

Proposition 6 directly derives from the previous equivalence. ∎