The Role of Multinational Production in Cross-Country Risk Sharing*

Natalia Ramondo†
University of Texas-Austin

Veronica Rappoport‡
Columbia Business School

April 8, 2008

Abstract

In this paper, we introduce the role of Multinational Production (MP) in cross-country risk sharing. We present a two-country, two-sector model with complete financial markets, and country-specific productivity shocks to the tradable sector. Firms can do MP by opening affiliates abroad which bear the productivity shock to the *host* country.

By treating MP simultaneously as a portfolio and production flow, we find that MP affects the pattern of world risk even under the existence of a full set of contingent claims. MP changes TFP in the host country, affecting the impact of country-specific productivity shocks. Moreover, risk considerations increase the incentives of firms from large countries to do MP while the opposite happens for smaller economies. As a result, the model has predictions on the composition of international portfolios across countries: large countries have positive (net) position of Direct Investment while small economies hold positive (net) position of other financial assets.

*We benefited from comments of participants at seminars in UC Berkeley, Columbia University, the Annual Meeting of the Society for Economic Dynamics (2007), Dartmouth International Trade Summer Camp (2007), FRB San Francisco, FRB Dallas, Harvard University, International Trade Workshop FRB Philadelphia (Fall 2007), U. Maryland, Penn State University, Stanford University, U. Texas-Austin, and Universidad de San Andres. For helpful comments, we thank George Alessandria, Hal Cole, Russell Cooper, Pierre-Olivier Gourinchas, Karen Lewis, Ellen McGrattan, Katheryn Russ, and Neng Wang.

†E-mail: nramondo@mail.utexas.edu

‡E-mail: ver2102@columbia.edu

1 Introduction

When analyzing the composition of countries' international portfolio, the literature on international risk sharing has focused on the distinction between risky and risk-free assets, without differentiating Foreign Direct Investment (FDI) from other risky positions. The vast majority of such macroeconomic models takes the market structure of goods and factors as given. Under such restriction, buying shares of foreign firms or doing FDI are indeed equivalent. However, the crucial difference between FDI and other international flows is that the former involves reallocation of production. Indeed, one of the most notable features of economic globalization has been the increasing importance of Multinational Production (MP) in international goods markets: by 2004, total sales of foreign affiliates of multinational firms represented 51% of world GDP, almost double the share of world exports.[1]

In this paper we introduce the role of MP both as a technology and portfolio flow in a risky environment. We find that the change in the host country's Total Factor Productivity (TFP) entailed by FDI flows has implications for the pattern of world aggregate risk, even in a context of frictionless financial markets. We also find that international risk patterns affect the location of multinational firms and the international portfolio composition of countries. In particular, "large" economies have more incentives to do MP –where "large" refers to the magnitude of the impact of country-specific shocks on world financial prices–. As a consequence, these countries tend to have a larger FDI position and, simultaneously, larger debt position.

Treating MP simultaneously as a portfolio and technology flow results in important and novel insights:

First, it fundamentally alters previous results on the relevance of MP in international risk-sharing. If the impact of MP on the host countries' TFP is ignored, MP flows only affect

[1] World Investment Report 2006, UNCTAD.

international risk patterns under imperfect financial markets. By contrast, in the framework proposed here, MP flows have a role in international risk diversification even if a complete set of state contingent claims exists, as it alters the international production structure. In other words, affiliates of multinational firms represent assets which returns are not spanned by existing securities.

Second, the interaction between these two roles of MP results in novel cross-country implications for the location of firms. We find that for "large" economies, these two roles of MP as a portfolio and technology flow are complements: risk considerations increase incentives of firms from "large" countries to do MP while the opposite happens for "smaller" economies. In the context of frictionless financial assets, risk neutral firms internalize the risk diversification motive of risk averse consumers through asset prices. Therefore, firms from the "large" economy create value by reallocating production towards the "small" economy, as they create assets that pay relatively more in those states of the world in which consumption is lower.

We present a two-country stochastic model with a full set of contingent financial assets. There are two sectors: tradable and non-tradable. The sources of uncertainty are country-specific shocks that affect the relative price of tradable to non-tradable goods. In the spirit of Melitz (2003), firms in the non-tradable sector are heterogenous in technology, compete monopolistically, and can do MP by opening affiliates abroad after paying an entry cost. Multinational firms are treated as technology entities, that is, affiliates replicate their parent firm's technology. Still, they face the *host* country specific shock and, as a result, MP profits co-move with host country risk.

Unlike the rest of the literature on international risk-sharing, we do not differentiate assets according to their risk. Rather, our analysis distinguishes the following two international assets: FDI, which involves changes in relative TFP across countries; and a financial portfolio of other risky and risk-free assets, that we reinterpret as fully contingent claims. The representative consumer holds these securities, and shares of national firms, some of which are multinationals.[2]

We recognize that countries are asymmetric and, as a result, country specific shocks have different impact on world output. We model this feature by assuming that countries differ in their productivity level. Then, a relative bad shock to the most productive economy has bigger impact on world output. This sole source of asymmetry introduces aggregate risk: the "small" economy cannot provide full insurance to the "large" economy. In this context, reallocation of production from the "large" to the "small" economy improves the capacity of the less productive country to provide insurance. The role of MP in reshaping international risk patterns results from the assumption that affiliate plants bear the host country specific shock. Then, an increase in the number of affiliates located in the "small" economy reduces the disparity in TFP across countries and, as a result, lowers the differences in the impact of country specific shocks on world output.

The existence of frictionless financial markets results in efficient consumption allocations and geographical distribution of multinational firms. We find that, indeed, prices of financial securities give multinationals from the "large" economy more incentives to do MP, while the opposite is true for firms from the "small"economy. Contingent claims that pay in states with relatively scarce world output are more expensive. Those states are characterized by a negative shock to the "large" economy. Since MP profits co-move with the shock in the host country, MP profits of multinational firms from the "small" country are higher when world output is relatively abundant, while profits of multinational firms from the "large" country are higher when world output is relatively scarce and the price of contingent claims is high. As a result, everything else equal, risk increases the value of multinational production for firms from the "large" economy. These results are reinforced when risk diversification is more valuable, that is, the larger the volatility of country-specific shocks or the higher the risk aversion of consumers.

[2]Results are not affected if national firms are initially owned by national consumers and later sold in the international market.

Our model builds on Grossman and Razin (1984, 1985) and the literature on trade and international risk sharing.[3] They introduce production risk into a model that determines the international pattern of trade and capital flows. They analyze the choice between risky and riskless production done by asymmetric economies and find, as we do, that risky production in the small economy is more valuable. We build on that result, and endogenize the location of firms in a similar risky environment. Our paper is also close to Svensson (1988). He characterizes trade in a set of assets, complete or incomplete, in a stochastic endowment economy. Using the law of comparative advantages, he compares the price of assets across states and across countries under autarky to determine the pattern of international trade of financial assets once financial markets are integrated. In the same spirit, in this paper, we compare the price of assets across states in a world without MP to characterize the geographical pattern of multinational firms once economies open to this type of technology flows.

To our knowledge, no study has analyzed the role of MP in international risk-sharing treating it simultaneously as a portfolio and technology flow.[4] On the one hand, the international trade literature has focused on the role of MP as a way of serving foreign consumers by replicating production facilities abroad (horizontal FDI), or splitting the production chain to take advantage of cheap input costs (vertical FDI).[5] This literature emphasizes the role of MP in the exchange of goods but does not address its implications in terms of international risk-sharing. On the other hand, the international business cycle literature has mainly treated MP as a portfolio flow abstracting from the changes in relative TFP across countries that this flow entails.[6] This

[3]See, for instance, Obstfeld and Cole (1991), Tesar (1993), and Backus and Smith (1993).

[4]Aizenman and Marion (2004), and Goldberg and Kolstad (1995) study the location of MP activities under uncertainty. Both frameworks and motivations are very different from ours. They do not address the change in aggregate risk that results from reallocation of production nor do they have financial assets that allow firms to optimally diversify risk.

[5]See Markusen (1984); Brainard (1997); Markusen and Venables (1998); Carr, Markusen, and Maskus (2001); Helpman, Melitz, and Yeaple (2004); Ramondo (2005), Burstein and Monge (2006), McGrattan and Prescott (2007), for horizontal FDI. See Helpman (1984), Antras (2003), Antras and Helpman (2004), for vertical FDI.

[6]See for example Backus and Smith (1993), Backus, Kehoe, and Kydland (1995), Baxter and Jermann (1997), Perri and Heathcote (2004), and Mendoza, Quadrini, and Rios-Rull (2006).

disconnection between international macroeconomics and trade misses interesting and relevant interactions.

Finally, our paper has predictions for the composition of international portfolios across countries. In risky environments, "large" countries are net sources of multinational firms and hence have positive (net) Foreign Direct Investment positions. As a counterpart, this model predicts negative net positions on financial assets other than FDI. The negative position on the rest of financial assets follows from two features of the model: first, firms in the "large" country borrow to finance the entry cost to multinational activities; and second, since the "large" economy has a higher share of multinational firms, the dividends from its national firms are internationally diversified. As a result, consumers from the "large" country do not rely on other international assets to diversify country-specific risk. In this sense, we contribute to the literature on international portfolio composition.[7]

The paper has the following structure. Section 2 presents the set-up of the model; Section 3 characterizes the equilibrium and describes the main mechanism of the model; Section 4 presents some numerical examples; and Section 5 concludes.

2 Model

This is a stochastic, two-period, two-country model with complete financial markets. There are two sectors: tradable and non-tradable. The tradable sector is subject to a country-specific productivity shock.

Firms in the non-tradable sector are heterogenous in productivity. They can do MP by opening affiliates abroad, after paying an entry cost. Affiliates inherit the productivity parameter

[7] Lewis (2006) findings are consistent with this result. She finds that international equity markets have become more highly correlated across countries and, as a result, the attainable diversification from foreign diversification is declining.

from the source firm but bear the shock to the *host* country. Hence, MP profits co-move with host country risk.

Consumers hold shares of national firms that include multinationals and Arrow-Debreu securities.

2.1 Set-up

There are two countries, Home and Foreign, of size L and L^*, respectively. Firms can do MP by opening affiliates abroad. Hence, it is relevant to distinguish between national (ownership criteria) and domestic (location criteria) variables. We indicate with an asterisk * those variables that are owned by Foreign consumers, irrespectively of location.

There are two periods: an initial period, *before* country-shocks are realized, in which trade in Arrow-Debreu securities and Foreign Direct Investment (FDI) take place; and a second period, *after* uncertainty is realized, in which production and consumption take place.

Let the vector $s \in S$ denote the state of the world economy in this second period, which is characterized by the realization of country productivity shocks. Assume that there is a finite discrete number of states: $S = \{s_1; s_2; ...; s_n\}$, each with probability $\Pr(s)$, $\sum_{s \in S} \Pr(s) = 1$, and $0 \leq \Pr(s) \leq 1$.

Each economy produces two types of goods: an aggregate CES non-tradable consumption good

$$C_{NT}(s) = \left[\int_{\omega \in \Omega} c(\omega)^{\frac{\eta-1}{\eta}} d\omega \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

with elasticity of substitution $\eta > 1$, and price index:

$$P_{NT}(s) = \left[\int_{\omega \in \Omega} p(\omega)^{1-\eta} d\omega \right]^{\frac{1}{1-\eta}}, \tag{2}$$

and a freely-traded homogenous consumption good $C_T(s)$ that is used as numeraire, $P_T = 1$.

Preferences. The representative consumer supplies L units of labor and maximizes the following expected utility from consumption:

$$U = \beta \sum_{s \in S} \Pr(s) \frac{C(s)^{1-\sigma}}{1-\sigma}, \tag{3}$$

where $\sigma > 1$

$$C(s) = \left[C_T(s)^{\frac{\rho-1}{\rho}} + C_{NT}(s)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \tag{4}$$

the elasticity of substitution between tradable and non-tradable sectors is assumed lower than one: $\rho < 1$.[8] The price index for $C(s)$ is:

$$P(s) = \left[1 + P_{NT}(s)^{1-\rho} \right]^{\frac{1}{1-\rho}}. \tag{5}$$

Total expenditure in each individual good ω is

$$x(\omega) = \left[\frac{p(\omega)}{P_{NT}(s)} \right]^{1-\eta} X_{NT}(s), \tag{6}$$

where $X_{NT}(s)$ is aggregate expenditure in the CES good:

$$X_{NT}(s) = \left(\frac{P_{NT}(s)}{P(s)} \right)^{1-\rho} X(s), \tag{7}$$

and $X(s)$ is aggregate expenditure. Total expenditure in the homogeneous good is:

$$X_T(s) = \left(\frac{1}{P(s)} \right)^{1-\rho} X(s). \tag{8}$$

[8] In a sample of 30 industrialized countries and developing nations, Stockman and Tesar (1995) find an elasticity of substitution between traded and non-traded goods of 0.44.

Technology. There is a continuum of firms of measure one, each producing a differentiated good ω. Each firm operates an only-labor constant returns to scale technology with productivity $z(\omega)$. The parameter $z(\omega)$ is known, and drawn from a country-specific distribution, $G(z)$ and $G^*(z)$, for Home and Foreign respectively, independently distributed across countries.

Additionally, countries are subject to a country-specific productivity shock to the tradable sector, $W(s)$ and $W^*(s)$, which is the only source of risk in this economy. Hence, the world aggregate state is $s = \{W, W^*\}$. We refer to this shock as wage-shock.

Firms can open affiliate plants abroad with the same productivity parameter $z(\omega)$ as the one they have at home. Hence, production functions for a Home firm producing good ω at Home and Foreign are, respectively:

$$y_d(\omega, s) = A \cdot z(\omega) \cdot l(\omega, s), \tag{9}$$

$$y_m(\omega, s) = A^* \cdot z(\omega) \cdot l^*(\omega, s). \tag{10}$$

where $y_d(\omega, s)$ and $y_m(\omega, s)$ are domestic and foreign output, while $l(\omega, s)$ and $l^*(\omega, s)$ are labor requirements. Since the only parameter that varies across differentiated goods is the firm-specific productivity $z(\omega)$, and goods enter symmetrically in preferences, we can rename each good ω by its productivity z. Total profits of a Home firm with productivity z are given by:

$$\pi(z, s) = \pi_d(z, s) + \tau(z)\pi_m(z, s), \tag{11}$$

where $\tau(z)$ is one if the firm does MP and zero otherwise, $\pi_d(z, s)$ denotes profits at Home, and $\pi_m(s, z)$ profits in Foreign.

Firms compete monopolistically. Hence, the price charged by a firm with productivity z at

Home is:

$$p(z,s) = \frac{\eta}{\eta - 1} \cdot \frac{W(s)}{A} \cdot \frac{1}{z}, \tag{12}$$

and in Foreign:

$$p(z,s) = \frac{\eta}{\eta - 1} \cdot \frac{W^*(s)}{A^*} \cdot \frac{1}{z}. \tag{13}$$

The tradable homogeneous consumption good is produced under constant returns to scale with an only-labor technology and productivity $W(s)$: $Y_h(s) = W(s) L_h(s)$. Provided that this good is produced everywhere, nominal wages at Home and Foreign are, respectively, $W(s)$ and $W^*(s)$.[9]

Assets Structure. The representative consumer in each country holds two types of assets: shares of firms, $\theta(z)$ and fully contingent bonds $B(s)$. Firms are assumed to be owned by national consumers $\theta(z) = 1$ for $z \in Z$ and $\theta^*(z) = 1$ for $z \in Z^*$.[10] The budget constraint is therefore:

$$\sum_{s \in S} q(s)P(s)C(s) = B_0 + \sum_{s \in S} q(s) \left\{ LW(s) + \int_{z \in Z} \pi(z,s)\, dG(z) \right\} \tag{14}$$

where $q(s)$ is the date-0 price of an Arrow-Debreu security that pays one unit of the numeraire in state s, and B_0 is initial net wealth. Finally $\pi(z,s)$ denotes profits of Home firms with technology z. From the consumer's optimization problem, the Euler equation for securities is:

$$q(s) = \frac{\beta \Pr(s)}{\lambda} \frac{u'(C(s))}{P(s)} = \frac{\beta \Pr(s)}{\lambda} u'_T(C(s)), \tag{15}$$

where $u'_T(C)$ corresponds to the marginal utility of consumption in tradable goods and λ is the multiplier on the Home consumer's budget constraint.

Foreign Direct Investment. Foreign Direct Investment (FDI) takes place in the following

[9] It will never be optimal to do MP in this sector.

[10] The results are not affected if national firms are initially owned by national consumers and sold in the international market.

way.[11] Firms decide whether to become multinationals or not before the realization of country shocks. If they decide to enter the foreign market, they pay a one time entry cost, f and f^*, for Home and Foreign, respectively. The value of doing MP for a firm with productivity z is given by the expected discounted stream of profits $\sum_{s \in S} q(s)\pi_m(z, s)$. Both countries are endowed with an initial stock of investment tradable good, Y_0 and Y_0^*. The MP entry cost is paid in units of this good, which international price is denoted by p_f. Therefore, the FDI decision is characterized by the following rule:

$$\begin{aligned}\sum_{s \in S} q(s)\pi_m(z, s) &\geq f^* p_f : \tau(z) = 1 \\ \sum_{s \in S} q(s)\pi_m(z, s) &< f^* p_f : \tau(z) = 0.\end{aligned}$$

Finally, the initial net wealth in the budget constraint (14) is given by

$$B_0 = p_f \left[Y_0 - f^* \int_{z \in Z} \tau(z) dG(z) \right]. \tag{16}$$

2.2 Equilibrium

We define the equilibrium in two steps. First, we characterize *national* equilibrium prices and quantities as functions of the state vector s, the number of firms doing MP, and aggregate expenditure at Home and Foreign, $X(s)$ and $X^*(s)$. In the second step, we define the *international* equilibrium.

[11] Note the distinction between FDI and MP: the first one refers to the Balance of Payment flow and in our model occurs only once, i.e. the initial setting-up of affiliates abroad; the second one refers to the productive activities of affiliates abroad, e.g. sales, profits, employment, and occurs every period.

2.2.1 National Equilibrium

As it is explained in the next subsection, the FDI decision follows a cut-off rule. We denote $\overline{z}$ (and $\overline{z}^*$ for Foreign) the productivity level for which firms with z above $\overline{z}$ become multinationals, and firms with z below $\overline{z}$ stay domestic:

$$\begin{aligned} \forall z &\geqq \overline{z} : \tau(z) = 1, \\ \forall z &< \overline{z} : \tau(z) = 0. \end{aligned}$$

The *national equilibrium* prices and quantities are characterized as functions of the state vector s, the cut-off rule for doing MP activities, $\overline{z}$ and $\overline{z}^*$, and aggregate expenditure at Home and Foreign, $X(s)$ and $X^*(s)$.

Define the following aggregate productivity indexes:

$$Z_d \equiv \int_{z_{min}}^{\infty} z^{\eta-1} dG(z), \tag{17}$$

$$Z_m \equiv \int_{\overline{z}}^{\infty} z^{\eta-1} dG(z), \tag{18}$$

and analogously for Foreign firms, Z_d^* and Z_m^*.

From (2), (12), and (13), price indices for the composite good, at Home and Foreign, are given by:

$$P_{NT}(s) = \frac{\eta}{\eta-1} \cdot \frac{W(s)}{A} \cdot (Z_d + Z_m^*)^{\frac{1}{1-\eta}}, \tag{19}$$

$$P_{NT}^*(s) = \frac{\eta}{\eta-1} \cdot \frac{W^*(s)}{A^*} \cdot (Z_d^* + Z_m)^{\frac{1}{1-\eta}}. \tag{20}$$

The only source of uncertainty in this model comes from the realization of the productivity shock in the tradable sector –i.e. wage shock-. Notice from equations (19) and (20) that the impact of such a shock on the relative price of tradable to non-tradable goods depends on the number of firms operating in that market, which is measured by the aggregate productivity indexes Z_d and Z_m^*. Indeed, the price elasticity to a wage-shock is given by:

$$\xi_{PW} \equiv \frac{\partial P}{\partial W}\frac{W}{P} = 1 - P^{\rho-1} \in (0,1)\,, \tag{21}$$

which decreases in the overall productivity in the non-tradable sector $A(Z_d + Z_m^*)^{\frac{1}{\eta-1}}$. In other words, the location of firms alters both average TFP in the host country and the impact of the country specific wage shocks on the relative price between tradable and non-tradable goods.[12]

Profits for an individual Home firm with productivity z at Home are given by:

$$\pi_d(z,s) = \frac{1}{\eta}\cdot\frac{z^{\eta-1}}{Z_d + Z_m^*}\cdot X_{NT}\left(s\right), \tag{22}$$

and in Foreign:

$$\pi_m(z,s) = \frac{1}{\eta}\cdot\frac{z^{\eta-1}}{Z_d^* + Z_m}\cdot X_{NT}^*(s). \tag{23}$$

Hence, aggregate profits for domestic and multinational firms from Home are given by:

$$\Pi_d\left(s\right) = \int_{z_{min}}^{\infty} \pi_d(z,s)dG(z) = \frac{1}{\eta}\cdot\frac{Z_d}{Z_d + Z_m^*}\cdot X_{NT}\left(s\right), \tag{24}$$

$$\Pi_m\left(s\right) = \int_{\overline{z}}^{\infty} \pi_m(z,s)dG(z) = \frac{1}{\eta}\cdot\frac{Z_m}{Z_d^* + Z_m}\cdot X_{NT}^*\left(s\right). \tag{25}$$

Analogous expressions characterize aggregate profits of Foreign firms.

Profits of multinational firms, $\Pi_m\left(s\right)$, follow the evolution of total expenditure in non-

[12]Notice that the real exchange rate is $P(s)/P^*(s)$. From (5), the real exchange rate is lower in states where $W(s)$ is lower relative to $W^*(s)$.

tradable goods in the host market , X^*_{NT}. Since consumption is a CES bundle of homogenous and composite goods with elasticity $\rho < 1$, the share of expenditure in the non-tradable sector $X^*_{NT}(s)/X^*(s)$ increases in states where $P^*_{NT}(s)$ is higher.[13] That is, for a given level of expenditure, MP profits of Home multinationals are higher in those states where the host wage shock, W^*, is large.

Equilibrium for each good z, is given by the feasibility constraint in state s:

$$y(z,s) = c(z,s). \tag{26}$$

From (6), (9), (10) and (26), aggregate labor demands in the non-tradable sector, for national and foreign firms at Home, are:

$$\begin{aligned} L_d(s) &= \frac{\eta-1}{\eta}\cdot\frac{1}{W(s)}\cdot\frac{Z_d}{Z_d+Z^*_m}\cdot X_{NT}(s), \\ L^*_m(s) &= \frac{\eta-1}{\eta}\cdot\frac{1}{W^*(s)}\cdot\frac{Z^*_m}{Z_d+Z^*_m}\cdot X_{NT}(s). \end{aligned}$$

Labor demand in the homogeneous good sector at Home follows from the labor resource constraint:

$$L = L_h(s) + L_d(s) + L^*_m(s). \tag{27}$$

Finally, Home's net exports are given by:

$$NX(s) = L_h(s)\,W(s) - X_T(s).$$

Combining equations (5), (19) and the labor resource constraint (27), we can rewrite net exports

[13] In the Cobb-Douglas case, $\rho = 1$, income and substitution effects cancel out and $X^*_{NT}(s)/X^*(s)$ remains constant across states.

as follows:

$$NX\left(s\right)=LW\left(s\right)-\left(1-\frac{\xi_{PW}}{\eta}\right)X\left(s\right), \tag{28}$$

where ξ_{PW} corresponds to the price elasticity to the wage-shock in (21). Analogous condition characterizes Foreign's net exports.

2.2.2 International Equilibrium

FDI occurs before uncertainty is realized. From (23), MP profits increase in z. Therefore:

$$\sum_{s\in S}q(s)\frac{\partial}{\partial z}\pi_m(z,s)>0. \tag{29}$$

That is, expected MP profits increase in z. The optimal MP entry decision is therefore given by a cut-off rule, characterized by a productivity level $\overline{z}$ for which firms are indifferent between becoming multinationals or not:

$$\begin{aligned}\sum_{s\in S}q(s)\pi_m(\overline{z},s) &= f^*\cdot p_f, && (30)\\ \sum_{s\in S}q(s)\pi_m(\overline{z}^*,s) &= f\cdot p_f, && (31)\end{aligned}$$

where p_f is the world price of the investment good, and it can be interpreted as the equilibrium price that clears the FDI market. As long as there exists a positive entry cost $f\cdot p_f$, only the most productive firms do MP. It follows from (29) that expected MP profits net of entry cost increase in z.

The *national* equilibrium prices and quantities can be all characterized as functions of the state vector s, the cut-off rule for doing MP activities, $\overline{z}$ and $\overline{z}^*$, and aggregate expenditure at Home and Foreign, $X(s)$ and $X^*(s)$. Replacing these functions in (14), the aggregate budget constraint can be re-written as the Balance of Payment condition. We can now close the model and define the *international* equilibrium as follows:

Definition. For a given initial wealth, Y_0 and Y_0^*, the *international* equilibrium is a vector $[X(s), X^*(s), B(s), B^*(s)]$, for each $s \in S$, a pair $\{\overline{z}, \overline{z}^*\}$, and prices $[p_f, \{q(s)\}_{s\in S}]$ such that:

1. Euler equation (15) is satisfied for both countries;

2. The zero profit conditions for MP in equations (30) and (31) are satisfied;

3. The Arrow-Debreu securities are in zero net supply:

$$B(s) + B^*(s) = 0;$$

4. The world resource constraint at the initial investment period is satisfied:

$$Y_0 + Y_0^* = [1 - G(\overline{z})]f^* + [1 - G^*(\overline{z}^*)]f; \tag{32}$$

5. The intertemporal budget constraint (14) is satisfied for Home and Foreign

6. The resource constraint for the homogeneous tradable good holds, for each s:

$$NX(s) + NX^*(s) = 0. \tag{33}$$

3 MP under Country-Specific Uncertainty

We describe the main mechanism of the model assuming the following. First, to emphasize the effects of MP on international risk diversification, we assume that country-specific wage shocks take only two values, are symmetric, and perfectly negatively correlated across countries:

$s \in \{s_1, s_2\}$, where $s_1 = \{W_L, W_H^*\}$ and $s_2 = \{W_H, W_L^*\}$, with $W_L = W_L^*$ and $W_H = W_H^*$. Second, we assume that the source of world aggregate risk comes from a single asymmetry across countries: Home's non-tradable sector is more productive than Foreign's ($A > A^*$). As we show next, this single asymmetry implies different impact of country-specific shocks on international markets. The remaining parameters are assumed equal across countries.

We focus on the deterministic equilibrium, and analyze a perturbation of it. The main results are derived by comparing equilibria with and without MP.

3.1 Complete Financial Markets without MP

In a world with complete financial markets without MP, Arrow-Debreu prices co-move with shocks to the more productive country, that is, these prices are more responsive to shocks to the Home country. The existence of a full set of Arrow-Debreu securities guarantees perfect international risk sharing: countries "share" tradable goods such that the ratio of their marginal utilities of consumption is constant across states. From the Euler Equation (equation 15),

$$\frac{u_T'(C\,(s))}{u_T'(C^*\,(s))} = \frac{u'(C\,(s))/P\,(s)}{u'(C^*\,(s))/P^*\,(s)} = \frac{\lambda}{\lambda^*},$$

where λ and λ^* correspond to the Lagrange multipliers on the budget constraint for Home and Foreign, respectively.[14],[15] The only source of non-diversifiable risk in the consumption of tradable goods derives from the assumption on asymmetric countries: when $A > A^*$, a shock to Home has a larger impact on international markets than a shock of equal magnitude to Foreign. If countries were symmetric ($A = A^*$), all risk would be diversifiable: the world supply of the tradable good would be constant across states, and marginal utilities across countries would

[14]The multipliers λ and λ^* are also the inverse of the welfare weights of the corresponding planner's problem, which is presented in the Appendix.

[15]The existence of non-tradable goods prevents Purchasing Power Parity from holding. Indeed, each state of nature s is characterized by a different real exchange rate –i.e. $P\,(s)\,/P^*\,(s)$ –.

be equalized in each state, $u_T'(C(s)) = u_T'(C^*(s))$. Hence, the impact of (symmetric) country shocks on world resources would be identical whether the shock hits Home or Foreign. With symmetric countries, not only the ratio but also the level of marginal utilities –and Arrow-Debreu prices- would be constant across states (from 15), $u_T'(C(s_1)) = u_T'(C(s_2))$.

However, with asymmetric countries where $A > A^*$, even if shocks are perfectly negatively correlated across countries, there is some undiversifiable risk derived from the asymmetric impact of country shocks on world markets. Foreign cannot fully insure Home, and the marginal utility of consumption in tradable goods is higher in states where the negative shock hits Home, $u_T'(C(s_1)) > u_T'(C(s_2))$. In other words, Arrow-Debreu prices are higher in $s_1 = \{W_L, W_H^*\}$ than in $s_2 = \{W_H, W_L^*\}$: $q(s_1) > q(s_2)$. Proposition 1 proves this result.

Proposition 1. *Assume $A > A^*$. Countries are otherwise identical. For $\frac{\sigma}{(\sigma-1)} < \frac{\eta}{1-\rho}$, in a world without MP, Arrow-Debreu prices are higher in s_1 than in s_2, where $s_1 = \{W_L, W_H^*\}$ and $s_2 = \{W_H, W_L^*\}$.*[16]

Proof. See Appendix. □

The mechanism behind Proposition 1, that is, the co-movement of Arrow-Debreu prices with shocks to the more productive country, follows from the different reaction of countries' net exports to country shocks. This reaction results from consumers deciding, in each state, the allocation of consumption between tradable and non-tradable goods, governed by the elasticity of substitution ρ; and the allocation of consumption across states, governed by the risk aversion σ. The elasticity of substitution η among non-tradable goods governs the size of the mark-up charged by firms, and therefore, affects the amount of labor they demand. Higher η implies

[16]Notice that we can still impose that the cross-derivative of the marginal utility of consumption of tradable goods (C_T) with respect to non-tradable goods (C_{NT}) is positive, as in Tesar (1993): $1-\rho > 1-1/\sigma > (1-\rho)/\eta$. Furthermore, for the range of empirical estimates for these parameters, both conditions hold: $\rho = 0.44$ (Stockman and Tesar, 1993), $\sigma = 2$ (Backus and Smith, 1992), and $\eta = 3.1$ (Broda and Weinstein 2006).

stronger reaction of labor demand to changes in prices. Intuitively, the condition in Proposition 1 requires consumers having relatively high risk aversion σ, low elasticity of substitution across tradable and non-tradable goods, ρ, and within non-tradable goods, η.[17] In other words, consumers prefer relatively more to stabilize consumption across states than to stabilize the composition of consumption between tradable and non-tradable goods within states. Interestingly, although preference parameters are assumed identical across countries, the consumers' response to symmetric shocks is different in each country: the impact of wage shocks on the relative price of non-tradable goods is lower for the more productive economy. Consequently, the composition of consumption (and labor allocation) between tradable and non-tradable goods is also less volatile at Home, within states. Then, when this economy is hit by a positive wage shock, relatively more tradable goods are freed into the international market than when Foreign is hit by a similar positive shock; the Arrow-Debreu price in this state decreases to induce a shift in the demand for tradable goods toward this "tradable-abundant" state. In other words, to smooth marginal utilities of consumption across states, Home relies more on sharing risk across states with Foreign, that is freeing tradable goods into the international market, than changing the composition of consumption between tradable and non-tradable goods within each state.

Lets analyze in more detail the different reactions of countries' net exports to country shocks. Consider the following perturbation to the deterministic equilibrium: $dW^* > 0$–a positive shock to Foreign wages–, and $dW = 0$. Define the elasticity of x with respect to y as $\xi_{xy} = \frac{\partial x}{\partial y}\frac{y}{x}$.

Differentiating the Euler Equation in (15) with respect to W^*, the elasticity of Arrow-Debreu prices to the shock W^* is:

$$\xi_{qW^*} = -\sigma\xi_{C^*W^*} - \xi_{P^*W^*} = -\sigma\xi_{CW^*}. \tag{34}$$

[17]With perfect competition ($\eta \to \infty$), labor demand reacts one-to-one to changes in relative prices, and the condition in Proposition 1 always holds.

Proposition 1 showed that, with $A > A^*$, $\xi_{qW} < \xi_{qW^*} < 0$.

In the deterministic case, the consumers' intertemporal budget constraint implies $NX = NX^* = 0$. From equation (28), for Foreign, we have:

$$NX^* = L^*W^* - X^* \left(1 - \frac{\xi_{P^*W^*}}{\eta}\right). \tag{35}$$

Further, feasibility for tradable goods implies $NX + NX^* = 0$, and differentiating with respect to W^*, we get:

$$\frac{dNX}{dW^*}W^* + \frac{dNX^*}{dW^*}W^* = 0. \tag{36}$$

The reaction of Foreign's net exports in equation (36) to W^* is then:

$$\frac{dNX^*}{dW^*}W^* = L^*W^* + \frac{d\xi_{P^*W^*}}{dW^*}W^*\frac{X^*}{\eta} - (L^*W^* - NX^*)\,\xi_{X^*W^*} > 0, \tag{37}$$

where $W^*d\xi_{P^*W^*}/dW^* = (1-\rho)\,\xi_{P^*W^*}\,(1-\xi_{P^*W^*}) > 0$ (from equation 21), and $\xi_{X^*W^*} = \left(\frac{\sigma-1}{\sigma}\right)\xi_{P^*W^*} - \frac{1}{\sigma}\xi_{qW^*} > 0$ (from the first equality in 34).

Foreign net exports react to the shock through three channels. The first term in equation (37) corresponds to the direct effect of an increase in productivity in the tradable sector. The second term captures the effect of the wage shock on labor demand in the non-tradable sector: higher wages translate in a higher relative price of non-tradable goods that, in turns, shift demand away the non-tradable sector and, consequently, free labor from this sector towards the tradable sector. These first two terms positively affect Foreign net exports. The third term counteracts: it corresponds to the increase in Foreign's absorption X^* that follows from an increase in both the aggregate price level P^*, and aggregate consumption C^*. This increase in C^* is induced by the decrease in Arrow-Debreu prices. Indeed, these prices decrease with the wage shock, $\xi_{qW^*} < 0$.

Analogously, the reaction of Home net exports to the Foreign shock W^* is given by:

$$\frac{dNX}{dW^*}W^* = -\left(LW - NX\right)\xi_{XW^*}X < 0, \tag{38}$$

where, from (34), $\xi_{XW^*} = \xi_{CW^*} = -(1/\sigma)\xi_{qW^*} > 0$, because W^* do not affect Home prices $-\xi_{PW^*} = 0$. Hence, the effect of the Foreign wage shock on Home net exports entirely depends on the impact on Home aggregate consumption.

Combining equations (36), (37), and the fact that in the deterministic equilibrium $NX^* = 0$, the elasticity of Arrow-Debreu prices with respect to W^* is:

$$\xi_{qW^*} = \sigma\frac{L^*W^*}{LW + L^*W^*}\left(-1 + \Psi_C^*\right), \tag{39}$$

where

$$\Psi_C^* \equiv \xi_{P^*W^*}\left[\frac{\sigma-1}{\sigma} - (1-\rho)\frac{(1-\xi_{P^*W^*})}{(\eta-\xi_{P^*W^*})}\right], \tag{40}$$

and $\Psi_C^* < 1$. Hence, $\xi_{qW^*} < 0$. In particular, the response of Arrow-Debreu prices to Foreign shocks, given by the elasticity in (39), increases (in absolute terms) with relative country size, $\frac{L^*W^*}{LW+L^*W^*}$, and for $\frac{\sigma-1}{\sigma} > \frac{1-\rho}{\eta}$, decreases (in absolute terms) with $\xi_{P^*W^*}$. Since $A > A^*$, we have $\xi_{P^*W^*} > \xi_{PW}$.[18] It follows that $\xi_{qW} < \xi_{qW^*} < 0$, as established in Proposition 1.

Summing up, a wage shock has larger impact on Arrow-Debreu prices when it hits the more productive economy. Equivalently, a state with a positive wage shock to Home, and a symmetric negative shock to Foreign implies a lower Arrow-Debreu price.

[18] $\Psi_C < \Psi_C^* < 1$.

3.2 Complete Financial Markets with MP

By reallocating technologies, and altering relative productivity across countries, MP reshapes the patterns of world risk. Proposition 2 shows that, if production reallocates from Home (the most productive country) to Foreign, the gap in the marginal utility of consumption of tradable goods across states is smaller in an economy with MP (denoted by MP) than in an economy without MP (denoted by C):

$$0 < u'^{MP}_T(C(s_1)) - u'^{MP}_T(C(s_2)) < u'^{C}_T(C(s_1)) - u'^{C}_T(C(s_2)).$$

Equivalently, the gap in Arrow-Debreu prices across states narrows.

Proposition 2. *Assume $A > A^*$. Countries are otherwise identical. For $\frac{\sigma-1}{\sigma} > \frac{1-\rho}{\eta}$, the gap in Arrow-Debreu prices across states narrows if production reallocates from Home to Foreign.*

Proof. See Appendix. □

MP acts by (i) increasing productivity in the non-tradable sector in Foreign, and (ii) increasing (decreasing) Foreign (Home) net exports in all states of nature.

Intuitively, an increase in the number of firms located in Foreign results in a less volatile consumption bundle of tradable and non-tradable goods (and hence labor allocation across sectors), within states. The counterpart is that Foreign's net exports react more to its own shock than in a world without MP, which results in a higher impact of Foreign shocks on Arrow-Debreu prices.

Consider again the following perturbation to the deterministic equilibrium: $dW^* > 0$, and $dW = 0$. The reaction of Foreign and Home's net exports is characterized, as in the case without MP, by equations (37) and (38). However, due to MP, net exports are not necessarily zero; MP entry costs and net MP profits have to be taken into account. Replacing in the intertemporal budget constraint in equation (14) the initial wealth from equation (16), the intertemporal budget constraint for Foreign, in the deterministic case, can be re-written as zero balance of payment:

$$NX^{*}+\left[\Pi_{m}^{*}-\Pi_{m}\right]+\frac{p_{f}}{\beta}\left\{Y_{0}-f\left[1-G\left(\overline{z}^{*}\right)\right]\right\}=0, \tag{41}$$

where Π_m and Π_m^* are aggregate MP profits for Home and Foreign, respectively, given by equation (25). Further replacing MP profits by (25), and using the zero profit condition for FDI in (31), we get:[19]

$$NX^{*}=-\pi_{m}\left(\overline{z}\right)\left\{\frac{Z_{m}^{*}}{\overline{z}^{*\eta-1}}-\frac{Z_{m}}{\overline{z}^{\eta-1}}-\left[\frac{Y_{0}}{f}-\left(1-G\left(\overline{z}\right)\right)\right]\right\}. \tag{42}$$

The difference between equation (35) and (42) is given by the existence of multinational firms. If Home firms do more MP than Foreign firms, then ($\overline{z}<\overline{z}^*$). Given the definition for Z_m in (18), then $Z_m > Z_m^*$, and from the feasibility condition for the investment good in (32), $2Y_0/f = 1-G(\overline{z})+[1-G(\overline{z}^*)$, it follows that $0<\frac{Z_m^*}{\overline{z}^{*\eta-1}}-\frac{Z_m}{\overline{z}^{\eta-1}}<\frac{Y_0}{f}-(1-G(\overline{z}))$. Then, Foreign's net exports are positive: tradable goods are shipped to Home under the form of net MP profits. Moreover, also from (32), we get that $d\overline{z}/d\overline{z}^*=-dG(\overline{z}^*)/dG(\overline{z})$. Differentiating (42), it can be proved that Foreign's net exports increase with the number of affiliates from Home located there, $dNX^*/d\overline{z}<0$ (and the corresponding reduction of affiliates from Foreign located at Home, $dNX^*/d\overline{z}^*>0$).

Using (36), (37) and (38), the impact of a Foreign shock on Arrow-Debreu prices is given by:

$$\xi_{qW^{*}}=\sigma\frac{L^{*}W^{*}}{LW+L^{*}W^{*}}\left[-1+\Psi_{MP}^{*}\left(1-\frac{NX^{*}}{L^{*}W^{*}}\right)\right], \tag{43}$$

where Ψ_{MP}^* is defined analogously to Ψ_C^* in equation (40), and NX^* is given by equation (28).

[19] Recall that we assume $f=f^*$ and $Y_0=Y_0^*$.

The key result here is that Arrow-Debreu prices are more responsive to a Foreign shock when there is more MP (lower $\overline{z}$) located there, $\partial \left|\xi_{qW^*}\right| / \partial \overline{z} < 0$.

Comparing equation (43) with the analogous equation for the case without MP, (39), it is clear that MP increases the impact of Foreign shocks on Arrow-Debreu prices through two channels:

1. The non-tradable sectors become more similar across countries when firms reallocate towards the least productive country. Aggregate productivity in the non-tradable sector is given by $A\left(Z_d + Z_m^*\right)^{\frac{1}{\eta-1}}$ for Home, and $A^*\left(Z_d^* + Z_m\right)^{\frac{1}{\eta-1}}$ for Foreign. These expressions include the country-specific productivity (A), and an aggregate index of firm-specific productivity, that includes domestic (Z_d) and foreign firms (Z_m^*) —as defined in equations (17) and (18). Recall that the price index for non-tradable goods in Foreign is given by:

$$P_{NT}^*(s) \quad = \quad \frac{\eta}{\eta-1} \cdot \frac{W^*(s)}{A^*} \cdot \left(Z_d^* + Z_m\right)^{\frac{1}{1-\eta}}.$$

   From the price elasticity in (21), we know that $\xi_{P^*W^*} = 1 - P^{*\rho-1}$. Thus, an increase in Z_m results in lower $\xi_{P^*W^*}$. Comparing Ψ_{MP}^* and Ψ_C^*, as defined in equation (40), it follows that $\Psi_C^* > \Psi_{MP}^*$.

2. From equation (41), an increase in the number of multinationals operating in Foreign increases Foreign's net exports.

Hence, we conclude that $\Psi_C^* > \Psi_{MP}^*(1 - NX^*/L^*W^*)$. Comparing (39) and (43), more MP from Home to Foreign (lower $\overline{z}$) increases the reaction of Arrow-Debreu prices to Foreign shocks, i.e. higher $|\xi_{qW^*}|$.

3.3 Incentives to do MP

In the previous subsection we assume that reallocation of production occurs from Home (the most productive country) to Foreign (the least productive). Proposition 3 shows that indeed firms from Home have more incentives to open affiliates in Foreign, and that such incentives increase with risk.

First, we show that in the deterministic economy, production reallocates from Home to Foreign. Since the elasticity of substitution between tradable and non-tradable goods is assumed lower than one ($\rho < 1$), profits of multinationals, that produce non-tradable goods, are larger in the economy with higher prices or, equivalently, lower productivity in the non-tradable sector. The following lemma formalizes the result.

Lemma 1. *Assume $A > A^*$. Countries are otherwise identical. For $\rho < 1$, in the deterministic case, firms from Home do more MP ($\overline{z} < \overline{z}^*$).*

Proof. See Appendix. □

Proposition 3. *Assume $A > A^*$. Countries are otherwise identical. For $\frac{\sigma}{(\sigma-1)} < \frac{\eta}{1-\rho}$, an identical mean preserving spread over $W(s)$ and $W^*(s)$ increases the number of firms from Home and reduces the number of firms from Foreign doing MP. As a result, the aggregate net expected discounted flow of MP profits for Home firms increases.*

Proof. See Appendix. □

In a risky environment, the incentives for Home firms to open affiliates in Foreign increase with risk, while the opposite it is true for Foreign firms. As analyzed in Section 2.2, with inelastic demand functions, MP profits co-move with host country risk. In particular, profits for Home affiliates located in Foreign are relatively larger in state $s_1 = \{W_L, W_H^*\}$, while profits for Foreign affiliates located in Home are relatively larger in state $s_2 = \{W_H, W_L^*\}$. Thus, MP profits for Home multinationals are higher when world output is relatively scarce, and Arrow-Debreu prices are higher. Hence, risk provides more incentives to open foreign affiliates to firms from the more productive country. Consider an identical mean preserving spread over $W(s)$ and $W^*(s)$. From Proposition 1, Arrow-Debreu prices are lower when a positive shock to $W(s)$ hits Home (and a symmetric negative shock to $W^*(s)$ hits Foreign). Thus, an increase in shocks volatility widens the gap of marginal utilities across states, $u_T'(\widetilde{C}(s_1) - u_T'(\widetilde{C}(s_2)) > u_T'(C(s_1)) - u_T'(C(s_2))$, where $\widetilde{C}$ denotes consumption after the mean preserving spread. In other words, more risk amplifies the gap in Arrow-Debreu prices across states, and therefore increases the value of insurance. The amount of Home firms doing MP increases while the one for Foreign decreases.

Net Asset Position

The counterpart of an increase in the net value of MP for Home is a reduction in the position on Arrow-Debreu securities, $\sum_{s \in S} q(s) B(s)$; the opposite is true for Foreign. Home's consumers demand for insurance is increasingly satisfied through MP profits, while Foreign consumers rely more on Arrow-Debreu securities. Indeed, from the intertemporal budget constraint for Home in (14), where initial wealth $B_0 = p_f \left[Y_0 - f^* \int_{z \in Z} \tau(z) dG(z)\right]$, the position on Arrow-Debreu securities is given by the demand for resources to finance MP entry costs. Consequently, as Home does more MP, it needs more resources to finance the opening of affiliates abroad. Thus, Home reduces its position on Arrow-Debreu securities, and increases its position on Direct Investment. Conversely, Foreign increases its position on Arrow-Debreu securities, and decreases its position on Direct Investment.

Since a mean preserving spread increases the demand for insurance, it also increases the number of Home firms doing MP, ($\widetilde{\overline{z}} < \overline{z}$), and the price of MP, i.e. the price of the investment good, ($\widetilde{p}_f > p_f$). As a counterpart, Home's position on Arrow-Debreu securities is lower in a riskier world,

$$\sum_{s \in S} \widetilde{q}(s)\, \widetilde{B}(s) - \sum_{s \in S} q(s)\, B(s) = \widetilde{p}_f \left[Y_0 - \left(1 - G\left(\widetilde{\overline{z}}\right)\right) f^* \right] - p_f \left[Y_0 - \left(1 - G\left(\overline{z}\right)\right) f^* \right] < 0.$$

Summarizing, more risk, captured by a mean preserving spread over W and W^*, increases the amount of (net) MP done by the most productive country (Home) and reduces its (net) Arrow-Debreu position.

Indeed, Home's overall net asset position is unambiguously improved when Direct Investment is measured as the expected discounted flow of profits due to the fact that only the most productive firms do MP. Since the value of doing MP is equal to the entry cost for the marginal firm with productivity $\overline{z}$ but bigger for all other firms with $z > \overline{z}$, the aggregate expected discounted flow of MP profits net of MP costs increases with the number of multinationals. Thus, the increase in the aggregate amount of resources needed to finance MP –debt– is less than the increase in the stream of aggregate expected MP profits. This manifests on Home's Balance of Payment as an overall increase in its net asset position: a more than proportional shift toward Direct Investment away from other financial assets.

Concluding, MP affects international risk patterns in a world with asymmetric countries even under the existence of complete financial markets. Technology flows entailed by MP affect the relative size of economies, and the impact of country shocks on world markets. In particular, as firms from the most productive country have more incentives to do MP, 1) countries become more similar in terms of the impact of their shocks on world output, reducing the gap in Arrow-Debreu prices across states, and 2) countries shift their international portfolios toward Direct Investment away from other financial assets. These results are reinforced when risk diversification is more valuable, that is, the larger the volatility of country-specific shocks, or the higher the risk aversion of consumers.

The mechanism described in this section relies on three crucial assumptions of the model: (i) countries are asymmetric; (ii) affiliates abroad bear the shock specific to the *host* country; and (iii) heterogenous firms can replicate their home technology abroad after paying a fixed entry cost.

Finally, allocations from this problem are efficient. As shown in the Appendix, the distribution of firms across countries, and consumption allocations across states coincide with those chosen by a social planner constrained by monopolistic competition in the non-tradable sector.

4 Numerical Example

We present a numerical example to illustrate the main results of the model explained in the previous section. We compare a two-country world with MP and no MP.

We assume that shocks follow a symmetric two-state process, and are perfectly negatively correlated across countries.

$$W(s_1) = \overline{W}(1-\Delta) \qquad W^*(s_1) = \overline{W}^*(1+\Delta^*),$$

$$W(s_2) = \overline{W}(1+\Delta) \qquad W^*(s_2) = \overline{W}^*(1-\Delta^*),$$

with $\overline{W} = \overline{W}^*$ and $\Delta = \Delta^*$.

The model presented in Section 2 has two periods. The second period can be interpreted as the infinite future, with shocks following a stationary Markov chain and future consumption discounted at the rate β. When shocks follow a stationary Markov chain, the problem presented in Section 2 can be re-written recursively. The (per-state) budget constraint is:

$$
\begin{aligned}
P(s)C(s) + \int_{z\in Z} \theta'(z)Q(z,s)dG(z) + \sum_{s'\in S} q(s'|s)B(s') \\
= LW(s) + B(s) + \int_{z\in Z} \theta(z)[\pi(z,s) + Q(z,s)]dG(z)
\end{aligned}
$$

where $B(s')$ corresponds to a state-contingent one-period Arrow-Debreu security, and $q(s'|s)$ is its price conditional on the realization of s, given by the following Euler equation:

$$
q(s'|s) = \frac{q(s')}{q(s)} = \beta \frac{u_T'(C(s'))}{u_T'(C(s))} \Pr(s'|s),
$$

and $Q(z,s)$ corresponds to the market price of a firm with productivity z:

$$
Q(z,s) = \sum_{s'\in S} q(s'|s)[\pi(z,s') + Q(z,s')].
$$

Similarly, the value of doing MP for a firm with productivity z is given by:

$$
Q_{MP}(z,s) = \sum_{s'\in S} q(s'|s)[\pi_m(z,s') + Q_{MP}(z,s')]. \tag{44}
$$

Arrow-Debreu securities can be reinterpreted as a portfolio position, denoted by $\widehat{B}(s)$, and its (stochastic) rate of return $R(s'|s)$ can be computed accordingly:

$$
\begin{aligned}
\widehat{B}(s) &= \sum_{s'\in S} q(s'|s)B(s') \\
R(s'|s) &= \frac{B(s') - \widehat{B}(s)}{\widehat{B}(s)}.
\end{aligned}
$$

We interpret the income from this portfolio position as income from assets other than Direct

Investment, and refer to it as "Other Assets".

For current utility, we use a CRRA function with risk aversion parameter σ, equal across countries. We assume that the productivity distribution for the non-tradable sector, in each country, is Pareto with parameters z_{min} and γ:

$$G(z) = 1 - \left(\frac{z_{min}}{z}\right)^{-\gamma},$$

where $z_{min} = z^*_{min}$ and $\gamma = \gamma^*$.

The preferences and Pareto distribution parameters are taken from the literature:

Parameters	Values	Source
σ	2	Backus, Kehoe and Kydland (1992)
ρ	0.44	Stockman and Tesar (1993)
η	3.1	Broda and Weinstein (2006)
β	0.98	to match $r = 2\%$
γ	4	Helpman, Melitz and Yeaple (2004)
z_{min}	1	normalization

As in the previous section, we focus on a single source of asymmetry across countries: $A > A^*$. We normalize $\overline{W} = \overline{W}^* = 1$, $L = L^* = 1$, and $f = f^* = 1$. We calibrate A and A^* so as to match some characteristics of the US (Home) and the Rest of the World (Foreign):

1. The relative productivity of non-tradable at Home $A/\overline{W}$ is calibrated to match the US share of non-tradable goods:
$$\frac{X_{NT}}{X} = 72\%.$$

2. The relative productivity in non-tradable goods between Home and Foreign, A/A^*, is calibrated to match the ratio of sales of affiliates from US abroad to sales of foreign affiliates

in the US, as recorded by BEA for the period 1985-2003, which requires $A > A^*$:[20]

$$\frac{Sales\ by\ US\ affiliates\ abroad}{Sales\ by\ Foreign\ affiliates\ in\ US} = 1.58.$$

3. The initial endowment of investment good Y_0 is assumed equal across countries and set relative to f to match sales of foreign affiliates in US as share of GDP equal to 20% (from BEA).

We analyze the effects of an identical mean-preserving spread on W and W^* with $\Delta = \Delta^*$.

In Figure (1), panel A, we show Arrow-Debreu prices in each states s_1 and s_2, in a world with and without MP, and $A > A^*$. A shock volatility $\Delta = \Delta^* = 6.7\%$ corresponds to a standard deviation of consumption at Home of 2% and 3.5% in Foreign, while $\Delta = \Delta^* = 15\%$ implies Home consumption volatility of 4.6%, and 8% for Foreign. As shown in Proposition 1, Arrow-Debreu prices are higher in s_1 (when Home is hit by the bad shock) than in s_2 (when Foreign is hit by the bad shock). The presence of MP narrows the gap in Arrow-Debreu prices across states (Proposition 2). For a shock volatility, Δ, consistent with a standard deviation in consumption of at Home of 2% (and 3.4% in Foreign), the existence of MP reduces the gap in Arrow-Debreu prices across states in 17.1%.

Panel B in Figure (1) shows net MP profits for Home multinationals across states and the average Net Value of FDI. Average across states are computed using the unconditional Arrow-Debreu prices as a probability measure:[21]

$$FDI - FDI^* = \sum_s q(s) \left[\int_{\bar{z}}^{\infty} Q_{MP}(z,s) dG(z) - \int_{\bar{z}^*}^{\infty} Q^*_{MP}(z,s) dG(z) \right].$$

[20] Recall, that in the model, sales are proportional to profits Π_m.

[21] Notice that in risky environment with risk-averse consumers, the stream of MP profits should be discounted using Arrow-Debreu prices that take into account risk-adjustments. The unconditional Arrow-Debreu prices correspond to the stationary limit of the Markov probabilities.

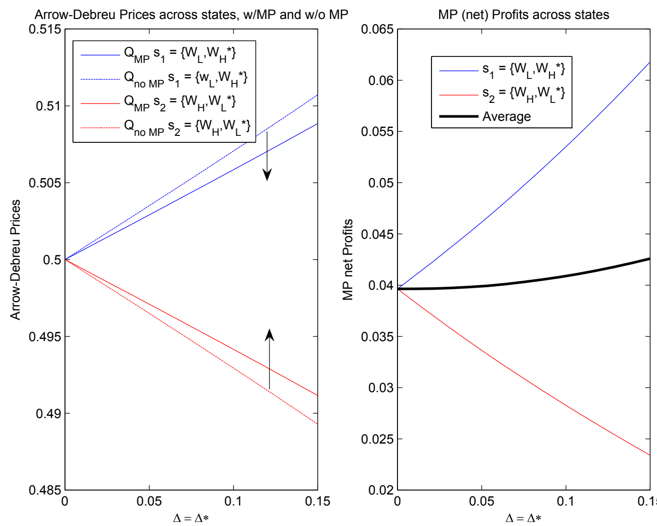

Figure 1:

Net MP profits for Home multinationals are higher in $s_1 = \{W_L, W_H^*\}$ than in $s_2 = \{W_H, W_L^*\}$ (the opposite characterizes Foreign multinationals), which coincides with the co-movement of Arrow-Debreu prices across states: $q(s_1) > q(s_2)$.

Table 1 shows the balance of Payment for Home, in a world with and without MP, with $\Delta = \Delta^* = 5\%$ that matches a consumption volatility of 1.5% at Home, and 2.6% in Foreign. Indeed, net exports deficit is much larger in a world with MP: it represents 3.1% of GDP, and

(% of GDP)	MP	no MP
Net Exports	-3.1	-0.1
Net Factor Payments	3.2	0.32
Net "Other Assets" Position	-47	0.96
Net Direct Investment Position†	195	–
Net Asset Position	147	0.96
% of Multinational Firms: Home	24	
% of Multinational Firms: Foreign	15.6	

(†): At cost value, net Direct Investment position represents 97% of GDP.

Table 1: Balance of Payments for Home.

only 0.1% with no MP. As expected, net factor payments are larger with MP: these are the sum of (net) returns to "other assets" (that is negative for Home), and net MP profits that are largely positive. To finance its multinational activities, Home has a (net) debt with Foreign that reaches 47% of GDP. Indeed, this debts disappears when there is no MP: the net position in "other assets" is 0.96% of GDP. The net direct investment position is positive and large when measured as in equation (44). The result is a positive overall net asset position of 147% of GDP. Finally, as shown in the previous section, Home has relatively more multinational firms than Foreign: 24% versus 15.6%.

Finally, Table 2 compares the standard deviation of consumption and its composition with and without MP, in Home and Foreign, for $\Delta = \Delta^* = 5\%$. As shown in Proposition 2, the existence of MP reduces the gap in the price of Arrow-Debreu securities across states. Arrow-Debreu prices are proportional to the marginal utility of consumption of tradable goods, which explains why the standard deviation of consumption in tradable goods decreases in a world with MP both in Home and in Foreign. In this numerical example, the existence of MP reduces standard deviation of consumption in tradable goods in 46% and 11%, for Home and Foreign, respectively.

The impact of MP on the volatility of consumption in non-tradable goods varies across countries. While the introduction of MP unambiguously decreases the standard deviation of non-tradable consumption in Foreign, the same does not necessarily hold for Home. To see this, consider a positive shock to wages at Home and a symmetric negative shock in Foreign: $dW = -dW^* > 0$. Combining equation (7) for demand for the non-tradable good $X_{NT}(s)$, and the elasticity of Arrow-Debreu prices to the shock in (34), we can compute the elasticity of consumption of non-tradable goods to the shock, around certainty, as follows:

$$\begin{aligned}
\xi_{NT,W} &= -\rho\left(1-\xi_{PW}\right)-\frac{1}{\sigma}\xi_{PW}-\frac{1}{\sigma}\left(\xi_{qW}-\xi_{qW^*}\right) \\
\xi_{NT^*,W^*} &= -\rho\left(1-\xi_{P^*W^*}\right)-\frac{1}{\sigma}\xi_{P^*W^*}+\frac{1}{\sigma}\left(\xi_{qW}-\xi_{qW^*}\right),
\end{aligned}$$

where $\xi_{NT,W}=\frac{dC_{NT}}{dW}\frac{W}{C_{NT}}$ and, from Proposition 1, $(\xi_{qW}-\xi_{qW^*})<0$.

The introduction of MP affects $\xi_{NT,W}$ and ξ_{NT^*,W^*} in two ways: First, it results in a larger number of firms operating in both countries (this effect is particularly important in Foreign where most new affiliates are located). From equation (21), this results in a lower price elasticities to the shock, (ξ_{PW} and $\xi_{P^*W^*}$), in both countries. And second, as shown in Proposition 2, the introduction of MP reduces the gap between ξ_{qW} and ξ_{qW^*}.

For $\rho<\frac{1}{\sigma}$, as in our calibration, the two factors contribute to reduce the impact of the shock in consumption of non-tradable goods in Foreign (-2% in our numerical example). In the case of Home, the two factors counteract without an overall prediction on the ultimate impact of MP on the volatility of consumption in non-tradable goods. In our numerical example, the standard deviation of consumption in non-tradable goods at Home is 3% larger in a world with MP than without MP, and results in a higher standard deviation of aggregate consumption C.

(σ_x/E_x)	MP	no MP
	(%)	
C	1.52	1.51
C^*	2.66	2.73
C_{NT}	2.13	2.07
C^*_{NT}	2.78	2.84
C_T	0.07	0.13
C^*_T	0.57	0.64

Table 2: Consumption Volatility.

5 Conclusions

This paper emphasizes the connection between production location and the pattern of international risk. In particular, the scope for international risk diversification is improved when production is reallocated towards economies with business cycles less correlated with the world risk pattern. Reallocation of production towards such economies may be triggered by a number of different factors, namely a reduction on trade cost, improvements in the investment opportunities in those countries, or, as analyzed in this paper, by "Multinational Production".

The main contribution of this paper is to uncover the dual role of MP as a technology and portfolio flow in international risk sharing. MP affects international risk diversification even with complete financial markets, as it alters host country's productivity, and the impact of country specific shocks on international goods and assets markets. Moreover, we find that risk affects the optimal location of multinational firms, and therefore, the international portfolio composition of countries. In particular, risk sharing considerations provide, endogenously, more incentives to firms from the *most productive* country to do MP and consequently: 1) brings countries closer together in terms of the impact of their country shocks on world output reducing differences between Arrow-Debreu prices across states; and 2) shifts its international portfolio composition toward Direct Investment away from other financial assets. These results are reinforced when risk diversification is more valuable, that is, the larger the volatility of country-specific shocks.

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6 Appendix: Proofs

6.1 Proof of Lemma 1

By contradiction. Assume $\overline{z} = \overline{z}^*$ in the zero profit conditions (30) and (31). Since $G(z) = G^*(z), f = f^*$, and $Y_0 = Y_0^*$, it follows that $\frac{Z_m^*}{\overline{z}^{*\eta-1}} = \frac{Z_m}{\overline{z}^{\eta-1}}$ and $f^*(1 - G(\overline{z})) = f(1 - G(\overline{z}^*)) = Y_0 = Y_0^*$. Replacing in equation (42), the aggregate expenditure functions on non-tradable goods in the two countries are given by:

$$
\begin{aligned}
X_{NT} &= \frac{\xi_{PW}}{\eta - \xi_{PW}} \eta L W \\
X_{NT}^* &= \frac{\xi_{P^*W^*}}{\eta - \xi_{P^*W^*}} \eta L^* W^*
\end{aligned}
$$

Countries are assumed to only differ in the non-tradable productivity: $A > A^*$. The price elasticity with respect to the shock negatively depends on the productivity in the non-tradable sector

$$
\frac{\partial \xi_{PW}}{\partial A} = -\frac{(1-\rho)}{A} \xi_{PW} (1 - \xi_{PW}) < 0
$$

which implies $\xi_{PW} < \xi_{P^*W^*}$. It follows that $X_{NT} < X_{NT}^*$ and therefore $\overline{z} < \overline{z}^*$, which is a contradiction. With the same argument it can also be ruled out that $\overline{z} > \overline{z}^*$. It follows that if $A > A^*$, then $\overline{z} < \overline{z}^*$.

6.2 Proof of Proposition 3

Notice that a mean preserving spread implies $\pi_m'(s_1) - \pi_m'(s_2) > \pi_m(s_1) - \pi_m(s_2)$, which combined with the widening in the gap in Arrow-Debreu prices across states results in an increase in the discounted MP profits of Home firms (and the opposite for Foreign's). That is, for all $z : \sum_{s \in S} q'(s) \pi'(z,s) > \sum_{s \in S} q(s) \pi'(z,s) > \sum_{s \in S} q(s) \pi(z,s)$.

Then, for the marginal firms in Home and Foreign, $\overline{z}$ and $\overline{z}^*$, the following inequalities are satisfied (from MP entry condition):

$$
\frac{\sum_{s \in S} q'(s) \pi'(\overline{z}, s)}{f^*} > \frac{\sum_{s \in S} q(s) \pi(\overline{z}, s)}{f^*} = \frac{\sum_{s \in S} q(s) \pi(\overline{z}^*, s)}{f} > \frac{\sum_{s \in S} q'(s) \pi'(\overline{z}^*, s)}{f}.
$$

which implies that a mean preserving spread lowers the cut-off level for Home firms ($\overline{z}' < \overline{z}$) and rises it for Foreign firms ($\overline{z}^{*\prime} > \overline{z}^*$). Therefore: $[1 - G(\overline{z}')] > [1 - G(\overline{z})]$, while the amount of

MP by Foreign firms decreases, $[1-G(\overline{z}^{*\prime})] < [1-G(\overline{z}^{*\prime})]$ Finally, it follows that

$$\sum_{s\in S} q'(s)\left(\Pi'_m(s)-\Pi^{*\prime}_m(s)\right) > \sum_{s\in S} q(s)\left(\Pi_m(s)-\Pi^*_m(s)\right)$$

6.3 Social Planner

The optimal allocation $\Gamma = \{C_T(s), C_T^*(s), C_{NT}(s), C_{NT}^*(s), L_h(s), L_h^*(s), \overline{z}, \overline{z}^*\}$ corresponds to the following program

$$\begin{array}{rcl} & & \max_{\Gamma} \sum_{s} \beta \Pr(s)\{\lambda u(C(s)) + \lambda^* u(C^*(s))\} \\ & & s.t. \\ C(s) & = & \left[C_T(s)^{\frac{\rho-1}{\rho}} + C_{NT}(s)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \\ C^*(s) & = & \left[C_T^*(s)^{\frac{\rho-1}{\rho}} + C_{NT}^*(s)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \\ C_{NT}(s) & = & \frac{\eta-1}{\eta} A(L-L_h(s))(Z_d+Z_m^*)^{\frac{1}{\eta-1}} \\ C_{NT}^*(s) & = & \frac{\eta-1}{\eta} A^*(L-L_h^*(s))(Z_d^*+Z_m)^{\frac{1}{\eta-1}} \\ C_T(s)+C_T^*(s) & = & W(s)L_h(s)+W^*(s)L_h^*(s) \\ Y_0+Y_0^* & = & [1-G(\overline{z})]f^* + [1-G^*(\overline{z}^*)]f \end{array}$$

where the term $\frac{\eta-1}{\eta}$ corresponds to the distortion imposed by monopolistic competition in the non-tradable sector. The existence of monopolistic competition introduces a gap $\left(\frac{\eta-1}{\eta}<1\right)$ between the marginal rate of substitution and the marginal rate of transformation of tradable to non-tradable goods:

$$\left(\frac{C_T(s)}{C_{NT}(s)}\right)^{\frac{1}{\rho}} = \frac{\eta}{\eta-1}\frac{W(s)}{A}(Z_d+Z_m^*)^{\frac{1}{1-\eta}}$$

This expression is equivalent to the ones derived for the decentralized equilibrium (7), (8), and the relative price between tradables and non-tradables in (19). Replacing, we can express aggregate consumption as a function of consumption in the tradable good:

$$C(s) = C_T(s)\left[1+\left[\frac{\eta}{\eta-1}\frac{W(s)}{A}(Z_d+Z_m^*)^{\frac{1}{1-\eta}}\right]^{1-\rho}\right]^{-\frac{\rho}{1-\rho}}$$

The efficient allocation involves perfect international risk sharing:

$$\frac{\lambda}{\lambda^*}=\frac{u'\left(C^*\left(s\right)\right)}{u'\left(C\left(s\right)\right)}\frac{\left[1+\left[\frac{\eta}{\eta-1}\frac{W(s)}{A}\left(Z_d+Z_m^*\right)^{\frac{1}{1-\eta}}\right]^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[1+\left[\frac{\eta}{\eta-1}\frac{W^*(s)}{A^*}\left(Z_d^*+Z_m\right)^{\frac{1}{1-\eta}}\right]^{1-\rho}\right]^{\frac{1}{1-\rho}}}=\frac{u_T'\left(C^*\left(s\right)\right)}{u_T'\left(C\left(s\right)\right)}$$

Notice that the condition above is equivalent to the decentralized condition for international risk sharing derived from the Euler Equation (15) and the price level (5).

Finally, the optimal entry decision for foreign and domestic firms is characterized by:

$$\sum_s \beta\lambda \Pr\left(s\right)u'\left(C\left(s\right)\right)\left[1+\left[\frac{\eta}{\eta-1}\frac{W\left(s\right)}{A}\left(Z_d+Z_m^*\right)^{\frac{1}{1-\eta}}\right]^{1-\rho}\right]^{-\frac{1}{1-\rho}}\frac{W\left(s\right)}{\eta-1}\left(1-L_h\left(s\right)\right)\frac{\overline{z}^{*\eta-1}}{\left(Z_d+Z_m^*\right)}=f\mu_0$$

Notice that in the decentralized problem, the following expressions characterize the labor in the non-tradable sector, profits for the marginal firm doing MP, and price level:

$$\begin{aligned}
\left(1-L_h\left(s\right)\right) &= \frac{\eta-1}{\eta}\frac{1}{W\left(s\right)}X_{NT}\left(s\right)\\
\pi_m\left(\overline{z}^*,s\right) &= \frac{1}{\eta}\cdot\frac{\overline{z}^{*\eta-1}}{Z_d+Z_m^*}\cdot X_{NT}(s).\\
P\left(s\right) &= \left[1+\left[\frac{\eta}{\eta-1}\frac{W\left(s\right)}{A}\left(Z_d+Z_m^*\right)^{\frac{1}{1-\eta}}\right]^{1-\rho}\right]^{\frac{1}{1-\rho}}
\end{aligned}$$

Then, using the Euler Equation for the Arrow-Debreu prices (15), the optimal entry condition of the social planner is equivalent to the following decentralized solution:

$$\begin{aligned}
\sum_s q\left(s\right)\pi_m^*\left(\overline{z}^*,s\right) &= f\mu_0\\
\sum_s q\left(s\right)\pi_m\left(\overline{z},s\right) &= f^*\mu_0
\end{aligned}$$

which corresponds to the free entry condition in the decentralized economy (30).