Minimum Wages and Positive Employment Effects in General Equilibrium

Tom Ahn **Peter Arcidiacono***

Duke University Duke University

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Abstract

We develop a general equilibrium search model with the following features: a) endogenous entry, and therefore zero expected profits by firms, b) endogenous labor supply by workers, and c) the possibility of having positive employment effects from an increase in the minimum wage. Total employment depends jointly on the numbers of searching firms and searching workers. An increase in the minimum wage induces some workers who were previously not searching to participate in the labor market. Welfare implications are similar to the classical analysis: workers who most want the minimum wage jobs are hurt by the increase in the minimum wage with workers who were marginally interested in the minimum wage jobs benefiting. We estimate the model using data from the CPS and show that small changes in the employment level are masking large changes in labor supply and labor demand.

Keywords: Minimum wages, search, unemployment

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*Department of Economics, Duke University Box 90097, Durham, NC 27708. Phone: 919-660-1816. Psarcidi@econ.duke.edu. We thank John Kennan, Michael Pries, Curtis Taylor, and seminar participants at Duke University, New York University, and Pennsylvania State University.

1 Introduction

Until recent years, the debate over the effects of the minimum wage on employment has been an econometric issue, with the understanding that the classical prediction of decreased employment due to an increase in the minimum wage held. The classical analysis operates from a simple labor supply and demand framework with the minimum wage serving as a price floor. An over-supply of workers and too little demand for labor by firms results in a decrease in employment and an increase in the unemployment rate. The consensus among published studies was that there were relatively modest effects on employment and income distribution due to an increase in the minimum wage.

However, empirical work by Card and Krueger (1994, 1995) suggest that, in some cases, an increase in the minimum wage leads to an increase in employment. There has been considerable controversy regarding these findings (See Welch (1995), Neumark and Wascher (1995), and Brown (1999)). Most notably, there has been an active debate between Neumark and Wascher and Card and Krueger on minimum wage employment at fast food restaurants in New Jersey and Pennsylvania. While Card and Krueger found modest positive employment effects due to a minimum wage hike, Neumark and Wascher found a small negative employment effect. Card and Krueger estimated the labor demand elasticity to be between 0.54 and 0.89. Neumark and Wascher estimated the labor demand elasticity to be between -0.21 and -0.22.

Labor monopsony models have been advanced as theoretical support for the Card and Krueger studies. Under labor monopsony models, the individual firm faces an upward sloping labor supply curve. Raising wages for one worker then implies raising wages for all workers. A binding minimum wage removes the tie between hiring another worker and raising the wages of everyone else. Hence, in a labor monopsony model it is possible to increase employment by increasing the minimum wage. However, until recently, these models have been treated with skepticism by economists who believe entry and exit by firms and workers should compete away monopsony rents (Boal and Hanson (1997)).

We develop a two-sided search model with endogenous labor supply and labor demand that can exhibit positive employment effects from an increase in the minimum wage. In the classical analysis, the number of searching workers has no effect on the number of matches. In a more general search model, the number of matches increases with the number of searching workers. Hence, increasing the minimum wage may induce search which can lead to higher employment levels even with the number of firms falling. However, these positive employment effects also lead to lower probabilities of matching at the individual level. As in Luttmer (1998) and Glaeser (1996), in expectation, those with the lowest reservation wages are hurt most by the increase in the minimum wage when workers are randomly allocated to jobs.

Our search model can therefore generate zero or positive employment growth, which duplicates the primary benefit of using labor monopsony models. However, unlike monopsony models, firms here have zero expected profits both before and after the minimum wage hike.

The search model shows that the effect of a minimum wage hike may appear small because the variable used to measure this effect, employment level, does not adequately capture the churning of the labor market. Individuals induced to enter the labor market result in more matches and may not lower the employment level. However, the new matches push out those who originally wanted minimum wage jobs. Therefore, there are possibly large negative welfare effects from a minimum wage hike, even if the employment level stays constant or increases.[1]

Our theoretical model makes four predictions about the effects of a minimum wage hike: a) probability of finding employment conditional on searching always decreases, b) employment level change is ambiguous, c) a larger number of higher reservation workers enter, and d) lower reservation workers may be worse off. We estimate a structural econometric model and simulate increases in the minimum wage to show that the predictions of the theory are supported. We use a twelve year band (1989 to 2000) sample of 16 to 19 year old white teenagers from the basic monthly outgoing rotation CPS files and minimum wage at the state level in each month of the sample band.

We find that the employment is indeed masking large changes in labor supply and demand. This particularly holds for workers who are relatively unattached to the labor force; the youngest individuals in our sample and those who come from wealthier families. We estimate the effect of a 10% increase in the minimum wage and find that it would only lower the employment level for sixteen to nineteen year-olds by -0.88%. However, the labor demand elasticity is estimated to be -2.9%, where the difference between the effect on the employment level and the demand elasticity can be explained by increased labor supply.

Flinn (2003) also estimates a structural model of how a minimum wage increase affects employment. His paper has the advantage of working in a continuous time search model as opposed to the static model presented here.

[1] We are aware of only one other paper that has firms earning zero expected profits and yet can still generate employment increases from a minimum wage hike. Lang and Kahn (1998) use a model with segmented labor markets and firms having preferences for workers of a particular type to obtain the result. Without a minimum wage, an equilibrium exists where second-class workers have high employment rates and low wages while preferred workers have just the opposite and are better off in expectation. For a minimum wage slightly above the equilibrium wage in the low wage market, Lang and Kahn show that an equilibrium exists where now some preferred workers search for minimum wage jobs, trading off higher wages for higher employment probabilities, and that this effect can more than compensate for the loss of jobs for the second-class workers

However, this comes at a cost. In our model, firms earn zero expected profits so the probability of receiving an offer responds to a change in the wage. Further, our model allows for the possibility of positive employment effects and heterogeneity in reservation values. This latter feature is particularly important in assessing winners and losers form a minimum wage hike.

The rest of the paper proceeds as follows. Section 2 shows the classical model and how it does and does not relate to the matching model. Section 3 develops the two-sided search model, with welfare and employment analysis in section 4. Section 5 describes the data that we use to estimate the model. The translation from the theoretical model to what is estimated is done in section 6. Section 7 presents the estimation results. Section 8 performs the policy simulations and Section 9 concludes.

2 The Classical Model

The classical analysis of the effects of a minimum wage can be found in most introductory economics textbooks. However, by first examining the classical model it is possible to see why in our model it is possible to have positive employment effects from an increase in the minimum wage while in the classical model it is not. Further the welfare implications of our model will turn out to be very similar to those of the classical model.

Figure 1 shows the implications of an increase in the minimum wage in the classical model. Employment here falls from Q^* to $\underline{Q}$. Note that the employment level only depends upon labor demand. How elastic, or inelastic, labor supply may be has no effect on the employment level. This is the primary difference between the classical model and matching models. Matching models rely on a 'matching function' which takes the number of searching worker and the number of searching firms and produces an employment level. The matching function in the classical model is the minimum of the number of searching firms and the number of searching workers which must be the number of searching firms when there is a binding minimum wage. However, other matching functions that depend upon both the number of searching firms and the number of searching workers can produce increases in the employment level because the increased labor supply may more than compensate for the decreased labor demand.

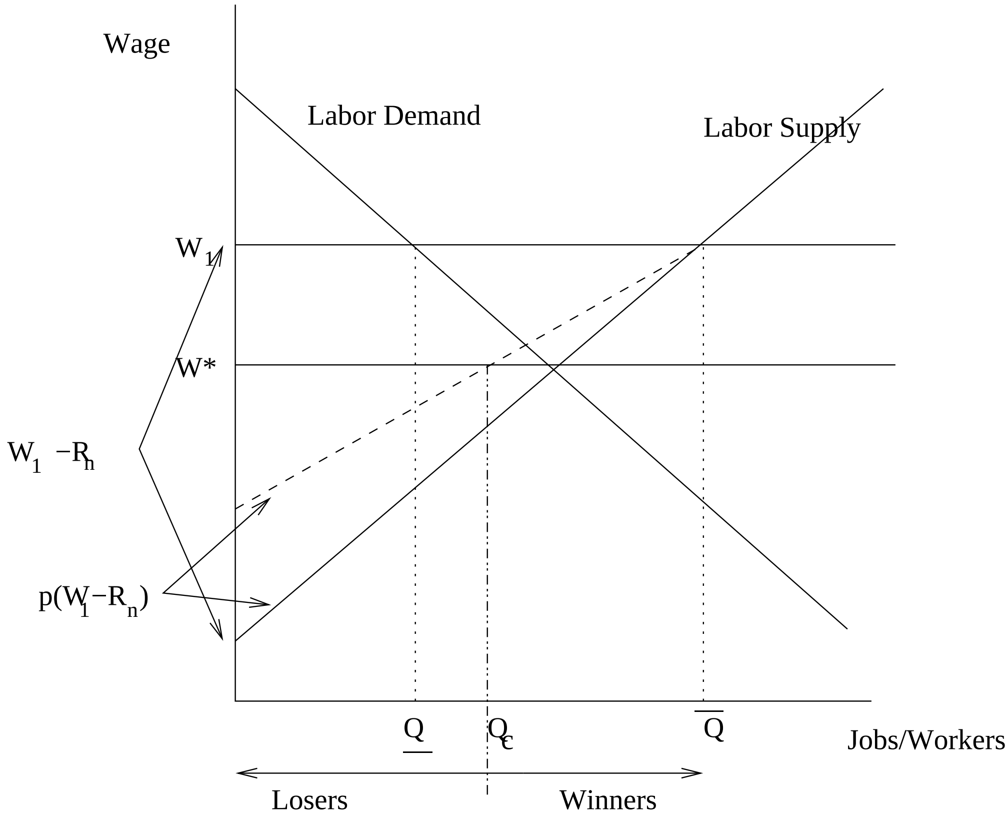
The classical model requires an additional assumption as to how jobs are assigned because there is an excess supply of workers. In Figure 1 we have assumed that the probability of employment is the same across searching workers. This probability of finding a minimum wage job is given by $\underline{Q}/\overline{Q}$ where $\overline{Q}$ is the number of individuals interested in working at the minimum wage. The area between the labor supply curve and the curves that kinks at $\overline{Q}$ gives the expected surplus over the reservation wage of the workers. Note that the expected surplus is smaller with the minimum wage increase for all workers below Q_c. These are the workers who were most interested in being employed and would be willing to trade a lower wage for a higher probability of employment.

The matching model described below has very similar welfare implications. If there are losers because of a minimum wage increase, it will be those individuals who were most interested in being employed. Winners are then those individuals who would either not be interested or only marginally interested in being employed at the market clearing wage.

3 The Matching Model

In this section we present a two-sided search model designed to highlight the effects of a minimum wage increase in the low wage market. There are $\overline{N}$ individuals available to search. Individuals are differentiated in their reservation values for not working. The ith individual has reservation value R_i, where R_i is drawn from the cumulative distribution function $F(R)$ and has support $[0, \infty)$ This reservation value can be leisure or any outside option for workers. For instance, we may assume that R_i is the value of schooling for teenagers, with the treatment effect of education varying across the population. Adept students expect to acquire more human capital in school and will therefore consider the

Figure 1: Classical Employment Losses From a Minimum Wage Increase†



minimum wage job as a less attractive option.

Denote p_i as the probability that the ith worker would find an acceptable match conditional on searching. Denote K as the search cost which is uniform across individuals and is paid whether an individual matches with a firm or not.[2] Individuals are risk neutral and the expected value of searching, V_i, is given by:

$$V_i = p_i[E(W|W > R_i) - R_i] - K, \tag{1}$$

where W is the wage. The number of searching workers, N, is endogenous and individuals search when $V > 0$

The number of firms, J, is endogenous. All firms are identical and therefore have identical probabilities of a successful match with a worker, q. Production from a match is given by S and firms pay a search cost, C, whether or not they find a match. Firms enter until all firms have zero expected profits. Expected profits are then given by:

$$qE(S - W|S - W > 0) - C = 0. \tag{2}$$

We assume that the surplus of a match is given by:

$$S_{ij} = \overline{S} - \zeta_{ij} \tag{3}$$

where $\overline{S}$ is the average match value. ζ_{ij} is then a match-specific component with zero mean, is drawn from the cumulative distribution function $G(\zeta)$, and has support $[\underline{\zeta}, \overline{\zeta}]$.

We now specify the wage-generating process. Matched pairs split S according to generalized Nash bargaining, with the caveat that a successful match pays at least the minimum wage, $\underline{W}$.[3] The worker's bargaining power is set at β, $\beta \in (0, 1)$. Wages are then given by:

$$W_{ij} = \max\{\beta(\overline{S} + \zeta_{ij}), \underline{W}\}. \tag{4}$$

[2]Heterogeneity in the values of the search cost has no effect on the qualitative results.

[3]See Flinn (2003) for a similar specification.

The splitting of the surplus in this manner can generate the spike observed at the minimum wage in the data. All matches where the worker's share of the surplus would normally be below the minimum wage will earn the same wage even if their match-specific components differ.

Note that Nash bargaining in the wage generating process does *not* include the worker's reservation value. This may seem incomplete as the usual bargaining framework defines surplus as the total value of the match minus the worker's outside option. However, reservation values here are very different than in the standard search literature. In this model, reservation values are not tied to the labor market in ways that are easily verifiable through such factors as unemployment rates. Rather, reservation values here refer to the differentiated values of leisure or education. We do not believe either of these factors influence wages in the low wage labor market.[4]

A successful match must ensure that both the firm and the worker prefer W_{ij} over not matching. Unproductive matches, whether because of low match values or unverifiable reservation values, significantly complicate the model. Let ζ_A give the expected value of ζ conditional on a match being acceptable and let π_A be the corresponding probability of an acceptable match conditional on matching. We make the following assumption which ensures that the wage generating process described above only produces successful matches:

A.1 $\qquad \pi_A \beta \zeta_A - \beta \underline{\zeta} < K$ for all $\{\pi_A, \zeta_A\}$

This assumption means that the match-specific component is relatively unimportant compared to the cost of searching– an assumption that may be more reasonable in the market for low wage workers than in other markets. If the search costs are sufficiently high relative to the match-specific component, conditional on searching, all matches will be accepted. In the appendix, we show

[4]If workers differed systematically in their reservation values but not in their productivities, groups of workers with low reservation values would be more highly sought after than their high reservation counterparts. Hence, if teenagers from rich families have higher reservation values than their low income counterparts, these teenagers have higher unemployment rates, something which is not observed in the data.

that this assumption does indeed generate only successful matches.

Lemma 1 *Regardless of reservation value, a worker who finds it optimal to search will accept any match.*

Lemma 2 *A firm which decides it is profit-maximizing to search will accept any match.*

To close the model we need to specify the matching function and the corresponding probabilities of finding a match. Although many matching functions allow for positive employment effects from an increase in the minimum wage, we use a Cobb-Douglas matching function as in Pissarides (1992) to illustrate the result because of its prevalence in the literature[5] and also because we use it in the empirical section. The number of matches is then given by:

$$x = \min(AJ^{\alpha}N^{1-\alpha}, J, N), \tag{5}$$

where $\alpha \in (0,1)$ and A is a normalizing constant. All workers and firms have the same probability of finding a match, $p = \frac{X}{N}$ and $q = \frac{X}{J}$. Proposition 1 then establishes that an equilibrium for this model exists.

Proposition 1 *Given equations (1) - (7), assumption A.1, $F(R)$, $G(\zeta)$, $\underline{W}$, K, C, α, $\overline{S}$, and $\overline{N}\}$, there exists an equilibrium in N and J.*

4 Implications of the Model

The model described above has a number of implications for a minimum wage increase. In this section we describe how a minimum wage increase affects the probability of matching and conditions under which a minimum wage increase positively affects the employment level. We further show conditions under which a minimum wage increases welfare for all searching workers and show that if these conditions are not met which workers are hurt.

[5] Indeed, most of the empirical literature is in agreement that there is a stable aggregate matching function of the Cobb-Douglas form and constant returns to scale in unemployment and job vacancies. See Petrongolo and Pissarides (2001) for a review.

We first show that a minimum wage hike will always lower the probability of an individual obtaining a minimum wage job, even if the employment level increases.

Proposition 2 $\frac{dp}{d\underline{W}} < 0$, *regardless of the signs of* $\frac{dN}{d\underline{W}}, \frac{dJ}{d\underline{W}}$, *and* $\frac{dx}{d\underline{W}}$.

The intuition comes from examining the expected zero profit condition of the firms. In particular, if the costs of the firm increase, the probability of finding a match for the firm must fall in order for the expected zero profit condition to hold. Since an increase in the match of the probability of the firm means a decrease in the match probability of the worker, we have the result. This holds whether or not the employment *level* has increased.

Although the probability of finding a job always falls with an increase in the minimum wage, the effect on the employment level is ambiguous. Proposition 4 outlines conditions on the labor demand and supply elasticities under which positive employment effects due to a minimum wage hike are possible.

Proposition 3 $\frac{dx}{d\underline{W}} \geq 0$ *if* $\alpha\varepsilon_{LD} + (1-\alpha)\varepsilon_{LS} \geq 0$, *where* ε_{LD} *is the elasticity of labor demand and* ε_{LS} *is the elasticity of labor supply.*

Proposition 3 explicitly demonstrates that the direction of growth of employment is jointly dependent on the elasticity of labor supply and demand. Furthermore, since both elasticities depend on J and N, which are endogenous, as well as $\underline{W}$, the model can exhibit positive or negative employment effects. This is because an increase in $\underline{W}$ will generally pull J and N in opposite directions, which then leads to labor demand and supply elasticities being pulled in opposite directions. This dual effect on the employment level helps to explain not only why different studies have found positive and negative employment effects, but also why the magnitude of the effects has been so small. Since J and N are moving in opposite directions, α, or the measure of sensitivity of the matching function to a relative increase in J/N, helps to determine which effect is larger. As $\underline{W}$ increases, ε_{LD} and ε_{LS} continue to offset each other, which translates to a small movement in the employment level.

The proposition above gave conditions which relied upon endogenous variables. We next demonstrate sufficient conditions in terms of the exogenous parameters of the model to generate positive employment effects. However, we cannot do this without explicitly defining the cumulative distribution function for the reservation values. We focus on the case where R is distributed uniform $[0, \overline{R}]$, though conditions under which the exogenous parameters generate positive employment effects with a minimum wage hike are easy to show for many possible distributions of the reservation values.

Proposition 4 *If R is distributed uniform $[0, \overline{R}]$ and $1 - \frac{\alpha}{1-\alpha}\frac{E(W)}{\overline{S}-E(W)} > 0$ then $\frac{dx}{d\underline{W}} > 0$.*

Although proposition 4 is only an existence condition, it is instructive to analyze what the expression reveals about the employment effect. The expression is more likely to be positive if $E(W)$ and α are small. A small $E(W)$ means less of the per-match surplus is given up to the worker. Less firms then pull out of the market in response to a minimum wage hike, which helps to increase the employment level. A small value for α means that the matching function is less sensitive to changes in J. We interpret this to mean that the labor market is not 'tight,' and employers face some trouble matching workers to already open positions. This is a good sketch of the fast food industry in the Card and Krueger studies. A significant percentage of managers at fast food establishments offer cash incentives to employees to refer new people for employment.

The model also has implications for the effect of a minimum wage increase on the welfare of workers. In particular, it is possible to have an increase in the minimum wage and have all workers be made better off.[6] Denote $E_1(W)$ and p_1 as the expected wage and probability of finding a match before the minimum wage increase. Denote $E_2(W)$ and p_2 as the corresponding values after the

[6] The same implications hold in the classical model. If the drop in the probability of finding a job is small relative to the increase in the wage, even low reservation workers are made better off.

minimum wage increase. All workers are made better off by the minimum wage increase if:

$$p_1 E_1(W) < p_2 E_2(W).$$

Workers have reservation values that are bounded below by zero. The two expressions above give the unconditional expected wage, implying that since the probability of finding a job is lower and yet unconditional expected wage is actually higher after the minimum wage increase all workers must be made better off. Proposition 5 shows the necessary and sufficient condition for this to be the case.

Proposition 5 *All workers benefit from a marginal increase in the minimum wage if and only if* $\left[1 - \frac{(1-\alpha)}{\alpha} \frac{E(W)}{\overline{S}-E(W)}\right] > 0$

Note that proposition 5 looks very similar to the conditions necessary for positive employment effects. However, positive employment effects depend upon the distribution of the reservation values. This is the not the case here as the relevant variable is not N but N/J. Given an expected wage, there is only one value for N/J that will satisfy the zero profit condition of the firms. Since in the zero profit condition N/J is raised to the power α, small α's mean large changes are necessary for N/J in order for the zero profit condition to be met. This is why the $\alpha/(1-\alpha)$ term in proposition 5 enters inversely here.

If the conditions for proposition 5 are not met, then some workers are made worse off by the increase in the minimum wage. In particular, as in the classical model discussed in section 2, it is those workers who most want the minimum wage jobs who are hurt by the increase. These are the workers who have the lowest reservation values and hence are more willing to trade a lower paying job for a higher probability of employment.

5 Data

In this section we present the data used in the econometric analysis. We use a twelve year band of the basic monthly outgoing rotation survey files of the

Current Population Survey (CPS) from 1989 to 2000. These twelve years are suitable for analysis because there were four federal minimum wage changes during this period. More importantly, fifteen states[7] changed their state minimum wage to outpace the federal wage, at various points in time. The range of observed minimum wage in the twelve years run from \$3.35 to \$6.50.[8] We select white teenage workers (16 years to 19 years) during non-summer months[9] to look at the employment and wage effects from an increase in the minimum wage. The CPS is especially useful for this analysis because the survey explicitly asks for hourly wage. From the CPS, we collect hourly wage, whether the individual is searching for work or not, whether the searching worker is employed or not, parent weekly income, years of parental education, and other demographic characteristics, such as whether the teenager is from a single parent household.

Parent weekly income is a proxy for the level of monetary support the teenager has from his parents. A higher level of monetary support from parents should lower the teenager's willingness to search for a minimum wage job. We restrict the analysis to teenagers who are identified as having positive parental income.[10] If a parent was surveyed to be employed, but did not enter an income amount, the household was treated as having a missing income.

The level of parental education is indicative of the extent to which the family regards education for the teenager as important. That is, a teenager with parents with a high level of education should be receiving encouragement to focus on studying. Therefore, the reservation value of a teenager with well-educated parents should be higher, compared to a teenager with parents with a lower level of education, all else equal. Using these two measures of reservation,

[7] There are actually seventeen states that paced ahead of the federal minimum wage, but we exclude Alaska and Hawaii from analysis.

[8] We look at state minimum wages because some of the larger and more populous states, such as California, New York, and Pennsylvania have state minimum wages above the federally mandated minimum wage and are raised on different schedules.

[9] We exclude June, July, and August.

[10] Restricting observations to positive parent income is important because it is difficult to capture the reservation wage for a teenager with no visible source of financial support. See McElroy (1985).

we hope to capture much of the concrete sources of reservation (outside support) as well as preference for education over minimum wage work.

Table 1 contains general descriptive statistics from the CPS portion of the data. Individuals between 16 and 19 years old inclusive, were included in the sample, and were identified as employed, unemployed but looking for work, or not searching for work. Observations with employed individuals earning less than the minimum of state or federal minimum wage were dropped, as well as individuals who reported earning more than $ 10 per hour, about 0.79% of the sample.[11]

Searchers are more likely to come from low income families with low parental education and are more likely to be in a single parent household. However, high income, highly educated, two parent families are more likely to actually find a job conditional on searching. This latter trend will be picked up in our model by these individuals being in states with better economies.

We also use the Monthly Labor Review to collect minimum wage at the state/month level. That is, from 1989 to 2000, we observe the minimum wage in each state, each month. Table 2 presents the minimum wage in each state, each month within the range of the collected CPS data. These minimum wages are nominal values. In the analysis, the wages and incomes are deflated to account for annual inflation.

[11]Deflated to 1988 dollars. The empirical estimates for wage distribution was not significantly altered when the ceiling for hourly earnings was dropped or raised.

Table 1: Description of Data

Variable	Mean	Std. Dev	Min	Max
Entire Sample (N: 120686)				
Age	17.36	1.102	16	19
Female	0.4757	0.4994	0	1
Pr(Search)	0.4808	0.0970	0	1
Single parent	0.1824	0.3862	0	1
Family size	3.802	1.167	2	15
Parent education (years)	13.74	2.869	0	21
Weekly parent income ($000's)	692.64	433.71	0	4131.05
Missing income	0.2668	0.4423	0	1
Searching Workers (N: 58025)				
age	17.62	1.077	16	19
Female	0.4745	0.4994	0	1
Pr(Employed—Search)	0.8387	0.0895	0	1
Single parent	0.1951	0.3963	0	1
Family size	3.767	1.155	2	14
Parent education (years)	13.58	2.649	0	21
Weekly parent income ($000's)	674.97	402.97	0	4093.15
Missing income	0.2346	0.4238	0	1
Employed Workers (N: 48668)				
Hourly wage	4.30	1.02	3.02	10
Age	17.66	1.068	16	19
Female	0.4835	0.4997	0	1
Single parent	0.1829	0.3866	0	1
Family size	3.790	1.153	2	14
Parent education (years)	13.66	2.601	0	21
Weekly parent income ($000's)	693.53	400.43	0	4093.15
Missing income	0.2377	0.4257	0	1

Table 2: State Minimum Wage from 1989 to 2000.

State	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
California	4.25	4.25	4.25	4.25	4.25	4.25	4.25	4.75	$5.00^{\diamond}$	$5.75^{\diamond}$	5.75	5.75
Connecticut	4.25	4.25	$4.27^{\dagger}$	4.27	4.27	4.27	4.27	4.77°	5.18^{*}	5.18	$5.65^{\ddagger}$	$6.15^{\ddagger}$
Iowa	3.35	$3.85^{\ddagger}$	$4.25^{\ddagger}$	$4.65^{\ddagger}$	4.65	4.65	4.65	4.75	5.15	5.15	5.15	5.15
Massachusetts	3.75	3.75	3.75	4.25	4.25	4.25	4.25	$4.75^{\ddagger}$	$5.25^{\ddagger}$	5.25	5.25	$6.00^{\ddagger}$
Maine	$3.75^{\ddagger}$	$3.85^{\ddagger}$	$4.25^{\dagger}$	4.25	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
Minnesota	$3.85^{\ddagger}$	$3.95^{\ddagger}$	$4.25^{\ddagger}$	4.25	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
New Hampshire	$3.65^{\ddagger}$	$3.75^{\ddagger}$	$3.85^{\ddagger}$	$4.25^{\dagger}$	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
New Jersey	3.35	$3.80^{\dagger}$	3.80	$5.05^{\dagger}$	5.05	5.05	5.05	5.05	5.15	5.15	5.15	5.15
New York	3.35	$3.80^{\dagger}$	3.80	$4.25^{\dagger}$	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
Oregon	3.85^{*}	$4.25^{\ddagger}$	$4.75^{\ddagger}$	4.75	4.75	4.75	4.75	4.75	$5.50^{\ddagger}$	$6.00^{\ddagger}$	$6.50^{\ddagger}$	6.50
Pennsylvania	$3.70^{\ddagger}$	$3.80^{\dagger}$	$4.25^{\dagger}$	4.25	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
Rhode Island	$4.25^{\triangledown}$	4.25	$4.45^{\dagger}$	4.45	4.45	4.45	4.45	4.75	5.15	5.15	$5.65^{\triangle}$	6.15^{*}
Vermont	$3.75^{\triangle}$	$3.85^{\triangle}$	$4.25^{\dagger}$	4.25	4.25	$4.50^{\ddagger}$	4.50	4.75	5.15	5.15	5.75°	5.75
Washington	$3.85^{\ddagger}$	$4.25^{\ddagger}$	4.25	4.25	4.25	$4.90^{\ddagger}$	4.90	4.90	5.15	5.15	$5.70^{\ddagger}$	$6.50^{\ddagger}$
Wisconsin	$3.65^{\triangle}$	$3.80^{\dagger}$	3.80	4.25	4.25	4.25	4.25	4.75	5.15	5.15	5.15	5.15
Other States	3.35	$3.80^{\dagger}$	$4.25^{\dagger}$	4.25	4.25	4.25	4.25	4.75°	5.15^{*}	5.15	5.15	5.15

‡ Minimum wage change on 1/1 or 1/2. $\diamond$ Minimum wage change on 3/1. $\dagger$ Minimum wage change on 4/1.

$^{\triangle}$ Minimum wage change on 7/1 or 7/2. $\triangledown$ Minimum wage change on 8/1. $*$ Minimum wage change on 9/1.

$\circ$ Minimum wage change on 10/1.

6 Parameterizing the Model

In this section we show how to estimate the structural model. Estimation proceeds in the three steps. In the first step we estimate the wage generating process from which we can calculate expected wages. Although we cannot observe $\overline{S}$, the average surplus value, from the wage generating process, we are able to estimate βS. The second step uses the zero profit conditions for the firms to estimate β (worker's share when the minimum wage does not bind) and α (sensitivity of the matching function to changes in the number of firms). Although we do observe the probability of a firm finding a match, we are able to rewrite the zero profit condition as a function of the individual's probability of finding a match. Given estimates from the previous two steps, we can calculate the expected wage and the probability of finding a match for the individual. Substituting these into the individual's valuation of search and parameterizing the reservation values then allows us to estimate the parameters of the utility function.

6.1 Parameterizing Wages

We assume that the surplus is log-normally distributed. Since wage is defined as: $W_{ij} = \min\{\beta S, \underline{W}\}$, the wage distribution is also distributed log-normally with censoring at the minimum wage. The CPS sample has multiple censoring points at different states and different years.

We regress log wages on age, state, month, and year indicator variables to take into account the ebbs and flows of the business cycle and allow productivity and bargaining power to be different across age and state. The estimated parameters are then used to calculate expected wages at the age, state, month, and year level. We can further calculate the βS's by forecasting the expected wage without the censoring at the minimum wage.

6.2 Parameterizing Firms

Although we have no information on the firm, we can infer the parameters of the profit function by rewriting the zero profit condition as a function of the individual's probability of finding a match. To see this, note that the probability of finding a match for firms and workers is given by:

$$q = A\left(\frac{N}{J}\right)^{1-\alpha} \qquad\qquad p = A\left(\frac{J}{N}\right)^{\alpha}$$

implying that we can write q as:

$$q = A^{\frac{\alpha}{\alpha-1}} p^{\frac{\alpha-1}{\alpha}}$$

Substituting for q as a function of p in the zero profit condition yields:

$$A^{\frac{\alpha}{\alpha-1}} p^{\frac{\alpha-1}{\alpha}} (\overline{S} - E(W)) - C = 0$$

Solving for p yields:

$$p = \delta(\overline{S} - E(W))^{\frac{\alpha}{1-\alpha}}$$

where:

$$\delta = C^{\frac{-\alpha}{1-\alpha}} A^{-1}$$

This zero profit condition is satisfied for every economy. That is, zero profits hold by age, state, month and year.

Given estimates from the log wage regression of $\beta\overline{S}$ and $E(W)$, we can estimate $1/\beta$, α, and a transformation of A and C using the search outcomes of the workers. In particular, positive search outcomes for workers are drawn with the probability given above. The likelihood function is then given by:

$$L_m = \prod_{i=1}^{I_m} \left(\delta\left(\frac{(\beta\overline{S})}{\beta} - E(W)\right)^{\frac{\alpha}{1-\alpha}}\right)^{m_i=1} \left(1 - \delta\left(\frac{(\beta\overline{S})}{\beta} - E(W)\right)^{\frac{\alpha}{1-\alpha}}\right)^{m_i=0}$$

where I_m is the number of searching workers and m_i indicates whether or not the ith worker was matched.

We then allow the β's to vary by state and age. Identification of the β's comes from the relationship between observed wages and the probability of employment in the data. If a particular state has a high expected wage, this can either be because of a strong economy or because worker bargaining power is high in the state. If a high expected wage translates into a high probability of finding a job, then we have evidence of the former. However, if we see a high expected wage and a high unemployment rate this must be because worker bargaining power is high.

6.3 Parameterizing the Individual

We now turn to the decision by individuals as to whether or not to search. Recall that an individual searches if:

$$p(E(W) - R_i) - K > 0.$$

With the estimates from the previous two stages it is possible to calculate expected wages and the probability of employment for each individual. We now need to parameterize the reservation values. In particular, we parameterize R_i such that all workers have positive reservation values:

$$R_i = \exp(X_i\gamma + \epsilon_i)$$

X_i is then a vector of demographic characteristics which affect the individual's outside option, the γ's are the coefficients to be estimated, and ϵ_i is the unobserved portion of the reservation value.

Substituting in and solving for ϵ_i shows that an individual will search when:

$$\epsilon_i < \log\left(E(W) - \frac{K}{p}\right) - X_i\gamma$$

We assume that the ϵ's are distributed N(0,1). Since we do not observe the ϵ's, the likelihood function is then given by:

$$L_s = \prod_{i=1}^{I} F\left(\log\left(E(W) - \frac{K}{p}\right) - X_i\gamma\right)^{s_i=1}\left(1 - F\left(\log\left(E(W) - \frac{K}{p}\right) - X_i\gamma\right)\right)^{s_i=0}$$

where I is the total number of potential searchers and F is the standard normal cdf.

Table 3: Estimates from the Firm's Zero Profit Condition

	Coefficient	Std. Error
$C^{\frac{-\alpha}{1-\alpha}} A^{-1}$	0.6009	0.0118
α	0.3242	0.0048
β (Alabama)	0.6553	0.0090
β adjustments		
California	0.0546	(0.0034)
Colorado	0.0293	(0.0044)
Nebraska	-0.0218	(0.0051)
North Dakota	-0.0183	(0.0051)
Age=17	-0.0205	(0.0023)
Age=18	-0.0032	(0.0022)
Age=19	0.0064	(0.0024)

7 Results

Having specified the estimation strategy, we now turn to the results. Table 3 shows the estimates of the parameters in the firm's zero profit condition. The β's for states which are discussed later are reported in the table with the rest of the state-specific β's given in the appendix. α, how sensitive the number of matches is to the number of searching firms, is estimated at 0.32. This is slightly lower than the 0.4 to 0.6 estimates typically found in the literature (Petrongolo and Pissarides 2001), though the estimation strategy is very different here. Estimates of the bargaining parameter, β, range from 0.63 to 0.71. These estimates are higher than those found by Flinn (2003) but in Flinn's model the bargaining parameter is also built into the the reservation value which affects the wage. Further, the estimates here are taken directly from the firm's zero profit condition.

Table 4 shows exactly what is identifying β by displaying β along with the

Table 4: Identification of β

State	β	Pr (Match)	S
West Virginia	0.7122	0.7032	5.29
California	0.7098	0.7750	6.09
Colorado	0.6846	0.8227	6.19
North Dakota	0.6370	0.8467	6.34
Nebraska	0.6335	0.8860	5.97

average probability of a searching worker finding a match and the average S value. The table is for states with the two highest β's (West Virginia and California), the two lowest (North Dakota and Nebraska), as well as one in the middle (Colorado). High β's are generally correlated with lower probabilities of a searching worker finding a match. At same time, there are large differences in the probability of a searching worker finding a match across states with similar β's. For example, West Virginia and California have essentially the same β but the probability of a worker matching is much higher in California. This reason for this is found in the last column: the economy is better in California.

With the estimates of the log wage regression and the parameters of the zero profit condition, we calculate the probability of matching and the expected wages conditional on matching. We then use these estimates to estimate the value of search with the results presented in Table 5. Consistent with the theory, the expected wage is positive and significant while searching is costly. The parameters characterizing the reservation values consistently have the expected sign and, with the exception of female, all statistically significant. In particular, higher parental education and parental income are both associated with lower probabilities of search while coming from a single parent family makes it more likely that an individual will enter the labor market. Reservation values are also higher for younger individuals.

Table 5: Estimates of the Search Parameters

	Coefficient	Std. Error
Expected Wage	0.3463	0.0054
$\frac{\text{Search Cost}}{\text{Pr(Match)}}$	-0.0839	0.0071
Reservation Values (exp()):		
Constant	0.3067	0.0062
Parental Edu.	-0.0188	0.0011
Parental Inc.	-0.0747	0.0065
Missing Inc.	-0.2428	0.0068
Two Parents	-0.0360	0.0062
Female	-0.0051	0.0056
Age=16	-0.6098	0.0086
Age=17	-0.2610	0.0087
Age=18	-0.1161	0.0095

Table 6: Changes in Labor Market Outcomes from a 10% Minimum Wage Increase

	Mean	Std. Dev.
Pr (Search) Before	0.4811	(0.1239)
Pr (Search) After	0.4910	(0.1206)
Pr (Match) Before	0.8269	(0.0502)
Pr (Match) After	0.8031	(0.0601)
Pct. Change in Employment	-0.88%	
Pct. Change in Pr (Search)	2.1%	
Pct. Change in Pr (Match)	-2.9%	
Pct. w/ Positive Employment Effects	15.4%	

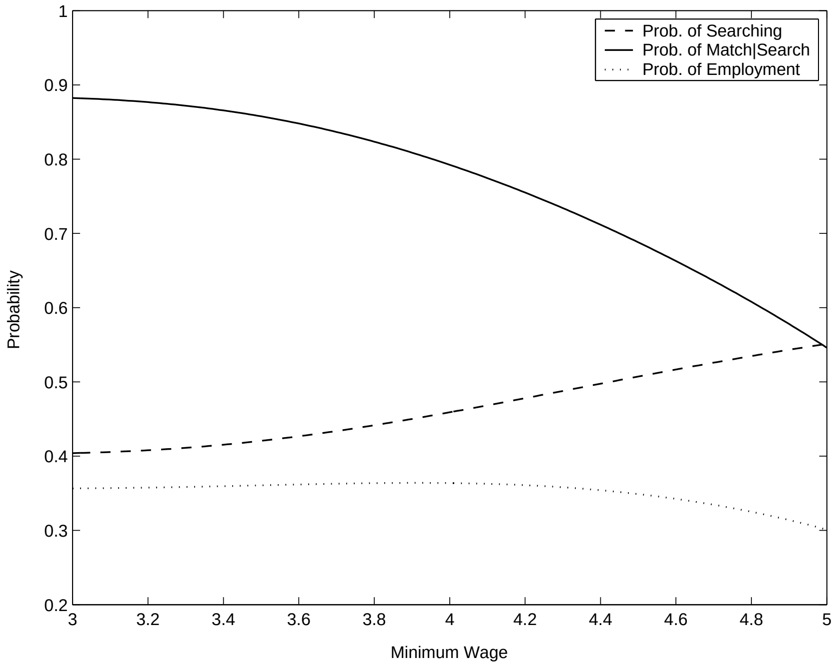
8 Simulations

Besides of the nonlinearities, it is difficult to interpret the magnitude of the coefficients. To characterize what the parameters mean we simulate how a 10% change in the minimum wage affects the decision to search, the probability of matching conditional on searching, and the employment level. The average effects are presented in Table 6.

The table shows that with a minimum wage increase the probability of searching increases. However, this is counteracted by a decrease in the probability of finding a job conditional on searching. The overall effect on the probability of employment is a modest -0.88% decrease. However, this is masking large changes in labor supply and demand that are effectively canceling out. Namely, a 10% increase in the minimum wage causes a 2.1% increase in the probability of searching and 2.9% decrease in the probability of employment conditional on searching.

To further examine how the labor market responds to increases in the minimum wage, we simulate the probability of searching, the probability of finding

Figure 2: Labor Market Outcomes as a Function of the Minimum Wage



a match, and the probability of employment for 16 year olds in Missouri in 1990 as a function of the minimum wage. The results of this simulation are displayed in Figure 2. The probability of employment is not particularly responsive to changes in the minimum wage. However, this is because the steep drops in the probability of matching conditional on searching are counteracted by the increased probability of searching.

While overall employment falls for all states as a result of the minimum wage increase, the last row of Table 6 shows that 15% of individuals see their expected employment rise as the increase in their probability of searching outweighs the decrease in the probability of finding a job. This effect is almost entirely constrained to sixteen year olds. Table 7 splits the sixteen year olds into two groups, those who and do not have higher expected employment probabilities, and examines the demographic characteristics across the two groups.

Over half the sixteen year old sample is more likely to be employed as a

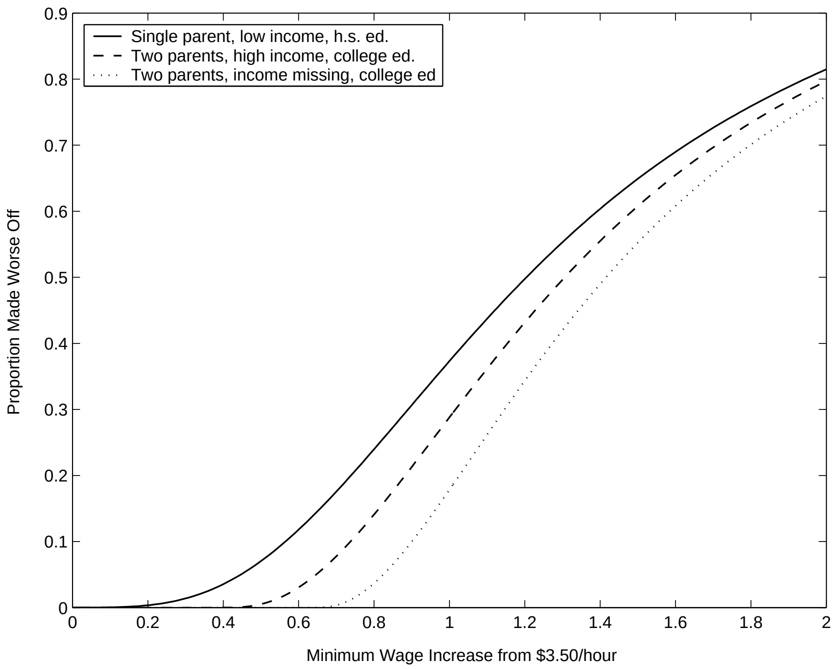
Table 7: Means by Positive and Negative Employment Effects for 16 Year Olds

	Mean — $\Delta E > 0$ (N=17,934)	Mean — $\Delta E < 0$ (N=16,387)
Parental Edu.	14.46	12.93
Weekly Parental Inc. ($000's)	0.8104	0.5941
Missing Inc.	0.3907	0.1315
Single Parent	0.1175	0.2584
Pr (Search) Before	0.3035	0.3494
Pr (Search) After	0.3158	0.3644
Pr (Match) Before	0.8316	0.7901
Pr (Match) After	0.8022	0.7519
Pct. Δ Employment	0.4%	0.5%
Pct. Δ Pr (Search)	4.1%	4.3%
Pct. Δ Pr (Match)	-3.5%	-4.8%

result of the minimum wage increase. There are striking differences across those who do and do not experience this positive employment effect. Namely, the group with positive employment effects have parents with education levels that are a year and a half higher, weekly parental income levels for those who report that are over 30% higher, and are half as likely to come from single parent families than those who do not experience positive employment effects. Those who experience positive employment effects also are likely to have come from stronger economies: the probability of finding a job conditional on searching is higher for this group.

However, whether a group has positive employment effects as a whole or not, is only tangentially connected to welfare. As shown in section 3, it is possible to have all workers benefit from a minimum wage increase. This occurs when the drop in employment is sufficiently small relative to the increase in the expected wage. Figure 3 examine the percentage of people who are made worse off by

Figure 3: Probability of Being Made Worse Off by a Minimum Wage Increase



increasing the minimum wage. The base minimum wage is set at $3.50/hr and is raised by up to $5.50/hr. The simulation is conducted on three types of individuals: one from a low income, low educated single parent family, two from highly educated two parent families with one having a high value for income and the other reporting a missing value for income. Minimum wage increases are at first beneficial to all. However, those who are of the low income type quickly see a high percentage who are made worse off by an increase in the minimum wage. This occurs for the high income groups as well, but occurs at higher minimum wages.

This figure shows that the employment *level* gains seen in Card and Krueger are likely to be concentrated in the segment of the population that have higher reservation values. The intuition is that lower reservation value workers were already searching for work at lower minimum wage levels. The wage hike induces entry from high reservation workers, and majority of the increase in employment levels is from more workers in the labor force. Note that high reservation workers have no advantage in finding a job compared to low reservation workers in this model. If higher reservation workers had an advantage, it would exacerbate the situation for low reservation wage workers, and we would expect to see the number of matches for low reservation workers decrease at a faster rate. Conditional on finding a job, lower reservation individuals are, of course, happier with the higher minimum wage. However, in expectation, low reservation workers have more to lose, and would trade off the higher wage for a better chance of matching with a firm.

9 Conclusion

This paper has developed a theory of two-sided search to explain the puzzling absence of an impact on employment levels when the minimum wage is increased. A minimum wage hike may induce some agents who were not looking for minimum wage work to enter the labor force, which increases the pool of searching workers. At the same time, firms may be induced to decrease the number of jobs available because of the decreased expected surplus from each generated match.

In the classical framework, the exit by firms would dictate a decrease in employment. However, more general matching functions such as those commonly used in the search literature can generate positive employment effects from an increase in the minimum wage. In particular, if employment depends upon *both* the number of searching workers and the number of searching firms, the increase in the number of searching workers may more than offset the decrease in the number of searching firms. Even if positive employment effects result from a minimum wage hike, however, the probability of any individual worker finding a job has fallen.

The welfare analysis suggests that workers who desire the minimum wage job the most are the ones who are hurt by the minimum wage hike, and these workers may move to avoid the effects of the policy. This suggests that zero or even positive employment growth from a minimum wage hike does not necessarily translate into a welfare improvement for individuals affected by the policy, and that mobility and search effects are masking a churning in the economy that is not fully captured by observing employment levels.

Simulations using the parameter estimates from the econometric model showed that the important theoretic predictions of the theoretic model hold. When minimum wage increases, probability of employment declines, the number of matches can increase, a greater number of high reservation workers search, and low reservation workers can be worse off. These results are driven by the differences in expected wage, probability of employment, and reservation values. A higher minimum wage translates to a higher expected wage, which induces entry by workers with higher reservation values and exit by firms. More searchers and less vacancies can result in more matches, but will always lead to a lower probability of employment.

The results of the simulations indicate that Card and Krueger were correct in their observation that moderate a minimum wage hike can result in no decrease to a small increase in employment levels. At the same time, the conventional wisdom that the introduction of an artificial price floor will cause a surplus because demand retracts and supply increases also remains valid. While it is possible in both the classical model and this model for all workers to be made better off in expectation by a minimum wage increase, if anyone is adversely affected it will be the working poor who most desperately want the job.

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Appendix

Proof of Lemma 1

We show that, given A.1, if a worker searches, he accepts all matches. Note that if a worker would accept all matches without a minimum wage, he will also accept all matches when a minimum wage exists. Hence, it is sufficient to show that a worker will accept will matches without a minimum wage. Let ζ_A be the expected value of ζ given some cutoff value ζ'. Let π_A be the probability that $\zeta > \zeta'$.

Case 1: $\beta\overline{S} > R$.

Suppose $\beta\overline{S} > R$ and $p\pi_A((\beta(\overline{S} + \zeta_A) - R) - K > 0$. Then:

$$p\pi_A(\beta(\overline{S} + \zeta_A) - R) - K > \pi_A\beta(\overline{S} + \underline{\zeta}) - R$$

$$\beta\pi_A(\zeta - \underline{\zeta}) > K + (1 - \pi_A)(\beta\overline{S} - R)$$

Note that the last term on the r.h.s. is positive. If the term inside the parenthesis is negative, we have a contradiction. If the term inside the parenthesis is positive, then with $\beta < 1$, $\pi_A(\zeta - \underline{\zeta}) > \beta\pi_A(\zeta - \underline{\zeta}) > K$ and we also have a contradiction.

Case 2: $\beta\overline{S} < R$

For the worker to search, $\pi_A\beta\zeta_A > K$. With $\underline{\zeta} < 0$ and $\beta < 1$, this implies $\pi_A(\zeta_A - \underline{\zeta})$ and we have a contradiction. QED.

Proof of Lemma 2

Again let ζ_A be the expected value of ζ given some cutoff value ζ'. Let π_A be the probability that $\zeta > \zeta'$.

Case 1: $\overline{S} + \zeta_A > \underline{W}$

Suppose $\overline{S} + \underline{\zeta} - \underline{W} < 0$ and $q\pi_A((1-\beta)(\overline{S}+\zeta_A)) - C > 0$. Then:

$$q\pi_A((1-\beta)(\overline{S}+\zeta_A)) - C > \pi_A(\overline{S} + \underline{\zeta} - \underline{W})$$

with $q < 1$ it must be that:

$$\pi_A(\zeta_A - \underline{\zeta} + \underline{W} - \beta(\overline{S}+\zeta_A)) > C$$

with $\underline{W} - \beta(\overline{S}+\zeta_A)$, it must be that: $\pi_A(\zeta_A - \underline{\zeta}) > C$, a contradiction.

Case 2: $\overline{S} + \zeta_A < \underline{W}$

Suppose $\overline{S} + \underline{\zeta} - \underline{W} < 0$ and $q\pi_A(\overline{S} + \zeta_A - \underline{W}) - C > 0$.

$$q\pi_A(\overline{S} + \zeta_A - \underline{W}) - C > \pi_A(\overline{S} + \underline{\zeta} - \underline{W})$$

with $q < 1$ it must be that:

$$\pi_A(\zeta_A - \underline{\zeta}) > C$$

a contradiction. QED.

Proof of Proposition 1

Note that conditional on any $N \in [0, \overline{N}]$, as $J \to \infty$, $q \to 0$. There then exists a $\overline{J}$ such that for all N if $J' > \overline{J}$, profits are negative. Since the partial derivative of π is negative with respect to J,

$$\frac{\partial \pi}{\partial J} = -\frac{q\alpha(\overline{S} - E(W))}{J} < 0$$

We know that for each value of N there is at most one value of J such that $\pi = 0$.

Similarly, define V as the search value. Since $\partial V/\partial N$ is also negative,

$$\frac{\partial V}{\partial N} = -\frac{p(1-\alpha)(E(W)-R)}{N} < 0$$

We know that for each J there is at most one value of N such that $V = 0$. We can then define the following mappings:

$$f_1 = \begin{cases} \pi(J,N) & \text{for } J \in (0,\overline{J}], N \in [0,\overline{N}] \\ \max\{\pi(0,N),0\} & \text{for } J = 0, N \in [0,\overline{N}] \end{cases}$$

$$f_2 = \begin{cases} \min\{V(J,\overline{N}),0\} & \text{for } J \in [0,\overline{J}], N = \overline{N} \\ V(J,N) & \text{for } J \in [0,\overline{J}], N \in (0,\overline{N}) \\ \max\{V(J,0),0\} & \text{for } J \in [0,\overline{J}], N = 0 \end{cases}$$

Then for each value of N, there exists a unique value of $J \in [0,\overline{J}]$ that satisfy $f_1 = 0$. Further, since π is continuous in N, this unique value is a continuous function of N. Similarly, for each J, there is a unique $N \in [0,\overline{N}]$ satisfying f_2 which is continuous in J. We can then use functions to define a continuous vector valued function mapping from $[0,\overline{J}] \times [0,\overline{N}]$ into itself. Then by Brouwer's fixed point theorem there exists a doublet $\{J^*, N^*\}$ where $f_1 = 0$ and $f_2 = 0$. QED.

Proof of Proposition 2

Consider the equilibrium before the minimum wage increase where expected wages are given by $E_1(W)$ and the probability of a firm matching is given by q_1. The firm's expected zero profit condition is:

$$q_1(\overline{S} - E_1(W)) - C = 0$$

Note that $E_1(W)$ is weakly increasing in the minimum wage. Note that if a minimum wage increase does not affect the expected wage the probability of matching for the firm will remain the same. However, if $E_2(W) > E_1(W)$ then $q_2 > q_1$ for the zero profit condition to still bind. Note further that the probabilities of firms and workers matching is given by:

$$q = A\left(\frac{N}{J}\right)^{\alpha} \qquad p = A\left(\frac{J}{N}\right)^{1-\alpha}$$

The expression for the firm implies that $\frac{N}{J}$ must increase for the zero profit condition to bind. But if this fraction increases then p must fall. QED.

Proof of Proposition 3

Differentiating the matching function with respect to the minimum wage yields:

$$\frac{dx}{d\underline{W}} = \alpha q \frac{dJ}{d\underline{W}} + (1-\alpha) p \frac{dN}{d\underline{W}}$$

Rewrite as:

$$\begin{aligned}
\frac{dx}{d\underline{W}} &= \alpha \frac{x}{J}\frac{dJ}{d\underline{W}} + (1-\alpha)\frac{x}{N}\frac{dN}{d\underline{W}} \\
&= x\left(\alpha \frac{\frac{dJ}{J}}{d\underline{W}} + (1-\alpha)\frac{\frac{dN}{N}}{d\underline{W}}\right) \\
&= \frac{x}{\underline{W}}\left(\alpha \frac{\frac{dJ}{J}}{\frac{d\underline{W}}{\underline{W}}} + (1-\alpha)\frac{\frac{dN}{N}}{\frac{d\underline{W}}{\underline{W}}}\right) \\
&= \frac{x}{\underline{W}}\left(\alpha\varepsilon_{LD} + (1-\alpha)\varepsilon_{LS}\right)
\end{aligned}$$

Therefore, for the employment effect to be positive ($\frac{dx}{d\underline{W}} > 0$), it must be that $(\alpha\varepsilon_{LD} + (1-\alpha)\varepsilon_{LS}) > 0$, where ε_{LD} is the elasticity of labor demand and ε_{LS} is the elasticity of labor supply. QED

Proof of Proposition 4

In order to obtain conditions under which employment is increasing in the minimum wage, we first need to know how J and N respond to changes in the minimum wage. The zero profit condition and the worker indifference condition are as follows:

$$\begin{aligned}
F_1 &= q(\overline{S} - E(W)) - C \\
F_2 &= (1-p)R^* + p \cdot E(W) - K - R^*
\end{aligned}$$

Rewrite F_2 such that:

$$F_2 = p(E(W) - R^*) - K$$

where $E(W) = E(\max\{\beta(\overline{S}+\zeta_{ij}), \underline{W}\})$. Note that $\frac{\partial E(W)}{\partial \underline{W}} = \frac{\partial E(\max\{\beta(\overline{S}+\zeta_{ij}),\underline{W}\})}{\partial \underline{W}}$, which equals 1 with some probability and 0 with some probability. Therefore, we define $\frac{\partial E(W)}{\partial \underline{W}} = \xi$, where $0 \leq \xi \leq 1$.

The Jacobian of the system of equations is then:

$$B = \begin{pmatrix} \frac{\partial F_1}{\partial J} & \frac{\partial F_1}{\partial N} \\ \frac{\partial F_2}{\partial J} & \frac{\partial F_2}{\partial N} \end{pmatrix}$$

and,

$$\begin{aligned}
\frac{\partial F_1}{\partial J} &= \frac{(\alpha-1)q(\overline{S}-E(W))}{J} < 0 \\
\frac{\partial F_1}{\partial N} &= \frac{(1-\alpha)q(\overline{S}-E(W))}{N} > 0 \\
\frac{\partial F_2}{\partial J} &= \frac{\alpha p(E(W)-R^*)}{J} > 0 \\
\frac{\partial F_2}{\partial N} &= -\frac{\alpha p(E(W)-R^*)}{N} - \frac{p\partial R^*}{\partial N} < 0 \\
\frac{\partial F_1}{\partial \underline{W}} &= -q\xi < 0 \\
\frac{\partial F_2}{\partial \underline{W}} &= p\xi > 0
\end{aligned}$$

By the implicit function theorem:

$$\begin{pmatrix} \frac{\partial J}{\partial \underline{W}} \\ \frac{\partial N}{\partial \underline{W}} \end{pmatrix} = -B^{-1} \begin{pmatrix} \frac{\partial F_1}{\partial \underline{W}} \\ \frac{\partial F_2}{\partial \underline{W}} \end{pmatrix}$$

where

$$B^{-1} = \frac{1}{Det(B)} \begin{pmatrix} \frac{\partial F_2}{\partial N} & -\frac{\partial F_1}{\partial N} \\ -\frac{\partial F_2}{\partial J} & \frac{\partial F_1}{\partial J} \end{pmatrix}$$

and $Det(B)$ can be written as:

$$\begin{aligned}
Det(B) &= \frac{\partial F_1}{\partial J}\frac{\partial F_2}{\partial N} - \frac{\partial F_1}{\partial N}\frac{\partial F_2}{\partial J} \\
&= \left\{\frac{\alpha p(E(W)-R(\gamma^*))}{J} \times \frac{(1-\alpha)q(\overline{S}-E(W))}{N}\right\} - \\
&\qquad \left\{\left(-\frac{\alpha p(E(W)-R^*)}{N} - \frac{p\partial R^*}{\partial N}\right) \times -\frac{(1-\alpha)q(\overline{S}-E(W))}{J}\right\} \\
&= -\frac{(1-\alpha)qp(\overline{S}-E(W))}{J}\frac{\partial R^*}{\partial N} < 0
\end{aligned}$$

$$
\begin{aligned}
\frac{dN}{d\underline{W}} &= \frac{\frac{\partial F_2}{\partial J}\frac{\partial F_1}{\partial \underline{W}} - \frac{\partial F_1}{\partial J}\frac{\partial F_2}{\partial \underline{W}}}{Det(B)} \\
&= \frac{\xi}{\frac{\partial R^*}{\partial N}}\left(1 - \frac{\alpha}{1-\alpha}\frac{E(W)-R^*}{\overline{S}-E(W)}\right)
\end{aligned}
$$

and,

$$
\begin{aligned}
\frac{dJ}{d\underline{W}} &= \frac{\frac{\partial F_2}{\partial N}\frac{\partial F_1}{\partial \underline{W}} - \frac{\partial F_1}{\partial N}\frac{\partial F_2}{\partial \underline{W}}}{Det(B)} \\
&= \frac{J}{N}\frac{\xi}{\frac{\partial R^*}{\partial N}}\left(1 - \frac{\alpha}{1-\alpha}\frac{E(W)-R^*}{\overline{S}-E(W)} - \frac{N\frac{\partial R^*}{\partial N}}{(1-\alpha)(\overline{S}-E(W))}\right) \\
&= \frac{J}{N}\frac{dN}{d\underline{W}} - \frac{J\xi}{(1-\alpha)(\overline{S}-E(W))}
\end{aligned}
$$

With the information on $dJ/d\underline{W}$ and $dN/d\underline{W}$, we can now substitute in for the labor demand and supply elasticities.

$$
\begin{aligned}
\alpha\varepsilon_{LD} + (1-\alpha)\varepsilon_{LS} &= \alpha\frac{dJ}{d\underline{W}}\frac{\underline{W}}{J} + (1-\alpha)\frac{dN}{d\underline{W}}\frac{\underline{W}}{N} \\
&= \underline{W}\left[\alpha\left(\frac{1}{N}\frac{dN}{d\underline{W}} - \frac{\xi}{(1-\alpha)(\overline{S}-E(W))}\right) + (1-\alpha)\frac{1}{N}\frac{dN}{d\underline{W}}\right] \\
&= \underline{W}\left[-\frac{\alpha}{1-\alpha}\frac{\xi}{\overline{S}-E(W)} + \frac{1}{N}\frac{dN}{d\underline{W}}\right] \\
&= \varepsilon_{LS} - \frac{\alpha}{1-\alpha}\frac{\xi\underline{W}}{\overline{S}-E(W)} \\
&= \frac{\xi\underline{W}}{N\frac{\partial R^*}{\partial N}}\left(1 - \frac{\alpha}{1-\alpha}\frac{E(W)-R^*}{\overline{S}-E(W)}\right) - \frac{\alpha}{1-\alpha}\frac{\xi\underline{W}}{\overline{S}-E(W)}
\end{aligned}
$$

Let R be distributed uniform $[0, \overline{R}]$ implying that $R^* = \frac{N}{N\overline{R}}$ and $\frac{\partial R^*}{\partial N} = \frac{1}{N\overline{R}}$. Substituting into the above and simplifying, we have:

$$
\alpha\varepsilon_{LD} + (1-\alpha)\varepsilon_{LS} = \frac{\xi\underline{W}\overline{N}}{N \cdot a}\left[1 - \frac{\alpha}{1-\alpha}\frac{E(W)-b}{\overline{S}-E(W)}\right] + \frac{\alpha}{1-\alpha}\frac{\xi\underline{W}}{\overline{S}-E(W)}\left[\frac{\overline{N}}{N} - 1\right]
$$

Since the last term is positive, setting $N = \overline{N}$ yields a sufficient condition on the parameters. Simplifying yields:

$$
1 - \frac{\alpha}{1-\alpha}\frac{E(W)}{\overline{S}-E(W)} > 0
$$

QED

Proof of Proposition 5

In order for all workers to benefit from an increase in the minimum wage it is sufficient to show that the workers with the lowest reservation values, zero, are made better off by the increase. The value of search for these workers can be written as:

$$V = A\left(\frac{N}{J}\right)^{\alpha-1} E(W) - K$$

Note that the zero profit condition for firms can be written as:

$$A\left(\frac{N}{J}\right)^{\alpha} (\overline{S} - E(W)) = 0$$

and that both of these conditions depend on N and J only through the ratio N/J. Further, the zero profit condition for the firm is an identity. Differentiating profits with respect to an increase in the minimum wage yields:

$$A\left(\frac{N}{J}\right)^{\alpha} \left((\overline{S} - E(W)) \left(\frac{N}{J}\right)^{-1} \frac{\left(d\frac{N}{J}\right)}{d\underline{W}} - \frac{dE(W)}{d\underline{W}} \right) = 0$$

Solving for $d(N/J)/d\underline{W}$ yields:

$$\frac{d\left(\frac{N}{J}\right)}{d\underline{W}} = \frac{N}{(\overline{S} - E(W))J} \frac{dE(W)}{d\underline{W}}$$

We now have all components necessary to sign $dV/d\underline{W}$ for those with a reservation value of zero. Differentiating V with respect to $\underline{W}$ yields:

$$E(W)A(\alpha - 1)\left(\frac{N}{J}\right)^{\alpha-2} \frac{d\left(\frac{N}{J}\right)}{d\underline{W}} + A\left(\frac{N}{J}\right)^{\alpha-1} \frac{dE(W)}{d\underline{W}}$$

substituting in for $d(N/J)/d\underline{W}$ and rewriting yields:

$$\frac{pdE(W)}{d\underline{W}} \left[1 - \frac{(1-\alpha)E(W)}{\alpha(\overline{S} - E(W))} \right]$$

Since $dE(W)/d\underline{W} > 0$, we have the result. QED

Table 8: β Adjustments Not Reported in the Text

	Coefficient	Std. Error
Arizona	0.0255	(0.0048)
Arkansas	0.0022	(0.0047)
Connecticut	0.0546	(0.0034)
Delaware	0.0305	(0.0045)
Florida	0.0101	(0.0047)
Georgia	-0.0049	(0.0053)
Idaho	0.0088	(0.0047)
Illinois	0.0058	(0.0046)
Indiana	0.0079	(0.0047)
Iowa	-0.0020	(0.0048)
Kansas	-0.0059	(0.0049)
Kentucky	0.0035	(0.0049)
Louisiana	-0.0167	(0.0056)
Maine	0.0325	(0.0045)
Maryland	0.0374	(0.0051)
Massachusetts	0.0508	(0.0030)
Michigan	0.0065	(0.0046)
Minnesota	0.0080	(0.0047)
Mississippi	-0.0084	(0.0054)
Missouri	-0.0182	(0.0052)
Montana	0.0050	(0.0048)
Nevada	0.0299	(0.0045)
New Hampshire	0.0382	(0.0046)
New Jersey	0.0492	(0.0041)
New Mexico	0.0380	(0.0055)
New York	0.0329	(0.0043)
North Carolina	-0.0012	(0.0046)
Ohio	0.0062	(0.0048)
Oklahoma	-0.0104	(0.0046)
Oregon	0.0287	(0.0047)
Pennsylvania	0.0228	(0.0043)
Rhode Island	0.0273	(0.0044)
South Carolina	0.0098	(0.0047)
South Dakota	-0.0165	(0.0051)
Tennessee	0.0006	(0.0048)
Texas	0.0313	(0.0045)
Utah	0.0017	(0.0049)
Vermont	0.0136	(0.0046)
Virginia	0.0170	(0.0046)
Washington	0.0433	(0.0044)
Wisconsin	0.0010	(0.0049)
Wyoming	0.0193	(0.0049)