Sorting on skills and preferences: Tinbergen Meets Sattinger*

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Abstract

The literature on labor market sorting is divided into two distinct classes of models. One class of models, led by Tinbergen (1956), focuses on the assignment of workers to jobs based on preferences. The second class of models, led by Sattinger (1979), focuses on the assignment of workers to jobs based on skills. This paper proposes an assignment model unifying both classes of models. Equilibrium in this model yields a wage function with both jobs and workers' attributes as arguments. The classical two-step procedure due to Rosen (1974) to estimate preference parameters is ruled

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out and new techniques are proposed. Possible extensions of the model beyond the labor market are also proposed.

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1 Introduction

Recent emerging empirical literature (e.g. Borghans et al. (2008)) has shown the importance of personality traits in economics and in particular for earnings (Bowles et al. (2001)). This literature shows that earnings are related to personality traits like risk aversion or conscientiousness. One possible explanation for these wage differentials would be that personality traits are linked to preferences for certain jobs attributes so that the correlation between personality and earnings reflects compensating wage differentials for jobs disamenities. Yet, another explanation would be that personality traits are linked to skills that enhance productivity on the job and hence lead to higher wages. For instance, the documented positive effect of conscientiousness on earnings could come about because conscientiousness enhances workers' productivity or because in equilibrium, more conscientious workers are mapped onto jobs whose attributes are associated with negative intrinsic utility (tax controller) and hence require a wage compensation.

The model presented in this paper is the first treatment of personality traits in assignment models. The model is concerned with the process by which heterogenous workers that are characterized by a vector of attributes t, including both skills and preferences, are assigned to heterogenous jobs characterized by a vector of attributes z , including both required skills and disamenities. This paper shows that this type of assignment models yields an equilibrium[1] that is defined by a mapping of workers attributes t onto jobs attributes z, a function say $t(z)$ or $z(t)$, together with a pricing function $w(z,t)$ that depends on workers' attributes and jobs attributes. This feature of the model has important implications for the estimation of preference (technology) parameters in hedonic models. The traditional two-step procedure proposed by Rosen (1974) to estimate preference parameters provides biased estimates unless all workers' attributes are observed in the data. In general, only some attributes are observed. Since jobs attributes map onto *all* workers' attributes through $z(t)$, unobserved workers' attributes will correlate with some jobs attributes and bias the estimation of the wage function in Rosen's first step. As a consequence, estimates of preferences in Rosen's second step will be a mixture of preference and technology parameters.

The model presented in this paper nests existing assignment models in the literature. This literature is divided into two distinct classes of models depending on the nature of the process governing assignment. One class of models, led by Tinbergen (1956), focuses on the assignment of workers to jobs based on preferences.

[1] The existence, uniqueness, purity and efficiency of this type of models have been treated elsewhere, e.g. Gretsky et al. (1992; 1999) deals with the so call *endowment* economy where z is endowed to the firm and Ekeland (2005a;2005b) and more recently Chiappori et al. (2007) that deal with the generalization to hedonic *production* economy where z is produced by firms with endowed attributes say y. Chiappori et al. (2007) have shown that hedonic models are equivalent to matching models and both belong to the general class of optimal transportation problems (Monge-Kantorovich). Under the assumptions made in this paper, in particular, about the shape of the profit and utility functions and given absolute continuous distribution of characteristics on both sides, an equilibrium exists, is unique and pure, see for instance Theorem 1, p. 3 in Ekeland (2005).

Within this class of models, jobs attributes z are seen as disamenity and workers derive intrinsic disutility from z. Although jobs with different attributes are unequally productive, output at a job with attribute z does not depend on workers' attributes t. Hence productivity is merely determined by job attributes and all workers are equally productive at all jobs. In this class of models, workers select jobs attributes to maximize their utility. The pricing function $w(z,t)$ does not depend on workers attributes but merely on jobs attributes, i.e. $w(z,t) = w(z)$, and is therefore interpreted as a compensating wage differential. As an example, the preference class of models indicates that risk loving workers will tend to become firemen as they command lower compensations for the risks taken on the job, but yet, assumes that risk loving workers would just make as good firemen as any other (risk averse) worker. While this class of models explains wage formation due to risk compensation, the model fails to explain wage formation due to productivity differential across workers.

In contrast, the second class of models, led by Sattinger (1979), focuses on the assignment of workers to jobs based on skills. Jobs attributes are seen as productive capacities and workers derive no intrinsic (dis-)utility from z. Both workers' and jobs attributes matter for productivity. Workers with certain attributes are more productive at certain jobs than others. In this class of models, workers select jobs attributes to maximize their wage and the wage function $w(z,t)$ does not depend on jobs attributes but merely on workers' attributes, i.e. $w(z,t) = w(t)$. For instance, the skills class of models indicates that conscientious workers will tend to become tax controllers as conscientiousness is an important factor of productivity on the job, but yet, assumes that conscientious workers derive the same disutility from being a tax controller as less conscientious workers. While this class of models explains wage formation due to differential productivity across workers, these models fail to explain wage formation due to preference compensation.

The model presented in this paper nests both Tinbergen's and Sattinger's models.[2] Under the assumption that all jobs attributes lead to intrinsic disutility and workers attributes do not affect productivity the model collapses to Tinbergen's model. Under the assumption that all workers attributes contribute to productivity and no job attributes lead to intrinsic disutility the model collapses to Sattinger's differential rents model.

There exist only few examples of closed form solutions for the hedonic price as a function of attributes, i.e. $w(z,t)$. The first was proposed by Tinbergen (1956). Assuming that i) workers derive intrinsic disutility from all their attributes, ii) price enters log linearly in the utility function, iii) intrinsic (dis-)utility is quadratic in jobs attributes z, iv) the supply of products is exogenous (the hedonic *endowment*

[2] Sattinger (1977) developed a compensating wage differential model where workers differ in terms of productivity and jobs in terms of the satisfaction workers receive from working at it, both unidimensional. Workers and jobs attributes are encompassed in the definition of job satisfaction and cannot be distinguished from each other. Moreover, all jobs have similar productivity. There is no complementarity between workers skills and jobs requirements.

economy model) and v) both workers and jobs attributes are normally distributed, Tinbergen showed that the *log* of the equilibrium price is a function quadratic in attributes z.[3] Assuming i), ii'), iii) and v) and relaxing iv) by allowing firms to produce job attributes and introducing production costs (the hedonic *production* economy model), Epple (1984) provided a closed form solution for the hedonic price function that is quadratic in z when production costs are also quadratic in z. Sattinger (1979 and 1980) provided closed for solutions when jobs and workers are differentiated along a single attribute (skills demanded and supplied) assuming that workers derive no intrinsic disutility from the level of skills demanded by their job, i.e. maximize their wage. This skills attribute affects productivity but does not derive intrinsic utility. The pricing function of interest in this model is the wage as a function of workers' skills. Since skills derive no intrinsic disutility and merely affect production, sorting in this model occurs on productive attributes rather than preference attributes as in Tinbergen and Epple. Sattinger's (1980) closed form solutions for the wage function are obtained when the distribution of jobs and workers are Pareto, production is multiplicative in attributes (i.e. Cobb-Douglas) and utility depends on wages only. This last assumption is characteristic of the differential rents model that precludes compensating wage differential for intrinsic disutility derived from the job. A closed form solution for the unified

[3]In fact, replacing ii) by ii') price enters linearly in the utility function, the *level* of the equilibrium price would be quadratic in z.

model is proposed in this paper. This solution is derived when workers' and jobs attributes are normally distributed and intrinsic dis-utility is quadratic in jobs attributes and productivity is quadratic in workers' attributes.

This paper relates to the general literature on hedonic models and not only on that segment focussing on the labor market. For instance, Epple's extension of Tinbergen's *endowment* economy to a *production* economy was originally written in a consumer/producer context, not a worker/firm context. In the consumer/producer model, the restriction that consumers' attributes do not affect the production of goods does not at first sight seem to be too strong. However, the generalization proposed in this paper is also relevant in that case. Think for instance of an economy where firms are endowed with a vector of attributes y. In this economy, to produce good z, firms need to hire a fixed number of workers, one and only one worker for simplicity. Suppose further that the attributes of that worker, say t', matter in the production process so that the costs (profits) of producing good z depend on t'. Firms need now to optimize not only on z but also on t'. The model presented in this paper can be seen as a special case in which workers consume their own product $t = t'$. The foundation of this model and its potential applications to other markets are discussed in section 4 of this paper.

The remaining structure of the paper is as follows. In the next section, I present the unified model for the hedonic *endowment* economy and a closed form solution for the wage function in the quadratic-normal setup. Section 3 discusses the implication for the estimation of preference parameters using Rosen's two-step approach when the true model is the unified model. Section 4 discusses first an extension of the model to the hedonic *production* economy, and then potential generalizations of the model to other markets than the labor market. Section 5 summarizes and concludes.

2 The unified hedonic *endowment* economy model

2.1 Setup

Consider a static labor market where workers match one-to-one with firms. Let each firm be endowed with a single machine. The supply of machines is therefore assumed exogenous to the model,[4] and the assumption that workers and firms match one-to-one therefore means that to produce output each machine must be operated by one and only one worker. Let a machine be characterized by a vector of attributes denoted by $z \in \mathbb{R}^{n_z}$. To fix ideas, machines attributes could be the level of physical strength required to operate the machine, the level of intellectual

[4] The assumption that firms are endowed with a machine z can be released by supposing that firms are endowed with a vector of attributes y (investments capacity, managers' attributes etc.) and "produce" their machine z. The distribution of machines is then endogenous to the model. This case corresponds to the hedonic *production* economy and is dealt with section 4 of this paper. The main results of the paper remain unchanged but the mechanic of the model simplifies significantly by assuming machines are endowed.

complexity involved in operating the machine, the level of noise generated by the machine, the degree of risk taken while operating the machine, etc. Let $f_z(z)$ and $F_z(z)$ be the PDF and CDF of z respectively and let F_z be absolutely continuous with respect to Lebesgue measure.

Similarly, suppose that workers are endowed with a vector of attributes $t \in \mathbb{R}^{n_t}$. These attributes could refer to cognitive ability such as physical strength, intellectual ability but also personality traits such as conscientiousness, risk aversion etc.. Let the distribution of t be exogenous and let $f_t(t)$ and $F_t(t)$ be its PDF and CDF respectively and let F_t be absolutely continuous with respect to Lebesgue measure. For simplicity, I assume that $n_z = n_t = n$ although this is not necessary to solve the equilibrium problem.[5]

In contrast to Tinbergen (1956), Epple (1984) and Ekeland et al. (2002 and 2004), the model does not require workers' attributes to be non productive. Let the output of each machine depend on its own attributes but also on the attributes of the worker operating this machine. Let $p(z, t; E)$ be a continuous function indicating the units of output produced by the pair (z, t) where E are technology parameters[6] common to all firms. An attribute t_j is a productive attribute at

[5] It should be noted here that the mass of workers is assumed to be equal to the mass of firms. The model could be accommodated to allow for different masses and would inevitably lead to unemployed workers or vacancies in equilibrium depending on whether the mass of workers exceeds that of firms. Although assignment models offer an interesting structure to analyze which agents are kept out of the market by the equilibrium pricing, the primary aim of this paper is to analyze wage formation when workers' attributes are both skills and personality traits. The assumption of equal mass does not seem to be restrictive with respect to this aim.

[6] Throughout the paper we use capital letters to denote matrices.

job z if and only if $\frac{\partial p(z,t;E)}{\partial t_j} \neq 0$. Note that some attributes may be productive at some jobs but not at others. While skills of different types will clearly affect productivity, some preferences may also affect productivity, for instance, a risk averse person might also tend to operate a machine slower, conscientious workers may take better care of their machine, etc...

Let $w(z,t)$ be the wage of a worker with attributes t when assigned to a machine with attributes z and let $r(z,t)$ be the rents of a firm owning machine with attributes z when employing a worker with attributes t. Note that, by definition, product is exhausted so that $p(z,t;E) - w(z,t) = r(z,t)$.

In contrast to Sattinger (1979), the model does not require that jobs attributes do not affect intrinsic disutility. Assume that utility $u \equiv u(c,t,z;A)$ is a continuous function where A are preference parameters common to all workers. Utility u depends on consumption c, equal to $w(z,t)$ by assuming no unearned income, and the job satisfaction derived from the attributes of the machine workers' are assigned to. More specifically, let $j(z,t;A)$ be a continuous function capturing job dissatisfaction defined as the tension between a worker's attributes and the attributes of her machine. The function j could take the specific form proposed by Tinbergen (1956), $j(z,t;A) = \frac{1}{2}(z-t)' A (z-t)$ where A is a positive definite matrix of parameters. We therefore have $u(c,t,z,A) = w(z,t) - j(z,t;A)$. A job attribute z_i may or may not provide intrinsic utility depending on whether $\frac{\partial j(z,t;A)}{\partial z_i}$

is different from 0 or not.

2.1.1 Equilibrium

Definition 1 *An equilibrium is a wage function $w(z,t)$ and a mapping function $t(z;A,E,w_z,w_t)$ so that i) firms' supply of machines with attributes z equals workers' demand for machine with attributes z everywhere on the support of z, ii) workers maximize utility and iii) firms maximize rents.*[7]

Utility maximizing workers seek for a machine with attributes z so that:

$$
\begin{aligned}
\frac{\partial u(w(z,t),j(z,t;A))}{\partial z} &\equiv \frac{\partial w(z,t)}{\partial z} - \frac{\partial j(z,t;A)}{\partial z} = 0 \\
&\Leftrightarrow \\
\frac{\partial w(z,t)}{\partial z} &= \frac{\partial j(z,t;A)}{\partial z} \qquad (1)
\end{aligned}
$$

Let $z(t;A,w_z)$ denote the implicit function that solves Equation 1 for z given parameters A and a function w_z where $w_z \equiv \frac{\partial w}{\partial z}$. This function indicates the optimal machine a worker with attributes t chooses given preference parameters

[7] *Existence, uniqueness, purity and efficiency of hedonic models have been studied elsewhere: Gretsky et al. (1992;1999), Ekeland (2005), and Chiappori et al. (2007), mimeo. Under the standing assumptions formulated in the setup above, an equilibrium allocation exists, is unique, and efficient. It will also be pure if the generalized Spence-Mirrlees condition is also satisfied. An equilibrium wage function exists, and is unique if every workers and every firms participate (full employment and no vacancies).*

A and the shape of the wage function and in particular the wage differential at z.

The second order condition for utility maximization reads as:

$$\begin{aligned}\frac{\partial^2 u(w(z,t), j(z,t;A))}{\partial z^2} &< 0\\ &\Leftrightarrow\\ \frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial^2 j(z,t;A)}{\partial z^2} &< 0\end{aligned}$$

Rents maximizing firms will look for a worker with attributes t so that:

$$\begin{aligned}\frac{\partial r(z,t)}{\partial t} &= \frac{\partial p(z,t;E)}{\partial t} - \frac{\partial w(z,t)}{\partial t} = 0\\ &\Leftrightarrow\\ \frac{\partial w(z,t)}{\partial t} &= \frac{\partial p(z,t;E)}{\partial t} \qquad (2)\end{aligned}$$

The second order condition for rents maximization reads as:

$$\frac{\partial^2 p(z,t;E)}{\partial t^2} - \frac{\partial^2 w(t)}{\partial t^2} < 0$$

Let $t(z; E, w_t)$ denote the implicit function that solves Equation 2 for t given parameters E and a function w_t where $w_t \equiv \frac{\partial w}{\partial t}$. This function indicates the optimal choice of worker for a firm with machine z given productivity parameters E and the shape of the wage function and in particular the differential at t.

If the equilibrium is pure, the two mapping functions $z(t; A, w_z)$ and $t(z; E, w_t)$ are invertible. We therefore have the restriction $t^{-1}(t; E, w_t) = z(t; A, w_z)$ and, for notational clarity, re-write the implicit function as $t(z; A, E, w_z, w_t)$ without loss of generality. Note that for the equilibrium mapping to be pure, i.e. $t(z^a; A, E, w_z, w_t) = t(z^b; A, E, w_z, w_t) \Longrightarrow z^a = z^b$, the generalized Spence-Mirrlees condition must be satisfied, see Ekeland (2005). Write the total surplus of a pair (z, t) as $s(z, t; E, A) \equiv p(z, t; E) - w(z, t) + u(t, z; A) = p(z, t; E) - j(z, t; A)$, the generalized Spence-Mirrlees condition reads as:

$$
\begin{aligned}
\frac{\partial s(z, t^a; E, A)}{\partial z} &= \frac{\partial s(z, t^b; E, A)}{\partial z} \Longrightarrow t^b = t^a \\
\frac{\partial s(z^a, t; E, A)}{\partial t} &= \frac{\partial s(z^b, t; E, A)}{\partial t} \Longrightarrow z^b = z^a
\end{aligned}
$$

For an equilibrium allocation to be reached, the supply of machines with attributes z should be equal to workers' demand for machines with attributes z for all z. This means that:

$$f_z(z)dz = f_t(t(z;A,E,w_z,w_t)) \left| \frac{\partial t(z;A,E,w_z,w_t)}{\partial z} \right| dz$$

Equilibrium will be reached by choosing the right shape for the function w and in particular the right differentials at t and z. Workers and firms will participate if their wage and rents are larger than their reservation levels. To close the model, the usual assumption (see Ekeland et al. (2002 and 2004) and Sattinger (1979) among others) is to fix a reservation value for the utility, say $\underline{u}$ and rent $\underline{r}$ so that u and r must be larger than their respective thresholds. Ekeland (2005) has shown that the wage function has a unique solution on the support of z. The equilibrium in this economy is therefore characterized by a wage function $w(z,t)$ and a mapping of workers' attributes onto jobs attributes $t(z;A,E,w_z,w_t)$ so that i) supply equals demand everywhere on the support of z , ii) workers maximize utility and iii) firms maximize rents.

2.2 Quadratic-normal example

2.2.1 The model

As noted earlier by Tinbergen (1956) and Epple (1984), when attributes on both sides of the labor market are normally distributed, i.e. $z -> N(\mu_z, \Sigma_z)$ and $t ->$

$N(\mu_t, \Sigma_t)$, linear mapping functions of the form $t = \pi_0 + \Pi_1 z$ equilibrate supply and demand. Indeed, the equilibrium condition $f_t(t)dt_1dt_2...dt_N = f_z(z)dz_1dz_2...dz_N$ given normally distributed attributes, is equivalent to equating the means, i.e. $\mu_t = \pi_0 + \Pi_1\mu_z$ and equating the variances, i.e. $\Sigma_t = \Pi_1'\Sigma_z\Pi_1$.

The solution for π_0 and Π_1 is:[8]

$$\begin{aligned}
\pi_0 &= \mu_t - \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z \\
\Pi_1 &= \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}
\end{aligned}$$

To see this, note first that $\Pi_1\left(\Pi_1^{-1}\right)' = \Pi_1^{-1}\Pi_1' = I$ where I is the identity matrix. Post-multiply both sides of the equation $\Sigma_t = \Pi_1'\Sigma_z\Pi_1$ by $\left(\Pi_1^{-1}\right)' = \Sigma_t^{-1/2}\Sigma_z^{1/2}$. This yields $\Sigma_t\Sigma_t^{-1/2}\Sigma_z^{1/2} = \Pi_1'\Sigma_z$. Pre-multiply both sides of this equation by Π_1^{-1}. This yields the identity $\Sigma_z^{1/2}\Sigma_t^{-1/2}\Sigma_t\Sigma_t^{-1/2}\Sigma_z^{1/2} = \Sigma_z$.

This implies that one could solve analytically the shape of the equilibrium wage function by looking for the job satisfaction function $j(.,.;A)$ and the production

[8] Note that the power p, $p \in R$, $p \neq 0$, of a square matrix A of size $n \times n$ is obtained as $A^pX = Xdiag(\lambda)$ where X is matrix of size $n \times n$ formed of the n eigenvectors of A and λ is the vector containing the corresponding eigenvalues. If in addition A is symmetric, then X is orthogonal so that $X'X = XX' = I$ and, post-multiplying both sides by X', the result simplifies to $A^p = Xdiag\left(\lambda\right)^pX'$. The matrix A^p will be real if and only if all eigenvalues λ are real and strictly positive that is if and only if A is positive definite. Since Σ_t and Σ_z are symmetric, the above result applies to $\Sigma_t^{-1/2}$ and $\Sigma_z^{1/2}$. So, $\Sigma_t^{-1/2}$ and $\Sigma_z^{1/2}$ will be real if and only if all eigenvalues of Σ_t and Σ_z respectively are real and positive. (See Bosch (1987))

function $p(.,.;E)$ so that the first order conditions for utility maximization and rents maximization would both yield linear relationships between t and z.

Suppose that as in Tinbergen (1956) job (dis-)satisfaction is defined as $j(z,t;A) = \frac{1}{2}(z-t)' A (z-t)$, where A is a positive definite matrix of preference parameters. Suppose further that productivity is given by $p(z,t;E) = b_0 + b'z + c't + \frac{1}{2}z'Bz + \frac{1}{2}t'Ct + t'Dz$ with $E = \{b_0, b, c, B, C, D\}$ and where b_0 is a positive constant, b and c are vectors filled with positive constants or zeros and B, C and D are matrices of parameters. The parameters contained in b and B indicate how productive a machine with attributes z is, independently of the attributes of the worker operating this machine, i.e. $\frac{\partial p(z,t;E)}{\partial z} = b + Bz$ The parameters contained in c and C indicate the extent to which workers' attributes affect productivity, independently of the attributes of the machine, i.e. $\frac{\partial p(z,t;E)}{\partial t} = c + Ct$ The parameters contained in D indicate the extent to which the attributes of machines complement or substitute workers attributes, i.e. $\frac{\partial p(z,t;E)}{\partial z \partial t} = D$.

Note that since t and z take on negative values with positive probability, i) the productivity of machines decreases with workers attributes for some machines with negative attributes, $\frac{\partial p}{\partial t} = c + Ct + Dz < 0$ for some z, and ii) the productivity of workers decreases with machines attributes for some workers with negative attributes, $\frac{\partial p}{\partial z} = b + Bz + Dt < 0$ for some t. However, the share of machines and the share of workers for which i) and ii) hold can be made arbitrarily small by varying the parameters of the distribution of z and t. Note further, that the generalized Spence-Mirrlees condition will be satisfied as long as $D + A \neq \boxed{0}$ where $\boxed{0}$ is a matrix of zeros. As long as $D + A \neq \boxed{0}$, equilibrium is pure for any distribution F_t and F_z so that the mapping function $t(z)$ is invertible with inverse $z(t) \equiv t^{-1}(t)$. The mapping function $t(z)$ is linear when the distribution of t and z is normal.

The first order conditions read now as:[9]

$$\begin{aligned}
\frac{\partial w(z,t)}{\partial z} &= A(z-t) \\
\frac{\partial w(z,t)}{\partial t} &= c + Ct + Dz
\end{aligned}$$

Note that the wage differential for jobs attributes is (positively) related to workers preference parameters A. However, the wage differential for workers attributes does not depend on A.

It is now easy to see that the first order conditions yield linear mapping of jobs attributes on workers attributes if and only if $\frac{\partial w(z,t)}{\partial t}$ is linear in t and $\frac{\partial w(z,t)}{\partial z}$ is

[9] The second order conditions are trivial and given by:

$$\begin{aligned}
A - \frac{\partial^2 w(z,t)}{\partial z^2} &> 0 \\
C + \frac{\partial^2 w(z,t)}{\partial t^2} &> 0
\end{aligned}$$

linear in z. This, in turns, implies that the equilibrium wage function is quadratic and reads as:

$$w(z,t) = \delta_0 + \delta' t + \lambda' z + \frac{1}{2} t' \Delta t + \frac{1}{2} z' \Lambda z \tag{3}$$

Note here that t and z are additively separable in the wage function. Is this a special case or is this a generic properties of the unified model??? (Work to be done to show this...)

Using equation 3 in equations 1 and 2 respectively and rearranging yields:[10]

$$\begin{aligned} \lambda + (\Lambda - A)\, z &= -At && (4) \\ \delta - c + (\Delta - C)\, t &= Dz && (5) \end{aligned}$$

If all jobs attributes provide intrinsic disutility, the matrix A is invertible and the equilibrium solution associated with equation 4 will be given by:

[10] This also means that the second order conditions require that the matrices $A - \Lambda$ and $C + \Delta$ are positive definite.

$$t = -A^{-1}\lambda - A^{-1}(\Lambda - A)z$$

The vector λ is identified as $\lambda = -A\pi_0$ and the matrix Λ as $\Lambda = A(I - \Pi_1)$. The case where not all jobs attributes provide intrinsic disutility is discussed in the following subsection.

If all of a worker's attributes are complementary to at least one job attribute in production, the matrix D is invertible and the first order condition to profits maximization gives:

$$z = D^{-1}(\delta - c) + D^{-1}(\Delta - C)t$$

By noting that $z = -\Pi_1^{-1}\pi_0 + \Pi_1^{-1}t$, the vector δ is identified as $\delta = c - D\Pi_1^{-1}\pi_0$ and the matrix Δ is identified as $\Delta = C + D\Pi_1^{-1}$. The case where not all workers attributes are complementary to at least one job attribute in production is discussed in the following next subsection.

To summarize, we have shown that the equilibrium parameters of the wage function λ, Λ, δ and Δ are retrieved from the distribution parameters of workers and firms attributes using the mapping function $t(z) = \pi_0 + \Pi_1 z$ where Π_1 is identified by the variances and covariances of workers and jobs attributes only and π_0 is identified from the means of workers and jobs attributes and from Π_1. The second order coefficients of the wage function Λ and Δ depend only on the matrix of variances and covariances of workers and jobs attributes. The first order coefficients λ and δ depend on the matrix variances and covariances and the means of workers and jobs attributes.

The equilibrium wage function has for parameters:

$$
\begin{aligned}
\lambda &= A\left(\mu_t - \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z\right) &(6)\\
\Lambda &= A\left(\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1} - I\right) &(7)\\
\delta &= -D\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)\left(\mu_t - \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z\right) + c &(8)\\
\Delta &= D\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right) + C &(9)
\end{aligned}
$$

The constant δ_0 is not identified. To close the model, the usual assumption (see Ekeland et al. (2002 and 2004) and Sattinger (1979) among others) is to fix a reservation value for the utility, say $\underline{u}$ and rent $\underline{r}$ so that u and r must be larger than their respective thresholds.

2.2.2 Special cases

If not all jobs attributes provide intrinsic disutility, the associated rows and columns of A will be filled with 0's. Suppose that z is partitioned as $z = \begin{pmatrix} z_p \\ z_s \end{pmatrix}$, where z_p is a vector of those jobs attributes that derive intrinsic disutility and z_s its complement containing attributes that derive no intrinsic disutility. We therefore have $A = \begin{pmatrix} A_{pp} & \boxed{0} \\ \boxed{0} & \boxed{0} \end{pmatrix}$.[11] The wage differential for jobs attributes z_s corresponding to attributes from which no intrinsic disutility is derived, will be 0, i.e. $\lambda = \begin{pmatrix} \lambda_p \\ \underline{0} \end{pmatrix}$, where $\underline{0}$ is a vector of 0, and $\Lambda = \begin{pmatrix} \Lambda_{pp} & \boxed{0} \\ \boxed{0} & \boxed{0} \end{pmatrix}$, where $\boxed{0}$ is a matrix of 0. We could solve for the wage differential of the remaining attributes by just looking at attributes t_p and z_p in equation 4.

Using equation 3 in equations 1 and 2 respectively and rearranging yields:

$$\lambda_p + (\Lambda_{pp} - A_{pp})\, z_p = -A_{pp} t_p \tag{10}$$

[11] A bit of notation is required here. A vector v of dimension N is partitioned into two subvectors v_s and v_p of size S and P respectively with $S + P = N$, so that $v = \begin{pmatrix} v_p \\ v_s \end{pmatrix}$. A matrix V of dimension $N \times N$ is partitioned into four submatrices, i.e. V_{pp}, V_{ps}, V_{sp} and V_{ss} of respective size $P \times P$, $P \times S$, $S \times P$ and $S \times S$ so that $V = \begin{pmatrix} V_{pp} & V_{ps} \\ V_{sp} & V_{ss} \end{pmatrix}$.

A_{pp} is invertible, i.e. all p attributes are associated with intrinsic disutility, the equilibrium solution associated with equation 4 will be given by:

$$t_p = -A_{pp}^{-1}\lambda_p - A_{pp}^{-1}(\Lambda_{pp} - A_{pp})z_p$$

This means that:

$$\begin{aligned}
\lambda &= \begin{pmatrix} \lambda_p \\ \underline{0} \end{pmatrix} = \begin{pmatrix} -A_{pp}\pi_{p0} \\ \underline{0} \end{pmatrix} \\
\Lambda &= \begin{pmatrix} \Lambda_{pp} & \boxed{0} \\ \boxed{0} & \boxed{0} \end{pmatrix} = \begin{pmatrix} A_{pp}\left(I_{pp} - \pi_{pp1}\right) & \boxed{0} \\ \boxed{0} & \boxed{0} \end{pmatrix}
\end{aligned}$$

Suppose that workers attributes t_p are complementary to no job attributes whereas attributes t_s are complementary to at least one job attribute. The matrix D is then equal to $\begin{pmatrix} \boxed{0} & \boxed{0} \\ \boxed{0} & D_{ss} \end{pmatrix}$. In this case, the right hand side of equation 5 is equal to zero. We could solve for the wage differential of the remaining attributes by just looking at attributes t_s and z_s in equation 5.

Using equation 3 in equations 1 and 2 respectively and rearranging yield:

$$\delta_s - c_s + (\Delta_{ss} - C_{ss})\, t_s = D_{ss} z_s$$

D_{ss} is invertible, i.e. all s attributes are complementary with at least one job attribute, the equilibrium solution associated with equation and the first order condition to profits maximization gives:

$$z_s = D_{ss}^{-1}(\delta_s - c_s) + D_{ss}^{-1}(\Delta_{ss} - C_{ss})\, t_s$$

This means that:

$$\begin{aligned} \delta &= \begin{pmatrix} \delta_p \\ \delta_s \end{pmatrix} = \begin{pmatrix} c_p \\ c_s - D_{ss}\pi_{s1}^{-1}\pi_{s0} \end{pmatrix} \\ \Delta &= \begin{pmatrix} C_{pp} & C_{ps} \\ & \\ C_{sp} & C_{ss} + D_{ss}\pi_{ss1}^{-1} \end{pmatrix} \end{aligned}$$

2.3 Relations to Tinbergen (1956) and Sattinger (1979)

Tinbergen (1956), Epple (1984) and Ekeland et al. (2002 and 2004) consider the case where workers attributes do not contribute to production, i.e. $\frac{\partial p(z,t;E)}{\partial t} = 0$, so that the first order condition to rents maximization in equation 2 indicate no wage differentials across workers' attributes. In the quadratic-normal example, this condition is met when $D = C = \boxed{0}$, $c' = \underline{0}$, so that equation 5 yields $\Delta = C = \boxed{0}$ and $\delta = c = \underline{0}$. Since $D = \boxed{0}$, the generalized Spence-Mirrlees condition for pure equilibrium is now satisfied for $A \neq -D = \boxed{0}$, the equilibrium will be pure in Tinbergen's model only if $A \neq \boxed{0}$.

The model proposed above admits Sattinger's differential rents model as a special case. This is the case when t and z are unidimensional and z carries no intrinsic disutility so that $\frac{\partial j(z,t;A)}{\partial z} = 0$ or $A = \boxed{0}$ in the quadratic-normal example. From equation 2 (equation 4 respectively in the quadratic-normal example) we then have $\Lambda = (A =)\boxed{0}$ and $\lambda = \underline{0}$ so that the wage function depends merely on t. As soon as t is loaded with intrinsic disutility, $A \neq \boxed{0}$, the slope of the rents function increases and the increase is more pronounced for higher z. Again, since $A = \boxed{0}$, the generalized Spence-Mirrlees condition is satisfied if $D \neq -A = \boxed{0}$.

3 Implications for empirical applications

Since the seminal work by Rosen (1974), the traditional approach to estimate preference parameters, i.e. parameters A of the function $j(z, t; A)$, has consisted of two steps. In the first step, using market data on wages and jobs attributes, one estimates the wage function to get $\widehat{w}(z)$ applying the functional form that fits best the data. In the second step, one uses the first order condition equation 1 together with the marginal wage as derived from the first step estimate, i.e. $\frac{\partial \widehat{w}(z)}{\partial z}$, to recover preference estimates $j(z,t;A)$. A similar argument holds for the estimation of technology parameters E.

Brown and Rosen (1982), Epple (1987), Bartik (1987) and Kahn and Lang (1988) have argued that $j(z,t;A)$ cannot be identified in a single cross-sectional data unless an arbitrary nonlinear marginal price is assumed. Recently, Ekeland et al. (2002 and 2004) have shown that the generic hedonic model yields a nonlinear marginal price function and proposed instrumental variable techniques to estimate the model in single cross-section.

This discussion, however, is about the estimation of the first order condition equation and relies essentially on the estimation of $\widehat{w}(z)$ in the first step. This first step estimation relies on the explicit assumption that the wage function does not depend on workers attributes t. This contrast with the unified model presented above that yields in equilibrium a wage function that depends on *both* workers and jobs attributes $w(z,t)$. This assumption is of crucial importance for the estimation of $\widehat{w}(z)$. To see this, remember that the equilibrium condition of supply and demand requires a mapping of workers attributes on jobs attributes $t(z)$. Using this mapping function to replace t in $w(z,t)$, the equilibrium wage function can be written as $\widetilde{w}(z) = w(z, z(t))$. If the true economy is the one depicted by the unified model, then $\widehat{w}(z)$ is an estimate of $\widetilde{w}(z)$ and not $w(z,t)|t$. This means that the estimated marginal wage derived from $\widehat{w}(z)$ is given by:

$$\begin{aligned} \frac{\partial \widehat{w}(z)}{\partial z} &= \frac{\partial \widetilde{w}(z)}{\partial z} \equiv \frac{\partial w(z,t(z))}{\partial z} \\ &= \frac{\partial w(z,t(z))}{\partial z} + t'(z)\frac{\partial w(z,t)}{\partial t} \end{aligned} \tag{11}$$

which yields

$$\frac{\partial \widehat{w}(z)}{\partial z} = \frac{\partial j(z,t(z);A)}{\partial z} + t'(z)\frac{\partial p(z,t(z);E)}{\partial t} \tag{12}$$

where the last step follows from the first order conditions 1 and 2.

Rosen's step 2 consists of estimating the first order condition for utility maximization and hence reads as $\frac{\partial \widehat{w}(z)}{\partial z} = \widehat{\frac{\partial j(z,t;A)}{\partial z}} + u$ where u is an error term. As can be seen from equation 12, the term $\widehat{\frac{\partial j(z,t;A)}{\partial z}}$ will be an estimate of $\frac{\partial j(z,t(z);A)}{\partial z} + t'(z)\frac{\partial p(z,t(z);E)}{\partial t}$. Clearly, unless workers attributes do not affect productivity and the equilibrium wage function does not depend on t, i.e. in which case $\frac{\partial w(z,t)}{\partial t} = 0$, $\widehat{\frac{\partial j(z,t;A)}{\partial z}}$ will be a biased estimate of $\frac{\partial j(z,t;A)}{\partial z}$, the magnitude of the biased will be given by $t'(z)\frac{\partial p(z,t;E)}{\partial t}$. In the unified model, the dependent variable of Rosen's second step is not the marginal wage associated with job attributes holding workers attributes constant but rather a mixture of the marginal wage associated with job attributes and the marginal wage associated with workers attributes. The right hand side of the second step equation is therefore a composite of preference parameters $j(z,t;A)$ and productivity parameters $p(z,t;E)$.

Note that this argument holds as long as some of a worker's attributes are unobserved. To see this, write $w(z,t) \equiv w_1(z,t^o(z),t^u(z))$, using the equilibrium mapping of workers attributes to jobs attributes, i.e. $t(z) = \langle t^o(z);t^u(z)\rangle$, $w_1(z,t^o(z),t^u(z))$. The first step regression estimates $\widehat{w}_1(z,t^o,t^u(z))$ when controlling for observable attributes of workers t^o. Deriving $\widehat{w}_1$ with respect to z yields $\frac{\partial \widehat{w}_1}{\partial z} = \frac{\partial w_1(z,t^o,t^u)}{\partial z} + t^{u\prime}(z)\frac{\partial w_1(z,t^o,t^u(z))}{\partial t^u}$.

3.1 Empirical methodology to estimate the unified model

Under construction....

Assuming z is scalar and $t =< t^o, t^u >$ where t^u is scalar, if it turns out that the generic model has additively separable wage functions of the form $w(z,t) = w^a(z) + w^b(t)$ and $E[w^a(z)|t]$ and $w^b(z)$ are non collinear, the IV method proposed by Ekeland et al. (2004) for the second step equation, could be applied on the first step equation. We could instrument $w^a(z)$ by t^o, where t^o are observed attributes of workers in a regression of wages on job attribute z. Note that no independence between observed t^o and unobserved t^u workers (or firms) attributes are required.

4 Extensions (under construction)

4.1 The unified hedonic *production* economy model

Suppose that instead of being endowed with a machine, firms can produce their own machine. For instance, firms could invest in less noisy machines, safer machines, machines requiring less physical strength to operate, high-tech machines etc. Suppose further that firms are endowed with a vector of attributes y, $y \in \mathbb{R}^{N_y}$. To fix ideas, these attributes could be related to investments capacities but also to the managers' attributes, again, either skills or preferences. Let $f_y(y)$ and $F_y(y)$ be the PDF and CDF of y respectively and let F_y be absolutely continuous with respect to Lebesgue measure.

Let the costs of producing a machine with attributes z for a firm with attributes y be given by the continuous function $c(y, z; G)$ with parameters G. It is still assumed that to produce output each machine needs to be operated by one and only one worker so that workers and firms match one-to-one. The profits of a firm with attributes y producing output with machine z and employing worker t are now given by:

$$r(z, t, y) = p(z, t; E) - w(z, t) - c(y, z; G)$$

The first order condition to utility maximization is unchanged and given by equation 1. We therefore have the mapping function $z(t; A, w_z)$ indicating the optimal machine demanded by a worker with attributes t when wage differential at z is given by w_z. However, firms are now maximizing profits by selecting the optimal combination of worker x and machine z. First order conditions for profit maximization read as:

$$\frac{\partial w(z,t)}{\partial z} = \frac{\partial p(z,t;E)}{\partial z} - \frac{\partial c(y,z;G)}{\partial z} \tag{13}$$

$$\frac{\partial w(z,t)}{\partial t} = \frac{\partial p(z,t;E)}{\partial t} \tag{14}$$

Let $z(t; E, w_t)$ denote the implicit function that solves Equation 14 for parameters E and function w_t where $w_t \equiv \frac{\partial w}{\partial t}$. This function indicates the optimal worker t to select for a firm supplying machine with attributes z when the wage differential at t is given by w_t. Let $z(y, t; G, w_z)$ denote the implicit function that solves Equation 13 for parameters E and G and function $w_z \equiv \frac{\partial w}{\partial z}$. This function indicates the optimal machine z to supply for a firm with attributes y employing worker with attributes t. Substituting $t(z; E, w_t)$ for t in $z(y, t; G, w_z)$ we obtain an implicit function $z(y; E, G, w_t, w_z)$ indicating the optimal machine $z^* = z(y; E, G, w_t, w_z)$ to supply for a firm with attributes y given productivity parameters E, costs parameters G and wage function w.

Assume further that the total surplus function $s(z,t;E,A) \equiv p(z,t;E) - c(y,z;G) - j(t,z;A)$ satisfies the generalized Spence-Mirrlees condition so that equilibrium is pure and the mapping functions $z(y;E,G,w_t,w_z)$ and $z(t;A,w_z)$ are invertible. Define these inverse functions as $y(z;E,G,w_t,w_z)$ and $t(z;A,w_t)$ respectively. Workers' demand for machines with attributes z is then given by $f_z(z)dz = f_t(t(z;A,w_t)) \left|\frac{\partial t(z;A,w_z)}{\partial z}\right| dz$ while firms' supply is given by $f_z(z)dz = f_y(y(z;E,G,w_t,w_z)) \left|\frac{\partial y(z;E,G,w_t,w_z)}{\partial z}\right| dz$. For an equilibrium to be reached, the supply of machines with attributes z should be equal to workers' demand for machines with attributes z for all z. This means that:

$$f_t(t(z;A,w_t)) \left|\frac{\partial t(z;A,w_z)}{\partial z}\right| dz = f_y(y(z;E,G,w_t,w_z)) \left|\frac{\partial y(z;E,G,w_t,w_z)}{\partial z}\right| dz$$

Equilibrium will be reached by choosing the right shape for the function w and in particular the right differentials at t and z. The equilibrium in this economy is therefore characterized by a wage function $w(z,t)$ and a mapping of workers attributes onto jobs attributes $t(z;A,w_z)$ and a mapping function of firms attributes onto jobs attributes and workers' attributes $y(z;E,G,w_t,w_z)$ so that i) supply equals demand everywhere on the support of z −provided all workers and firms receive more than their reservation levels− , ii) workers maximize utility and iii) firms maximize profits (rents minus costs of producing z).

4.2 Generalization to other markets

This paper relates to the general literature on hedonic models and on only on that segment focussing on the labor market. For instance, Epple's extension of Tinbergen's endowed economy to a production economy was originally written in a consumer/producer context, not a worker/firm context. In the consumer/producer model, the restriction that consumers' attributes do not affect the production of goods does not at first sight seem to be too strong. However, the generalization proposed in this paper is also relevant in that case. Think for instance of an economy where firms with endowed vector of attributes (capital) y. In this economy, to produce good z, firms need to hire a fixed number of workers, one worker for simplicity. Suppose further that the attributes of that worker matter in the production process so that the costs (profits) of producing good z depend on the attributes of the worker, say t'. Firms need now to optimize not only on z but also on t'. The model presented in this paper can be seen as a special case in which workers consume their own product $t = t'$. The foundation of this model and its potential applications to other markets are discussed in section 4 of this paper.

To be continued...

5 Conclusion

This paper unifies the two classes of models within the sorting literature. The model nests both Tinbergen's model of sorting on job preferences and Sattinger's model of sorting on productivity. Under the assumption that all job attributes lead to intrinsic disutility but workers attributes do not affect productivity the model collapses to Tinbergen's model. Workers care about their job satisfaction but are equally productive at all jobs. This means that the equilibrium wage function does not depend on workers attributes but merely on jobs attributes. Opposite to this, under the assumption that all workers attributes contribute to productivity but no job attributes lead to intrinsic disutility the model collapses to Sattinger's differential rents model. Workers do not care about job satisfaction, only about their wage, but workers with different attributes are unequally productivity. This means that the equilibrium wage function does not depend on jobs attributes but merely on workers attributes. In the more general case depicted by the unifying model, workers do care about job satisfaction and productivity depends on workers attributes. As a result, the equilibrium wage function has both workers and jobs attributes as arguments. An example of closed form solution is provided.

The implication of the unified model is that the wage function admits both sets of attributes as arguments. This paper also shows that the traditional two-step procedure proposed by Rosen (1974) to estimate hedonic wage function provides biased estimate. While most of the literature has focused on problems involved in the estimation of the second step of the procedure, the unified model indicates that the traditional first step, consisting of estimate the wage function using data on wages and job attributes is flawed. Indeed, the dependent variable of Rosen's second step is not the marginal wage associated with job attributes holding workers attributes constant but rather a mixture of the marginal wage associated with job attributes and the marginal wage associated with workers attributes. The method estimates therefore a composite of preference parameters A of the function $j(z, t; A)$ and productivity parameters E of the function $p(z, t; E)$.

References

BARTIK, T. J. (1987): "Estimating Hedonic Demand Parameters with Single Market Data: The Problems Caused by Unobserved Tastes," *The Review of Economics and Statistics*, 69(1), 178–80.

BORGHANS, L., A. L. DUCKWORTH, J. H. HECKMAN, AND B. TER WEEL (2008): "Economics and Psychology of Personality Traits," *Journal of Human Resources*, forthcoming.

BOSCH, A. J. (1987): "Note on the Factorization of a Square Matrix Into Two Hermitian or Symmetric Matrices," *SIAM Review*, 29(3), 463–468.

Bowles, S., H. Gintis, and M. Osborne (2001): "The Determinants of Earnings: A Behavioral Approach," *Journal of Economic Literature*, 39(4), 1137–1176.

Brown, J. N., and H. S. Rosen (1982): "On the Estimation of Structural Hedonic Price Models," *Econometrica*, 50(3), 765–768.

Epple, D. (1984): "Closed Form Solutions to a Class of Hedonic Models," *unpublished manuscript*.

——— (1987): "Hedonic Prices and Implicit Markets: Estimating Demand and Supply Functions for Differentiated Products," *Journal of Political Economy*, 95(1), 59–80.

Kahn, S., and K. Lang (1988): "Efficient Estimation of Structural Hedonic Systems," *International Economic Review*, 29(1), 157–66.

Rosen, S. (1974): "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition," *Journal of Political Economy*, 82, 34–55.

Sattinger, M. (1977): "Compensating wage differences," *Journal of Economic Theory*, 16(2), 496–503.

——— (1979): "Differential Rents and the Distribution of Earnings," *Oxford Economic Papers*, 31(1), 60–71.

——— (1980): *Capital and the Distribution of Labor Earnings*. Amsterdam: North-Holland.