Marginal Jobs, Heterogeneous Firms, & Unemployment Flows*

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Abstract

Much recent research has sought to explain the cyclical amplitude of unemployment fluctuations in the US. This paper shows that amplification of the cyclical variation of unemployment can be obtained from adding two very simple features to an otherwise standard model of the aggregate labor market, namely downward sloped short run labor demand and endogenous job destruction. This generalized model is able to match more closely the cyclicality of both job finding and employment to unemployment flows observed in US data. Contrary to standard models, the model can generate amplification while maintaining realistic surplus to employment relationships. In addition, we uncover a novel source of amplification of cyclical shocks that is generated by the interaction of countercyclical unemployment inflows and job creation.

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Much recent research has sought to explain the cyclical amplitude of unemployment fluctuations in the US. Shimer (2005) has shown that a leading model of the aggregate labor market, the Mortensen-Pissarides (MP) model, cannot explain this cyclical volatility. A common solution to this problem proposed in subsequent literature has been to invoke rigidity in the wages of newly hired workers (see among others Shimer, 2004; and Hall, 2005). However, the empirical validity of such an assumption has been questioned by Haefke, Sonntag, & van Rens (2007), and by Pissarides (2007).[1] This paper takes a different approach. We show that amplification of the cyclical volatility of unemployment can instead be obtained simply by adding two very conventional features to the standard search model, namely downward sloped short run demand for labor and endogenous job destruction.

The motivation for these additional features is simple. First, downward sloped labor demand is motivated by the fact that other production inputs, notably capital, are not fully flexible at cyclical frequencies.[2] Second, the inclusion of endogenous job destruction is informed by empirical evidence that part of the cyclical upswing in unemployment in times of recession is accounted for by increased flows from employment to unemployment.[3]

However, incorporating these two conventional features simultaneously is not a trivial exercise. We show that it is also not a daunting one. In particular, downward sloped labor demand implies that firms face a non-linear production technology which poses a number of theoretical challenges. First, this complicates wage setting because the surplus generated by each of the employment relationships within a firm is not the same (e.g. "the" marginal worker generates less surplus than infra-marginal workers). In section 1, we derive a very intuitive and explicit wage bargaining solution for this environment, something that has been considered challenging in recent research (see Cooper, Haltiwanger & Willis, 2007; and

[1] This echoes the earlier results of Baker, Gibbs, & Holmstrom (1994) who observe substantial wage flexibility among new hires in data for a particular firm (see especially their Figure II).

[2] Another motivation for downward sloped labor demand is the existence of imperfect product market competition. For a model with this feature (but with exogenous job destruction) see Rotemberg (2006).

[3] See Perry (1972); Marston (1976); Blanchard & Diamond (1990); Elsby, Michaels, & Solon (2007); Fujita & Ramey (2007); Pissarides (2007); Shimer (2007); and Yashiv (2006).

Hobijn & Sahin, 2007). In particular, the solution is a very natural generalization of the wage bargaining solution in the standard MP model. The simplicity of our solution is therefore a useful addition to the literature.[4]

The wage bargaining solution then enables us to characterize the properties of the optimal labor demand policy of an individual firm in the presence of idiosyncratic firm heterogeneity. An interesting by-product of this exercise is that the optimal labor demand solution in the generalized model is analogous to that of a model of kinked hiring costs in the spirit of Bentolila & Bertola (1990), but where the hiring cost is endogenously determined by frictions in the labor market. Thus, the correspondence between the two major approaches to the economics of aggregate labor markets – search and matching models and employment adjustment cost models – sharpens in the process of generalizing the standard search model.

A second analytical challenge in models with a non-linear production technology and idiosyncratic heterogeneity is that aggregation of microeconomic behavior is not straightforward, because a representative firm interpretation of the model doesn't exist. To address this, in section 2 we develop a method for aggregating the behavior of individual firms that holds for a wide class of optimal labor demand policies at the microeconomic level. In particular, we are able to solve for the equilibrium distribution of employment across firms, which in turn allows us to determine the level of the aggregate (un)employment stock. In addition, we also provide a related method that allows us to solve for aggregate unemployment flows (hires and separations) implied by microeconomic behavior. Together, these characterize the aggregate equilibrium of the model economy.

These aggregation results allow us to quantitatively evaluate our model, which we turn to in section 3. In particular, we show that a standard calibration of our generalized model

[4] Related models with endogenous separations such as Cooper et al. (2007) and Hobijn & Sahin (2007) have set worker bargaining power to zero in order to derive wages. Acemoglu & Hawkins (2006) characterize wages in a model with exogenous separations, but they focus on a time to hire aspect to job creation, which leads to a more challenging bargaining problem. Our solution is analogous to the wage bargaining solutions derived by Smith (1999), Cahuc & Wasmer (2001), and Krause & Lubik (2007) for models with exogenous job destruction.

can more closely match the observed cyclical variation of both the job finding rate and the employment to unemployment transition rate in the US, and is a substantial improvement on the basic MP model. A potential concern in models, such as this, that incorporate countercyclical job destruction has been that they often cannot generate the observed procyclicality of vacancies (Shimer, 2005; Mortensen & Nagypal, 2007b). Importantly, we find that our model makes considerable progress in this regard: Our calibration of the model generates approximately 3/4 of the observed comovement between vacancies and output per worker. Moreover, we suggest that the remaining procyclicality is likely due to procyclical job-to-job flows that are observed in the data (Fallick & Fleischman, 2004) but are abstracted from in the present paper.

The common factor that generates both the procyclicality of the job finding rate and of vacancies in the model is the procyclicality of desired job creation. To uncover the processes underlying this, in section 4 we derive a simple approximation to the decline in job creation following an adverse aggregate shock in the generalized model, analogous to the method employed by Shimer (2005) and Mortensen & Nagypal (2007a,b). This exercise reveals two sources of amplification. The first generalizes a well-known result that the standard MP model is consistent with observed unemployment cyclicality if the average flow surplus to employment relationships is sufficiently small.[5] We show that an analogous result occurs in the generalized model if a weighted average of the average and *marginal* flow surplus is sufficiently small. However, because downward sloped labor demand implies that the marginal surplus will be smaller than the average surplus, the generalized model can deliver amplification of the job creation response to aggregate shocks at the same time as preserving a sizeable average surplus from employment relationships.[6] This is important because recent

[5] Intuitively, a small surplus to employment relationships implies that small reductions in the productivity of labor (as in a recession) quickly exhaust the surplus and lead firms to cut back substantially on hiring. See Mortensen & Pissarides (1994), Hagedorn & Manovskii (2005), and Costain & Reiter (2005).

[6] One might imagine that a symmetric logic holds on the supply side of the labor market if there is heterogeneity in workers' valuations of leisure so that "the" marginal worker obtains a low surplus from employment. Interestingly, Mortensen & Nagypal (2007a) argue that this is not the case. They show that

research has suggested that the average surplus required for the standard model to match the observed cyclicality of the job finding rate is too small (Mortensen & Nagypal, 2007a).[7]

Our results also suggest a second, more novel source of amplification that is generated by the interaction of heterogeneous firms and downward sloped labor demand. Following a reduction in aggregate labor demand, low productivity firms wish to shed more workers, and high productivity firms wish to hire fewer workers. Thus inflows into the unemployment pool rise, and outflows from the unemployment pool fall, *ceteris paribus*, causing unemployment to rise. To restore equilibrium in the model, hiring firms must be convinced to hire enough workers to equate inflows to outflows once more. The model achieves this by allowing the labor market to slacken, so that unemployed workers become more abundant, and hiring (suitable) workers becomes less costly for firms. With downward sloped labor demand, increased hiring retards the productivity of additional employment relationships, and so the labor market must slacken further, and unemployment must rise more, in order to return the economy back to equilibrium once again.

Section 5 of the paper discusses the broader implications of our analysis. We argue that the model developed in the paper provides a rich, yet analytically tractable model of the aggregate labor market in the short run. As such, we believe that this model will provide a useful laboratory for the cyclical analysis of aggregate labor markets in future empirical and theoretical research. In addition, we suggest that, by developing a model with a well-defined concept of a firm, the analytical results derived here are a natural complement to recent research that has investigated the empirical implications of search frictions using establishment level data (Cooper, Haltiwanger & Willis, 2007; Davis, Faberman, & Haltiwanger, 2007). Fi-

if firms cannot differentiate workers' types when making hiring decisions, they will base their decision on the average, rather than the marginal, valuation of leisure among the unemployed. The same is unlikely to be true of the model studied here, since firms presumably know their production technology when making hiring decisions.

[7] A small average surplus also jars with widespread evidence for the prevalence of long term employment relationships in the US economy, which researchers have taken to imply substantial rents to ongoing matches (Hall, 1982; Stevens, 2005).

nally, we emphasize the wider applicability of the aggregation results we derive as a method of characterizing aggregate labor market equilibrium in models without a representative firm interpretation. Our aggregation results can be applied in exactly the same manner to other popular models of non-linear microeconomic behavior, such as non–convex adjustment cost models (Caballero, Engel, & Haltiwanger, 1997; Cooper, Haltiwanger, & Willis, 2004). Our hope is that this will further our understanding of the macroeconomic implications of non-linearities in firm level labor demand for the aggregate labor market.

1 The Firm's Problem

In what follows we consider a model in which there is a mass of firms, normalized to one, and a mass of potential workers equal to the labor force, L.[8] In order to hire unemployed workers, firms must post vacancies. However, frictions in the labor market limit the rate at which unemployed workers and hiring firms can meet. As is conventional in the search and matching literature, these frictions are embodied in a matching function, $M = M(U, V)$, that regulates the number of hires, M, that the economy can sustain given that there are V vacancies and U unemployed workers. We assume that $M(U, V)$ exhibits constant returns to scale.[9] Vacancies posted by firms are therefore filled with probability $q = M/V = M(U/V, 1)$ each period. Likewise, unemployed workers find jobs with probability $f = M/U = M(1, V/U)$. Thus, the ratio of aggregate vacancies to aggregate unemployment, $V/U \equiv \theta$, is a sufficient statistic for the job filling (q) and job finding (f) probabilities in the model. Taking these flow probabilities as given, firms choose their optimal level of

[8] Assuming a fixed number of firms is important for the model to depart from the standard MP model. Free entry would yield an economy of infinitesimal firms that converges to the MP limit. In principle, one could allow for costly firm entry as a middle ground. We abstract from this in part for simplicity. But our choice is also informed by evidence in Davis and Haltiwanger (1992). They find that, in manufacturing, establishment births account for 20% of job creation on average each year. While this is not a small number, the majority of job creation is clearly accounted for by incumbent establishments. For a model that explores the impact of firm entry on job creation, see Garibaldi (2006).

[9] See Petrongolo & Pissarides (2001) for a summary of empirical evidence that suggests this is reasonable.

employment, to which we now turn.

1.1 Labor Demand

We consider a discrete time, infinite horizon model in which firms use labor, n, to produce output according to the production function, $y = pxF(n)$ where $F' > 0$ and $F'' \leq 0$. The latter is a key generalization of the standard MP model that we consider: When $F'' < 0$, the marginal product of labor will decline with firm employment, and thereby will generate a downward sloped demand for labor at the firm level. p represents the state of aggregate labor demand, whereas x represents shocks that are idiosyncratic to an individual firm. We assume that the evolution of the latter idiosyncratic shocks is described by the c.d.f. $G(x'|x)$.

A typical firm's decision problem is completely analogous to that in Mortensen & Pissarides (1994), and is as follows. Firms observe the realization of their idiosyncratic shock, x, at the beginning of a period. Given this, they then make their employment decision. Specifically, they may choose to separate from part or all of their workforce, which we assume may be done at zero cost. Any such separated workers then join the unemployment pool in the subsequent period. Alternatively, firms may hire workers by posting vacancies, $v \geq 0$, at a flow cost of c per vacancy. If a firm posts vacancies, the matching process then matches these up with unemployed workers inherited from the previous period. After the matching process is complete, production and wage setting are performed simultaneously.

It follows that we can characterize the expected present discounted value of a firm's profits, $\Pi(n_{-1}, x)$, recursively as:[10]

$$\Pi(n_{-1}, x) = \max_{n,v} \left\{ pxF(n) - w(n,x)\,n - cv + \beta \int \Pi(n, x')\, dG(x'|x) \right\} \tag{1}$$

[10] We adopt the convention of denoting lagged values with a subscript, $_{-1}$, and forward values with a prime, $'$.

where $w(n,x)$ is the bargained wage in a firm of size n and productivity x. A typical firm seeks a level of employment that maximizes its profits subject to a dynamic constraint on the evolution of a firm's employment level. Specifically, firms face frictions that limit the rate at which vacancies may be filled: A vacancy posted in a given period will be filled with probability $q < 1$ prior to production. Thus, the number of hires an individual firm achieves is given by:

$$\Delta n \mathbf{1}^{+} = qv \tag{2}$$

where Δn is the change in employment, and $\mathbf{1}^{+}$ is an indicator that equals one when the firm is hiring, and zero otherwise. Substituting the constraint, (2), into the firm's value function, we obtain:

$$\Pi(n_{-1},x) = \max_{n}\left\{pxF(n) - w(n,x)\,n - \frac{c}{q}\Delta n\mathbf{1}^{+} + \beta\int \Pi(n,x')\,dG(x'|x)\right\} \tag{3}$$

Note that the value function is not fully differentiable in n: There is a kink in the value function around $n = n_{-1}$. This reflects the (partial) irreversibility of separation decisions in the model. While firms can shed workers costlessly, it is costly to reverse such a decision because hiring (posting vacancies) is costly. In this sense, the labor demand side is formally analogous to the kinked employment adjustment cost model of the form analyzed in Bentolila & Bertola (1990), except that the per–worker hiring cost, $c/q(\theta)$, is endogenously determined.

In order to determine the firm's optimal employment policy, we take the first-order conditions for hires and separations (i.e. conditional on $\Delta n \neq 0$):

$$pxF'(n) - w(n,x) - w_n(n,x)\,n - \frac{c}{q}\mathbf{1}^{+} + \beta D(n,x) = 0, \text{ if } \Delta n \neq 0 \tag{4}$$

where $D(n,x) \equiv \int \Pi_n(n,x')\,dG(x'|x)$ reflects the marginal effect of current employment decisions on the future value of the firm. Equation (4) is quite intuitive. It states that the marginal product of labor ($pxF'(n)$) net of any hiring costs ($\frac{c}{q}\mathbf{1}^{+}$), plus the discounted expected future marginal benefits from an additional unit of labor ($\beta D(n,x)$) must equal the marginal cost of labor ($w(n,x) + w_n(n,x)\,n$). To provide a full characterization of the firm's optimal employment policy, it remains to characterize the future marginal benefits from current employment decisions, $D(n,x)$, and the wage bargaining solution, $w(n,x)$, to which we now turn.

1.2 Wage Setting

The existence of frictions in the labor market implies that it is costly for firms and workers to find alternative employment relationships. As a result, there exist quasi–rents over which the firm and its workers must bargain. The assumption of constant marginal product in the standard MP model has the tractable implication that these rents are the same for all workers within a given firm. It follows that firms can bargain with each of their workers independently, because the rents of each individual employment relationship are independent of the rents of all other employment relationships.

However, because we allow for the possibility of downward–sloped labor demand ($F'' < 0$), these rents will depend on the number of workers within a firm. Intuitively, the rent that a firm obtains from "the" marginal worker will be lower than the rent obtained on all infra–marginal hires due to diminishing marginal product. An implication of the latter is that the multilateral dimension of the firm's bargain with its many workers becomes important: The rents of each individual employment relationship within a firm are no longer independent.

To take this into account, we adopt the bargaining solution of Stole & Zwiebel (1996) which generalizes the Nash solution to a setting with downward–sloped labor demand.[11] Stole & Zwiebel present a game where the bargained wage is the same as the outcome of simple Nash bargaining over the *marginal* surplus. The game that supports this simple result is one in which a firm negotiates with each of its workers in turn, and where the

[11] This approach was first used by Cahuc & Wasmer (2001) to generate a wage equation for the exogenous job destruction case.

breakdown of a negotiation with any individual worker leads to the renegotiation of wages with all remaining workers.[12]

In accordance with timing of decisions each period, wages are set after employment has been determined. Thus, hiring costs are sunk at the time of wage setting, and the marginal surplus, which we denote as $J(n,x)$, is equal to the marginal value of labor gross of the costs of hiring:

$$J(n,x) = pxF'(n) - w(n,x) - w_n(n,x)\,n + \beta D(n,x) \tag{5}$$

The surplus from an employment relationship for a worker is the additional utility a worker obtains from working in her current firm over and above the utility she obtains from unemployment. The value of employment in a firm of size n and productivity x, $W(n,x)$, is given by:

$$W(n,x) = w(n,x) + \beta\mathbb{E}\left[sU' + (1-s)\,W(n',x')\,|n,x\right] \tag{6}$$

While employed, a worker receives a flow payoff equal to the bargained wage, $w(n,x)$. She loses her job with (endogenous) probability s next period, upon which she flows into the unemployment pool and obtains the value of unemployment, U'. With probability $(1-s)$, she retains her job and obtains the expected payoff of continued employment in her current firm, $W(n',x')$. Likewise, the value of unemployment to a worker is given by:

$$U = b + \beta\mathbb{E}\left[(1-f)\,U' + fW(n',x')\right] \tag{7}$$

Unemployed workers receive flow payoff b, which represents unemployment benefits and/or the value of leisure to a worker. They find a job next period with probability f, upon which they obtain the expected payoff from employment, $W(n',x')$.

[12]The intuition for the Stole & Zwiebel result is as follows. If the firm has only one worker, the firm and worker simply strike a Nash bargain. If a second worker is added, the firm and the additional worker know that, if their negotiations break down, the firm will agree to a Nash bargain with the remaining worker. By induction, then, the firm approaches negotiations with the nth worker as if that worker were marginal. Therefore, the wage that solves the bargaining problem is that which maximizes the marginal surplus.

Wages are then the outcome of a Nash bargain between a firm and its workers over the marginal surplus, with worker bargaining power denoted as η:

$$(1-\eta)\left[W(n,x)-U\right]=\eta J(n,x) \tag{8}$$

Given this, we are able to derive a wage bargaining solution with the following simple structure:

Proposition 1 *The bargained wage, $w(n,x)$, solves the differential equation*[13]

$$w(n,x)=\eta\left[pxF'(n)-w_n(n,x)\,n+\beta f\frac{c}{q}\right]+(1-\eta)\,b. \tag{9}$$

The intuition for (9) is quite straightforward. As in the MP model, wages are increasing in the worker's bargaining power, η, the marginal product of labor, $pxF'(n)$, workers' job finding probability, f, the marginal costs of hiring for a firm, c/q, and workers' flow value of leisure, b. There is an additional term, however, in $w_n(n,x)\,n$. To understand the intuition for this term, consider a firm's negotiations with a given worker. If these negotiations break down, the firm will have to pay its remaining workers a higher wage. The reason is that fewer workers imply that the marginal product of labor will be higher in the firm, which will partially spillover into higher wages ($w_n n < 0$). The more powerful this effect is (the more negative is $w_n n$), the more the firm loses from a given breakdown of negotiations with a worker, and the more workers can extract a higher wage from the bargain.

In what follows, we will adopt the simple assumption that the production function is of the Cobb-Douglas form, $F(n)=n^{\alpha}$ with $\alpha\leq 1$. Given this, the differential equation for

[13] An attractive feature of this solution is its similarity to the solution obtained by Cahuc & Wasmer (2001) for the exogenous job destruction model. It is also consistent with Acemoglu & Hawkins' (2006) Lemma 2, except that it holds both in and out of steady state.

the wage function, (9), has the following simple solution:

$$w(n,x) = \eta\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} + \beta f\frac{c}{q}\right] + (1-\eta)\,b \tag{10}$$

Setting $\alpha = 1$ yields the discrete time analogue to the familiar wage bargaining solution for the MP model.

1.3 The Firm's Optimal Employment Policy

Now that we have obtained a solution for the bargained wage at a given firm, we can combine this with the firm's first–order condition for employment and thereby characterize the firms optimal employment policy, which specifies the firm's optimal employment as a function of its state, $n(n_{-1},x)$. Thus, combining (4) and (9) we obtain:

$$(1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f\frac{c}{q} - \frac{c}{q}\mathbf{1}^{+} + \beta D(n,x) = 0 \tag{11}$$

Given (11) we are able to characterize the firm's optimal employment policy as follows:

Proposition 2 *The optimal employment policy of a firm is of the form*

$$n(n_{-1},x) = \begin{cases} R_v^{-1}(x) & \text{if } \; x > R_v(n_{-1}) \\ n_{-1} & \text{if } \; x \in [R(n_{-1}), R_v(n_{-1})] \\ R^{-1}(x) & \text{if } \; x < R(n_{-1}) \end{cases} \tag{12}$$

where the functions $R_v(\cdot)$ and $R(\cdot)$ satisfy

$$(1-\eta)\left[\frac{pR_v(n)\,\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f\frac{c}{q} + \beta D(n,R_v(n)) \equiv \frac{c}{q} \tag{13}$$

$$(1-\eta)\left[\frac{pR(n)\,\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f\frac{c}{q} + \beta D(n,R(n)) \equiv 0. \tag{14}$$

The firm's optimal employment policy will be similar to that depicted in Figure 1. It is characterized by two reservation values for the firm's idiosyncratic shock, $R(n_{-1})$ and $R_v(n_{-1})$. Specifically, for sufficiently bad idiosyncratic shocks ($x < R(n_{-1})$ in the figure), firms will shed workers until the first-order condition in the separation regime, (14), is satisfied. Moreover, for sufficiently good idiosyncratic realizations ($x > R_v(n_{-1})$ in the figure), firms will post vacancies and hire workers until the first-order condition in the hiring regime, (13), is satisfied. Finally, for intermediate values of x, firms freeze employment so that $n = n_{-1}$. This occurs as a result of the kink in the firm's profits at $n = n_{-1}$, which arises because hiring is costly to firms, while separations are costless.

To complete our characterization of the firm's optimal employment policy, it remains to determine the marginal effect of current employment decisions on future profits of the firm, $D(n,x)$. It turns out that we can show that $D(n,x)$ has the following recursive structure:

Proposition 3 *The marginal effect of current employment on future profits, $D(n,x)$, is given by*

$$D(n,x) = d(n,x) + \beta \int_{R(n)}^{R_v(n)} D(n,x')\, dG(x'|x) \tag{15}$$

where

$$d(n,x) \equiv \int_{R(n)}^{R_v(n)} \left\{ (1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f\frac{c}{q} \right\} dG(x'|x) + \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x'|x). \tag{16}$$

Equation (15) is a contraction mapping in $D(n,\cdot)$, and therefore has a unique fixed point.

The intuition for this result is as follows. Because of the existence of kinked adjustment costs (costly hiring and costless separations) the firm's employment will be frozen next period with positive probability. In the event that the firm freezes employment next period ($x' \in [R(n), R_v(n)]$), the current employment level persists into the next period and so do the marginal effects of the firm's current employment choice. Proposition 3 shows that these

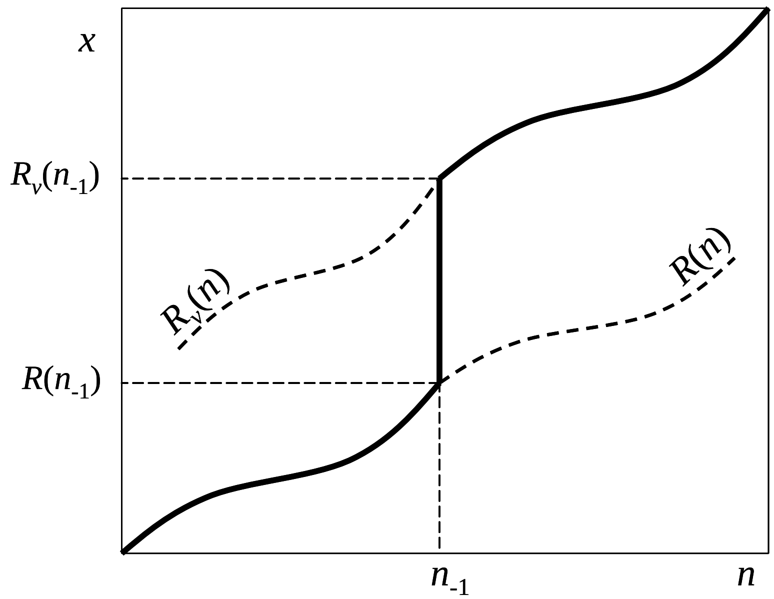

Figure 1: Optimal Employment Policy of a Firm

marginal effects persist into the future in a *recursive* fashion. Propositions 2 and 3 thus summarize the microeconomic behavior of firms in the model.[14]

To get a sense for how the microeconomic behavior of the model works, we next derive the response of an individual firm's employment policy function to changes in (exogenous) aggregate productivity, p, and the (endogenous) aggregate vacancy–unemployment ratio, θ. To do this, we assume that the evolution of idiosyncratic shocks is described by:

$$x' = \begin{cases} x & \text{with probability} \quad 1-\lambda \\ \tilde{x} \overset{c.d.f.}{\sim} \tilde{G}(x) & \text{with probability} \quad \lambda \end{cases} \tag{17}$$

Thus, idiosyncratic shocks display some persistence ($\lambda < 1$) with innovations drawn from the distribution function $\tilde{G}$. Given this, we can establish the following result:

Proposition 4 *If idiosyncratic shocks, x, evolve according to (17), then the effects of the*

[14] It is straightforward to show that equations (10) to (16) reduce down to the discrete time analogue to the Mortensen & Pissarides (1994) model when $\alpha = 1$.

aggregate state variables p and θ on a firm's optimal employment policy are

$$\frac{\partial R_v}{\partial p} < 0; \frac{\partial R}{\partial p} < 0; \frac{\partial R_v}{\partial \theta} > 0; \text{ and } \frac{\partial R}{\partial \theta} > 0 \iff n \text{ is sufficiently large.} \tag{18}$$

The intuition behind these marginal effects is quite simple. First, note that increases in aggregate productivity, p, shift a firm's employment policy function downwards in Figure 1. Thus, unsurprisingly, when labor is more productive, a firm of a given idiosyncratic productivity, x, is more likely to hire workers, and less likely to shed workers. Second, increases in the vacancy–unemployment ratio, θ, unambiguously reduce the likelihood that a firm of a given idiosyncratic productivity will hire workers (R_v increases for all n). The reason is that higher θ implies a lower job–filling probability, q, and thereby raises the marginal cost of hiring a worker, c/q. Moreover, higher θ implies a tighter labor market and therefore higher wages (from (9)) so that the marginal cost of labor rises as well. Both of these effects cause firms to cut back on hiring. Finally, increases in the vacancy–unemployment ratio, θ, will reduce the likelihood of shedding workers for small firms, but will raise it for large firms. This occurs because higher θ has countervailing effects on the separation decision of firms. On the one hand, higher θ reduces the job–filling probability, q, rendering separation decisions less reversible (since future hiring becomes more costly), so that firms become less likely to destroy jobs. On the other hand, higher θ implies a tighter labor market, higher wages, and thereby a higher marginal cost of labor, rendering firms more likely to shed workers. The former effect is dominant in small firms because the likelihood of their hiring in the future is high.

2 Aggregation and Steady State Equilibrium

2.1 Aggregation

Since we are ultimately interested in the equilibrium behavior of the aggregate unemployment rate, in this section we take on the task of aggregating up the microeconomic behavior of section 1 to the macroeconomic level. This exercise is non–trivial because each firm's employment is a non–linear function of the firm's lagged employment, n_{-1}, and its idiosyncratic shock realization, x. As a result, there is no representative firm interpretation that will aid aggregation of the model.

To this end, we are able to derive the following result which characterizes the steady state aggregate employment stock and flows in the model:

Proposition 5 *If idiosyncratic shocks, x, evolve according to (17), the steady state c.d.f. of employment across firms is given by*

$$H(n) = \frac{\tilde{G}[R(n)]}{1 - \tilde{G}[R_v(n)] + \tilde{G}[R(n)]}. \tag{19}$$

Thus, the steady state aggregate employment stock is given by

$$N = \int n dH(n) \tag{20}$$

and the steady state aggregate number of separations, S, and hires, M, is equal to

$$S = \lambda \int [1 - H(n)] \tilde{G}[R(n)] dn = \lambda \int H(n) \left(1 - \tilde{G}[R_v(n)]\right) dn = M. \tag{21}$$

Proposition 5 is useful because it provides a tight link between the solution for the microeconomic behavior of an individual firm and the macroeconomic outcomes of that behavior. Specifically, it shows that once we know the optimal employment policy function of an individual firm (that is, the functions $R(n)$ and $R_v(n)$) then we can directly obtain solutions for the aggregate employment stock and flows. An important feature to note about Proposition 5 is its generality. Specifically, it allows one to generate analytically the steady state aggregate employment stock and labor flows for any given employment policy function at the microeconomic level, not just that derived above. In addition, the expressions for aggregate employment and flows are straightforward to compute numerically.

The three components of Proposition 5 are also quite intuitive. The steady state distribution of employment across firms, (19), is obtained by setting the flows into and out of the mass $H(n)$ equal to each other. The inflow into the mass comes from firms who reduce their employment from above n to below n. There are $[1 - H(n)]$ such firms, and since they are reducing their employment, it follows from (12) that each firm will reduce its employment below n with probability equal to $\Pr[x < R(n)] = \lambda\tilde{G}[R(n)]$. Thus, the inflow into $H(n)$ is equal to $\lambda[1 - H(n)]\tilde{G}[R(n)]$. Similarly, one can show that the outflow from the mass is equal to $\lambda H(n)\left(1 - \tilde{G}[R_v(n)]\right)$. Setting inflows equal to outflows yields the expression for $H(n)$ in (19).[15] Given this, the expression for aggregate employment, (20), follows directly.

The intuition for the final expression for aggregate flows in Proposition 5, (21), is as follows. Recall that the mass of firms whose employment switches from above some number n to below n is equal to $\lambda[1 - H(n)]\tilde{G}[R(n)]$. Equation (21) states that the aggregate number of separations in the economy is equal to the *cumulative* sum of these downward switches in employment over n. To get a sense for this, consider the following simple discrete example. Imagine an economy with two separating firms: one that switches from three employees to one, and another that switches from two employees to one. It follows that two firms have switched from > 2 employees to ≤ 2 employees, and one firm switched from > 1 to ≤ 1 employee. Thus, the cumulative sum of downward employment switches is three, which is also equal to the total number of separations in the economy.

[15] This mirrors the mass-balance approach used in Burdett & Mortensen (1998) to derive the equilibrium wage distribution in a search model with wage posting.

2.2 Steady State Equilibrium

Given (19), (20), and (21), the conditions for aggregate steady state equilibrium can be obtained as follows. First note that each firm's optimal policy function, summarized by the functions $R(n)$ and $R_v(n)$ in Proposition 2, depends on two aggregate variables: The (exogenous) state of aggregate productivity, p; and the (endogenous) ratio of aggregate vacancies to aggregate unemployment, $V/U \equiv \theta$, which uniquely determines the flow probabilities q and f.

In the light of Proposition 5, we can characterize the aggregate steady state of the economy for a given p in terms of two relationships. The first, the *job creation condition*, is simply equation (20), which we re–state here in terms of unemployment, making explicit its dependence on the aggregate vacancy–unemployment ratio, θ:

$$U(\theta)_{JC} = L - \int n dH(n;\theta) \tag{22}$$

(22) simply specifies the level of aggregate employment that is consistent with the inflows to (hires) and outflows from (separations) aggregate employment being equal as a function of θ. The second steady state condition is the *Beveridge Curve* relation. This is derived from the difference equation that governs the evolution of unemployment over time:

$$\Delta U' = S(\theta) - f(\theta) U \tag{23}$$

(23) simply states that the change in the unemployment stock over time, $\Delta U'$, is equal to the inflow into the unemployment pool – the number of separations, S – less the outflow from the unemployment pool – the job finding probability, f, times the stock of unemployed workers, U. In steady state, aggregate unemployment will be stationary, so that we obtain

the steady state unemployment relation:

$$U(\theta)_{BC} = \frac{S(\theta)}{f(\theta)} \tag{24}$$

The steady state value of the vacancy–unemployment ratio, θ, is co–determined by (22) and (24).

3 An Illustrative Simulation

Shimer (2005) demonstrated that the standard MP model cannot generate enough cyclical amplitude in the job finding rate to match that observed in US data. A natural question is whether the generalized model analyzed here can alleviate this problem. To this end, we perform the following illustrative numerical exercise. We normalize aggregate productivity, p, to 1 and calculate the steady state response of the job finding rate, f, and the unemployment inflow rate, $s \equiv S/N$, to a 1% reduction in p. We examine the steady state response of the model as an approximation to the true dynamic response of the model based on the results of Shimer (2005), Mortensen & Nagypal (2007a), and Rotemberg (2006), who show that such an approximation is very close in models of the aggregate labor market. To see why, note that we can rewrite the difference equation for the evolution of unemployment, (23), as:

$$\Delta U' = -f(U - U_{BC}) \tag{25}$$

where U_{BC} is defined in (24) and is the steady state unemployment level. In US data, the monthly outflow rate from unemployment is on the order of $f = 0.45$ (see Shimer, 2007), implying that around half of the gap between actual and steady state unemployment is closed over the course of a month. That is, deviations of unemployment from steady state are very short-lived, and thus steady state responses to aggregate shocks are very good, and intuitive, approximations to the true dynamic response of the model.

Calibration. The empirical moments that we seek to match are summarized in Table 1.[16] In particular, we want to get sense of the cyclical variation in the job finding rate implied by the model. To assess this, we calibrate the model as follows. We take a time period to be equal to one week, which in practice acts as a good approximation to the continuous time nature of unemployment flows (see Hagedorn & Manovskii, 2005). We then assume that the matching function is of the conventional Cobb-Douglas form, $M = \mu U^{\phi} V^{1-\phi}$.[17] We set $\phi = 0.6$ based on the estimates reported in Petrongolo & Pissarides (2001). We target a weekly unemployment outflow probability of $f = 0.1125$, to be approximately consistent with a monthly outflow hazard of 0.45. In addition, we follow Pissarides (2007) and target a mean value of the vacancy–unemployment ratio of $\theta = 0.72$. Noting from the matching function that $f = \mu\theta^{1-\phi}$, the latter implies that $\mu = 0.129$ on a weekly basis.

Idiosyncratic productivity shocks, x, are assumed to evolve according to (17). Following Mortensen & Pissarides (1994), we assume for simplicity that the latter distribution is uniform on the interval $[\gamma, 1]$, so that $\tilde{G}(x) = \frac{x-\gamma}{1-\gamma}$. Given this setup, it is possible to solve for firms' optimal employment policy in closed form, the details of which are in Appendix A. We then use the aggregation results of Proposition 5 to derive numerically the aggregate job creation, (22), and Beveridge curve, (24), conditions, and thereby solve the model.

Given this, we then target a mean weekly inflow probability of $s = 0.0075$ to be consistent with data in Table 1. Following Mortensen & Nagypal (2007a), we also target the empirical elasticity of s with respect to aggregate output per worker, Y/N. Specifically, we use Shimer's (2007) estimate of the employment to unemployment transition rate to measure s

[16] The elasticity for the job finding rate reported in Table 1 differs from the value of 2.34 reported in Mortensen & Nagypal (2005) because they base their calculations on Table 1 of Shimer (2005a) which reports summary statistics for the monthly job finding *probability*, rather than for the hazard, which is what matters for unemployment flows.

[17] An issue that can arise when using a Cobb–Douglas matching function in a discrete time setting is that the flow probabilities f and q are not necessarily bounded above by 1. This issue does not arise here due to the short time period of one week.

Table 1: Summary Statistics and Target Moments (*in italics*), Weekly

Outcome	Level	Elasticity w.r.t. $\frac{Y}{N}$	Source
f	*0.1125*	2.95	Shimer (2007)
s	*0.0075*	*–2.48*	Shimer (2007)
V	–	3.68	Shimer (2005)
$b/\left(\frac{Y}{N}\right)$	*0.7*	–	Mortensen & Nagypal (2007a)
$c/\mathbb{E}$ [Workers' Wage]	*0.27*	–	Silva & Toledo (2007)
$\mathbb{E}$ [New Hires' Wage]	–	*0.94*	Haefke et al. (2007)

Notes: Following Mortensen & Nagypal (2007a), elasticities are obtained by regressing the log deviation from trend of f and s on the log deviation from trend of non-farm business output per worker obtained from the Bureau of Labor Statistics. Following Shimer (2005), series are detrended using a Hodrick–Prescott filter with smoothing parameter 10^5. The series for f and s are respectively the job-finding rate and the employment to unemployment transition rate derived in Shimer (2007).

to derive an elasticity with respect to output per worker of –2.48.[18]

In addition to these, we also target the workers' opportunity cost of employment, b, and the flow cost of a vacancy, c, as follows. As suggested by Mortensen & Nagypal (2007a) and Hall & Milgrom (2008), we target b to be approximately 0.7 of output per worker, Y/N. Moreover, we target c to generate per worker hiring costs c/q approximately equal to 14% of quarterly worker compensation in accordance with Silva & Toledo (2007) who used the Saratoga Institute's (2004) estimate of the labor costs of posting vacancies. In the context of the model, this implies a value of c approximately equal to 0.27 of the average worker's wage.[19]

Finally, we target the elasticity of average wages of newly hired workers to be equal to 0.94, based on the results of Haefke et al. (2007).[20] We target the elasticity of the average

[18] We are grateful to Robert Shimer for posting his estimates of flow transition rates among labor market states from the CPS Gross Flows data on his webpage.

[19] We want to equate the per worker hiring cost c/q to 14% of quarterly wages, $0.14 \cdot [12 \cdot \mathbb{E}(w)]$, since there are 12 weeks per quarter. Then note that the implied weekly job filling probability is given by $q = \mu\theta^{-\phi} = 0.129 \cdot 0.72^{-0.6} = 0.16$. Piecing this together yields $c/\mathbb{E}(w) = 0.16 \cdot 0.14 \cdot 12 = 0.27$.

[20] We target an elasticity of 0.94 based on Haefke et al.'s baseline results. It is worth bearing in mind that this is at the upper end of the range of estimates presented in their paper.

wages of newly hired workers rather than the elasticity of average wages of all workers for two reasons. First, it is well known empirically that the wages of workers in ongoing relationships are rigid (see among others Card & Hyslop, 1997), which is at odds with the assumption of Nash wage setting that we employ here.[21] Second, it is also well known that it is the flexibility of wages of new hires, rather than of ongoing workers, that is relevant to the cyclicality of the job finding rate implied by search and matching models of the labor market such as the one studied here (Shimer, 2004; Hall, 2005).

We thus have six moments that we seek to match, and seven model parameters: α (production function, $F(n) = n^{\alpha}$), b (flow value of leisure), c (flow cost per vacancy), η (worker bargaining power), L (potential labor force),[22] λ (arrival probability of idiosyncratic shocks), and γ (lower support of idiosyncratic shock distribution). We therefore set α to be equal to the conventional 2/3,[23] and evaluate the steady state response of the model over a grid of values for the remaining six parameters. Given the results of this exercise, we pick the parameter values that most closely match the six target moments italicized in Table 1.

The parameter results of this numerical exercise are reported in Table 2, and the implied model outcomes are in Table 3. The results are very encouraging: The model is able to match quite closely the target moments in italics in Table 1.[24] Of particular interest is that the model can also match the observed elasticity of the job finding rate with respect to output per worker even though this moment was not targeted when calibrating the model.

[21] Indeed, in the calibration that follows, the Nash wage setting assumption implies an elasticity of average worker wages with respect to output per worker of approximately 1. This overstates the cyclicality of ongoing wages observed in the data, which display an elasticity with respect to output per worker of approximately 0.5 (see Solon, Barsky, & Parker, 1994; Pissarides, 2007).

[22] Taken literally, L represents the labor force as a fraction of the number of the number of firms in the model economy. In reality, however, L is more accurately described as the labor force as a fraction of the number of production units in the economy. The latter may correspond to a small firm, a small division within a large firm etc. For this reason, we do not calibrate L directly.

[23] Strictly speaking, labor's share will be more than 2/3 in the model due to surplus sharing. We use a value of $\alpha = 2/3$ for simplicity.

[24] Although the fit with the target moments is good, it is not perfect. This is due to some coarseness in the grid over which we evaluated the model. However, if anything, we err on the side of generating less amplification since we overstate the flow surplus (b is a smaller fraction of output per worker) and overstate the flexibility in the wages of new hires relative to the target moments.

Table 2: Calibrated Parameters (Weekly)

Parameter	Meaning	Value	Reason
ϕ	Matching elasticity	0.6	Petrongolo & Pissarides (2001)
μ	Matching efficiency	0.129	Pissarides (2007)
α	$F(n) = n^{\alpha}$	0.67	Labor share $\approx 2/3$
β	Discount factor	0.999	Quarterly interest rate = 0.012
b	Value of leisure	0.4	Match target moments in italics in Table 1
c	Flow vacancy cost	0.12	
η	Worker bargaining power	0.4	
L	Labor force	2.25	
λ	x: arrival rate	0.045	
γ	x: lower support	0.5	

This makes substantial progress relative to the standard MP model. To see this, as a benchmark for comparison Table 3 also reports the outcomes from calibrating the standard MP model (the case with $\alpha = 1$) to match the target moment outcomes obtained for the $\alpha = 2/3$ case.[25] A striking feature of this comparison is that the standard MP model with $\alpha = 1$ generates a much lower elasticity for the job finding rate. This confirms the results of previous literature that details the inability of the standard MP model to generate enough cyclicality in job creation. Shimer's (2005a) calibration of the standard MP model yields an elasticity of f equal to 0.48. Mortensen & Nagypal (2007a) favor a different calibration of the standard MP model that yields an elasticity of f equal to 1.56 (see their section 3.2). Pissarides' (2007) calibration of the standard model with endogenous job destruction obtains an elasticity of f equal to 1.54. Thus, the standard MP model, with or without endogenous job destruction, appears to be able explain up to one half of the observed elasticity of the job finding rate. The results of Table 3 suggest that the generalized model studied here

[25] It is possible to rewrite the MP model in terms of $b/(Y/N)$ and $c/\mathbb{E}[w]$. We impose the conditions $b/(Y/N) = 0.67$ and $c/\mathbb{E}[w] = 0.26$. We then set $\mu = 0.128$ to target $f = 0.112$; $\eta = 0.625$ to target an elasticity of new hire's wages of 1.1; $\gamma = 0.825$ to target a mean level of $s = 0.0078$; and $\lambda = 0.03$ to target an elasticity of s equal to -2.46.

Table 3: Model Outcomes (Weekly)

Outcome	Mean Level		Elasticity w.r.t. $\frac{Y}{N}$	
	$\alpha = 2/3$	$\alpha = 1$	$\alpha = 2/3$	$\alpha = 1$
f	0.112	0.112	3.24	1.29
s	0.0078	0.0079	–2.46	–2.47
V	–	–	2.80	–0.28
$b/\left(\frac{Y}{N}\right)$	0.67	0.67	–	–
$c/\mathbb{E}$ [Workers' Wage]	0.26	0.26	–	–
$\mathbb{E}$ [New Hires' Wage]	–	–	1.1	1.05

Parameter values for $\alpha = 2/3$ case as in Table 2.

can plausibly account for *all* of the observed cyclical comovement between f and output per worker.

Cyclicality of Vacancies. Until now we have ignored the cyclicality of vacancies generated by our generalized model. Readers of Shimer (2005), however, will recall that the standard MP search and matching model also fails to match the observed cyclical volatility in the vacancy rate in the US. Specifically, as shown in Table 1, the empirical elasticity of the vacancy rate with respect to output per worker in the US derived by Shimer (2005) is equal to 3.68. The implied elasticity from Shimer's calibration of the standard MP model is $0.995 \times 0.027/0.020 = 1.34$ (see Shimer, 2005, Table 3). Moreover, the calibration of the standard MP model with $\alpha = 1$ in Table 3 reveals that the model with endogenous job destruction performs even worse on this dimension, yielding a very mild *counter*cyclical vacancy elasticity of -0.28. This arises because countercyclical job destruction leads to an offsetting increase in hires in times of recession to maintain balance between unemployment inflows and outflows, and thereby stymies the procyclicality of vacancies (Shimer, 2005).

The analogous elasticity generated by our simulation of the generalized model studied here is 2.80. This is clearly a substantial improvement over the standard model, especially given that the generalized model incorporates countercyclical job destruction. However, there remains a question of why the generalized model, which matches the cyclicality of job finding and employment to unemployment transition rates so well, cannot fully explain the cyclicality of vacancies. We believe that the answer is that a complete understanding of the cyclical behavior of vacancies requires an understanding of the processes underlying job-to-job employment flows, a phenomenon that we abstract from in our analysis of unemployment flows. To see why, note the following identity that relates the job-filling rate, q, vacancies (or job openings), V, and the *numbers* (not the hazard rates) flowing from unemployment to employment, UE, and from job to job, EE:

$$qV = UE + EE \tag{26}$$

Log differentiation of this identity yields:

$$d\log q + d\log V = \varphi d\log UE + (1-\varphi)\, d\log EE \tag{27}$$

where φ is the share of total hires that originates from unemployment. Recent research has shown that job-to-job flows (EE) are substantially procyclical and account for approximately 60% of total hires using Current Population Survey data from 1994 onwards (Fallick & Fleischman, 2004). Equation (27) shows that the procyclicality of EE flows must therefore contribute substantially to the procyclicality of vacancies observed in the data. For this reason, we feel that the elasticity of vacancies obtained from the generalized model without on-the-job search may in fact be quite reasonable: If we were able to match the empirical elasticity, we would implicitly be *over*-explaining the procyclicality of vacancies. For the same reason, however, we also feel that extending the model to account for job-to-job flows is an important task for future research.

4 Understanding Amplification

Approximating the Cyclicality of Job Creation. Figure 2 plots the response of the Job Creation, (22), and Beveridge Curve, (24), conditions to a 1% decline in p for the simulation detailed in Tables 2 to 4. The figure reveals that allowing for downward sloped labor demand amplifies the response of the vacancy-unemployment ratio to aggregate disturbances primarily through movements in the Job Creation condition. This naturally raises the question of why the JC condition moves so much. The following result provides a sense for where this amplification comes from by taking a log–linear approximation to a firm's marginal surplus around mean employment:

Proposition 6 *For small λ, the horizontal shift in the JC condition induced by a change in aggregate productivity, p, is given approximately by*

$$\left.\frac{d\ln\theta}{d\ln p}\right|_{JC} \approx \frac{(1-\eta)\,\tilde{p}}{\omega\phi\left[(1-\eta)\left(\tilde{p}-b\right)-\eta\beta c\theta\right]+\eta\beta c\theta}, \tag{28}$$

where ω is the steady state employment share of hiring firms, and $\tilde{p} \equiv \rho\overline{apl} + (1-\rho)\,\overline{mpl}$ where $\overline{apl}$ and $\overline{mpl}$ are respectively the average and marginal product of labor of the average–sized firm, and $\rho \equiv \frac{\alpha\eta}{1-\eta(1-\alpha)}$.

Corollary 1 *The elasticity of the vacancy–unemployment ratio to aggregate productivity in the $\alpha = 1$ case (Mortensen & Pissarides, 1994) is approximately equal to*

$$\frac{d\ln\theta}{d\ln p} \approx \frac{(1-\eta)\,p}{\phi\left[(1-\eta)\left(p-b\right)-\eta\beta c\theta\right]+\eta\beta c\theta}. \tag{29}$$

Equation (29) extends results presented in Mortensen & Nagypal (2007a) for the standard MP model with exogenous job destruction to the endogenous job destruction case. It echoes Mortensen & Nagypal's results in that it shows that the cyclical response of the vacancy-unemployment ratio, θ, is amplified in the endogenous job destruction case when the average flow surplus to employment relationships, $p - b$, is small. Intuitively, when the flow surplus is small, small reductions in aggregate productivity, p, can easily exhaust that surplus and lead firms to cut back substantially on hiring. Thus, to incentivize firms to hire once more and thereby restore equilibrium, the model must allow the labor market to slacken, and labor market tightness to fall, substantially.

Equation (28) generalizes this result to the case of downward sloped labor demand and endogenous job destruction. Inspection of (29) and (28) reveals that there are two ways that the addition of downward sloped labor demand can potentially yield amplification of the response of labor market tightness (and thereby of the job finding rate, $f = \mu\theta^{1-\phi}$) to changes in aggregate productivity. The first is that the effective surplus that matters for amplification is now given by by $\tilde{p} - b$, and this is smaller than the average flow surplus. The reason is that the effective flow surplus, $\tilde{p} - b$, is now a weighted average of the average and marginal flow surplus. When the demand for labor slopes downward, the marginal surplus will be smaller than the average surplus, because infra–marginal employment relationships are more productive. This provides a sense for why the numerical exercise above is able to generate greater volatility in θ even when the average flow surplus is relatively large: It is because the *marginal* flow surplus is relatively small in the simulation.

It's important to stress that while the model implies a small marginal surplus, it does allow for a substantial average flow surplus to employment relationships. This stands in marked contrast to the standard MP model. For instance, Mortensen and Nagypal (2007a) report that their preferred estimate of the average flow match surplus is $\frac{1}{b/(Y/N)} - 1 = \frac{1}{0.73} - 1 = 37$ percent, which is comparable to, though still somewhat less than, what we obtain (see Table 3). They find that under this calibration, the standard MP model is able to account for only about one half of the amplification generated by the model developed here. When endogenous job destruction is added to the standard ($\alpha = 1$) model, Pissarides (2007) finds that it is still unable to reconcile the observed volatility of f with a substantial

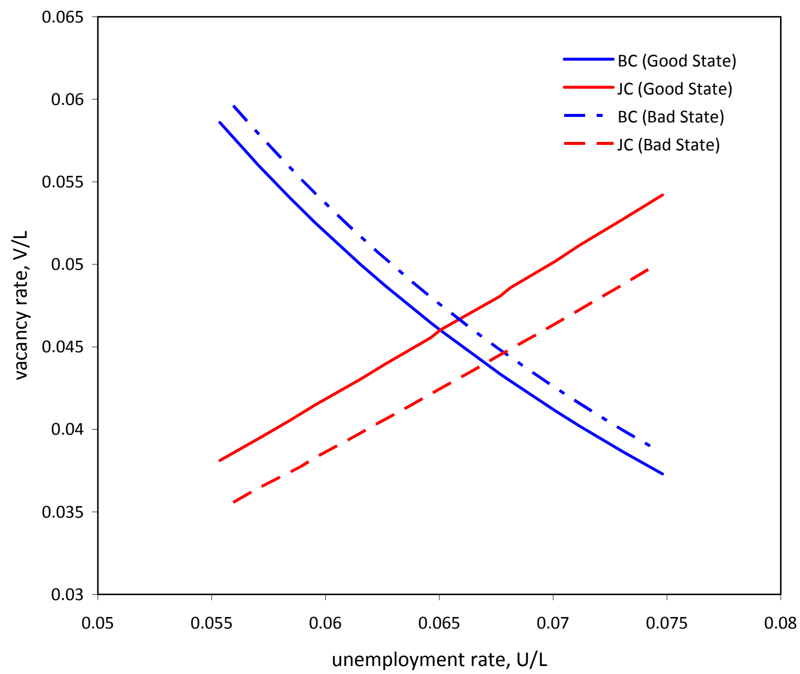

Figure 2: Job Creation and Beveridge Curve in the Simulated Model

average surplus.[26]

Equation (28) also suggests that there is an additional effect at work in the form of the variable ω, the steady state employment share of hiring firms. To understand the significance of this term, note that in the standard MP model with flat labor demand (the special case where $\alpha = 1$), ω is equal to 1. When $\alpha = 1$, a firm that reduces its employment will shed all of its workers since, if one worker is unprofitable at a firm, all workers are unprofitable. Similarly, when $\alpha = 1$, if it is profitable to hire one worker, it is profitable to hire any number of workers. Thus shedding firms have zero employment, and all of steady state employment is accounted for by hiring firms in the standard MP model.

The latter is a useful point of contrast with the model with downward sloped labor demand and endogenous job destruction. Because of downward sloped labor demand, shedding

[26]The worker's surplus in our simulation is also substantial compared to previous calibrations using the standard MP model. Our calibration implies workers obtain a $(\mathbb{E}[w] - b)/b = 15$ percent flow surplus from employment over unemployment.

firms do not reduce their employment to zero because reducing employment replenishes the marginal product of labor in those firms. Likewise, hiring firms' desired employment level is bounded because additional hiring depletes the marginal product of labor. Hence ω will be less than unity, and inspection of (28) and (29) reveals that this will lead to greater amplification relative to the standard MP model.[27]

The intuition for this effect is related to the interaction of downward sloped labor demand and heterogeneous firms. Following a reduction in aggregate productivity, shedding firms wish to shed more workers, and hiring firms wish to hire fewer workers. Thus, inflows into the unemployment pool rise, and outflows from the unemployment pool fall, *ceteris paribus*, and unemployment rises. To return the model to steady state, hiring firms must be convinced to hire enough workers to equate inflows to outflows once more. The model achieves this by allowing the job filling probability, $q(\theta)$, to rise (and labor market tightness, θ, to fall) so that hiring becomes less costly for firms. However, when the demand for labor slopes downward, additional hiring reduces the marginal product of labor, making additional employment relationships less attractive to hiring firms. As a result, the job filling probability, $q(\theta)$ must rise (and hence θ must fall) more to convince these firms to increase hiring and return the economy back to steady state once more.

A natural question in the light of this is how much of the cyclicality of job creation generated by the model can be attributed to small surplus to marginal employment relationships on the one hand, and to the interaction of endogenous job destruction with job creation on the other. To get a sense for this, Table 4 reports outcomes for a special case of the generalized model that preserves downward sloped labor demand ($\alpha = 2/3$), but which switches off endogenous job destruction.[28] As before, we calibrate this exogenous job destruction

[27] The reader may worry whether $(1-\eta)(\tilde{p} - b) - \eta\beta c\theta$ is positive or not. To see that it is, note that we can rewrite it as $(1-\eta)\left(\frac{p\alpha x n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right) - \eta\beta c\theta$, and observe from equations (13) and (14) that it is, in fact, the marginal flow surplus of a firm, and therefore must be positive.

[28] This special case of the model is formally analogous to the model analyzed in Cahuc & Wasmer (2001) and Krause & Lubik (2007). Krause & Lubik find much less cyclicality in the job finding rate in their calibration. This is because the flow surplus to employment relationships is very large in their parameterization (b as a

Table 4: Endogenous vs. Exogenous Job Destruction, $\alpha = 2/3$

Outcome	**Mean Level**		**Elasticity w.r.t. $\frac{Y}{N}$**	
	Endog. JD	Exog. JD	Endog. JD	Exog. JD
f	0.112	0.112	3.24	2.48
s	0.0078	0.0078	–2.46	0
V	–	–	2.80	3.88
$b/\left(\frac{Y}{N}\right)$	0.67	0.67	–	–
$c/\mathbb{E}$ [Workers' Wage]	0.26	0.26	–	–
$\mathbb{E}$ [New Hires' Wage]	–	–	1.1	1.1

version of the model to match the target moment outcomes generated by the generalized model in Table 3.[29] This exercise yields an elasticity of the job finding rate equal to 2.48, which is lower than the value of 3.24 derived for the model with endogenous job destruction. This suggests that, while the majority of the procyclicality in job finding generated by the model is attributable to small marginal surplus, the addition of endogenous job destruction enhances this result, and this second effect is also not small.[30]

5 Summary and Discussion

This paper has shown that the addition of two very simple features – downward sloped short run labor demand and endogenous job destruction – to an otherwise standard model of the aggregate labor market can help explain the observed cyclical variation in the job finding rate, the employment to unemployment transition rate, and vacancies observed in US data. We show that this is driven by two effects. First, cyclicality in job creation is generated by the fact that marginal employment relationships generate a low surplus in the short run. Small

fraction of output per worker is approximately equal to 0.1).

[29] Specifically, it is possible to rewrite the model with exogenous job destruction in terms of $b/(Y/N)$ and $c/\mathbb{E}[w]$. We then impose the conditions $b/(Y/N) = 0.67$ and $c/\mathbb{E}[w] = 0.26$, and choose μ and η to match $f = 0.112$ and an elasticity of wages of 1.1. Setting $\mu = 0.128$ and $\eta = 0.517$ achieves this.

[30] It is also true that the model with exogenous job destruction yields greater procyclicality in vacancies. However, as shown in Shimer (2005) and Mortensen & Nagypal (2007b), this is simply a side-effect of the (counterfactual) lack of any countercyclical movement in separations in that version of the model.

aggregate disturbances quickly exhaust this surplus and lead to substantial reductions in hiring. Importantly, however, due to downward sloped labor demand, low marginal surplus is nevertheless consistent with a sizeable surplus to the average employment relationship, contrary to the standard search model. Second, increased job destruction in recessions must be soaked up by increased hiring in equilibrium. With downward sloped labor demand, increased hiring diminishes the value of additional employment relationships to firms. As a result, hiring firms are less willing to soak up the separations of shedding firms, and unemployment rises more in the wake of a recession. Calibration of the model to available moments suggests that, while the majority of the cyclicality in job creation is attributable to small marginal surplus, both of these mechanisms appear to be quantitatively significant.

In the course of establishing these results, we also provide a rich, yet analytically tractable model of the aggregate labor market in the short run. As such, we believe that this model will provide a useful laboratory for the cyclical analysis of aggregate labor markets in future empirical and theoretical research. A number of avenues arise naturally in the light of this. First, the model has a well-defined concept of a firm and so lends itself to estimation using establishment level data. As a result, the analytical framework developed here will complement recent research efforts that have sought to solve and estimate search models using numerical methods (e.g. Cooper, Haltiwanger & Willis, 2007).

A second natural extension relates to the aggregation results derived in the analysis of section 2. Specifically, we provide a very simple and general approach to deriving both the aggregate unemployment stock and flows in non-linear models without representative firms such as the one studied here. An obvious extension is to apply these results to other popular models of non-linear microeconomic behavior. For example, recent research has emphasized the importance of non–convex adjustment costs in explaining the empirical properties of labor demand at the micro level (see for example Caballero, Engel, & Haltiwanger, 1997, and Cooper, Haltiwanger, & Willis, 2004). From the perspective of the analysis of section

2, non–convex adjustment costs simply lead to another form for the optimal employment policy of a firm, which can be aggregated in exactly the same manner. Our hope, therefore, is that this will enable future work that further explores the implications of non-linearities in firm level labor demand for the aggregate labor market.

6 References

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7 Appendix

A Solution of the Simulated Model

Optimal Employment Policy. We follow Mortensen & Pissarides (1994) and assume idiosyncratic shocks evolve according to (17), with $\tilde{x} \sim U[\gamma, 1]$ so that $\tilde{G}(x) = \frac{x-\gamma}{1-\gamma}$. In this case, we can rewrite the recursion for the function $D(n,x)$ in Proposition 3 as:

$$\begin{aligned} D(n,x) &= (1-\lambda)\chi(x) + \lambda \int_{R(n)}^{R_v(n)} \chi(x')\, d\tilde{G}(x') + \lambda \int_{R_v(n)}^{\infty} \frac{c}{q} d\tilde{G}(x') \\ &\quad + \beta(1-\lambda) D(n,x) + \beta\lambda \int_{R(n)}^{R_v(n)} D(n,x')\, d\tilde{G}(x') \end{aligned} \tag{30}$$

where $\chi(x) \equiv (1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f \frac{c}{q}$. We then conjecture that the function $D(n,x)$ is of the form $D(n,x) = d_0 + d_1 \chi(x)$. Substituting this into the latter, and equating coefficients, we obtain the following solution for $D(n,x)$:

$$\begin{aligned} D(n,x) &= \frac{1-\lambda}{1-\beta(1-\lambda)}\chi(x) \\ &\quad + \frac{\lambda}{1-\beta(1-\lambda)} \frac{R_v - R}{[1-\beta(1-\lambda)](1-\gamma) - \beta\lambda(R_v - R)} \chi\left(\frac{R+R_v}{2}\right) \\ &\quad + \frac{1-R_v}{[1-\beta(1-\lambda)](1-\gamma) - \beta\lambda(R_v - R)} \lambda \frac{c}{q} \end{aligned} \tag{31}$$

Note also that differencing the first-order conditions (13) and (14) in this case implies that:

$$R_v(n) - R(n) \equiv \delta(n) = [1-\beta(1-\lambda)] \frac{c/q}{\psi p\alpha n^{\alpha-1}} \tag{32}$$

where $\psi \equiv \frac{1-\eta}{1-\eta(1-\alpha)}$. Using these two results, and after some tedious algebra, one obtains the following closed–form solution for $R_v(n)$:

$$R_v(n) = \frac{(1-\eta)b + \eta\beta c\frac{f}{q} + \left(1-\beta-\beta\lambda\frac{\gamma}{1-\gamma}\right)\frac{c}{q} - \beta\lambda[1-\beta(1-\lambda)]\frac{1}{1-\gamma}\frac{1}{2}\left(\frac{c}{q}\right)^2 \frac{1}{\psi p\alpha n^{\alpha-1}}}{\psi p\alpha n^{\alpha-1} - \beta\lambda\frac{1}{1-\gamma}\frac{c}{q}} \tag{33}$$

To derive $R(n)$, simply note that, by definition, $R(n) = R_v(n) - \delta(n)$. The aggregate employment stock and flows are then obtained directly from applying the results of Proposition 5.

Average Product and Average Marginal Product. The average product of labor implied by the model is given by $APL = \mathbb{E}\left[pxn^{\alpha-1}\right]$. Note that:

$$\mathbb{E}\left[xn^{\alpha-1}\right] = \int\left[\int xdG\left(x|n\right)\right]n^{\alpha-1}dH\left(n\right)$$

Moreover, the optimal employment policy implies that, given n, x must lie in the interval $\left[R\left(n\right), R_v\left(n\right)\right]$, but is otherwise independently distributed. Thus:

$$\int xdG\left(x|n\right) = \frac{\int_{R(n)}^{R_v(n)} xdG\left(x\right)}{G\left[R_v\left(n\right)\right] - G\left[R\left(n\right)\right]} = \frac{1}{2}\left[R\left(n\right) + R_v\left(n\right)\right] \tag{34}$$

where the last equality follows from the assumption of uniform idiosyncratic shocks in the simulation. Thus:

$$APL = \mathbb{E}\left[pxn^{\alpha-1}\right] = p\int \frac{1}{2}\left[R\left(n\right) + R_v\left(n\right)\right]n^{\alpha-1}dH\left(n\right) \tag{35}$$

Moreover, the average marginal product of labor is simply given by $\mathbb{E}\left[MPL\right] = \mathbb{E}\left[px\alpha n^{\alpha-1}\right] = \alpha APL$.

Average Wages. It follows from equation (9) that the average wage across firms is given by:

$$\bar{w}_f = \frac{\eta}{1-\eta\left(1-\alpha\right)}\mathbb{E}\left[MPL\right] + \eta\beta f\frac{c}{q} + \left(1-\eta\right)b \tag{36}$$

To obtain the average wage across workers, which we denote $\bar{w}_w$, note that $\bar{w}_w = \mathbb{E}\left[\frac{n}{\mathbb{E}(n)}w\left(n,x\right)\right]$ where $w\left(n,x\right)$ is the wage in a given firm defined in (9). That is, it is the employment-weighted average of wages across firms. Thus:

$$\bar{w}_w = \frac{\eta}{1-\eta\left(1-\alpha\right)}\frac{1}{\mathbb{E}\left(n\right)}\mathbb{E}\left[px\alpha n^{\alpha}\right] + \eta\beta f\frac{c}{q} + \left(1-\eta\right)b \tag{37}$$

This has a very similar structure to the average wage across firms. It follows that:

$$\bar{w}_w = \frac{\eta p\alpha}{1-\eta\left(1-\alpha\right)}\frac{1}{\mathbb{E}\left(n\right)}\int\frac{1}{2}\left[R\left(n\right) + R_v\left(n\right)\right]n^{\alpha}dH\left(n\right) + \eta\beta f\frac{c}{q} + \left(1-\eta\right)b \tag{38}$$

Finally, the average wage of new hires, which we denote $\bar{w}_m$, is equal to a hiring–weighted average of wages across hiring firms. Noting from (12) that idiosyncratic productivity of hiring firms is given by $x = R_v\left(n\right)$, we have that:

$$\bar{w}_m = \mathbb{E}\left[\mathbb{E}\left(w\left(n,x\right)|n > n_{-1}, n_{-1}\right)\right] = \int\int_{n_{-1}} w\left(n, R_v\left(n\right)\right)\frac{dG\left[R_v\left(n\right)\right]}{1-G\left[R_v\left(n_{-1}\right)\right]}dH\left(n_{-1}\right) \tag{39}$$

B Proofs

Conjecture 1 *The optimal employment policy function is of the form specified in (12).*

We will later verify in the proof of Proposition 2 that the Conjecture is consistent with the solution for the wage equation obtained in Proposition 1.

Proof of Proposition 1. Note first that, under the Conjecture, we can write the marginal surplus to a firm recursively as:

$$J(n,x) = pxF'(n) - w(n,x) - w_n(n,x)\,n + \beta\int_{R_v(n)}^{\infty}\frac{c}{q}dG(x') + \beta\int_{R(n)}^{R_v(n)} J(n,x')\,dG(x') \quad (40)$$

In addition, we can write the value to a worker of unemployment as:

$$U = b + \beta\left\{(1-f)\,U' + f\int_0^{\infty}\int_{R_v(n)}^{\infty} W\left(R_v^{-1}(x'),x'\right)\frac{dG(x')}{1-G(R_v(n))}dH(n)\right\} \quad (41)$$

Upon finding a job, which occurs with probability f, the new job must be in a firm which is posting vacancies. This implies that the idiosyncratic productivity of the firm, $x' > R_v(n)$, and that the level of employment in the hiring firm, $n = R_v^{-1}(x')$. Moreover, since firms differ in size, there is a distribution of employment levels, $H(n)$, over which an unemployed worker will take expectations when evaluating the expected future benefits of being hired. It is useful to rewrite the worker's value of unemployment as:

$$U = b + \beta\left\{U' + f\int_0^{\infty}\int_{R_v(n)}^{\infty}\left[W\left(R_v^{-1}(x'),x'\right) - U'\right]\frac{dG(x')}{1-G(R_v(n))}dH(n)\right\} \quad (42)$$

Then note that, due to Nash sharing, the worker's surplus in an expanding firm, $W\left(R_v^{-1}(x'),x'\right) - U' = \frac{\eta}{1-\eta}J\left(R_v^{-1}(x'),x'\right)$, and moreover that, by the first-order condition for a hiring firm (see (4)), $J\left(R_v^{-1}(x'),x\right) = c/q$. Thus, we obtain the simple result:

$$U = b + \beta U' + \beta f\frac{\eta}{1-\eta}\frac{c}{q} \quad (43)$$

The value of employment to a worker can be written as:

$$\begin{aligned} W(n,x) \;=\; & w(n,x) + \beta\left\{\int_0^{R(n)}\left[\tilde{s}U' + (1-\tilde{s})\,W\left(R^{-1}(x'),x'\right)\right]dG(x')\right. \quad (44) \\ & \left. + \int_{R(n)}^{R_v(n)} W(n,x')\,dG(x') + \int_{R_v(n)}^{\infty} W\left(R_v^{-1}(x'),x'\right)dG(x')\right\} \end{aligned}$$

An employed worker's expected future payoff can be split into three regimes. If the firm sheds workers next period ($x' < R(n)$) then the worker may separate from the firm. We denote by $\tilde{s}$ the probability that a worker separates from a firm conditional on the firm shedding workers. If the worker separates, she transitions into unemployment and receives a payoff U'. Otherwise she continues to be employed in a firm of size $n' = R^{-1}(x')$. Note that Nash sharing implies that $W\left(R^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} J\left(R^{-1}(x'), x'\right)$, and that, by the first-order condition, $J\left(R^{-1}(x'), x'\right) = 0$. Thus, $W\left(R^{-1}(x'), x'\right) = U'$. In the event that a firm freezes employment next period ($x' \in [R(n), R_v(n)]$) then Nash sharing implies that $W(n, x') - U' = \frac{\eta}{1-\eta} J(n, x')$. Finally, in the event that the firm hires next period, $W\left(R_v^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} \frac{c}{q}$. Thus, we have that:

$$W(n, x) = w(n, x) + \beta U' + \beta \frac{\eta}{1-\eta} \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x') + \beta \frac{\eta}{1-\eta} \int_{R(n)}^{R_v(n)} J(n, x')\, dG(x') \quad (45)$$

Subtracting the value of unemployment to a worker from the latter, we obtain the following description of the worker's surplus:

$$W(n, x) - U = w(n, x) - b + \beta \frac{\eta}{1-\eta} \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x') + \beta \frac{\eta}{1-\eta} \int_{R(n)}^{R_v(n)} J(n, x')\, dG(x') - \beta f \frac{\eta}{1-\eta} \frac{c}{q} \quad (46)$$

Under Nash, this must be equal to $\frac{\eta}{1-\eta} J(n, x)$, where $J(n, x)$ is as derived in (40) so that we have:

$$w(n, x) = \eta \left[pxF'(n) - w_n(n, x)\, n + \beta f \frac{c}{q} \right] + (1-\eta)\, b \quad (47)$$

as required. ■

Proof of Proposition 2. Given the wage function in (9), it follows that the firm's objective, (3), is continuous in (n_{-1}, x) and concave in n. Thus, it follows from the Theorem of the Maximum that the firm's optimal employment policy function is continuous in (n_{-1}, x). Given this, it follows that the employment policy function must be of the form stated in Proposition 2. This verifies that the Conjecture stated at the beginning of the appendix holds. ■

Proof of Proposition 3. First, note that one can re-write the continuation value conditional on each of the three possible continuation regimes:

$$\Pi(n, x') = \begin{cases} \Pi^{-}(n, x') & \text{if} \quad x' < R(n) \\ \Pi^{0}(n, x') & \text{if} \quad x' \in [R(n), R_v(n)] \\ \Pi^{+}(n, x') & \text{if} \quad x' > R_v(n) \end{cases} \quad (48)$$

where superscripts $^{-/0/+}$ refer to whether their are separations, a hiring freeze, or hires tomorrow. Thus we can write[31]:

$$\int \Pi(n, x')\, dG(x'|x) = \int_0^{R(n)} \Pi^{-}(n, x')\, dG + \int_{R(n)}^{R_v(n)} \Pi^{0}(n, x')\, dG + \int_{R_v(n)}^{\infty} \Pi^{+}(n, x')\, dG \quad (49)$$

[31] Henceforth, "dG" without further elaboration is to be taken as "$dG(x'|x)$".

Taking derivatives with respect to n, recalling the definition of $D(\cdot)$, and noting that, since $\Pi(n,x')$ is continuous, it must be that $\Pi^{-}(n,R(n)) = \Pi^{0}(n,R(n))$ and $\Pi^{0}(n,R_v(n)) = \Pi^{+}(n,R_v(n))$, yields:

$$D(n,x) = \int_{0}^{R(n)} \Pi_n^{-}(n,x')\,dG + \int_{R(n)}^{R_v(n)} \Pi_n^{0}(n,x')\,dG + \int_{R_v(n)}^{\infty} \Pi_n^{+}(n,x')\,dG \qquad (50)$$

Finally, using the Envelope conditions in Lemma 1 below, and substituting into (50) we obtain (15) and (16) in the main text:

$$\begin{aligned} D(n,x) &= \int_{R(n)}^{R_v(n)} \left\{ (1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f\frac{c}{q} \right\} dG(x'|x) \\ &\quad + \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x'|x) + \beta \int_{R(n)}^{R_v(n)} D(n,x')\,dG(x'|x) \\ &\equiv (\mathbf{C}D)(n,x) \end{aligned} \qquad (51)$$

To verify that $\mathbf{C}$ is a contraction mapping, we confirm that Blackwell's sufficient conditions for a contraction hold here (see Stokey & Lucas, 1989, p.54). To verify monotonicity, fix $(n,x) = (\bar{n},\bar{x})$, and take $\hat{D} \geq D$. Then note that:

$$\int_{R(\bar{n})}^{R_v(\bar{n})} \hat{D}(\bar{n},x')\,dG(x'|\bar{x}) - \int_{R(\bar{n})}^{R_v(\bar{n})} D(\bar{n},x')\,dG(x'|\bar{x}) = \int_{R(\bar{n})}^{R_v(\bar{n})} \left[\hat{D}(\bar{n},x') - D(\bar{n},x')\right] dG(x'|\bar{x}) \geq 0 \qquad (52)$$

Since $(\bar{n},\bar{x})$ were arbitrary, it thus follows that $\mathbf{C}$ is monotonic in D. To verify discounting, note that:

$$[\mathbf{C}(D+a)](n,x) = (\mathbf{C}D)(n,x) + \beta a\left[G(R_v(n)|x) - G(R(n)|x)\right] \leq (\mathbf{C}D)(n,x) + \beta a \qquad (53)$$

Since $\beta < 1$ it follows that $\mathbf{C}$ is a contraction. It therefore follows from the Contraction Mapping Theorem that $\mathbf{C}$ has a unique fixed point. ■

Lemma 1 *The value function defined in (3) has the following properties:*

$$\begin{aligned} \Pi_n^{-}(n,x') &= 0 \\ \Pi_n^{0}(n,x') &= (1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f\frac{c}{q} + \beta D(n,x') \\ \Pi_n^{+}(n,x') &= c/q \end{aligned} \qquad (54)$$

Proof of Lemma 1. First, note that standard application of the Envelope Theorem implies that $\Pi_n^{-}(n,x') = 0$ and $\Pi_n^{+}(n,x') = c/q$. It is only slightly less obvious what happens when $\Delta n' = 0$, i.e. when the employment is frozen next period. In this case, $n' = n$ and this implies that:

$$\Pi^0(n,x') = px'F(n) - w(n,x')n + \beta \int \Pi(n,x'')\,dG(x''|x') \tag{55}$$

It therefore follows that:

$$\Pi^0_n(n,x') = px'F'(n) - w(n,x') - w_n(n,x')n + \beta \int \Pi_n(n,x'')\,dG(x''|x') \tag{56}$$

Since, by definition $D(n,x') \equiv \int \Pi_n(n,x'')\,dG(x''|x')$, the statement holds as required. ■

Proof of Proposition 4. First note that if x evolves according to (17), then we can rewrite the recursion for $D(n,x)$ as:

$$\begin{aligned} D(n,x) &= \frac{1-\lambda}{1-\beta(1-\lambda)}\chi(x) + \frac{\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} \chi(x')\,d\tilde{G}(x') \\ &\quad + \frac{\lambda}{1-\beta(1-\lambda)}\int_{R_v(n)}^{\infty} \frac{c}{q}\,d\tilde{G}(x') + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} D(n,x')\,d\tilde{G}(x') \end{aligned} \tag{57}$$

where $\chi(x) \equiv (1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta c\theta$. It follows that the LHS of the first–order conditions, (13) and (14) are increasing in x, because $\chi(x)$ is increasing in x. Thus, to establish that $\partial R_v/\partial p < 0$ and $\partial R/\partial p < 0$, simply note that the function $D(n,x)$ is also increasing in p and thus the LHS of (13) and (14) are increasing in p.

To ascertain the marginal effects of θ we first need to establish the marginal effect of θ on the function $D(n,x)$. Rewriting $f/q = \theta$ and $q = q(\theta)$ in (57), differentiating with respect to θ, and using the first–order conditions, (13) and (14), to eliminate terms we obtain:

$$D_\theta = -\eta\beta c\frac{1-\lambda(1-p^0)}{1-\beta[1-\lambda(1-p^0)]} - \frac{c}{q}\frac{q'(\theta)}{q}\frac{\lambda p^+}{1-\beta[1-\lambda(1-p^0)]} \tag{58}$$

where $p^0 \equiv \tilde{G}(R_v(n)) - \tilde{G}(R(n))$, $p^+ \equiv 1 - \tilde{G}(R_v(n))$, and $p^- \equiv \tilde{G}[R(n)]$. Note that D_θ is independent of x. Differentiating the first–order condition for a hiring firm, (13), with respect to θ we obtain:

$$-\eta\beta c + \frac{c}{q}\frac{q'(\theta)}{q} + \beta D_\theta = -\frac{\eta\beta c}{1-\beta[1-\lambda(1-p^0)]} + \frac{c}{q}\frac{q'(\theta)}{q}\frac{1-\beta(1-\lambda p^-)}{1-\beta[1-\lambda(1-p^0)]} < 0 \tag{59}$$

since $q'(\theta) < 0$. Thus it follows that $\partial R_v/\partial\theta > 0$. Likewise, differentiating the first–order condition for a shedding firm, (14), with respect to θ we obtain:

$$-\eta\beta c + \beta D_\theta = -\frac{\eta\beta c}{1-\beta[1-\lambda(1-p^0)]} - \beta\frac{c}{q}\frac{q'(\theta)}{q}\frac{\lambda p^+}{1-\beta[1-\lambda(1-p^0)]} \tag{60}$$

Thus, $\partial R/\partial\theta > 0 \iff n > R_v^{-1}\tilde{G}^{-1}\left(1 + \frac{\eta}{\varepsilon_{q\theta}}\frac{f}{\lambda}\right)$ where $\varepsilon_{q\theta} \equiv \frac{d\ln q}{d\ln\theta}$. ■

Proof of Proposition 5. *Proof of (19) and (20):* See main text.

Proof of (21): First note that a necessary condition for a firm to shed workers is that it receives an idiosyncratic shock, which occurs with probability λ. In this event, the number of separations in a firm that is shedding workers is equal to $[n_{-1} - R^{-1}(x)]$, since separating firms set employment, $n = R^{-1}(x)$. Now imagine, counterfactually, that all firms shared the same lagged employment level, n_{-1}. Then, the aggregate number of separations in the economy would equal:

$$\Lambda(n_{-1}) = \lambda \int_{n_{\min}}^{R(n_{-1})} \left[n_{-1} - R^{-1}(x)\right] d\tilde{G}(x) \tag{61}$$

where $n_{\min}$ is the lower support of employment. Using the change of variables, $x = R(n)$, and integrating by parts:

$$\Lambda(n_{-1}) = \lambda \int_{n_{\min}}^{n_{-1}} (n_{-1} - n) \frac{d\tilde{G}[R(n)]}{dn} dn = \lambda \int_{n_{\min}}^{n_{-1}} \tilde{G}[R(n)]\, dn \tag{62}$$

Now, of course, the true aggregate number of separations is equal to $S = \int \Lambda(n_{-1})\, dH(n_{-1})$, where $H(\cdot)$ is the c.d.f. of employment. Denoting $n_{\max}$ as the upper support of $H(\cdot)$, further integration by parts reveals that:

$$S = \Lambda(n_{\max}) - \lambda \int \tilde{G}[R(n_{-1})]\, H(n_{-1})\, dn_{-1} = \lambda \int [1 - H(n)]\, \tilde{G}[R(n)]\, dn \tag{63}$$

as required. A similar method reveals that the aggregate number of hires in the economy, $M = \lambda \int H(n) \left(1 - \tilde{G}[R_v(n)]\right) dn$. It follows from the steady state condition for the distribution for employment, (19), that separations, S, are equal to hires, M. ■

Lemma 2 *If idiosyncratic shocks evolve according to (17), and the matching function is of the form $M(U,V) = \mu U^{\phi} V^{1-\phi}$, then the marginal firm surplus defined in (40) is given by*

$$\begin{aligned} J &= \frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)} \left[x + \frac{\beta\lambda p^0}{1-\beta(1-\lambda)-\beta\lambda p^0} \mathcal{E}(n)\right] \\ &\quad - \frac{(1-\eta)\, b}{1-\beta(1-\lambda)-\beta\lambda p^0} - \beta \frac{c}{q} \frac{\eta f - \lambda p^+}{1-\beta(1-\lambda)-\beta\lambda p^0}, \end{aligned} \tag{64}$$

and the marginal effects of n, p and θ on J are given by

$$\begin{aligned} J_n &= -\frac{1-\alpha}{n} \frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)} \left[x + \frac{\beta\lambda p^0}{1-\beta(1-\lambda)-\beta\lambda p^0} \mathcal{E}(n)\right] \\ J_p &= \frac{1}{p} \frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)} \left[x + \frac{\beta\lambda p^0}{1-\beta(1-\lambda)-\beta\lambda p^0} \mathcal{E}(n)\right] \\ J_\theta &= -\beta \frac{c}{q} \frac{1}{\theta} \frac{\eta f - \phi\lambda p^+}{1-\beta(1-\lambda)-\beta\lambda p^0}, \end{aligned} \tag{65}$$

where $\psi \equiv \frac{1-\eta}{1-\eta(1-\alpha)}$*,* $\mathcal{E}(n) \equiv \mathbb{E}(x'|x' \in [R(n), R_v(n)])$*, and* p^0*,* p^+ *are as defined in the Proof to Proposition 4.*

Proof. Since firms only receive an idiosyncratic shock with probability λ each period, we can use the recursion for $J(n,x)$, (40), to write:

$$\begin{aligned} J(n,x) &= \frac{1}{1-\beta(1-\lambda)}\left[\psi px\alpha n^{\alpha-1} - (1-\eta)b - \eta\beta c\theta\right] \\ &+ \frac{\beta\lambda}{1-\beta(1-\lambda)}\frac{c}{q}\int_{R_v(n)} d\tilde{G} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J(n,x')\,d\tilde{G} \qquad (66) \end{aligned}$$

We then conjecture that $J(n,x)$ is of the form $j_0 + j_1 x$. Substituting this assumption into the latter, and equating coefficients yields:

$$\begin{aligned} j_0 &= -\frac{(1-\eta)b}{1-\beta(1-\lambda)} - \beta\frac{c}{q}\frac{\eta f - \lambda p^+}{1-\beta(1-\lambda)} + \frac{\beta\lambda p^0}{1-\beta(1-\lambda)}\left[j_0 + j_1\mathcal{E}(n)\right] \\ j_1 &= \frac{\psi p\alpha n^{\alpha-1}}{1-\beta(1-\lambda)} \qquad (67) \end{aligned}$$

Solving for j_0 we obtain the required solution for $J(n,x)$. Likewise, we can obtain recursions for the marginal effects of n and θ:

$$\begin{aligned} J_n(n,x) &= -\frac{1}{1-\beta(1-\lambda)}\frac{1-\alpha}{n}\psi px\alpha n^{\alpha-1} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_n(n,x')\,dG \\ J_p(n,x) &= \frac{1}{1-\beta(1-\lambda)}\psi x\alpha n^{\alpha-1} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_p(n,x')\,d\tilde{G} \\ J_\theta(n,x) &= -\frac{\eta\beta c + \beta\lambda\frac{c}{q^2}q'(\theta)\int_{R_v(n)} dG}{1-\beta(1-\lambda)} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_\theta(n,x')\,dG \qquad (68) \end{aligned}$$

Again using the method of undetermined coefficients, and noting that the Cobb Douglas matching function implies $q = \mu\theta^{-\phi} \implies \frac{c}{q^2}q'(\theta) = -\frac{c}{q}\frac{\phi}{\theta}$, yields the required solutions for J_n, J_p and J_θ. ■

Proof of Proposition 6. Total differentiation of the JC condition, $U(\theta) = L - \mathbb{E}(n)$, yields $\frac{d\theta}{dp} = -\frac{\mathbb{E}(\partial n/\partial p)}{\mathbb{E}(\partial n/\partial \theta)}$. In steady state, the probabilities of raising, freezing, and cutting employment will all be constants. Denoting these probabilities as $\mathbf{p}^+, \mathbf{p}^0$, and $\mathbf{p}^-$ respectively, it follows that we can write:

$$\mathbb{E}\left(\frac{\partial n}{\partial \xi}\right) = \mathbf{p}^+\mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n > 0\right) + \mathbf{p}^0\mathbb{E}\left(\frac{\partial n_{-1}}{\partial \xi}\right) + \mathbf{p}^-\mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n < 0\right) \qquad (69)$$

for any variable ξ. Note further that in steady state $\mathbb{E}(\partial n/\partial \xi) = \mathbb{E}(\partial n_{-1}/\partial \xi)$ so that we obtain the result that:

$$\mathbb{E}\left(\frac{\partial n}{\partial \xi}\right) = \pi \mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n > 0\right) + (1-\pi)\,\mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n < 0\right) \tag{70}$$

where $\pi \equiv \frac{\mathbf{p}^{+}}{1-\mathbf{p}^{0}}$. Thus, we can rewrite the marginal effect of a change in p on θ as:

$$\frac{d\theta}{dp} = -\frac{\pi \mathbb{E}\left(\frac{\partial n}{\partial p}|\Delta n > 0\right) + (1-\pi)\,\mathbb{E}\left(\frac{\partial n}{\partial p}|\Delta n < 0\right)}{\pi \mathbb{E}\left(\frac{\partial n}{\partial \theta}|\Delta n > 0\right) + (1-\pi)\,\mathbb{E}\left(\frac{\partial n}{\partial \theta}|\Delta n < 0\right)} \tag{71}$$

Then note that the first–order conditions for optimal labor demand set the marginal firm surplus, $J(n,x)$ as follows:

$$J(n,x) = \begin{cases} c/q(\theta) & \text{if} \quad \Delta n > 0 \\ 0 & \text{if} \quad \Delta n < 0 \end{cases} \tag{72}$$

It is immediate from Lemma 2 that $\frac{\partial n}{\partial p} = -\frac{J_p}{J_n} = \frac{1}{1-\alpha}\frac{n}{p}$ regardless of whether $\Delta n > 0$ or $\Delta n < 0$. Thus it remains to derive $\frac{\partial n}{\partial \theta}$ in each case. Log–linearizing the function J around n, p, x, and θ, we obtain:

$$\log J \approx \varepsilon_{Jn} \log n + \varepsilon_{Jp} (\log p + \log x) + \varepsilon_{J\theta} \log \theta + const. \tag{73}$$

Using this and totally differentiating the first–order conditions for optimal labor demand with respect to n and θ, we obtain:

$$\varepsilon_{Jn} d\log n + \varepsilon_{J\theta} d\log\theta \approx \begin{cases} -d\log q(\theta) & \text{if} \quad \Delta n > 0 \\ 0 & \text{if} \quad \Delta n < 0 \end{cases} \tag{74}$$

Given the Cobb Douglas matching function assumption, $q(\theta) = \mu\theta^{-\phi}$, and it follows that $d\log q(\theta) = -\phi d\log\theta$. Thus:

$$\frac{\partial n}{\partial \theta} = \frac{\partial \log n}{\partial \log \theta}\frac{n}{\theta} \approx \begin{cases} \frac{\phi - \varepsilon_{J\theta}}{\varepsilon_{Jn}}\frac{n}{\theta} & \text{if} \quad \Delta n > 0 \\ -\frac{\varepsilon_{J\theta}}{\varepsilon_{Jn}}\frac{n}{\theta} & \text{if} \quad \Delta n < 0 \end{cases} \tag{75}$$

Substituting this into (71), we obtain:

$$\frac{d\log\theta}{d\log p}|_{JC} \approx -\frac{1}{1-\alpha}\frac{\varepsilon_{Jn}}{\omega\phi - \varepsilon_{J\theta}} \tag{76}$$

where $\omega \equiv \frac{\pi \mathbb{E}(n|\Delta n > 0)}{\mathbb{E}(n)}$ is the steady state share of employment in hiring firms. In what follows, we evaluate the approximation (73) to the marginal surplus around mean employment, $\bar{n} \equiv \mathbb{E}(n)$, and mean productivity conditional on mean employment, $x = \mathcal{E}(\bar{n}) \equiv$

$\mathbb{E}\left(x'|x' \in \left[R\left(\bar{n}\right), R_v\left(\bar{n}\right)\right]\right)$. Thus, using the results of Lemma 2 it follows that we can write:

$$J_n = -\frac{1}{\bar{n}} \frac{(1-\alpha)\,\psi p \alpha \bar{n}^{\alpha-1}}{1-\beta(1-\lambda)-\beta\lambda p^0} \mathcal{E}\left(\bar{n}\right)$$

and:

$$J\left[1-\beta(1-\lambda)-\beta\lambda p^0\right] = \psi p \mathcal{E}\left(\bar{n}\right)\alpha\bar{n}^{\alpha-1} - (1-\eta)\,b - \beta\frac{c}{q}\left[\eta f - \lambda p^+\right] \tag{77}$$

where $\psi \equiv \frac{1-\eta}{1-\eta(1-\alpha)}$. Substituting back into the aggregate elasticity of θ with respect to p, we obtain:

$$\frac{d\log\theta}{d\log p}|_{JC} \approx \frac{\psi p \mathcal{E}\left(\bar{n}\right)\alpha\bar{n}^{\alpha-1}}{\omega\phi\left[\psi p \mathcal{E}\left(\bar{n}\right)\alpha\bar{n}^{\alpha-1} - (1-\eta)\,b - \eta\beta c\theta\right] + \eta\beta c\theta - (1-\omega)\,\phi\beta\frac{c}{q}\lambda p^+} \tag{78}$$

Noting that the marginal product of labor in the average–sized firm is equal to $p\mathcal{E}\left(\bar{n}\right)\alpha n^{\alpha-1}$, and assuming λ is sufficiently small, we obtain:

$$\frac{d\log\theta}{d\log p}|_{JC} \approx \frac{(1-\eta)\,\tilde{p}}{\omega\phi\left[(1-\eta)\left(\tilde{p}-b\right) - \eta\beta c\theta\right] + \eta\beta c\theta} \tag{79}$$

where $\tilde{p} \equiv \rho p \mathcal{E}\left(\bar{n}\right)\bar{n}^{\alpha-1} + (1-\rho)\,p\mathcal{E}\left(\bar{n}\right)\alpha\bar{n}^{\alpha-1}$ and $\rho \equiv \frac{\alpha\eta}{1-\eta(1-\alpha)}$, as required. ■