THE BASIC PUBLIC FINANCE OF PUBLIC-PRIVATE PARTNERSHIPS

EDUARDO M.R.A. ENGEL, RONALD D. FISCHER AND ALEXANDER GALETOVIC[1]

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Abstract

When should public-private partnerships (PPPs) be preferred to public provision and how should they be structured? We answer these questions within a simple public finance framework. We show that PPPs are not a means for substituting cheap private funding for distortionary finance. Private firms must be productively more efficient than the public sector for PPPs to be desirable from a public finance point of view.

We also study the contract that trades off optimally demand risk, user-fee distortions and the opportunity cost of public funds, under the assumption that the private sector is more efficient. This contract is characterized by two thresholds: a minimum income guarantee and a cap on the firm's revenues. When demand is high, the contract ends when the present value of user fee revenue reaches the upper threshold; when demand is low, the contract lasts indefinitely and the government subsidizes the firm in the amount needed to attain the minimum income guarantee. Government finances are unaffected by the project in intermediate demand states. Even though income guarantees and revenue sharing arrangements are often observed in practice, they differ in fundamental ways from those derived in this paper. Finally, we show how the optimal contract can be implemented with a competitive auction that does not require knowledge of the upfront investment in the project or firms' risk aversion.

Key words: cost of public funds, Demsetz auction, subsidies, income guarantee, profit cap.

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[1]Engel: Department of Economics, Yale University, 28 Hillhouse Ave., New Haven, CT 06511. Fischer: Center for Applied Economics (CEA), Department of Industrial Engineering, University of Chile, Av. República 701, Santiago, Chile. Galetovic: Facultad de Ciencias Económicas y Empresariales, Universidad de Los Andes, Av. San Carlos de Apoquindo 2200, Santiago, Chile. E-mails: eduardo.engel@yale.edu, rfischer@dii.uchile.cl, agaletovic@uandes.cl.

1 Introduction and motivation

There is an increasing interest in public-private partnerships (PPPs) around the world.[2] In a typical project of this type, a private firm builds and finances the infrastructure and then collects user fees for many years. Eventually, the franchise ends and the infrastructure reverts to the state.[3]

PPPs have been used to finance toll-roads, provide sanitation services, sports stadiums, train lines, seaports and airports, and even to develop so-called orphan drugs, i.e., neglected disease drug development projects.[4] They have been hallowed as a third way between public provision and privatization, potentially combining the strengths of both. But the new trend raises many questions. When is private financing of infrastructure projects desirable? When is public financing optimal? When are public subsidies warranted? More generally, when is a public-private partnership best and how should the corresponding contract be designed? This paper provides a public-finance framework to answer these questions.

We study a model where a risk neutral government must contract a risk-averse firm to build and finance an infrastructure project with uncertain demand that requires a large upfront investment.[5] The firm (or *franchise holder*) can be compensated with a combination of subsidies that are paid out of the general budget, and user fees. At one extreme are arrangements where subsidies are the only source of income for the firm. This is the "traditional approach" or public model, where firms build the infrastructure project and then hand it over to a public agency. At the other extreme is the case where all of the firm's income comes from user fees. A variety of public-private financing arrangements are possible in between.

Our first result is that the usual justification for PPPs—relieving public budgets and substituting cheap private funding for distortionary tax finance—is suspect. To see why, note that it implies that the firm should finance as much as possible of the project's construction cost and,

[2]For example, articles in the Financial Times mentioning this concept increased twenty-fold over the last decade, from 50 in 1995 to 1,153 in 2004.

[3]The term "Public-Private Partnership" does not have an unambiguous meaning and definitions abound. In this paper we will have in mind an infrastructure project such that (i) assets are possibly temporarily owned by the private firm; (ii) both the private firm and the government are residual claimants, often in ambiguous terms; and (iii) there is substantial public planning involved.

[4]The case of PPPs in the transportation sector is particularly compelling. Growing congestion, budgetary problems, and a major decrease in toll collection costs have led more than 20 U.S. states to pass legislation permitting the operation of public-private partnerships (PPPs) to build, finance and operate toll-roads, bridges and tunnels. See "Paying on the Highway to Get Out of First Gear." New York Times, April 28, 2005. Congestion costs in the top U.S. metro areas have grown steadily, reaching $63.1 billion in 2003, 60% higher (in real terms) than a decade earlier (see Schrank and Lomax, 2005).

[5]As in principal-agent models, the less risk averse party —in our case the government— is assumed to be risk neutral. Assuming a risk averse firm is a shortcut for agency problems preventing risk diversification, see Appendix D in the working paper version of Engel, Fischer and Galetovic (2001) for a model along these lines.

consequently, the government should subsidize as little as possible. Yet this argument overlooks an essential point. At the margin, extending the concession term has an opportunity cost, since the government foregoes the revenues generated by the project during this period and this revenue could have been used to reduce distortionary taxation. Hence, the opportunity cost of $1 in user fees is the shadow cost of public funds. For this reason, if the public and the private sector are equally efficient, user fees and subsidies are perfect substitutes at the margin and a continuum of revenues/subsidy combinations implement the optimum.

We then show that private participation is warranted only if firms are productively more efficient and can deliver the infrastructure at a lower cost. This is not terribly surprising, because it is the standard argument in favor of privatization. Nevertheless, the optimal contract has quite specific features, most of which seem to be absent from contracts observed in practice.

To begin, the optimal contract remunerates the firm as much as possible with revenues from user fees. Because the public sector spends inefficiently (otherwise a PPP is not warranted) one would like to subsidize as little as possible. It follows that no subsidies are granted if user revenues are sufficient to pay for the infrastructure in all states. On the contrary, if a subsidy is warranted in some state, then the concession should last indefinitely, thus minimizing the subsidy payment.

Given a perfectly inelastic stochastic demand structure, optimal contracts can be classified into three groups, depending on the size of the upfront investment. For small projects, defined as those where user fees are enough to pay for the infrastructure in all states of demand, the firm receives full insurance and the franchise term is finite and flexible. Franchises last longer in states where demand is lower.

For large projects, defined as those where user fees cannot finance the project in any state of demand, demand-contingent subsidies are paid in all states and the contract lasts indefinitely. Again, the firm is provided full insurance. This implies that assigning the project to the firm that demands the lowest subsidy, as is often done in practice, is not only suboptimal but also leads to subsidies that, on average, are higher than those of the optimal contract.

The most interesting case is that of intermediate size projects, defined as those with both demand states where the project can finance itself, and states where this is not the case. For these projects, the optimal contract is characterized by two thresholds: a minimum income guarantee and a cap on the firm's revenues.[6] When demand is high, the contract ends when the present value of user fee revenue reaches the upper threshold; when demand is low the government subsidizes the firm in the amount needed to attain the minimum income guarantee. Government finances are unaffected by the project in intermediate demand states. Even

[6] Large and small projects may be viewed as particular cases of this type of contract as well.

though income guarantees and revenue sharing arrangements are often observed in practice, they differ in fundamental ways from those derived in this paper.

The optimal contract can be implemented with a competitive auction that does not require knowledge of the upfront investment in the project or the firm's degree of risk aversion. The government announces the probability density that characterizes demand uncertainty and the parameter that summarizes the government's inefficiency. Firms submit a minimum income guarantee and a revenue cap, these numbers are combined into a scalar bid using the information mentioned above, and the contract is assigned to the firm with the lowest bid.

The third set of results relaxes the assumption of infinitely inelastic demand and considers optimal pricing of infrastructure services provided with a PPP. In general, prices should be set above the marginal cost of production even if no scale economies are present. The reason is that user fees substitute for distortionary taxation at the margin, both during and after the franchise. Thus it pays to distort pricing a little to reduce the need of distortionary taxation. Optimal prices are even higher in states where subsidies are paid, because they substitute for inefficient subsidies at the margin. In these states it pays to distort beyond what is warranted by the cost of public funds.

What is the economics behind these results? The optimal contract trades off three margins. First, the planner can distort user fees to raise revenue to cover the up front investment. A second margin is the extent to which the firm is forced to bear demand risk.[7] And third, the government may use subsidies to insure the firm and reduce user fee distortions, but it must collect distortionary taxes and bear the inefficiencies of public spending to do so. In principle, these three margins suggest a complicated optimal combination of distortions; in practice the solution is quite simple and has the same structure as that described above for the perfectly inelastic demand case.

If user fees that distort as much as the shadow cost of public funds can pay for the infrastructure in all states, the firm receives full insurance, no subsidies are paid, and the infrastructure is priced optimally to substitute for public funds at the margin. This is the case of a project of small size, relative to its demand. At the opposite extreme, if subsidies are paid in all states, the firm receives full insurance and the infrastructure is priced optimally, to substitute for public subsidies at the margin. This is the case of a large project. Most projects lie in between, with subsidies being paid in low but not in high-demand states. The inefficiency of public spending introduces a wedge between the marginal opportunity cost of public funds and the marginal opportunity cost of public spending. Thus, it pays to depart from optimal pricing in some states, and to introduce some risk.

[7]As is standard in principal agent theory, we assume the less risk averse party—the government—is risk neutral.

In Engel, Fischer and Galetovic (2001) we imposed a “self-financing constraint” that ruled out subsidies by assumption, and studied the optimal private provision of infrastructure projects solving a Ramsey problem with variable franchise lengths. We go beyond that paper by allowing the government to grant subsidies, so that any combination along the public-private continuum is now possible. This allows us to answer the question of when the private provision of infrastructure is desirable, thereby deriving several new insights for the optimal contract in this case.

This paper contributes to a growing literature on PPPs. There are many definitions, but most authors would probably agree on three characteristics shared by most of these arrangements. First, projects are bundled together, and the same private firm or consortium builds a large sunk infrastructure, operates it, and sells the service directly either to the public or the government or a combination of both. Second, in a PPP the public and the private sector share risks. Third, the concession lasts for a long but limited term (e.g. 20 or 30 years).

One strand of the literature studies under what circumstances a PPP is productively more efficient than traditional procurement. Martimort and Pouyet (2006) and Bentz et al. (2005) show that bundling makes sense when infrastructure provision and operation moderates moral hazard and adverse selection. From an incomplete contracts perspective, Hart (2003) shows that PPPs are more efficient if quality of service can be accurately described in the initial contract. Then control rights can be allocated to the private firm thus providing performance incentives. By contrast, when asset quality is relatively easy to describe but service quality is not, then the traditional, unbundled model is better. We complement this literature by showing that productive efficiency is not only sufficient for a PPP to be desirable, but also necessary. We show that traditional financial arguments in favor of PPPs, in particular that they allow governments to build and fund infrastructure without increasing public debt, are suspect at least and probably wrong—future revenue lost by ceding user-fee flows to private firms (or additional expenditures needed to pay for the service) exactly offset the investment savings made by the current government. We also show that another public finance argument given in support of PPPs, namely that the substitute private finance for distortionary taxation, is also wrong.

Our paper also sheds light on the ongoing debate about privatizations and PPPs (see, for example, Daniels and Trebilcock (1996, 2000), Gerrard (2001), Savas (2000), and Starr (1988)). We find that PPPs are very different from a privatization. In a privatization assets and cash flows are fully transferred to a private firm and the links between the project and the public budget are permanently severed. By contrast, in a PPP the link is never severed, even when the project is fully paid for by users. Moreover, we show that with the optimal risk-sharing contract the intertemporal public budget bears the same risk as when the project is undertaken directly by the government. Nevertheless, it is always convenient to finance the project as much as possible with user fees.

Clarifying what makes a PPP different from a privatization also gives some hints on how PPPs should be accounted for in the public budget. As pointed out by IMF (2004, p. 24), there is not yet a comprehensive fiscal accounting and reporting standard specifically for PPPs. Eurostat (2004) recommends that the assets involved in a public-private partnership should be classified as non-government assets when the private partner bears the construction risk and, in addition, at least one of either availability or demand risk.

Our intertemporal framework suggests that it is misleading to think of demand risk as a fixed quantity to be allocated between the government and the private firm. To see why, assume a fixed-term PPP such that the government fully subsidizes the project, pays a fixed amount per person that uses the infrastructure and demand is random over the life of the PPP. Because the term is fixed, both the amount that the government will pay and the private firm will receive are random. Hence, relative to traditional provision, where the government pays the same amount in all states of demand, both the private firm and the public budget bear more risk.

Our intertemporal framework also suggests that using the allocation of demand risk to define whether a PPP should be included in the public budget may lead to mistakes. For example, when user fees are enough to fully finance the infrastructure the optimal risk-sharing contract is such that the private firm bears no risk, and yet the link between the public budget and the project is not severed because at the margin each dollar of revenue foregone substitutes for one dollar obtained with distortionary taxation.

Last, this paper is also related to the literature on franchise bidding pioneered by Chadwick (1859) and Demsetz (1968), according to which competition for a monopoly infrastructure project will reproduce the competitive outcome (see Stigler [1968], Posner [1972], Riordan and Sappington [1987], Spulber [1989, ch. 9], Laffont and Tirole [1993, chs. 7 and 8], Harstad and Crew [1999] for important papers within this tradition, and Williamson [1976, 1985] for a critique). We contribute to this literature by including cases where projects cannot self-finance and government subsidies are necessary to make them feasible, and show how the optimal contract can be implemented with an auction.

The remainder of the paper is organized as follows. We begin with a simple model with perfectly inelastic demand and only two states of nature (Section 2). This simplification allows us to study the basic public finance problem without the additional complications introduced by multiple states and a demand that responds to prices. The model is generalized in various directions in section 3, increasing the number of demand states, incorporating price-responsive demand and allowing for moral hazard. Section 4 concludes.

2 Benchmark model and the irrelevance result

To better isolate the novel elements in our results, it is convenient to start with a perfectly inelastic demand that is unaffected by the firm's effort. In the next section we consider a price-responsive demand and moral hazard.

The model

A benevolent social planner hires a private firm to build a socially desirable infrastructure project with exogenous technical characteristics. The firm can be compensated with user fee revenues and subsidies. The planner's objective is to maximize the expected present value of social welfare, considering the shadow cost of the public funds needed to pay for the subsidies, subject to finding a firm that is willing to build the project.[8] When the franchise ends, the project reverts to the government and any future revenues are used to reduce distortions elsewhere in the economy.

Demand uncertainty is summarized by a probability density over the present value of user fee revenue generated by the infrastructure over its entire lifetime, $f(v)$, with c.d.f. $F(v)$. This density is common knowledge to firms and the planner, and is bounded from below by $v_{\min}$ and from above by $v_{\max}$. There is a fixed price per unit of service equal to P, constant across demand states.

There are neither maintenance nor operation costs,[9] the upfront investment does not depreciate, and there are many firms that can build the project at cost $I > 0$.[10] All firms are identical, risk-averse expected utility maximizers, with preferences represented by the strictly concave utility function u.[11]

[8]This objective function assumes that the income of users is uncorrelated with the benefit of using the project, so that if users spend a small fraction of their incomes on the services of the project they will value the benefits produced by the project as if they were risk neutral. See Arrow and Lind (1970).

[9]This assumption is relaxed in Section 3. In any case, there are two reasons why ignoring maintenance and operations costs is not a serious limitation. First, for most of the infrastructure projects of interest, the main costs are upfront, and maintenance and operation costs are relatively smaller (consider highways, dams, sport stadiums and rail lines). Second, and more important, if maintenance and operations costs are proportional to demand for the project, which is a good approximation in the case of highways and rail lines, then our framework extends trivially to the case with maintenance and operations costs, by substituting the price net of estimated maintenance costs for the price in what follows.

[10]That is, we ignore construction cost uncertainty and instead focus on demand uncertainty, which is considerably larger for many PPP projects.

[11]This should be interpreted as a reduced form for an agency problem that prevents the firm from diversifying risk. See Appendix D in the working paper version of Engel, Fischer and Galetovic (2001) for a model along these lines.

The planner's problem

It is often claimed that private involvement in infrastructure provision is desirable because private firms have access to funds at a lower social cost—they do not raise funds through distortionary taxation. By contrast, governments must resort to distortionary taxation to finance infrastructure projects. Is this argument enough to make the case for PPPs?

It is useful to consider first the problem solved by a planner who knows I. The planner can subsidize the project in the amount $S(v)$ in state v, where by "subsidy" we mean any cash transfer from the government to the private firm.[12] It may represent the payment made upfront to procure the project in the traditional fashion, it could also be a cash transfer under a Build-Operate-and-Transfer (BOT) contract to supplement sales revenue from the project ('minimum income guarantees'). In any case, distortionary taxes that cost $\lambda > 1$ must be raised to pay one dollar of subsidy.

We also assume that the planner collects user fees after the contract ends. Each dollar raised in user fees is then used to reduce distortionary taxation elsewhere in the economy. Therefore a second margin available to the planner is the contract length in each state of demand, that is, how user fee revenue is split between the firm and the planner. We denote by $R(v)$ the present value of user fees collected by the firm in state v, the government then receives $v - R(v)$ and we have $0 \leq R(v) \leq v$. Furthermore, assuming that willingness to pay is positive at all points in time, we have that $R(v) = v$ only when the contract lasts indefinitely.

Since private participation is voluntary, the planner solves the following problem:

$$\max_{\{R(v),S(v)\}} \int [CS(v) + \alpha PS(v)] f(v) dv \tag{1}$$

$$\text{s.t.} \int u(R(v) + S(v) - I) f(v) dv \geq u(0), \tag{2}$$

$$0 \leq R(v) \leq v, \tag{3}$$

$$S(v) \geq 0, \tag{4}$$

where $CS(v)$ and $PS(v)$ denote the present value of consumer and producer surplus in state v, α measures the importance the planner gives to the latter relative to the former, and the firm's outside option—the level of utility attained when not undertaking the project—is $u(0)$. By using the same probability density in the planner's objective function and the firm's participation constraint, we are abstracting from possible differences between the planner's and the firm's risk-free discount rates.

When consumer and producer surplus are weighed equally, we have $\alpha = 1$. Assuming $\alpha < 1$ is appropriate when a dollar in the hands of users is socially more valuable than in the pocket

[12] $S(v)$ then denotes the present value of this subsidy.

of the firm (as in Laffont and Tirole [1993]), say because the firms providing the infrastructure are foreign. In what follows we assume that $0 \le \alpha \le 1$.[13]

The assumptions and notation we introduced above imply that

$$PS(v) = R(v) + S(v) - I, \tag{5}$$

$$CS(v) = (\lambda - 1)[v - R(v)] + [v - R(v) - \lambda S(v)] = \lambda v - \lambda R(v) - \lambda S(v). \tag{6}$$

The terms on the right hand side of (5) are the difference between the firm's income and the upfront investment it finances. The terms after the first equality sign in (6) are justified as follows: $[v - R(v)]$ is total revenue collected by the government, in present value. This allows the government to reduce distortionary taxation, thereby saving resources to society in the amount $(\lambda - 1)[v - R(v)]$. Next, $[v - R(v) - \lambda S(v)]$ is the difference between users' valuation of the infrastructure and the total amount they transfer to the firm in state v. Of course, the firm receives only $R(v) + S(v)$, precisely the amount that appears in the firm's participation constraint (2).

Substituting (5) and (6) in (1) shows that the planner's objective function can be replaced by:

$$-(\lambda - \alpha) \int [R(v) + S(v)] f(v) dv,$$

where we have dropped the terms αI and λv from the objective function because they do not depend on the planner's choice variables.[14] Since $\lambda > 1 \ge \alpha$, it follows that maximizing the objective function (3) is equivalent to minimizing:

$$\int [R(v) + S(v)] f(v) dv. \tag{7}$$

The Irrelevance Result

It follows from (7) that the per-dollar cost of paying for the project with sales revenues or subsidies is the same. Thus, social welfare only depends on *total* transfers to the firm, not on how these transfers are split up between subsidies and user fee revenue. This is the fundamental insight behind the following result:

Proposition 1 (Irrelevance of the Public Cost of Funds Argument) *Any combination of user fee and subsidy schedules that satisfies constraints (3) and (4) and such that $R(v) + S(v) = I$ for all v solves the planner's problem described in (1)–(4).*

[13]We actually use the weaker condition $\alpha < \lambda$.

[14]The latter term will be important in our analysis of effort in Section 3.2.

Proof: See the Appendix. ∎

What is the economics of this result? The standard reasoning in favor of private provision of infrastructure points out that subsidies are an expensive means of financing projects, because they are paid with distortionary taxes. Yet the multiplicity of optimal subsidy-sales revenue combinations indicates that distortionary taxation ($\lambda > 1$) is not sufficient to make private provision preferable. For one possible solution is that $R(v) \equiv 0$ and $S(v) \equiv I$; this is the "traditional approach" to infrastructure financing where the government pays for the project upfront. At the other extreme is limited-term privatization, where the firm invests I, collects user-fee revenues equal to I in present value, and no subsidies are paid.[15] In addition, there is a continuum of intermediate solutions, which we refer to as public-private partnerships, where the government provides (partial) financing. What does the standard reasoning overlook?

An essential aspect of infrastructure projects is that the government foregoes user fee revenue under a PPP arrangement. Indeed, increasing by one dollar user fees collected by the firm Hence, the opportunity cost of paying the firm with \$1 out of subsidies, λ, is exactly the same as paying it with one dollar out of sales revenue. Indeed, increasing by one dollar user fees collected by the firm not only costs consumers this additional dollar, but also reduces resources the government could have used to reduce distortions elsewhere in the economy by $(\lambda - 1)$. The rich set of optimal combinations of state-contingent subsidies and contract lengths reflects the fact that user fees and subsidies are perfect substitutes in the planner's objective function.

A similar argument shows that the planner will choose to satisfy the firm's participation constraint with equality. Extending the franchise term to collect an extra dollar provides additional resources to the firm valued by the planner at $\alpha \leq 1$. This benefit comes at a cost λ to users, by the same argument given above. Since $\lambda > \alpha$, the planner leaves no rents to the firm.

Evaluating shadow fees and availability contracts

In industrialized countries, PPPs often take the form of fixed term contracts where users pay no fees for the infrastructure service. If the firm is compensated via so-called "shadow fees," that is, per user fees paid directly by the government, then the firm is forced to bear risk and the resulting contract is not optimal. By contrast, if the firm is paid a yearly sum by the government conditional on delivering the agreed quality of service, as is the case with "availability contracts," then the firm bears no risk, and the contract will be optimal if the firm receives no

[15]For this to be possible we need that $v_{\min} \geq I$, for otherwise the project cannot be financed with user fees in low demand states.

rents.[16] Proposition 1 provides an argument to discourage fixed term contracts with shadow fees, while encouraging availability contracts.[17]

Corollary 1 *When no user fees can be charged (say because of political constraints), $S(v) = I$ for all states v characterizes the unique optimal contract.*[18] *Hence shadow fees which make payments contingent on the use of the infrastructure for a fixed and finite term T are never optimal. By contrast, availability contracts that leave no rents for the firm are optimal.*

3 Why PPPs? Productive efficiency

The analysis above shows that the justification of private participation in infrastructure cannot rest on the often claimed "fact" that it relieves strained budgets and reduces distortionary taxation. One of the main arguments in favor of PPPs is that governments are unable to spend efficiently, perhaps because of political economy considerations or outright corruption. On the other hand, many argue that the experience with infrastructure PPPs has been unsatisfactory and that the traditional model may be more cost efficient after all. This controversy is, of course, about productive efficiency. In this section we explore the implications of differences in productive efficiency for the optimal contract.

Modeling productive efficiency

To model productive efficiency we let ζ denote the number of dollars needed by the government to achieve with a subsidy what a private firm achieves by spending one dollar. Private firms are more efficient when $\zeta > 1$, while the traditional model is better when $\zeta < 1$.

The parameter ζ can be interpreted in many ways. If the government provides the subsidies in kind, for example by building part of the infrastructure project, this parameter captures the government's productive efficiency, relative to that of private firms.[19] On the other hand, if subsidies are monetary transfers from the government to the firm, then ζ may be a shortcut for inefficiencies associated with this transfer process.

[16] The above result remains valid if yearly payments by the government also include operational costs incurred by the firm to satisfy demand during that period. What is central for our result is whether there exists a relation between demand realization and payment for the upfront investment.

[17] Shadow fees may be more attractive when the firm can exert costly effort to influence demand. The case with moral hazard is considered in Section 5.2

[18] This assumes that $v_{\min} \geq I$. If this is not the case, none of the optimal contracts is feasible.

[19] For example, $\zeta > 1$ if public sector unions obtain higher wages for equal work, compared to their private sector counterparts. Or $\zeta < 1$ if the public sector has a technological advantage, such as economies of scope, and can build at a lower cost specific parts of the project.

While producer surplus remains unchanged, introducing productive efficiency means that the $\lambda S(v)$ term in (6) must be replaced by $\lambda\zeta S(v)$. The firm's participation constraint does not change, and the function minimized by the planner now becomes:

$$\int [(\lambda - \alpha)R(v) + (\lambda\zeta - \alpha)S(v)] f(v) dv. \tag{8}$$

Optimal contract when the government is more efficient

Subsidies are more expensive than user fee revenue as a means of financing the infrastructure project when $\zeta > 1$, since $\lambda\zeta - \alpha > \lambda - \alpha$ in this case, the opposite happens when $\zeta < 1$. The case $\zeta = 1$ corresponds to the Irrelevance Result considered in the preceding section. The following result shows that the traditional approach to infrastructure financing is better if $\zeta < 1$.

Proposition 2 (Optimal contract when the government is more efficient) *Let $\zeta < 1$. Then for all v we have $R(v) = 0$ and $S(v) = I$. That is, when $\zeta < 1$, all income received by the firm in the optimal contract comes from subsidies. The traditional approach to infrastructure financing is strictly preferred to a PPP contract in this case.*

Proof: See the Appendix. ∎

Optimal contract when the private sector is more efficient

For the remainder of the paper we consider the case $\zeta > 1$, that is, private firms are more efficient. Clearly, $\zeta > 1$ is *not* a sufficient argument against subsidizing a project, for its social value may exceed I and user fee income may be insufficient to pay for it in low demand states. In those cases, subsidies are warranted to make the project privately attractive.

To derive the optimal contract for this case, we begin by recalling that user-fee financing dominates subsidy financing when $\zeta > 1$, since the cost of transferring one dollar to the firm via a subsidy is higher than allowing it to collect an additional dollar in user fees—the cost of the former to society is $\lambda - \alpha$ while the cost of the latter is $\lambda\zeta - \alpha$. It follows that the planner only resorts to subsidies after exhausting user fees, that is:

$$S(v) > 0 \Longrightarrow R(v) = v, \tag{9}$$

or, equivalently:

$$R(v) < v \Longrightarrow S(v) = 0. \tag{10}$$

Denoting by $\mu > 0$ the multiplier associated with the firm's participation constraint (2), we have that the problem's Lagrangian is:[20]

$$\mathscr{L} = \int [(\lambda - \alpha)R(v) + (\lambda\zeta - \alpha)S(v)]\, f(v)dv \; - \mu \int u(R(v) + S(v) - I) f(v)dv.$$

The FOC with respect to $R(v)$ for a state v where the contract length is finite leads to:

$$u'(R(v) - I) = \frac{\lambda - \alpha}{\mu}, \tag{11}$$

while the FOC with respect to $S(v)$ for a state where subsidies are paid out leads to

$$u'(v + S(v) - I) = \frac{\lambda\zeta - \alpha}{\mu}, \tag{12}$$

where in both cases we have used that revenue financing dominates subsidy financing (see [9]).

Define M and m via:

$$u'(M - I) = \frac{\lambda - \alpha}{\mu}, \tag{13}$$

$$u'(m - I) = \frac{\lambda\zeta - \alpha}{\mu}, \tag{14}$$

respectively. Since $\zeta > 1$ we have $m < M$. Also, the definitions above imply that:

$$u'(m - I) = \frac{\lambda\zeta - \alpha}{\lambda - \alpha} u'(M - I), \tag{15}$$

It follows from (11) and (13) that no subsidies are paid out and the contract length is finite in states with $v > M$. The contract lasts until the firm collects M in present value and the government collects $v - M$ after the contract ends. We refer to this group as *high demand* states,

Similarly, from (12) and (14) we have that subsidies are paid out, in the amount of $m - v$, in states with $v < m$. The contract lasts indefinitely in these states; we refer to them as *low demand* states.

There exists a third group of demand states that completes this taxonomy, those where $m \leq v < M$. The contract lasts indefinitely in these states, for otherwise they would be high demand states. And no subsidies are paid out by the government, for otherwise they would be low demand states. It follows that $R(v) = v$ and $S(v) = 0$ in this group. For obvious reasons these are referred to as *intermediate demand* states.

[20] As argued above, the firm's participation constraint will hold with equality since $\lambda > \alpha$.

Proposition 3 (Taxonomy of Demand States) *The optimal contract is characterized by a minimum income guarantee, m, and revenue cap, M, with $m < M$, as follows:*

1. *If $v > M$, the firm collects M in present discounted user fees while the government collects the remaining $v - M$. No subsidies are paid and the contract length is finite.*

2. *If $m \leq v \leq M$, the contract lasts indefinitely and no subsidies are paid. Total revenues accrued to the firm in present value equals v and the government budget is unaffected by the contract.*

3. *If $v < m$, the franchise lasts indefinitely and the government grants a subsidy of $m - v$ to the firm.*

The above states are described as high, intermediate and low demand states, respectively.

The economic intuition behind this proposition is the following: the opportunity cost of the marginal dollar transferred to the firm is different in high and low demand states. When the contract length is finite—as happens in high demand states—the social cost of the last dollar paid to the firm must be weighed against the opportunity cost of decreasing distortionary taxation elsewhere in the economy, $\lambda - \alpha$. By contrast, in low demand states the last dollar paid to the firm comes from government subsidies and therefore costs society $\lambda\zeta - \alpha$. As we discuss in more detail below, the gap between both margins induces the planner to have the firm bear risk.

To complete our characterization of the optimal contract, we discuss next how the constants m and M are determined. Consider first the case where the present value of user fees can finance the project in all demand states, that is, $v_{\min} \geq I$. It is easy to see that in this case the optimal contract has $M = I$.[21] The intuition is that $R(v) = I$ for all v—which is feasible in this case—is the best option, since it finances the project out of user fees in all states of demand and the firm bears no risk.[22] Average revenue obtained by the firm is I in this case. It could not be less without violating the participation constraint and, had the firm been forced to bear risk it would have certainly been more. All states are high demand states when $v_{\min} \geq I$.

Next we consider the case where user fees are not enough to finance the project in any demand state, that is, $v_{\max} < I$. We argue that $m = I$ in this case, by discarding the possibilities that $m > I$ and $m < I$. If $m > I$, all states are low demand, and the firm's participation constraint

[21]Since $m < M$, it follows that no subsidies are paid out for all feasible values of m, and therefore this threshold is irrelevant to pin down the optimal contract.

[22]The formal proof is similar to that of Proposition 1.

holds with slack, so this cannot be optimal. By contrast, if $m < I$, the firm's participation constraint cannot be satisfied, since revenue in all demand states will be below I. It follows that $m = I$ while M is irrelevant. The optimal contract subsidizes the firm in all demand states so as to ensure total revenue equal to the cost of the project.

We refer to a project with $v_{\min} \geq I$ as a high demand *project*, while a project with $v_{\max} < I$ is a low demand *project*. Projects in between are intermediate demand projects. We have shown that all states are low demand for a low demand project, and all states are high demand for a high demand project. We show below that intermediate demand projects typically have high, intermediate and low demand states.

We have also shown that the firm bears no risk in the optimal contract for low and high demand projects. The reason is that, in both cases, the social costs of transferring an additional dollar to the firm is the same in all states of demand—$(\lambda - \alpha)$ for a high demand project, $(\lambda\zeta - \alpha)$ for a low demand project. And when financing costs are the same across states, there is no reason to have the firm bear risk.

Proposition 4 (Optimal Contract for High and Low Demand Projects) *The optimal contract for high and low demand projects stipulates that $R(v) + S(v) = I$ for all v. Given demand realization v, the government collects $v - I$ if the project is high demand, while it pays a subsidy of $I - v$ if the project is low demand.* ∎

To characterize m and M for an intermediate demand project, we first show that, by contrast with high and low demand projects, the contract with full insurance ($m = M = I$) can be improved upon. To see this, consider decreasing m to $I - \Delta m$, and spending the money that is freed up increasing M to $I + \Delta M$. Lowering the guarantee frees up resources in the amount of $F(I)\Delta m$, which finance an increase in M of $F(I)\Delta m/(1 - F(I))$.[23] Society is made better off in the process, since each dollar saved in guarantees is valued $(\lambda\zeta - \alpha)/(\lambda - \alpha) > 1$ times as much as each dollar foregone in user fee revenue. Thus the planner's objective function improves by $\lambda(\zeta - 1)F(I)\Delta m$. At the same time, the firm's expected utility only decreases by an expression of the order of $(\Delta m)^2$ due to increased risk. It follows that the optimal values of m and M satisfy $m < I < M$. The following proposition characterizes the optimal values of both thresholds:

Proposition 5 (Optimal Contract for Intermediate Demand Projects) *Consider an intermediate demand project: $v_{\min} < I < v_{\max}$, and denote $\bar{\zeta} \equiv (\lambda\zeta - \alpha)/(\lambda - \alpha)$.*[24] *Assume $u'(v_{\min} - I) >$*

[23] We are using here that $0 < F(I) < 1$, that is, that we have an intermediate demand project.

[24] Note that $\bar{\zeta} > 1 \Longleftrightarrow \zeta > 1$ and $\bar{\zeta} < 1 \Longleftrightarrow \zeta < 1$. Furthermore, $\zeta = \bar{\zeta}$ when $\alpha = 0$.

$\bar{\zeta}u'(v_{\max}-I)$.[25] *Then the optimal contract is characterized by quantities m and M, with $v_{\min} < m < I < M < v_{\max}$, such that states with $v > M$ are high demand states, states with $m \leq v \leq M$ are intermediate demand states and states with $v < m$ are low demand states.*[26] *The values of m and M are characterized by*

$$u'(m-I) = \bar{\zeta}u'(M-I), \tag{16}$$

and the firm's participation constraint

$$F(m)u(m-I) + \int_m^M u(v-I)f(v)dv + (1-F(M))u(M-I) = u(0). \tag{17}$$

Proof: See the Appendix. ∎

For intermediate demand projects, the optimal contract combines a minimum income guarantee with a cap on profits, and the firm bears risk. Since user fee revenue can finance the road in some states, while subsidies are used in others, the margin that should be used to evaluate the social cost of the last dollar collected by the firm varies across states.

Evaluating income guarantees and profit sharing arrangements

Minimum income guarantees are routine in many types of PPPs, so one may ask whether observed contracts follow the prescriptions laid out in Proposition 5. The answer is no. Most contracts that are observed in practice are fixed term, that is, they fix the term of the franchise for, say, 30 years. These contracts would be closer to the optimal contract derived above if they lasted longer in low demand states where guarantees are paid out.

Even though less common than minimum income guarantees, profit sharing arrangements in high demand scenarios are also observed in practice. Yet these arrangements usually involve having the government and the firm split revenues, in excess of a given threshold, in fixed proportions. By contrast, the above proposition suggests assigning all the revenue in excess of a

[25] This condition ensures that $m > v_{\min}$ and $M < v_{\max}$, so that condition (16) below holds with equality. Two possibilities arise if $u'(v_{\min}-I) < \bar{\zeta}u'(v_{\max}-I)$. First, if $\int u(v-I)f(v)dv > u(0)$, the optimal contract involves no subsidies ($m < v_{\min}$) and M is determined from

$$\int_{v_{\min}}^M u(v-I)f(v)dv + (1-F(M))u(M-I) = u(0).$$

By contrast, the optimal contract involves no revenue cap when $\int u(v-I)f(v)dv < u(0)$ and in this case the minimum income guarantee is determined by

$$F(m)u(m-I) + \int_m^{v_{\max}} u(v-I)f(v)dv = u(0).$$

[26] See Proposition 3 for the definition of high, intermediate and low demand states.

given threshold to the government. In particular, our results taxing windfall profits at a rate of 100%.

The rationale behind real-world guarantees and revenue sharing schemes is only to reduce risk. By contrast, the rationale behind the optimal contract in Proposition 5 is to optimally trade off insurance on one hand, and the use of public and private public funds on the other. This is why the contract lasts indefinitely when subsidies (i.e. guarantees) are granted; the term is of variable length in high-demand states; and franchise income in high-demand states is higher than in low-demand states.

Comparative statics

The difference between the cost of subsidies and the cost of public funds introduces a wedge that justifies having the firm bear risk. The firm bears more risk—which is reflected in a larger gap between M and m—when ζ is larger or the firm is less risk averse. Also, as I grows the risk premium demanded by a firm with decreasing absolute risk aversion increases, which explains why the thresholds m and M increase faster than I. The following proposition formalizes this intuition:

Proposition 6 (Comparative Statics) *Denote the probability density describing uncertainty about users' willingness to pay by $f(v)$, $v_{\min} \leq v \leq v_{\max}$, with c.d.f. $F(v)$. The statements below apply to the range of values of ζ and I for which $v_{\min} < m < M < v_{\max}$.*

Denote by $m(\zeta)$ the minimum income guarantee, and by $M(\zeta)$ the profit cap, both as a function of the inefficiency parameter ζ. Denote $CARA(c) \equiv -u''(c)/u'(c)$. We then have that the risk borne by the firm increases with the social cost of subsidies, ζ. Furthermore, $(\lambda\zeta - \alpha)(M'(\zeta) - m'(\zeta))/\lambda$ takes a value between $1/CARA(m-I)$ and $1/CARA(M-I)$.

Denote by $m(I)$ and $M(I)$ the thresholds that define the optimal contract as a function of the upfront investment I and let and $C(I) \equiv CARA(M-I)/CARA(m-I)$. We then have that m and M grow faster than I. Moreover, for a firm with decreasing absolute risk aversion, the wedge between M and m increases with I, while it does not depend on I for a firm with constant absolute risk aversion.

Proof: See the Appendix. ∎

4 Implementation

The informational requirements needed to implement the optimal contract might seem formidable. Somewhat surprisingly, this is not the case. We show next how to implement the optimal contract with a competitive auction when the planner does not know I nor the firms' degree of risk aversion.

4.1 High revenue projects

Consider first a high demand project, that is, a project that can pay its way with user fees in all states. Then an auction where the bidding variable is the total present value of user fee revenues (PVR) collected by the firm over the life of the contract, β, implements the optimal contract. This follows from noting that rents will be dissipated in a competitive auction, so that β will satisfy:

$$\int u(\beta - I) f(v) dv = u(0). \tag{18}$$

Hence the winning bid will be $\beta = I$, which corresponds to the optimal contract derived in the preceding section. Denote by $T(v)$ the time it takes for user fee revenue accumulated in state v to reach I. The contract term is shorter when demand is high, that is, when $T(v)$ is small. Users pay the firm the same amount in all states of nature and thus the firm faces no risk.[27] Furthermore, the planner can implement the optimal contract using a PVR auction even if she does not know I, $f(v)$ or the firm's degree of risk aversion. All the planner needs to know is that the project can finance itself in all states of demand, that is, that $v \geq I$ with probability one..[28]

4.2 Low revenue projects

Consider next a low demand project, that is, a project where the optimal contract involves subsidies in all demand states. A PVR auction will implement the optimal contract in this case as well, as long as the government subsidizes the difference between the winning bid and the present value of user fees collected. Informational requirements are small again, since the planner only needs to know that the project is low revenue, that is, that $v < I$ with probability one, and a means of verifying revenue in each state.

We summarize both cases so far as follows:

Proposition 7 *The optimal contract can be implemented with a PVR auction, or a simple extensions thereof, for both high and low demand projects. Furthermore, bidders reveal their (common) value of I in the auction and informational requirements are weak.* ∎

[27] Uncertainty in I, which may be important in some projects, cannot be eliminated with a variable term contract.
[28] This case is considered in Engel, Fischer and Galetovic (1997).

4.3 Evaluating Lowest Subsidy Auctions

Low demand projects are often auctioned to the firm that requires the smallest subsidy, that is, the planner sets a fixed franchise term T and a user fee P, and firms bid on the subsidy they require to build, operate and maintain the infrastructure. We assume that user fee revenue accrued by time t in state v is equal to $\gamma(t,v)v$, with γ strictly increasing in its first argument and $\lim_{t\to\infty}\gamma(t,v)=1$. Assuming a competitive auction, so that ex-ante rents are dissipated, the winning bid S then satisfies:

$$\int u\big(\gamma(T,v)v+S-I\big)f(v)dv=u(0),$$

which means that the firm will be forced to bear risk.[29] It follows that

$$S>I-\int\gamma(T,v)vf(v)dv.$$

and, since $\gamma(T,v)\leq 1$,

$$S>I-\mu_v,$$

where μ_v is the mean of $f(v)$.

By contrast, with a PVR auction the equilibrium outcome satisfies $S(v)=I-v$ and expected expenditures are equal to:

$$\mathrm{E}[S]=I-\mu_v.$$

The problem with the minimum subsidy auction is that it leads to subsidies that are the same in all states of demand, which forces the firm to bear risk. By contrast, the optimal contract has state-contingent subsidies that ensure that the firm bears no risk. This leads to the counterintuitive result according to which subsidies paid out on average when the winner is the firm that bids the lowest subsidy are higher than those paid out with a PVR auction. The firm is forced to bear risk in the latter case, therefore demanding higher revenue on average, and a higher subsidy.

Proposition 8 (Sub-optimality of Least Subsidy Auctions) *A minimum subsidy auction of a fixed term contract is not optimal. Furthermore, for low demand projects this auction does not minimize the average subsidy paid out by the government.*

[29]The only exception is when $\gamma(T,v)v$ does not depend on v, which is highly unlikely, since usually this expression is increasing in v.

4.4 General Case

Next we consider the case where the planner does not know whether the project is high, intermediate or low demand. We also assume that the planner does not know the degree of the firm's risk aversion. However, the planner knows the probability density $f(v)$. We show next how to implement the optimal contract via a simple competitive auction.

Proposition 9 (Optimality of two threshold auction) *The following two-threshold auction implements the optimal contract: The government announces the probability density that characterizes uncertainty on the expected discounted user fee revenue flow from the project, $f(v)$, and the parameter $\bar{\zeta}$ that summarizes the wedge between the shadow cost of public funds in general and subsidies in particular. Firms bid on the minimum income guarantee, m, and the cap on their user fee revenue, M. The firm that bids the lowest value of the scoring function*

$$W(M,m) = M(1-F(M)) + \int_0^M v f(v)dv + \bar{\zeta}\int_0^m (m-v)f(v)dv \tag{19}$$

wins the contract.

Proof: Denote by m^* and M^* the thresholds that solve the planner's problem, and let $W^* = W(M^*, m^*)$.[30] These thresholds satisfy the firms' participation constraint and therefore are among the bids that firms can make. A bid that leads to a larger value of the expression in (19) therefore cannot be the equilibrium outcome. And a combination of m and M that leads to a lower value cannot be the outcome either, for it would contradict optimality of m^* and M^*. ∎

What is the economic intuition underlying this result? First note that the planner's objective function does not involve the construction cost I, which the planner ignores. The objective function only depends on the probability distribution of the present value of revenue obtained in a contract that lasts indefinitely and the distortions associated with government expenditures, as summarized by $\bar{\zeta}$. By auctioning the project to the firm that maximizes the planner's objective function, the planner induces the firm to solve society's problem without knowing the cost of the project or the firms' degree of risk aversion.

In the case of a high demand project, the two-threshold auction described above is equivalent to a present-value-of-revenue auction. When all states are high demand, any bid with $M = I$ and $m \le I$ will win the auction. No subsidies are paid out in this case and the contract length decreases monotonically with demand. Similarly, in the case of a low demand project, any bid with $m = I$ and $M \ge I$ will win the contract, since this time the upper threshold is not

[30] This covers the cases of high and low demand projects, where one of the thresholds is not binding.

binding. In this case the two threshold auction reduces to the extension of the PVR auction described above. Yet the two threshold auction is considerably more general than a PVR auction, since it also can be used for intermediate demand projects or, more importantly, for projects where the planner does not know whether the project is low, intermediate or high demand.

5 Extensions

This section extends our results in two directions. We consider a price-responsive demand and show that, once prices are chosen optimally, the results we obtained with perfectly inelastic demand follow through. Second, we incorporate moral hazard, by assuming that demand depends on the effort of the firm, but effort is costly and must be rewarded. The planner now has two reasons for having the firm bear risk: to elicit effort to attract demand and to avoid costly subsidy financing. We show that the firm faces more risk than when only one of this motives is present. As the effect of effort decreases, the optimal contract tends to the contracts studied in previous sections.

5.1 Price-responsive demand

In this section we consider a price responsive demand, thereby incorporating allocative efficiency. We also allow for a cost of providing the service, beyond the upfront investment considered so far. The results obtained in the previous section extend naturally to this setting. Once prices are set optimally, the contract continues being a two-threshold contract. In high demand states, profits are capped by the upper threshold, with residual profits accruing to the government. By contrast, the government transfers resources to the firm in low demand states to ensure discounted profits attain the lower threshold. The optimal contract does not affect government finances in intermediate demand states where users discounted willingness to pay falls between both thresholds. Also, the optimal contract can be implemented via a two-threshold auction.

What is new in this extension, is that prices now must be set optimally. With price responsive demand, prices provide an additional margin the government can use to collect funds. The distortions created by this margin must be commensurate with the distortions created by alternative revenue collection options. In high demand states, where the contract term is finite, the pricing decision is substituting for conventional financing sources, and hence determined by the shadow cost of public funds, λ. By contrast, the public cost of subsidies, $\lambda\zeta$, is the relevant margin in low demand states, since the firm receives government subsidies in these states. Not surprisingly, for intermediate demand states the relevant shadow cost of funds lies between λ

and $\lambda\zeta$. We show how to exactly pin down this cost.

The planner's problem

There exists a continuum of observable and enforceable demand states, indexed by θ and described by a probability density $g(\theta)$. For tractability, we assume that the demand schedule becomes known immediately after the project is built and remains constant over time. The results that follow extend easily to the case where the demand schedule grows at an exogenous rate that may vary over time and with θ.[31]

Present discounted demand for the infrastructure service in state θ, as function of price P, is given by $Q(P,\theta)$. The discounted cost of producing Q units, $C(Q,\theta)$, is increasing and convex in Q. It follows that the firm's discounted cash flow is:

$$\Pi(P,\theta) \equiv PQ(P,\theta) - C(Q,\theta).$$

As before, the investment, I, and state-contingent subsides, $S(\theta)$, are paid out up-front and therefore not included in the above expression.

When the planner gives weight $\eta \geq 0$ to producer surplus, discounted welfare with price P in state θ is:

$$H(P,\eta,\theta) \equiv \mathrm{CS}(P,\theta) + \eta\Pi(P,\theta), \tag{20}$$

where $\mathrm{CS}(P,\theta)$ is consumer surplus. Again, welfare is net of the upfront investment and subsidies, so we refer to H as *net welfare*. We next use the function H to pose the planner's problem.

For every demand state θ, the planner chooses two prices, the user fee paid during the contract and the user fee collected by the government after the contract ends. These prices are denoted by $P^F(\theta)$ and $P^G(\theta)$, where the superscripts F and G stand for *firm* and *government*, respectively. The planner also sets the optimal contract length for each θ, so that a fraction $\gamma(\theta)$, of discounted profits accrues to the firm with the remainder being collected by the government. Therefore the planner chooses functions $P^F(\theta)$, $P^G(\theta)$, $\gamma(\theta)$ and $S(\theta)$, that solve:

$$\max \int [H(P^F(\theta), \alpha, \theta)\gamma(\theta) + H(P^G(\theta), \lambda, \theta)(1-\gamma(\theta)) - (\lambda\zeta - \alpha)S(\theta)]\, g(\theta)\, d\theta \tag{21}$$

$$\text{s.t.} \int u\left(\Pi(P^F(\theta), \theta)\gamma(\theta) + S(\theta) - I\right) g(\theta))\, d\theta = u(0), \tag{22}$$

$$0 \leq \gamma(\theta) \leq 1,$$

$$S(\theta) \geq 0.$$

[31] By contrast, the problem becomes considerably harder when demand is allowed to evolve arbitrarily over time.

The first term in the integrand of (21) is the planner's discounted welfare during the contract—the planner weighs profits generated by the infrastructure during this period by α, since these profits accrue to the firm. By contrast, the second term reflects welfare after the contract ends—during this period user fees are collected by the government and substitute for funds collected elsewhere in the economy, explaining why the planner's weight on profits now is λ. The third term in the objective function subtracts the cost of subsidies, which reflect the difference between social cost of one dollar of subsidy, $\lambda\zeta$, and the weight the planner gives to an additional dollar in the firms hands, α. The interpretation of the constraints is straightforward.

Optimal contract

What is new, compared to section 3, is that now the determination of optimal prices is not trivial anymore. The net welfare function defined above is useful to characterize these prices. We denote the user fee that maximizes $H(P,\eta,\theta)$ by $P^*(\eta,\theta)$ When $\eta = 1$, the function defined in (20) is the standard total welfare function, and $P^*(1,\theta)$, is often referred to as the *congestion fee*.

We assume that $P^*(\eta,\theta)$ increases with η for a fixed value of θ. That is, the price that maximizes the planner's net welfare function increases with the relative importance of producer's surplus.[32] From the first order condition that characterizes $P^*(\eta, v)$ we have:

$$\eta = -\frac{\mathrm{CS}_1(P^*(\eta,\theta),\theta)}{\Pi_1(P^*(\eta,\theta),\theta)}, \tag{23}$$

where CS_1 and Π_1 denote the partial derivatives of CS and Π with respect to P. As η grows, $P^*(\eta,\theta)$ approaches the monopoly price for state θ, denoted by $P^M(\theta)$. We also assume that $\Pi(P,\theta)$ is concave and strictly increasing in P in the range $[P^*(1,\theta), P^M(\theta)]$.

We consider first the optimal user fee after the contract ends. Since the infrastructure project generates no profits for the firm during this period, the planner chooses a price that creates a distortion commensurate with the social cost of raising funds elsewhere in the economy. Thus $P^G(\theta) = P^*(\lambda,\theta)$.

Next we turn to the optimal price during the contract. As in the case with infinitely inelastic demand, it is always more convenient to finance the firm with user fee revenue than with subsidies, so that

$$S(\theta) > 0 \implies T(\theta) = \infty.$$

It follows that in states where the contract length is finite, the relevant margin when choosing

[32]See Engel, Fischer and Galetovic (2001) for an example with congestion fees where this property is derived from first principles. Of course, demand needs to be relatively inelastic.

the optimal price is the planner's shadow cost of funds, λ. By contrast, when $S(\theta) > 0$, the relevant margin is the social cost of subsidies, $\lambda\zeta$.

Proposition 10 *If in state θ the contract ends in finite time, $P^F(\theta) = P^G(\theta) = P^*(\lambda,\theta)$. By contrast, if it lasts indefinitely and subsidies are paid out, then $P^F(\theta) = P^*(\lambda\zeta,\theta)$.*

Proof: See the Appendix. ∎

An argument similar to the one we gave in the case with infinitely inelastic demand, shows that there are three possible types of demand states. First, those where the contract length is positive and finite and there are no subsidies ($T < \infty, S = 0$); second, those where the term is infinite but there are no subsidies ($S = 0, T = \infty$); and finally, those where the franchise term is infinite and there are subsidies ($T = \infty, S > 0$). The first type is *high demand*, the second is *intermediate demand* and the third *low demand.* Also, the firm's profits are the same across high demand states, and across low demand states. As before, we denote these common values by M and m and have $M > m$.

By contrast with section 3, now users' willingness to pay varies (in a non-trivial way) with the price being charged. This is reflected by how states are classified into high, intermediate and low demand. State θ is high demand if:

$$\Pi(P^*(\lambda,\theta)) > M, \tag{24}$$

while it is low demand if

$$\Pi(P^*(\lambda\zeta, v)) < m. \tag{25}$$

Intermediate demand states then satisfy:

$$m \leq \Pi(P^*(\lambda,\theta)) < \Pi(P^*(\lambda\zeta,\theta)) \leq M. \tag{26}$$

The assumptions we made—that $P^*(\eta,\theta)$ is increases in η and $\Pi(P,\theta)$ is increasing and concave for $P \in (P^*(1,\theta), P^M(\theta))$—ensure that $\Pi(P^*(\lambda,\theta)) < \Pi(P^*(\lambda\zeta,\theta))$ and therefore the above taxonomy of states is well defined.

A high demand project is a project where in all demand states θ we have $\Pi(P^*(\lambda,\theta)) \geq I$. For these projects, the optimal contract provides full insurance, no subsidies are involved and the contract length is finite. The firm has discounted profits of I in all states, so that $M = I$ and m is irrelevant. All states are high demand and the optimal user fee in state θ is determined by the shadow cost of public funds and equal to $P^*(\lambda,\theta)$.

By contrast, the project is low demand when all states satisfy $\Pi(P^*(\lambda\zeta,\theta)) < I$. In this case the optimal contract also provides full insurance to the firm ($m = I$ and M is irrelevant) and government subsidies are involved in all demand states.

In between high and low demand projects are intermediate demand projects, which are characterized by having a state θ such that

$$\Pi(P^*(\lambda,\theta)) \; < \; I \; < \; \Pi(P^*(\lambda\zeta,\theta)).$$

We show shortly that the firm bears risk for these projects. Before doing so, we describe how P^F is chosen in intermediate demand states.

Proposition 11 *If θ satisfies (26), then $P^F = P^*(\eta,\theta)$ with $\eta \in (\lambda\,,\,\lambda\zeta)$ characterized by:*

$$\frac{\eta-\alpha}{\lambda\zeta-\alpha}u'(m-I) \;\; = \;\; u'\left(\Pi(P^*(\eta,v))-I\right) \;\; = \;\; \frac{\eta-\alpha}{\lambda-\alpha}u'(M-I). \tag{27}$$

Proof: See the Appendix. ∎

It is time to take stock:

Proposition 12 *For high and low demand projects, the firm receives profits equal to I in all demand states. The user fee is $P^*(\lambda,\theta)$ in the former case and $P^*(\lambda\zeta,\theta)$ in the latter.*

For intermediate demand projects, the optimal contract is characterized by thresholds m and M, with $m < M$, as follows:[33]

- *In states where $\Pi(P^*(\lambda,\theta)) > M$, the contract length is finite and the user fee is $P^*(\lambda,\theta)$ both during and after the contract. The firm's discounted profits are equal to M and the government collects funds in the amount of $\Pi(P^*(\lambda,v)) - M$.*

- *For states where $\Pi(P^*(\lambda\zeta,\theta)) < m$, the user fee is $P^*(\lambda\zeta)$, the contract lasts indefinitely, and the firm collects a subsidy in the amount needed to complete discounted profits of m.*

- *For states where $\Pi(P^*(\lambda,\theta)) \leq M$ and $\Pi(P^*(\lambda\zeta,\theta)) \geq m$, the optimal contract lasts indefinitely and no government subsidies are involved. The user fee $P^*(\eta(\theta),\theta)$ is determined by solving for η in:*

$$\frac{\eta-\alpha}{\lambda\zeta-\alpha}u'(m-I) \;\; = \;\; u'\left(\Pi(P^*(\eta,\theta))-I\right) \;\; = \;\; \frac{\eta-\alpha}{\lambda-\alpha}u'(M-I). \tag{28}$$

In section 3 we identified demand states with user's willingness to pay for the project, and therefore were able to set $v = \theta$. The results obtained in this section suggest that when demand

[33] As before, we assume $u'(v_{\min} - I) > \bar{\zeta}u'(v_{\max} - I)$. If this is not the case, then the optimal policy is described along the lines of footnote 25.

responds to prices, demand uncertainty can be conveniently summarized by the distribution of the flow profits generated by the project for two particular user fees: $P^*(\lambda)$ and $P^*(\lambda\zeta)$.

We denote the joint density of $\Pi(P^*(\lambda),\theta),\theta)$ and $\Pi(P^*(\lambda\zeta,\theta),\theta)$. by $f(w_\lambda, w_{\lambda\zeta})$, and the corresponding marginal c.d.f.s by $F_\lambda(w_\lambda)$ and $F_{\lambda\zeta}(w_{\lambda\zeta})$. Figure 1 depicts a partition of $(w_\lambda, w_{\lambda\zeta})$-space into high, intermediate and low demand states, for $m = 0.3$ and $M = 0.7$. Since $w_{\lambda\zeta}$ is always larger than w_λ, the joint density only has mass above the 45-degree line. The lower-left triangle depicts demand states where user fees add up to less than m in present value and subsidies are handed out. By contrast, user fee revenue in states in the upper-right triangle adds up to more than M and the government obtains revenue in these states.

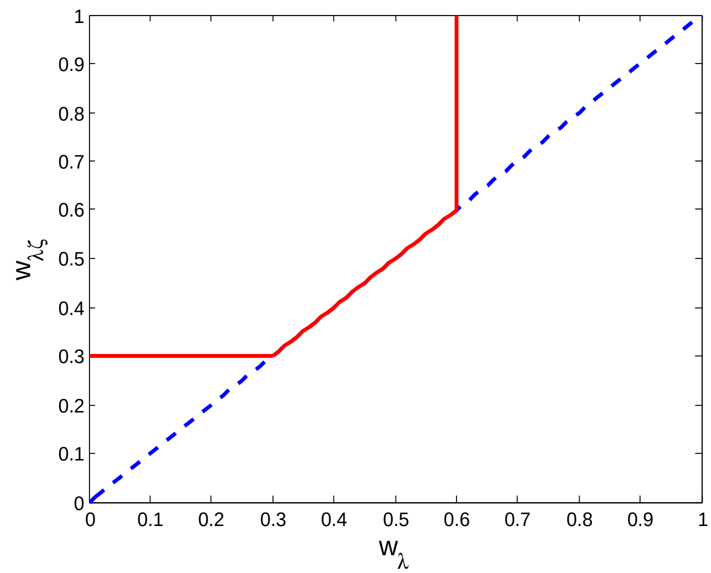

Figure 1: Partition of $(w_\lambda, w_{\lambda\zeta})$-space into high, intermediate and low demand states

The above characterization of uncertainty can be used to find m and M for an intermediate demand project:

Proposition 13 *For an intermediate demand project, m and M are characterized by the firm's participation constraint:*

$$F_{\lambda\zeta}(m)u(m-I)+\int_0^M\int_m^\infty u\left(\Pi(P^*(\eta(w_{\lambda\zeta},w_\lambda),w_{\lambda\zeta},w_\lambda))-I\right)f(w_{\lambda\zeta},w_\lambda)dw_{\lambda\zeta}dw_\lambda+(1-F_\lambda(M))u(M-I)=u(0), \tag{29}$$

and

$$u'(m-I)=\bar{\zeta}u'(M-I), \tag{30}$$

where $\bar{\zeta} = (\lambda\zeta - \alpha)/(\lambda - \alpha)$

Proof: The first expression is obtained from (22) and the fact that the optimal policy is of the two-threshold type. The second expression follows from (28). The appendix includes an alternative derivation of the second expression that provides additional insights. ∎

Implementation

The optimal contract can be implemented via a competitive auction. As before, the planner does not need to know the upfront cost of the project or the firms utility function. Firms bid on the lower and upper thresholds m and M and the contract is adjudicated to the firm that bids the highest value of aggregate welfare. Of course, aggregate welfare can be split up into the contribution of high, intermediate and low demand states, leading to:[34]

$$W(M,m) = \mathscr{W}_{\text{high}} + \mathscr{W}_{\text{int}} + \mathscr{W}_{\text{low}},$$

with

$$\begin{aligned}
\mathscr{W}_{\text{high}} &= \int_M^\infty [\text{CS}(w) + \alpha M + \lambda(w - M)]\, dF_\lambda(w), \\
\mathscr{W}_{\text{int}} &= \int_0^M \int_m^\infty \text{CS}((P^*(\eta(w_{\lambda\zeta}, w_\lambda), w_{\lambda\zeta}, w_\lambda)) f(w_{\lambda\zeta}, w_\lambda)\, dw_{\lambda\zeta}\, dw_\lambda, \\
\mathscr{W}_{\text{low}} &= \int_0^m [\text{CS}(w) + \alpha m + \lambda\zeta(w - M)]\, dF_{\lambda\zeta}(w).
\end{aligned}$$

Even though more information on demand is needed to set up the auction in this case, reasonably good approximations can be obtained if the government provides the distribution of the present value of profits under two particular set of use fees: those corresponding to the shadow cost of subsidies for the project, $P^*(\lambda\zeta)$, and those reflecting the shadow cost of funds elsewhere in the economy, $P^*(\lambda)$.

5.2 Moral hazard

We embed the model of Section 3 in a simple moral hazard framework. Costly effort, that affects users' willingness to pay, is exerted by the firm before demand is realized. The density $f(v, \epsilon)$ summarizes uncertainty over the present discounted value of user fee revenue when the firm

[34] See the Appendix for the derivation of the expressions that follow.

chooses effort level ϵ. We assume the monotone likelihood ratio property (MRLP) holds, so that

$$L(v,\epsilon) \equiv \frac{\frac{\partial f}{\partial \epsilon}(v,\epsilon)}{f(v,\epsilon)}$$

is increasing in v for all ϵ. That is, effort tilts demand toward higher realizations.

The utility of the firm, $U(y,\epsilon)$, is separable into net income and effort, so that $U(y,\epsilon) = u(y) - k\epsilon$, $k > 0$, where y denotes the present value of user fees collected by the firm and $\epsilon \geq 0$ the firm's effort. The problem facing the planner is to choose effort level ϵ, and revenue and subsidy schedules $R(v)$ and $S(v)$, to solve:

$$\min \int [(\lambda-\alpha)R(v) + (\lambda\zeta - \alpha)S(v) - \lambda v] f(v,\epsilon)dv + \alpha k\epsilon, \tag{31}$$

subject to

$$\begin{aligned} \int u(R(v)+S(v)-I)f(v,\epsilon)dv &\geq u(0)+k\epsilon, &(32)\\ \epsilon &= \text{argmax}_{\epsilon'} \int u(R(v)+S(v)-I)f(v,\epsilon')dv - k\epsilon', &(33)\\ 0 \leq R(v) &\leq v, &(34)\\ S(v) &\geq 0. &(35) \end{aligned}$$

Comparing (31) with (8) we see that two terms have been added to the planner's objective function. First, $\alpha k\epsilon$ appears because the planner weighs the firm's cost of exerting effort by α. Second, a term proportional to users' willingness to pay appears now, this term varies with effort and cannot be ignored anymore. Constraints (32) and (33) are the firm's participation and incentive compatibility constraints, respectively.

Under standard assumptions (e.g., strict concavity of the agent's utility as a function of ϵ) we can use the First Order Approach to examine the properties of the solution and it follows that the firm's incentive compatibility constraint can be replaced by:

$$\int u(R(v)+S(v)-I)L(v,\epsilon)f(v,\epsilon)dv = k. \tag{36}$$

Denoting by $\mu > 0$ and $\tau > 0$ the multipliers associated with (32) and (36), we have that the problem's Lagrangian is:

$$\begin{aligned}\mathscr{L} = &\int [(\lambda-\alpha)R(v) + (\lambda\zeta-\alpha)S(v) - \lambda v] f(v,\epsilon)dv + \alpha k\epsilon \\ &- \mu\left[\int u(R(v)+S(v)-I)f(v,\epsilon)dv - k\epsilon\right] - \tau \int u(R(v)+S(v)-I)L(v,\epsilon)f(v,\epsilon)dv.\end{aligned}$$

The first order condition w.r.t. to ϵ, combined with (36), provides an expression for τ:

$$\tau = \frac{\int [(\lambda-\alpha)R(v) + (\lambda\zeta-\alpha)S(v) - \lambda v]\, L(v,\epsilon) f(v,\epsilon) dv + \alpha k}{\int u(R(v)+S(v)-I)\frac{\partial^2 f}{\partial e^2}(v,\epsilon) dv}. \tag{37}$$

The Case $\zeta = 1$

The planner's problem described above shows that, in this case the distinction between user fees and revenue is irrelevant and the optimal policy can be described exclusively in terms of total revenue, $T(v) \equiv R(v) + S(v)$. As before, the Irrelevance Result holds, and the higher cost of public funds does not justify PPPs.

The FOC w.r.t. $T(v)$ leads to:

$$u'(T(v)-I) = \frac{\lambda-\alpha}{\mu+\tau L(v,\epsilon)}, \tag{38}$$

and the MLRP implies that $T(v)$ is strictly increasing in v.

In what follows we assume that:

$$G(v,\epsilon) = u'(v-I)[\mu+\tau L(v,\epsilon)]$$

is strictly decreasing in v for all feasible ϵ.[35] To derive this condition from first principles is not trivial, since μ and τ are multipliers that vary with the problem's parameters and, at least in principle, can take any positive value. Appendix C finds sufficient conditions for the problem's deep parameters, in the case of absolute risk aversion and an exponential distribution, under which $\partial G/\partial v < 0$.

Define by M via:

$$u'(M-I) = \frac{\lambda-\alpha}{\mu+\tau L(M,\epsilon)}.$$

It then follows that the firm collects less than v in states $v < M$ and more than v in states $v > M$. By contrast with the results obtained in Section 2, the firm's average revenue now is above I for two reasons: the firm must be compensated for exerting costly effort and now it must bear risk.

The Case $\zeta > 1$

The FOC with respect to $R(v)$ for a state v where the contract length is finite ('high demand' state) leads to:

$$u'(R(v)-I) = \frac{\lambda-\alpha}{\mu+\tau L(v,\epsilon)}, \tag{39}$$

[35] As discussed in Appendix C, all we really need is a weaker single-crossing condition.

while the FOC with respect to $S(v)$ for a state where subsidies are paid out ('low demand' state) yields

$$u'(v+S(v)-I)=\frac{\lambda\zeta-\alpha}{\mu+\tau L(v,\epsilon)}. \tag{40}$$

Denote by M and m the values of v that solve:

$$u'(v-I) = \frac{\lambda-\alpha}{\mu+\tau L(v,\epsilon)}, \tag{41}$$

$$u'(v-I) = \frac{\lambda\zeta-\alpha}{\mu+\tau L(v,\epsilon)}, \tag{42}$$

respectively. It then follows from $u''<0$, (39), (40), and the assumption that $G(v,\epsilon)$ is decreasing in v, that states with $v \geq M$ are high demand states, while states $v \leq m$ are low demand states. As before, values of v in between both thresholds correspond to intermediate demand states where the contract lasts indefinitely but the government budget is unaffected by the contract. The optimal contract continues being a two threshold contract, yet the firm's total revenue (user fees plus subsidies) is strictly increasing in v, while before it was constant across high (and low) demand states.

It follows from the MLRP and the definition of m and M that if v_H denotes a high demand state and v_L a low demand state, then:

$$u'(v_L-I) = \bar{\zeta}\frac{\mu+\tau L(v_H,\epsilon)}{\mu+\tau L(v_L,\epsilon)}u'(v_H-I). \tag{43}$$

When effort matters, the marginal utility gap between high- and low-demand states is larger than in the case without moral hazard. Even at the thresholds m and M, (43) implies that now

$$u'(m-I) > \bar{\zeta}u'(M-I),$$

while without effort we had an equality in the above expression (see (16)). Of course, the utility gap is even larger when a high and a low demand state that are not at the threshold are considered.

Before, the optimal contract had the firm bear risk to save on costly subsidies. Now the planner has an additional reason for having the firm bear risk: providing incentives to exert socially optimal levels of effort. The increase in the marginal utility gap between high and low demand states reflects this additional motive; the difference between M and m will generally be larger. For example, for CARA utility with coefficient of absolute risk aversion A:

$$M-m=\frac{1}{A}\log(\bar{\zeta})+\frac{1}{A}\log\left(\frac{\mu+\tau L(M,\epsilon)}{\mu+\tau L(m,\epsilon)}\right),$$

which, because of the MLRP, is larger than the corresponding expression when there are no incentives:

$$M - m = \frac{1}{A}\log(\bar{\zeta}).$$

It is time to take stock:

Proposition 14 *Consider the extension of the model in Section 3 where the distribution of users' willingness to pay increases with effort, and assume the MLRP holds. Then the optimal contract is characterized by two thresholds: M and m, with $M > m$. Demand states can be classified into high, intermediate and low demand, according to whether $v > M$, $m \leq v \leq M$ or $v < m$. In high demand states, the contract length is finite, no subsidies are paid to the firm and the value of $R(v)$ is determined from (39). In intermediate demand states the contract lasts indefinitely and the firm receives no subsidies.*[36] *Finally, in low demand states the contract lasts indefinitely and the firm receives the subsidy determined by (40).*

By contrast with the case without moral hazard, once effort matters the firm's total income increases monotonically with demand realization, even in low and high demand states. Both the minimum income guarantee and the cap on the firm's profits now are state contingent.

The optimal contract involves both a state dependent income guarantee in low demand states, and a state dependent threshold above which the government collects all user fees. We continue having that the contract length is state contingent in high demand states, and that the contract lasts longer in demand states where subsidies are handed out. Both of these characteristics are generally not observed in actual PPP contracts.

6 Conclusion

As the worldwide enthusiasm about privatizations waned, PPPs began to boom. One reason governments like PPPs is that they allow them to transfer assets to private firms without transferring ownership, thus avoiding criticism from those who oppose privatizations. But contrary to a privatization, most PPPs involve a risk sharing agreement, often specified in ambiguous terms, whereby both parties remain residual claimants, thus maintaining a direct link between the public budget and the infrastructure project.

This paper developed a simple analytical framework to study this link and its normative implications. Our model highlights that PPPs allow governments to make intertemporal transfers. A PPP may liberate public funds today, but it does so at the cost of fewer public revenues or

[36]Note that there are no intermediate demand states when there are no efficiency differences between the government and the private sector ($\zeta = 1$).

higher subsidies in the future. At the margin, the marginal sacrifice of resources invested in the project equals the opportunity cost of public funds—exactly as in the traditional model—. Clearly then, the advantages from PPPs are not financial.

We have also shown that a PPP is justified only if the private firm is productively more efficient. This should not be surprising after all, but it has many implications. One is that whenever government provision is more efficient there is no case for a PPP. Moreover, if PPP is warranted and able to pay its way, it should look like a privatization, in that no risks should be shared. Last, if a PPP is warranted but unable to pay its way completely out of user fees, concessions should last as long as possible so as to minimize the subsidy transfer to the firm.

To conclude, let us briefly discuss the place of public-private partnerships in fiscal policy and the public budget.

When a government runs an optimal fiscal policy, the size of the public sector is optimal: at the margin, the social return of resources invested in public projects is the same as those invested by the private sector. Now assume that one of these "public" projects is a PPP. What are its properties? According to our analysis, they can only be optimal if the private party is productively more efficient—otherwise the project should have been executed by the public sector. In addition, as the private party is more efficient in the use of resources, subsidies are minimized, if the project is awarded competitively and if the private firm is willing to do the project. This implies that the PPP should last indefinitely in those states of demand where a subsidy is necessary. If, on the other hand, no subsidies are required, risk is reduced by endogenously adjusting the term of the franchise and equalizing revenues in present value over different demand states. Finally, whether or not the project receives subsidies, it should pass a social cost-benefit evaluation ensuring that using the resources in the project is at least as valuable as using them in the marginal public or private project.

The important case when a PPP is able to fully pay its way deserves special mention, for it is closest to standard privatization, but nonetheless quite different. As we have seen, in this case the franchise holder should receive the same revenue in present value, regardless of the state of demand. Hence, the PPP adds no additional risk to the public intertemporal budget, compared to the case when the project is publicly executed. However, because the franchise eventually ends, the state receives revenues from the project and bears demand risk. For this reason, the link between the project and the public budget is never severed with a PPP. In standard privatization, by contrast, the public sector transfers all risk to the private sector against a one-time payment—there is no longer a link to the public budget.

How should PPPs be included in the budget? So far PPPs have largely sidestepped public budgets. One reason is that intertemporal budgetary accounting is still in its infancy. And perhaps many do not even view such accounting as necessary, given that it is still widely believed that PPPs are essentially a sort of imperfect privatization. In practice, these shortcomings are bound to cause trouble, because there is a variety of anecdotal evidence showing that governments have used PPPs to sidestep normal budgetary constraints. A PPP allows the current government to spend in infrastructure without including this expenditure in the current budget. Moreover, it allows the government to grant subsidies to be paid in the future—hence the popularity of minimum income guarantees.[37] Our results suggest that the risks associated with PPPs are a further reason to develop intertemporal budgets which could include them. At the margin, the sacrifice of resources invested in a PPP equals the opportunity cost of public funds, exactly as in the traditional model.

[37] See Engel et al (2006) for details.

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Appendix

A When is a project socially valuable?

A.1 Model

We define producer surplus as

$$PS(v) = R(v) + s(v) - I$$

and consumer surplus as

$$CS(v) = \mathscr{E}(v) + (\lambda - 1)\left[\beta v - R(v)\right] + \left[v - R(v) - \lambda\zeta S(v)\right].$$

$\mathscr{E}(v)$ is the externality generated by the project and β indicates what fraction of the private willingness to pay can be collected charging user fees. Hence

$$\begin{aligned} CS(v) + \alpha PS(v) &= \mathscr{E}(v) + (\lambda - 1)\left[\beta v - R(v)\right] + \left[v - R(v) - \lambda\zeta S(v)\right] + \alpha(R(v) + S(v) - I). \\ &= \mathscr{E}(v) + \left[(\lambda - 1)\beta + 1\right] v - (\lambda - \alpha)R(v) - (\lambda\zeta - \alpha)S(v) - \alpha I \end{aligned}$$

Let γv be the maximum fraction of consumer willingness to pay that can be transfered to the firm.[38]And let $\left\{R^*(v), S^*(v)\right\}$ solve

$$\max_{\{R(v),S(v)\}} \int \left[CS(v) + \alpha PS(v)\right] f(v)dv$$

$$\text{s.t.} \int u(R(v) + S(v) - I) f(v)dv \geq u(0),$$

$$0 \leq R(v) \leq v \cdot \min\left\{\beta, \gamma\right\}$$

$$S(v) \geq 0.$$

Then the project's social value is

$$SV \equiv \int \left[\mathscr{E}(v) + \left[(\lambda - 1)\beta + 1\right] v - (\lambda - \alpha)R^*(v) - (\lambda\zeta - \alpha)S^*(v) - \alpha I\right] f(v)dv. \tag{44}$$

We can now use (44) to explore when is $SV > 0$.

A.2 Social value of a project

A high-demand project In this case $I \leq v_{\min} \cdot \min\left\{\beta, \gamma\right\}$, $R^*(v) = I$ and $S^*(v) = 0$ for all v. Then a high-demand project is socially worthwhile if and only if

$$SV = \int \left[\mathscr{E}(v) + \left[(\lambda - 1)\beta + 1\right] v - \lambda I\right] f(v)dv \geq 0.$$

[38]This is to model, for example, a franchise with a maximum term (e.g. 50 years).

If, in addition $\beta = 1$, then the condition is

$$SV = \int \left[\mathscr{E}(v) + \lambda(v - I)\right] f(v) dv \geq 0. \tag{45}$$

Note that with a high-demand project, its social value does not depend on α. Also, because $\lambda > 1$, the social value of the project increases with β. Last, when $\beta = 1$ expression (45) has a simple interpretation: the social value of the project is the sum of the the private surplus $v - I$, augmented by the fact that this surplus allows the government to reduce distortionary taxation, and the externality, this summed over all states of nature.

A low-demand project In this case $I > v_{\max} \cdot \min\{\beta, \gamma\}$, $S^*(v) + R^*(v) = I$ and $R^*(v) = v \cdot \min\{\beta, \gamma\}$ for all states v. Hence, after some algebraic manipulation,

$$SV = \int \left[\mathscr{E}(v) + \left[(\lambda - 1)\beta + 1\right] v - \lambda I - \lambda(\zeta - 1) S^*(v)\right] f(v) dv \geq 0.$$

If, in addition, $\beta = 1$, then the condition is

$$SV = \int \left[\mathscr{E}(v) + \lambda(v - I) - \lambda(\zeta - 1) S^*(v)\right] f(v) dv \geq 0.$$

This expression is the same as before but for the fact that now the project must bear an additional cost in states where subsidies are paid—$\lambda(\zeta - 1) S^*(v)$. Also, because

$$S^*(v) = I - R^*(v) = I - v \cdot \min\{\beta, \gamma\}$$

the social value of the project is (locally) increasing in γ when $\min\{\beta, \gamma\} = \gamma$.

An intermediate-demand project In this case, $R^*(v) = M > I$, and $S^*(v) = 0$ in high-demand *states*. Hence, social surplus in such a state is

$$\mathscr{E}(v) + \left[(\lambda - 1)\beta + 1\right] v - (\lambda - \alpha) M - \alpha I$$

In intermediate-demand states $R^*(v) = v \cdot \min\{\beta, \gamma\}$ and $S^*(v) = 0$. Hence, social surplus is

$$\mathscr{E}(v) + \left[(\lambda - 1)\beta + 1\right] v - \lambda v \cdot \min\{\beta, \gamma\} + \alpha(v \cdot \min\{\beta, \gamma\} - I)$$

Last, in low-demand states $R^*(v) = v \cdot \min\{\beta, \gamma\}$ and $R^*(v) + S^*(v) = m < I$. Hence, social surplus is

$$\mathscr{E}(v) + \left[(\lambda - 1)\beta + 1\right] v - \lambda\left[R^*(v) + \zeta S^*(v)\right] + \alpha(m - I)$$

This implies that the condition is

$$
\begin{aligned}
SV = \int_{M}^{v_{\max}} & \left[\mathscr{E}(v) + \left[(\lambda-1)\beta+1\right] v - \lambda M - \alpha(M-I)\right] f(v)dv \\
& + \int_{m}^{M} \left[\mathscr{E}(v) + \left[(\lambda-1)\beta+1-\lambda\min\{\beta,\gamma\}\right] v + \alpha(v\cdot\min\{\beta,\gamma\}-I)\right] f(v)dv \\
& \quad + \int_{v_{\min}}^{m} \left[\mathscr{E}(v) + \left[(\lambda-1)\beta+1\right] v - \lambda m + \lambda(\zeta-1)S^*(v) + \alpha(m-I))\right] f(v)dv \geq 0.
\end{aligned}
$$

With $\beta = \gamma = 1$, it reduces to

$$
\begin{aligned}
SV = \int_{M}^{v_{\max}} & [\mathscr{E}(v) + \lambda(v-M) - \alpha(M-I)] f(v)dv + \int_{m}^{M} [\mathscr{E}(v) + \alpha(v-I)] f(v)dv \\
& + \int_{v_{\min}}^{m} \left[\mathscr{E}(v) - \lambda\zeta S^*(v) + \alpha(m-I))\right] f(v)dv \geq 0.
\end{aligned}
$$

B Proofs of Propositions

Proof of Propositions 1 and 2

Since u is concave, applying Jensen's inequality to the firm's participation constraint leads to

$$u(\int [R(v)+S(v)]f(v)dv - I) \geq \int u(R(v)+S(v)-I)f(v)dv = u(0).$$

And since u is strictly increasing, the above inequality implies that

$$\mathrm{E}[R] + \mathrm{E}[S] \geq I,$$

where $\mathrm{E}[R] = \int R(v)f(v)dv$ denotes the expected revenue before demand is realized and $\mathrm{E}[S]$ denotes expected government expenditure on subsidies.

It follows that if the solution to

$$
\begin{aligned}
\min_{R\geq 0, S\geq 0} \quad & (\lambda-\alpha)\mathrm{E}[R] + (\lambda\zeta-\alpha)\mathrm{E}[S] \\
s.t. \quad & \mathrm{E}[R] + \mathrm{E}[S] \geq I,
\end{aligned}
\tag{46}
$$

satisfies (2)–(4), then it solves the original planner's problem as well.

Hence, if $\zeta = 1$, any combination of revenue and subsidy schedules that satisfies (3), (4), and $R(v) + S(v) = I$ for all v, solves the planner's problem. And, when $\zeta < 1$, it is trivial to see that the optimal solution to (46) is $R \equiv 0$ and $S \equiv I$ which indeed satisfies (2)–(4). ∎

Proof of Proposition 5

Having established the form of the optimal contract, the planner's problem is equivalent to finding m and M that minimize

$$M(1-F(M))+\int_0^M vf(v)dv+\bar{\zeta}F(m)\int_0^m (m-v)f(v)dv, \tag{47}$$

subject to the firm's participation constraint (17). Noting that (17) implicitly defines M as a function of m, we have that:

$$M'(m)=-\frac{F(m)u'(m-I)}{(1-F(M))u'(M-I)}. \tag{48}$$

A similar calculation shows that the rate at which M and m have to change to keep the objective function (47) unchanged is given by

$$M'(m)=-\frac{\bar{\zeta}F(m)}{1-F(M)}. \tag{49}$$

Equating both expressions for $M'(m)$ leads to (16) and completes the proof.[39] ∎

Proof of Proposition 6

With the assumptions and notation introduced in the main text we prove that:

$$\begin{aligned}
M'(\zeta) &= \frac{\lambda F(m)}{(\lambda\zeta-\alpha)F(m)\mathrm{CARA}(M-I)+(\lambda-\alpha)(1-F(M))\mathrm{CARA}(m-I)},\\
m'(\zeta) &= -\frac{\lambda(\lambda-\alpha)(1-F(M))}{(\lambda\zeta-\alpha)[(\lambda\zeta-\alpha)F(m)\mathrm{CARA}(M-I)+(\lambda-\alpha)(1-F(M))\mathrm{CARA}(m-I)]},\\
M'(\zeta)-m'(\zeta) &= \frac{\lambda[(\lambda\zeta-\alpha)F(m)+(\lambda-\alpha)(1-F(M))]}{(\lambda\zeta-\alpha)[(\lambda\zeta-\alpha)F(m)\mathrm{CARA}(M-I)+(\lambda-\alpha)(1-F(M))\mathrm{CARA}(m-I)]}.
\end{aligned}$$

It follows that risk borne by the firm increases with the social cost of subsidies, ζ. Furthermore, $(\lambda\zeta-\alpha)(M'(\zeta)-m'(\zeta))/\lambda$ takes a value between $1/\mathrm{CARA}(m-I)$ and $1/\mathrm{CARA}(M-I)$.

We also show that:

$$\begin{aligned}
m'(I) &= 1+\frac{\bar{\zeta}C(I)\int_m^M u'(v-I)f(v)dv}{[\bar{\zeta}C(I)F(m)+1-F(M)]u'(m-I)},\\
M'(I) &= 1+\frac{\int_m^M u'(v-I)f(v)dv}{[\bar{\zeta}C(I)F(m)+1-F(M)]u'(M-I)},\\
M'(I)-m'(I) &= \bar{\zeta}(1-C(I))\frac{\int_m^M u'(v-I)f(v)dv}{[\bar{\zeta}C(I)F(m)+1-F(M)]u'(m-I)}.
\end{aligned}$$

It follows that m and M grow faster than I. Also, for a firm with decreasing absolute risk aversion, the wedge between M and m increases with I, while it does not depend on I for a firm with constant absolute risk aversion.

[39] The above proof assumes that $F(m)>0$ and $F(M)<1$. Footnote 25 outlines the proof when this is not the case.

Proof: Implicit differentiation of (16) with respect to ζ and a bit of algebra leads to:

$$M'(\zeta) = \frac{\lambda}{(\lambda\zeta - \alpha)\mathrm{CARA}(M-I)} + \frac{\mathrm{CARA}(m-I)}{\mathrm{CARA}(M-I)} m'(\zeta).$$

Implicitly differentiating (17) with respect to $\bar{\zeta}$ leads to:

$$M'(\zeta) = -\frac{(\lambda\zeta - \alpha)F(m)}{(\lambda - \alpha)(1 - F(M))} m'(\zeta).$$

Both expressions above lead to the comparative statics results for $\bar{\zeta}$.

Implicit differentiation of (16) with respect to I leads to:

$$\frac{m'(I) - 1}{M'(I) - 1} = \bar{\zeta} C(I).$$

Implicit differentiation of (17) with respect to I leads to:

$$F(m)u'(m-I)[m'(I) - 1] + \int_m^M u'(v-I) f(v)\,dv + (1 - F(M))u'(M-I)[M'(I) - 1] = 0.$$

The three comparative statics expressions in I now follow easily. ∎

Proof of Proposition 10

Proof: It follows immediately from the planner's objective function that $P^G(\theta) = P^*(\lambda,\theta)$ when $\gamma < 1$, that is, when the contract length is finite.

To derive the expressions for $P^F(\theta)$, consider first the case where the contract length is finite. We fix the firm's profits, and choose the price that maximizes the planner's welfare, that is, we solve:

$$\begin{aligned} \max_{P,\gamma} \quad & \gamma H(P,\alpha) + (1-\gamma) H^*(\lambda) \\ \text{s.t.} \quad & \gamma \Pi(P) = K, \end{aligned}$$

where we have dropped θ from our notation, $H^*(\lambda) \equiv H(P^*(\lambda))$ and P stands for P^F. Using the constraint to get rid of γ in the objective function leads to the following equivalent problem:

$$\max_P \frac{\mathrm{CS}(P) - H^*(\lambda)}{\Pi(P)}.$$

The corresponding first order condition leads to:

$$H^*(\lambda) = \mathrm{CS}(P) - \frac{\mathrm{CS}'(P)}{\Pi'(P)} \Pi(P)$$

and it follows from (23) that $P = P^*(\lambda)$ is optimal in this case.

Next we consider the case where $S > 0$ and maximize the planner's objective function over

P and S, keeping fixed the firm's total profits:

$$\begin{array}{ll} \max\limits_{P,S} & H(P,\alpha) - (\lambda\zeta - \alpha)S \\ \text{s.t.} & \Pi(P) + S = K. \end{array}$$

This time we use the constraint to get rid of S in the objective function, which leads to:

$$\max_{P} \; H(P,\alpha) + (\lambda\zeta - \alpha)\Pi(P),$$

which, by the definition of H, is equivalent to choosing the user fee that maximizes $H(P,\lambda\zeta)$. It follows that $P^F = P^*(\lambda\zeta)$ in this case. ∎

Proof of Proposition 11

We consider two intermediate demand states, θ_1 and θ_2, and find the optimal price in each state subject to a fixed expected utility for the firm. That is, we solve:

$$\begin{array}{ll} \max\limits_{P_1,P_2} & H(P_1,\alpha,\theta_1)f(\theta_1) + H(P_2,\alpha,\theta_2)f(\theta_2) \\ \text{s.t.} & u\,(\Pi(P_1,\theta) - I)\,f(\theta_1) + u\,(\Pi(P_2,\theta) - I)\,f(\theta_2) = K. \end{array}$$

The Lagrangian for this problem is

$$\mathscr{L}(P_1,P_2) = H(P_1,\alpha,\theta_1)f(\theta_1) + H(P_2,\alpha,\theta_2)f(\theta_2) + \mu[u_1' f(\theta_1) + u_2' f(\theta_2)],$$

where $u_i' = u(\Pi(P_i,v) - I)$, $i = 1,2$, and μ denotes the multiplier for the firm's participation constraint.

Using the first order conditions in P_1 and P_2 to get rid of μ then leads to:

$$\frac{u_1'}{u_2'} = \frac{\frac{\mathrm{CS}_1(P_1,\theta)}{\Pi_1(P_1,\theta_1)} + \alpha}{\frac{\mathrm{CS}_1(P_2,\theta_2)}{\Pi_1(P_1,\theta_1)} + \alpha}.$$

Define η_1 and η_2 via $P_1 = P^*(\eta_1,\theta_1)$ and $P_2 = P^*(\eta_2,\theta_2)$. Since θ_1 and θ_2 are intermediate demand states and $\Pi(p^*(\eta),\theta)$ is increasing in η, we have that $\eta_i \in (\lambda,\lambda\zeta)$, $i = 1,2$. The above expression combined with (23) implies that:

$$\frac{u_1'}{u_2'} = \frac{\eta_1 - \alpha}{\eta_2 - \alpha}.$$

A similar argument, with an intermediate and a low (high) demand state instead of two intermediate states, leads to the second (third) equality in (27). ∎

Proof of Proposition 13

We use Figure 1 to extend (47) and (48) to the more general setting considered here and in this way prove (30). We show that the planner substitutes m and M at a rate:

$$M'(m) = -\frac{\bar{\zeta} F_{\lambda\zeta}(m)}{1 - F_{\lambda}(M)}, \tag{50}$$

while the rate at which m and M are substituted along the firm's participation constraint satisfies:

$$M'(m) = -\frac{F_{\lambda\zeta}(m)\, u'(m-I)}{(1 - F_{\lambda}(M))\, u'(M-I)}. \tag{51}$$

Equating both rates of substitution leads to (30).

Consider the impact on the firm's participation constraint of an increase of m by Δm. Demand states than originally enjoyed a minimum income guarantee of m see this guarantee increase by Δm, thereby increasing the firm's expected utility by $F_{\lambda\zeta}(m)u'(m-I)\Delta m$. We also have a small fraction of states—those with $v_{\lambda\zeta} \in [m, m+\Delta m]$—that now have a guarantee and did not have one before. And the user-fee in these states is somewhat smaller once they have a minimum income guarantee. In any case, the contribution of these marginal states to the firm's expected utility is of second order in Δm and can therefore be ignored.

A similar argument shows that a decrease of M by ΔM leads to a decrease of the firm's expected utility of $(1 - F_{\lambda}(M) u'(M-I)\Delta M$, where again we ignore higher order terms in ΔM. Equating to zero the expected utility change associated with an increase in m and a decrease of M leads to (51).

To derive (50) we first use our two-threshold characterization of the optimal contract to simplify the planner's objective function (21). In high demand states we have $\gamma\Pi(P^*(\lambda)) = M$ and therefore

$$[\alpha\gamma + \lambda(1-\gamma)]\Pi = \lambda\Pi - (\lambda - \alpha)M.$$

We use this expression to get rid of γ in the expression for welfare in high demand states:

$$\mathscr{W}_{\text{high}} = \text{CS}(P^*(\lambda)) \;+\; \alpha M \;+\; \lambda(\Pi(P^*(\lambda)) - M). \tag{52}$$

In low demand states we have $\Pi + S = m$, which allows us to get rid of S in the planner's welfare function for these states:

$$\mathscr{W}_{\text{low}} = \text{CS}(P^*(\lambda\zeta)) \;+\; \alpha m \;+\; \lambda\zeta(\Pi(P^*(\lambda\zeta)) - m) \tag{53}$$

Finally, in intermediate demand states we have:

$$\mathscr{W}_{\text{int}} = \text{CS}(P^*(\eta)) \;+\; \alpha\Pi(P^*(\eta)), \tag{54}$$

with $\eta \in (\lambda, \lambda\zeta)$ determined based on Proposition 11.

Consider next the effect on total welfare of an increase of Δm in m and a decrease of ΔM in M. Comparing (52)–(54) it is clear that the change in welfare due to marginal firms—those close to m or M—is second order, since $\eta \simeq \lambda\zeta$ for firms with $w_{\lambda\zeta}$ close to m and $\eta \simeq \lambda$ for firms with w_{λ} close to M. It follows that, as in the previous case, the first order aggregate change in welfare is due to inframarginal low demand states and inframarginal high demand states. The subsidy paid out in the former states increases significantly, leading to a welfare reduction of $(\lambda\zeta-\alpha)F_{\lambda\zeta}(m)\Delta m$. And user fees freed up by the decrease in M allow the government to reduce distortions elsewhere in the economy, increasing welfare by $(\lambda-\alpha)(1-F_{\lambda}(M))\Delta M$. Equating to zero the total change in welfare leads to (50) and completes the proof. ∎

C Moral Hazard and Single-Crossing Property

The distribution of users' willingness to pay follows an exponential distribution with mean θ that increases with effort ϵ. The firm has constant risk aversion A. In this appendix we find conditions on θ, k and A so that the optimal contract derived in Section 5.2 is of the two threshold type.

The problem

We partition willingness to pay outcomes into three sets:

- $\mathcal{H}$: outcomes where it is optimal to have a finite contract term and therefore no subsidies,
- $\mathcal{L}$: outcomes where subsidies are called for and therefore the contract lasts indefinitely,
- $\mathcal{I}$: outcomes where the contract lasts indefinitely but no subsidies are involved.

With the notation introduced in Section 5.2, let:

$$G(v,\epsilon) = u'(v-I)[\mu+\tau L(v,\varepsilon)]. \tag{55}$$

The first order conditions (39) and (39) and $u''<0$ imply that

$$\begin{aligned}
\mathcal{H} &= \{v : G(v,\epsilon) < \lambda-\alpha\},\\
\mathcal{I} &= \{v : \lambda-\alpha \le G(v,\epsilon) \le \lambda\zeta-\alpha\},\\
\mathcal{L} &= \{v : G(v,\epsilon) > \lambda\zeta-\alpha\},
\end{aligned}$$

where ϵ is set equal to the value that maximizes the planner's objective function, which is assumed positive.

We want to show that there exist constants m and M, with $m<M$, such that $\mathcal{H}$, $\mathcal{I}$ and $\mathcal{L}$ are characterized by $v>M$, $m\le v\le M$ and $v<m$, respectively.[40] When $L\equiv 0$ in (55), this follows directly from $u''<0$. Yet once effort matters, we must show that, for all feasible values of ϵ we have that $G(v,\epsilon)$ crosses the horizontal lines $\lambda\zeta-\alpha$ and $\lambda-\alpha$ only once and from above. The problem is not trivial because μ and τ are multipliers that vary with the problem's parameters and, in principle, can take any positive values.

Basic setup

[40] Since we do not know the value of ϵ a priori, this must hold for all feasible values of ϵ.

The distribution of users' willingness to pay follows an exponential distribution with mean $\theta(\epsilon)$, with $\theta'(\epsilon) > 0$:

$$f(v,\epsilon) = \frac{1}{\theta}e^{-v/\theta}.$$

It follows that:

$$L(v,\epsilon) = \frac{\theta'}{\theta}\left[\frac{v}{\theta} - 1\right]$$

and therefore

$$\frac{\partial L(v,\epsilon)}{\partial v} = \frac{\theta'}{\theta^2} > 0.$$

and the MLRP holds.

Since the firm has constant risk aversion, denoted by A in what follows, the firm's participation and incentive compatibility constraints lead to:

$$\int u'(T(v) - I)f(v,\epsilon)dv = 1 - kA\epsilon, \tag{56}$$

$$\int u'(T(v) - I)L(v,\epsilon)f(v,\epsilon)dv = -kA. \tag{57}$$

(58)

A useful identity

The first order conditions(39) and (39) imply that, for all v we have:

$$\lambda - \alpha \leq u'(T(v) - I)[\mu + \tau L(v,\epsilon)] \leq \lambda\zeta - \alpha.$$

Integrating over v we then have:

$$\mu\int u'(T(v) - I)f(v,\epsilon)dv + \tau\int u'(T(v) - I)L(v,\epsilon)f(v,\epsilon)dv = C,$$

with $\lambda - \alpha \leq C \leq \lambda\zeta - \alpha$. Substituting (56) and (57) in the expression above leads to:

$$(1 - kA\epsilon)\mu = \tau kA + C. \tag{59}$$

Since mu, τ and C are positive, this expression implies that

$$\epsilon < \frac{1}{kA}.$$

Thus, as expected, optimal effort is smaller when the firm is more risk averse or the cost of effort is higher.

It also follows from (59) that:

$$\frac{\mu}{\tau} = \frac{kA}{1 - kA\epsilon} + \frac{C}{\tau(1 - 1 - kA\epsilon)},$$

and since $kA\epsilon < 1$ and $C > 0$, this implies that:

$$\frac{\mu}{\tau} > kA. \tag{60}$$

Sufficient condition

A straightforward calculation shows that:

$$\frac{\partial G}{\partial \epsilon}(v,\epsilon) = -e^{-A(v-I)}\left[A\mu - \tau\frac{\theta'}{\theta^2}(1 + A\theta) + A\tau\frac{\theta'}{\theta^2}v\right].$$

It follows that $G(v,\epsilon)$ is decreasing in v over the entire range of possible values if and only if

$$A\mu > \tau\frac{\theta'}{\theta^2}(1 + A\theta),$$

that is, if and only if

$$\frac{\mu}{\tau} > \frac{\theta'}{\theta}\left(1 + \frac{1}{A\theta}\right). \tag{61}$$

From (60) and (61) it follows that

$$kA > \frac{\theta'}{\theta}\left(1 + \frac{1}{A\theta}\right) \tag{62}$$

is sufficient to ensure that the optimal policy is of the two-threshold type.

We have shown the following

Proposition 15 *When the firm has constant risk aversion A and the distribution of users' willingness to pay, v, follows an exponential distribution with mean $\theta(\epsilon)$, $\theta' > 0$, we have that*

$$kA > \frac{\theta'}{\theta}\left(1 + \frac{1}{A\theta}\right)$$

ensures that there exist constants m and M, $m < M$, such that the contract length is finite and the government collects user fees from the project when $v > M$, the contract last indefinitely and involves subsidies for the firm when $v < m$, and the contract lasts indefinitely but involves no transfers from the government when $m \le v \le M$.

It follows from (62) that large values of k and A, or small values of θ' are needed to ensure that the single-crossing properties hold. This is consistent with the idea that effort matters, but not too much.