Equilibrium Tuition, Applications, Admissions and Enrollment in the College Market*

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Abstract

I develop and estimate a structural equilibrium model of the college market. Students, having heterogeneous abilities and preferences, make college application decisions, subject to uncertainty and application costs. Colleges, observing only noisy measures of student ability, choose tuition and admissions policies to compete for more able students. Tuition, applications, admissions and enrollment are joint outcomes from a subgame perfect Nash equilibrium. I estimate the structural parameters of the model using data from the National Longitudinal Survey of Youth 1997, via a three-step procedure to deal with potential multiple equilibria. In counterfactual experiments, I use the model first to examine the extent to which college enrollment can be increased by expanding the supply of colleges, and then to assess the importance of various measures of student ability.

Keywords: College market, tuition, applications, admissions, enrollment, discrete choice, market equilibrium, multiple equilibria, estimation

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1 Introduction

Both the level of college enrollment and the composition of college student bodies continue to be issues of widespread scholarly interest as well as the source of much public policy debate. In this paper, I develop and structurally estimate an equilibrium model of the college market. It provides insights into the determination of the population of college enrollees and permits quantitative evaluation of the effects of counterfactual changes in the features of the college market. The model interprets the allocation of students in the college market as an equilibrium outcome of a decentralized matching problem involving the entire population of colleges and potential applicants.[1] As a result, counterfactuals that directly involve only a subset of the college or student population can produce equilibrium effects for all market participants. My paper thus provides a mechanism for assessing the market equilibrium consequences of changes in government policies on higher education.

While the idea of modeling college matching as a market equilibrium problem is not new, this paper makes advances relative to the current literature by simultaneously modeling three aspects of the college market that are plausibly regarded as empirically important and incorporating them into the empirical analysis. The three aspects are: 1) Application is costly to the student. Besides application fees, a student has to spend time and effort gathering and processing information and preparing application materials. Moreover, she also incurs nontrivial psychic costs such as the anxiety felt while waiting for admissions results. 2) Students differ in their abilities and preferences for colleges.[2] 3) While trying to attract and select more able students, colleges can only observe noisy measures of student ability, such as student test scores and essays. As a result, both sides of the market face uncertainties: for the student, admissions are uncertain, which, together with the cost of application, leads to a non-trivial portfolio problem for her: how many and which, if any, colleges to apply to? For the college, the yield of each admission and the quality of a potential enrollee are both uncertain. The inference of these has to account for students' strategies. Colleges' policies are also interdependent because students' application portfolios and their enrollment depend on the policies of all colleges.

I model three stages of the market. First, colleges simultaneously announce their tuition. Second, students make application decisions and colleges simultaneously choose their admissions policies. Third, students make their enrollment decisions. My model incorporates tuition, applications, admissions and enrollment as joint outcomes from a subgame perfect Nash equilibrium (SPNE). SPNE in this model need not be unique. Multiplicity may arise

[1] In this paper, colleges refer to four-year colleges; students (potential applicants) refer to high school graduates.

[2] Throughout the paper, student ability refers to her readiness for college.

from two sources: 1) multiple common self-fulfilling expectations held by the student about admissions policies, and 2) the strategic interplay among colleges.[3]

To estimate the model with potentially multiple equilibria, I extend the estimation strategy of Moro (2003) and estimate the model in three steps.[4] The first two steps recover all the structural parameters involved in the application-admission subgame without having to impose any equilibrium selection rule. In particular, each application-admission equilibrium can be uniquely summarized in the set of probabilities of admission to each college for different types of students. The first step, using simulated maximum likelihood, treats these probabilities as parameters and estimates them along with fundamental student-side parameters in the student decision model, thereby identifying the equilibrium that generated the data. The second step, based on a simulated minimum distance estimation procedure, recovers the college-side parameters by imposing each college's optimal admissions policy. Step three recovers the remaining parameters by matching colleges' optimal tuition with the data tuition.

To implement the empirical analysis, I use data from the National Longitudinal Survey of Youth 1997 (NLSY97), which provides detailed information on student applications, admissions, financial aid and enrollment. Tuition information comes from the Integrated Postsecondary Education Data System.

Some of my major findings are as follows: first, students not only attach different values to the same college, but also rank various colleges and the non-college option differently. That is, there is not a single best college for all, nor is attending college better than the non-college option for all. As a result, my first counterfactual experiment finds that increasing the supply of colleges has very limited effect on college attendance. In particular, when the lower-ranked public colleges are expanded, at most 2.1% more students can be drawn into colleges, although the enlarged colleges adopt an open admissions policy and lower their tuition to almost zero. Therefore, neither tuition cost nor the number of available slots is a major obstacle to college access. A large group of students, mainly low-ability students, prefer the outside option over any of the college options.

Second, there are significant amounts of noise in various types of ability measures and different types of measures complement one another. In particular, relative to measures such

[3] Models with multiple equilibria do not have a unique reduced form and this indeterminacy poses practical estimation problems. In direct maximum likelihood estimation of such models, one should maximize the likelihood not only with respect to the structural parameters but also with respect to the types of equilibria that may have generated the data. The latter is a very complicated task and can make the estimation infeasible.

[4] Moro (2003) estimates a statistical discrimination model in which only one side of the market is strategic. I show how the extended strategy can be used to estimate a model in which both sides of the market are strategic, and hence, the second source of multiple equilibria arises.

as student essays, test scores (SAT) are more effective in distinguishing the lowest-ability students from the rest, but are less effective in singling out high-ability students.[5] My second counterfactual experiment assesses the importance of the non-test-score measures of student ability by eliminating them in the admissions process. Without such additional information, colleges draw on higher tuition to help screen students. Enrollee ability drops in all colleges, especially in the top college groups. Despite the increased tuition, the gain from being mixed with others outweighs the loss for the lowest-ability students. However, all the other students suffer and the average student welfare decreases by $\$1,325$. The highest-ability students suffer the most with a loss of about $\$5,000$.

The third counterfactual experiment examines the equilibrium impacts of dropping SAT in the admissions process, as urged by some critics. Among other reasons, these critics blame SAT for inhibiting the access to college education for students from low-income families, who typically have low test scores. As a result of dropping SAT, the fraction of low-income students does increase in top colleges. In particular, the mean family income among enrollees drop by over $\$10,000$ in top private colleges. However, this is accompanied by increased tuition and decreased enrollee ability in these colleges.

Although this paper is the first to estimate a market equilibrium model that incorporates tuition setting, applications, admissions and enrollment, it builds on various studies on similar topics. For example, Manski and Wise (1983) use nonstructural approaches to study various stages of the college admissions problem separately in a partial equilibrium framework. Most relevant to this paper, they find that applicants do not necessarily prefer the highest quality school.[6] Arcidiacono (2005) develops and estimates a structural model to address the effects of college admissions and financial aid rules on future earnings. In a dynamic framework, he models student's application, enrollment and choice of college major and links education decisions to future earnings.

While an extensive empirical literature focuses on student decisions, little research has examined the college market in an equilibrium framework. One exception is Epple, Romano and Sieg (2006). In their paper, students differ in family income and ability (perfectly measured by SAT) and make a single enrollment decision.[7] Taking, as given, its endowment and gross tuition level, each college chooses its financial aid and admissions policies to maximize the quality of education provided to its students. Their model provides an equilibrium

[5] In this paper, SAT is used as a generic term for tests such as SAT and ACT.

[6] Some examples of papers that focus on the role of race in college admissions include Bowen and Bok (1998), Kane (1998) and Light and Strayer (2002).

[7] In their paper, the application decision is not modeled. It is implicitly assumed that either application is not necessary for admission, or all students apply to all colleges. Accordingly, their empirical analysis is based on a sample of first-year college students.

characterization of colleges' pricing strategies, where colleges with higher endowments enjoy greater market power and provide higher-quality education. With complete information, no uncertainty and no unobserved heterogeneity, their model predicts that students with the same SAT and family income would have the same admission, financial aid and enrollment outcomes. The authors assume measurement errors in SAT and family income, which are found to be large in order to accommodate data variations.[8]

This paper departs from Epple, Romano and Sieg (2006) in several respects: 1) The college market is subject to information friction and uncertainty: colleges can only observe noisy measures of student ability, and they do not observe student preferences. As a result, colleges are faced with complex inference problems in making their admissions decisions. Meanwhile, application becomes a non-trivial problem for the student, as is manifested by the popularity of various application guide programs. Both colleges and students will adjust their behavior according to how much information is available on the market. Consequently, evaluating the severity of the information friction is important for predicting the equilibrium effects of various counterfactual education policies. 2) Student application decisions differ substantially. For example, over 50% of high school graduates do not apply to any college. However, the college market includes not only those who do apply, but all potential college applicants. Alternative education policies will affect not only where applicants are enrolled, but also who will apply at the first place. Therefore, to evaluate the effects of these policies, it is necessary to understand the application decisions (including non application) made by all students and how these decisions interact with colleges' decisions. 3) Students have different abilities and preferences for colleges, which are unobservable to econometricians. Arguably, such heterogeneity may be the key force underlying data variations unexplained by observables. Hence it is important to incorporate them in the model. As the first two structural papers that study college market equilibrium, Epple, Romano and Sieg (2006) and this paper complement one another: the former provides a more comprehensive view on colleges' pricing strategy, while the latter endogenizes student application as part of the equilibrium in a frictional market.

Theoretically, I build on the work by Chade, Lewis and Smith (2011), who model the decentralized matching of students and two colleges. Students, with heterogeneous abilities, make application decisions subject to application costs and noisy evaluations. Colleges compete for better students by setting admissions standards for student signals.[9] As part of its contribution, my paper quantifies the significance of the two key elements of Chade, Lewis

[8] The authors note that "the model may not capture some important aspects of admission and pricing." (page 911)

[9] Nagypál (2004) analyzes a model in which colleges know student types, but students themselves can only learn their type through normally distributed signals.

and Smith (2011): information friction and application costs. Moreover, I extend Chade, Lewis and Smith (2011) to account for some elements that are important, as acknowledged by the authors, to understand the real-world problem. On the student side, first, students are heterogeneous in their preferences for colleges as well as in their abilities, both of which are unknown to the colleges. Second, I allow for two noisy measures of student ability. One measure, as the signal in Chade, Lewis and Smith (2011), is subjective and its assessment is known only to the college. A typical example of this type of measure is the student essay. The other measure is the objective test score, which is known both to the student and to the colleges she applies to, and may be used strategically by the student in her applications.[10] Third, in addition to the admission uncertainty caused by noisy evaluations, students are subject to post-application shocks. These shocks incorporate new information for the student before she makes her enrollment decision. For example, the amount of financial aid she can obtain is not known with certainty upon application. Moreover, during the months between application and enrollment, a student may learn more about the colleges and she may also experience unexpected family and/or job prospects.[11] Empirically, these shocks are necessary to explain "seemingly sub-optimal" behaviors such as an applicant choosing not to attend any college after being admitted. On the college side, I model multiple colleges, which compete against each other via tuition as well as admissions policies.[12]

The rest of the paper is organized as follows: Section 2 lays out the model. Section 3 explains the estimation strategy, followed by a brief discussion of identification. Section 4 describes the data. Section 5 presents empirical results, including parameter estimates and model fit. Section 6 describes the counterfactual experiments. The last section concludes the paper. The appendix contains some details and additional tables.

2 Model

2.1 Primitives

This subsection lays out the environment of the college market that features costly application and incomplete information.

[10] For example, a low-ability student with a high SAT score may apply to top colleges to which she would not otherwise apply; a high-ability student with a low SAT score may apply less aggressively than she would otherwise.

[11] For enrollment in the fall semester starting from September, the typical application deadline is in January.

[12] As a price of these extensions, it is infeasible to obtain an analytical or graphical characterization of the equilibrium as in Chade, Lewis and Smith (2011).

2.1.1 Players

There are J colleges, indexed by $j = 1, 2, ...J$. In the following, J will also denote the set of colleges. A college's payoff depends on the total expected ability of its enrollees and its tuition revenue. To maximize its payoff, each college has the latitude to choose its tuition and admissions policies, subject to its fixed capacity constraint κ_j, where $\kappa_j > 0$ and $\sum_{j \in J} \kappa_j < 1$, the total measure of students.

There is a continuum of students, making college application and enrollment decisions. Students differ in their family backgrounds (B), SAT, abilities and preferences for colleges.[13] The various components of student characteristics are drawn from some joint distribution, unknown to the econometrician, who observes neither students' abilities nor their preferences.

2.1.2 Application Cost

Application is costly to the student. The cost of application, denoted as $C(\cdot)$, is a non-parametric function of the number of applications sent. $C(n+1) \geq C(n) > 0$, for any $n \in \{1, ..., J-1\}$.

2.1.3 Financial Aid

A student may obtain financial aid that helps to fund her attendance in any college, and she may also obtain college-specific financial aid. The amounts of various financial aid depend on the student's family background and SAT, via financial aid functions $f_j(B, SAT)$, for $j = 0, 1...J$, with 0 denoting the general aid and j denoting college j-specific aid.[14] In reality, although guidelines are available for students to calculate the expected financial aid she might obtain, the exact amount remains uncertain to her. To capture this uncertainty, I allow the final realizations to be subject to post-application shocks $\eta \in \mathbb{R}^{J+1}$. η is i.i.d. $N(0, \Omega_\eta)$, where Ω_η is a diagonal matrix with $\sigma^2_{\eta_j}$ denoting the variance of shock η_j. The realized financial aid for student i is given by

$$f_{ji} = \max\{f_j(B_i, SAT_i) + \eta_{ji}, 0\} \text{ for } j = 0, 1, ..J.$$

[13] SAT can be low(1), medium(2) or high(3).

[14] Ideally, a more complete model would endogenize tuition, applications, admissions, enrollment and financial aid. Unfortunately, this involves great complications that will make the empirical analysis intractable. As a compromise, Epple, Romano and Sieg (2006) abstract from application decisions and hence the effects of college policies on the pool of applicants, so that they can better focus on college's financial aid strategies. I carry out my analysis in a way that complements their work: I endogenize application decisions and allow colleges to choose gross tuition while leaving financial aid exogenous, which indirectly affects colleges' policies via its effects on students' application and enrollment decisions. The development of a modeling framework that combines the strengths of these two papers is a great challenge.

2.1.4 Student Preference

Student characteristics such as their abilities and preferences are unobservable to the econometrician. They are modeled as follows: students are of different types (T), where T is correlated with (SAT, B) and distributed according to $P(T|SAT, B)$. A student type T consists of both ability and non-ability related characteristics, with $T \equiv (A, Z)$. The first component, A, represents a student's ability, which can be low (1), medium (2) or high (3). Meanwhile, some students may prefer big (public) universities that offer greater diversity and a wider range of student activities; while some may prefer small (private) colleges where they can get more personal attention from professors. Such heterogeneity is captured by $Z \in \{1, 2\}$.

Students' preferences for colleges may differ systematically across types. In addition, each student may still have her own idiosyncratic tastes for colleges that are not representative of her type. For example, a student may prefer a particular type of colleges because her parents used to attend such colleges. To capture both the systematic and the idiosyncratic preference heterogeneities, students' preferences for colleges are modeled as a J-dimensional random vector drawn from $N(\overline{u}_T, \Omega_\epsilon)$, where $\overline{u}_T$ is the mean preference for colleges among type-T students and Ω_ϵ is a diagonal matrix with $\sigma^2_{\epsilon_j}$ denoting the variance of students' idiosyncratic tastes for college j. In this way, a student's preferences for different colleges are allowed to be correlated in a nonparametric fashion via her type-specific preference $\overline{u}_T$.

Given tuition profile $t \equiv \{t_j\}_{j=1}^{J}$, the ex-post value of attending college j for student i is given by

$$u_{ji}(t) = (-t_j + f_{0i} + f_{ji}) + (\overline{u}_{jT_i} + \epsilon_{ji}), \tag{1}$$

where t_j is tuition for attending college j. The first parenthesis of (1) summarizes student i's net monetary cost to attend college j. Her expected payoff, net of effort cost, is captured by: $\overline{u}_{jT_i}$, type T_i-specific preference, and ϵ_{ji}, her idiosyncratic taste for college j. That is, $\epsilon_i \tilde{} N(0, \Omega_\epsilon)$.

An outside option is always available to the student and its net expected value is normalized to zero. During the months between application and enrollment, a student may experience some unforeseen events that increase or decrease the value of her outside option. For example, she may receive a good job offer that dominates the option of attending college. Such uncertainties are captured by a post-application random shock ζ, which is i.i.d. $N(0, \sigma^2_\zeta)$, and the ex-post value of the outside option is $u_{0i} = \zeta_i$.

2.1.5 College Payoff

A college's payoff depends on the ability of its enrollees and its tuition revenue. The payoff for college $j \in J$ is given by:

$$\pi_j = \sum_{a=1}^{3} \omega_a n_{ja} + M(t_j; m_j) \sum_{a=1}^{3} n_{ja}, \tag{2}$$

where ω_a is the value of ability $A = a$, with $\omega_{a+1} > \omega_a$ and ω_1 normalized to 1. n_{ja} is the measure of j's enrollees with $A = a$. The first term in (2) is college j's total enrollee ability. The second term in (2) is college j's payoff from its total tuition revenue. The continuous function $M(\cdot; m_j)$ captures college j's preference for tuition relative to its preference for enrollee ability, which may be different across colleges via the parameter m_j.

2.1.6 Timing

First, colleges simultaneously announce tuition levels, to which they commit. Second, students make their application decisions, and all colleges simultaneously choose admissions policies. Finally, students learn about admission results and post-application shocks, and then make enrollment decisions.[15]

2.1.7 Information Structure

Upon student i's application, each college she applies to receives a signal $s \in \{1, 2, 3\}$ (low, medium, high) drawn from the distribution $P(s|A_i)$, the realization of which is known only to the college. For $A < A'$, $P(s|A')$ first order stochastically dominates $P(s|A)$.[16] Conditional on the student's ability, signals are i.i.d. across the colleges she applies to.

$P(s|A)$, the distributions of characteristics, preferences, payoff functions and financial aid functions are public information. An individual student's SAT score is known both to her and to the colleges she applies to. A student has private information about her type T, her idiosyncratic taste ϵ and her family background B.[17] To ease notation, let $X \equiv (T, B, \epsilon)$.

[15] This paper excludes early admissions, which is a very interesting and important game among top colleges. See, for example, Avery, Fairbanks and Zeckhauser (2003), and Avery and Levin (2010). For college applications in general, however, early admissions account for only a small fraction of the total applications. For example, in 2003, 17.7% of all four-year colleges offered early decision. In these colleges, the mean percentage of all applications received through early decision was 7.6%. *Admission Trends Survey* (2004), National Association for College Admission Counseling.

[16] That is, if $A < A'$, then for any $s \in \{1, 2, 3\}$, $\Pr(s' \leq s|A) \geq \Pr(s' \leq s|A')$.

[17] One might argue that neither party knows student ability. However, it is reasonable to assume that students have information advantages. As high school graduates, students have been evaluated repeatedly. Although these evaluations may be noisy, students eventually learn their ability by observing these signals over time. It is feasible and interesting to extend the current model to a case where both parties are uncertain

After application, the student observes her post-application shocks. The following table summarizes, in addition to the public information, what information is available to the student and college j when they make decisions.

Information Set

	Student	College j
Application-Admission	$SAT, X = (T, B, \epsilon)$	SAT, s_j
Enrollment	SAT, X, ζ, η	$-$

For any individual applicant, college j observes only her SAT and the signal she sends to j, which are the basis on which the college makes admissions decisions. For the student, the admission probability is a function of her SAT and ability (instead of signal), because she cannot observe her signal but her ability governs her signal distribution.

2.2 Applications, Admissions and Enrollment

In this subsection, I solve the student's problems backwards and the college's admissions problem, taking as given the tuition levels announced in the first stage of the game.

2.2.1 Enrollment Decision

Knowing her post-application shocks and admission results, student i chooses the best among her outside option and admissions on hand, i.e., $\max\{u_{0i,}\{u_{ji}(t)\}_{j\in O_i}\}$, where O_i denotes the set of colleges that have admitted student i. Let

$$v(O_i, X_i, SAT_i, \eta_i, \zeta_i|t) \equiv \max\{u_{0i,}\{u_{ji}(t)\}_{j\in O_i}\} \tag{3}$$

be the optimal ex-post value for student i, given admission set O_i; and denote the associated optimal enrollment strategy as $d(O_i, X_i, SAT_i, \eta_i, \zeta_i|t)$.

2.2.2 Application Decision

Given her admissions probability $p_j(A_i, SAT_i|t)$ to each college j, which depends on her ability and SAT, the value of application portfolio $Y \subseteq J$ for student i is

$$V(Y, X_i, SAT_i|t) \equiv \sum_{O\subseteq Y} \Pr(O|A_{i,}SAT_i, t)E\left[v(O, X_i, SAT_i, \eta_i, \zeta_i|t)\right] - C(|Y|), \tag{4}$$

about student ability. In this paper, I focus on the special case, which captures the main idea of information asymmetry of the type that I consider.

where the expectation is over shocks (η_i, ζ_i), and $|Y|$ is the size of portfolio Y.

$$\Pr(O|A_{i,}SAT_i, t) = \prod_{j \in O} p_j(A_i, SAT_i|t) \prod_{k \in Y \setminus O} (1 - p_k(A_i, SAT_i|t))$$

is the probability that the set of colleges $O \subseteq Y$ admit student i. The student's application problem is

$$\max_{Y \subseteq J} \{V(Y, X_i, SAT_i|t)\}. \tag{5}$$

Let the optimal application strategy be $Y(X_i, SAT_i|t)$.

2.2.3 Admissions Policy

Given tuition, a college chooses its admissions policy to maximize its expected payoff, subject to its capacity constraint. Its optimal admissions policy must be a best response to other colleges' admissions policies while accounting for students' strategic behavior. In particular, observing only signals and SAT scores of its applicants, the college has to infer: first, the probability that a certain applicant will accept its admission, and second, the expected ability of this applicant *conditional on her acceptance of the admission*, both of which depend on the strategies of all other players.[18] For example, whether or not a student will accept college j's admission depends on whether she also applies to other colleges (which is unknown to college j), and if so, whether or not she will be accepted by each of those colleges. In addition, college j needs to integrate out the post-application shocks that may occur to the student.

In this paragraph, I describe how an optimal admissions policy is implemented. The rest of Section 2.2.3 formalizes the problem, which can be skipped by readers not interested in the details. To implement its admissions policy, college j will first rank its applicants with different (s, SAT) by their expected ability conditional on their acceptance of j's admissions. All applicants with the same (s, SAT) are identical to the college and hence are treated equally. Everyone in an (s, SAT) group will be admitted if 1) this (s, SAT) group is ranked highest among the groups whose admissions are still to be decided, 2) their marginal contribution to the college is positive, and 3) the expected enrollment of this group is no larger than college j's remaining capacity, where j's remaining capacity equals κ_j minus the sum of expected enrollment of groups ranked above. A random fraction of an (s, SAT) group is admitted if 1) and 2) hold but 3) fails, where the fraction equals the remaining capacity divided by the expected enrollment of this group. As a result, a typical set of admissions policies for the

[18] Conditioning on acceptance is necessary to make a correct inference about the student's ability because of the potential "winner's curse": the student might accept college j's admission because she is of low ability and is rejected by other colleges.

ranked (s, SAT) groups, $\{e_j(s, SAT|t)\}$, would be $\{1, ..., 1, \varepsilon, 0, ..., 0\}$, with $\varepsilon \in (0,1)$ if the capacity constraint is binding, and $\{1, ..., 1\}$ if the capacity constraint is not binding or just binding.

The following formally derives a college's optimal admissions policy, readers not interested in the details can skip to Section 2.2.4. Given tuition profile t, students' strategies $Y(\cdot), d(\cdot)$ and other colleges' admissions policies e_{-j}, college j solves the following problem:

$$\begin{aligned} &\max_{e_j(\cdot|t)}\{\sum_{s,SAT} e_j(s, SAT|t)\alpha_j(s, SAT|t, e_{-j}, Y, d)\gamma_j(s, SAT|\cdot)\mu_j(s, SAT|\cdot) \\ &\qquad + M(t_j; m_j)\sum_{s,SAT} e_j(s, SAT|t)\alpha_j(s, SAT|\cdot)\mu_j(s, SAT|\cdot)\} \\ &s.t. \sum_{s,SAT} e_j(s, SAT|t)\alpha_j(s, SAT|\cdot)\mu_j(s, SAT|\cdot) \leq \kappa_j \\ &\qquad e_j(s, SAT|t) \in [0,1], \end{aligned} \tag{6}$$

where $e_j(s, SAT|t)$ is college j's admissions policy for its applicants with (s, SAT), $\alpha_j(s, SAT|t, e_{-j}, Y, d)$ is the probability that such an applicant will accept college j's admission, $\gamma_j(s, SAT|t, e_{-j}, Y, d)$ is the expected ability of such an applicant conditional on her accepting j's admission, and $\mu_j(s, SAT|t, e_{-j}, Y, d)$ is the measure of j's applicants with (s, SAT).[19] Therefore, the first line of (6) is college j's expected total enrollee ability, and the second line is its expected total tuition revenue. The first order condition for problem (6) is

$$\gamma_j(s, SAT|\cdot) + M(t_j; m_j) - \lambda_j + \nu_0 - \nu_1 = 0,$$

where λ_j is the multiplier associated with capacity constraint, i.e., the shadow price of a slot in college j. ν_0 and ν_1 are adjusted multipliers associated with the constraint that $e_j(s, SAT|t) \in [0,1]$.[20]

If it admits an applicant with (s, SAT) and the applicant accepts the admission, college j must surrender a slot from its limited capacity, thus inducing the marginal cost λ_j. The marginal benefit is the expected ability of such an applicant conditional on her accepting j's admission plus her tuition contribution. Balancing between the marginal benefit and the marginal cost, the solution to college j's admissions problem is characterized by:

$$e_j(s, SAT|t) \begin{cases} = 1 & \text{if } \gamma_j(s, SAT|\cdot) + M(t_j; m_j) - \lambda_j > 0 \\ = 0 & \text{if } \gamma_j(s, SAT|\cdot) + M(t_j; m_j) - \lambda_j < 0 \\ \in [0,1] & \text{if } \gamma_j(s, SAT|\cdot) + M(t_j; m_j) - \lambda_j = 0 \end{cases}, \tag{7}$$

[19] Appendix A1 provides details on how to calculate $\alpha_j(\cdot)$ and $\gamma_j(\cdot)$.

[20] ν_0, ν_1 are the multipliers associated with $\alpha_j(s, SAT|\cdot)\mu_j(s, SAT|\cdot)e_j(s, SAT|t) \in [0,1]$.

$$\sum_{s,SAT} e_j(s,SAT|t)\alpha_j(s,SAT|\cdot)\mu_j(s,SAT|\cdot) \leq \kappa_j, \tag{8}$$

and

$$\lambda_j \begin{cases} \geq 0 \text{ if } (8) \text{ is binding} \\ = 0 \text{ if } (8) \text{ is not binding} \end{cases}.$$

2.2.4 Link Among Various Players

The probability of admission to each college for different (A, SAT) groups of students, $\{p_j(A, SAT|t)\}$, summarizes the link among various players. Knowledge of p makes the information about admissions policies $\{e_j(s, SAT|t)\}$ redundant. Students' application decisions are based on p. Likewise, based on p_{-j}, college j can make inferences about its applicants and therefore choose its admissions policy. The relationship between p and e is given by:[21]

$$p_j(A, SAT|t) = \sum_s P(s|A)e_j(s, SAT|t). \tag{9}$$

2.2.5 Application-Admission Equilibrium

Definition 1 *Given tuition profile t, an application-admission equilibrium, denoted as $AE(t)$, is $(d(\cdot|t), Y(\cdot|t), e(\cdot|t))$, such that*
(a) $d(O, X, SAT, \eta, \zeta|t)$ is an optimal enrollment decision for every (O, X, SAT, η, ζ);
(b) Given $e(\cdot|t)$, $Y(X, SAT|t)$ is an optimal college application portfolio for every (X, SAT), i.e., solves problem (5);
(c) For every j, given $(d(\cdot|t), Y(\cdot|t), e_{-j}(\cdot|t))$, $e_j(\cdot|t)$ is an optimal admissions policy for college j, i.e., solves problem (6).

2.3 Tuition Policy

Before the application season begins, colleges simultaneously announce their tuition levels, understanding that their announcements are binding and will affect the application-admission subgame. Although from the econometrician's point of view the subsequent game could admit multiple equilibria, I assume that the players agree on the equilibrium selection rule.[22] Let $E(\pi_j|AE(t))$ be college j's expected payoff under $AE(t)$. Given t_{-j} and the

[21] The role of p as the link among players and the mapping (9) are of great importance in the estimation strategy to be specified later.

[22] The way in which the equilibrium selection rule is reached is beyond the scope of this paper.

equilibrium profiles $AE(\cdot)$ in the following subgame, college j's problem is

$$\max_{t'_j \geq 0}\{E\left(\pi_j | AE(t'_j, t_{-j})\right)\}. \tag{10}$$

Independent of its preference on tuition, each college considers the strategic role of its tuition in the subsequent $AE(t'_j, t_{-j})$. On the one hand, low tuition makes the college more attractive to students and more competitive in the market. On the other hand, high tuition serves as a screening tool and leads to a better pool of applicants if high-ability students are less sensitive to tuition than low-ability students.[23] Together with the monetary incentives of tuition revenue, such trade-offs determine the college's optimal tuition level.

2.4 Subgame Perfect Nash Equilibrium

Definition 2 *A subgame perfect Nash equilibrium for the college market is* $(t^*, d(\cdot|\cdot), Y(\cdot|\cdot), e(\cdot|\cdot))$ *such that:*
(a) For every t, $(d(\cdot|t), Y(\cdot|t), e(\cdot|t))$ *constitutes an* $AE(t)$*, according to Definition 1*;
(b) For every j*, given* t^*_{-j}, t^*_j *is optimal for college* j*, i.e., solves problem* (10) .

In the appendix, I show the existence of equilibrium for a simplified version of the model with two colleges. Numerically, I have found equilibrium in the full model throughout my empirical analyses.

3 Estimation Strategy and Identification

3.1 Estimating the Application-Admission Subgame

First, I fix the tuition profile at its equilibrium (data) level and estimate the parameters that govern the application-admission subgame. To save notation, I suppress the dependence of endogenous objects on tuition.

The estimation is complicated by potential multiple equilibria in the subgame and the fact that econometricians do not observe the equilibrium selection rule.[24] One way to deal with this complication is to impose some equilibrium selection rule assumed to have been used by the players and to consider only the selected equilibrium. However, for models like the

[23] This is a possible scenario. However, in the estimation, I do not impose any restriction on the relationship between student ability and their sensitivity to prices.

[24] The problem of possible multiple equilibria is a difficult, yet frequent problem in structual equilibrium models. For example, the model by Epple, Romano and Sieg (2006) also admits multiple equilibria, and the authors assume unique equilibrium in their estimation and other empirical analyses.

one in this paper, there is not a single compelling selection rule (from the econometrician's point of view).[25] I use a two-step strategy to estimate the application-admission subgame without having to impose any equilibrium selection rule.

Each application-admission equilibrium is uniquely summarized in the admissions probabilities $\{p_j(A, SAT)\}$, which provide sufficient information for players to make their unique optimal decisions. In the student decision model, $\{p_j(A, SAT)\}$ are taken as given just like all the other parameters are. Step One treats $\{p_j(A, SAT)\}$ as parameters and estimates them along with structural student-side parameters, thereby identifying the equilibrium that generated the data.[26] Step two imposes colleges' optimal admissions policies, which yield a new set of admissions probabilities. Under the true college-side parameters, these probabilities should match the equilibrium admissions probabilities estimated in the first step.

3.1.1 Step One: Estimate Fundamental Student-Side Parameters and Equilibrium Admissions Probabilities

I implement the first step via simulated maximum likelihood estimation (SMLE): together with estimates of the fundamental student-side parameters $\left(\widehat{\Theta}_0\right)$, the estimated equilibrium admissions probabilities $\widehat{p}$ should maximize the probability of the observed outcomes of applications, admissions, financial aid and enrollment, conditional on observable student characteristics, i.e., $\{(Y_i, O_i, f_i, d_i | SAT_i, B_i)\}_i$. Θ_0 is composed of 1) preference parameters $\Theta_{0u} = [\{\overline{u}_j(T)\}, \{\sigma_{\epsilon_j}\}]'$, 2) application cost parameters $\Theta_{0C} = [C(1), ..., C(J)]'$, 3) financial aid parameters Θ_{0f}, 4) the standard deviation of the shock to the outside option $\Theta_{0\zeta} = \sigma_\zeta$ and 5) the parameters involved in the distribution of types Θ_{0T}.

Suppose student i is of type T. Her contribution to the likelihood, denoted by
$L_{iT}(\Theta_{0u}, \Theta_{0C}, \Theta_{0f}, \Theta_{0\zeta}, p)$, is composed of the following parts:
$L_{iT}^{Y}(\Theta_{0u}, \Theta_{0C}, \Theta_{0f}, \Theta_{0\zeta}, p)-$ the contribution of applications Y_i,
$L_{iT}^{O}(p)-$ the contribution of admissions $O_i|Y_i$,
$L_{iT}^{f}(\Theta_{0f})-$ the contribution of financial aid $f_i|O_i$, and
$L_{iT}^{d}(\Theta_{0u}, \Theta_{0f}, \Theta_{0\zeta})-$ the contribution of enrollment $d_i|(O_i, f_i)$.

[25] See, for example, Mailath, Okuno-Fujiwara and Postlewaite (1993), who question the logical foundations and performances of many popular equilibrium selection rules.

[26] Given admissions probabilities, students' application strategies are independent, which yields a unique equilibrium in the student-side problem. This may not hold if students directly value the quality of their peers. With peer effects, multiple equilibria may coexist in both the student-side and the college-side problem, inducing substantial complications into the model. The existence of peer effects has been controversial in the higher-education literature. (See, for example, Sacerdote (2001), Zimmerman (2003), Arcidiacono and Nicholson (2005) and Dale and Krueger (1998)). In this paper, I focus on the interactions between colleges and students and the competition among colleges, leaving the inclusion of interactions among students for future research.

Hence,

$$L_{iT}(\cdot) = L_{iT}^{Y}(\cdot)L_{iT}^{O}(\cdot)L_{iT}^{f}(\cdot)L_{iT}^{d}(\cdot).$$

Now, I will specify each part in detail. Conditional on (T, SAT_i, B_i), there are no unobservables involved in the probabilities of $O_i|Y_i$ and $f_i|O_i$. The probability of $O_i|Y_i$ depends only on ability and SAT, and is given by

$$L_{iT}^{O}(p) \equiv \Pr(O_i|Y_i, A, SAT_i) = \prod_{j\in O_i} p_j(A, SAT_i) \prod_{k\in Y_i\backslash O_i} [1 - p_k(A, SAT_i)].$$

Let $J_i^f \subseteq \{0, O_i\}$ be the sources of observed financial aid for student i, where 0 denotes general aid. The probability of the observed financial aid depends only on SAT and family background:

$$\begin{aligned} L_{iT}^{f}(\Theta_{0f}) &\equiv \Pr(f_i|O_i, SAT_i, B_i) \\ &= \begin{cases} \prod_{j\in J_i^f} \phi(\frac{f_{ji}-f_j(SAT_i,B_i)}{\sigma_{\eta_j}})^{I(f_{ji}>0)}\Phi(\frac{-f_j(SAT_i,B_i)}{\sigma_{\eta_j}})^{I(f_{ji}=0)} & \text{if } J_i^f \neq \emptyset \\ 1 & \text{otherwise} \end{cases}, \end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal density and cumulative distribution, respectively, and $I(\cdot)$ is the indicator function.

The choices of Y_i and $d_i|(O_i, f_i)$ both depend on the unobserved idiosyncratic tastes ϵ. Let $G(\epsilon, \zeta, \{\eta_j\}_{j\in\{0,O_i\}\backslash J_i^f})$ be the joint distribution of idiosyncratic taste, outside option shock and unobserved financial aid shocks,

$$\begin{aligned} & L_{iT}^{Y}(\Theta_{0u}, \Theta_{0C}, \Theta_{0f}, \Theta_{0\zeta}, p)L_{iT}^{d}(\Theta_{0u}, \Theta_{0f}, \Theta_{0\zeta}) \equiv \\ & \int I(Y_i|T, SAT_i, B_i, \epsilon)I(d_i|O_i, T, SAT_i, B_i, \epsilon, \zeta, \{\eta_j\}_{j\in\{0,O_i\}\backslash J_i^f}, \{f_{ji}\}_{j\in J_i^f}) \\ & \quad dG(\epsilon, \zeta, \{\eta_j\}_{j\in\{0,O_i\}\backslash J_i^f}). \end{aligned}$$

The multi-dimensional integration has no closed-form solution and is approximated by a kernel smoothed frequency simulator (McFadden (1989)).[27]

To obtain the likelihood contribution of student i, I integrate over the unobserved type:

$$L_i(\Theta_0, p) = \sum_{T} P(T|SAT_i, B_i; \Theta_{0T})L_{iT}(\Theta_{0u}, \Theta_{0C}, \Theta_{0f}, \Theta_{0\zeta}, p). \tag{11}$$

[27] See the appendix for details.

Finally, the log likelihood for the entire random sample is

$$\widetilde{L}(\Theta_0, p) = \sum_i \ln(L_i(\Theta_0, p)). \tag{12}$$

3.1.2 Step Two: Estimate College-Side Parameters

The college-side parameters to be estimated in Step Two, denoted Θ_2, are signal distribution $P(s|A)$, capacity constraints κ and values of abilities ω. These are estimated via simulated minimum distance estimation (SMDE). Based on $\widehat{\Theta}_0$, I simulate a population of students and obtain their optimal application and enrollment strategies under $\widehat{p}$. The resulting equilibrium enrollment in each college group should equal its expected capacity.[28] These equilibrium enrollments, together with $\widehat{p}$, serve as targets to be matched in the second-step estimation.

The estimation explores each college's optimal admissions policy: taking student strategies and $\widehat{p}_{-j}$ as given, college j chooses its admissions policy e_j. This leads to the admissions probability to college j for each (A, SAT) type, according to equation (9). Ideally, the admissions probabilities derived from Step Two should match $\widehat{p}$ from Step One, and the capacity parameters in Step Two should match equilibrium enrollments. The estimates of the college-side parameters minimize the weighted sum of the discrepancies. Let $\widehat{\Theta}_1 = [\widehat{\Theta}_0', \widehat{p}']'$; the objective function in Step Two is

$$\min_{\Theta_2} \{q(\widehat{\Theta}_1, \Theta_2)' \widehat{W} q(\widehat{\Theta}_1, \Theta_2)\}, \tag{13}$$

where $q(\cdot)$ is the vector of the discrepancies mentioned above, and $\widehat{W}$ is an estimate of the optimal weighting matrix. The choice of W takes into account that $q(\cdot)$ is a function of $\widehat{\Theta}_1$, which are point estimates with variances and covariances.[29]

3.2 Step Three: Tuition Preference

Given other colleges' equilibrium (data) tuition t^*_{-j}, I solve college j's tuition problem (10).[30] Under the true tuition preference parameters m, the optimal solution should match the

[28] It is implicitly assumed that the tuition weights m are such that, at the data tuition level, the marginal benefit from admitting a student is non-negative, i.e., $\gamma_j(s, SAT|\cdot) + M(t_j; m_j) \geq 0$ for any (s, SAT). Under this assumption, capacity constraints are binding in the realized equilibrium because the data admissions rates are below 100% for all college groups.

[29] The standard errors of the parameter estimates in the second step and the third step account for the estimation errors in the previous step(s).

[30] See Appendix C3 for details.

tuition data.[31] The objective in Step Three is

$$\min_m \{(t^* - t(\widehat{\Theta}, m))'(t^* - t(\widehat{\Theta}, m))\},$$

where t^* is the data tuition profile, $t(\cdot)$ consists of each college's optimal tuition, and $\widehat{\Theta} \equiv [\widehat{\Theta}_0, \widehat{\Theta}_2]$ is the vector of fundamental parameter estimates from the previous two steps. I obtain the variance-covariance of $\widehat{m}$ using the Delta method, which exploits the variance-covariance structure of $\widehat{\Theta}$.

3.3 Identification

Given the policies on tuition, admissions and financial aid, students with the same observable characteristics may make different application decisions due to their unobserved types and idiosyncratic tastes. With the latter assumed to be normal, the student-side model can be viewed as a finite mixture of multinomial probits. In the appendix, I prove formally the identification of a mixed probit model with two types, which shares the same logic for the identification in the more general case of mixed multinomial probits with multiple types.[32] In this subsection, I provide a more intuitive discussion about the identification of student types. Discussions about the identification of some specific key parameters will be provided along with the estimation results. Interested readers can also find more formal and detailed discussions in the appendix.

As is true in most structural models, functional form assumptions and exclusion restrictions facilitate the identification. However, the most important source of identification is the dynamics of the model, which help to identify student types through realizations of admissions and financial aid as well as through student application and enrollment decisions. For example, someone with a strong preference to attend college but low ability will diversify her risks by sending out more applications, but may be rejected by most of the college groups she applies to. Besides the sizes of application portfolios, the contents of these portfolios also inform us about types. In the model, a student's preferences for different colleges are correlated via her type-specific preference parameters. Consider students with the same SAT and

[31] Given that there is only a single college market, there are only four tuition observations on which to base the estimation of the colleges' objective functions. Therefore, pursing a conventional estimation approach is not sensible. Instead, I treat the four nonlinear best response functions as exact, which implies that the econometrician observes all factors involved in a college's tuition decision, and saturate the model. This approach also enables me to recover the tuition preference parameters without solving the full equilibrium of the model. As is shown below, the fit to the tuition data is quite good, although there is no statistical criterion that can be applied.

[32] The proof builds on Meijer and Ypma (2008), who show the identification for a mixture of two continuous univariate distributions that are normal.

family background, hence the same expected net tuition and ability. Without heterogeneity along the Z dimension of student type, i.e., the dimension that captures students' preferences for public relative to private colleges, these students differ only in their i.i.d. idiosyncratic tastes. As a result, there should not be any systematic difference between their application portfolios. However, in the data when these students send out multiple applications, they are more likely to concentrate either on public colleges or on private colleges, rather than applying to a mixture of both. The patterns of such concentration, therefore, inform us about the distribution of Z and its effects on students' preferences.

4 Data

4.1 NLSY Data and Sample Selection

In NLSY97, a college choice series was administered in years 2003-2005 to respondents from the 1983 and 1984 birth cohorts who had completed either the 12th grade or a GED at the time of interview. Respondents provided information about each college to which they applied, including name and location; any general financial aid they may have received; whether each college to which they applied had accepted them for admission, along with financial aid offered. Information was asked about each application cycle.[33] In every survey year, the respondents also reported on the college(s), if any, they attended during the previous year. Other available information relevant to this paper includes SAT/ACT score and financial-aid-relevant family information (family income, family assets, race and number of siblings in college at the time of application).

The sample I use is from the 2303 students within the representative random sample who were eligible for the college choice survey in at least one of the years 2003-2005. To focus on first-time college application behavior, I define applicants as students whose first-time college application occurred no later than 12 months after they became eligible. Under this definition, 1756 students are either applicants or non-applicants.[34] I exclude applications for early admission. I also drop observations where some critical information, such as the identity of the college applied to, is missing. The final sample size is 1646.

[33] An application cycle includes applications submitted for the same start date, such as fall 2004.

[34] I exclude students who were already in college before their first reported applications. If a student is observed in more than one cycle, I use only her/his first-time application/non-application information.

4.2 Aggregation of Colleges

Two major constraints make it necessary to aggregate colleges. One is computational feasibility: with a large number of colleges, solving the student optimal portfolio problem and/or computing the equilibrium poses major computational challenges.[35] Another major constraint is sample size: without some aggregation, the number of observations for each option would be too small to obtain precise parameter estimates. Consequently, I aggregate colleges into groups and treat each group as one college in the estimation. By doing so, I focus on the main features of the data, while abstracting from some idiosyncratic factors such as regional preferences that may be important at a disaggregate level but are less likely to be important at a more aggregate level.

The aggregation goes as follows: first, I divide all four-year colleges into private and public categories, and then I use the within-category rankings from U.S. News and World Report 2003-2006 for further division.[36] Table 1 shows the detailed grouping: I group the top 30 private universities and top 20 liberal arts colleges into Group 1, the top 30 public universities into Group 2, and all other four-year private (public) colleges into Group 3 (Group 4).

With this aggregation, the paper captures the majority of students' behavior: 60% of applicants in the sample applied to no more than *one* college within a group. Meanwhile, cross-group application is a significant phenomenon in the data, suggesting the importance of competitions across college groups. Table 1.2 shows, conditional on applying to the college group in the row, the fraction of applicants who applied to each of the college groups in the column. For example, 32.7% of students who applied to the top private college group (Group 1) also applied to the top public college group (Group 2). Moreover, when an applicant applied to both groups within the public/private category, in over 95% of the cases, she applied to colleges that are far apart in ranking.[37]

[35] The choice set for the student application problem grows exponentially with the number of colleges. Moreover, a fixed point has to be found for each college's tuition and admissions policies in order to solve for the equilibrium.

[36] The report years I use correspond to the years when most of the students in my sample applied to colleges, and the rankings had been very stable during that period.

[37] Among the applicants who applied to both groups within the public/private category, I define a student as a "close applicant" if the ranking distance is less than 10 between the best lower-ranked college and the worst top college she applied to. Among Group 1-and-Group 3 applicants, 10% are close applicants. Among Group 2-and-Group 4 applicants, none are close applicants.

Table 1.1 Aggregation of Colleges

	Group 1	Group 2	Group 3	Group 4
Num. of colleges (Potential^a)	51	32	1921	619
Num. of colleges (Applied^b)	37	32	312	292
Capacity^c (%)	1.0	4.6	11.2	24.4

Group 1: Top private colleges; Group 2: Top public colleges;

Group 3: Other private colleges; Group 4: Other public colleges.

a. Total number of colleges in each group (IPEDS).

b. Number of colleges applied to by some students in the sample.

c. Capacity = Num. of students in the sample enrolled in group j/sample size.

Table 1.2 Applications|Applied to a Certain Group

%	Group 1	Group 2	Group 3	Group 4
Group 1	100.0	32.7	70.9	40.0
Group 2	12.2	100.0	39.9	52.7
Group 3	13.0	19.6	100.0	47.2
Group 4	4.1	14.5	26.4	100.0

Conditional on applying to the group in the row,

the fraction that applied to each group in the column.

I adjust the empirical definitions of application, admission and enrollment to accommodate the aggregation of colleges. A student is said to have applied to group j if she applied to any college within group j; is said to have been admitted to group j if she was admitted to any college in group j; and is said to have enrolled in group j if she enrolled in any college in group j. I also use the within-group average tuition as the group tuition, based on the tuition information from the Integrated Postsecondary Education Data System (IPEDS).

4.3 Summary Statistics

Table 2 summarizes characteristics among non-applicants, applicants and attendees. Clear differences emerge between non-applicants and applicants: the latter are much more likely to be female, white, with higher SAT scores and with higher family income. Conditional on applying, attendees and non-attendees do not significantly differ. Similar patterns have been found in other studies using different data.[38]

[38] For example, Arcidiacono (2005), using data from the National Longitudinal Study of the Class of 1972, and Howell (2010), using data from National Education Longitudinal Study of 1988 report similar patterns.

Table 2 Student Characteristics

	Non-Applicants	Applicants	Attendees
Female	43.2%	53.0%	54.1%
Black	17.7%	13.3%	12.1%
Family Income[a]	39835 (32361)	68481 (51337)	70605 (51279)
$SAT^b = 1$	79.8%	16.5%	13.7%
$SAT = 2$	17.0%	59.7%	60.6%
$SAT = 3$	3.2%	23.8%	25.7%
Observations	899	747	678

a. in 2003 dollars, standard deviations are in parentheses.

b. SAT=1 if SAT or ACT equivalent is lower than 800 (Obs: 840).[39]

SAT=2 if SAT or ACT equivalent is between 800 and 1200 (Obs: 599).

SAT=3 if SAT or ACT equivalent is above 1200 (Obs: 207).

Table 3 summarizes the distribution of application portfolio sizes. Fifty-five percent of students did not apply to any four-year college. Among applicants, 67% applied to only one group, and only 7% of applicants applied to three groups or more. Table 4 shows group-specific application rates and admissions rates. The application rate, defined as the fraction of applicants that apply to a certain group, increases as one goes from Group 1 to Group 4.[40] However relative to their capacities (shown in Table 1.1), top colleges receive disproportionately higher fractions of applications than lower-ranked colleges. For example, Group 4 is almost 25 times as large as Group 1, but the application rate for Group 4 is only 10 times as high as that for Group 1. Consistently, the admissions rate increases monotonically from 58% in Group 1 to 96% in Group 4.

Table 3 Distribution of Portfolio Sizes (%)

Size= 0	Size= 1	Size= 2	Size= 3	Size= 4
54.6	31.0	11.2	2.9	0.3

[39] Students who did not take the SAT or ACT test are categorized into SAT=1 group, since the other observable characteristics of these students and the outcomes of their applications, admissions and enrollment are very similar to those with low SAT/ACT scores.

[40] Application rates across groups will not necessarily add up to 100%, since some students applied to multiple college groups.

Table 4 Application & Admission: All Applicants

(%)	Group 1	Group 2	Group 3	Group 4
Application Rate	7.4	19.8	40.3	72.0
Admission Rate	58.2	76.4	91.7	95.7

Num of all applicants: 747

Application rate=num. of group j applicants/num. of all applicants

Admission rate=num. of students admitted to group j/num. of group j applicants

Table 5 shows the final distribution of students who obtained at least one admission. Over 80% of them attended lower-ranked colleges, with Group 4 accommodating 56%. Only 2% attended colleges in the top-ranked private Group 1. A significant fraction (6%) of students who had been offered admissions chose the outside option. Given that application is costly, such behavior cannot be optimal unless there are post-application shocks that make students reconsider the value of attending college.

Table 5 Final Allocation of Admitted Students (%)

Group 1	Group 2	Group 3	Group 4	Outside
2.2	10.6	25.6	55.7	6.0

Num. of students with at least one admission: 720.

Table 6 summarizes tuition and financial aid. Private colleges are four to five times as costly as public colleges of similar ranking. Within the public/private category, the higher-ranked colleges are more costly. Relative to students admitted to top groups, a higher fraction of students admitted to lower-ranked groups receive college financial aid. Conditional on obtaining some aid, the amount of aid is monotone in the tuition cost. As shown in the last column, 40% of admitted students receive some outside financial aid that helps to fund college attendance in general, but the average amount of general aid is lower than that of any college-specific aid.

Table 6 Tuition and Financial Aid

	Group 1	Group 2	Group 3	Group 4	General
Tuition[a]	27009	5347	17201	3912	–
Fraction of Aid Recipients[b]	42.4%	32.8%	67.1%	46.6%	39.9%
Mean Aid for Recipients	12836	8968	11347	5345	4326

a. Tuition and aid are measured in 2003 dollars.

b. Num. of aid recipients in the sample/num. of admitted students in the sample

5 Empirical Results

This section presents first the estimates of some key structural parameters and then the model fit. More detailed results are in Appendix F2.

5.1 Parameter Estimates

5.1.1 Student Preference

Table 7 Preference Parameter Estimates

(\$1,000)	Group 1	Group 2	Group 3	Group 4
$\overline{u}_j(A=1,Z=1)$	−233.9 (79.8)	−287.0 (18.9)	−217.0 (8.1)	−120.0 (4.5)
$\overline{u}_j(A=2,Z=1)$	−222.4 (43.6)	−97.7 (9.3)	−20.9 (3.2)	81.5 (1.1)
$\overline{u}_j(A=3,Z=1)$	−57.5 (3.5)	59.7 (6.4)	−52.0 (6.1)	11.0 (4.6)
$\overline{u}_j(A=1,Z=2)$	−73.9	−309.8	−61.3	−244.7
$\overline{u}_j(A=2,Z=2)$	−62.4	−120.4	134.8	−43.3
$\overline{u}_j(A=3,Z=2)$	124.2	−6.9	37.0	−104.9
$\overline{\psi}_j(A\in\{1,2\})$	160.0 (40.9)	−22.8 (10.1)	155.7 (4.2)	−124.8 (6.8)
$\overline{\psi}_j(A=3)$	181.7 (26.7)	−66.6 (7.5)	89.0 (9.6)	−115.9 (21.1)
σ_{ϵ_j}	115.0 (1.2)	91.6 (3.8)	77.9 (1.9)	43.6 (1.5)

$\overline{u}_j(A,Z=2) = \overline{u}_j(A,Z=1) + \overline{\psi}_j(A)$, with the restriction that $\overline{\psi}_j(1) = \overline{\psi}_j(2)$.

The restriction cannot be rejected at 10% significance level.

There is significant heterogeneity in students' preferences for colleges, both across types and within each type. Rows 1 to 3 of Table 7 show the mean values (in \$1,000) attached to colleges by type $Z=1$ students with $A=1$ to $A=3$, respectively. Rows 4 to 6 show those values for type $Z=2$ students. $\overline{\psi}_j(A)$'s shown in the next two rows are the additional values attached to each college group by $Z=2$ type relative to $Z=1$ type, conditional on ability. That is, $\overline{u}_j(A,Z=2)=\overline{u}_j(A,Z=1)+\overline{\psi}_j(A)$.

For an average student of the lowest ability ($A=1$), attending college is a much worse option than her outside option. This explains why the majority of (low family income, low SAT) students, who are most likely to be of low ability, do not apply to any college in the data. Due to their low family income, these students would obtain very generous financial aid if they were admitted to any college. Moreover, from an individual student's point of view, there is a nontrivial probability that such a student would be admitted, at least, to the lower-ranked colleges. Given the apparent "unclaimed" benefits for these students, their decisions not to apply inform us that their valuations of colleges must be low.[41]

[41] Another potential, but probably minor reason for non-application among these students is borrowing constraint. For example, Cameron and Heckman (1998) and Keane and Wolpin (2001), find that borrowing

For students of the two higher ability levels, their valuations of colleges are not universally monotone in ability: on average, $A = 3$ students value top colleges more and lower-ranked colleges less than $A = 2$ students do. Since these preference parameters reflect the expected benefits, net of effort costs, of attending colleges, such non-monotone patterns are not completely surprising. For example, it is reasonable to believe that the effort costs required in top colleges are higher than those required in lower-ranked colleges, and that these costs decrease with student ability. Considering the effort costs and the probabilities of success in different colleges, a mediocre student might be better off attending a lower-ranked college.

Holding ability constant, $Z = 2$ type value private colleges more and public colleges less than $Z = 1$ type. Private colleges and public colleges have different features that may fit some students better than others. For example, private colleges are usually smaller than public colleges, which may be an advantage for some students but a disadvantage for others.

By introducing types, the model explains the systematic differences in the behaviors among students with similar observable characteristics. The residual non-systematic differences in student choices are accounted for by their idiosyncratic preferences, where there are significant dispersions (σ_{ϵ_j}).[42] In sum, not only do students attach different values to the same college, but they also rank colleges differently. For example, attending an elite college is not optimal for all students.[43] Instead, each option (including the outside option) offered in the college market best caters to some groups of students.

5.1.2 Application Costs

As shown in Table 8, the cost for the first application is about $\$6,400$, but as the number of applications increases, the marginal cost rapidly decreases, suggesting the existence of some economies of scale. Put into context, the application cost is about 6% of the value of attending college net of tuition for the median applicant, and 5% for the median college attendee.

Table 8 Application Costs

($\$1,000$)	$n = 1$	$n = 2$	$n = 3$	$n = 4$
$C(n)$	6.4 (0.3)	7.9 (0.2)	8.3 (0.2)	8.5 (0.2)
$C(n) - C(n-1)$	6.4	1.5	0.4	0.2

constraints have a negligible impact on college attendance, based on which I assume no borrowing constraint.

[42] For example, although Group 1 colleges are worth only $\$124,188$ for an average student of $(A = 3, Z = 2)$ type, this value becomes $\$271,618$ at the 90^{th} percentile. Table F2.1 in the appendix illustrates the importance of within-type taste dispersion by showing the mean evaluations of colleges among all students, applicants and attendees, from a simulated example.

[43] This is consistent with findings from some other studies, for example, Dale and Krueger (2002).

In interpreting these costs, on the one hand, one must remember that they incorporate all factors that make application costly, i.e., all student-side barriers to applying for colleges other than their ability and preferences. On the other hand, student ability and preferences are far more important in explaining the application patterns found in the data. As an example, in the student decision model, if one fixes all the other parameters, including the marginal application costs $C(n) - C(n-1)$ for $n > 1$, and reduces the cost for the first application to $\$1,500$, the fraction of non-applicants remains at 51%, as compared to 55% in the data and in the baseline model. For many students, application costs are irrelevant to their decisions. For example, average low-ability students, who derive negative utilities from colleges, will not apply even if application is costless. However, idiosyncratic student tastes place some students at the margin of applying and not applying, as is true in the data, where observationally equivalent students may have very different application behaviors. The estimates of application costs adjust such that the "right" fraction of marginal students decide to apply.

5.1.3 Ability Measures

Based on the ability distribution parameter estimates, each row of Table 9.1 shows the distribution of SAT scores given ability. Ability-1 students are most likely to score low in SAT, and rarely score high in SAT. Based on SAT, it is relatively easier to distinguish low-ability students from the others. However, SAT is less useful in distinguishing medium-ability and high-ability types. For example, most students of both types obtain medium SAT scores.

Table 9.1 SAT and Ability: Simulation

	$P(SAT=1\|A)$	$P(SAT=2\|A)$	$P(SAT=3\|A)$
$A=1$	0.79	0.18	0.03
$A=2$	0.16	0.63	0.21
$A=3$	0.04	0.55	0.41

Table 9.2 reports parameter estimates for the distribution of signals conditional on ability. Signals, such as student essays, can effectively distinguish the highest ability students from the others: the former are much more likely to send the highest signal, and almost never send out the lowest signal. Ability-2 students are most likely to send a medium signal, and they distinguish themselves from Ability-1 students primarily by their reduced probability of sending out the lowest signal. However, their chance of obtaining the highest signal is almost the same as Ability-1 students. As a result, it is hard to distinguish the two lower-ability types based on their signals.

Table 9.2 Signal Distribution

	$P(s=1\|A)$	$P(s=2\|A)$	$P(s=3\|A)$
$A=1$	0.22 (0.07)	0.39	0.39 (0.09)
$A=2$	0.03 (0.005)	0.58	0.39 (0.08)
$A=3$	0.000001 (0.05)	0.28	0.71 (0.06)

5.2 Model Fit

Given the parameter estimates, I first fix the tuition profile at the data level and simulate the student-side partial equilibrium model (PE) and the application-admission equilibrium model (AE). Then I endogenize tuition and simulate the whole subgame perfect Nash equilibrium model (SPNE).[44]

Table 10: Model vs. Data

Distribution of Portfolio Sizes (%)

Size	Data	PE	AE	SPNE
0	54.6	54.9	55.1	55.7
1	30.9	29.6	30.9	31.5
2	11.2	11.8	10.7	9.6
3	2.9	3.3	3.0	2.9
4	0.3	0.2	0.3	0.2
χ^2 Stat		2.95	0.47	5.64

PE: Partial Equilibrium Model

AE: Application-Admission Equilibrium

SPNE: Market Equilibrium Model

$\chi^2_{4,0.05}= 9.49$

Table 10 shows the fit for the distribution of portfolio sizes: all three models fit the data well, with SPNE slightly understating the fraction of multiple applications. Table 11 displays the fit of application and admissions rates among applicants. The first set of rows shows that all three models closely match application rates, except that the SPNE model under-predicts the application rate for Group 4. The fit for admissions rates is shown in the second set of rows: PE closely matches the admissions rates for all groups. AE and SPNE under-predict the admissions rate for Group 1 and over-predict that for Group 3. Table 12 displays the fits of student allocation. The first set of columns shows the allocation for all

[44] This section shows model fits for the whole sample. Model fits by race, by SAT and by family income are also good. They are available from the author.

students, and the second set of columns shows that for students with at least one admission, all models closely fit the allocation patterns, with SPNE fit being the best.

Finally, Table 13 contrasts SPNE predicted tuition levels with the data. The model fits Group 4's tuition almost perfectly, but it under-predicts College 2's tuition and over-predicts College 3's tuition by about 10%.[45]

Table 11 Model vs. Data

Application & Admission: All Applicants (%)

Application Rate	Data	PE	AE	SPNE
Group 1	7.4	7.6	7.1	7.4
Group 2	19.8	21.1	19.9	20.2
Group 3	40.3	41.4	41.2	41.9
Group 4	72.0	72.5	70.8	67.0*
Admission Rate				
Group 1	58.2	54.2	44.1*	43.6*
Group 2	76.4	80.2	81.9	82.0
Group 3	91.7	90.9	95.3*	98.6*
Group 4	95.7	95.0	95.0	97.1

$^{*}\ \chi^2 > \chi^2_{1,0.05}$

Table 12 Model vs. Data

Final Allocation of Students (%)

	All Students				Students With Some Admission			
	Data	PE	AE	SPNE	Data	PE	AE	SPNE
College 1	1.0	1.1	1.0	1.0	2.2	2.7	2.2	2.2
College 2	4.6	4.5	4.3	4.5	10.6	10.6	10.1	10.5
College 3	11.2	10.7	11.3	11.1	25.6	24.9	26.4	25.8
College 4	24.4	23.5	24.0	24.3	55.7	54.8	55.9	56.3
Outside	58.8	60.2	59.4	59.1	6.0	7.0	5.3	5.1
χ^2 Stat.		2.11	0.54	0.12		1.93	1.54	1.45

$\chi^2_{4,0.05} = 9.49$

[45] The deviation of the SPNE tuition from data tuition comes mainly from the SPNE structure. Table F3 in the appendix shows each college's tuition as the best response to others' equilibrium (data) tuition (i.e., the fit for the third-step estimation), which closely matches the data.

Table 13: Model vs. Data

Tuition

	Group 1	Group 2	Group 3	Group 4
Data	27009	5347	17201	3912
SPNE	26162	4555	19173	3925

6 Counterfactual Experiments

With the estimated model, which fits the data reasonably well, I conduct three counterfactual experiments. Comparisons are made between the baseline SPNE and the new SPNE, simulated using the same set of random draws.[46]

6.1 Creating More Opportunities

In the first counterfactual experiment, I examine to what extent the government can further expand college access by increasing the supply of colleges. I increase the capacity of the lower-ranked public colleges (Group 4) by growing magnitudes while keeping the capacities of other groups fixed.[47] The response of college enrollment to the increase in supply is shown in Figure 1. At the beginning, there is a one-to-one response of college enrollment to the increase in supply. Then, enrollment reaches a satiation point where there is neither excess demand nor excess supply of college slots in Group 4 and the equilibrium outcomes remain the same thereafter. The following tables report the case when Group 4's supply is at the satiation point.

Table 14.1 shows changes in tuition. To attract enough students, Group 4 cuts its tuition from $3,925 to an almost negligible level of $136. Its private counterpart, Group 3, also lowers its tuition by about 9%.[48] However, the two top groups increase their tuition. To better understand the difference in colleges' tuition adjustments, we need to jointly consider their reactions in tuition and admission policies.

[46] In simulating the baseline model and the counterfactual experiments, I tried a wide range of initial guesses in my search for equilibrium. For each model, I find only one equilibrium.

[47] Similar results hold in analogous experiments with Group 3's capacity. I increase the supply of lower-ranked colleges because they accommodate most college attendees and are most relevant to the overall access to college education.

[48] Colleges do not have to fill their capacities, and they can charge high tuition and leave some slots vacant. However, under the current situation and the estimated parameter values, it is not optimal for them to do so.

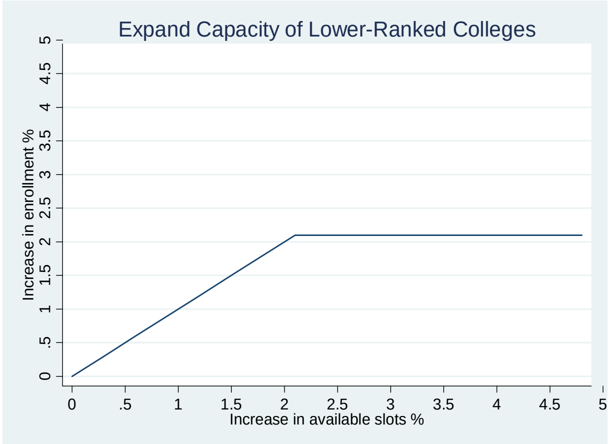

Figure 1: Enrollment & Expansion of Lower-Ranked Groups

Table 14.1 Increasing Supply

Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26162	4555	19173	3925
New SPNE	27549	6473	17394	136

Table 14.2 Increasing Supply

Admission Rates

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	43.6	82.0	98.6	97.1
New SPNE	47.7	99.0	99.1	100.0

Table 14.2 indicates that admissions rates increase in all colleges and reach (almost) 100% except for Group 1. The major driving forces for the increased admissions rates are likely to differ across college groups. For lower-ranked groups, higher admissions rates and lower tuition reflect their efforts to enroll enough students. Top groups increase their admissions rates mainly because they are faced with a better self-selected applicant pool: the increased tuition in top groups pushes, and the tuition and admissions policies in lower-ranked colleges pull lower-ability applicants toward lower-ranked groups.

Table 14.3 Increasing Supply

Attendance

%	Base SPNE	New SPNE	All Open&Free
All	40.9	43.0	51.1
$A = 1$	1.9	3.2	14.9
$A = 2$	94.7	97.7	99.4
$A = 3$	86.4	90.3	98.6

Table 14.3 shows the allocation effect. The first row displays the attendance rate over all students: regardless of the 100% admissions rate and the dramatically lowered tuition in Group 4, only 2.1% more students are drawn into colleges. Since the supply of colleges in Group 4 exceeds demand if its capacity is further increased, this 2.1% increase represents the upper limit to which the government can increase college attendance by increasing the supply of Group 4 colleges. To further understand these equilibrium results, I conduct a partial equilibrium experiment where all colleges are open and free. This is an extreme situation with unlimited supply of colleges. The attendance rate is reported in the last column of Table 14.3: only 51%, or 10% more students, would attend colleges under this condition. Therefore, neither college capacity nor tuition is a major barrier to college access. The vast majority of students who do not attend colleges under the base SPNE prefer the outside option over any college option. Among them, most are of low ability. In fact, as indicated in the last three rows of Table 14.3, only 2% of the lowest-ability students attend college in the base SPNE, and fewer than 15% of them would attend college even if colleges were free and open. In contrast, the majority of students of higher ability attend college in the base SPNE, and almost all of them would attend college if colleges become free and open. The major limit on college access, therefore, is ability and associated preferences.[49]

6.2 Ignoring Signals

In some countries, such as China, college admissions are based almost entirely on scores in a nation-wide test.[50] Although such a system may save resources invested in the admissions process, such as the human resource employed in reading thousands of student essays, it ignores a valuable source of information about student ability. In the second counterfactual experiment, I assess the consequences of ignoring signals in the admissions process.

Table 15.1 shows the changes in tuition under the new SPNE. All colleges increase their tuition, with the top public colleges making the most dramatic increase of 64%. As reflected

[49] This finding is in line with earlier research. See, for example, Cameron and Heckman (1998) and Keane and Wolpin (2001).

[50] Exceptions apply to, for example, students with special talents.

by the estimated student preference parameters, higher-ability students have higher preferences for colleges and hence greater willingness to pay. Knowing this, colleges draw on higher tuition to screen students when the information on ability provided by signals becomes unavailable. In response to the higher tuition and hence lower net returns from attending colleges, fewer students apply and applicants apply to fewer colleges, as shown in Table 15.2.

Table 15.1 Ignore Signals

Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26162	4555	19173	3925
New SPNE	27476	7470	20211	4885

Table 15.2 Ignore Signals

Distribution of Portfolio Sizes

	Size= 0	Size= 1	Size= 2	Size= 3	Size= 4
Base SPNE	55.7	31.5	9.6	2.9	0.2
New SPNE	56.0	33.2	8.6	2.2	0.1

Table 15.3 shows the changes in admission rates. With SAT as the single criterion for admissions, all *applicants* with the highest SAT are admitted to all colleges. In some colleges, higher admission rates for higher SAT applicants come at the expense of reduced admissions for those with lower SAT scores. For example, in the top private colleges, less than 8% of medium SAT applicants are admitted as compared to 28% in the baseline case. However, the overall admission rates increase in all colleges as a result of reduced applications. Although higher tuition levels help to screen students, all colleges experience a drop in their enrollee ability (Table 15.4). As one might expect, the effect is more obvious in the top groups than in the lower-ranked groups.

Table 15.3 Ignore Signals

Admission Rates

%	All		SAT= 1		SAT= 2		SAT= 3	
	Base	New	Base	New	Base	New	Base	New
Group 1	43.6	55.6	N/A^*	N/A	28.2	7.6	66.7	100.0
Group 2	82.0	91.5	N/A	N/A	72.8	86.8	99.4	100.0
Group 3	98.6	100.0	91.7	100.0	100.0	100.0	100.0	100.0
Group 4	97.1	98.2	96.2	88.3	97.7	100.0	97.8	100.0

* Not applicable since application is zero.

Table 15.4 Ignore Signals

Fraction of High-Ability Students

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	89.9	84.1	11.7	7.2
New SPNE	87.2	82.5	11.5	7.1

Finally, changes in student welfare are reported in Table 15.5. Overall, student welfare decreases by $1,325. Although the increase in tuition affects all students negatively, the changes in admissions policies have different effects on students with different abilities. The highest-ability students lose the most, with a loss of $5,000. They are the ones who have the most to gain from signals, through which they can most effectively distinguish themselves from others. The medium-ability students benefit from being mixed with the highest-ability ones, but suffer from being mixed with the lowest-ability ones. The only winners are the lowest-ability students: their gain from being mixed with others outweighs the loss from increased tuition.

Table 15.5 Ignore Signals

Mean Student Welfare

$	Base SPNE	New SPNE	Change
All	41402	40077	−1325
$A = 1$	677	752	75
$A = 2$	98630	95907	−2723
$A = 3$	84673	79692	−4981

6.3 Dropping SAT

Although SAT remains one of the most important criteria for admissions to most colleges, there have been concerns that SAT has been overemphasized. For example, critics have been blaming SAT as inhibiting the access to higher education for students from low-income families, who typically have low SAT scores. In response, starting from 1969, a number of liberal arts colleges have either joined, or have been important influences on, the SAT Optional Movement, which urges dropping SAT in the admissions process.[51] In a 2001 speech to the American Council on Education, Richard Atkinson, the president of the University of California, also urged dropping the SAT as a college admissions requirement.[52] The

[51] Bowdoin College and Bates College were among the first to institute SAT-optional programs in 1969 and 1984, respectively. In 2006, 27 of the top 100 liberal arts colleges did not require SAT or ACT. For a complete list of colleges that do not require SAT or ACT in admission, see http://fairtest.org/university/optional.

[52] In response to threats by the University of California to drop the SAT as an admission requirement, the College Entrance Examination Board announced the restructuring of the SAT, to take effect in March 2005.

following experiments examine the impacts on the college market of such a policy.[53] Two subcases are considered: in the first, only top private colleges (Group 1) drop SAT and base admissions only on signals; in the second, all colleges do so. Tables 16.1-16.4 show, respectively, the changes in tuition, admissions rates, the fraction of high-ability students and the mean income level in each college group.

When Group 1 alone drops SAT, it increases its tuition by $\$1,762$ to help screen students. In addition, it also lowers its admission rate from 44% to 37%. The reactions from the other groups are very mild. As a result, Group 1 experiences a significant drop in its student ability: the fraction of high-ability students drops from 90% to 77%. All the other groups see a slight increase in their student ability. The average family income among students in Group 1 drops by about $\$11,500$. Without using financial tools, the top private colleges increase the presence of low-income students on their campuses, which is a major motive behind the SAT Optional Movement. This goal is served, however, at the price of increased tuition and decreased student ability in Group 1.

Table 16.1 Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26162	4555	19173	3925
Drop SAT (Group1)	27924	4405	19323	4155
Drop SAT (All)	30561	6805	19886	4038

Table 16.2 Admission Rates

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	43.6	82.0	98.6	97.1
Drop SAT (Group1)	37.2	81.0	98.5	97.6
Drop SAT (All)	40.6	84.0	99.1	97.2

Table 16.3 Fraction of High-Ability Students

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	89.9	84.1	11.7	7.2
Drop SAT (Group1)	76.6	84.7	12.0	7.3
Drop SAT (All)	78.0	82.1	11.9	7.5

[53] The results are better interpreted as the upper bounds for the impacts because colleges may consider other measures of ability that are known to both sides of the market, such as high school ranking.

Table 16.4 Mean Income

$	Group 1	Group 2	Group 3	Group 4
Base SPNE	93802	90610	65916	64657
Drop SAT (Group 1)	82343	90988	65934	64831
Drop SAT (All)	83205	87560	66206	64587

When all college groups drop SAT, they all increase their tuition to enhance self-selection among students. The reaction is especially strong in the two top groups: Group 2 increases its tuition by almost 50%, and Group 1 increases its tuition by more than it does in the first subcase.[54] The student bodies in all colleges become more diverse: lower-ranked colleges obtain more high-ability students, while the top colleges, especially Group 1, obtain more low-ability and low-family-income students. In the baseline equilibrium, low-SAT students are discouraged from applying to top colleges because they know their SAT before application and they know with low SAT, they have (near) zero probability of being admitted. Dropping SAT, therefore, opens the door to top colleges for high-ability students who happen to obtain low SAT scores, as desired by the current SAT-optional colleges. However, dropping SAT also opens the door for low-ability students. These students have a fair chance (39%) of sending a high signal and a significant number of them have strong preference for top colleges, especially for the top private group.

7 Conclusion

In this paper, I have developed and structurally estimated an equilibrium model of the college market. It provides a first step toward a better understanding of the college market by jointly considering tuition setting, applications, admissions and enrollment. In the model, students are heterogeneous in their abilities and preferences. They face uncertainty and application costs when making their application decisions. Colleges, observing only noisy measures of student ability, compete for more able students via tuition and admissions policies. I have estimated the structural model via a three-step estimation procedure to cope with the complications caused by potential multiple equilibria. The empirical results suggest that the model closely replicates most of the patterns in the data.

My empirical analyses suggest that, first, there is substantial heterogeneity in students' preferences for colleges. As a result, increasing the supply of colleges has very limited effect on college attendance: neither tuition cost nor college capacity is a major obstacle to college

[54] This implies that, in Subcase 1, the seeming "lack of reactions" from the other college groups restricts Group 1 from taking more dramatic actions.

access; a large fraction of students, mainly low-ability students, prefer the outside option over any college option. Second, there are significant amounts of noise in various types of ability measures and different types of measures complement one another. Dropping SAT, as urged by some critics, indeed increases the fraction of low-income students in top colleges, but it is accompanied by lower enrollee ability and higher tuition in these colleges.

The methods developed in this paper and the main empirical findings are promising for future research. Building on Epple, Romano and Sieg (2006) and this paper, a model that endogenizes applications, admissions and financial aid would provide a more comprehensive view of the college market. Given the fast development of computational capacities, this extension may become feasible in the near future.

Building on Arcidiacono (2005) and this paper, a model that studies the strategic interactions between colleges and students and links them to students' labor market outcomes would also be an important extension. Such an extension would enrich the current paper by making explicit the factors underlying student preferences. This will become feasible as information on students' labor market outcomes becomes available from future surveys of the NLSY97.

Finally, studying the long-run equilibrium would lead to a better understanding of the trend of college tuition and attendance. In a long run equilibrium model, one can be more explicit about why colleges value student ability. For example, higher-ability students are more likely to do better in the job market, which enhances the college's prestige and attractiveness to future applicants. Moreover, one can also endogenize college capacities. One approach to implement this extension is to introduce a cost function for college education, assuming free entry to the market. Equilibrium of the model would then depend on the form of the cost function. Estimation of such a model would require additional data on college expenses and non-tuition revenues, as well as application and admissions data over multiple years.

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APPENDIX:

A. Model Details:

A1. College Admission Problem: $\alpha_j(s, SAT|t, e_{-j}, Y, d)$ and $\gamma_j(s, SAT|t, e_{-j}, Y, d)$

All objects defined in A1 depend on $\{t, e_{-j}, Y, d\}$. To save notation, the dependence is suppressed. Let $\Pr(accept|X, SAT, \eta, \zeta, j)$ be the probability that a student with characteristics (X, SAT, η, ζ) who applies to Group j accepts j's admission. Let $F(X, \eta, \zeta|s, SAT, j)$ be the distribution of (X, η, ζ) conditional on (s, SAT) and application to j. The probability that an applicant with (s, SAT) accepts j's admission is:

$$\alpha_j(s, SAT|\cdot) = \int \Pr(accept|X, SAT, \eta, \zeta, j) dF(X, \eta, \zeta|s, SAT, j).$$

Let $\Pr(O_{-j}|A, SAT) \equiv \prod_{l \in O \backslash j} p_l(A, SAT) \prod_{k \in Y \backslash O} (1 - p_k(A, SAT))$ be the probability of admission set O for a student with (A, SAT), with college j admitting her for sure,

$$\Pr(accept|X, SAT, \eta, \zeta, j) = \sum_{O_{-j} \subseteq Y(X,SAT) \backslash \{j\}} \Pr(O_{-j}|A, SAT) I(j = d(X, SAT, \eta, \zeta, O)).$$

That is, the student will accept j's admission if j is the best post-application choice for her. The distribution $F(X, \eta, \zeta|s, SAT, j)$ is given by

$$dF(X, \eta, \zeta|s, SAT, j) = \frac{P(s|A) I(j \in Y(X, SAT)) dH(X, \eta, \zeta|SAT)}{\int P(s|A) I(j \in Y(X, SAT)) dH(X, \eta, \zeta|SAT)},$$

where $H(X, \eta, \zeta|SAT)$ is exogenous and equal to the product of type distribution, the distribution of ex-post shocks and the distribution of family backgrounds conditional on SAT. Finally, the expected ability of applicant (s, SAT) conditional on acceptance is

$$\gamma_j(s, SAT|\cdot) = \frac{\int A \times \Pr(accept|X, SAT, \eta, \zeta, j) dF(X, \eta, \zeta|s, SAT, j)}{\alpha_j(s, SAT|\cdot)}.$$

A2. Proof of existence in a simplified model.

Assume there are two colleges $j \in \{1, 2\}$ and a continuum of students divided into two ability levels. The utility of the outside option is normalized to 0. The utility of attending college 1 is $u_1(A)$ for all with ability A, and that of attending college 2 is $u_2(A) + \epsilon$, where ϵ is i.i.d. idiosyncratic taste. There are two SAT levels and two signal levels. There is no ex-post shock. Some notations to be used: for an (A, SAT) group, let the fraction of students that do not apply to any college be $\theta^0_{A,SAT}$, the fraction of those applying to college j only be $\theta^j_{A,SAT}$ and the fraction applying to both be $\theta^{12}_{A,SAT}$. For each (A, SAT) group, $\theta_{A,SAT} \in \Delta$, a 3-simplex. For all four (A, SAT) groups, $\theta \in \Theta \equiv \Delta^4$. On the college side, each college chooses admissions policy $e_j \in [0, 1]^4$, where 4 is the number of (s, SAT) groups faced by the college.

Proposition 1 *For any given tuition profile t, an application-admission equilibrium exists.*

Proof. Step 1: The application-admission model can be decomposed into the following sub-mappings:
Taking the distribution of applicants, and the admissions policy of the other college as given, college j's problem (6) can be viewed as the sub-mapping

$$M_j : \Theta \times [0, 1]^4 \rightrightarrows [0, 1]^4,$$

for $j = 1, 2$. Taking college admissions policies as given, the distribution of students is obtained via the sub-mapping

$$M_3 : [0, 1]^4 \times [0, 1]^4 \to \Theta.$$

An equilibrium is a fixed point of the mapping:

$$\begin{aligned} M &: \Theta \times [0, 1]^4 \times [0, 1]^4 \rightrightarrows \Theta \times [0, 1]^4 \times [0, 1]^4 \\ s.t.\ \theta &\in M_3(e_1, e_2) \\ e_j &\in M_j(\theta, e_k)\ j, k \in \{1, 2\}, j \neq k. \end{aligned}$$

Step 2: Show that Kakutani's Fixed Point Theorem applies in mapping M and hence an equilibrium exists.
1) The domain of the mapping, being the product of simplexes, is compact and non empty.
2) It can be shown that the correspondence $M_j(\cdot,\cdot)$ is compact-valued, convex-valued and upper-hemi-continuous, for $j = 1, 2$. In particular, the $(s, SAT)'$th component of $M_j(\theta, e_k)$ is characterized by (7) and (8), where $\gamma_j(s, SAT) + M(t_j; m_j) - \lambda_j$ is continuous in (θ, e_k).
3) Aggregate individual optimization into distribution of students θ.
Generically, each student has a unique optimal application portfolio as the solution to (5). For given (A, SAT), there exist $\epsilon^*(e) \geq \epsilon^{**}(e)$, both continuous in e, such that:
For $\epsilon \geq \epsilon^*(e)$, $Y(A, SAT, \epsilon) = \begin{cases} \{2\} \text{ if } C(2) - C(1) > k_1(e) \\ \{1,2\} \text{ otherwise} \end{cases}$;
for $\epsilon \in [\epsilon^{**}(e), \epsilon^*(e)), Y(A, SAT, \epsilon) = \{1, 2\}$; and
for $\epsilon < \epsilon^{**}(e)$, $Y(A, SAT, \epsilon) = \begin{cases} \{1\} \text{ if } C(1) \leq k_2(e) \\ \emptyset \text{ otherwise} \end{cases}$,
where $k_1(e)$ and $k_2(e)$ are continuous in e. Therefore, the (A, SAT) population can be mapped into a distribution $\theta_{A,SAT} \in \Delta$, and this mapping is continuous in e. Because the mapping from $[0, 1]^4 \times [0, 1]^4$ into the individual optimal portfolio is a continuous function, and the mapping from the individual optimization to Θ is continuous, the composite of these two mappings, M_3, is single-valued and continuous.[55]
Given 1)-3), Kakutani's Fixed Point Theorem applies.[56] ■

Since for every t, $AE(t)$ exists in the subsequent game, an SPNE exists if a Nash equilibrium exists in the tuition setting game. Let $\bar{t}_j$ denote some large positive number, such that for any t_{-j}, the optimal $t_j < \bar{t}_j$. $\bar{t}_j$ exists because the expected enrollment, hence college j's payoff goes to 0, as t_j goes to ∞. Define the strategy space for college j as $[0, \bar{t}_j]$, which is nonempty, compact and convex. The objective function of college j is continuous in t, since the distribution of applicants, and hence the total expected ability, is continuous in t. Given certain regularity conditions, the objective function is also quasi-concave in t_j. The general existence proof for Nash equilibrium applies.

B. Data Details

B1. The NLSY97 consists of a sample of 8984 youths who were 12 to 16 years old as of December 31, 1996. There is a core nationally representative random sample and a supplemental sample of blacks and Hispanics. Annual surveys have been completed with

[55] In the case of four schools, ϵ becomes a 3-dimension vector, as are the cutoff tastes. To show continuity, we change one dimension of ϵ at a time while keeping the other dimensions fixed.

[56] When there are $J > 2$ schools, Step 1 of the proof can be easily extended. In Step 2, ϵ, and hence the cutoffs, will be of $J - 1$ dimensions. Obtaining an analytical solution to these cutoffs is much more challenging.

most of these respondents since 1997.

B2. Empirical Definition of Early Admission:

1) Applications were sent earlier than Nov. 30th, for attendance in the next fall semester and

2) The intended college has early admissions/ early decision/ rolling admissions/ priority admissions policy,[57] and

3) Either a. one application was sent early and led to an admission or

b. some application(s) was (were) sent early but rejected, and other application(s) was (were) sent later.

B3. Since 1983, U.S. News and World Report has been publishing annual rankings of U.S. colleges and is the most widely quoted of its kind in the U.S.[58] Each year, seven indicators are used to evaluate the academic quality of colleges for the previous academic year.[59]

C. Details on Estimation

C1. Details on SMLE:

(1) Approximate the following integration via a kernel smoothed frequency simulator [60]

$$\int I(Y_i|T, SAT_i, B_i, \epsilon)I(d_i|O_i, T, SAT_i, B_i, \epsilon, \zeta, \eta)dG(\epsilon, \zeta, \eta). \tag{14}$$

For each student (SAT_i, B_i), I draw shocks $\{(\epsilon_{ir}, \zeta_{ir}, \eta_{ir})\}_{r=1}^R$ from their joint distribution $G(\cdot)$. These shocks are the same across T for the same student i, but are i.i.d. across students. All shocks are fixed throughout the estimation. Let u_{jir} be the ex-post value of college j for student$_{ir}$ with $(T, SAT_i, B_i, \epsilon_{ir}, \zeta_{ir}, \eta_{ir})$, let $v_{ir} = \max\{0, \{u_{jir}\}_{j\in O_i}\}$, let $V_{ir}(Y)$ be the ex-ante value of portfolio Y for this student, and $V_{ir}^* = \max_{Y\subseteq J}\{V_{ir}(Y)\}$. (14) is then

[57] The data source for college early admission programs is 1) Christopher et al. (2003), and 2) web information posted by individual colleges.

[58] The exception is 1984, when the report was interrupted.

[59] These indicators include: assessment by administrators at peer institutions, retention of students, faculty resources, student selectivity, financial resources, alumni giving, and (for national universities and liberal arts colleges) "graduation rate performance", the difference between the proportion of students expected to graduate and the proportion who actually do. The indicators include input measures that reflect a school's student body, its faculty, and its financial resources, along with outcome measures that signal how well the institution does its job of educating students.

[60] I describe the situation where I do not observe any information about the student's financial aid. For students with some financial aid information, the observed financial aid replaces the random draw of the corresponding financial aid shock.

approximated by:

$$\frac{1}{R}\sum_{r=1}^{R}\frac{\exp[(V_{ir}(Y_i)-V_{ir}^*)/\tau_1]}{\sum_{Y\subseteq J}\exp[(V_{ir}(Y)-V_{ir}^*)/\tau_1]}\frac{\exp[(u_{d_i ir}-v_{ir})/\tau_2]}{\sum_{j\in O_i}\exp[(u_{jir}-v_{ir})/\tau_2]},$$

where τ_1, τ_2 are smoothing parameters. When $\tau \to 0$, the approximation converges to the frequency simulator.

(2) Solving the optimal application problem for student $(T, SAT_i, B_i, \epsilon_{ir})$:

$$V_i(Y) = \sum_{O\subseteq Y}\Pr_i(O)E_{(\eta,\zeta)}\max\{u_{0ir,}\{u_{jir}\}_{j\in O}\} - C(|Y|).$$

The Emax function has no closed-form expression and is approximated via simulation. For each $(T, SAT_i, B_i, \epsilon_{ir})$, draw M sets of shocks $\{(\eta_m, \zeta_m)\}_{m=1}^M$. For each of the M sets of $(T, SAT_i, B_i, \epsilon_{ir}, \eta_m, \zeta_m)$, calculate $\max\{u_{0irm,}\{u_{jirm}\}_{j\in O}\}$, where u_{jirm} denotes u_{jir} evaluated at the shock (η_m, ζ_m). The Emax is the average of these M maximum values.

C2. Details on the Second-Step SMDE:

(1) Targets to be matched: for each of the Groups $2, 3$ and 4, there are 9 admissions probabilities to be matched $\{p_j(A, SAT)\}_{(A,SAT)\in\{1,2,3\}\times\{1,2,3\}}$. For Group 1, there are 6 admissions probabilities to be matched. Since no one in $SAT = 1$ group applied to Group 1, $\{p_1(A, SAT = 1)\}_{A\in\{1,2,3\}}$ are fixed at 0. The other four targets are the equilibrium enrollments simulated from the first step. In all, there are 37 targets to be matched using college-side parameters: $\{P(s|A)\}, \{\kappa_j\}_j$, ten of which are free.

(2) Optimal Weighting Matrix:

Let Θ^* be the true parameter values. The first-step estimates $\widehat{\Theta}_1$, being MLE, are asymptotically distributed as $N(0, \Omega_1)$. It can be shown that the optimal weighting matrix for the second-step objective function (13) is $W = Q_1\Omega_1 Q_1'$, where Q_1 is the derivative of $q(\cdot)$ with respect to $\widehat{\Theta}_1$, evaluated at $\left(\widehat{\Theta}_1, \Theta_2^*\right)$. The estimation of W involves the following steps:

1) Estimate the variance-covariance matrix $\widehat{\Omega}_1$: in the case of MLE, this is minus the outer product of the score functions evaluated at $\widehat{\Theta}_1$. The score functions are obtained via numerically taking partial derivatives of the likelihood function with respect to each of the first step parameters evaluated at $\widehat{\Theta}_1$.

2) Obtain preliminary estimates $\widetilde{\Theta}_2 \equiv \arg\min_{\Theta_2}\{q(\widehat{\Theta}_1, \Theta_2)'\widetilde{W}q(\widehat{\Theta}_1, \Theta_2)\}$, where $\widetilde{W}$ is any positive-definite matrix. The resulting $\widetilde{\Theta}_2$ is a consistent estimator of Θ_2^*.

3) Estimate Q_1 by numerically taking derivative of $q(\cdot)$ with respect to $\widehat{\Theta}_1$, evaluated at $\left(\widehat{\Theta}_1, \widetilde{\Theta}_2\right)$. In particular, let Δ_m denote a vector with zeros everywhere but the

m'th entry, which equals a small number ε_m. At each $\left(\widehat{\Theta}_1 + \Delta_m, \widetilde{\Theta}_2\right)$, I simulate the student decision model and calculate the targets for the second-step estimation. Then holding student applications fixed, I solve for college optimal admissions and calculate the distance vector $q\left(\widehat{\Theta}_1 + \Delta_m, \widetilde{\Theta}_2\right)$. The m'th component of Q_1 is approximated by $[q\left(\widehat{\Theta}_1 + \Delta_m, \widetilde{\Theta}_2\right) - q\left(\widehat{\Theta}_1, \widetilde{\Theta}_2\right)]/\varepsilon_m$.

C3. Details on the Third-Step: Solving College j's Tuition Problem

Given $\widehat{\Theta}$, t^*_{-j} and some m, I examine college j's expected payoff at each trial tuition level t'_j and obtain the optimal tuition associated with this m. This procedure requires computing the series of application-admission equilibria $AE\left(\cdot, t^*_{-j}\right)$, which can only be achieved through simulation. To do so, I use an algorithm motivated by the rule of "continuity of equilibria," which requires, intuitively, that $AE(t'_j, t^*_{-j})$ be close to $AE(t_j, t^*_{-j})$ when t'_j is close to t_j. Specifically, I start from the equilibrium at the data tuition level $\left(t^*_j, t^*_{-j}\right)$, which is numerically unique for nontrivial initial beliefs ($p >> 0$). $AE(t^*)$ is found to be unique numerically in my search for equilibrium starting from 500 different combinations of nontrivial initial beliefs. Then, I gradually deviate from t^*_j, for $\left(t^"_j, t^*_{-j}\right)$, I start the search for new equilibrium, i.e., the fixed point of admissions policies $e\left(.|\left(t^"_j, t^*_{-j}\right)\right)$, using, as the initial guess, the equilibrium $e\left(.|\left(t'_j, t^*_{-j}\right)\right)$ associated with the most adjacent $\left(t'_j, t^*_{-j}\right)$. The resulting series of $AE\left(., t^*_{-j}\right)$ is used to solve college j's tuition problem.

D. Detailed Functional Forms:

D1. Type Distribution:

$P(T|SAT, B) = \Pr(A = a|SAT, B)P(Z|A) = \Pr(A = a|SAT, y)P(Z|A)$, where y is family income, $\Pr(A = a|SAT, y)$ is an ordered logistic distribution and $P(Z|A)$ is non-parametric. For $a = 1, 2, 3$

$$\begin{aligned}\Pr(A &= a|SAT, y) = \frac{1}{1 + e^{-cut_a + \alpha_1 y_i + \alpha_2 I(SAT_i=2) + \alpha_3 I(SAT_i=3)}} \\ &- \frac{1}{1 + e^{-cut_{a-1} + \alpha_1 y_i + \alpha_2 I(SAT_i=2) + \alpha_3 I(SAT_i=3)}}\end{aligned}$$

where $cut_0 = -\infty$ and $cut_3 = +\infty$.

D2. College Payoff from Tuition Revenue:

$$M(t_j; m_j) = m_{j1}t_j + m_{j2}t_j^2$$

A non-linear preference over tuition is assumed because most colleges are non-profit, and a non-linear preference allows for possibly satiated preference over tuition. For example, state governments (alumni) may have objections to very high tuition levels, and adjust their funding (donations) to colleges accordingly. A concave preference for tuition can be derived if high tuition crowds out non-tuition revenues.[61,62]

D3. Financial Aid Functions:

$$\begin{aligned} f_0(SAT_i, B_i) &= \beta_0^0 + \beta_1^0 I(race_i = black) + \beta_2^0 I(SAT_i = 2) + \beta_3^0 I(SAT_i = 3) \\ &\quad + \beta_4^0 y_i + \beta_5^0 asset_i \\ f_{0i} &= \max\{f_0(SAT_i, B_i) + \eta_{0i}, 0\}, \end{aligned}$$

where $\eta_{0i}\tilde{}i.i.d.N(0,\sigma_{f_0}^2)$. for $j = 1, 2..4$

$f_j(SAT_i, B_i) =$

$$\begin{aligned} \beta_0^1 \quad + \beta_1^1 I(race_i = & \; black) + \beta_2^1 I(SAT_i = 2) + \beta_3^1 I(SAT_i = 3) + \beta_4^1 y_i + \beta_5^1 asset_i \\ + \beta_6^1 I(nsib > & \; 0) + \beta_7^1 I(SAT_i = 2) I(j \in public) + \beta_8^1 I(SAT_i = 3) I(j \in public) \\ + \beta_9^1 I(j = & \; 2) + \beta_{10}^1 I(j = 3) + \beta_{11}^1 I(j = 4) \end{aligned}$$

$$f_{ji} = \max\{f_j(SAT_i, B_i) + \eta_{ji}, 0\}$$

where $nsib$ denotes the number of siblings in college at the time of i's application and $\eta_{ji}\tilde{}i.i.d.N(0,\sigma_{f_1}^2)$.

E. Identification

E1. Type distribution and type-specific utilities

In the following, I will prove the identification of a mixed probit model with two types. The identification of the more general case of mixed multinomial probits with multiple types would require more complicated algebraic analyses but would nevertheless follow the same logic.

E1.1 Identification of a mixed probit model with two types

Assume there are two unobserved types of individuals $A \in \{1, 2\}$, and $\Pr(A = 1) = \lambda$. Let the continuous variable $z \in Z \subseteq R$ be an observed individual characteristics and $f(\cdot)$ be a differentiable function of z. Let $y \in \{0, 1\}$ be the observed discrete choice, which relates

[61] In particular, if non-tuition revenue for college j adjusts to tuition level according to $\upsilon_{0j} - \upsilon_{1j} t_j - \upsilon_{2j} t_j^2$, where υ_{0j} is the maximum level of college j's non-tuition revenue; let the tradeoff between ability and total (tuition and non-tuition) revenue be m_{0j}, then the tuition weight parameters in the model can be derived as $m_{1j} = m_{0j}(1 - \upsilon_{1j})$ and $m_{2j} = -m_{0j}\upsilon_{2j}$.

[62] I also tried some other functional forms. For example, a linear preference over tuition is rejected because it predicts much higher tuition levels than are observed in the data.

to the latent variable y^* in the following way:

$$y(z) = 1 \text{ if only if } y^*(z) \equiv f(z) + u_1 I(A=1) + u_2 I(A=2) + \epsilon > 0$$

where $\epsilon\tilde{}i.i.d.N(0,1)$. The model implies that

$$P(z) \equiv \Pr(y(z)=1) = \lambda\Phi(f(z)+u_1) + (1-\lambda)\Phi(f(z)+u_2) \tag{15}$$

Theorem 1 *Assume that 1) $\lambda \in (0,1)$, 2) there exists an open set $Z^* \subseteq Z$ such that for $z \in Z^*$, $f'(z) \neq 0$. Then the parameters $\theta = (\lambda, u_1, u_2)'$ in (15) are locally identified.*

Proof. The proof draws on the well-known equivalence of local identification with positive definiteness of the information matrix. In the following, I will show the positive definiteness of the information matrix for model (15) .
Step 1. Claim: *The information matrix $I(\theta)$ is positive definite if and only if there exist no $w \neq 0$, such that $w'\frac{\partial P(z)}{\partial \theta} = 0$ for all z.*
The log likelihood of an observation (y, z) is

$$L(\theta) = y\ln(P(z)) + (1-y)\ln(1-P(z)).$$

The score function is given by

$$\frac{\partial L}{\partial \theta} = \frac{y - P(z)}{P(z)(1-P(z))}\frac{\partial P(z)}{\partial \theta}.$$

Hence, the information matrix is

$$I(\theta|z) = E\left[\frac{\partial L}{\partial \theta}\frac{\partial L}{\partial \theta'}|z\right] = \frac{1}{P(z)(1-P(z))}\frac{\partial P(z)}{\partial \theta}\frac{\partial P(z)}{\partial \theta'}.$$

Given $P(z) \in (0,1)$, it is easy to show that the claim holds.
Step 2. Show $w'\frac{\partial P(z)}{\partial \theta} = 0$ for all $z \Longrightarrow w = 0$.
$\frac{\partial P(z)}{\partial \theta}$ is given by:

$$\begin{aligned}
\frac{\partial P(z)}{\partial \lambda} &= \Phi(f(z)+u_1) - \Phi(f(z)+u_2) \\
\frac{\partial P(z)}{\partial u_1} &= \lambda\phi(f(z)+u_1) \\
\frac{\partial P(z)}{\partial u_2} &= (1-\lambda)\phi(f(z)+u_2)
\end{aligned}$$

Suppose for some w, $w'\frac{\partial P(z)}{\partial\theta}=0$ for all z :

$$w_1[\Phi(f(z)+u_1)-\Phi(f(z)+u_2)]+w_2\lambda\phi(f(z)+u_1)+w_3(1-\lambda)\phi(f(z)+u_2)=0$$

Take derivative with respect to z evaluated at some $z\in Z^*$

$$\begin{aligned} w_1[\phi(f(z)+u_1)-\phi(f(z)+u_2)]f'(z)+w_2\lambda\phi'(f(z)+u_1)f'(z) & \\ +w_3(1-\lambda)\phi'(f(z)+u_2)f'(z) &= 0. \end{aligned} \tag{16}$$

Define $\gamma(z)=\frac{\phi(f(z)+u_1)}{\phi(f(z)+u_2)}$, divide (16) by $\phi(f(z)+u_2)$:

$$w_1[\gamma(z)-1]-w_2\lambda(f(z)+u_1)\gamma(z)-w_3(1-\lambda)(f(z)+u_2)=0$$

$$\gamma(z)[w_1-w_2\lambda(f(z)+u_1)]-[w_1+w_3(1-\lambda)(f(z)+u_2)]=0 \tag{17}$$

Since $\gamma(z)$ is a nontrivial exponential function of z, (17) hold for all $z\in Z^*$ only if both terms in brackets are zero for each $z\in Z^*$, i.e.

$$\begin{aligned} w_1-w_2\lambda(f(z)+u_1) &= 0 \\ w_1+w_3(1-\lambda)(f(z)+u_2) &= 0. \end{aligned} \tag{18}$$

Take derivative of (18) again with respect to z, evaluated at $z\in Z^*$:

$$\begin{aligned} w_2\lambda f'(z) &= 0 \\ w_3(1-\lambda)f'(z) &= 0. \end{aligned}$$

Since $\lambda\in(0,1)$ and $f'(z)\neq 0$ for some z, $w=0$. ■

E1.2 Relating the proof to the model

In the previous proof, type distribution is assumed to be a single parameter λ. In practice, I assume a logistic distribution of types based on observables. In particular, I assume that only SAT and family income (a 5-year average) enter the type distribution, i.e., SAT and family (permanent) income summarize all information that correlates with ability. The expected financial aid net of tuition serves the role of the $f(\cdot)$ function in the previous proof. It depends on SAT and all family-background variables. For example, conditional on family permanent income, family assets (which serves the role of the z variable in the previous proof) vary with factors, such as housing prices and stock prices, that are not correlated with ability.[63]

E2. Other student-side parameters

Given the identification results from the mixture of probits, I now discuss the major sources for the identification of other student-side parameters. However, readers should be reminded that all parameters are jointly identified.

E2.1 The probabilities of admissions $\{p_j(A, SAT)\}$

In the data, we observe the admission rates for students given their SAT and family income (y), which is generated via the following equation:

$$\Pr(Admssion\ to\ j|SAT, y) = \sum_A \Pr(A|SAT, y) p_j(A, SAT).$$

In the model, students with the same SAT but different family income (y) will have different admission rates only because they differ in their abilities. That is, family income affects one's admission rates only via its effect on ability. Given the identification of $\Pr(A|SAT, y)$, the correlation between family income and admissions helps to identify $p_j(A,SAT)$.

E2.2 σ_ϵ and σ_ζ

The standard deviation of the i.i.d. idiosyncratic tastes σ_ϵ is identified from the variation in expected financial aid across students within a college, given that student utility is measured in monetary units and that the coefficient on net tuition is normalized to one. The fraction of admitted students who choose not to attend any college serves as the major identification source for σ_ζ, the standard deviation of the outside option shock.

E2.3 Application costs $C(\cdot)$

$C(\cdot)$ is identified mainly from the distribution of the sizes of student application portfolios. As an illustration, consider the case where a student with ability A and idiosyncratic taste ϵ is deciding whether or not to apply. She will apply if

$$p(A, SAT)(\overline{u}_A + f(SAT, B) + \epsilon) - C(1) > 0,$$

or

$$\epsilon > \frac{C(1)}{p(A, SAT)} - \overline{u}_A - f(SAT, B),$$

where $f(SAT, B)$ is the expected financial aid net of tuition. The right-hand side, as a whole, is identified as the type-specific utility from applying in the mixture of probits. Given ability,

[63] This exclusion restriction is sufficient but not necessary for identification. For example, I could allow family assets to enter type distribution as a categorical variable, and to enter the financial aid function as a continuous variable. The within-category variation in assets would be enough for identification.

students with different SAT (hence different family income) will have different probabilities of admission. That is, given ability, $\frac{C(1)}{p(A,SAT)}$ moves with SAT, while $\overline{u}_A$ keeps constant. As a result, $C(1)$ and $\overline{u}_A$ can be separately identified by the correlation between application and SAT among students, whose (SAT, family income) combinations predict them to be of the same ability.[64] The latter relation is identified as the type distribution in the mixture of probits.

E3. Ability values ω :

ω is not point identified, even after normalizing ω_1. The reasoning is as follows: each college j faces discrete (s, SAT) groups of applicants and its admissions policy depends on the rankings of these groups in terms of their conditional expected abilities. These relative rankings remain unchanged for a range of ω's, as do colleges' decisions and the model implications. Knowing that ω is not point identified, I set up a grid of ω's and implement the second step estimation given each of these ω's. The best fit occurs with ω's around $[1, 2, 3]'$; therefore, I fix $\widehat{\omega} = [1, 2, 3]'$. At other values of ω around $[1, 2, 3]'$, the estimates for the other parameters in steps two and three will change accordingly. However, the counterfactual experiment results are robust.[65]

F. Additional Tables

F1. Data:

Table F1.1 Fraction of Applicants by $(SAT, Income)$

%	*Low Income*	*Middle Income*	*High Income*
$SAT = 1$	10.3	16.4	20.0
$SAT = 2$	63.7	74.5	78.8
$SAT = 3$	68.4	81.6	94.4

Low Income : if family income is below 25th percentile (group mean $10,017)

Middle Income : if family income is in 25-75th percentile (group mean $45,611)

High Income : if family income is above 75th percentile (group mean $110,068)

Table F1.2 Portfolio Size by $(SAT, Income)$

	Low Income	*Middle Income*	*High Income*
$SAT = 1$	0.11	0.20	0.23
$SAT = 2$	0.86	0.97	1.22
$SAT = 3$	1.05	1.16	1.54

[64] See Appendix F1 for application patterns by (SAT, income).

[65] Appendix F4 shows the counterfactual experiment results with alternative ω's around $[1, 2, 3]'$.

Table F1.3 Admission Rates by SAT

%	Group 1	Group 2	Group 3	Group 4
SAT= 1	N/A^*	33.3	82.6	87.8
SAT= 2	45.0	72.6	95.5	96.0
SAT= 3	65.7	89.1	93.8	100.0

Num of applicants: SAT1 (123), SAT2 (446), SAT3 (178)

* Not applicable since application is zero.

Group 1: Top private; Group 2: Top public;

Group 3: Other private; Group 4: Other public.

F2. Parameter Estimates

Table F2.1 College Value: A Simulated Example

($\$1,000$)	All		Applicants		Attendees	
$\widetilde{u}_1(A=1,Z=1)$	-234.1	(115.1)	85.9	(30.8)	99.9	(34.7)
$\widetilde{u}_1(A=2,Z=1)$	-222.8	(115.5)	117.1	(43.7)	157.1	(44.9)
$\widetilde{u}_1(A=3,Z=1)$	-57.7	(115.6)	134.4	(58.6)	159.0	(58.6)
$\widetilde{u}_1(A=1,Z=2)$	-74.1	(115.1)	108.9	(50.6)	126.1	(50.8)
$\widetilde{u}_1(A=2,Z=2)$	-62.8	(115.5)	133.9	(57.0)	158.9	(58.8)
$\widetilde{u}_1(A=3,Z=2)$	124.0	(115.6)	187.1	(82.3)	211.3	(79.1)

Each row represents the mean and the standard deviation of Group 1's value for all students, Group 1's applicants and its attendees within a given type.

Table F2.2 Ordered Logit Ability Distribution

cut_1	cut_2	Family Income/1000	SAT= 2	SAT= 3
2.48	5.41	0.01	2.81	3.69
(0.15)	(0.22)	(0.002)	(0.16)	(0.23)

cut_1, cut_2 are the cutoff parameters for the ordered logit.

Table F2.3 Z Type Distribution By Ability

	$A=1$	$A=2$	$A=3$
$\Pr(Z=1\|A)$	0.84 (0.07)	0.74 (0.02)	0.63 (0.08)
$\Pr(Z=2\|A)$	0.16	0.26	0.37

$Z=1$: the type that values public colleges over private colleges.

78% of all students are of type $Z=1$, .

lower-ability students are more likely to be of type $z=1$.

Table F2.4 Ability Distribution: Simulation

%	$A=1$	$A=2$	$A=3$
All	57.2	33.9	8.9
$Z=1$	60.9	31.9	7.2
$Z=2$	43.5	41.3	15.2

Simulation based on the estimates in Tables F2.3 and F2.4.
Ability distribution among all students and by Z type.

Table F2.5 Financial Aid and Ex-post Shock to Outside Option

	General aid		College-Specific Aid	
	Coefficient	Std. Error	Coefficient	Std. Error
Constant	-4907.8	(817.1)	-13664.3	(1756.3)
Black	1490.7	(915.2)	3277.2	(1033.2)
Family Income/1000	-25.3	(10.7)	-46.1	(9.2)
Family Assets/1000	-4.1	(2.7)	-4.5	(2.4)
$SAT=2$	3993.1	(854.5)	8141.6	(1837.6)
$SAT=3$	6081.6	(1079.3)	15227.5	(1843.6)
Sibling in College[a]			4336.6	(897.9)
$(SAT=2)\times public$			-4068.0	(2487.1)
$(SAT=3)\times public$			-7821.9	(2563.7)
Group 2			3993.8	(2870.7)
Group 3			9511.5	(1811.9)
Group 4			6855.0	(2278.0)
σ_η (aid shock)	8034.1	(169.3)	9758.8	(285.9)
σ_ζ (outside shock)		10433.4	(2916.1)	

a. Whether the student has some siblings in college at the time of application.

Table F2.6 Capacities (%)

κ_1	κ_2	κ_3	κ_4
0.96 (0.15)	4.59 (0.13)	10.82 (0.09)	24.56 (0.21)

Table F2.7 Tuition Weights

$j \in \{1,3\}$ private		$j \in \{2,4\}$ public	
m_{j1}	m_{j2}	m_{j1}	m_{j2}
0.067 (0.002)	-0.001 (0.0004)	0.007 (0.003)	-0.0006 (0.0001)

Tuition is measured in thousands of dollars.

College's tuition preference: $M(t_j; m_j) = m_{j1}t_j + m_{j2}t_j^2$.

m is restricted to be the same within the public/private category.

F3. Model Fit

Table F3 Tuition Fit in Step-3

	Top Priv.	Top Pub.	Low Priv.	Low Pub.
Data	27009	5347	17201	3912
Best Response	27579	4954	18010	3921

F4. Robustness Check: Counterfactual Experiments With Alternative ω[66]

F4.1 Creating Opportunity

Table F4.1.1 Increasing Supply

Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26940	4773	19907	4392
New SPNE	27534	6890	18176	98

Table F4.1.2 Increasing Supply

Admission Rates

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	44.1	82.7	99.0	98.2
New SPNE	47.3	95.3	99.8	100.0

Table F4.1.3 Increasing Supply

Attendance Rate

Base SPNE	New SPNE
40.9	43.0

[66] This subsection shows the results for $\omega = [1, 1.4, 2]'$. For other ω's around $[1, 2, 3]$, the results are similarly robust.

F4.2 Ignoring Signals

Table F4.2.1 Ignore Signals

Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26162	4555	19173	3925
New SPNE	27415	7478	20223	5386

Table F4.2.2 Ignore Signals

Distribution of Portfolio Sizes

	Size= 0	Size= 1	Size= 2	Size= 3	Size= 4
Base SPNE	55.7	31.5	9.6	2.9	0.2
New SPNE	56.0	33.1	8.5	2.3	0.1

Table F4.2.3 Ignore Signals

Admission Rates

%	All		$SAT=1$		$SAT=2$		$SAT=3$	
	Base	New	Base	New	Base	New	Base	New
Group 1	43.6	57.7	N/A	N/A	28.2	7.5	66.7	100.0
Group 2	82.0	88.4	N/A	N/A	72.8	81.9	99.4	100.0
Group 3	98.6	99.9	91.7	99.3	100.0	100.0	100.0	100.0
Group 4	97.1	99.0	96.2	93.6	97.7	100.0	97.8	100.0

Table F4.2.4 Ignore Signals

Ability Distribution Within Each Destination

%	Group 1	Group 2	Group 3	Group 4	Outside
Base SPNE					
$A=1$	3.7	0.2	7.4	0.9	94.9
$A=2$	6.5	15.7	80.9	91.9	3.0
$A=3$	89.9	84.1	11.7	7.2	2.1
New SPNE					
$A=1$	4.2	0.2	8.1	0.9	94.8
$A=2$	8.9	17.4	80.2	92.0	2.8
$A=3$	86.9	82.4	11.6	7.1	2.4

Table F4.2.5 Ignore Signals

Mean Student Welfare

\$	Base SPNE	New SPNE	Change
All	41402	39712	−1690
$A = 1$	677	747	30
$A = 2$	98630	95132	−3498
$A = 3$	84673	78594	−6079

F4.3 Dropping SAT

Table F4.3.1 Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26162	4555	19173	3925
Drop SAT (Group1)	27654	4976	20082	4877
Drop SAT (All)	28889	6989	20109	4358

Table F4.3.2 Admission Rates

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	43.6	82.0	98.6	97.1
Drop SAT (Group1)	36.7	81.0	99.5	98.3
Drop SAT (All)	38.1	83.0	100.0	97.9

Table F4.3.3 Fraction of High-Ability Students

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	89.9	84.1	11.7	7.2
Drop SAT (Group1)	76.6	84.5	11.9	7.3
Drop SAT (All)	77.0	82.0	11.8	7.5

Table F4.3.4 Mean Income

\$	Group 1	Group 2	Group 3	Group 4
Data	89663	95422	72547	64247
Base SPNE	93802	90610	65916	64657
Drop SAT (Group 1)	82463	91002	65974	64524
Drop SAT (All)	82730	87638	66176	64471