THE VALUE OF STOCK OPTIONS TO NON-EXECUTIVE EMPLOYEES

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Abstract

This study empirically investigates the value employees place on stock options using information from the option exercise behavior of individuals. Employees hold options for another period if the value from holding them and reserving the right to exercise them later is higher than the value of exercising them immediately and collecting a profit equal to the stock price minus the exercise price. This simple model implies the hazard describing employee exercise behavior reveals information about the value to employees of holding options another time period. We show the parameters of this model are identified with data on multiple option grants per employee and we apply this model to the disposition of options received in the 1990s by a sample of over 2000 middle-level managers from a large, established firm outside of manufacturing. Exercise behavior is modeled using a random effects probit model of monthly exercise behavior that is estimated using simulated maximum likelihood estimation methods. Our estimates show there is substantial heterogeneity (observed and unobserved) among employees in the value they place on their options. Surprisingly, we find most employees value their options at a value greater than the option's Black-Scholes value. We discuss the implications of our work in light of a recent Financial Accounting Standards Board (FASB) proposal to require firms to expense options.

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The Value of Stock Options to Non-Executive Employees

Over the past 15 years there has been dramatic growth in the use of stock options for senior-level executives (e.g. Hall and Leibman, 1998 and Murphy, 1999) and by the mid-1990s substantial growth in the use of options for non-executive employees had begun that was only partially dampened by the market adjustment in 2001. Using Black-Scholes to value options, stock options to managers and employees who are not among the top 5 highest paid employees in the firm has grown from less than 85 percent of total options granted to employees in the mid-1990s to over 90 percent by 2002 (Hall and Murphy, 2003). A variety of explanations have been offered for the rapid and widespread growth in employee stock options including providing incentives to employees, motivating employees to sort, employee retention, and financing constraints (Bergman and Jenter, 2004, Core and Guay, 2001, Kedia and Mozumdar, 2002, Lazear, 2001, and Oyer and Schaefer, 2004a,b).

Understanding the specific reasons for the use of employee stock options by firms requires knowledge of the costs and benefits to both firms and employees of options relative to cash compensation. Because the payoff to an employee from stock options depends on the firm's stock price which is beyond the complete control of even the most senior manager, option based compensation exposes risk averse employees to uncertainty that they must be compensated for to make them indifferent between an option and cash compensation costing the firm an equal amount. Economically rational explanations for the use of stock options requires that the risk premium a firm pays to employees to accept options instead of cash is offset by the value of the benefits the firm expects to receive from granting the options. These benefits from options may be reflected in lower agency costs, lower turnover, increased commitment by workers to the firm, etc.

Understanding where firms use stock options requires estimates of both the dollar value employees place on options and the magnitude of the benefits firms receive by granting stock options to employees. In this study we take a step toward making this evaluation by empirically estimating the value a sample of middle managers place on the stock options they received from their employer.

Previous research emphasizes the distinction between the cost of options to firms and the likely lower value employees place on options due to employee risk aversion and the characteristics of employee stock option grants (Lambert, Larcker, and Verecchia, 1991, and Hall and Murphy, 2000, 2002, 2003). However, this research has not produced an estimate of the value employees place on options inferred from the observed behavior of a sample of option holders. Hall and Murphy (2002) calculate the value executives place on options by making assumptions about the shape of an executive's utility function, their level of risk aversion and the share of their wealth invested in their firm's stock. While this research provides valuable insight into the *possible* trade-offs between options and cash compensation, their calculations are a function of the assumptions they make about executives and are not inferred from the behavior of employees who have received options.

In this study we empirically estimate the dollar value employees place on employee stock options (ESO) using a method that requires few assumptions about the form of the utility function or the level of employee risk aversion. Our analysis is based on the observation that an employee will choose to hold an option for another period (e.g. day, week, or month) if the utility of the income she would receive (e.g. stock price – exercise price) by immediately exercising the option is less than the value from holding the option and reserving the right to exercise it on a later date. Conversely, if she exercises in the current period, then we know the value from not exercising the option is less than what she gains now by exercising the option and receiving an amount equal to the stock price minus the exercise price. Our model implies the decision to exercise or hold an option for another period provides the critical piece of information needed to infer the value employees place on an option at any point in time when the option could be exercised for a profit (e.g., stock price – exercise price > 0).

The rest of the paper is organized as follows. In section I we introduce the theoretical model used to estimate the value employees place on options. Section II discusses model identification and estimation methods. The data are described in Section III. Section IV presents the results and Section V summarizes our research and discusses the implications our findings for additional research and the current public policy debate regulating the information firms are required to disclose about the cost of employee stock options.

I. A Model of Employee Exercise Decisions

Like previous efforts to model the value of stock options to employees, the starting point for this research is the pioneering work of Black and Scholes (1972) and Merton (1973) who describe how rational investors holding a diversified portfolio value tradable stock options. The famous diagram shown in Figure 1 summarizes the basic relationship between the value of a call option and the firm's stock price and the option's exercise price. Mathematically, the Black-Scholes value (BSV) of the option in Figure 1 is equal to the expected value of the stock price on the expiration date of the option minus the option's exercise price discounted at the risk free interest rate. Stock returns each period are assumed to be normally and independently distributed so the expected stock price on the expiration date is described by the mean of a truncated log normal distribution multiplied times the probability the stock price is greater than the exercise price. They show the value of holding an option is a function of six variables- the risk free interest rate, the expiration date of the option, the variance in the firm's stock returns, the firm's dividend rate, the option exercise price and the current stock price. The BSV declines or shifts in toward the kinked intrinsic value line (e.g., Max(0, Stock price-Exercise Price)) as the option expiration date approaches, the firm's dividend rate increases, the risk free interest rate increases or the variance in the firm's stock returns declines.

The Black-Scholes model predicts that a diversified investor will never exercise an American call option prior to the moment before it expires because at any earlier date the expected gain from holding the option until the expiration date is greater than the intrinsic value of the option. In other words, the possibility of a stock price increase between the current period and the expiration date makes the Black-Scholes value greater than the profit that could be made by exercising the option at any earlier time and receiving the option's intrinsic value (Stock Price-Exercise Price). It is important to note that the model makes no prediction about how long an individual investor will own a market traded option. The model only predicts that an investor will sell the option rather than exercise an option to increase liquidity or rebalance his portfolio.

Applying Black-Scholes to ESO is thought to give a poor estimate of the value of the option to an employee who receives it as part of her compensation package. The empirical evidence cited to support this conclusion is that employees frequently exercise ESO "early" or well before the option's expiration date (Huddart and Lang, 1996; Carpenter 1998). In the firm we study 86 percent of employees exercised their options prior to the month before the options expired and half of the sample exercised some of their options at least 27 months prior to their expiration.[1] In a simulation study, Hall & Murphy (2002) show risk averse executives who also have a substantial portion of their portfolio invested in the firm may exercise "early" to lock-in gains from large stock price increases and diversify their portfolio. Heath, Huddart and Lang (1999) find that employees tend to exercise options when the firm's stock price exceeds a target or referent price based on recent stock price highs. Since an employee must typically forfeit her options if she leaves the firm, early exercise decisions may also be caused by voluntary or involuntary employee turnover (Carpenter 1998).

The observation that employees frequently exercise ESOs "early" compared to the Black-Scholes prediction for market traded options is because employees cannot sell the options they receive from their employer and must exercise the options if they wish to liquidate their position to diversify their portfolio, meet a budget constraint or prepare for an anticipated departure from the firm. This fundamental feature of ESOs is the basis for our model because it implies there is substantial heterogeneity among employees in the value of holding an option and information about the option's value is revealed each period by observing whether or not a vested option is exercised. If an option is not exercised in a period then a researcher can conclude the value to the employee of holding the option and reserving the right to exercise it in a later period is greater than the value of exercising the option and receiving the intrinsic value (stock price-exercise price). On the other hand, if an employee exercises an option in a period, then the analyst knows the value of holding the option another period is less than what is gained by exercising

[1] These data are for the first large ESO grant awards made to the middle level managers included in this study.

the option and receiving a payment equal to the stock price minus the exercise price. Thus, the decision to exercise or hold an option for another period is an indicator of the value an employee places on an option at a point in time and, therefore, the exercise hazard rate reveals information about the option's value.

Formally, define the following terms: SP_t = stock price at time "t" where "t" indexes the time since the option vested, $EP_{k,j}$ = exercise price for option "k" granted to employee "j" , $EVF_{k,j,t}$ = Employee Value Function or the the utility (in dollars) to person "j" in period "t" from holding option "k" another time period. For vested options held by an employee, in each period "t" we assume the employee decides whether to hold the option another period or to exercise the option by comparing the utility from exercising the option with the utility of holding the option for another period. Since $SP_t - EP_{k,j}$ equals the certain cash payment the employee receives from exercising the option, the option will be exercised if this cash payment is greater than the monetary value to the employee of holding the option another period:

(1a) $(SP_t - EP_{k,j}) > EVF_{k,j,t}$

and the option will be held another period if

(1b) $(SP_t - EP_{k,j}) \leq EVF_{k,j,t}$.

Let $EVF_{k,j,t}$ be a linear function of a set of observable characteristics of the employee, the option and the market (X) and an unobserved randomly distributed error term that is indexed by option grant, person and time:

(2) $EVF_{k,j,t} = X_{k.j,t}\beta + v_{k,j,t}$, where $\upsilon \sim N(0,\sigma^2_v)$.

Equations (1) and (2) describe the probability an option will be exercised in period "t" conditional on the exogenous variables. Let $I_{k,j,t} = 1$ if option "k" held by employee "j" is exercised in period "t", and zero otherwise. The probability $I_{k,j,t} = 1$ is:

(3a) $\Pr(I_{k,j,t} = 1) = \Pr((SP_t - EP_{k,j}) > EVF_{k,j,t})$

(3b) $\Pr(I_{k,j,t} = 1) = \Pr[(SP_t - EP_{k,j}) > X_{k,j,t}\beta + v_{k,j,t}]$

(3c) $\Pr(I_{k,j,t} = 1) = \Pr[v_{k,j,t} < (SP_t - EP_{k,j}) - X_{k,j,t}\beta)]$

(3d) $\Pr(I_{k,j,t} = 1) = \Phi[((SP_t - EP_{k,j}) - X_{k,j,t}\beta))/\sigma_v]$,

where $\Phi(.)$ is the CDF for a standard normal variable.

The right hand side of equation (3d) shows the probability an option is exercised in period t depends on the difference between the value from exercising the option immediately ($SP_t - EP_{k,j}$) and holding the option for another period. This is shown in Figure 2. The probability the option is exercised is the shaded area or the probability that $v_{k,j,t}$ is sufficiently small to cause EVF to be less than SP-EP.

For model identification and estimation reasons that are explored in the next section, the error term in the preceding model is assumed to be normally distributed. This assumption also implies the value to an employee of holding an option another period could assume either very "large" or "small" values that are inconsistent with previous predictions made about the value employees place on options. As noted earlier, previous research (e.g. Lambert, Larcker, and Verecchai, 1991) has assumed the value of an option to an employee is *less* than the option's Black-Scholes value because of the design of employee stock option plans and the risk aversion of employees. Employees receiving stock options are likely to be "over-invested" in their firm relative to an outside investor because they cannot sell the options and are heavily invested in the firm through firm specific human capital and implicit or explicit deferred compensation (pensions) that depends on the success and survival of the firm. This lack of diversification and the possibility of having to exercise "early" because of turnover risk leads to the prediction that employees value options at less than their BSV.

On the other hand, employees may have great difficulty using Black-Scholes as a benchmark in their personal valuations because of the limited information employees have about options and option valuation theory. There is no market information that employees can use to aid their valuation because the 10 year term of a typical ESO is far longer than market traded options that may exist for the firm. Employees also have no way of determining the value an outside investor or lender would place on the option since employees cannot sell their options or use them as collateral for a loan. Without market information, it is unlikely the typical employee will be knowledgeable about option pricing theory or able to solve an option pricing problem that won a Nobel Prize in economics for Merton and Scholes. These arguments suggest the unconstrained model described by Equations (1)-(3) is a plausible empirical specification for the EVF where employees must make valuation judgments with very little market information about the option's value.

The one piece of market information employees do observe is the firm's stock price. Previous research shows *any* rational model employees use to value their options predicts the value is less than the firm's stock price. Given the choice between an option and share of stock priced at SP, an employee is always better off owning the share of stock (Brealey, Myers & Marcus 1995). If SP ≤ EP on the expiration date the option is worth nothing but the share of stock is still worth SP>0. If SP > EP the share of stock is also worth more than the option as the option is worth SP-EP and the share of stock is worth SP. Thus, a rational employee will never value an option at more than the stock's current price. This prediction is used to specify an alternative constrained specification for EVF where EVF cannot exceed the firm's stock price. The distribution of EVF when truncated at SP is

(4a) $EVF_{k,j,t} = X_{j,t}C + v_{j,t}$

(4b) $f(\nu \mid EVF_{k,j,t} < SP_t) = (1/\sigma_v)\, f((EVF_{k,j,t} - X_{j,t}C)/\sigma_v)\, \Phi((SP_t - X_{j,t}C)/\sigma_v)$

The probability an option is exercised in period t conditional on (SP-EP), X and v is:

(5) $$\Pr(EVF_{k,j,t} < (SP_t - EP_{k,j})) = \Phi\left(\frac{(SP_t - EP_{k,j})}{\sigma_v} - X_{j,t}\frac{C}{\sigma_v} \right) / \ \Phi\left(\frac{SP_t}{\sigma_v} - X_{j,t}\frac{C}{\sigma_v} \right).$$

and the probability an option is not exercised in period t is $1 - \Pr(EVF_{k,j,t} > (SP_t - EP_{k,j}))$. We refer to this model as a truncated probit model. We estimate both the constrained and unconstrained models and then evaluate which model best fits the data.

II. Model Identification and Other Econometric Issues

In the models developed above exercise decisions each period are based on whether or not the EVF is greater or less then a threshold value equal to the option's intrinsic value, (SP-EP). In most studies where the dependent variable is a discrete, binary outcome, only the standardized coefficients, β/σ, are identified because the threshold determining which outcome is observed is unobserved and arbitrarily set to zero. While the standardized coefficients for either the truncated or untruncated model will predict exercise decisions, inferring the dollar value of an option from exercise behavior requires estimates of the unstandardized coefficients, the βs and Cs. Because the exercise decision is based on the value of EVF relative to an *observed* threshold, the unstandardized EVF coefficients are identified if options data have different exercise prices.

A natural way to parameterize the EVF is to make it a function of the Black-Scholes value of the option because it reflects rational expectations about the expected value of the option on its expiration date, or:

(6) $EVF_{k,j,t} = \alpha(BSV_{k,t}) + v_{k,j,t}$

In this specification the probability an option is exercised in period t equals:

(7) $\Pr(EVF_{k,j,t} < (SP_t - EP_{k,j})) = \Phi(\frac{(SP_t - EP_{k,j})}{\sigma_v} - \frac{\alpha}{\sigma_v} BSV_{k,t})$

This equation shows α and σ_v are separately identified because the parameter on the variable (SP-EP) equals $1/\sigma_v$ and the parameter on BSV equals α/σ_v. Alternatively, if all the options in the dataset have the same exercise price then in any period "t" (SP-EP) is a constant, making σ_v unidentified which, in turn, means α is unidentified and only the standardized coefficient on BSV, α/σ_v , can be estimated.[2] Thus, option data with multiple exercise prices is required to estimate the "structural" parameters of the EVF described by Equation (1).

In most ESO plans the exercise price and expiration date for options granted to employees on a specific date are identical for all employees. The exercise price is typically set equal to the firm's stock

[2] There are other settings where researchers have been able to identify σ in a probit model. For example, empirical studies of the fair wage beliefs of arbitrations under final offer arbitration identify σ from variation in the final offers presented to arbitrators. See Ashenfelter & Bloom (1984), Olson & Jarley (1991).

price on the grant date and the options expire on the same day 6-10 years later. Exercise data with only one grant and expiration date creates a second identification problem for Eq. (7). If the dataset includes options that have the same exercise price and expiration date, then all the options have the same Black-Scholes value in any period. Thus, even if (SP-EP) were excluded from the probit equation, EVF cannot be a function of the BSV of the option without variation in the time remaining until the options expire. In this study we overcome this identification problem by using data on options granted to a common group of employees on multiple dates. If all of the options expire 10 years from their grant dates, on any calendar day "t" the time left until the options expire will vary and this variation produces variation in the Black-Scholes value of the options on day "t" and allows us to estimate how exercise decisions are affected by difference in the Black-Scholes value of the option. To summarize, it is possible with data on grants with different exercise prices and expiration dates to identify the EVF in period "t" and permit the EVF to be a function of the Black-Scholes value of the option in period "t" using only information on whether or not options are exercised in period "t".

We now provide a more formal proof of how α and σ_v in Eq. (7) are identified using exercise activity in period t for options that have different exercise prices and expiration dates. Assume a firm provides one option to each employee under two option grant plans – plan "A" and "B". Options in the two plans are granted on two different dates, at different exercise prices and all options expire after 10 years. In period "t" when the options are "above water" (e.g., SP>EP), data are collected on the fraction of unexercised options that are exercised in period "t" from each option plan. Thus, we have the following data: $(SP_t\text{-}EP_A)$, $(SP_t\text{-}EP_B)$, BS_A, BS_B, P_A and P_B where P_j is the fraction of unexercised options from grant j exercised in period t. Equation (7) shows the relationships between these variables is described by the following two equations:

$$P_A = \Phi\left(\frac{(SP_t - EP_A)}{\sigma_v} - \frac{\alpha}{\sigma_v} BS_A \right) \text{ and } P_B = \Phi\left(\frac{(SP_t - EP_B)}{\sigma_v} - \frac{\alpha}{\sigma_v} BS_B \right).$$

The two unknown parameters, α and σ_v, can be solved from these two equations.

The preceding discussion describes the data necessary to estimate the parameters of the EVF using data on exercise activity from a single period after options from the two grants have vested. To take advantage of the data we have on exercise decisions over an 8 year exercise window we model the time until an employee first exercises at least one option from grant k by dividing the 8 year exercise window into 96 months and estimate discrete time hazard models that describes the month in the exercise window that an option from a grant is first exercised. The probability an option from grant k=1 to employee "j" is first exercised in period M_1 and an option from grant k=2 is first exercised in period M_2 is equal to the probability an option from the first grant is not exercised in periods 1 through M_1-1 and then exercised in period M_1 and the probability an option from the second grant is not exercised in periods 1 through M_2-1 and then exercised in month M_2. If $D_{k,j}$ is the month an option from grant k is first exercised, then this sequence of events occurs with the following probability

(8) $\Pr(D_{1,j} = M_1, D_{2,j}=M_2) = \Pr[(SP_1-EP_{1,j,1}) < EVF_{1,j,1},\ (SP_2-EP_{1,j,2}) < EVF_{1,j,2},\ldots(SP_{M-1}-EP_{k,j,M1-1}) < EVF_{k,j,M1-1}, (SP_M-EP_{k,j,M1}) \geq EVF_{k,j,M1}\ , (SP_1-EP_{2,j,1}) < EVF_{2,j,1},\ (SP_2-EP_{2,j,2}) < EVF_{2,j,2},\ldots(SP_{M2-1}-EP_{2,j,M2-1}) < EVF_{2,j,M2-1}, (SP_M-EP_{2,j,M2}) \geq EVF_{2,j,M2}]$

Several adjustments were made to Eq. (8) to account for months when the model is unidentified because nothing can be learned about exercise behavior. In months in the interval $[1,M_i]$ when an option is "below water", (e.g., SP-EP < 0), nothing can be learned about the value of holding an option another period because nobody will exercise their options at a loss.[3] Also, in month 96 when options from a grant expire rational employees will always exercise their outstanding options if a profit can be made (SP > EP). Therefore, month 96 is excluded from the likelihood function and the contribution to the likelihood function for grants not exercised in periods 1-95 is treated as right censored in month 95. The last adjustment was for option grants that had not expired but no options from the grant had been exercised by an individual at the end of the study period. These grants were also treated as right censored at the last observed time period (i.,e., the last term in Eq (8) was $(SP_M-EP_{k,j,M}) < EVF_{k,j,M.}$).

[3] In our sample the options were always "above water" during the study period so this constraint was never binding.

The major methodological issues yet to be discussed are the assumptions made about the structure of the error term in Equation (1) and the related issue of how the model was estimated. We could have assumed the v's are normally and independently distributed across option grants, time periods and employees. This assumption greatly simplifies the estimation because Eq. (8) reduces to an independent Bernoulli model (Heckman 1981a) equal to the product of M_1+M_2 individual binary probit functions or:

(9) $\Pr(D_{1,j} = M_1, D_{2,j}=M_2) = \Pr[(SP_1-EP_{1,j,1}) < EVF_{1,j,1}] \times \Pr[(SP_2-EP_{1,j,2}) < EVF_{1,j,2}] \times\ldots$

$\Pr[(SP_{M1-1}-EP_{1,j,M1-1}) < EVF_{1,j,M1-1}] \times \Pr[(SP_{M1}-EP_{1,j,M1}) \geq EVF_{1,j,M1}] \times \Pr[(SP_1-EP_{2,j,1}) < EVF_{2,j,1}]$

$\times\ldots\ldots\Pr[(SP_{M2}-EP_{2,j,M2}) \geq EVF_{2,j,M2}]$

This model is easily estimated by "pooling" the data across option grants, employees and months then estimate the parameters β and σ using a standard probit program.

Previous research on modeling state dependence in discrete time processes shows the independent Bernoulli model of the exercise hazard function is likely to be incorrectly specified because the error terms each period are correlated across observations (Heckman 1981a). Previous research suggests this correlation reflects, among other factors, employee risk aversion. More risk averse employees are more likely to have negative values of v or a "low" EVF (conditional on the observables) and seek to "lock-in" options gains by exercising options earlier relative to less risk averse individuals with "high" values of υ (Hall & Murphy, 2002). These predictions generate a positive correlation between the error terms in Eq. (8) across time periods and option grants for an individual that will produce biased estimates of the exercise decision process (Heckman 1981b).

A more realistic but still tractable assumption for the error term is a random effects (RE) specification where $v_{k,j,t} = u_j + \varepsilon_{k,j,t.}$ and u and ε are each normally distributed. Constant unobserved individual effects are captured by u_j and $\varepsilon_{k,j,t}$ captures purely random factors independent across time periods, individuals and option grants. For an individual, the diagonal elements of the error variance-covariance matrix equal $(\sigma^2_u + \sigma^2_\varepsilon)$, the off-diagonal covariance terms equal σ^2_u,and across individuals the covariance terms are equal to zero. The constant covariance terms for an individual reflects the unobserved individual effects like risk aversion and causes the error terms to have a constant correlation equal to $\sigma^2_u/(\sigma^2_u + \sigma^2_\varepsilon)$ across time periods and grants for an individual (Heckman, 1981a). To estimate this correlation and control for the constant, unobserved individual heterogeneity requires data for a sample of individuals that receive options grants on several occasions over the study period. In the firm we study most employees received options from two grants on two different dates in the early 1990s.

One way of estimating the model that controls for the unobserved individual component is to estimate a fixed effect model that includes a set of dummy variables for each person. While this strategy provides unbiased estimates in a linear model, it does not give consistent estimates for nonlinear models such as a probit model (Heckman 1981b). The alternative to the fixed effect model which does produce consistent estimates is a random effect (RE) probit model (Heckman, 1981a). Since the probability described by Eq. 8 is person j's contribution to the likelihood function, estimating the parameters of the model by MLE requires the integration of an $M_1 + M_2$ dimensioned multivariate normal distribution. Good numerical approximations exist for calculating this integral with two or three time periods and for up to, perhaps, 4-5 dimensions, Gaussian quadature procedures produce accurate estimates of the underlying probability (Moffit & Butler, 1982).[4] In this study option exercise decisions from a grant can be exercised anytime during the 96 month exercise window and most employees in our sample hold options from two different option grants. Thus, we need to estimate probabilities for a multivariate normal distribution with more dimensions than can be feasibly calculated using existing numerical methods.

Research over the last 20 years (Lerman & Manski 1981, Geweke 1991, Hajivassiliou and McFadden 1990, Keane 1994) and the speed of desktop computers makes simulation estimation methods a feasible way to estimate our model. The GHK (Geweke, Hajivassiliou & Keane) estimator appears to be the preferred estimator when the error structure is relatively unrestricted. However, the random effects

[4] This is the method used by the xtprobit command in STATA. As the Stata v. 7.0 manual on the xtprobit command notes "The quadrature formula requires that the integrated function be well approximated by a polynomial. If the number of time periods becomes large (as panel size gets large), [the function] is no longer well-approximated by a polynomial."

specification permits a simpler and faster simulation method described by Train (2003, chpt 5).[5] If an analyst could observe u_j, then

(10) $\Pr(D_{1,j} = M_1, D_{2,j}=M_2 | u_j=u^*_j) =$

$$\prod_{t=1,M1}(1-I_{1,j,t})[1-\Phi(((SP_t - EP_{1,j}) - X_{1,j,t}\beta - u_j^*)/\sigma_\varepsilon)] + I_{1,j,t}[\Phi(((SP_t - EP_{1,j}) - X_{1,j,t}\beta - u_j^*)/\sigma_\varepsilon)]x$$
$$\prod_{t=1,M2}(1-I_{2,j,t})[1-\Phi(((SP_t - EP_{2,j}) - X_{2,j,t}\beta - u_j^*)/\sigma_\varepsilon)] + I_{2,j,t}[\Phi(((SP_t - EP_{2,j}) - X_{2,j,t}\beta - u_j^*)/\sigma_\varepsilon)]$$

An estimate by simulation of $\Pr(D_{1,j} = M_1, D_{2,j}=M_2)$ for a particular value of $\tilde{\sigma}_u$ is obtained by: (a) creating a 1 x N matrix of values randomly drawn from $N(0,\tilde{\sigma}_u^2)$, (b) calculating Eq. (10) for each of the N possible values of u_j and (c) taking the mean of the N values from step (b) as an estimate of Eq. (8) (d) repeating steps (a)-(c) for each observation within each iteration of the maximum likelihood routine when an estimate of Eq (8) is required.[6]

For the untruncated model a "standard" RE probit model was estimated by setting $\sigma_\varepsilon=1$ so the coefficient for variable X_i was "standardized" or equal to $-\beta_i/\sigma_\varepsilon$, the coefficient on (SP-EP) equals $1/\sigma_\varepsilon$ and the within person correlation between the error terms is $\sigma^2_u/(1+\sigma^2_u)$. Since σ_ε is identified in our data because of sample variation in the options' intrinsic value, the unstandardized coefficients (β_is), σ^2_u and σ^2_ε were estimated. This was done to obtain the standard errors for the parameters estimates of the EVF. When the truncated probit model was estimated Eq. (10) was appropriately modified using Eq. 5. We assumed the error truncation was on the ε component of the error term so that $\Pr(D_{1,j} = M_1, D_{2,j}=M_2)$ was calculated by the steps a-d where u was drawn from an untruncated normal distribution.

III. The Data and Empirical Specification

We have collected a set of very specific information on stock option grants made to a sample of 2180 middle level managers in a large, established firm outside of manufacturing that has many tens of

[5] See also Guilkey & Murphy (1993).

[6] We used an N of 80. Estimating the models by simulated maximum likelihood took 5-10 hours of computing time on a 2.4 GHz Pentium 4 single processor computer. The estimation was done using Gauss™ v. 6.00.

thousands of employees, billions of dollars in sales and locations throughout the United States. [7] The data we use are from several stock option grant programs where a group of middle level managers were given the opportunity to receive options as part of a stock purchase plan where participants were eligible to receive options proportional to their salary. As discussed above, an important feature of the data required to identify the model are data on options granted at different exercise prices. In our sample employees received options at 13 different exercise prices on 13 different days with the majority of the grants occurring on two calendar dates where one exercise price was almost twice the magnitude of the other exercise price.[8] We use data on the exercise decisions made by employees for options received in up to the first two grants an employee participated in during the 1990s where the options from the grants had vested before the fall of 2003. Table 1 contains descriptive statistics on the sample of option grants. A total of 3712 options grants were received by the 2180 employees and all but 1127 of these grants were exercised during the study period (e.g., were not right censored). An interesting feature of the experience in this firm is that not all options from a grant were exercised in the same month. In about 58 percent of the option grants where we observe the first exercise date (e.g., the exercise time is not right censored), the employee exercised all of the options in the grant and options exercised on the first exercise day account for 77 percent of the total options that could have been exercised. In this study we model only the time until the first option from a grant is exercised. In future work we intend to model the decision to exercise all or part of a grant and the time to the second exercise date for employees that exercised only part of their options on the first exercise date.

In addition to the terms of each option grant (exercise price, vesting date, expiration date and number of options in the grant), we also have information on the employee's salary and tenure with the firm. A final important feature of the sample is that it excludes managers who joined the firm during the 1990s and managers who received options during the 1990s but left the firm before the Fall of 2003.

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[7] A condition for obtaining data from the firm included a promise that we would not reveal the identity of the firm. Therefore, we cannot provide a more detailed description of the firm or make the data available to other researchers.

[8] In all cases the options were granted "at the money." That is, the exercise price was equal to the firm's stock price on the day of the grant.

Thus, this analysis focuses on the exercise decisions of long-tenured, stable employees who were not exercising options in anticipation of leaving the firm. Excluding option recipients who left the firm during the study period simplifies the analysis because we don't have to jointly model turnover and exercise decisions.[9]

Figures 3a and 3b report data on the exercise decisions made by employees in the sample. Figure 3a is the survival curve for the time to first exercise date for the 3712 option grants. Twenty-five percent of the grants are exercised by the 34th month following vesting, the median exercise time is 69 months and the 75th percentile of the exercise distribution is 90 months. Figure 3b plots the smoothed hazard rate over the 96 month period and shows a relatively stable but low hazard rate during the first 72 months and then a sharp increase in the final year as unexercised options are exercised before they expire. Over the first 72 months of the exercise window an average of 1.11 percent of unexercised option grants were exercised for the first time in each month.

The exogenous variables in the model determining the EVF fall into three categories; characteristics of the option grant, characteristics of the employee, and financial market indicators that may influence employee exercise decisions. The characteristics of the option include the BSV of the option, BSV squared and cubed, and the number of shares the individual received in the option grant. The closed-form solution for the Black-Scholes value of the option was computed for each month the option was at risk of being exercised using the maximum closing stock price during the month.[10] The number of shares in the option grant was included to capture the possibility that an employee holding more options may exercise earlier to "lock-in" gains and diversify their wealth.

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[9] Firms often report that they provide options to improve employee retention. Modeling option exercise decisions and turnover is difficult. While options might reduce turnover, employees that are planning to leave the firm can be expected to exercise vested options before their departure. This creates a positive correlation between exercise decisions and the probability of turnover in the "near term." Modeling exercise decisions and turnover behavior jointly would require a more elaborate competing risk framework and data on employees who did not receive options.

[10] The other parameters used to calculate the Black-Scholes value in each month in the exercise window are the usual values – the firm's observed dividend rate, a risk free rate of return equal to the yield on 10 year treasury bonds and the standard deviation of yearly firm returns.

While the time until the option expires is included in the Black-Scholes formula, Figures 3a and 3b suggest time since vesting should be included separately to fully capture the spike in exercise activity as the expiration date approaches. Therefore, the model includes a linear time trend (T) measuring months since the option vested and the following six splines: Max(0, T-12), Max(0,T-24), Max(0,T-36), Max(0,T-48), Max(0,T-60) and $(\text{Max}(0,T\text{-}60))^2/100$. The EVF function also includes a dummy variable equal to "1" in the first month the option grant is vested to capture the possibility of a spike in exercise activity caused by the two year vesting period and unobserved individual heterogeneity. For each month an option's intrinsic value and BSV were calculated using the maximum stock price in the month.

The market condition variable included in the EVF was the monthly return to holding a portfolio equal to the Dow Jones Industrial Average and the monthly return for the firm. If employees compare the return to holding stock in the firm with market alternatives, we predict the probability an option is exercised will increase with the return on the overall market (holding the firm's return constant). The individual characteristics in the EVF were the employee's tenure with the firm and the natural log of his/her monthly salary. The tenure variable was calculated each month and the salary variable was adjusted annually to reflect salary changes over the study period. Since the analysis compares the maximum profit that can be made by exercising an option grant in a month with the value an individual places on holding the grant another period for each month over potentially a 10 year period, for each calendar month the (SP-EP), the Black-Scholes value and the employee's salary were converted to real dollars using the CPI.

IV. Empirical Results

Table 2 reports estimates of the parameters of the EVF obtained by estimating the month when options from a grant were first exercised during the 96 month exercise window. The columns in Table 2 report estimates for different specifications of the EVF and exercise decisions. The columns labeled "Hazard Function" report standardized probit coefficients, $-\beta_i/\sigma_\varepsilon$, where β_i is the coefficient on variable i in the EVF and the coefficient on (SP-EP) equals $-1/\sigma_\varepsilon$. These coefficients describe how the probability an option grant is exercised in a month changes with a change in X_i given that it had not been exercised earlier. Positive coefficients in these models are interpreted as increasing the probability an option is exercised given an increase in the variable. Columns (5) and (6) are labeled "Correlated Error EVF" and report estimates for the Employee Value Function described by Eq. (2). A positive coefficient implies an increase in the variable raises the value to the employee of holding the option another period and *reduces* the exercise probability. Because all of the parameters of the EVF are identified, the estimates in columns (3) and (5) are equivalent because the column (3) coefficients equal the column (5) coefficients divided by $(-1/\sigma_\varepsilon)$.

We hypothesize that employees have heterogeneous views about the value of holding an option another period and those employees who decide to exercise their options value them at an amount less than both the Black-Scholes and the intrinsic values of the option. This variation in option valuation implies everyone does not value the option at exactly the Black-Scholes value as one might expect if ESO could be traded. On the other hand, it may still be the case that E(EVF) equals the option's Black-Scholes value. This hypothesis is tested in columns (1) and (2) where we estimate the untruncated probit specification and constrain the EVF to be only a function of the option's BSV. If E(EVF) = BSV, then the coefficient on BSV will equal one; a dollar increase in the BSV leads to a predicted dollar increase in EVF. The point estimates in column (1) implies a dollar increase in the BS value of the option raises the predicted EVF by \$.91 (e.g., (1/-.282)*-.2575). Constraining β_{BSV} to equal one in the EVF equation is equivalent to constraining the coefficient on the Black-Scholes variable to be equal and opposite in sign to the coefficient on (SP – EP). This constraint is imposed in column 2 and a likelihood ratio test comparing columns (1) and (2) decisively rejects the constraint ($p < .0001$). Therefore, the data decisively reject the hypothesis that the E(EVF) equals the option's Black-Scholes value. That is to say, the employees we study do not value their employe stock options at the level characterized by Black-Scholes.

Columns 3-5 add the complete set of exogenous variables to the employee value function for the untruncated probit specification. Columns 3 and 4 report standardized probit coefficients for exercise decisions in a period. Column 3 reports the random effects probit specification estimated by the simulation method and column 4 shows estimates for a "standard" binary probit model that assumes the error terms are independent across employees, options and time (Eq. 9). Comparing the RE estimates in column 1 with the estimates in either column 3 or 4 shows the specifications with the more complete set of variables provides a much better fit compared to a model where EVF is only a linear function of the BSV. Many of the variables, including the squared and cubic BSV terms, are statistically significant at the .05 level in columns 3 and 5. The difference in the values of the log-likelihood function between the models in columns 1 and 3 clearly rejects the hypothesis that the coefficients on these additional variables are jointly equal to zero ($\chi^2 = 512.1$).

Comparing the Untruncated and Truncated Model Estimates

The estimate of the truncated probit specification that imposes the constraint that the EVF < SP is reported in column 6. Except for the coefficients on some of the time variables, many of the point estimates of the truncated model parameters are substantially smaller than the untruncated model estimates shown in column 5. The coefficient on returns to the Dow Jones Industrial Average in the truncated model is about one-third of the value for the untruncated model, the effect of the number of shares in the grant for the truncated model is about 10 percent of the coefficient in the untruncated model and the impact of the first month in the exercise window on the EVF for the truncated model is about 50 percent of the value for the untruncated model. The two models also provide very different estimates of the error variances. The estimated standard deviation of the unobserved individual heterogeneity error component is $1.50 for the truncated model and $4.95 for the untruncated model and the estimated standard deviation for the purely random component is $2.97 for the truncated model and $7.47 for the untruncated model.

The truncated model is not nested in the untruncated model so a standard likelihood ratio test cannot be used to determine which model provides the best and most parsimonious fit to the data. Instead, we compare the models two ways: a likelihood ratio test for comparing non-nested models developed by Vuong (1989) and a graphic comparison of how well the two models match the exercise behavior in the data. The Vuong statistic identifies which model is closer to the "true" model that generated the data and is equal to a normalized value of the log differences of the likelihood functions for each observation. Define this statistic as $G(\sum \text{Log}(L_{T,i}/L_{NT,i}))$ where L_T and L_{NT} equal the values of the likelihood function for observation i in, respectively, the truncated and untruncated models. Under the null hypothesis that the two models are equally close to the true model this statistic is distributed N(0,1) where a negative (positive) value means the untruncated (truncated) model is closer to the true model than the truncated (untruncated) model. The test statistic for our data is -2.58 which means the null hypothesis that the two models are equivalent is rejected with a p-value of .005 in favor of the untruncated model. We were somewhat surprised by this as the truncated model seemed more likely to explain the behavior of a rational investor.

The second method used to compare the two sets of estimates is a graphic comparison of how well each model fits the data by comparing the actual survival curve for the sample with the predicted survival curves generated from the two specifications. The predicted survival probability for period T is simply the probability an option is not exercised in period T given that it had not been exercised in periods 1 through (T-1). In the absence of unobserved heterogeneity, the predicted survival probabilities are easily calculated using the estimated parameters and the sample data. Such a procedure cannot be used for these RE probit models because of the constant unobserved individual heterogeneity. Calculating the predicted survival probabilities in the presence of unobserved heterogeneity poses the same problem encountered when calculating the exercise probabilities in the likelihood function for the RE model. Let $S^m_{k,j,t}$ equal the estimated survival probability for period t from model m for an option from grant k to individual j. This estimated survival probability equals

$$S^m_{k,j,t} = \Pr[\ (SP_1-EP_{k,j,1}) < X_{k,j,1}b^m + u_j,\ (SP_2-EP_{k,j,2}) < X_{k,j,2}b^m + u_j\ ,\ (SP_3-EP_{k,j,3}) < X_{k,j,3}b^m + u_j,$$
$$\ldots.(SP_{M-1}-EP_{k,j,t}) < X_{k,j,t}\, b^m + u_j],$$

where b^m is the estimated parameter vector β in Equation (2) for model m. If u_j was observed this equation would simplify to the product of t normally and independently distributed CDFs similar to Eq. 9 and is easily calculated. However, because u_js are unobserved a numerical solution does not exist and we follow a procedure comparable to the simulation estimation procedure used to estimate the RE model. For each individual a random error component, $\widetilde{u}_j$, was drawn from a $N(0,\sigma^2_u)$ distribution to give a predicted survival probability for period T conditional on $u_j = \widetilde{u}_j$ or

$$(11)(S^m{}_{k,j,T} \mid \widetilde{u}_j) = \Pr((SP_1 - EP_{k,j,1}) < (X_{j,i,1}b^m + \widetilde{u}j))x\Pr((SP_2 - EP_{k,j,2}) < X_{j,i,2}b^m + \widetilde{u}_j))x$$
$$\Pr((SP_3 - EP_{k,j,3}) < (X_{j,i,3}b^m + \widetilde{u}j))x....\Pr((SP_T - EP_{k,j,T}) < (X_{j,i,T}b^m + \widetilde{u}_j))$$

If person j exercised option k in month T or data were right censored in month T, Eq. (11) was calculated for periods 1 through T, producing an estimated survival curve for periods 1 through T given a randomly drawn value for u_j. These steps were replicated 50 times by drawing 50 different values of $\widetilde{u}_j$ for each person j to produce 50 different values for Eq (11) for each option grant and time period for which data are available for person j. The estimated average survival probability for person j, grant k in period t from model m is equal to the average of the 50 simulated conditional survival probabilities:

$$S^m k,j,t = (1/50)x \sum_{n=1,50} (S^m k,j,t \mid u_j = \widetilde{u}_n).$$

Applying this equation to each of the T periods gives the estimated survival curve for model m for option k held by individual j. An estimate of the survival curve was obtained for each person in the sample for each of the two models using this procedure. Then for each model the estimated aggregate sample survival probability for period t was calculated as the mean of $S^m_{k,j,t}$ over all subjects who were still at risk of exercising their options in period t.

Figure 4 shows the results from the calculations described in the preceding paragraph and plots the actual sample survival curve and the estimated survival curves for each of the two models. These figures are consistent with the Vuong test and show the untruncated model comes much closer than the truncated model to fitting the data. Figure 4a shows the untruncated model consistently under-predicts the exercise probability over the first half of the exercise window. By about month 48 the difference in the survival probabilities is about 9 points and this gap remains relatively constant until about month 80. Figure 4b shows the truncated model substantially over-predicts the exercise probabilities and, compared to the untruncated model, is much further from the actual survival curve. At month 48 the actual survival probability is .65 and the truncated model predicts that only about 30 percent of the option grants had not yet had an exercised option. It is clear from both the Vuong test and Figure 4 that the untruncated probit model is the preferred specification.

Results for the Untruncated Model

We focus the remainder of our discussion on the estimates in columns 3-5 because of the superior fit of the untruncated relative to the truncated model. A comparison of the RE specifications in columns (3) and (5) with the independent probit estimates in column 4 also reveals a clear preference for the RE model. The estimates in column 5 show the estimated standard deviation of u is 4.95 and significant. This parameter yields a within person error correlation of .31 (i.,e., $\sigma^2_u / (\sigma^2_u + \sigma^2_\varepsilon)$) and shows the exercise decision in one period for an individual is positively correlated with the decision in the next (or previous) period because the decision in each period is affected by constant unobserved individual characteristics captured by u. Over the two option grants that we have for most of our sample, this positive correlation implies employees who exercised options "early" from their first grant are also likely to be "early" exercisers of options from their second grant for reasons not controlled for by the observables in our model. The factors captured by u may reflect differences in employee risk aversion and liquidity constraints that cause individuals to systematically differ in their willingness to hold an option.

The unobserved heterogeneity captured by u has a substantial impact on the distribution of exercise activity in the exercise window. Controlling for the observables in the model, the "early" exercisers in the sample will most likely be employees with "low" values of u because the exercise probability, $\Phi(((SP_t - EP_{k,t}) - (X_{j,i,1}b^m + u_j))/\sigma_\varepsilon)$, is larger for an individual with a large negative value for u. As time progresses through the exercise window this selection bias (Heckman 1979) means the sample of individuals who have not yet exercised options will, on average, have large positive values for u and lower exercise probabilities relative to early exercisers with small u_js.

One way to illustrate the impact of this unobserved person heterogeneity on the distribution of exercise times is to compare the exercise probabilities for each period in the exercise window for two hypothetical samples. One sample consists of a sample of individual who are "average" on both observable (X) and unobservable (u) characteristics. In other words, $X = \overline{X}$ and u = 0 for all individuals in the sample. The other sample consists of individuals who are average on observables characteristics $(X = \overline{X})$ but differ along unobserved dimensions captured by u. In other words, in this sample the E(u) = 0 and u for each individual is drawn from a $N(0,\sigma_u)$ distribution. A simulation similar to the one used to construct Figure 4 was conducted to obtain the average exercise probability each period for each of the two samples.[11] Figure 5 shows the ratio of the predicted exercise probability for individuals with unobserved heterogeneity to predicted exercise probability for the sample where there is no variation in u. Early in the exercise window the exercise probability for the sample with unobserved heterogeneity is several times greater than the homogeneous sample. However, after about month 48 the exercise probability is smaller for survivors with unobserved heterogeneity relative to the sample with no individual heterogeneity because sample selection causes the sample with heterogeneity causes E(u >0) while in the other sample u continues to equal 0 for all of the survivors. These differences in aggregate exercise probabilities for the two samples generates differences in the distribution of exercise times in the two samples and illustrates the importance of accounting for unobserved individual heterogeneity when explaining when employees exercise their stock options.

The estimates in columns 3 and 5 show the EVF and their exercise decisions are strongly related to what happens to the stock market during the month; an increase in the return to holding a portfolio that

[11] For the sample without heterogeneity values of ε were randomly chosen for each period and the predicted exercise period was the first period XB+ε > (SP-EP). This exercise was repeated 10,000 times to produce a distribution of exercise probabilities. For the sample with heterogeneity 100 values of u_j were drawn from $N(0,\sigma_\varepsilon)$ and then for each of the 100 values of j 100 values of ε were drawn from the estimated distribution of ε for each period to produce 100 exercise decisions/person. The sample exercise probabilities for each period were calculated from the simulated decisions for the 100 individuals x 100 decisions/individual.

matches the Dow Jones Industrial Average increases the exercise probability by lowering the value the employee places on holding the option. A 5 percentage point increase (SD = 4.7%) in the DJIA lowers the predicted value EVF by about $.19. The predicted change in the probability the option is exercised is equal to

$$\frac{\partial \Pr(I=1)}{\partial DJIA} = \phi(\overline{X}b)(.4981*.05)$$

where ϕ is the density function for the standard normal distribution. The mean hazard rate in the sample is .012 which implies a value of $\phi(\overline{X}\beta)$ equal to .03123. At this value, the predicted change in the exercise probability from a 5 percent increase in the DJIA equals .0008 points or a 6.5 percent increase in the probability.

The coefficients on the number of shares in the grant and the employee's salary are consistent with the hypothesis that employees are risk averse or liquidity constrained and likely to exercise their options earlier to diversify their portfolio or generate cash when they hold more options or have a lower salary. The exercise hazard for the first exercise time increases and the EVF declines as the number of shares in the grant increases. The estimates imply 500 additional options (about 1 SD) lowers the value of holding the options by $.62 and raises the exercise hazard by .0026 points or a 22 percent increase evaluated at the mean. The coefficients on the employee's ln(real salary) shows this variable has a statistically significant effect on exercise behavior. A standard deviation increase in log earnings (.17) raises the EVF by $.46 and lowers the exercise probability by .002 points or 16 percent. The estimates show no relationship between employee tenure and the EVF.

The impact of changes in the firm's stock price on exercise decisions and the E(EVF) is difficult to evaluate by simply inspecting the coefficients in Table 2 because the stock price is part of both the gain from exercising an option and the option's BSV. To illustrate how E(EVF) changes with respect to the stock price the predicted value of E(EVF) was calculated at different stock prices and at different times in the event window holding the characteristics of the individual constant. In these calculations we set $u_j = 0$ and salary, tenure and number of options equal to their sample means Figures 6a-6f plot these values along with the BSV and the intrinsic values for an "at the money" option with an exercise price of $10. Two features of these figures are noteworthy. First, the expected value employees place on options is quite large and substantially greater than the BSV. In the month after the options vest the estimate of the E(EVF) is about $15 greater than the option's intrinsic value with the gap between the EVF and either the BSV or intrinsic value declining only slightly as the stock price increases. At a stock price of $14 the BSV value is $8.45, the intrinsic value is $4 and E(EVF) is $26.33. At a stock price of $24 the values for the BSV and EVF are, respectively, $18.01 and $33.08. The second noteworthy finding is that the EVF shifts very little over the first 48 months of the exercise window. For example, at a stock price of $30 the E(EVF)s are $38.76, $37.92, and $38.75 in months 1, 24 and 48. By month 72 the predicted value of EVF had declined slightly to $35.60 and in months 84 and 94 the values were, respectively, $30.67 and $23.46. Thus, two months before the options expire at a stock price of $30 the E(EVF) is less than the stock price but still greater than the BSV of $20.10. The stability of the gap between the E(EVF) and the option's intrinsic value (for a fixed stock price) over most of the exercise window implies the exercise probability changes very little for an average employee as the expiration date approaches. Over the last 12 months of the option's life the expected value of holding the option shifts toward the option's intrinsic value as the expected additional gains from continuing to hold an option declines and the exercise probability increases.

The values of E(EVF) shown in Figure 6 imply that a large fraction of employees value their options by an amount substantially greater than the option's BSV or the firm's current stock price. One way to quantify these effects is to calculate the predicted probability EVF is greater than BSV and SP for the average individual. For our firm the mean historical monthly stock return is about 1.5 percent. Therefore, for an option granted "at the money" with an exercise price of $10, the expected stock price 48 months into the exercise window is $29.21 (e.g., $\$10*(1.015)^{(24+48)}$) and the BSV is $21.34. Using the estimates in columns 3 and 5 of Table 2 and the characteristics of an average individual, the E(EVF) is about $37.66 and the predicted exercise probability given the option had not been previously exercised is about .0067. The estimate of E(EVF) implies a .986 probability that the average individual values the option at more than the BSV and a .87 probability the option's value to the individual exceeds the firm's stock price. We described earlier why the absence of a market for employee stock options may cause employees to value options at a level greater than the BSV but that a rational employees should not value an option at more than the firm's stock price. These estimates and our rejection of a model that constrained EVF to be less than the firm's stock price suggest many employees in our sample over-value their options relative to what finance theory predicts a rational employee should do. This may be another example and setting where individuals make judgments inconsistent with the predictions of economics and rational decision-making (Kahneman & Tversky, 1979).

Figures 6a-6f show the probability of exercising an option increases as the firm's stock price increases because the gap between E(EVF) and the firm's stock price declines as the stock price increases. For example, at month 48 (E(EVF)-SP) is \$10.27 at SP = \$20 and \$8.75 at SP = \$30. This change in the EVF-SP gap causes the predicted probability of exercising an option to go from .0033 at \$20 to .006 at a \$30 stock price or an 82 percent increase in the exercise probability. The positive relationship between the stock price and exercise probability implies employees will exercise "earlier" when the firm's stock is doing well in an effort to "lock-in" profits and the exercise hazard will vary depending on how well the firm is performing during the term of the options. A useful way to illustrate how changes in a firm's stock prices influences exercise decisions is to plot the survival curve for an "average employee" confronted with either a "high" or "low" stock return path over the 96 months. Based on historical monthly return data for the firm, monthly stock returns are assumed to be distributed $N(.015, .075^2)$. Following the assumptions of the Black-Scholes model which assumes monthly returns are normally and independently distributed, we can describe the distribution of the firm's stock price 10 years from the option grant date. The 10^{th} and 90^{th} percentile of this log normal price distribution is produced, respectively, by constant monthly returns of .0063 and .024 percent. We label the security price paths generated by these returns as the "low" and "high" return price path. Since the model predicts exercise decisions using the maximum stock price in the month, an estimate of the maximum stock price in each of the 96 months an option is vested is also needed. Historical data for the firm shows the maximum monthly stock price is, on average, 5 percent more than the average price in the month. Therefore, the maximum price in a month was set 5 percent above the predicted monthly price generated by the "low" and "high" return price path. The maximum stock price for each month was combined with the estimates in column 4 of Table 2 to generated predicted hazard rates and survival curves for each price path. The results from this exercise are shown in Figure 7 and show very different exercise patterns for the two price paths. When confronted with the "high" price path the model predicts half of a sample of "average" workers will exercise by the 29th month while half of the workers facing the low price path will exercise *after* the 90th month. These estimates suggest that when employees face rising stock prices they will exercise options substantially sooner in an effort to "lock-in" gains.

V. Discussion and Conclusion

For thirty years, economists and others have valued stock options using the pioneering work of Merton (1973) and Black and Scholes (1973). Their techniques are ideal for pricing stock options to diversified investors who are free to trade their options. However, when firms provide stock options to their employees they place limits on how quickly employees can exercise their options in an effort to encourage employees to behave in ways consistent with the interests of shareholders. For this reason, Lambert, Larcker, and Verrecchia (1991) and others have recognized that the value employees place on options may be different from the value placed on options by outside investors. To the best of our knowledge, ours is the first study that uses actual employee exercise behavior to estimate the value employees place on holding employee stock options. We show it is possible to identify the value of options to employees using a simple theoretical model, a discrete time statistical hazard model of exercise behavior and carefully collected data.

The model that we use is built around the recognition that the prediction from the Black-Scholes model that market traded options will never be exercised prior to their expiration date cannot be used to make a prediction about how employees will behave when deciding when to exercise their untradeable employee stock options. Black-Scholes makes no prediction about how long individuals will own market traded options. It only predicts individuals will sell rather than exercise their options to liquidate their options because the options can always be sold for more than the profit that can be made by exercising them (e.g., BSV > max(0,SP-EP)). Since employees can't sell their options, exercising their options is their only choice if they wish to profitably dispose of their options. This leads to our model which predicts an employee will choose to hold an option for another period if the utility of the income from exercising the option now is less than the value of holding the option and reserving the right to exercise it at a later date.

The data required to identify and estimate the value to employees of holding an option, the Employee Value Function (EVF), from a hazard model of the option exercise decisions must have two characteristics: (a) the data must include options granted to employees at different exercise prices and expiration dates, and (b) employees in the dataset must receive options from multiple grants. Data condition (a) permits a researcher to separately identify how changes in the benefit from holding the option another period changes compared to the gains from immediately exercising the option. These identifying conditions permit us to estimate the E(EVF), the variance around this expected value and how variation in the EVF and the (Stock price – exercise price) determine when options are exercised.

An important point emphasized in previous research (Lambert, Larcker, and Verecchia, 1991, and Hall and Murphy, 2003) is that employee exercise decisions are likely to be influenced by risk aversion and household budget constraints. In our data and in most data we think researchers are likely to have access to these theoretically important variables are likely to be unobserved or very crudely measured. These data limitations imply the exercise probability in one period for options from one grant for an individual will be correlated with the exercise probabilities for options from a second grant. Failure to account for this unobserved heterogeneity will produce biased estimates of exercise behavior. Data condition (b), multiple option grants per employees, allows us to estimate the exercise hazard and the EVF and control for unobserved employee heterogeneity that might influence exercise behavior over successive time periods using a random effects probit model that is estimated by simulation methods.

The model is estimated using proprietary data on options granted to over 2000 middle-level managers employed by a large, established firm outside of manufacturing. Our estimates show the expected value to employees from continuing to hold their options after the vesting date is significantly related to a variety of individual and market characteristics. Options are exercised earlier when the overall stock market is doing well relative to the firm's stock price, when the employee has a larger number of options in his/her grant or when an employee has a lower salary. The exercise probability increases as the firm's stock price goes up because the increase in the utility from exercising the option immediately is greater than the increase in the utility from continuing to hold the options. We also find exercise decisions are heavily influenced by unobserved employee characteristics which causes a significant (.3) correlation between the value an employee places on options across different option grants. Our estimates are inconsistent with both the widely held view that employees value options at less than the option's Black-Scholes value and that employees behave rationally and value options at less than the firm's stock price.

We now turn briefly to a discussion of some of the implications of our results for public policy regulating the information firms are required to disclose about the cost of employee stock options and the information firms should be providing employees about the options they receive. Accounting for stock options has been a topic of considerable discussion for the past decade. On March 31, 2004 the Financial Accounting Standards Board (FASB, 2004) proposed new rules that, if adopted, will require that firms report employee stock options as an expense at the time the options are granted. The new rules require that firms use Black-Scholes or a "Black-Scholes like" formula to expense options and encourages firms to include in their cost estimates the expected exercise pattern of employees because this pattern affects the cost of options to the firm. Firms are encouraged to look at the past exercise decisions of employees when determining the expected exercise pattern for newly granted options. Our finding that the distribution of exercise times is a function of the path of a firm's stock price (See Figure 7) implies that a firm's recent history of exercise activity will accurately predict exercise activity for a new set of grants only if future stock price path is the same as recent history. Since this will almost certainly not be the case, a considerably more sophisticated approach must be taken to properly estimate the expected cost to the firm of providing stock options to employees.

As mentioned above, one additional analysis we have undertaken uses our methods and results is to compare the cost of options to the firm with the value employees place on the options. Our finding that options in this firm are valued by employees at a level considerably above the Black-Scholes level suggest the value of these options to employees is substantially greater than the cost of the options to the firm. Thus, if this firm were to decide to curtail the use of options for middle managers because of the new FASB rule, it would likely lead to significant employee dissatisfaction that could, perhaps, be offset only by paying employees substantially more than what the firm is spending on options. In this firm these options appear to be a source of firm "competitive advantage." This advantage, however, appears to be generated by employee beliefs about the value of their options that is inconsistent with rational decision-making. This suggests the firm should undertake efforts to better educate its employees about how to value their options.

Although the theory and empirical methods in this paper are completely general, the estimates use data from a single firm, a particular type of employee and a particular period of time. All of the discussion in the previous two paragraphs assumes our findings are robust and generalizeable to other firms and employees. The estimate of σ_ε fixes the position of the EVF relative to both the stock price and the BSV. Therefore, it is important to investigate the robustness of this parameter estimate because it affects the conclusion drawn about the rationality of employee beliefs about the value of their options. We plan to extend this work by studying other firms and groups of employees in an effort to determine how (or if) the determinants of the EVF change across different settings. For example, do executives value options differently from non-executives?

We also intend to explore a number of other issues using the data from the current firm. These issues include estimating models that permit a more general error structure in the EVF equation, estimating state dependence (citation) in exercise activity and a comparison of the discrete-time probit hazard estimates with estimates from more traditional continuous time hazard models. This study focuses solely on the first exercise date for options from a grant. Since about half of the first exercise decisions are decisions where less than 100 percent of the options in the grant are exercised, we intend to model the partial/complete exercise decision of employees and the timing of the second exercise decision when less than 100 percent of the options are exercised on the first exercise date. The options employees received that are used in this study were part of an employee stock purchase plan. This allows us to estimate another value for EVF using the decisions made by employees to participate in the plan. These estimates can then be compared with the values reported here or a model of both the probability of participating and the Pr(exercising| participation) can be estimated simultaneously. Finally, we hope to obtain additional data from the firm that will allow us to investigate whether employees value options differently based on different demographic characteristics (gender, age, marital status, geographic location). A great deal more work can be done to better understand the value of stock options to employees. We think this study is a useful step along this path.

Figure 1

Black-Scholes Value of an Option

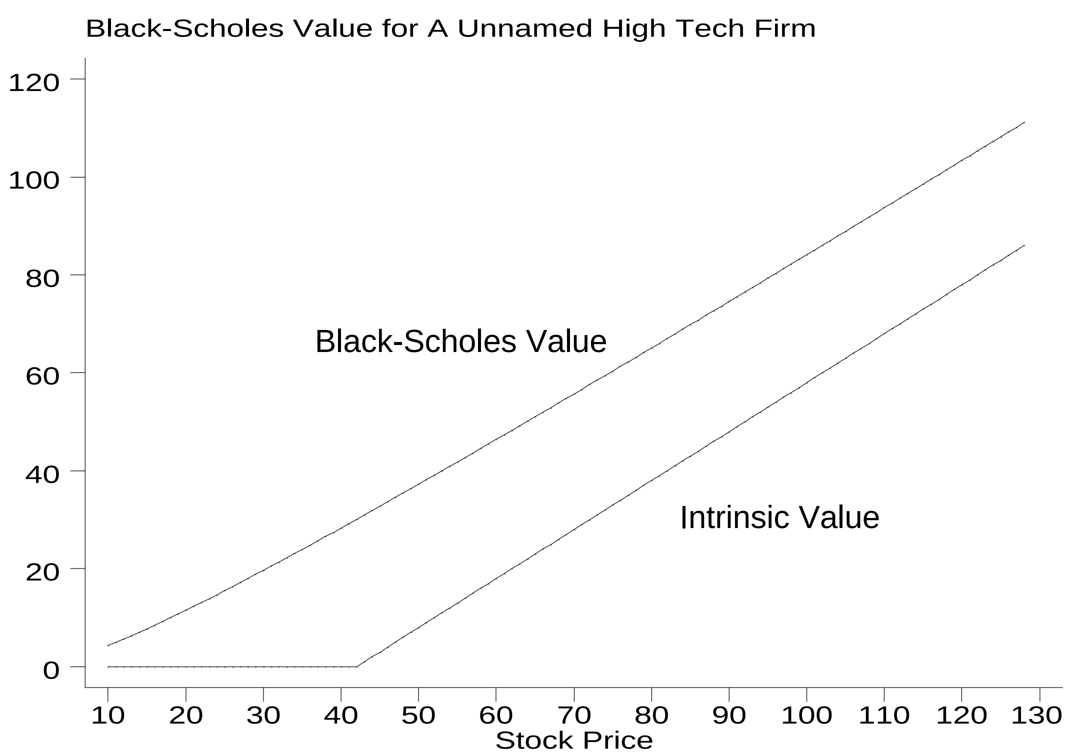
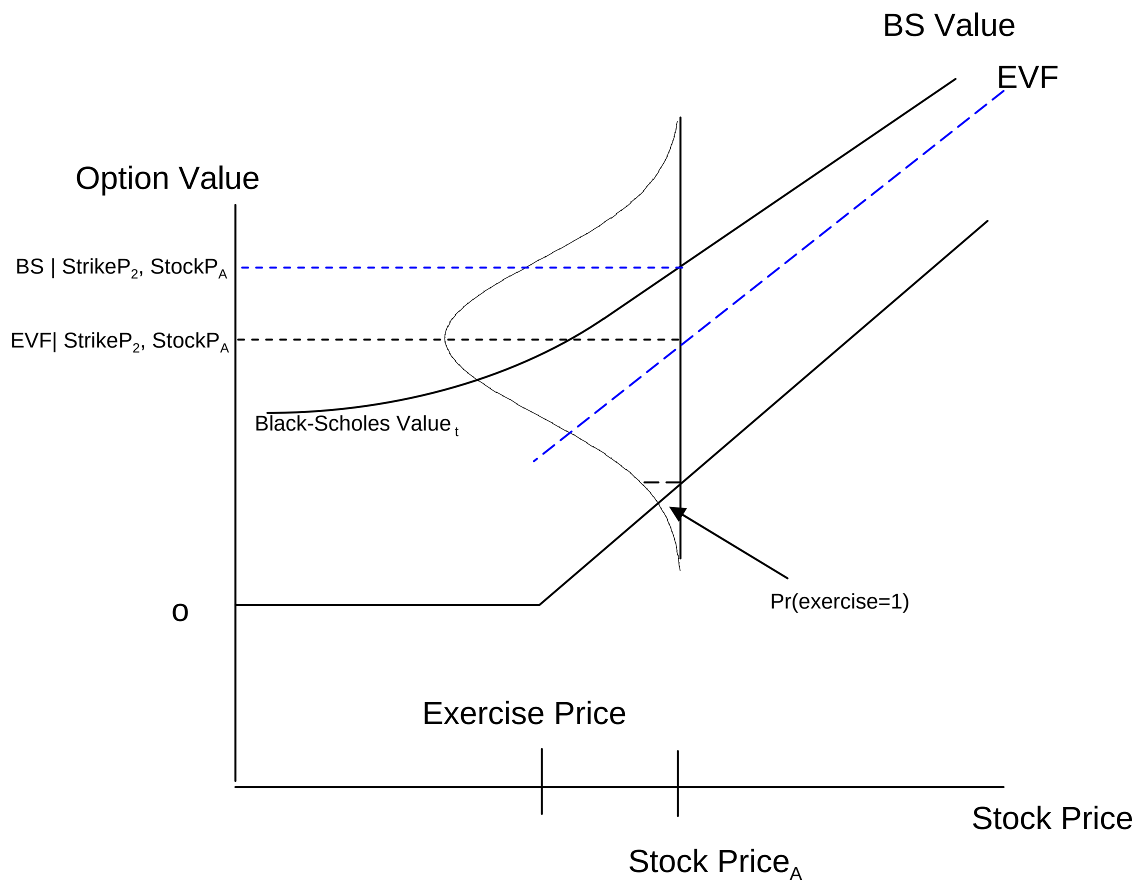


Figure 2



BS Value
EVF
Option Value
BS | StrikeP2, StockPA
EVF| StrikeP2, StockPA
Black-Scholes Value t
o
Pr(exercise=1)
Exercise Price
Stock Price
Stock PriceA

Figure 3a

Sample Exercise Activity

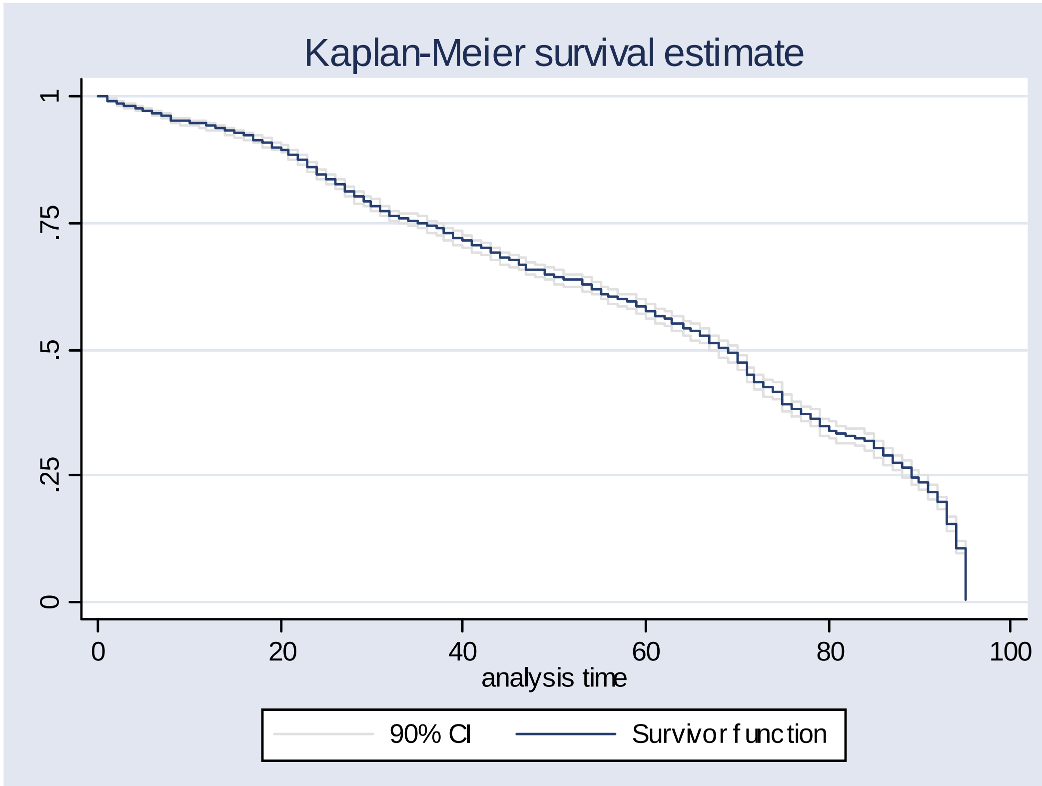


Figure 3b

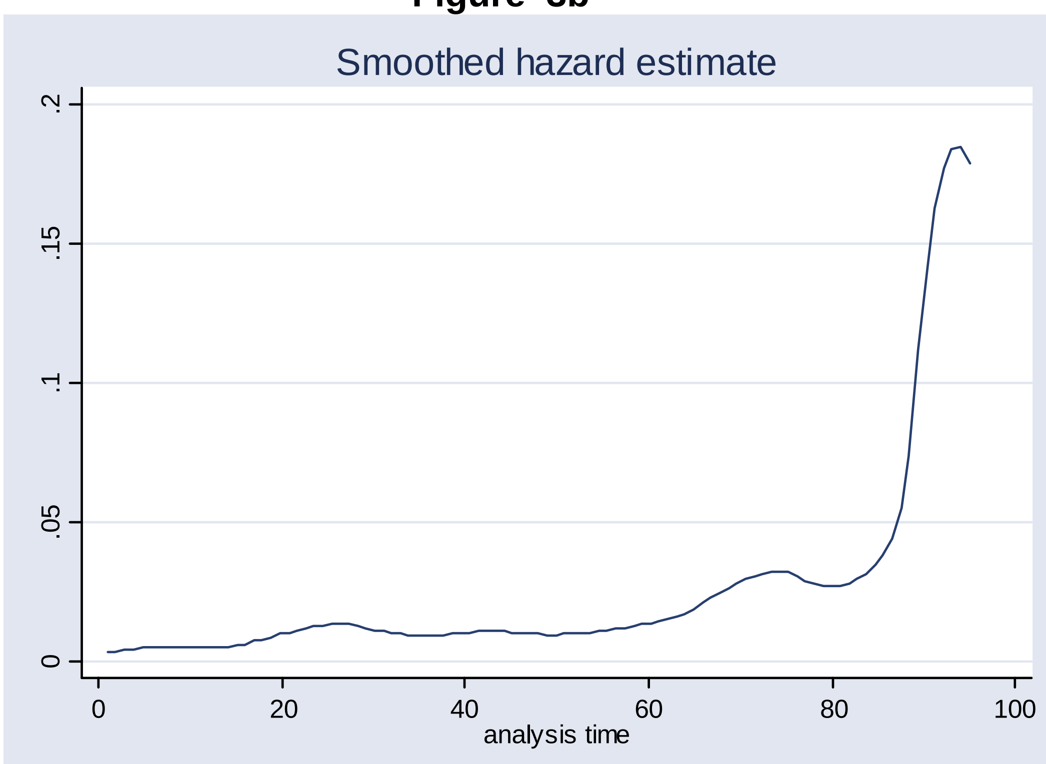
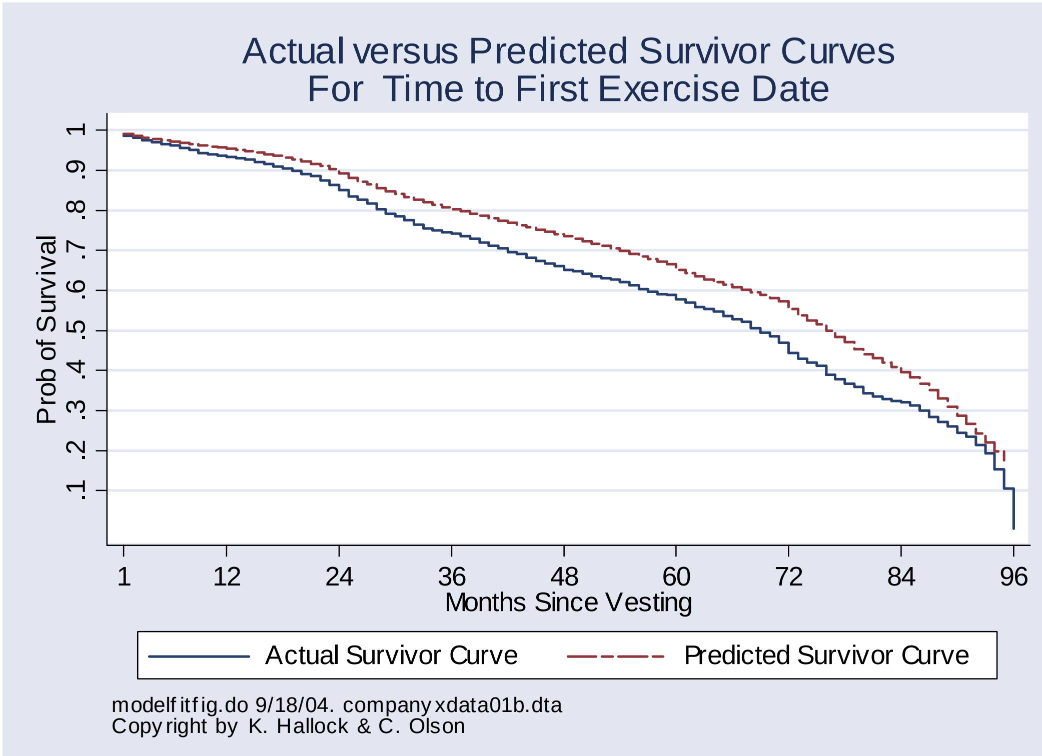


Figure 4


Actual versus Predicted Survivor Curves
For Time to First Exercise Date
Prob of Survival
.1 .2 .3 .4 .5 .6 .7 .8 .9 1
1 12 24 36 48 60 72 84 96
Months Since Vesting
Actual Survivor Curve
Predicted Survivor Curve
modelfitfig.do 9/18/04. companyxdata01b.dta
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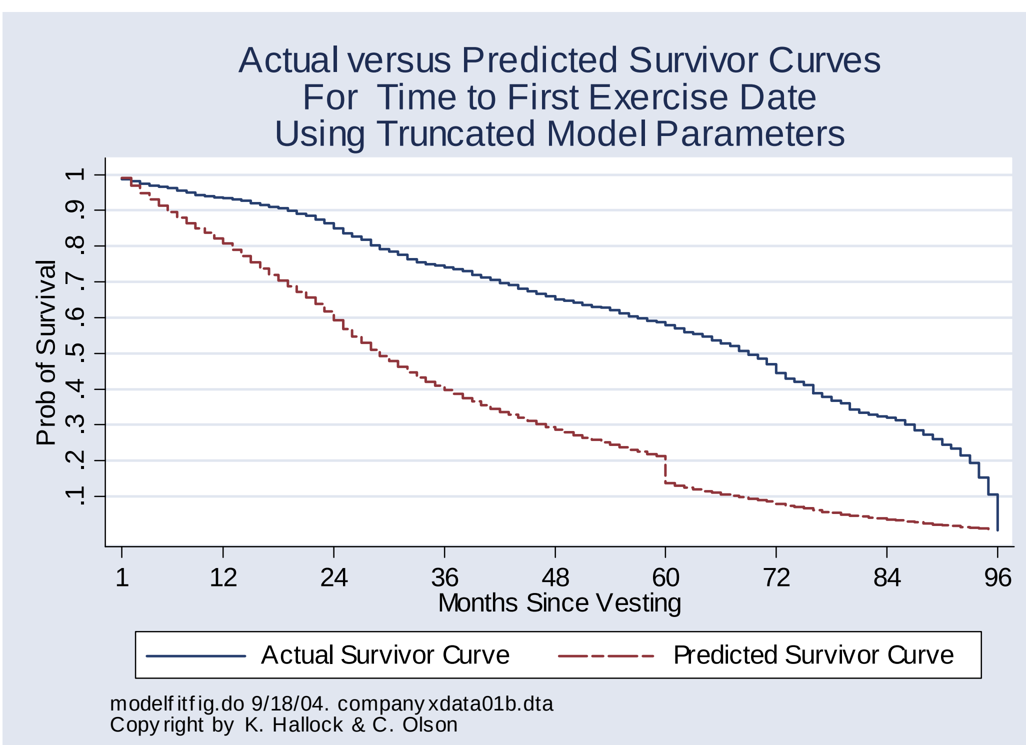
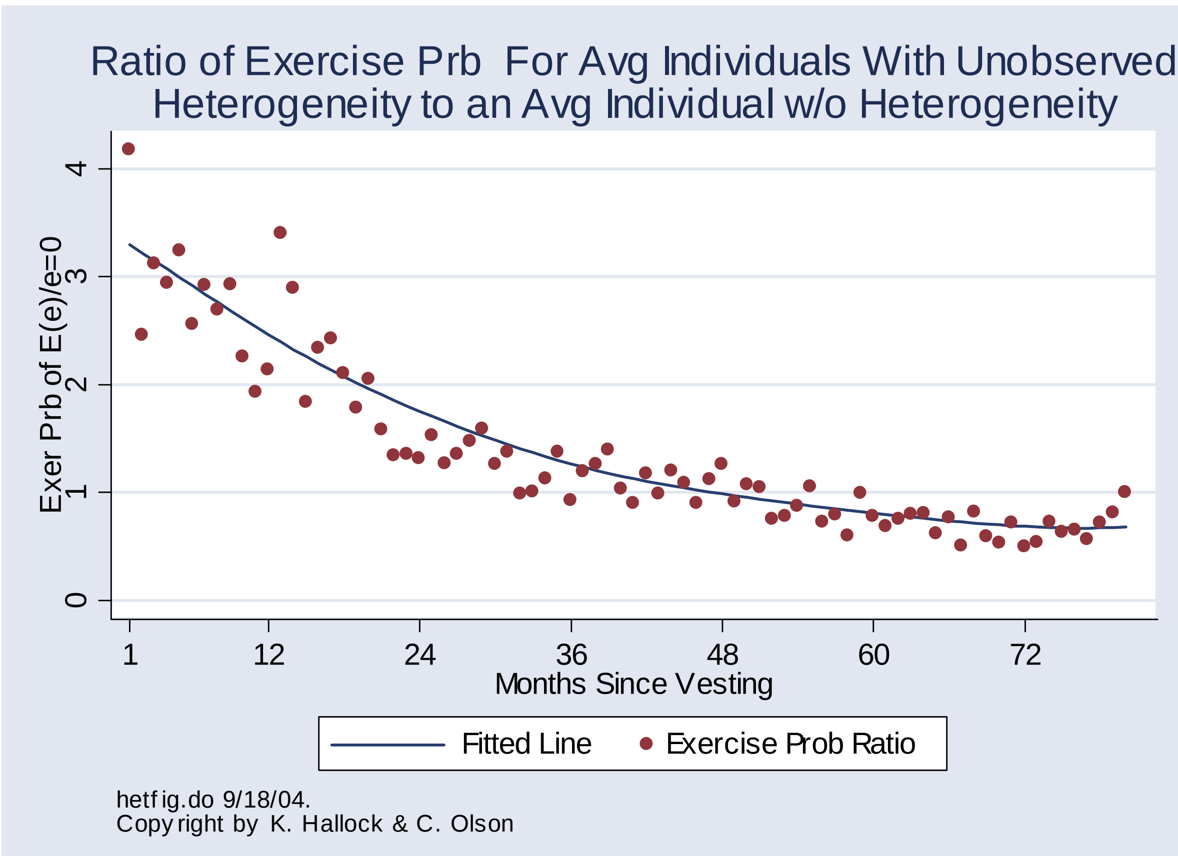

Actual versus Predicted Survivor Curves
For Time to First Exercise Date
Using Truncated Model Parameters
Prob of Survival
.1 .2 .3 .4 .5 .6 .7 .8 .9 1
1 12 24 36 48 60 72 84 96
Months Since Vesting
Actual Survivor Curve
Predicted Survivor Curve
modelfitfig.do 9/18/04. companyxdata01b.dta
Copyright by K. Hallock & C. Olson

Figure 5



Ratio of Exercise Prb For Avg Individuals With Unobserved Heterogeneity to an Avg Individual w/o Heterogeneity
Exer Prb of E(e)/e=0
0
1
2
3
4
1
12
24
36
48
60
72
Months Since Vesting
Fitted Line
Exercise Prob Ratio
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Figure 6

Predicted Values of the EVF At Different Stock Prices and Time in the Exercise Window

Figure 6a

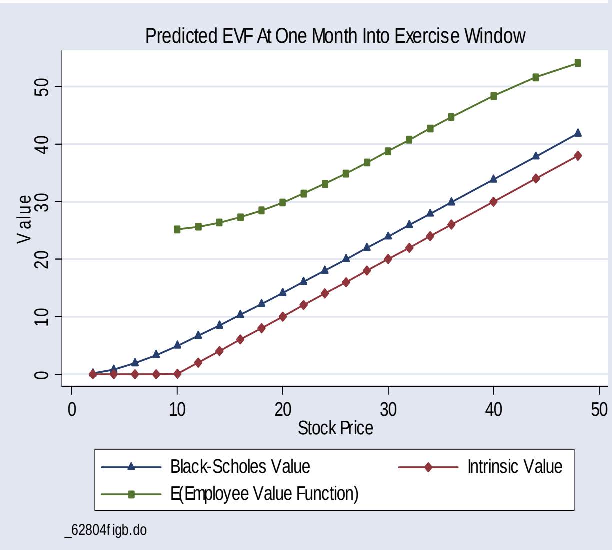


Figure 6b

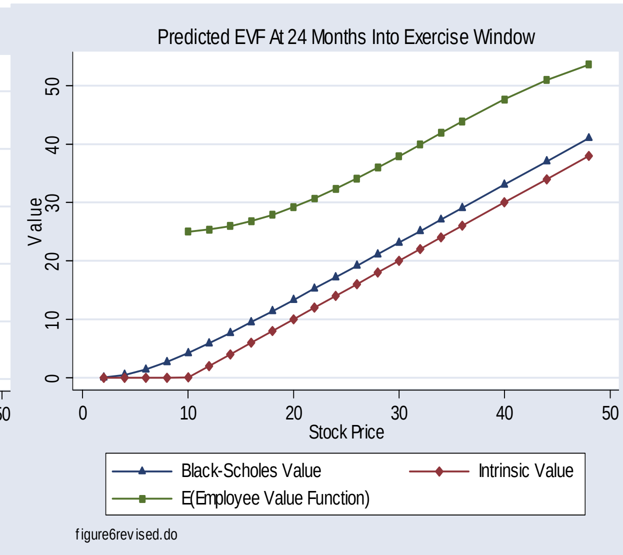


Figure 6c

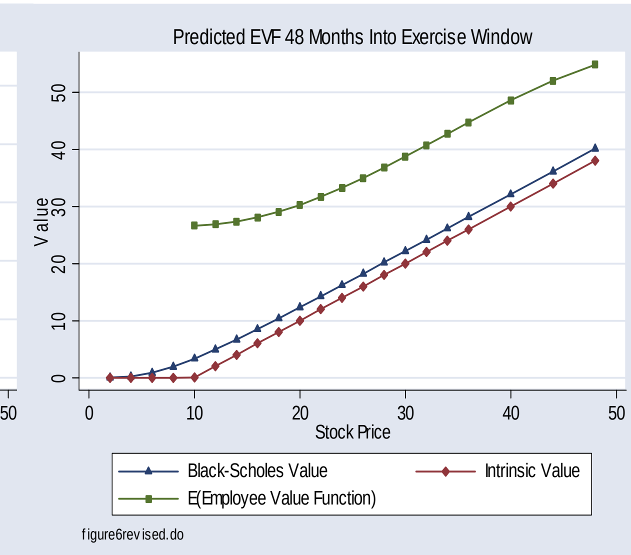


Figure 6d

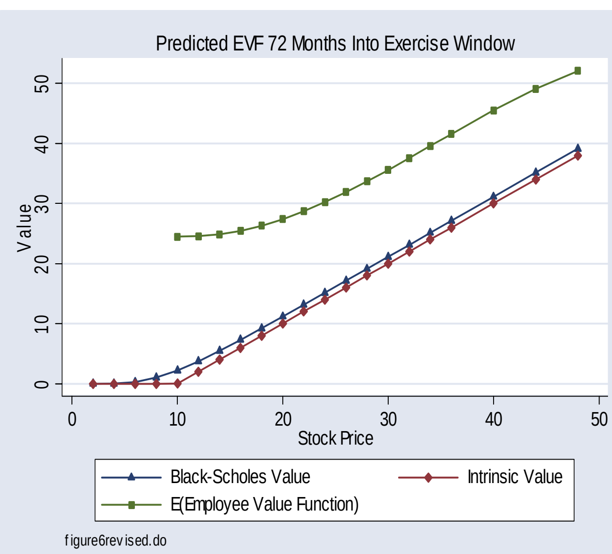


Figure 6e

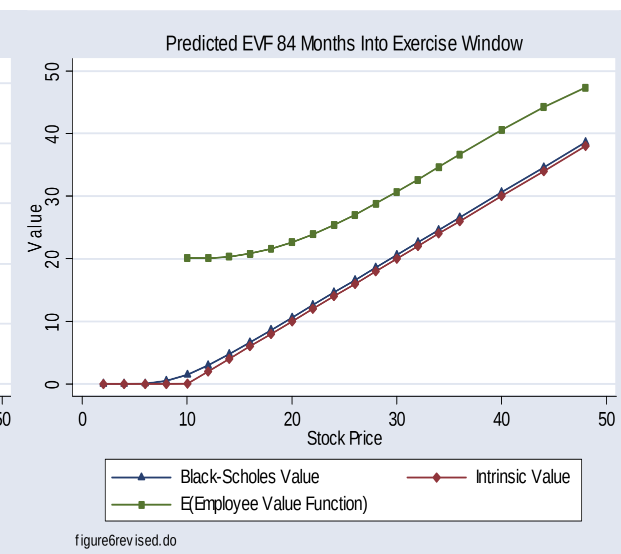


Figure 6f

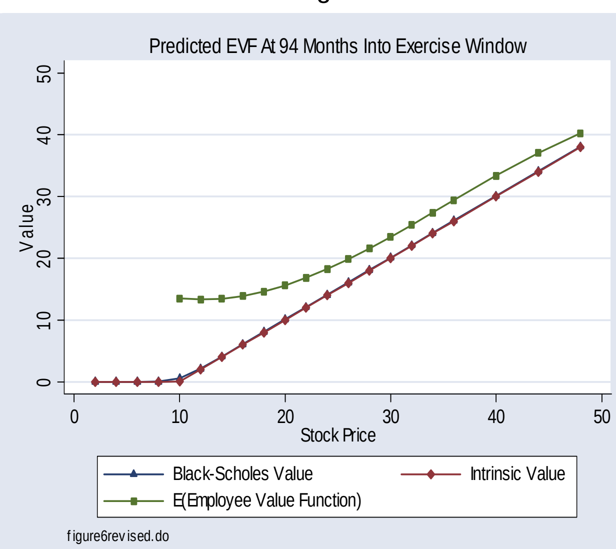
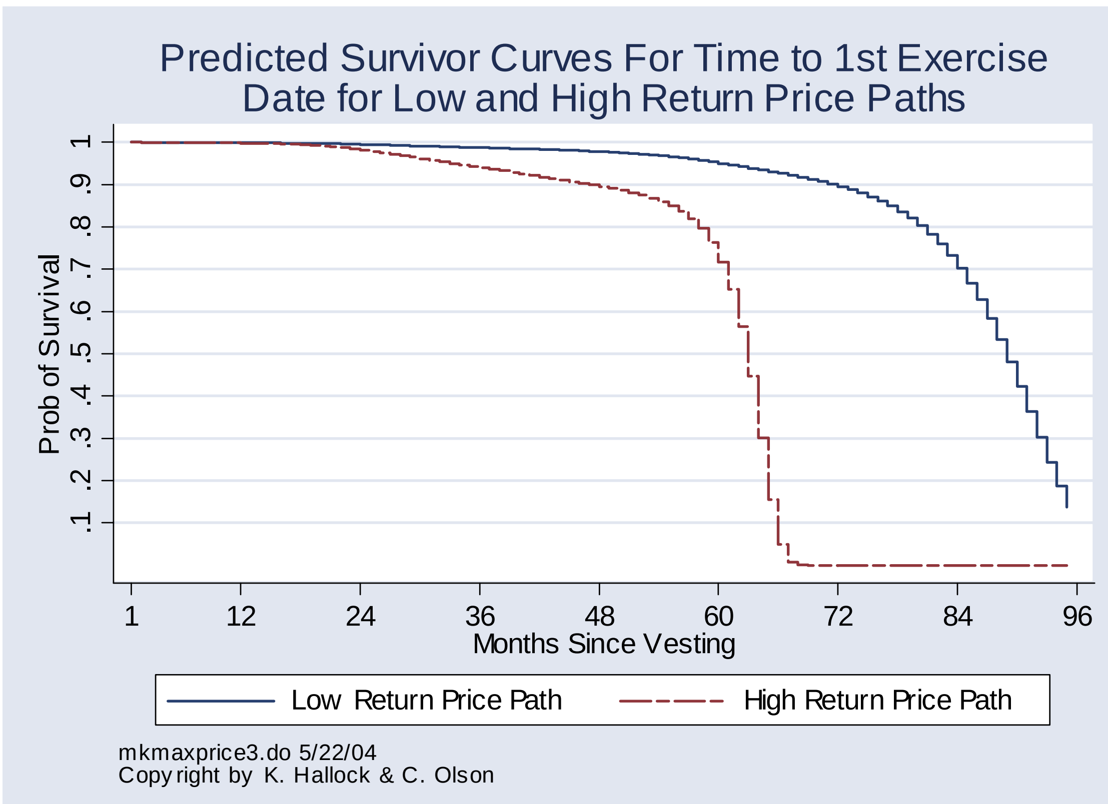


Figure 7



Predicted Survivor Curves For Time to 1st Exercise
Date for Low and High Return Price Paths
Prob of Survival
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
1
12
24
36
48
60
72
84
96
Months Since Vesting
Low Return Price Path
High Return Price Path
mkmaxprice3.do 5/22/04
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Table 1

Summary Statistics on Exercise Decisions

Number of Employees Receiving Options	2180
Number of Option Grants	3712
Number of option grants where time to first exercise date is censored	1127
Mean options/grant	1302
Mean hazard rate/month	0.0128
25th Percentile of Time to 1st exercise date (months)	34
Median time to first exercise date (months)	69
75th Percentile of Time to 1st exercise date (months)	90
Options exercised on first exercise date as fraction of options in the grant	0.765
Fraction of first exercise decisions where 100% of options in grant were exercised	0.576

Table 2

Alternative Estimates of The Exercise Hazard Rate and Employee Value Function

	---------- EVF is Untruncated --------------					EVF Truncated At SP
	(1)	(2)	(3)	(4)	(5)	(6)
	Hazard Function	Hazard Function	Correlated Error Probit Hazard Function	Independent Error Probit Hazard Function	Correlated Error EVF	Correlated Error EVF
Constant	-2.4018 (0.0300)	-1.9096 (0.0192)	-0.6444 (0.7998)	-1.2259 (0.4327)	4.8144 (0.1679)	6.5396 (2.2322)
Real (Stock price - Exercise price)	0.2820 (0.0121)		0.1338 (0.0438)	0.1344 (0.0325)		
Real Black-Scholes Value	-0.2575 (0.0118)		0.0297 (0.0384)	0.0138 (0.0329)	-0.2222 (0.2024)	0.4353 (0.0478)
Real (SP-EP)- Real BS Value		0.2146 (0.0113)				
(Real Black-Scholes Value)^2/100			-0.6432 (0.0690)	-0.6080 (0.0608)	4.8044 (0.8821)	1.9280 (0.2020)
(Real Black-Scholes Value)^3/1000			0.0838 (0.0092)	0.0799 (0.0082)	-0.6261 (0.1209)	-0.2274 (0.0271)
Monthly Mean Firm Returns			0.0363 (0.1246)	0.1400 (0.1135)	0.2709 (0.9250)	0.2185 (0.3242)
Month;y Return DJIA			0.4981 (0.2000)	0.4800 (0.1817)	-3.7208 (1.6292)	-1.2496 (0.5293)

Table 2 (Continue)

No Shares in Grant (1000s)			0.1667	0.0910	-1.2470	-0.1355
			(0.0266)	(0.0151)	(0.3053)	(0.0688)
Ln(Real Monthly Wage)			-0.3586	-0.2142	2.6795	0.2840
			(0.0941)	(0.0507)	(0.8620)	(0.2410)
Firm Tenure (Years)/100			-0.3678	-0.1879	2.7474	-0.8893
			(0.2858)	(0.1372)	(2.1985)	(0.7668)
=1 if first month options are vested			0.2419	0.2280	-1.8070	-0.8850
			(0.0891)	(0.0776)	(0.7018)	(0.2212)
Months Since Vested			-0.0115	-0.0161	0.0837	-0.0174
			(0.0073)	(0.0064)	(0.0581)	(0.0177)
Max(0, Months Since Vested-12)			0.0423	0.0358	-0.3145	0.0096
			(0.0118)	(0.0103)	(0.1102)	(0.0272)
Max(0, Months Since Vested-24)			-0.0487	-0.0471	0.3675	0.1030
			(0.0077)	(0.0069)	(0.0862)	(0.0209)
Max(0, Months Since Vested-36)			0.0172	0.0208	-0.1262	-0.0300
			(0.0075)	(0.0067)	(0.0604)	(0.0195)
Max(0, Months Since Vested-48)			0.0212	0.0179	-0.1606	-0.0994
			(0.0083)	(0.0075)	(0.0607)	(0.0210)
Max(0,Months Since Vested-60)			-0.0402	-0.0364	0.2951	0.1332
			(0.0091)	(0.0082)	(0.0810)	(0.0261)
Max(0, Months Since Vested-60)^2			0.1879	0.1567	-1.4066	-0.5597
			(0.0170)	(0.0153)	(0.2636)	(0.0646)
ln(Var(u)/Var(e))	-1.4488	-3.3954	-0.8223		3.2000	
	(0.0903)	(0.2527)	(0.0822)		(0.3404)	

Table 2 (Continue)

Ln(SD of ε)					2.011 (0.168)	1.0893 (0.1648)
σ(ε) (error iid time, person grants)	3.5461		7.4738	7.4405	7.4700	2.9722
σ(u) (fixed individual random error)			4.9528	-------	4.9530	1.4999
ρ_{uv}			0.3051		0.3054	0.2030
-Log L	-12509.13	-12879.13	-11997.03	------	-11996.93	-12024.01

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