Labor Market Friction, Firm Heterogeneity, and Aggregate Employment and Productivity*

Rasmus Lentz
University of Wisconsin–Madison
NBER, CAM, and LMDG

Dale T. Mortensen
Northwestern University
Aarhus University, NBER, and IZA

June 18, 2010
Preliminary Draft

Abstract

The paper is based on a synthesis of a "product variety" version of the firm life cycle model developed by Klette and Kortum (2004) and an equilibrium search model of the labor market with job to job flows introduced by Mortensen (2003). In the construction, a continuum of intermediate product and service varieties are produced with labor that serve as inputs in the production of a final good. Intermediate goods producers generally differ with respect to their productivity. New firms enter and continuing firms grow by developing new product varieties. The time required to match workers and jobs in the model depends on the total search effort of workers and the total number of vacancies. Workers can search both while employed and unemployed. Wages are set continuously as the outcome of a bargaining problem over current output. A job separation occurs if either a worker quits or a job is destroyed. We show that a general equilibrium solution to the model exists and that the equilibrium is broadly consistent with observed dispersion in firm productivity, wages, and the relationship between them as well as patterns of worker flows. The model implies that frictions, both in the labor market and in the firm growth process, can be important determinants of aggregate productivity as well as aggregate employment.

*Financial supported of this research includes grants from the U.S. National Science Foundation and the Danish National Research Foundation. The research assistance of Jesper Bagger is also gratefully acknowledged.

1 Introduction

Firm productivity differentials are large and persistent. Average wages paid by firms are positively correlated with firm productivity and more productive firms are larger and are more likely to export. Empirical evidence supports the view that workers move from lower to higher paying jobs. These differences imply that the reallocation process may be an important determinate of aggregate productivity as well as employment. The purpose of this paper is to develop a tractable equilibrium model that explains these and other stylized facts relating worker flows, wages, and productivity across firms. The equilibrium solution to the model also provides a framework for studying the determinants of the distribution of productivity across firms as well as the level of aggregate employment and productivity when frictions in the labor market are present.[1]

The paper is based on a synthesis of a "product variety" version of the firm life cycle model developed by Klette and Kortum (2004), the Melitz (2003) model of heterogenious firms, and an equilibrium search model of the labor market in the spirit of that introduced in Mortensen (2003). Households value future streams of consumption and leisure. A continuum of intermediate product and service varieties are produced with labor input. These serve as inputs in the production of a final good which can be used either for consumption or investment.

Both potential and continuing firms invest in costly R&D. Potential firms that innovate enter the economy as operating firms. Continuing firms grow by creating and developing new product lines. The creator of a new variety has a cost advantage in supplying the product which generates the surplus needed to motivate the investment required to innovate. The firm's derived demand for labor is limited by the demand for the firm's portfolio of products. Existing products are destroyed at an exogenous rate and a firm dies when all of its products lines are destroyed.

Time is required to match workers and jobs in the model. Workers search at endogenous intensities both while employed and unemployed. Wages are set as the outcome of a strategic bargaining problem over match surplus. A job separation occurs if either a worker quits or a job is destroyed. Firms with vacancies also invest in recruiting workers. The rates at which workers and vacancies meet are determined by a matching function that depends on aggregate worker search and recruiting effort.

[1] In this respect, the paper is closely related to Lagos (2006).

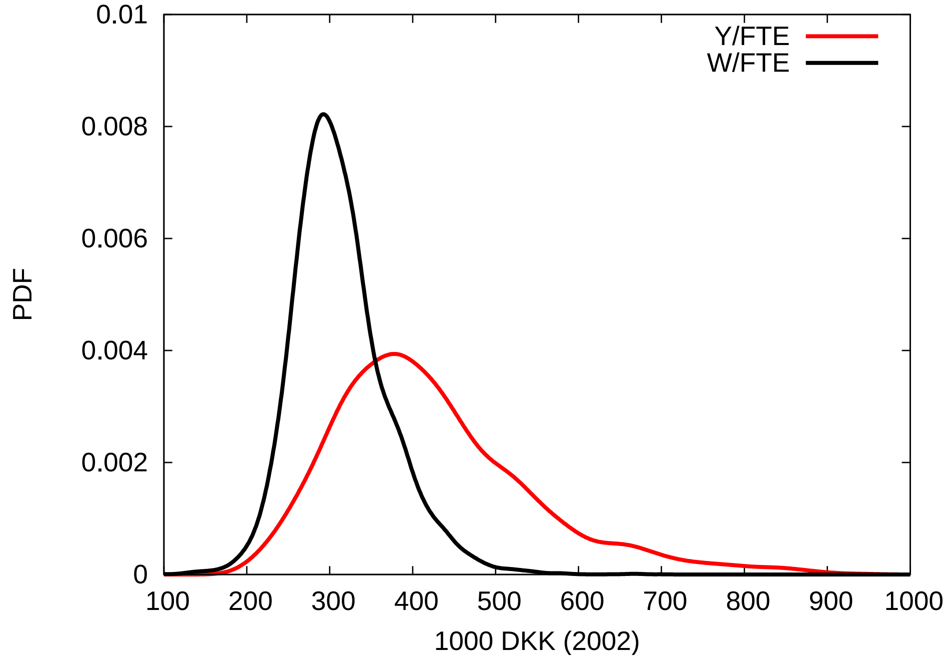
In general steady state equilibrium, consumption and leisure, the total number of product lines supplied by each firm type, the number of worker employed by each firm type, and the number of employed workers are all stationary. The steady state equilibrium distributions of both labor productivity and wages across firms and the employment rate reflect these steady state conditions as well as optimality requirements. A steady state equilibrium is defined as a consumption flow, a firm entry rate, a product creation rate and a measure of products supplied for each firm type, a firm recruiting strategy, and a worker search strategy that satisfy optimality and the steady state conditions. The existence of at least one steady state equilibrium solution is established. The equilibrium solution illustrates the importance of both labor market frictions and the cost of firm growth as determinants of aggregate productivity and employment.

2 Danish Wage and Productivity Dispersion

Both productivity and wage dispersion are large and persistent in the micro data. (Early work documenting productivity dispersion is reviewed in Bartelsman and Doms (2000). See Davis and Haltiwanger (1991) on wage dispersion.) Foster et al. (2001) present reduced form evidence that workers are reallocated from less to more productive firms as well. Finally, Lentz and Mortensen (2008) estimate a structural model of firm dynamics, closely related to that studied here, using Danish longitudinal firm data. It implies that about half of productivity growth can be attributed to reallocation within firm birth cohort as reflected in the higher relative growth rates of more productive firms.

Nagypál (2004) finds that over half of all U.S. prime age full time workers who separate in a month are reemployed with another employer in the next month and that 70% of those who don't leave the labor force experience such a job-to-job movement. That workers who quit to move to another employer typically receive an increase in earnings has long been know. (Bartel and Borjas (1981) and Mincer (1986) represent early work on the finding.) Recently structural models of job to job movements have added more details. For example, Jolivet et al. (2006) show that an estimated off-the-shelf search on-the-job model does a good job of explaining the observed extent to which the distribution of wages offered to new employees is stochastically dominated by the distribution of wages earned in 9 out of 11 OECD countries. Christensen et al. (2005) estimate a structural model that allows for an endogenous choice of search intensity using

Figure 1: Danish Firm Wage and Productivity Distributions



Danish matched worker-employer data. All of these studies supports the basic view that wage dispersion exits in the sense that different firms pay similar workers differently and that workers respond to these differences by moving from lower to higher paying employers.

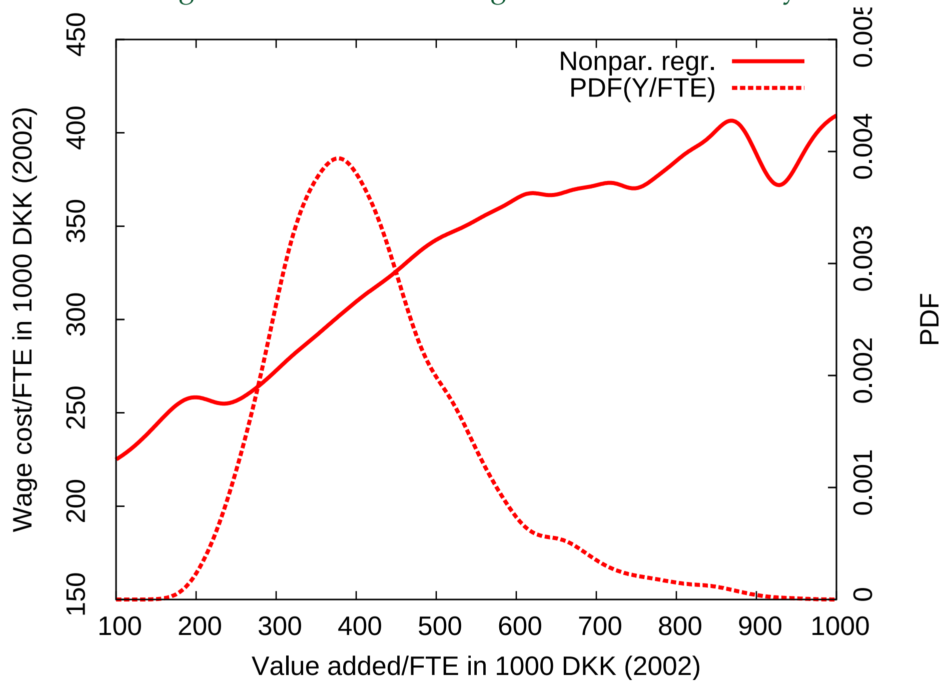
This paper is based on evidence that wage dispersion is induced by productive firm heterogeneity through "rent sharing." Given search friction, match rents are larger at more productive firms. Hence, if the wage is determined by some form of rent sharing, then we should see a positive cross section relationship between the average wage paid and firm productivity.

In this section, we present Danish evidence for wage and productivity dispersion as well as a positive association between them drawn from an longitudinal files on the value added (Y), full time equivalent (FTE) employment (N) and the wage bill (W) paid by privately owned Danish firms. These firm accounting data are collected annually in a survey conducted by Statistics Denmark and are supplemented from tax records. The survey is a rolling panel and the sampling of firms is based on firm size and revenue.[2]

In Figure 1, cross firm probability density functions for wage cost per worker, as reflected in

[2]Only firms with 5 or more employees and revenue above 500M DKK are included. All firms with labor force at or above 50 are included in the survey. The sampling proportions for 20-49 workers is 50%, for 10-19 is 20%, and for 5-9 is 10%.

Figure 2: Danish Firm Wage vs Firm Productivity



the ratio of the wage bill to FTE employment (W/N), and for average productivity per worker, as measured by the ratio of valued added to employment (Y/N), are plotted for the year 2002.[3] Note that dispersion in both the average wage paid and labor productivity are large. Specifically, four fold productivity differentials lie within the 5-95 percentile support of the data as well as over 100% differences in the average wages paid. The relative positions of the two distributions suggest that average labor productivity exceeds the average wage paid, as one should expect.

In Figure 2, a non-parametric regression of firm labor productivity on average firm wage cost is plotted. This evidence suggests that the relationship is close to linear.

The strong positive relationship between firm wage and productivity supports "rent-sharing" theories of the wage determination. Of course, the linear nature of the relationship is implied by the simple bilateral bargaining model assumed in this paper. But, there are alternative models of rent sharing that might be incorporated instead. The dynamic monopsony model proposed by Burdett and Mortensen (1998) implies a strictly concave relationship. Indeed, Mortensen (2003) shows that the degree of concavity is much larger than that observed in these data. Postel-Vinay and Robin (2002) and Cahuc et al. (2006) develop and estimate a structural model in

[3] At the moment, we have data for 1999-2002 inclusive. All the properties characteristics of the data pointed out here are virtually identical for each of the other years as well.

which the growth in worker wages with experience can be attributed to Bertrand competition between a worker's current and a potential alternative employer. Although this wage setting process is more complicated than either bilateral bargaining or dynamic monopsony, it may also be consistent with the quantitative relationships reported here.

3 The Model

3.1 Household Preferences

There is a continuum L of identical households each composed of a unit measure of individuals who are either employed or not.. There is a single final good used for either consumption or investment and produced by a competive market sector using a continuum of intermediate goods. Each intermediate good is supplied by a single firm. Employed workers produce intermediate goods and those not employed produce a substitute for the final good at rate b. Household utility is the expected present value of consumption where the fixed discount rate r is the equilibrium rate of interest.

3.2 Intermediate Product Demand

A single final good is supplied by a competitive sector and the aggregate quantity produced is a function of a continuum of intermediate inputs each supplied by a monopolist. Specifically, final output is determined by the following constant returns to scale Dixit-Stigliz production function,

$$Y = \frac{A}{K}\left[\int_0^K \alpha(j)^{\frac{1}{\sigma}} x(j)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $x(j)$ is the quantity of intermediate product j available, K is the measure of intermediate goods, and σ is the elasticity of substitution between inputs. The demand for each input given final output is

$$x(j) = \alpha(j) Y \left(\frac{A}{K}\right)^{\sigma-1} p(j)^{-\sigma},$$

where $p(j)$ is the price of input j expressed in units of the final output. Each intermediate good is produced with labor and supplied by a single firm. Specifically, $x(j) = q(j)n(j)$ where $q(j)$ is the productivity of each worker employed to produce product j and $n(j)$ is employment. The demand and productivity pair parameters (α, q) define the supplier of product j in the model.

Given this pair of parameters, the supplying firm's revenue is

$$px = R_n(\alpha, q) = (\alpha Y)^{\frac{1}{\sigma}} \left(\frac{Aqn}{K}\right)^{\frac{\sigma-1}{\sigma}}. \tag{2}$$

Equation (2) makes clear that in this model with monopolistic pricing, demand and productivity shocks work in the same way by shifting the revenue function. It is useful to rewrite revenue as a function of a single random variable, $\mu = \alpha^{\frac{1}{\sigma}} (Aq)^{\frac{\sigma-1}{\sigma}}$,

$$R_n(\mu) = \mu Y^{\frac{1}{\sigma}} \left(\frac{n}{K}\right)^{\frac{\sigma-1}{\sigma}}. \tag{3}$$

In the following variation in μ is surpressed until needed.

3.3 Bargaining, Search, and Vacancy Creation

When an individual worker meets an employer, the two bargain over their match surplus. Given that the value of the worker's current employment status is the threat point in the bilateral bargaining game, the generalized Nash solution allocates the share $\beta \in (0,1)$ to the worker and the residual to the employer. A match is formed if its value exceeds the worker's outside option.

For example, if the worker is unemployed, then match surplus to be divided is $X - U$ where X is the match value and U is the value of unemployment. Given the Nash solution to the bargaining problem, the worker's value of unemployment is

$$rU = b + \lambda_0 \beta \int_U^\infty (X - U) dF(X), \tag{4}$$

where b is the flow value of home production, λ_0 is the rate a which an unemployed worker meets vacancies, and $F(X)$ is the offer c.d.f., the distribution of match values over vacancies.

In the model, employed workers have the opportunity to transit voluntarily to alternative employment at frequency λ_1 and an exogenous transition occurs with frequency λ_2. Consistent with this interpretation, a worker actually moves from one job to the other in the first case only if the transition is value improving but not necessarily in the second. As in Jolivet et al. (2006), the principal purpose of allowing for "involuntary" job-to-job transitions is to account for the fact that the significant fraction of workers in the data experience wage loss when making job-to-job transitions. More generally, involuntary transitions also account for job-to-job movements made for non-economic reasons.

When employed workers can move from one employer to another, the nature of the employment relationship is complicated. For example, an employer has an incentive to match an employee's outside offer in order to retain the worker ex-post, but such behavior encourages rent seeking behavior by the worker ex-post. For this reason, it can be optimal from the employer's point of view to precommit not to respond to alternative employment opportunities of employees, a fact established in Postel-Vinay and Robin (2004). As Mortensen () and Diamond and Maskin (1979) point out, an optimal employment contract in this case generally requires that the worker compensate the employer for the future income loss in the event of a quit for this same reason. Such a "bonding" assumption is not unlike an alternative arrangement where the match surplus is "sold" to the worker in return for an up front payment to the firm. Either way, one obtains efficient separation. In the model environment at hand, outside offers arrive at an exogenously given rate λ_1 which eliminates from the model the kind of rent seeking behavior that precommitment as in Postel-Vinay and Robin (2004) is supposed to discourage. Consequently, matching outside offers is an optimal strategy for the firm and also implements efficient separation. We will therefore adopt a bargaining setup that resembles that of Dey and Flinn (2005) and Cahuc et al. (2006) where firms match outside offers, and the employment contract specifies that the worker receives the share $\beta \in (0,1)$ of the surplus value of the match, which can be viewed as the outcome of a bilateral bargaining problem between worker and employer when they meet and the worker's outside option is full surplus extraction from the outside employer.

Under this bargaining assumption, a worker voluntarily moves from job-to-job if and only if the total value of the new match exceeds that of the old. Hence, the worker's value of employment at the beginning of a new job is $W = A + \beta(X - A)$ where X is the value of the marginal match with the new employer when employed and A is equal to the value of the previous match if the worker was hired voluntary and the value of unemployment if either the worker was previously unemployed or the move was involuntary.

The firm's recruiting strategy determines the number of vacancies posted contingent on the current level of employment in a firm. An optimal strategy maximizes the employer's profit, the product of the employer's share of the match surplus and the rates at which workers of different types are met summed over the types of worker as characterized by employment status. As all unemployed workers but only those employed at smaller match values accept, the number of vacancies posted by a firm with n employees is the solution to the optimization problem,

$$\pi(\Delta V_{n+1}) = \max_{v \geq 0} \left\{ (1-\beta) v \left[(\eta_0 + \eta_2)(\Delta V_{n+1} - U) + \eta_1 \int_U^{\Delta V_{n+1}} (\Delta V_{n+1} - X) dG(X) \right] - c_0(v) \right\}, \tag{5}$$

where $\Delta V_{n+1} = V_{n+1} - V_n$ is the value to the firm of adding a worker, $G(X)$ denotes the fraction of workers employed at a firm with match value X or less, and $c_0(v)$ represents the cost of posting v vacancies, an increasing convex function. The parameters η_0, η_1, and η_2 are rates per vacancy posted at which employers meet unemployed workers, employed workers who have the option of turning down an outside offer, and employed workers who move for reasons exogenous to the model.

As all offers are acceptable to an unemployed worker and one who otherwise transitions to unemployment but only those employed worker in a match with a lower value that the match will voluntarily accept an offer, the employment size contingent hire frequency is

$$h_n = h(\Delta V_{n+1}) = [\eta_0 + \eta_1 G(\Delta V_{n+1}) + \eta_2] \, v(\Delta V_{n+1}), \tag{6}$$

where $v(X)$ is the solution to the optimization problem defined in (5) when $X = \Delta V_{n+1}$. Note that the optimal number of vacancies does not depend directly on either productivity or market size. In short, the value of the match is a sufficient statistic.

3.4 The Value of the Marginal Match

Firms are composed of product lines, each of which faces a destruction risk equal to δ_1. Each product line has its own labor force; direct reallocation across lines within the firm is no less expensive than worker reallocation through the labor market. Hence, the employing unit in the model is the product line, a coalition composed of an employer and its labor force.

The size of the labor force employed in a product line, denoted as n, is a discrete variable defined on the non-negative integers. As the model is formulated in continuous time and hires and separations are individual decisions, only a one unit change in n can occur in any instant if the line continues into the future. Only in the case of product destruction, an event which occurs infrequently at rate δ_1, is this rule violated. In this case, all workers are laid off. Hence, employment in a product line is a birth-death stochastic process with transition rate from n to $n+1$ equal its hire frequency, from $n-1$ to n equal to the frequency with which workers quit, and from any n to $n = 0$ with frequency δ_1.

Define V_n as the expected present value of the future income of the members of a product line composed of the employer and the n employees. If new workers are hired at contingent frequency h_n, then

$$\begin{aligned}
rV_n(\mu) &= R_n(\mu) + \pi(\Delta V_{n+1}) + \delta_1(nU - V_n(\mu)) + \delta_0 n(U - \Delta V_n) \\
&\quad + \lambda_1 n \int_{\Delta V_n}^{\infty} [\Delta V_n + \beta(X - \Delta V_n) + V_{n-1} - V_n]\, dF(X) \\
&\quad + \lambda_2 n \int_{U}^{\infty} [U + \beta(X - U) + V_{n-1} - V_n]\, dF(X) \\
&= R_n(\mu) + \pi(\Delta V_{n+1}) + \delta_1(nU - V_n(\mu)) + (\delta_0 + \lambda_2)\, n(U - \Delta V_n) \\
&\quad + \lambda_1 n\beta \int_{\Delta V_n}^{\infty} (X - \Delta V_n) dF(X) + \lambda_2 n\beta \int_{U}^{\infty} (X - U) dF(X),
\end{aligned} \tag{7}$$

where $\Delta V_n = V_n - V_{n-1}$ is the value of the marginal worker to the firm, λ_1 is the rate at which employed workers generate outside offers, λ_2 represents the quit rate of those moving from another job for other reasons, δ_0 is the transition rate to unemployment, δ_1 denotes the product destruction rate, and $F(X)$ is again the offer distribution.

It is useful to reformulate Equation (7) in terms of the coalition match surplus, $S_n(\mu) = V_n(\mu) - nU$. For this purpose, $\tilde{F}(S) = F(S + U)$ and $\tilde{G}(S) = G(S + U)$. By use of these definitions, a worker's valuation of unemployment can written as,

$$rU = b + \lambda_0 \beta \int_0^{\infty} S d\tilde{F}(S).$$

It then follows that,

$$\begin{aligned}
rS_n(\mu) &= R_n(\mu) + \tilde{\pi}(\Delta S_{n+1}) - \delta_1 S_n - (\delta_0 + \lambda_2)\, n\Delta S_n \\
&\quad + \lambda_1 n\beta \int_{\Delta S_n}^{\infty} (X - \Delta S_n) d\tilde{F}(X) + \lambda_2 n\beta \int_0^{\infty} X d\tilde{F}(X) - nb - \lambda_0 n\beta \int_0^{\infty} S d\tilde{F}(S) \\
&= R_n(\mu) - nb - (\lambda_0 - \lambda_2)\, n\beta E[S] + \tilde{\pi}(\Delta S_{n+1}) - \delta_1 S_n - (\delta_0 + \lambda_2)\, n\Delta S_n \\
&\quad + \lambda_1 n\beta \int_{\Delta S_n}^{\infty} (X - \Delta S_n) d\tilde{F}(X),
\end{aligned} \tag{8}$$

where $E[S] = \int_0^{\infty} S d\tilde{F}(S)$ and

$$\tilde{\pi}(\Delta S_{n+1}) = \max_{v \geq 0} \left\{ (1 - \beta) v \left[(\eta_0 + \eta_2)\, \Delta S_{n+1} + \eta_1 \int_0^{\Delta S_{n+1}} (\Delta S_{n+1} - X) d\tilde{G}(X) \right] - c_0(v) \right\}.$$

By equation (8) one obtains the expression for the marginal coalition surplus,

$$
\begin{aligned}
(r+\delta_1+n(\delta_0+\lambda_2))\,\Delta S_n(\mu) = {} & \Delta R_n - b - (\lambda_0-\lambda_2)\,\beta E[S] + \tilde{\pi}(\Delta S_{n+1}) - \tilde{\pi}(\Delta S_n) \\
& + (\delta_0+\lambda_2)(n-1)\,\Delta S_{n-1} \\
& + \lambda_1 n\beta \int_{\Delta S_n}^{\infty} (X-\Delta S_n)\,d\tilde{F}(X) \\
& - \lambda_1 (n-1)\,\beta \int_{\Delta S_{n-1}}^{\infty} (X-\Delta S_{n-1})\,d\tilde{F}(X),
\end{aligned} \tag{9}
$$

were $\Delta R_n = R_n - R_{n-1}$ is the revenue product of the marginal worker.

Define $m(\mu)$ as the upper product line size for a given μ realization. It is defined by $\Delta S_{m(\mu)}(\mu) = 0$. Since $\Delta S_n(\mu)$ is decreasing in n, there is no option value from hiring at the upper firm size bound. Hence, by equation (9),

$$
\begin{aligned}
0 = {} & \Delta R_m - b - (\lambda_0-\lambda_2)\,\beta E[S] + (\delta_0+\lambda_2)(m-1)\,\Delta S_{m-1} + \lambda_1 m\beta E[S] \\
& - \lambda_1 (m-1)\,\beta \int_{\Delta S_{m-1}}^{\infty} (X-\Delta S_{m-1})\,d\tilde{F}(X) \\
& \Updownarrow \\
\Delta R_m = {} & b + (\lambda_0-\lambda_2-\lambda_1)\,\beta E[S] - \lambda_1(m-1)\,\beta\left[E[S] - \int_{\Delta S_{m-1}}^{\infty} (X-\Delta S_{m-1})\,d\tilde{F}(X)\right] \\
& - (\delta_0+\lambda_2)(m-1)\,\Delta S_{m-1}.
\end{aligned} \tag{10}
$$

Since ΔR_m is monotonically decreasing in ΔS_{m-1} one can bound the expression for ΔR_m by,

$$
\Delta R_m \le \Delta R_{\hat{m}} = b + (\lambda_0-\lambda_1-\lambda_2)\,\beta E[S]. \tag{11}
$$

Equation (16) states that ignoring labor hoarding incentives, labor is hired to the point where the marginal revenue product of an additional worker equals a searching worker's payoff flow, $b + (\lambda_0-\lambda_1-\lambda_2)\,\beta E[S]$. The last term reflects the efficiency of off the job search relative to on the job search. We denote this level by $\hat{m}(\mu)$. Generally, $m(\mu) > \hat{m}(\mu)$ reflecting that firms may choose to hoard labor in anticipation of the costly replacement of future losses of workers to quits or exogenous separation.

3.5 The Wage Function

The future wage stream for any new employee must equal the surplus value of the worker's previous employment status, represented as A, plus the share β of the difference between it and the value of the new match. Consequently, the value of employment given that the worker is hired when there are n employees in the firm is, $W_n^0 = A + \beta(\Delta V_n - A)$, where A is the value of unemployment if the worker is hired while unemployed or in the process of an involuntary job-to-job move and the value of the worker's previous match otherwise. Although there are a continuum of payment streams that are consistent with W_n^0, it is unique under the additional restriction that the wage paid at every date must be the same for all current employees with the same market experience as embodied in the value of their employment status when hired. Both legal requirements and fairness considerations provide reasons for this equal pay condition.

By the fact that separation is efficient, A is necessarily less than the value of the match at its inception, however as time passes in the match, the marginal value of a worker in the coalition may drop and possibly below A. However, efficient separation requires that a worker's continuation value cannot exceed a worker's marginal value in her coalition. We make the assumption that when $A > \Delta V_n$, the employment contract awards the worker with the full surplus of the match, ΔV_n. Hence, we obtain that conditional on labor market experience as represented by the marginal value of the last employment position, and the number of employees in the product line, the worker's continuation value is given by,

$$W_n = \min\left[\Delta V_n, A + \beta(\Delta V_n - A)\right]. \tag{12}$$

The asset equation for W_n is given by,

$$\begin{aligned} rW_n = {} & w_n + (\delta_0 + \delta_1)(U - W_n) + h_n \Delta W_{n+1} - (n-1)(\delta_0 + s_n)\Delta W_n \\ & + \lambda_1 \int_{\Delta V_n}^{\infty} [\Delta V_n + \beta(X - \Delta V_n) - W_n] dF(X) \\ & + \lambda_1 \int_{A}^{\Delta V_n} (X + \beta(\Delta V_n - X) - W_n) dF(X) \\ & + \lambda_2 \int_{U}^{\infty} [U + \beta(X - U) - W_n] dF(X), \end{aligned}$$

where

$$s_n = s_n(\Delta V_n) = \lambda_1[1 - F(\Delta V_n)] + \lambda_2. \tag{13}$$

Given that equation (12) holds the worker surplus can be written as,

$$
\begin{aligned}
(r+\delta_0+\delta_1+\lambda_2)(W_n-U) = w_n - b + h_n\Delta W_{n+1} - (n-1)(\delta_0+s_n)\Delta W_n \\
+ \lambda_1\beta\int_{\Delta V_n}^{\infty}(X-\Delta V_n)dF(X) \\
+ \lambda_1\left[1-F(\Delta V_n)\right](\Delta V_n - W_n) \\
+ \lambda_1(1-\beta)\int_{A}^{\Delta V_n}(X-A)\,dF(X) \\
+ (\lambda_2-\lambda_0)\,\beta\int_{U}^{\infty}(X-U)dF(X).
\end{aligned}
$$

Define $\tilde{A} = A - U$ and $\tilde{W}_n = W_n - U$. Following the approach in equation (8), this can then be expressed in terms of worker and marginal coalition surplus,

$$
\begin{aligned}
(r+\delta_0+\delta_1+\lambda_2)\tilde{W}_n = w_n - b + h_n\Delta\tilde{W}_{n+1} - (n-1)(\delta_0+s_n)\Delta\tilde{W}_n \\
+ \lambda_1\beta\int_{\Delta S_n}^{\infty}(S-\Delta S_n)d\tilde{F}(S) \\
+ \lambda_1\left[1-\tilde{F}(\Delta S_n)\right]\left(\Delta S_n - \tilde{W}_n\right) \\
+ \lambda_1(1-\beta)\int_{\tilde{A}}^{\Delta S_n}\left(S-\tilde{A}\right)d\tilde{F}(S) \\
+ (\lambda_2-\lambda_0)\,\beta E\left[S\right],
\end{aligned}
$$

where

$$
\tilde{W}_n = \min\left[\Delta S_n, \tilde{A} + \beta\left(\Delta S_n - \tilde{A}\right)\right]. \tag{14}
$$

It follows that

$$
\begin{aligned}
w_n = {} & b + (\lambda_0-\lambda_2)\,\beta E\left[S\right] \\
& + \left[r+\delta_1+n(\delta_0+s_n)+h_n\right]\tilde{W}_n - h_n\tilde{W}_{n+1} - (n-1)(\delta_0+s_n)\tilde{W}_{n-1} \\
& - \lambda_1\left[\beta\int_{\Delta S_n}^{\infty}Sd\tilde{F}(S) + (1-\beta)\left[\left(1-\tilde{F}(\Delta S_n)\right)\Delta S_n + \int_{\tilde{A}}^{\Delta S_n}\left(S-\tilde{A}\right)d\tilde{F}(S)\right]\right].
\end{aligned} \tag{15}
$$

Equation (17) is a straightforward evaluation given a numerical solution for ΔS_n. Equation (9) provides one strategy for solving for the marginal coalition surplus.

The model has interesting implications for wage dynamics. As emphasized in both Dey and Flinn (2005) and Postel-Vinay and Robin (2002) one may observe both wage increases and decreases between jobs in this type of model. However, unlike these two previous examples, wages can both decrease and increase within the job in the model. Wages will increase as a result of an arrival of an outside offer that increases $\tilde{A}$ but is below the marginal coalition surplus of the worker's current job. However, wages will also rise and fall as changes in the coalitions labor force size impact the coalition's marginal match surplus. In the early life of a product line, the expansion of its labor force size will tend to imply decreasing wage profiles which will be impacted in the opposite direction as workers improve their bargaining position as a result of outside offer arrivals.

3.6 The Case of Perfect Substitutes ($\sigma = \infty$)

Solving the model for the value of the marginal match and the associated optimal recruiting strategy is complicated in the general case in which intermediate good are imperfect substitutes. However, doing so in the limiting case of perfect substitutes is quite straight forward. Since the qualitative properties of the model are likely to be similar, it is useful to study the special case.

The critical property of the special case is linearity of revenue in employment. That is $R_n = Aqn/K$ in the limit from equation (2). Therefore, $\Delta R = Aq/K$ is a constant which together with equation (8) implies that the surplus value of the marginal worker is also constant. Indeed, its value, denoted as $\Delta S(q)$ where the dependence on productivity$q$ is emphasized, is the unique solution to

$$(r + \delta_0 + \delta_1 + \lambda_2)\Delta S(q) = \Delta R - b - (\lambda_0 - \lambda_2)\beta E[S] + \lambda_1 \beta \int_{\Delta S(q)}^{\infty} (S - \Delta S(q)) d\tilde{F}(S), \tag{16}$$

the present value of output per worker, Aq/K, minus the unmatched worker's flow value, plus the expected gain in joint value attributable to the on-the-job search. It is straight forward to verify that $\Delta S(q)$ is monotonically increasing in q . In the case of equally efficient matched and unmatched search, $\lambda_1 + \lambda_2 = \lambda_0$, the marginal product line ($\Delta S(q) = 0$) has marginal revenue $\Delta R = b$. Any product line with productivity below that of the marginal product line will not offer any vacancies. If unmatched search is more efficient than matched search then the marginal product line will have productivity $q > b$.

By use of the wage equation (17) and the expression for the marginal coalition surplus, equation (16), the wage can be stated as,

$$w = \Delta R - (1 - \beta) \left\{ [r + \delta_1 + \delta_0 + s(q)] \left(\Delta S(q) - \tilde{A} \right) + \lambda_1 \int_{\tilde{A}}^{\Delta S(q)} \left(S - \tilde{A} \right) d\tilde{F}(S) \right\}, \tag{17}$$

where

$$s(q) = \lambda_1 \left[1 - \tilde{F}(\Delta S(q))\right] + \lambda_2.$$

Because the value of the marginal match is a constant, the wage is also independent of the labor force size. It is equal to the marginal match productivity, q, minus the flow value to the firm which is the $(1-\beta)$ fraction of the match surplus minus the worker's history dependent bargaining position, $\tilde{A}$, multiplied by the effective discount rate of the match plus a compensation to the firm for the possibility that outside offer arrivals may improve the worker's bargaining position while remaining in the match.

The FONC for an optimal recruiting effort, the problem defined on the right side of (5), implies that

$$c_0'(v(q)) = (1-\beta)\left[(\eta_0 + \eta_2)\,\Delta S(q) + \eta_1 \int_0^{\Delta S(q)} (\Delta S(q) - S) d\tilde{G}(S)\right].$$

As the value of the marginal match is increasing in productivity and the return to recruiting effort is increasing in that value, more productive firms post more vacancies. In this case, neither the frequency with which workers are hired nor the quit rate depend on labor force size. Indeed, given this fact, the hire frequency

$$h(q) = \left[\eta_0 + \eta_1 \tilde{G}(\Delta S(q)) + \eta_2\right] v(q) \tag{18}$$

is increasing in worker productivity while the quit rate $s(q)$ is decreasing in q. As the employment process is mean reverting and tends toward the value of n that equates hires and quit flows, more productive product lines are larger on average.

4 Product Innovation

Firm size as reflected in the number of products supplied is generated by a research and development process as introduced in Klette and Kortum (2004). At any point in time, a firm has k product lines can grow only by creating new product varieties. Investment in R&D is required to create new products. Specifically, the firm's R&D investment flow generates new product arrivals at frequency γk where γ represents the firm's innovation rate per product line. The total R&D investment cost expressed in terms of output is $c_1(\gamma)k$ where $c_1(\gamma)$ is assumed to be strictly increasing and convex function. The assumption that the total cost of R&D investment is linearly homogeneous in the new product arrival flow, γk, and the number of existing product, k, "captures the idea that a firm's knowledge capital facilitates innovation," in the words of Klette and Kortum (2004). The specification assumption also implies that Gibrat's law holds in the sense that innovation rates are size independent contingent on type, a property needed to match the data on firm growth. Finally, every product is subject to destruction risk with exogenous frequency δ_1. Given this specification, the number of products supplied by any firm is a stochastic birth-death process characterized by "birth rate" γ and "death rate" δ_1.

The productivity of a product line, q, and the size of the market, α, are independent random variables realized when a new product is created. The distribution of market size is assumed independent of type. Firm type heterogeneity enters the model through type dependence of the distribution of product line productivity, q. Consequntly, the distribution of the random variable $\mu = \alpha^{\frac{1}{\sigma}} (Aq)^{\frac{\sigma-1}{\sigma}}$ is also type dependent. Let $\Gamma_\tau(\mu), \tau \in \{1,2....,T\}$, denote the cumulative distribution of μ for any firm of type τ. We assure that the magnitude of the type index reflects the productivity rank of the type in the sense of first order stochastic dominance. That is, $\tau > \tau'$ implies that product lines of firms of τ are likely to have greater employment conditional revenue than firms of type τ', i.e., $\Gamma_\tau(\mu) \leq \Gamma_{\tau'}(\mu)$ for all $\mu \in [0, \bar{\mu}]$.

4.1 Value of R&D

Let Ψ represent the value of research per existing product line. The innovation frequency per product line is the choice variable γ. If an innovation arrives in next instant, the employer will have a product line with no workers, which generates a revenue flow equal to the net profit associated with attempting to fill the first job, which is $\pi(\Delta V_1)$. As every product line faces destruction risk δ_1, its present value at the moment of discovery is $\pi(\Delta V_1)/(r+\delta_1)$. In addition, the option to create still another, which again has value Ψ, arrives with each product created. Because the option to create a new product is lost if the existing product line is destroyed and the market size and cost of production are realized only after the product is created, the value of a new product for firm of type τ solves

$$(r+\delta_1)\Psi_\tau = \max_\gamma \left\{ \gamma \left(\int \frac{\pi(\Delta V_1(\tilde{\mu}))d\Gamma_\tau(\tilde{\mu})}{r+\delta_1} + \Psi_\tau \right) - c_1(\gamma) \right\}. \tag{19}$$

Note that the contribution of a product line to the expected present value of future income of the coalition composed of the firm and all currently employed worker is $\Delta V_0 = \pi(\Delta V_1)/(r+\delta_1)$

from equation (7). Because there are no workers yet in the line, only the future return to recruiting the first worker is relevant and that accrues to the employer. Hence, one can show that the innovation frequency optimal for the firm, the solution to the problem specified on the right side of (19), is optimal for the coalition as well.

At a point in time, the coalition is supplying k different products. Let $V_{n_k}(\mu_k)$ represent the value of the k^{th} product line were n_k represents the number of employees and μ_k characterizes the revenue function of the product line. Given these specifics, it solves equation (7) when $R_n = R_{n_k}(\mu_k)$ as defined in equation (2). Let $(\vec{n},\vec{\mu},k)$ represent the state of the coalition where $\overrightarrow{x} = (x_1,...,x_k)$ represents a vector of length k. Under the assumption that the innovation rate is chosen to maximize the value of the coalition given that it is of type τ, denoted $\Lambda_\tau(\vec{n},\vec{\mu},k)$, is the solution to the following continuous time Bellman equation,

$$\begin{aligned} r\Lambda_\tau(\vec{n},\vec{\mu},k) = k\max_{\gamma\geq 0}&\left\{\gamma\int\left[\Lambda_\tau\big((\vec{n},0),(\vec{\mu},\tilde{\mu}),k+1\big) - \Lambda_\tau(\vec{n},\vec{\mu},k)\right]d\Gamma_\tau(\tilde{\mu}) - c_1(\gamma)\right\} \\ &+ (r+\delta_1)\sum_{j=1}^{k} V_{n_j}(\mu_j) + \delta_1\sum_{j=1}^{k}\Big(\Lambda_\tau(\vec{n}_{\langle j\rangle},\vec{\mu}_{\langle j\rangle},k-1) - \Lambda_\tau(\vec{n},\vec{\mu},k)\Big), \end{aligned} \tag{20}$$

where $\vec{x}_{\langle j\rangle}$ is an $k-1$ vector constructed from $\vec{x}$ by deleting its j^{th} element. The first term on the right is the expected gain associated with the creation of a new product, the second term is simply the employer's gross revenue flow from its current product portfolio, and the last is the expected capital loss in value attributable to the destruction of an existing product.

Proposition 1. *The value of a firm-worker coalition takes the form*

$$\Lambda_\tau(\vec{n},\vec{\mu},k) = \sum_{j=1}^{k} V_{n_j}(\mu_j) + k\Psi_\tau. \tag{21}$$

Proof. Insert the conjecture into equation (20) to obtain

$$\begin{aligned} r\left[\sum_{j=1}^{k} V_{n_j}(\mu_j) + k\Psi_\tau\right] &= k\max_{\gamma\geq 0}\left\{\gamma\int\left(V_0(\tilde{\mu}) + \Psi_\tau\right)d\Gamma_\tau(\tilde{\mu}) - c_1(\gamma)\right\} \\ &\quad + (r+\delta_1)\sum_{j=1}^{k} V_{n_j}(\mu_j) - \delta_1\sum_{j=1}^{k}\Big(V_{n_j}(\mu_j) - \Psi_\tau\Big) \\ &\Updownarrow \\ (r+\delta_1)k\Psi_\tau &= k\max_{\gamma\geq 0}\left\{\gamma\int\left(\frac{\pi(\Delta V_1(\tilde{\mu}))}{r+\delta_1} + \Psi_\tau\right)d\Gamma_\tau(\tilde{\mu}) - c_1(\gamma)\right\}. \end{aligned}$$

Hence, equation (21) holds by (7) where Ψ_τ is the solution to (19). Since, equation (20) can be reformulated as a contraction, the form in equation (21) is also the unique solution. □

4.2 Innovation Frequency

Equation (7) defines the value of innovation activity per product line for a type τ firm. It is the present value of the maximal difference between two parts, the product of the return to innovation and the chosen innovation rate less the R&D investment required to sustain that rate. The return is the sum of two parts, the expected present value of the profit of a new product line plus that value of the opportunity to create another product facilitated by the existence of a new product line, the value of the knowledge embodied in the creation of a new product. From equation (20), the innovation frequency satisfies

$$c_1'(\gamma_\tau) = E_\tau V_0(\tilde{\mu}) + \Psi_\tau \tag{22}$$

when positive where E_τ represents the expectation operator conditional on firm type. Note that the choice is independent of both the number of products currently supplied and of the number of workers employed to supply each line.

Proposition 2. *If the cost of R&D, $c_1(\gamma)$, is increasing, strictly concave and $c_1(0) = c_1'(0) = 0$, intermediate products are imperfect substitutes, and the measure of products supplied, K, is sufficiently large, then the optimal product creation rate is positive and less than the product destruction rate, δ.*

Proof. If a solution to (22) exists, then the value of an additional product line is defined by

$$\Psi_\tau = \max_{\gamma \geq 0} \left\{ \frac{\gamma E_\tau V_0(\tilde{\mu}) - c_\gamma(\gamma)}{r + \delta_1 - \gamma} \right\}. \tag{23}$$

Hence, the first order condition for the optimal innovation rate can be written as

$$f(\gamma) = (r + \delta_1 - \gamma)\left(E_\tau V_0(\tilde{\mu}) - c_\gamma'(\gamma)\right) + \gamma E_\tau V_0(\tilde{\mu}) - c_\gamma(\gamma) = 0 \tag{24}$$

and the second order condition requires $f'(\gamma) = -(r + \delta_1 - \gamma)c''(\gamma) \leq 0$ at a maximal solution. As $f'(0) = (r + \delta_1)E_\tau V_0(\tilde{\mu}) > 0$, the first order condition has a unique solution satisfying $0 < \gamma < \delta_0$ and the sufficient second order condition is satisfied if $f'(\delta_1) = (r + \delta_1)E_\tau V_0(\tilde{\mu}) - rc_\gamma(\delta_1) - c_\gamma(\delta_1) < 0$. As $V_0(\tilde{\mu})$ is bounded above by the largest value of a new product line and that bound converges to zero as $K \to \infty$ by equations (2) and (7) where $\sigma > 1$, the claim follows. $\square$

4.3 Entry

Assume that entry requires a successful innovation, that the cost of innovation activity by a potential entrant is $c_1(\gamma)$, and that firm type is unknown to an entrepreneur prior to entry. The entry rate is the product $v = m\gamma_0$ where m is a given measure of entrepreneurs and γ_0 is the frequency with which any one of them creates new product. As the optimal innovation rate by a potential entrant maximizes expected value, equal to $\gamma_0 \sum_\tau [E_\tau V_0(\tilde{\mu}) + \Psi_\tau] \phi_\tau - c(\gamma_0)$ where ϕ_τ is the exogenous probability of being a type τ firm, the optimal choice is defined

$$v = m\gamma_0 = m \arg\max_{\gamma \geq 0} \left\{ \left(\sum_\tau [E_\tau V_0(\tilde{\mu}) + \Psi_\tau] \phi_\tau \right) \gamma - c_1(\gamma) \right\}. \tag{25}$$

5 Steady State Market Equilibrium

5.1 The Meeting Process

The aggregate rate at which workers and vacancies meet is determined by increasing concave and homogeneous of degree one matching function of aggregate vacancies and search effort. Aggregate search effort is equal to $\lambda_0 u + (\lambda_1 + \lambda_2)(1-u)$ where the parameters λ_0 and $\lambda_1 + \lambda_2$ reflect the search intensities of unemployed and employed workers respectively. Hence, the aggregate meeting rate is $m(\theta)(u + (a_1 + a_2)(1-u))L$ where by an appropriate normalization

$$\lambda_0 = m(\theta) \text{ and } \lambda_i = a_i \lambda_0,\ i \in \{1,2\} \tag{26}$$

represent the meeting rates, u is the unemployment rate, and L is the fixed measure of the households supplying labor to the market.

By assumption the function $m(\theta)$ is increasing and concave. The rate a vacancy meets some worker is $\eta = m(\theta)/\theta$. The assumption that workers of each type are met at rates proportional to their relative search intensities implies that the meeting rates by worker type are

$$\begin{aligned}
\eta_0 &= \left(\frac{u}{u + (1-u)(a_1 + a_2)} \right) \frac{m(\theta)}{\theta} \\
\eta_1 &= \left(\frac{(1-u)a_1}{u + (1-u)(a_1 + a_2)} \right) \frac{m(\theta)}{\theta} \\
\eta_2 &= \left(\frac{(1-u)a_2}{u + (1-u)(a_1 + a_2)} \right) \frac{m(\theta)}{\theta},
\end{aligned} \tag{27}$$

5.2 Product Line Size and Product Distributions

The offer c.d.f. $F(X)$ is the fraction of vacancies posted by product lines with marginal match values less than or equal to X. As the number of vacancies posted by each product line depends on the number of worker employed, the size of the market, and the productivity of the line, one needs to calculate the distribution of employment over product lines of each type. Let $P_n(q,\alpha)$ represent the fraction of product lines with employment equal to n. As the flow out of the state $n = 0$ equal hires plus product destruction and the flow into the state is equal to the flow of newly created products,

$$\sum_\tau (v\phi_\tau + \gamma_\tau)\,\Gamma'_\tau(\mu) + \left(\delta_0 + s_1(\mu)\right) P_1(\mu) = \left[\delta_1 + h_0(\mu)\right] P_0(\mu),$$

where $\Gamma'_\tau(\mu)$ is the μ p.d.f.. Since only transition from $n-1$ to n, $n+1$ to n, and n to zero can occur in an instant, all the other measures satisfy the difference equation

$$\left(\delta_0 + s_{n+1}(\mu)\right)(n+1)P_{n+1}(\mu) + h_{n-1}(\mu)P_{n-1}(\mu) = \left[\delta_1 + h_n(\mu) + \left(\delta_0 + s_n(\mu)\right)n\right] P_n(\mu) \tag{28}$$

where the function $s_n(\mu)$ and $h_n(\mu)$ are those defined by equations (6) and (13).

The measure of products supplied by the set of type τ firms evolves according to the law of motion, $\dot{K}_\tau = v\phi_\tau + \gamma_\tau K_\tau - \delta_1 K_\tau$ where v is the innovation rate of new entrants, ϕ_τ is the fraction of entrant who are of type τ, and γ_τ is the innovation rate of type τ firms per product line, and δ_1 is the product destruction rate. In other words, the net rate of change in the measure is equal to the sum of the flows of products supplied by new entrants and continuing firms respectively less the flow of product lines currently supplied by the type that are destroyed. Hence, in steady state, the measure of products supplied by type τ firms and the aggregate measure of products are

$$K_\tau = \frac{v\phi_\tau}{\delta_1 - \gamma_\tau} \text{ and } K = \sum_\tau K_\tau. \tag{29}$$

5.3 Offer and Employment Distributions

Given these constructs and the fact that the optimal number of vacancies posted depends only on the marginal value, the market steady state distribution of vacancies over match value offers is

$$F(X) = \frac{\sum_\tau K_\tau E_\tau \left[\sum_{n=0}^\infty \mathbb{I}\left[\Delta V_n(\tilde{\mu}) \leq X\right] v_n(\tilde{\mu})\, P_n(\tilde{\mu})\right]}{\sum_\tau K_\tau E_\tau \left[\sum_{n=0}^\infty v_n(\tilde{\mu})\, P_n(\tilde{\mu})\right]}. \tag{30}$$

where $v_n(\mu)$ is the number of vacancies posted and $\Delta V_n(\mu)$ is the value of the marginal match in a product line characterized by μ when employment is n, $\mathbb{I}(\cdot)$ is the indicator function equal to unity if the argument is true and zero otherwise, K_τ is the number of products supplied by firms of type τ, and $E_\tau\{\cdot\}$ is the expectation operator taken with respect to μ distribution. Similarly, the distribution of employment over match values is given by

$$G(X) = \frac{\sum_\tau K_\tau E_\tau \left[\sum_{n=0}^{\infty} \mathbb{I}\left[\Delta V_n(\tilde{\mu}) \le X\right] n P_n(\tilde{\mu})\right]}{(1-u)L}, \tag{31}$$

5.4 Market Tightness and Unemployment

Of course, the ratio of the aggregate vacancies to aggregate search effort, market tightness, is

$$\theta = \frac{\sum_\tau E_\tau \left\{\sum_{n=0}^{\infty} v_n(\tilde{\mu}) P_n(\tilde{\mu})\right\} K_\tau}{(u + (a_1 + a_2)(1-u))L}. \tag{32}$$

Finally, because unemployed workers find jobs at the rate $m(\theta)$ and lose them at rate equal to $\delta_0 + \delta_1$, the steady state unemployment rate, that which balances inflow and outflow, is the solution to

$$\frac{u}{1-u} = \frac{\delta_0 + \delta_1}{\delta_0 + \delta_1 + m(\theta)}. \tag{33}$$

5.5 Aggregate Output

Given the production function specified in equation (1) and the fact that input j is supplied in quantity $x(j) = q(j)n(j)$ where $q(j)$ is the productivity and $n(j)$ is employment of product j, final market output in steady state is produced at rate

$$Y = \frac{1}{K}\left[\sum_\tau \sum_{n=1}^{\infty} E_\tau \left\{\tilde{\mu} n^{\frac{\sigma-1}{\sigma}} P_n(\tilde{\mu})\right\} K_\tau\right]^{\frac{\sigma}{\sigma-1}} \tag{34}$$

by the law of large numbers. Note that equation (19) yields the following upper bound on final good output

$$\overline{Y} = AL\overline{q} \tag{35}$$

which is equal to what output would be if the entire labor force were employed in a product line with productivity equal to the sup of the upper supports, $\overline{q}$, of the distributions of productivity conditional on type.

5.6 Definition and Existence

Definition 1. A value of unemployment, U, a vacancy posting strategy, value function, and employment size probability for a product line, $v_n(\mu)$, $V_n(\mu)$, and $P_n(\mu)$ for all $\mu \in [0, \bar{\mu}]$ and

$n \in \{0,1,2,...\}$, an innovation rate, γ_τ, and a measure of products supplied, K_τ, for each firm type $\tau \in \{1,2,...,T\}$, a firm entry innovation rate v, unemployment rate u, a productivity distribution over employment $G(q)$, a wage offer distribution function $F(w)$, and a labor market tightness θ, an aggregate final product flow Y that satisfy equations (4), (5), (**??**), (28), (23), (29), (25), (33), (31), (30), (32), and (34).

The first two of the following set of assumptions are standard regularity conditions and the third simplifies the existence proof without any essential loss of economic content.

Assumption 1: The matching function is increasing, concave, and homogeneous of degree one so that $\lambda(\theta)$ is positive, increasing, and concave. The boundary conditions, $\lim_{\theta \to 0} \lambda(\theta) = \lim_{\theta \to \infty} \eta(\theta) = 0$, also hold.

Assumption 2: The cost functions, $c_i(\cdot)$, $i \in \{0,1\}$, are both continuous, strictly convex, and satisfy $c_i(0) = c_i'(0) = 0$.

Proposition 3. *A steady state equilibrium exists with a strictly positive value of market tightness.*

Proof. In progress. □

References

Bartel, Ann P. and George J. Borjas (1981). Wage growth and job turnover: An empirical analysis. In S. Rosen (Ed.), *Studies in Labor Markets*. Chicago: University of Chicago Press.

Bartelsman, Eric J. and Mark Doms (2000). Understanding productivity: Lessons from longitudinal microdata. *Journal of Economic Literature* 38, no. 3: 569–594.

Burdett, Kenneth and Dale T. Mortensen (1998). Wage differentials, employer size, and unemployment. *International Economic Review* 39, no. 2: 257–273.

Cahuc, Pierre, Fabien Postel-Vinay, and Jean-Marc Robin (2006). Wage bargaining with on-the-job search: Theory and evidence. *Econometrica* 74, no. 2: 323–64.

Christensen, Bent Jesper, Rasmus Lentz, Dale T. Mortensen, George Neumann, and Axel Werwatz (2005). On the job search and the wage distribution. *Journal of Labor Economics* 23, no. 1: 31–58.

Davis, Steven J. and John Haltiwanger (1991). Wage dispersion within and between manufacturing plants. *Brookings Papers on Economic Activity: Microeconomics* 1991, no. 1991: 115–180.

Dey, Matthew S. and Christopher J. Flinn (2005). An equilibrium model of health insurance provision and wage determination. *Econometrica* 73, no. 2: 571–627.

Diamond, Peter A. and Eric Maskin (1979). An equilibrium analysis of search and breach of contract, i: Steady states. *Bell Journal of Economics* 10, no. 1: 282–316.

Foster, Lucia, John Haltiwanger, and C.J. Krizan (2001). Aggregate productivity growth: Lessons from microeconomic evidence. In Charles R. Hulten, Edwin R. Dean, and Michael J. Harper (Eds.), *New Developments in Productivity Analysis*. Chicago: University of Chicago Press.

Jolivet, Grégory, Fabien Postel-Vinay, and Jean-Marc Robin (2006). The empirical content of the job search model: Labor mobility and wage distributions in europe and the US. *European Economic Review* 50, no. 4: 877–907.

Klette, Tor Jakob and Samuel Kortum (2004). Innovating firms and aggregate innovation. *Journal of Political Economy* 112, no. 5: 986–1018.

Lagos, Ricardo (2006). A model of TFP. *Review of Economic Studies* 73, no. 4: 983–1007.

Lentz, Rasmus and Dale T. Mortensen (2008). An empirical model of growth through product innovation. *Econometrica* 76, no. 6: 1317–73.

Melitz, Marc J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica* 71, no. 6: 1695–1725.

Mincer, Jacob (1986). Wage changes in job changes. In Ronald G. Ehrenberg (Ed.), *Research in Labor Economics*, pp. 171–97. Greenwich, Connecticut: JAI Press.

Mortensen, Dale T. (2003). *Wage Dispersion: Why Are Similar Workers Paid Differently?* Cambridge, Massachusetts and London, England: MIT Press.

Nagypál, Eva (2004). Worker reallocation over the business cycle: The importance of job-to-job transitions. *Northwestern University Working Paper*.

Postel-Vinay, Fabien and Jean-Marc Robin (2002). Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica* 70, no. 6: 2295–2350.

Postel-Vinay, Fabien and Jean-Marc Robin (2004). To match or not to match? optimal wage policy with endogenous worker search intensity. *Review of Economic Dynamics* 7, no. 2: 297–331.