Structural Labor Supply Models when Budget Constraints are Nonlinear

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Abstract

Despite the advantages of structural labor supply methods, many recent papers cite the arguments of MaCurdy et al. as a reason for avoiding structural methods, and instead use simple estimation methods such as differences in differences. This paper examines the role of economic assumptions in structural labor supply methods and how some of the assumptions may be relaxed. We first show the sources of inconsistency in the local linearization method. We then examine the standard approach generally attributed to Hausman, and show that this approach relies on the convexity of preferences in the construction of the likelihood function. We show that the criticisms of MaCurdy et al. can be reinterpreted as showing where in the estimation method the assumption of maximization of convex preferences is enforced. We provide a formal argument that if observed preferences are nonconvex, but the estimation method does not allow for nonconvexity, then estimated parameters may not satisfy the Slutsky restrictions, as has often been found in empirical work. Our simulations show that this event occurs for a substantial range of parameter values. Other Monte Carlo experiments show the sensitivity of estimates to functional form assumptions, the stochastic specification, and the hours endowment. We discuss how other generalizations of the Hausman method dispense with utility maximization altogether. Finally, we show that the standard methods in the literature do not permit estimation of parameters consistent with nonconvex preferences, and describe methods that allow for less restrictive assumptions.

1 Introduction

The effect of taxation on labor supply is of key interest to both policy makers and economists. Labor supply responses to income taxes, and the taxes implicit in social insurance and welfare programs, determine the effects of these policies on incomes, revenues, welfare and budgetary costs. In these contexts, structural methods, though controversial, are advantageous in many cases. Structural methods are often needed to separate out income and substitution effects and calculate deadweight losses. Such methods are also well-suited to simulate the effects of many potential changes in tax and transfer policies that change budget sets in complicated ways.

As is well known, when the tax schedule is nonlinear in income, estimation of labor supply parameters is difficult. In such a case, an individual's marginal tax rate, and hence their after tax wage rate, is not exogenous, but rather is a function of an individual's hours of work. This endogeneity of the after tax wage rate and nonlinearity of the choice problem has led to the development of several methods to estimate labor supply parameters.

Prior to the 1980s, the prevailing method, which will be referred to as local linearization, was to create a linear budget constraint tangent to the actual budget constraint at the level of hours at which an individual was observed. Individuals' hours of work were then regressed on the wage and nonlabor income associated with their respective budget constraints, with instrumental variables often used to attempt to correct for the endogeneity of the wage and income measures in this regression.[1]

Beginning with Burtless and Hausman (1978), and continuing with Hausman (1979, 1980, 1981, 1985a), a method was proposed which explicitly took account of the entire budget constraint generated by a nonproportional tax system, and estimated labor supply parameters using maximum likelihood techniques. The introduction of this method stimulated an outpouring of empirical research on labor supply using some variant of this approach.[2]

In the 1990s, however, two papers sharply changed researchers' views of the usefulness of the Hausman method.[3] In MaCurdy, Green and Paarch (1990) and MaCurdy (1992), it was shown that the likelihood function employed by the Hausman method implicitly enforced that estimated parameters imply a positive Slutsky term at budget set kink points. These papers, which we refer to as the MGP critique, argued that, in order for parameters to satisfy Slutsky positivity at all kinks in the data, the uncompensated substitution effect was essentially constrained to be positive, and the income effect was essentially constrained to be negative (MaCurdy, 1992). It is further argued that "[these] constraints arise not as a consequence of economic theory, but instead as a requirement to create a properly defined statistical model." (MaCurdy, et al. 1990)

Because of the apparent restrictions on parameters in the Hausman method, the results in MaCurdy et al. (1990) and MaCurdy (1992) have led several researchers to avoid using sophisticated techniques that explicitly take into account the budget constraint generated by

[1] See Pencavel (1986) for a survey of studies that use local linearization methods.

[2] For surveys of these studies, see Hausman (1986) and Blundell and MaCurdy (1999).

[3] Of course, MaCurdy et al. (1990) and MaCurdy (1992) were not the only criticisms of the Hausman method. See, for example, Heckman (1982) and Pencavel (1986) for other critiques of the usefullness of the Hausman method.

tax and transfer policies, in favor of simpler methods, like difference in differences estimators.[4] However, since policy changes are rarely of the form that would enable us to separately identify income and substitution effects, in using such techniques to estimate labor supply, the ability to do some of the most important policy evaluations and simulations is sacrificed.

In this paper, we discuss the various methods that have been used to estimate structural labor supply parameters. We clarify some past results, and provide new results on where economic assumptions, in particular, convexity of preferences, enter into the different methods. We show how the incorrect imposition of these assumptions may have led to some of the puzzling empirical results in the literature, and describe ways in which they might be relaxed in practice.[5]

We first show that in plausible circumstances, local linearization will generally yield inconsistent estimates, even when instrumental variables are used. Although this point has been mentioned in the literature,[6] it has not been shown formally, and is not well understood.

We then develop several results new to the literature on the relationship between the economic assumptions made in the derivation of estimation methods, the parameters that may result, and the interpretation of such parameters. We outline the assumptions that are implicit in the Hausman method, including convexity of preferences, and review the restrictions these assumptions imply about parameters of the labor supply function. We then describe the MGP critique, and show that the MGP critique pointed out where in the Hausman method the assumption of maximization of convex preferences was enforced.

We further show that several generalizations of the Hausman method that have been proposed only increase the parameter space to include parameters inconsistent with utility maximization altogether. However, we show that if data generating preferences are actually nonconvex, then relying on the assumption of convexity may result in estimated parameters that are inconsistent, or constrained to be consistent, with utility maximization, a pattern that has often been found. We also provide simulations that demonstrate this claim. As a result, before relaxing the assumption that individuals maximize utility, as others have done, it is advisable to first relax the much stronger assumption that preferences are convex.

We then explore the ability of various estimation methods to allow for the relaxation of the assumption that preferences are convex. We show that local linearization, the Hausman method, the MGP method, and even some nonparametric methods cannot be modified to allow for the estimation of parameters consistent with nonconvex preferences. We then describe estimation using a direct utility function, which avoids making some of the assumptions in the aforementioned methods, and which can be used to estimate parameters consistent with nonconvex preferences. Lastly, we present Monte Carlo evidence on the

[4] See, for example, Blundell, Duncan and Meghir (1998), Fortin and Lacroix (1994), Eissa (1995) and others.

[5] A few caveats must be mentioned. This research, like the other papers in this literature, is strictly a partial equilibrium analysis of labor market behavior using a static model of labor supply to infer preference parameters. As such, we ignore lifecycle considerations, the preferences of employers as to the number of hours worked, imperfect perception of tax rules, and constraints on hours that individuals may work. However we address the current problem first, since estimation methods with those extensions often use this static model as a foundation.

[6] See, for example, passing references in Moffitt (1990, p. 136) and Ericson and Flood (1996, p. 434).

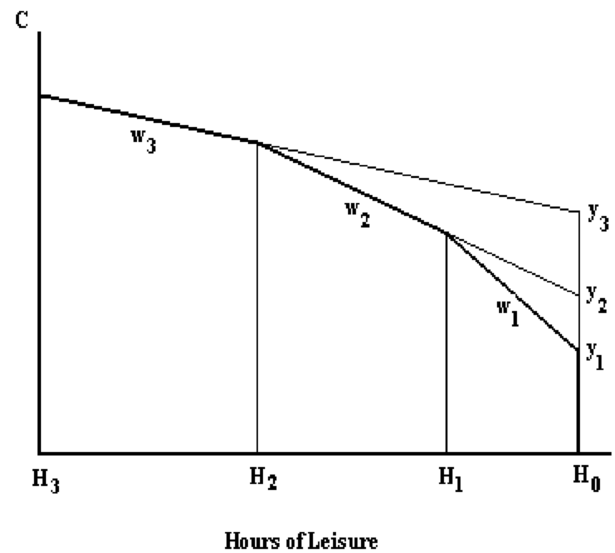

Figure 1: Nonlinear Budget Constraint

performance of this method relative to that of the Hausman method in a variety of settings.

This paper proceeds as follows. In Section 2, we review the local linearization method and show conditions under which commonly used instruments are invalid. In Section 3, we describe the assumptions implicit in the Hausman method and show how these assumptions imply that the labor supply function satisfies Slutsky positivity. We also clarify the MGP critique, and show how it may be reinterpreted as showing where the assumption of convex preferences was enforced on estimated parameters by requiring that Slutsky positivity hold. In Section 4, we provide an argument and a numerical simulation which show that data consistent with nonconvex preference maximization might lead to parameters that violate Slutsky positivity if convexity is assumed, and hence be inconsistent with utility maximization. In Section 5, we demonstrate that commonly used methods cannot be adapted to estimate parameters consistent with nonconvex preferences. In Section 6, we discuss estimation using a direct utility function that allows estimated preferences to be nonconvex. In Section 7, we perform Monte Carlo experiments comparing the performance of this method to that of the Hausman method. These Monte Carlo experiments show that this method improves on the performance of the Hausman method in some cases, and show the importance of functional form, stochastic, and hours endowment specifications when employing either method. Section 8 concludes.

2 The Inconsistency of Local Linearization

In order to understand the controversy surrounding the Hausman method, it is useful to understand the method that preceded it, local linearization.

Consider the piecewise linear budget constraint in Figure 1. In the tax system illustrated in this figure, there are three tax brackets, with tax rates $\{t_1, t_2, t_3\}$. The tax rate on labor income between hours of work H_{j-1} and H_j is t_j, and so the after tax wage rate over this segment of the budget constraint is $w_j = W(1 - t_j)$, where W is the exogenous before tax wage rate.[7] If segment j is extended to the vertical axis, the intercept of this extended segment at 0 hours of work is referred to as virtual income, and is denoted y_j. Denote this budget constraint as $B(\{w_1, ..., w_J\}, \{y_1, ..., y_J\}, h)$

Suppose that for a given individual, i, utility over consumption and hours of work, $U(C, h)$, is maximized on the piecewise linear budget constraint at h^*, where $H_j < h^* < H_{j+1}$. As was noted in Hall (1973), if the individual has convex preferences and the budget constraint is convex, then utility would be maximized at the same level of hours if the budget constraint were $C \leq w_j h + y_j$.

The local linearization method exploits this fact, and uses the after-tax wage w_j and virtual income y_j from the segment of the budget constraint on which the individual is observed as the key explanatory variables in a regression. Letting this segment be denoted j', the regression is of the form

$$h = f(w_{j'}, y_{j'}) + u. \tag{1}$$

where the subscripts for individual i are suppressed to reduce unnecessary clutter. So, for each individual, one identifies the hours at which the individual is observed working, and the after tax wage and virtual income associated with this level of hours. Hours of work are then regressed on these wage and income measures. For example, Hall (1973) uses a variant of this approach in his heavily cited paper.

However, estimating such an equation by ordinary least squares ignores a serious reverse causality problem, in that the after-tax wage and virtual incomes included in the equation are determined by the number of hours that an individual works. Individuals with a greater taste for work will tend to work more hours, which, in the case of a progressive income tax system, will lead to a lower net wage and higher virtual income being imputed for these individuals. Thus, the error term in (1) will be correlated with the wage and virtual income variables.[8] As a result, several researchers have used instrumental variables (IV) to correct for this reverse causality.[9] Usually, the instruments used are the wage and nonlabor income associated with the budget segment at a given level of hours in all individuals budget constraints[10], but demographic characteristics have also been used[11].

There are several problems with the use of local linearization to estimate labor supply equations, however, even when instrumental variables are used. First, note that the above discussion ignores individuals who are observed at a kink point. If an individual is observed at a kink point, it is unclear what net wage and virtual income should be imputed for that

[7] Note that throughout this paper, we assume that an individual's gross wage is exogenous, and hence invariant to both the number of hours the individuals works and the individual's taste for work.

[8] See Moffitt (1990) for a discussion of local linearization and the rationale behind using IV in this setting.

[9] Note that the above discussion assumes that an individual's before tax wage is exogenous, and so it is the unobserved taste for work that creates the endogeneity of the right hand side variables in the estimation equation.

Of course, there are other reasons why the right hand side variables could be endogenous, in that an individual's gross wage could be a function of the hours they work. This is not the problem that instrumental variables, in this setting, is meant to correct for, and so this discussion does not take this possibility into account.

[10] See, for example, Rosen (1976) and Hausman and Wise (1976).

[11] See, for example, Flood and MaCurdy (1993).

observation. Second, the approach generally ignores the participation decision, and focuses solely on marginal changes in hours. In some settings, for example when studying the labor supply behavior of adult males, this may not be a bad approximation. For other groups, such as married women, this would clearly be undesirable. Third, local linearization is not applicable when budget constraints are nonconvex or preferences are nonconvex.

Finally, and most fundamentally, the nature of the problem makes seemingly plausible instruments invalid when hours are measured with error, which is likely the case. There are passing references to this point in the literature, for example in Moffitt (1990) and Ericson and Flood (1996), but it has not been shown formally. Since this point is not well understood, and since some recent papers have argued for a return to local linearization techniques[12], we formally demonstrate the inconsistency of local linearization here.

For IV to be consistent, the instruments used must be correlated with the regressors, but uncorrelated with the error term. However, the error term in this specification will generally be a function of preference parameters, and wage and income variables associated with various budget segments, making it correlated with variables that might otherwise seem like suitable instruments.

To demonstrate this point, note that the individual's utility maximization problem is

$$\begin{aligned} & \max U(C,h,v) \\ \text{s.t. } C & \leq B(\{w_1,...,w_J\},\{y_1,...,y_J\},h), \end{aligned} \tag{2}$$

where the unobserved value of leisure for a given individual is denoted as v, in which a higher value of v denotes a greater taste for work. In the absence of measurement error, we would observe

$$h = h(\{w_1,...,w_J\},\{y_1,...,y_J\},v). \tag{3}$$

In order to break this function out into separate terms, suppose that $f(w_j,y_j)+v$ is the solution to

$$\begin{aligned} & \max U(C,h,v) \\ \text{s.t. } C & \leq w_j h + y_j. \end{aligned} \tag{4}$$

In this case, $h(\cdot)$ may be rewritten as

$$\begin{aligned} & h(\{w_1,...,w_J\},\{y_1,...,y_J\},v) \\ = & \sum_j [f(w_j,y_j)+v]\, 1\left(H_{j-1} < f(w_j,y_j)+v < H_j\right), \end{aligned} \tag{5}$$

where $1(\cdot)$ denotes the indicator function. This form of the labor supply function takes account of the fact that, as noted above, if the individual is observed between H_{j-1} and H_j, it must be that $f(w_j,y_j)+v$ is also between those points. Letting j^* be such that $H_{j^*-1} < f\left(w_{j^*},y_{j^*}\right)+v < H_{j^*}$, so that j^* is the segment of the budget constraint on which utility is maximized, in the absence of measurement error, we would observe

$$h = f(w_{j^*},y_{j^*})+v. \tag{6}$$

[12] See, for example, Blundell and MaCurdy (1999).

On the other hand, if hours observations are contaminated by measurement, ε, we observe

$$h = f(w_{j^*}, y_{j^*}) + v + \varepsilon. \tag{7}$$

Now, let j^{obs} be defined as $H_{j^{obs}-1} < h \leq H_{j^{obs}}$, so that using local linearization, we would impute the after tax wage and nonlabor income associated with segment j^{obs} of the budget constraint. Suppose, then, that we use local linearization methods to estimate

$$h = f(w_{j^{obs}}, y_{j^{obs}}) + u. \tag{8}$$

For IV to be consistent, the instruments must be uncorrelated with u. To examine under which cases this might be so, consider the constituent parts of this error term.

As a point of reference, consider the choice of an individual in the absence of heterogeneity in preferences, where $v = 0$. In this case, the individual would choose to work hours $f(w_{j'}, y_{j'})$, where $H_{j'-1} \leq f(w_{j'}, y_{j'}) \leq H_{j'}$.

Suppose, then, there is heterogeneity in tastes for work, but no measurement error. For a sufficiently large v in absolute value, the individual's utility maximizing segment will be different from that of an individual with $v = 0$, and the larger is the value of v on the real line, the larger is j^*. Nevertheless, the individual would be observed on the utility maximizing segment, and so $j^{obs} = j^*$. As a result, the estimated equation is the form

$$h = f(w_{j^*}, y_{j^*}) + u, \tag{9}$$

which can be rewritten as

$$h = f(w_{j'} + (w_{j^*} - w_{j'}), y_{j'} + (y_{j^*} - y_{j'})) + u. \tag{10}$$

where the error term u is of the form

$$u = v. \tag{11}$$

In this case, taste for work will be correlated with $(w_{j^*} - w_{j'})$ and $(y_{j^*} - y_{j'})$. For example, under a progressive tax system, a higher value of v implies that $(w_{j^*} - w_{j'})$ will be more negative and $(y_{j^*} - y_{j'})$ will be more positive. As a result,

$$Cov((w_{j^*} - w_{j'}), v) < 0,\ Cov((y_{j^*} - y_{j'}), v) > 0,$$

which implies

$$Cov((w_{j^*} - w_{j'}), u) < 0, Cov((y_{j^*} - y_{j'}), u) > 0. \tag{12}$$

This is the standard rationale invoked when using instrumental variables. For individuals with greater tastes for work, we will impute a smaller w_{j^*} and larger y_{j^*}, and so the error term will be correlated with the regressors.

Suppose we use the gross wage, W, and gross nonlabor income, Y, as instruments. Clearly, these instruments would be correlated with w_{j^*} and y_{j^*}. In addition, although it will generally be the case that $E[v|W, Y, w_{j^*}, y_{j^*}] \neq 0$[13], under the assumption that v is distributed independently of W and Y, it will also be the case that $E[v|W, Y] = 0$ which implies $E[u|W, Y] = 0$. Hence, so long as the taste for work is distributed independently of the gross wages and nonlabor incomes, these instruments will be valid.

However, hours of work are likely measured with error.[14] In this case, for a sufficiently

[13] In other words, given that desired hours were on a segment with slope w_{j^*} and intercept y_{j^*}, the values of W and Y tell us about the range from which v was drawn, and the expectation of v in this range is not necessarily 0.

[14] See Bound et al. (1994) for evidence on the extent to which hours of work are measured with error.

large ε, an individual will be observed on a segment, j'', which is different from the utility maximizing segment. In this case, $j^{obs} = j''$, and the estimated equation is

$$h = f(w_{j''}, y_{j''}) + u, \tag{13}$$

which can be rewritten as

$$h = f(w_{j'} + (w_{j^*} - w_{j'}) + (w_{j''} - w_{j^*}), y_{j'} + (y_{j^*} - y_{j'}) + (y_{j''} - y_{j^*})) + u, \tag{14}$$

The error term, however, is now

$$u = [f(w_{j^*}, y_{j^*}) - f(w_{j''}, y_{j''})] + v + \varepsilon. \tag{15}$$

Again, the taste for work will be correlated with $(w_{j^*} - w_{j'})$ and $(y_{j^*} - y_{j'})$. In addition, in this case, measurement error will be correlated with $(w_{j''} - w_{j^*})$ and $(y_{j''} - y_{j^*})$.

Consider again the use of gross wage and nonlabor income as instruments. Again, these instruments will be correlated with $w_{j''}$ and $y_{j''}$. However, these instruments will now generally be correlated with the error term. The reasoning is as follows. Suppose that individuals with higher gross wages tended to have smaller sets of hours within which the after tax wage is the same.[15] Even though the distribution of ε is assumed to be the same for all individuals, high wage individuals' observed hours will correspond to a segment other than their utility maximizing segment at a greater rate. Since it will generally be the case that $f(w_k, y_k) \neq f(w_l, y_l)$ for $k \neq l$, and $[f(w_{j^*}, y_{j^*}) - f(w_{j''}, y_{j''}) | j'' > j^*]$ will generally be different in absolute value from $[f(w_{j^*}, y_{j^*}) - f(w_{j''}, y_{j''}) | j'' < j^*]$ thus unlikely to be offsetting. Hence, the greater prevalence of $j'' \neq j^*$ for higher wage individuals implies that $[f(w_{j^*}, y_{j^*}) - f(w_{j''}, y_{j''})]$ will vary systematically with the gross wage, and so $E[[f(w_{j^*}, y_{j^*}) - f(w_{j''}, y_{j''})] | W] \neq 0$ in general. A similar argument would apply if the tax brackets tended to get bigger or smaller as income increased, or if the changes in the after-tax wage between brackets $(w_{j+1} - w_j)$ became bigger or smaller. In all cases, it would generally be that $E[[f(w_{j^*}, y_{j^*}) - f(w_{j''}, y_{j''})] | Y] \neq 0$. As a result, it will generally be the case that $E[u | W, Y] \neq 0$. Hence, such instruments would be invalid, and estimates resulting from such an approach will be inconsistent.

Further, other instruments that have been proposed, including the net wage and virtual income at a fixed point in the budget constraint, or demographic variables, are probably correlated with the gross wage and nonlabor income variables. Hence, these instruments are likely correlated with the error term by the same argument above, and they too are generally invalid.

Thus, in addition to the other problems with local linearization, even in its more sophisticated implementations, local linearization will likely lead to biased and inconsistent estimates. This feature will often make local linearization methods undesirable to use in labor supply estimation.

[15] This would be the case, for example, in a tax system like that in the U.S. in which the marginal tax rate is t_j on net income, $Wh - Y - D$, between $I_{j-1}$and I_j, so that $H_j = \frac{I_j - (Y-D)}{W}$, where I_j is the income level at the start of the jth tax bracket and D is deductions. In this case, since $H_{j+1} - H_j = \frac{I_{j+1} - I_j}{W}$, the width of each tax bracket on the hours axis is decreasing in W.

3 The Hausman Method and the MGP Critique

Unlike local linearization, the Hausman method explicitly controls for the endogeneity of after tax wage rate and virtual income which are functions of the number of hours worked, and takes account of the possibility that an individual's observed hours are not their utility maximizing hours. In this section, then, we discuss the derivation of the Hausman method likelihood function, and review the consumer theory relevant to the assumptions made in the derivation of the likelihood. Given these results from consumer theory, we argue that, in constraining parameters to satisfy Slutsky positivity (as was noted in MaCurdy et al. (1992) and MaCurdy (1990), and which we discuss here), the Hausman method constrained parameters to be consistent with the maximization of convex preferences, an assumption already made in the derivation of the likelihood. Further, these results from consumer theory imply that several attempts in the literature to modify the Hausman method amounted to generalizing the Hausman method to allow estimated parameters to be inconsistent with utility maximization altogether. We argue that other assumptions should be weakened before abandoning utility maximization, and suggest such an assumption in Section 4.

3.1 Derivation of the Hausman Method

The stochastic elements in the Hausman method are posited to consist of two components, a term capturing unobserved heterogeneity in tastes for work, and a measurement error term[16]. The derivation of the likelihood function then exploits the fact that, if preferences are convex, there exists a simple algorithm that identifies the optimal hours of work, given the values of the parameters and the heterogeneity term. Observed hours differ from these optimal hours by the measurement error term.

The algorithm that identifies the optimal hours of work on the budget constraint is important for the discussion that follows, and so it is repeated here. Suppose the budget constraint is as in Figure 1. Let v denote an unobserved component in preferences, and let $h_j = h(w_j, y_j, v)$ be the optimal labor supply if an individual with heterogeneity, v, were maximizing utility over consumption and hours of work, $U(C, h, v)$, subject to the budget constraint defined by $C = w_j h + y_j$.

If preferences are convex, then the following algorithm (using the notation in MaCurdy (1992)) identifies desired hours, h^*, on the piecewise linear budget set.

[16] The second term has also been interpreted as a optimization error term. This interpretation of the second error term would be more complicated than it first seems as one might want to allow individuals to make their choice as to what hours to aim at realizing that they may not achieve their desired hours. We should also note if the measurement error arises because one uses a wage measure defined as total earnings divided by hours. In this latter case, the hours error contaminates the wage measure, and so the budget constraint is measured with error. This is not an insurmountable problem, as the maximum likelihood procedure could be augmented to deal with this problem. However, few (if any) researchers have attempted to make this correction.

$$h^* = \begin{cases} H_0 & if & h_1 \leq H_0 \text{ (lower limit)} \\ h(w_1, y_1, v) & if & H_0 < h_1 < H_1 \text{ (segment 1)} \\ H_1 & if & h_1 \geq H_1 \text{ and } h_2 \leq H_1 \text{ (kink 1)} \\ h(w_2, y_2, v) & if & H_1 < h_2 < H_2 \text{ (segment 2)} \\ H_2 & if & h_2 \geq H_2 \text{ and } h_3 \leq H_2 \text{ (kink 2)} \\ h(w_3, y_3, v) & if & H_2 < h_3 < H_3 \text{ (segment 3)} \\ H_3 & if & h_3 \geq H_3 \text{ (upper limit)} \end{cases} \tag{16}$$

Later, in Section 5.1, we discuss the necessity of the assumption that preferences are convex for the Hausman method to be properly specified, in that this algorithm will be certain to identify desired hours *only if* preferences are convex. Although convexity of preferences is mentioned in some of the Hausman method papers,[17] the implications of the approach's fundamental reliance on the assumption is not discussed. However, if preferences are indeed convex, the following derivation of the likelihood follows.

Observed hours are assumed to be $h_i = h^* + \varepsilon_i$, where ε_i denotes measurement error.[18] The likelihood of observing individual i, then, is:

$$\begin{aligned} P(h_i) = \quad & P\left[\begin{array}{c} h(w_1, y_1, v_i) \leq H_0, \\ h_i = H_0 + \varepsilon_i \end{array}\right] && \begin{array}{c}\text{Optimal hours} \\ \text{below } H_0, \\ \text{observed at } h_i\end{array} \\ & + \textstyle\sum_{j=1}^{3} P\left[\begin{array}{c} H_{j-1} < h(w_j, y_j, v_i) < H_j, \\ h_i = h(w_j, y_j, v_i) + \varepsilon_i \end{array}\right] && \begin{array}{c}\text{Optimal hours} \\ \text{at } h(w_j, y_j, v_i), \\ \text{observed at } h_i\end{array} \\ & + \textstyle\sum_{j=1}^{2} P\left[\begin{array}{c} h(w_j, y_j, v_i) \geq H_j, \\ h(w_{j+1}, y_{j+1}, v_i) \leq H_j, \\ h_i = H_j + \varepsilon_i \end{array}\right] && \begin{array}{c}\text{Optimal hours} \\ \text{at } H_j, \\ \text{observed at } h_i\end{array} \\ & + P\left[\begin{array}{c} h(w_3, y_3, v_i) \geq H_3, \\ h_i = H_3 + \varepsilon_i \end{array}\right] && \begin{array}{c}\text{Optimal hours} \\ \text{above } H_3, \\ \text{observed at } h_i\end{array} \end{aligned} \tag{17}$$

A popular specification for the hours of work function has been the linear labor supply function, where $h(w_j, y_j, v) = c + \alpha w_j + \beta y_j + v$. For ease of exposition, this form will be assumed in the discussion that follows, though the key results are generalized to an arbitrary differentiable labor supply function in Appendix A.

3.2 Consumer Theory Related to the Hausman Method and the MGP Critique

In order to understand the MGP critique, it is useful to recall the consumer theory that applies to the critique. In Hurwicz and Uzawa (1971), several theorems on the relationship

[17] For example, see Hausman (1979, p. 172) and Hausman (1985a, p.1257). Burtless and Hausman (1978) and Hausman (1981) appear to make no mention about the required form of preferences.

[18] This statement is somewhat of a simplification, in that there are often separate conditions under which an individual is observed working 0 hours. These conditions are not important for the discussion that follows, however, and are left out for the sake of simplicity.

between maximization of utility subject to a linear budget constraint and the properties of the demand function are proven. For convenience, they are adapted here to the setting of labor supply choice.

Theorem 1 *Let preferences over consumption, C, and hours of work, h, be complete and transitive, and let $h(w,y)$ be the solution to the maximization of these preferences subject to $C \leq wh + y$. If $C = wh(w,y) + y$, and $h(w,y)$ is single valued and differentiable, then the Slutsky substitution term associated with $h(w,y)$, $\frac{\partial h}{\partial w} - \frac{\partial h}{\partial y}h$, is positive.*

Proof. See Hurwicz and Uzawa (1971), p. 119-123 ■

The above theorem states that, under certain conditions, any labor supply function generated by maximizing complete and transitive preferences subject to a linear budget constraint will have a positive Slutsky term when evaluated at any hours that could be the utility maximizing choice.[19] Since applications of the Hausman method, along with other methods, invariably assumed a single valued and differentiable labor supply function, for the resulting parameters to be consistent with the assumption of convex preference maximization (or the maximization of any preferences, for that matter), they would need to satisfy Slutsky positivity by Theorem 1.

The next theorem, and the following corollary, show that under certain conditions, if the Slutsky term of a labor supply function is positive, then there exists a utility function representing convex preferences that, when maximized subject to a linear budget constraint, will generate the labor supply function.

Theorem 2 *Let $h(w,y)$ be single valued and differentiable with bounded derivatives, and let $C = wh(w,y) + y$. If the Slutsky substitution term associated with $h(w,y)$, $\frac{\partial h}{\partial w} - \frac{\partial h}{\partial y}h$, is positive, then $\exists\ U(C,h)$ s.t. $h(w,y)$ is the solution to*

$$\begin{aligned} & \max U(C,h) \\ s.t.\ C &\leq wh + y \end{aligned} \tag{18}$$

Proof. See Hurwicz and Uzawa (1971), p. 124-130. ■

Corollary 1 *Under the same assumptions as in Theorem 2, $U(C,h)$ is quasiconcave.*

Proof. See Hurwicz and Uzawa (1971), p. 131. ■

Again, recall that the Hausman style estimation method involves the estimation of a single valued and differentiable labor supply function. Thus, if the estimated parameters satisfy Slutsky positivity, then they are consistent with the maximization of convex preferences.

It is also useful to note that even if preferences are nonconvex, the corresponding labor supply function will still satisfy Slutsky positivity at points where the labor supply function is continuous. At points of discontinuity, a discrete version of Slutsky positivity will still hold. The following theorem makes this statement precise.

[19] Although the Slutsky term is negative in the case of a consumption bundle choice subject to a linear budget constraint, in the case of labor supply decisions the analogous expression is positive. Hence, in this paper, the condition usually referrred to as Slutsky negativity is denoted as Slutsky positivity.

Theorem 3 *Let preferences over consumption, C, and hours of work, h, be described by the utility function $U(C,h)$, and let $h(w,y)$ be the solution to the maximization of these preferences subject to $C \leq wh + y$. If $U(C,h)$ represents preferences that are monotonic, but nonconvex, then where demands are continuous, the Slutsky substitution term, $\frac{\partial h}{\partial w} - \frac{\partial h}{\partial y} h$, must be positive, and where demands are discontinuous, a discrete version of the Slutsky substitution term must be positive.*

Proof. See Appendix B. ■

3.3 The MGP Critique

In two papers, MaCurdy et al. (1990) and MaCurdy (1992), argue that the Hausman method imposes that estimated parameters satisfy Slutsky positivity. The reason for this restriction is straightforward. MaCurdy et al. show that in order for all probabilities in the likelihood function to be nonnegative[20], for each kink point observed in an individuals budget constraint, the level of hours at which that kink point is located must have a nonnegative probability of being the individual's desired hours.[21]

The reason for this restriction is as follows. Starting with the heuristic for the likelihood above, and assume that ε and ν are independently distributed and that a given ν corresponds to one and only one feasible desired hours choice. Then the term above that corresponds to the optimal hours being at the kink point can be rewritten using the functional form for the labor supply function assumed above[22]

$$\sum_{j=1}^{2} P\left[\begin{array}{c} c + \alpha w_j + \beta y_j + v_i \geq H_j, \\ c + \alpha w_{j+1} + \beta y_{j+1} + v_i \leq H_j, \end{array}\right] P\left[h_i = H_j + \varepsilon_i\right] \tag{19}$$

which can further be rewritten as

$$\sum_{j=1}^{2} P\left[\begin{array}{c} v_i \geq H_j - c - \alpha w_j - \beta y_j, \\ v_i \leq H_j - c - \alpha w_{j+1} - \beta y_{j+1}, \end{array}\right] P\left[h_i = H_j + \varepsilon_i\right] \tag{20}$$

or

$$\sum_{j=1}^{2} P\left[H_j - c - \alpha w_{j+1} - \beta y_{j+1} \geq v_i \geq H_j - c - \alpha w_j - \beta y_j\right] P\left[h_i = H_j + \varepsilon_i\right] \tag{21}$$

To implement this likelihood, if v_i is distributed according to the CDF $G(v_i)$, the expression above is typically rewritten as

$$\sum_{j=1}^{2} P\left[h_i = H_j + \varepsilon_i\right] \int_{H_j - c - \alpha w_j - \beta y_j}^{H_j - c - \alpha w_{j+1} - \beta y_{j+1}} dG(v_i) \tag{22}$$

[20]Coherency of the likelihood function also requires that probabilities not exceed 1 and that the union of all events has probability 1. It is the nonnegativity restriction, however, that is the focus of MaCurdy et al.

[21]For a dissent to this critique, see Blomquist (1995).

[22]The MaCurdy critique generalizes to nonlinear labor supply functions, though the exact restrictions implied depend somewhat on the functional form and stochastic specification assumed. MaCurdy et al. (1990) contains a generalization of this point for any labor supply function, increasing in v, that is derived from the maximization of a quasi-concave utility function. In Appendix A, we generalize the argument used in this section to an arbitrary labor supply function.

MaCurdy et al. then argue that, in order for this probability to be nonnegative, the upper limit of integration must be greater than or equal to the lower limit of integration, and so it must be the case that

$$H_j - c - \alpha w_{j+1} - \beta y_{j+1} \geq H_j - c - \alpha w_j - \beta y_j \tag{23}$$

Using that $y_{j+1} = y_j + (w_j - w_{j+1})H_j$,[23] (23) can be rewritten to yield

$$\alpha - \beta H_j \geq 0. \tag{24}$$

Of course, (24) is just the Slutsky compensated wage effect consistent with a linear labor supply function, $\frac{\partial h}{\partial w} - \frac{\partial h}{\partial y}h$, evaluated at the kink point, H_j. If the likelihood function is undefined when this restriction is violated, all estimates of α and β must satisfy this restriction.

One might ask why doesn't the term (22) disappear if inequality imbedded in (19) as summarized in (23) does not hold? However, suppose that $H_j - c - \alpha w_{j+1} - \beta y_{j+1} < H_j - c - \alpha w_j - \beta y_j$. In this case, if v_i is continuously distributed over $[-\infty, \infty]$, which is typically the case for the assumed distribution for v_i, there exists some v_i such that

$$H_j - c - \alpha w_{j+1} - \beta y_{j+1} < v_i < H_j - c - \alpha w_j - \beta y_j$$

which implies that

$$\begin{aligned} H_j - c - \alpha w_j - \beta y_j &> v_i, \\ H_j - c - \alpha w_{j+1} - \beta y_{j+1} &< v_i \end{aligned}$$

or

$$\begin{aligned} c + \alpha w_j + \beta y_j + v_i &< H_j, \\ c + \alpha w_{j+1} + \beta y_{j+1} + v_i &> H_j \end{aligned}$$

This last inequality implies that there will be some values of v_i where both

$$H_{j-1} < c + \alpha w_j + \beta y_j + v_i < H_j$$

and

$$H_{j+1} > c + \alpha w_{j+1} + \beta y_{j+1} + v_i > H_j$$

If this is the case, a given value of v_i corresponds to more than one feasible desired hours choice, violating our assumptions. Hence, the conclusion of the MGP critique is still correct, that in order for the likelihood to be coherent, it must be that Slutsky positivity holds at all kink points.

MaCurdy et al. note that there are a number of kinks in each individual's budget constraint, and that the location of these kinks differ across individuals. Hence, the condition above, they argue, amounts to requiring that the parameters are such that Slutsky positivity

[23] This equation follows from the definition of the virtual incomes. See MaCurdy (1992, p. 244) for example.

is enforced globally, which in the linear labor supply case amounts to constraining $\alpha > 0$ and $\beta < 0$. It is posited in MaCurdy (1992) that such a constraint is a likely explanation for the consistently larger elasticities that were estimated when the Hausman method was used.

However, given the results of Theorems 2 and Corollary 1 above, however, constraining parameters to globally satisfy Slutsky positivity amounts to constraining those parameters to be consistent with the maximization of globally convex preferences. Further, recall that it is assumed in the derivation of the Hausman likelihood function that individuals are maximizing preferences that are globally convex.[24] Hence, the MGP critique essentially points out where in the Hausman likelihood function the assumption of convex preference maximization is enforced on estimated parameters.

It is important to note that the restrictions implied by the MGP critique depend somewhat on the assumed functional form for labor supply and the stochastic specification. For example, in the case of a CES utility function and the stochastic specification given in Appendix E, the Slutsky condition is enforced by construction whenever one only considers kink points less than the maximum possible hours (as one would naturally do). No additional restrictions are imposed on any parameters.

3.4 Relation to Findings in the Literature

Constraining estimates to be consistent with the maximization of globally convex preferences is not necessarily undesirable. For example, if one strongly believes a priori that preferences are convex, one may want to enforce this restriction in estimation. It is troubling, however, if this constraint is found to be binding.

In fact, MaCurdy et al. (1990), as well as Blomquist and Hannson-Brusewitz (1990), Colombino and Del Boca (1990), and Triest (1990) find that, when the Hausman method is used, the statistical constraints are binding on the parameters of interest.[25] Further, MaCurdy et al. (1990) propose an estimation method which relaxes this restriction on the Slutsky term, and find that estimates from this method violate Slutsky positivity.

The method in MaCurdy et al. incorporates the same underlying structural model, and invokes the same assumption of convex preferences, but replaces the true budget constraint with a twice differentiable approximation.[26] In this method, all probabilities in the likelihood function can be nonnegative even when parameters violate Slutsky positivity, as long as the Slutsky term is not too negative. Since this likelihood function is still defined in some cases in which the Slutsky term is negative, then, given Theorem 1 above, the estimated labor supply function is allowed to be inconsistent with utility maximization, even over a

[24] See Footnote 17.

[25] There is, however, some controversy as to why MaCurdy et. al. found the constraints to be binding in their estimation. For example, in a recent paper, Eklöf and Sacklén (1999) argue that the wage measure used in MaCurdy et al. is contaminated by division bias, which led to the nonnegativity constraint binding.

[26] A weakness of the MaCurdy method is its requirement that the budget set be convex. Convexity of the budget set is required to guarantee that there is a unique level of hours, h, that maximizes utility for a given specification of heterogeneity of preferences, v. This, in turn, implies that the Implicit Function Theorem may be applied to yield v as a function of h, which is then used in the estimation. Thus, if the actual budget set is nonconvex, the MaCurdy method requires the creation of a convexified approximation to the budget set.

range of hours in which data are observed.[27] As such, it allows estimated parameters to be inconsistent with the assumptions that underlie the structural model and assumed data generating process.

Using the same dataset that they used for their Hausman method estimation, MaCurdy et al. then estimate labor supply parameters using their differentiable budget constraint method, and find that the estimated parameters violate Slutsky positivity. Further, estimated parameters satisfy Slutsky positivity only when they are constrained to do so. Hence, MaCurdy et al. use these results to argue that the Slutsky restriction implicit in the Hausman method is a binding restriction in their data, and that parameter estimates from the Hausman method satisfy Slutsky positivity only because they are constrained to do so.

The results in MaCurdy et al., then, demonstrate the presence of the constraint in the Hausman method, and argue that it is binding in practice. However, this result leaves the researcher in a quandary as to how to proceed. The MaCurdy et al. method is really only a generalization of the Hausman method in that it expands the parameter space over which all probabilities are nonnegative to include parameters which are inconsistent with utility maximization. Hence, if the MaCurdy method's unconstrained estimates violate Slutsky positivity, as was found, then they are not useful for welfare calculations and some policy simulations, since they are inconsistent with utility maximizing behavior.

Blomquist and Hannson-Brusewitz (1990) argue that this problem may be rectified by modifying the data generating process to one which is consistent with utility maximizing behavior if Slutsky positivity is satisfied, and is consistent with some form of non-utility maximizing behavior if Slutsky positivity is violated. They then argue that one can interpret observations for which estimated parameters satisfy Slutsky positivity as resulting from the maximization of convex preferences. However, for the portion of the sample for which estimates violate Slutsky positivity, they do not know what behavioral model corresponds to their data generating process, once again rendering the estimates useless in welfare analyses and some policy simulations.

It is important to emphasize, however, that the modifications to the Hausman method suggested in the above papers only generalize the Hausman method by expanding the parameter space to include parameters which violate Slutsky positivity for some observed individuals, and hence are inconsistent with utility maximization altogether. But, since these papers do not address why the constraint in the Hausman method would be binding, the estimation strategies proposed in these papers suggest modifications to the Hausman method to fix a problem whose cause is not fully known. If there are other assumptions that we may be more willing to drop than the assumption of utility maximization, and if the imposition of these other assumptions may cause problems for the Hausman method, then clearly the weakening of these assumptions should be tried first. In the following section, we identify such an assumption.

[27] To see this, use the contrapositive of Theorem 3.1 above. If parameters violate Slutsky positivity, then they are not consistent with utility maximization.

4 Nonconvexities as a Possible Source of Problems

As noted in the previous section, the MGP critique argues that the Hausman method forces estimates to exhibit Slutsky positivity, and further argues that these restrictions are binding. However, MaCurdy et al. do not have an explanation as to why the constraints are found to be binding.

One possible explanation is offered in Eklöf and Sacklén (1999), who argue that in MaCurdy et al.'s study, the hours variable is measured with error, and this error contaminates the wage measure, which is calculated by dividing annual earnings by annual hours. They argue that this, in turn, biases the wage coefficient downward, which causes the Slutsky constraint to be binding.[28] Although this explanation may be consistent with MaCurdy et al.'s finding that the wage coefficient had to be constrained to be positive to insure nonnegative probabilities, it is inconsistent with results in other studies, such as Triest (1990), which finds that the income coefficient had to be constrained to be negative to insure nonnegative probabilities. Hence, this explanation is not fully satisfactory.

In this section, we argue for an alternative explanation of these results. Namely, we argue that if the data are of a form consistent with individuals maximizing nonconvex preferences subject to a nonlinear budget constraint, but one uses a method such as the Hausman method or local linearization, which estimate the parameters of a labor supply function under the assumption that preferences are convex, such a method may yield estimates which are either constrained to satisfy Slutsky positivity, or which violate Slutsky positivity.

One should be clear at the outset that we are not arguing that if preferences are nonconvex, the Slutsky compensated wage effect is negative; on the contrary, even if preferences are nonconvex, the compensated wage effect will have the usual sign. Rather, we are arguing that data of a form consistent with nonconvex preference maximization may cause the aforementioned methods to yield estimated parameters that either violate Slutsky positivity, and hence wrongly exhibit negative compensated wage elasticities, or be constrained to satisfy Slutsky positivity.

We also hasten to note that we do not, in this paper, give a rationale as to why preferences over consumption and hours of work might appear to be nonconvex. An argument as to why data generating preferences in a structural labor supply model may be nonconvex, as well as the effects of the source of the nonconvexity on identification and policy analyses, are addressed in Heim and Meyer (2003).

With that in mind, the implication of the argument that we make in this section is that assuming convexity of preferences, when false, can lead to estimates that violate the more rudimentary assumption of utility maximization, even when this assumption is actually true. As a result, before considering non-utility-maximizing generalizations of estimation methods, one should allow for the possibility that preferences are nonconvex when estimating labor supply parameters.

[28] In a Monte Carlo study of the robustness of the Hausman method to various forms of error, Blomquist (1996) finds that a form of the Hausman method performs quite well when measurement error in the wage rate is present. However, in these experiments, there is no spurious correlation between hours and wages in the simulated data. The contaminated wage rate is used to construct the budget constraint, and used as the wage rate in the estimation, but observed hours come from the uncontaminated data.

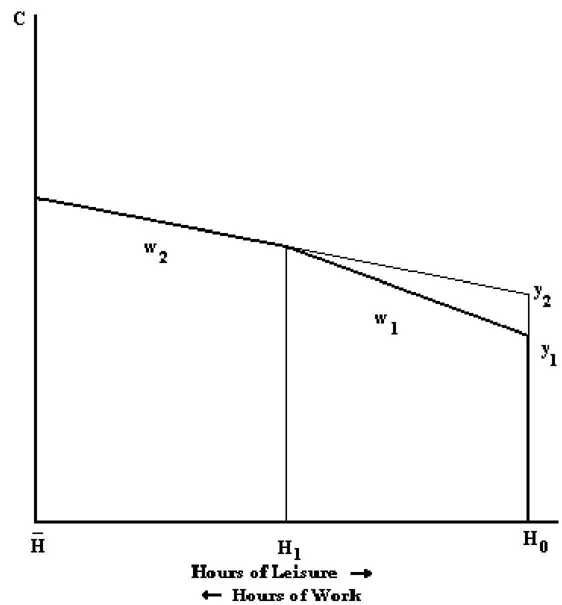

Figure 2: Budget Constraint with One Kink Point for Group g

4.1 Model Assumptions

For simplicity, suppose all individuals face a budget constraint of the general type depicted in Figure 2. Let a group of individuals face the same budget constraint that consists of a single kink at H_1. Let the slope of this budget constraint be w_1 over $[H_0, H_1]$, and w_2 over $[H_1, \overline{H}]$. Finally, let the virtual income associated with the segment over $[H_0, H_1]$ be y_1, and the virtual income associated with the segment over $[H_1, \overline{H}]$ be y_2.[29] Let individual i who faces this budget constraint be observed working hours h_i.

Let the labor supply equation that is being estimated be $h(v) = c + \alpha w + \beta y + v$[30], where c, α, and β are the parameters to be estimated, and v is the stochastic element. Suppose that we used the following assumptions to infer parameters using the observed distribution of data:

$$\text{Assumption 1} \; : \; \text{For } i \text{ s.t. } h_i < H_1, \quad \begin{array}{c}\text{it must be that} \\ c + \alpha w_1 + \beta y_1 + v_i < H_1.\end{array} \tag{25}$$

$$\text{Assumption 2} \; : \; \text{For } i \text{ s.t. } h_i = H_1, \quad \begin{array}{c}\text{it must be that} \\ c + \alpha w_1 + \beta y_1 + v_i \geq H_1 \\ \text{and } c + \alpha w_2 + \beta y_2 + v_i \leq H_1.\end{array} \tag{26}$$

$$\text{Assumption 3} \; : \; \text{For } i \text{ s.t. } h_i > H_1, \quad \begin{array}{c}\text{it must be that} \\ c + \alpha w_2 + \beta y_2 + v_i > H_1.\end{array} \tag{27}$$

$$\text{Assumption 4} \; : \; \text{The distribution of } v_i \text{ is continuous} \tag{28}$$

[29]This figure is identical to Figure 1 in MaCurdy (1992), with the exception that this figure only incorporates two tax brackets, whereas MaCurdy's incorporates three.

[30]For a generalization of this argument to an arbitrary labor supply function, see Appendix A.

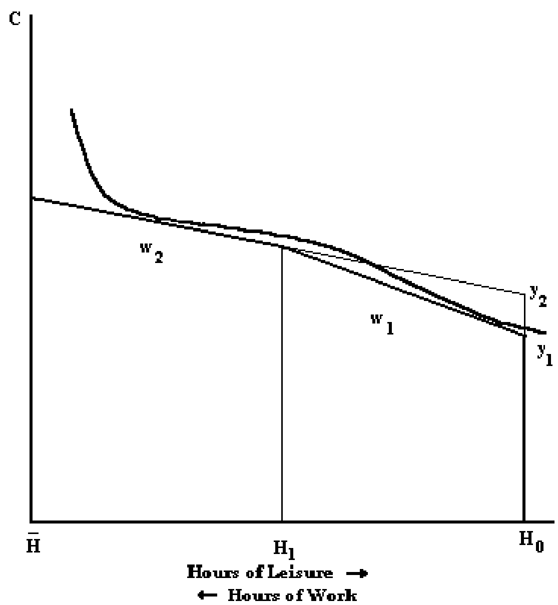

Figure 3: Nonconvex Preferences Maximized Subject to a Budget Constraint with One Kink Point

Note that these assumptions are implicit in the Hausman method when there is no measurement error (See Equation (16) above), and that such assumptions, or variants thereof, are correct if individuals have convex preferences. In such a case, one can interpret $c+\alpha w_1+\beta y_1+v_i$ as the hours of work that the individual would choose on a linearized budget set tangent to the segment below H_1, $c+\alpha w_2+\beta y_2+v_i$ as the hours of work that the individual would choose on a linearized budget set tangent to the segment above H_1, and use the algorithm in Hausman (1979) to find the individual's desired hours on the nonlinear budget constraint.

Suppose, then, that we attempted to infer parameters, c, α and β, and a distribution for v, that satisfied Assumptions 1-4, given an observed distribution of data. Obviously, an estimation method does not use such deductive logic to infer estimated parameters, but the parameters obtained in such a thought experiment may be informative as to the type of parameters that would result when using an estimation method that incorporates these assumptions. In the following subsection, then, we examine the implications of these assumptions when analyzing data generated by individuals maximizing nonconvex preferences.

4.2 Parameters if the Data are Consistent with Nonconvex Preference Maximization

Suppose that the data are such that for some budget constraints with kink points, H_1, we observe a bi-modal distribution of hours with most individuals either working hours substantially below or above the kink point. Such data would be consistent with individuals having nonconvex indifference curves of the general form depicted in Figure 3, and with heterogeneity in taste for work shifting those indifference curves so that some individuals would choose to work below the kink point, and others above the kink point, but few individuals would choose to work near the kink point.[31]

Suppose, then, that we used Assumption 1-4 to infer parameters c, α, β, and a distribution for v from such data. To examine the type of parameters that would be consistent with these assumptions, first note that since no individuals are observed at the kink, Assumption 2 does not apply. For individuals working a number of hours hours less than H_1, the parameters would satisfy $c+\alpha w_1+\beta y_1+v < H_1$ due to Assumption 1. For individuals observed working a number of hours greater than H_1, the parameters would satisfy $c + \alpha w_2 + \beta y_2 + v > H_1$ due to Assumption 3. Given Assumption 4, that the distribution of v is continuous, there must be some individuals with the same v in both groups.[32] Thus, both inequalities must be satisfied for some v. As a result, the combination of these conditions implies that parameters would satisfy

$$c + \alpha w_1 + \beta y_1 + v < c + \alpha w_2 + \beta y_2 + v. \tag{29}$$

The inequality in (29) can be rewritten as

$$\alpha(w_1 - w_2) < \beta(y_2 - y_1) \tag{30}$$

which, again using $y_2 = y_1 + (w_1 - w_2)H_1$, may be further rewritten as

$$\alpha(w_1 - w_2) < \beta(w_1 - w_2)H_1. \tag{31}$$

Since $w_1 > w_2$, parameters would satisfy

$$\alpha - \beta H_1 < 0. \tag{32}$$

Thus, in data consistent with individuals maximizing nonconvex preferences, in which individuals are not observed working near a kink point, but are observed working on either side of it, parameters consistent with the assumptions would exhibit a *negative* Slutsky compensated wage effect.

Of course, not all data arising from individuals maximizing nonconvex preferences on a nonlinear budget constraint would be of the form described above. In those cases, parameters consistent with the assumptions above may well satisfy Slutsky positivity.

However, if the data were of the form above, parameters inferred from the data using Assumptions 1-4 would violate Slutsky positivity and, given Theorem 1, be inconsistent with utility maximization. This result suggests the possibility that making the assumption that preferences are convex, implying that behavior can be modelled using an algorithm such as that in Assumptions 1-4, can have unfortunate effects on labor supply parameter estimates if those assumptions are wrong. In particular, if such a method were used on data consistent with nonconvex preference maximization, estimated parameters may be inconsistent with utility maximization altogether, even though utility maximization was indeed what generated the data. Since the assumptions that are described above are implicit in the Hausman method, it is certainly possible that parameters estimated using the Hausman method on

[31] Note that if preferences were convex, instead of observing a gap around the kink point, we would expect a mass point in the distribution of observed hours at the kink point.

[32] Suppose there are no such individuals. Then there would be a v in between the highest v that satisfies the first inequality and the lowest v that satisfies the second inequality. But, then we would observe such an individual working a level of hours in the gap between the two groups, which we do not.

such data would either violate (if the constraint wasn't enforced), or be constrained to satisfy, Slutsky positivity. A similar argument could be made for estimates coming from the use of local linearization on such data.

4.3 A Numerical Simulation

In this subsection, we present a numerical simulation that illustrates the theoretical argument presented above, which claims that nonconvex data generating preferences can lead the Hausman method to yield parameter estimates that are constrained to satisfy Slutsky positivity.

To keep the numerical example as close as possible to the theoretical argument above, we use a simplified tax system, in which all individuals have only one kink in their piecewise linear budget constraint. As such, we use a tax system in which every individual takes a standard deduction of $1000, and in which the tax rate is .2 on taxable income up to $11,500, and .4 thereafter.

We then generate a simulated sample of individuals' wages and incomes.[33] The mean gross hourly wage in the sample used in Hausman (1981) was approximately $6, and so the wages in the simulated sample were drawn from a $N(6, 1.25)$ distribution, truncated at $0.[34] The mean gross nonlabor income in the Hausman sample was approximately $1400, and so the nonlabor incomes in the simulated sample were drawn from a $N(1400, 250000)$ distribution, again truncated at $0. With this sample, the mean individual has a kink point at 1927 hours. Thus, individuals working full time hours would be at a level of hours above the kink point, and individuals working around part time hours or below would be at a level of hours below the kink point.

In order to empirically verify the above result, one must first have some specification for utility for which indifference curves are nonconvex in such a way that there is a "dent" in indifference curves, as is illustrated in Figure 3. Unfortunately, although many functional forms have been used in the economics literature[35], we could not find a utility function whose indifference curves had the shape desired. Hence, a new functional form had to be developed which has these properties.

To construct such a utility function, we augment a well known utility function, the quadratic utility function, by replacing the leisure argument with a (3,2) Pade function[36] in hours. Namely, the functional form utilized is

$$\begin{aligned} U(C, h, v_i; \beta, \gamma, \delta) &= \beta\left(\frac{\delta_3 h^3 + \delta_2 h^2 + \delta_1 h + \delta_0}{\beta_2 h^2 + \beta_1 h + 1}\right)^2 + \gamma C\left(\frac{\delta_3 h^3 + \delta_2 h^2 + \delta_1 h + \delta_0}{\beta_2 h^2 + \beta_1 h + 1}\right) \quad (33) \\ &\quad +(\delta + v_i)\left(\frac{\delta_3 h^3 + \delta_2 h^2 + \delta_1 h + \delta_0}{\beta_2 h^2 + \beta_1 h + 1}\right) + C. \end{aligned}$$

[33] The reason for simulating a sample, instead of using the data used in the Monte Carlo experiments that are described in Section 7, is that that sample contains observations for which nonlabor income is above $12,500, and so those individuals would have no kink points in their budget constraints.

[34] In the sample used, no observations required truncation.

[35] See, for example, Stern (1986), who summarizes the properties of various functional forms that have been used in labor supply estimation.

[36] A (p, q) Pade function consists of a ratio of a polynomial of degree p divided by a polynomial of degree q.

where C denotes consumption, h denotes hours of work, η, ρ, $\overline{C}$, δ_3, δ_2, β_2, β_1, δ_1, and δ_0 are preference parameters, and v_i is a term representing unobserved heterogeneity in taste for work.

Although this functional form seems complex at first blush, it does have a straightforward theoretical interpretation. To understand this, note that the above utility function may be rewritten in an equivalent form as

$$\begin{aligned} U(C,h,v_i;\beta,\gamma,\delta) &= \beta\left(\overline{H}-h-\gamma_1 h-\frac{\alpha_2 h^2+\alpha_1 h+\alpha_0}{\beta_2 h^2+\beta_1 h+1}\right)^2 \\ &\quad +\gamma C\left(\overline{H}-h-\gamma_1 h-\frac{\alpha_2 h^2+\alpha_1 h+\alpha_0}{\beta_2 h^2+\beta_1 h+1}\right) \\ &\quad +(\delta+v_i)\left(\overline{H}-h-\gamma_1 h-\frac{\alpha_2 h^2+\alpha_1 h+\alpha_0}{\beta_2 h^2+\beta_1 h+1}\right)+C. \end{aligned} \tag{34}$$

Using the insights and terminology from Heim and Meyer (2003), then, it is easily seen that this functional form is the functional form consistent with observable preferences for an individual with standard quadratic utility preferences facing unobserved time costs of work that take the form $F_2(h)=\gamma_1 h+\frac{\alpha_2 h^2+\alpha_1 h+\alpha_0}{\beta_2 h^2+\beta_1 h+1}$.

Depending on the parameterization of the (3,2) Pade function, this functional form can represent standard quadratic utility preferences (if $\delta_3=\delta_2=\beta_2=\beta_1=0$, $\delta_1=1$, and $\delta_0=\overline{H}$), augmented quadratic utility preferences that are convex, or augmented quadratic utility preferences that are nonconvex. To see how this specification results in preferences that are nonconvex, and which have "dented" indifference curves, consider the form in (34). In this form, subtracting the ratio of two polynomials has the effect of taking the standard quadratic utility indifference curves and stretching or compressing them along the hours axis, depending on the magnitude and sign of the derivative of this ratio. Over the range of hours in which the ratio increases, the standard quadratic utility indifference curves are compressed, and over the range of hours in which the ratio decreases, the standard quadratic utility indifference curves are expanded.

Although the properties of this utility function can vary greatly depending on the parametrization, here we only note the properties of the parameterization used in the numerical example. So, let $\alpha_2>0$, $\beta_2>0$, $\alpha_1<0$, and $\beta_1<0$. In this case, the numerator of the ratio initially decreases, and then increases, and the denominator of the ratio does the same. Thus, either the ratio increases, then decreases, then increases toward an asymptote, or the ratio first decreases, then increases, then decreases, then increases toward the asymptote. This results in indifference curves that are initially compressed along the hours axis, then expanded, then compressed in the first case, and expanded, then compressed, then expanded, then compressed in the second.

The parameters used in the numerical example fall into this final case. They are $\beta=-.00001$, $\gamma=-.0001$, $\delta=.597$, $\delta_3=.25e-5$, $\delta_2=-.0038$, $\delta_1=2.15$, $\delta_0=140$, $\beta_2=1x10^{-6}$, and $\beta_1=-.0018$. These indifference curves are depicted in Figure 4. These preferences are such that, given the tax system described above, an individual with the mean wage and nonlabor income could have desired hours either above or below the kink point, depending on their draw of v_i, but there exists a span of hours around the kink point in which the individual's desired hours could not fall. This condition is also true for many

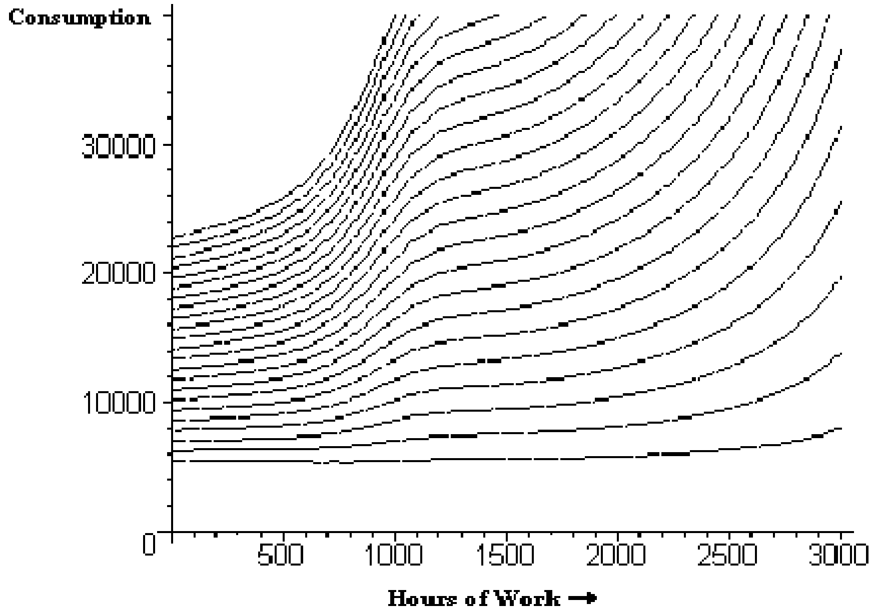

Figure 4: Sample Indifference Curves for the Nonconvex Preferences Used in the Numerical Simulations

other individual's observed in the simulated data, and so these parameters capture the spirit of the argument presented in preceding section.

To generate the data, we drew a value of v_i for each simulated individual from a $N(0, \sigma_v^2)$ distribution, and a value of ε_i from a $N(0, \sigma_\varepsilon^2)$ distribution. In the base specification, we let $\sigma_v = .3$ and $\sigma_\varepsilon = 200$. We check the robustness of these results to the specification of these stochastic parameters below.

We then used a program written in Gauss 4.0 to maximize each simulated individual's utility function subject to their budget constraint, yielding each simulated individual's desired hours.[37] A measurement error term was then added to each of the simulated individual's desired hours to yield their observed hours. Figures 5 and 6 illustrate the distribution of observed hours in the 1976 PSID, which was used in the Hausman and MaCurdy et al. studies, and the simulated observed hours distribution. As can be seen from these figures, the parameters above resulted in an hours distribution with a degree of clumping around 2000 and which looks relatively realistic. Sample statistics from the observed hours distribution are presented in Table 1.

We then read these simulated data into two Hausman style estimation programs written in Stata 7. The first program estimated the labor supply function consistent with the standard quadratic utility function, which is of the form

$$h_{ij}\left(v_i, \beta, \gamma, \delta\right) = \frac{-w_j - \gamma y_j - (\delta + v_i)}{2(\gamma w_j + \beta)}. \tag{35}$$

[37] To solve each individual's utility maximization problem, we first use a grid search with nodes spaced out by 10 hours to bracket the global optimum, and then use a bracketing algorithm (See Judd (1998), p. 95-6) to identify the utility maximizing hours accurate to 4 decimal places.

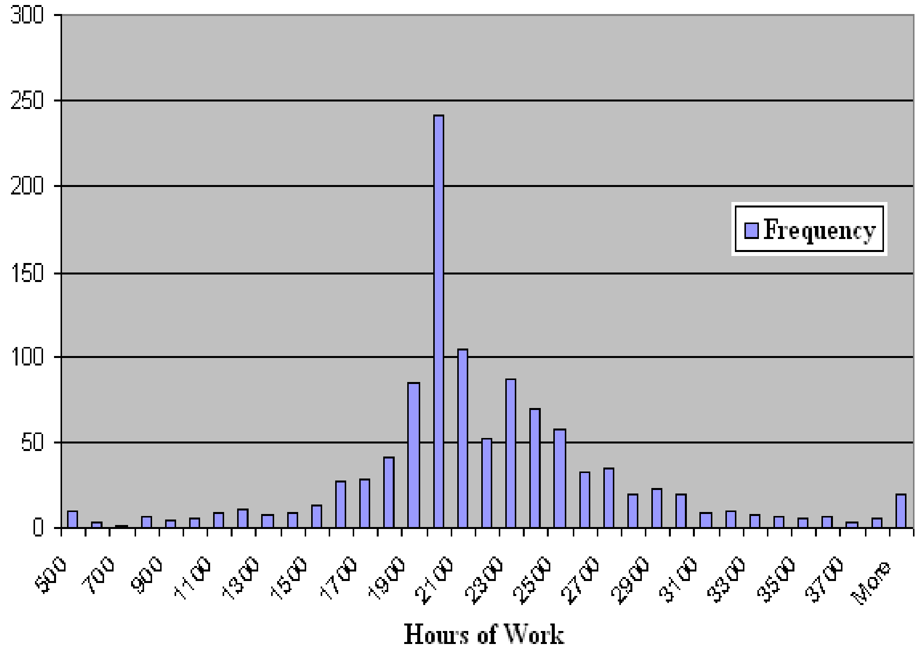

Figure 5: 1976 PSID Prime Age Male Hours Distribution

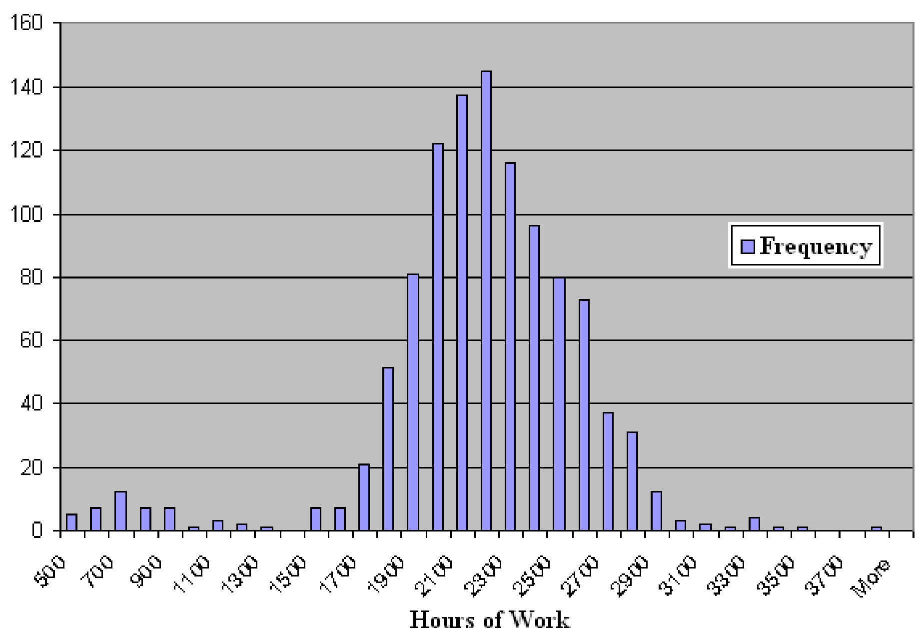

Figure 6: Simulated Observed Hours Distribtion

Table 1: Sample Statistics - Simulated Sample

Variable	N	Mean	Std. Dev.	Min.	Max.
W	1074	6.03	1.25	2.15	10.09
Y	1074	1411.54	500.10	0	3060.90
H_1	1074	1927.55	464.03	1112.82	5460.62
hours	1074	2128.58	420.44	336.07	3747.39

Details of this estimation method, and the likelihood function that it employs, are presented in the Appendix E.

The resulting parameter estimates are presented in the first row of the top panel of Table 2, where it can be seen that the estimates in the Hausman style estimation are bound by the Slutsky constraint implicit in this method. To see this, note that in the quadratic utility function specification, if β and γ are negative (which they are in this case), the Slutsky constraint is of the form $-1 - \gamma H_1 \geq 0$. Note that, for $H_1 = 5460.62$, which is the largest kink point observed in this simulated data,$-1 - \widehat{\gamma} H_1 \cong 0$. Hence, the estimated parameters are again bound by the Slutsky constraint, since if $\widehat{\gamma}$ were any more negative, the Slutsky constraint would not be satisfied for at least one individual observed in the simulated data.

In turn, the estimated parameters that are being bound at the constraint suffer from a large bias. For the true parameters, the true uncompensated wage elasticity (evaluated at a wage of \$3.60 and nonlabor income of \$3540, which corresponds to the second segment of the mean individual's budget constraint) is $-.0350$, and the income elasticity is $-.0347$.[38] For the estimated parameters that were bound to satisfy Slutsky positivity, however, the estimated uncompensated wage elasticity was .1383, and the estimated income elasticity was $-.1100$. Thus, this constraint has the effect of biasing the uncompensated wage elasticity to a positive and larger (in absolute value) amount, and biasing the income elasticity upwards in absolute value.

To further illustrate this point, we also applied the Hausman method, using a linear labor supply function. The results from this exercise are presented in the first row of the bottom panel of Table 2. Again, the estimated parameters are bound by the Slutsky constraint. To see this, recall that, in order for the Slutsky constraint to be satisfied in this specification, it must be that $\alpha - \beta H_1 \geq 0$ for all individuals. Note that, for $H_1 = 1112.82$, which is the smallest kink point observed in this simulated data, $\widehat{\alpha} - \widehat{\beta} H_1 \cong 0$. Hence, the estimated parameters are again bound by the Slutsky constraint, since if $\widehat{\alpha}$ were any more negative, or $\widehat{\beta}$ were any less negative, the Slutsky constraint would not be satisfied for at least one individual observed in the simulated data. The estimated parameters imply an uncompensated wage elasticity of $-.0831$ and an income elasticity of $-.0734$, both biased downward from the actual elasticities.

We have done some robustness testing to examine whether this result is an aberration, or whether this example is one of a larger set nonconvex preferences that cause the Slutsky constraint to bind in either the quadratic utility or linear labor supply specification when

[38] Since these nonconvex preferences do not yield a closed form expression for the labor supply function, these elasticities were calculated numerically by maximizing these preferences subject to several budget constraints, and then using a finite difference method.

Table 2
Results from Monte Carlo Experiments
Estimates from Hausman Method when Data Generating Preferences are Nonconvex

	True Parameters		Quadratic Utility Specification											
Specification	St Dev v	St Dev e	Log L	Beta	Gamma	Delta	St Dev v	St Dev e	Wage Elast.	Absolute Bias	% Bias	Income Elast.	Absolute Bias	% Bias
1	0.3	200	-8164.411	-7.726E-04	-1.831E-04	2.482	0.200	618.90	0.1383	0.1733	495%	-0.1078	-0.0731	-211%
2	0.3	0	-8027.964	-7.370E-04	-1.831E-04	2.401	0.205	652.14	0.1418	0.1768	505%	-0.1105	-0.0758	-219%
3	0.3	100	-8129.654	-7.362E-04	-1.831E-04	2.325	0.195	612.76	0.1419	0.1769	505%	-0.1106	-0.0759	-219%
4	0.3	400	-8314.647	-1.042E-03	-1.831E-04	3.705	0.253	647.83	0.1164	0.1514	433%	-0.0907	-0.0560	-161%
5	0.3	800	-8736.083	-7.487E-04	-1.831E-04	2.379	1.826	583.76	0.1406	0.1756	502%	-0.1096	-0.0749	-216%
6	0.0	200	-8027.964	-7.370E-04	-1.831E-04	2.401	0.205	652.14	0.1418	0.1768	505%	-0.1105	-0.0758	-219%
7	0.5	200	-8611.311	-4.998E-04	-1.831E-04	1.148	1.243	621.29	0.1708	0.2058	588%	-0.1332	-0.0985	-284%
8	0.8	200	-8958.209	-6.416E-04	-1.831E-04	1.569	1.652	844.85	0.1522	0.1872	535%	-0.1186	-0.0839	-242%
9	1.0	200	-9020.784	-4.160E-04	-1.831E-04	0.698	2.210	872.65	0.1841	0.2191	626%	-0.1435	-0.1088	-314%
			Linear Labor Supply Specification											
			Log L	Constant	Alpha	Beta	St Dev v	St Dev e	Wage Elast.	Absolute Bias	% Bias	Income Elast.	Absolute Bias	% Bias
1	0.3	200	-8008.575	2437.01	-47.52	-0.043	3.13	418.23	-0.0972	-0.0622	-178%	-0.0194	0.0153	44%
2	0.3	0	-7181.438	2505.40	-59.02	-0.053	-29.16	180.31	-0.1207	-0.0857	-245%	-0.0241	0.0106	31%
3	0.3	100	-7936.825	2567.20	-66.63	-0.060	-0.81	397.92	-0.1363	-0.1013	-289%	-0.0273	0.0074	21%
4	0.3	400	-8258.194	2411.53	-42.50	-0.038	3.62	527.35	-0.0869	-0.0519	-148%	-0.0174	0.0173	50%
5	0.3	800	-8718.386	2356.04	-37.42	-0.034	3.28	852.27	-0.0765	-0.0415	-119%	-0.0153	0.0194	56%
6	0.0	200	-7181.438	2505.40	-59.02	-0.053	-29.16	180.31	-0.1207	-0.0857	-245%	-0.0241	0.0106	31%
7	0.5	200	-8584.835	2297.82	-31.31	-0.028	-11.72	717.85	-0.0640	-0.0290	-83%	-0.0129	0.0218	63%
8	0.8	200	-8955.885	2250.11	-28.26	-0.025	0.88	1073.37	-0.0578	-0.0228	-65%	-0.0116	0.0231	67%
9	1.0	200	-9026.183	2095.17	-4.80	-0.004	-0.60	1299.19	-0.0098	0.0252	72%	-0.0020	0.0327	94%

Notes: The estimated equation in each replication is the labor supply function associated with the quadratic utility function, or the linear labor supply function.

using the Hausman method. The results of these experiments suggest that the latter is the case. It turns out that there are other values for the $\beta's$ and $\delta's$ that represent nonconvex preferences which, when data are generated in the above described manner, cause the Slutsky constraint to bind. We have evaluated the generality of this result to different measurement error and heterogeneity specifications. Estimated coefficients from these simulations are presented in Table 2. For all values of σ_ε, the Slutsky constraint is binding in the quadratic utility specification of the Hausman method. In addition, the Slutsky constraint binds in all but two of the linear labor supply specifications, with the exceptions being when $\sigma_\varepsilon = 0$ and $\sigma_\varepsilon = 1000$. The α coefficients for these specification, however, are still negative. When σ_v is varied, on the other hand, all specifications resulted in binding Slutsky constraints. Thus, it appears that these results are quite robust to the specification of heterogeneity and measurement error distributions. We should also note that wage and income elasticities are subject to substantial biases in many of these specifications.

4.4 Implications of Theoretical Argument and Numerical Simulation

In the previous subsections, we argued, both theoretically and with the use of a numerical simulation, that data generated by the maximization of nonconvex preferences can cause the Hausman method to estimate parameters that are constrained to be consistent with Slutsky positivity. Further, in practice, several studies have estimated parameters which violated or were constrained to satisfy Slutsky positivity. Therefore, given the results above, before allowing for the violation of utility maximization in labor supply estimation, as was done in MaCurdy et al. and Blomquist and Hannson-Brusewitz, it seems preferable to instead use an estimation method that does not require the assumption that preferences are convex in its construction, and which allows for the estimation of parameters of both convex and nonconvex preferences.

In the next section, however, we show that the commonly used methods, and further any method that relies on a result by Hall (1973) to estimate the parameters of a labor supply function, cannot be adapted for such a purpose, and so other methods must be employed.

5 Unadaptability of Local Linearization, and the Hausman and MaCurdy Methods

The local linearization, Hausman, and MaCurdy methods each utilize the result in Hall (1973), mentioned previously, that in the presence of non-proportional taxation, a person who has convex preferences will choose the same consumption-hours bundle on a nonlinear budget constraint that they would choose if they faced a linear budget constraint tangent to the actual budget constraint at the chosen bundle. As a result, desired labor supply on a nonlinear budget constraint can be written as a function of the set of desired hours of work that would be chosen if the worker faced various linear budget constraints tangent to the nonlinear budget constraint, and the likelihood function or regression model can be written in terms of such a labor supply function.

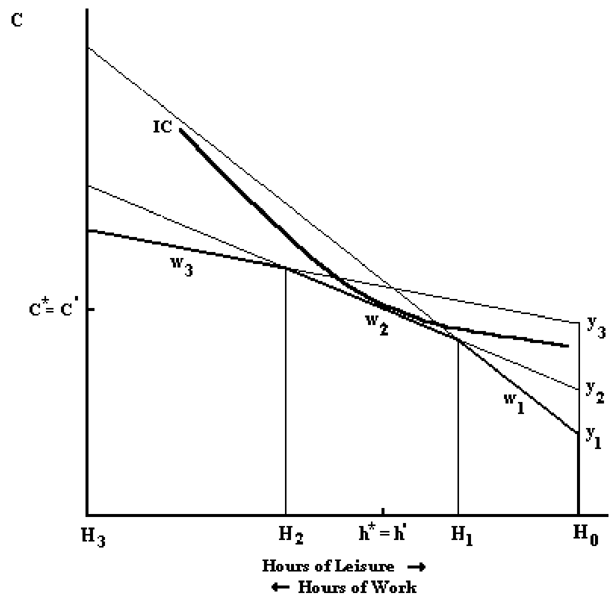

Figure 7: Graphical Demonstration of Proposition 1

In this section, however, we show that the result used in Hall does not always apply when preferences are nonconvex. This stems from the fact that when preferences are nonconvex, but the budget constraint is nonlinear, the optimal consumption-leisure bundle may lie in the interior of the convex hull of the upper contour set. The following propositions, then, examine under what conditions on the utility function, $U(C, h)$, the Hall result holds, and thus can be applied to infer the desired hours of work on a nonlinear budget constraint.

Formally, let $(C^*, h^*) = \arg\max_{C,h}\{U(C, h) : C \leq f(W, Y, h)\}$, where C is a composite consumption good, h is hours of work, w is the wage, y is nonlabor income, and $f(\cdot)$ denotes a nonlinear budget constraint. Let w^* be the wage and y^* be the level of virtual income, defined such that $w^* = \frac{\partial f(h^*)}{\partial h}$ and $y^* = C^* - w^*h^*$. The following proposition shows that if preferences are continuous, locally non-satiated, and strictly convex[39], then the result used in Hall holds, and hence the estimation methods commonly used are applicable.

Proposition 1. *Let $U(C, h)$ represents continuous, locally non-satiated, convex preferences over consumption and hours of work. Then for (C', h') such that*

$$(C', h') = \arg\max_{C,h}\{U(C, h) : C \leq w^*h + y^*\},$$

$(C', h') = (C^*, h^*)$.
Proof. See Appendix C. ■

For the intuition behind Proposition 1, see Figure 7. Clearly, since all portions of the

[39] Although Hausman (1981) and MaCurdy et. al. (1990) assume strict convexity of preferences (or, equivalently, strict quasiconcavity of the utility function), generalized versions of these results hold for the case of weakly convex preferences.

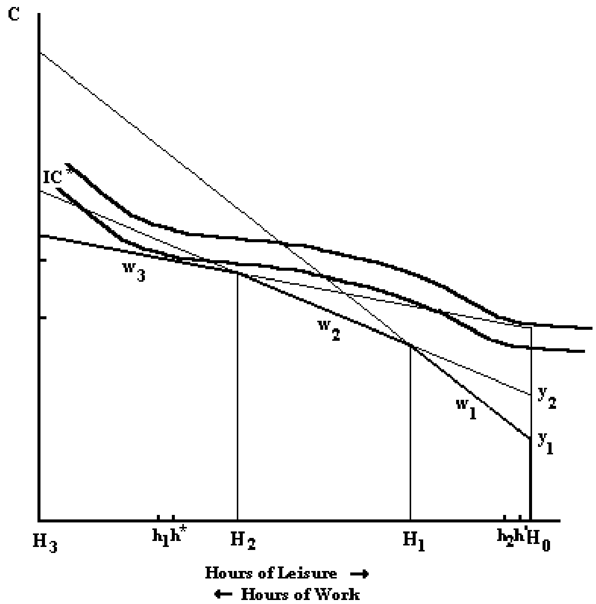

Figure 8: Graphical Demonstration of Proposition 2

actual budget constraint are tangent to or below the linearized budget constraint, and all portions of the highest indifference curve are above the linearized budget constraint, the optimal hours of work will be the same for both.

Thus, if preferences are strictly convex, the application of the Hall result is a valid one. Furthermore, if preferences are nonconvex, but the chosen consumption-hours bundle on the nonlinear budget constraint lies on the boundary of the convex hull of the upper contour set, it is easy to see that the result in Hall once again applies.

If, however, preferences are nonconvex, and the optimal consumption-hours bundle with the nonlinear budget constraint is not on the convex hull of the upper contour set, then the result used in Hall does not apply.

Proposition 2. *Let $U(C,h)$ represents continuous, locally non-satiated, nonconvex preferences over consumption and leisure, but let (C^*,h^*) defined above lie inside the boundary of the convex hull of $\{(C,h) : U(C,h) \geq U(C^*,h^*)\}$. Then for (C',h') such that $(C',h') = \arg\max\{U(C,h) : C \leq w^*h + y^*\}$, $(C',h') \neq (C^*,h^*)$.*

Proof. See Appendix C. ■

The intuition behind this proposition can be seen in Figure 8. Since part of indifference curve IC^* lies below the linearized budget constraint, this indifference curve is not the highest feasible indifference curve. Instead, utility will be maximized along IC', where the choice of hours is different than on IC^*, and hence $h^* \neq h'$.

Thus, if preferences are nonconvex, it is not necessarily true that the consumption-hours bundle chosen on the actual nonlinear budget constraint is the same as that which would be chosen if the individual faced a linear budget constraint tangent to the actual budget constraint. Since local linearization methods, the Hausman method, and the MaCurdy method all rely on the Hall result holding, the result of Proposition 2 suggests that these methods will not be adaptable to the case of nonconvex preferences.

The unadaptability of local linearization is straightforward. An individual with nonconvex preferences would not necessarily choose the same hours of work on the actual budget constraint as they would choose if they were faced with a linear budget constraint tangent to the actual budget constraint at their observed hours of work. As a result, no simple function of only the observed after tax wage and virtual income could possibly determine the desired hours of work for the individual.

The Hausman and MaCurdy methods are more complex in their use of the Hall result, but the inapplicability of the Hall result in some cases when preferences are nonconvex also renders them unable to estimate parameters consistent with nonconvex preferences. For the Hausman method, the result in Proposition 2 implies that the Hausman algorithm, which identifies the desired hours on a nonlinear budget constraint, may fail when preferences are nonconvex. In the MaCurdy method, the result in Proposition 2 implies that an equation using an implicit function relied upon in the derivation of the likelihood may fail to hold when preferences are nonconvex. These claims are proven in Appendix D.

Furthermore, although the discussion in this section has dealt with specific estimation methods, it is clear that any method that attempts to apply the Hall result when formulating an estimation method will not be adaptable to the case of nonconvex preferences, and that this is true regardless of how flexibly one specifies the labor supply equation. For example, the recent work of Blomquist and Newey (2000) applied nonparametric techniques to labor supply estimation in this setting, but since their method also invoked the Hall result, it too cannot be used to estimate parameters consistent with nonconvex preferences.

6 Relaxing the Assumption of Convexity by Estimating a Direct Utility Function

In this section, we outline a method that can be used to estimate labor supply parameters in the presence of a nonconvex budget set without appealing to the Hall result.

To do so, instead of specifying the labor supply function, we specify a direct utility function. Since we are working with the direct utility function, we need not appeal to the Hall result in order to identify an individual's desired hours on the actual budget set, but must simply find the hours choice that maximizes utility. Of course, the parameterization of the utility function must be flexible enough so that it may represent either convex or nonconvex preferences, but given such a specification, an estimation method may be constructed in a straightforward manner.

One way of implementing such a method would be a straightforward adaptation of the methods in Keane and Moffitt (1998) or Hoynes (1996), as was done in a recent paper by van Soest et al. (2002).[40] Suppose that there exists a sufficiently flexible specification of the utility function, $U(C, h; \beta)$, so that parameters, β, could make the utility function represent either convex or nonconvex preferences. For example, one could use the functional form

[40]Neither Keane and Moffitt (1998) nor Hoynes (1996) note the possibility to extending their estimation methods to allow preferences to be nonconvex, but there is nothing inherent in the methods that precludes them from doing so.

described above in Section 4. Approximate individual i's budget constraint by a set of discrete consumption and hours pairs, $\{C_{ik}, h_{ik}\}_{k=1}^{K}$. The utility of each discrete point, then, is this level of utility plus a random term, ε_{ik}, so that

$$U_i(C_{ik}, h_{ik}; \beta) = U(C_{ik}, h_{ik}; \beta) + \varepsilon_{ik} \tag{36}$$

The probability of observing the individual working h_{ik}^* hours, then, is

$$P(h_{ik}^*) = P\left[U(C_{ik}, h_{ik}^*; \beta) + \varepsilon_{ik} > U(C_{ij}, h_{ij}; \beta) + \varepsilon_{ij} \ \forall \ j \neq k\right] \tag{37}$$

The parameters, β, are then be chosen to maximize the likelihood of observing the sample.

There are some problems with this approach. First, this method requires that the labor supply choice be discretized, when clearly this is a simplification. More importantly, however, in order for such a model to be computationally feasible, the assumption is often made that the errors are distributed i.i.d. However, this assumption quickly becomes untenable as the number of hours choice increases. On the other hand, modelling the dependence between error terms probably makes the model computationally infeasible.

Instead, we follow a different approach. The approach that we use does not require that one use a discrete approximation to the budget constraint, nor that the budget constraint be piecewise linear or twice differentiable. This method does, however, involve the execution of a computationally intensive maximization procedure.

Again, suppose there exists a specification of the utility function, $U(C, h; \beta)$, so that the utility function may represent both convex and nonconvex preferences, and where v_i is a term that reflects unobservable heterogeneity in tastes for leisure. Let the budget constraint be given by $C \leq B(h, W, Y)$, where C is consumption, h are hours of work, W is the wage, Y is unearned income, $B(\cdot)$ is an arbitrary budget set, and β are the parameters of interest. Let v_i have a cumulative distribution function, known up to parameters σ_v, of $G(v_i; \sigma_v)$.[41] Individual i now solves

$$\begin{aligned} & \max U(C, h, v_i; \beta) \\ s.t.\ C \ & \leq \ B(W_i, Y_i, h) \end{aligned} \tag{38}$$

Desired hours for individual i, given parameters β, are now represented by

$$h_i^*(v_i; \beta) = \arg\max_{h_i} U(B(W_i, Y_i, h), h, v_i; \beta) \tag{39}$$

In this case, observed hours are related to desired hours in the manner

$$h_i = \begin{cases} h_i^*(v_i; \beta) + \varepsilon_i & if \quad h_i^*(v_i, \beta) > 0 \text{ and } h_i^*(v_i, \beta) + \varepsilon_i > 0 \\ 0 & if \quad \begin{cases} h_i^*(v_i, \beta) = 0 \\ \text{or } h_i^*(v_i, \beta) > 0 \text{ and } h_i^*(v_i, \beta) + \varepsilon \leq 0. \end{cases} \end{cases} \tag{40}$$

and the likelihood for individual i is now

$$l_i = \int_{v_i} \begin{Bmatrix} 1\left(h_i^*\left(v_i; \beta\right) = 0\right) \\ +1\left(h_i^*\left(v_i; \beta\right) > 0\right)\left[1 - F(h_i^*(v_i; \beta); \sigma_\varepsilon)\right] \\ \times[f(h_i - h_i^*(v_i; \beta); \sigma_\varepsilon) 1\left(h_i^*\left(v_i; \beta\right) > 0\right)]^{1(h_i>0)} \end{Bmatrix}^{1(h_i=0)} dG(v_i; \sigma_v) \tag{41}$$

[41] To aid in computation, $G(v_i; \sigma_v)$ may be a discrete distribution.

where $1(\cdot)$ denotes the indicator function.

It is possible to make this approach computationally feasible by employing two numerical methods. First, to perform the integration in (41) over the distribution of v_i, one can use Gaussian quadrature techniques, which approximate the likelihood with a weighted sum. Second, to solve for $h_i^*(v_i;\beta)$, which now must only be evaluated at discrete points, one can perform a line search over $[0,\overline{H}]$ by first using a grid search method to bracket the utility maximizing hours, and then use a bracketing technique to solve for $h_i^*(v_i;\beta)$ accurate to four decimal places.

7 Monte Carlo Comparison of the Hausman Method to a Method that Estimates Parameters of a Direct Utility Function

In order to compare the performance of the Hausman method and the method described above, we perform Monte Carlo Experiments for the two methods using two specifications. All of these Monte Carlo experiments take place in a setting that is generally favorable to the Hausman method, in which the assumed and true data generating processes are the same, preferences are convex over the hours range in which data are observed, and the true budget constraint is piecewise linear and convex.

In the first set of experiments, the data generating preferences are those consistent with a linear labor function. Since many of the papers reviewed in Hausman (1985b) and Blundell and MaCurdy (1999) use a linear labor supply specification when using the Hausman method, it is clearly important to examine how the method describe above compares to the Hausman method under this specification. This specification exhibits the bias that may result from the Hausman method due to its requirement that parameters satisfy Slutsky positivity at all kink points, but also shows that a method that estimates parameters of a direct utility function may suffer from a similar bias, not due to any constraint in the likelihood that parameters must satisfy Slutsky positivity, but rather due to peculiarities of the utility function consistent with a linear labor supply function. These experiments illustrate the importance of functional form choice, even when the direct utility function is being estimated.

In the second set of experiments, the data are assumed to be generated by a quadratic utility function. This specification is one that has gained in popularity recently (see, for example, Ransom (1987), Lacroix and Fortin (1992), Keane and Moffitt (1998), Blundell et al. (1999) and others). It turns out that this specification demonstrates a clear bias that results from the Hausman method's requirement that parameters satisfy Slutsky positivity at all kink points observed in the data. This problem arises if one makes the common assumption that the support of the heterogeneity term is infinite, that desired hours increase with the heterogeneity term linearly over the entire range of hours, and that the hours endowment can take an arbitrary value. In this case, estimating a direct utility function does not suffer from such a large bias, because, as will be seen, this method prohibits desired hours from increasing linearly with the heterogeneity term if such hours lie in a range where preferences are nonmonotonic or nonconvex. These experiments, then, illustrate that the choice of stochastic specification and hours endowment are not inconsequential choices, but rather

Table 3: Sample Statistics: Hausman (1981) Sample

Variable	**N**	**Mean**	**Std. Dev.**	**Min.**	**Max**
Annual Hours of Work	1084	2148.54	584.57	0.00	5096.00
Nonlabor Income	1084	1387.54	1618.69	0.00	29120.00
Gross Wage	1082	6.27	1.84	0.67	9.99

Table 4: 1975 Income Tax Table

Standard Deduction for Married Filing Jointly		$1,900
Marginal Tax Rate is	on Net Income Above	But Below
14%	$0	$1,000
16%	$1,000	$4,000
19%	$4,000	$8,000
22%	$8,000	$12,000
25%	$12,000	$16,000
28%	$16,000	$20,000
32%	$20,000	$24,000
36%	$24,000	$28,000
39%	$28,000	$32,000
42%	$32,000	$36,000
45%	$36,000	$40,000
48%	$40,000	$44,000
50%	$44,000	–

can have large effects on the resulting estimated parameters.

7.1 Source of Wage and Income Data, Parameters, and Generation of Simulated Data

Data for the wages and non-labor incomes in these Monte Carlo experiments come from the 1975 and 1976 Panel Study of Income Dynamics. In order to closely match the samples that have been previously used, we attempt to replicate the sample used in Hausman (1981) by using the sample selection criteria outlined in Eklöf and Sacklén (2000). Sample statistics are presented in Table 3. We also follow Hausman's method for creating hours, wage and income variables. As such, hours worked are defined as reported average hours per week times weeks per year. The gross wage comes from a direct report of the wage rate, with topcoded and missing wages replaced by predicted wages from a Tobit wage equation. Finally, the nonlabor income variable is constructed by attributing an eight percent return to housing equity.

For each individual in a given sample, using their gross wage and nonlabor income variables, we calculate parameters of each individual's piecewise linear budget constraint using the federal income tax schedule for tax year 1975, which is summarized in Table 4. Taking account of only federal taxes not only simplifies the programming in these simulations, but also ensures that the budget constraint is convex due to the progressive nature of the income tax schedule, which greatly simplifies implementation of the Hausman method.

Data are generated as follows. For each individual, a value of ε_i and v_i are drawn from their respective distributions. The individual's simulated desired hours, h_i^*, and observed hours, are then derived using the relevant algorithms outlined in Appendix E.

We perform simulations of each of the methods using a variety of parameters to examine the behavior of each of the methods under various conditions. We initially attempt to use, with some modifications due to the differing models that are being estimated, the parameters estimated in Hausman (1981). We also draw from the results of other studies in order to examine whether the results differ depending on the parametrization.

For the linear labor supply function specification, the estimated labor supply function is

$$h = c + \alpha w + \beta y + v, \tag{42}$$

and the corresponding utility function used in the method that estimates a direct utility function is

$$U(C,h) = \frac{1}{\beta}\left(h - \frac{\alpha}{\beta}\right)\exp - \left[1 + \frac{\beta\left(C + \frac{c+v}{\beta} - \frac{\alpha}{\beta^2}\right)}{\frac{\alpha}{\beta} - h}\right]. \tag{43}$$

Since α is simply the uncompensated wage effect in this specification and β is simply the income effect, we are able to use estimates of wage and income effects directly when specifying the parameters used in the experiments.

To choose these parameter values, we draw from various sources. Hausman (1981) used a linear specification for the labor supply function of the form and estimated the constant to be $2,419.5$, the uncompensated wage effect to be .2, and a distribution of the income effect with a mean of $-.166$ and a median of $-.120$. In comparison, in a summary of related studies in Blundell and MaCurdy (1999), the authors compile results from various studies which report uncompensated wage elasticities ranging from 0 to .25, with a mean of .085 and a median of .08. These values translate in our sample to uncompensated wage effects ranging from 0 to 85.6, with a mean of 29.1 and a median of 27.4. The total income elasticities (defined as $\frac{\partial h}{\partial y}\overline{w}$) that are reported, on the other hand, range from 0 to -1.03, with a mean of $-.158$ and a median of $-.07$. Again, for our sample, these values translate to income effects ranging from 0 to $-.166$, with a mean of $-.025$ and a median of $-.011$. As such, we use values of α ranging from .2 to 100, and values of β ranging from $-.0166$ to $-.166$. For c, we use the Hausman estimated value of 2419.5. To complete the specification of the data generating process, however, we must identify values to use for the standard deviations of the heterogeneity and measurement error distributions. For these, we turn to Triest (1990), who, in estimating the linear labor supply specification, obtains estimates of $\sigma_v = 234.5$ and $\sigma_\varepsilon = 498.5$.

For the quadratic utility specification, when using the method that estimates parameters of the direct utility function, the estimated utility function is

$$U(C,h) = \beta h^2 + \gamma Ch + (\delta + v)h + C. \tag{44}$$

The corresponding labor supply function estimated when using the Hausman method takes the form

$$h = \frac{-w - \gamma y - (\delta + v)}{2(\gamma w + \beta)}. \tag{45}$$

Therefore, we must translate the wage and income effects listed above into the parameters of this function. To do this translation, we solve for parameters such that, when evaluated at a wage of \$4.50 (approximately the mean after tax wage) and a nonlabor income of \$2200 (approximately the mean virtual income across all budget constraints), the uncompensated wage effect ranges from .2 to 100, and the income effect ranges from $-.0166$ to $-.12$[42]. To remain consistent with the linear labor supply specification, the value of σ_v is chosen so that the effect of v measured in hours in the labor supply equation in (45) has a standard deviation of 234.5 when evaluated at a wage of 4.5 and the relevant parameters.[43] Finally, we use the value of $\sigma_\varepsilon = 498.5$ estimated by Triest.

The likelihood functions used in these experiments, as well as the numerical methods used to make estimation of a direct utility function computationally feasible, are described in more detail in Appendix E. In all of the experiments, we constrain all terms of the likelihood functions to be nonnegative.

7.2 Results

7.2.1 Linear Labor Supply

In Table 5, we present the results of Monte Carlo experiments examining the behavior of the Hausman method and the alternative method under the linear labor supply specification.

The first row of each group of results reports the true data generating parameters for the group. The second row presents results from the Hausman method Monte Carlo experiments in which the sample size is increased by a factor of 10 by replicating the sample described above and drawing different stochastic elements for each replicated individual. This row, then, provides evidence on the behavior of the Hausman method when the sample is considerably larger than samples previously used in this literature. The third row of each group presents results from Hausman method Monte Carlo experiments using the original sample of 1074 individuals. This row can be used to examine the small sample properties of the Hausman method. In the fourth and last row of each group, we present the results when we estimate parameters of the direct utility function using Gaussian quadrature and line search techniques.

From these Monte Carlo experiments, it is clear that when the true uncompensated wage elasticity is small, there is considerable small sample bias in both methods. For example, using the original sample of 1074 individuals, when the true uncompensated wage elasticity is .0005 in Specification 1, the estimated uncompensated wage elasticity displays a bias that is between .023 and .015 in the two methods. When the true uncompensated

[42] Note that we do not use an income effect of -.166 due to the peculiar shape that indifference curves take under the quadratic utility function specification when the income effect is this large.

[43] In other words, we solve for σ_v in the equation

$$234.5 = \frac{\sigma_v}{2\,(4.5\gamma + \beta)}.$$

Table 5
Results from Monte Carlo Experiments
Comparison of Hausman Method and Estimating Parameters of a Direct Utility Function - Linear Labor Supply Specification

Specification	Method	Reps	N	Constant	Alpha	Beta	St Dev v	St Dev e	Wage Elast.	Income Elast.	% Constrained
1	True Parameters			2419.50	0.200	-0.01660	234.50	498.5	0.0005	-0.0098	
	Hausman	10	10740	2416.15	2.389	-0.01851	264.02	448.94	0.0063	-0.0109	10%
	Hausman	100	1074	2391.53	9.025	-0.02091	226.13	460.31	0.0237	-0.0123	27%
	Direct Utility Function	100	1074	2402.12	6.059	-0.01970	232.87	497.77	0.0159	-0.0116	
2	True Parameters			2419.50	0.200	-0.16600	234.50	498.5	0.0006	-0.1126	
	Hausman	10	10740	2416.24	2.304	-0.16757	275.00	476.98	0.0070	-0.1136	30%
	Hausman	100	1074	2390.94	9.034	-0.17047	223.89	482.89	0.0274	-0.1156	23%
	Direct Utility Function	100	1074	2395.37	8.057	-0.17024	234.25	496.20	0.0244	-0.1154	
3	True Parameters			2419.50	2.000	-0.01660	234.50	498.5	0.0052	-0.0097	
	Hausman	10	10740	2417.61	3.593	-0.01802	273.87	446.42	0.0094	-0.0106	30%
	Hausman	100	1074	2394.79	10.015	-0.02069	233.03	453.40	0.0263	-0.0122	22%
	Direct Utility Function	100	1074	2397.37	9.008	-0.01979	232.90	497.86	0.0236	-0.0116	
4	True Parameters			2419.50	2.000	-0.16600	234.50	498.5	0.0060	-0.1121	
	Hausman	10	10740	2418.16	3.567	-0.16730	274.04	477.49	0.0108	-0.1130	30%
	Hausman	100	1074	2395.34	9.802	-0.17030	220.10	484.30	0.0296	-0.1150	21%
	Direct Utility Function	100	1074	2397.71	9.318	-0.17058	230.71	497.37	0.0281	-0.1152	
5	True Parameters			2419.50	20.000	-0.01660	234.50	498.50	0.0508	-0.0094	
	Hausman	10	10740	2423.95	19.938	-0.01749	210.33	463.45	0.0506	-0.0099	0%
	Hausman	100	1074	2407.29	24.789	-0.01945	231.38	440.22	0.0630	-0.0111	2%
	Direct Utility Function	100	1074	2410.49	23.324	-0.01814	235.41	494.80	0.0593	-0.0103	

Notes: The estimated equation in each replication is the linear labor supply function, or its associated utility function. The first row presents the true parameters for the group of Monte carlo experiments. The second row of each group presents results from Hausman method replications when the sample size is increased by a factor of 10. The third row presents results from all replications of the Hausman method when the original sample is used. The fourth row presents results from a method that estimates parameters of the corresponding direct utility function.

Table 5 Cont.
Results from Monte Carlo Experiments
Comparison of Hausman Method and Estimating Parameters of a Direct Utility Function - Linear Labor Supply Specification

Specification	Method	Reps	N	Constant	Alpha	Beta	St Dev v	St Dev e	Wage Elast.	Income Elast.	% Constrained
6	True Parameters			2419.50	20.000	-0.16600	234.50	498.50	0.0583	-0.1084	
	Hausman	10	10740	2423.48	19.969	-0.16643	267.81	480.39	0.0582	-0.1087	0%
	Hausman	100	1074	2412.33	23.239	-0.16813	240.01	471.11	0.0678	-0.1098	11%
	Direct Utility Function	100	1074	2411.55	23.518	-0.16894	210.82	500.37	0.0686	-0.1103	
7	True Parameters			2419.50	50.000	-0.01660	234.50	498.50	0.1206	-0.0090	
	Hausman	10	10740	2423.44	50.256	-0.01764	281.28	419.45	0.1212	-0.0095	0%
	Hausman	100	1074	2408.39	54.736	-0.01950	206.59	444.50	0.1320	-0.0105	0%
	Direct Utility Function	100	1074	2411.77	52.861	-0.01851	231.91	497.02	0.1275	-0.0100	
8	True Parameters			2419.50	50.000	-0.16600	234.50	498.50	0.1378	-0.1024	
	Hausman	10	10740	2423.70	49.906	-0.16653	255.93	486.74	0.1375	-0.1027	0%
	Hausman	100	1074	2412.88	53.032	-0.16766	247.42	467.23	0.1461	-0.1034	0%
	Direct Utility Function	100	1074	2411.53	53.096	-0.16773	226.87	496.77	0.1463	-0.1035	
9	True Parameters			2419.50	100.000	-0.01660	234.50	498.50	0.2756	-0.0102	
	Hausman	10	10740	2424.41	99.926	-0.01744	307.10	447.51	0.2754	-0.0108	0%
	Hausman	100	1074	2407.97	104.703	-0.01891	239.51	435.52	0.2885	-0.0117	0%
	Direct Utility Function	100	1074	2410.45	103.815	-0.01852	223.16	500.08	0.2861	-0.0114	
10	True Parameters			2419.50	100.000	-0.16600	234.50	498.50	0.2756	-0.1024	
	Hausman	10	10740	2423.90	99.993	-0.16665	265.38	486.18	0.2756	-0.1028	0%
	Hausman	100	1074	2411.31	103.209	-0.16721	229.39	482.72	0.2844	-0.1032	0%
	Direct Utility Function	100	1074	2409.49	104.515	-0.16897	228.59	496.74	0.2880	-0.1042	

Notes: The estimated equation in each replication is the linear labor supply function, or its associated utility function. The first row presents the true parameters for the group of Monte carlo experiments. The second row of each group presents results from Hausman method replications when the sample size is increased by a factor of 10. The third row presents results from all replications of the Hausman method when the original sample is used. The fourth row presents results from a method that estimates parameters of the corresponding direct utility function.

wage elasticity is .0052 in Specification 3, the bias in the uncompensated wage elasticity estimated by the Hausman method decreases slightly to .021. When a direct utility function is estimated, this bias reaches only decreases to .018. The bias generally decreases as the true uncompensated wage elasticity increases, but is still quite large, with a bias of .011 when the true uncompensated wage effect is .1206 in Specification 7, and a bias of .013 when the true elasticity is .2756 in Specification 9.

The bias in the Hausman method does decrease dramatically, however, as sample size increases. When the sample size is increased by a factor of 10, the bias in the uncompensated wage elasticity almost disappears when the true uncompensated wage elasticity is between .05 and .2756, and even when the true elasticity is only .0005, the bias in this elasticity is considerably smaller, at .0058.

In the far right column, we report the percent of replications in which the constraint in the Hausman method, that Slutsky positivity be satisfied at all kink points, is binding. When $\alpha = .2$ and $\beta = -.166$ in Specification 2, for example, we find that the constraint is binding at the maximum likelihood estimates in 23% of the replications. When the sample size is increased tenfold, this event still occurs 30% of the time. Interestingly, given a value for α, when the true value of β is smaller in absolute value, this event is less prevalent. In addition, for larger values of α, the constraint is binding to a lesser extent. When $\alpha = 50$ or larger, no replications result in parameters at which the constraint is binding.

In small samples, estimates of the income elasticity are biased to a much lesser extent than the estimates of the uncompensated wage elasticities, even when the true income elasticity is close to 0. However, the bias still can be substantial. For example, in specification 1, the bias in the income elasticity in the Hausman method is -.0025 when the original sample is used, and this drops to -.0011 when the sample size is increased tenfold. In the experiments estimating a direct utility function on the original sample, the bias in the income elasticity is -.0018.

The bottom two lines for each specification provide a direct comparison of the two methods. Generally, estimating a direct utility function does about as well as the Hausman method. In some specifications of parameters, estimating a direct utility function using line search and Gaussian quadrature yields estimates that suffer from less bias, but the improvement is often small.

The failure of estimation of a direct utility function to perform decidedly better than the Hausman method, despite the fact that there is no implicit constraint that parameters satisfy Slutsky positivity, derives from the following. When $\alpha < 0$ or $\beta > 0$ in this specification, indifference curves have a peculiar shape, and in such a case, utility is maximized at the extremes ($h_i^* = 0$ or $h_i^* = 5800$) for most individuals in the data.[44] Hence, it is very

[44] The reason for this is as follows. When $a < 0$ and $\beta < 0$, the first term is everywhere negative and the third is everywhere positive, but the second term switches from negative to positive as hours increase past $\frac{\alpha}{\beta}$, and so utility switches from positive to negative. Utility in this case could thus only be maximized where $h < \frac{a}{\beta}$, and examination of graphs of indifference curves over this range suggest that utility will only be maximized at $h = 0$ or $\frac{\alpha}{\beta}$.

Similar logic applies when $\alpha > 0$ and $\beta > 0$, but in this case, utility appears to be maximized either at $\frac{\alpha}{\beta}$ or 5800 hours.

Finally, when $a < 0$ and $\beta > 0$ utility is everywhere positive, and so not discontinuous at $h = \frac{\alpha}{\beta}$, but indifference curves appear to be concave, resulting in utility being maximized in the majority of cases at unlikely that these parameters will be the parameters at which the likelihood function is maximized. However, contrary to the Hausman method case above, the optimal parameters will be parameters consistent with Slutsky positivity not because they are forced to do so, but rather because parameters that violate Slutsky positivity would be a bad fit to the data.

Thus, these experiment illustrate the importance of functional form choice in doing structural labor supply estimation. The Slutsky constraint in the Hausman method when a linear labor supply specification is used essentially constrains α to be positive and β to be negative, but estimating parameters of the associated direct utility function does not get around this, because of peculiarities in this functional form.

Nevertheless, in the linear labor supply specification, both methods appear to do about as well. In the specification below, however, the results obtained from the two methods are dramatically different.

7.2.2 Quadratic Utility

In Table 6, we present the results of Monte Carlo experiments comparing estimates using the Hausman method and estimates of parameters of a direct utility function using line search and Gaussian quadrature techniques under the quadratic utility specification. While the desired and actual hours do not depend on the assumed hours endowment, the estimated parameters using the Hausman method do differ sharply depending on the assumed hours endowment as these simulations show.

In this table, the first row of each group of results reports the true data generating parameters for the group. The second and third rows of each group presents results from 100 replications from the Hausman method Monte Carlo experiments using the original sample of 1074 individuals. The second row reports estimates where the hours endowment is 5800 per year, while the third row reports estimates were the endowment is 4000 per year. In the fourth and last row of each specification, we report estimates obtained by maximizing the direct utility function.

Unlike the previous specification, the Hausman method suffers from a large bias in most of the specifications when choices are determined by a quadratic utility function, whereas the parameters that are obtained by estimating a direct utility function using line search and Gaussian quadrature suffers from little bias in comparison. For the discussion that follows, it is useful to keep in mind the form that the Slutsky constraint takes in this specification. As noted in Appendix E, the constraint takes the form $1 + \widehat{\gamma} H_j \geq 0$, where H_j is the largest kinkpoint considered in the data. This largest kinkpoint will likely be very close to the hours endowment. Thus, $\widehat{\gamma}$ is restricted to be greater than or equal to $-1/H_j$. For example, when the hours endowment is taken to be 5800 hours, this constraint implies that $\widehat{\gamma} \geq -.0001724$.

In almost all of the Hausman method specifications, this constraint is binding in the vast majority of the replications. This, in turn, biases the estimated wage and income elasticities to a great extent. For example, in first row with a 5800 hours endowment for specification 2, the estimated income elasticity is biased downward in absolute value by .0174, when the true income elasticity was -.0455, and the estimated uncompensated wage elasticity is biased upward by .1762, when the true wage elasticity was .0004. Although this group presents

either 0 or 5800 hours.

Table 6
Results from Monte Carlo Experiments
Comparison of Hausman Method and Estimating Parameters of a Direct Utility Function - Quadratic Utility Specification

Specification	Method	Reps	N	Beta	Gamma	Delta	St Dev v	St Dev e	Wage Effect	Income Effect	Wage Elast.	Income Elast.	Comp. Wage Elast.	% Constrained
1	True Parameters			-5.8069E-03	-2.2665E-04	25.992	3.2018	498.70	0.20	-0.0166	0.0004	-0.0075	0.16524	
	Hausman (Max Hours = 5800)	100	1074	-6.1436E-03	-1.7166E-04	26.402	2.5402	512.74	17.69	-0.0124	0.0362	-0.0056	0.202471965	97%
	Hausman (Max Hours = 4000)	100	1074	-6.9101E-03	-2.1897E-04	30.694	3.6681	458.81	2.31	-0.0139	0.0047	-0.0063	0.14769129	33%
	Direct Utility Function	100	1074	-5.5259E-03	-2.2339E-04	24.718	2.7764	496.85	1.31	-0.0171	0.0027	-0.0078	0.17519316	
2	True Parameters			-1.1358E-04	-2.2717E-04	0.952	0.5327	498.70	0.20	-0.1000	0.0004	-0.0455	0.9909	
	Hausman (Max Hours = 5800)	100	1074	-6.1906E-04	-1.7250E-04	2.073	0.7180	478.02	86.36	-0.0618	0.1766	-0.0281	1.000586251	100%
	Hausman (Max Hours = 4000)	100	1074	-1.2620E-04	-2.2716E-04	1.015	0.2897	529.03	0.22	-0.0989	0.0005	-0.0450	0.980124139	9%
	Direct Utility Function	100	1074	-1.1362E-04	-2.2641E-04	0.941	0.5062	502.66	1.68	-0.1000	0.0034	-0.0454	0.997202928	
3	True Parameters			7.5729E-05	-2.2719E-04	0.119	0.1140	498.70	0.20	-0.1200	0.0004	-0.0545	1.1889	
	Hausman (Max Hours = 5800)	100	1074	-3.2888E-04	-1.7224E-04	0.820	0.0406	663.05	109.67	-0.0780	0.2243	-0.0355	1.265809105	99%
	Hausman (Max Hours = 4000)	100	1074	1.3939E-04	-2.2049E-04	-0.276	0.3716	539.65	17.49	-0.1293	0.0358	-0.0588	1.35850099	6%
	Direct Utility Function	100	1074	7.7463E-05	-2.2612E-04	0.093	0.4284	501.32	2.69	-0.1203	0.0055	-0.0547	1.202748676	
4	True Parameters			-4.5712E-03	-1.7842E-04	19.5026	2.5204	498.70	20.00	-0.0166	0.0409	-0.0075	0.25434	
	Hausman (Max Hours = 5800)	100	1074	-6.3244E-03	-1.6001E-04	26.8455	2.4539	484.68	21.01	-0.0114	0.0430	-0.0052	0.206962372	72%
	Hausman (Max Hours = 4000)	100	1074	-6.2935E-03	-1.7566E-04	27.0466	2.9801	453.41	16.03	-0.0124	0.0328	-0.0056	0.194875657	5%
	Direct Utility Function	100	1074	-4.2819E-03	-1.7757E-04	18.2354	2.3154	497.90	21.52	-0.0175	0.0440	-0.0079	0.269831163	
5	True Parameters			-1.0870E-04	-2.1739E-04	0.7174	0.5098	498.70	20.00	-0.1000	0.0409	-0.0455	1.08	
	Hausman (Max Hours = 5800)	100	1074	-5.3513E-04	-1.7226E-04	1.6984	0.6630	493.28	92.37	-0.0657	0.1889	-0.0299	1.066418555	99%
	Hausman (Max Hours = 4000)	100	1074	-1.0922E-04	-2.1725E-04	0.7235	0.2323	534.59	20.29	-0.0999	0.0415	-0.0454	1.080754473	0%
	Direct Utility Function	100	1074	-1.0844E-04	-2.1684E-04	0.7091	0.4827	502.57	21.18	-0.1000	0.0433	-0.0455	1.085280499	
6	True Parameters			7.2993E-05	-2.1898E-04	-0.0474	0.4279	498.70	20.00	-0.1200	0.0409	-0.0545	1.278	
	Hausman (Max Hours = 5800)	100	1074	1.0896E-04	-1.7241E-04	-1.1378	0.6808	424.20	181.01	-0.1293	0.3702	-0.0588	2.09426588	100%
	Hausman (Max Hours = 4000)	100	1074	2.8636E-04	-2.2107E-04	-0.927	0.3450	500.41	19.28	-0.1560	0.0394	-0.0709	1.631376445	0%
	Direct Utility Function	100	1074	6.7096E-05	-2.1846E-04	-0.0282	0.4207	501.38	21.17	-0.1193	0.0433	-0.0542	1.275848026	

Notes: The estimated equation in each replication is the quadratic utility function, or its associated labor supply function. The first row presents the true parameters for the group of Monte Carlo experiments. The second row presents results from all replications of the Hausman method when the original sample is used, and the maximum number of hours is set to 8000. The third and fourth row presents results from this same sample when the maximum number of hours is set to 5800 hours and 4000 hours, respectively. The fourth row presents results from a method that estimates the parameters of the corresponding direct utility function.

Table 6 Cont.

Results from Monte Carlo Experiments

Comparison of Hausman Method and Estimating Parameters of a Direct Utility Function - Quadratic Utility Specification

Specification	Method	Reps	N	Beta	Gamma	Delta	St Dev v	St Dev e	Wage Effect	Income Effect	Wage Elast.	Income Elast.	comp w elast	% Constrained
7	True Parameters			-3.4566E-03	-1.3492E-04	13.650	1.9059	498.70	50.00	-0.0166	0.1023	-0.0075	0.38934	
	Hausman (Max Hours = 5800)	100	1074	-4.0547E-03	-1.2801E-04	16.147	2.0982	457.17	47.16	-0.0138	0.0965	-0.0063	0.34905078	13%
	Hausman (Max Hours = 4000)	100	1074	-3.9712E-03	-1.2878E-04	15.797	2.1349	456.76	47.62	-0.0141	0.0974	-0.0064	0.35435211	0%
	Direct Utility Function	100	1074	-3.4522E-03	-1.3443E-04	13.645	1.8201	500.44	50.34	-0.0166	0.1030	-0.0075	0.39056406	
8	True Parameters			-1.0204E-04	-2.0408E-04	0.398	0.4786	498.70	50.00	-0.1000	0.1023	-0.0455	1.215	
	Hausman (Max Hours = 5800)	100	1074	-4.4177E-04	-1.7250E-04	1.265	0.3769	531.35	98.93	-0.0708	0.2024	-0.0322	1.14622408	100%
	Hausman (Max Hours = 4000)	100	1074	-1.0534E-04	-2.0393E-04	0.415	0.2822	521.09	50.20	-0.0997	0.1027	-0.0453	1.21262917	0%
	Direct Utility Function	100	1074	-9.8767E-05	-2.0349E-04	0.376	0.4514	502.82	51.57	-0.1003	0.1055	-0.0456	1.22499284	
9	True Parameters			6.9204E-05	-2.0761E-04	-0.279	0.4057	498.70	50.00	-0.1200	0.1023	-0.0545	1.413	
	Hausman (Max Hours = 5800)	100	1074	-1.5122E-04	-1.7231E-04	-0.019	0.4185	505.63	130.48	-0.0930	0.2669	-0.0423	1.50766209	99%
	Hausman (Max Hours = 4000)	100	1074	-5.6619E-06	-2.0295E-04	-0.035	0.3234	551.20	58.23	-0.1104	0.1191	-0.0502	1.35525898	0%
	Direct Utility Function	100	1074	7.2407E-05	-2.0678E-04	-0.305	0.3866	502.65	52.53	-0.1205	0.1074	-0.0548	1.42919603	
10	True Parameters			-2.4578E-03	-9.5932E-05	2.9985	1.3552	498.50	100.00	-0.0166	0.2045	-0.0075	0.61434	
	Hausman (Max Hours = 5800)	100	1074	-2.7112E-03	-8.8888E-05	9.3794	1.3398	474.05	97.86	-0.0143	0.2002	-0.0065	0.58177653	0%
	Hausman (Max Hours = 4000)	100	1074	-2.7113E-03	-8.8912E-05	9.3803	1.3520	473.98	97.83	-0.0143	0.2001	-0.0065	0.58168618	0%
	Direct Utility Function	100	1074	-2.4333E-03	-9.6333E-05	8.3248	1.3125	499.15	100.48	-0.0168	0.2055	-0.0076	0.61851378	
11	True Parameters			-9.2593E-05	-1.8519E-04	-0.2593	0.4343	498.50	100.00	-0.1000	0.2045	-0.0455	1.44	
	Hausman (Max Hours = 5800)	100	1074	-1.6084E-04	-1.7172E-04	-0.1974	0.1755	547.86	130.92	-0.0920	0.2678	-0.0418	1.49964234	97%
	Hausman (Max Hours = 4000)	100	1074	-1.2246E-04	-1.7999E-04	-0.2212	0.1703	555.72	111.56	-0.0965	0.2282	-0.0439	1.4575321	0%
	Direct Utility Function	100	1074	-7.8133E-05	-1.8534E-04	-0.3104	0.3834	506.51	101.13	-0.1016	0.2069	-0.0462	1.46086466	
12	True Parameters			6.3694E-05	-1.9108E-04	-0.4745	0.3734	498.50	100.00	-0.1200	0.2045	-0.0545	1.638	
	Hausman (Max Hours = 5800)	100	1074	-1.4886E-05	-1.7126E-04	-0.5156	0.4103	510.76	156.87	-0.1090	0.3209	-0.0495	1.78505712	91%
	Hausman (Max Hours = 4000)	100	1074	1.1354E-05	-1.8342E-04	-0.3737	0.2940	567.51	118.53	-0.1127	0.2424	-0.0512	1.64872312	0%
	Direct Utility Function	100	1074	6.5390E-05	-1.9060E-04	-0.4893	0.3553	502.46	101.83	-0.1203	0.2083	-0.0547	1.64902462	

Notes: The estimated equation in each replication is the quadratic utility function, or its associated labor supply function. The first row presents the true parameters for the group of Monte Carlo experiments. The second row presents results from all replications of the Hausman method when the original sample is used, and the maximum number of hours is set to 8000. The third and fourth row presents results from this same sample when the maximum number of hours is set to 5800 hours and 4000 hours, respectively. The fourth row presents results from a method that estimates the parameters of the corresponding direct utility function.

results for which the bias is extremely large, it is not an outlier. In fact, in seven of the twelve specifications, the estimated uncompensated wage elasticity is biased by more than .1.

Estimating parameters of the direct utility function, on the other hand, results in elasticity estimates that are much less biased. The estimated income elasticity tends to be biased less than .005, and the uncompensated wage elasticity tends to be biased by less than .03. Although these estimates are by no means unbiased, the bias is clearly much smaller than that found when using the Hausman method.

Two differences between these methods seem to be driving these dramatically different results. As background information, note that over $\left[0, \frac{-1}{\gamma}\right]$, which is the range in which all of the simulated data are observed, the theoretical likelihood functions are the same; they are just written in different forms. Nevertheless, the numerical methods that are used to evaluate the likelihood are different, in that the Hausman method uses an exact calculation of the likelihood, whereas estimating parameters of a direct utility function requires the use of a discrete approximation of the likelihood using Gaussian quadrature to make the method computationally feasible. This approximation over most of the range of the data, however, does not seem to be what is driving the difference.

The first difference that seems to affect the estimated parameters is that, because the method that estimates parameters of the direct utility function calculates the likelihood using a discrete approximation to the likelihood function, the method implicitly puts no weight on desired hours that would be the desired hours only if the individual's draw of v_i had been more than several standard deviations above the mean. Given the stochastic specification in this group of Monte Carlo results, however, the Hausman method requires that there be a positive probability that desired hours be at any kink point, no matter how extreme its value (even those above $\frac{-1}{\gamma}$, which is usually about 7 standard deviations above the mean). Since, in this specification, these hours have a positive, though extremely small, probability of being the desired hours, Slutsky positivity must be satisfied at the kink points that lie in this range, something that is not required when estimating parameters of the direct utility function.

The second difference results from the fact that for hours over the range $\left[\frac{-1}{\gamma}, \infty\right)$, the likelihood functions are different. In this stochastic specification of the Hausman method, with desired hours on a linearized budget constraint increasing linearly with v_i, which has infinite support, implies that there must be a positive probability of desired hours falling at any kink points observed in the data. Hence, parameters are constrained to be such that Slutsky positivity holds at such hours, which implies that estimated parameters must satisfy

$$1 + \gamma H_j \geq 0 \tag{46}$$

or

$$H_j \leq \frac{-1}{\gamma} \tag{47}$$

for any H_j observed in the data. If this inequality were not satisfied, then terms in the likelihood corresponding to the probability of desired hours falling at kink points beyond $\frac{-1}{\gamma}$ would be negative. Thus in requiring all terms in the likelihood to be nonnegative, γ is constrained to be no more negative than approximately -.0001724.

In a method in which a direct utility function is estimated using line search and Gaussian quadrature techniques, however, when

$$U = \beta h^2 + \gamma C h + (\delta + v_i) h + C \tag{48}$$

is maximized subject to the budget constraint, the curvature of this utility function is such that utility is never maximized at a level of hours greater than $\frac{-1}{\gamma}$. Thus, if there are any kink points beyond $\frac{-1}{\gamma}$ on the hours axis, there is simply zero probability that these kink points are the individual's desired hours. This, in turn, puts no requirements on the sign of the Slutsky term when evaluated at these hours.

Thus, in this specification, estimating a direct utility function greatly reduces the amount of bias in the estimated parameters. This, in turn, illustrates the importance of the choice of stochastic specification and hours endowment when using the Hausman method. Parameters in that method were constrained to satisfy Slutsky positivity at any kink points observed in the data below 5800 hours, because the hours endowment was set at 5800 hours, and because the chosen stochastic specification implied that any of those hours must have positive probabilities of being chosen. If the hours endowment were set much lower, this would reduce the range over which kink points were observed in the data, which in turn would relax the Slutsky constraint somewhat, possibly reducing the bias that the Hausman method suffers. Further, if the stochastic specification were changed so that these extreme levels of hours had zero probability of being chosen, this would alter the likelihood function of the Hausman method to be more in line with that of estimating a direct utility function, again possibly reducing the bias. Nevertheless, the experiments presented here suggest that the choice of stochastic specification and hours endowment, previously thought to be unimportant choices to make, may not be so.

7.3 Summary of Monte Carlo Experiments

Thus, it appears that, in moving to a method in which a direct utility function is estimated using line search and Gaussian quadrature techniques, we have not lost any precision in the estimates compared to those found using the Hausman method. Further, these Monte Carlo experiments have illustrated the importance of functional form, stochastic specification, and hours endowment choices.

Obviously, the estimation of a direct utility function is much more computationally intensive than is the Hausman method. However, there are some clear benefits. First, the use of such a method may provide guidance as to when choices of functional form, stochastic specification, and hours endowments are important, and when they are not. Second, it may be implemented using functional forms for utility that do not have closed form expressions for the related labor supply function. Finally, and most importantly in the context of this paper, such a method can be implemented with a functional form in which parameters may represent nonconvex preferences.

8 Conclusion

In this chapter, we have reviewed various methods that have been used to estimate labor supply parameters in the presence of nonlinear budget constraints. We noted the weaknesses in the local linearization methods, even when instrumental variables are used. We examined the Hausman method, and noted that the MGP critique pointed out where assumptions made in the construction of the likelihood were enforced. We then provided an argument why data consistent with nonconvex preference maximization can lead to parameters that violate Slutsky positivity when using one of the standard methods, and presented a numerical example that demonstrated this claim. We further showed that it is not possible to adapt these methods to allow for the estimation of parameters consistent with nonconvex preferences. Finally, we suggested a method that can be used to estimate such preferences, and evaluated its performance in comparison to the Hausman method using Monte Carlo experiments. These Monte Carlo experiments illustrated that in moving to a method in which a direct utility function is estimated using line search and Gaussian quadrature techniques, no precision is lost. In addition, they demonstrated the importance of functional form assumptions when using either method, and the importance of assumptions about the stochastic specification and hours endowments when using the Hausman method.

How seriously should one take the possibility of nonconvex preferences? That issue is addressed in the next chapter of this dissertation. In it, we discuss the plausibility of nonconvex preferences in the setting of structural labor supply models. We argue that, besides the possibility of preferences over consumption and leisure being nonconvex, even when these underlying preferences are convex, data generating preferences in a structural labor supply model may still be nonconvex because of the way structural models are customarily formulated.

This is because the variables used in structural estimation are usually not consumption and leisure, but rather monetary outlays and hours of work. We show that, in ignoring costs of work that vary with the number of hours worked, when translating preferences over consumption and leisure into preferences over monetary outlays and hours of work, work costs that are unaccounted for in budget and time constraints in the structural model are subsumed into data generating preferences. We then establish a necessary condition on the form of the work costs functions for these data generating preferences over monetary outlays and hours of work to be nonconvex. Finally, we argue that an examination of the likely form of work costs suggests that nonconvex preferences are plausible in this setting.

Hence, we argue that the one should take seriously the possibility that preferences are nonconvex, and use a method that allows for such a possibility when estimating labor supply. In this paper, we show how to allow for nonconvexities in preferences in labor supply estimation, and note the likely consequences if one does not.

9 Appendix A

The presentation of two sets of arguments in the paper was simplified by the use of linear labor supply. We generalize those arguments here. The first argument is the explanation of the MGP critique in Section 3.2. The second argument is the Section 4 explanation of how nonconvexities in preferences may lead to optimal parameters that would violate the Slutsky constraint. Both of these arguments may be generalized with an appeal to the mean value theorem, under the assumption that the estimated labor supply function is continuous and differentiable. In this case, let the desired hours of labor supply function (when unobservable heterogeneity $v = 0$) on segment j, S_j, be given by $h(w_j, y_j, X, \theta)$, where w_j and y_j are the after tax wage and virtual income associated with S_j, X are other independent variables that are constant regardless of the number of hours worked, and θ are the parameters to be estimated.

The argument in Section 3.2 can be generalized as follows: There must be some set, V_j, of unobservable heterogeneity, v for which $h(w_j, y_j, X, \theta) \geq H_j$ and $h(w_{j+1}, y_{j+1}, X, \theta) \leq H_j$. These two together imply that,

$$h(w_j, y_j, X, \theta) \geq h(w_{j+1}, y_{j+1}, X, \theta) \tag{49}$$

which implies that

$$h(w_j, y_j, X, \theta) - h(w_{j+1}, y_{j+1}, X, \theta) \geq 0 \tag{50}$$

Using the mean value theorem, we have that, for some $(\widehat{w}, \widehat{y})$ such that $(\widehat{w}, \widehat{y}) = t\,(w_j, y_j) + (1-t)\,(w_{j+1}, y_{j+1})\,,\, t \leq 1$,

$$\frac{\partial h(\widehat{w}, \widehat{y}, X, \theta)}{\partial w}(w_j - w_{j+1}) + \frac{\partial h(\widehat{w}, \widehat{y}, X, \theta)}{\partial y}(y_j - y_{j+1}) \geq 0 \tag{51}$$

Using the fact that $y_{j+1} = y_j + (w_j - w_{j+1})H_j$, we have that

$$\frac{\partial h(\widehat{w}, \widehat{y}, X, \theta)}{\partial w} - \frac{\partial h(\widehat{w}, \widehat{y}, X, \theta)}{\partial y}H_j \geq 0 \tag{52}$$

which is the Slutsky term evaluated at $(\widehat{w}, \widehat{y})$ and H_j. Further, the linear case is a special case of this result, in which $\frac{\partial h(\widehat{w}, \widehat{y}, X, v, \theta)}{\partial w} = \alpha$ and $\frac{\partial h(\widehat{w}, \widehat{y}, X, v, \theta)}{\partial y} = \beta \;\forall\; (\widehat{w}, \widehat{y})$.

10 Appendix B

In this appendix we prove Theorem 3 of Section 3.2.

Theorem 3. *Let preferences over consumption, C, and hours of work, h, be described by the utility function $U(C, h)$, and let $h(w, y)$ be the solution to the maximization of these preferences subject to $C \leq wh + y$. If $U(C, h)$ represents preferences that are monotonic, but nonconvex, then where demands are continuous, the Slutsky substitution term, $\frac{\partial h}{\partial w} - \frac{\partial h}{\partial y}h$, must be positive, and where demands are discontinuous, a discrete version of the Slutsky substitution term must be positive.*

Proof. Let

$$h(w, y) \in \arg\max_{C,h} \; \{U(C, h) \text{ s.t. } C \leq wh + y\}\,,$$

where $U(C, h)$ represents continuous, locally nonsatiated preferences that are possibly nonconvex. Let the level of utility at this hours level be U_0. Consider a discrete change in the wage from w to $w + \Delta w$, and a corresponding change in nonlabor income from y to $y + \Delta y$, so that the individual's utility maximizing utility level remains the same on the new budget constraint. Finally, let $h(w + \Delta w, y + \Delta y)$ be the utility maximizing choice on this compensated budget set, or

$$h(w + \Delta w, y + \Delta y) \in \arg\max_{C,h} \{U(C, h) \text{ s.t. } C \leq (w + \Delta w)h + (y + \Delta y)\}.$$

Because the marginal rate of substitution of the convex hull of the indifference curve $U = U_0$ (only the convex hull of the indifference curve is relevant here in the case of linear constraints) is nondecreasing in h, $\Delta w\,[h(w + \Delta w, y + \Delta y) - h(w, y)] \geq 0$, so that $\frac{h(w+\Delta w,y+\Delta y)-h(w,y)}{\Delta w} \geq 0$. This term is a discrete version of the compensated substitution effect.

Next, note that

$$\frac{h(w + \Delta w, y + \Delta y) - h(w, y)}{\Delta w} = \frac{h(w + \Delta w, y + \Delta y) - h(w + \Delta w, y)}{\Delta w} + \frac{h(w + \Delta w, y) - h(w, y)}{\Delta w},$$

where

$$h(w + \Delta w, y) \in \arg\max_{C,h} \{U(C, h) \text{ s.t. } C \leq (w + \Delta w)h + y\}.$$

Now, consider the linear budget constraint with slope w and intercept y that is tangent to the indifference curve $U = U_0$ at $h(w, y)$, and the linear budget constaint with slope $w + \Delta w$ and intercept $y + \Delta y$ that is tangent to the same indifference curve at $h(w + \Delta w, y + \Delta y)$. These two line segements intersect at an hours level between $h(w, y)$ and $h(w + \Delta w, y + \Delta y)$. Letting the hours level at that intersection point be h_c, we have that $\Delta y = -\Delta w h_c$, where $h(w + \Delta w, y + \Delta y) \geq h_c \geq h(w, y)$ if $\Delta w > 0$ (and signs are reversed if $\Delta w < 0$). Using this fact yields

$$\frac{h(w + \Delta w, y + \Delta y) - h(w, y)}{\Delta w} = -\frac{h(w + \Delta w, y + \Delta y) - h(w + \Delta w, y)}{\Delta y} h_c + \frac{h(w + \Delta w, y) - h(w, y)}{\Delta w}.$$

This last equation is a discrete version of the Slutsky equations that holds when demand functions are possibly discontinuous. The expression in this case is written in terms of discrete differences and is evaluated at an hours level between the two hours levels. Thus,

$$\frac{h(w + \Delta w, y) - h(w, y)}{\Delta w} - \frac{h(w + \Delta w, y + \Delta y) - h(w + \Delta w, y)}{\Delta y} h_c \geq 0.$$

If we take the limit as $\Delta w \to 0$ and $\Delta y \to 0$, we get (where the limit exists)

$$\frac{\partial h}{\partial w} - \frac{\partial h}{\partial y} h(w, y) \geq 0.$$

Of course, for w, y such that there are multiple tangencies, the limit may not exist, since at these points a change in wage and income may result in a discrete jump in hours. ■

11 Appendix C

In this appendix, we provide proofs for several propositions that appear in the main text of the paper.

Proposition 1. *Let $U(C,H)$ represents continuous, locally non-satiated, convex preferences over consumption and hours of work. Then for (C',h') such that $(C',h') = \arg\max_{C,h}\{U(C,h) : C \leq w^*h + y^*\}$, $(C',h') = (C^*,h^*)$.*

Proof. Suppose not, that $(C',h') \neq (C^*,h^*)$. Then it must be that $U(C',h') > U(C^*,h^*)$, and $C' = w^*h' + y^*$. Then, by local nonsatiation, there must exist some $(C",h")$ s.t. $U(C'',h'') = U(C^*,h^*)$ and $C'' < w^*h'' + y^*$. But, since preferences are convex, for every pair $\{(C,h) : U(C,h) \geq U(C^*,h^*)\}$, it must be that (C,h) is an element of the intersections of the upper half spaces that contain this set. Clearly, since $C \geq w^*h + y^*$ is one of such half spaces, then it must be that $C'' \geq w^*h'' + y^*$.$\Rightarrow\Leftarrow$ ■

Proposition 2. *Let $U(C,h)$ represents continuous, locally non-satiated, nonconvex preferences over consumption and leisure, but let (C^*,h^*) defined above lie inside the boundary of the convex hull of $\{(C,h) : U(C,h) \geq U(C^*,h^*)\}$. Then for (C',h') such that $(C',h') = \arg\max\{U(C,h) : C \leq w^*h + y^*\}$, $(C',h') \neq (C^*,h^*)$.*

Proof. Since preferences are nonconvex, and (C^*, L^*) lies inside the boundary of the convex hull of $\{(C,h) : U(C,h) \geq U(C^*,h^*)\}$, then $\exists$ (C'',h'') such that $C'' < w^*h'' + y^*$ and $U(C'',h'') = U(C^*,h^*)$. Then, by local nonsatiation, $\exists$ (C''',h''') such that $C''' = w^*h''' + y^*$ and $U(C''',h''') > U(C^*,h^*)$. Hence, $(C^*,h^*) \neq \arg\max\{U(C,h) : C \leq w^*h + y^*\}$. ■

12 Appendix D

In this appendix, we formally show why the Hausman and MaCurdy methods may not be adapted to estimate preferences consistent with the maximization of nonconvex preferences.

12.1 Unadaptability of Hausman Method

In the following proposition we show that the algorithm implicit in the Hausman method used to identify desired hours on a nonlinear budget constraint may fail to yield the actual desired hours for individuals with nonconvex preferences. As a result, the likelihood in (17) will be misspecified.

Again, let a piecewise linear budget constraint be characterized by a set $\{w_j, y_j\}$, $j = 1..N$, where w_j is the after tax wage rate for hours of work between kink points H_{j-1} and H_j, and y_j is the associated virtual income, and denote S_j as the segment of the budget constraint between H_{j-1} and H_j. When budget constraints are convex, the Hausman method uses an algorithm that derives the desired hours of work as follows. Let $h(w,y)$ denote the labor supply correspondence derived by maximizing a utility function $U(C,h)$ subject to a linear budget constraint $C \leq wh + y$. Then, denote Hausman desired hours of work, h^H, as the desired hours of work that are derived through the following algorithm:

$$h^H = \begin{cases} h(w_j,y_j) \text{ if } h(w_j,y_j) \in S_j & \text{for some } j \\ H_j \text{ if } h(w_j,y_j) > H_j \text{ and } h(w_{j+1},y_{j+1}) < H_j & \text{for some } j \end{cases} \tag{53}$$

Let h^* be an element of the set of solutions to

$$h^* \in \{\arg\max U(C,h) \text{ s.t. } C \leq y + \sum_{j=1}^{J}(1-t_j)w \left[\begin{array}{c} (H_j - H_{j-1}) \cdot 1(h \geq H_j) \\ +(h - H_{j-1}) \cdot 1(H_j > h \geq H_{j-1}) \end{array} \right] \quad (54)$$

That is, h^* is the hours of work chosen by a utility maximizing agent faced with a piecewise linear budget constraint. The Hausman method utilizes the idea that, if preferences are strictly convex, then $h^H = h^*$. As a result, given parameters and stochastic elements, the likelihood that the sample is generated by the utility maximization in (54) is identical to the likelihood that the sample is generated by people using the Hausman algorithm to choose their desired hours of work in (53).

However, the following proposition demonstrates that if preferences are nonconvex, then the desired hours generated by the Hausman algorithm, h^H, are not always equal to the actual desired hours, h^*.

Proposition 3. *Let $U(C,h)$ represent nonconvex preferences, and derive $h(w,y)$ by maximizing $U(C,h)$ subject to the budget constraint $C \leq wh + y$. Let h^* be the hours that maximize $U(C,h)$ on a piecewise linear budget constraint. If h^* occurs (1) on the interior of segment S_j and (2) on the interior of the convex hull of the upper contour set of the indifference curve, then $h^H \neq h^*$.*

Proof. Define C_1 and h_1 such that $U(C_1,h_1) = U(C^*,h^*)$, where

$$h_1 = \arg\max_h \left\{ h : (C,h) \in \begin{array}{c}\text{boundary of} \\ \text{convex hull}\end{array} \{(C,h) : U(C,h) \geq U(C^*,h^*)\};\ h < h^* \right\} \quad (55)$$

and define C_2 and h_2 such that $U(C_2,h_2) = U(C^*,h^*)$, where

$$h_2 = \arg\min_h \left\{ h : (C,h) \in \begin{array}{c}\text{boundary of} \\ \text{convex hull}\end{array} \{(C,h) : U(C,h) \geq U(C^*,h^*)\};\ h > h^* \right\} \quad (56)$$

Again, h_1 and h_2 bracket h^*. (See Figure 6) Let $\overline{w} = \frac{C_2 - C_1}{h_2 - h_1}$. Since the tangency of the indifference curve and the actual budget constraint occurs on the interior of segment S_j and on the interior of the convex hull of the upper contour set of the indifference curve, then either $\overline{w} \in (w_j, w_{j+1})$, or $\overline{w} \in (w_{j-1}, w_j)$. If $\overline{w} \in (w_j, w_{j+1})$, then $h_j(w_j, y_j) > h_2 > H_j$, and $h_{j+1}(w_{j+1}, y_{j+1}) < h_1 < H_j \Longrightarrow h^H = H_j$. If $\overline{w} \in (w_{j-1}, w_j)$, then $h_{j-1}(w_{j-1}, y_{j-1}) > h_2 > H_{j-1}$, and $h_j(w_j, y_j) < h_1 < H_{j-1} \Longrightarrow h^H = H_{j-1}$. Since $h^* \in S_j = (H_{j-1}, H_j)$, $h^* \neq h^H$. ■

Refer back to Figure 6. In this case, $h(w_2, y_2)$ is clearly greater than H_2, and $h(w_3, y_3)$ is clearly less than H_2. Hence, the Hausman algorithm would yield Hausman desired hours $h^H = H_2$, when this is clearly not the actual optimal level of hours.[45]

[45] Although the above propositions deal with convex budget sets, the arguments apply equally well to the Hausman method used in nonconvex budget sets. In Hausman (1985), a generalization of previously used methods is presented, in which a nonconvex piecewise linear budget set is decomposed into a union of a finite number of convex budget sets. The desired hours of work on each of the convex budget sets is derived, and then the indirect utility at each of these choices is compared, to yield the desired hours of work on the actual nonconvex budget set. Since this method requires the use of the Hausman algorithm for convex budget sets described above, any problems in applying the Hausman algorithm to nonconvex preferences when the budget set is convex are present in the nonconvex budget set case as well.

Hence, the desired hours inferred by Hausman algorithm are not always equal to true desired hours when $h(w, y)$ is derived from the maximization of nonconvex preferences, regardless of how flexible the specification of $h\left(w, y\right)$. Since a likelihood function derived from the application of this algorithm will not calculate the correct likelihood for the sample, the Hausman method cannot be generalized to estimate parameters consistent with nonconvex preferences.

12.2 Unadaptability of MaCurdy Method

In this section, we show that the MaCurdy method also cannot be generalized to estimate parameters consistent with nonconvex preferences. MaCurdy et al. define a budget constraint as consisting of two functions, $w(h)$ and $y(h)$, where $w(h)$ is the slope of the budget constraint at hours of work h, and $y(h)$ is the virtual income associated with a linear budget constraint tangent to the actual budget constraint at hours of work h. They then note that if $h > 0$ and preferences are convex, then hours worked must satisfy the implicit equation $h = h^s(w(h), y(h), v)$, where $h^s(w(h), y(h), v)$ is the worker's choice of hours if he were faced with a linear budget constraint with slope $w(h)$ and virtual income $y(h)$, and v denotes individual heterogeneity. Derivation of the likelihood involves solving this implicit equation analytically for h as a function of v and other variables and parameters, and using the Implicit Function Theorem to transform the equation to $v = v^s\left(h, w(h), y(h)\right)$. The density of positive hours in the likelihood function then takes this function as an argument.

Now, however, suppose that $h^s(w(h), y(h), v)$ is derived from the maximization of nonconvex preferences represented by the utility function $U(C, h^s; v)$ subject to the budget constraint $C \leq w(h)h^s + y(h)$.[46] The following proposition demonstrates that when preferences are nonconvex, the above implicit equation may fail to hold, and so the likelihood function cannot be derived in the manner described above.

Corollary 2. *Let $U(C, h; v)$ represents continuous, locally non-satiated, nonconvex preferences over consumption and hours of work, and let (C^*, h^*) be the utility maximizing levels of consumption and hours of work on the nonlinear budget constraint. Let (C^*, h^*) lie inside the boundary of the convex hull of $\{(C, h) : U(C, h; v) \geq U(C^*, h^*; v)\}$. Then $h^* \neq h^s(w(h^*), y(h^*), v)$.*

Proof. Note that $h^s(w(h^*), y(h^*), v) = \arg\max_h \{U(C, h, v) : C \leq w(h^*)h + y(h^*)\}$. Applying Proposition 2 yields the result. ■

To understand the intuition behind this corollary, see Figure 9. In this figure, the wage and virtual income associated with the linear budget constraint tangent to the differentiable budget constraint at h^* are $w(h^*)$ and $y(h^*)$. The optimal hours of work on this linear budget constraint, however, is $h^s \neq h^*$.

Thus, the method in MaCurdy et al. also cannot be adapted to allow estimated parameters be consistent with nonconvex preferences.

[46]Since $U(C, h^s; v)$ represents nonconvex preferences, $h^s(w(h), y(h), v)$ will again likely be a complicated correspondence that is discontinuous in its arguments.

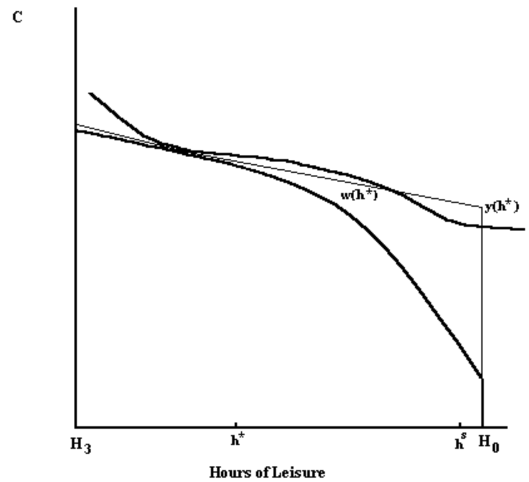

Figure 9: Graphical Demonstration of Corollary 2

13 Appendix E - Likelihood Functions

The assumed data generating process for each of the specifications that follow is described by the following. Individual i is assumed to solve

$$\begin{aligned} & \max U(C, h, v_i; \beta) \\ \text{s.t. } C \leq & \; B(h, W_i, Y_i). \end{aligned} \tag{57}$$

where $U(C_i, h_i, v_i; \beta)$ takes one of the forms outlined below. If U is everywhere increasing in C, then desired hours for individual i, given parameters β, may be written as

$$h_i^*(v_i; \beta) = \arg\max_{h_i} U(B(h_i, W_i, Y_i), h_i, v_i; \beta). \tag{58}$$

It is assumed that, for individuals that are observed to be working, we observe $h_i = h_i^*(v_i; \beta) + \varepsilon_i$, where ε_i denotes optimization or measurement error. It is further assumed (as in Hausman (1981)) that an individual is observed working 0 hours if desired hours are 0, or if desired hours are positive but ε_i is sufficiently negative so as to induce the individual not to work. Thus, observed hours are assumed to be related to desired hours in the manner

$$h_i = \begin{cases} h_i^*(v_i; \beta) + \varepsilon_i & if \quad h_i^*(v_i, \beta) > 0 \text{ and } h_i^*(v_i, \beta) + \varepsilon_i > 0 \\ 0 & if \quad \begin{cases} h_i^*(v_i, \beta) = 0 \\ \text{or } h_i^*(v_i, \beta) > 0 \text{ and } h_i^*(v_i, \beta) + \varepsilon \leq 0. \end{cases} \end{cases} \tag{59}$$

The assumed data generating process in the Hausman method utilizes an algorithm to simplify the expression in (58). Formally, let $h_{ij}(v_i, \beta)$ be the optimal labor supply if an individual with heterogeneity v_i were maximizing utility over consumption and hours of work, $U(C, h, v_i; \beta)$, subject to the budget constraint defined by $C = w_{ij}h + y_{ij}$. The particular specifications of $h_{ij}(v_i, \beta)$ are outlined in the discussion that follows. If $U(C, h, v_i; \beta)$ is strictly quasiconcave over $\left[0, \overline{H}\right]$, and the budget constraint is piecewise linear and convex, then the expression in (58) may be rewritten as

$$h_i^*(v_i,\beta) = \begin{cases} H_{i0} & if \quad h_{i1}(v_i,\beta) \leq H_{i0} \text{ (lower limit)} \\ h_{ij}(v_i,\beta) & if \quad H_{i,j-1} < h_{ij}(v_i,\beta) < H_{ij} \text{ (segment } j) \\ H_{ij} & if \quad h_{ij}(v_i,\beta) \geq H_{ij} \text{ and } h_{i,j+1}(v_i,\beta) \leq H_{ij} \text{ (kink } j) \\ H_{iJ} & if \quad h_{iJ}(v_i,\beta) \geq H_{iJ} \text{ (upper limit).} \end{cases} \tag{60}$$

Thus, in the Explicit Utility Maximization method, the assumed data generating process is described by (58) and (59), whereas in the Hausman method, the assumed data generating process is described by (59) and (60).

13.1 Linear Labor Supply

In the linear labor supply specification, the utility function is assumed to take the form

$$U(C,h,v_i;c,a,\beta) = \frac{1}{\beta}\left(h - \frac{\alpha}{\beta}\right)\exp - \left[1 + \frac{\beta\left(C + \frac{c+v_i}{\beta} - \frac{\alpha}{\beta^2}\right)}{\frac{\alpha}{\beta} - h}\right], \tag{61}$$

As such, $h_{ij}(v_i,\beta)$ takes the form

$$h_{ij}(v_i,c,\alpha,\beta) = c + aw_j + \beta y_j + v_i.$$

It is also assumed that $v_i \tilde{} N(0,\sigma_v^2)$ and $\varepsilon \tilde{} N(0,\sigma_\varepsilon^2)$.

Under these assumptions, letting

$$\widetilde{h}_{ij}(c,\alpha,\beta) = c + aw_j + \beta y_j,$$

the Hausman method likelihood of observing individual i working h_i hours is[47]

$$\begin{aligned} L(h_i|h_i > 0) = {} & \sum_{j=0}^{J-1} \frac{1}{\sigma_{\varepsilon v}} \phi\left(\frac{h_i - \widetilde{h}_{i,j+1}}{\sigma_{\varepsilon v}}\right) \left[\begin{array}{c} \Phi\left(\frac{\frac{H_{i,j+1}-\widetilde{h}_{i,j+1}}{\sigma_v} - \rho\frac{h_i-\widetilde{h}_{i,j+1}}{\sigma_{\varepsilon v}}}{\sqrt{1-\rho^2}}\right) \\ -\Phi\left(\frac{\frac{H_{ij}-\widetilde{h}_{i,j+1}}{\sigma_v} - \rho\frac{h_i-\widetilde{h}_{i,j+1}}{\sigma_{\varepsilon v}}}{\sqrt{1-\rho^2}}\right) \end{array} \right] \\ & + \sum_{j=1}^{J} \frac{1}{\sigma_\varepsilon} \phi\left(\frac{h_i - H_{ij}}{\sigma_\varepsilon}\right) \left[\Phi\left(\frac{H_{ij} - \widetilde{h}_{i,j+1}}{\sigma_v}\right) - \Phi\left(\frac{H_{ij} - \widetilde{h}_{ij}}{\sigma_v}\right)\right] \\ & + \frac{1}{\sigma_{\varepsilon v}} \phi\left(\frac{h_i - \widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v}}\right) \left[1 - \Phi\left(\frac{\frac{H_{iJ}-\widetilde{h}_{i,J+1}}{\sigma_v} - \rho\frac{h_i-\widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v}}}{\sqrt{1-\rho^2}}\right)\right]. \end{aligned} \tag{62}$$

[47] Note that this likelihood function is the one found in Moffitt (1986) with some corrections.

where $\sigma_{\varepsilon v}=\sqrt{\sigma_v^2+\sigma_\varepsilon^2}$ and $\rho=\sigma_v/\sigma_{\varepsilon v}$. The likelihood of observing individual i working 0 hours is

$$\begin{aligned}
L(h_i|h_i &= 0)=\Phi\left(\frac{-\widetilde{h}_{i1}}{\sigma_v}\right) \qquad (63)\\
&+\sum_{j=1}^{J}\left[\Phi\left(\frac{H_j-\widetilde{h}_{ij}}{\sigma_v},\frac{-\widetilde{h}_{ij}}{\sigma_{\varepsilon v}},\rho\right)-\Phi\left(\frac{H_{j-1}-\widetilde{h}_{ij}}{\sigma_v},\frac{-\widetilde{h}_{ij}}{\sigma_{\varepsilon v}},\rho\right)\right]\\
&+\sum_{j=1}^{J}\Phi\left(\frac{-H_j}{\sigma_\varepsilon}\right)\left[\Phi\left(\frac{H_j-\widetilde{h}_{i,j+1}}{\sigma_v}\right)-\Phi\left(\frac{H_j-\widetilde{h}_{i,j+1}}{\sigma_v}\right)\right]\\
&+\left[\Phi\left(\frac{-\widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v}}\right)-\Phi\left(\frac{H_J-\widetilde{h}_{i,J+1}}{\sigma_v},\frac{-\widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v}},\rho\right)\right].
\end{aligned}$$

where $\Phi(m,n,\rho)$ denotes the cumulative distribution function for standard bivariate normal variables with correlation ρ, cumulative over $(-\infty,m]\times(-\infty,n]$. The overall likelihood of observing the sample, then, is

$$\prod_{i=1}^{N}L(h_i|h_i>0)^{1(h_i>0)}L(h_i|h_i=0)^{1(h_i=0)}, \qquad (64)$$

where $1(\cdot)$ denotes the indicator function. In this specification, all probabilities in the likelihood are positive iff $\alpha-\beta H_j\geq 0\forall H_j$.

When estimating parameters of a direct utility function, the likelihood takes the form

$$L=\prod_i\left[\begin{array}{c}\left(\int\limits_{v_{i0}(\beta)}^{\infty}\frac{1}{\sigma_\varepsilon}\phi\left(\frac{h_i-h_i^*(v_i;\beta)}{\sigma_\varepsilon}\right)d\Phi(\frac{v_i}{\sigma_v})\right)^{1(h_i>0)}\\ \times\left(\Phi\left(\frac{v_{i0}(\beta)}{\sigma_v}\right)+\int\limits_{v_{i0}(\beta)}^{\infty}\Phi\left(\frac{-h_i^*(v_i;\beta)}{\sigma_\varepsilon}\right)d\Phi(\frac{v_i}{\sigma_v})\right)^{1(h_i=0)}\end{array}\right], \qquad (65)$$

where $v_{i0}(\beta)$ is defined as v_i s.t. $h_i^*(v_i;\beta)=0$. Since the integrals in (65) have no closed form solution, Gaussian quadrature techniques are used to evaluate the likelihood, in which the expression in (65) is replaced by a weighted sum of the form

$$L=\prod_{i=1}^{N}\sum_{k=1}^{K}w_k\left[\begin{array}{c}\left(\frac{1}{\sigma_\varepsilon}\phi(\frac{h_i-h_i^*(v_k;\beta)}{\sigma_\varepsilon})1\left(h_i^*(v_k;\beta)>0\right)\right)^{1(h_i>0)}\\ \times\left(1\left(h_i^*(v_k;\beta)\leq 0\right)+\Phi\left(\frac{-h_i^*(v_i;\beta)}{\sigma_\varepsilon}\right)1\left(h_i^*(v_k;\beta)>0\right)\right)^{1(h_i=0)}\end{array}\right], \qquad (66)$$

where $\{w_k\}_{k=1}^{K}$ are the weights and $\{v_k\}_{k=1}^{K}$ are the nodes at which the likelihood is evaluated, and $1(\cdot)$ denotes the indicator function. In the Monte Carlo experiments discussed in the paper, we use a ten node specification.

We solve for h_i^* using a bracketing algorithm.[48] Although bracketing algorithms are slow compared to other algorithms for well behaved functions, the advantage of such an algorithm

[48] For more info on bracketing algorithms, see Judd (1998), p. 95-96.

is that it does not require use of derivatives, and is guaranteed to find a maximum regardless of the curvature of the objective function.[49] Further, this problem is greatly simplified by the fact that this is a one dimensional maximization problem over a finite interval of the real line between 0 and $\overline{H}$.

13.2 CES Utility Specification

In the CES utility specification, the utility function is assumed to take the form

$$U(C,h,v_i;\eta,\rho,\overline{c},\overline{H}) = (1-\eta)(C-\overline{c}-v_i)^r + \eta(\overline{H}-h)^r. \tag{67}$$

As such, $h_{ij}(v_i,\beta)$ takes the form

$$h_{ij}\left(v_i,\eta,e,k,\overline{c},\overline{H}\right) = \overline{H} - \frac{y_j + w_j\overline{H} - \overline{c} - v_i}{w_j + kw_j^e},$$

where $e = \frac{1}{1-r}$ and $k = \left(\frac{\eta}{1-\eta}\right)^e$. It is further assumed that $v_i\tilde{}N(0,\sigma_v^2)$ and $\varepsilon\tilde{}N(0,\sigma_\varepsilon^2)$.

Under these assumptions, letting

$$\widetilde{h}_{ij} = \overline{H} - \frac{y_j + w_j\overline{H} - \overline{c}}{w_j + kw_j^e},$$

the Hausman method likelihood of observing individual i working h_i hours is

$$\begin{aligned} L(h_i|h_i > 0) = & \sum_{j=0}^{J-1}\frac{1}{\sigma_{\varepsilon v_j}}\phi\left(\frac{h_i-\widetilde{h}_{i,j+1}}{\sigma_{\varepsilon v_{j+1}}}\right)\left[\begin{array}{c}\Phi\left(\frac{\frac{H_{i,j+1}-\widetilde{h}_{i,j+1}}{\sigma_{v_{j+1}}}-\rho_j\frac{h_i-\widetilde{h}_{i,j+1}}{\sigma_{\varepsilon v_{j+1}}}}{\sqrt{1-\rho_{j+1}^2}}\right)\\ -\Phi\left(\frac{\frac{H_{ij}-\widetilde{h}_{i,j+1}}{\sigma_{v_{j+1}}}-\rho_{j+1}\frac{h_i-\widetilde{h}_{i,j+1}}{\sigma_{\varepsilon v_{j+1}}}}{\sqrt{1-\rho_{j+1}^2}}\right)\end{array}\right] \\ & +\sum_{j=1}^{J}\frac{1}{\sigma_\varepsilon}\phi\left(\frac{h_i-H_{ij}}{\sigma_\varepsilon}\right)\left[\Phi\left(\frac{H_{ij}-\widetilde{h}_{i,j+1}}{\sigma_{v_{j+1}}}\right)-\Phi\left(\frac{H_{ij}-\widetilde{h}_{ij}}{\sigma_{v_j}}\right)\right] \\ & +\frac{1}{\sigma_{\varepsilon v_{J+1}}}\phi\left(\frac{h_i-\widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v_{J+1}}}\right)\left[1-\Phi\left(\frac{\frac{H_{iJ}-\widetilde{h}_{i,J+1}}{\sigma_{v_{J+1}}}-\rho\frac{h_i-\widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v_{J+1}}}}{\sqrt{1-\rho_{J+1}^2}}\right)\right]. \end{aligned} \tag{68}$$

where $\sigma_{v_j} = \frac{\sigma_v}{w_j+kw_j^e}$, $\sigma_{\varepsilon v_j} = \sqrt{\sigma_{v_j}^2+\sigma_\varepsilon^2}$ and $\rho = \sigma_{v_j}/\sigma_{\varepsilon v_j}$. The likelihood of observing

[49] Of course, it is not guaranteed to find a global maximum if the utility function is not strictly quasiconcave or the budget constraint is not convex. When this method is applied to more general preferences and budget constraints, multiple starting points must be used in order to be confident that the maximum that the algorithm finds is indeed the global maximum.

individual i working 0 hours is

$$\begin{aligned} L(h_i|h_i &= 0) = \Phi\left(\frac{-h_{i1}}{\sigma_{v_1}}\right) \\ &+\sum_{j=1}^{J}\left[\Phi\left(\frac{H_j-\widetilde{h}_{ij}}{\sigma_{v_j}},\frac{-\widetilde{h}_{ij}}{\sigma_{\varepsilon v_j}},\rho_j\right)-\Phi\left(\frac{H_{j-1}-\widetilde{h}_{ij}}{\sigma_{v_j}},\frac{-\widetilde{h}_{ij}}{\sigma_{\varepsilon v_j}},\rho_j\right)\right] \\ &+\sum_{j=1}^{J}\Phi\left(\frac{-H_j}{\sigma_\varepsilon}\right)\left[\Phi\left(\frac{H_j-\widetilde{h}_{i,j+1}}{\sigma_{v_{j+1}}}\right)-\Phi\left(\frac{H_j-\widetilde{h}_{i,j}}{\sigma_{v_j}}\right)\right] \\ &+\left[\Phi\left(\frac{-\widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v_{J+1}}}\right)-\Phi\left(\frac{H_J-\widetilde{h}_{i,J+1}}{\sigma_{v_{J+1}}},\frac{-\widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v_{J+1}}},\rho_{J+1}\right)\right]. \end{aligned} \tag{69}$$

where $\Phi(m,n,\rho)$ denotes the cumulative distribution function for standard bivariate normal variables with correlation ρ, cumulative over $(-\infty,m]\times(-\infty,n]$. The overall likelihood of observing the sample, then, is

$$\prod_{i=1}^{N} L(h_i|h_i>0)^{1(h_i>0)}L(h_i|h_i=0)^{1(h_i=0)}, \tag{70}$$

where $1(\cdot)$ denotes the indicator function. In this specification, all probabilities in the likelihood are positive iff $(w^e_{j+1}-w^e_j)\left(H_j-\overline{H}\right)\geq 0\ \forall\ _j$. In the usual case where $e>0$ and $w_{j+1}<w_j$ we have $\overline{H}\geq H_j\ \forall\ _j$.

When estimating parameters of a direct utility function, the likelihood takes a form analogous to the expressions in (65) and (66).

13.3 Quadratic Utility Specification

In the quadratic utility specification, the utility function is assumed to take the form

$$U(C,h,v_i;\beta,\gamma,\delta)=\beta h^2+\gamma Ch+(\delta+v_i)h+C, \tag{71}$$

where $\beta,\gamma<0$. As such, $h_{ij}(v_i,\beta)$ takes the form

$$h_{ij}(v_i,\beta,\gamma,\delta)=\frac{-w_j-\gamma y_j-(\delta+v_i)}{2(\gamma w_j+\beta)}.$$

It is further assumed that $v_i\tilde{}N(0,\sigma_v^2)$ and $\varepsilon\tilde{}N(0,\sigma_\varepsilon^2)$.

Under these assumptions, letting

$$\widetilde{h}_{ij}=\frac{-w_j-\gamma_j y_j-\delta}{2(\gamma w_j+\beta)},$$

the Hausman method likelihood of observing individual i working h_i hours is

$$
\begin{aligned}
L(h_i|h_i &> 0) = \sum_{j=0}^{J-1} \frac{1}{\sigma_{\varepsilon v_j}} \phi\left(\frac{h_i - \widetilde{h}_{i,j+1}}{\sigma_{\varepsilon v_{j+1}}}\right) \left[\begin{array}{c} \Phi\left(\frac{\frac{H_{i,j+1}-\widetilde{h}_{i,j+1}}{\sigma_{v_{j+1}}} - \rho_j \frac{h_i - \widetilde{h}_{i,j+1}}{\sigma_{\varepsilon v_{j+1}}}}{\sqrt{1-\rho_{j+1}^2}}\right) \\ -\Phi\left(\frac{\frac{H_{ij}-\widetilde{h}_{i,j+1}}{\sigma_{v_{j+1}}} - \rho_{j+1} \frac{h_i - \widetilde{h}_{i,j+1}}{\sigma_{\varepsilon v_{j+1}}}}{\sqrt{1-\rho_{j+1}^2}}\right) \end{array}\right] \qquad (72) \\
&+ \sum_{j=1}^{J} \frac{1}{\sigma_\varepsilon} \phi\left(\frac{h_i - H_{ij}}{\sigma_\varepsilon}\right) \left[\Phi\left(\frac{H_{ij} - \widetilde{h}_{i,j+1}}{\sigma_{v_{j+1}}}\right) - \Phi\left(\frac{H_{ij} - \widetilde{h}_{ij}}{\sigma_{v_j}}\right)\right] \\
&+ \frac{1}{\sigma_{\varepsilon v_{J+1}}} \phi\left(\frac{h_i - \widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v_{J+1}}}\right) \left[1 - \Phi\left(\frac{\frac{H_{iJ}-\widetilde{h}_{i,J+1}}{\sigma_{v_{J+1}}} - \rho \frac{h_i - \widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v_{J+1}}}}{\sqrt{1-\rho_{J+1}^2}}\right)\right].
\end{aligned}
$$

where $\sigma_{v_j} = \frac{\sigma_v}{|2(\gamma w_j + \beta)|}$, $\sigma_{\varepsilon v_j} = \sqrt{\sigma_{v_j}^2 + \sigma_\varepsilon^2}$ and $\rho = \sigma_{v_j}/\sigma_{\varepsilon v_j}$. The likelihood of observing individual i working 0 hours is

$$
\begin{aligned}
L(h_i|h_i &= 0) = \Phi\left(\frac{-\widetilde{h}_{i1}}{\sigma_{v_1}}\right) \qquad (73) \\
&+ \sum_{j=1}^{J} \left[\Phi\left(\frac{H_j - \widetilde{h}_{ij}}{\sigma_{v_j}}, \frac{-\widetilde{h}_{ij}}{\sigma_{\varepsilon v_j}}, \rho_j\right) - \Phi\left(\frac{H_{j-1} - \widetilde{h}_{ij}}{\sigma_{v_j}}, \frac{-\widetilde{h}_{ij}}{\sigma_{\varepsilon v_j}}, \rho_j\right)\right] \\
&+ \sum_{j=1}^{J} \Phi\left(\frac{-H_j}{\sigma_\varepsilon}\right) \left[\Phi\left(\frac{H_j - \widetilde{h}_{i,j+1}}{\sigma_{v_{j+1}}}\right) - \Phi\left(\frac{H_j - \widetilde{h}_{i,j}}{\sigma_{v_j}}\right)\right] \\
&+ \left[\Phi\left(\frac{-\widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v_{J+1}}}\right) - \Phi\left(\frac{H_J - \widetilde{h}_{i,J+1}}{\sigma_{v_{J+1}}}, \frac{-\widetilde{h}_{i,J+1}}{\sigma_{\varepsilon v_{J+1}}}, \rho_{J+1}\right)\right].
\end{aligned}
$$

where $\Phi(m, n, \rho)$ denotes the cumulative distribution function for standard bivariate normal variables with correlation ρ, cumulative over $(-\infty, m] \times (-\infty, n]$. The overall likelihood of observing the sample, then, is

$$\prod_{i=1}^{N} L(h_i|h_i > 0)^{1(h_i>0)} L(h_i|h_i = 0)^{1(h_i=0)}, \qquad (74)$$

where $1(\cdot)$ denotes the indicator function. In this specification, all probabilities in the likelihood are positive iff $1 + \gamma H_j \geq 0 \forall H_j$.

When estimating parameters of a direct utility function, the likelihood takes a form analogous to the expressions in (65) and (66).

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