Equilibrium Search and Tax Credit Reform*

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Abstract

This paper develops an empirical equilibrium job search model with wage posting, and uses it to analyse the labour market impact of the British Working Families' Tax Credit (WFTC) reform. The model allows for a rich characterisation of the labour market, with hours responses, accurate representations of the tax and transfer system, and both worker and firm heterogeneity. Firms differ in their productivity, while workers differ in their opportunity costs of employment as well as their transitional parameters and the tax schedule that they face. All workers compete within the same labour market, and the arrival rates of job offers is endogenized by complementing the model with aggregate matching functions. The model is estimated with pre-reform longitudinal survey data using a semi-parametric estimation technique, and the impact of actual tax reform policies is simulated. The model predicts that WFTC and the contemporaneous reforms increased employment, with lone mothers experiencing the largest increase. In the main simulations, equilibrium effects are found to play a relatively minor role in this reform; the changes to employment and earnings are primarily due to the direct effect of changing job acceptance behaviour. It is also demonstrated that the equilibrium effects of the same reforms may be much larger if a labour market solely comprised of lone mothers, one of the main beneficiaries of WFTC, is considered.

Keywords: Labour market equilibrium, job search, wage dispersion, unemployment, monopsony, incidence, tax credits

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1 Introduction

Over the past two decades earned income tax credit programmes have grown substantially in the UK, US and many other countries.[1] These programmes are typically motivated by a desire from policy makers to increase labour market participation among target groups, and to alleviate in-work poverty. While the effect of these policies on labour supply has been studied extensively, much less is known regarding the incidence of these tax credit programmes and the broader general equilibrium impact. The objective of this paper is to develop an empirical equilibrium job search model that provides us with an appropriate framework to address these issues, and to apply it in our analysis of the British Working Families' Tax Credit (WFTC) reform.

Tax credit programmes, such as EITC in the US and WFTC (together with its predecessors and recent successor) in the UK, have received considerable attention in the economics literature. The employment impacts of these programmes have been evaluated in a number of empirical studies, which have typically adopted either a difference-in-differences (see for example, Eissa and Liebman, 1996) or structural discrete choice model approach (for example, Blundell et al., 2000). While the magnitudes of the estimated or simulated impacts do differ somewhat across these studies, there appears to be a general consensus that tax credit programmes like WFTC have had positive effects on the employment of lone mothers, while there is somewhat less agreement concerning the effect of these reforms on men and women in couples.[2]

Despite the wealth of labour supply studies, there is surprisingly little evidence regarding the equilibrium impact of these reforms. Exceptions include the recent tax credit incidence studies of Azmat (2006b), Leigh (2009), and Rothstein (2008, 2009). Using data from the federal expansion in the mid-1990s, Rothstein (2008) examined the effect of the US EITC on labour market participation and equilibrium wages. Using his central elasticity estimates, and assuming a distinct labour market for single women, he simulated a $1 expansion in EITC payments and found that pre-tax earnings of eligible workers fell by $0.30 while the earnings of ineligible workers fell by $0.43.

[1] See Hotz and Scholz (2003) for EITC in the US, and Blundell and Hoynes (2004) for the British WFTC and its predecessors.

[2] Studies which have evaluated the employment impact of WFTC include Azmat (2006a), Blundell et al. (2004a), Blundell and Shephard (2009), Brewer et al. (2006), Francesconi and van der Klaauw (2004), Gregg and Harkness (2003), and Leigh (2007). These will be discussed in more detail in Section 5.5.

Leigh (2009) exploited state variation in EITC supplements over the period 1989–2002 to examine the effect of the tax credit on wages, and found that a one percent increase in the generosity of the programme reduced gross hourly wages by 0.5% for high school dropouts, 0.2% for those with a high school diploma, and had no effect on college graduates. Azmat (2006b) examined the incidence of the British WFTC and found evidence to suggest that firms discriminate by cutting the wage of claimant workers relative to similarly skilled non-claimant workers, and that there are spill-over effects on the wages of similarly skilled non-eligible workers.

The policy context of this paper is the UK earned income tax credit reform in October 1999, when the government replaced the existing Family Credit (FC) programme – the main form of in-work support for lower income families with children – with Working Families' Tax Credit. While WFTC represented a continued expansion of in-work programmes of support, it was considerably more generous than its predecessor, offering higher credits and a lower withdrawal/taper rate. Since the presence of dependent children is a central eligibility criteria for the receipt of WFTC, families without children (who act as a control group in the quasi-experimental employment impact studies) may also find themselves affected by the reform, if say, wages were to adjust. Consequently, the government's view regarding the desirability of such reforms – and more generally how such programmes should be designed – may depend crucially upon the nature and quantitative importance of these equilibrium effects.

In order to address these important issues we require a model in which both employment and the distribution of wages emerges as an equilibrium outcome. While a competitive model of the labour market is able to generate wage dispersion if individuals differ in their marginal productivity of labour, and may also generate equilibrium price effects following adjustments in labour supply (see for example, the model presented in Rothstein, 2008), there is empirical evidence that suggests that labour markets are characterised by substantial search frictions (see for example, van den Berg and Ridder, 2003). Departing from the competitive paradigm may have important implications for our understanding of tax credit reforms. In particular, if firms set wages then the presence of search frictions gives firms some degree of monopsony power. Thus, one possible equilibrium effect of the reform is that firms may attempt to extract a greater share of the rent by lowering the wages that they offer. If this mechanism is important, then the effective transfer to eligible individuals may be reduced, while non-eligible workers may be made worse off.

The equilibrium job search literature allows us to capture these and other effects in a dynamic and imperfectly competitive economy that is characterized by search frictions.[3] Here it is the competition between firms that is the fundamental determinant of wages, with the extent of this competition limited by the presence of search frictions. We consider a model with ex-ante wage posting – firms set wages before meeting potential workers, which workers can then either accept or reject. Manning (2003) argues that while wage posting is not appropriate in all contexts, it is likely to provide a good characterisation of wage determination in many applications. This is likely to be particularly true when we are focussing on low-skilled labour markets, as we examine in this paper. Hall and Krueger (2008) present recent US survey evidence which suggests that while other forms of wage formation are also important, wage posting is much more prevalent in less skilled occupations (see also the discussion in Manning, 2003, chapter 5).[4]

It is instructive to consider briefly the conditions under which a dispersed offer distribution may be supported in an equilibrium with wage posting. Diamond (1971) showed that if workers are homogeneous, and workers are unable to search while employed, then the equilibrium wage offer distribution would collapse to a degenerate distribution equal to the common reservation wage. The literature offers two standard ways of generating wage dispersion as an equilibrium outcome. The first is to assume that workers themselves are heterogeneous in their opportunity cost of employment (Albrecht and Axell, 1984; Eckstein and Wolpin, 1990). Since this ex-ante heterogeneity translates into heterogeneity in reservation wages, firms now face a trade-off between offering a low wage and just attracting workers with low reservation wages, or offering

[3]Our analysis remains partial equilibrium to the extent that capital is ignored, as is the product market and the possible effect of the reform on outcomes such as education and fertility. However, the model is equilibrium in the sense that employment, the distribution of wages, and the arrival rate of job offers are determined within the model.

[4]Lise et al. (2005) simulate the general equilibrium effect of a wide scale implementation of the Canadian Self-Sufficiency Project (SSP) in a model with ex-post worker-firm bargaining. They find substantial general equilibrium effects, which reverse the positive cost-benefit conclusions of their partial equilibrium evaluation. Kolm and Tonin (2006) consider the impact of introducing an in-work benefit in an analytical framework using a Pissarides (2000) search model. Here general equilibrium effects reinforce the employment impact of the programme through job creation.

a high wage and attracting workers with both low and high reservation wages. In equilibrium firms will be indifferent between offering these wages, so that a dispersed wage distribution may emerge.

An alternative approach that generates wage dispersion as an equilibrium outcome maintains the homogeneous worker assumption, but allow workers to continue searching for jobs when employed (Burdett and Mortensen, 1998). For employed workers, their reservation wage will equal their current wage. This therefore generates ex-post heterogeneity among ex-ante identical agents so that wage dispersion can be supported in equilibrium. If all firms are equally productive, then the model implies increasing wage offer and earnings densities. This apparent empirical failing can be overcome to some extent by allowing for heterogeneity in firm productivity. Building upon the analysis of Mortensen (1990), which demonstrated that more productive firms offer higher wages, Bontemps et al. (2000) allowed for a continuous distribution of firm productivity and showed that the model can induce empirical wage distributions by allowing for an appropriately skewed distribution of firm productivity.

To our knowledge, this is the first time that an empirical equilibrium job search model has been used to analyse a policy reform of this kind. Our analysis advances the existing literature in several dimensions, with our model designed to capture key features of the UK labour market and the possibility of rich equilibrium effects following reforms such as WFTC. Firstly, since one of the objectives of the reform was to increase labour supply, it is necessary to have a model with reservation wage heterogeneity so that at least some workers may accept some wage offers and reject others. Such a model was presented in Bontemps et al. (1999) which allowed for continuous distributions of firm productivity and worker opportunity costs with the restriction that the job arrival rate is independent of employment status. This over-identifying restriction simplifies the analysis as it implies that the optimal strategy of unemployed workers is independent of the equilibrium wage offer distribution. This restriction led to a poor fit of the duration data in their application, as empirically job arrival rates for unemployed workers are often estimated to exceed that of the employed. Relaxing this job arrival rate restriction will be an important part of our analysis.

A pertinent feature of the UK labour market is that certain groups of individuals – most notably women with children – work part-time at some point in their lifetime. Since the presence of children is a central eligibility criteria for receipt of WFTC, it is therefore necessary to incorporate hours into our model. While the use of the canonical labour supply model may be pervasive, there is a body of empirical work that challenges the view that individuals are able to freely choose their hours of work at a fixed hourly wage (Altonji and Paxson, 1988; Lundberg, 1985; Martinez-Granado, 2005; Moffitt, 1984; Stewart and Swaffield, 1997). Blundell et al. (2008) use British Household Panel Survey data to study the impact of a series of in-work benefit reforms in the UK during the 1990s, and found that the introduction of WFTC had positive effects on hours worked, but this increase was largely driven by women who changed job.

That jobs sequentially arrive as wage-hours packages is an assumption that will be maintained throughout this paper. This is important not only in its ability to describe that apparent lack of hours flexibility within jobs, but also in its ability to potentially capture the so-called part-time penalty. Manning and Petrongolo (2008) document recent British evidence for female workers. We do not attempt to provide rich micro-foundations for this, but rather assume it is a purely technological description of firms – firms are able to offer either full-time jobs, or part-time jobs, but not both.

Within this framework we incorporate detailed representations of the UK tax and transfer system. Since we are interested in how WFTC affected different groups of workers, we allow for demographic heterogeneity to influence the key structural parameters of our model as well as the tax and transfer schedules. Crucially – and in contrast to the segmented markets approach adopted by van den Berg and Ridder (1998) – we assume that workers of all types operate within the same labour market. This allows us to capture rich equilibrium effects and to study possible unintended consequences of the tax credit reform within our model.

We develop a three step semi-non-parametric estimation technique similar to that proposed by Bontemps et al. (1999, 2000), and estimate our model using UK Labour Force Survey data shortly before WFTC was introduced. Using the structural parameters we then simulate the equilibrium impact of WFTC, holding the structural parameters and the distribution of firm productivity fixed. This allows us to investigate the impact of the reform on employment, hours of work, unemployment durations, and the entire distribution of wages. Recognizing the role that firms may play as both wage setters and job creators, we follow Mortensen (2000) by complementing our model with aggregate matching functions, which then allows us to endogenize rate at which workers encounter job offers at the macroeconomic level. We find that the introduction of WFTC, together with other accompanying changes to the tax and transfer system between April 1997 and April 2002, increased employment for most groups, with lone mothers experiencing the largest increase. Our main simulations suggest that while equilibrium considerations do play a role in this particular reform, the changes in employment and earnings are dominated by the direct effect of changing job acceptance behaviour. We also show how the type of equilibrium effects considered have the potential to be much more important for tax reforms which are less targeted. We demonstrate that the equilibrium effects of the same reforms may be much larger if we consider a labour market solely comprised of lone mothers, one of the main beneficiaries of WFTC.

The paper proceeds as follows. Section 2 briefly describes the WFTC reform, while Section 3 presents our model and describes the optimal strategies of firms and workers. In Section 4 we describe the estimation and identification of our model, derive the likelihood function and discuss our data and estimation results. In Section 5 we present our main simulation exercises and compare our predictions to the actual employment changes observed over the relevant period, as well as the findings from other studies. Finally, Section 6 concludes.

2 The Working Families' Tax Credit reform

The UK has a long history of in-work benefits, starting with the introduction Family Income Support (FIS) in 1971. Over the years, these programmes became more generous, and in October 1999, Working Families' Tax Credit was introduced, replacing a similar, but less generous, tax credit programme called Family Credit. Both WFTC and FC shared a similar eligibility structure, requiring recipients work for at least 16 hours per week, and with the credit tapered away with household earnings above a threshold. Both also offered a further credit when recipients worked

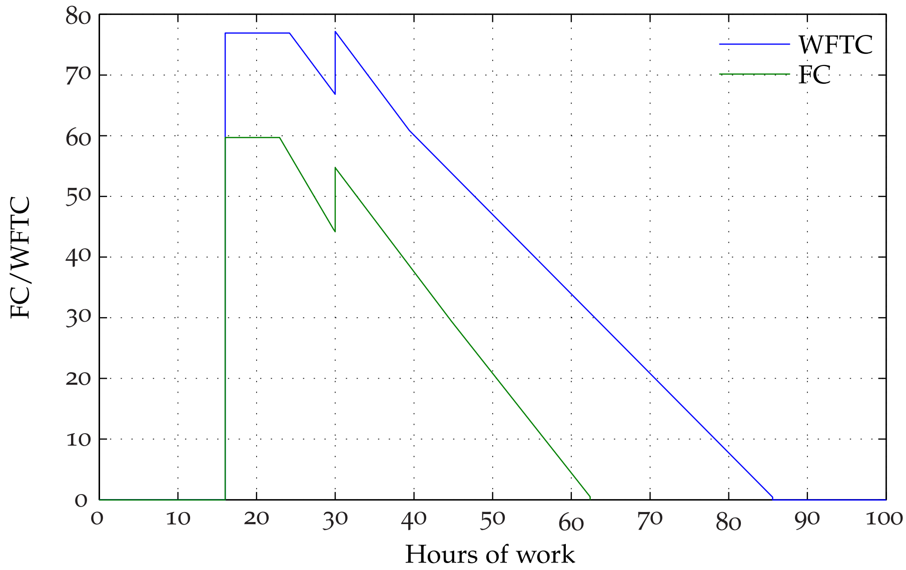


Figure 1: Tax Credit awards under FC and WFTC. FC refers to Family Credit as of April 1997. WFTC refers to Working Families' Tax Credit as of April 2002. Figure assumes a lone parent with a single child aged 10, and a constant hourly wage rate of £3.50. All incomes expressed in April 1997 prices.

at least 30 hours a week. There were essentially five ways in which WFTC increased the level of in-work support relative to the FC system: (i) it offered higher credits, especially for families with younger children; (ii) the increase in the threshold meant that families could earn more before it was phased out; (iii) the tax credit withdrawal rate was reduced from 70% to 55%; (iv) it provided more support for formal childcare costs through a new childcare credit; (v) all child maintenance payments were disregarded from income when calculating tax credit entitlement. These changes meant that the largest potential beneficiaries of WFTC were those families who were just at the end of the FC benefit withdrawal taper (see Figure 1). The combined increases in generosity and caseload resulted in nominal spending on WFTC rising to £4.6 billion by 2000/1 and £6.3 billion by 2002/3 compared to £2.4 billion on FC in 1998/9. Table 1 presents the main parameters of FC and WFTC.[5]

When analysing the effect of programmes such as WFTC it is necessary to take an integrated view of the tax system. This is because tax credit income is counted as income when calculating

[5]There were also important administrative changes between FC and WFTC. In particular, while FC was paid direct to recipients as a cash benefit, WFTC was typically paid by employers through the wage packet.

Table 1: Parameters of FC/WFTC

	April 1999 (FC)	October 1999 (WFTC)	June 2000 (WFTC)	June 2002 (WFTC)
Basic Credit	49.80	52.30	53.15	62.50
Child Credit				
under 11	15.15	19.85	25.60	26.45
11 to 16	20.90	20.90	25.60	26.45
over 16	25.95	25.95	26.35	27.20
30 hour credit	11.05	11.05	11.25	11.65
Threshold	80.65	90.00	91.45	94.50
Taper rate	70% after income tax and National Insurance	55% after income tax and National Insurance	55% after income tax and National Insurance	55% after income tax and National Insurance
Childcare	Expenses up to £60 (£100) for 1 (more than 1) child under 12 disregarded when calculating income	70% of total expenses up to £100 (£150) for 1 (more than 1) child under 15	70% of total expenses up to £100 (£150) for 1 (more than 1) child under 15	70% of total expenses up to £135 (£200) for 1 (more than 1) child under 15

Notes: All monetary amounts are in pounds per week and expressed in nominal terms. Minimum tax credit award is 50p per week in all years above.

entitlements to other benefits. Families in receipt of such benefits would gain less from the WFTC reform than otherwise-equivalent families not receiving these benefits. Furthermore, there were other notable changes to the tax system affecting families with children that coincided with the expansion of tax credits, therefore making the overall labour market impact more difficult to predict. In particular, there were increases in the generosity of Child Benefit (a cash benefit available to all families with children regardless of income), as well as notable increases in the child additions in Income Support (a welfare benefit for low income families working less than 16 hours a week). For many families with children, these increases in out-of-work income meant that despite the increased generosity of in-work tax credits, replacement rates remained relatively stable. There were also changes to the tax and transfer system that affected families both with and without dependent children during the lifetime of WFTC: a new 10% starting rate of income tax was introduced; the basic rate of income tax was reduced from 23% to 22%; there was a real rise in the point at which National Insurance (payroll tax) becomes payable.[6]

[6]There were also important non-tax related reforms over this period, which we do not consider in our analysis.

3 The model

This section lays out the theoretical model that we use to study the impact of the WFTC reform. We begin by setting out the core assumptions, and derive the optimal job acceptance strategies of workers. We then characterise the steady state flows and close the model by deriving the optimal wage setting and job creating behaviour of firms. A summary of the subsequent notation used is presented in Appendix C.

3.1 Model assumptions

The economy consists of a continuum of individuals with a population size normalized to unity. These individuals may differ with regards to observable demographic characteristics (for example, gender, marital status and the presence of children) that are finite in number and indexed by $i \in \mathcal{I}$. In our analysis these will be treated as being fixed and time invariant, and we let n_i denote the fraction of type i individuals, with $\sum_i n_i = 1$. As shall become clear, while all such individuals will operate in the same labour market, these demographic characteristics will be allowed to influence both the tax and transfer system (reflecting the conditioning performed by the UK tax authorities) as well as the transitional parameters of the model. Time is continuous and individuals live forever with the constant discount rate $\rho_i > 0$. There is no access to either saving or borrowing technology. These individuals (or workers) can be either employed or unemployed, and we use e and u to index these respective states. Jobs are characterised by a wage rate w and required hours of work h. We assume that the required hours of work within a job is a technological characterisation of firms, with firms offering either part-time jobs (hours h_0) or full-time jobs (hours $h_1 > h_0$), but not both.[7] The particular hours sector that firms operate in is

Various "New Deal" programmes were introduced which aimed to improve both the incentives and the ability of the long-term unemployed to obtain employment (see Blundell et al., 2004b). Furthermore, a national minimum wage was introduced in April 1999, at a rate of £3.00 per hour for those aged 18-21 (the development rate) and at the higher rate of £3.60 per hour for those aged 22 and over. Since their introduction, both the main and the development rates have been subject to a number of above-inflation increases.

[7] The framework we develop generalizes to more than two hours choices, and can also be applied in the context of other non-wage amenities. See Hwang et al. (1998) for an analysis of non-wage amenities in an equilibrium search framework.

exogenously determined.[8] Mirroring the conditioning that is performed by the UK government, hours can directly affect the tax schedule, with $T_i^h(wh)$ denoting the (potentially negative) net taxes paid by a employed worker with observable characteristics i at earnings wh and hours sector $h \in \{0, 1\}$. Similarly, the net transfer paid to an unemployed worker with characteristics i is given by $-T_i^u$. We assume that the tax schedule $T_i^h(wh)$ is continuously differentiable.

There is an exogenous distribution of firm productivity in both part-time and full-time sectors. The cumulative distribution of part-time firm productivity is given by Γ_0 on support $[\underline{p}_0, \overline{p}_0]$ while the cumulative distribution of full-time firm productivity is Γ_1 on support $[\underline{p}_1, \overline{p}_1]$. While workers are assumed to be equally productive at a given firm, they differ in their unobserved opportunity cost of employment b which has the cumulative distribution function H_i on support $[\underline{b}_i, \overline{b}_i]$. To simplify some of the exposition we assume that $\underline{b}_i$ is sufficiently low so that in equilibrium all firms are active in the labour market. In the presences of taxes, the flow utility of an unemployed individual is given by $b - T_i^u$, whereas for employed workers flow utility is assumed linear in net-income and a monetary disutility of work: $wh - T_i^h(wh) - C_i^h$, with C_i^h denoting the monetary disutility of work at hours h. From the outset we shall impose the location normalization $C_i^0 = 0$ for all i.

Individuals encounter part-time (full-time) job offers at the exogenous (to the worker) rate λ_{ui}^0 (λ_{ui}^1) when unemployed, and λ_{ei}^0 (λ_{ei}^1) when employed. To maintain focus on the decisions faced by workers, we postpone any discussion concerning how these arrival rates may depend upon the overall state (or tightness) of the labour market, but return to this issue in Section 3.5.[9] Employment spells end at rate δ_i regardless of whether individuals are employed in part-time of full-time jobs. Here and throughout this paper, we place no restrictions on the relative magnitude of these quantities, but note that the assumption that the job destruction and job arrival rates when employed are independent of whether individuals are currently engaged in

[8]Note that we are implicitly assuming an indivisibility in the production technology; two part-time workers are not a substitute for a single full-time worker.

[9]A more realistic approach would also endogenize the job offer arrival rates λ_{ji}^h at the micro-level by relating them directly to an endogenously determined worker search effort, as in Christensen et al. (2005). In our analysis we do not allow the search effort of workers to vary with their current wage or to directly respond to any changes in the tax system. Non-hours labour supply responses to tax reforms, such as search effort, may be quantitatively important as emphasized by the new-tax responsiveness literature (Feldstein, 1995). However, incorporating this into our model is non-trivial and is left as an extension for future research.

part-time or full-time work should be considered an over-identifying restriction.

Regardless of observed characteristics i or unobserved characteristics b, all workers are assumed to sample part-time (full-time) wage offers from the distribution F_0 (F_1), with the lowest wage offer denoted $\underline{w}_0$ ($\underline{w}_1$) and the highest wage offer $\overline{w}_0 \geq \underline{w}_0$ ($\overline{w}_1 \geq \underline{w}_1$). This implies that, while the government may be able to condition taxes and transfers on demographic characteristics i, firms are unable to do so. We justify this assumption by the presence of anti-discrimination laws, such as the Equal Pay Act 1970, Sex Discrimination Act 1975, and various Employment Equality Regulations, which outlaw such practices. Note that this assumption does not imply that workers of all types will have the same distribution of earnings. This is because workers may differ in their job acceptance behaviour, and may also gravitate towards higher paying jobs at different rates. As noted in the introduction, assuming that workers operate in the same labour market permits interesting spill-over effects, so that workers who are not targeted by a particular tax reform (in our case, childless families) can still be indirectly affected by it.

We assume that there is wage posting: employers post wages prior to forming matches with potential employees, who can then either accept or reject the wage offer. Furthermore, firms may post job vacancies that affect the rate at which these firms encounter workers. Wages are assumed constant throughout an individual's employment spell within a given firm. In contrast to the papers surveyed above, jobs are no longer completely characterized by the wage that they offer, so the behaviour of individuals will now be summarized by a slightly more complicated reservation wage strategy which will depend on the required hours of work. These strategies are discussed in the following section.

3.2 Worker strategies

In this section we describe the optimal strategies of unemployed and employed workers; formal derivations are presented in Appendix A. To proceed we define $q_i(w)$ such that the value to a type i individual holding a full-time job paying wage w is the same as the value of a part-time job paying wage $q_i(w)$. Since the job destruction rate and the arrival rates for both full-time and part-time jobs are assumed independent of current hours of work, it can be shown that in order to determine $q_i(w)$ it sufficient to compare the instantaneous utility flows between part-time and full-time employment. In other words $q_i(w)$ solves:

$$wh_1 - T_i^1(wh_1) - C_i^1 = q_i(w)h_0 - T_i^0(q_i(w)h_0) \tag{1}$$

which has a unique solution provided that conditional on hours of work marginal tax rates are always strictly less than one (an assumption that we maintain throughout), in which case we have $dq_i(w)/dw > 0$. So while employed workers with wage w will accept any wage offer w' from their current hours sector that is (by convention) strictly greater than their current wage, a full-time worker would find a part-time job offer acceptable if and only if $q_i(w') > w$. Similarly, a part-time worker would find such a full-time wage offer acceptable if and only if $q_i^{-1}(w') > w$.

Unemployed workers also follow a reservation wage strategy, that will again vary depending on whether or not a full-time or part-time offer is received. We let $\phi_i(b)$ denote the reservation wage for *full-time* work conditional on observed type i and unobserved leisure cost b. This takes a similar form to the standard reservation wage equation with on-the-job search (see, for example, Mortensen and Neumann, 1988), but is here modified both by the presence of taxes and because workers are now sampling offers from two distributions; in particular, taxes act to discount future earnings by the net-of-tax rate. In Appendix A we show that $\phi_i(b)$ is the solution to:

$$\phi_i(b)h_1 - T_i^1(\phi_i(b)h_1) - C_i^1 = b - T_i^u + h_1 \int_{\phi_i(b)}^{\infty} B_i(w)dw \tag{2}$$

where:

$$B_i(w) \equiv \frac{\left(1 - T_i^{1\prime}(wh_1)\right)\left[(\kappa_{ui}^0 - \kappa_{ei}^0)\overline{F}_0(q_i(w)) + (\kappa_{ui}^1 - \kappa_{ei}^1)\overline{F}_1(w)\right]}{1 + \rho_i/\delta_i + \kappa_{ei}^0\overline{F}_0(q_i(w)) + \kappa_{ei}^1\overline{F}_1(w)}$$

and where $\kappa_{ji}^h = \lambda_{ji}^h/\delta_i$, with $j \in \{u, e\}$ and $h \in \{0, 1\}$. From our discussion above, it follows that the lowest acceptable wage offer for a part-time job is simply given by $q_i(\phi_i(b))$. Before we proceed note that in the case where $\kappa_{ui}^h = \kappa_{ei}^h$ for $h \in \{0, 1\}$, we have $B_i(w) = 0$ for all (w, i) so that the optimal strategy of unemployed workers is independent of the equilibrium wage offer distributions. This is the case analysed in Bontemps et al. (1999). For reference, we summarize

Table 2: Worker acceptance strategies

	Full-time offer	Part-time offer
Unemployed	$w' \geq \phi_i(b)$	$w' \geq q_i(\phi_i(b))$
Employed, full-time	$w' > w$	$w' > q_i(w)$
Employed, part-time	$w' > q_i^{-1}(w)$	$w' > w$

Notes: Assumes a current wage w and either a full-time or part-time wage offer w'. $q_i(w)$ is as defined in equation 1; $\phi_i(b)$ is as defined in equation 2.

the job acceptance strategy of all individuals in Table 2.

3.3 Steady state flows

This section derives a number of steady state objects by using flow equations. For now, we treat the distributions of wage offers and their arrival rates as being given, but will later show how they emerge as an equilibrium outcome.

3.3.1 Distribution of reservation wages

In deriving steady state flows it is necessary to consider the distribution of (full-time) reservation wages for each demographic group (denoted A_i) which is related to the distribution of work opportunity costs according to $A_i(w) = H_i(\phi_i^{-1}(w))$. We denote the distribution of full-time reservation wages amongst the stock of unemployed and employed workers as A_{ui} and A_{ei} respectively, and these are related to A_i according to:

$$A_i(\phi) = u_i A_{ui}(\phi) + (1 - u_i) A_{ei}(\phi). \tag{3}$$

In order to describe the equilibrium of the labour market, it is necessary to determine the distribution of reservation wages amongst the unemployed A_{ui}, so that we are able to describe the flows from the unemployment pool into employment at a given wage. This also allows us to determine the equilibrium unemployment rate. In steady state we require that the flow of layoffs must exactly equal the flow out of the unemployment pool to either part-time or full-time employment. Since unemployed workers accept any wage that is at least as high as their reservation wage we have,

$$\delta_i(1-u_i)A_{ei}(\phi) = \lambda^0_{ui}u_i \int_{-\infty}^{\phi} \overline{F}_0(q_i(w))dA_{ui}(w) + \lambda^1_{ui}u_i \int_{-\infty}^{\phi} \overline{F}_1(w)dA_{ui}(w). \tag{4}$$

Notice that the wage offer distributions (F_0 and F_1) are the only quantities in the above that do not vary with observable worker type i. By differentiating equation 4 we obtain a relationship between the densities of employed and unemployed worker reservation wages, which when combined with equation 3 allows us to derive the following expression for the distribution of ϕ amongst the unemployed:

$$u_i A_{ui}(\phi) = \int_{-\infty}^{\phi} \frac{dA_i(w)}{1+\kappa^0_{ui}\overline{F}_0(q_i(w)) + \kappa^1_{ui}\overline{F}_1(w)}. \tag{5}$$

If we define $\underline{w}_i \equiv \min\{\underline{w}_1, q_i^{-1}(\underline{w}_0)\}$ and $\overline{w}_i \equiv \max\{\overline{w}_1, q_i^{-1}(\overline{w}_0)\}$, and let $\phi \to \infty$ in equation 5 then we obtain the following expression for the steady-state unemployment rate for workers of a given demographic type:

$$u_i = \frac{1}{1+\kappa^0_{ui}+\kappa^1_{ui}}A_i(\underline{w}_i) + \int_{\underline{w}_i}^{\overline{w}_i} \frac{dA_i(w)}{1+\kappa^0_{ui}\overline{F}_0(q_i(w)) + \kappa^1_{ui}\overline{F}_1(w)} + (1-A_i(\overline{w})) \tag{6}$$

which decomposes the unemployment rate into three parts; those individuals who accept all job offers, those who accept some and reject others, and those who reject all job offers.

3.3.2 Between jobs

In what follows we need to consider the fraction of workers currently employed in part-time and full-time jobs, and denote these respective quantities as m_{0i} and m_{1i} so that $m_{0i} + m_{1i} = 1 - u_i$. We also let $G_{1i}(w)$ denote the fraction of full-time workers of type i with a wage no greater than w, and similarly define $G_{0i}(w)$. We shall now determine these objects. Since individuals will only accept jobs that offer a value strictly greater than the value associated with their current wage-hours package, the number of individuals who leave a full-time job paying wage w (whether through their job being destroyed, or receiving a higher valued offer from a part-time or full-time firm) must exactly equal the number of individuals who accept such a job. Hence,

$$m_{1i}g_{1i}(w)\left[\delta_i + \lambda_{ei}^0\overline{F}_0(q_i(w)) + \lambda_{ei}^1\overline{F}_1(w)\right] \\ = f_1(w)\left[\lambda_{ui}^1 u_i A_{ui}(w) + \lambda_{ei}^1 m_{0i}G_{0i}(q_i(w)) + \lambda_{ei}^1 m_{1i}G_{1i}(w)\right]. \quad (7)$$

Similarly, we can derive an analogous expression for part-time jobs paying wage w:

$$m_{0i}g_{0i}(w)\left[\delta_i + \lambda_{ei}^0\overline{F}_0(w) + \lambda_{ei}^1\overline{F}_1(q_i^{-1}(w))\right] \\ = f_0(w)\left[\lambda_{ui}^0 u_i A_{ui}(q_i^{-1}(w)) + \lambda_{ei}^0 m_{0i}G_{0i}(w) + \lambda_{ei}^0 m_{1i}G_{1i}(q_i^{-1}(w))\right]. \quad (8)$$

Both equations 7 and 8 will also feature later when we discuss identification and present our semi-non-parametric estimation technique. Integrating equation 7 and equation 8 (the LHS of both equations are integrated by parts) we obtain the following alternative representations of these flow equations:

$$m_{1i}G_{1i}(w)\left[\delta_i + \lambda_{ei}^0\overline{F}_0(q_i(w)) + \lambda_{ei}^1\overline{F}_1(w)\right] + \lambda_{ei}^0 m_{1i}\int_{-\infty}^{w} G_{1i}(x)dF_0(q_i(x)) \\ = \lambda_{ui}^1 u_i \int_{-\infty}^{w} A_{ui}(x)dF_1(x) + \lambda_{ei}^1 m_{0i}\int_{-\infty}^{w} G_{0i}(q_i(x))dF_1(x) \quad (9)$$

and:

$$m_{0i}G_{0i}(w)\left[\delta_i + \lambda_{ei}^0\overline{F}_0(w) + \lambda_{ei}^1\overline{F}_1(q_i^{-1}(w))\right] + \lambda_{ei}^1 m_{0i}\int_{-\infty}^{w} G_{0i}(x)dF_1(q_i^{-1}(x)) \\ = \lambda_{ui}^0 u_i \int_{-\infty}^{w} A_{ui}(q_i^{-1}(x))dF_0(x) + \lambda_{ei}^0 m_{1i}\int_{-\infty}^{w} G_{1i}(q_i^{-1}(x))dF_0(x). \quad (10)$$

While expressions for G_{0i} and G_{1i} from above are both individually complicated, an appropriately weighted average term, that features prominently in our analysis, has a considerably simpler form. To derive this we add equation 9 (evaluated at w) and equation 10 (evaluated at $q_i(w)$) so that we are able to eliminate the terms where we integrate over the cross-sectional distributions of part-time and full-time wages. This delivers the following average condition:

$$\begin{aligned}(m_{1i}G_{1i}(w) + m_{0i}G_{0i}(q_i(w))) &\left[\delta_i + \lambda_{ei}^0 \overline{F}_0(q_i(w)) + \lambda_{ei}^1 \overline{F}_1(w)\right] \\ &= \lambda_{ui}^0 u_i \int_{-\infty}^{w} A_{ui}(x) dF_0(q_i(x)) + \lambda_{ui}^1 u_i \int_{-\infty}^{w} A_{ui}(w) dF_1(x). \quad (11)\end{aligned}$$

Using equation 4 we note that the RHS of equation 11 can be written as:

$$\begin{aligned}\delta_i(1-u_i)A_{ei}(w) - u_i A_{ui}(w) &\left[\lambda_{ui}^0 \overline{F}_0(q_i(w)) + \lambda_{ui}^1 \overline{F}_1(w)\right] \\ &= \delta_i A_i(w) - u_i A_{ui}(w) \left[\delta_i + \lambda_{ui}^0 \overline{F}_0(q_i(w)) + \lambda_{ui}^1 \overline{F}_1(w)\right] \quad (12)\end{aligned}$$

so by combining equation 12 with equation 11 and dividing through by δ_i we then obtain:

$$\begin{aligned}(m_{1i}G_{1i}(w) + m_{0i}G_{0i}(q_i(w))) &\left[1 + \kappa_{ei}^0 \overline{F}_0(q_i(w)) + \kappa_{ei}^1 \overline{F}_1(w)\right] \\ &= A_i(w) - u_i A_{ui}(w) \left(1 + \kappa_{ui}^0 \overline{F}_0(q_i(w)) + \kappa_{ui}^1 \overline{F}_1(w)\right) \quad (13)\end{aligned}$$

which can be substituted into equations 7 and 8 to eliminate the averaged earnings distributions and yield expressions for the earnings densities g_{0i} and g_{1i}. These may then be integrated to obtain the respective cumulative distribution functions (G_{0i} and G_{1i}) and employment shares (m_{0i} and m_{1i}).

3.4 Firm behaviour

In this section we characterize the optimal behaviour of firms. In addition to choosing a wage policy, we allow firms to exercise a role as job creators. The opening of job vacancies allows firms of a given productivity level to increase their visibility in the labour market, with changes in the supply of job vacancies affecting the arrival rate of job offers at the macroeconomic level through an aggregate matching function (see Section 3.5).

The flow cost of a full-time productivity p firm with v job vacancies is given by $c_1(p, v)$. We assume that this function is strictly convex in v with $c_1(p, 0) = 0$. The profit flow of a firm with strategy (w, v) is given by $(p - w)h_1 L_1(w, v) - c(p, v)$ where $L_1(w, v) \equiv \sum_i n_i l_{1i}(w, v)$ is the steady-state employment of such a firm, and where $l_{1i}(w, v)$ is the steady-state employment of a type i worker. Letting V_1 denote the aggregate stock of full-time job vacancies, $l_{1i}(w, v)$ solves the flow equation:

$$(\delta_i + \lambda_{ei}^0 \overline{F}_0(q_i(w)) + \lambda_{ei}^1 \overline{F}_1(w)) l_{1i}(w, v) = \frac{v}{V_1} \left[\lambda_{ui}^1 u_i A_{ui}(w) + \lambda_{ei}^1 m_{0i} G_{0i}(q_i(w)) + \lambda_{ei}^1 m_{1i} G_{1i}(w) \right]$$

which balances the number of workers who enter and exit employment, and reflects the assumption that vacancies affect the sampling probability of firms. Given that v enters the above expression for $l_{1i}(w, v)$ multiplicatively, it is convenient in what follows to write $l_{1i}(w, v) = \bar{l}_{1i}(w) v / V_1$, and similarly let $\overline{L}_1(w) \equiv \sum_i n_i \bar{l}_{1i}(w)$, with:

$$\bar{l}_{1i}(w) = \frac{\kappa_{ei}^1 A_i(w) + \left[\kappa_{ui}^1 (1 + \kappa_{ei}^0 \overline{F}_0(q_i(w))) - \kappa_{ei}^1 (1 + \kappa_{ui}^0 \overline{F}_0(q_i(w))) \right] u_i A_{ui}(w)}{\left(1 + \kappa_{ei}^0 \overline{F}_0(q_i(w)) + \kappa_{ei}^1 \overline{F}_1(w)\right)^2} \tag{14}$$

which is obtained by substituting equation 13 in the flow equation for $l_{1i}(w, v)$ to eliminate the cross-sectional wage distributions. Each full-time firm chooses its optimal wage policy $K_1(p)$ and optimal recruiting policy $v_1(p)$ to maximize its steady state profit flow,[10] taking the arrival rate of job offers, together with the behaviour of other firms (both part-time and full-time) and workers as given. Hence:

$$(K_1(p), v_1(p)) = \underset{(w,v)}{\arg\max}\ \pi_1(p, w) \frac{v}{V_1} - c_1(p, v)$$

where:

$$\pi_1(p, w) = (p - w) h_1 \overline{L}_1(w). \tag{15}$$

The optimal vacancy level $v_1(p)$ equates the marginal cost of an additional vacancy to the expected profit flow from such a vacancy. That is:

$$\left. \frac{\partial c_1(p, v)}{\partial v} \right|_{v = v_1(p)} = \frac{\pi_1(p, K_1(p))}{V_1} \tag{16}$$

[10]This implicitly assumes that firms have a zero rate of time preference.

Rather than working directly with the first order conditions for the optimal choice of wages, we use the envelope theorem to write $\pi_1(p) = \pi_1(p, K_1(p))$ as follows:

$$\begin{aligned} \pi_1(p) &= \pi_1^*(\underline{p}_1) + h_1 \int_{\underline{p}_1}^{p} \overline{L}_1(K_1(y))dy \\ &= (\underline{p}_1 - \underline{w}_1^*)\overline{L}_1(\underline{w}_1^*) + h_1 \int_{\underline{p}_1}^{p} \overline{L}_1(K_1(y))dy \end{aligned}$$

where $\underline{w}_1^*$ maximizes equation 15 for the lowest productivity full-time firm ($p = \underline{p}_1$). Setting the above equal to 15 (evaluated at $w = K_1(p)$) we obtain:

$$K_1(p) = p - \left[\pi_1^*(\underline{p}_1) + h_1 \int_{\underline{p}_1}^{p} \overline{L}_1(K_1(y))dy \right] \times \frac{1}{h_1 \overline{L}_1(K_1(p))} \tag{17}$$

which is a form that we exploit when we numerically solve for the equilibrium of our model. Of course, directly analogous expressions holds for the wage policy function and recruiting efforts of part-time firms. Note that equation 17 (and the corresponding expression for part-time firms) will depend upon the entire distribution of both part-time and full-time wage offers. Once these wage policy functions have been calculated, it is also necessary to verify whether the second-order conditions of firms in both sectors hold, so that the candidate equilibrium is indeed implementable.

3.5 Equilibrium

In order to close the model we endogenize the arrival rate of job offers by complementing it with aggregate matching functions as in Mortensen (2000, 2003). Here, the total flow of matches M_h in each sector $h \in \{0, 1\}$ depends on the total stock of vacancies V_h:

$$V_0 = \int_{\underline{p}_0}^{\overline{p}_0} v_0(p) d\Gamma_0(p) \quad \text{and} \quad V_1 = \int_{\underline{p}_1}^{\overline{p}_1} v_1(p) d\Gamma_1(p) \tag{18}$$

(with $v_h(p)$ given in equation 16) and the total intensity adjusted search effort of workers S_h:

$$S_h = \sum_i n_i \left[s_{ui}^h u_i + s_{ei}^h (1 - u_i) \right] \tag{19}$$

where s_{ji}^h denotes the exogenous search effort of workers for each i, $j \in \{u, e\}$ and $h \in \{0, 1\}$. The matching function in hours sector h is then written as:

$$M_h \left(V_h, \sum_i n_i \left[s_{ui}^h u_i + s_{ei}^h (1 - u_i) \right] \right) \tag{20}$$

where M_h is assumed to be increasing in both its arguments, concave, and linearly homogeneous. The job offer arrival rates for each worker are then related to the flows of matches according to:

$$\lambda_{ji}^h = \frac{s_{ji}^h M_h}{\sum_i n_i (s_{ui}^h u_i + s_{ei}^h (1 - u_i))}. \tag{21}$$

The market equilibrium of the economy is now defined in Definition 1.

Definition 1 *A market equilibrium in the economy is defined by $\{F_0, F_1, v_0, v_1\}$ such that simultaneously:*

1. *The arrival rates of job offers $\{\lambda_{ui}^0, \lambda_{ei}^0, \lambda_{ui}^1, \lambda_{ei}^1\}_{i \in \mathcal{I}}$ are given by equation 21*

2. *The distribution of wage offers in the economy is:*

$$F_0(K_0(p)) = \int_{\underline{p}_0}^{p} \frac{v_0(p) d\Gamma_0(p)}{V_0} \quad \textit{and} \quad F_1(K_1(p)) = \int_{\underline{p}_1}^{p} \frac{v_1(p) d\Gamma_1(p)}{V_1}$$

   with V_0 and V_1 as defined in equation 18.

3. *Each worker of type (b, i) follows the strategy described in Table 2.*

4. *The strategy of each type-p firm is to choose a vacancy level and wage that maximizes profits given*

the job offer arrival rates, strategies of other firms' and workers':

$$
\begin{aligned}
(K_1(p), v_1(p)) &= \arg\max_{(w,v)} \pi_1(p,w)\frac{v}{V_1} - c_1(p,v) \\
(K_0(p), v_0(p)) &= \arg\max_{(w,v)} \pi_0(p,w)\frac{v}{V_0} - c_0(p,v)
\end{aligned}
$$

where $\pi_h(p,w)$ is as defined in equation 15.

That the wage offer distributions (see part 2 of Definition 1) are equal to a vacancy weighted distribution of firm productivity follows from the observation that more productive firms pay higher wages in equilibrium. See for example, Mortensen (2003) for a simple proof.

4 Estimation

In this section we discuss the structural estimation of our model using longitudinal survey data. We first derive the log-likelihood function, and then proceed to discuss the identification of our model and the three step estimation procedure that we adopt. We then discuss our application of the UK tax and transfer system and the data used in estimation. Results are presented in Section 4.6.

4.1 Likelihood function

We now derive the likelihood contribution for individuals in different labour market positions, and with different initial transitions. The derivation closely follows that of Bontemps et al. (1999), and here we continue to use u and e to index the respective states of unemployment and employment, and 0 and 1 to denote part-time and full-time jobs. Note that we do not use any information beyond the first observed transition. In what follows elapsed and residual durations are given by:

$$
\begin{aligned}
t_{ub} &= \text{elapsed unemployment duration} \\
t_{uf} &= \text{residual unemployment duration} \\
d_{ub} &= \text{1 if unemployment duration left-censored, otherwise 0} \\
d_{uf} &= \text{1 if unemployment duration right-censored, otherwise 0} \\
t_{eb} &= \text{elapsed employment duration} \\
t_{ef} &= \text{residual employment duration} \\
d_{eb} &= \text{1 if employment duration left-censored, otherwise 0} \\
d_{ef} &= \text{1 if employment duration right-censored, otherwise 0}
\end{aligned}
$$

while earned and accepted wages are denoted as follows:

$$
\begin{aligned}
w_u &= \text{full-time wage accepted by unemployed individuals} \\
q_i(w_u) &= \text{part-time wage accepted by unemployed individuals} \\
d_u &= \text{1 if } w_u \text{ unobserved, otherwise 0} \\
w_e &= \text{full-time wage of employees at date of first interview} \\
q_i(w_e) &= \text{part-time wage of employees at date of first interview} \\
d_e &= \text{1 if } w_e \text{ unobserved, otherwise 0}
\end{aligned}
$$

current employment is indexed by:

$$
\begin{aligned}
h_e^0 &= \text{1 if employed work in the part-time sector, otherwise 0} \\
h_e^1 &= \text{1 if employed work in the full-time sector, otherwise 0}
\end{aligned}
$$

and initial transitions are indexed by:

$$\begin{aligned}
v_u^0 &= \text{1 if unemployed accept a part-time job, otherwise 0} \\
v_u^1 &= \text{1 if unemployed accept a full-time job, otherwise 0} \\
v_e^0 &= \text{1 if employed accept a part-time job, otherwise 0} \\
v_e^1 &= \text{1 if employed accept a full-time job, otherwise 0}
\end{aligned}$$

4.1.1 Unemployed workers

Using the above notation, we now derive the likelihood contribution for unemployed workers of type i. For unemployed workers who exit unemployment to a full-time job paying w_u, or a part-time job paying $q_i(w_u)$ (in either case we must have $d_u = 0$ and $d_{uf} = 0$), the likelihood contribution is given by:

$$\begin{aligned}
&(\lambda_{ui}^0 + \lambda_{ui}^1)^{2-d_{ub}} \exp\left[-\left(\lambda_{ui}^0 + \lambda_{ui}^1\right)(t_{ub} + t_{uf})\right] \frac{A_i(\underline{w}_i)}{1 + \kappa_{ui}^0 + \kappa_{ui}^1} \\
&\quad \times \frac{(\lambda_{ui}^0 f_0(q_i(w_u)))^{v_u^0} (\lambda_{ui}^1 f_1(w_u))^{v_u^1}}{\lambda_{ui}^0 + \lambda_{ui}^1} + \left\{ \int_{\underline{w}_i}^{w_u} (\lambda_{ui}^0 \overline{F}_0(q_i(b)) + \lambda_{ui}^1 \overline{F}_1(b))^{2-d_{ub}} \right. \\
&\qquad \times \exp\left[-\left(\lambda_{ui}^0 \overline{F}_0(q_i(b)) + \lambda_{ui}^1 \overline{F}_1(b)\right)(t_{ub} + t_{uf})\right] \\
&\qquad\quad \left. \times \frac{(\lambda_{ui}^0 f_0(q_i(w_u)))^{v_u^0} (\lambda_{ui}^1 f_1(w_u))^{v_u^1}}{\lambda_{ui}^0 \overline{F}_0(q_i(b)) + \lambda_{ui}^1 \overline{F}_1(b)} \underbrace{\frac{dA_i(b)}{1 + \kappa_{ui}^0 \overline{F}_0(q_i(b)) + \kappa_{ui}^1 \overline{F}_1(b)}}_{u_i dA_{ui}(b)} \right\}
\end{aligned}$$

where we integrate over the distribution of possible reservations wages in the above.

If we do not observe a wage accepted by the unemployed ($d_u = 1$), but we nonetheless have $d_{ub} + d_{uf} < 2$, then it still must be the case that the full-time reservation wage of such an individual is no greater than $\overline{w}_i$. It therefore follows their likelihood contribution is:

$$
\begin{aligned}
&(\lambda_{ui}^0+\lambda_{ui}^1)^{2-d_{ub}-d_{uf}} \exp\left[-(\lambda_{ui}^0+\lambda_{ui}^1)(t_{ub}+t_{uf})\right] \frac{A_i(\underline{w}_i)}{1+\kappa_{ui}^0+\kappa_{ui}^1} \\
&\qquad \times \left[\frac{(\lambda_{ui}^0)^{v_u^0}(\lambda_{ui}^1)^{v_u^1}}{\lambda_{ui}^0+\lambda_{ui}^1}\right]^{1-d_{uf}} + \left\{\int_{\underline{w}_i}^{\overline{w}_i} (\lambda_{ui}^0\overline{F}_0(q_i(b))+\lambda_{ui}^1\overline{F}_1(b))^{2-d_{ub}-d_{uf}}\right. \\
&\qquad\qquad \times \exp\left[-\left(\lambda_{ui}^0\overline{F}_0(q_i(b))+\lambda_{ui}^1\overline{F}_1(b)\right)(t_{ub}+t_{uf})\right] \\
&\qquad\qquad \times \left.\left[\frac{(\lambda_{ui}^0\overline{F}_0(q_i(b)))^{v_u^0}(\lambda_{ui}^1\overline{F}_1(b))^{v_u^1}}{\lambda_u^0\overline{F}_0(q_i(b))+\lambda_{ui}^1\overline{F}_1(b)}\right]^{1-d_{uf}} \frac{dA_i(b)}{1+\kappa_{ui}^0\overline{F}_0(q_i(b))+\kappa_{ui}^1\overline{F}_1(b)}\right\}.
\end{aligned}
$$

Finally, if we have both $d_u = 1$ and $d_{ub}+d_{uf} = 2$, then individuals are never observed in the employment state so we must also consider the probability that such individuals have a full-time reservation wage that is greater than $\overline{w}_i$. The likelihood contribution then becomes:

$$
\begin{aligned}
&\exp\left[-(\lambda_{ui}^0+\lambda_{ui}^1)(t_{ub}+t_{uf})\right] \frac{A_i(\underline{w}_i)}{1+\kappa_{ui}^0+\kappa_{ui}^1} \\
&\qquad + \left\{\int_{\underline{w}_i}^{\overline{w}_i} \exp\left[-\left(\lambda_{ui}^0\overline{F}_0(q_i(b))+\lambda_{ui}^1\overline{F}_1(b)\right)(t_{ub}+t_{uf})\right]\right. \\
&\qquad\qquad \times \left.\frac{dA_i(b)}{1+\kappa_{ui}^0\overline{F}_0(q_i(b))+\kappa_{ui}^1\overline{F}_1(b)} + (1-A_i(\overline{w}_i))\right\}.
\end{aligned}
$$

4.1.2 Employed workers

The likelihood contribution of a type i individual working full-time (part-time) at wage w_e $(q_i(w_e))$ is given by:

$$
\begin{aligned}
&\{m_{0i}g_{0i}(q_i(w_e))\}^{h_e^0}\{m_{1i}g_{1i}(w_e)\}^{h_e^1}\left[\delta_i+\lambda_{ei}^0\overline{F}_0(q_i(w_e))+\lambda_{ei}^1\overline{F}_1(w_e)\right]^{2-d_{eb}-d_{ef}} \\
&\qquad \times \exp\left[-\left(\delta_i+\lambda_{ei}^0\overline{F}_0(q_i(w_e))+\lambda_{ei}^1\overline{F}_1(w_e)\right)(t_{eb}+t_{ef})\right] \\
&\qquad\qquad \times \left[\frac{\delta_i^{1-v_e^0-v_e^1}(\lambda_{ei}^0\overline{F}_0(q_i(w_e)))^{v_e^0}(\lambda_{ei}^1\overline{F}_1(w_e))^{v_e^1}}{\delta_i+\lambda_{ei}^0\overline{F}_0(q_i(w_e))+\lambda_{ei}^1\overline{F}_1(w_e)}\right]^{1-d_{ef}}.
\end{aligned}
$$

Note that in the above we do not use any information on the wage accepted following a job-to-job transition. The reason for adopting such a limited information approach is that the model we have developed does not permit transitions associated with lower job values.[11] Finally, if the wage of an employed worker were missing ($d_e = 1$), then the likelihood contribution is simply given by m_{1i} if they are a full-time worker, m_{0i} if working part-time, or $1 - u_i$ if hours of work is also unobserved.[12]

4.2 Identification

To understand identification it is useful to first consider a special case of our model in the absence of a tax system, where the distribution of opportunity costs collapses to a degenerate distribution (i.e. workers are homogeneous), and where there is a single sector in the economy. This is the model analysed in Bontemps et al. (2000). Conditional on transitional parameters, identification of the wage offer distribution follows directly from a steady state relationship between the wage offer and earnings distributions (a simpler form of equation 9 presented here) that permits a simple inversion. Moreover, in such a setting all job offers will be accepted by all unemployed workers so that the accepted wage distribution will coincide with the wage offer distribution. This special case of our more general model is therefore over identified.

Regardless of its source, once we allow for heterogeneity in the reservation wage of unemployed workers the distribution of accepted wages will no longer equal the wage offer distribution. This is because workers are selective in the wages that they are willing to accept, so that the distribution of accepted wages (which will stochastically dominate the wage offer distribution) will be a convolution of two distributions. We are still able to establish non-parametric identification in this case because we observe as many distributions (starting wages and cross-sectional earnings) as distributions that we wish to recover. If we observe further distributions, such as the distribution of wages that the employed receive in their next job, then we once again will have

[11]The model can be extended to allow for job-to-job transitions associated with lower values by introducing a *reallocation shock* as in Jolivet et al. (2006). These shocks are draws from the wage offer distributions for which the only alternative to acceptance is to become unemployed. The presence of reservation wage heterogeneity would mean that some individuals may wish to exercise the unemployment option upon receiving such a shock.

[12]By construction of the data, if the hourly wage is known then so too is the hours of work.

over identification.[13] These ideas are presented more formally in Appendix B, and are closely related to the estimation procedure that we now present.

4.3 Three step estimation procedure

We estimate our model using a three step procedure similar to that proposed by Bontemps et al. (1999, 2000). While we no longer have a simple inversion between the observed earnings distributions and the unobserved wage offer distributions, it nonetheless remains possible to perform an inversion by iterating on the relevant flow equations. More specifically:

1. We estimate $\{\underline{w}_0, \overline{w}^0\}$ as the sample minimum and maximum values of w_e amongst part-time jobs ($h = h_0$) and $\{\underline{w}_1, \overline{w}^1\}$ as the sample minimum and maximum values of w_e amongst full-time jobs ($h = h_1$). None of these estimates condition upon worker type. We then calculate estimates of the unconditional earnings densities in each sector using non-parametric (kernel) techniques. We denote these estimated densities as $\hat{g}_0$ and $\hat{g}_1$.

2. We assume a parametric form for the distribution of opportunity costs $H_i(b)$ with a finite parameter vector $\theta = \{\theta_i\}_{i \in \mathcal{I}}$. Since workers are assumed to sample wage offers from the same distributions F_0 and F_1 regardless of their demographic type i, we weight equations 7 and 8 by n_i and sum across types to obtain appropriately averaged equations of the form:

$$f_1(w) = \frac{\sum_i n_i m_{1i} g_{1i}(w)}{\sum_i n_i \bar{l}_{1i}(w)} \tag{22}$$

and:

$$f_0(w) = \frac{\sum_i n_i m_{0i} g_{0i}(w)}{\sum_i n_i \bar{l}_{0i}(w)} \tag{23}$$

with $\bar{l}_{1i}(w)$ (defined in equation 14) and the similarly defined $\bar{l}_{0i}(w)$ both depending upon the set of transitional parameters, the reservation wage distribution, and the distribution of full-time and part-time wage offers. We then replace the numerators of equations 22

[13] This is related to the approach taken by Barlevy (2008) and Barlevy and Nagaraja (2006) who using record-value theory demonstrate identification of the wage offer distribution by tracking the wage growth of workers as a function of past mobility.

and 23 by $m_1\hat{g}_1(w)$ and $m_0\hat{g}_0(w)$ respectively, where here we have $m_1 = \sum_i n_i m_{1i}$ and $m_0 = \sum_i n_i m_{0i}$. To recover the part-time and full-time offer distributions that induce our estimates of the unconditional empirical earnings distributions, we provide an initial guess of f_0 and f_1 and then repeatedly (and simultaneously) iterate on these two equations, exploiting the conditional linearity seen above. At each iteration step we scale the densities by a normalization factor to ensure that we have proper distribution functions, and then verify that these normalization factors converge to 1. Conditional on the set of transitional parameters and distribution of opportunity costs, we then obtain consistent estimates of the offer distributions which we denote $\hat{F}_0$ and $\hat{F}_1$. These estimates (together with the corresponding density functions, $\hat{f}_0$ and $\hat{f}_1$) are then substituted into the likelihood function, and are also used to calculate the conditional employment shares and earnings densities: $u_i(\hat{F}_0, \hat{F}_1)$, $m_{hi}(\hat{F}_0, \hat{F}_1)$, and $g_{hi}(\hat{F}_0, \hat{F}_1)$ for all $i \in \mathcal{I}$ and $h \in \{0, 1\}$. Note that we are able to assess the fit of our model by its ability to explain differences in the part-time and full-time earnings distributions across demographic types through variation in the tax schedule, transitional parameters and opportunity cost distribution (see the discussion in Section 4.6).

3. Given a parametric form for the matching functions M_h and the vacancy cost functions $c_h(p, v)$, we obtain the implied distribution of firm productivity and the supply of job vacancies by first rewriting the first order conditions from the firms maximization problem as:

$$p_1 = K_1^{-1}(w_1) = w_1 + \overline{L}_1(w_1)/\overline{L}_1'(w_1)$$

$$p_0 = K_0^{-1}(w_0) = w_0 + \overline{L}_0(w_0)/\overline{L}_0'(w_0)$$

and then using equations 16, 20, and 21, together with the two relationships $F_1(K_1(p)) = \int_{\underline{p}_1}^{p} v_1(p)/V_1 d\Gamma_1(p)$ and $F_0(K_0(p)) = \int_{\underline{p}_0}^{p} v_0(p)/V_0 d\Gamma_0(p)$ from Definition 1. If the discount rate ρ_i is assumed known, then the distribution of opportunity costs H_i can then be recovered using equation 2.

We construct confidence intervals by bootstrapping the entire three stage estimation procedure. The advantages of this three step procedure versus a completely parametric approach, are essentially threefold. Firstly, it is considerably easier to perform this numerical inversion than it is to solve the full model at every evaluation of the likelihood function. Second, it permits greater flexibility than simple parametric forms for the productivity distribution. Thirdly, the likelihood function is continuous in all the parameters. Conversely, the main disadvantage of this approach compared to a completely parametric specification, is that it does not guarantee a monotonically increasing relationship between wages and productivity (in which case the empirical distribution of wages can not be an equilibrium outcome from our model), and in general it may not be possible to constrain the structural parameters of the model to achieve such monotonicity.

4.4 Applying the UK tax and transfer system

Our empirical application seeks to accurately represent the main features of the UK tax and transfer system so that we may consider the impact of the WFTC reform.[14] We allow for both income tax and National Insurance (payroll tax), council tax and council tax benefit (a local property based taxation and the corresponding benefit received by lower income families), Income Support and income-based Job-seekers Allowance, FC/WFTC, child benefit (a cash benefit available to all families with children irrespective of income), free school meals, and additional tax allowances given to couples and families with children. The underlying tax and transfer schedules are calculated prior to estimation using FORTAX (Shephard, 2009), and reflect the complex interactions between the tax and transfer system, varying accurately with earnings, hours of work and demographic characteristics.[15]

To economize on the number of groups that we need to consider in our analysis (and potentially structural parameters to estimate), we make a number of further assumptions regarding the set of demographic types $\mathcal{I}$. Specifically, we do not allow taxes and transfers to vary by the age of the claimant or by the age of any children. Taxes and transfers are calculated as if the claimant were at least 25 years old, and as if any children are aged 10 years. Families with more than two

[14]Recent surveys of the UK tax and transfer system are provided by Adam and Browne (2006) and O'Dea et al. (2007).

[15]A potentially important benefit that we do not consider is housing benefit. While this is modelled by FORTAX, the Labour Force Survey data that we use in our empirical application (see Section 4.5) does not contain any information on rents. Since tax credit income results in housing benefit entitlement being withdrawn, families in receipt of housing benefit would gain less from the tax credit reform than otherwise equivalent families not in receipt of housing benefit.

children are treated as there were only two children. Since some benefits have asset tests, we also assume that no families in our sample are affected by them. All families are assigned average band C council tax.[16]

The model developed in Section 3 assumed the presence of a single economic decision maker. This presents difficulties for our empirical application because benefits and in-work tax credits are assessed on family income in the UK. A complete treatment of couples is beyond the scope of this paper (see Guler et al., 2009 for a characterization of the reservation wage strategy of couples with income pooling). Rather than providing a detailed characterisation of the household decision making process, we take an admittedly limited approach, by conditioning upon the current employment status and (discretized) earnings of the individuals' partner. We then subsume partner earnings in the tax-schedule, but allow this tax schedule to accurately vary with the earnings of both individuals. In our empirical application we discretize the empirical distribution of partner earnings (conditional on gender and the presence and number of children) into ten groups, including non-employment (zero earnings); actual partner earnings are then replaced with those observed at either the 10$^{\text{th}}$, 20$^{\text{th}}$, ..., or 90$^{\text{th}}$ percentile point of the relevant empirical distribution.

The above categorization requires that we consider 64 different worker types in our empirical analysis. Conditional on hours of work, the resultant tax schedules for each of these groups i as a function of the wage rate will be a piecewise linear function, with possible discontinuities. We first remove these (small) discontinuities by appropriately modifying parameters of the tax and transfer system.[17] The modified marginal tax rate schedule for fixed hours is then approximated by a differentiable function using the method proposed by MaCurdy et al. (1990). The smoothed tax schedule is then obtained by integration.[18]

[16]The Labour Force Survey data does not contain information on the council tax band that households are subject to; band C is the most common band.

[17]Given it is not possible for working families to receive Income Support with our choice of discrete hours (see Section 4.5) this essentially involves setting the minimum payment for all benefits to zero, and starting employee National Insurance payments at slightly lower earnings to remove the entry fee discontinuity.

[18]With K tax brackets, the marginal tax rate approximation at hours h and earnings wh for a type i individual is given by $MTR_i^h(wh) = \sum_{k=1}^{K}[\Phi_{i(k)}^h(wh) - \Phi_{i(k+1)}^h(wh)]\tau_{i(k)}^h(wh)$, where $\tau_{i(k)}^h$ is the marginal tax rate at the k^{th} bracket and $\Phi_{i(k)}^h$ is the normal cumulative distribution function with a mean equal to the value of the k^{th} tax bracket and with variance σ_{ki}^2. The value of σ_{ki} determines how quickly the marginal rates change in the neighbourhood of the break points, with a small value fitting the underlying step function more closely. We set $\sigma_{ki} = 20$ which produces a relatively smooth tax schedule, but our results are not sensitive to this choice.

4.5 Data

We estimate our model using a sub-sample of the UK Labour Force Survey (LFS). The LFS is a quarterly survey of around 60,000 households in Great Britain, with these households followed for five successive quarters or "waves". When individuals first enter the survey they are in wave one, so that in any given quarter, there are roughly equal proportions of individuals in each interview wave. This rolling panel structure means that there is approximately an 80% overlap in the samples for successive quarters. The short panel dimension of the LFS is of some concern, as relatively few transitions and accepted wages are observed. While alternative panel data sets, such as the British Household Panel Survey (BHPS), provide us with a much more extensive panel, the resultant sample sizes are unfortunately too small, especially if we wish to capture demographic heterogeneity in our model.

The LFS provides us with very rich information concerning the respondents labour market status. Crucially, we observe employment status and spell durations, together with hours and earnings information (in the first and fifth waves since 1997) for workers. Our pre-reform estimation is performed using data shortly before WFTC was introduced. We follow individuals who are observed in the first quarter of 1997 until (at the latest) the first quarter of 1998. We calculate incomes and construct the likelihood function (see Section 4.1) as if individuals always faced the April 1997 system during this period so that the environment is stationary. While we may observe long elapsed spell durations, we nonetheless impose left censoring for durations greater than 24 months as it would be difficult to justify the assumption that they were generated from the same steady state.

We classify individuals as being employed if they have a job, and non-employed if they do not. Since we do not distinguish between the states of unemployment and non-participation, our definition of non-employment is therefore much broader than the standard ILO definition of unemployment. Amongst the employed, women who report working for less than 30 hours per week are classified as being part-time workers, while those working at least 30 hours per week classified as full-time workers. In our model, all part-time workers are treated as if they worked for 20 hours per week, whereas all full-time workers are treated as if they worked 40 hours per week. These hours points correspond well to the respective conditional averages. Empirically, very few men are observed to work part-time, so we treat all male workers as working 40 hours per week regardless of their reported hours of work.[19] In both cases, we calculate gross wages using the reported hours of work, but then proceed to calculate incomes as if they were working at the relevant discrete hours point.

Individuals who are aged below 21 or above 55 are excluded from our sample, as are individuals who are in receipt of disability related benefits, or are either self-employed or in full-time education. Given the assumption that workers are equally productive at any given firm, we additionally restrict our sample to those individuals whose highest qualification is O-level (or equivalent) or below, and assume that any higher educated individuals operate in a separate labour market. After sample selection, we have roughly 23,000 observations. Table 3 presents some summary statistics.

While the tax and transfer schedules potentially vary with each observable type $i \in \mathcal{I}$, we only allow the structural parameters of the model to vary with a subset of demographic types. For couples we do not allow the parameters to vary with the earnings and labour market status of their partner; for parents we do not allow them to vary with the number of their children. The distribution of work opportunity costs H_i is assumed to be Normally distributed, with mean μ_i and variance σ_i^2. This gives us 47 parameters to estimate.

4.6 Estimation Results and Model Fit

Given our maximum likelihood parameter estimates (Table 4), the implied wage policy functions $K_0(p)$ and $K_1(p)$ that are obtained from the first order conditions to the firms' profit maximization problem are found to be monotonically increasing so that the estimated empirical distribution of wages can be an equilibrium outcome from our model. That is, the theoretical model is not rejected by the data.[20] These wage policy functions are presented in Figure 2. The first notable

[19]The derivation of worker behaviour and the flow equations are less complicated when there is a single sector. Nonetheless, the relevant steady-state quantities and distributions can be obtained from our earlier exposition when the job arrival rates in the part-time sector approaches zero. Note also that with a single hours sector it is not possible to identify C_i^1 so we normalize it to be zero for these groups.

[20]Monotonicity is violated for a small proportion of the bootstrap samples. In order to construct bootstrap confidence intervals for the policy responses we therefore first apply a rearrangement procedure (see Chernozhukov et al.,

Table 3: Descriptive Statistics

	Unemployed				Employed						
	$\#N_u$	$u \to h_0$	$u \to h_1$	$\#w_u$	$\#N_e^0$	$\#N_e^1$	$e \to h_0$	$e \to h_1$	$e \to u$	$\#w_e^0$	$\#w_e^1$
single men	1481	–	135	72	–	2560	–	136	112	–	1573
married men, no kids	441	–	43	32	–	1931	–	67	49	–	1391
married men, kids	888	–	85	54	–	3077	–	131	108	–	2163
single women	1031	28	37	36	372	1828	11	72	65	254	1237
lone mothers	1793	85	16	73	676	408	31	14	62	488	306
married women, no kids	579	21	16	25	578	1215	15	47	62	442	916
married women, kids	1713	100	25	78	1444	808	59	35	115	1083	608

Notes: $\#N_u$ refers to the number of unemployed observations in a given category; $\#N_e^0$ and $\#N_e^1$ respectively refer to the number of part-time and full-time employment observations. $\#w_u$ refers to the number of observed accepted wages from unemployment; $\#w_e^0$ and $\#w_e^1$ refer to the number of cross-sectional wage observations in part-time and full-time employment. $i \to j$ refers to the numbers of observed transitions from state i to state j, with states u, e, h_0 and h_1, denoting unemployment, overall employment, part-time employment, and full-time employment respectively.

Table 3: (continued)

	Part-time wages							Full-time wages						
	P_{10}	P_{25}	P_{50}	P_{75}	P_{90}	mean	SD	P_{10}	P_{25}	P_{50}	P_{75}	P_{90}	mean	SD
single men	–	–	–	–	–	–	–	3.43	4.16	5.39	7.03	9.28	5.94	2.55
married men, no kids	–	–	–	–	–	–	–	4.02	4.93	6.32	8.29	10.74	6.97	2.90
married men, kids	–	–	–	–	–	–	–	3.74	4.90	6.34	8.46	11.12	6.97	2.99
single women	2.72	3.23	3.85	4.90	6.43	4.27	1.69	3.34	4.04	5.29	6.98	9.26	5.84	2.49
lone mothers	2.70	3.18	3.72	4.66	6.12	4.13	1.62	3.47	4.13	5.28	6.79	8.44	5.68	2.20
married women, no kids	2.81	3.37	3.95	4.96	6.45	4.35	1.66	3.40	4.14	5.09	6.55	8.48	5.63	2.24
married women, kids	2.87	3.37	4.00	5.20	6.77	4.46	1.69	3.31	4.00	5.03	6.45	8.06	5.45	2.03

Notes: All wages are hourly and are expressed in April 1997 prices. P_{10}, P_{25}, P_{50}, P_{75}, and P_{90} respectively refer to the 10^{th}, 25^{th}, 50^{th}, 75^{th}, and 90^{th} percentiles of the cross-sectional hourly wage distribution; SD refers to the standard deviation.

Table 4: Maximum Likelihood Structural Parameter Estimates

	$1/\delta_i$	$1/\lambda_{ui}^0$	$1/\lambda_{ui}^1$	$1/\lambda_{ei}^0$	$1/\lambda_{ei}^1$	μ_i	σ_i	C_i^1
single men	94.5	–	19.7	–	32.6	18.7	87.8	–
	[88.4,102.3]		[15.6,24.2]		[25.9,38.6]	[-12.7,35.5]	[57.9,135.2]	
married men, no kids	195.4	–	14.5	–	23.9	49.6	66.7	–
	[176.4,217.8]		[10.8,18.7]		[19.8,29.0]	[37.2,60.3]	[53.8,83.2]	
married men, kids	177.3	–	21.1	–	19.3	37.6	48.1	–
	[163.7,190.7]		[17.5,24.8]		[15.5,23.5]	[24.3,51.4]	[31.5,65.2]	
single women	141.9	42.5	38.8	117.2	54.2	-39.8	126.3	24.0
	[128.0,157.0]	[27.5,60.5]	[25.9,56.8]	[62.6,375.8]	[43.1,68.2]	[-156.7,-13.0]	[84.1,248.6]	[13.0,33.1]
lone mothers	66.1	54.0	337.7	118.4	55.2	41.7	28.9	37.0
	[60.1,72.6]	[43.0,81.5]	[188.1,664.7]	[74.1,230.9]	[41.3,72.5]	[36.3,45.7]	[12.9,41.7]	[33.2,42.1]
married women, no kids	171.8	23.4	68.0	147.8	74.7	4.7	71.6	36.7
	[154.2,192.2]	[16.5,32.9]	[39.1,133.1]	[100.4,250.4]	[60.3,92.2]	[-10.1,17.8]	[60.3,85.6]	[25.5,48.0]
married women, kids	99.4	29.2	280.5	37.8	115.6	36.8	36.8	27.7
	[92.1,106.4]	[23.4,35.9]	[174.9,416.5]	[31.0,46.0]	[93.6,135.9]	[33.1,39.9]	[31.9,41.5]	[23.6,34.2]

Notes: All durations are monthly. Incomes are measured in pounds per week in April 1997 prices. The distribution of work opportunity costs H_i is assumed to be Normal, with mean μ_i and variance σ_i^2. The 5^{th} and 95^{th} percentiles of the bootstrap distribution of parameter estimates are presented in brackets, and are calculated using 500 replications.

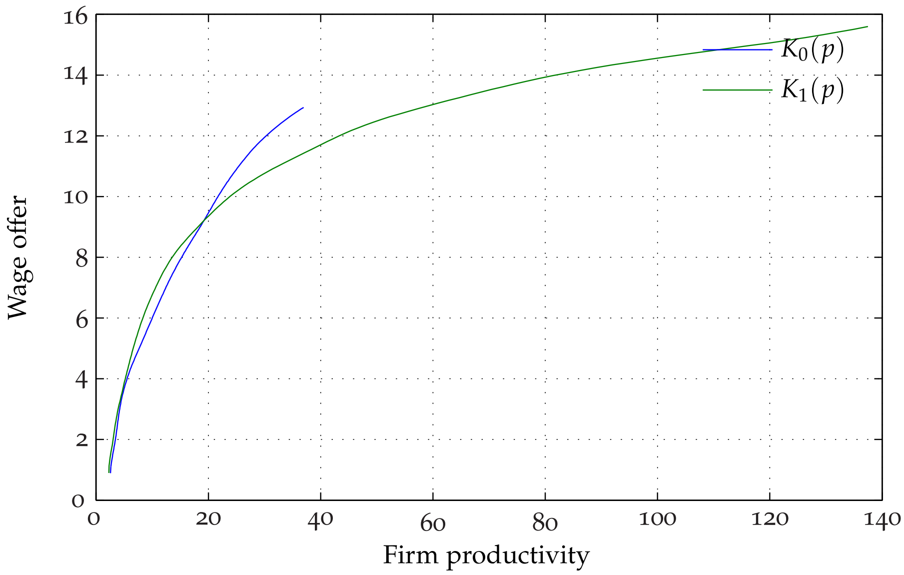


Figure 2: Wage policy function (pre-reform). Figure shows how optimal wage policy $K_h(p)$ varies with firm productivity and hours sector under April 1997 tax and transfer system. Figure is truncated at firm productivity greater than $K_1^{-1}(G_1^{-1}(0.99))$.

feature that is evident in this figure is that the wage policy function for full-time firms becomes very flat as the productivity of firms increases. This implies that high productivity firms have a very high degree of monopsony power. Second, the extent of monopsony power is much lower for part-time firms at high wages. When wages are high, the disutility of work C_i^1 becomes small relative to earnings so that part-time firms must offer much higher wages if they are to attract workers from full-time jobs. This additional layer of competition is clearly important for the optimal wage policy of firms and will become apparent in our simulation exercises. The underlying distribution of firm productivity is shown in Figure 3, while the wage offer distributions are shown in Figure 4; the latter figure shows that there is a larger concentration of part-time firms offering relatively low wages, while the distribution amongst full-time firms is more dispersed and with a longer tail.

The structural parameter estimates suggest that there is considerable heterogeneity across the different demographic groups. The job destruction rate is highest for lone mothers ($\hat{\delta}_i = 0.015$) with this estimate implying that jobs are exogenously destroyed on average every 66 months

2007). These violations are not a large concern as they typically occur for very high productivity full-time firms where the productivity density is very low.

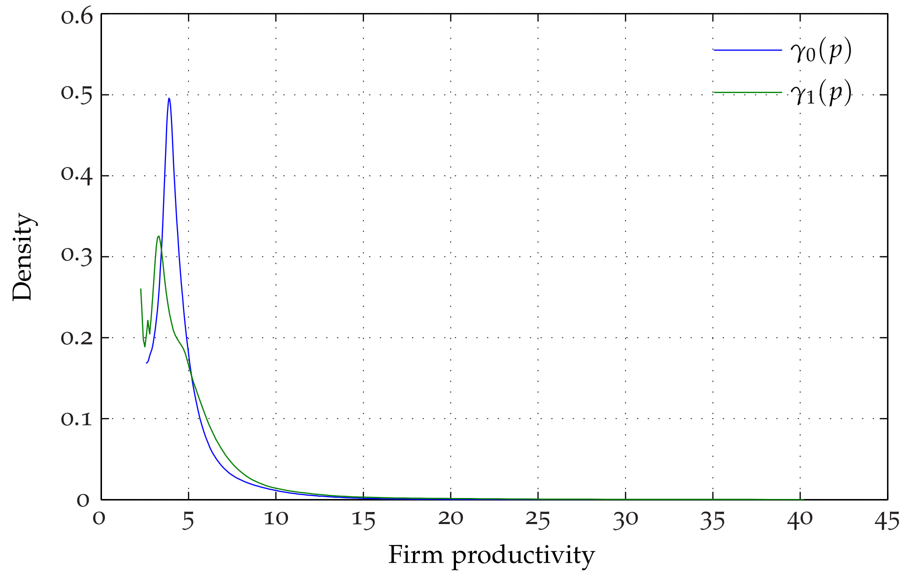


Figure 3: Productivity distribution. Distribution of firm productivity is obtained from pre-reform estimation by setting $v_h(p) = 1$ for all (h, p). Figure is truncated at firm productivity greater than $K_1^{-1}(G_1^{-1}(0.95))$.

Figure 4: Wage offer distribution (pre-reform). Figure shows distribution of wage offers for part-time and full-time firms under April 1997 tax and transfer system.

($= 1/0.015$). The destruction rates are lowest for married men and married women without dependent children, where they are estimated to be around three times as small. The arrival rates of job offers also varies considerably across the different groups. Job offers arrive most frequently for men: for unemployed married men without children we obtain $\hat{\lambda}^1_{ui} = 0.069$ which implies that offers arrive on average every 14 months ($= 1/0.069$); the unemployed arrival rate is estimated slightly lower for single men and married men without children (0.051 and 0.047 respectively). Of course, not all these job offers will be acceptable to all workers. The estimated total job offer arrival rates $\hat{\lambda}^0_{ui} + \hat{\lambda}^1_{ui}$ for unemployed childless women is similar to the values of $\hat{\lambda}^1_{ui}$ for men. However, for lone mothers this total rate is estimated to be around three times as small; for married women with children it is around one and a half times as small. Note that the arrival rate of full-time offers for unemployed mothers is especially low with $\hat{\lambda}^1_{ui} \approx 0.003$. While our model could potentially explain the proportion of mothers working part-time by a high value of C^1_i (discussed below), this would also require that the accepted wages of full-time working mothers be much higher than those accepted in part-time jobs. We do not observe this in our data.

For a number of groups, the estimated job offer arrival rate when employed ($\hat{\lambda}^h_{ei}$) is similar to that then unemployed ($\hat{\lambda}^h_{ui}$) and in some cases we can not formally reject the null hypothesis that they are the same. While this is similar to the finding of van den Berg and Ridder (1998), it contrasts with Bontemps et al. (2000) which found (using French Labour Force Survey data) that job offers typically arrive ten times as frequently for the unemployed compared to the employed. In our estimation we find that $\hat{\lambda}^1_{ui}$ is around 1.6 times higher than $\hat{\lambda}^1_{ei}$ for childless men, but we can not reject the hypothesis that $\lambda^1_{ui} = \lambda^1_{ei}$ for married men with children. Amongst women, we estimate that λ^1_{ui} is around 1.4 times larger than λ^1_{ei} for single women, very similar for married women without children (no significant difference), but $\hat{\lambda}^1_{ei}$ is much larger than $\hat{\lambda}^1_{ui}$ for both lone mothers (six times larger) and married women with children (more than twice as large). For all groups of women we estimate $\hat{\lambda}^0_{ei} < \hat{\lambda}^0_{ui}$, but we can not reject the hypothesis that they are equal for married mothers.

The monetary disutility of full-time work C^1_i is estimated to be equal to around £24 per week for single women, and is somewhat higher for lone mothers and married women (up to around £37 per week), but none of the differences across groups are especially large. We obtain considerable dispersion in the unobserved leisure cost for all groups, and this translates into dispersion in reservation wages. The distribution of (full-time) reservation wages is shown in Table 5. The table shows the proportion of workers of each type whose reservation wage is below given percentiles of the (full-time) wage offer distribution and reflects uncertainty in all distributions and structural parameters. For all worker types i we obtain $\hat{A}_i(\underline{\hat{w}}_1) < 1$, so that unemployed workers are indeed selective in the wage offers that they are willing to accept. This feature also implies a negative duration dependence in the exit rate out of unemployment. Furthermore, the value of $\hat{A}_i(\hat{\overline{w}}_1)$ is very close to one for all groups so that essentially all individuals would be willing to accept the highest full-time wage offer.

Since the wage offer distributions are common to workers of all types i, any difference in employment states and earnings distributions must be explained by variation in the transitional parameters, opportunity cost distribution, and the tax and transfer system. Overall, we obtain a good fit to the data. The difference in the empirical and predicted states for the main demographic groups is small and never exceeds more than around 2 percentage points (see Table 6). Similarly, we do well in replicating the observed distribution of wages (see Figure 5); for most groups the fit is very good, but the model does appear to have some difficulty fitting the full-time earnings distribution for married women with children (Figure 5(g)). Finally, we note that the fit is less satisfactory if we compare the empirical and predicted employment states of individuals in couples conditional on the earnings of their partner. More specifically, our model tends to under-predict the non-employment rates for individuals with a non-working partner. In other words, our model is not able to fully explain employment patterns within couples of a given demographic type solely by variation in the tax and transfer system.

5 Simulating WFTC and contemporaneous reforms

In this section we simulate the real impact of all changes to the tax and transfer system between April 1997 (the system operating in our pre-reform sample period) and April 2002. As discussed

Table 5: Reservation Wage Distribution

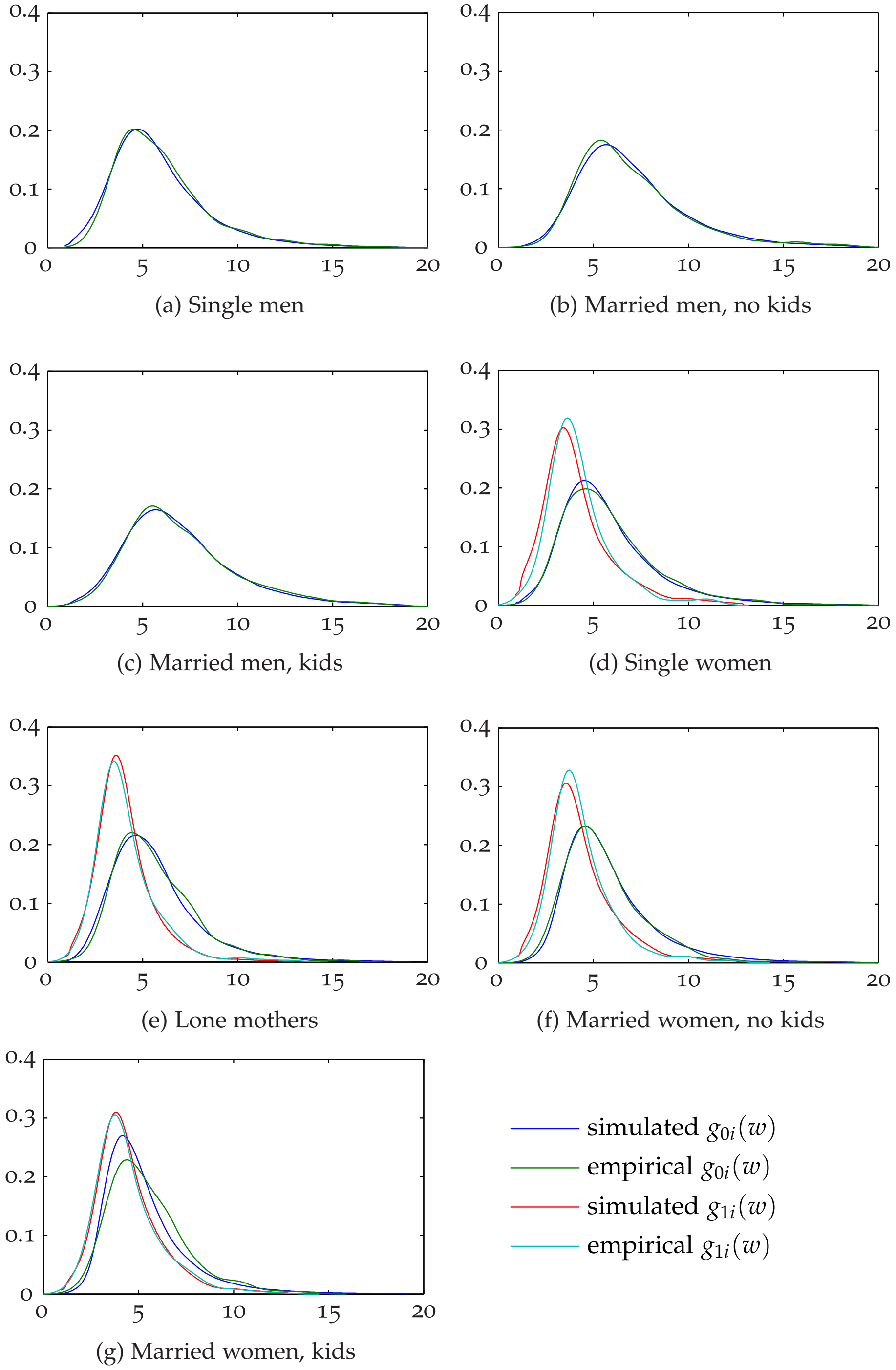
	Percentile of full-time offer distribution $\hat{F}_1(w)$					
	0	20	40	60	80	100
single men	0.243	0.404	0.523	0.644	0.788	1.000
	[0.086,0.441]	[0.183,0.572]	[0.323,0.654]	[0.496,0.735]	[0.711,0.856]	[1.000,1.000]
married men, no kids	0.145	0.340	0.503	0.651	0.811	1.000
	[0.077,0.237]	[0.200,0.443]	[0.350,0.597]	[0.515,0.723]	[0.713,0.857]	[1.000,1.000]
married men, kids	0.465	0.621	0.702	0.768	0.847	1.000
	[0.398,0.540]	[0.563,0.672]	[0.656,0.750]	[0.729,0.813]	[0.817,0.892]	[1.000,1.000]
single women	0.438	0.571	0.651	0.723	0.806	1.000
	[0.175,0.660]	[0.318,0.720]	[0.464,0.755]	[0.603,0.790]	[0.741,0.859]	[0.995,1.000]
lone mothers	0.232	0.473	0.584	0.659	0.754	1.000
	[0.063,0.413]	[0.245,0.632]	[0.427,0.734]	[0.529,0.830]	[0.655,0.932]	[1.000,1.000]
married women, no kids	0.183	0.479	0.668	0.784	0.882	1.000
	[0.086,0.331]	[0.265,0.627]	[0.472,0.764]	[0.657,0.844]	[0.813,0.919]	[1.000,1.000]
married women, kids	0.270	0.548	0.699	0.793	0.870	1.000
	[0.193,0.393]	[0.380,0.658]	[0.548,0.776]	[0.689,0.844]	[0.812,0.901]	[1.000,1.000]

Notes: Table shows the fraction of individuals whose full-time reservation wage is below various percentiles p of the full-time wage offer distribution, $A_i(\hat{F}_1^{-1}(p))$, and is calculated using the maximum likelihood estimates from Table 4. The 5th and 95th percentiles of the bootstrap distribution are presented in brackets, and are calculated using 500 replications.

Table 6: Empirical and Predicted Employment States

	Empirical			Predicted		
	u_i	m_{0i}	m_{1i}	u_i	m_{0i}	m_{1i}
single men	0.366	–	0.634	0.346	–	0.654
	[0.354,0.379]		[0.621,0.646]	[0.334,0.356]		[0.644,0.666]
married men, no kids	0.186	–	0.814	0.178	–	0.822
	[0.175,0.200]		[0.800,0.825]	[0.167,0.191]		[0.809,0.833]
married men, kids	0.224	–	0.776	0.211	–	0.789
	[0.214,0.236]		[0.764,0.786]	[0.202,0.220]		[0.780,0.798]
single women	0.319	0.115	0.566	0.307	0.119	0.574
	[0.305,0.332]	[0.106,0.125]	[0.552,0.580]	[0.294,0.320]	[0.111,0.128]	[0.561,0.587]
lone mothers	0.623	0.235	0.142	0.610	0.244	0.146
	[0.609,0.638]	[0.222,0.248]	[0.131,0.153]	[0.597,0.623]	[0.232,0.256]	[0.135,0.158]
married women, no kids	0.244	0.244	0.512	0.241	0.254	0.505
	[0.230,0.259]	[0.228,0.258]	[0.495,0.530]	[0.227,0.255]	[0.239,0.268]	[0.488,0.523]
married women, kids	0.432	0.364	0.204	0.419	0.370	0.211
	[0.419,0.445]	[0.351,0.377]	[0.193,0.214]	[0.408,0.432]	[0.358,0.383]	[0.199,0.221]

Notes: Predicted states are calculated using the maximum likelihood estimates from Table 4. Employment states may not sum to one due to rounding. The 5^{th} and 95^{th} percentiles of the bootstrap distribution of employment states are presented in brackets, and are calculated using 500 replications.



(a) Single men

(b) Married men, no kids

(c) Married men, kids

(d) Single women

(e) Lone mothers

(f) Married women, no kids

(g) Married women, kids

Figure 5: Simulated and empirical earnings by group. Horizontal axis refers to hourly wage rate in April 1997 prices; Vertical axis refers to wage density. Empirical distributions are calculated using a Gaussian kernel with a bandwidth of 0.6.

in Section 2, this captures the introduction of WFTC together with other changes to the tax and transfer system, including increases in the generosity of out-of-work support for families with children. We do this by using the estimated structural parameters of our model and examining how the equilibrium changes when we impose a different tax and transfer system.

In the simulations we present in this section, we assume a vacancy cost function for each sector $h \in \{0,1\}$ of the form $c_h(v,p)$=$c_h(p)v^2/2$, together with a Cobb-Douglas matching function $M_h(V_h, S_h) = V_h^{\theta_h} S_h^{1-\theta_h}$, where $S_h \equiv \sum n_i(s_{ui}^h u_i + s_{ei}^h(1-u_i))$ is the total search intensity in sector h. Our main results assume $\theta_0 = \theta_1 = 1/2$ but we do discuss sensitivity with respect to this parameter. Before we proceed we note that with a fixed distribution of firms' productivity and a vacancy cost function that is quadratic in v, our simulation exercises are invariant to the parameterization of $c_h(p)$ provided that $c_h(p) > 0$.[21] Without loss of generality, we therefore assume that $v_h(p) = 1$ for all p and $h \in \{0,1\}$ in the pre-reform period and recover the values of $c_h(p)$ that are consistent with this being an equilibrium. This also implies that $\Gamma_h(p) = F_h(K_h(p))$ under the base system.

To highlight the relative importance that this set of reforms has on job acceptance behaviour and the behaviour of firms, we present our results in two stages. Firstly, we consider the impact of the reform holding the distribution of job offers and their arrival rate constant; secondly we additionally allow firms to respond optimally by changing their wage policy and recruiting effort. We refer to the first channel as the *direct impact* of the reform, and the second channel as the *equilibrium impact* of the reform.

5.1 Direct impact

We present both the direct and equilibrium impact of the reform on employment states in Table 7, and we first discuss the direct effect. The table shows that the (non- tax credit) reforms had a small positive effect (around 1 percentage point) on the employment of both singles and couples

[21]This is because the marginal cost of a job-vacancy becomes linear in $c_h(p)v$. If we assume an alternative $c_h(p)$ we will identify a new $v_h(p)$ such that $c_h(p)vV_h$ is unchanged (see equation 16), and similarly identify a new value of $\gamma_h(p)$ such that distribution of wage offers is preserved, and the search intensities s_{ji}^h such that the arrival rate of offers is maintained. The equilibrium effect of tax reforms is invariant to the choice of $c_h(p)$ as any effect of the reform on $v_h(p)$ is also scaled by $\gamma_h(p)$.

without children. This increase is mainly due to small reductions in the real value of IS/income-based JSA for families without children, together with reductions in income-tax (the introduction of a new lower starting rate, and a penny reduction is the basic rate – see Section 2) which act to raise the value of holding low wage jobs and so lower reservation wages. Since these changes are only small, there is little impact on durations.

Perhaps unsurprisingly, the largest predicted impact of these reforms is on the employment rate of lone mothers, where we predict an increase of 5.6 percentage points. Despite both full-time and part-time reservation wages falling,[22] this employment increase is almost entirely due to a movement into full-time work. This is partly because the lower withdrawal rate of WFTC compared to FC results in full-time incomes increasing by more than part-time incomes over a large range of wages for this group.[23] As such the indifference condition $q_i(w)$ changes so that more part-time workers accept full-time jobs through on-the-job search, while the acceptance of part-time jobs by full-time workers also declines. Note also that the much higher arrival rate of full-time offers relative to part-time offers amongst the employed ($\lambda^1_{ei} \gg \lambda^0_{ei}$) is important for the quantitative impact.

In Figure 6a we show the impact of the reforms on the (monthly) unemployment exit rate: $d_{ui}(b) = \lambda^0_{ui}\overline{F}_0(q_i(\phi_i(b))) + \lambda^1_{ui}\overline{F}_1(\phi_i(b))$; the figure also shows the distribution of b in the stock of the unemployed under the base system. The figure illustrates that the reforms increase the exit rate by a considerable amount relative to the pre-reform level, with corresponding large reductions in unemployment durations.[24] Similarly, Figure 6b shows the impact on the separation rates of employed lone mothers in part-time and full-time jobs (respectively, $d^0_{ei}(w) = \delta_i + \lambda^0_{ei}\overline{F}_0(w) + \lambda^1_{ei}\overline{F}_1(q_i^{-1}(w))$ and $d^1_{ei}(w) = \delta_i + \lambda^0_{ei}\overline{F}_0(q_i(w)) + \lambda^1_{ei}\overline{F}_1(w)$).[25] Apart from at high

[22] Any reform that lowers reservation wages necessarily relies on parametric identification, as the distribution of reservation wages is only non-parametrically identified on the support of pre-reform wages.

[23] At very low wages the reforms makes part-time work relatively more desirable as tax credit income is counted as income when determining eligibility to other benefits. Consequently, at these low wages other benefits may be withdrawn at full-time hours following the reform, but not with the lower earnings associated with part-time hours. At moderate wages (discussed above) the reduction in the taper rate dominates so that full-time work becomes more desirable. At high wages (where individuals become eligible for tax credits at full-time hours following the reform), part-time incomes increase by more than full-time incomes. At very high wages individuals are not-affected by the tax credit reform, so there are only small changes in the indifference condition due to the other smaller changes to the tax and transfer system.

[24] The expected unemployment duration conditional on b is given by the $1/d_{ui}(b)$.

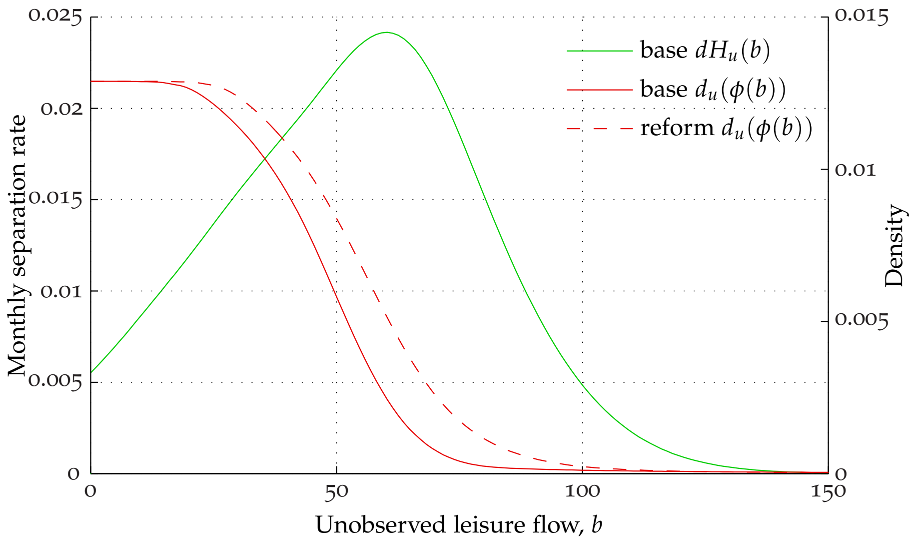
[25] The distribution of lone mothers' earnings is shown in Figure 5e earlier.

wage rates, the separation rate in part-time jobs increases, while it either falls or remains effectively unchanged in full-time jobs over the entire support of wages. Due to the increased flows to the full-time sector, the average duration of a part-time job falls by around 5 months, while there is also a half-month reduction for full-time jobs. This latter reduction reflects a compositional change following the inflow of workers into low paying jobs where the job separation rate is higher.

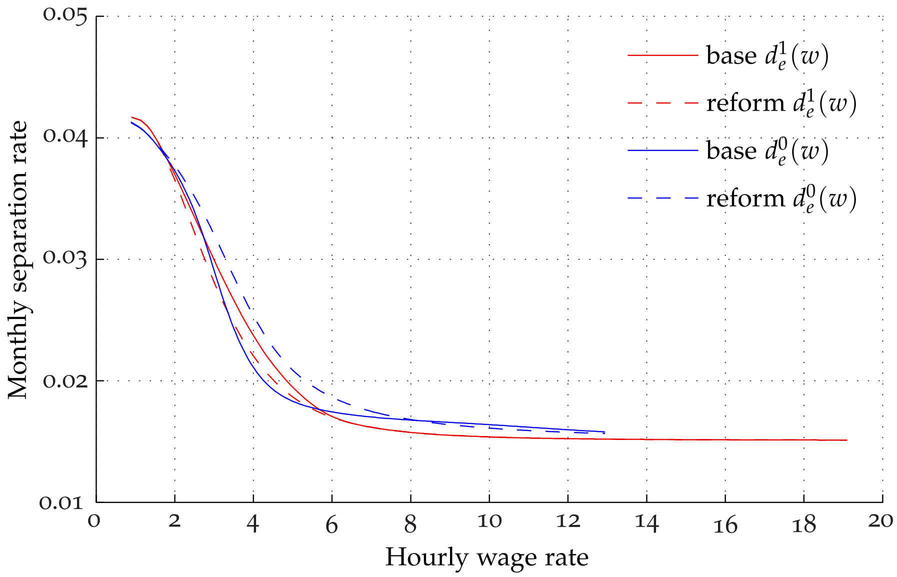
For couples with children the impact of these reforms is more complicated: individuals in couples with a high earning partner are effectively unaffected by the reform as entitlement for tax credits depends upon family income; those with a non-working or very low earning partner partner respond positively with increases in their unemployment exit rate, much like lone mothers; in intermediate cases, movement into work can taper away tax credit awards which may induce negative labour supply responses (particularly among the newly eligible families where there are large relative reductions in the unemployment exit rate). On balance, these factors lead to a small decrease in the labour supply of married women with children (a 1.3 percentage point decrease), but increase the employment rate of married men with children by a little under 3 percentage points. Among married women, the decrease in labour supply comes primarily through a reduction in those working part-time. Since men have no choice of part-time hours, there is no change in the job separation function for married fathers. For married mothers, while there are only small changes in these functions (and job durations) on average, there are much more pronounced changes once we condition on partner earnings.

Note that the simulations performed for couples hold constant the distribution of partner earnings. To understand the importance of this, we use our structural parameter estimates to perform dynamic simulations whereby we allow both individuals in a couple to sample wage offers, but otherwise maintain the same individualistic behaviour described in Section 3.[26] That is, we allow each individual to receive wage offers, but then sequentially condition on the current wage and employment state of their partner (subsumed in the tax schedule as before) when job acceptance decisions are made. In contrast to the optimal joint search behaviour analysed in

[26]There is a slight inconsistency here as our sample selection was performed at the level of the individual and not of the family.



(a) Unemployment exit rate



(b) Job separation rate

Figure 6: Lone mother separation rates. Figure shows monthly separation rates for lone mothers under the April 1997 (base) and April 2002 (reform) tax and transfer systems; reform simulations refer to the direct impact only. Panel (a) shows the exit rates from unemployment and is truncated at weekly leisure flows less (greater) than 0 (150); panel (b) shows the job separation rate from part-time and full-time jobs. See text for definitions of d_u, d_e^0 and d_e^1.

Guler et al. (2009), no voluntary quits are permitted. From this simulation exercise we obtain direct employment impacts which are essentially the same as those presented in Table 7 and discussed above. Given this finding, the remainder of our analysis will continue to present results which condition on partner earnings in the base system.[27]

Before we discuss the equilibrium effect of the reforms, we briefly discuss the impact on wages. Note that selection effects alone imply that earnings will change even though the distribution of wage offers is held fixed. This highlights the fact that attempting to estimate the incidence of earned income tax credit programmes by comparing changes in observed wages amongst eligible and non-eligible groups is potentially misleading without carefully controlling for these selection effects. Indeed, selection alone implies some large reductions in full-time average wages. Our simulations imply that lone mothers experience a 7% reduction in average full-time wages and a 1.5% increase in part-time wages, while married men with children see their earnings fall by 1%; the changes for other groups is negligible. Since it is only lone mothers who experience sizeable changes in their earnings, the unconditional (across worker type i) distributions change very little due to these selection effects.

5.2 Equilibrium impact

In Table 7 we also present the equilibrium impact of the reform. The first immediate thing to note is that the impacts are extremely similar to those obtained from the direct impact. That is, equilibrium considerations do not appear to be very important for *this particular set of reforms*. Looking more closely we can see that equilibrium considerations tend to increase employment in full-time jobs, and decrease employment in part-time jobs. To understand these subtle changes to employment it is useful to consider how the optimal strategy of firms' has changed. Figure 7 shows how $v_h(p)$ changes following the reform. Full-time firms are predicted to increase their recruiting effort over much of the distribution, with the increase most pronounced in the middle; only among very high (where the density of firms is particularly low – see Figure 3 earlier) and low productivity firms is a decrease predicted. In contrast, the largest increases among part-time

[27]Our dynamic simulations were conducted using a population of 100,000 families of each type $i \in \mathcal{I}$, with behaviour simulated over a period of 1,000 years with a monthly time unit.

Table 7: Employment Impact of Reforms

	Direct Impact			Equilibrium Impact		
	Δu_i	Δm_{0i}	Δm_{1i}	Δu_i	Δm_{0i}	Δm_{1i}
single men	-0.010	–	0.010	-0.012	–	0.012
	[-0.013,-0.007]		[0.007,0.013]	[-0.015,-0.009]		[0.009,0.015]
married men, no kids	-0.008	–	0.008	-0.009	–	0.009
	[-0.009,-0.007]		[0.007,0.009]	[-0.010,-0.008]		[0.008,0.010]
married men, kids	-0.029	–	0.029	-0.030	–	0.030
	[-0.042,-0.021]		[0.021,0.042]	[-0.043,-0.022]		[0.022,0.043]
single women	-0.008	-0.000	0.008	-0.008	-0.003	0.011
	[-0.010,-0.004]	[-0.001,0.001]	[0.005,0.011]	[-0.011,-0.005]	[-0.005,-0.002]	[0.008,0.014]
lone mothers	-0.056	-0.005	0.061	-0.053	-0.011	0.064
	[-0.068,-0.043]	[-0.016,0.002]	[0.049,0.075]	[-0.066,-0.041]	[-0.022,-0.002]	[0.052,0.079]
married women, no kids	-0.009	-0.002	0.012	-0.009	-0.006	0.015
	[-0.011,-0.008]	[-0.005,0.001]	[0.009,0.014]	[-0.010,-0.008]	[-0.011,-0.002]	[0.012,0.020]
married women, kids	0.013	-0.012	-0.001	0.015	-0.015	-0.000
	[0.009,0.016]	[-0.014,-0.009]	[-0.003,0.001]	[0.010,0.020]	[-0.019,-0.011]	[-0.002,0.003]

Notes: All employment responses are expressed in percentage points. Changes may not sum to zero due to rounding. The direct impact considers all changes to the tax and transfer system between April 1997 and April 2002, holding the wage offer distributions and arrival rates at their pre-reform levels. The equilibrium impact allows the wage offer distribution and arrival rates to change.

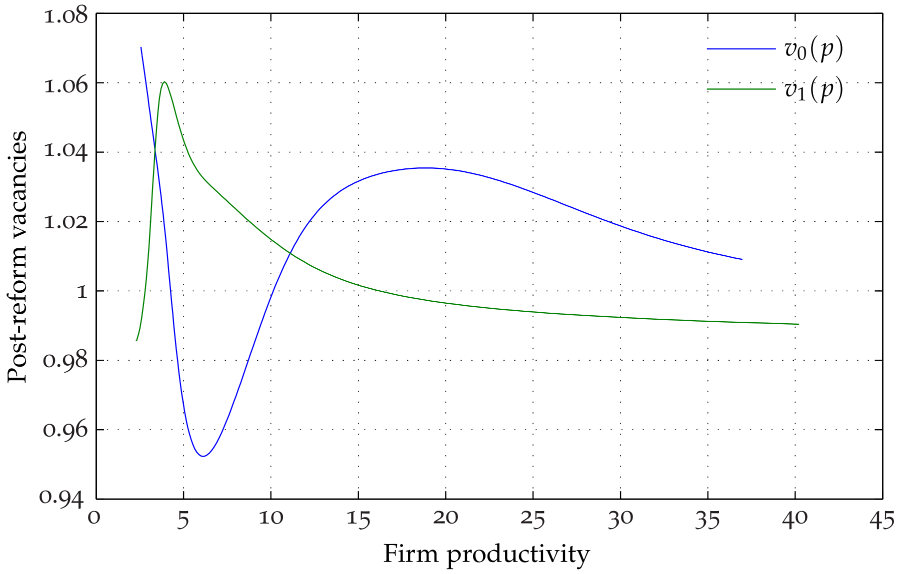


Figure 7: Change in recruiting policy function. Figure shows the level of vacancies $v_h(p)$ under with April 2002 tax and transfer system. In pre-reform period we set $v_h(p) = 1$ for all (h, p), so values greater (less) than one correspond to increases (decreases) in recruiting effort. Figure is truncated at firm productivity greater than $K_1^{-1}(G_1^{-1}(0.95))$ under base system.

firms is for those with the lowest productivity, while the upper half of the distribution tend to decrease recruiting effort. Overall these changes imply that V_0 is effectively unchanged, whereas V_1 increases by around three percent. Given these changes, together with the changes in the total search intensities S_0 and S_1, the flow of part-time matches M_0 is reduced by a negligible amount, while the flow of full-time matches M_1 increases by just 1.6 percent. This increase in full-time matches will benefit all workers, regardless of whether or not they were directly affected by the introduction of WFTC. These additional changes only have a very small impact on the separation rates for all types of workers.

The effect that the reforms have on the optimal wage policy of firms is difficult to predict *a priori* due to the changing competition both within and between sectors. Fixing the optimal strategies of part-time firms at their pre-reform levels, the direct effect on full-time firms acts to increase employment $L_1(w)$ across the distribution of firm productivity. Despite this, changes in the indifference condition $q_i(w)$ mean that some relatively low productivity full-time firms react by increasing the wages that they offer in order to attract workers from low-wage part-time firms, while higher productivity firms decrease wages. Meanwhile, despite the negligible reductions in overall part-time employment, the direct effect of the reforms still increases $L_0(w)$ for a number of firms and these firms respond to this by lowering their wage offers. On balance these changes mean that the equilibrium effects increase full-time employment further, whilst decreasing part-time employment. The full equilibrium effect on the distribution of wage offers is shown in Figure 8. The figure shows that there is a small, but noticeable, general shift in the distribution of part-time wage offers towards lower wages. In the full-time sector most changes can be seen to occur in the lower half of the distribution, with a greater proportion of firms posting wages that are close to the median wage offer. The overall effect of the reform on the distribution of part-time and full-time earnings is shown in Figure 9. While the direct impact of the reforms on the overall distribution of earnings is relatively minor given the small change in part-time employment, equilibrium effects do appear to have a noticeable impact on the shape of the part-time earnings distribution as the figure illustrates. To understand these changes further we note that the reason why the changes in the distribution of full-time offers does not have a larger impact on the full-time earnings distribution is because we estimate somewhat higher job offer arrival rates for full-time jobs amongst the employed. This means that workers gravitate to the higher paying full-time jobs much more quickly than in the part-time sector.

We now comment upon the sensitivity of our results to our calibration of the matching functions. Given that the equilibrium effect of the tax reforms is dominated by the direct labour supply effect, it is perhaps unsurprising that our results are not especially sensitive to the choice of θ_h. Higher values of θ_h makes the flow of matches more sensitive to changes in $v_h(p)$ which then acts to increase slightly employment in full-time firms, decrease employment in part-time firms, and raise employment overall. Similarly, lower values of θ_h have the opposite effect. The quantitative importance of these changes is relatively modest: for example, relative to our baseline results with $\theta_0 = \theta_1 = 0.5$, increasing θ_h to 0.9 for $h \in \{0, 1\}$ only increases the employment rate of lone-mothers by a further 0.4 percentage points, while decreasing θ_h to 0.1 just decreases it by a further 0.1 percentage points; other groups experience similar changes.

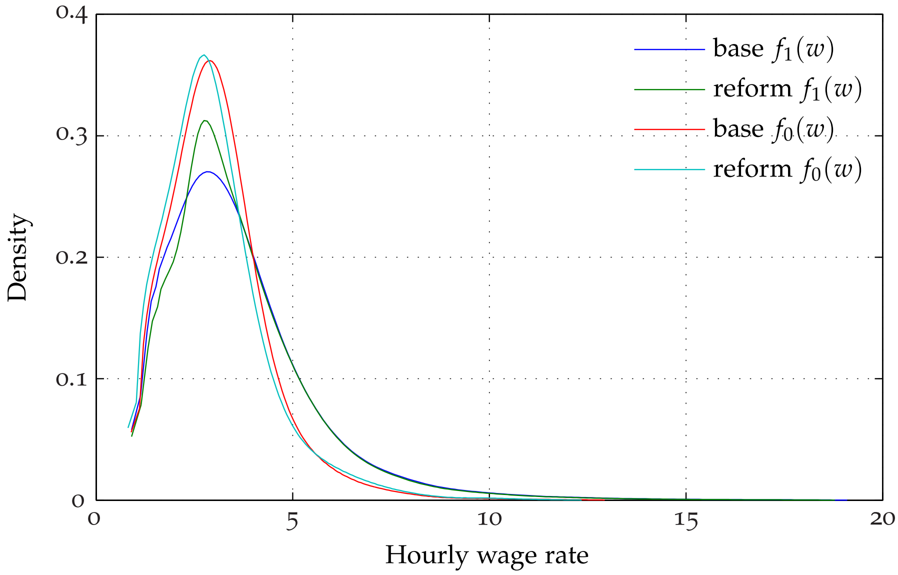


Figure 8: Change in wage offer distribution. Figure shows distribution of wage offers for part-time and full-time firms under the April 1997 (base) and April 2002 (reform) tax and transfer systems.

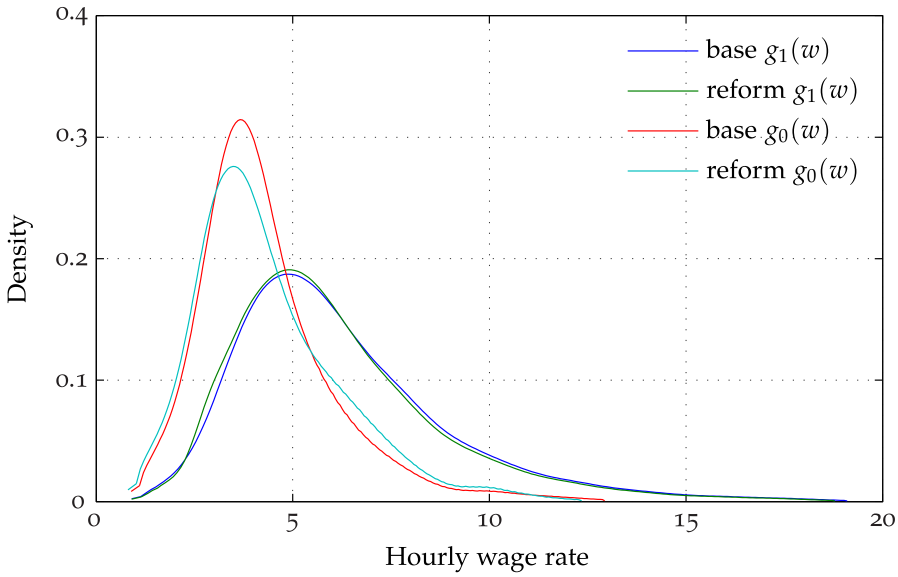


Figure 9: Change in earnings distribution. Figure shows distribution of earnings for part-time and full-time workers under the April 1997 (base) and April 2002 (reform) tax and transfer systems.

Table 8: Empirical and Predicted Employment Changes

	Empirical			Predicted		
	Δu_i	Δm_{0i}	Δm_{1i}	Δu_i	Δm_{0i}	Δm_{1i}
single men	-0.030	–	0.030	-0.012	–	0.012
married men, no kids	-0.021	–	0.021	-0.009	–	0.009
married men, kids	-0.021	–	0.021	-0.030	–	0.030
single women	0.001	0.003	-0.003	-0.008	-0.003	0.011
lone mothers	-0.052	0.027	0.024	-0.053	-0.011	0.064
married women, no kids	-0.001	-0.013	0.014	-0.009	-0.006	0.015
married women, kids	-0.021	-0.006	0.028	0.015	-0.015	-0.000

Notes: Predicted changes are calculated using the maximum likelihood estimates from Table 4 and simulating the equilibrium effect of replacing the April 1997 system with the April 2002 system. Empirical changes refer to the observed changes in our data over this period using the sample selection as described in Section 4.5. Changes may not sum to zero due to rounding.

5.3 Aggregation

The very selected nature of sample (see Section 4.5) implies that the labour market responses presented in Table 7 can not be applied to the whole population. The education selection criteria in particular, has the largest impact on the representativeness of the sample, although the impact this has on lone mothers is much smaller relative to other demographic groups. Using survey frequency weights, and assuming that the employment rates of individuals excluded from our sample are unaffected by the reform, our simulations imply that overall employment increases by 135,000. The majority of this increase is though the increased employment of lone mothers (60,000) and married men with children (50,000).

5.4 Post-reform comparison

The simulations that we presented in the previous section allowed us to examine the *ceteris paribus* labour market impact of the set of tax reforms between April 1997 and April 2002. Before comparing these predictions to those which were obtained in previous evaluations of WFTC, we first briefly compare them to the actual changes in the labour market. That is, we ask to what extent did the tax reforms contribute to the observed labour market changes. The simulated and empirical changes are presented in Table 8.

Most groups experienced an increase in employment over this period with the exception of women without children where essentially no change was observed. Men without children experienced an increase in employment of between two and three percentage points over this period, and while we did predict a small increase in employment for such individuals, these changes represent a more than doubling of the impact. This therefore suggests that other changes over this period (including robust productivity growth, changes in the distribution of partner earnings, a national minimum wage, and various "New Deal" programmes) may have had a more important effect on employment for this group. The employment increase for married men with children is very similar to that for married men without children, despite very different simulated impacts. To reconcile these findings we therefore require that the other changes in the economy have had an opposite effect on the employment of married men with children.

Lone mothers experienced the largest employment increase by far, with employment increasing by 5.2 percentage points. While the overall impact here line up remarkably well with the simulations, the hours responses differ: the empirical contribution to the increase is approximately evenly split between movements into both part-time and full-time employment; this contrasts with our simulations which suggested that it was exclusively due to a movement into full-time work. Despite predicting small increases in employment for single women and married women without children we effectively see no change for these groups, despite modest employment growth among men. One possible explanation for this is that the labour market is perhaps not as "integrated" as we have modelled, with childless women possibly being more effected by changes in wage offers following the reform than childless men. Of course, to be a credible explanation this would require substantially larger equilibrium effects than we obtained in the previous section (see the discussion in Section 5.6). Finally, despite predicting reductions in the employment of married women with children we again observe a modest increase, which again suggests that other changes in the economy boosted employment rates.

5.5 Other evaluations of WFTC

We now compare the labour market responses from our study to those obtained in previous WFTC evaluations. Since our analysis only considers individuals with low levels of educational attainment (see Section 4.5), our sample is typically more selective than in the studies cited in this section. As such, the comparisons here are considered more indicative than exact, with the results for lone mothers being most comparable due to lower average education levels for this group. In terms of the employment impact, which is sometime the only outcome considered in these evaluations, the results for lone mothers are broadly similar to those obtained in these other studies. This is perhaps unsurprising given theat we did not find evidence of strong equilibrium effects in our analysis.

The most common method that has been used when analysing the impact of WFTC on employment outcomes is difference-in-differences. Such existing evaluations (including Azmat, 2006a; Blundell et al., 2004a; Francesconi and van der Klaauw, 2004; Gregg and Harkness, 2003; Leigh, 2007) have largely (but not exclusively) focussed upon the impact on lone mothers and essentially involve comparing the changing employment outcomes of lone mothers, with single women without children. As discussed in Section 2, the introduction of WFTC was accompanied by a number of other changes to the tax and benefit system. Given this choice of control group, at best these evaluations will be informative about the effect of the set of reforms that only affected parents. Since we predict that the changes to Income Tax and National Insurance acted to increase slightly the labour supply of non-parents, the relevant statistic to compare from our study for lone mothers is an impact of 4.5 percentage points $(= 5.3 - 0.8)$. Before proceeding further, we note that if equilibrium effects were quantitatively important, then the usual stable unit treatment value assumption (or SUTVA) would be violated.

The headline impacts of these estimates on lone mothers (or lone parents) varies somewhat, lying between around 1 and 7 percentage points. The differences across these studies appears to be largely attributable to two factors: (i) the period considered in the estimation; (ii) attempts to control for pre-programme differences in employment trends between treatment and control groups (see Brewer and Shephard, 2004 for a time series). The first factor is important in any comparison because those studies which focus on the period immediately around the introduction of WFTC (such as Leigh, 2007), find considerably smaller impacts. This is unsurprising both because WFTC grew in generosity following its introduction in 1999 (see Table 1 earlier), and individuals may require time to obtain an acceptable job. The second issue is more problematic for these studies as it suggests that the usual common trends assumption invoked in difference-in-differences may be violated.Unsurprisingly, studies which assume that this pre-programme differential employment growth would have stopped had WFTC not been introduced typically find larger effects than those which achieve identification through a particular parametric differential time trend specification (using an otherwise similar specification, Blundell et al., 2004a report a 4 percentage point increase, compared to the 2 percentage point impact reported in Azmat, 2006a).

Francesconi and van der Klaauw (2004) offer an interesting interpretation of this pre-reform employment growth, attributing it to an "anticipation effect" so that the pre-programme growth in employment is due to the programme itself. Such effects are possible because the main details of WFTC were announced over a year earlier in the government's March 1998 Budget. Under such an assumption, Francesconi and van der Klaauw estimate that WFTC increased employment by around 7 percentage points, which is somewhat higher than the other evaluations cited here (as well as our simulations for this group). Such effects would be a qualitative implication of a non-stationary labour market search model with anticipation, although other forms of non-stationarity (such as the arrival rate of job offers depending upon calender time) would also be consistent with these trends (see van den Berg, 1990).

An alternative evaluation methodology that has also been adopted involves the estimation and simulation of a static discrete choice labour supply model (Blundell et al., 2000; Blundell and Shephard, 2009; Brewer et al., 2006). These models, which assume away the presence of equilibrium effects, identify preferences by relating changing employment patterns to changing financial work incentives assuming a constant hourly wage and some finite set of work alternatives. Using cross-sectional data from 1997 to 2002, Blundell and Shephard (2009) predicted that the reforms over this period increased employment by around 4 percentage points amongst lone mothers.[28] This study, like other ex-post evaluations, relies upon variation in data caused by the reform itself to obtain this impact.[29] In particular, it explains some of this employment growth by reductions in the "cost" of receiving tax credits.

5.6 Why aren't equilibrium effects more important?

The analysis performed in section 5.2 suggests that equilibrium effects may be small. We now explore the extent to which this may be due to the integrated nature of the labour market and the targeted nature of the reforms. As noted previously, lone mothers are the main beneficiaries of the tax credit reforms, and our analysis suggests that labour supply responses are by far the greatest for this group. However, even amongst our sample of workers with low education, they only represent a little over 10% of the sample. While allowing all workers to compete within the same market was a very natural characterisation of the UK labour market, and one which permitted spillover effects, it does severely limit the potential for equilibrium effects following a targeted reform like WFTC if firms are constrained to have a single wage policy.[30]

To understand the importance of our assumptions regarding market segmentation, we re-estimate our model on a sample comprised solely of lone mothers, and perform our simulation exercises as before. Since the model is re-estimated, there are some differences in the direct impact. In particular, the positive employment impact of the reform is now more evenly split between increases in full-time and part-time employment. This is largely due to estimated differences in the wage offer distributions, and because the arrival rates of part-time and full-time wage offers when employed are now estimated to be very similar (in our previous estimation results, full-time offers arrived more than twice as frequently as part-time offers amongst employed lone mothers – see Table 4). An implication of this is that selection effects reduce full-time wages

[28] Using a similar model, Brewer et al. (2006) reported a similar employment increase for lone mothers, together with a small reduction in the employment of both men and women in couples with children (around half a percentage point).

[29] An ex-ante evaluation using a similar model was provided by Blundell et al. (2000). This predicted a 2 percentage point increase in the employment of lone mothers, together with a small decline for married women with children and essentially no change for men in couples. These results are not comparable to the employment responses that we simulate here as only the "immediate" reform (that is, WFTC in October 1999) was considered.

[30] In a model with worker and firm bargaining, wages essentially become individualistic so that the potential for equilibrium effects is much larger. Lise et al. (2005) used such a model in their analysis of the Canadian Self-Sufficiency Project, and found substantial equilibrium effects.

by 5%, and reduce part-time wages by 3%.

Once we allow for equilibrium responses we obtain much larger increases in the flows of matches relative to when the labour market is not segmented: M_0 increases by 5% while M_1 increases by almost 18%. Similarly, we obtain much stronger reductions in full-time wage offers. As a result of these equilibrium responses, full-time (part-time) earnings now fall by 14% (4%) relative to the pre-reform average. Accompanying these changes is a 1 percentage point reduction in full-time employment, with little change in part-time employment. Overall, equilibrium considerations have a non-negligible impact in this experiment, and act to reduce the positive employment impact of the reforms.

6 Conclusion

This paper has developed an empirical equilibrium job search model with wage posting, and has used it to analyse the impact of the British Working Families' Tax Credit reform. Our model extends the existing literature in a number of important dimensions: we allow for hours-of-work responses; accurate non-linear tax-schedules; worker and firm heterogeneity (without restrictive arrival rate assumptions); and allow workers of different types to all operate within the same labour market. We close our model by endogenizing the rate of job offer arrivals through aggregate matching functions, and also propose a semi-non-parametric estimation procedure.

We estimate our model using data from before WFTC was introduced, and use our structural parameter estimates to simulate the labour market impact of actual tax reforms. Our analysis suggests that WFTC, together with other reforms to the tax and transfer system between April 1997 and April 2002, had a positive effect on the employment of most groups; only amongst married women with children do we predict a fall in employment. Perhaps unsurprisingly, the increase in employment is found to be strongest for lone mothers, where a 5 percentage point increase is predicted. Our simulations suggest that while equilibrium considerations do play a role, the changes in employment and earnings for most groups are dominated by the direct effect of changing job acceptance behaviour. And while the tax reforms do appear to be able to explain some of the actual changes in employment over the relevant period, for some groups other changes in the economy appear more important.

Even though equilibrium effects may not appear very important for this particular set of reforms, it does not imply that they should always be ignored. Recalling that WFTC is only available to low income families with children, equilibrium effects have the potential to be much more important for tax reforms which are less targeted. We demonstrate that the equilibrium effects of the same reforms may be much larger if we consider a labour market solely comprised of lone mothers, one of the main beneficiaries of WFTC.

We believe that this paper represents an important first step in using empirical equilibrium job search models to evaluate the impact of tax reform policies. Despite performing our empirical analysis on individuals with low education levels, it is likely that differences in worker ability persist within this group. A natural extension could therefore involve incorporating heterogeneity in worker productivity which necessitates a more careful modelling of firm production technologies. Furthermore, given that the tax and transfer systems of many countries (including the UK) depend upon family income to some extent, a more detailed characterisation of the behaviour of couples (building upon the analysis of Guler et al., 2009) would allow us to explore the impact of tax policies on household labour supply allocations. Finally, given the importance of labour supply in our simulation exercises, incorporating micro-level endogenous search intensity (as in Christensen et al., 2005) would create a further dimension along which individuals can respond to changing financial incentives. While each of these represent non-trivial extensions, it does suggest a very exciting agenda of future research.

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Appendix

A Worker strategies

In this appendix we derive the optimal strategies of employed and unemployed workers that were presented in Section 3.2.[31] Using the same notation as in the main text, the value of unemployment V_{ui} must satisfy:

$$\begin{aligned}\rho_i V_{ui} = b - T_i^u \quad &+ \quad \lambda_{ui}^0 \mathbb{E}_{w\sim F_0} \max\left\{V_{ei}^0(w) - V_{ui}, 0\right\} \\ &+ \quad \lambda_{ui}^1 \mathbb{E}_{w\sim F_1} \max\left\{V_{ei}^1(w) - V_{ui}, 0\right\}\end{aligned} \tag{A-1}$$

where $V_{ei}^0(w)$ and $V_{ei}^1(w)$ are the values of part-time and full-time employment when receiving wage w. For workers who are employed in a part-time job ($h = h_0$) we have:

$$\begin{aligned}\rho_i V_{ei}^0(w) = wh_0 - T_i^0(wh_0) + \lambda_{ei}^0 \mathbb{E}_{x\sim F_0} \max\left\{V_{ei}^0(x) - V_{ei}^0(w), 0\right\} \\ + \lambda_{ei}^1 \mathbb{E}_{x\sim F_1} \max\left\{V_{ei}^1(x) - V_{ei}^0(w), 0\right\} + \delta_i(V_{ui} - V_{ei}^0(w))\end{aligned}$$

and for workers employed in a full-time job ($h = h_1$):

$$\begin{aligned}\rho_i V_{ei}^1(w) = wh_1 - T_i^1(wh_1) - C_i^1 + \lambda_{ei}^0 \mathbb{E}_{x\sim F_0} \max\left\{V_{ei}^0(x) - V_{ei}^1(w), 0\right\} \\ + \lambda_{ei}^1 \mathbb{E}_{x\sim F_1} \max\left\{V_{ei}^1(x) - V_{ei}^0(w), 0\right\} + \delta_i(V_{ui} - V_{ei}^1(w)).\end{aligned}$$

To proceed we define $q_i(w) : \Re \to \Re$ such that $V_{ei}^1(w) = V_{ei}^0(q_i(w))$. This means that a type i worker is indifferent between holding a full-time job with wage w and a part-time job with wage

[31] For notational simplicity, here we do not explicitly write the value functions or the resultant reservation wages as a function of the work opportunity b.

$q_i(w)$. It therefore follows that the value of a full-time job may be written as:

$$\rho_i V_{ei}^1(w) = wh_1 - T_i^1(wh_1) - C_i^1 + \lambda_{ei}^0 \int_{q_i(w)}^{\overline{w}_0} (V_{ei}^0(x) - V_{ei}^1(w)) dF_0(x) + \lambda_{ei}^1 \int_w^{\overline{w}_1} (V_{ei}^1(x) - V_{ei}^1(w)) dF_1(x) + \delta_i (V_{ui} - V_{ei}^1(w)).$$

We now wish to obtain the envelope condition $V_{ei}^{1\prime}(w)$. To do this we first perform integration by parts on the above to obtain:

$$\rho_i V_{ei}^1(w) = wh_1 - T_i^1(wh_1) - C_i^1 + \lambda_{ei}^0 \int_{q_i(w)}^{\overline{w}_0} \overline{F}_0(x) dV_{ei}^0(x) + \lambda_{ei}^1 \int_w^{\overline{w}_1} \overline{F}_1(x) dV_{ei}^1(x) + \delta_i (V_{ui} - V_{ei}^1(w)) \quad \text{(A-2)}$$

which when differentiated with respect to w yields:

$$\rho_i V_{ei}^{1\prime}(w) = h_1(1 - T_i^{1\prime}(wh_1)) - \lambda_{ei}^0 \overline{F}_0(q_i(w)) V_{ei}^{0\prime}(q_i(w)) q_i'(w) - \lambda_{ei}^1 \overline{F}_1(w) V_{ei}^{1\prime}(w) - \delta_i V_{ei}^{1\prime}(w).$$

Noting that $V_{ei}^{1\prime}(w) = V_{ei}^{0\prime}(q_i(w)) q_i'(w)$ we may simplify the above equation to obtain:

$$h_1(1 - T_i^{1\prime}(wh_1)) = (\delta_i + \rho_i + \lambda_{ei}^0 \overline{F}_0(q_i(w)) + \lambda_{ei}^1 \overline{F}_1(w)) V_{ei}^{1\prime}(w) \quad \text{(A-3)}$$

and performing a similar set of calculations for part-time jobs we arrive at the analogous expression:

$$h_0(1 - T_i^{0\prime}(wh_0)) = (\delta_i + \rho_i + \lambda_{ei}^0 \overline{F}_0(w) + \lambda_{ei}^1 \overline{F}_1(q_i^{-1}(w))) V_{ei}^{0\prime}(w). \quad \text{(A-4)}$$

Note also that equating equation A-2 (evaluated at wage w) with the analogous expression for part-time employment (evaluated at wage $q_i(w)$) implies that $q_i(w)$ is the solution to:

$$wh_1 - T_i^1(wh_1) - C_i^1 = q_i(w)h_0 - T_i^0(q_i(w)h_0)$$

which is equation 1 from the main text. We obtain this simple expression because, conditional on being in employment, the arrival rates for both full-time and part-time jobs are assumed independent of the individuals current hours of work so that it is only necessary to compare the instantaneous utility flows.[32] We can now calculate the reservation wage for unemployed workers. Let us denote ϕ_i as the lowest acceptable wage offer for *full-time* work. Since $V_{ui} = V^1_{ei}(\phi_i) = V^0_{ei}(q_i(\phi_i))$, the lowest acceptable wage offer for part-time work is then $q_i(\phi_i)$. We can therefore write equation A-1 as:

$$\begin{aligned}
\rho_i V_{ui} &= b - T^u_i + \lambda^0_{ui}\int_{q_i(\phi_i)}^{\overline{w}_0}(V^0_{ei}(w) - V_{ui})dF_0(w) + \lambda^1_{ui}\int_{\phi_i}^{\overline{w}_1}(V^1_{ei}(w) - V_{ui})dF_1(w) \\
&= b - T^u_i + \lambda^0_{ui}\int_{q_i(\phi_i)}^{\overline{w}_0}\overline{F}_0(w)dV^0_{ei}(w) + \lambda^1_{ui}\int_{\phi_i}^{\overline{w}_1}\overline{F}_1(w)dV^1_{ei}(w).
\end{aligned}$$

Substituting our expressions for $V^{0\prime}_{ei}(w)$ and $V^{1\prime}_{ei}(w)$ from equation A-4 and equation A-3 in to the above:

$$\begin{aligned}
\rho_i V_{ui} = b - T^u_i + \lambda^0_{ui}\int_{q_i(\phi_i)}^{\overline{w}_0}&\frac{h_0(1 - T^{0\prime}_i(wh_0))\overline{F}_0(w)}{\delta_i + \rho_i + \lambda^0_{ei}\overline{F}_0(w) + \lambda^1_{ei}\overline{F}_1(q_i^{-1}(w))}dw \\
&+ \lambda^1_{ui}\int_{\phi_i}^{\overline{w}_1}\frac{h_1(1 - T^{1\prime}_i(wh_1))\overline{F}_1(w)}{\delta_i + \rho_i + \lambda^0_{ei}\overline{F}_0(q_i(w)) + \lambda^1_{ei}\overline{F}_1(w)}dw.
\end{aligned}$$

By definition of the reservation wage we can set the above equal to $\rho_i V^1_{ei}(\phi_i)$ (from equation A-2) to obtain the following implicit equation defining ϕ_i in terms of the structural parameters of our model:

$$\begin{aligned}
\phi_i h_1 - T^1_i(\phi_i h_1) - C^1_i = b - T^u_i + (\lambda^0_{ui} - \lambda^0_{ei})\int_{q_i(\phi_i)}^{\overline{w}_0}&\frac{h_0(1 - T^{0\prime}_i(wh_0))\overline{F}_0(w)}{\delta_i + \rho_i + \lambda^0_{ei}\overline{F}_0(w) + \lambda^1_{ei}\overline{F}_1(q_i^{-1}(w))}dw \\
&+ (\lambda^1_{ui} - \lambda^1_{ei})\int_{\phi_i}^{\overline{w}_1}\frac{h_1(1 - T^{1\prime}_i(wh_1))\overline{F}_1(w)}{\delta_i + \rho_i + \lambda^0_{ei}\overline{F}_0(q_i(w)) + \lambda^1_{ei}\overline{F}_1(w)}dw
\end{aligned}$$

dividing both the numerator and denominator of the integral terms by δ_i, and performing a simple change of variable, we then obtain the simplified expression presented in equation 2 in the main text.

[32] In the more general case, this indifference condition would depend upon the distributions of wage offers.

B Identification

In Section 4.2 we discussed the identification of our model, and we now illustrate these ideas more formally. Here we set out to show that conditional on the set of transitional parameters, the observed distributions of part-time and full-time wages, together with the distributions of wages accepted by the unemployed are sufficient to separately identify the wage offer and reservation wage distributions. Once these are known, the structure of the model then permits identification of the opportunity cost and productivity distributions. In what follows, we let $G^U_{1i}(w)$ and $G^U_{0i}(w)$ denote the respective cumulative distribution functions of wages first accepted by type i unemployed workers in full-time and part-time jobs. Since individuals will accept any wage offer that is at least as high as their reservation wage, $G^U_{1i}(w)$ will be given by:

$$\begin{aligned} G^U_{1i}(w) = \int_{-\infty}^{w} \Pr(W_1 < w | W_1 > x) dA_{ui}(x) &= \int_{-\infty}^{w} \frac{F_1(w) - F_1(x)}{\overline{F}_1(x)} dA_{ui}(x) \\ &= A_{ui}(w) - \overline{F}_1(w) \left[\int_{\underline{w}}^{w} \frac{dA_{ui}(x)}{\overline{F}_1(x)} + A_{ui}(\underline{w}) \right] \end{aligned}$$

similarly the fraction of part-time jobs accepted that pay no more than $q_i(w)$ can be shown to be given by:

$$G^U_{0i}(q_i(w)) = A_{ui}(w) - \overline{F}_0(q_i(w)) \left[\int_{\underline{w}}^{w} \frac{dA_{ui}(x)}{\overline{F}_0(q_i(x))} + A_{ui}(\underline{w}) \right].$$

If we combine the above two expressions with the respective density functions of accepted wages, $g^U_{hi}(w) \equiv G^{U\prime}_{hi}(w)$ for $h \in \{0, 1\}$, we can write:

$$A_{ui}(w; F_0) = G^U_{0i}(q_i(w)) + \frac{\overline{F}_0(q_i(w)) g^U_{0i}(q_i(w))}{f_0(q_i(w))} \tag{A-5}$$

$$A_{ui}(w; F_1) = G^U_{1i}(w) + \frac{\overline{F}_1(w) g^U_{1i}(w)}{f_1(w)} \tag{A-6}$$

which therefore demonstrates that the distribution of reservation wages amongst the unemployed on support $[\underline{w}_i, \overline{w}_i]$ is identified given knowledge of the wage offer functions F_0 and F_1.[33] Furthermore, the requirement that $A_{ui}(w; F_1) = A_{ui}(w; F_0)$ allows us to identify the work cost parameter

[33] Recall from Section 3.3.1 that $\underline{w}_i \equiv \min\{\underline{w}_1, q_i^{-1}(\underline{w}_0)\}$ and $\overline{w}_i \equiv \max\{\overline{w}_1, q_i^{-1}(\overline{w}_0)\}$.

C_i^1.

Substituting equations A-5 and A-6 into equations 7 and 8 from the main text, we can eliminate the unobserved reservation wage distribution to obtain the following differential equations governing the evolution of the two wage offer distributions:

$$F_1'(w) = m_{1i}g_{1i}(w)R_i^1(w; F_0, F_1) \tag{A-7}$$

$$F_0'(q_i(w)) = m_{0i}g_{0i}(q_i(w))R_i^0(w; F_0, F_1) \tag{A-8}$$

where:

$$R_i^1(w; F_0, F_1) \equiv \frac{(1 + \kappa_{ei}^0 \overline{F}_0(q_i(w)) + \kappa_{ei}^1 \overline{F}_1(w)) - u_i g_{1i}^U(w)\kappa_{ui}^1 \overline{F}_1(w)}{\kappa_{ui}^1 u_i G_{1i}^U(w) + \kappa_{ei}^1 \left(m_{0i}G_{0i}(q_i(w)) + m_{1i}G_{1i}(w)\right)}$$

and:

$$R_i^0(w; F_0, F_1) \equiv \frac{(1 + \kappa_{ei}^0 \overline{F}_0(q_i(w)) + \kappa_{ei}^1 \overline{F}_1(w)) - u_i g_{0i}^U(q_i(w))\kappa_{ui}^0 \overline{F}_0(q_i(w))}{\kappa_{ui}^0 u_i G_{0i}^U(q_i(w)) + \kappa_{ei}^0 \left(m_{0i}G_{0i}(q_i(w)) + m_{1i}G_{1i}(w)\right)}.$$

Equations A-7 and A-8 define a system of differential equations, which together with the initial conditions $F_1(\underline{w}_i) = 0$ and $F_0(q_i(\underline{w}_i)) = 0$, establishes non-parametric identification of both wage offer functions conditional on the set of transitional parameters. Identification of the underlying opportunity cost distribution and the productivity distributions then follows as described in Section 4.3.

C Notation summary

Indexing

$i \in \mathcal{I}$	individual observed type
$h \in \{0, 1\}$	hours of work
$j \in \{u, e\}$	employment state
w	wages
p	firm productivity
v	job vacancies
b	unobserved utility flow for unemployed workers

Workers

n_i	fraction of type i workers
ρ_i	worker discount rate
C_i^h	additive disutility flow of working h hours
s_{ji}^h	exogenous worker search intensity
$\phi_i(b)$	full-time work reservation wage
$q_i(w)$	part-time work indifference wage
$H_i(b)$	cumulative distribution of unobserved utility flow b
$A_i(w)$	cumulative distribution of reservation wages
$A_{ui}(w)$	cumulative distribution of reservation wages amongst unemployed
$A_{ei}(w)$	cumulative distribution of reservation wages amongst employed

Transitional parameters

λ_{ji}^h	job arrival rates
δ_i	job destruction rate
κ_{ji}^h	defined as λ_{ji}^h/δ_i

Employment states

u_i	unemployment rate
m_{0i}	part-time work rate
m_{1i}	full-time work rate

Taxes

$T_i^h(wh)$	net taxes at hours h and earnings wh
$T_i^{h\prime}(wh)$	marginal tax rate at hours h and earnings wh
T_i^u	net taxes when unemployed

Wage distributions

$F_h(w)$	distribution of wage offers
$\overline{F}_h(w)$	defined as $1 - F_h(w)$
$G_{hi}(w)$	cumulative distribution of wages amongst employed
$g_{hi}(w)$	density of wages amongst employed
$[\underline{w}_h, \overline{w}_h]$	support of wage distributions
$\underline{w}_i$	defined as $\min\{\underline{w}_1, q_i^{-1}(\underline{w}_0)\}$
$\overline{w}_i$	defined as $\max\{\overline{w}_1, q_i^{-1}(\overline{w}_0)\}$

Firms

$\Gamma_h(p)$	cumulative distribution of firm productivity
$\gamma_h(p)$	density of firm productivity
$[\underline{p}_h, \overline{p}_h]$	support of firm productivity
$K_h(p)$	optimal wage policy
$v_h(p)$	optimal recruiting policy
$c(p, v)$	vacancy flow cost
$l_{hi}(w, v)$	steady state employment of worker i
$L_h(w, v)$	total steady state employment: $\sum_i n_i l_{hi}(w)$
$\bar{l}_{hi}(w)$	defined such that $l_{hi}(w, v) = \bar{l}_{hi}(w) v / V_h$
$\overline{L}_h(w)$	defined as $\sum_i n_i \bar{l}_{hi}(w)$
$\pi(p, v)$	steady state profit flow: $(p - w) h_h \overline{L}_h(w)$

Matching technology

V_h	aggregate stock of job vacancies
S_h	total search intensity
$M_h(S_h, V_h)$	aggregate matching function
θ_h	matching function elasticity

Further notation from appendix

V_{ui}	value of unemployment
$V_{ei}^{h}(w)$	value of employment
$G_{hi}^{U}(w)$	cumulative distribution of wages accepted by unemployed
$g_{hi}^{U}(w)$	density of wages accepted by unemployed