Consumption and Retirement: Evaluating Social Security Reform with a Life–cycle Model*

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Abstract

This paper estimates the parameters of a life–cycle model in order to evaluate the effect of (a potential reform of) the Social Security system on retirement and consumption choices. We assume intratemporally nonseparable preference orderings and endogenous retirement. The specification predicts a change in consumption at retirement; and we use the empirical magnitude of the change, together with desired retirement age, to identify key parameters such as the curvature of the utility function. We then simulate the possible long–run effect of a Social Security reform in which individuals no longer face the OASI payroll tax after 62, and their subsequent earnings have no bearing on their Social Security benefits. This simulation indicates that retirement ages would rise by approximately 2 years on average; an increase we view as warranting further attention.

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1. Introduction

Although American longevity is increasing, over the last 50 years male retirement ages in the U.S. have generally moved downward, if at all. One explanation for this pair of facts is that leisure is a luxury good so that demand for it rises with living standards (see, e.g., Costa, 1998). Another is that taxes and government programs such as Social Security have tended to encourage earlier retirement. In view of the second possibility, this paper considers the potential effects of a simple but, to our knowledge, novel reform of Social Security: at age 62, each person's benefits for all prospective retirement ages would be determined from the current formula; however, other than collecting the benefits, the individual's connections to the Social Security System would then close. In other words, at age 62 the OASI tax would cease to apply (as would benefit adjustments for subsequent earnings) — so that individuals who continued to work would receive a 10.6 percent boost to their pay.

Our analysis is based on the familiar life–cycle model (e.g., Diamond [1965], Tobin [1967], Auerbach and Kotlikoff [1987], Modigliani [1986], Hubbard *et al.* [1995], Altig *et al.* [2001], and many others). We present a formulation in which jobs require full-time work; thus, each household's options include choice of retirement age and a lifetime trajectory of consumption/saving decisions. The benefit to a household of late retirement is greater lifetime earnings; the cost is forgone leisure. As in Auerbach and Kotlikoff [1987], a household derives a flow of services from its consumption expenditure and leisure. The service flow, in turn, yields utility through a conventional isoelastic utility function. Although our "basic model" ignores health considerations, we present a second formulation with a stochastic, but insurable, chance of disability.

We estimate our life–cycle model's parameters using pseudo–panel consumption data from the Consumer Expenditure Survey (CEX) and lifetime earning and retirement data from the Health and Retirement Study (HRS). We pursue what we believe is a novel estimation strategy. The model's preference ordering is intratemporally nonseparable in consumption and leisure, and it predicts a change in consumption expenditure at a household's retirement. A number of recent empirical studies have described a drop in household consumption expenditure at the time of retirement (e.g., Banks *et al.* [1998], Bernheim *et al.* [2001], Hurd and Rohwedder [2003], Haider and Stephens [2004], Aguiar and Hurst [2004], and others); we use the magnitude of the drop, which this paper measures from CEX data, as well as age of retirement, which we measure from the HRS, to identify our key parameters.

Our analysis predicts that discontinuing the Social Security payroll tax after, say, age 62, could lead households to postpone their retirement by perhaps as much as 2 years, on average. These results thus suggest that the social security system has important effects on the labor supply of older Americans. Our plan is distinguished from most current proposals involving changes in the funding of the system or changes in the early or normal retirement ages in that it emphasizes efficiency over direct solvency concerns. Our plan would not directly remedy or even reduce the current solvency problems of the Social Security System, but our proposed reforms could mitigate inefficiencies from tax distortions to private labor supply decisions — and longer careers could contribute to the nation's income–tax base, could tend to raise GDP and GDP per capita, and could augment households' lifetime resources from earnings.

This paper joins a large literature which aims to evaluate the effects of social security systems on labor supply. See Feldstein and Liebman [2002] for a review. By applying an explicit life–cycle model we differ from large portions of this literature which seek reduced form estimates. Implementing a structural model allows us, most importantly, to evaluate the *lifecycle* effects of both the existing social security system and counterfactual reforms on retirement and consumption. Our analysis is distinguished from other recent structural lifecycle studies of retirement, such as Bound *et al.* [2005], Blau [2005] and French [2005], by its emphasis on a novel reform, by its use of lifecycle consumption (rather than wealth data at older age) data, and by its effort to build a rich but tractable model that permits analytic rather than numerical insights.

The organization of this paper is as follows. Section 2 describes our basic model and its re-formulation with stochastic disability. Section 3 discusses our pseudo–panel data on consumption expenditure, our HRS data on lifetime earnings and retirement ages, and it displays our parameter estimates. Section 4 presents simulated outcomes for the Social Security reform outlined above. Section 5 concludes.

2. Model

This section presents our basic model and derives two optimality conditions on which our empirical analysis depends. Then it elaborates the framework to include the possibility of disability.

2.1 Basic Model. We have an overlapping generations model. We restrict this analysis to couples. A household begins when its male reaches age S. He marries at age S_0 and has children $k = 1, ..., K$ at age S_k. Males die at age T^M; females at age T^F. Set

$$T \equiv \max \{T^M , T^F\} .$$

A key feature of our model is intratemporally nonseparable preferences. A household's current utility depends on its current service flow from market consumption and leisure. We assume the service flow is a Cobb–Douglas function of household consumption, c, and leisure, ℓ, per capita:

$$f(c\,,\,\ell) \equiv [c]^{\alpha} \cdot [\ell]^{1-\alpha}\,, \quad \alpha \in (0,1)\,.$$

For couples, the man and woman work full time until retirement and retire when the male is age R. Males work in the labor force. Females divide their work hours between home production and the labor force. We normalize $\ell = 1$ post retirement; prior to retirement,

$$\ell = \bar{\ell} \in (0\,,\,1)\,.$$

A household's utility flow is an isoelastic function of its service flow:

$$\frac{[f(c\,,\,\ell)]^{\gamma}}{\gamma}\,, \quad \gamma < 1\,.$$

Our treatment of life–cycle changes in family composition follows Tobin [1967]. For household i at age t define

$$\chi^S(i\,,\,t) \equiv \begin{cases} 1\,, & \text{if age–}t\text{ household includes a spouse,} \\ 0\,, & \text{otherwise.} \end{cases}$$

If household i at age t has K "kids" of ages 0-22, define

$$\chi^K(i\,,\,t) \equiv K\,.$$

The number of "equivalent adults" in the household when it is age t is

$$n_{it} \equiv 1 + \chi^S(i\,,\,t) \cdot \xi^S + \chi^K(i\,,\,t) \cdot \xi^K\,, \tag{1}$$

where ξ^S and ξ^K are positive parameters. Economies of scale in household operation and/or the public–good nature of household consumption might well leave $\xi^S < 1$ and $\xi^K < 1$. The utility flow of household i at age t is

$$\begin{aligned} u(c_{it}) &= \frac{1}{\gamma} \cdot n_{it} \cdot [\frac{c_{it}}{n_{it}}]^{\alpha\cdot\gamma} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\,, && \text{for } t \in [S\,,\,R)\,, \\ v(c_{it}) &= \frac{1}{\gamma} \cdot n_{it} \cdot [\frac{c_{it}}{n_{it}}]^{\alpha\cdot\gamma} + \phi(i)\,, && \text{for } t \in [R\,,\,T]\,. \end{aligned} \tag{2}$$

In other words, flow utility depends upon consumption per equivalent adult and leisure per adult, weighted by number of equivalent adults. Following Bound*et al.* [2005], Gustman–Steinmeier [1986], Bernheim *et al.* [2001], and others, we allow additively separable inter-household differences in taste for leisure: household i gains extra utility $\phi(i)$ from leisure after retirement. However, this paper assumes that the population mean of these extra terms is 0.

Household i solves the following life–cycle problem:

$$\max_{R_i\,,\,c_{it}} \int_S^{R_i} e^{-\rho\cdot t}\cdot u(c_{it})\,dt + \varphi\big(a_{i,R_i} + B_i(R_i)\cdot e^{r\cdot R_i}\,,\,R_i\big) \tag{3}$$

$$\text{subject to:}\quad \dot{a}_{it} = r\cdot a_{it} + y_{it} - c_{it}\,,$$

$$a_{iS} = 0\,,$$

where ρ is the subjective discount rate; the household's adult male supplies e^M_{it} "effective hours" in the labor market per hour of work time; the adult female supplies e^F_{it} "effective hours" for home or market production; the wage rate per effective hour is w; the income–tax rate is τ; the Social Security and Hospital Insurance tax rate is τ^{ss}; household net worth (i.e., "assets") is a_{it}; and,

$$y_{it} \equiv \begin{cases} (1-\bar{\ell})\cdot[e^M_{it} + e^F_{it}]\cdot w\cdot(1-\tau-\tau^{ss})\,, & \text{for } S \le t < R_i, \\ 0\,, & \text{otherwise.}\end{cases}$$

"Effective hours" change with age, reflecting an individual's cumulative experience and economywide technological progress. The function $\varphi(.)$ comes from

$$\varphi\big(A + B_i(R_i)\cdot e^{r\cdot R_i}\,,\,R_i\big) \equiv \max_{c_{it}} \int_{R_i}^{T} e^{-\rho\cdot t}\cdot v(c_{it})\,dt \tag{4}$$

$$\text{subject to:}\quad \dot{a}_{it} = r\cdot a_{it} - c_{it}\,,$$

$$a_{iR_i} = A + B_i(R_i)\cdot e^{r\cdot R_i} \quad \text{and} \quad a_{iT} \ge 0\,,$$

where the age–0 present value of capitalized Social Security, Medicare, and private defined–benefit pension benefits are $B_i(R_i)$. A household maximizes (3)-(4) taking r, w, τ, τ^{ss}, e^M_{it}, e^F_{it}, and $B(.)$ as given. Social Security benefits only begin at age $\max\{R_i\,,\,62\}$; Medicare benefits begin at age 65. Social Security and private–pension benefits depend upon retirement age. Social Security benefits are taxed at rate $\tau/2$, private–pension benefits at rate τ, and Medicare benefits are not taxed.

One aspect of (3)-(4) that is slightly nonstandard is that the criterion and the asset constraint are only piecewise (twice) continuously differentiable — which the following lemma shows that one can easily handle by seeking a solution to first–order conditions in which the costate variable has a continuous time path.

Lemma 1: *Suppose that discontinuities in n_{it} and labor supply at retirement make criterion (3)-(4) and right–hand side of the asset equation discontinuous at ages t_j. Define a present–value Hamiltonian with costate variable λ:*

$$\mathcal{H} \equiv \begin{cases} e^{-\rho \cdot t} \cdot u(c_{it}) + \lambda_{it} \cdot [r \cdot a_{it} + y_{it} - c_{it}] \,, & \textit{for } t \in [S \,, R_i) \,, \\ e^{-\rho \cdot t} \cdot v(c_{it}) + \lambda_{it} \cdot [r \cdot a_{it} - c_{it}] \,, & \textit{for } t \in [R_i \,, T] \,. \end{cases} \tag{5}$$

Then for a given R_i, the following conditions are necessary and sufficient for an optimum:

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \quad \textit{all} \quad t \,, \tag{i}$$

$$\dot{\lambda}_{it} = - \frac{\partial \mathcal{H}}{\partial a} \quad \textit{all} \quad t \,, \tag{ii}$$

$$\dot{a}_{it} = r \cdot a_{it} + y_{it} - c_{it} \quad \textit{all} \quad t \neq R_i \,, \tag{iii}$$

$$a_{iS} = 0 \,, \quad a_{iR_i+} = a_{iR_i-} + B_i(R_i) \cdot e^{r \cdot R_i} \,, \quad \textit{and} \quad a_{iT} = 0 \,. \tag{iv}$$

Proof: See Appendix.

Our empirical analysis rests on two features of solutions to (3)-(4). The first involves changes in c_{it} at, and after, retirement. Solution of (3)-(4) yields

Proposition 1: *Let household i choose to retire at age $R = R_i$. Suppose that discontinuities in n_{it} and labor supply at retirement make the criterion and right–hand side of the asset equation discontinuous at monotone increasing ages t_j, $j = 1, ..., J$. Let $t_0 \equiv S$ and $t_{J+1} \equiv T$. Then a solution of (3)-(4) has*

$$\frac{\dot{c}_{it}}{c_{it}} = \frac{r - \rho}{1 - \alpha \cdot \gamma} \,, \tag{6}$$

$$c_{it+} = M_{ij} \cdot c_{it-} \,, \quad M_{ij} \equiv \frac{n_{it+}}{n_{it-}} \,, \quad t = t_j \,, \, j = 1, ..., J, \; \textit{but } t \neq R \,, \tag{7}$$

$$c_{iR+} = M_{ij} \cdot c_{iR-} \,, \quad M_{ij} \equiv [\bar{\ell}]^{-\frac{(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}} \,, \quad t = t_j = R \,. \tag{8}$$

Letting $M_{i0} = 1$, initial household consumption is

$$c_{iS} = \psi(i \,, R_i) \equiv \frac{\int_S^{R_i} e^{-r \cdot t} \cdot y_{it} \, dt + e^{-r \cdot R_i} \cdot B_i(R_i)}{\sum_{j=0}^{J} \left[\prod_{k=0}^{j} M_{ik}\right] \cdot \int_{t_j}^{t_{j+1}} e^{-r \cdot t} \cdot e^{\frac{r-\rho}{1-\alpha\cdot\gamma} \cdot t} \, dt} \,. \tag{9}$$

Proof: See Appendix.

For this paper, the result of greatest interest is (8). The intuition for (8) is as follows. The inputs of a bivariate neoclassical production function are complementary in the sense that more of one raises the other's marginal product. If $u(.)$ were linear, a household would desire to raise its consumption c_{it} at retirement to take advantage of this complementarity. If $u(.)$ departs from linearity, two competing forces arise at retirement: input complementarity makes a household want to increase c_{it}; the household's desire to "smooth" its service flow intertemporally together with the increase in leisure upon cessation of work lead the household to want to decrease c_{it}. Condition (8) shows that the first force predominates for $\gamma \in (0\,,\,1)$, and the second for $\gamma < 0$. The sign and magnitude of the drop in household expenditure at retirement is one of the two key elements in our strategy for estimating α and γ.

The second feature of solutions to (3)-(4) on which our empirical analysis depends comes from the first–order condition for R_i. We have

Proposition 2: *Given a solution to (3)-(4), at $R = R_i \in (S\,,\,T)$ we have*

$$\left[\alpha \cdot [n_{iR}]^{1-\alpha\cdot\gamma} \cdot [c_{iR-}]^{\alpha\cdot\gamma-1} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\right] \cdot [y_{iR-} - c_{iR-} + c_{iR+} + B_i'(R) \cdot e^{r\cdot R}] =$$
$$\frac{1}{\gamma} \cdot [n_{iR}]^{1-\alpha\cdot\gamma} \cdot \left[[c_{iR+}]^{\alpha\cdot\gamma} - [c_{iR-}]^{\alpha\cdot\gamma} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\right] + \phi(i)\,. \tag{10}$$

Proof: See Appendix.

The intuitive idea of (10) is as follows. The left–hand side assesses the advantage of a marginal increase in retirement age R: y_{iR-} measures additional earnings contingent upon postponing retirement; $c_{iR+} - c_{iR-}$ registers the fact that if desired consumption declines after retirement, earnings may stretch farther; and, $B_i'(R) \cdot e^{r\cdot R}$ measures incremental pension–benefit gains. We multiply the sum of these dollar advantages by the costate variable to convert to units of utility. The right–hand side captures the marginal cost of postponing retirement, namely, the difference in the flow of utility after versus before retirement. Omitting $\phi(i)$, condition (8) shows the latter difference would always be positive.[1]

[1] Using (8), the sign on the right–hand side of (10) is

$$\text{sgn}(\frac{1}{\gamma}) \cdot \text{sgn}([\bar{\ell}]^{\frac{-(1-\alpha)\cdot\gamma\cdot\alpha\cdot\gamma}{1-\alpha\cdot\gamma}} - [\bar{\ell}]^{(1-\alpha)\cdot\gamma}) \cdot \text{sgn}([c_{iR-}]^{\alpha\cdot\gamma}) =$$
$$\text{sgn}(\frac{1}{\gamma}) \cdot \text{sgn}(1 - [\bar{\ell}]^{\frac{(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}}) \cdot \text{sgn}([\bar{\ell}]^{\frac{-(1-\alpha)\cdot\gamma\cdot\alpha\cdot\gamma}{1-\alpha\cdot\gamma}}) =$$

2.2 Discussion. In this section we digress to discuss in greater detail two important features of our model: discrete labor supply choices and intratemporally non-separable preferences.

Households in our analysis must either work full time or retire. While in practice employers do offer part–time jobs, the rate of pay is, on average, substantially lower than that for full–time work.[2] As Rust and Phelan [1997] write,

> The finding that most workers make discontinuous transitions from full–time work to not working, and the finding that the majority of the relatively small number of 'gradual retirees' reduce their annual hours of work by taking on a sequence of lower wage partial retirement 'bridge jobs' rather than gradually reducing hours of work at their full-time pre-retirement 'career job' suggests the existence of explicit or implicit constraints on the individual's choice of hours of work. [p.786]

An indivisible work day is also consistent with the fact that U.S. data show little trend in male work hours or participation rates after 1940, except for a trend toward earlier retirement 1940-80 (e.g., Pencavel [1986], Blundell and MaCurdy [1999], and Burkhauser *et al.* [1999]).

While intratemporal additivity is the most common specification for utility in the life–cycle literature, our nonseparable specification is similar to a number of other papers, including Auerbach and Kotlikoff [1987].[3] An intratemporally additive utility function leaves any routine drop in consumption at retirement unexplained. Auerbach and Kotlikoff employ a CES aggregator for service flows, assuming an elasticity of substitution for $f(.)$ of 0.75 in their "base case;" Cooley and Prescott [1995], for example, use the same functional forms as we do. Although deviating from the Cobb–Douglas function would not affect Lemma 1 or the substance of Proposition 2, it would complicate expressions (6) and (8) in Proposition 1. Whether our data (see below) could successfully distinguish between an elasticity of substitution for the inputs of $f(.)$ of, say, 0.75, and 1.00, remains, at this point,

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$$\mathrm{sgn}(\frac{1}{\gamma}) \cdot \mathrm{sgn}(1 - [\bar{\ell}]^{\frac{(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}})\,.$$

Recall that $\bar{\ell} \in (0,1)$ and $\alpha \in (0,1)$. If $\gamma \in (0,1)$, the sign of both terms in the last product is positive. If $\gamma < 0$, each is negative.

[2] Reasons for the wage penalty for part-time work include daily fixed costs of startup and shutdown, scheduling and coordination problems, employer concern for timely return on training investments, and the fixed–cost nature of some employee benefits (e.g., Hurd [1996]).

[3] E.g., King *et al.* [1988], Hurd and Rohwedder [2003], and Cooley and Prescott [1995].

a topic for future research.

2.3 Specification with Disability. This section augments the basic model to include a stochastic chance of disability. In the basic model, at its inception a household determines its retirement age. In practice, however, the possible onset of disability may complicate planning. This paper considers only the case with actuarially fair insurance.

Assume that a household's health status is either "not disabled" or "disabled," that a disabled household cannot work, and that disability is an absorbing state. Once a household becomes disabled, it remains disabled until it reaches its (previously) chosen retirement age R, at which point we reclassify it as retired. Let $p(t)$ = probability that a household becomes disabled at age t. Assume

$$\int_S^T p(t)\,dt = 1\,.$$

Define

$$P(s) \equiv 1 - \int_S^s p(t)\,dt = \int_s^T p(t)\,dt\,, \tag{11}$$

so that $P(s)$ is the probability of becoming disabled after age s.

At age $t < R$, a nondisabled household purchasing term disability insurance during the interval $[t\,,\,t+dt)$ would pay an insurance premium with annual rate $p(t)\cdot X_{it}/P(t)$ — in other words, it would pay total premiums $p(t)\cdot X_{it}\,dt/P(t)$, to receive (current–dollar) lump–sum benefit X_{it} in the event of disability. Whether disabled or not, household i receives capitalized sum $B_i(R_i)\cdot e^{r\cdot R_i}$, in current dollars, at its chosen retirement age R_i; thus, retirement benefits implicitly include a disability–insurance component in our framework, and disability insurance need only tide a household over until its retirement age.

Disabled households benefit from full-time leisure; presumably disability lowers their utility as well. Provided the latter implies an additively separable term in the flow utility function, the term does not affect household behavior, since the onset of disability is an exogenous event. For simplicity, we therefore omit it from the analysis.

Behavior after retirement is the same as before; hence, problem (4) remains as above. If household i happens to become disabled at age $D = D_i < R_i = R$ and has insurance payout X_{iD}, its cumulative utility for ages $t \in [D\,,\,T]$ is

$$\bar{\varphi}\big(A + X_{iD}\,,\,D\,,\,R\big) \equiv \max_{\bar{c}_{it}} \int_D^R e^{-\rho\cdot t}\cdot v(\bar{c}_{it})\,dt + \varphi\big(\bar{a}_{iR-} + B_i(R)\cdot e^{r\cdot R}\,,\,R\big) \tag{12}$$

$$\text{subject to:} \quad \dot{\bar{a}}_{it} = r \cdot \bar{a}_{it} - \bar{c}_{it} \, ,$$

$$\bar{a}_{iD+} = A + X_{iD} \, ,$$

$$\bar{a}_{iR} \geq 0 \, .$$

We are now ready to set out a household's complete life–cycle problem. Continue to let $D = D_i$ and $R = R_i$. At its inception, household i solves

$$\max_{R\,,\,c_{it},X_{it}} \int_S^R p(D) \cdot [\int_S^D e^{-\rho \cdot t} \cdot u(c_{it}) \, dt + \bar{\varphi}(a_{iD-} + X_{iD} \, , \, D \, , \, R)] \, dD +$$
$$[1 - \int_S^R p(t) \, dt] \cdot \int_S^R e^{-\rho \cdot t} \cdot u(c_{it}) \, dt +$$
$$[1 - \int_S^R p(t) \, dt] \cdot \varphi\big(a_{iR+} + B_i(R) \cdot e^{r \cdot R} \, , \, R\big) \tag{13}$$

$$\text{subject to:} \quad \dot{a}_{it} = r \cdot a_{it} + y_{it} - c_{it} - \frac{p(t) \cdot X_{it}}{P(t)} \, ,$$

$$a_{iS} = 0 \, .$$

The criterion's first term captures utility for ages 0 to R if the household becomes disabled at age $D < R$, the second component captures utility over the same ages if the household reaches its chosen retirement age R without first becoming disabled, and the last term captures utility from retirement to death if the household reaches the former without becoming disabled. Recall that when $D < R$, $\bar{\varphi}(.)$ incorporates the household's utility from R to T as well as from D to R.

We can simplify the criterion of (13). Looking at the criterion's first integral, we have

$$\int_S^R p(D) \cdot \int_S^D e^{-\rho \cdot t} \cdot u(c_{it}) \, dt \, dD =$$
$$\int_S^R \int_t^R p(D) \cdot e^{-\rho \cdot t} \cdot u(c_{it}) \, dD \, dt =$$
$$\int_S^R [P(t) - P(R)] \cdot e^{-\rho \cdot t} \cdot u(c_{it}) \, dt \, ,$$

where the middle step uses Fubini's theorem. This enables us to rewrite the whole criterion as

$$\int_S^R [P(t) \cdot e^{-\rho \cdot t} \cdot u(c_{it}) + p(t) \cdot \bar{\varphi}(a_{it-} + X_{it}, t, R)]\, dt +$$
$$P(R) \cdot \varphi\big(a_{iR-} + B_i(R) \cdot e^{r \cdot R}\,,\, R\big)\,. \tag{14}$$

We can then establish an analogue to Proposition 1.

Proposition 3: *Consider the model with disability. Let household i choose to retire at age $R = R_i$. Suppose that discontinuities in n_{it} and labor supply at retirement make the criterion and right–hand side of the asset equation discontinuous at ages t_j, $j = 1, ..., J$. Let $t_0 \equiv S$ and $t_{J+1} \equiv T$. Then a solution of (3)-(4) has*

$$\frac{\dot{c}_{it}}{c_{it}} = \frac{r - \rho}{1 - \alpha \cdot \gamma}\,, \tag{14}$$

$$c_{it+} = \frac{n_{it+}}{n_{it-}} \cdot c_{it-}\,, \quad t = t_j\,, \quad j = 1, ..., J, \quad \textit{but} \quad t \neq R\,, \tag{15}$$

$$c_{iR+} = [\bar{\ell}]^{-\frac{(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}} \cdot c_{iR-}\,. \tag{16}$$

If the household becomes disabled at age $D < R$, its change in consumption at that age is

$$c_{iD+} = [\bar{\ell}]^{-\frac{(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}} \cdot c_{iD-}\,. \tag{17}$$

Let M_{ij} be the consumption jump at t_j, $j = 1, ..., J$, and let $M_{i0} = 1$. When disability occurs at age $D < R$, drop the breakpoint at age R, add instead one at D with the corresponding consumption jump being as in (17), and reorder subscripts j if necessary so that $t_j < t_{j+1}$. Since one breakpoint occurs at age $\min\{D\,,\, R\}$, write $t_j = t_j(D)$ when $D < R$ and $t_j = t_j(R)$ otherwise. Then the initial consumption of household i is

$$c_{iS} = \bar{\psi}(i\,,\, R) \equiv$$
$$\frac{\int_S^R p(D) \cdot [\int_S^D e^{-r \cdot t} \cdot y_{it}\, dt]\, dD + P(R) \cdot \int_S^R e^{-r \cdot t} \cdot y_{it}\, dt + e^{-r \cdot R} \cdot B_i(R)}{DEN}\,, \tag{18}$$

$$DEN \equiv \int_S^R p(D) \cdot \Big[\sum_{j=0}^{J} [\prod_{k=0}^{j} M_{ik}] \cdot \int_{t_j(D)}^{t_{j+1}(D)} e^{-r \cdot t} \cdot e^{\frac{r-\rho}{1-\alpha\cdot\gamma}\cdot t}\, dt\Big]\, dD +$$
$$P(R) \cdot \sum_{j=0}^{J} [\prod_{k=0}^{j} M_{ik}] \cdot \int_{t_j(R)}^{t_{j+1}(R)} e^{-r \cdot t} \cdot e^{\frac{r-\rho}{1-\alpha\cdot\gamma}\cdot t}\, dt\,.$$

Proof: See Appendix.

The new feature of Proposition 3 is the change in consumption upon pre-retirement disability, namely, condition (17). The intuition for (17) is as follows. Although the possibility of disability reduces lifetime resources (cf., (9) and (18)), households adopt full insurance. This does not mean that the household perfectly insures its *income* against disability. Rather the household chooses a level of insurance such that its continuation utility, net of the separable disutility of disability is preserved. Thus, the need to pay insurance premiums causes lifetime consumption to be lower, but the actual onset of disability causes a household no further financial hardship. The latter fact implies that a household chooses the same consumption change at the onset of disability as at the arrival of its planned retirement age in the absence of disability.

The analogue to Proposition 2 determines the utility–maximizing retirement age for household i:

Proposition 4: *Given a solution to (4) and (12)-(13), at $R = R_i \in (S\,,T)$ we have*

$$\big[\alpha \cdot [n_{iR}]^{1-\alpha\cdot\gamma} \cdot [c_{iR-}]^{\alpha\cdot\gamma-1} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\big] \cdot [y_{R-} - c_{iR-} + c_{iR+} + B_i'(R) \cdot e^{r\cdot R} - \frac{p(R)\cdot X_{iR}}{P(R)}] =$$

$$\frac{1}{\gamma} \cdot [n_{iR}]^{1-\alpha\cdot\gamma} \cdot \big[[c_{iR+}]^{\alpha\cdot\gamma} - [c_{iR-}]^{\alpha\cdot\gamma} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\big] + \phi(i)\,, \tag{19}$$

when $R_i \leq D_i$. Furthermore,

$$X_{iR} = y_{iR-} - c_{iR-} + c_{iR+}\,. \tag{20}$$

Proof: See Appendix.

The logic of Proposition 4 resembles Proposition 2, namely, condition (19) balances extra wages and retirement benefits against the utility gains from more leisure. What is new is that only earnings net of the corresponding disability–insurance cost constitute an advantage for postponing retirement. This does not imply that the possibility of disability necessarily leads to earlier retirement than in the basic model — as we have seen, the chance of disability lowers lifetime consumption (see the numerator of (18)); and, dividing (19) by $[c_{iR+}]^{\alpha\cdot\gamma}$ and using (16), we can see that $1/c_{iR-}$ remains on the left–hand side of (19). In other words, although the chance of disability creates a substitution effect that tends to promote earlier retirement, an income effect that tends to promote longer working lives arises as well.

2.4 Estimation equations. This section derives the pair of equations that we use for estimation. Since the model with disability includes the basic model as a special case, we work with the former.

Our first equation comes from Proposition 3. Consider household i. When it is age s, it has experienced a set of ages, say, $\kappa(s\,,\,i)$ with breakpoints from family composition changes and either retirement or pre-retirement disability. Let the consumption-level adjustment factors corresponding to the breakpoints be M_{ik}, as in Proposition 3. Then Proposition 3 shows that the household's current consumption is

$$c_{is} = \bar{\psi}(i\,,\,R_i)\cdot[\prod_{k\in\kappa(s\,,\,i)} M_{ik}]\cdot e^{\frac{r-\rho}{1-\alpha\cdot\gamma}\cdot(s-S)}$$

(see (18)). Let D_i be the household's age of disability. Define

$$\chi^{RD}(i\,,\,t) \equiv \begin{cases} 1\,, & \text{if } t \geq \min\{R_i\,,\,D_i\}, \\ 0\,, & \text{otherwise.} \end{cases}$$

Noting that $\kappa(s\,,\,i) \subseteq \kappa(s+1\,,\,i)$, we have

$$\ln(c_{i,s+1}) - \ln(c_{i,s}) = \sum_{k\in\kappa(s+1,i)-\kappa(s,i)} \ln\big(M_{ik}\big) + \frac{r-\rho}{1-\alpha\cdot\gamma} \approx$$
$$\frac{r-\rho}{1-\alpha\cdot\gamma} + \xi^S\cdot[\chi^S(i\,,\,s+1) - \chi^S(i\,,\,s)] + \xi^K\cdot[\chi^K(i\,,\,s+1) - \chi^K(i\,,\,s)] +$$
$$[\bar{\ell}]^{\frac{(1-\alpha)\cdot\gamma}{\alpha\cdot\gamma-1}}\cdot[\chi^{RD}(i\,,\,s+1) - \chi^{RD}(i\,,\,s)]\,,$$

where the approximation comes from a first–order Taylor series.

Our consumption data (see below) provides repeated, population–representative cross sections; so, we estimate from a pseudo panel:

$$\sum_{i'} \ln(c_{i',s+1}) - \sum_{i} \ln(c_{i,s}) =$$
$$\frac{r-\rho}{1-\alpha\cdot\gamma} + \xi^S\cdot[\sum_{i'} \chi^S(i'\,,\,s+1) - \sum_{i} \chi^S(i\,,\,s)] +$$
$$\xi^K\cdot[\sum_{i'} \chi^K(i'\,,\,s+1) - \sum_{i} \chi^K(i\,,\,s)] +$$
$$\frac{(1-\alpha)\cdot\gamma\cdot\ln(\bar{\ell})}{\alpha\cdot\gamma-1}\cdot[\sum_{i'} \chi^{RD}(i'\,,\,s+1) - \sum_{i} \chi^{RD}(i\,,\,s)] + \upsilon_{s+1} - \upsilon_s\,, \qquad (21)$$

where υ is a regression error representing measurement error in the consumption data.

Proposition 4 yields our second estimation equation: for a household i that did not become disabled prior to its chosen retirement age $R = R_i$, we have

$$\begin{aligned}&\big[\alpha \cdot [n_{iR}]^{1-\alpha\cdot\gamma} \cdot [c_{iR-}]^{\alpha\cdot\gamma-1} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\big] \cdot \\ &\qquad \big[[y_{R-} - c_{iR-} + c_{iR+}] \cdot [1 - \frac{p(R)}{P(R)}] + B_i'(R) \cdot e^{r\cdot R}\big] - \\ &\frac{1}{\gamma} \cdot [n_{iR}]^{1-\alpha\cdot\gamma} \cdot \big[[c_{iR+}]^{\alpha\cdot\gamma} - [c_{iR-}]^{\alpha\cdot\gamma} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\big] = \\ &\phi(i)\,. \end{aligned} \tag{22}$$

The random effect $\phi(i)$, reflecting inter–household heterogeneity of preferences for leisure, becomes our regression error.

3. Data and Estimation

As previewed in the introduction, this paper estimates (21) from CEX data and (22) from the HRS.

3.1 CEX Data. Our primary data source for estimating (21) is the U.S. Consumer Expenditure Survey (CEX). It is the most comprehensive source of disaggregate consumption data for the U.S. The CEX obtains diary information on small purchases from one set of households; for a second set of households, it conducts quarterly interviews that catalog major purchases. The survey also collects demographic data and data on value of the respondent's house. At any given time, the sample consists of approximately 5,000 (7,000 after 1999) households each of which remains in the survey for at most 5 quarters. The survey was conducted at multi–year intervals prior to 1984, and annually thereafter. This paper uses the CEX surveys from 1984-2001.[4]

Table 1 compares National Income and Product Account (NIPA) personal consumption for 1985, 1990, 1995, and 2000 with weighted totals from the CEX.[5] We exclude household expenditures on pensions and life insurance from the CEX: the former constitute saving, and our concept of earnings is net of insurance. Looking at the last row of

[4] The web site http://stats.bls.gov.gov/csxhome.htm presents aggregative tables, codebooks, etc., for the CEX. This paper uses raw CEX data from the ICPSR archive, and we gratefully acknowledge the assistance of the BLS in providing stub files of changing category definitions.

[5] We abstract from the empirical difference between consumption and expenditure (e.g., Aguiar and Hurst [2004]). Except in the case of housing, this paper draws no distinction between consumer durable stocks and flows.

Table 1, total consumption measured in the CEX is only about 50-70 percent as large as the NIPA equivalent, with the discrepancy higher in later years.[6] This paper assumes that the NIPA numbers are accurate; that item–nonresponse and other measurement errors of the survey typically make CEX totals too low; and, that the relative magnitude of survey errors does not systematically vary with age. Thus, for each year we scale CEX consumption categories, uniformly across ages, to match NIPA amounts. Appendix II describes in detail three additional adjustments concerning the treatment of housing services, health care, and personal business expenditures.

Table 1. Consumer Expenditure Amount ÷ NIPA Amount (percent)[a]

Category	1985	1990	1995	2000
food	73.5	69.6	64.9	62.3
apparel	22.0	60.0	55.4	49.5
personal care	73.7	65.8	61.7	70.2
shelter:	82.9	82.4	81.4	81.0
own home	74.1	69.7	73.0	71.4
other	102.1	112.2	102.5	107.6
household operation	76.0	82.6	78.6	71.4
transportation	111.7	109.0	110.7	105.4
medical care	27.6	23.2	20.1	19.3
recreation	61.8	55.5	50.9	45.0
education	65.1	61.2	58.8	57.4
personal business:	14.8	12.2	9.9	6.8
brokerage fees	0.0	0.0	0.0	0.0
other	48.5	37.5	33.0	23.4
miscellaneous	120.0	80.0	68.0	67.5
total	66.7	64.3	59.7	56.0

a. Source: http://www.bea.doc.gov/bea/dn/nipaweb/AllTables.asp, Section 2, Table 2.4.

Deflating with the NIPA personal consumption deflator, and using survey weights, we derive an adjusted consumption amount for each age s and year t, say, C_{st}. Due

[6] There is a particularly large gap for "apparel" in 1985. The 1984 and 1985 data files omit a number of apparel subcategories. We assume this does not create biases with respect to age — so that our scaling procedure below is sufficient to eliminate the problem.

to the construction of the CEX from separate interview and diary surveys, we do not have consumption figures for individual households; however, we use our weighted–average consumption figures C_{st} for $age \times time$ cells to form a pseudo panel. The number of interviewed households per cell below varies from 127 to 981. The left–hand side variable for (21) in our estimation process is

$$\Delta \ln(C_{st}) \equiv \ln(C_{s+1,t+1}) - \ln(C_{st}) \, .$$

We organize the CEX data so that a household's age is the age of the wife for a married couple, and the age of the single household head in other cases.

The CEX provides information on whether the household is married. The latter provides our regressor $n^S \in \{0\, ,\, 1\}$. Although the CEX also reports number of children ages 0-17, we need more flexibility; hence, we construct our own measure of children per household as follows. Using Census data on births per woman at age $s \in \{15, ..., 49\}$ in year $t \in \{1920, ..., 2001\}$, we simulate the number of children of each age for women of separate ages i in 1984,...,2001. As stated, our data set assigns household observations to each age cell according to the age of the adult woman for all but single male households. We append numbers of children to each cell on the basis of the ages and birth dates of women.

Similarly, the CEX survey questions on retirement are not satisfactory for our purposes. The CEX interview questionnaire only asks whether the respondent is "retired" if he or she had zero weeks of work in the last twelve months; therefore, we turn to the March Current Population Survey 1984-2001 for our $\chi_s(R_i)$ variable.[7] We consider a household retired if the head is at least 50 years old and answers that he or she is out of the labor force at the time of the March survey. For male-female couples, the household is retired for our purposes if the male is at least 50 and out of the labor market (in the March survey). We focus on male behavior because males were more attached to the labor force in the cohorts of our data, and because our analysis abstracts from a detailed model of decision making within dual earner households and home–work/market–work choices.

3.2 Estimation of (21). Although our goal is to estimate (21)-(22) together with cross–equation constraints, at this point we estimate them individually.

Table 2 presents regression results for (21). We use households of age 25-80 for 1984-2001, so that our differences $\Delta \ln(C_{st})$ cover ages 25-79 and years 1984-2000. We exclude earlier ages out of concern for men and women assigned in the data to their parents'

[7] The average median retirement age 1984-2001 in the CEX data is 64-65, whereas it is about 62 in the Current Population Survey over the same period. See also our HRS results in Section 5 below.

households (indeed we treat children 0-22 as under parental care). The dependent variable is $\Delta \ln(C_{st})$. Independent variables include separate time dummies for 1984, 1985,..., 1999; a constant, β_0; change from (s,t) to $(s+1,t+1)$ in weighted average number of households with spouse, with coefficient β_1; change in weighted average number of households retired, with coefficient β_2; and, change in weighted average number of children 0-22, with coefficient β_3. In light of (21), coefficient interpretations are

$$\beta \equiv (\beta_0, \beta_1, \beta_2, \beta_3) \equiv (\frac{r-\rho}{1-\alpha \cdot \gamma}, \xi^S, -\frac{\gamma \cdot (1-\alpha) \cdot \ln(\bar{\ell})}{1-\alpha \cdot \gamma}, \xi^C)\,. \tag{23}$$

Table 2 presents our estimates.

Table 2. Estimated Coefficients for Equation (21): Consumer Expenditure Survey Data 1984-2000[a]

Parameter	CEX Female Ages:		
	Age 25-79	Age 30-79	Age 35-79
β_0	0.0259	0.0261	0.0249
(S.E.)	(0.0010)	(0.0012)	(0.0018)
T Stat.	27.1764	21.2747	14.0274
β_1	0.4351	0.4441	0.4336
S.E.	(0.0460)	(0.0528)	(0.0606)
T Stat.	9.4498	8.4139	7.1553
β_2	-0.1595	-0.1602	-0.1387
S.E.	(0.0313)	(0.0332)	(0.0390)
T Stat.	-5.0914	-4.8198	-3.5529
β_3	0.1346	0.1370	0.1290
S.E.	(0.0078)	(0.0103)	(0.0139)
T Stat.	17.2704	13.3255	9.2761
Summary Statistics			
R^2	0.0822	0.0712	0.0745
Observations	935	850	765
Mean Sq Error	0.0082	0.0087	0.0091

a. Year dummies for 1984, 1985, ... , 1999 not reported.

All columns of Table 2 are similar. Consider the middle one. The estimate of β_0 implies an average lifetime growth rate for per capita consumption of 2.6%/yr. That suggests that between, say, ages 25 and 62, in the absence of retirement a household's consumption per equivalent adult would rise by a factor of 2.62. In Auerbach and Kotlikoff [1987], for instance, the corresponding factor is about 1.54; in Gokhale *et al.* [2001], it is 1.74; in Tobin [1967], it is 13.33. For an infinite–lived representative agent model (e.g., Cooley and Prescott [1995]), the growth rate of consumption in a steady–state equilibrium would, of course, match the growth rate of GDP.

Our estimate of β_1 suggests that the addition of a spouse raises household consumption by 44 percent. This closely agrees with the U.S. Social Security System's award to retired households of 50 percent extra benefits for a spouse. The estimate is consistent with substantial returns to scale for larger households. Many papers in the literature implicitly set $\xi^S = 1.0$, and Table 2 suggests that such a calibration may be misleading.

Our estimate of β_2 suggests a 16 percent drop in consumption at retirement. This is consistent with, though at the smaller end of, estimates in Bernheim *et al.* [2001], Banks *et al.* [1998], and Hurd and Rohwedder [2003] and the retirement brochures cited in Laitner [2001].[8]

Table 2's estimate of β_3 suggests an increase in household consumption of 14 percent for each child age 0-22. Since two parents correspond to 1.44 "equivalent adults," a child adds about 20 percent as much as each parent. Mariger [1986] estimates that children consume 30 percent as much as adults; Attanasio and Browning [1995, p. 1122] suggest 58 percent; Gokhale *et al.* [2001] assume 40 percent; most of the analysis in Auerbach and Kotlikoff [1987] implicitly weights children at zero; Tobin [1967] assumes teens consume 80 percent as much as adults, and minor children 60 percent.

One might worry that economywide shocks could affect both general retirement behavior and consumption. Our regressions include year dummies (and they have little effect on the coefficients of primary interest). Because macroeconomic shocks conceivably affect consumption differently at different ages, we also perform GLS on a version of (21) in which we average every variable at each age over all years. Although the t statistics drop as the number of degrees of freedom falls, the coefficient estimates reconfirm Table 2: for ages 25-79, the new coefficient estimates are, respectively, 0.0258, 0.4481, -0.1625, 0.1332; for ages 30-79, they are 0.0253, 0.4318, -0.1589, 0.1298; and, for ages 35-79, they are 0.0230, 0.3025, -0.1507, 0.1137.

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[8] Earlier drafts of this paper with less disaggregate treatment of medical expenditures (recall Appendix II) implied declines of 20 percent or more, but we believe that our current specification is more accurate.

3.3 Retirement Data. Expression (23) shows that coefficients from our first regression provide 4 restrictions for 5 underlying parameters; we employ expression (22) to complete our estimation. The HRS is our main additional data source, though we calibrate other parts of our life–cycle framework as follows.

We assume a constant gross–of–income tax real interest rate of 5%/yr.[9]

We disregard government transfer payments other than Social Security. Our income tax rate τ comes from government spending on goods and services less indirect taxes (already removed from profits, and implicitly absent from wages and salaries below). Dividing by national income, the average over 1952–2003 is 14.28%/year.[10]

We assume a payroll tax of 15.3% per year. One–half of Social Security benefits are subject to the income tax. In the calculations below, the Social Security benefit formula, including the ceiling on taxable annual earnings, follows the history of the U.S. system.

We assume that adults work 40 hours per week until retirement and 0 hours per week after retirement. With 16×7 waking hours per week, we set[11]

$$\bar{\ell} = \frac{16 \times 7 - 40}{16 \times 7} = .6429\,.$$

Although the next draft will use demographic profiles from individual HRS households, here our retirement–age calculation uses a representative household consisting of a husband, wife, and two children. The wife reaches age 65 in the year 2000; the husband is 2 years older; the husband starts work at age 22 and marries at 24, with the latter being the

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[9] Our real interest rate comes from a ratio of factor payments to capital over the market value of private net worth. For the numerator, NIPA Table 1.13 gives corporate business income, indirect taxes, and total labor compensation. The first less the other two is our measure of corporate profits; the ratio of profits to profits plus labor remuneration is "profits share." We multiply the latter times corporate, noncorporate, and nonprofit–institution income less indirect taxes. We add household–sector income (NIPA Table 1.13) less indirect taxes and labor remumeration. Finally, we subtract personal business expenses (brokerage fees, etc. from NIPA Table 2.5.5, rows 61–64). The denominator is U.S. Flow of Funds household and non–profit institution net worth (Table B.100, row 19), less government liabilities (Table L106c, row 20). We average beginning and end of year figures. The average ratio 1952–2003 is .0504. For comparison, Auerbach and Kotlikoff [1987] use 6.7%/year, Altig *et al.* [2001] 8.3%/yr., Cooley and Prescott [1995] 7.2%/yr., and Gokhale *et al.* [2001] use post–tax rates of 4%/yr. and 6%/yr.

[10] Auerbach and Kotlikoff [1987], for example, use 15%/year.

[11] See also Cooley and Prescott [1995] — who, on the basis of time–use studies, determine that households devote 1/3 of waking hours to work.

time at which the household begins; both children are born two years after the marriage; and, both children leave home at age 22.[12] Following U.S. mortality tables, the husband dies at the end of age 74; the wife dies at the end of age 80. All of our calculations use 2000 dollars; our price index is the NIPA consumption deflator.

We derive earnings profiles and retirement ages from the original HRS survey cohort, consisting of households in which the respondent is age 51-61 in 1992. A majority of participant households signed a permission waiver allowing the HRS to link to their Social Security Administration (SSA) earnings history. Each history runs 1951-1991; the HRS itself covers 1992, 1994, 1996, 1998, 2000, and 2002. In this paper, we use the HRS and linked SSA records. For men, we estimate a so–called earnings dynamics model regressing log earnings on a quartic in age and dummy variables for time. Our regression error has an individual effect as well as a random term. The data is right censored at the Social Security tax cap prior to 1980 and the Medicare tax cap 1980-1991, and our likelihood function takes this into account. Assuming full–time work until retirement (and no work thereafter), we predict an average earnings profile for a man who reaches 65 in 1998. Table A1, Appendix III, presents predicted annual earnings. For women, we use the same type of statistical model to predict earnings above the censoring limit. To allow for women's part–time work and absence at some ages from the labor force, we compute average earnings in all other cases from the actual data. We then use the regression time dummies to deduce an average earnings profile for women who reach 65 in 2000. Again, see Table A1. Finally, since HRS earnings are net of employer benefits (including health insurance, pension contributions, and employer Social Security tax), we multiply household earnings for each year by the ratio of NIPA total compensation to NIPA wages and salaries.

We derive Social Security benefits after retirement from the statutory benefit formula for 2000. Table A1, Appendix III, shows household benefits if the male retires at age 62 and his wife retires simultaneously, at age 60. Given our treatment of consumption, we must also incorporate a stream of Medicare benefits after age 65, less participant SMI cost. To do this, for each adult 65 and older, we add to household resources Medicare benefits equaling the SMI annual premium for 2000 (i.e., \$546) multiplied by the ratio of HI and SMI total expenditures to SMI premiums for 2000 (i.e., 10.7282, less 1). Again, see Table A1.

Table 3 presents data on the conditional probability of disability at age t, $p(t)/P(t)$. As stated, male earnings are households' principal resource in our data; hence, we focus on male disability and retirement. We employ HRS data on age of onset of work limiting disabilities and data by age on work status "disabled" and work status "retired." For our

[12] Notice that assuming two children per household produces a rough match with recent, slow U.S. population growth.

purposes, we define the fraction of males too disabled to work at age t as

$$f(t) \equiv \{\text{number retired and work limited or work status "disabled"}\}/\{\text{total number}\}\,.$$

(We employ HRS individual weights.) Since disability is an absorbing state in our model, we set

$$F(t) = \max\{f(t)\,,\,F(t-1)\}\,.$$

Using a five–period average, we then numerically evaluate

$$p(t) \equiv F'(t)\,.$$

Table 3. Conditional Probability of Work–Preventing Disability: HRS Data 1992-2002[a]

Age	$p(t)/P(t)$	Age	$p(t)/P(t)$	Age	$p(t)/P(t)$
25	.0000	42	.0016	59	.0245
26	.0000	43	.0016	60	.0365
27	.0001	44	.0017	61	.0436
28	.0002	45	.0023	62	.0469
29	.0002	46	.0021	63	.0554
30	.0002	47	.0027	64	.0507
31	.0002	48	.0038	65	.0478
32	.0003	49	.0045	66	.0406
33	.0004	50	.0060	67	.0297
34	.0005	51	.0083	68	.0145
35	.0006	52	.0091	69	.0134
36	.0005	53	.0108	70	.0134
37	.0006	54	.0132	71	.0134
38	.0009	55	.0137	72	.0134
39	.0011	56	.0161	73	.0134
40	.0013	57	.0187	74	.0134
41	.0014	58	.0214		

a. See text.

In general, the conditional probabilities are low. Somewhat surprisingly, they peak around the mean age of retirement (see below) rather than rising monotonically with age. In fact, below we find that the term $[1 - p(t)/P(t)]$ in (22) has little quantitative impact upon our results.

Expression (22) also includes a term $B'(R)$ registering change in capitalized pension benefits with respect to retirement age. Our calculations below do reflect Social Security benefits.[13] In the case of private pensions, employers may or may not use defined benefit plans to influence employee retirement ages (e.g., Ippolito [1997]). If they do, we need to incorporate data on $B'(R)$ for defined benefit pensions in (22); if they do not, we can exclude them — in fact, their inclusion would affect the interpretation of our simulation results. This draft omits private pensions from $B'(R)$. Future work will attempt to assess their possible role.[14]

<u>3.4 Estimation of (22).</u> We use the HRS as our source for empirical retirement ages. Measuring the average (male) age of retirement is not entirely straightforward because the retirement date of many respondents is not observed.

We use a censored regression to extract an unbiased estimate. Of the 6214 men who appear in one or more waves of the HRS, 727 retire before age 50 or provide insufficient information for our analysis. Excluding these, the sample is 5487. Among the latter, 3661 retire by the last available HRS wave, 2002. Their mean retirement age is 60.25. The remaining 1826 men either (i) died before retiring, (ii) left the survey before retiring, or (iii) continued to work as of the last interview in 2002. Our analysis treats the 1826 men as right–censored at their age of death or last interview. The regression equation is

$$R_i = \mu + \eta_i \, . \tag{24}$$

Table 4 presents our Tobit estimate of μ, 62.57.

For our specification with chance of disability, the table's bottom section treats as right–censured retirees who classify their status as "disabled" when they retire. In other words, we assume chosen retirement age is later than actual termination age for this group. The new estimate of μ is one year higher.

Given estimates $\widehat{\beta}$ from (21) and a value for α, we can solve (22) for the desired retirement age, say, $R = g(\alpha\, , \beta)$ of our representative household (see above) when $\phi(i) = 0$. In this draft, using Table 4's estimate of R, say, $\widehat{\mu}$, we invert to determine an estimate of α satisfying

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[13] In particular, since the benefit formula depends on average, indexed earnings excluding the five lowest years, later retirement may raise the average.

[14] Indeed, a unique feature of the HRS is its inclusion of pension plan descriptions — e.g., Gustman *et al.* [2000].

Table 4. Average Retirement Age HRS Males[a]			
Coefficient	Value	S.E.	T–Stat
Censored if died or left survey prior to retirement, or not retired by last survey date			
μ	62.57	.0831	752.92
σ_η	5.58	.0669	
Summary Statistics			
Log Likelihood	-12722.884		
Uncensored Obs.	3661		
Censored Obs.	1826		
Additionally censored if disabled at retirement			
μ	63.53	.0882	720.29
σ_η	5.49	.0712	
Summary Statistics			
Log Likelihood	-10876.872		
Uncensored Obs.	3056		
Censored Obs.	2431		

a. See text.

$$\widehat{\mu} = g(\alpha\,,\,\widehat{\beta})\,. \tag{25}$$

Given such an estimate of α, (23) yields estimates for γ and ρ, and the delta method and covariance matrices for $\widehat{\beta}$ and $\widehat{\mu}$ generate confidence intervals.

Table 5 presents our structural–parameter estimates. The table's top section uses the basic model from Section 2.1 together with the Table 4 retirement age that ignores disability. The bottom uses Section 2.3's specification with stochastic disability together with the estimate of μ from the bottom of Table 4. In the second case, we utilize the conditional probabilities from Table 3 in (22).

Table 5's estimates of γ vary from -.47 to -.58; the estimates of α vary from .26 to .27. These correspond to estimates of an IES for services, $1/(1-\gamma)$, of 0.63 to 0.68 and an IES for consumption itself, $1/(1-\alpha\cdot\gamma)$, of 0.86 to 0.89. The estimates of the subjective time discount rate, ρ, fall between .012 and .014. All estimates of γ, α, and ρ are statistically different from zero at the 5 percent significance level. Augmenting the model to include a

Table 5. Structural Parameters[a]			
Para–meter	Consumption–Regression Sample:		
	Female Age 25-79	Female Age 30-79	Female Age 35-79
Basic Model — see Section 2.1			
γ	-0.5622	-0.5657	-0.4786
(S.E.)	(0.1255)	(0.1339)	(0.1518)
[95% Confid.]	[-0.8083,-0.3162]	[-0.8282,-0.3032]	[-0.7762,-0.1810]
α	0.2631	0.2635	0.2619
(S.E.)	(0.0033)	(0.0037)	(0.0027)
[95% Confid.]	[0.2566,0.2695]	[0.2562,0.2708]	[0.2532,0.2707]
ρ	0.0132	0.0129	0.0148
(S.E.)	(0.0016)	(0.0020)	(0.0027)
[95% Confid.]	[0.0099,0.0164]	[0.0090,0.0169]	[0.0095,0.0202]
$\alpha \cdot \gamma$	-0.1479	-0.1490	-0.1254
(S.E.)	(0.0917)	(0.0979)	(0.1157)
[95% Confid.]	[-0.3277,0.0319]	[-0.3409,0.0428]	[-0.3522,0.1015]
Model with Stochastic Chance of Disability — see Section 2.3			
γ	-0.5793	-0.5829	-0.4924
(S.E.)	(0.1303)	(0.1391)	(0.1574)
[95% Confid.]	[-0.8348,-0.3238]	[-0.8556,-0.3103]	[-0.8008,-0.1839]
α	0.2769	0.2774	0.2759
(S.E.)	(0.0036)	(0.0041)	(0.0048)
[95% Confid.]	[0.2699,0.2840]	[0.2694,0.2854]	[0.2664,0.2853]
ρ	0.0129	0.0126	0.0146
(S.E.)	(0.0017)	(0.0021)	(0.0028)
[95% Confid.]	[0.0095,0.0162]	[0.0085,0.0167]	[0.0090,0.0201]
$\alpha \cdot \gamma$	-0.1604	-0.1617	-0.1358
(S.E.)	(0.0950)	(0.1015)	(0.1198)
[95% Confid.]	[-0.3467,0.0259]	[-0.3607,0.0373]	[-0.3707,0.0990]

a. See text.

stochastic chance of disability makes virtually no difference for our parameter estimates.

Our estimates of γ, α, and ρ are similar to a number of calibrations in the literature. For example, Auerbach and Kotlikoff's [1987] favorite calibration has $\gamma = -3$, α (roughly) $= .4$, and $\rho = .015$; Altig *et al.* [2001] use $\gamma = -3$, α (roughly) $= .5$, and $\rho = .004$; and, Cooley and Prescott [1995] set $\gamma = 0$, $\alpha = .36$, and $\rho = .053$.

Our results may also be compared with estimates that have identified the IES from expected changes interest rates. Using aggregate consumption data Hall [1988], Cambell and Mankiw [1989], and Patterson and Pesaran [1992], for example, estimate the IES for consumption to be very nearly zero. Micro studies tend to estimate larger intertemporal elasticities. Banks *et al.* [1998], for instance, estimate the average IES for consumption to be approximately 0.5. In another example, Attanasio and Weber [1993] estimate an IES for consumption of approximately 0.75 from micro data.[15] Although we use a very different source of variation to identify the IES, our estimates are similar to, if on the larger end of, those obtained in micro studies from the change in consumption growth with expected changes in interest rates.

4. Social Security Reform

This section investigates the consequences of a Social Security reform in which the OASI tax, and benefit adjustments based on new earnings, cease at age 62. Individuals who avoid disabiilty could retire at any age; however, those who continue working after age 62 would enjoy a 10.6 percent increase in their aftertax wage. Individuals could start collecting Social Security benefits at age 62 or after, with an actuarially fair adjustment for postponed receipt.

Table 6 presents simulation outcomes for different parameter estimates. Our model is not general equilibrium in nature — wages and interest rates are exogenous. Furthermore, this section does not study transitions after reform. Rather, we compare the behavior of a representative household under the existing Social Security System with the same household living its entire life under a reformed System.

The first two rows simulate our proposed reform, using each combination of parameter estimates from Table 5. The basic model assumes the timing of retirement is a choice for all agents. The second model includes stochastic disability which precludes work. The

[15] Barsky *et al.* [1997] use hypothetical questions to estimate an IES distribution for their sample. They find an average IES of 0.2, with less than 20% of respondents having an IES greater than 0.3. Others who have attempted to estimate a distribution of intertemporal elasticities of substitution find evidence that the IES is increasing with wealth (e.g., Blundell *et al.* [1994]).

Table 6. Months Increase in Retirement Age After Reform[a]

Specification	Consumption–Regression Sample: Female Age 25-79	Female Age 30-79	Female Age 35-79
Basic Model	38	38	39
Model with Disability	23	23	24
Addendum			
Months Increase After Elimination of Social Security System			
Basic Model	10	10	10
Model with Disability	7	7	7
Months Increase After Elimination of Social Security System But Institution of Lump–Sum Legacy Tax			
Basic Model	40	39	41
Model with Disability	25	25	28

a. See text.

increases in career length are quite large — slightly more than 3 years in all cases, for the basic model and approximately 2 years for the model with disability.[16] In the first cell of the basic model, for example, the average age of retirement changes from 62 years and 7 months to 65 years and 9 months. The remainder of this section seeks to explore intuitive reasons for this result.

Consider Proposition 2. Disregard the error term $\phi(i)$ and omit the subscript i. From (10) and (8), define

$$\psi(R) \equiv \frac{y_{R-}}{c_{R-}} + \frac{B'(R) \cdot e^{r \cdot R}}{c_{R-}} - \frac{c_{R-} - c_{R+}}{c_{R-}} - \frac{1}{\alpha \cdot \gamma} \cdot \left[[\bar{\ell}]^{\frac{-(1-\alpha)\cdot\gamma\cdot\alpha\cdot\gamma}{1-\alpha\cdot\gamma}} - [\bar{\ell}]^{(1-\alpha)\cdot\gamma} \right] .$$

[16] The effects on the simulation of allowing disability are mechanical. Simulations (not shown here) show that households who avoid disability until retirement have almost identical average retirement dates as those for whom disability is not possible.

At the utility–maximizing retirement age, this expression is 0. If the expression is positive, a later retirement age yields more utility; if it is negative, retiring sooner is preferred. Using footnote 1, we have

$$\operatorname{sgn}\big(\psi(R)\big) =$$
$$\operatorname{sgn}\Big(\frac{y_{R-}}{c_{R-}} + \frac{B'(R)\cdot e^{r\cdot R}}{c_{R-}} - \big[[\bar{\ell}]^{\frac{-(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}} - 1\big] - \frac{1}{\alpha\cdot\gamma}\cdot\big[1-[\bar{\ell}]^{\frac{\gamma\cdot(1-\alpha)}{1-\alpha\cdot\gamma}}\big]\cdot[\bar{\ell}]^{\frac{-(1-\alpha)\cdot\gamma\cdot\alpha\cdot\gamma}{1-\alpha\cdot\gamma}}\Big).$$

Hence, as we consider the effect of policy changes on the desired retirement age, we can limit our attention to

$$\frac{y_{R-}}{c_{R-}} \quad \text{and} \quad \frac{B'(R)\cdot e^{r\cdot R}}{c_{R-}}. \tag{25}$$

Consider the first cell of Table 6. Under the current Social Security System, for our representative household $y_{R-} \approx 64000$, $c_{R-} \approx 58000$, and $B'(R)\cdot e^{r\cdot R} \approx 2500$. In other words, in terms of Social Security cumulative benefits, the advantage of working one more year is relatively small — the effect on average earnings in the benefit formula is small, and the formula allocates benefits progressively on the margin. Furthermore, the first ratio in (25) is approximately unity.

Under the proposed reform, we take away the OASI tax of 10.6 percent just before retirement and terminate adjustments in the benefit formula; hence, the first ratio in (25) rises 10.6% and the second, formerly about .04, drops to 0. The sum of the two terms then rises .066, $\psi(R)$ swings positive, and R rises. We have two substitution effects of opposite sign, and the positive one predominates. Since the OASI tax disappears late in the representative agent's career, there is virtually no income effect on behavior.

In the third row, we compare the original representative household to an identical one in an economy that never had a social security system. Again, the comparison need only consider the two terms in (25). For the new household, the lack of an OASI tax makes y_{R-} 10.6 percent higher. Since the household has never faced the tax, its lifetime resources are about 10.6 percent higher as well — tending to make c_{R-} 10.6 percent higher. However, Social Security benefits are absent for the new household. An important point is that the present value of benefits is lower than the present value of OASI taxes for the original household, in large part because of so–called legacy costs of the Social Security System. In fact, in our calculations legacy costs are slightly more than half of the present value of OASI taxes. In our comparison, therefore, for the new household, the numerator of the first term in (25) is 10.6% higher than that for a household under the existing Social Security System but the denominator is only about 4.2% higher. On balance, the first term rises about 6.4%. The second term is absent for the new household. A comparison then shows the new household's ψ at the original $R = 62.57$ to be about 2.4% higher. The gain in desired work years from not having a Social Security System at all turns out to be relatively small.

The fifth row provides a more interesting contrast. Again, we consider our same representative household placed in an economy without a social security system. However, this case assumes the new environment does have the "legacy costs" of our System, and they take the form of a lump–sum tax. In terms of (25), in the new environment y_{R-} is 10.6% higher due to the absence of the OASI tax, c_{R-} is unchanged from the original household because lost Social Security benefits and the legacy tax fully counterbalance higher lifetime resources from the missing OASI tax, and the second term is absent. The gain in work life is now virtually the same as row 1.[17]

5. Conclusion

This paper estimates coefficients for a life–cycle model with endogenous retirement. We use pseudo–panel consumption data from the CEX and panel data on earnings, health, and retirement from the HRS. We assume intratemporally nonseparable preference orderings familiar from a segment of the life–cycle literature. The specification predicts a change in consumption at retirement, and this paper uses the magnitude of the change, together with desired retirement age, to identify key parameters such as the intertemporal elasticity of substitution. We consider specifications with and without a stochastic chance of disability, though the latter ends up making only minor changes in our estimates and results.

Section 4 employs the model to simulate the possible long–run effect of a Social Security reform in which individuals terminate their connections with the Social Security System at age 62. In other words, individuals no longer face the OASI payroll tax after 62, and their subsequent earnings have no bearing on their Social Security benefits. Although we do not study transitional dynamics, such a reform would seem to have far fewer transitional complications than many proposals. Section 4 finds that average retirement ages would tend to rise by nearly 2 years. This is, we believe, a large enough change to warrant attention.

[17] Slight differences arise from rows 1-2 to 5-6 because in 1-2, existing Social Security revenues from the OASI tax on workers 62 and over are lost — whereas in 5-6 the representative household's lost benefits and lump–sum legacy tax fully counterbalance lost OASI–tax revenues.

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Appendix I: Proofs for Section 2

Proof of Lemma 1: Let R_i be given. Begin with problem (4). Suppose it has one breakpoint, $t_1 \in (R_i\,,\, T)$. Solving the subproblem for $t \geq t_1$ — which is standard — we have (i)-(iv). Call the subproblem's maximized criterion $\Phi(a_{it_1}\,,\, t_1)$. Next, solve

$$\max_{c_{it}} \int_{R_i}^{t_1} e^{-\rho \cdot t} \cdot u(c_{it}, i, t, R_i)\, dt + \Phi(a_{it_1}\,,\, t_1)$$

with the same constraints as (4). This is a standard problem: we have (i)-(iii) and

$$\lambda_{it_1-} = \frac{\partial \Phi(a_{it_1}\,,\, t_1)}{\partial a}\,. \qquad (v)$$

(See, for example, Kamien and Schwartz [1981].) Since a_{it} is continuous by nature, it only remains to show that λ_{it} is continuous at t_1. But, the envelope theorem shows

$$\frac{\partial \Phi(a_{it_1}\,,\, t_1)}{\partial a} = \lambda_{it_1+}\,. \qquad (vi)$$

Combining (v)-(vi) establishes continuity of the costate at t_1. Induction on the number of breakpoints, say, J', in (4) establishes continuity for any J'. The logic of (v)-(vi), with $\Phi(.) = \varphi(.)$, establishes continuity of the costate at $t = R_i$. The same arguments apply for $t < R_i$. ▮

Proof of Proposition 1: Suppose we have a solution of (3)-(4). Fix the R_i. The optimal consumption path must solve (3)-(4) conditional on this R_i. Follow Lemma 1. From (ii), $\dot{\lambda}_{it} = -r \cdot \lambda_{it}$. Then for $t \in (t_j\,,\, t_{j+1})$, we have

$$e^{-\rho \cdot t} \cdot [n_{it}]^{1-\alpha \cdot \gamma} \cdot [c_{it}]^{\alpha \cdot \gamma - 1} \cdot [\ell_{it}]^{(1-\alpha)\cdot \gamma} = \lambda_{it} \qquad \text{from Lemma 1, (i)}$$

$$\Longleftrightarrow (\alpha \cdot \gamma - 1) \cdot \frac{\dot{c}_{it}}{c_{it}} = \rho - r\,, \qquad \text{since } t \in (t_j\,,\, t_{j+1})$$

establishing (6). For $t = t_j$, $j = 1, ..., J$, Lemma 1 shows λ_{it} is continuous; so,

$$e^{-\rho \cdot t} \cdot [n_{it-}]^{1-\alpha \cdot \gamma} \cdot [c_{it-}]^{\alpha \cdot \gamma - 1} \cdot [\ell_{it}]^{(1-\alpha)\cdot \gamma} = \lambda_{it} =$$
$$e^{-\rho \cdot t} \cdot [n_{it+}]^{1-\alpha \cdot \gamma} \cdot [c_{it+}]^{\alpha \cdot \gamma - 1} \cdot [\ell_{it}]^{(1-\alpha)\cdot \gamma}\,, \qquad \text{from Lemma 1, (i)}$$

establishing (7). For $t = R_i$, by the same logic, since $\ell_{it+} = 1$,

$$e^{-\rho \cdot t} \cdot [n_{it}]^{1-\alpha \cdot \gamma} \cdot [c_{it-}]^{\alpha \cdot \gamma - 1} \cdot [\ell_{it-}]^{(1-\alpha) \cdot \gamma} = \lambda_{it} = e^{-\rho \cdot t} \cdot [n_{it}]^{1-\alpha \cdot \gamma} \cdot [c_{it+}]^{\alpha \cdot \gamma - 1} \,, \qquad \text{from Lemma 1, (i)}$$

establishing (8). Integrating budget constraint (iii) from $t = S$ to T gives (9). ▮

Proof of Proposition 2: For any $R = R_i$, define a Hamiltonian as in (5). It can serve for both (3) and (4). Lemma 1 shows

$$\lambda_{iR} = \frac{\partial \varphi(a_{iR} + B_i(R) \cdot e^{r \cdot R} \, , \, R)}{\partial a} \,. \tag{vii}$$

Using (vii) and Kamien and Schwartz [1981],

$$\frac{\partial \varphi(a_{iR} + B_i(R) \cdot e^{r \cdot R} \, , \, R)}{\partial R} = \lambda_{iR} \cdot [B_i'(R) \cdot e^{r \cdot R} + r \cdot B_i(R) \cdot e^{r \cdot R}] - \mathcal{H}(c_{iR+}, a_{iR+}, \lambda_{iR}, R) \,. \tag{viii}$$

As household i chooses $R = R_i$ in (3), we have a "free endpoint problem." Kamien and Schwartz show that the necessary condition for an optimal $R \in (S \, , \, T)$ is

$$\mathcal{H}(c_{iR-}, a_{iR-}, \lambda_{iR}, R) + \frac{\partial \varphi(a_{iR} + B_i(R) \cdot e^{r \cdot R} \, , \, R)}{\partial R} = 0 \,. \tag{ix}$$

Hence, for an optimal $R \in (S \, , \, T)$, (viii)-(ix) imply

$$e^{-\rho \cdot R} \cdot u(c_{iR}) + \lambda_{iR} \cdot [r \cdot a_{iR-} + y_{R-} - c_{iR-}] + \lambda_{iR} \cdot [B_i'(R) \cdot e^{r \cdot R} + r \cdot B_i(R) \cdot e^{r \cdot R}] - e^{-\rho \cdot R} \cdot v(c_{iR}) - \lambda_{iR-} \cdot [r \cdot a_{iR+} - c_{iR+}] = 0 \,.$$

Recall that $a_{iR+} = a_{iR-} + B_i(R) \cdot e^{r \cdot R}$. Hence, the preceding simplifies to

$$e^{-\rho \cdot R} \cdot u(c_{iR}) + \lambda_{iR} \cdot [y_{R-} - c_{iR-} + c_{iR+}] + \lambda_{iR} \cdot [B_i'(R) \cdot e^{r \cdot R}] - e^{-\rho \cdot R} \cdot v(c_{iR}) = 0 \,. \tag{x}$$

As (i), Lemma 1, shows that

$$\lambda_{it} = e^{-\rho \cdot t} \cdot \frac{\partial u(c_{it})}{\partial c} \quad \text{for} \quad t < R \,,$$

condition (x) establishes (10). ▮

Proof of Proposition 3: Fix $R = R_i$ for the remainder of this proof. Set up Hamiltonians for, respectively, disability problem (12), retirement problem (4), and lifetime problem with possible disability (13):

$$\mathcal{D} \equiv e^{-\rho \cdot t} \cdot v(\bar{c}_{it}) = \bar{\Lambda}_{it} \cdot [r \cdot \bar{a}_{it} - \bar{c}_{it}] \,, \quad t \geq D \,,$$

$$\mathcal{R} \equiv e^{-\rho \cdot t} \cdot v(c_{it}) + \Lambda_{it} \cdot [r \cdot a_{it} - c_{it}] \,, \quad t \geq R \,,$$

$$\mathcal{H} \equiv P(t) \cdot e^{-\rho \cdot t} \cdot u(c_{it}) + p(t) \cdot \bar{\varphi}(a_{it-} + X_{it}, t, R) +$$
$$\lambda_{it} \cdot [r \cdot a_{it} + y_{it} - c_{it} - \frac{p(t) \cdot X_{it}}{P(t)}] \,, \quad t < R \,.$$

The costate variables are $\bar{\Lambda}_{it}$, Λ_{it}, and λ_{it}, respectively.

Step 1. At demographic breakpoints, the analysis follows the proof of Proposition 1 exactly.

Step 2. We have

$$\frac{\partial \mathcal{R}}{\partial c_{iR}} = 0 \Rightarrow e^{\rho \cdot R} \cdot \frac{\partial v(c_{iR})}{\partial c} = \Lambda_{iR} \,, \qquad \text{F.O.C. for (4)}$$

$$\Lambda_{iR} = \frac{\partial \varphi(a_{iR} + B_i(R) \cdot e^{r \cdot R} \,, \, R)}{\partial a_{iR}} \,, \qquad \text{envelope theorem}$$

$$\lambda_{iR} = P(R) \cdot \frac{\partial \varphi(a_{iR} + B_i(R) \cdot e^{r \cdot R} \,, \, R)}{\partial a_{iR}} \,, \qquad \text{F.O.C. for (13)}$$

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \Rightarrow P(R) \cdot e^{-\rho \cdot R} \cdot \frac{\partial u(c_{iR})}{\partial c} = \lambda_{iR} \,. \qquad \text{F.O.C. for (13)}$$

These four equations together establish (16).

Step 3. We have

$$\frac{\partial \mathcal{H}}{\partial X_{iD}} = 0 \Rightarrow p(D) \cdot \frac{\partial \bar{\varphi}(a_{iD-} + X_{iD} \,, \, D \,, \, R)}{X_{iD}} = \lambda_{iD} \cdot \frac{p(D)}{P(D)}$$
$$\Rightarrow P(D) \cdot \frac{\partial \bar{\varphi}(a_{iD-} + X_{iD} \,, \, D \,, \, R)}{X_{iD}} = \lambda_{iD} \,, \qquad \text{F.O.C. for (13)}$$

$$\frac{\partial \mathcal{H}}{\partial c_{iD}} = 0 \Rightarrow P(D) \cdot e^{-\rho \cdot D} \cdot \frac{\partial u(c_{iD})}{c_{iD}} = \lambda_{iD} \, , \qquad \text{F.O.C. for (13)}$$

$$\frac{\partial \mathcal{D}}{\partial \bar{c}_{iD}} = 0 \Rightarrow e^{-\rho \cdot D} \cdot \frac{\partial v(\bar{c}_{iD})}{c_{iD}} = \bar{\Lambda}_{iD} \, , \qquad \text{F.O.C. for (12)}$$

$$\frac{\partial \bar{\varphi}(a_{iD-} + X_{iD} \, , \, D \, , \, R)}{X_{iD}} = \bar{\Lambda}_{iD} \, . \qquad \text{envelope theorem}$$

These four equations together establish (17).

Step 4. The numerator of (18) is the expected present value of the household's lifetime earnings and retirement benefits. (One could subtract disability–insurance premiums and add expected disability–insurance benefits, but they would exactly balance.) The denominator times c_{iS} is the expected present value of lifetime consumption. ∎

Proof of Proposition 4: Use the notation from the proof of Proposition 3. Analogous to the proof of Proposition 2, we have

$$\mathcal{H}(c_{iR}, a_{iR}, \lambda_{iR}, R) + \frac{\partial [P(R) \cdot \varphi\big(a_{iR} + B_i(R) \cdot e^{r \cdot R} \, , \, R\big)]}{\partial R} = 0$$
$$\Longleftrightarrow \mathcal{H}(.) + P(R) \cdot \frac{\partial \varphi(.)}{R} - p(R) \cdot \varphi(.) = 0 \, , \qquad (xi)$$

$$\frac{\partial \varphi\big(a_{iR} + B_i(R) \cdot e^{r \cdot R} \, , \, R\big)}{\partial R} =$$
$$\Lambda_{iR} \cdot [B_i'(R) \cdot e^{r \cdot R} + r \cdot B_i(R) \cdot e^{r \cdot R}] - \mathcal{R}\big(c_{iR}, a_{iR} + B_i(R) \cdot e^{r \cdot R}, \lambda_{iR}, R\big) \, . \qquad (xii)$$

Combining (xi)-(xii),

$$P(R) \cdot e^{-\rho \cdot R} \cdot u(c_{iR}) + p(R) \cdot \bar{\varphi}(a_{iR} + X_{iR} \, , \, R) + \lambda_{iR} \cdot [r \cdot a_{iR-} + y_{iR} - c_{iR-} - \frac{p(R) \cdot X_{iR}}{P(R)}] +$$
$$P(R) \cdot \Lambda_{iR} \cdot [B_i'(R) \cdot e^{r \cdot R} + r \cdot B_i(R) \cdot e^{r \cdot R}] -$$
$$P(R) \cdot \big[e^{-\rho \cdot R} \cdot v(c_{iR}) + \Lambda_{iR} \cdot [r \cdot a_{iR-} + r \cdot B_i(R) \cdot e^{r \cdot R} - c_{iR+}]\big] -$$
$$p(R) \cdot \varphi\big(a_{iR-} + B_i(R) \cdot e^{r \cdot R} \, , \, R\big) = 0 \, . \qquad (xiii)$$

The proof of Proposition 3 shows

$$\lambda_{iR} = P(R) \cdot \Lambda_{iR} \, .$$

By construction,

$$\bar{\varphi}(a_{iR-} + X_{iR} \, , \, R) = \varphi\big(a_{iR-} + B_i(R) \cdot e^{r \cdot R} \, , \, R\big) \, .$$

First–order conditions for (13) imply

$$P(R) \cdot e^{-\rho \cdot R} \cdot u'(c_{iR}) = \lambda_{iR} \, .$$

So, (xiii) simplifies to

$$P(R) \cdot e^{-\rho \cdot R} \cdot u'(c_{iR}) \cdot [y_{iR-} - c_{iR-} + c_{iR+} - \frac{p(R) \cdot X_{iR}}{P(R)} + B_i'(R) \cdot e^{r \cdot R}] =$$
$$P(R) \cdot e^{-\rho \cdot R} \cdot [v(c_{iR+}) - u(c_{iR-})] \, , \qquad (xiv)$$

which establishes (19).

Proposition 3 shows that term disability insurance for $[t \, , \, t + dt)$, where the interval ends with retirement, should cover lost earnings, corrected for changing consumption needs in the disabled state; hence,

$$\frac{p(R)}{P(R)} \cdot X_{iR} \, dt = \frac{p(R)}{P(R)} \cdot [y_{iR-} - c_{iR-} + c_{iR+}] \, dt \, .$$

This completes the proof. ▮