Preliminary and incomplete (empirical work will be presented at seminar):

Marriage Matching, Risk Sharing and Family Labor Supplies: An Empirical Framework*

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September 18, 2006

Abstract

The paper integrates marriage matching with the collective model of spousal labor supplies with full risk sharing. It derives observable implications of how marriage market conditions affect spousal labor supplies. In addition to sex ratio which is an indirect measure, it provides a direct measure of changes in marriage market conditions. The framework also clarifies the identifying assumptions necessary to estimate causal effects of marriage market conditions on spousal labor supplies. The empirical section of the paper tests for marriage market effects on spousal labor supplies using data from a panel of US cities.

*Seitz thanks the Social Sciences and Humanities Research Council of Canada for financial support.

1 Introduction

There is a robust negative correlation between the average hours of work of wives or women and the sex ratio (ratio of men to women) in a society. Researchers have interpreted this negative correlation as supporting the hypothesis that marriage market conditions affect intra-household allocations.[1] Changes in sex ratio may be due to changes in labor market conditions. So the estimated effect of changes in sex ratio on spousal labor supplies confound marriage market and labor market effects.[2]

Building on the literature on intra-household allocations within *existing* marriages[3], and the literature on empirical marriage matching models[4], this paper provides an empirical framework to explicitly integrate marriage matching with intrahousehold allocations. We integrate the marriage matching framework of Choo Siow (2006a; henceforth CS) with the collective model of spousal labor supplies with full risk sharing. Within such a framework, we derive observable implications of how marriage market conditions affect spousal labor supplies. In addition to sex ratio which is an indirect measure, we provide a direct measure of changes in marriage market conditions. The framework also clarifies the identifying assumptions necessary to estimate causal effects of marriage market conditions on spousal labor supplies.

In the empirical section of the paper, we test for marriage market effects on spousal labor supplies using data from a panel of cities.

The CS framework models the marriage market as Walrasian. This paper relaxes CS's transferable utilities assumption. A male of type i who wants to enter into a type π marriage[5] with a female of type j can do so if he agrees to accept the bargaining power allocated to him in the marriage. Similarly, a female of type j who wants to enter into a type π marriage with a male of type i can do so if she agrees to accept the bargaining power allocated to her. Following Walrasian economies, the equilibrium division of spousal bargaining power adjust to clear the marriage market.

After marriage, following the collective model with full risk sharing, each household chooses commodities to maximize a utilitarian welfare function of the two spouses with utility weights (bargaining power) determined by marriage market clearing. As per the collective

[1] E.g. Angrist 2002; Chiappori, Fortin, and Lacroix 2002; Francis 2005; Grossbard-Schechtman 1993; Seitz 2005, South and Trent. The evidence for male labor supplies are less clear.

[2] Angrist uses an instrumental variable strategy to disentangle these two effects. His work is dicussed more carefully later.

[3] The study of intra-household allocations began with Becker's rotten kid theorem, the early work of Manser and Brown (1980) and McElroy and Horney (1981) within a bargaining framework, and Chiappori (1988, 1992) in the collective framework. Other contributions include...

[4] E.g. Choo Siow, Costa and Kahn, Dagsvik, Gunther, et. al., Pollak and Compton, Wong, etc.

[5] In Choo Siow 2006b, partners may choose to marry or cohabit with each other. Here, we focus on non-specialized marriages where both spouses work versus specialized marriages where only the husband works. Other living arrangements are feasible.

model, these choices will have non-parametric implications for spousal labor supplies. This paper will consider three different collective models with decreasing order of detail: (1) A static collective model of spousal labor supplies with full risk sharing between the spouses. (2) A collective model of dynamic spousal labor supplies with full risk sharing between the spouses. (3) The standard static collective model of spousal labor supplies without uncertainty.

The contributions of our paper are as follows:

1. We show how bargaining power in the marriage market manifests itself in spousal labor supplies.
2. We demonstrate existence of a rational expectation marriage matching equilibrium for such a society.
3. Within a society, the marriage matching model can fit any observed marriage matching distribution.
4. Using multi-society data, the framework provides new tests of the link between changes in marriage market conditions and changes in spousal labor supplies. This test complements existing tests of changes in sex ratios and spousal labor supplies.
5. Our empirical evidence shows that ....

This paper builds on a large literature and is closely related to recent work. Thus it is useful to delay the literature review until Section 10.

2 The model

Consider a society in which there are I types of men, $i = 1, .., I$, and J types of women, $j = 1, .., J$. All type i men have the same preferences and ex-ante opportunities; and all type j women also have the same preferences and ex-ante opportunities. That is, the type of an individual is defined by his or her preferences and ex-ante opportunities.

Let m_i be the number of type i men and f_j be the number of type j women. M and F are the vectors of the numbers of each type of men and women respectively.

This paper considers a static model of marital, consumption and labor supply choices. First, individuals choose whether to marry and who to marry if they marry. At the time of their marital choices, wages and non-labor income for each marital choice are random variables. After their marital choices, wages and non-labor income for each household are realized and households choose labor supplies and consumption.

All men and unmarried women have positive hours of work. Married women choose whether to participate in the labor force, and conditional on participation, how many hours to work. The participation status of a wife is known as of the time the marriage decision is made. Thus a marriage is characterized by the triplet $\{i, j, \pi\}$ where π is determined by whether the wife works or not, where $\pi = 1$ if the wife works and $\pi = 0$ if the wife does not work. Since individuals can choose the type of marriage, they can choose whether to enter a *specialized* marriage, where one spouse works in the market and one remains at home, versus a *non-specialized* marriage, where both spouses work. If a man chooses not to marry, $\pi = .$ and his spouse is $j = 0$. If a woman chooses not to marry, $\pi = .$ and her spouse is $i = 0$.

If a type i man want to match with a type j woman in a type π marriage, he must agree to transfer to her τ_{ij}^{π} amount of non-labor income. The transfer will usually be *dependent* on the realizations of non-labor income and wages that the family faces. These transfers are used to clear the marriage market. Following the collective model terminology, we call τ_{ij}^{π} a sharing rule.

After marriage, wages, non-labor income for the family are realized. Upon realization of those random variables, the transfer, agreed to before marriage, is also realized. Finally in each $\{i, j, \pi\}$ marriage, the husband takes his own wage, non-labor income and the transfer to his wife as given and makes his own labor supply and consumption decisions to maximize his own utility. The wife takes her wage if any, her transfer and makes her own labor supply and consumption decisions to maximize her utility.

2.1 Preferences

For analytic convenience, we assume that every spouse have egotistical preferences, that their utilities only depend on their own labor supplies and consumption. We also ignore explicit expenditure in household public goods.[6]

Let C_{ijG}^{π} be the consumption of woman G of type j matched to a type i man in a type π marriage. H_{ijG}^{π} is her labor supply and L_{ijG}^{π} is her leisure. We normalize the total amount of time for each individual to 1. So $1 = H_{ijG}^{\pi} + L_{ijG}^{\pi}$. For women who do not work, $H_{ijG}^{0} = 0$. Her utility function is:

$$U_{ij}^{\pi}(C_{ijG}^{\pi}, L_{ijG}^{\pi}, \varepsilon_{ijG}^{\pi}) = \widehat{Q}_{ij}^{\pi}(C_{ijG}^{\pi}, L_{ijG}^{\pi}) + \Gamma_{ij}^{\pi} + \varepsilon_{ijG}^{\pi}, \tag{1}$$

$\widehat{Q}_{ij}^{\pi}(.)$ depend on i, j, π, which allows for differences in home production technologies across different types of marriages. Even with the same individual budget constraint, variations in $\widehat{Q}_{ij}^{\pi}(.)$ will allow the model to fit observed labor supply behavior for different types of marriages.

[6] Chiappori, et. al. extends the collective model to include expenditures on public good.

For women who work, $\widehat{Q}_{ij}^{\pi}$ is in the class of utility functions from the parameters which determine $\widehat{Q}_{ij}^{\pi}$ may be recovered by estimating their Marshallian labor supply functions.

The invariant gain to an i, j, π marriage for the woman, Γ_{ij}^{π}, shifts her utility according to the type of marriage and allows the model to fit the observed marriage matching patterns in the data. It may vary across different types of marriages and societies due to technological differences in different types of marriages, legal and cultural differences across societies. The important restriction is that Γ_{ij}^{π} does not affect her marginal utilities from consumption or labor supply.

Finally, we assume ε_{ijG}^{π} is a type I extreme value random variable that is realized before marital decisions are made. The rationale for the extreme value assumption will be made clear later. The realizations of this random variable across different women of type j in the same society will produce different marital choices for different type j women in period one. Given her marital choice, ε_{ijG}^{π} also has no impact on her consumption and labor supply decisions.

The specification of a representative man's problem is similar to that of women. Let c_{ijg}^{π} be the consumption of man g of type j matched to a type j woman in a type π marriage in society x. Denote his labor supply and leisure by h_{ijg}^{π} and l_{ijG}^{π} respectively. His total time satisfy $1 = l_{ijG}^{\pi} + h_{ijG}^{\pi}$. If he chooses not to marry, then $\pi = .$ and $j = 0$. The utility function for males is described by:

$$u_{ij}^{\pi}(c_{ijg}^{\pi}, h_{ijg}^{\pi}, \varepsilon_{ijg}^{\pi}) = \widehat{q}_{ij}^{\pi}(c_{ijg}^{\pi}, l_{ijg}^{\pi}) + \gamma_{ij}^{\pi} + \varepsilon_{ijg}^{\pi}, \tag{2}$$

$\widehat{q}_{ij}^{\pi}(.)$ depends on i, j, π, will allow the model to fit observed labor supply behavior for different types of marriages. $\widehat{q}_{ij}^{\pi}$ is in the class of utility functions from the parameters which determine $\widehat{q}_{ij}^{\pi}$ may be recovered by estimating their Marshallian labor supply functions.

The invariant gain to an i, j, π marriage for the man, γ_{ij}^{π}, shifts his utility by i, j, π and allows the model to fit the observed marriage matching patterns in the data. It may vary across different types of marriages and societies due to technological differences in different types of marriages, legal and cultural differences across societies. The important restriction is that γ_{ij}^{π} does not affect his marginal utilities from consumption and labor supply.

Finally, we assume ε_{ijg}^{π} is a type I extreme value random variable that is realized before marital decisions are made. The realizations of this random variable across different men of type i in the same society will produce different marital choices for different type i men in period one. Given his marital choice, ε_{ijg}^{π} also has no impact on his consumption and labor supply decisions.

2.2 Private budget constraints

Consider a particular husband g and his wife G in a type $\{i, j, \pi\}$ marriage. Total non-labor family income is A^{π}_{ijgG} which is a random variable. The wage for a working woman is also a random variable W^{1}_{ijgG}. For families whose wives do not work, $W^{0}_{ijgG} = 0$ and $L^{0}_{ijgG} = 1$. The male's wage is another random variable w^{π}_{ijgG}. A^{π}_{ijgG}, W^{π}_{ijgG} and w^{π}_{ijgG} are realized in the second period, after the marriage decision.

Since we hold the type of marriage, $\{i, j, \pi\}$, fixed in this section, for expositional ease, we will dispense with the i, j, π subscripts when it does not cause confusion.

Let τ_{gG} be the state contingent transfer that the wife gets in the second period after wages and non-labor income are realized. The private budget constraint of the wife is:

$$W_{gG}L_G + C_G \leq \tau_{gG} + W_{gG} \tag{3}$$

and the private budget constraint of the husband is:

$$w_{gG}l_g + c_g \leq A_{gG} - \tau_{gG} - w_{gG} \tag{4}$$

Adding the private budget constraints yields the family budget constraint:

$$w_{gG}l_g + c_g + W_{gG}L_G + C_G \leq A_{gG} + W_{gG} + w_{gG}. \tag{5}$$

2.3 Efficient Risk Sharing

This section has three objectives. First, we want to derive a shadow price for any type of marriage so that it can be used to equilibrate the excess demand for that type of marriage. Second, we derive the sharing rule and some of its properties. Third, we derive a well known implication of efficient risk sharing, ex-post efficiency, to anticipate a testable restriction of efficient risk sharing.

Consider an $\{i, j, \pi\}$ marriage between female G and male g. Because wages and non-labor income, W^{π}_{ijgG}, w^{π}_{ijgG}, and A^{π}_{ijgG}, are random variables whose values are realized after marriage, in the second period, the spouses can share income risk in the first period.

The continuous joint distribution of A^{π}_{ijgG}, W^{π}_{ijgG} and w^{π}_{ijgG} with bounded support is characterized by the parameter vector $\underline{Z}$. Let $S^{\pi}_{ijgG} = \{W^{\pi}_{ijgG}, w^{\pi}_{ijgG}, A^{\pi}_{ijgG}\}$ and $f(S^{\pi}_{ijgG}|\underline{Z})$ be the cumulative distribution function of S^{π}_{ijgG}. $\underline{Z}$ is known to individuals before their marriage decisions.

Let $\mathbf{E}$ be the expectations operator. Following the collective model with full risk sharing, we pose the efficient risk sharing spousal arrangement as a planner solving the following problem:

$$\max_{\{C,c,L,l\}} \beta_{ij}^{\pi}\mathbf{E}(\widehat{Q}(C_{ijgG}^{\pi}, L_{ijgG}^{\pi})|\underline{Z}) + (1-\beta_{ij}^{\pi})\mathbf{E}(\widehat{q}(c_{ijgG}^{\pi}, l_{ijgG}^{\pi})|\underline{Z}) \tag{P1}$$

subject to (i) $W_{ijgG}^{\pi}L_{ijgG}^{\pi} + w_{ijgG}^{\pi}l_{ijgG}^{\pi} + C_{ijgG}^{\pi} + c_{ijgG}^{\pi} \leq A_{ijgG}^{\pi} + W_{ijgG}^{\pi} + w_{ijgG}^{\pi} \; \forall \; S_{ijgG}^{\pi}$

(ii) $L_{ijgG}^{\pi} = 0 \; \forall \; S_{ijgG}^{\pi}$ if $\pi = 0$ (6)

In problem (P1), the planner chooses family consumption and labor supplies to maximize the weighted sum of the wife's and the husband's expected utilities subject to their family budget constraint. $\beta_{ij}^{\pi} \in [0,1]$ is the weight allocated to the wife's expected utility and $(1-\beta_{ij}^{\pi})$ is the weight allocated to the husband's expected utility. As in the collective model literature, β_{ij}^{π} depends on $\underline{Z}$, marriage market conditions, and other factors affecting the gains to marriage in which the individuals live.

Since the risk sharing arrangement takes the marriage as given, we dispense with the i, j, π subscripts when it does not cause confusion.

(P1) is equivalent to each spouse solving their own individual utility maximization problem subject to a judiciously chosen private budget constraint. In this equivalent problem, each spouse take their private budget constraint as given, i.e. (3) for the wife and (4) for the husband. Given the realization of W_{gG} and τ_{gG}, the working wife solves P2:

$$Q(W_{gG}, W_{gG} + \tau_{gG}) = \max_{L_{gG}} \widehat{Q}(\tau_{gG} + W(1 - L_{gG}), L_{gG}) \tag{P2}$$

Let $H(W_{gG}, \tau_{gG}) = 1 - L_{gG}$ be her Marshallian labor supply. As is well known from static labor supply theory, her labor supply is decreasing in the transfer that she receives:

$$H_{\tau_{gG}} < 0 \tag{7}$$

The non-working wife's indirect utility is:

$$Q(0, \tau_{gG}) = \widehat{Q}(\tau_{gG}, 1) \tag{8}$$

Given the realization of w_{gG}, A_{gG} and τ_{gG}, the husband solves P3:

$$q(w_{gG}, A_{gG} + w_{gG} - \tau_{gG}) = \max_{l_{gG}} \widehat{q}(A_{gG} - \tau_{gG} + w_{gG}(1 - l_{gG}), l_{gG}) \tag{P3}$$

Let $h(w_{gG}, A_{gG} - \tau_{gG}) = 1 - L_{gG}$ be his Marshallian labor supply. As is well known from static labor supply theory, his labor supply is increasing in the transfer that he has to provide:

$$h_{\tau_{gG}} > 0 \tag{9}$$

We leave to the reader to follow Wilson 1968 and Chiappori 1999 to show that P1 is equivalent to the wife choosing τ_{gG} to solve P4[7]:

$$\max_{\tau(S_{gG})} \beta \mathbf{E}(Q(W_{gG}, W_{gG} + \tau(S_{gG}))|\underline{Z}) + (1-\beta)\mathbf{E}(q(w_{gG}, A_{gG} + w_{gG} - \tau(S_{gG}))|\underline{Z}) \quad \text{(P4)}$$

Let

$$p = \frac{1-\beta}{\beta} \tag{10}$$

There is a one to one mapping between β and p. When $\beta = \frac{1}{2}$ or $p = 1$, the couple have equal bargaining power.

In solving P4, for every S_{gG}, her optimal choice of $\tau(S_{gG}, p, \underline{Z})$ will satisfy:

$$\frac{Q_Y(W_{gG}, W_{gG} + \tau(S_{gG}, p, \underline{Z}))}{q_y(w_{gG}, A_{gG} + w_{gG} - \tau(S_{gG}, p, \underline{Z}))} = \frac{1-\beta}{\beta} = p \tag{11}$$

Equation (11) is of course a well known implication of efficient risk sharing and we dispense with discussing its economic interpretation.

Using (11), we can find how the sharing rule changes with p, S_{gG} and z, $z \in \underline{Z}$:

$$\begin{aligned}
\tau_p &= \frac{q_y}{Q_{YY} + pq_{yy}} < 0 & (12)\\
\tau_A &= \frac{pq_{yy}}{Q_{YY} + pq_{yy}} > 0 & (13)\\
\tau_W &= \frac{-(Q_{YW} + Q_{YY})}{Q_{YY} + pq_{yy}} & (14)\\
\tau_w &= \frac{p(q_{yw} + q_{yy})}{Q_{YY} + pq_{yy}} & (15)\\
\tau_z &= 0 & (16)
\end{aligned}$$

(16) says that the optimal sharing rule is not a function of $\underline{Z}$, the determinants of the wages and non-labor income distributions. Such a result is not unexpected because there is full risk sharing and variations in $\underline{Z}$ do not affect p, the relative bargaining power of the husband.

(12) plays an important role in our empirical framework. It says that the transfer is decreasing in p, the relative bargaining power of the husband. Together with (7) and (9),

[7] Check that for a judicious choice of $\tau(S)$, the first order condtions to problems P1 and P4 are the same.

(12) implies that the wife's labor supply is increasing in p whereas the husband's labor supply is decreasing in p:

$$\frac{\partial H_{gG}}{\partial p} > 0 \tag{17}$$

$$\frac{\partial h_{gG}}{\partial p} < 0 \tag{18}$$

Since for every $\{i, j, 1\}$ couple, the husband's labor supply is decreasing in p and the wife's labor supply is increasing in p, for $\{i, j, 1\}$ couples, mean husband's labor supply is decreasing in p and mean wife's labor supply is increasing in p.

(12) and (11) also imply

$$\frac{\partial \mathbf{E}(Q(W_{gG}, W_{gG} + \tau(S_{gG}, p))|\underline{Z})}{\partial p} = -p\frac{\partial \mathbf{E}(q(w_{gG}, A_{gG} + w_{gG} - \tau(S_{gG}, p))|\underline{Z})}{\partial p} < 0 \tag{19}$$

The expected utility of the wife is decreasing in p. Conversely, the expected utility of the husband is increasing in p. (19) is unsurprising because p is the relative utility weight of the husband as shown in (10). So an increase in p makes $\{i, j, \pi\}$ marriages more attractive to type i males but less attractive to type j females. p is a non-negative scalar and is specific to $\{i, j, \pi\}$ marriages. Thus p can function as a shadow price that can equilibrate the excess demand for $\{i, i, \pi\}$ type marriages.

(19) will play a critical role in making our marriage matching model empirically operational.

The final point in this section is to point out a well known implication of the efficient risk sharing model with spousal labor supplies data when both spouses work, $\pi = 1$. A necessary condition for solving P1 is that given realized wages and non-labor income, i.e. S_{gG}, the planner solves problem P7:

$$\max_{C_{gG}, c_{gG}, L_{gG}, l_{gG}} \beta\widehat{Q}(C_{gG}, L_{gG}) + (1-\beta)\widehat{q}(c_{gG}, l_{gG}) \tag{P7}$$

$$\text{subject to } W_{gG}L_{gG} + w_{gG}l_{gG} + C_{gG} + c_{gG} \leq A_{gG} + W_{gG} + w_{gG} \tag{20}$$

Problem P7 is a unitary model of the family faced with wages W_{gG}, w_{gG}, and non-labor income A_{gG}. Thus we cannot reject a unitary model of the family for $\{i, j, \pi\}$ couples in the same society, by observing their spousal labor supplies behavior if they share risk efficiently.[8]

Problem P7 is also useful because it is a standard consumer choice problem. In particular, we know the spousal labor supplies functions must satisfy Slutsky symmetry, a testable restriction with spousal labor supply data when both spouses work, $\pi = 1$. Although this implication can be empirically tested, we will not do that in this paper.

[8]This point is well known. Hayashi, Altonji and Kotlikoff, Lich Tyler, Mazzacco, Ogaki.

2.4 Marriage decision problems in the first period

In the first period, agents decide whether to marry and who to marry if they choose to marry. Consider a particular woman G of type J. Recall that she can choose between I types of men, two types of marriages (specialized or not) and whether or not to marry. She can choose between $I*2+1$ choices. Her expected utility in an $\{i,j,\pi\}$ marriage is:

$$\overline{V}(i,j,\pi,p_{ij}^{\pi},\varepsilon_{ijG}^{\pi}) = \mathbf{E}(Q(W_{ijgG}^{\pi},W_{ijgG}^{\pi}+\tau(S_{ijgG}^{\pi},p_{ij}^{\pi}))|\underline{Z})+\Gamma_{ij}^{\pi}+\varepsilon_{ijG}^{\pi} \tag{21}$$

Given the realizations of all the ε_{ijG}^{π}, she will choose the marital choice which maximizes her expected utility. Let $\underline{\varepsilon_{jG}} = [\varepsilon_{0jG},..,\varepsilon_{ijG}^{0},..,\varepsilon_{ijG}^{1},..,\varepsilon_{IjG}^{1}]$. The expected utility from her optimal choice will satisfy:

$$V^{*}(\underline{\varepsilon_{jG}}) = \max[\overline{V}(0,j,\varepsilon_{0jG}),..,\overline{V}(i,j,0,p_{ij}^{0},\varepsilon_{ijG}^{0}),..,\overline{V}(i,j,1,p_{ij}^{1},\varepsilon_{ijG}^{1}),..] \tag{22}$$

The problem facing men in the first stage is analogous to that of women. A man g of type i in an $\{i,j,\pi\}$ marriage, with ε_{ijg}^{π}, attains an expected utility of:

$$\overline{v}(i,j,\pi,p_{ij}^{\pi},\varepsilon_{ijg}^{\pi}) = \mathbf{E}(q(w_{ijgG}^{\pi},A_{ijgG}^{\pi}+w_{ijgG}^{\pi}-\tau(S_{ijgG}^{\pi},p_{ij}^{\pi}))|\underline{Z})+\gamma_{ij}^{\pi}+\varepsilon_{ijg}^{\pi} \tag{23}$$

Given the realizations of all the ε_{ijg}^{π}, he will choose the marital choice which maximizes his expected utility. He can choose between $J*2+1$ choices. Let $\underline{\varepsilon_{ig}} = [\varepsilon_{i0g},..,\varepsilon_{ijg}^{0},..,\varepsilon_{ijg}^{1},..,\varepsilon_{ijg}^{1}]$. The expected utility from his optimal choice will satisfy:

$$v^{*}(\underline{\varepsilon_{ig}}) = \max[\overline{v}(i,0,\varepsilon_{i0g}),..,\overline{v}(i,j,0,p_{ij}^{0},\varepsilon_{ijg}^{0}),..\overline{v}(i,j,1,p_{ij}^{1},\varepsilon_{ijg}^{1})..] \tag{24}$$

3 The Marriage Market

Our model of the marriage market follows CS. Let

$$\overline{Q}_{ij}^{\pi}(p_{ij}^{\pi},\underline{Z}) = \mathbf{E}(Q(W_{ijgG}^{\pi},W_{ijgG}^{\pi}+\tau(S_{ijgG}^{\pi},p_{ij}^{\pi}))|\underline{Z}) \tag{25}$$

$$\overline{q}_{ij}^{\pi}(p_{ij}^{\pi},\underline{Z}) = \mathbf{E}(q(w_{ijgG}^{\pi},A_{ijgG}^{\pi}+w_{ijgG}^{\pi}-\tau(S_{ijgG}^{\pi},p_{ij}^{\pi}))|\underline{Z}) \tag{26}$$

Assume that there are lots of men and women of each type, and each woman is solving (22) and each man is solving (24). Because $\varepsilon_{ijG}^{\pi t}$ are i.i.d. extreme value random variables, McFadden (1974) showed that for every type of woman j:

$$\frac{\overline{\mu}_{ij}^{\pi}}{f_j} = \frac{\exp(\Gamma_{ij}^{\pi} + \overline{Q}_{ij}^{\pi}(p_{ij}^{\pi}, \underline{Z}))}{\sum_{k=0}^{I} \exp(\Gamma_{kj}^{\pi} + \overline{Q}_{kj}^{\pi}(p_{kj}^{\pi}, \underline{Z}))}, \quad i = 0, 1, .., J \tag{27}$$

where $\overline{\mu}_{ij}^{\pi t}$ is the number of (i, j, π, t) marriages demanded by j type females and $\overline{\mu}_{0j}$ is the number of type j females who choose to remain unmarried.

(27) implies:

$$\ln \overline{\mu}_{ij}^{\pi} - \ln \mu_{0j} = (\Gamma_{ij}^{\pi} - \Gamma_{0j}) + \overline{Q}_{ij}^{\pi}(p_{ij}^{\pi}, \underline{Z}) - \overline{Q}_{0j}(\underline{Z}) \ , \quad i = 1, .., I \tag{28}$$

CS calls the left hand side of (28), $\ln \overline{\mu}_{ij}^{\pi} - \ln \mu_{0j}$, the net gains to a j type woman in an $\{i, j, \pi\}$ marriage relative to remaining umarried.

Similarly, for every type of man i,

$$\frac{\underline{\mu}_{ij}^{\pi}}{m_i} = \frac{\gamma_{ij}^{\pi} + \overline{q}_{ij}^{\pi}(p_{ij}^{\pi}, \underline{Z})}{\sum_{k=0}^{J} \exp(\gamma_{ik}^{\pi} + \overline{q}_{ik}^{\pi}(p_{ik}^{\pi}, \underline{Z}))}, \quad j = 0, 1, .., I \tag{29}$$

which implies:

$$\ln \underline{\mu}_{ij}^{\pi} - \ln \mu_{i0} = (\gamma_{ij}^{\pi} - \gamma_{i0}) + \overline{q}_{ij}^{\pi}(p_{ij}^{\pi}, \underline{Z}) - \overline{q}_{i0}(\underline{Z}), \quad j = 1, .., J, \tag{30}$$

where $\underline{\mu}_{ij}^{\pi}$ is the number of $\{i, j, \pi\}$ marriages demanded by j type males and $\underline{\mu}_{i0}$ is the number of type i males who choose to remain unmarried.

CS calls the left hand side of (30), $\ln \overline{\mu}_{ij}^{\pi} - \ln \mu_{i0}$, the net gains to a i type man in an $\{i, j, \pi\}$ marriage relative to remaining umarried.

Marriage market clearing requires the supply of wives to be equal to the demand for wives for each type of marriage:

$$\underline{\mu}_{ij}^{\pi} = \overline{\mu}_{ij}^{\pi} = \mu_{ij}^{\pi} \ \forall \{i > 0, j > 0, \pi, \} \tag{31}$$

Imposing marriage market clearing (31), sum 28) and (30) to get:

$$\ln \frac{\mu_{ij}^{\pi}}{\sqrt{\mu_{i0}\mu_{0j}}} = \frac{(\Gamma_{ij}^{1} - \Gamma_{0j}) + (\gamma_{ij}^{\pi} - \gamma_{i0}) + \overline{Q}_{ij}^{\pi}(p_{ij}^{\pi}, \underline{Z}) + \overline{q}_{ij}^{\pi}(p_{ij}^{\pi}, \underline{Z}) - \overline{Q}_{0j}(\underline{Z}) - \overline{q}_{i0}(\underline{Z})}{2} \tag{32}$$

CS calls $\ln \mu_{ij}^{\pi}(\mu_{i0}\mu_{0j})^{-\frac{1}{2}}$ the total gains to an (i, j, π) marriage relative to the couple remaining unmarried. Assuming transferable utilities, CS argued the total gains to marriage should be invariant across societies when preferences do not change across societies. As the rhs of (32) shows, even if preferences for consumption and leisure are invariant across societies, the total gains depend on the society that i and j live in. There are three reasons why the total gains in this model depend on the society. First, as the society changes, $\underline{Z}$, the parameters which govern the distribution of wages and non-labor income change. Second, as p_{ij}^{π} changes due to changes in M and F or $\underline{Z}$, the induced changes in spousal utilities do not cancel in (32). Third, legal and cultural differences in the marriage markets across societies will affect $(\Gamma_{ij}^{1} - \Gamma_{0j})$, $(\gamma_{ij}^{\pi} - \gamma_{i0})$.

There are feasibility constraints that the stocks of married and single agents of each gender and type cannot exceed the aggregate stocks of agents of each gender in the society:

$$f_j = \mu_{0j} + \sum_{i,\pi} \mu_{ij}^{\pi} \tag{33}$$

$$m_i = \mu_{i0} + \sum_{j,\pi} \mu_{ij}^{\pi} \tag{34}$$

We can now define a rational expectations equilibrium. There are two parts to the equilibrium, corresponding to the two stages at which decisions are made by the agents. The first corresponds to decisions made in the marriage market; the second to the intra-household allocation. In equilibrium, agents make marital status decisions optimally, the sharing rules clear each marriage market, and conditional on the sharing rules, agents choose consumption and labor supply optimally. Formally:

Definition 1. *A rational expectations equilibrium consists of a distribution of males and females across individual type, marital status, and type of marriage $\{\hat{\mu}_{0j}, \hat{\mu}_{i0}, \hat{\mu}_{ij}^{\pi}\}$, a set of decision rules for marriage $\{\widehat{V}^{*}(\underline{\varepsilon_{jG}}), \widehat{v}^{*}(\underline{\varepsilon_{ig}})\}$, a set of decision rules for spousal consumption and leisure*
$\{\hat{C}_{ijgG}^{\pi}, \hat{c}_{ijgG}^{\pi}, \hat{L}_{ijgG}^{\pi}, \hat{l}_{ijgG}^{\pi}\}$, and a set of shadow prices $\{p_{ij}^{\pi}\}$ such that:

1. *The decision rules $\{\widehat{V}^{*}(\underline{\varepsilon_{jG}}), \widehat{v}^{*}(\underline{\varepsilon_{ig}})\}$ solve (22) and (24);*

2. *All marriage markets clear implying (31), (33), (34) hold;*

3. *For an $\{i, j, \pi\}$ marriage with wages and non-labor income realizations $\{S_{ijgG}^{\pi}\}$, the decision rules $\{\hat{C}_{ijgG}^{\pi}, \hat{c}_{ijgG}^{\pi}, \hat{L}_{ijgG}^{\pi}, \hat{l}_{ijgG}^{\pi}\}$ solve (P2), (P3).*

Theorem 1. *A rational expectations equilibrium exists.*

Sketch of proof: We have already demonstrated (1) and (3). So what needs to be done is to show that there is a set of shadow prices, $\{p^{\pi}_{ij}\}$ which clears the marriage market. Let $\underline{p}$ be the vector of shadow prices for society x. For every marriage market $\{i, j, \pi\}$ excluding $i = 0$ or $j = 0$, define the excess demand function for marriages by men:

$$E^{\pi}_{ij}(\underline{p}) = \underline{\mu}^{\pi}_{ij}(\underline{p}) - \overline{\mu}^{\pi}_{ij}(\underline{p}) \tag{35}$$

The demand and supply functions (27) and (29), for every marriage market $\{i, j, \pi\}$, satisfy the weak gross substitute property. So the excess demand functions also satisfy the weak gross substitute property. We can use Mas-Colell, Winston and Green (1995: p. 646, exercise 17.F.16^{C}) to provide a proof of existence of market equilibrium when the excess demand functions satisfy the weak gross substitute property.[9] For convenience, we reproduce their proof in our context in Appendix A.

In the proof, we need:

$$\begin{aligned} E^{\pi}_{ij}(\underline{p}) &> 0 \text{ as } \underline{p} \to \infty && \text{(Condition A1)} \\ E^{\pi}_{ij}(\underline{p}) &< 0 \text{ as } \underline{p} \to 0 && \text{(Condition A2)} \end{aligned}$$

That is, the utility functions q and Q must be such that as $\underline{p}$ approaches 0, men will not want to marry. And as $\underline{p}$ approaches ∞, women will not want to marry.

4 Marriage market identification with approximately equal bargaining power

Consider the marriage market $\{i, j, \pi\}$. As derived in Section 3, marriage market clearing, equation (32), implies:

$$\ln \frac{\mu^{\pi}_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} = \frac{(\Gamma^{\pi}_{ij} - \Gamma_{0j}) + (\gamma^{\pi}_{ij} - \gamma_{i0}) + \overline{Q}^{\pi}_{ij}(p^{\pi}_{ij}, \underline{Z}) + \overline{q}^{\pi}_{ij}(p^{\pi}_{ij}, \underline{Z}) - \overline{Q}_{0j}(\underline{Z}) - \overline{q}_{i0}(\underline{Z})}{2} \tag{36}$$

p^{π}_{ij} is the equilibrium shadow price which clears the marriage market. $p^{\pi}_{ij} = 1$ is an important benchmark. In this case, the particular marriage market clears with equal bargaining power between the spouses, i.e. $\beta^{\pi}_{ij} = \frac{1}{2}$.

Using a first order Taylor series expansion around $p^{\pi}_{ij} = 1$,

[9] The proof does not rely on Walras Law or that excess demand is homogenous of degree zero in $\underline{p}$, both of which our model does not satisfy.

$$\overline{q}^{\pi}_{ij}(p^{\pi}_{ij},\underline{Z}) \simeq \overline{q}^{\pi}_{ij}(1,\underline{Z})+(p^{\pi}_{ij}-1)\frac{\partial \overline{q}^{\pi}_{ij}(p^{\pi}_{ij},\underline{Z})}{\partial p^{\pi}_{ij}}|_{p^{\pi}_{ij}=1} \tag{37}$$

$$\overline{Q}^{\pi}_{ij}(p^{\pi}_{ij},\underline{Z}) \simeq \overline{Q}^{\pi}_{ij}(1,\underline{Z})+(p^{\pi}_{ij}-1)\frac{\partial \overline{Q}^{\pi}_{ij}(p^{\pi}_{ij},\underline{Z})}{\partial p^{\pi}_{ij}}|_{p^{\pi}_{ij}=1} \tag{38}$$

$$= \overline{Q}^{\pi}_{ij}(1,\underline{Z})-(p^{\pi}_{ij}-1)\frac{\partial \overline{q}^{\pi}_{ij}(p^{\pi}_{ij},\underline{Z})}{\partial p^{\pi}_{ij}}|_{p^{\pi}_{ij}=1} \tag{39}$$

The last line, (39), obtains because of (19), an implication of efficient spousal risk sharing.

Using (37) and (39), (**??**) becomes:

$$\ln \frac{\mu^{\pi}_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} = \frac{(\Gamma^{\pi}_{ij}-\Gamma_{0j})+(\gamma^{\pi}_{ij}-\gamma_{i0})+\overline{Q}^{\pi}_{ij}(1,\underline{Z})+\overline{q}^{\pi}_{ij}(1,\underline{Z})-\overline{Q}_{0j}(\underline{Z})-\overline{q}_{i0}(\underline{Z})}{2} \tag{40}$$

$$= T^{\pi}_{ij}(\Gamma^{\pi}_{ij},\Gamma_{0j},\gamma^{\pi}_{ij},\gamma_{i0},\underline{Z}) \tag{41}$$

The right hand side of (40) is independent of p^{π}_{ij}, the equilibrium shadow price. It only depends on i,j,π, the types of spouses involved as well as the type of marriage that they are engaged in.

Because μ^{π}_{ij}, μ_{i0}, μ_{0j} are observed, we can estimate the total gains to marriage, $T^{\pi}_{ij}(\Gamma^{\pi}_{ij},\Gamma_{0j},\gamma^{\pi}_{ij},\gamma_{i0},\underline{Z})$. If two different societies have different $\underline{Z}'s$ and or $\Gamma^{\pi}_{ij},\Gamma_{0j},\gamma^{\pi}_{ij},\gamma_{i0}$, they will have different total gains, T^{π}_{ij}. Thus we do not expect two different societies to have the same total gains.

Equation (40) is familiar from CS where it was derived under the hypothesis of transferable utilities without post marital uncertainty.

That $\ln \mu^{\pi}_{ij}(\mu_{i0}\mu_{0j})^{-\frac{1}{2}}$ measures the total gains to a $\{i,j,\pi\}$ type marriage in an efficient spousal risk sharing marriage market model is important because it shows that transferable utilities is not necessary to obtain equation (40). As discussed in CS, T^{π}_{ij} is an intuitive measure of total gains because it says that the more $\{i,j,\pi\}$ marriages there are relative to the geometric average of the unmarrieds, the larger is the total gains to that type of marriage.

As discussed in CS, (40) does not have any overidentifying assumption. There is no way to test the marriage matching model using (40). (40) is derived under the assumption that bargaining power between the spouses are approximately equal.

Finally, we have assumed that individuals can freely choose their hours of work. If workers are rationed in their hours of work, the labor supplies models proposed in this paper are misspecified. In particular $\overline{q}^{\pi}_{ij}$ and $\overline{Q}^{\pi}_{ij}$ will be misspecified. But as (40) shows, we can still identify marriage market parameters because $\overline{q}_{ij}^{\pi}$ and $\overline{Q}_{ij}^{\pi}$ do not need to be separately identified.

5 Multi-markets restrictions (INCOMPLETE)

Let the equilibrium shadow prices be $\{p_{ij}^{\pi}(\Gamma, \gamma, \underline{Z}, M, F)\}$. The equilibrium quasi supply by women for $\{i, j, \pi\}$ marriages satisfies:

$$\ln \frac{\mu_{ij}^{\pi}}{\mu_{0j}} = (\Gamma_{ij}^{\pi} - \Gamma_{0j}) + \overline{Q}_{ij}^{\pi}(p_{ij}^{\pi}(\Gamma, \gamma, \underline{Z}, M, F), \underline{Z}) - \overline{Q}_{0j}(\underline{Z}) \tag{42}$$

Then we have the following comparative static:

$$\frac{\partial \ln \frac{\mu_{ij}^{\pi}}{\mu_{0j}}}{\partial \omega} = \frac{\partial(\Gamma_{ij}^{\pi} - \Gamma_{0j}^{\pi})}{\partial \omega} + \frac{\partial \overline{Q}_{ij}^{\pi}}{\partial p_{ij}^{\pi}} \frac{\partial p_{ij}^{\pi}}{\partial \omega} + \frac{\partial(\overline{Q}_{ij}^{\pi} - \overline{Q}_{0j})}{\partial \underline{Z}} \frac{\partial \underline{Z}}{\partial \omega} \tag{43}$$

(43) decomposed the change in net gains to marriage for wives into three components. The first component is the change utility due to the change in net invariant gains for wives. The second component is the utility change from the change in relative bargaining power. The third component is the utility change from the changes in wage and non-labor income distributions.

The equilibrium quasi demand by men for $\{i, j, \pi\}$ marriages satisfy:

$$\ln \frac{\mu_{ij}^{\pi}}{\mu_{i0}} = (\gamma_{ij}^{\pi} - \gamma_{i0}) + \overline{q}_{ij}^{\pi}(p_{ij}^{\pi}(\Gamma, \gamma, \underline{Z}, M, F), \underline{Z}) - \overline{q}_{i0}(\underline{Z}) \tag{44}$$

Then we have the following comparative static for a scalar parameter $\omega \in \{\Gamma, \gamma, \underline{Z}, M, F\}$:

$$\frac{\partial \ln \frac{\mu_{ij}^{\pi}}{\mu_{i0}}}{\partial \omega} = \frac{\partial(\gamma_{ij}^{\pi} - \gamma_{i0}^{\pi})}{\partial \omega} + \frac{\partial \overline{q}_{ij}^{\pi}}{\partial p_{ij}^{\pi}} \frac{\partial p_{ij}^{\pi}}{\partial \omega} + \frac{\partial(\overline{q}_{ij}^{\pi} - \overline{q}_{i0})}{\partial \underline{Z}} \frac{\partial \underline{Z}}{\partial \omega} \tag{45}$$

The interpretation of (45) is similar to that given for wives.

Using (19), (45) and (43),

$$\frac{\partial(\ln \frac{\mu_{ij}^{\pi}}{\mu_{0j}} - \ln \frac{\mu_{ij}^{\pi}}{\mu_{i0}})}{\partial \omega} = \frac{\partial((\Gamma_{ij}^{\pi} - \Gamma_{0j}^{\pi}) - (\gamma_{ij}^{\pi} - \gamma_{0j}))}{\partial \omega} + (\frac{\partial(\overline{Q}_{ij}^{\pi} - \overline{Q}_{0j}) - (\overline{q}_{ij}^{\pi} - \overline{q}_{i0})}{\partial \underline{Z}}) \frac{\partial \underline{Z}}{\partial \omega} + (1 + p_{ij}^{\pi}) \frac{\partial \overline{Q}_{ij}^{\pi}}{\partial p_{ij}^{\pi}} \frac{\partial p_{ij}^{\pi}}{\partial \omega}$$

(46)

(46) says that the change in the difference in net spousal gains is equal to three terms. The first term is the change in the difference in spousal invariant gains. The second term is the change in the difference in spousal utilities from a change in the wages and non-labor income distributions. The third term is proportional to the change in the wife's utility from a change in her husband's relative bargaining power, p^{π}_{ij}. Since $(1 + p^{\pi}_{ij}) > 0$ and $\frac{\partial \overline{Q}^{\pi}_{ij}}{\partial p^{\pi}_{ij}} < 0$, if p^{π}_{ij} increases, the wife's net gain will fail relative to her husband and vice versa.

Note that the difference in net spousal gains is equal to the log of the ratio of the number of unmarried type i men to unmarried type j women:

$$\ln \frac{\mu^{\pi}_{ij}}{\mu_{0j}} - \ln \frac{\mu^{\pi}_{ij}}{\mu_{i0}} = \ln \frac{\mu^{\pi}_{i0}}{\mu_{0j}} \tag{47}$$

(47) says that the net spousal gain of the wife increases relative to her husband if the number of unmarried type i men increases relative to the number of unmarried type j women, and vice versa.

The empirical content of (46) is as follows. Consider two societies, r and r'. Using (47), the difference in difference in net spousal gains between the two societies is:

$$\Delta \ln \frac{\mu_{i0}}{\mu_{0j}} = \ln \frac{\mu^{\pi r'}_{i0}}{\mu^{r'}_{0j}} - \ln \frac{\mu^{\pi r}_{i0}}{\mu^{r}_{0j}} \tag{48}$$

Using (46), (48) becomes:

$$\begin{aligned} \Delta \ln \frac{\mu_{i0}}{\mu_{0j}} &= \Delta((\Gamma^{\pi}_{ij} - \Gamma^{\pi}_{0j}) - (\gamma^{\pi}_{ij} - \gamma_{0j})) + (1 + p^{\pi r}_{ij}) \frac{\partial \overline{Q}^{\pi}_{ij}}{\partial p^{\pi}_{ij}} \Delta p^{\pi}_{ij} && (49) \\ &\quad + \frac{\partial((\overline{Q}^{\pi}_{ij} - \overline{Q}_{0j}) - (\overline{q}^{\pi}_{ij} - \overline{q}_{i0}))}{\partial Z} \Delta Z && (50) \end{aligned}$$

Given base society r and i, j, choose the partner city, r', such that the wage distributions of the umarrieds, $i0$ and $0j$, are the same in both cities. We are assuming that if the wage distributions of the unmarrieds are the same across the two cities, the wage distributions of the marrieds and the distributions of non-labor income are also the same across the two cities for i, j types. In other words, between the two cities r and r', $\Delta Z = 0$.

(49) reduces to:

$$\Delta \ln \frac{\mu_{i0}}{\mu_{0j}} = \Delta((\Gamma^{\pi}_{ij} - \Gamma^{\pi}_{0j}) - (\gamma^{\pi}_{ij} - \gamma_{0j})) + (1 + p^{\pi r}_{ij}) \frac{\partial \overline{Q}^{\pi}_{ij}}{\partial p^{\pi}_{ij}} \Delta p^{\pi}_{ij} \tag{51}$$

Because the invariants gains to marriage for both spouses are likely to be positively correlated across societies, we expect $\Delta((\Gamma^{\pi}_{ij}-\Gamma^{\pi}_{0j})-(\gamma^{\pi}_{ij}-\gamma_{0j}))$ to be approximately zero. In this case, $\Delta \ln \frac{\mu_{i0}}{\mu_{0j}}$ primarily captures changes in bargaining power between the two societies.

If $\Delta p^{\pi}_{ij} > 0$, then the second term in (51) is positive because $p^{\pi}_{ij} > 0$ and $\partial \overline{Q}^{\pi}_{ij}/\partial p^{\pi}_{ij}$ is also positive. We know from (17) and (18) that the wife's labor supply is positively correlated with p^{π}_{ij} and the husband's labor supply is negatively correlated with p^{1}_{ij}. Let $\overline{h^{\pi r}_{ij}}$ and $\overline{H^{\pi r}_{ij}}$ be the mean hours of work of husbands and wives in $\{i, j, \pi\}$ marriages in society r respectively. Then we expect $\Delta\overline{h^{\pi}_{ij}}$ to be positively correlated with Δp^{π}_{ij} and $\Delta\overline{H^{1}_{ij}}$ to be negatively correlated with Δp^{1}_{ij}.

The second test uses the difference in differences methodology. We continue to assume the cities are matched by Z. Consider two pairs of marital matches, $\{i, j, \pi\}$ and $\{i', j', \pi\}$, where $i, j \neq i', j'$. Here we assume that:

$$\Delta((\Gamma^{\pi}_{ij}-\Gamma^{\pi}_{0j})-(\gamma^{\pi}_{ij}-\gamma_{0j})) = \Delta((\Gamma^{\pi}_{i'j'}-\Gamma^{\pi}_{0j'})-(\gamma^{\pi}_{i'j'}-\gamma_{0j'})) \tag{52}$$

In other words, the difference in the difference in net invariant spousal gains between the two cities are the same for $\{i, j, \pi\}$ and $\{i', j', \pi\}$ matches. Assuming (52), we can net out common city pair effects from our estimates of $\Delta \ln \frac{\mu_{i0}}{\mu_{0j}}$. What should be left is the second term in (51) which should just reflect the change in bargaining power between the city pairs. Empirically, we regress $\Delta\overline{h^{\pi}_{ij}}$ or $\Delta\overline{H^{1}_{ij}}$ on $\Delta \ln \frac{\mu_{i0}}{\mu_{0j}}$ and include city pairs dummies.

Finally, in the third test, we use the difference in differences methodology to compare changes in marriage matching patterns and spousal labor supplies over time for a city. Let r and r' denote the same city r at two points in time, t and t'. The assumption that the wage and non-income distributions in city r are the same across time is not tenable. In addition to assumption (52), that changes in net invariant gains for wives between t and t' are the same for $\{i, j, \pi\}$ and $\{i', j', \pi\}$ matches, we will also add the assumption:

$$\frac{\partial(\overline{Q}^{\pi}_{ij}-\overline{Q}_{0j})}{\partial \underline{Z}}\Delta\underline{Z} = \frac{\partial(\overline{Q}^{\pi}_{i'j'}-\overline{Q}_{0j'})}{\partial \underline{Z}}\Delta\underline{Z} \tag{53}$$

(53) says that the change in expected utility from consumption and leisure, due to changes in the wage and non-labor income distributions between time t and t', is the same for $\{i, j, \pi\}$ and $\{i', j', \pi\}$. Such an assumption will be false if there is non-neutral technical change which favors particular groups of individuals.

If (52) and (53) hold, we can apply the argument in the previous test to again net out common city pair effects from our estimates of $\Delta \ln \frac{\mu_{i0}}{\mu_{0j}}$. The difference between the previous test and the current is that under the current test, r and r' do not have the same wage and non-labor income distributions.

6 Multi period marriages

This section extends our model to allow for multi period marriages. After marriage, married individuals have to make multi period labor supplies and savings decisions. Wages and non-labor income will evolve stochastically over their marriage. We assume that there is complete contracting between the spouses. With fixed spousal utility weights and time separable current period utility functions, the analysis of intertemporal and intratemporal allocations is well known. So what we will deal with here is to show that this complete contracting model fits with our marriage matching model.

We will discuss a multi period non-specialized marriage. Consider a gG couple in an $\{i, j, 1\}$ marriage of length T and the husband's relative utility weight in marriage is p. Let $S_t = \{W_t, w_t, A_t\}$ where A_t is non-labor or non-interest income. S_t is a first order Markov process. Let ψ_t denote the savings of the couple at the beginning of period t of the marriage.

The family planner's Bellman equation in period t is:

$$\omega(\psi_t, S_t, t, p) = \max_{\{C_t, c_t, L_t, l_t\}} \widehat{Q}(C_t, L_t) + p\widehat{q}(c_t, l_t) + \lambda \mathbf{E}\{\omega(\psi_{t+1}, S_{t+1}, t+1, p)|S_t\} \tag{54}$$

$$\text{subject to } \psi_{t+1} = (1+r)(\psi_t + A_t + W_t(1-L_t) + w_t(1-l_t) - C_t - c_t) \tag{55}$$

$\widehat{Q}(.,.)$ and $\widehat{q}(.,.)$ are the wife's and husband's current period utilities respectively. λ is the discount factor and r is the interest rate on savings. $\mathbf{E}$ is the expectations operator. Equation (55) is the asset accumulation equation.

We reformulate (54) as a two stage budgeting problem:[10]

$$\omega(\psi_t, S_t, t, p) = \max_{X_t, \tau_t} Q(W_t, \tau_t) + pq(w_t, Y_t - X_t - \tau_t) + \lambda \mathbf{E}\{\omega(\psi_{t+1}, S_{t+1}, t+1, p)|S_t\} \tag{DP2}$$

$$\text{subject to (i) } Y_t = \psi_t + A_t + W_t + w_t \tag{56}$$

$$\text{(ii) } \psi_{t+1} = (1+r)(Y_t - X_t) \tag{57}$$

Y_t is full family income in period t. In period t, the planner has to choose total savings for the family, X_t. Of the remaining income, $Y_t - X_t$, τ_t is allocated to the wife for her to spend on current consumption and leisure. $Y_t - X_t - \tau_t$ is allocated to the husband for him to spend on current consumption and leisure. $Q(.,.)$ and $q(.,.)$ has the same properties as in Section 2.3. The first order condition with respect to τ_t is:

[10] This reformulation is common in studies of intertemporal labor supplies (E.g. Blundell and Walker 1986).

$$Q_Y(W_t, \tau_t) = pq_Y(w_t, Y_t - X_t - \tau_t) \tag{58}$$

Differentiability of the value function follows standard arguments. Then the first order condition with respect to X_t is:

$$pq_Y(w_t, Y_t - X_t - \tau_t) = \lambda(1+r)\mathbf{E}\{\omega_\psi(\psi_{t+1}, S_{t+1}, t+1, p)|S_t\} \tag{59}$$

By the envelope theorem,

$$\mathbf{E}\{\omega_\psi(\psi_{t+1}, S_{t+1}, t+1, p)|S_t\} = \mathbf{E}\{pq_Y(w_{t+1}, Y_{t+1} - X_{t+1} - \tau_{t+1})|S_t\} \tag{60}$$

Substituting (60) into (59),

$$q_Y(w_t, Y_t - X_t - \tau_t) = \lambda(1+r)\mathbf{E}\{q_Y(w_{t+1}, Y_{t+1} - X_{t+1} - \tau_{t+1})|S_t\} \tag{61}$$

(61) is the Euler equation for describing optimal savings.

The expected discounted utilities of the wife and the husband from consumption and leisure by entering the marriage is:

$$\overline{Q}(p) = \mathbf{E}\{\sum_1^T \lambda^t Q(W_t, \tau_t)|\underline{Z}\} \tag{62}$$

$$\overline{q}(p) = \mathbf{E}\{\sum_1^T \lambda^t q(w_t, Y_t - X_t - \tau_t)|\underline{Z}\} \tag{63}$$

The changes in expected discounted utilities when p increases are:

$$\overline{Q}_p = \mathbf{E}\{\sum_1^T \lambda^t Q_Y(W_t, \tau_t)\frac{\partial \tau_t}{\partial p}|\underline{Z}\} \tag{64}$$

$$\overline{q}_p = \mathbf{E}\{\sum_1^T \lambda^t[-q_y(w_t, Y_t - X_t - \tau_t)(\frac{\partial X_t}{\partial p} + \frac{\partial \tau_t}{\partial p}) + \lambda(1+r)q_y(w_{t+1}, Y_{t+1} - X_{t+1} - \tau_{t+1})\frac{\partial X_t}{\partial p}|\underline{Z}\} \tag{65}$$

$$= \mathbf{E}\{\sum_1^T \lambda^t[-q_y(w_t, Y_t - X_t - \tau_t)\frac{\partial \tau_t}{\partial p}|\underline{Z}\} \tag{66}$$

(66) obtains by substituting the Euler equation (61) into (65).

Using (64), (66) and (58),

$$\overline{Q}_p = -p\overline{q}_p \tag{67}$$

(67) is the multi period equivalent of (19). We can use the multi period discounted expected utilities instead of (25) and (26) to derive the demand for marriages in Section 3. With (67), the excess demand function for marriages, (35), has the same properties required for a rational expectations equilibrium with multi period marriages.

The multi period spousal labor supply model discussed here is standard in the literature, discussed under the unitary framework. Empirical unitary models of intertemporal spousal labor supplies include Blundell and Walker (1986), Hyslop (2001), Lundberg (1985). Their evidence suggest that there is non-trivial spousal earnings risk sharing. But as we discussed in the static case, complete risk sharing implies the unitary model after contracting.

7 Dynamic marriage matching

When individuals are long lived, they also have to choose when to marry if they want to marry. Choo Siow (2006) extends the static CS model into a dynamic marriage matching model where finitely lived individuals choose who and when to marry. The important assumption in that extension is the assumption of complete contracting in marriage and exogenous divorce behavior. Since we use complete contracting in our multi period marriage model in the previous section, we should be able to use Choo Siow (2006) to extend our multi period marriage model to allow for dynamic marriage matching decisions.

8 One period marriage without uncertainty

Most of literature on the collective model deals with a static model of intrahousehold allocations without uncertainty. That is, wages and non-labor income are known as of the time the individuals enter into the marriage. Our marriage matching framework can accommodate this case.

Let observed wages, non-labor income and labor supplies be equal to true wages, non-labor income and labor supplies plus measurement error:

$$W^{\pi}_{ijgG} = W^{\pi}_{ij} + \varepsilon^{W\pi}_{ijgG} \tag{68}$$

$$w^{\pi}_{ijgG} = w^{\pi}_{ij} + \varepsilon^{w\pi}_{ijgG} \tag{69}$$

$$A^{\pi}_{ijgG} = A^{\pi}_{ij} + \varepsilon^{A\pi}_{ijgG} \tag{70}$$

$$L^{\pi}_{ijgG} = L^{\pi}_{ij} + \varepsilon^{L\pi}_{ijgG} \tag{71}$$

$$l^{\pi}_{ijgG} = l^{\pi}_{ij} + \varepsilon^{l\pi}_{ijgG} \tag{72}$$

$\varepsilon^{W\pi}_{ijgG}$, $\varepsilon^{w\pi}_{ijgG}$, $\varepsilon^{L\pi}_{ijgG}$, $\varepsilon^{l\pi}_{ijgG}$and $\varepsilon^{A\pi}_{ijgG}$ are measurment errors which are uncorrelated with the true values. Marriages are still identified by $\{i, j, \pi\}$. Thus we can still use β^{π}_{ij}, the utility weight of the wife to clear the marriage market. Given β^{π}_{ij}, instead of problem P1, the planner will now solve:

$$\max_{\{C,c,L,l\}} \beta^{\pi}_{ij}\widehat{Q}(C^{\pi}_{ij}, L^{\pi}_{ij}) + (1 - \beta^{\pi}_{ij})\widehat{q}(c^{\pi}_{ij}, l^{\pi}_{ij}) \tag{P1a}$$

$$\text{subject to (i) } W^{\pi}_{ij}L^{\pi}_{ij} + w^{\pi}_{ij}l^{\pi}_{ij} + C^{\pi}_{ij} + c^{\pi}_{ij} \leq A^{\pi}_{ij} + W^{\pi}_{ij} + w^{\pi}_{ij} \ \forall\ S^{\pi}_{ij}$$

$$\text{(ii) } L^{\pi}_{ij} = 0 \ \forall\ S^{\pi}_{ij} \text{ if } \pi = 0 \tag{73}$$

(11), appropriately reinterpreted, continues to hold which is what is critical for marriage market clearing. Thus as long as we can identify the type of an individual and the types of marriages that the individual can enter into, i.e. $\{i, j, \pi\}$, the empirical tests that we develop in this paper remain valid.

Thus our empirical results should be interpreted with care. *Even if our empirical results is consistent with our model predictions, they do not shed light on whether there is efficient risk sharing within the family or not.*

It is also convenient at this point to discuss empirical tests of the static collective model using spousal labor supplies such as CFL. In their paper, they estimate restricted spousal labor supplies models where the restrictions are derived from a static collective model. They instrument spousal wages with education, father's education, age, city size, religion. Different values of these instruments define different types of individuals in different regions. There is no instrument which captures the transitory component of wages.[11] Our interpretation of their empirical results is that they provide evidence of efficient bargaining between different types of spouses. Their empirical results are not informative about whether there is efficient risk sharing with the household as we suppose, or whether there is not as they supposed. In order to empirically distinguish between whether there is efficient risk sharing or not, one would need an instrument for transitory wage shocks when one estimates spousal labor supplies equations.

Our static formulation of the collective model in this section is related to Del Boca and Flinn's formulation. Their paper also studies marriage matching and intrahousehold allocations. Like us, they have a two period model in which individuals marry in the first period and then choose intrahousehold allocations in the second period. Our papers differ in the following significant way. In the second period, after marriage matches are determined, spouses bargain over intrahousehold allocations or choose intrahousehold allocations non-cooperatively without any reference to marriage market conditions.[12] This assumption is

[11] Although age changes for an individual over time, the changes are deterministic. Also, the previous section shows that our model extends to multi-period marriages.

[12] They motivate their assumption by arguing that there is no commitment in marriage and the threat of

completely different from our full commitment assumption, where a couple in the second period fullfill their first period commitments.

9 The Stone Geary case

[Eugene and Shannon: It will be useful to show whether (**??**) and (**??**) are satisfied.]

10 Literature review (incomplete)

Beginning with Grossbard-Schectman 1993, researchers have investigated the impact of sex ratios on female labor supplies (...). Since the sex ratio is defined at the society level, it is convenient to abstract from individual level data.[13] Let $\overline{H_{ij}^r}$ denote the average or log of the average hours of work of wives in $\{i, j\}$ marriages in society r. Let S_{ij}^r denote the sex ratio (ratio of type i men to type j women) in society r. Consider the typical average female labor supply regression:

$$\overline{H_{ij}^r} = \phi_0 + \phi_S S_{ij}^r + \underline{X_{ij}^r}' \underline{\phi_x} + u_{ij}^r,\ r = 1, .., R, i = 1, .., I; j = 1, .., J \tag{74}$$

$\underline{X_{ij}^r}$ is a vector of control variables and u_{ij}^r is the error term of the regression. Depending on the data set, $\underline{X_{ij}^r}$ may include location dummies, time dummies, and individual type dummies. In general, (74) is estimated with ordinary least squares. A robust finding from this literature is that $\phi_S < 0$, that an increase in the sex ratio decreases hours of work of women. Previous researchers have argued that this finding supports the hypothesis that marriage market conditions affect intra-household allocation of resources.

Arguing that the sex ratio is endogenous, Angrist instruments S_{ij}^r with immigration flows of $\{i, j\}$ individuals into society r. If there is an increase in the demand for male workers, i, relative to female workers, j, in city r, the sex ratio may increase as well as the wages of males relative to females. Independent of marriage market effects, changes in relative wages will affect spousal labor supplies. Thus reduced from estimates of (74) using OLS confound both labor market effects and marriage market effects. If immigration flows are correlated with labor market conditions, then Angrist's instrument do not completely solve the endogeneity problem.

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divorce is not salient for most marriages (Lundberg and Pollak).

[13] Abstracting from individual data is of course not innocuous if we want to distinguish between collective and unitary models of the family.

Instead of (74), tests in this paper investigates, for pairs of cities:

$$\Delta \overline{H_{ij}^r} = \phi_0 + \phi_n \Delta \ln \frac{\mu_{i0}^\pi}{\mu_{0j}^\pi} + \Delta \underline{X_{ij}}' \underline{\phi_x} + u_{ij}^r, \; r = 1,..,R; i = 1,..,I; j = 1,..,J \tag{75}$$

$$\Delta \overline{H_{ij}^r} = \phi_0 + \phi_S \Delta S_{ij} + \Delta \underline{X_{ij}}' \underline{\phi_x} + u_{ij}^r, \; r = 1,..,R; i = 1,..,I; j = 1,..,J \tag{76}$$

$\Delta \underline{X_{ij}}$ are city pairs dummies. For some tests, we choose pairs of cities in the same region with the same wage distributions for unmarried individuals. (75) directly tests whether changes in net spousal gains affect spousal labor supplies, holding region and the wage distributions constant. (76) is our reduced form version of (74). Our estimate of ϕ_s in (76), which holds region and the wage distributions constant, is an attempt to estimate the effect of variation in the sex ratio on spousal labor supplies, holding labor effects constant.

When we do not hold the wage distributions of the unmarrieds the same across city pairs, as when we compare a city at two points in time, we control for city and year effects.

11 Appendis A: Proof of existence of equilibrium

To conserve on notation, we dispense with the π and superscript. We know:

$$\frac{\partial \underline{\mu}_{ij}}{\partial p_{ij}} > 0 \tag{77}$$

$$\frac{\partial \underline{\mu}_{ij}}{\partial p_{ik}} < 0, \; k \neq j \tag{78}$$

$$\frac{\partial \underline{\mu}_{kl}(\beta)}{\partial p_{ij}} = 0; \; k \neq i, l \neq j \tag{79}$$

$$\frac{\partial \overline{\mu}_{ij}}{\partial p_{ij}} < 0 \tag{80}$$

$$\frac{\partial \overline{\mu}_{ij}}{\partial p_{kj}} > 0, \; k \neq i \tag{81}$$

$$\frac{\partial \overline{\mu}_{kl}(\beta)}{\partial p_{ij}} = 0; \; k \neq i, l \neq j \tag{82}$$

Recall that $p_{ij} = (1 - \beta_{ij})(\beta_{ij})^{-1}$ where $\beta_{ij} \in [0, 1]$ is the utility weight of the wife in an $\{i, j\}$ marriage. Let β be a matrix with typical element β_{ij} and the IxJ matrix function $E(\beta)$ be:

$$E(\beta) = \underline{\mu}(\beta) - \overline{\mu}(\beta) \tag{83}$$

An element of $E(\beta)$, $E_{ij}(\beta)$, is the excess demand for j type wives by i type men given β.

An equilibrium exists if there is a β^* such that $E(\beta^*) = 0$.

Assume that there exists a function $f(\beta) = \alpha E(\beta) + \beta$, $\alpha > 0$ which maps $[0,1]^{I*J} \rightarrow [0,1]^{I*J}$ and is non-decreasing in β. Tarsky's fixed point theorem says if a function $f(\beta)$ maps $[0,k]^N \rightarrow [0,k]^N$, $k > 0$, and is non-decreasing in β, there exists $\beta^* \in [0,k]^N$ such that $\beta^* = f(\beta^*)$. Let $f(\beta) = \alpha E(\beta) + \beta$, $k = 1$ and $N = I * J$, and apply Tarsky's theorem to get $\beta^* = \alpha E(\beta^*) + \beta^* \Rightarrow E(\beta^*) = 0$.

Thus the proof of existence reduces to showing $f(\beta)$ which has the required properties.

We know from (77) to (82) that:

$$\frac{\partial E_{ij}(\beta)}{\partial \beta_{ij}} < 0 \tag{84}$$

$$\frac{\partial E_{ik}(\beta)}{\partial \beta_{ij}} > 0 \tag{85}$$

$$\frac{\partial E_{kj}(\beta)}{\partial \beta_{ij}} > 0 \tag{86}$$

$$\frac{\partial E_{kl}(\beta)}{\partial \beta_{ij}} = 0;\ k \neq i, l \neq j \tag{87}$$

(84) to (87) imply that $E(\beta)$ satisfies the Weak Gross Substitutability (WGS) assumption.

We now show that the WGS property of $E(\beta)$ implies that we can construct $f(\beta)$, such that $f(\beta)$ maps $[0,1]^{I*J} \rightarrow [0,1]^{I*J}$ and is non-decreasing in β. The proof follows the solution to exercise 17.F.16^C of Mas-Colell, Whinston and Green given in their solution manual. (N.B. Unlike them, we do not start with Gross Substitution, we begin from WGS, but it turns out to be sufficient for Tarsky's conditions)

For notational convenience, now onwards we'll treat the matrix function $E(\beta)$, as a vector function.

Let $N = I * J$ and 1_N be a $N \times 1$ vector of ones. $E(\beta) : [0,1]^N \rightarrow R^N$ is continuously differentiable and satisfies $E(0_N) >> 0_N$ and $E(1_N) << 0_N$ (Conditions A1 and A2).

For every $\beta \in [0,1]^N$ and any n, if $\beta_n = 0$, then $E_n(\beta) > 0$.

For every $\beta \in [0,1]^N$ and any n, if $\beta_n = 1$, then $E_n(\beta) < 0$.

If $\beta = \{0_N, 1_N\}$, the facts follow from Conditions A1 and A2. Otherwise, they are due to Conditions A1 and A2, and (84) to (87), i.e. WGS.

For each n, define $C_n = \{\beta \in [0,1]^N : E_n(\beta) \geq 0\}$ and $D_n = \{\beta \in [0,1]^N : E_n(\beta) \leq 0\}$.

Then $C_n \subset \{\beta \in [0,1]^N : \beta_n < 1\}$ and $D_n \subset \{\beta \in [0,1]^N : \beta_n > 0\}$.

Then by continuity, the following two minima, $\min((1-\beta_n)/E_n(\beta) : \beta \in C_n)$ and $\min(-\beta_n/E_n(\beta) : \beta \in D_n)$, exist and are positive. Let $\underline{\beta}_n > 0$ be smaller than those two minima. Then, for all $\alpha \in (0, \underline{\beta}_n)$ and any $\beta \in [0,1]^N$, we have $0 \leq \alpha E_n(\beta) + \beta_n \leq 1$.

For each n, define $L_n = \max\{|\partial E_n(\beta)/\partial \beta_n| : \beta \in [0,1]^N\}$. Then, for all $\alpha \in (0, 1/L_n)$,

$$\begin{aligned}
\frac{\partial(\alpha E_n(\beta) + \beta_n)}{\partial \beta_n} &= \alpha \frac{\partial E_n(\beta)}{\partial \beta_n} + 1 \geq -\alpha L_n + 1 > 0 \\
\frac{\partial(\alpha E_n(\beta) + \beta_n)}{\partial \beta_m} &= \alpha \frac{\partial E_n(\beta)}{\partial \beta_m} \geq 0; n \neq m, \text{ follows from (84) to (87).}
\end{aligned}$$

Now let $K = \min\{\underline{\beta}_1, .., \underline{\beta}_N, 1/L_1, .., 1/L_N\}$, choose $\alpha \in (0, K)$, then $f(\beta) = \alpha E(\beta) + \beta \in [0,1]^N$ and $\partial f(\beta)/\partial \beta_n \geq 0$ for every $\beta \in [0,1]^N$, and any n. Hence Tarsky's conditions are satisfied.