### Gender Occupational Segregation in an Equilibrium Search Model\*

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#### Abstract

This paper studies an equilibrium search model in which wage and hours of work are job attributes and workers have different preferences for hours of work. In particular, there are women and men in the labor market, and the marginal disutility of an additional hour is higher for women than men. Employers have different production technologies, and they post a tied salary/hours offer that maximizes their steady-state profit (or utility) flow. Simulations show that women and men are concentrated in short and long hour jobs, respectively. There are fewer women on the job when employers have a taste for discrimination against women. These employers make their job offer less appealing to women by requiring more working hours. On the other hand, when women have a disamenity value to working on a job, women choose not to work in that job because of a loss in utility. The prediction on segregation is similar to the case with employer discrimination.

<sup>\*</sup>This article is a revised version of a chapter from my Ph.D. dissertation at Northwestern University (Usui, 2002). I am grateful to Joseph Altonji, Dale Mortensen, and Christopher Taber for their guidance. I also thank Rebecca Blank, Kerwin Charles, Kevin Lang, Tsunao Okumura, Jee-Hyeong Park, Steven Spurr, Randal Watson and participants in the labor lunch seminar at the University of Michigan, Wayne State University, and the Society of Labor Economists Meeting for their helpful comments and suggestions. All errors are my own.

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### 1 Introduction

In the model with perfect matching, when workers differ in their preferences for the non-wage component of a job (for instance, hours of work), workers who are more averse to longer hours of work are employed in short-hour jobs and those who are less averse are in long-hour jobs. The labor market is completely segregated by types of workers, and employers offer a package of wages and hours that maximize their profits for hiring one type of worker. However, when there are frictions in the labor market, workers are unable to locate themselves in their desired jobs and they move between jobs to obtain a higher occupational utility. Therefore, employers offer a package of wages and hours of work that accounts for the probability of employing different types of workers.<sup>1</sup>

This paper analyzes an equilibrium search model in which employers post a tied salary/hours offer to maximize their steady-state profit (or utility) flow. There are both women and men in the labor market, and it is assumed that the marginal disutility of an additional hour is higher for women than it is for men. Women are reluctant to work long hours relative to men, since women tend to bear the primary family responsibilities of caring for children and relatives.<sup>2</sup> In the model, women and men search for jobs, moving between unemployment and employment and between jobs to locate themselves in jobs that give them a higher utility. Employers have different production technologies, where their input in production is hours of work. Employers with identical job productivity post one tied salary/hours offer to all potential employees. I also assume that they cannot refuse to make that offer to women or men. There are a couple of reasons as to why employers usually post only one job description to fill a job vacancy. It is illegal to make job offers that differ systematically by gender. It can also be too costly for employers to monitor employees if employees work different hours on the same job. The primary purpose of this paper is to analyze a tied salary/hours offer

<sup>&</sup>lt;sup>1</sup>Altonji and Blank (1999) survey recent literature on occupational segregation. Johnson and Stafford (1998) provide a simple framework to understand the factors that affect gender occupational segregation. In their model, wages are determined when individuals are indifferent between working in predominantly female and predominantly male jobs. However, job mobility pattern in the National Longitudinal Survey of Youth (NLSY) indicates that women and men tend to move to more predominantly male jobs when they quit their jobs, but they tend to move to more predominantly female jobs when laid off (Usui, 2005). Thus, workers are not perfectly matched to their desired jobs.

<sup>&</sup>lt;sup>2</sup>Usui (2005) finds that the probability of reporting overemployment is higher for women than for men in the Panel Study of Income Dynamics (PSID). The overemployment measure is created by using the variables indicating worker's hours constraints on the job, as in the studies of Ham (1982, 1986), Kahn and Lang (1992, 1995), and Altonji and Paxson (1988).

when employers hold a taste for discrimination against women but are constrained to making a single job offer.

Using simulations, I illustrate the offer that maximizes the employer's steady-state profit (or utility) flow conditional on the job offers by all the other employers and on the job search behavior of workers. I show that employers with a higher marginal productivity of an additional hour require more hours, and their offers are tailored to men's preferences because men are more heavily represented in their jobs. Next, I consider a Becker (1971)-type employer discrimination where employers have a disamenity value to employing women. The employers suffer a utility loss when hiring women, and so they increase their hours to alter the packages away from women's preferences. This reinforces segregation. I show that employers can obtain a higher profit by being more discriminating. Discriminating employers regard women's productivity as low and pay less wages to them. As a result, the wage rate for both women and men is lowered because employers are constrained to post only one job offer. Women with a higher reservation utility find it better for them to be unemployed, but men remain on the job and suffer the wage loss. Since the wage rate is lower, employers decide to require more work hours. The discriminating employers, who have monopsony power, can realize a greater monetary profit when the decline in the number of female employees is small.

I then consider an employee discrimination model where a woman acts as if there were nonpecuniary costs for working in certain jobs. The nonpecuniary costs vary across jobs. Women choose not to work in jobs that where they will suffer a loss in utility. Consequently, there are more men on these jobs. The predictions on segregation are similar whether discrimination is related to productivity (employer discrimination) or whether it is related to women's utility (employee preference). However, the prediction on employers' profitability is different. In the case for employee discrimination, employers lower their profits because they tailor their job packages to attract more workers.

Lastly, I consider a case in which employee discrimination against women increases with the fraction of men on the job, since male cultures in the workplace can adversely affect women's job satisfaction. According to Mansfield et al. (1991), women in traditionally male occupations reported significantly less satisfaction and more stress at work than women in traditionally female occupations. In my study using job satisfaction data in the National Longitudinal Survey of Youth (NLSY), women who quit and move to jobs where there are more men, report that co-workers are less friendly and physical surroundings are less pleasant;

but men report the opposite (Usui, 2006). When I set up the discrimination coefficient to be positively related to the composition of men on the job, there can be multiple equilibria. Discrimination is absent in one equilibrium, and it exists in the other. In the discriminatory equilibrium, the predictions on segregation and employers' profitability are similar to the case where the employee discrimination parameter is set exogenously.

The job offer distributions described above are computed using a three-step algorithm. The algorithm is based on the idea that employers post a tied salary/hours offer that accounts for the difference in preference by gender, and the mix of women and men who typically choose the particular job type. In the first step, employers choose hours so that the marginal productivity of an additional hour equals the weighted average of the marginal disutility for women and men, where the weights reflect the gender composition of the particular job type in equilibrium. In the second step, employers choose a salary to maximize the steady-state profit (or utility) flow given the hours determined in the first step. The tied salary/hours offer here is not necessarily an optimal offer. Employers prefer workers with lower turnover rate, and value them more than the fraction of these workers on the job. In the third step, I start from the above tied salary/hours offer, and search for the tied salary/hours offer that maximizes the employer's steady-state profit (or utility) flow.

Almost all of the literature on the equilibrium search treats the wage as the only relevant job attribute. However, Lang and Majumdar (2004) and Hwang, Mortensen, and Reed (1998) are exceptional studies that allow jobs to have an additional attribute other than the wage. Lang and Majumdar (2004) (hereafter denote this paper by LM) consider a nonsequential model in which employers are homogeneous but workers are heterogeneous in preference for job amenities. Employers do not know the types of workers they face, and make a take-it-or-leave-it offer. Each employer trades off the salary/hours package against the possibility that the offer may be rejected. LM develops a strategy to compute the equilibrium tied salary/hours profile. LM shows that salary need not increase as the level of disamenity rises, which contradicts compensating differentials for negative job-characteristics.

My model has two differences from LM. First, I allow jobs to differ in production technologies. I show that jobs with larger marginal productivity of an additional hour require more hours and pay a higher salary. High productivity jobs may offer overall better job packages because they have a greater opportunity cost of going unfilled. Second, I characterize a sequential search model where workers move between jobs to obtain a higher utility. The

optimal tied salary/hours offer accounts for the different turnover rates between women and men. Employers prefer workers with a lower turnover rate and weight these workers more than the fraction of the workers on the job would suggest. In a nonsequential search model in which employers of the same type post a single job offer, the weights in determining the optimal hours reflect the gender composition of the job.

Hwang, Mortensen, and Reed (1998) (hereafter denote this paper by HMR) analyze a hedonic wage offer when workers have identical preference for job amenity in a sequential search model. Specifically, HMR extends the Burdett and Mortensen (1998) model to the case where a job consists of a wage and an amenity, and the employers differ in the cost of producing the amenity. HMR show by simulation that jobs which offer better amenity can pay higher wages, which contradicts the theory of compensating differentials. Cost-efficient employers offer better amenities, and they may offer overall higher-valued job packages. Following the framework of HMR, I construct a model in which employers post one tied salary/hours offer and workers differ in preferences for job amenity.

The paper proceeds as follows: Section 2 sets up the model. Section 3 presents the algorithm to compute the equilibrium tied salary/hours offer. Section 4 displays the simulation results. The paper concludes in Section 5.

### 2 The Model

The model extends Hwang, Mortensen, and Reed (1998)'s model to the case where women and men have different preferences for hours of work.

Men, women, and employers. There are men and women in the labor market. Let g = m stand for men and g = f stand for women. The measure of men and women in the labor force is  $n_m$  and  $n_f$ , respectively. Workers are either employed or unemployed.<sup>3</sup> Workers value a job by its salary and hours of work. The utility of a job for a type g worker is:

$$v^{g}(S, H) = S + \xi^{g}\phi(H),$$

where S is salary, H is hours of work,  $\phi' < 0$ ,  $\phi'' < 0$ , and,

<sup>&</sup>lt;sup>3</sup>As in many other studies, I do not distinguish between the states of unemployment and nonemployment (not in the labor force).

$$\xi^g = \begin{cases} \xi^m & \text{for men} \\ \xi^f & \text{for women.} \end{cases}$$

Men and women are homogeneous in productivity, but they differ in their preferences for work hours. Specifically, the marginal disutility of an additional hour is larger for women than men,

$$0 < \xi^m < \xi^f.$$

An unemployed worker receives a flow utility of  $b^g$  from non-market activities and searches for a job. There is a continuous distribution of heterogeneity in the flow utility of being unemployed, and so  $b^g$  varies among each gender. Its distribution within the type g worker is denoted by  $K^g$ , and let  $\underline{b}^g$  and  $\overline{b}^g$  be the infimum and supremum of its support.

Example: The functional form for the utility of a job used in the simulation exercise is:

$$v^{g}\left(S,H\right) = S - \frac{\xi^{g}}{T - H},\tag{1}$$

where  $0 < \xi^m < \xi^f$ , and T > 0. The distribution of  $b^g$  follows gamma distribution.

The measure of jobs is 1, and there is a continuous distribution of heterogeneity in job productivity  $\Gamma$ . The production function is  $\rho_j(H)$  for a type j job and satisfies  $\rho'_j > 0$  and  $\rho''_j < 0$ . I assume that employers of the same type offer only one tied salary/hours package to all of their potential employees. It is also assumed that the cost per period of posting a vacancy is high enough so that employers will not wait to employ a worker who can provide a higher profit. (In the model of employer discrimination, an employer will offer a package to women and not hold off for a man to fill the position.) Therefore, employers treat each potential match as a separate profit opportunity.

Example: In the simulation exercise, the job productivity distribution  $\Gamma$  follows Pareto distribution. The functional form for the production function is:

$$\rho_{i}(H) = -a_{i}(H - T)^{2} + c_{i}, \tag{2}$$

where  $a_j > 0$ ,  $0 \le H \le T$ , and  $(a_j, c_j)$  is distributed along  $(\underline{a}, \underline{c})$  and  $(\overline{a}, \overline{c})$ . I refer to a job with productivity parameters  $(\underline{a}, \underline{c})$  as the least productive job, a job with productivity

parameters  $\left(\frac{\underline{a}+\overline{a}}{2}, \frac{\overline{c}+\underline{c}}{2}\right)$  as the mid-productivity jobs, and a job with productivity parameters  $(\overline{a}, \overline{c})$  as the most productive job.

Employers post a tied salary/hours offer that is valued as greater than either  $\underline{b}^m$  for men or  $\underline{b}^f$  for women, because otherwise the offer will not attract any worker.

**Discrimination.** I consider two types of discrimination: (1) Becker-type employer discrimination where employers have a disamenity value for employing women and (2) employee discrimination where women have a disamenity value for working at certain jobs. It is possible to consider that these types of discrimination persist because employers (or employees) have culture-based gender concepts. In the model of employer discrimination, type j employers suffer a utility loss of  $d_j^{ER}$  for employing women but not for men, so the employer's utility per worker is  $\rho_j(H) - S - d_j^{ER}$  for women and  $\rho_j(H) - S$  for men.

In the model of employee discrimination, I first set the discrimination coefficient exogenously. Women suffer a utility loss of  $d_j^{EE}$  for working in type j jobs, so women's utility of a job is  $v^f(S, H) = S + \xi^f \phi(H) - d_j^{EE}$ . Next, the discrimination coefficient is set as a function of the gender composition of jobs. Women's utility of a job becomes  $v^f(S, H, \theta) = S + \xi^f \phi(H) - d^{EE}(\theta)$  where  $\theta$  is the fraction of men on a job and  $\frac{\partial d^{EE}(\theta)}{\partial \theta} < 0$ .

Labor market setup. Workers search for job offers, which arrive at rates  $\lambda_U$  when unemployed and  $\lambda_E$  when employed. Men and women obtain job offers that are random drawings from a distribution of job offers. The utility distribution of job offers is  $F^m$  for men and  $F^f$  for women. An employed worker faces job separation with an arrival rate of  $\delta$ .

Workers maximize the expected steady-state discounted present utility. The discounted present value of being unemployed  $V_0^g$  for the type g worker solves the following asset pricing equation:

$$rV_0^g = b^g + \lambda_U \left[ \int \max \left\{ V_0^g, V_1^g \left( \widetilde{v}^g \right) \right\} dF^g \left( \widetilde{v}^g \right) - V_0^g \right],$$

where r is the discount rate and  $V_1^g(v^g)$  is the discounted present value of being employed at a job with flow utility  $v^g$  for a type g worker. The flow value of unemployment equals the flow utility when unemployed plus the expected capital gain of job search when unemployed. The value of employment at a job with flow utility  $v^g$ ,  $V_1^g(v^g)$ , solves,

$$rV_{1}^{g}(v^{g}) = v^{g} + \lambda_{E} \left[ \int \max \left\{ V_{1}^{g}(v^{g}), V_{1}^{g}(\widetilde{v}^{g}) \right\} dF^{g}(\widetilde{v}^{g}) - V_{1}^{g}(v^{g}) \right] - \delta \left( V_{1}^{g}(v^{g}) - V_{0}^{g} \right).$$

The flow value of a job valued as  $v^g$  to a worker equals the current flow value of a job plus the expected capital gain of on the job search. As  $V_1^g(\cdot)$  is increasing in  $v^g$ , whereas  $V_0^g$  is independent of it, there exists a reservation utility  $R^g$  such that  $V_1^g(v^g) \geq V_0^g$  as  $v^g \geq R^g$  where  $V_1^g(R^g) = V_0^g$ . From the above two equations, and by integration by parts,

$$R^{g} = b^{g} + (\lambda_{U} - \lambda_{E}) \int_{R^{g}}^{\infty} \frac{1 - F^{g}(v^{g})}{r + \delta + \lambda_{E} [1 - F^{g}(v^{g})]} dv^{g}$$

$$= b^{g} + (\kappa_{U} - \kappa_{E}) \int_{R^{g}}^{\infty} \frac{1 - F^{g}(v^{g})}{r / \delta + 1 + \kappa_{E} [1 - F^{g}(v^{g})]} dv^{g},$$
(3)

where  $\kappa_U = \lambda_U/\delta$  and  $\kappa_E = \lambda_E/\delta$  represent the ratios of arrival rates to the job separation rate. When unemployed, the optimal job acceptance strategy is to accept all jobs having a value greater than or equal to  $R^g$ . In contrast, when employed, the optimal job acceptance strategy is to accept all jobs having a greater value than the current one. In the following analysis, I assume that the arrival rates of job offers are independent of the employment status, i.e.,  $\kappa_U = \kappa_E = \kappa$ . Then, the reservation utility equals the utility flow of being unemployed  $b^g$ :  $R^g = b^g$ . The optimal strategy of unemployed workers does not depend on  $F^g$ , and so unemployed workers will not wait (or be impatient) to accept a better (or worse) offer.

**Steady-state level of employment.** In steady-state, the flow of workers into employment equals the flow from employment to unemployment. Let  $u^g(x|F^g)$  denote the steady-state measure of unemployed workers whose reservation utility is less than or equal to x conditional on the utility distribution of the job offer  $F^g$ . Then,

$$u^{g}\left(x|F^{g}\right) = \int_{b^{g}}^{x} \left(\frac{\delta n^{g}}{\delta + \lambda_{U}\left[1 - F^{g}\left(b^{g}\right)\right]}\right) dK^{g}\left(b^{g}\right),$$

since the unemployment rate of workers with a utility flow of  $b^g$  is  $\frac{\delta}{\delta + \lambda_U[1 - F^g(b^g)]}$  and the density of these workers is  $n^g dK^g(b^g)$ .

Let the steady-state utility distribution of job offers received by type g employed workers be  $G^g$ . Then, the steady-state measure of employed workers receiving utility no greater than  $v^g$  is:  $G^g(v^g)\left\{n^g - u^g\left(\overline{b^g}|F^g\right)\right\}$  where  $u^g\left(\overline{b^g}|F^g\right)$  is the total unemployment of the type g worker in the economy. The flow of unemployed workers into jobs valued as  $v^g$  is:  $\lambda_U \int_{\underline{b}g}^{v^g} \left[F^g(v^g) - F^g(x)\right] du^g(x|F^g)$ . The flow of employed workers who move out of these jobs into higher valued jobs is:  $\lambda_E \left[1 - F^g(v^g)\right] G^g(v^g) \left\{n^g - u^g\left(\overline{b^g}|F^g\right)\right\}$ , and the flow of those who move out to unemployment is  $\delta G^g(v^g) \left\{n^g - u^g\left(\overline{b^g}|F^g\right)\right\}$ . In steady-state, the flow of workers into jobs valued no greater than  $v^g$  equals the flow from these jobs to unemployment or higher  $v^g$  jobs. Thus, in the steady-state the proportion of employed workers receiving utility no greater than  $v^g$  is:

$$G^{g}\left(v^{g}\right)\left\{n^{g}-u^{g}\left(\overline{b^{g}}|F^{g}\right)\right\} = \frac{\lambda_{U}\int_{\underline{b}g}^{v^{g}}\left[F^{g}\left(v^{g}\right)-F^{g}\left(x\right)\right]du^{g}\left(x|F^{g}\right)}{\delta+\lambda_{E}\left[1-F^{g}\left(v^{g}\right)\right]}.$$

Let  $l^g(v^g, F^g)$  represent the steady-state number of the type g worker available to an employer offering  $v^g$  given the utility distribution of job offer  $F^g$ . Since it is assumed that the job offer arrival rates are the same whether a worker is unemployed or employed ( $\kappa_E = \kappa_U = \kappa$ ),  $l^g(v^g, F^g)$  can be written as,

$$l^{g}(v^{g}, F^{g}) = \frac{dG^{g}(v^{g})}{dF^{g}(v^{g})} \left\{ n^{g} - u^{g} \left( \overline{b^{g}} | F^{g} \right) \right\}$$
$$= \frac{\kappa n^{g} K^{g}(v^{g})}{\left\{ 1 + \kappa \left[ 1 - F^{g}(v^{g}) \right] \right\}^{2}},$$

when  $F^g$  is continuous (see the appendix 6.1 for derivation).  $l^g(v^g, F^g)$  is continuous and is strictly increasing on the support of  $F^g$ . There are two reasons as to why employers that offer a higher value of  $v^g$  attracts more type g workers. First, a job with a higher value of  $v^g$  attracts more unemployed workers whose reservation utility is high. Second, a job with a higher value of  $v^g$  attracts more workers currently employed in jobs with a lower value of  $v^g$ , and also retains these workers.

The distribution of tied salary/hours offer in equilibrium. I characterize the equilibrium job offers in the model of employer discrimination. The case without discrimination

is depicted by eliminating the discrimination coefficient, and the case with employee discrimination is depicted by replacing the women's utility function by  $v^f(S, H) = S + \xi^f \phi(H) - d_j^{EE}$ .

The employer's steady-state utility given the tied salary/hours offer can be expressed as  $\left[\rho_{j}\left(H\right)-S\right]l^{m}\left(v^{m},F^{m}\right)$  for men and  $\left[\rho_{j}\left(H\right)-S-d_{j}^{ER}\right]l^{f}\left(v^{f},F^{f}\right)$  for women. Conditional on the job packages offered by all the other employers and on the search behavior of workers, a type j employer posts a tied salary/hours offer that maximizes its steady-state utility flow. The optimal job offer solves the following problem:

$$\pi_{j} = \max_{(S,H)} \left[ \rho_{j} (H) - S \right] l^{m} (v^{m}, F^{m}) + \left[ \rho_{j} (H) - S - d_{j}^{ER} \right] l^{f} (v^{f}, F^{f})$$

$$= \max_{(S,H)} \left[ \rho_{j} (H) - S \right] \frac{\kappa n^{m} K^{m} (v^{m})}{\left\{ 1 + \kappa \left[ 1 - F^{m} (v^{m}) \right] \right\}^{2}} + \left[ \rho_{j} (H) - S - d_{j}^{ER} \right] \frac{\kappa n^{f} K^{f} (v^{f})}{\left\{ 1 + \kappa \left[ 1 - F^{f} (v^{f}) \right] \right\}^{2}}.$$

$$(4)$$

The first-order conditions for interior solutions for Equation (4) are,

$$\frac{\partial \pi_{j}}{\partial S} = -l^{m} \left( v^{m}, F^{m} \right) - l^{f} \left( v^{f}, F^{f} \right) + \left[ \rho_{j} \left( H \right) - S \right] \frac{\partial \left[ l^{m} \left( v^{m}, F^{m} \right) + l^{f} \left( v^{f}, F^{f} \right) \right]}{\partial S} - d_{j}^{ER} \frac{\partial l^{f} \left( v^{f}, F^{f} \right)}{\partial S} = 0,$$
and

$$\frac{\partial \pi_{j}}{\partial H} = \rho_{j}'\left(H\right)\left[l^{m}\left(v^{m}, F^{m}\right) + l^{f}\left(v^{f}, F^{f}\right)\right] + \left[\rho_{j}\left(H\right) - S\right] \frac{\partial\left[l^{m}\left(v^{m}, F^{m}\right) + l^{f}\left(v^{f}, F^{f}\right)\right]}{\partial H} - d_{j}^{ER} \frac{\partial l^{f}\left(v^{f}, F^{f}\right)}{\partial H} = 0.$$

The sufficient second-order conditions are,

$$\frac{\partial^{2} \pi_{j}}{\partial S \partial S} = \left[ \rho_{j} \left( H \right) - S \right] \frac{\partial^{2} \left[ l^{m} \left( v^{m}, F^{m} \right) + l^{f} \left( v^{f}, F^{f} \right) \right]}{\partial S \partial S} - d_{j}^{ER} \frac{\partial^{2} l^{f} \left( v^{f}, F^{f} \right)}{\partial S \partial S} - 2 \frac{\partial \left[ l^{m} \left( v^{m}, F^{m} \right) + l^{f} \left( v^{f}, F^{f} \right) \right]}{\partial S} < 0,$$

and

$$\begin{split} \frac{\partial^{2}\pi_{j}}{\partial H\partial H} &= \left[\rho_{j}\left(H\right) - S\right] \frac{\partial^{2}\left[l^{m}\left(v^{m}, F^{m}\right) + l^{f}\left(v^{f}, F^{f}\right)\right]}{\partial H\partial H} - d_{j}^{ER} \frac{\partial^{2}l^{f}\left(v^{f}, F^{f}\right)}{\partial H\partial H} \\ &+ 2\rho_{j}'\left(H\right) \frac{\partial\left[l^{m}\left(v^{m}, F^{m}\right) + l^{f}\left(v^{f}, F^{f}\right)\right]}{\partial H} + \rho_{j}''\left(H\right) \left[l^{m}\left(v^{m}, F^{m}\right) + l^{f}\left(v^{f}, F^{f}\right)\right] < 0. \end{split}$$

A market equilibrium is defined such that: the equilibrium utility distributions of the job offer,  $F^m$  and  $F^f$ , satisfy the above first-order conditions and the second-order conditions for all jobs, and the worker's reservation utility satisfies  $R^g(b^g) = b^g$  where g = m for men and g = f for women.

# 3 The Algorithm to Solve for the Distribution of Tied Salary/Hours Offer in Equilibrium

I numerically show the characteristics of the equilibrium, since it is impossible to derive the analytical characteristics of the equilibrium utility distribution of job offers. I restrict the offers to the case where the same types of employers are constrained to post identical offers. (If employers of the same type are allowed to post different offers, employers with a given job type may choose different strategies because different tied salary/hours offers could yield an identical profit.) Below I provide an algorithm to solve for the equilibrium distribution of job offers, (S, H). In posting an offer, employers account for the gender difference in job acceptance and turnover rates. In contrast to Hwang, Mortensen and Reed (1999) where workers have identical preferences for hours of work and where high productivity jobs offer a higher utility, this model shows that a high productivity job is not necessarily associated with a higher utility  $v^g$ . High productivity jobs require longer hours of work. Women may place lower value on these types of jobs (even though the salary is higher), because they are more averse to working long hours.<sup>4</sup> Men, on the other hand, place a higher value on these jobs. Employers tailor their tied salary/hours offer to the workers they can hire more and retain longer.

The algorithm works in three steps. First, hours are determined so that the marginal productivity of an additional hour equals the weighted average of the marginal disutility for women and men, where the weights reflect the gender composition of the particular job type in equilibrium. Second, salary is determined to maximize the steady-state profit (or utility) flow given the hours determined in the first step. The tied salary/hours offer derived from this two-step procedure may not necessarily be an optimal offer that maximizes the employer's

<sup>&</sup>lt;sup>4</sup>As shown in the appendix 6.2, there is a positive relationship between output per worker  $\rho_j$  and salary. The employers with larger output pay higher salaries to their employees because they have a greater opportunity cost of remaining unfilled.

profit (or utility). In a sequential search model, employers have a preference for workers with a lower turnover rate and may be willing to offer a package that would retain such workers.<sup>5</sup> Employers may place a greater value than the gender composition of jobs on workers whose turnover rate is lower. In the third step, starting from the tied salary/hours offer derived using the first two steps, I search for an optimal offer that satisfies the first-order conditions and second-order conditions for Equation (4).

#### Step 1: Hours Choice

Employers set hours of work so that the marginal productivity of an additional hour equals the weighted average of the marginal disutility of an additional hour for women and men. The weights reflect the gender composition of the particular job type in equilibrium. Let  $\theta_j^*$  be the fraction of men on a type j job in equilibrium. Then H solves,

$$\max_{\{H\}} \quad \theta_{j}^{*} \left[ \rho_{j} \left( H \right) + v^{m} \left( S, H \right) \right] + \left( 1 - \theta_{j}^{*} \right) \left[ \rho_{j} \left( H \right) + v^{f} \left( S, H \right) - d_{j}^{ER} \right],$$

thus,

$$\rho'_{j}(H) + \left[\theta_{j}^{*}\xi^{m}\phi'(H) + \left(1 - \theta_{j}^{*}\right)\xi^{f}\phi'(H)\right] = 0.$$

Using the functional forms for  $v^{g}(S, H)$  and  $\rho_{j}(H)$  that are given in Equations (1) and (2), the hours are determined as:

$$H_{j} = T - \left(\frac{\theta_{j}^{*} \xi^{m} + (1 - \theta_{j}^{*}) \xi^{f}}{2a_{j}}\right)^{1/3}.$$
 (5)

#### Step 2: Salary Choice

Let  $\rho_j = \rho_j(H_j)$ , where  $H_j$  is the hours determined in the first step. In the second step, employers choose a salary that maximizes their steady-state utility flow given the hours determined in the first step,

$$\pi_{j} = \max_{\{S\}} (\rho_{j} - S) l^{m} (S + \xi^{m} \phi(H_{j}), F^{m}) + (\rho_{j} - S - d_{j}^{ER}) l^{f} (S + \xi^{f} \phi(H_{j}), F^{f}).$$

The first-order condition for an interior solution is:

<sup>&</sup>lt;sup>5</sup>When there is no possibility for workers to search while employed, workers only leave their jobs through an exognous job separation at an arrival rate of  $\delta$ . There is no incentive for employers to alter their offer to retain these workers.

$$\frac{\partial \pi_j}{\partial S} = -l^m \left( v^m, F^m \right) - l^f \left( v^f, F^f \right) + \left( \rho_j - S \right) \frac{\partial \left[ l^m \left( v^m, F^m \right) + l^f \left( v^f, F^f \right) \right]}{\partial S} - d_j^{ER} \frac{\partial l^f \left( v^f, F^f \right)}{\partial S} = 0,$$

and the sufficient second-order condition is:

$$\frac{\partial^{2} \pi_{j}}{\partial S \partial S} = \left(\rho_{j} - S\right) \frac{\partial^{2} \left[l^{m} \left(v^{m}, F^{m}\right) + l^{f} \left(v^{f}, F^{f}\right)\right]}{\partial S \partial S} - d_{j}^{ER} \frac{\partial^{2} l^{f} \left(v^{f}, F^{f}\right)}{\partial S \partial S} - 2 \frac{\partial \left[l^{m} \left(v^{m}, F^{m}\right) + l^{f} \left(v^{f}, F^{f}\right)\right]}{\partial S} < 0.$$

**Two-Step Calculation:** I compute the tied salary/hours offer and the fraction of men on jobs from Step 1 and Step 2. The algorithm is to first choose the initial guess  $\theta^0$ . Then compute the hours using the first step and the salary using the second step. Based on the solution derived from the two steps, I calculate the fraction of men on jobs  $\theta^1 = \frac{l^m(v^m, F^m)}{l^m(v^m, F^m) + l^f(v^f, F^f)}$ . I update the fraction of men on jobs to  $\theta^1$ , and repeat the two-step procedure till  $(S, H, \theta)$  converges for each job. Note that the discrimination parameter does not directly appear in the hours determination in Equation (5), but discrimination can affect hours of work through the fraction of men on jobs  $\theta$ .

#### Step 3: Direction Method

The optimal tied salary/hours offer satisfies the first-order conditions,  $\frac{\partial \pi}{\partial S} = 0$  and  $\frac{\partial \pi}{\partial H} = 0$ , and the second-order conditions,  $\frac{\partial^2 \pi}{\partial S \partial S} < 0$  and  $\frac{\partial^2 \pi}{\partial H \partial H} < 0$  for all jobs. The solution derived from the above two-step calculation will not necessarily be optimal because turnover behavior differs between women and men. Employers tailor their package to the group that remains on the job longer. Essentially, the solution from the two-step procedure satisfies  $\frac{\partial \pi}{\partial S}|_{two-step} = 0$  but  $\frac{\partial \pi}{\partial H}|_{two-step} \gtrsim 0$  for  $\theta \gtrsim 1/2$ . (I numerically show this in Section 4.) Optimal hours are longer than the hours determined in the two-step calculation in jobs where men are more prevalent, but hours are shorter in jobs where women are more prevalent.

**Optimal Calculation:** Starting from the solutions derived in the two-step calculation, I update hours by  $H^{k+1} = H^k + \lambda \frac{\partial \pi}{\partial H^k}$  where  $\lambda$  is the negative of the inverse of the second-order derivative. Then, I solve for the salary and the fraction of men on jobs. I repeat this

procedure till  $(S, H, \theta)$  in all jobs converges and satisfies the first-order conditions and the second-order conditions for Equation (4).<sup>6</sup>

### 4 Simulation Exercises

In this section, I use simulations to illustrate the equilibrium distribution of job offers. The purpose of the simulation is to obtain qualitative comparative statistics results; it is not intended to assess the quantitative magnitude.

Table 1 lists the parameter values used in the simulations. The measures of men and women are  $n_m = .5$  and  $n_f = .5$ , respectively. I vary the preference parameter  $\xi^g$  and the arrival rate of jobs  $\kappa$  as listed in Table 1. The time endowment T for a worker is 70. The distribution of the utility flow of being unemployed  $K^g(b^g)$  follows a gamma distribution. Following the study by Bontemps et al. (1999), the distribution of productivity across jobs is Pareto:  $\Gamma(x) = \frac{\alpha \beta^{\alpha}}{x^{\alpha+1}}$  where  $x \geq \beta$ ,  $\beta = 3000$ , and  $\alpha = 2.15$ , 2.5, or 2.8 (depending on the exercises). I choose one hundred jobs at regular intervals along  $x \in [3000, 7000]$  and normalize the productivity distribution. The equilibrium job offers are derived for each of these jobs. The parameters on job productivity and discrimination are also listed in Table 1. The summary results for the simulation exercises are presented in Table 2. It reports the utility of a job for men and women, salary, hours of work, the fraction of men working on a job, employer's profit, employer's profit per worker, and the number of male and female employees for the least productive, mid-productive, and the most productive jobs. For the major simulation results, I report the results for all jobs in Figures 1, 2, and 3.

### 4.1 Models without Discrimination

First, I derive the equilibrium distribution of job offers when women and men are identical in all aspects. Next, I consider a case where the job offer arrival rate is lower for women than men. Lastly, I consider a case where women are more averse to working longer hours than men.

<sup>&</sup>lt;sup>6</sup>Due to the fact that, in the two-step calculation, discrimination affects the hours decision only through the composition of men on jobs  $\theta$ ; the discrepancies in the solutions between the two-step calculation and optimal calculation are greater when discrimination is present in the model.

<sup>&</sup>lt;sup>7</sup>The shape parameter of the gamma distribution is set as  $\gamma = 5$  and the scale parameter as  $\mu = 8$ . The parameter choice for  $\underline{b}^g$  is listed in Table 1.

#### Case 1: Women and men are identical in all aspects.

Employers with a larger marginal productivity of an additional hour require more working hours and pay a higher salary. Women and men place greater value on high productivity jobs. They quit and move to these jobs at the same rate; and so women and men are equally distributed across all job types ( $\theta = .5$ ).

#### Case 2: The arrival rate of job offer is lower for women than men.

When women are discouraged from searching for jobs because of discrimination or because of stronger family responsibilities, the arrival rate of job offer can be lower for them. In this case, women and men equally place greater value on high productivity jobs. However, women are slower in moving to these jobs because they receive fewer job offers than men. The fraction of men on a job increases with job productivity from  $\theta = .434$  for the least productive job, to  $\theta = .750$  for the most productive. In comparison with Case 1 (where women and men have identical job offer arrival rates), employers obtain a higher profit in low productivity jobs, but a lower profit in high productivity jobs (in Case 2). This is because low productivity jobs can retain female employees, whereas high productivity jobs have a lower probability of meeting potential female employees.

## Case 3: The marginal disutility of an additional hour is higher for women than men, $\xi^m < \xi^f$ . (Figure 1)

Figure 1 displays the simulation results. Men place a greater value on high productivity jobs, but women place a lower value on these jobs. The composition of men on jobs  $\theta$  increases sharply with job productivity from .062 to .941. In contrast to Cases 1 and 2, the solutions between the optimal and two-step calculations are different. The first-order condition with respect to hours in the two-step calculation  $\left(\frac{\partial \pi}{\partial H}|_{two-step}\right)$  is positive when  $\theta > .5$ , but it is negative when  $\theta < .5$ . Hence, the optimal hours are longer in high productivity jobs where men are more prevalent, but they are shorter in low productivity jobs where there are more women. High productivity jobs require more hours, and their offers are tailored to men's preferences. Conversely, low productivity jobs require fewer hours, which appeal to women.

### 4.2 Employer Discrimination

The rest of the simulation exercise incorporates discrimination. It is always assumed that the marginal disutility of an additional hour is higher for women than men, and that women and men have an identical job offer arrival rate.

### Case 3ER: A small change in the fraction of men on a job with discrimination. (Figure 2)

I use the same preference and job productivity parameters as in Case 3. The discrimination parameter is set as  $d_j^{ER} = 1.1j$ , where  $0 \le j \le 100$ ; so the disamenity value of employing women increases with job productivity.<sup>8</sup> Figure 2 displays the simulation results. Discriminating employers suffer a loss in utility by employing women. Therefore, they make their offers uninviting to women by requiring more working hours while not considerably increasing the salary. As the job value for women declines, the fraction of men somewhat increases. This increase in the fraction of men on a job is small, because the job offers have already been tailored toward men in the absence of discrimination.

Next, I examine the profitability of employers. The employer's profit is defined as  $[\rho_j(H) - S]$   $[l^m(v^m, F^m) + l^f(v^f, F^f)]$ , which excludes the discrimination coefficient. Becker (1971) predicted that discriminating employers may be competed out of business with free entry or constant returns to scale, because discrimination is indulged at a positive cost to them. In this monopsony model that is based on Burdett and Mortensen (1998), discriminating employers can obtain a higher profit. The mid-productivity jobs obtain a profit of 39.87, and the high productivity jobs 173.3. Their profits in the absence of discrimination are 36.83 and 170.8, respectively. Due to the discrimination coefficient in women's productivity, employers regard women's productivity as being low. As a result, the wages (= S/H) are less, and work hours are more. Constrained to make only one offer to heterogeneous groups, employers hire both males and females at a lower wage rate and require them to work longer. Profit per employee is, therefore, greater. Men remain on the job, but women with a higher reservation utility find it better to remain unemployed. The overall decline in female and male employee size is small, and therefore it is possible for discriminating employers to obtain

<sup>8</sup> Specifically, the discrimination coefficient is  $d^{ER} = 0$  for the least productive job, and it is  $d^{ER} = 110$  for the most productive job.

<sup>&</sup>lt;sup>9</sup>The least productive jobs do not hold a taste for discrimination against women, and their profit is 6.869 which is the same as in Case 3.

a higher overall profit.

## Case 4ER: A large change in the fraction of men on a job with discrimination. (Figure 3)

I use different parameter values as in Case 3ER to show that there can be a greater difference between the fraction of men on jobs with discrimination and without the presence of discrimination. Figure 3 displays the simulation results. Job offers are made unappealing to women by increasing the working hours. In high productivity jobs, the value of work for women drops, whereas for men it slightly increases. Some women prefer to remain unemployed, as the job value is less than their reservation utility. The number of females employed on the job therefore declines, and the fraction of men on the job increases.

A higher profit is obtained in mid-productive jobs, compared to the case when discrimination is absent from the model (Columns 6 and 7 in Table 2). Female employees (constituting 18.5% of the employees) are considered to be less productive, so the employers significantly decrease wages and increase work hours. Consequently, a higher profit per employee is obtained without a decline in total employee size. On the other hand, profit in high productivity jobs is less. Since women make up only 5.9% of the employees in the absence of discrimination, the job offers are already tailored to men's preferences; thus, the change in the tied salary/hours offer is less when discrimination is present. However, there is a large reduction in the number of female employees.

# Case 5ER: Women and men are equally distributed across all job types with discrimination and without the presence of discrimination, $\theta = .5$ .

More productive jobs are preferred by both women and men, when no discrimination is present (Column 8 in Table 2). When the discrimination parameter is set as  $d_j^{ER} = .5j$ , where  $0 \le j \le 100$ , the results are shown in Column 9 of Table 2. Discriminating employers require more hours of work, causing a decline in the job value for women. Women and men are equally distributed across all job types, as women still prefer more productive jobs.

Discriminating employers obtain a higher profit in comparison to the case when discrimination is absent from the model. Women, who comprise 50% of their employees, are viewed as less productive. Identical job offers are presented to both genders, and workers are required to work longer hours with low wages.

### 4.3 Employee Discrimination

#### 4.3.1 The Discrimination Parameter is Exogenous

## Case 3EE: Gender occupational segregation exists, when there is no discrimination.

I use the same preference and job productivity parameters as in Case 3. The discrimination parameter is set as  $d_j^{EE} = .7j$ , where  $0 \le j \le 100$ . Women do not choose the high productivity jobs because they incur a disamenity value  $d_j^{EE}$  while also working longer hours. The utility of work for women declines in the high productivity jobs, and the composition of men on the job increases. This increase is small, however, as the job offers are already tailored toward men.

The employers realize less profit, compared to the case when discrimination is absent. Women dislike working in jobs in which they are treated poorly, and so employers have to offer packages that attract more workers to maintain the employee numbers. The utility of work for women in high productivity jobs is -418.4 (the disamenity value  $d^{EE} = 70$  is included in computing the utility of work in Column 5 of Table 2). The utility women derive from just the salary/hours package is -348.4 (=  $v^f + d^{EE}$ ), which is greater than the utility women receive in the absence of discrimination, -374.5.

### Case 5EE: Women and men are equally distributed across all job types in the absence of discrimination.

Women and men prefer more productive jobs, and are equally distributed across all job types, when no discrimination is present (Column 8 in Table 2). Now, the discrimination parameter is set as  $d_j^{EE} = 5j$ , where  $0 \le j \le 100$ . The utility of work for women largely decreases in high productivity jobs, due to the large disamenity value incurred. Men, however, being offered higher salary and longer hours, find these same positions most appealing. The fraction of men on jobs increases as job productivity increases from  $\theta = .062$  to .941.

Jobs where there are more men  $(\theta > .5)$  have been tailored to require more hours, which leads to a greater output per worker  $\rho_j(H)$  and a larger profit per worker. The number of women employed decreases with longer working hours and the disamenity value. This decrease is greater than the increase in profit per worker, so the overall profit of the employers is less. On the other hand, in jobs where there are more female employees  $(\theta < .5)$ , fewer

working hours are required. This leads to less output per worker  $\rho_j(H)$  and a smaller profit per worker. However, because women find these jobs more attractive, the number of female employees increases. This increase is larger than the reduction in the profit per worker. Therefore, the employers' overall profit increases. The total effect of discrimination on welfare is the following: With discrimination, the overall profit of all employers, on average, is lower by -9.464; when discrimination is present, women are worse off, on average, by -386.9 and men gain utility by 65.91.

### 4.3.2 The Discrimination Parameter is a Function of the Gender Composition of Jobs

I assume a functional relationship between the discrimination coefficient  $d^{EE}$  and the gender composition of jobs  $\theta$ , since women's disamenity value to working on jobs can be related with the fraction of men on the jobs. The algorithm to solve for the equilibrium solution is to first choose the initial guess on  $\theta$ , and then use the two-step calculation and optimal calculation to solve for  $(S, H, \theta)$  that converges.

### Case 6: Gender occupational segregation exists, when there is no discrimination.

I use the same parameter values as in Case 3, except that the discrimination parameter is set as  $d^{EE}(\theta) = 100 (\theta - 1/2)$ . When the fraction of men on the job  $\theta$  is greater (or less) than 1/2, the discrimination parameter  $d^{EE}$  is greater (or less) than 0. Thus, women are treated poorly in jobs where there are more men, but better where there are more women. The predictions on segregation and welfare are similar to the case when the discrimination parameter is exogenous (Case 3EE). Women suffer a utility loss from working on high productivity jobs where the hours requirement is longer and where there are more men. Consequently, the number of women employed on these jobs drops, leading to a lower profit for the employers.

### Case 7: Women and men are equally distributed across all job types in the absence of discrimination.

I use the same parameter values as in Case 5 and set the discrimination parameter as  $d^{EE}(\theta) = 900 (\theta - 1/2)$ . When the initial guess on  $\theta$  takes values close to .5 on all jobs, the distribution of job offers converges to a case where discrimination is absent. Hence, women and men are equally distributed across all job types,  $\theta = .5$  (Part 1: Column 12 in Table 2). On the other hand, when the initial guess on  $\theta$  takes values that increase with job

productivity, the job offers converge to a discriminatory equilibrium (Part 2: Column 13 in Table 2). In this case, the predictions on segregation and welfare are similar to those when the discrimination parameter is exogenous.

### 5 Conclusions

In this paper, I analyze an equilibrium search model in which salary and hours of work are job attributes and workers differ in their preferences for hours of work. The simulations indicate that employers with a larger marginal productivity of an additional hour require more hours. When women are more averse to longer hours of work than men, females predominate in the less productive jobs which offer fewer hours and pay a lower salary. When employers have a taste for discrimination against women, they require more working hours and exclude women from their jobs. Employers can control the types of workers they hire by distinguishing the job-amenity, since different types of workers have different preferences for a particular amenity. On the other hand, when women have a disamenity value for working on jobs in which they are treated poorly, employers adjust their offers to have more men on the job. The prediction on segregation is similar between employer and employee discrimination. The implication on welfare is somewhat different. Employers can obtain a higher profit with employer discrimination. This occurs because employers regard women to be less productive, and lower their wage rate and require more working hours, while not considerably lowering the number of employees they hire. In the Burdett and Mortensen (1998)'s equilibrium search model, employers have monopsony power, and so employers can continue obtaining a higher profit. On the other hand, employers lower their profit in the model of employee discrimination, since they have to post a job offer that attract more workers.

As for future research, there are two extensions. First is to relax the assumption that the distribution of productivity across jobs is fixed and allow free entry. Second is to allow for heterogeneity in firm preferences so that the same type of employers have different taste in regard to discrimination against women.

### 6 Appendix

### 6.1 Derivation of the steady-state employment size

$$l^{g}(v^{g}, F^{g}) = \frac{dG^{g}(v^{g}) \left\{ n^{g} - u^{g} \left( \overline{b^{g}} | F^{g} \right) \right\}}{dF^{g}(v^{g})}$$

$$= \frac{dG^{g}(v^{g}) \left\{ n^{g} - u^{g} \left( \overline{b^{g}} | F^{g} \right) \right\}}{dv^{g}} \frac{dv^{g}}{dF^{g}(v^{g})}$$

$$= \frac{dv^{g}}{dF^{g}(v^{g})} \cdot \left[ \frac{\kappa_{U} \int_{\underline{b^{g}}}^{v^{g}} \frac{dF^{g}(v^{g})}{dv^{g}} du^{g} \left( x | F^{g} \right) \left\{ 1 + \kappa_{E} \left[ 1 - F^{g} \left( v^{g} \right) \right] \right\}}{\left\{ 1 + \kappa_{E} \left[ 1 - F^{g} \left( v^{g} \right) \right] \right\}^{2}} + \frac{\kappa_{U} \kappa_{E} \frac{dF^{g}(v^{g})}{dv^{g}} \int_{\underline{b^{g}}}^{v^{g}} \left[ F^{g} \left( v^{g} \right) - F^{g} \left( x \right) \right] du^{g} \left( x | F^{g} \right)}{\left\{ 1 + \kappa_{E} \left[ 1 - F^{g} \left( v^{g} \right) \right] \right\}^{2}} \right]}$$

$$= \frac{\kappa_{U} \int_{\underline{b^{g}}}^{v^{g}} \left[ 1 + \kappa_{E} \left[ 1 - F^{g} \left( v^{g} \right) \right] \right] du^{g} \left( x | F^{g} \right)}{\left\{ 1 + \kappa_{E} \left[ 1 - F^{g} \left( v^{g} \right) \right] \right\}^{2}}.$$

Since it is assumed that  $\kappa_U = \kappa_E = \kappa$ , use  $\{1 + \kappa_U [1 - F^g(x)]\} du^g(x|F^g) = n^g dK^g(x)$  which is derived from the first steady-state condition. Then,  $l^g(v^g, F^g)$  is simplified to:

$$l^{g}(v^{g}, F^{g}) = \frac{\kappa_{U} \int_{\underline{b}^{g}}^{v^{g}} \left[1 + \kappa_{U} \left[1 - F^{g}(x)\right]\right] du^{g}(x|F^{g})}{\left\{1 + \kappa_{E} \left[1 - F^{g}(v^{g})\right]\right\}^{2}}$$

$$= \frac{\kappa_{U} \int_{\underline{b}^{g}}^{v^{g}} n^{g} dK^{g}(x)}{\left\{1 + \kappa_{E} \left[1 - F^{g}(v^{g})\right]\right\}^{2}}$$

$$= \frac{\kappa_{U} n^{g} K^{g}(v^{g})}{\left\{1 + \kappa_{E} \left[1 - F^{g}(v^{g})\right]\right\}^{2}}$$

$$= \frac{\kappa n^{g} K^{g}(v^{g})}{\left\{1 + \kappa \left[1 - F^{g}(v^{g})\right]\right\}^{2}}.$$

### 6.2 The relationship between salary and output

Let the optimal hours be  $H^*$ . Then, the level of output is  $\rho_j = \rho_j(H^*)$ . The employers choose salary that maximize their utility flow,

$$\pi_{j} = \max_{\{S\}} (\rho_{j} - S) l^{m} (v^{m} (S, H^{*}), F^{m}) + (\rho_{j} - S - d_{j}^{ER}) l^{f} (v^{f} (S, H^{*}), F^{f}).$$

Using the envelope theorem,

$$\frac{\pi_{j}^{*}\left(\rho_{j}, S^{*}\right)}{\partial \rho_{j}} = l^{m}\left(v^{m}\left(S^{*}, H^{*}\right), F^{m}\right) + l^{f}\left(v^{f}\left(S^{*}, H^{*}\right), F^{f}\right).$$

Then,

$$\frac{\partial^{2} \pi_{j}^{*}\left(\rho_{j}, S^{*}\right)}{\partial \rho_{i} \partial S} = \frac{\partial \left[l^{m}\left(v^{m}\left(S^{*}, H^{*}\right), F^{m}\right) + l^{f}\left(v^{f}\left(S^{*}, H^{*}\right), F^{f}\right)\right]}{\partial S}.$$
 (6)

The first-order condition with respect to salary is,

$$\frac{\partial \pi_{j}}{\partial S} = -\left[l^{m}\left(v^{m}\left(S, H^{*}\right), F^{m}\right) + l^{f}\left(v^{f}\left(S, H^{*}\right), F^{f}\right)\right]$$

$$+\left(\rho_{j}-S\right)\frac{\partial\left[l^{m}\left(v^{m}\left(S,H^{*}\right),F^{m}\right)+l^{f}\left(v^{f}\left(S,H^{*}\right),F^{f}\right)\right]}{\partial S}-d_{j}^{ER}\frac{\partial l^{f}\left(v^{f}\left(S,H^{*}\right),F^{f}\right)}{\partial S}=0.$$

Using Equation (6), the above first-order condition can be rewritten as,

$$\frac{\partial \pi_{j}}{\partial S} = -\left[l^{m}\left(v^{m}\left(S, H^{*}\right), F^{m}\right) + l^{f}\left(v^{f}\left(S, H^{*}\right), F^{f}\right)\right] + \left(\rho_{j} - S\right) \frac{\partial^{2} \pi_{j}^{*}\left(\rho_{j}, S^{*}\right)}{\partial \rho_{j} \partial S} - d_{j}^{ER} \frac{\partial l^{f}\left(v^{f}\left(S, H^{*}\right), F^{f}\right)}{\partial S} = 0.$$

Therefore,

$$\frac{\partial^{2} \pi_{j}^{*}\left(\rho_{j}, S^{*}\right)}{\partial \rho_{j} \partial S} = \frac{l^{m}\left(v^{m}\left(S, H^{*}\right)\right) + l^{f}\left(v^{f}\left(S, H^{*}\right)\right) + d_{j}^{ER} \frac{\partial l^{f}\left(v^{m}\left(S, H^{*}\right)\right)}{\partial S}}{\rho_{j} - S} > 0. \tag{7}$$

The first-order condition with respect to salary,  $\frac{\partial \pi_j}{\partial S} = 0$ , implies:  $\frac{\partial^2 \pi_j}{\partial S \partial S} dS + \frac{\partial^2 \pi_j}{\partial S \partial \rho_j} d\rho_j = 0$ . So,

$$\frac{\partial S}{\partial \rho_j} = -\frac{\frac{\partial^2 \pi_j}{\partial S \partial \rho_j}}{\frac{\partial^2 \pi_j}{\partial S \partial S}} > 0.$$

 $\frac{\partial S}{\partial \rho_j}$  is positive, since the denominator of the right-hand side of the above equation is the second-order condition which is negative,  $\frac{\partial^2 \pi_j}{\partial S \partial S} < 0$ , and the numerator is positive from Equation (7).

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Table 1
Parameter Values Used in the Simulation Exercises

_	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Case 1	Case 2	Case 3		Case 3EE		Case 4ER			Case 5EE		Case 7
			w/o disc.		Employee	w/o disc.		w/o disc.		Employee		
-				disc.	disc.		disc.		disc.	disc.	disc.	disc.
Preference parameter												
$\xi^m$ for men	3800	3800	3000	3000	3000	3000	3000	250	250	250	3000	250
$\xi^f$ for women	3800	3800	22500	22500	22500	24000	24000	8000	8000	8000	22500	8000
Arrival rate of jobs												
$\kappa$ for men	3	3	3	3	3	3	3	3	3	3	3	3
$\kappa$ for women	3	1	3	3	3	3	3	3	3	3	3	3
Discrimination coefficient												
d <sup>ER</sup> =				1.1 <i>j</i>			.75j		.5 <i>j</i>			
d <sup>EE</sup> =					.7 <i>j</i>					5 <i>j</i>	100 (θ-1/2)	900 (θ-1/2)
Production parameters												
<u>.</u> <u>a</u>	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.4	0.4	0.4	0.05	0.4
<u>a</u> _ a	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.5	0.5	0.5	0.3	0.5
<u>c</u>	600	600	600	600	600	600	600	550	550	550	600	550
<u>c</u> c	1100	1100	1100	1100	1100	1110	1110	1000	1000	1000	1100	1000
α	2.8	2.8	2.8	2.8	2.8	2.15	2.15	2.5	2.5	2.5	2.8	2.5
β	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
Infimum of the utility flow of being unemployed												
$b^m$ for men	150	150	150	150	150	150	150	100	100	100	150	100
<u>b</u> f for women	150	150	-500	-500	-500	-500	-500	-900	-900	-900	-500	-900

Notes: The utility of the job is  $v^g = S - \xi^g/(T-H)$  where S is salary, H is hours of work, g = m for men and g = f for women, and T = 70 is the time endowment. (The measure of men and women in the labor market is  $n^m = n^f = \frac{1}{2}$ .)  $\kappa$  is the arrival rate of the job offer relative to the job separation rate,  $d_j^{ER}$  ( $0 \le j \le 100$ ) is the employer's disamenity value to employing women,  $d_j^{EE}$  ( $0 \le j \le 100$ ) is the women's disamenity value to working in jobs,  $\theta$  is the fraction of men on jobs,  $\theta$  and  $\theta$  are the parameters related to job productivity (refer to Equation (2) in the text for the functional form), and  $\theta$  are the parameters for the Pareto distribution of productivity across jobs.

Table 2
Summary of Results in the Simulation Exercises

	_	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
		Case 1	Case 2	Case 3	Case 3ER	Case 3EE	Case 4	Case 4ER	Case 5	Case 5ER	Case 5EE
				w/o disc.	<b>Employer</b>	Employee	w/o disc.	Employer	w/o disc.	<b>Employer</b>	<b>Employee</b>
					disc.	disc.		disc.		disc.	disc.
	Job (j)			(Figure 1)	(Figure 2)			(Figure 3)			
Utility for men, v <sup>m</sup>	L	414.6	410.8	363.0	363.0	363.0	355.1	355.1	398.9	398.9	350.0
,	М	549.2	538.8	579.2	578.9	579.2	582.9	585.8	593.4	604.5	659.1
	Н	689.5	669.0	729.4	728.0	730.3	772.5	773.3	769.6	767.4	849.2
Utility for women, v <sup>f</sup>	L	414.6	410.8	40.80	40.80	40.80	15.54	15.54	-49.73	-49.73	-11.66
,	М	549.2	538.8	-260.6	-389.2	-295.2	-277.7	-390.0	127.0	81.85	-370.0
	Н	689.5	669.0	-374.5	-413.0	-418.4	-406.2	-461.4	286.4	230.1	-730.6
Salary, S	L	527.6	523.8	412.5	412.5	412.5	403.6	403.6	413.3	413.3	361.7
	М	720.4	710.0	708.4	727.8	708.4	705.8	725.2	608.5	621.4	684.4
	Н	894.8	874.4	899.2	903.6	896.3	940.9	949.7	785.2	784.7	884.0
Hours of work, H	L	36.38	36.38	9.476	9.476	9.476	8.158	8.158	52.72	52.72	48.57
	Μ	47.80	47.80	46.78	49.86	46.78	45.60	48.48	53.38	55.17	60.09
	Н	51.50	51.50	52.34	52.91	51.92	52.18	52.99	53.96	55.58	62.82
Fraction of men on a	L	0.500	0.434	0.062	0.062	0.062	0.062	0.062	0.500	0.500	0.062
job, <i>θ</i>	М	0.500	0.649	0.850	0.850	0.850	0.815	0.815	0.500	0.500	0.835
	Н	0.500	0.750	0.941	0.942	0.942	0.941	0.966	0.500	0.500	0.941
Profit,	L	3.168	4.524	6.869	6.869	6.869	8.235	8.235	3.429	3.429	7.449
$(\rho(H) - S)(I^{m} + I^{f})$	М	57.13	55.14	36.83	39.87	36.83	30.76	33.31	51.24	67.03	33.78
	Н	307.4	245.9	170.8	173.3	168.2	117.7	114.2	258.7	333.7	143.8
Profit per worker,	L	15.88	19.67	4.294	4.294	4.294	5.151	5.151	17.27	17.27	4.658
ρ(H) - S	М	41.44	51.92	45.39	49.15	45.39	43.15	46.74	40.14	52.51	44.19
	Н	102.5	123.0	107.2	108.8	105.7	73.89	73.56	86.22	111.2	90.21
Employment size for	L	0.100	0.100	0.100	0.100	0.100	0.099	0.099	0.099	0.099	0.099
men, I <sup>m</sup>	М	0.689	0.689	0.689	0.689	0.689	0.581	0.581	0.638	0.638	0.638
,	Н	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500
Employment size for	L	0.100	0.130	1.500	1.500	1.500	1.500	1.500	0.099	0.099	1.500
women, I <sup>f</sup>	М	0.689	0.373	0.122	0.122	0.122	0.132	0.132	0.638	0.638	0.126
- ,	Н	1.500	0.500	0.094	0.092	0.092	0.093	0.053	1.500	1.500	0.094

Notes: The table displays outcomes (in the row headings) for the least productive job (labeled as "L"), the mid-productive job (labeled as "M"), and the most productive jobs (labeled as "H"). Refer to Table 1 for the parameter values used in the simulation.

Table 2 (continued)

		(11)	(12)	(13)
		Case 6	Case 7	Case 7
		<b>Employee</b>	Employee	<b>Employee</b>
		disc.	disc.	disc.
	Job (j)		Part 1	Part 2
Utility for men, v <sup>m</sup>	L	363.0	398.9	350.0
•	М	579.2	593.4	659.1
	Н	729.8	769.6	849.2
Utility for women, $v^f$	L	84.56	-49.73	382.5
•	М	-295.5	127.0	-424.0
	Н	-407.5	286.4	-627.5
Salary, S	L	412.5	413.3	361.7
	М	708.4	608.5	684.4
	Н	898.0	785.2	884.0
Hours of work, H	L	9.476	52.72	48.57
	М	46.78	53.38	60.09
	Н	52.16	53.96	62.82
Fraction of men in a	L	0.062	0.500	0.062
job, $\theta$	М	0.850	0.500	0.835
	Н	0.942	0.500	0.941
Profit,	L	6.869	3.429	7.449
$(\rho(H) - S)(I^{m} + I^{f})$	М	36.83	51.24	33.78
	Н	169.7	258.7	143.8
Profit per worker,	L	4.294	17.27	4.658
ρ(H) - S	М	45.39	40.14	44.19
	Н	106.6	86.22	90.21
Employment size for	L	0.100	0.099	0.099
men, I <sup>m</sup>	М	0.689	0.638	0.638
	Н	1.500	1.500	1.500
Employment size for	L	1.500	0.099	1.500
women, I <sup>f</sup>	М	0.122	0.638	0.126
-	Н	0.093	1.500	0.094

Figure 1
Case 3: Marginal disutility of an additional hour is higher for women than men

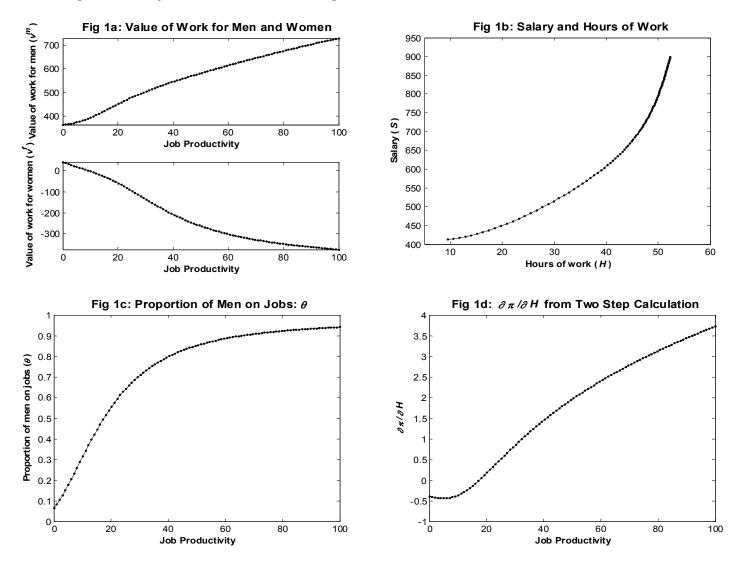


Figure 2
Case 3ER: Employer discrimination. Comparison between the case with discrimination and the case without discrimination.

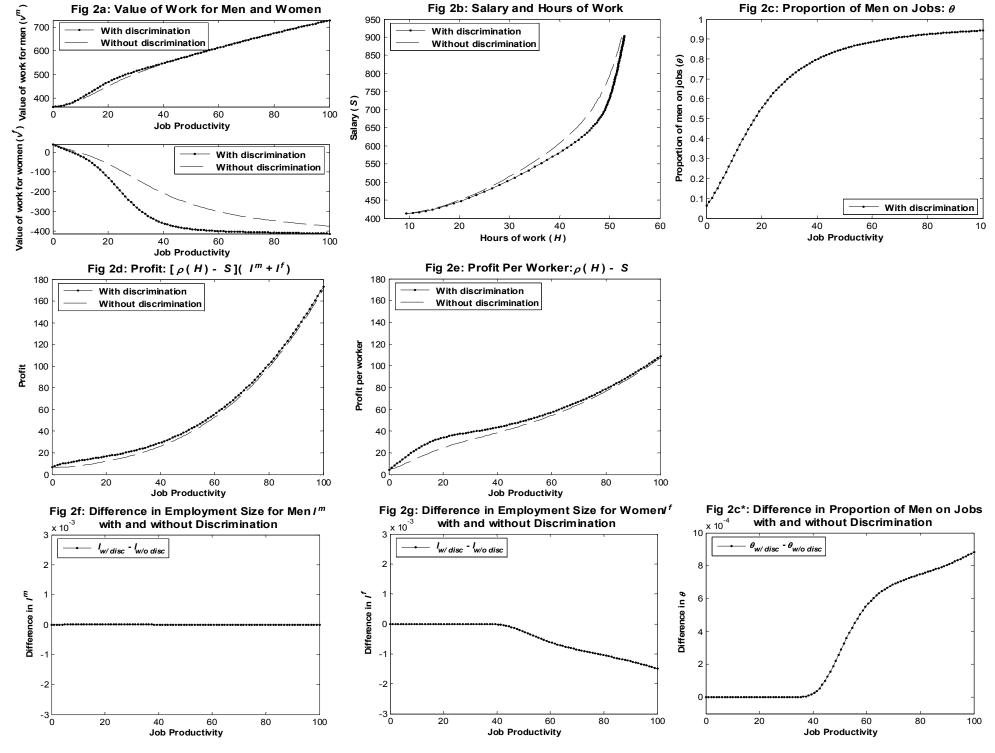
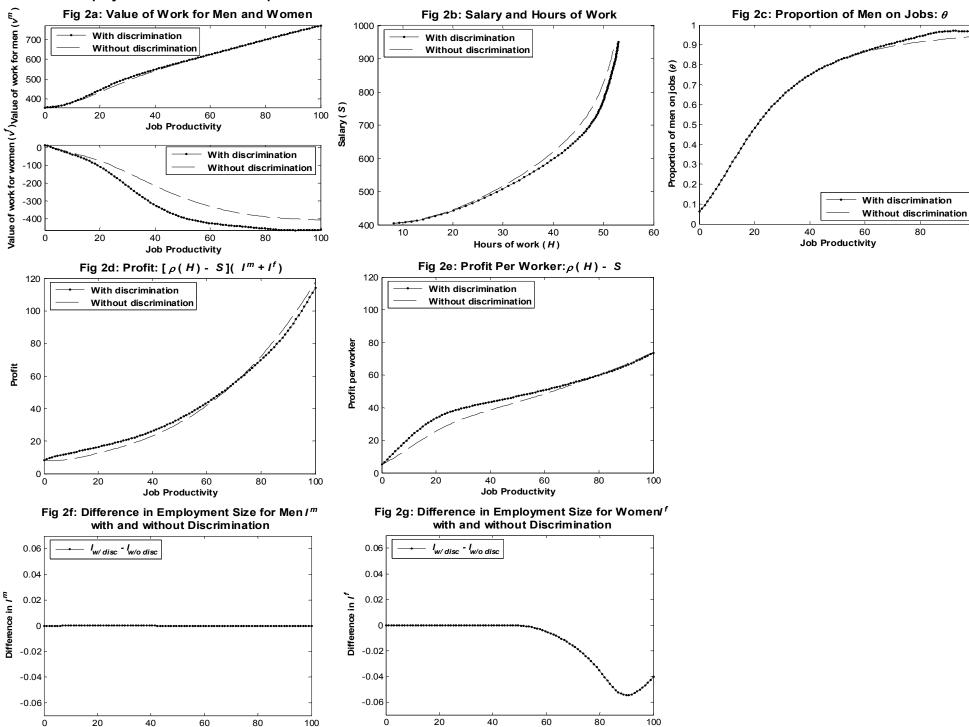


Figure 3
Case 4ER: Employer discrimination. Comparison between the case with discrimination and the case without discrimination

Job Productivity



Job Productivity