Insurance and Opportunities: The Welfare Implications of Rising Wage Dispersion

Jonathan Heathcote*, Kjetil Storesletten†, and Giovanni L. Violante‡

September 2005

Abstract

This paper provides an analytical characterization of the welfare effects of changes in cross-sectional wage dispersion, using a class of tractable heterogeneous-agent economies with various insurance market structures. We express welfare effects both in terms of changes in the observable joint distribution over individual wages, consumption and hours, and in terms of the underlying parameters defining preferences and the changing nature of wage risk. Our analysis reveals an important trade-off for welfare calculations. On the one hand, as wage uncertainty rises, so does the cost associated with missing insurance markets. On the other hand, greater wage inequality presents opportunities to increase aggregate productivity by concentrating market work among more productive workers. This productivity gain means that improving the degree of insurance against wage risk offers larger welfare gains than redistributive policies that reduce individual wage variability. In a calibration exercise, we find that the observed rise in wage dispersion in the United States over the past three decades implies a welfare loss roughly equivalent to a 2.5% decline in lifetime consumption. This number is the net effect of a welfare *gain* of around 5% from an endogenous increase in labor productivity, coupled with a *loss* of around 7.5% associated with greater dispersion in consumption and leisure.

JEL Classification Codes: D31, D58, D91, E21, J22, J31.

Keywords: Insurance, Labor supply, Productivity, Wage dispersion, Welfare.

*Georgetown University, and CEPR; jhh9@georgetown.edu

†University of Oslo, Frisch Centre (Oslo), IIES (Stockholm), and CEPR; kjstore@econ.uio.no

‡New York University, and CEPR; glv2@nyu.edu

1 Introduction

There has been a sharp increase in wage inequality in the United States over the past thirty years.[1] What are the welfare consequences of this phenomenon? This is an important question, since cross-sectional wage inequality and individual wage volatility over the life-cycle are large. For example, the variance of the growth rate of individual wages in the United States is over *100 times* larger than the variance of the growth rate of average wages.[2]

We develop a tractable class of dynamic heterogeneous-agent economies with partial insurance against idiosyncratic labor productivity (wage) risk. The framework delivers analytical characterizations of the welfare effects associated with a change in wage dispersion. Specifically, we are able to derive two sets of closed-form expressions for welfare effects, one in terms of structural parameters characterizing risk and preferences, and another in terms of changes in observable moments of the cross-sectional distribution over wages, consumption and hours worked.

Agents in our models are subject to stochastic idiosyncratic shocks to their labor market productivity and thus to their market wage. Welfare costs depend crucially on the set of assets available to insure against these shocks. In our benchmark "incomplete markets" economy, the process for idiosyncratic wages has two components: an uninsurable piece, and a component that may be fully insured via trade in state-contingent securities. To better understand the role played by access to explicit insurance markets we also consider two alternative market structures: complete markets, where all wage inequality is insurable, and autarky, where all trade between agents is ruled out.

Several authors have examined the welfare consequences of changes in earnings or income risk.[3] We focus instead on wage risk and endogenize labor supply because the ability to adjust hours can mitigate the welfare cost of rising wage inequality via two alternative channels. First, agents may vary hours worked inversely with fluctuations in individual wages, thereby reducing fluctuations in earnings. Alternatively, agents may choose to work more hours in periods when individual wages are high, thereby increasing average earnings per hour. A

[1]For surveys on the causes of the changes in inequality, see Katz and Autor (1999), Acemoglu (2002), Aghion (2003), and Hornstein, Krusell and Violante (2004).

[2]This number is calculated from the PSID, 1967-1996. The variance of the mean wage growth over the period is 0.0012 and the cross-sectional variance of individual wage growth, averaged over the period, is 0.161. See section 7 for details on the sample selection.

[3]See e.g. Attanasio and Davis (1996); Blundell and Preston (1998); and Krebs, Krishna and Maloney (2004).

negative wage-hour correlation may be observed if agents cannot smooth income by other means, such as by purchasing explicit insurance against wage risk. Conversely the wage-hour correlation will be positive if wage inequality can be insured directly within financial markets. Thus the model highlights an interesting interaction between the asset market structure and the role of endogenous labor supply in absorbing idiosyncractic wage shocks.[4]

We consider two standard specifications for preferences: one in which consumption and leisure enter the utility function in a Cobb-Douglas fashion, and one in which preferences are separable between consumption and hours worked. For each market structure and preference specification we derive intuitive analytic solutions for equilibrium allocations and expected lifetime utility as functions only of preference parameters and of the variances of the insurable and uninsurable components in individual labor productivity. These transparent expressions enable us to answer two sets of questions relating to welfare.

First, what are the welfare costs of rising wage dispersion, holding constant the asset market structure? We show that the welfare change associated with rising wage dispersion in the benchmark incomplete-markets economy is a weighted average of the effects under autarky and complete markets, where the weights correspond to the relative increases in insurable versus uninsurable risk. Second, what are the welfare costs of market incompleteness, defined as the difference between expected lifetime utility in the incomplete-markets economy versus the complete markets economy, holding constant the wage-generating process?

A key finding of the paper is that, with flexible labor supply, the closed-form expressions for welfare reveal two important offsetting forces: an increase in idiosyncratic wage risk increases the need for *insurance*, but also presents an *opportunity* to increase the level of aggregate productivity, measured as output per hour worked, by concentrating work effort among more productive workers. To help clarify this trade-off, we follow Benabou (2002) and Flodén (2001) and decompose the overall welfare effects into the relative contributions of changes in aggregate consumption and leisure on the one hand, and changes in the cross-sectional dispersion of these variables on the other.

We find that the sign of the welfare effect associated with increased wage dispersion depends on the market structure and on agents' willingness to substitute consumption and leisure inter-temporally. In particular, when markets are complete, a rise in (insurable) wage dispersion unambiguously increases welfare, since wages co-move perfectly with hours, and

[4]Low (2005) explores the implications of this interaction for the life-cycle profiles of consumption, hours and asset holdings.

increased wage dispersion increases aggregate productivity in proportion to the labor supply elasticity. In autarky, a rise in (uninsurable) wage dispersion generally implies welfare losses. Surprisingly, however, welfare gains are possible even in this case, if preferences are such that agents are willing to tolerate larger fluctuations in consumption and leisure in exchange for higher average earnings per hour.

Another key result of the paper is that welfare effects can alternatively be expressed as simple functions of various moments of the cross-sectional joint distribution over wages, hours and consumption. For example, in the separable-preferences case, the welfare effect from a rise in wage dispersion can be expressed in terms of observables as the sum of the changes in (i) the covariance between log-wages and log-hours, (ii) the variance of log-consumption weighted by the coefficient of relative risk-aversion, and (iii) the variance of log-hours weighted by the inverse of the labor supply elasticity. The advantage of the representation based on these moments is that we do not have to take a stand on the fraction of wage dispersion that is insurable – this information is effectively embedded in the endogenous joint distribution over consumption, hours and wages. Thus, we can in principle estimate welfare effects directly from data simply by computing the relevant moments in repeated cross-sections and assigning values to the risk aversion and labor elasticity parameters. However, this observables-based approach requires high quality data on consumption and hours, while the model-based approach outlined above does not. We therefore view these two alternative approaches as complementary.

In the quantitative part of the paper we compute the welfare costs of rising wage dispersion and the welfare costs of missing markets for a set of commonly used preference parameters. For example, with Cobb-Douglas preferences and a coefficient of relative risk aversion equal to 2, the welfare cost of the observed rise in labor market risk in the U.S. over the past 30 years in the incomplete-markets economy is 2.5% of lifetime consumption. This number is the combination of a welfare loss of 7.5% due to larger uninsurable fluctuations in individual consumption and hours, and a welfare gain of 5% from an increase in aggregate labor productivity.

For the same preferences, households would be willing, ex-ante, to give up almost 40% of their expected lifetime consumption in exchange for access to complete markets. One might suspect that this welfare gain stems from reducing inequality in the cross-sectional distributions for consumption and leisure. However, we find that two thirds of the welfare gains from completing markets take the form of higher average productivity. Thus our analysis highlights an important cost of missing markets that has been largely overlooked to date, namely the loss in aggregate labor productivity that arises when low productivity agents work too much (because lack of insurance makes them inefficiently poor) while high productivity agents work too little (because lack of insurance makes them inefficiently rich).

Finally, we compute the welfare effects from eliminating individual wage risk, and compare them to the gains from insuring this risk. In the presence of endogenous labor supply, eliminating risk always leads to smaller welfare gains than insuring risk because it breaks the positive assortative matching between productivity and hours that characterizes the efficient allocation. Computing the welfare gain from eliminating idiosyncratic risk (through some form of redistributive scheme) is the cross-sectional equivalent of the calculations underlying the vast literature on the welfare costs of business cycles fluctuations (for a survey, see Lucas 2003). We find that complete wage compression - eliminating all individual productivity fluctuations - delivers a net welfare gain. This gain is only about half the size of the gain from completing markets, but is at least two orders of magnitude larger than Lucas' estimate of the upper bound for potential welfare gains from stabilizing business cycles (0.1 percent of permanent consumption).

The main contribution of our paper is to clarify what drives the welfare effects of changes in the wage process. However, our simple framework can also shed light on the *quantitative* findings of richer models with more complex interaction between wages and the wealth distribution. In particular, when properly calibrated, our model delivers quantitatively similar results to Kubler and Schmedders (2001), Krueger and Perri (2003), Pijoan-Mas (2004), and Heathcote, Storesletten and Violante (2004). The advantage of our approach is that welfare effects can be solved for in closed form (rather than via numerical solution and simulation), and consequently the role of preference parameters, wage risk parameters and market structure are all transparent.

The rest of the paper is organized as follows. Section 2 describes the model economies, and Section 3 describes our welfare measures. Sections 4 and 5 characterize equilibrium allocations and the particular welfare expressions that obtain under the two alternative preference specifications we consider. In section 6, we express the welfare effects in terms of observable cross-sectional moments. Section 7 describes the calibration to the United States, and reports our quantitative results. Finally, in Section 8, we make some concluding remarks.

2 The Economy

Demographics and preferences: The economy is populated by a unit mass of infinitely-lived agents. Each agent has the same time-separable utility function $U(\mathbf{c}, \mathbf{h})$ over streams of consumption $\mathbf{c} = \{c_t\}_{t=1}^{\infty}$ and hours worked $\mathbf{h} = \{h_t\}_{t=1}^{\infty}$,

$$U(\mathbf{c}, \mathbf{h}) = (1-\beta) \sum_{t=1}^{\infty} \beta^{(t-1)} u(c_t, h_t),$$

where $\beta \in (0,1)$ is the agents' discount factor. We will consider two alternative specifications for the period utility function. In the first, consumption and leisure enter in a Cobb-Douglas fashion. In the second, period utility is separable between consumption and hours worked.

Production and individual labor productivity: Production takes place through a constant-returns-to-scale aggregate production function with labor as the only input. The labor market and the goods market are perfectly competitive, so individual wages equal individual productivity. Since we do not focus on growth or short-term fluctuations, we assume the hourly rental rate per efficiency unit to be constant and normalized to unity.

Individuals' wage rates vary stochastically over time and are independently and identically distributed across the agents in the economy. We assume that an individual's log wage at a point in time has two orthogonal components: an 'uninsurable' component $\alpha \in A \subseteq \mathbb{R}$, and an 'insurable' component $\varepsilon \in E \subseteq \mathbb{R}$:

$$\log w(\alpha, \varepsilon) = \alpha + \varepsilon. \tag{1}$$

In order to fix ideas and simplify the expressions for our subsequent welfare results, we will interpret the uninsurable component as a non-stochastic fixed effect for the agent, and we will interpret the insurable component as a purely transitory shock that is *iid* over time. Thus, at the beginning of period $t = 1$, each agent draws a pair (α, ε). Then for every $t > 1$, each agent draws a new value for ε. All shocks are publicly observable. The assumption that insurable shocks are *iid* is purely for ease of exposition, we can generalize it to any stochastic process. In section 4.2.1 we explain how the analysis can be extended to allow for a richer specification of the process for the uninsurable component, incorporating permanent shocks to wages, while still retaining analytical tractability.

We assume that ε and α are drawn from normal distributions, with

$$\varepsilon \sim N\left(-\frac{v_\varepsilon}{2}, v_\varepsilon\right), \qquad \alpha \sim N\left(-\frac{v_\alpha}{2}, v_\alpha\right).$$

As a result,

$$\log w \sim N\left(-\frac{v}{2}, v\right), \tag{2}$$

where $v = v_\varepsilon + v_\alpha$ is the variance of the log-normal productivity shock. Note that equation (2) implies that the population mean wage (in levels) is equal to one, i.e. $E[w] = 1$. Thus the mean wage will be invariant to dispersion when we study comparative statics with respect to the variance of wages, v. Let ϕ_v denote the normal density function with mean $-\frac{v}{2}$ and variance v.

Market structure: We compare competitive equilibria under three alternative asset market structures: autarky (AUT), complete markets (CM) and incomplete markets (IM). Each market structure decentralizes an equal-weight planner's problem. There is an intuitive mapping between the number of assets that may be traded in the particular market structure, and the planner's ability to transfer resources between agents in the corresponding planner's problem.

A common assumption for all market structures is that the available assets come in zero net supply and that agents have zero financial wealth before markets open for trade. Hence, aggregate wealth is zero in every period.[5] We will assume that all assets traded are one-period-lived, and define equilibria sequentially (i.e., with Arrow securities).

We will show that under each market structure, equilibrium choices for an agent in period t depend only the agent's fixed effect α and on the agent's current draw for the transitory shock ε. Thus we let the functions $c_m(\alpha, \varepsilon)$ and $h_m(\alpha, \varepsilon)$ denote the decision rules for consumption and hours as functions of these state variables under market structure $m \in \{CM, IM, AUT\}$. In particular, individual wealth is not a state variable, so one does not need to keep track of the equilibrium distribution of assets. This feature is what makes our model tractable compared to the standard Bewley model with a single bond.[6]

We now present the three alternative market structures and the associated sequential budget constraints.

1. *Autarky (AUT):* In this economy no financial instruments are traded, and households simply consume their labor income every period. In any period, the budget constraint

[5]It is easy to see that our economy is isomorphic to an economy with capital, where all claims on capital are foreign-owned, capital is perfectly mobile internationally, and the world interest rate is $R = 1/\beta$.

[6]See Bewley (1984), Imrohoroglu (1989), Huggett (1993), Aiyagari (1994) and Rios-Rull (1994) for details.

for a household with fixed effect α and a draw of the transitory shock ε is simply

$$c_{AUT}(\alpha,\varepsilon) = w(\alpha,\varepsilon)\, h_{AUT}(\alpha,\varepsilon). \tag{3}$$

This corresponds to a planner's problem in which each agent lives alone on an island, and the planner is unable to transfer resources across islands.

2. *Complete markets (CM):* In the complete-markets economy, at every date households are free to trade contracts contingent on every possible realization of the individual productivity shocks next period. Insurance markets open in period 0 *before* α is realized, at which time the household budget constraint is given by

$$\int_A \int_E p_0(\alpha,\varepsilon)\, b_0(\alpha,\varepsilon)\, d\alpha d\varepsilon = 0, \tag{4}$$

where $b_0(\alpha,\varepsilon)$ and $p_0(\alpha,\varepsilon)$ are respectively the quantity and the price of one period state-contingent bonds that pay one unit of output in period 1 if the realization of the individual fixed effect is equal to α and the transitory shock is equal to ε. The right-hand side of the period 0 constraint is zero since no labor income is received in that period and initial individual wealth is zero. From period 1 onwards there is no more uncertainty regarding the fixed effect α and the Arrow securities traded need only be contingent on the transitory shock. Hence the budget constraint for an agent with individual state (α,ε) becomes

$$c_{CM}(\alpha,\varepsilon) + \int_E p(\zeta)\, b(\zeta;(\alpha,\varepsilon))\, d\zeta = b(\varepsilon;(\alpha,\varepsilon_{-1})) + w(\alpha,\varepsilon)\, h_{CM}(\alpha,\varepsilon), \tag{5}$$

where $b(\zeta;(\alpha,\varepsilon))$ denotes the quantity of a bond, purchased by an individual with fixed effect α and transitory shock ε, that pays one unit of output in the next period if the realization of the transitory shock is ζ; $p(\zeta)$ denotes the price of this state-contingent bond. An arbitrarily loose constraint on borrowing rules out Ponzi schemes. This decentralized economy corresponds to a planner's problem in which all agents have equal weights and live on the same island, and the planner is free to dictate hours worked for each agent and to redistribute aggregate output.

3. *Incomplete markets (IM):* This is our intermediate benchmark, which we interpret as an approximate description of actual economies. Here, households have access to perfect insurance against the transitory ε-shocks and no insurance against the permanent α-shocks. In other words, markets open *after* α is realized.[7] This corresponds to a

[7] In this sense, this market-structure is ex-ante incomplete, but ex-post complete, where ex-ante and ex-post refer to the realization of the fixed effect.

planner's problem in which agents are segregated across islands by their fixed effect α. Within each $\alpha-$island, the planner is unconstrained regarding how to allocate work effort across agents with different values for the transitory shock ε, and can allocate resulting output freely across agents on the island. However, the planner cannot transfer resources across islands.

In the decentralized version of this economy, budget constraints are exactly as in the complete markets economy for $t > 0$ (see equation 5), except that the initial budget constraint is given by

$$\int_E p_0(\varepsilon)\, b_0(\varepsilon;\alpha)\, d\varepsilon = 0.$$

One literal interpretation of this economy is one of explicit insurance against some risks (such as short spells of unemployment or illness) but no insurance against others (such as being endowed with low productive ability or being born to poor or uneducated parents). An alternative, and perhaps more natural interpretation, is that our model is an approximation of the "Bewley model" in which a single non-contingent bond is traded. Since borrowing and saving through a risk-free asset allows for near-perfect smoothing of transitory shocks, but provide virtually no insurance against permanent productivity differences, our economy will closely approximate the Bewley model. However, while that class of models requires numerical solutions, equilibrium allocations in our economies can be characterized analytically, as shown below. In Section 7 we revisit the comparison between these two classes of economies.

2.1 Solving for equilibrium allocations

A competitive equilibrium requires individual optimization and market clearing in all markets, i.e. that the net demand for all assets is zero and that aggregate consumption equals aggregate labor earnings.

In order to find the equilibrium allocations of consumption and labor supply we solve the corresponding planner's problem for each market structure. These problems and the solutions to them are described in detail in Appendix A. Focussing on planner's problems has the advantage that asset prices do not appear in the constraint set, and one can abstract from portfolio choices.

Since there is no interaction between agents in the autarky economy, it is immediate that allocations in the competitive equilibrium and the solution to the corresponding planner's problem coincide: irrespective of how the planner weights the welfare of different agents, it is efficient for the planner to equate each agent's marginal rate of substitution between hours worked and consumption in preferences to the agent's marginal rate of transformation between hours and output.

In the economies with asset trade, we compute first the planner allocations of consumption and hours, and the implied equilibrium prices. We then verify that all the conditions characterizing equilibrium under the particular market structure are satisfied given these allocations and prices. In particular, we check that (i) agents' intra-temporal first order conditions for labor supply and inter-temporal first order conditions for asset purchases are satisfied, (ii) agents' budget constraints are satisfied, and (iii) the goods market and all asset markets clear.

Before characterizing allocations under both Cobb-Douglas and separable preferences, we first describe how we perform our welfare analysis.

3 Qualitative welfare analysis

We compare and rank allocations using the following utilitarian social welfare function:

$$\begin{aligned} \mathcal{W} &= (1-\beta)E_0\left[\sum_{t=1}^{\infty}\beta^{(t-1)}u\left(c_m(\alpha,\varepsilon_t),h_m(\alpha,\varepsilon_t)\right)\right] \\ &= \int_A\int_E u\left(c_m\left(\alpha,\varepsilon\right),h_m\left(\alpha,\varepsilon\right)\right)\phi_{v_\varepsilon}(\varepsilon)\phi_{v_\alpha}(\alpha)d\varepsilon d\alpha \qquad (6) \end{aligned}$$

This expression for welfare has two interpretations. First, it is the value for a utilitarian planner who weights all agents equally. Second, it is the expected lifetime utility for an agent at time 0 “under the veil of ignorance”; *i.e.* expected lifetime utility before uncertainty is realized.

As discussed in the introduction, we assess the welfare costs associated with labor market uncertainty from two distinct perspectives. First, for a given insurance market structure, what are the welfare costs of a rise in labor market risk? Second, for a given level of risk, what are the welfare gains from completing markets?

The welfare implications of rising labor market risk: We begin by fixing the market structure of the economy and measuring the welfare implications from increasing wage dispersion. Suppose the variances of permanent and transitory shocks rise from the baseline values v_α and v_ε respectively to $\widehat{v}_\alpha$ and $\widehat{v}_\varepsilon$. Let $\Delta v_\alpha = \widehat{v}_\alpha - v_\alpha$ and $\Delta v_\varepsilon = \widehat{v}_\varepsilon - v_\varepsilon$. Let ω_m denote the associated welfare gain under market structure m, expressed in units of the "equivalent compensating variation" in lifetime consumption under the baseline wage variance:

$$\begin{aligned} & \int_A \int_E u\left((1+\omega_m)c_m(\alpha,\varepsilon), h_m(\alpha,\varepsilon)\right)\phi_{v_\varepsilon}(\varepsilon)\phi_{v_\alpha}(\alpha)d\varepsilon d\alpha \\ = & \int_A \int_E u\left(\widehat{c}_m(\alpha,\varepsilon), \widehat{h}_m(\alpha,\varepsilon)\right)\phi_{\widehat{v}_\varepsilon}(\varepsilon)\phi_{\widehat{v}_\alpha}(\alpha)d\varepsilon d\alpha. \end{aligned} \tag{7}$$

Here $c_m(\alpha,\varepsilon)$ and $h_m(\alpha,\varepsilon)$ denote the optimal policies for consumption and hours worked in an economy with shock variances v_α and v_ε. Similarly, $\widehat{c}_m(\alpha,\varepsilon)$ and $\widehat{h}_m(\alpha,\varepsilon)$ denote the optimal policies in the economy with variances $\widehat{v}_\alpha$ and $\widehat{v}_\varepsilon$.

Decomposing welfare changes: A theme of our paper is that increases in wage dispersion can impact aggregate productivity (by changing the covariance between hours and individual productivity, i.e. wages) in addition to affecting the amount of risk that agents face. We are therefore interested in decomposing the overall welfare effect ω_m into two pieces capturing respectively the welfare change associated with changes in the size of the aggregate pie due to additional risk - which we label the *level effect* - and the welfare change associated with changes in how evenly the pie is distributed - which we label the *volatility effect*.

Formally, our strategy for identifying these two components closely follows that outlined by Flodén (2001), who in turn builds on earlier work by Benabou (2002). Let capital letters denote population averages. We define the *level effect* associated with an increase in wage dispersion (in units of consumption) as the value for ω_m^{lev} that solves:

$$u\left(\left(1+\omega_m^{lev}\right)C_m, H_m\right) = u\left(\widehat{C}_m, \widehat{H}_m\right). \tag{8}$$

Next, for an agent behind the veil of ignorance, define the *cost of uncertainty* (in terms of consumption) as the value for p_m that solves

$$u\left((1-p_m)\,C_m, H_m\right) = \int_A \int_E u\left(c_m(\alpha,\varepsilon), h_m(\alpha,\varepsilon)\right)\phi_{v_\varepsilon}(\varepsilon)\phi_{v_\alpha}(\alpha)d\varepsilon d\alpha. \tag{9}$$

Note that the cost of uncertainty is a measure of the utility difference between drawing a lottery over $c_m(\alpha,\varepsilon)$ and $h_m(\alpha,\varepsilon)$ versus receiving the expected values for consumption and leisure associated with this lottery. Analogously, we can define the cost of uncertainty associated with the higher variances $\widehat{v}_\alpha$ and $\widehat{v}_\varepsilon$, which we denote $\widehat{p_m}$.

We then define the *volatility effect* associated with an increase in wage dispersion as

$$\omega_m^{vol} = \frac{1 - \widehat{p_m}}{1 - p_m} - 1. \tag{10}$$

Thus the volatility effect is the percentage change in the cost of uncertainty associated with the increase in wage dispersion.[8]

For both types of preferences, we will establish that the two components approximately sum to the total welfare change, i.e.

$$\omega_m \simeq \omega_m^{lev} + \omega_m^{vol}.$$

The welfare gains from completing markets: Similarly, we measure the welfare gain associated with completing insurance markets, for given levels of permanent and transitory risk v_α and v_ε, as the percentage increase in consumption in the incomplete-markets (or autarkic) economy required to achieve the same welfare as in the economy with complete markets. In particular, we define the welfare gain as the value for χ_m that solves

$$\begin{aligned} & \int_A \int_E u\left((1+\chi_m)c_m(\alpha,\varepsilon), h_m(\alpha,\varepsilon)\right) \phi_{v_\varepsilon}(\varepsilon)\phi_{v_\alpha}(\alpha) d\varepsilon d\alpha \\ = & \int_A \int_E u\left(c_{CM}(\alpha,\varepsilon), h_{CM}(\alpha,\varepsilon)\right) \phi_{v_\varepsilon}(\varepsilon)\phi_{v_\alpha}(\alpha) d\varepsilon d\alpha. \end{aligned} \tag{11}$$

Strictly speaking, the solutions for ω_m and χ_m allow one to compare welfare across two economies with different levels of inequality. One can interpret these as the welfare gain from rising inequality (or completing markets) when the transition between steady state distributions of wages is instantaneous, as it is in the economies we consider.

4 Cobb-Douglas preferences

First, we consider preferences that are Cobb-Douglas between consumption and leisure. In this case

$$u(c,h) = \frac{\left(c^\eta (1-h)^{1-\eta}\right)^{1-\theta}}{1-\theta}.$$

[8] Flodén (2001) actually further decomposes the volatility effect (which he calls the distributional effect) into two sub-components that he labels the inequality effect and the insurance effect. The idea is that policy changes both redistribute resources across agents, and change the amount of risk that agents face. The welfare cost of increased inequality for a government contemplating a change in policy depends on the governments attitude to inequality aversion. By contrast, the cost of additional risk just depends on the curvature in preferences. In our analysis, we focus on welfare comparisons from behind a veil of ignorance. Behind this veil, all agents are equal. Thus there is no inequality effect from increased wage dispersion, and our volatility effect is identical to Flodén's uncertainty effect.

where $\eta \in (0,1)$ determines the relative taste for consumption versus leisure. Cobb-Douglas preferences are widely used in the macro literature, since they are consistent with balanced growth, irrespective of the choice for θ. In labor economics, this specification is often advocated because there is some empirical evidence of non-separability between consumption and leisure; see, for example, Heckman (1974).

The share parameter η is generally pinned down by the share of disposable time agents devote to market work, implying that the single parameter θ governs both the intertemporal elasticity of substitution for consumption and the corresponding elasticity for hours worked. In particular, the intertemporal elasticity of substitution for consumption is given by $1/\theta$.

The coefficient of relative risk aversion is

$$\bar{\gamma} \equiv \gamma(\theta, \eta) \equiv -\frac{c u_{cc}}{u_c} = 1 - \eta + \eta\theta. \tag{12}$$

The Frisch elasticity of labor supply depends on hours worked, and is given by

$$\phi(\theta, \eta, h) = \frac{\lambda(1-h)}{h},$$

where $\lambda \equiv (1 - \eta + \eta\theta)/\theta$ defines the Frisch elasticity for leisure.[9] It is useful to define a "non-stochastic Frisch elasticity" of labor supply in a non-stochastic representative-agent economy, in which case $h = \eta$:

$$\bar{\phi} \equiv \frac{\lambda(1-\eta)}{\eta}. \tag{13}$$

4.1 Allocations with Cobb-Douglas preferences [10]

Autarky (AUT)– In autarky, consumption equals earnings every period. Individual hours worked are chosen optimally, and using the budget constraint (3) and the appropriate intratemporal first order condition, it is straightforward to solve for the equilibrium choices for consumption and hours worked:

$$\begin{aligned} c_{AUT}(\alpha, \varepsilon) &= \eta \exp(\alpha + \varepsilon), \\ h_{AUT}(\alpha, \varepsilon) &= \eta. \end{aligned} \tag{14}$$

[9] The Frisch elasticity of labor supply (leisure) measures the elasticity of hours worked (leisure) to transitory changes in wages, keeping the marginal utility of consumption constant.

[10] See Appendix A for derivations of the expressions for equilibrium allocations reported below.

Note that allocations depend only on the current period wage, $w(\alpha,\varepsilon) = \exp(\alpha+\varepsilon)$ and not on the two shocks separately. Under the Cobb-Douglas specification, income and substitution effects from uninsurable wage changes exactly offset, so hours are constant.

Complete markets (CM)– In the complete markets economy equilibrium allocations for consumption and hours are given by:

$$\begin{aligned} c_{CM}(\alpha,\varepsilon) &= \eta \exp\left(\lambda(1-\lambda)\frac{v}{2}\right)\exp\left((1-\lambda)(\alpha+\varepsilon)\right), \\ h_{CM}(\alpha,\varepsilon) &= 1-(1-\eta)\exp\left(\lambda(1-\lambda)\frac{v}{2}\right)\exp\left(-\lambda(\alpha+\varepsilon)\right). \end{aligned} \tag{15}$$

Because of non-separability, equalizing the marginal utility of consumption across agents in the Cobb-Douglas case does not in general imply equalizing consumption. For $\lambda < 1$ (which is equivalent to $\theta > 1$), consumption and leisure are substitutes in the sense that high productivity individuals who in the complete-market allocation enjoy relatively little leisure are compensated with relatively high consumption. When $\theta = 1$ the Frisch elasticity of leisure is also equal to 1, in which case consumption is constant and equal to η, while hours respond strongly to wage changes.

It is interesting to note that with Cobb-Douglas preferences, average consumption and average hours worked in complete markets are:

$$E\left[c_{CM}(\alpha,\varepsilon)\right] = \eta, \tag{16}$$

$$E\left[h_{CM}(\alpha,\varepsilon)\right] = 1-(1-\eta)\exp(\lambda v). \tag{17}$$

Thus with Cobb-Douglas preferences, it is efficient to take advantage of more variable shocks by reducing average hours and increasing leisure rather than by increasing average consumption.

Equilibrium asset prices are given by:

$$p_0(\alpha,\varepsilon) = \beta\phi_{v_\alpha}(\alpha)\phi_{v_\varepsilon}(\varepsilon) \qquad \forall\alpha\in A, \forall\varepsilon\in E, \tag{18}$$

$$p(\zeta) = \beta\phi_{v_\varepsilon}(\zeta) \qquad \forall\zeta\in E. \tag{19}$$

Note that prices are simply probabilities, discounted with the subjective discount factor β. This reflects the fact that agents are perfectly able to insure idiosyncratic risk, so the price of risk is zero. Consequently, there is no motive for precautionary savings and the interest rate on a risk free bond is equal to $1/\beta$.

Equilibrium asset purchases are given by

$$\begin{aligned} b_{0,CM}(\alpha,\varepsilon) &= c_{CM}(\alpha,\varepsilon) - w(\alpha,\varepsilon)\, h_{CM}(\alpha,\varepsilon) + \\ &\quad \frac{\beta}{1-\beta}\int_E \left[c_{CM}(\alpha,\zeta) - w(\alpha,\zeta)\, h_{CM}(\alpha,\zeta)\right]\phi_{v_\varepsilon}(\zeta)d\zeta, \\ b_{CM}(\zeta;(\alpha,\varepsilon)) &= b_{0,CM}(\alpha,\zeta). \end{aligned}$$

The expressions for purchases of state contingent claims are such that financial assets plus the expected present value of labor income in each possible realization of the state is equal to the present value of equilibrium consumption. Note that the market price of the portfolio purchased at date zero is zero, while the price of the portfolio purchased at future dates may not be zero. In particular, if the realization of α is low, so that the present value of future labor income is lower than the present value of consumption, there will be a large initial payout from the Arrow securities indexed to α – *i.e.* $b_{0,CM}(\alpha,\varepsilon)$ will be a large positive number. From this point onwards, the low$-\alpha$ agent will effectively consume the interest from this initial payoff and maintain a constant wealth level.[11]

Incomplete markets (IM)– Equilibrium allocations in the incomplete markets economy are given by

$$\begin{aligned} c_{IM}(\alpha,\varepsilon) &= \eta\exp(\alpha)\exp\left(\lambda(1-\lambda)\frac{v_\varepsilon}{2}\right)\exp\left((1-\lambda)\varepsilon\right), \\ h_{IM}(\alpha,\varepsilon) &= 1-(1-\eta)\exp\left(\lambda(1-\lambda)\frac{v_\varepsilon}{2}\right)\exp(-\lambda\varepsilon). \end{aligned} \tag{20}$$

In this case, hours worked are increasing in the insurable transitory shock ε (as in the complete markets economy) but are independent of the uninsurable permanent shock α (as under autarky). Because offsetting income and substitution effects imply that cross-sectional differences in the permanent shock do not translate to differences in hours worked, they must show up in consumption, which is directly proportional to α. Current consumption is increasing in current ε if and only if $\lambda < 1 \Leftrightarrow \theta > 1$. The interpretation for this result follows immediately from the discussion for the complete markets case.

Within each group of fixed-effects α, average consumption is constant and equal to $\eta\exp(\alpha)$. Aggregate economy-wide consumption is equal to η and invariant to changes in wage inequality, while aggregate hours worked is declining in v_ε.

[11]By combining all the one-period claims indexed to ε one can construct a non-contingent bond in zero net supply, which suffices for agents with low (high) α to maintain a constant positive (negative) wealth.

Equilibrium prices in the first period are $p_0(\varepsilon) = \beta\phi_{v_\varepsilon}(\varepsilon)$, and for $t > 0$ prices are given by (19). Purchases of state contingent bonds are such that for each possible realization for ζ, the quantity of bonds that pays out plus the equilibrium value of labor income is equal to the equilibrium value for consumption:

$$b_{IM}(\zeta; (\alpha, \varepsilon)) = c_{IM}(\alpha, \zeta) - w(\alpha, \zeta)\, h_{IM}(\alpha, \zeta).$$

Note that the market price of buying this portfolio is zero at each date, so in this sense the agent's wealth is constant and equal to zero. To understand, recall that the main role of wealth in a standard buffer-stock-saving model where agents can borrow and lend but cannot insure shocks directly is to facilitate self-insurance against risk (Carroll 1997, Deaton 1991, Zeldes 1989). For the agents in our model, wealth accumulation cannot smooth the ex-ante uncertainty associated with fixed individual effects. Moreover, with full insurance against transitory (insurable) risk, agents have no incentive to accumulate wealth to buffer those shocks either.

4.2 Welfare with Cobb Douglas preferences

We now analyze the welfare effects of rising inequality for each of the three market structures described above.

Proposition 1: *With Cobb-Douglas preferences, the (approximate) welfare change from a rise in labor market risk equal to Δv, where $\Delta v = \Delta v_\varepsilon + \Delta v_\alpha$, is given by the following expressions under the three market structures we consider:*

$$\omega_{AUT} \simeq -\bar{\gamma}\frac{\Delta v}{2} = \underbrace{0}_{\omega_{AUT}^{lev}} \underbrace{-\ \bar{\gamma}\Delta v/2}_{\omega_{AUT}^{vol}},$$

$$\omega_{CM} \simeq \bar{\phi}\frac{\Delta v}{2} = \underbrace{\bar{\phi}\Delta v}_{\omega_{CM}^{lev}} \underbrace{-\ \bar{\phi}\Delta v/2}_{\omega_{CM}^{vol}},$$

$$\omega_{IM} \simeq \bar{\phi}\frac{\Delta v_\varepsilon}{2} - \bar{\gamma}\frac{\Delta v_\alpha}{2} = \underbrace{\bar{\phi}\Delta v_\varepsilon}_{\omega_{IM}^{lev}} \underbrace{-\bar{\phi}\Delta v_\varepsilon/2 - \bar{\gamma}\Delta v_\alpha/2}_{\omega_{IM}^{vol}}$$

Proof: *See Appendix B.*

In the Proof of Proposition 1 we carefully derive the exact closed-form solution for the welfare effects ω_m for all three market structures. However, these expressions are cumbersome and not particularly transparent. Through a set of log-approximations of the class

$\ln(1+x) \simeq x$ and $e^x \simeq 1+x$, one obtains the simple and useful solutions stated in Proposition 1. The linearity of ω_m in Δv_α and Δv_ε is a feature of the approximation. In Section 7.2 we discuss in more detail the quality of our approximations.

Autarky– In autarky, there is always a welfare loss associated with greater wage inequality. This loss is equal to the expression computed by Lucas (1987) for the welfare costs of aggregate consumption fluctuations in an economy with inelastic labor supply. Looking at the decomposition, there is no level effect from a change in wage dispersion, and the overall welfare change is thus equal to the negative volatility effect. The level effect is zero because with Cobb-Douglas preferences, the income and substitution effects associated with wage changes always exactly offset, and thus hours worked in autarky are always equal to η. Since hours are wage invariant, aggregate earnings and consumption are both equal to η times the average wage, which is held constant as we vary wage dispersion. Given that hours are wage invariant, it is no surprise that Lucas' result for an economy with exogenous labor supply extends to our economy. The larger is the risk aversion parameter $\bar{\gamma}$, the larger are the welfare costs associated with increased uninsurable risk. See Panel (A) in Figure 1.

Complete markets– Perhaps surprisingly, under complete markets, increasing productivity dispersion strictly increases welfare. The source of this gain can be understood by considering the decomposition between the level effect and the volatility effect. The level effect is proportional to the non-stochastic Frisch elasticity. The source of this result comes from the endogeneity of labor supply: an unconstrained planner can achieve better allocative efficiency with larger dispersion, without any loss in terms of consumption smoothing, by commanding longer hours from high-productivity workers and higher leisure from less productive workers. This result is closely related to the standard result from classical consumer theory that the indirect utility function of a static consumer is quasi-convex in prices, so a mean-preserving spread of the price distribution raises welfare (see, for example, Mas Colell 1995, page 59). In this case the indirect utility function of the planner in the planner's problem that corresponds to the complete markets equilibrium is quasi-convex in productivities w_{it}. Recall that with Cobb-Douglas preferences, aggregate labor productivity increases by virtue of a fall in average hours worked, rather than an increase in aggregate output.

Why is there a negative volatility effect in the complete markets economy notwithstanding full insurance? The reason is that to exploit greater dispersion in productivity across workers, the planner must increase dispersion in hours. Since utility is concave in leisure, this is welfare reducing. At the margin, the welfare gain for the planner from additional specialization in terms of increased average labor productivity is exactly offset by the loss associated with greater dispersion in leisure.

Panel (B) in Figure 1 plots ω_{CM} as a function of $\bar{\phi}$, for Δv normalized to one.

Incomplete markets– Realistic insurance market structures lie, arguably, strictly between complete markets and autarky. Hence, one can think of ω_{CM} and ω_{AUT} as, respectively, an upper bound and a lower bound on the welfare consequences of a rise in wage inequality. In the more realistic incomplete markets economy, the welfare gain can be expressed as

$$\omega_{IM} = \omega_{CM} \frac{\Delta v_\varepsilon}{\Delta v} + \omega_{AUT} \frac{\Delta v_\alpha}{\Delta v},$$

i.e. exactly as a weighted average between the welfare gain in complete markets and the welfare loss in autarky, with weights equal to the share of the rise in wage dispersion due to insurable and uninsurable shocks. Relative to an economy with inelastic labor supply, flexibility to adjust hours worked can reduce the welfare cost (or increases the welfare gain) from an increase in wage inequality because it allows for a more efficient division of labor in response to additional insurable productivity dispersion.

Exploiting the welfare analysis for changes to the variance of wages, we now compare the measure of household welfare defined in (6) across economies with different market structures, given the same stochastic process for idiosyncratic labor market risk.

Proposition 2: *With Cobb-Douglas preferences, the (approximate) welfare gains from completing the markets in an economy with uninsurable labor market risk equal to* v_α *are:*

$$\chi_{IM \to CM} \simeq \left(\bar{\phi} + \bar{\gamma}\right) \frac{v_\alpha}{2}, \tag{21}$$

This expression is very intuitive in light of the welfare expressions from the previous section. In particular, one way to think about what it means to complete markets is (i) there is a reduction $\Delta v_\alpha = -v_\alpha$ in the variance of uninsurable risk and (ii) there is a corresponding increase $\Delta v_\varepsilon = v_\alpha$ in the variance of insurable risk.[12] The parametric expression multiplying the variance has two separate components. The first term captures specialization, whereby more productive households work relatively harder and less productive households enjoy

[12]It follows immediately that the welfare gain of completing markets, starting from autarky, is given by equation (21), but with v_α replaced by v.

more leisure. This contribution to welfare is increasing in the non-stochastic Frisch elasticity $\bar{\phi}$. The second term – proportional to the coefficient of relative risk aversion $\bar{\gamma}$ – captures the value of the additional insurance provided by increased risk-sharing.

Figure 2 documents how the welfare gain from completing the set of assets traded varies with the risk aversion coefficient $\bar{\gamma}$ and with the willingness to substitute hours intertemporally $\bar{\phi}$. Recall that the two are both functions of (η, θ), so they vary together. In panel (B), we plot them for various values of θ, with η fixed. Interestingly, the shape of the welfare effect in panel (A) is non-monotone. Initially, as the Frisch elasticity falls ($1/\bar{\phi}$ rises), the welfare gain gets smaller, since the gain associated to being better able to exploit production opportunities declines. However, panel (B) shows that risk aversion and the Frisch elasticity are inversely related. At some point, risk aversion becomes so high than the welfare gain from completing the markets begins to rise as we reduce $\bar{\phi}$ because additional insurance becomes very valuable to shelter costly consumption fluctuations.

4.2.1 Robustness of the Welfare Calculations with Permanent Shocks

We now extend the model to incorporate persistent wage shocks by adding a unit root process to the uninsurable component of the individual wage. In order to ensure that cross-sectional wage dispersion remains bounded in the presence of permanent shocks to wages, we assume that households in the economy survive from one period to the next with constant probability $\delta < 1$. A new generation with unit mass enters the economy each period. Upon birth agents draw their initial realization of the permanent component α_0. At each successive age t they draw a transitory insurable shock ε_t and an innovation to the uninsurable unit root component of the wage, π_t. The shocks ε_t and π_t are assumed to be orthogonal and normally distributed. Thus the process for wages is

$$\log(w_t) = \alpha_t + \varepsilon_t$$

where

$$\begin{aligned}
\alpha_t &= \alpha_{t-1} + \pi_t, \quad t \geq 1 \\
\alpha_0 &\sim N\left(-\frac{v_0}{2}, v_0\right) \\
\varepsilon_t &\sim N\left(-\frac{v_\varepsilon}{2}, v_\varepsilon\right) \\
\pi_t &\sim N\left(-\frac{v_\pi}{2}, v_\pi\right).
\end{aligned}$$

Expected lifetime utility is now given by

$$E\sum_{t=1}^{\infty}\beta^{t-1}\delta^{t-1}u\left(c_t,h_t\right).$$

As in the simpler model, the unconditional cross-sectional wage variance is $v=v_\varepsilon+v_\alpha$, but the cross-sectional variance of the uninsurable component is now given by

$$v_\alpha=v_0+\frac{1}{1-\delta}v_\pi.$$

In the decentralized version of this economy households can in principle self-insure against permanent risk by bundling securities indexed to the transitory shocks to create non-contingent bonds. In Heathcote, Storesletten and Violante (2005) we prove that the equilibrium interest rate is such that the (negative) intertemporal saving motive exactly offsets the (positive) precautionary saving motive. Thus in equilibrium agents always hold zero wealth (as in Constantinides and Duffie, 1996; and Krebs, 2003).

With Cobb-Douglas preferences, the equilibrium allocations in the IM model are exactly as in (20). The expected welfare change for a newborn agent associated with changes Δv_0, Δv_π and Δv_ε in the variances of fixed effects, permanent innovations, and transitory shocks is

$$\omega^{IM}\simeq\frac{\bar{\phi}}{2}\Delta v_\varepsilon-\frac{\bar{\gamma}}{2}\left[\Delta v_0+\frac{1}{1-\beta\delta\exp\left(-\frac{1}{2}\bar{\gamma}\left(1-\bar{\gamma}\right)v_\pi\right)}\Delta v_\pi\right]. \tag{22}$$

This expression is qualitatively similar to the expression in Proposition 1, and the derivation follows exactly the same logic. The novel term involving the exponential is the "lifetime multiplier" on the (change in the) variance of the persistent shock, Δv_π. To understand this term, it is helpful to first set $\beta=1$ and consider the log-separable case where $\bar{\gamma}=1$. Then, the term in the square parenthesis becomes Δv_α and (22) exactly coincides with the welfare change formula without permanent shocks established in Proposition 1.

To understand the role of the risk aversion parameter $\bar{\gamma}$ in the multiplier, it is helpful to temporarily abstract from labor supply and from transitory shocks. In this case expected period utility at age t is given by $E\left[u(w_t)\right]=E\left[u\left(\exp(\alpha_t)\right)\right].$ When utility is logarithmic, $E\left[\log(\exp(\alpha_t))\right]=E\left[\alpha_t\right]$, which declines by $-\frac{v_\pi}{2}$ (the mean of the innovation π_t) between ages $t-1$ and t. If agents are more risk averse than log, then $E\left[u(w_t)\right]=E\left[\phi\left(\alpha_t\right)\right]$, where ϕ is a concave function. Thus when $\bar{\gamma}>1$, expected period utility falls with age at a faster rate. The fact that increasing risk aversion effectively magnifies the welfare effects of uninsurable wage risk at older ages explains why the lifetime multiplier on persistent shock is larger than $1/(1-\beta\delta)$.[13]

Apart from the logic of the cumulation of the welfare effects from permanent shocks, the economics behind this welfare calculation is essentially the same as in the simpler model. Thus, in what follows, we return to wage process with only fixed individual effects and transitory shocks.

5 Separable preferences

We now consider preferences that are separable between consumption and hours worked. Separability is a common assumption in the micro literature that estimates elasticities for consumption and labor supply (for a survey, see Browning, Hansen and Heckman, 1999). In this case

$$u(c,h) = \frac{c^{1-\gamma}}{1-\gamma} - \frac{h^{1+\sigma}}{1+\sigma}, \tag{23}$$

where $\gamma,\ \sigma \in [0,+\infty)$. The coefficient of relative risk aversion is simply γ, while the intertemporal elasticity of substitution for consumption is $1/\gamma$, as in the Cobb-Douglas case. The Frisch elasticity for labor supply is simply $1/\sigma$. In contrast with Cobb-Douglas preferences, note that separability allows for a lot of flexibility in distinguishing between agents' willingness to substitute consumption and hours intertemporally.[14]

5.1 Allocations with separable preferences [15]

Autarky (AUT)– When preferences are separable between consumption and hours worked, allocations are given by

$$\begin{aligned} c_{AUT}(\alpha,\varepsilon) &= \exp\left(\frac{1+\sigma}{\gamma+\sigma}(\alpha+\varepsilon)\right), \\ h_{AUT}(\alpha,\varepsilon) &= \exp\left(\frac{1-\gamma}{\gamma+\sigma}(\alpha+\varepsilon)\right). \end{aligned} \tag{24}$$

[13] The condition that guarantees that expected lifetime utility is bounded is precisely

$$\beta\delta < \exp\left(\bar{\gamma}(1-\bar{\gamma})v_\pi/2\right).$$

This condition also guarantees that welfare effects are always well defined.

[14] It is straightforward to generalize the analysis to a model with preferences displaying varying distaste for work relative to the taste for consumption, namely $u(c,h) = c^{1-\gamma}/(1-\gamma) - \psi \cdot h^{1+\sigma}/(1+\sigma)$, where ψ measures the strength of this distaste. As we discuss below, such a generalization has no impact on the subsequent welfare results, so we proceed with the simpler specification ($\psi = 1$).

[15] See Appendix A for derivations of the expressions for equilibrium allocations reported below.

As with Cobb-Douglas preferences, allocations depend only on the current period wage, $w(\alpha, \varepsilon) = \exp(\alpha + \varepsilon)$ and not on the two shocks separately. Whether hours increase or decrease with individual productivity depends on the relative strength of substitution versus income effects. With separable preferences, the income effect dominates the substitution effect if the consumption risk aversion parameter γ is larger than one.

Complete markets (CM)– Equilibrium allocations for consumption and hours worked in the complete markets economy are given by:

$$\begin{aligned} c_{CM}(\alpha, \varepsilon) &= \bar{c} = \exp\left(\frac{1+\sigma}{\gamma+\sigma}\left(\frac{v}{2\sigma}\right)\right), \\ h_{CM}(\alpha, \varepsilon) &= \exp\left(\frac{1+\sigma}{\gamma+\sigma}\left(\frac{-\gamma v}{2\sigma^2}\right)\right)\exp\left(\frac{\alpha+\varepsilon}{\sigma}\right). \end{aligned} \tag{25}$$

These allocations are easy to interpret. First, since utility is separable in consumption and hours worked, agents insure fully against fluctuations in consumption, so consumption is constant across states and over time. This consumption level is equal to average labor earnings. Hours worked are increasing in individual productivity and there is no distinction between permanent and transitory shocks in the labor supply decision, exactly as in autarky, but for the opposite reason: all shocks are equally insured. The Frisch elasticity $1/\sigma$ determines the responsiveness of individual hours to the individual wage.

Note that average consumption is increasing in the variance of wages, in contrast to the Cobb-Douglas preferences case. Expressions for asset purchases and prices are as in the Cobb-Douglas case.

Incomplete markets (IM)– When preferences are separable between consumption and hours worked, equilibrium allocations in the incomplete markets economy are given by:

$$\begin{aligned} c_{IM}(\alpha, \varepsilon) &= \exp\left(\frac{1+\sigma}{\gamma+\sigma}\left(\frac{v_\varepsilon}{2\sigma}\right)\right)\exp\left(\frac{1+\sigma}{\gamma+\sigma}\alpha\right), \\ h_{IM}(\alpha, \varepsilon) &= \exp\left(\frac{1+\sigma}{\gamma+\sigma}\left(\frac{-\gamma v_\varepsilon}{2\sigma^2}\right)\right)\exp\left(\frac{1-\gamma}{\gamma+\sigma}\alpha\right)\exp\left(\frac{\varepsilon}{\sigma}\right). \end{aligned} \tag{26}$$

These are closely related to the expressions for the autarkic and complete markets economies. Individual consumption is independent of the realization of the transitory shock, since that can be fully insured, as in complete markets, but is rescaled by the individual permanent effect α, as under autarky. Hours worked are increasing in the transitory shock ε, since these shocks have a substitution effect but no income effect given that they are perfectly insured. Permanent shocks do have an income effect, and hours increase with α if and only if $\gamma < 1$.

5.2 Welfare with separable preferences

We are now ready to state a pair of Propositions equivalent to Propositions 1 and 2.

Proposition 1a: *With separable preferences, the (approximate) welfare change from a rise in labor market risk equal to Δv, where $\Delta v = \Delta v_\varepsilon + \Delta v_\alpha$, is given by the following expressions under the three market structures we consider:*

$$\omega_{AUT} \simeq \left[\frac{1-\gamma}{\sigma+\gamma} - \gamma\frac{1+\sigma}{\sigma+\gamma}\right]\frac{\Delta v}{2} = \underbrace{\left(\frac{1-\gamma}{\gamma+\sigma}\right)\Delta v}_{\omega_{AUT}^{lev}} \underbrace{-\left[\frac{1-\gamma}{\gamma+\sigma} + \gamma\left(\frac{1+\sigma}{\sigma+\gamma}\right)\right]\frac{\Delta v}{2}}_{\omega_{AUT}^{vol}},$$

$$\omega_{CM} \simeq \frac{1}{\sigma}\frac{\Delta v}{2} = \underbrace{\frac{1}{\sigma}\Delta v}_{\omega_{CM}^{lev}} \underbrace{-\frac{1}{\sigma}\Delta v/2}_{\omega_{CM}^{vol}},$$

$$\begin{aligned}\omega_{IM} &\simeq \frac{1}{\sigma}\frac{\Delta v_\varepsilon}{2} + \left[\frac{1-\gamma}{\sigma+\gamma} - \gamma\left(\frac{1+\sigma}{\sigma+\gamma}\right)\right]\frac{\Delta v_\alpha}{2} \\ &= \underbrace{\left(\frac{1-\gamma}{\gamma+\sigma}\right)\Delta v_\alpha + \frac{1}{\sigma}\Delta v_\varepsilon}_{\omega_{IM}^{lev}} \underbrace{-\left[\frac{1-\gamma}{\gamma+\sigma}\frac{\Delta v_\alpha}{2} + \gamma\left(\frac{1+\sigma}{\sigma+\gamma}\right)\right]\frac{\Delta v_\alpha}{2} - \frac{1}{\sigma}\frac{\Delta v_\varepsilon}{2}}_{\omega_{IM}^{vol}}.\end{aligned}$$

Proof: *See Appendix C.*

Autarky– Note first that as $\sigma \to \infty$, the welfare cost of rising productivity fluctuations in autarky becomes

$$\omega_{AUT} \simeq -\frac{\gamma}{2}\Delta v,$$

the expression computed by Lucas (1987) for the welfare costs of aggregate consumption fluctuations in an economy with inelastic labor supply. Note also that $\partial\omega_{AUT}/\partial\sigma < 0$. Thus, for a given coefficient for risk aversion, introducing flexible labor supply always (weakly) reduces the welfare cost of uninsurable wage fluctuations, in contrast to the Cobb-Douglas case above. Precisely how labor supply effectively substitutes for the presence of missing insurance markets depends on the value for γ. When $\gamma > 1$, the income effect from a positive wage shock dominates the substitution effect, so agents increase work effort in bad times. In this case, flexible labor supply is used to improve consumption smoothing at the expense of productivity (the level effect ω_{AUT}^{lev} is negative). When $\gamma < 1$, the substitution effect dominates the income effect, and agents increase work effort in good times. In this case, flexible labor supply actually increases consumption volatility, but it is still beneficial because agents are relatively unconcerned about fluctuations in consumption, and concentrating work effort in high wage periods raises average output per hour (the level effect ω_{AUT}^{lev} is positive). There is only one (knife-edge) case when flexibility fails to mitigate the welfare cost of additional wage risk, namely when $\gamma = 1$ and labor supply is constant across all agents (see equation (24)).

Perhaps the most surprising result in Proposition 1a is that, when risk aversion is sufficiently small and labor supply elasticity sufficiently high, a rise in v has a positive welfare effect (see panel (B) in Figure 3) even in autarky.[16] For low levels of risk-aversion, $\gamma < 1/(2+\sigma)$, agents willingly substitute labor supply intertemporally to raise average productivity, and are relatively unconcerned about the resulting fluctuations in consumption. The complete-markets result (discussed below) sheds some further light on this. When $\gamma = 0$ (the risk-neutrality case), it is easy to see that $\omega_{AUT} = \omega_{CM} > 0$.

Complete Markets– As with Cobb-Douglas preferences, under complete markets, increasing productivity dispersion strictly increases welfare, as long as the labor supply elasticity is positive (σ finite). Once again, the intuition is simply that an unconstrained planner can achieve better allocative efficiency with larger dispersion by having more productive agents specialize in market work.

Panel (A) in Figure 3 plots ω_{CM} as a function of σ for Δv normalized to one. The larger the Frisch elasticity, the greater the opportunities for exploiting the heterogeneity in labor productivity, and the larger the welfare gains from increased wage dispersion. As an example, consider the case with unit elasticity of labor supply ($\sigma = 1$). Then an increase in wage dispersion translates into a rise in welfare of half its size. Note that with inflexible labor supply ($\sigma \to \infty$), rising wage inequality has no welfare implications since hours worked is the same for all agents.

Incomplete Markets– The welfare loss under incomplete markets is a convex combination of the loss under the other two market structures. In light of our discussion of the autarky case, we conclude that for flexible labor supply to mitigate the welfare cost of increases in *uninsurable* wage risk under incomplete markets, it must be the case that $\gamma \neq 1$,

[16] Recall that in the Cobb-Douglas case, increases in wage dispersion in autarky are always welfare-reducing.

implying that preferences are inconsistent with balanced growth.

We now revisit the welfare gains associated with expanding the set of insurance assets that may be traded.

Proposition 2a: *With separable preferences, the (approximate) welfare gains from completing the markets in an economy with uninsurable labor market risk equal to v_α are:*

$$\chi_{IM \to CM} \simeq \left[\frac{1}{\sigma} + \frac{\gamma - 1}{\sigma + \gamma} + \gamma \left(\frac{1 + \sigma}{\sigma + \gamma} \right) \right] \frac{v_\alpha}{2}.$$

As in the Cobb-Douglas case, there are two sources of welfare gains from completing markets. The first is associated to the allocative efficiency gain associated with elastic labor supply: more productive households work relatively harder and less productive households enjoy more leisure. The second is the gain from the additional insurance provided by increased risk sharing.

Figure 4 plots χ for different values of σ and γ in their admissible range $(0, \infty)$ and for v normalized to 1. Notice first that the welfare gain of completing the markets is always weakly positive and strictly increasing in γ, the degree of risk-aversion. A few benchmarks are of interest. First, for $\gamma = 0$ (risk-neutrality), the welfare gain is exactly zero, since consumption fluctuations are not costly for individuals. Second, in the absence of flexible labor supply ($\sigma \to \infty$), the welfare gain is $\chi \simeq \gamma/2 \cdot v_\alpha$. Intuitively, inflexible labor supply implies that labor productivity fluctuations translate one for one into consumption fluctuations. Third, when $\gamma = 1$ and $\sigma = 1$, $\chi \simeq v_\alpha$, so the welfare gain from completing markets exactly equals the variance of uninsurable components of wages.

Other things equal, greater flexibility in adjusting hours must always be welfare-improving. But is additional labor supply flexibility more useful when markets exist to pool wage risk or when they do not? On the one hand, a higher Frisch elasticity increases the value of the gain via specialization in labor supply that can be achieved when more contingent claims are traded. On the other a high elasticity reduces the (positive) value of increasing explicit insurance through financial markets, since agents can effectively adjust hours to self-insure against 'uninsurable' shocks. It turns out that which effect dominates depends on the particular combination of γ and σ.

For large values of the labor supply elasticity ($1/\sigma > 1$), the specialization effect dominates, and χ is increasing in $1/\sigma$. For $1/\sigma \leq 1$, whether or not χ is increasing in $1/\sigma$ depends on whether $\gamma \leq 2\sigma/(\sigma-1)$. For large values of risk-aversion γ, there is always a region where agents are relatively unwilling to adjust hours inter-temporally in which χ becomes smaller as the labor supply elasticity rises. The intuition is that given high aversion to consumption fluctuations, an increase in the willingness to substitute hours intertemporally might have a large positive impact on welfare under autarky by effectively improving self-insurance, thereby reducing the gain from expanding insurance markets (recall that when $\gamma > 1$, the income effect dominates the substitution effect in autarky and low-productivity households work harder). By contrast, for $\gamma \leq 2$, χ is always increasing in $1/\sigma$.[17]

Finally, note that all the approximated welfare expressions in Propositions 1a and 2a, as well as the exact ones in the Appendix, would remain unchanged even when generalizing preferences to allow for varying taste for leisure, as discussed in footnote 14.[18] This implies that our results would be robust to an important class of preference heterogeneity.

6 Measuring welfare changes using cross-sectional data

In this section we show that it is possible to provide an alternative representation of welfare changes and their two components (level and volatility effects) as functions only of preference parameters and second moments of the joint cross-sectional distribution of wages, hours and consumption. This representation is particularly useful for quantifying the welfare effects of rising inequality, which is the focus of section 7.

Proposition 3: *The (approximate) welfare change ω of a rise in wage dispersion in the Cobb-Douglas case can be expressed as:*

$$\omega \simeq \underbrace{\Delta cov(\log w, \log h)}_{\omega^{lev}} \underbrace{-\frac{\bar{\gamma}}{2}\Delta var(\log c) - \frac{1}{2\bar{\phi}}\Delta var(\log h) + \frac{\bar{\gamma}-1}{2}\Delta cov(\log c, \log h)}_{\omega^{vol}},$$

and, in the separable preferences case, as

$$\omega \simeq \underbrace{\Delta cov(\log w, \log h)}_{\omega^{lev}} \underbrace{-\frac{\gamma}{2}\Delta var(\log c) - \frac{\sigma}{2}\Delta var(\log h)}_{\omega^{vol}}.$$

[17] The condition under which the welfare costs of completing the markets is increasing in the Frisch elasticity has an intuitive interpretation. In particular, when $1/\sigma \leq 1$, the variance of log-hours worked is larger in complete markets relative to autarky if and only if $\gamma \leq 2\sigma/(\sigma-1)$, the same condition we derived above under which χ is increasing in the Frisch elasticity.

[18] The taste for leisure, ψ, would enter the allocations in (24)-(26) multiplicatively, but would completely drop out of the welfare expressions.

These expressions are the same for all market structures $m \in \{AUT, CM, IM\}$.

Proof: *See Appendix D.*

The key step in the proof involves using our analytical solutions for the equilibrium allocations to infer a relationship between observable cross-sectional moments and the variances of the uninsurable and insurable shocks $(\Delta v_\alpha, \Delta v_\varepsilon)$. This relationship allows us to translate the welfare expressions of Propositions 1 and 1a into the observables-based expressions above. In recognizing that theory implies a tight link between observable measures of dispersion and the fraction of underlying risk that is insurable we build on the work by Blundell and Preston (1998) and Blundell, Pistaferri and Preston (2004).[19]

Consider first the separable case, which is somewhat simpler. The welfare change can always be written as a sum of three terms. The first term is the change in the covariance between hours and wages: a better assignment of individual hours to individual productivities improves the level of aggregate welfare. The second and third terms capture the volatility cost of a rise in wage dispersion: the increase in the variance of log consumption translates into a welfare cost proportional to the risk-aversion coefficient and the increase in the variance of log hours translates into a welfare cost that is inversely proportional to the Frisch elasticity, $1/\sigma$. This representation holds for every market structure. Of course, in complete markets $var_{CM}(\log c) = 0$.

The first three terms of the welfare expression for the Cobb-Douglas case are exactly as in the separable case, where $\bar{\gamma}$ and $\bar{\phi}$ are the analogues of γ and $1/\sigma$, respectively. The Cobb-Douglas preference specification is non-separable in consumption and leisure, so, it should not be surprising that the welfare change contains an extra term proportional to the change in the covariance between consumption and hours worked. When $\theta > 1$ (which implies $\bar{\gamma} > 1$), consumption and leisure are substitutes, thus households gain from a rise in the comovement between consumption and hours in the wake of an increase in wage dispersion.

We conclude this section by stating another interesting mapping between our welfare components and observable variables. From Proposition 3 the level effect ω^{lev} can be expressed as the change in the covariance between hours and wages, irrespectively of market structure and preferences. However, it turns out that ω^{lev} is also equal to the change in

[19]One attractive feature of our market structure is that the mapping between the cross-sectional moments and the parameters defining the wage process can be described in closed form and does not require any approximations.

aggregate labor productivity in the economy. In other words, the source of potential welfare gains from an increase in wage dispersion is a rise in aggregate productivity. Recall that in the separable preferences case, this productivity gain takes place through a rise in aggregate production (consumption), while in the Cobb-Douglas case, average consumption is constant, but average hours worked fall.

Proposition 4: *Under both preference specifications, and in each market structure m, the level effect of a rise in wage dispersion ω^{lev} also (approximately) equals the percentage change in aggregate labor productivity.*

Proof: *See Appendix E.*

7 Quantitative welfare analysis

As documented in a vast empirical literature (see Katz and Autor, 1999; Eckstein and Nagypal 2004, for surveys), cross-sectional wage inequality has increased substantially in the United States since the early 1970s. Our framework allows us to quantify 1) the welfare change due to this rise in wage dispersion, and 2) how much U.S. households would benefit, at the current level of labor market risk, from the availability of a full set of insurance markets. Moreover, one can separately quantify the "volatility" component $\left(\omega^{vol}\right)$ from the level effect related to the "opportunities" offered by productivity dispersion in the presence of flexible labor supply $\left(\omega^{lev}\right)$.

7.1 Calibration

The first step in the calibration is to choose values for the preference parameters. Next, we discuss the measurement of the insurable and uninsurable components.

Preference parameters: The welfare expressions for separable preferences discussed in section 5 depend only on two parameters (γ, σ). Estimates for the risk-aversion coefficient γ (or, identically, for the inverse of the intertemporal labor supply elasticity) between one and three are typical in the empirical consumption literature (see Attanasio 1999, for a survey), so we set $\gamma = 2$. Domeij and Flodén (2002) sample the empirical literature on male labor supply and conclude that the typical estimates of uncompensated wage elasticities for male labor supply range between 0.1 and 0.3. However, they argue that these estimates are typically downward biased because the standard estimation methods ignore the possibility that borrowing constraints may bind. By simulation, they show that the unbiased estimates can be up to twice as large. Moreover, estimates of this elasticity for females are, in general, 3-4 times as large as those for men (see Blundell and MaCurdy, 1999, Table 2). We therefore set the Frisch elasticity to 0.5, corresponding to a value of $\sigma = 2$.

With Cobb-Douglas preferences, the Frisch labor supply elasticity and the coefficient of risk-aversion are not independent, as they are both function of the pair of parameters (θ, η), as discussed in section 4.2. Moreover, the parameter η has a natural counterpart in the fraction of the time endowment devoted to work activities. Following the macroeconomic literature on business cycles, we set $\eta = 1/3$ (see e.g. Cooley, 1995).[20] We then set $\theta = 4$ so the implied coefficient of risk-aversion $\bar{\gamma}$ equals two, as in the separable case. As a by-product, we obtain a Frisch elasticity $\bar{\phi}$ exactly equal to one – a higher number than under the separable case, but perhaps not too implausible in light of our earlier comment on the higher elasticity for female labor supply.[21] The fact that with Cobb-Douglas preferences it is not possible to simultaneously allow for a realistic average fraction of time spent working and a realistic (low) elasticity for labor supply is a serious drawback for quantitative work.

We recognize that there is disagreement regarding appropriate values for preference parameters, and that some may object to our particular choices. One advantage of our intuitive closed-form expressions for welfare is that one can quickly compute different estimates based on alternative parameterizations. Figures 1-4 document the sensitivity of our results to a wide range of alternative parameter values.

Measurement of wage dispersion: The next step of the calibration requires quantifying processes for insurable and uninsurable risk and how they have changed. As discussed in Section 2, we assume that insurable and uninsurable risk are given by the variances of the permanent and transitory components before and after the surge in wage dispersion $(v_\alpha, v_\varepsilon)$

[20]More precisely, from the first-order condition for individual i hours worked in the non-stochastic version of the model, we obtain:

$$\frac{1-h_i}{h_i} = \frac{1-\eta}{\eta}.$$

[21]Note that under Cobb-Douglas preferences, the Frisch labor supply elasticity is bounded below by $(1-\eta) = 0.67$. In particular:

$$\lim_{\theta\to\infty} \bar{\phi} = \left(\frac{1-\eta+\eta\theta}{\theta}\right)\left(\frac{1-\eta}{\eta}\right) = 1-\eta.$$

This is why we chose to equate the coefficient of risk aversion across alternative preference specifications, rather than the Frisch elasticity for labor supply.

and $(\widehat{v}_\alpha, \widehat{v}_\varepsilon)$, respectively. From the 1968-1997 waves of the *Panel Study of Income Dynamics* (PSID), we have selected a sample of roughly 2,400 observations/year including every head of household (males and females) aged between 20 and 59 with positive earnings (not top-coded and not below half of the current minimum wage). We computed hourly wages as annual earnings divided by annual hours worked and found that the variance of log wages rose from 0.25 to 0.35 over this time period. Next, we estimated a simple permanent/transitory model for the variance of log wages, exactly the process specified in equation (1). The estimated variance of the transitory/insurable component v_ε starts around 0.08 and levels off 30 years later at around 0.13. The variance of the permanent/uninsurable component v_α starts at a value around 0.17 and rises to 0.22 in the mid 1990s.[22]

In light of these results, we set $\Delta v_\varepsilon = \Delta v_\alpha = 0.05$ when evaluating the welfare implications of rising dispersion. Moreover, focusing on the levels of labor market uncertainty of the 1990s, we set $v_\alpha = 0.22$ and $v_\varepsilon = 0.13$.[23]

7.2 Results

We summarize our results in Table 1. To gauge the quality of our approximations relative to the exact welfare expressions contained in the Appendix, we also report, in parentheses, the values implied by the approximated welfare expressions described in Propositions 1, 1a, 2, and 2a. Below the total welfare change, we report the decomposition of our approximated welfare change into volatility and level effects.

[22] Our findings can be summarized as follows: (i) the transitory component accounts for roughly 1/3 of the total dispersion; (ii) the rise in wage dispersion is accounted equally by the two components. These results are in line with the existing literature. See, among others, Gottschalk and Moffitt (1994), Katz and Autor (1999), Heathcote, Storesletten and Violante (2004). The latter paper also contains a more detailed discussion of the data.

[23] By using the statistical wage decomposition into permanent and transitory shocks estimated on U.S. data, we implicitly assume that the individual productivity process is exogenous and independent of the market structure. An alternative view is that the depth of financial markets might affect human capital accumulation decisions and hence influence the nature of the productivity shock process. For simplicity, we have abstracted from such interaction here (see Huggett, Yaron and Ventura, 2004, for recent progress in that direction).

Table 1: Welfare Changes (% of lifetime consumption)

Welfare change of rise in wage dispersion						Welfare gain from completing markets	
Cobb-Douglas Preferences							
ω_{CM}		ω_{AUT}		ω_{IM}		$\chi_{IM \to CM}$	
+5.13% (+5.00%)		-9.52% (-10.00%)		-2.47% (-2.50%)		+39.1% (+33.0%)	
Volat.	Level	Volat.	Level	Volat.	Level	Volat.	Level
-5.00%	+10.00%	-10.00%	0	-7.50%	+5.00%	+11.0%	+22.0%
Separable Preferences							
ω_{CM}		ω_{AUT}		ω_{IM}		$\chi_{IM \to CM}$	
+2.54% (+2.50%)		-8.29% (-8.75%)		-3.06% (-3.13%)		+29.2% (+24.8%)	
Volat.	Level	Volat.	Level	Volat.	Level	Volat.	Level
-2.50%	5.00%	-6.25%	-2.50%	-4.38%	+1.25%	+8.3%	+16.5%

Welfare changes of rising dispersion: We begin from the extreme market structures: the estimated welfare changes in complete markets and autarky are lower and upper bounds for the true welfare loss. In the complete-markets economy there is a sizeable increase in welfare when wage dispersion increases. The welfare gain is larger under the Cobb-Douglas specification (5% of lifetime consumption) due to the higher labor supply elasticity relative to the separable case. Note that the volatility effect is negative, even under complete markets, due to the increase in the dispersion of hours. However, the positive level effect always dominates.

The welfare loss in autarky is approximately 10%. The welfare loss in autarky is slightly smaller with separable preferences, just as the welfare gain in complete markets is smaller with the separable specification. Recall that, with Cobb-Douglas preferences, hours worked are constant in autarky, whereas adjustments in hours worked are used as a vehicle of self-insurance when preferences are separable (provided $\gamma > 1$), which reduces the welfare cost of additional wage risk.

Putting together these first two results, and recalling that ω_{IM} is a weighted average of ω_{CM} and ω_{AUT}, it is not surprising that the welfare losses of the observed rise in wage dispersion for the incomplete markets economy are quite similar across the two cases, between 2.5% and 3% of lifetime consumption. With Cobb-Douglas preferences, the welfare loss due to the volatility component is 7.5% of lifetime consumption, and the welfare gain due to the better productive opportunities (the aggregate labor productivity gain) is 5%. With separable preferences both components are smaller in absolute value.

Welfare gains from completing the markets: With Cobb-Douglas preferences, a household living in autarky values the availability of a complete set of insurance markets against her labor market risk at 69% of her lifetime consumption. Starting from an incomplete-markets economy, this number drops to 39% of lifetime consumption. This is the value of being able to completely insure the permanent wage component. With separable preferences, these estimates are smaller, respectively 52% and 29%. The striking feature of these results is that, in all cases, the gains associated with better productive opportunities in complete markets are twice as big as the gains from reduced dispersion. Recall that, since $\gamma > 1$, households with low permanent (uninsurable) wage components work fewer hours than those with high permanent components.[24] Efficiency dictates positive assortative matching between wages and hours. Our calculations indicate that the aggregate productivity loss due to this inefficient assignment is substantial, around 20%.

Relation to estimates obtained within numerically-solved incomplete-markets models: As a first point of comparison, we consider an economy identical to our benchmark incomplete-markets model, except that instead of having access to a complete set of state-contingent claims providing perfect insurance against transitory wage shocks, agents trade only a non-contingent bond (e.g, Bewley, 1983; Imrohoroglu, 1989; Huggett, 1993; Aiyagari, 1994; Rios-Rull, 1994). At the aggregate level bonds are in zero net supply. In this economy, the welfare change associated with an increase in wage dispersion will depend on how much borrowing is allowed, and on the discount factor β.[25] We set the borrowing constraint close to the "natural" limit (see, for example, Aiyagari, 1994) which ensures that interest payments never exceed earnings given maximum labor effort. We adopt the Cobb-Douglas preference specification, and set $\beta = 0.97$, which implies a final steady state interest rate of 3.05%.[26] We compute expected welfare changes for individuals with mean wealth (zero) drawn at random from the unconditional wage distribution. Given our end-of-sample estimates for wage dispersion, the welfare gain for such agents from being dropped at random into the

[24]With Cobb-Douglas preferences, hours are independent of the uninsurable component of the wage.

[25]Using an endowment economy, Levine and Zame (2002) show that when agents have infinite horizons and CRRA preferences, they achieve arbitrarily good insurance against non-permanent shocks when trading only a non-contingent bond, as the discount rate goes to zero.

[26]To solve this model numerically, we assume that both the uninsurable component of the wage α and the insurable component ε are drawn from symmetric two-point distributions. Given this two-point distribution, the welfare change from increased wage dispersion in our benchmark incomplete-markets economy is -2.37% compared to the -2.47% loss reported in Table 1 for the continuous Normal distribution. More details on the numerical implementation are available upon request.

incomplete markets economy is equivalent to a permanent 0.88% increase in consumption. The expected welfare change in the Bewley economy associated with a rise in wage dispersion is a 2.77% loss, compared to a 2.37% loss with incomplete-markets. Increases in wage risk are slightly more costly in the model with a single bond in part because greater wage risk reduces debt-repayment ability when wages are low, and thus tightens the borrowing constraint. Nonetheless, the two models deliver remarkably similar answers to our main welfare question.

In previous work (Heathcote, Storesletten and Violante, 2004), we computed the welfare effects from the rise in U.S. wage dispersion in an overlapping-generations incomplete-markets production economy. That model incorporated various features from which we abstract in the incomplete-markets developed in this paper, including a hump-shaped age profile for labor productivity, persistent wage shocks, mortality risk, unemployment risk, borrowing constraints and a public pension system. We computed a welfare loss of 2.5% when we averaged across all cohorts and weighted them equally. Thus our simple estimates in Table 1 are extremely close to the outcome of this computational experiment in a much richer model.

Other authors have used numerical methods to compute the welfare costs of market incompleteness. In an infinitely-lived-agent version of the Bewley-Aiyagari-Huggett economy Kubler and Schmedders (2001) estimate it at 6.5% in an endowment economy. Pijoan-Mas (2004) calculates it to be on the order of 16% of lifetime consumption in a production economy with flexible labor supply. Since fixed effects implicitly remain uninsured under both interpretations of what it means for markets to be complete, the right comparison in our setup would be moving from autarky to incomplete markets, i.e. providing insurance against all risk except the fixed-effects. From our welfare expressions, it is easy to see that in the separable preferences specification, setting $\gamma = 1.5$ and $\sigma = +\infty$ (the Kubler-Schmedders parameterization) we obtain $\omega_{AUT \to IM} = 7.5\%$ and setting $\gamma = 0.98$ and $\sigma = 0.61$ (the parametrization used by Pijoan-Mas), we arrive at $\omega_{AUT \to IM} = 18.5\%$.

Once again, the welfare estimates are remarkably similar. We conclude that our simpler and more transparent framework can shed additional light on work within the Bewley framework that currently predominates in the study of distributional issues.

7.2.1 Estimating welfare changes using cross-sectional moments

Section 6 of the paper offered an alternative strategy for computing welfare effects, based on preference parameters and observable cross-sectional moments of the joint wage, hours and consumption distribution. The main advantage of this representation relative to the model-based representation used in Table 1 is that it does not require taking a stand on the statistical process for wages, i.e. one need not specify a mapping from the degree of statistical persistence of the shocks to whether or not the shocks are insurable.[27] We therefore also report welfare estimates using the observables-based welfare expressions.

Measurement of cross-sectional moments: From the same PSID data discussed above, we measured an increase in the cross-sectional variance of log-hours worked of 0.01 (from 0.08 to 0.09), and an increase in the covariance between hours and wages of roughly 0.012 (from -0.021 to -0.009). According to Slesnick (2001), the rise in the variance of log-consumption over the past 20 years has been small, around 0.01 (from 0.20 in 1980 to 0.21 in 1995). Krueger and Perri (2005) and Attanasio, Battistin and Ichimura (2003) argue that consumption inequality rose by about 0.05. Since there are important measurement issues that are far from being settled in this literature, we simply adopt a mid-point estimate of 0.03 for our calculations. Finally, Krueger and Perri (2003) report that the covariance between hours and consumption declined by 0.007 (from 0.037 to 0.030) over the period of interest.[28]

Results: Since it is reasonable to think of the actual economy (the source of the above cross-sectional moments) as the incomplete markets model, we only compute the welfare change due to a rise in dispersion for this economy. For the separable preferences case (assuming $\gamma = \sigma = 2$), one can easily plug in the changes in the variances of hours, consumption, and in the covariance between hours and wages to obtain $\omega^{IM} = -2.05\%$. A similar computation for the Cobb-Douglas case (for which we also need the change in the covariance between hours and consumption) yields $\omega^{IM} = -2.65\%$. Both values are quite close

[27] At the same time, the model-based decomposition used in Table 1 has some important advantages over its observables-based counterpart. First, in cross-sectional data, wages are measured better than consumption (see Attanasio, Battistin, and Ichimura (2004) for a recent discussion of measurement error in consumption in the CEX). Moreover, insofar as one is interested specifically in the effects of changes in wage dispersion on welfare, directly using data on consumption and hours inequality might be misleading because the observed changes in these distributions over a period of time can be the result of a mix of factors –not exclusively of changes in wage dispersion.

[28] The levels of these variances and covariances are potentially affected by measurement error. As long as the measurement error 1) is multiplicative in levels, 2) is orthogonal to the true value, and 3) its variance is constant over the period, then the *changes* in these measured cross-sectional moments will not be affected.

to the welfare estimates of Table 1, an encouraging result, given the diametrically different approaches.[29]

From Propositions 3 and 4 it follows that the degree to which a society is able to allocate labor efficiently – labor productivity – has the simple empirical representation $cov(\log h, \log w)$, irrespectively of preferences or market structure. In our PSID sample, labor productivity, measured as the ratio of aggregate earnings divided by aggregate hours, increased by 13% from 1975 to 1995. Thus, better allocative matching of workers and hours due to increased inequality can alone account for almost a tenth of the increase in aggregate labor productivity over this period.

7.3 Insuring risk versus eliminating risk

In computing the welfare cost of business cycles, Lucas (1987) compared welfare associated with the actual U.S. time series for aggregate consumption to welfare associated with the trend for the actual path.[30] Thus he calculated the hypothetical welfare gain from eliminating aggregate risk. We now estimate the welfare gains from eliminating idiosyncratic risk by making the same actual to trend comparison as Lucas, but at the individual rather than the aggregate level. Thus we set every individual's wage at every date equal to its unconditional expected value. An appropriate wage compression or redistribution policy can in principle achieve this outcome. In particular, wage risk can be eliminated in our model by a system of wage taxes and subsidies that guarantees each worker an after-tax hourly wage rate equal to average labor productivity, which equals one. Thus, the tax (subsidy) rate paid by a worker with current pre-tax wage w is given by $\tau(w) = 1 - 1/w$.[31]

In Propositions 2 and 2a we characterized the welfare gains from insuring individual risk. In a model with exogenous labor supply, there would be no difference between insuring and

[29] Krueger and Perri (2003) propose to evaluate welfare effects using empirical data only, and it is natural to compare their findings to our observable-based expressions. However, when they construct their data, they abstract from the level effect. They assume Cobb-Douglas preferences and $\bar{\phi} = \bar{\gamma} = 4/3$. Given $\Delta var(\log h) = 0.01$, their measurement of $\Delta cov(\log h, \log c) = -0.007$ and $\Delta var(\log c) = 0.01$, the volatility effect would be $\omega^{vol} = -1.2\%$, which is close to their estimated welfare loss of -1.6%.

[30] More recently, Storesletten et. al. (2001), Krusell and Smith (1999), and Krebs (2003) have made similar calculations in models with heterogeneous agents. See Lucas (2003) for a survey.

[31] To verify that this system of wage taxes and subsidies is feasible we need to check that it is revenue-neutral. Since every agent faces the same after-tax wage, each agent works the same number of hours per period and enjoys the same level of consumption. Per-capita consumption will equal per-capita after-tax income, which in turn is (constant) hours times the after-tax wage, which is equal to one. Since average labor productivity is also equal to one, output per-capita will equal consumption per-capita. It follows immediately that the tax-subsidy scheme is revenue-neutral. Note that this tax system requires the observability of total individual productivity, but not of the two separate components.

eliminating idiosyncratic labor income risk. Both changes would lead to income and consumption being equalized across individuals, with no changes in aggregate quantities. With endogenous labor supply, increasing risk-sharing is not the same thing as reducing risk. Eliminating idiosyncratic risk will still remove cross-sectional heterogeneity in consumption and hours worked. However, as we have emphasized repeatedly, the complete markets allocation induces a positive correlation between wages and hours worked, which is efficient because it raises average labor productivity.

In the context of our model, eliminating wage risk amounts to reducing to zero the variances of both components of the wage process. Thus the welfare effects can be read directly from the expressions for ω_{IM} in Propositions 1 and 1a by setting $\Delta v_\varepsilon = -v_\varepsilon$ and $\Delta v_\alpha = -v_\alpha$.

Proposition 5: *With Cobb-Douglas preferences, the (approximate) welfare change from eliminating labor market risk is given by:*

$$\kappa_{IM} \simeq -\bar{\phi}\frac{v_\varepsilon}{2} + \bar{\gamma}\frac{v_\alpha}{2} = \underbrace{-\bar{\phi}v_\varepsilon}_{\kappa_{IM}^{lev}} \underbrace{+\bar{\phi}\frac{v_\varepsilon}{2} + \bar{\gamma}\frac{v_\alpha}{2}}_{\kappa_{IM}^{vol}} \tag{27}$$

With separable preferences, the corresponding welfare change is:

$$\kappa_{IM} \simeq -\frac{1}{\sigma}\frac{v_\varepsilon}{2} - \left[\frac{1-\gamma}{\sigma+\gamma} - \gamma\left(\frac{1+\sigma}{\sigma+\gamma}\right)\right]\frac{v_\alpha}{2}.$$

It is instructive to compare quantitatively the welfare gains from eliminating labor market risk to those from insuring risk reported in Table 1. Under the parameterization of the previous subsection, in the Cobb-Douglas case we obtain $\kappa_{IM} = 0.155$, compared to $\chi_{IM\to CM} = 0.33$. Thus, eliminating risk implies a welfare gain equivalent to roughly half of the welfare gain associated with completing markets. The corresponding numbers for the separable preferences case are $\kappa_{IM} = 0.16$ and $\chi_{IM\to CM} = 0.25$.

Why are the welfare gains from eliminating risk so much smaller than those from insuring risk? Eliminating the uninsured part of wage dispersion is welfare-improving, since this reduces consumption and leisure dispersion. However, eliminating the volatility in the insurable component of wages is detrimental since it breaks the positive assortative matching between productivity and hours that characterizes the efficient allocation. In a similar vein, Cho and Cooley (2001) noted that if aggregate hours are pro-cyclical, then eliminating aggregate business cycle risk will reduce average labor productivity.

The fact that eliminating the insurable component of wage risk is welfare-reducing leaves open the theoretical possibility that the welfare change from eliminating all idiosyncratic wage risk through some redistributive policy might be negative. This is not the case, however, in our calibration to the United States, because the empirical wage-hours covariance is rather small, while the volatility effect is large given plausible choices for the preference parameters.[32]

8 Concluding remarks

The main contribution of the paper is the analytical characterization of the welfare effects from an increase in the dispersion of labor productivity. The analysis is conducted within a tractable class of dynamic heterogeneous-agents economies with market structures ranging from autarky to complete markets. We applied our framework to the recent observed surge in cross-sectional wage dispersion in the United States and quantified the welfare loss associated with higher labor market uncertainty.

For both Cobb-Douglas and separable preferences, welfare changes have a representation in terms of observable second moments (variances and covariances) of the joint equilibrium distribution of wages, hours worked and consumption. Further work should investigate the generality of this representation across other families of preferences.

We emphasized the distinction between insuring risk and eliminating risk through redistributive policies (i.e. market-provided insurance vs. social insurance). Overall, eliminating idiosyncratic wage risk implies a welfare gain that is several orders of magnitude larger than most estimates of the welfare gains from eliminating aggregate business cycle risk. However, the welfare gains from eliminating wage risk are only around half as large as the gains that would accrue from perfectly insuring wage risk. From a policy perspective, the immediate implication is that the government should prioritize developing the legal and institutional frameworks that will allow new insurance markets to develop. Sargent (2001) and Shiller (2003) discuss a range of proposals along these lines.[33]

[32] In Heathcote, Storesletten and Violante (2004) we find the empirical correlation between hours and wages (after correcting for measurement error in hours) in the PSID to be close to zero. We argue that this is finding is consistent with a large fraction of wage risk being uninsurable, and with preferences that exhibit strong income effects in labor supply, so that hours respond negatively to positive uninsurable shocks to wages.

[33] For example, Shiller's six proposals are labeled "livelihood insurance", "home equity insurance", "macro markets", "income-linked loans", "inequality insurance" and "intergenerational social security".

Informational frictions and imperfect commitment may limit the amount of insurance that can ultimately be provided either through explicit financial markets, or through social insurance policies designed to compress the wage distribution. For example, Krueger and Perri (2005) study a calibrated endowment economy in which debt contracts can be only imperfectly enforced. They show that a rise in income dispersion might increase welfare because it reduces the value of default and thereby increases the set of insurance contracts that can be supported in equilibrium. The implications of rises in labor market risk in a private information environment have not yet been addressed. Looking further ahead, an important challenge in introducing these sorts of frictions is to do so in a way that maintains tractability, thereby allowing for a transparent characterization of the various mechanisms at work.

References

[1] Acemoglu, D. (2002); "Technical Change, Inequality and the Labor Market," *Journal of Economic Literature* 40 (1), 7-72.

[2] Aghion, P. (2002); "Schumpeterian Growth Theory and the Dynamics of Income Inequality," *Econometrica* 70 (3), 855-82.

[3] Aiyagari, R. (1994); "Uninsured Idiosyncratic Risk and Aggregate Savings," *Quarterly Journal of Economics* 109 (3), 659-684.

[4] Attanasio, O. (1999); "Consumption," Handbook of Macroeconomics Volume 1B, Chapter 11, Amsterdam: Elsevier Science

[5] Attanasio, O. and S. Davis (1996); "Relative Wage Movements and the Distribution of Consumption," *Journal of Political Economy* 104(6), 1227-62.

[6] Benabou, R. (2002); "Tax and Education Policy in a Heterogeneous Agent Economy: What Levels of Redistribution Maximizes Growth and Efficiency?," *Econometrica*, 70, 481-517.

[7] Bewley, T. (1983): "A Difficulty with the Optimum Quantity of Money," *Econometrica*, 51, 1485-1504.

[8] Blundell, R. and MaCurdy, T. (1999), "Labor Supply: A Review of Alternative Approaches," in A. Ashenfelter and D. Card, eds, Handbook of Labor Economics, vol. 3, Amsterdam: Elsevier Science.

[9] Blundell, R., L. Pistaferri, and I. Preston (2004);"Consumption Inequality and Partial Insurance," mimeo Stanford University.

[10] Blundell, R., and I. Preston (1998); "Consumption Inequality and Income Uncertainty," *Quarterly Journal of Economics* 113 (2), 603-640.

[11] Browning, M., L.P. Hansen and J. Heckman (1999), "Micro Data and General Equilibrium Models," Handbook of Macroeconomics, Volume 1A, Amsterdam: Elsevier Science.

[12] Carroll, C. (1997); "Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis," *Quarterly Journal of Economics*, 112(1), 1-56.

[13] Cho, J.O. and T.F. Cooley (2001); "Business Cycle Uncertainty and Economic Welfare," mimeo Stern School of Business, New York University.

[14] Constantinides G. and D. Duffie (1996); "Asset Pricing with Heterogenous Consumers," *Journal of Political Economy*, 104(2), 219-240.

[15] Cooley, T. (1995); *Frontiers of Business Cycle Research*, Princeton University Press.

[16] Deaton, A. (1991); "Saving and Liquidity Constraints," *Econometrica*, vol. 59(5), 1221-48.

[17] Domeij, D. and M. Flodén (2001); "The Labor-Supply Elasticity and Borrowing Constraints: Why Estimates are Biased," Working Paper Series in Economics and Finance 480, Stockholm School of Economics

[18] Eckstein, Z., and E. Nagypal (2004); "U.S. Earnings and Employment Dynamics 1961-2002: Facts and Interpretations," mimeo, Northwestern University.

[19] Flodén, M. (2001); "The Effectiveness of Government Debt and Transfers as Insurance," *Journal of Monetary Economics*, Elsevier, vol. 48(1), 81-108.

[20] Gottschalk, P., and R. Moffitt (1994); "The Growth of Earnings Instability in the U.S. Labor Market," *Brookings Papers of Economic Activity* 2, 217-272.

[21] Heathcote, J., K. Storesletten, and G. L. Violante (2004); "The Macroeconomic Implications of Rising Wage Inequality in the United States," mimeo, NYU.

[22] Heathcote, J., K. Storesletten, and G. L. Violante (2005); "Labor Supply and Consumption with Partial Insurance" mimeo, NYU.

[23] Heckman, J. (1974); "Life-Cycle Consumption and Labor Supply: An Explanation of the Relationship between Income and Consumption over the Life-Cycle," *American Economic Review* 64, 188-194.

[24] Hornstein A., P. Krusell and G.L. Violante (2004); "The Effects of Technical Change on Labor Market Inequalities," in P. Aghion and S. Durlauf (eds.), Handbook of Economic Growth, Amsterdam: Elsevier Science, forthcoming.

[25] Huggett, M. (1993); "The Risk Free Rate in Heterogeneous-Agents Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 17, 953-969.

[26] Huggett, M., A. Yaron, and G. Ventura (2004); "Human capital and Earnings Distribution Dynamics," mimeo, Georgetown University.

[27] Imrohoruglu, A. (1989); "Cost of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political Economy*, 97(6), 1364-1383.

[28] Katz, L. M., and D. Autor (1999); "Changes in the Wage Structure and Earnings Inequality," in O. Ashenfelter and D. Card (eds.), Handbook of Labor Economics, Vol. 3, Amsterdam: Elsevier Science.

[29] Krebs, T. (2003); "Growth and Welfare Effects of Business Cycles in Economies with Idiosyncratic Human Capital Risk", *Review of Economic Dynamics*, 6, 846-868.

[30] Krebs, T., P. Krishna and W. Maloney (2004), "Trade Policy, Income Risk, and Welfare," mimeo, Brown University.

[31] Krueger, D. and F. Perri (2003); "On the Welfare Consequences of the Increase in Inequality in the United States," in M. Gertler and K. Rogoff (eds.), NBER Macroeconomics Annual 2003, Cambridge, MA: MIT Press.

[32] Krueger, D. and F. Perri (2005); "Does Income Inequality Lead to Consumption Inequality? Evidence and Theory," *Review of Economic Studies*, forthcoming.

[33] Krusell, P. and A. Smith, Jr.(1999); "On the Welfare Effects of Eliminating Business Cycles," *Review of Economic Dynamics*, 2(1), 245-272.

[34] Kubler F. and K. Schmedders (2001); "Incomplete Markets, Transitory Shocks, and Welfare," *Review of Economic Dynamics*, 4(4), 747-766.

[35] Levine D.K. and W.R. Zame (2002); "Does Market Incompleteness Matter?," *Econometrica* 70(5), 1805-1839.

[36] Low, H. (2005); "Self-Insurance in a Life-Cycle Model of Labor Supply and Savings," *Review of Economic Dynamics*, forthcoming.

[37] Lucas, R. E., Jr. (1987); Models of Business Cycle, Oxford: Basil Blackwell.

[38] Lucas, R. E., Jr. (2003); "Macroeconomic Priorities," *American Economic Review* 93 (1), 1-14.

[39] Mas-Colell A., M.D. Whinston and J.R. Green (1995); Microeconomic Theory, Oxford: Oxford University Press.

[40] Pijoan-Mas, J. (2004); "Precautionary Savings or Working Longer Hours?," mimeo, CEMFI.

[41] Rios-Rull, V. (1994); On the Quantitative Importance of Market Incompleteness," *Journal of Monetary Economics*, 34, 463-496.

[42] Sargent, T. (2001); Discussion of "Can Market and Voting Institutions Generate Optimal Intergenerational Risk Sharing", by A. Rangel and R. Zeckhauser, in J.Y. Campbell and M. Feldstein (eds.), Risk Aspects of Investment-Based Social Security Reform, Chicago: The University of Chicago Press.

[43] Shiller, R.J. (2003); The New Financial Order: Risk in the 21st Century. Princeton: Princeton University Press.

[44] Slesnick (2001); Consumption and Social Welfare: Living Standards and their Distribution in the United States. Cambridge, UK: Cambridge University Press.

[45] Storesletten, K., C. Telmer, and A. Yaron (2001); " The welfare cost of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk," *European Economic Review*, 45(7), 1311-1339.

[46] Zeldes S. (1989); "Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence", *Quarterly Journal of Economics*, Vol. 104(2), 275-298.

9 Appendix

Appendix A: Derivation of Equilibrium Allocations

It is useful to start from the incomplete markets case and then generalize our expressions to the complete-markets and autarky economies. Consider a typical "component-planner" who chooses efficiently consumption and hours worked for all agents on a particular $\alpha-$island. First, note that only an agent's current transitory shock ε (and the island-wide value for α) can be relevant for consumption and labor allocations. This follows from the fact that the planner weights all agents identically, and preferences are time-separable. Thus the component planner's problem is given by

$$\max_{\{c(\alpha,\varepsilon)h(\alpha,\varepsilon)\}_{\varepsilon\in E}} \int_E u\left(c\left(\alpha,\varepsilon\right)h\left(\alpha,\varepsilon\right)\right)\phi_{v_\varepsilon}(\varepsilon)d\varepsilon \tag{28}$$

subject to

$$\int_E \left[w\left(\alpha,\varepsilon\right)h\left(\alpha,\varepsilon\right)-c\left(\alpha,\varepsilon\right)\right]\phi_{v_\varepsilon}(\varepsilon)d\varepsilon=0. \tag{29}$$

Cobb-Douglas preferences: In the Cobb-Douglas case, the planner's first order condition for hours may be written as

$$c\left(\alpha,\varepsilon\right)\frac{1-\eta}{\eta}-w\left(\alpha,\varepsilon\right)=-w\left(\alpha,\varepsilon\right)h\left(\alpha,\varepsilon\right).$$

Substituting the left hand side of this latter equation into the resource constraint (equation 29) and collecting terms gives

$$\int_E c\left(\alpha,\varepsilon\right)\phi_{v_\varepsilon}(\varepsilon)d\varepsilon=\eta\int_E w\left(\alpha,\varepsilon\right)\phi_{v_\varepsilon}(\varepsilon)d\varepsilon=\eta\exp(\alpha). \tag{30}$$

The first order condition for consumption is

$$\mu=\eta c\left(\alpha,\varepsilon\right)^{\eta(1-\theta)-1}\left(1-h\left(\alpha,\varepsilon\right)\right)^{(1-\eta)(1-\theta)}, \tag{31}$$

where μ is the Lagrange multiplier on the resource constraint for the component planner. Using the intratemporal first order condition to substitute out for leisure in equation (31) and rearranging gives

$$c\left(\alpha,\varepsilon\right)=\left(\frac{\eta}{\mu}\right)^{1/\theta}\left(\frac{1-\eta}{\eta}\right)^{(1-\eta)(1-\theta)/\theta}w\left(\alpha,\varepsilon\right)^{-(1-\eta)(1-\theta)/\theta}. \tag{32}$$

Integrating across the population

$$\int_E c\left(\alpha,\varepsilon\right)\phi_{v_\varepsilon}(\varepsilon)d\varepsilon=\left(\frac{\eta}{\mu}\right)^{1/\theta}\left(\frac{1-\eta}{\eta}\right)^{(1-\eta)(1-\theta)/\theta}\int_E w\left(\alpha,\varepsilon\right)^{-(1-\eta)(1-\theta)/\theta}\phi_{v_\varepsilon}(\varepsilon)d\varepsilon. \tag{33}$$

Note that

$$\begin{aligned}\int_E w\left(\alpha,\varepsilon\right)^{-(1-\eta)(1-\theta)/\theta}\phi_{v_\varepsilon}(\varepsilon)d\varepsilon &= \int_E\left(\exp(\alpha)\exp(\varepsilon)\right)^{-(1-\eta)(1-\theta)/\theta}\phi_{v_\varepsilon}(\varepsilon)d\varepsilon\\ &= \exp\left(\frac{(1-\eta)(\theta-1)}{\theta}\alpha\right)\exp\left((1-\eta)(1-\theta)\frac{1-\eta+\eta\theta}{\theta^2}\frac{v_\varepsilon}{2}\right),\end{aligned}$$

where the last step exploits the fact that ε is log-normally distributed.

Combining equations (30) and (33) implies

$$\left(\frac{\eta}{\mu}\right)^{1/\theta}\left(\frac{1-\eta}{\eta}\right)^{(1-\eta)(1-\theta)/\theta} = \eta^{-1}\exp\left(\frac{(1-\eta)(\theta-1)}{\theta}\alpha\right)\exp\left((1-\eta)(1-\theta)\frac{1-\eta+\eta\theta}{\theta^{2}}\frac{v_{\varepsilon}}{2}\right)\exp(-\alpha) \tag{34}$$

Substituting this term into equation (32) to solve for consumption and then using the first order condition for hours (equation 9) to solve for hours yields efficient allocations only as a function of the primitive parameters, as described in equations (20) in the text.

Separable preferences: When preferences are separable between consumption and hours, the first-order conditions for the planner imply

$$\begin{aligned} c(\alpha,\varepsilon) &= \mu^{-1/\gamma}, \\ h(\alpha,\varepsilon) &= c(\alpha,\varepsilon)^{-\gamma/\sigma}\exp\left(\frac{\alpha+\varepsilon}{\sigma}\right). \end{aligned} \tag{35}$$

Note immediately from equations (35) that when preferences are separable, the planner gives all agents on an $\alpha-$island the same level of consumption.

Using the planner's resource constraint (equation 29), it is straightforward to solve for μ :

$$\mu = \exp\left(-\gamma\left(\alpha+\frac{v_{\varepsilon}}{2\sigma}\right)\frac{1+\sigma}{\sigma+\gamma}\right).$$

Substituting this expression into equations (35) yields efficient allocations only as a function of the primitive parameters, as described in equations (26) in the text.

Allocations in the autarky and complete markets economies: The expressions for equilibrium allocations in the complete markets and autarky economies are essentially special cases of the expressions for the incomplete markets economy derived above. Consider the expressions for consumption and labor supply under incomplete markets in equations (20) (for Cobb-Douglas preferences) and (26) (for separable preferences). Let $s = (u, i)$ denote the values of the uninsurable component of the wage and the insurable component of the wage that constitute the individual state variables for a particular agent, and let $S = (v_u, v_i)$ denote the population variances of the uninsurable and insurable components that define the aggregate state of the economy. In the incomplete markets economy, $s = (\alpha, \varepsilon)$ and $S = (v_\alpha, v_\varepsilon)$. The complete markets economy is a special case of the incomplete markets economy in which $s = (0, \alpha+\varepsilon)$ and $S = (0, v_\alpha + v_\varepsilon)$. In particular, one can arrive at the corresponding complete markets allocations (15) and (25) simply by taking the incomplete markets allocations and applying this mapping. Similarly, the autarky economy is a special case of the incomplete markets economy in which $s = (\alpha+\varepsilon, 0)$ and $S = (v_a + v_\varepsilon, 0)$.

Endowing the planner with a savings technology: Note that v_ε and v_α are time invariant, since we are considering a stationary environment. Thus in the incomplete markets economy the marginal utility of consumption is constant through time within each $\alpha-$island. It follows immediately we could endow the component-planner with the ability to transfer resources through time via a technology offering gross return β^{-1} without affecting allocations. Similar arguments apply to the complete markets economy.

Appendix B: Proof of Proposition 1

Incomplete markets– First, it is helpful to compute expected welfare. When preferences are non-separable between consumption and hours worked, unconditional expected period utility is given by

$$\begin{aligned}\mathcal{W} &= E\left[u\left(c_{IM}(\alpha,\varepsilon),h_{IM}(\alpha,\varepsilon)\right)\right] = \int\limits_A\int\limits_E u\left(c_{IM}(\alpha,\varepsilon),h_{IM}(\alpha,\varepsilon)\right)\phi_{v_\varepsilon}(\varepsilon)\phi_{v_\alpha}(\alpha)d\varepsilon d\alpha \\ &= E\left[\frac{\left(c_{IM}(\alpha,\varepsilon)^{\eta}\left(1-h_{IM}(\alpha,\varepsilon)\right)^{1-\eta}\right)^{1-\theta}}{1-\theta}\right].\end{aligned}$$

Recalling that the wage $w=\exp(\alpha+\varepsilon)$, and using the intra-temporal first-order condition, we can write expected utility as

$$\begin{aligned}\mathcal{W} &= \left[\frac{\left(c_{IM}(\alpha,\varepsilon)^{\eta}\left(\frac{1-\eta}{\eta}\left(\exp\left(-\left(\alpha+\varepsilon\right)\right)\right)c_{IM}\left(\alpha+\varepsilon\right)\right)^{1-\eta}\right)^{1-\theta}}{1-\theta}\right] \\ &= \frac{1}{1-\theta}\left(\frac{1-\eta}{\eta}\right)^{(1-\eta)(1-\theta)}E\left[\exp\left(-\left(1-\eta\right)\left(1-\theta\right)\left(\alpha+\varepsilon\right)\right)c_{IM}\left(\alpha,\varepsilon\right)^{(1-\theta)}\right].\end{aligned}$$

Substituting the equilibrium expressions for $c_{IM}(\alpha,\varepsilon)$ (see equations 20), expected utility becomes:

$$\mathcal{W}=\kappa E\left[\exp\left(\eta\left(1-\theta\right)\alpha\right)\left(\exp\left(-\left(1-\eta\right)\left(1-\theta\right)\varepsilon\right)\exp\left(\frac{\left(1-\theta\right)\left(1-\eta\right)\left(\theta-1\right)}{\theta}\left(\varepsilon_i+\frac{1-\eta+\eta\theta}{\theta}\frac{v_\varepsilon}{2}\right)\right)\right)\right],$$

where $\kappa\equiv\frac{1}{1-\theta}\left(\frac{1-\eta}{\eta}\right)^{(1-\eta)(1-\theta)}\eta^{1-\theta}$.

Recall now that, since α is normally distributed with mean $\frac{-v_\alpha}{2}$ and variance v, $x\alpha$ is still normal with mean $\frac{-xv_\alpha}{2}$ and variance x^2v_α. Therefore, $E\left(\exp\left(x\alpha\right)\right)=\exp\left(\frac{-xv_\alpha}{2}+\frac{x^2v_\alpha}{2}\right)=\exp\left(x\left(x-1\right)\frac{v_\alpha}{2}\right)$ And similarly for ε. Using these properties to compute the expectation above, and rearranging terms yields:

$$\mathcal{W}=\kappa\exp\left(\frac{\left(1-\eta+\eta\theta\right)\left(1-\eta\right)\left(1-\theta\right)}{\theta}\frac{v_\varepsilon}{2}-\left(1-\eta+\eta\theta\right)\left(1-\theta\right)\eta\frac{v_\alpha}{2}\right). \tag{36}$$

Now, recall that ω_{IM}, the cost of a rise in the variance of the two components of the wage process from v_ε to $\widehat{v}_\varepsilon=v_\varepsilon+\Delta v_\varepsilon$ and from v_α to $\widehat{v}_\alpha=v_\alpha+\Delta v_\alpha$, is defined by equation (7), which we reproduce here:

$$\int\limits_A\int\limits_E u\left((1+\omega_{IM})c_{IM}(\alpha,\varepsilon),h_{IM}(\alpha,\varepsilon)\right)\phi_{v_\varepsilon}(\varepsilon)\phi_{v_\alpha}(\alpha)d\varepsilon d\alpha=\int\limits_A\int\limits_E u\left(\widehat{c}_{IM}(\alpha,\varepsilon),\widehat{h}_{IM}(\alpha,\varepsilon)\right)\phi_{\widehat{v}_\varepsilon}(\varepsilon)\phi_{\widehat{v}_\alpha}(\alpha)d\varepsilon d\alpha. \tag{37}$$

To compute the welfare effect of changing wage dispersion ω^{IM}, we can plug in the expression for expected utility in (36) and rewrite equation (37) as

$$\begin{aligned}&\left(1+\omega^{IM}\right)^{\eta(1-\theta)}\kappa\exp\left(\frac{\left(1-\eta+\eta\theta\right)\left(1-\eta\right)\left(1-\theta\right)}{\theta}\frac{v_\varepsilon}{2}-\left(1-\eta+\eta\theta\right)\left(1-\theta\right)\eta\frac{v_\alpha}{2}\right) \\ =\ &\kappa\exp\left(\frac{\left(1-\eta+\eta\theta\right)\left(1-\eta\right)\left(1-\theta\right)}{\theta}\frac{\hat{v}_\varepsilon}{2}-\left(1-\eta+\eta\theta\right)\left(1-\theta\right)\eta\frac{\hat{v}_\alpha}{2}\right).\end{aligned}$$

Collecting terms, we obtain

$$\left(1+\omega^{IM}\right)^{\eta(1-\theta)} = \exp\left(\frac{(1-\eta+\eta\theta)(1-\eta)(1-\theta)}{\theta}\frac{\Delta v_\varepsilon}{2} - (1-\eta+\eta\theta)(1-\theta)\eta\frac{\Delta v_\alpha}{2}\right),$$

which implies the exact expression for ω^{IM}:

$$1+\omega^{IM} = \exp\left(\frac{1-\eta}{\eta}\frac{1-\eta+\eta\theta}{\theta}\frac{\Delta v_\varepsilon}{2} - (1-\eta+\eta\theta)\frac{\Delta v_\alpha}{2}\right)$$

Thus, taking logarithms of both sides, and using a log-approximation of the type $\ln(1+x) \simeq x$ on the left-hand side of the above equation, we arrive at

$$\begin{aligned}
\omega^{IM} &\simeq \frac{1-\eta}{\eta}\frac{1-\eta+\eta\theta}{\theta}\frac{\Delta v_\varepsilon}{2} - (1-\eta+\eta\theta)\frac{\Delta v_\alpha}{2} \\
&= \bar{\phi}\frac{\Delta v_\varepsilon}{2} - \bar{\phi}\theta\frac{\eta}{1-\eta}\frac{\Delta v_\alpha}{2},
\end{aligned}$$

which is the expression stated in Proposition 1.

Complete markets and autarky–With the expression for the welfare cost under incomplete markets in hand, it is straightforward to derive the corresponding expressions under complete markets and under autarky. In particular, denote the aggregate state in the incomplete markets economy $S = (v_\alpha, v_\varepsilon)$ where the first element of S.denotes the variance of the uninsurable component of the wage, and the second element denotes the variance of the insurable component. The complete markets economy is a special case of the incomplete markets economy in which $S = (0, v_\alpha + v_\varepsilon)$. Thus the complete markets expression for ω_{CM}, the welfare cost of an increase in wage inequality, is the same as that for the incomplete markets economy described above except that the term Δv_α in the expression ω_{IM} drops out, and the term Δv_ε is replaced with $\Delta v_\alpha + \Delta v_\varepsilon$. Similarly, the autarky economy is a special case of the incomplete markets economy in which $S = (v_a + v_\varepsilon, 0)$.

Decomposition into level effect and volatility effect– We derive the decomposition for the IM case. The derivations for the other market structures follow easily. Aggregate consumption and leisure allocations in IM are given by:

$$\begin{aligned}
C &= E\left[c(\alpha,\varepsilon)\right] = \eta, \\
H &= E\left[h(\alpha,\varepsilon)\right] = 1-(1-\eta)\exp\left(\lambda v_\varepsilon\right).
\end{aligned}$$

With these expressions in hand, it is straightforward to compute the level affect associated with an increase in the variances of the two components of the wage from v_α and v_ε to $\widehat{v}_\alpha$ and $\widehat{v}_\varepsilon$ applying the definition in equation (8) to the separable specification for preferences:

$$\frac{\left(\left(1+\omega_{IM}^{lev}\right)^\eta C^\eta (1-H)^{1-\eta}\right)^{1-\theta}}{1-\theta} = \frac{\left(\widehat{C}^\eta\left(1-\widehat{H}\right)^{1-\eta}\right)^{1-\theta}}{1-\theta},$$

where $\widehat{C}_{IM}$ and $\widehat{H}_{IM}$ denote average consumption and hours worked given the more volatile process for wages. Since average consumption is invariant to wage dispersion,

$$\begin{aligned}
\left(1+\omega_{IM}^{lev}\right)^\eta (1-H)^{1-\eta} &= \left(1-\widehat{H}\right)^{1-\eta}, \\
1+\omega_{IM}^{lev} &= \exp\left(\frac{1-\eta}{\eta}\lambda\Delta v_\varepsilon\right), \\
\omega_{IM}^{lev} &\simeq \overline{\phi}\Delta v_\varepsilon,
\end{aligned}$$

which yields the level effect of Proposition 1.

At this point, it is useful to recall that Flodén (2001) demonstrates that if $u(xc,h)$ is "homogenous" in the sense that $u(xc,h) = g(x)\,u(c,h)$, then

$$1+\omega_m = \left(1+\omega_m^{lev}\right)\left(1+\omega_m^{vol}\right) \Rightarrow \omega_m \simeq \omega_m^{lev} + \omega_m^{vol}, \tag{38}$$

up to second-order terms. Clearly, the Cobb–Douglas specification satisfies this homogeneity property. Therefore, given ω_{IM}^{lev}, the expression for ω_{IM}^{vol} can be derived residually, exploiting equation (38).

Appendix C: Proof of Proposition 1a

Incomplete markets– As for the Cobb-Douglas case, first it is helpful to compute expected welfare. When preferences are separable between consumption and hours worked, unconditional expected period utility is given by

$$\begin{aligned}\mathcal{W} &= E\left[u\left(c_{IM}(\alpha,\varepsilon), h_{IM}(\alpha,\varepsilon)\right)\right] = \int\limits_A\int\limits_E u\left(c_{IM}(\alpha,\varepsilon), h_{IM}(\alpha,\varepsilon)\right)\phi_{v_\varepsilon}(\varepsilon)\phi_{v_\alpha}(\alpha)d\varepsilon d\alpha \\ &= E\left[\frac{1}{1-\gamma}c_{IM}(\alpha,\varepsilon)^{1-\gamma} - \frac{1}{1+\sigma}h_{IM}(\alpha,\varepsilon)^{1+\sigma}\right].\end{aligned}$$

Using the equilibrium expressions for $c_{IM}(\alpha,\varepsilon)$ and $h_{IM}(\alpha,\varepsilon)$ (see equations 26), expected utility is given by:

$$\begin{aligned}\mathcal{W} = &\frac{1}{1-\gamma}\exp\left(\frac{(1+\sigma)(1-\gamma)}{\gamma+\sigma}\frac{v_\varepsilon}{2\sigma}\right)E\left[\exp\left(\frac{(1+\sigma)(1-\gamma)}{\gamma+\sigma}\alpha\right)\right] - \\ &\frac{1}{1+\sigma}\exp\left(\frac{-(1+\sigma)^2}{\gamma+\sigma}\frac{\gamma v_\varepsilon}{2\sigma^2}\right)E\left[\exp\left(\frac{(1+\sigma)(1-\gamma)}{\gamma+\sigma}\alpha\right)\exp\left(\frac{(1+\sigma)}{\sigma}\varepsilon\right)\right].\end{aligned}$$

Now using the fact that α and ε are normally distributed, the terms inside the expectation signs are log-normally distributed and one can easily compute that the expectation terms are given by

$$\begin{aligned}E\left[\exp\left(\frac{(1+\sigma)(1-\gamma)}{\gamma+\sigma}\alpha\right)\right] &= \exp\left(\kappa(\kappa-1)\frac{v_\alpha}{2}\right), \\ E\left[\exp\left(\frac{(1+\sigma)(1-\gamma)}{\gamma+\sigma}\alpha\right)\exp\left(\frac{(1+\sigma)}{\sigma}\varepsilon\right)\right] &= \exp\left(\kappa(\kappa-1)\frac{v_\alpha}{2}\right)\exp\left(\frac{(1+\sigma)}{\sigma^2}\frac{v_\varepsilon}{2}\right),\end{aligned}$$

where $\kappa = \frac{(1+\sigma)(1-\gamma)}{\gamma+\sigma}$.

Collecting terms, the expression for expected utility reduces to

$$\begin{aligned}\mathcal{W} &= \exp\left(\kappa(\kappa-1)\frac{v_\alpha}{2}\right)\exp\left(\kappa\frac{v_\varepsilon}{2\sigma}\right)\left(\frac{1}{1-\gamma} - \frac{1}{1+\sigma}\right) \\ &= \exp\left(\kappa(\kappa-1)\frac{v_\alpha}{2}\right)\exp\left(\kappa\frac{v_\varepsilon}{2\sigma}\right)\frac{1}{\kappa}.\end{aligned} \tag{39}$$

To compute the welfare effect of changing wage dispersion ω^{IM}, note that the left hand side of equation (37) can be expressed as

$$\exp\left(\kappa(\kappa-1)\frac{v_\alpha}{2}\right)\exp\left(\kappa\frac{v_\varepsilon}{2\sigma}\right)\left(\frac{(1+\omega_{IM})^{1-\gamma}}{1-\gamma} - \frac{1}{1+\sigma}\right).$$

Thus ω_{IM} is implicitly defined by

$$\left(\frac{(1+\omega_{IM})^{1-\gamma}}{1-\gamma}-\frac{1}{1+\sigma}\right)=\exp\left(\kappa(\kappa-1)\frac{\Delta v_{\alpha}}{2}\right)\exp\left(\kappa\frac{\Delta v_{\varepsilon}}{2\sigma}\right)\frac{1}{\kappa}$$

or

$$1+\omega_{IM}=\left[\frac{1-\gamma}{1+\sigma}+\frac{\gamma+\sigma}{1+\sigma}\exp\left(\kappa(\kappa-1)\frac{\Delta v_{\alpha}}{2}\right)\exp\left(\kappa\frac{\Delta v_{\varepsilon}}{2\sigma}\right)\right]^{\frac{1}{1-\gamma}}$$

This welfare expression is exact, but rather involved. However, ω_{IM} can be very closely approximated by a much simpler expression. In particular, we use a log-approximation of the type $\ln(1+x)\simeq x$ on the left-hand side of the equation, and the approximation $\exp(x)\simeq 1+x$ on the right-hand side, which gives

$$\begin{aligned}
\omega_{IM} &\simeq \frac{1}{1-\gamma}\ln\left[\frac{1-\gamma}{1+\sigma}+\frac{\gamma+\sigma}{1+\sigma}\left(1+\kappa(\kappa-1)\frac{\Delta v_{\alpha}}{2}\right)\left(1+\kappa\frac{\Delta v_{\varepsilon}}{2\sigma}\right)\right]\\
&= \frac{1}{1-\gamma}\ln\left[1+\frac{\gamma+\sigma}{1+\sigma}\left(\kappa\frac{\Delta v_{\varepsilon}}{2\sigma}+\kappa(\kappa-1)\frac{\Delta v_{\alpha}}{2}+\kappa^{2}(\kappa-1)\frac{\Delta v_{\alpha}}{2}\frac{\Delta v_{\varepsilon}}{2\sigma}\right)\right]\\
&\simeq \frac{1}{1-\gamma}\ln\left[1+\frac{\gamma+\sigma}{1+\sigma}\left(\kappa\frac{\Delta v_{\varepsilon}}{2\sigma}+\kappa(\kappa-1)\frac{\Delta v_{\alpha}}{2}\right)\right]\\
&\simeq \frac{1}{\kappa}\left(\kappa\frac{\Delta v_{\varepsilon}}{2\sigma}+\kappa(\kappa-1)\frac{\Delta v_{\alpha}}{2}\right)\\
&= \frac{\Delta v_{\varepsilon}}{2\sigma}+(\kappa-1)\frac{\Delta v_{\alpha}}{2}\\
&= \frac{\Delta v_{\varepsilon}}{2\sigma}+\left[\frac{(1-\gamma)-\gamma(1+\sigma)}{\gamma+\sigma}\right]\frac{\Delta v_{\alpha}}{2}
\end{aligned}$$

where the last expression is the one we report in the text in Proposition 1.

Complete markets and autarky– The derivation of the welfare change under complete markets and autarky follows exactly the logic outlined above for the Cobb-Douglas case.

Decomposition into level effect and volatility effect– We derive the decomposition for the IM case. The derivations for the other market structures follow easily. Aggregate consumption and leisure allocations in IM are given by:

$$C = E[c(\alpha,\varepsilon)]=\exp\left(\frac{(1+\sigma)}{(\gamma+\sigma)}\frac{v_{\varepsilon}}{2\sigma}\right)\exp\left(\frac{(1+\sigma)(1-\gamma)}{(\gamma+\sigma)^{2}}\frac{v_{\alpha}}{2}\right), \tag{40}$$

$$H = E[h(\alpha,\varepsilon)]=\exp\left(\frac{1-2\gamma-\sigma}{(\gamma+\sigma)\sigma}\frac{v_{\varepsilon}}{2}\right)\exp\left(\frac{(1-\gamma)(1-2\gamma-\sigma)}{(\gamma+\sigma)^{2}}\frac{v_{\alpha}}{2}\right). \tag{41}$$

With these expressions in hand, we compute the level affect associated with an increase in the variances of the two components of the wage from v_{α} and v_{ε} to $\widehat{v}_{\alpha}$ and $\widehat{v}_{\varepsilon}$ applying the definition in equation 8 to the separable specification for preferences:

$$\frac{\left(1+\omega_{IM}^{lev}\right)^{1-\gamma}C^{1-\gamma}}{1-\gamma}-\frac{H^{1+\sigma}}{1+\sigma}=\frac{\widehat{C}^{1-\gamma}}{1-\gamma}-\frac{\widehat{H}^{1+\sigma}}{1+\sigma},$$

where $\widehat{C}_{IM}$ and $\widehat{H}_{IM}$ denote average consumption and hours worked given the more volatile process for wages. Substituting in the expressions for aggregate variables and collecting terms gives

$$\begin{aligned}\left(1+\omega_{IM}^{lev}\right)^{1-\gamma} \quad = \quad & \frac{1-\gamma}{1+\sigma}\exp\left(-\frac{(1+\sigma)}{\sigma}\frac{v_\varepsilon}{2}\right)\exp\left(-\kappa\frac{v_\alpha}{2}\right) \\ & -\frac{1-\gamma}{1+\sigma}\exp\left(\frac{\kappa}{\sigma}\frac{\Delta v_\varepsilon}{2}-\frac{(1+\sigma)}{\sigma}\frac{\widehat{v}_\varepsilon}{2}\right)\exp\left(\left(\frac{(1-\gamma)\kappa}{(\gamma+\sigma)}\frac{\Delta v_\alpha}{2}-\kappa\frac{\widehat{v}_\alpha}{2}\right)\right) \\ & +\exp\left(\kappa\frac{\Delta v_\varepsilon}{2\sigma}\right)\exp\left(\frac{(1-\gamma)\kappa}{(\gamma+\sigma)}\frac{\Delta v_\alpha}{2}\right)\end{aligned}$$

where $\kappa=\frac{(1+\sigma)(1-\gamma)}{\gamma+\sigma}$.

Applying the approximation $\exp(x)\simeq 1+x$ for $x\simeq 0$ to the right hand side of this expression and collecting terms gives

$$\frac{\left(1+\omega_{IM}^{lev}\right)^{1-\gamma}}{1-\gamma}\simeq\frac{1}{1-\gamma}+\frac{1}{\sigma}\Delta v_\varepsilon+\frac{1-\gamma}{(\gamma+\sigma)}\Delta v_\alpha.$$

Multiplying both sides by $(1-\gamma)$, and taking logs,

$$(1-\gamma)\ln\left(1+\omega_{IM}^{lev}\right)\simeq\ln\left(1+(1-\gamma)\left[\frac{1}{\sigma}\Delta v_\varepsilon+\frac{1-\gamma}{(\gamma+\sigma)}\Delta v_\alpha\right]\right).$$

Using the approximation $\ln(1+x)\simeq x$ on both sides of this expression gives

$$\omega_{IM}^{lev}\simeq\frac{1}{\sigma}\Delta v_\varepsilon+\frac{1-\gamma}{(\gamma+\sigma)}\Delta v_\alpha,$$

which is the expression reported in the text in Proposition 1a.

We now compute the volatility component of the welfare effect. The first step is to calculate a certainty equivalent value for consumption $c(H_{IM})$, such that the utility associated with consuming $c(H_{IM})$ and working H_{IM} hours is equal to expected equilibrium utility:

$$u(c(H_{IM}),H_{IM})=\int\limits_A\int\limits_E u\left(c_{IM}(\alpha,\varepsilon),h_{IM}(\alpha,\varepsilon)\right)\phi_{v_\varepsilon}(\varepsilon)\phi_{v_\alpha}(\alpha)d\varepsilon d\alpha.$$

Given the separable specification for preferences, and equations (39) and (41) for expected utility and aggregate hours, the expression for certainty equivalent consumption can be rewritten in terms of the aggregate state (v_α,v_ε) and preference parameters:

$$\begin{aligned}c(H)^{1-\gamma} \quad = \quad & \frac{1}{1+\sigma}(\gamma+\sigma)\exp\left(\frac{(1+\sigma)(1-\gamma)}{(\gamma+\sigma)}\left(\frac{1-2\gamma-\gamma\sigma}{\gamma+\sigma}\frac{v_\alpha}{2}+\frac{1}{\sigma}\frac{v_\varepsilon}{2}\right)\right)+ \\ & \frac{1}{1+\sigma}(1-\gamma)\exp\left(\frac{(1+\sigma)(1-2\gamma-\sigma)}{(\gamma+\sigma)}\left(\frac{(1-\gamma)}{(\gamma+\sigma)}\frac{v_\alpha}{2}+\frac{1}{\sigma}\frac{v_\varepsilon}{2}\right)\right).\end{aligned}$$

Applying the approximation $\exp(x)\simeq 1+x$ for $x\simeq 0$ and collecting terms gives

$$\begin{aligned}c(H)^{1-\gamma} \quad \simeq \quad & \frac{(1-\gamma)\left[(1-2\gamma-\gamma\sigma)(\gamma+\sigma)+(1-\gamma)(1-2\gamma-\sigma)\right]}{(\gamma+\sigma)^2}\frac{v_\alpha}{2}+ \\ & +\frac{1}{\sigma}\frac{(1-\gamma)^2}{(\gamma+\sigma)}\frac{v_\varepsilon}{2}+1.\end{aligned}$$

Using the approximation $1+x \simeq \exp(x)$ for $x \simeq 0$ and raising both sides of the equation to the power $\frac{1}{1-\gamma}$ gives

$$c(H) \simeq \exp\left(\frac{(1-2\gamma-\gamma\sigma)(\gamma+\sigma)+(1-\gamma)(1-2\gamma-\sigma)}{(\gamma+\sigma)^2}\frac{v_\alpha}{2}+\frac{1}{\sigma}\frac{(1-\gamma)}{(\gamma+\sigma)}\frac{v_\varepsilon}{2}\right). \tag{42}$$

Applying the definition in equation (9), the cost of uncertainty p is the solution to

$$u((1-p)C,H) = u(c(H),H)$$

which, given separable preferences, implies $1-p = c(H)/C$. Substituting in equations (40) and (42) gives

$$1-p \simeq \exp\left(\frac{-(1+\sigma\gamma)}{\gamma+\sigma}\frac{v_\alpha}{2}-\frac{1}{\sigma}\frac{v_\varepsilon}{2}\right) = \exp\left(-\frac{1-\gamma}{\gamma+\sigma}\frac{v_\alpha}{2}-\gamma\frac{1+\sigma}{\gamma+\sigma}\frac{v_\alpha}{2}-\frac{1}{\sigma}\frac{v_\varepsilon}{2}\right).$$

Using the definition for the cost of volatility in equation (10) gives

$$\begin{aligned}
1+\omega_{IM}^{vol} &= \frac{1-\widehat{p}}{1-p} \simeq \exp\left(-\frac{1-\gamma}{\gamma+\sigma}\frac{\Delta v_\alpha}{2}-\gamma\frac{1+\sigma}{\gamma+\sigma}\frac{\Delta v_\alpha}{2}-\frac{1}{\sigma}\frac{\Delta v_\varepsilon}{2}\right) \\
&\simeq 1-\frac{1-\gamma}{\gamma+\sigma}\frac{\Delta v_\alpha}{2}-\gamma\frac{1+\sigma}{\gamma+\sigma}\frac{\Delta v_\alpha}{2}-\frac{1}{\sigma}\frac{\Delta v_\varepsilon}{2}.
\end{aligned}$$

where the last expression is the one reported in Proposition 1a.

Appendix D: Proof of Proposition 3

Cobb-Douglas preferences: We begin by establishing a useful approximated relationship between $\log(1-h)$ and $\log h$:

$$\begin{aligned}
\log(1-h) &= \log E(1-h) + \log\frac{1-h}{E(1-h)} \\
&\simeq \log E(1-h) + \frac{1-h}{E(1-h)} - 1 \\
&= \log E(1-h) - \frac{E(h)}{E(1-h)}\left(\frac{h}{E(h)}-1\right) \\
&\simeq \log E(1-h) - \frac{E(h)}{E(1-h)}\log\left(\frac{h}{E(h)}\right) \\
&= \log E(1-h) + \frac{E(h)}{E(1-h)}\log E(h) - \frac{E(h)}{E(1-h)}\log(h) \\
&= const - \frac{1-(1-\eta)\exp\left(\frac{1-\eta+\eta\gamma}{\gamma}v_\varepsilon\right)}{(1-\eta)\exp\left(\frac{1-\eta+\eta\gamma}{\gamma}v_\varepsilon\right)}\log(h),
\end{aligned}$$

where the first two approximations are accurate for small deviations of $(1-h)$ and h from their respective means. The final row uses the fact that, from equation (20) in the main text,

$$E(1-h) = 1-E(h) = (1-\eta)\exp\left(\frac{1-\eta+\eta\gamma}{\gamma}v_\varepsilon\right).$$

Simplifying, we arrive at

$$\log(1-h) \simeq const - \frac{\eta}{1-\eta}\log(h), \tag{43}$$

where this last approximation is accurate when $v_\varepsilon \simeq 0$. Note that in autarky, this expression holds as an identity.

By virtue of this approximate relationship, it is straightforward to see that

$$cov(\log w, \log h) \simeq -\frac{1-\eta}{\eta} cov(\log w, \log(1-h)) = \left(\frac{1-\eta}{\eta}\right)\lambda(\theta,\eta) v_\varepsilon = \bar{\phi} v_\varepsilon.$$

Recall, from Proposition 1, that the level effect in the Cobb-Douglas case is $\omega_{IM}^{lev} = \bar{\phi}\Delta v_\varepsilon$. Thus, the level effect equals the change in the covariance between hours and wages.

To write the volatility effect ω_{IM}^{vol} in terms of observables, note first that from the equilibrium allocations in (20) and from (43), one can easily derive that

$$\begin{aligned}
var(\log h) &\simeq \left(\frac{1-\eta}{\eta}\right)^2 \lambda(\theta,\eta)^2 v_\varepsilon = \bar{\phi}^2 v_\varepsilon, \\
var(\log c) &= v_\alpha + (1-\lambda)^2 v_\varepsilon, \\
cov(\log h, \log c) &\simeq -\frac{1-\eta}{\eta} cov(\log(1-h), \log c) = \left(\frac{1-\eta}{\eta}\right)\lambda(1-\lambda) v_\varepsilon.
\end{aligned}$$

From the expression of the volatility effect in Proposition 1,

$$\begin{aligned}
\omega_{IM}^{vol} &= -\bar{\phi}\frac{\Delta v_\varepsilon}{2} - \bar{\gamma}\frac{\Delta v_\alpha}{2} \\
&= -\frac{1}{\bar{\phi}}\left[\bar{\phi}^2 \Delta v_\varepsilon\right] - \frac{\bar{\gamma}}{2}\left[var(\log c) - (1-\lambda)^2 v_\varepsilon\right] \\
&= -\frac{1}{\bar{\phi}}\Delta var(\log h) - \frac{\bar{\gamma}}{2}\left[var(\log c) - \left(\frac{\eta}{1-\eta}\right)\left(\frac{1-\lambda}{\lambda}\right)\Delta cov(\log h, \log c)\right] \\
&= -\frac{1}{\bar{\phi}}\Delta var(\log h) - \frac{\bar{\gamma}}{2}\left[var(\log c) - \left(\frac{\bar{\gamma}-1}{\bar{\gamma}}\right)\Delta cov(\log h, \log c)\right] \\
&= -\frac{1}{\bar{\phi}}\Delta var(\log h) - \frac{\bar{\gamma}}{2} var(\log c) + \frac{(\bar{\gamma}-1)}{2}\Delta cov(\log h, \log c).
\end{aligned}$$

The second row uses the definition of the variance of log-consumption, the third row the definition of the variance of log-hours and of the covariance between log-consumption and log-hours. The fourth row requires some algebra and the definition of λ and $\bar{\gamma}$ in equation (12). The final row contains the expression of Proposition 3.

Separable preferences: We can write the welfare change of a rise in inequality under complete markets ω_{IM} as:

$$\begin{aligned}
\omega_{IM} &\simeq \frac{1}{\sigma}\frac{\Delta v_\varepsilon}{2} + \left[\frac{1-\gamma}{\sigma+\gamma} - \gamma\frac{1+\sigma}{\sigma+\gamma}\right]\frac{\Delta v_\alpha}{2} \\
&= \frac{1}{\sigma}\Delta v_\varepsilon + \left(\frac{1-\gamma}{\sigma+\gamma}\right)\Delta v_\alpha - \frac{1}{\sigma}\frac{\Delta v_\varepsilon}{2} - \left[\frac{(1-\gamma)+\gamma(1+\sigma)}{\sigma+\gamma}\right]\frac{\Delta v_\alpha}{2}.
\end{aligned}$$

From Proposition 1a, we know that $\omega_{IM}^{lev} = \frac{1}{\sigma}\Delta v_\varepsilon + \left(\frac{1-\gamma}{\sigma+\gamma}\right)\Delta v_\alpha$, and from the equilibrium allocations, it is easy to verify that

$$cov(\log w, \log h) = \frac{1}{\sigma}v_\varepsilon + \frac{1-\gamma}{\sigma+\gamma}v_\alpha,$$

which proves that the level effect $\omega_{IM}^{lev} = \Delta cov\left(\log w, \log h\right)$.

Now, consider the volatility effect ω_{IM}^{vol}:

$$\begin{aligned}
\omega_{IM}^{vol} &= -\frac{1}{\sigma}\frac{\Delta v_\varepsilon}{2} - \left[\frac{(1-\gamma)+\gamma\left(1+\sigma\right)}{\sigma+\gamma}\right]\frac{\Delta v_\alpha}{2} \\
&= -\sigma\left(\frac{1}{\sigma^2}\frac{\Delta v_\varepsilon}{2}\right) - \left[\left(\sigma+\gamma\right)\frac{(1-\gamma)+\gamma\left(1+\sigma\right)}{\left(\sigma+\gamma\right)^2}\right]\frac{\Delta v_\alpha}{2}
\end{aligned} \tag{44}$$

$$\begin{aligned}
&= -\sigma\left(\frac{1}{\sigma^2}\frac{\Delta v_\varepsilon}{2}\right) - \left[\frac{\sigma\left(1-\gamma\right)^2+\gamma\left(1+\sigma\right)^2}{\left(\sigma+\gamma\right)^2}\right]\frac{\Delta v_\alpha}{2} \\
&= -\sigma\left[\frac{1}{\sigma^2}\frac{\Delta v_\varepsilon}{2} + \left(\frac{1-\gamma}{\sigma+\gamma}\right)^2\frac{\Delta v_\alpha}{2}\right] - \gamma\left(\frac{1+\sigma}{\sigma+\gamma}\right)^2\frac{\Delta v_\alpha}{2}.
\end{aligned}$$

From the equilibrium allocations in (26), it is easy to derive that

$$\begin{aligned}
var\left(\log c\right) &= \left(\frac{1+\sigma}{\sigma+\gamma}\right)^2 v_\alpha, \\
var\left(\log h\right) &= \frac{1}{\sigma^2}v_\varepsilon + \left(\frac{1-\gamma}{\sigma+\gamma}\right)^2 v_\alpha.
\end{aligned}$$

Substituting these two expressions in the last row of (44) concludes the proof for the separable case.

Appendix E: Proof of Proposition 4

Cobb-Douglas preferences: We show that the equivalence between level effect and the change in average productivity holds for the incomplete markets case. The results for the other market structures are easy to recover, in light of this proof. Let π_{IM} be the average productivity in IM in the economy with variances $(v_\alpha, v_\varepsilon)$, i.e.

$$\pi_{IM} \equiv \frac{E\left(c_{IM}\left(\alpha,\varepsilon\right)\right)}{E\left(h_{IM}\left(\alpha,\varepsilon\right)\right)} = \frac{\eta}{1-(1-\eta)\exp\left(\lambda v_\varepsilon\right)} \simeq \frac{\eta}{1-(1-\eta)(1+\lambda v_\varepsilon)} = \frac{1}{1-\bar{\phi}v_\varepsilon},$$

where the log-approximation holds for λv_ε small. Let $\hat{\pi}_{IM}$ be the average productivity in IM in the economy with variances $\left(\hat{v}_\alpha, \hat{v}_\varepsilon\right)$. Then, the ratio of labor productivities across the two economies is

$$\frac{\hat{\pi}_{IM}}{\pi_{IM}} = \frac{1-\bar{\phi}v_\varepsilon}{1-\bar{\phi}\hat{v}_\varepsilon} \Rightarrow \Delta\log\pi_{IM} \simeq \bar{\phi}\Delta v_\varepsilon = \omega_{IM}^{lev}.$$

Separable preferences: Using the same notation as for the Cobb-Douglas case:

$$\begin{aligned}
\pi_{IM} &\equiv \frac{E\left(c_{IM}\left(\alpha,\varepsilon\right)\right)}{E\left(h_{IM}\left(\alpha,\varepsilon\right)\right)} = \frac{\exp\left(\frac{1+\sigma}{\gamma+\sigma}\left(\frac{v_\varepsilon}{2\sigma}\right)\right)\exp\left(\frac{1+\sigma}{\gamma+\sigma}\left(\frac{1-\gamma}{\gamma+\sigma}\right)\frac{v_\alpha}{2}\right)}{\exp\left(\frac{1+\sigma}{\gamma+\sigma}\left(\frac{-\gamma v_\varepsilon}{2\sigma^2}\right)\right)\exp\left(\frac{1-\gamma}{\gamma+\sigma}\left(\frac{1-\gamma}{\gamma+\sigma}-1\right)\frac{v_\alpha}{2}\right)\exp\left(\frac{1-\sigma}{\sigma^2}\frac{v_\varepsilon}{2}\right)} \\
&= \exp\left[\left(\frac{1+\sigma}{\gamma+\sigma}+\frac{\gamma\left(1+\sigma\right)}{\sigma\left(\gamma+\sigma\right)}-\frac{1-\sigma}{\sigma}\right)\frac{v_\varepsilon}{2\sigma}\right] \\
&\quad\cdot\exp\left[\left(\left(1-\gamma\right)\frac{\left(1+\sigma\right)-\left(1-2\gamma-\sigma\right)}{\left(\gamma+\sigma\right)^2}\right)\frac{v_\alpha}{2}\right] \\
&= \exp\left(\frac{v_\varepsilon}{\sigma}\right)\exp\left(\frac{1-\gamma}{\gamma+\sigma}v_\alpha\right),
\end{aligned}$$

where the second and third rows require some simple rearranging and simplifying. Then, the productivity gain following a rise in wage dispersion is

$$\Delta \log \pi_{IM} = \frac{\Delta v_{\varepsilon}}{\sigma} + \left(\frac{1-\gamma}{\gamma+\sigma}\right) \Delta v_{\alpha} = \omega_{IM}^{lev}.$$

Figure 1: Cobb-Douglas Preferences

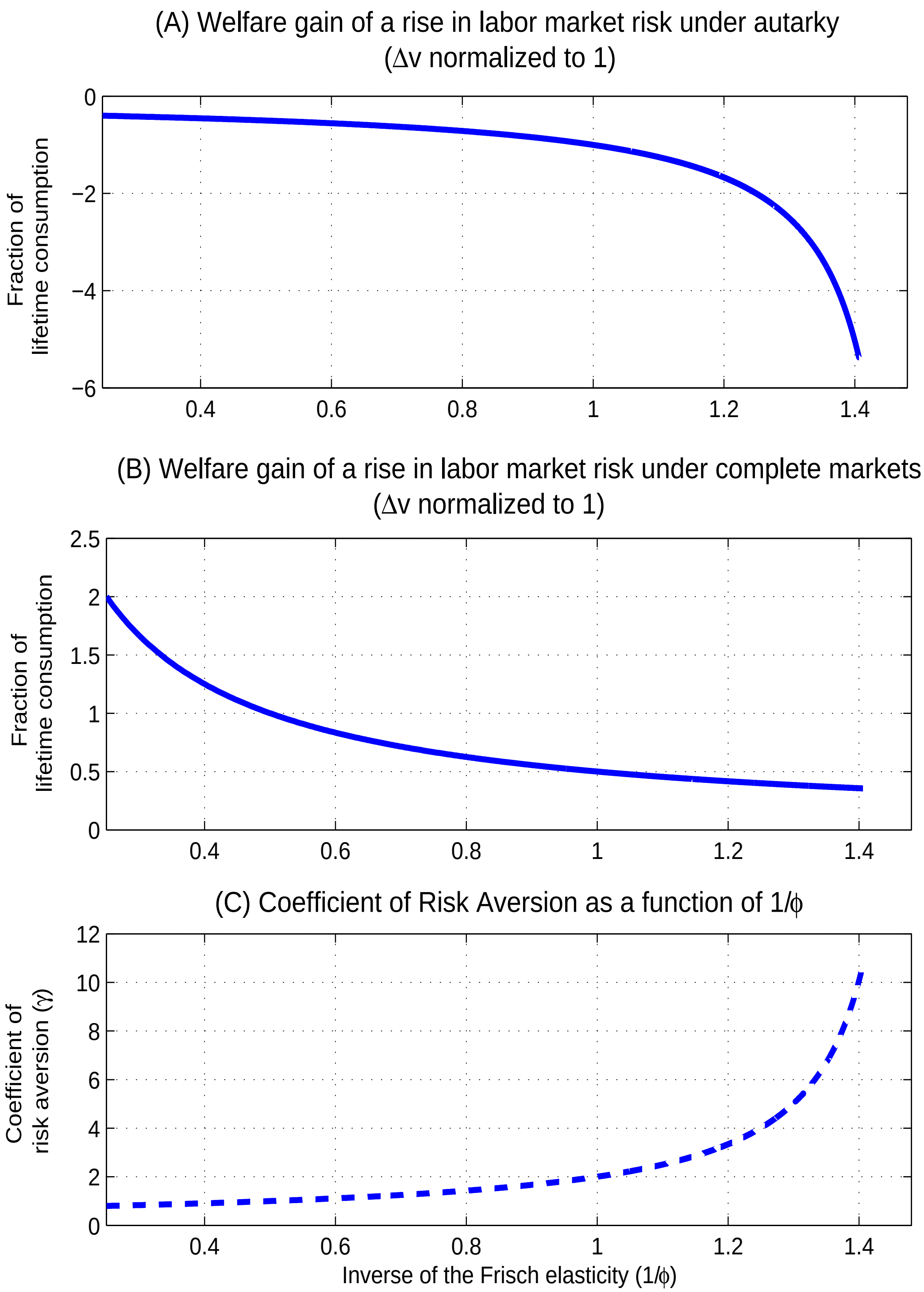

Figure 2: Cobb-Douglas Preferences

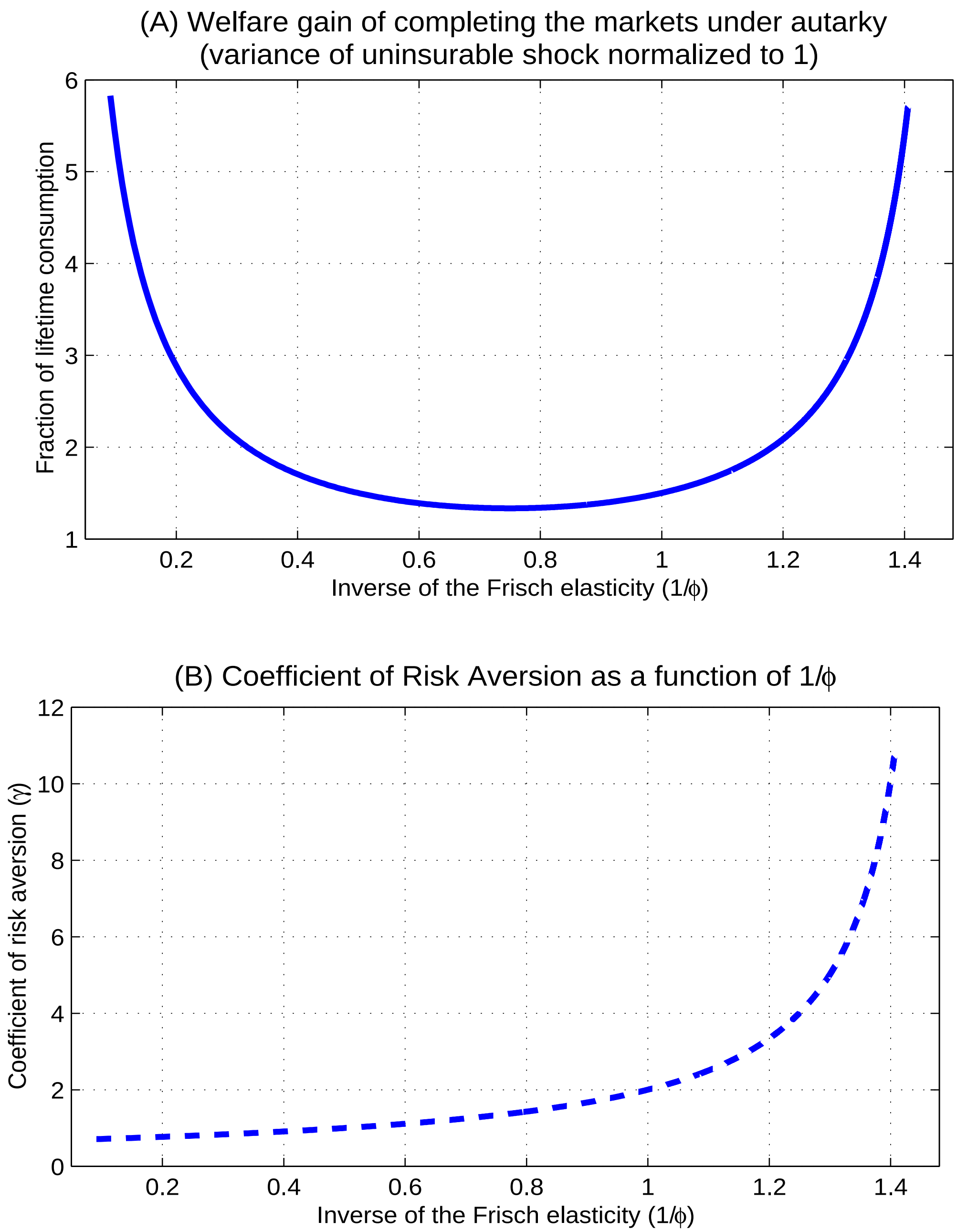

Figure 3: Separable Preferences

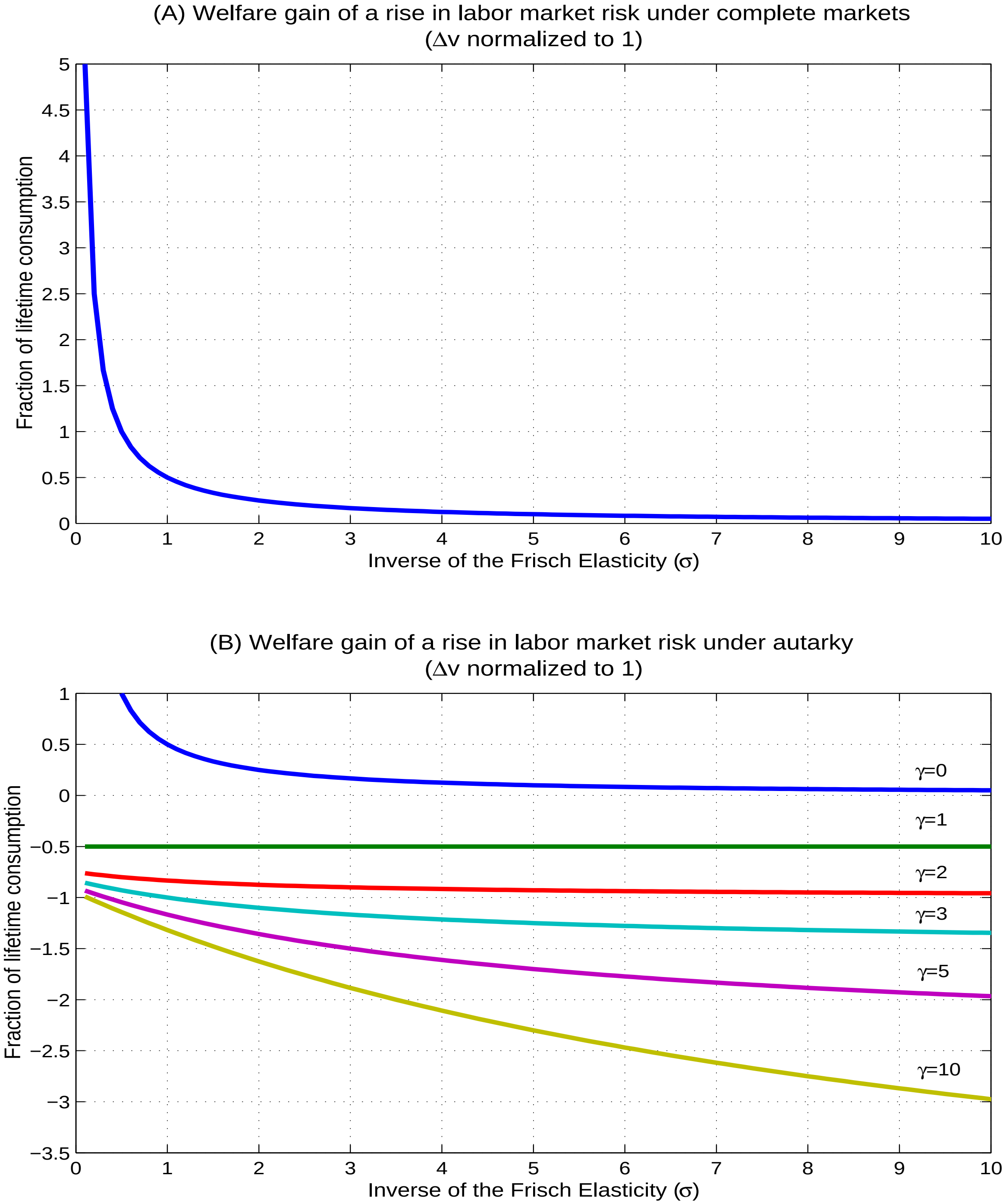

Figure 4: Separable Preferences