

Investment, Valuation, and Growth Options*

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Abstract

We develop a model in which the opportunity for a firm to upgrade its technology to the frontier (at a cost) leads to growth options in the value of the firm. Variation in the technological frontier leads to variation in firm value that is unrelated to current cash flow and investment, though variation in firm value anticipates future upgrades and investment. We simulate this model and show that in situations in which growth options are important, regressions of investment on Tobin's Q and cash flow yield small positive coefficients on Q and larger coefficients on cash flow, consistent with the empirical literature. We also show that when growth options are important, the volatility of firm value can substantially exceed the volatility of cash flow, as empirically documented by Shiller (1981) and West (1988).

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1 Introduction

A firm's value should measure the expected present value of future payouts to claimholders. This insight led Keynes (1936) and Brainard and Tobin (1968) to the ideas underlying Q theory—that the market value of installed capital (relative to uninstalled capital) summarizes the incentive to invest. This insight, while theoretically compelling, has met with mixed empirical success. Although Tobin's Q is typically correlated with investment in empirical studies, the relationship is sometimes weak and often dominated by the direct effect of cash flow on investment. Moreover, the measured volatility of firms' market values greatly exceeds the volatility of the fundamentals that they supposedly summarize, creating the “excess volatility” puzzle documented by Leroy and Porter (1981), Shiller (1981), and West (1988).

While these findings might be interpreted as irrationality in valuation, or as evidence that the stock market is a “sideshow” for real investment and value, we show that these phenomena can arise in an optimizing model with growth options. We examine the model developed in Abel and Eberly (2002), where the firm has a standard production function, with frictionless use of factor inputs (capital and labor). The only deviation from a frictionless model is that the firm must pay a fixed cost in order to upgrade its technology to the frontier. The frontier technology evolves exogenously and stochastically, and the firm pays the fixed cost to install the frontier technology when the frontier technology is sufficiently more productive than the firm's current technology. Once the new technology is installed, its productivity is fixed. The firm may upgrade again in the future by paying the fixed cost whenever it chooses.

The salient feature of this simple structure is the generation of “growth options” in the value of the firm. Even though the frontier technology is uninstalled and does not affect current cash flows, the firm has the option to upgrade its technology. Importantly, the value of this option fluctuates independently of current cash flow and creates a wedge between the firm's value and its current cash flow. Since the firm's investment is frictionless, it only depends on current conditions, which are summarized by current cash flow. Thus, instantaneous investment is most closely related to cash flow. However, eventually the firm will upgrade its technology, causing a burst of investment (since the marginal product of capital rises), creating an eventual link between valuation and investment. This generates a correlation between investment and Tobin's Q .

Discretely sampled data reflects both of these effects on investment. We would thus expect both cash flow and Tobin’s Q to be correlated with investment in discretely sampled data. Investment regressions including both Tobin’s Q and cash flow are often used as a diagnostic of the Q theory of investment and as a test for financing constraints. In the model examined here, both Q and cash flow are correlated with investment, but there are no adjustment costs (as in Q theory) and no financing constraints. By simulating the current model, allowing for discretely sampled data and also for time aggregation, the presence of growth options can result in a small regression coefficient on Q and a large effect of cash flow on investment. The former is often interpreted as an indicator of large capital adjustment costs—while in the current structure there are no adjustment costs at all. Similarly, following Fazzari, Hubbard, and Peterson (1988), a positive coefficient on cash flow when controlling for Q in an investment regression is often interpreted as evidence of financing constraints. Empirically, this cash flow effect is especially strong in subsamples of firms with characteristics consistent with restricted access to financing. In our model, these empirical characteristics (no dividend payout or no bond rating, for example) would suggest that these firms have important growth options, rather than necessarily facing restricted financing.

The presence of growth options causes fluctuations in firm valuation that are not matched by current variation in cash flows. Instead, this volatility can be driven by variation in the frontier technology. The independent variation in the growth options can thus also generate “excess volatility” in firm valuation relative to its fundamental cash flows. Such excess volatility has been empirically documented at least since Leroy and Porter (1981) (who examined equity prices relative to earnings) and Shiller (1981), who examined equity prices relative to dividends. Both of these studies required stationarity of the underlying processes, an assumption that was relaxed by West (1988), who also found excess volatility of equity prices relative to dividends. West found that equity prices were from four to twenty times too volatile relative to the variance implied by a present-value-of-dividends model and the observed volatility of dividends.

We begin Section 2 by laying out the model developed in Abel and Eberly (2002) and calculating the value of the firm and optimal investment used in the simulations. We then show how optimal investment behaves during the two regimes: during continuous investment periods between consecutive technology upgrades, investment is driven by cash flow, while Q predicts technology upgrades and the associated “gulps”

of investment. In Section 3 we simulate the model and quantitatively evaluate the model’s implications for the comovements among investment, Tobin’s Q , and cash flow. We also show how growth options may account for the effects of Tobin’s Q and cash flow that have been estimated in the empirical literature. Finally, in Section 4, we calculate the relative variances of (log changes in) firm value and cash flow and show that when growth options are important, the model can generate considerable “excess volatility.” Section 5 offers concluding comments.

2 A Model of the Firm with Growth Options

This section briefly describes the structure and solution of the model in Abel and Eberly (2002). The model is solved in two steps. Since capital is costlessly adjustable, we first solve for optimal factor choice and operating profit for a given level of the firm’s technology. Once these values are derived in Section 2.1, we analyze the firm’s optimal upgrade decisions in Section 2.2. We then solve for the value of a firm that has access to the frontier technology and upgrades optimally. Using the value of the firm and the optimal capital stock, we calculate the average value of the capital stock, or Tobin’s Q . Then in Section 2.3, we analyze the relationship among investment, Tobin’s Q and cash flow.

2.1 Operating Profits and Static Optimization

Let the firm’s revenue, net of the cost of flexible factors other than capital, be given by $(A_t Y_t)^{1-\gamma} K_t^\gamma$, where A_t is the level of technology, Y_t is the level of demand (which may also represent wages or the prices of other flexible factors), and K_t is the capital stock.¹ The firm has decreasing returns to scale in production or market power in the output market, so that $0 < \gamma < 1$. Define the user cost factor as $u_t \equiv r + \delta_t - \mu_p$, where r is the discount rate, δ_t is the depreciation rate of capital at time t ,² and p_t is the price of capital, which grows deterministically at rate μ_p . Operating profits,

¹The fact that that A_t and Y_t are raised to the $1 - \gamma$ power in the revenue function reflects a convenient normalization that exploits the fact that if a variable x_t is a geometric Brownian motion, then x_t^α is also a geometric Brownian motion.

²We allow the depreciation rate to be stochastic to motivate the stochastic user cost of capital. Specifically, since the user cost factor is $u_t \equiv r + \delta_t - \mu_p$, the increment to the user cost factor, u_t , equals the increment to the depreciation rate, $du_t = d\delta_t$.

which are net revenue minus the user cost of capital, are given by

$$\pi_t = (A_t Y_t)^{1-\gamma} K_t^\gamma - u_t p_t K_t \quad (1)$$

where $u_t p_t$ is the user cost of a unit of capital. Maximizing operating profits in equation (1) with respect to K_t yields³ the optimal capital stock

$$K_t = \frac{A_t X_t}{u_t p_t} \frac{\gamma}{1-\gamma}, \quad (2)$$

and the optimized value of operating profits

$$\pi_t = A_t X_t, \quad (3)$$

where

$$X_t \equiv Y_t \left(\frac{\gamma}{u_t p_t} \right)^{\frac{\gamma}{1-\gamma}} (1-\gamma) \quad (4)$$

summarizes the sources of non-technology uncertainty about operating profits. We assume that X_t follows a geometric Brownian motion

$$dX_t = mX_t dt + sX_t dz_X, \quad (5)$$

where the drift, m , and instantaneous variance, s^2 , depend on the drifts and instantaneous variances and covariances of the underlying processes for Y_t , u_t , and p_t . We assume that the user cost factor, u_t , follows a driftless Brownian motion, with instantaneous variance σ_u^2 .⁴

³Differentiating the right-hand side of equation (1) with respect to K_t , and setting the derivative equal to zero yields

$$\gamma \left(\frac{A_t Y_t}{K_t} \right)^{1-\gamma} = u_t p_t. \quad (*)$$

Solving this first-order condition for the optimal capital stock yields

$$K_t = A_t Y_t \left(\frac{\gamma}{u_t p_t} \right)^{\frac{1}{1-\gamma}}. \quad (**)$$

Substituting equation (**) into the operating profit function in equation (1) yields optimized operating profits

$$\pi_t = u_t p_t K_t \left(\frac{1-\gamma}{\gamma} \right) = A_t Y_t \left(\frac{\gamma}{u_t p_t} \right)^{\frac{\gamma}{1-\gamma}} (1-\gamma). \quad (***)$$

Use the definition of X_t in equation (4) to rewrite equation (**) as equation (2) and equation (***) as equation (3).

⁴If Y_t , u_t , and p_t are geometric Brownian motions, then the composite term X_t also follows a geometric Brownian motion. Specifically, let the instantaneous drift of the process for Y_t be μ_Y

Since in the next section we will examine the relationship between investment and cash flow, note that the firm's cash flow before investment expenditure is given by $C_t \equiv (A_t Y_t)^{1-\gamma} K_t^\gamma$. Equations (1), (3) and equation (***) in footnote (3) imply that

$$C_t \equiv \frac{\pi_t}{1-\gamma} = \frac{A_t X_t}{1-\gamma}. \quad (6)$$

Empirically, cash flow is usually normalized by the replacement cost of the capital stock, $p_t K_t$, so we define the cash flow-capital stock ratio,

$$c_t \equiv \frac{C_t}{p_t K_t} = \frac{u_t}{\gamma} = \frac{1}{\gamma} (r + \delta_t - \mu_p), \quad (7)$$

which is proportional to the user cost factor when the capital stock is optimally chosen.⁵

During periods when the firm does not change its technology, the capital stock is chosen continuously to maximize operating profits. We call this *continuous investment*, which is obtained by calculating the change in the capital stock by applying Ito's Lemma to the expression for the optimal capital stock in equation (2). When no upgrade occurs, $dA_t = 0$ so the growth in the capital stock is given by

$$\frac{dK_t}{K_t} = \frac{dX_t}{X_t} - \frac{du_t}{u_t} + (\sigma_u^2 - \rho_{Xu} s \sigma_u - \mu_p) dt. \quad (8)$$

Equation (8) shows the ratio of net investment to the capital stock when no upgrade occurs. Gross investment, I_t , is net investment, dK_t , plus depreciation, $\delta_t K_t dt$. Using the definition of gross investment yields the gross investment rate

$$\frac{I_t}{K_t} = \delta_t dt + \frac{dK_t}{K_t} = (\delta_t + m + \sigma_u^2 - \rho_{Xu} s \sigma_u - \mu_p) dt + s dz_X - \sigma_u dz_u. \quad (9)$$

To relate the drift term in equation (9) to cash flow per unit of capital, use equation (7) to substitute $\gamma c_t - r$ for $\delta_t - \mu_p$ to obtain

$$\frac{I_t}{K_t} = (\gamma c_t - \Gamma) dt + s dz_X - \sigma_u dz_u, \quad (10)$$

and its instantaneous variance be σ_Y^2 . Then given our specification of the processes for u_t and p_t , $m \equiv \mu_Y - \frac{\gamma}{1-\gamma} \left[\mu_p - \frac{1}{2} \frac{\sigma_u^2}{1-\gamma} + \rho_{Yu} \sigma_Y \sigma_u \right]$ and $s dz_X = \sigma_Y dz_Y - \frac{\gamma \sigma_u}{1-\gamma} dz_u$, where $\rho_{Yu} \equiv \frac{1}{dt} E(dz_Y dz_u)$ is the correlation between the shocks to Y_t and u_t . In addition, $s^2 = \sigma_Y^2 + \left(\frac{\gamma \sigma_u}{1-\gamma} \right)^2 - 2 \frac{\gamma}{1-\gamma} \rho_{Yu} \sigma_Y \sigma_u$; $s \rho_{Xu} = \rho_{Yu} \sigma_Y - \frac{\gamma}{1-\gamma} \sigma_u$; and $s \rho_{X\hat{A}} = \rho_{Y\hat{A}} \sigma_Y - \frac{\gamma}{1-\gamma} \rho_{u\hat{A}} \sigma_u$, where $\rho_{ij} \equiv \frac{1}{dt} E(dz_i dz_j)$.

⁵Use the first equality in equation (***) in footnote 3 to substitute for π_t in the definition of C_t . Using the definition of the cash flow-to-capital ratio, this yields $c_t \equiv \frac{C_t}{p_t K_t} = \frac{u_t p_t K_t}{\gamma p_t K_t} = \frac{u_t}{\gamma}$.

where $\Gamma \equiv r - m - \sigma_u^2 + \rho_{X_u} s \sigma_u$ is constant.

The technology variable, A_t , represents the firm's currently installed technology. The firm also has the choice to upgrade to the available technology, \widehat{A}_t , which evolves exogenously according to the geometric Brownian motion

$$d\widehat{A}_t = \mu \widehat{A}_t dt + \sigma \widehat{A}_t dz_{\widehat{A}}. \quad (11)$$

The instantaneous correlation between the innovations to X_t and \widehat{A}_t is $\frac{1}{dt} E(dz_X dz_{\widehat{A}}) \equiv \rho_{X\widehat{A}}$, and we assume that $\mu > \frac{1}{2}\sigma^2$.⁶

2.2 Optimal Upgrades and the Value of the Firm

The calculations above are performed conditional on the level of installed technology, A_t . Now consider the firm's decision about when to upgrade to the frontier technology, \widehat{A}_t . The cost of upgrading to the frontier technology, \widehat{A}_t , at time t , is $\theta_t \widehat{A}_t X_t$, where $\theta \geq 0$ is a constant. Because upgrading incurs a fixed cost (the cost depends only on exogenous variables), it will not be optimal to upgrade continuously. The firm optimally determines discrete times τ_j , $j = 0, 1, 2, \dots$ at which to upgrade.

Begin with a firm that does not own any capital. This firm rents the services of capital at each point in time, paying a user cost of $u_t p_t$ per unit of capital at time t . The value of this firm is the expected present value of operating profits less the cost of technology upgrades. Let $\Psi(A_t, X_t, \widehat{A}_t)$ be the expected present value of operating profits, net of upgrade costs, from time t onward, so

$$\Psi(A_t, X_t, \widehat{A}_t) = \max_{\{\tau_j\}_{j=1}^{\infty}} E_t \left\{ \int_0^{\infty} A_{t+s} X_{t+s} e^{-rs} ds - \sum_{j=1}^{\infty} \theta \widehat{A}_{\tau_j} X_{\tau_j} e^{-r(\tau_j-t)} \right\}, \quad (12)$$

where \widehat{A}_{τ_j} is the value of the available frontier technology when the upgrade occurs at time τ_j . We require that (1) $r - m > 0$ so that a firm that never upgrades has finite value; (2) $r - m - \mu - \rho_{X\widehat{A}} s \sigma > 0$ so that a firm that continuously maintains $A_t = \widehat{A}_t$ has a value that is bounded from above;⁷ and (3) $(r - m)\theta < 1$ so that the upgrade cost is not large enough to prevent the firm from ever upgrading.⁸

⁶The assumption that $\mu > \frac{1}{2}\sigma^2$ guarantees that the expected first passage time to the upgrade threshold is finite. We also assume initial conditions $X_0, \widehat{A}_0, u_0, p_0 > 0$.

⁷The condition $r - m - \mu - \rho_{X\widehat{A}} s \sigma > 0$ implies that even if the firm could maintain $A_t = \widehat{A}_t$ for all t without facing any upgrade costs, its value would be finite. Therefore, the value of a firm that faces upgrade costs would be bounded from above if it maintained $A_t = \widehat{A}_t$ for all t .

⁸See footnote 11 for the properties of the upgrade trigger \bar{u} .

Using the Bellman equation, we calculate the value of the firm when it is not upgrading, and then use the boundary conditions that hold when the firm upgrades its technology. These conditions are discussed in more detail in Abel and Eberly (2002). The required return on the firm, $r\Psi_t$, must equal current operating profits plus its expected capital gain. When the firm is not upgrading its technology, A_t is constant, so the equality of the required return and the expected return can be written as (omitting time subscripts)

$$\begin{aligned} r\Psi &= \pi + E(d\Psi) \\ &= AX + mX\Psi_X + \frac{1}{2}s^2X^2\Psi_{XX} + \mu\hat{A}\Psi_{\hat{A}} + \frac{1}{2}\sigma^2\hat{A}^2\Psi_{\hat{A}\hat{A}} + \rho_{X\hat{A}}s\sigma X\hat{A}\Psi_{X\hat{A}}. \end{aligned} \quad (13)$$

Direct substitution verifies that the following function satisfies the differential equation in equation (13)

$$\Psi(A_t, X_t, \hat{A}_t) = \frac{A_t X_t}{r - m} + BA_t X_t \left(\frac{\hat{A}_t}{A_t} \right)^\phi, \quad (14)$$

where B is an unknown constant and the parameter $\phi > 1$ is the positive root^{9,10} of the quadratic equation

$$f(\zeta) \equiv r - m - (\mu + \rho_{X\hat{A}}s\sigma - \frac{1}{2}\sigma^2)\zeta - \frac{1}{2}\sigma^2\zeta^2 = 0. \quad (15)$$

The boundary conditions imposed at times of technological upgrading determine the constant B and the rule for optimally upgrading to the new technology. The first boundary condition requires that at the time of the upgrade, the value of the firm increases by the amount of the fixed cost. The second boundary condition requires that the value of the firm is maximized with respect to the choice of τ_j , the upgrade time. In Abel and Eberly (2002) we show that solving these two equations yields an expression for the constant B as a function of the upgrade threshold, \bar{a} ,

$$B = \frac{\bar{a}^{-\phi}}{(\phi - 1)(r - m)} > 0, \quad (16)$$

⁹Notice that $f(0) > 0$, $f(1) > 0$, and $f''(\zeta) < 0$, so that the positive root of this equation exceeds one.

¹⁰An additional term including the negative root of the quadratic equation also enters the general solution to the differential equation. However, the negative exponent would imply that the firm's value goes to infinity as the frontier technology approaches zero. We set the unknown constant in this term equal to zero and eliminate this term from the solution.

where $a \equiv \frac{\hat{A}}{A}$ is the value of the frontier technology relative to the installed technology, and \bar{a} is the threshold value of this ratio at which an upgrade is optimally undertaken. The boundary conditions also yield a single nonlinear equation characterizing the threshold for optimal upgrades

$$g(a; \theta) \equiv a - 1 - \frac{1 - a^{1-\phi}}{\phi - 1} - a\theta(r - m) = 0 \quad \text{for } a = \bar{a}. \quad (17)$$

Notice that this expression depends only on the relative technology, $a \equiv \frac{\hat{A}}{A}$, and constant parameters. Therefore, the relative technology a must have the same value whenever the firm upgrades its technology; we defined this boundary value above as \bar{a} , so $g(\bar{a}; \theta) = 0$. It is straightforward to verify that $\bar{a} \geq 1$, with strict inequality when $\theta > 0$ and that $\frac{d\bar{a}}{d\theta} > 0$ when $\theta > 0$.¹¹ The firm upgrades A_t to the available technology when \hat{A}_t reaches a sufficiently high value; specifically, the firm upgrades when $\hat{A}_t = \bar{a} \times A_t \geq A_t$. The size of the increase in A_t that is needed to trigger an upgrade is an increasing function of the fixed cost parameter θ .

Substituting equation (16) into the value of the firm in equation (14) yields

$$\Psi \left(A_t, X_t, \hat{A}_t \right) = \frac{A_t X_t}{r - m} \left[1 + \frac{1}{\phi - 1} \left(\frac{a_t}{\bar{a}} \right)^\phi \right] > \frac{A_t X_t}{r - m}. \quad (18)$$

The value of the firm in equation (18) can be interpreted as the product of two terms: the expected present value of operating profits evaluated along the path of no future upgrades, $\frac{A_t X_t}{r - m}$, multiplied by a term that exceeds one and captures the value of growth options associated with expected future technological upgrades. If the frontier technology were permanently unavailable, so that the firm would have to maintain the current level of technology, A_t , forever, then the value of the firm would simply be $\frac{A_t X_t}{r - m}$. However, since the firm has the option to adopt the frontier technology, the value of firm is increasing in the relative value of the frontier technology, a_t , as well as in current operating profits, $A_t X_t$.

As noted above, $\Psi \left(A_t, X_t, \hat{A}_t \right)$ gives the value of a firm that never owns capital but rents the services of capital at each point of time. The value of a firm that owns

¹¹To see that $\bar{a} \geq 1$, use $\phi > 1$ and $(r - m)\theta < 1$ to note that $\lim_{a \rightarrow 0} g(a; \theta) > 0$, $g(1; \theta) = -\theta(r - m) < 0$, $\lim_{a \rightarrow \infty} g(a; \theta) > 0$, and $g''(a; \theta) > 0$. Thus $g(a; \theta)$ is a convex function of a with two distinct positive roots, $0 < \underline{a} < 1 < \bar{a}$, when $\theta > 0$, with $\frac{\partial g(\underline{a}; \theta)}{\partial a} < 0$ and $\frac{\partial g(\bar{a}; \theta)}{\partial a} > 0$. The smaller root, $\underline{a} < 1$, can be ruled out since it implies that the firm reduces the value of its technology whenever it changes technology. Since $\frac{\partial g(\bar{a}; \theta)}{\partial \theta} = -(r - m)a < 0$, the implicit function theorem implies that $\frac{d\bar{a}}{d\theta} > 0$ when $\theta > 0$. When $\theta = 0$ there is a unique positive value of a that solves equation (17); specifically, $\bar{a} = 1$ when $\theta = 0$.

capital K_t at time t is simply equal to the sum of $p_t K_t$ and $\Psi(A_t, X_t, \hat{A}_t)$. Thus, if V_t is the value of the firm at time t , equation (18) implies that

$$V_t = p_t K_t + \frac{A_t X_t}{r - m} \left[1 + \frac{1}{\phi - 1} \left(\frac{a_t}{\bar{a}} \right)^\phi \right]. \quad (19)$$

Tobin's Q is the ratio of the value of the firm, V_t , to the replacement cost of the firm's capital stock, $p_t K_t$. To calculate Tobin's Q , first use equation (6) to substitute $(1 - \gamma) C_t$ for $A_t X_t$ in equation (19), and then divide both sides of equation (19) by the replacement cost of the capital stock, $p_t K_t$, to obtain

$$Q_t \equiv \frac{V_t}{p_t K_t} = 1 + \frac{(1 - \gamma) C_t}{p_t K_t (r - m)} \left[1 + \frac{1}{\phi - 1} \left(\frac{a_t}{\bar{a}} \right)^\phi \right]. \quad (20)$$

This can be simplified by substituting the cash flow-capital stock ratio from equation (7) into equation (20) to obtain

$$Q_t = 1 + \frac{1 - \gamma}{r - m} c_t \left[1 + \frac{1}{\phi - 1} \left(\frac{a_t}{\bar{a}} \right)^\phi \right]. \quad (21)$$

Tobin's Q exceeds 1 because of the rents represented by the operating profits, π_t . The excess of Tobin's Q over 1 can be decomposed into the product of two terms. The first is the expected present value of operating profits per unit of capital (measured at replacement cost), calculated along the path of no future technological upgrades so that A_t is held fixed indefinitely. The second term reflects the value of the expected future upgrades. It is an increasing function of the value of the frontier technology relative to the installed technology, measured by a_t .

When a firm upgrades its technology, its capital stock jumps upward because the marginal product of capital jumps upward. We refer to this jump in the capital stock as an "investment gulp."¹² The investment gulp that accompanies the upgrade at time τ_j is calculated using the expression for the optimal capital stock in equation (2) to obtain¹³

$$\frac{K_{\tau_j}^+}{K_{\tau_j}^-} = \frac{A_{\tau_j}^+}{A_{\tau_j}^-} = \frac{\hat{A}_{\tau_j}}{\hat{A}_{\tau_j-1}} = \bar{a}. \quad (22)$$

¹²Hindy and Huang (1993) use the term "gulps" of consumption to describe jumps in the cumulative stock of consumption. We borrow their term to apply to jumps in the stock of capital, which is the cumulation of past (net) investment.

¹³The superscript "+" denotes the instant of time immediately following τ_j , and the superscript "-" denotes the instant of time immediately following τ_j .

This increase in the capital stock occurs instantaneously and is a component of investment for any interval of time that contains τ_j . Whenever the firm upgrades its technology, its capital stock jumps instantly by a factor \bar{a} .

Technological upgrades occur when the level of the frontier technology, \hat{A}_t , becomes high enough relative to the installed technology, A_t , to compensate for the cost of upgrading to the frontier. The ratio of the frontier technology to the installed technology, a_t , is a sufficient statistic for the upgrade decision. If a_t is below the threshold value, \bar{a} , the firm does not upgrade. When a_t reaches \bar{a} , the firm upgrades its technology to the frontier. However, the frontier technology, and hence a_t , is unobservable to an outside observer. Tobin's Q , however, provides an observable measure of a_t that can help predict the timing of upgrades and gulps. In particular, we rearrange equation (21) to produce an expression for a_t as a function of the observable variables Q_t and c_t ,

$$a_t = \left[\frac{Q_t - 1}{c_t} \frac{r - m}{1 - \gamma} - 1 \right]^{\frac{1}{\phi}} (\phi - 1)^{\frac{1}{\phi}} \bar{a} > 0. \quad (23)$$

Equation (23) implies that the expected time until an upgrade is decreasing in Q_t . High values of Tobin's Q thus predict imminent technology upgrades and the associated gulps of investment.

2.3 Investment, Tobin's Q and Cash Flow

Over finite intervals of time, investment consists of two components: continuous investment and gulps of investment. Continuous investment refers to the continuous variation in the optimal capital stock (equation (2)) that arises from continuous variation in X_t , p_t , and u_t . Gulps of investment are associated with technological upgrades.

To assess the relationship among continuous investment, Q , and cash flow, first rewrite equation (10) using equation (7) and the Brownian motion for u_t (so that $du_t = \sigma_u dz_u$) to obtain

$$\frac{I_t}{K_t} = (\gamma c_t - \Gamma) dt + s dz_X - \gamma dc_t, \quad (24)$$

where dc_t is the innovation to cash flow (scaled by the value of the capital stock). Notice that the investment rate is an increasing and linear function of the level of cash flow, c_t , but a decreasing function of the innovation to cash flow, dc_t . Furthermore,

the investment rate is independent of Q_t . A high *level* of cash flow is associated with a high value of the user cost factor, specifically a high rate of depreciation. The high rate of depreciation implies that gross investment, which includes replacement investment, will be high. An *increase* in cash flow is associated with an increase in the user cost factor, which reduces the optimal capital stock and causes investment to be low.

While continuous investment is independent of Q , it is clear from equation (23) that Q has a role to play in predicting technological upgrades and the associated gulps of investment. Upgrades occur when the frontier technology relative to the installed technology, measured by a , is sufficiently high. Although this value is unobservable, from equation (23), a is an increasing function of Q . Hence, high values of Q should also be associated with technological upgrades and gulps of investment. Combining continuous investment and investment gulps, as we do in our simulations, thus yields a role for both Q and cash flow in explaining investment.

3 Investment, Tobin's Q , and Cash Flow: Simulation Results

We now quantitatively examine the magnitudes of the effects of Q and cash flow on investment. We simulate the model by first choosing a baseline set of parameters. We solve for the optimal upgrade threshold, \bar{a} , given these parameters and then generate a quarterly series of normally-distributed shocks for each of the random variables, u , \hat{A} , and Y , in the model. We generate a simulated panel of data, corresponding to 500 firms over 80 quarters (roughly the size of the Compustat data set often used in empirical work). To generate heterogeneity among firms, we simulate a pre-sample period of 100 quarters. At the beginning of the pre-sample period, every firm has $a_t = 1$, i.e., $A_t = \hat{A}_t$. During the pre-sample period, each firm faces its own set of shocks to \hat{A}_t and optimally upgrades A_t . For each firm, the value of a at the end of the pre-sample period is used as the initial value of a during the simulations. Using the solution for \bar{a} and the path of \hat{A} , we solve for optimal upgrades and the path of the installed technology, A . We also calculate the composite variable X to summarize the non-technology sources of uncertainty, and then solve for the variables of interest: the capital stock and investment, cash flow, firm value, and Tobin's Q . After generating 500 time series of 80 quarters, we run the regressions indicated in

each of the tables. We repeat this process 100 times to assess the variability of the estimated regression parameters. Table 2 reports the means and standard deviations of the regression parameters across the 100 replications.

Table 1 reports basic features of the model under alternative parameterizations. The first row reports the results for the baseline parameters; the remaining rows give the analytic features of the model as we vary parameter values. The first column of results reports the upgrade threshold \bar{a} . The next two columns report the value of Q associated with $a = \bar{a}$ (before upgrading) and $a = 1$ (after upgrading) for each of the parameterizations.

Table 1: Features of the Model and Parameter Variation			
deviation from baseline:	threshold, \bar{a}	Q before upgrade	Q after upgrade
none	1.55	5.19	3.73
$\sigma = 0.40$	1.55	4.52	3.31
$\sigma = 0.20$	1.53	3.58	2.72
$\mu = .085, \sigma = 0.40$	1.49	2.49	2.03
$\mu = .025, \sigma = 0.20$	1.33	1.65	1.52
$\sigma_u = 0.03$	1.59	3.10	2.35
$\theta = 0.25$	1.35	5.19	4.13
$\theta = 0.10$	1.20	5.19	4.51
$\theta = 0.10, \rho_{u\hat{A}} = .25$	1.19	3.17	2.83
$\theta = 0.05, \rho_{u\hat{A}} = -0.2$	1.14	24.4	21.5
$\theta = 0.05, \rho_{Y\hat{A}} = 0.2$	1.14	14.1	12.5
$\theta = 0.05, \rho_{u\hat{A}} = -0.1,$ $\rho_{Y\hat{A}} = 0.1$	1.14	17.8	15.7

The baseline parameterization is $r = 0.18, \gamma = 0.8, \theta = 0.5, \mu = 0.13, \sigma = 0.5,$
 $\mu_Y = 0.065, \sigma_Y = 0.20, \sigma_u = 0.06, \mu_p = 0.02,$ and $\rho_{Y\hat{A}} = \rho_{u\hat{A}} = \rho_{Yu} = 0.$
These values imply the parameters governing X are $m = 0.021$ and $s = 0.312.$
The calculation of \bar{a} is described in the text. The calculations of Q before and
after upgrade use equation (21), assuming $\delta = 0.1,$ and $a = \bar{a}$ (before
adjustment) and $a = 1$ (after adjustment).

The first four rows after the baseline of Table 1 explore the implications of changing the parameters μ and σ , which govern the frontier technology. While lowering the variance of the frontier does not change the upgrade threshold, \bar{a} , very much, there is a large effect on Tobin's Q . As the variance falls, the option becomes less valuable, and Tobin's Q falls, since Q measures the present value of future operating profits – including future technological improvements. When the option is important, Q can be quite large.

The next row of the table returns the frontier technology to its baseline and shows the effect of decreasing σ_u . In the structure of the model, σ_u operates through its effect on the parameters m and s , which govern X . Specifically, a decrease in σ_u , as in the table, decreases m .¹⁴ A reduction in the growth rate of X increases the effective discount rate, $r - m$, applied by the firm. This tends to increase the threshold for upgrading, \bar{a} , and also to decrease Q .

The middle rows (rows 6 and 7 after the baseline) of Table 1 examine the consequences of reducing the fixed cost, θ , of upgrading. Not surprisingly, the threshold, \bar{a} , for upgrading is reduced. Tobin's Q evaluated after the upgrade ($a = 1$) rises since on average capital is more valuable in a firm with smaller adjustment costs. An interesting feature of these results is that the value of Q at the time of upgrade is invariant to the size of the fixed cost; in these parameterizations, it is equal to 5.19 for all values of the fixed cost. To see why this is the case, recall the expression for Q in equation (21). Evaluated at $a = \bar{a}$, this expression yields $Q = 1 + \frac{1-\gamma}{r-m} c_t \left(\frac{\phi}{\phi-1} \right)$, which is not a function of the fixed cost.¹⁵ Thus, the value of Q at the time of upgrades does not depend on the fixed cost.

The final rows of the table allow for cross-correlations among the various stochastic processes in the model; these effects will be useful in our assessment of regression coefficients and variance bounds below. In particular, we allow for correlation between the components of X and the frontier technology, \hat{A} . Keeping a low fixed cost (10%, which will be convenient later) and allowing a positive correlation between the user cost factor, u , and \hat{A} has only a small (negative) effect on the upgrade threshold, but a substantial negative effect on Q . In this case, positive innovations to the frontier

¹⁴The decrease in σ_u also increases s . This affects the optimal upgrade threshold and upgrade behavior only if X is correlated with the frontier technology, which is not the case in this parameterization.

¹⁵The constant ϕ is the solution to the quadratic equation (15) which is independent of the fixed cost, θ .

technology occur when the user cost of capital is high; this comovement reduces the average value of capital to the firm. When a lower fixed cost (5%) is combined with a negative correlation (-0.2) between u and \hat{A} , the upgrade threshold falls slightly, but the value of capital skyrockets: Q evaluated after adjustment (at $a = 1$) equals 21.5. A similar, but smaller, effect occurs when the low fixed cost (again 5%) is combined with a positive correlation between demand, Y , and \hat{A} : Q evaluated after adjustment (at $a = 1$) equals 12.5. This occurs because positive innovations to the frontier technology are associated with positive innovations to demand; this favorable comovement increases the value of capital. In the final row, where the frontier technology is correlated with both u (negatively) and Y (positively), again the value of capital is high.

Table 2 reports the estimated coefficients on Tobin's Q and the cash flow-to-capital ratio, c_t , in investment regressions in our simulated data. The first column reports the parameterization, using a subset of the variations shown in Table 1. The columns headed "no time aggregation" directly examine the data simulated at a quarterly frequency (though all rates are annualized for comparison purposes). The first two columns, labeled "univariate", report the results of univariate regressions of the investment rate, $\frac{I_t}{K_t}$, on Q_t and c_t , respectively. The next two columns report the results of a multivariate regression of $\frac{I_t}{K_t}$ on both Q_t and c_t together. In the quarterly regressions, gross investment I_t is the increase in the capital stock from quarter t to quarter $t+1$ plus depreciation, i.e., $K_{t+1} - (1 - \delta_{t+1})K_t$, where δ_{t+1} is the depreciation rate. The last four columns repeat the univariate and multivariate regressions for time-aggregated data. Specifically, we aggregate the quarterly data to annual data. Gross investment during year t is the sum of the four quarters of gross investment during year t . The explanatory variables for the annual investment-capital ratio are the value of the Q in the final quarter of the previous year, and the sum of cash flow during the four quarters of the previous year normalized by the capital stock during the first quarter of the previous year.

The first two columns of results in Table 2 report coefficients from regressing simulated investment on Q , and then on the cash flow-to-capital ratio in univariate regressions. In all of the simulations, the coefficient on cash flow is between 0.65 and 0.99, and most of the cash flow coefficients are close to 0.8. Recall that in the expression for continuous investment in equation (24), the coefficient on c_t is γ , which is 0.8 in all of these simulations.¹⁶ The coefficient on Q is, in all cases, positive and

¹⁶When $\gamma = 0.6$ and the rest of the parameters are set at their baseline values, the coefficient on

smaller than the coefficient on cash flow. The largest coefficient on Q in the univariate quarterly regressions is 0.66 and occurs when the frontier technology grows relatively slowly and steadily; this is the parameterization in which growth options have the least value. At the bottom of the table, where growth options are very valuable, the coefficient on Q is as small as 0.01.

When Q and cash flow are simultaneously included in the investment regressions (reported in columns 3 and 4 of the results in Table 2), the coefficient on Q is virtually unchanged from the univariate regressions¹⁷ and the coefficient on c_t uniformly falls relative to the univariate regressions, often by a substantial amount. In one case, in which the frontier technology grows slowly and steadily ($\mu = 0.025$ and $\sigma = 0.2$) so growth options are less important, the cash flow coefficient even becomes negative. Recall from equation (21) that Q_t is a linear function of c_t , though the slope of this linear relation changes when a_t changes. Thus, investment depends on both Q_t (through gulps) and c_t (through continuous investment), but Q_t and c_t are colinear. This colinearity reduces the coefficient on c_t but has no effect on the coefficient on Q_t , relative to the univariate regressions. This asymmetry may occur because the investment gulps that occur when technology is upgraded are much larger than increases in the capital stock resulting from continuous investment between upgrades. These gulps are predicted by Q_t , but c_t has no predictive power for these gulps.

When multivariate regressions appear in the empirical literature, the estimated coefficient on Q is typically very small and the estimated coefficient on cash flow is typically much larger. This pattern is found in the bottom rows of Table 2, where growth options are important.

We examine the impact of time aggregation in the final four columns of Table 2, which report the results of regressions run on data aggregated to annual frequency, as described earlier. Time aggregation has essentially no effect on the estimated coefficients on Q . For both univariate and multivariate regressions, the coefficient on Q is almost identical for quarterly data (expressed at annual rates) and for annual data.¹⁸ The major impact of time aggregation is on the coefficient on cash flow.

cash flow in a univariate regression is 0.59 with a standard deviation of 0.04.

¹⁷The only exception is the case in which $\mu = 0.025$ and $\sigma = 0.2$, in which the coefficient on Q grows by about 1/3.

¹⁸The sole exception is the case in which $\mu = 0.025$ and $\sigma = 0.2$, but even in this case the effect of time aggregation is very small, reducing the coefficient from 0.66 to 0.62 in the univariate case, and from 0.86 to 0.64 in the multivariate case.

Time aggregation reduces the estimated cash flow coefficients in both the univariate and multivariate regressions. Despite the drop in cash flow coefficients that results from time aggregation, for situations in which growth options are important (rows near the bottom of the table), the cash flow coefficients remain larger than the Q coefficients in the multivariate regressions. That is, in cases in which growth options are important, the regressions are consistent with findings in the empirical literature.

A potential concern about the cases in which growth options are important is that, as we noted in Table 1, the values of Q are high in these cases. For example, Q can exceed twenty even after an upgrade in one case where growth options are particularly valuable. If this value is taken seriously, this firm's empirical counterpart would be excluded as an outlier from most empirical studies. While this is a valid concern, it takes the cash flows and growth options examined above as the firm's only source of value. If these account for one component (a plant, division, or line of business, etc.) of the firm's value, while the remainder of the firm is a more conventional generator of cash flows, then this concern is diminished. The Q of the combined firm would be the capital-weighted average of the Q s of each division, which tends to reduce the measured Q for the overall firm.

Large cash flow effects are not typically observed for all firms in empirical samples. Indeed, this is the identification strategy used in the literature. Firms that are *a priori* expected to be liquidity constrained, such as smaller firms or those that do not pay dividends or have no bond rating, are selected into a "constrained" subsample, which tends to exhibit larger cash flow effects and smaller Q coefficients. This strategy, originally employed by Fazzari, Hubbard, and Peterson (1988), is broadly employed as an identification strategy in the empirical literature using investment regressions to test for financing constraints. It is well understood that this identification strategy is compromised if the sample splitting criteria are correlated with characteristics of the firm that generate a cash flow sensitivity that is not driven by financing constraints. Our model provides a rationale for such a correlation that is not generated by financing constraints. The results in Table 2 show that the cash flow effects are strongest and the Q effects weakest where the growth options are most valuable, which would tend to be among more quickly growing and more volatile firms. These characteristics also tend to be associated with small, non-dividend-paying firms without bond ratings, since dividends and bond issues tend to be associated with larger, more established and stable firms (see Fama and French (2001)). Thus, the finding of stronger cash flow effects among small, non-dividend paying, non-rated firms may suggest that these

firms have relatively valuable growth options, rather than that they necessarily face financing constraints.

Table 2: Estimated Coefficients on Q and Cash Flow								
deviation from baseline:	no time aggregation				time aggregation (annual)			
	univariate		multivariate		univariate		multivariate	
	Q	cashflow	Q	cashflow	Q	cashflow	Q	cashflow
none	0.12 (0.006)	0.80 (0.08)	0.12 (0.006)	0.36 (0.07)	0.12 (0.007)	0.54 (0.07)	0.12 (0.008)	0.18 (0.07)
$\sigma = 0.40$	0.16 (0.007)	0.80 (0.09)	0.16 (0.007)	0.05 (0.08)	0.15 (0.008)	0.52 (0.07)	0.15 (0.008)	0.07 (0.07)
$\mu = .085,$ $\sigma = 0.40$	0.35 (0.02)	0.81 (0.07)	0.34 (0.02)	0.14 (0.07)	0.33 (0.02)	0.47 (0.06)	0.32 (0.03)	0.08 (0.06)
$\mu = .025,$ $\sigma = 0.20$	0.66 (0.04)	0.80 (0.06)	0.86 (0.07)	-0.41 (0.12)	0.62 (0.04)	0.41 (0.05)	0.64 (0.05)	-0.04 (0.07)
$\sigma_u = 0.03$	0.28 (0.01)	0.81 (0.13)	0.28 (0.01)	0.22 (0.11)	0.27 (0.01)	0.62 (0.08)	0.26 (0.01)	0.12 (0.08)
$\theta = 0.25$	0.11 (0.005)	0.79 (0.08)	0.10 (0.005)	0.39 (0.06)	0.11 (0.006)	0.56 (0.05)	0.11 (0.007)	0.21 (0.06)
$\theta = 0.10$	0.10 (0.005)	0.82 (0.07)	0.09 (0.005)	0.44 (0.07)	0.10 (0.007)	0.58 (0.08)	0.10 (0.007)	0.24 (0.07)
$\theta = 0.10,$ $\rho_{u\hat{A}} = .25$	0.20 (0.009)	0.99 (0.06)	0.19 (0.01)	0.29 (0.07)	0.20 (0.01)	0.61 (0.05)	0.20 (0.01)	0.15 (0.06)
$\theta = 0.05,$ $\rho_{u\hat{A}} = -0.2$	0.01 (0.001)	0.65 (0.07)	0.01 (0.001)	0.58 (0.06)	0.02 (0.001)	0.54 (0.07)	0.02 (0.001)	0.33 (0.06)
$\theta = 0.05,$ $\rho_{Y\hat{A}} = 0.2$	0.03 (0.001)	0.80 (0.08)	0.03 (0.001)	0.51 (0.07)	0.03 (0.002)	0.60 (0.07)	0.03 (0.002)	0.28 (0.06)
$\theta = 0.05,$ $\rho_{u\hat{A}} = -0.1,$ $\rho_{Y\hat{A}} = 0.1$	0.02 (0.001)	0.71 (0.06)	0.02 (0.001)	0.52 (0.06)	0.02 (0.002)	0.57 (0.07)	0.02 (0.002)	0.30 (0.06)

The baseline parameterization is $r = 0.18, \gamma = 0.8, \theta = 0.5, \mu = 0.13, \sigma = 0.5,$
 $\mu_Y = 0.065, \sigma_Y = 0.20, \sigma_u = 0.06, \mu_p = 0.02, \rho_{Y\hat{A}} = \rho_{u\hat{A}} = \rho_{Yu} = 0.$ These
values imply the parameters governing X are $m = 0.021$ and $s = 0.31.$

4 Variance Bounds

Equity prices empirically exhibit “excess volatility” relative to the dividends on which they are a claim. This observation was formalized by Leroy and Porter (1981) and most provocatively by Shiller (1981), though assuming that equity prices and dividends were trend stationary. West (1988) showed that equities were indeed more volatile than justified by a dividend-discount model even allowing for non-stationarity. West found that the estimated variance of the stock price innovation is four to twenty times (depending on the data set and estimation procedure) higher than the theoretical upper bound associated with dividend volatility. The model examined in this paper could, in principle, address this puzzle, since growth options generate variation in the value of the firm that is unrelated to the firm’s current profitability. This variation might induce “excess volatility” in the firm’s valuation compared to its underlying cash flows.

Two issues, one theoretical and the other empirical, must be addressed in evaluating this potential explanation of excess volatility. First, our model produces excess volatility in the firm’s value during the continuous investment regime, but the opposite occurs at the time of a technological upgrade. To see this, rewrite the expression for the value of the firm from equation (19), using the optimal capital stock from equation (2) and the definition of cash flow, C_t , to obtain

$$V_t = \frac{A_t X_t}{u_t} \frac{\gamma}{1 - \gamma} + \frac{A_t X_t}{r - m} \left[1 + \frac{1}{\phi - 1} \left(\frac{a_t}{\bar{a}} \right)^\phi \right] = C_t \left\{ \frac{\gamma}{u_t} + \frac{(1 - \gamma)}{r - m} \left[1 + \frac{1}{\phi - 1} \left(\frac{a_t}{\bar{a}} \right)^\phi \right] \right\}. \quad (25)$$

The value of the firm is proportional to its current cash flow, but the proportionality factor varies with the user cost factor, u_t , and the state of the technological frontier, a_t .¹⁹ These additional sources of variation may contribute to apparent excess volatility. Since V and C are nonstationary, we follow West (1988) and take differences to induce stationarity. In his model, however, arithmetic differences were assumed sufficient to induce stationarity, while in our structure (with geometric Brownian motion), log differences are required. The variance of $\Delta \ln V$ depends on the variance of $\Delta \ln C$, as well as the variances of u_t and a_t , and importantly, the covariances

¹⁹The literature on excess volatility has argued that variation in discount rates is not sufficient to explain the magnitude of the excess volatility in equity valuations compared to dividends. These arguments could apply to variation in r and u_t in the current model, but do not apply to variation in a_t .

among these processes. While the variances of u_t and a_t increase the volatility of $\Delta \ln V$ compared to $\Delta \ln C$, the covariances can either reinforce this effect (if they are positive) or have an opposing effect if they are negative. The latter effect can arise if we assume negative correlations among the underlying stochastic processes, which we explore numerically in Table 3. Even without this negative correlation, however, there are two sources of negative correlation when the underlying processes are mutually independent. First, the user cost factor, u_t , is negatively correlated with cash flow. Specifically, the composite variable X depends inversely on the user cost factor, which tends to induce a negative correlation between X and u .²⁰ This tendency carries over from X to cash flow, since $C = AX/(1 - \gamma)$. Perhaps even more important is the comovement of these variables during an upgrade. When the firm upgrades its technology, the user cost factor remains unchanged, but cash flow jumps *upward* with the discrete increase in the installed technology, A , while a jumps *downward* from \bar{a} to one. This effect will be larger the greater is \bar{a} , and can be quantitatively important. Thus, after aggregating over regimes of continuous investment and upgrades, it is not clear that the volatility of the firm's value will exceed the volatility of its cash flow (both in log differences). Greater volatility of firm value relative to its cash flow should be observed during continuous investment regimes (subject to the above caveat regarding the covariance of X and u), but could be reversed by the negative covariance of cash flow and the relative technology during upgrades.

The second important issue to be confronted when assessing variance bounds in this model is that the model generates neither stock prices nor dividends, which are usually the empirically measured variables in the excess volatility literature. The model is set in perfect markets, so neither capital structure nor dividends are determined (since neither affect the value of the firm). This issue cannot be explicitly addressed without leaving the perfect markets paradigm, which is beyond the scope of the paper (and also outside the spirit of the current exercise—to examine the implications of growth options without other market imperfections). In order to examine volatility bounds in our model, we assume that the firm has no debt, and hence the value of the firm, V , is equal to its equity value. Our calculations thus provide a floor on the equity variance, since leverage would only increase the variance of the value of equity. If dividends are smoother than cash flows, then the variance of cash

²⁰As stated in footnote 4, $s\rho_{Xu} = \rho_{Yu}\sigma_Y - \frac{\gamma}{1-\gamma}\sigma_u$. Therefore, if $\rho_{Yu} = \rho_{Y\hat{A}} = \rho_{\hat{A}u} = 0$, the correlation ρ_{Xu} is negative.

flows that we calculate provides an upper bound for the dividend variance.²¹ In this case, the ratio of the variances of V to C (in log differences) that we calculate is a lower bound on the variance ratio for stock prices versus dividends.

Table 3 reports the standard deviation of the log change in value, $\Delta \ln V$, the standard deviation of the log change in cash flow, $\Delta \ln C$, and the variance ratio $\frac{\text{var}(\Delta \ln V)}{\text{var}(\Delta \ln C)}$. The volatilities of quarterly changes are reported in the first three columns of results, and the volatilities of annual changes are reported in the final three columns of results. For all of the parameter configurations in Table 3, and for both quarterly and annual data, the variance ratio, $\frac{\text{var}(\Delta \ln V)}{\text{var}(\Delta \ln C)}$, exceeds one. The largest values of the variance ratio appear in the rows at the bottom of the table where growth options are the most important. The increase in the variance ratio in these rows, relative to the baseline in the first row, results almost entirely from an increase in the volatility of the value of the firm as growth options become more important. The volatility of cash flows in these rows is virtually the same as in the baseline.

Comparing quarterly and annual volatilities, the standard deviations of annual $\Delta \ln V$ are double the standard deviations of quarterly $\Delta \ln V$, which is what we would expect if quarterly $\Delta \ln V$ are i.i.d. over time. However, for the change in cash flow, $\Delta \ln C$, the annual standard deviations are less than twice as large as quarterly standard deviations. Therefore, the variance ratio, $\frac{\text{var}(\Delta \ln V)}{\text{var}(\Delta \ln C)}$, increases when we move from quarterly data to annual data. The highest variance ratios in the table are for annual data in the final three rows of the final column, where growth options are important. The highest value of $\frac{\text{var}(\Delta \ln V)}{\text{var}(\Delta \ln C)}$, 3.04, falls about 25% short of the lower end of West's range, but, as discussed earlier, the variance ratios reported in Table 3 can be interpreted as lower bounds on the variance ratio for equities versus dividends, if dividends are smoother (i.e., have lower variance) than cash flow.²²

An additional consideration is that the results of Shiller (1981) and West (1988)

²¹If dividends are literally a smoother version of cash flows (and both must integrate to the same value), as in Lintner (1956), then the variance of cash flow should exceed the variance of dividends. Recent work, such as Brav, et al (2003), tends to confirm that dividends are smoothed relative to cash flows.

²²Also, our results are not directly comparable to West's since he compares the variances of innovations in the level (rather than the log) of equities versus dividends. West (1988) uses levels since the analytic results regarding the variance bounds hold for arithmetic differences. Empirically, however, there is evidence that geometric differencing is necessary to induce stationarity. West argues (pages 53-54) based on a Monte Carlo simulation that the use of levels rather than logs does not significantly affect his calculations and conclusions regarding the relative variances.

are estimated using aggregate equity indexes and aggregate dividend measures. The volatilities they calculate therefore depend on the covariance of equity movements and dividends across firms. If we were to extend our model to include common interest rate shocks across firms, these shocks would increase the variance ratio of firm value relative to cash flow.

Table 3: Volatility of Growth of Firm Value and Cash Flow						
deviation from baseline:	no time aggregation			time aggregation (annual)		
	$sd(\Delta \ln V)$	$sd(\Delta \ln C)$	$\frac{var(\Delta \ln V)}{var(\Delta \ln C)}$	$sd(\Delta \ln V)$	$sd(\Delta \ln C)$	$\frac{var(\Delta \ln V)}{var(\Delta \ln C)}$
none	0.199	0.169	1.38	0.402	0.290	1.91
$\sigma = 0.40$	0.194	0.176	1.28	0.403	0.300	1.80
$\mu = .085,$ $\sigma = 0.40$	0.182	0.166	1.21	0.370	0.281	1.73
$\mu = .025,$ $\sigma = 0.2$	0.175	0.160	1.20	0.352	0.266	1.75
$\sigma_u = 0.03$	0.148	0.134	1.22	0.304	0.234	1.68
$\theta = 0.25$	0.201	0.167	1.45	0.405	0.289	1.96
$\theta = 0.10$	0.202	0.165	1.49	0.407	0.288	2.00
$\theta = 0.10,$ $\rho_{u\hat{A}} = .25$	0.181	0.163	1.23	0.367	0.281	1.70
$\theta = 0.05,$ $\rho_{u\hat{A}} = -0.2$	0.257	0.167	2.36	0.514	0.295	3.04
$\theta = 0.05,$ $\rho_{Y\hat{A}} = 0.2$	0.237	0.166	2.04	0.476	0.292	2.66
$\theta = 0.05,$ $\rho_{u\hat{A}} = -0.1,$ $\rho_{Y\hat{A}} = 0.1$	0.243	0.166	2.15	0.487	0.292	2.80
The baseline parameterization is $r = 0.18, \gamma = 0.8, \theta = 0.5, \mu = 0.13, \sigma = 0.5,$ $\mu_Y = 0.065, \sigma_Y = 0.20, \sigma_u = 0.06, \mu_p = 0.02, \rho_{Y\hat{A}} = \rho_{u\hat{A}} = \rho_{Y_u} = 0.$ These values imply the parameters governing X are $m = 0.021$ and $s = 0.31.$						

5 Comments and Conclusions

The value of a firm, measured as the expected present value of payouts to claimholders, summarizes a variety of information about the current and expected future cash flows of the firm. Empirical studies, however, typically have difficulty confirming the information content of valuation measures, such as Tobin's Q . Regressions of investment on Tobin's Q and cash flow often find only a weak effect of Q and find instead an important role for cash flow in explaining investment. Moreover, there is strong evidence of excess volatility of equity values relative to their underlying dividends. We show that growth options can address both of these phenomena. The key feature of the model driving the results is volatility in the value of the firm that is not associated with current cash flows. These movements, driven by the frontier technology in our model, affect the value of the firm immediately, but do not affect the current marginal product of capital. Hence, this source of volatility moves V and Q , but does not affect the optimal capital stock nor investment. This reduces the correlation between investment and Q , yet leaves intact the correlation between current investment and current cash flow.

Our simulations show that the model can generate empirically realistic investment regressions when the growth option component of the firm is fairly important. Specifically, when growth options are important, investment regressions on Q and cash flow yield small positive coefficients on Q and larger positive coefficients on cash flow. This finding is noteworthy because empirical findings of a large cash flow coefficient are often interpreted as evidence of financing constraints. However, capital markets in our model are perfect, so there are no financing constraints.

Similarly, when growth options are a large component of value, the model also generates excess volatility of firm value relative to cash flow. The variance ratio of the log change in firm value relative to log change in cash flow can reach three in the parameterizations we consider, which provides a lower bound (if dividends have a lower variance than cash flows) for comparison to empirical estimates. These empirical estimates of excess volatility (of equity values relative to dividends) are in the range of four to twenty.

An avenue for further work is to allow for factor adjustment costs. In the current model, both capital and labor are costlessly adjustable. This leads to a high standard deviation of the investment rate, which is consistent with plant-level, but not firm-level data (see Doms and Dunne (1998)). This could again be addressed by

explicitly incorporating adjustment costs for capital. Another approach to matching the firm-level data on investment would be to model the behavior of plants, and then to aggregate the behavior plants into firms. This aggregation would reduce some of the investment spikes associated with technology upgrades at individual plants. Aggregating further to economy-wide valuation and dividends would allow us to address the results of Shiller (1981) and West (1988). If all firms face common movements in interest rates, their values would covary more than their dividend covary, and thus the variance ratio of value relative to dividends would increase with aggregation.

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