Optimal Fiscal Policy in Debt Constrained Economies*

Mark Aguiar
University of Rochester and NBER

Manuel Amador
Stanford University and NBER

PRELIMINARY AND INCOMPLETE

February 27, 2011

Abstract

We study optimal fiscal policy in a small open economy (SOE) with sovereign default risk. The SOE's government uses linear taxation to fund exogenous expenditures and uses public debt to inter-temporally allocate tax distortions. We characterize a class of environments in which the tax on *labor* goes to zero in the long run, while the tax on capital income may be non-zero, reversing the standard prediction of the Ramsey tax literature. The zero labor tax is an optimal long run outcome if the private agents are impatient relative to the international interest rate. We also extend the results to economies where the borrowing constrained is imposed on the citizens directly, rather than on their governments. We show that a similar result holds in a closed economy with imperfect inter-generational altruism.

*Email: mark@markaguiar.com and amador.manuel@gmail.com. We thank V.V. Chari and Iván Werning for fruitful comments and suggestions. This research was supported by a grant from IGC. Manuel Amador acknowledges NSF support.

1 Introduction

Economies frequently pursue policies that lead to fiscal crises, usually typified by sustained deficits that eventually lead to an inability to increase or roll-over debt (without paying an historically abnormal premium) and an associated sharp increase in tax rates and decline in government expenditure. The recent experience of Greece, Ireland, and Portugal, are only the latest examples of such crises. See Reinhart and Rogoff (2009) for many more examples from the historical record. Policies that run up debt, and eventually encounter borrowing constraints, may be the rational response of citizens (and their politicians) who face a world interest rate that is below their subjective rate of time preference. However, the normative question of whether the observed policies of front loading consumption and back loading taxes is indeed optimal in such an environment has not been thoroughly studied. To this end, this paper studies optimal fiscal policy in economies that are debt constrained, with a specific interest in relatively "impatient" economies for which the debt constraints are particularly relevant.

We consider optimal fiscal policy in an extended Ramsey-style framework. The canonical Ramsey formulation of optimal fiscal policy is quite simple: a government funds fiscal expenditures using linear taxes, and chooses the sequence of taxes that maximizes the welfare of the representative private citizen. A well known result in the canonical framework is that capital taxes should be zero in the long run if the economy converges to a steady state (Judd, 1985; Chamley, 1986; Atkeson et al., 1999), and that taxes on labor income should be "smoothed" using government debt (Lucas and Stokey, 1983; Ljungqvist and Sargent, 2004). That is, if a steady state exists, the government will rely solely on labor taxes.[1] This prediction is robust to dropping the Ramsey assumption of full commitment, as shown by Dominguez (2007) and Reis (2008). There are a number of alternative environments in which capital is taxed in the steady state, but our focus is not solely the role of capital taxes in a steady state, but also the role of labor taxes.

This paper explores several variations of the canonical framework. At their core, each variation shares the fact that the inter-temporal marginal rate of substitution (MRS) of private agents may differ from the marginal rate of transformation (MRT) in the long run. In particular, agents are impatient relative to the inter-temporal price of resources. The greater impatience may reflect higher mortality in developing economies, imperfect altruism, or simple preference heterogeneity (with large/rich countries being rich because

[1] If labor is a type of capital, as in environments in which human capital can be accumulated, then labor taxes may also go to zero for the same reason that capital taxes go to zero. See Jones et al. (1997).

they have patient agents). Our primary scenario is a small open economy (with or without capital) in which private agents discount at a rate that differs from the world interest rate. If a country (or the agents individually) faces a borrowing constraint, the economy will converge to a steady state in which agents' value inter-temporal tradeoffs differently than international financial markets: In the steady state, agents would like to pursue a declining path of consumption given the world interest rate, but are constrained from doing so. Our main result is that in such a steady state, the tax on labor income is driven to zero. That is, the optimal response to impatience and borrowing constraints is to *front load* taxes, driving labor taxes to zero in the limit.

The result is quite general in that the exact mechanism which drives a wedge between the MRS and the MRT is unimportant. An alternative scenario to a debt constrained small open economy is a closed economy in which the government discounts future consumption differently than the private agents. For example, if time indexes different generations, and private agents have imperfect altruism, the government may value inter-temporal tradeoffs differently than private agents (e.g., as considered by Phelan, 2006 and Farhi and Werning, 2007, in models of private information). We can think of a Pareto frontier in which each generation receives a Pareto weight, but this weight may differ from the value private agents place on future generations. The steady state of such an economy involves a subsidy to capital to sustain the required consumption of future generations. In this steady state, the labor tax is zero.

The intuition of the result begins with the fact that the economy ultimately faces a borrowing constraint. The class of borrowing constraints we consider are motivated by the need for debt to be "self-enforcing;" that is, utility is at least as great by paying back the debt than it is from defaulting. The appeal of self-enforcing constraints stems from the practical limits of enforcing international debt contracts. We model the constraints in a fairly general way, but at their essence they involve placing a lower bound on equilibrium utility. Impatience absent a borrowing constraint involves long run immiseration, an outcome that will be inconsistent with realistic enforcement mechanisms.

To the extent possible, the optimal response to impatience is to front load consumption and leisure. The flip side of this is to exhaust the economy's borrowing capacity. For a given level of utility, an undistorted allocation of labor maximizes output, and therefore maximizes the amount of debt that can be serviced. In particular, the borrowing constraint places a floor on utility, while the efficient allocation ensures that this utility is delivered in a way that maximizes long run debt payments, freeing up resources for early consumption. The zero labor tax in the long run is simply the mechanism through which fiscal policy exhausts the front-loading capacity of the economy's debt constraint. The closed economy result has a similar intuition. The optimal fiscal policy delivers the desired level of inter-generational altruism in a way that minimizes the need to pass-on physical capital.

In our framework, there is an aggregate borrowing constraint which involves both private and public debt, as both state variables determine the relative benefits of repayment versus default. As noted above, the optimal fiscal policy maximizes the long run aggregate debt of the economy. However, this does not necessarily imply large public debt positions. Indeed, the fact that labor taxes are zero in the long run requires the fiscal authority to fund government expenditures from net claims on private agents (and foreigners) plus any net tax receipts from capital income. In our simulated examples, the government runs fiscal surpluses on the transition to the steady state. It is the private sector that is indebted in the long run, not the fiscal authority. Private debt is consistent with efficient output, while fiscal debt limits the economy's debt servicing capacity due to the need for distortionary taxes.

The normative implications of our model therefore stand in stark contrast to many observed fiscal trajectories. The optimal policies studied in this paper involve front loading labor taxes, and, correspondingly, reducing the available tax revenue to fund long run fiscal expenditures and to pay interest on public debt. Viewed through our model, observed fiscal crises typified by large pubic debt positions and sharp increases in labor tax rates are not consistent with the optimal response to impatience and debt constraints.

We stress that we hew fairly closely to the standard Ramsey framework and its close variants. That is, taxation is linear by assumption. We do allow for limited commitment and consider taxes supported by reputational equilibria, as well as incorporate alternative political economy frictions. However, we do not address issues of private information, heterogeneity, and incompleteness of asset markets. It is well known in these models that optimal capital taxes may not be set to zero in the long run. We leave for future research whether our other normative results, including the zero labor tax, carry over to such environments.

The remainder of the paper is organized as follows: Section 2 presents the model environment; section 3 characterizes the optimal fiscal policy in an open economy setting; section 4 introduces borrowing constraints at the individual agent level (to be completed); section 5 discusses a closed economy model; section 6 presents a quantitative analysis of the optimal policy; and section 7 concludes. The appendix contains all proofs.

2 Environment

In this section we describe the environment faced by households, firms, and the government. The key departure from the standard framework is discussed in the final subsection. We focus on a deterministic environment and discuss the extension to a stochastic economy in the appendix (to be added). In this section, we focus on a small open economy. We characterize the closed economy model in section 5.

2.1 Households

The representative private agent has time-separable preferences with utility over consumption c (our numeraire) and labor n represented by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \tag{1}$$

with $\beta \in (0,1)$. We impose that consumption and leisure are normal goods, together with the assumptions:

Assumption 1. *The utility function u satisfies the following conditions: (i) $u : X \to \mathbb{R}$ where $X \equiv (0,\infty) \times (0,\bar{n})$ with $0 < \bar{n} \leq \infty$; (ii) u is twice differentiable with $u_c > 0$, $u_n < 0$, $u_{cc} < 0$, $u_{nn} < 0$ and $u_{cc}u_{nn} - (u_{cn})^2 > 0$ for all $c, n \in X$; (iii) $\lim_{c\to 0} u_c = \infty$ and $\lim_{c\to\infty} u_c = 0$ for all $n \in (0,\bar{n})$; (iv) $\lim_{n\to 0} u_n = 0$ and $\lim_{n\to\bar{n}} u_n = -\infty$ for all $c \in (0,\infty)$; (v) consumption and leisure are normal goods, $u_{cc}u_n - u_{cn}u_c \geq 0$ and $u_{nn}u_c - u_{cn}u_n \geq 0$ for all $c, n \in X$; and (vi) utility satisfies the following boundedness assumptions on preferences: $u_c(u_{cc}c/u_c - u_{cn}c/u_n)$, and $u_c(u_{cn}n/u_c - u_{nn}n/u_n)$ are bounded functions in $(c,n) \in (\epsilon_c, \infty) \times (0, \epsilon_n)$ for some $\epsilon_c > 0$, $\epsilon_n \in (0,\bar{n})$.*

The first five assumptions are standard. The final boundedness assumption insures that certain key expressions remain well behaved as consumption becomes large or labor approaches zero. This assumption holds for several of the preferences commonly used in the macroeconomics literature. For example, it is satisfied for the balanced-growth preferences of the form: $u(c,n) = (c^\gamma(1-n)^{1-\gamma})^{1-\sigma}/(1-\sigma)$ with $\gamma \in (0,1)$ and $\sigma > 0$; as well as for the power-separable specification: $u(c,n) = c^{1-\sigma}/(1-\sigma) - \psi n^{1+\gamma}/(1+\gamma)$ with $\sigma, \gamma, \psi > 0$.

Agents provide labor in a competitive labor market at a wage w_t, and labor is immobile across borders. Without loss of generality, we assume labor taxes are levied on the firms, so w_t represents wages after taxes.

Let r_t denote the net interest rate (before-taxes) received by consumers on their financial assets from time $t-1$ to t. Let r_t^k denote the rental rate of the domestic capital stock own by consumers, and δ its depreciation rate. Let ϕ_t^k represents the residence-based tax on capital income received in time t, where "residence-based" refers to the fact that domestic agents pay this tax regardless of the source of the capital income or its location. We introduce "source-based" taxation below. No arbitrage implies that the after tax return is equalized:

$$1+(1-\phi_t^k)r_t = 1+(1-\phi_t^k)(r_t^k-\delta).$$

It follows that:

$$r_t^k = r_t + \delta. \tag{2}$$

We define the (after-tax) period-0 price of consumption at time t as:

$$p_t = \prod_{s=1}^{t} \frac{1}{1+(1-\phi_s^k)r_s} \tag{3}$$

We normalize $p_0 = 1$. We let a_t denote the wealth of agents, including both financial wealth and capital holdings, net of capital income taxes. To be precise about the timing, if x is invested at the end of period $t-1$, the pre-tax wealth in period t is $(1+r_t)x$, and the after-tax wealth is $a_t = (1+r_t)x - \phi_t r_t x = (1+(1-\phi_t)r_t)x$. The flow budget constraint of the consumer can be expressed:

$$a_{t+1} = (1+(1-\phi_t^k)r_t)(a_t - c_t + w_t n_t + T_t)$$

where $T_t \geq 0$ denotes a non-negative lump-sum transfer from the government at time t. In the benchmark model, we assume private agents are not borrowing constrained. As discussed below, we impose these constraints on the sovereign, and the sovereign imposes them on agents via capital taxation. We discuss the alternative environment in which the agents face an explicit constraint in Section XX [to be added]. Beginning with from an initial wealth a_0 and imposing a No Ponzi condition, the present-value budget constraint of the consumer is:

$$\sum_{t=0}^{\infty} p_t \left(c_t - w_t n_t - T_t\right) \leq a_0. \tag{4}$$

Note that our notation implies that a_0 is net of period-0 capital taxes. As is well known, distortionary taxation can be avoided with a large enough initial capital levy. By starting from an a_0, which is after the initial capital tax is levied, such that distortionary taxes are still required, we are implicitly following the standard practice of assuming a bound on the initial capital levy. To be explicit, assume $\phi_0^k = 0$, which is without loss of generality given that a_0 is treated as an arbitrary initial condition.[2]

The private agent's problem is to choose sequences $\{c_t\}$ and $\{n_t\}$ to maximize (1) subject to (4). This is a standard convex problem, whose solution can be characterized by the multiplier on the budget constraint, θ, with the following first order conditions, which are necessary and sufficient:

$$\beta^t u_c(c_t, n_t) = \theta p_t \tag{5}$$

$$-\beta^t u_n(c_t, n_t) = \theta p_t w_t. \tag{6}$$

where $\theta > 0$ implies that the budget constraint holds with equality.

2.2 Firms

The representative firm operates a constant returns to scale production function $F(k, n)$ and hires labor and rents capital in competitive factor markets to maximize profits. It pays a linear source-based tax τ_t^n on its wage bill and a source-based tax τ_t^k on its rental payments. The firms problem in period t is therefore

$$\max_{k_t, n_t} \left\{ F(k_t, n_t) - (1 + \tau_t^n) w_t n_t - (1 + \tau_t^k) r_t^k k_t \right\}.$$

where w_t is the wage and r_t^k the rental rate.

The first order conditions, necessary and sufficient, are:

$$F_k(k_t, n_t) = (1 + \tau_t^k) r_t^k \tag{7}$$

$$F_n(k_t, n_t) = (1 + \tau_t^n) w_t. \tag{8}$$

[2]This is without loss of generality for the consumer's problem. For the government's problem introduced below, we can adjust period-0 fiscal requirements to reflect any initial capital tax revenue, making the zero tax without loss of generality for that problem as well.

2.3 Government Budget Constraint

The government has to fund a sequence of expenditures g_t, which we take to follow a deterministic and exogenous process. [3]

The relevant inter-temporal price for the government is the before tax price, q_t:

$$q_t \equiv \prod_{s=1}^{t} \frac{1}{1+r_s} \tag{9}$$

with $q_0 = 1$. Therefore, the government's budget constraint is:

$$\sum_{t=0}^{\infty} q_t \left(g_t + T_t - \tau_t^n w_t n_t - \tau_t^k r_t^k k_t - \frac{\phi_t^k r_t}{1+(1-\phi_t^k) r_t} a_t \right) \leq -b_0 \tag{10}$$

where b_0 is the initial debt of the government. Recall that a_t is after-tax period-t wealth, so the initial amount invested at the end of period $t-1$ is $a_t/(1+(1-\phi_t^k)r_t)$, which is the tax base for residence-based capital taxation in the expression above. Note as well that we are allowing free disposal of government income. .

2.4 Competitive Equilibrium

We are interested in allocations that can be supported as an equilibrium outcome. For the open-economy benchmark economy, we assume the world interest rate is given by r_t^*. No arbitrage implies that $r_t = r_t^*$. The agents' and government's budget constraints imply an aggregate resource constraint for the economy:

$$\sum_{t=0}^{\infty} q_t \left(c_t + g_t + (r_t^\star + \delta) k_t - F(k_t, n_t) \right) \leq A_0. \tag{RC*}$$

where A_0 is the initial wealth of the economy: $A_0 = a_0 - b_0$. Note that for the open economy case, the relevant state variable is total wealth. In particular, firms can rent capital from foreigners and domestic agents can trade their physical capital for foreign assets, and so the initial physical capital stock k_0 is an equilibrium outcome and not a predetermined variable.

[3] It would be straightforward to make g_t and endogenous choice with some (potentially time varying) utility value. As our main result holds for arbitrary sequences of government expenditure (subject to boundedness conditions on the equilibrium allocation), we simply treat public expenditures as a primitive.

We define a competitive equilibrium in the standard fashion:

Definition 1. *An* ***open economy competitive equilibrium*** *from an initial position* (A_0, a_0) *consists of sequences of prices* $\{r_t, w_t, r_t^k\}$*; taxes* $\{\phi_t^k, \tau_t^n, \tau_t^k\}$ *with* $\phi_0^k = 0$*; non-negative lump-sum transfers* $\{T_t\}$*; and quantities* $\{c_t, n_t, k_t\}$ *such that* ***(i)*** *intertemporal prices correspond to the small open economy assumption:* $r_t = r_t^\star$ *for all* t*, and* r_t^k*,* p_t*,* q_t *satisfy equations and (2), (3), (9);* ***(ii)*** *households optimize given prices and taxes subject to their budget constraint, that is, there exists* $\theta \geq 0$ *such that (4), (5) and (6) are satisfied;* ***(iii)*** *firms maximize profits given prices and taxes, that is, equations (7) and (8) hold;* ***(iv)*** *the labor market clears;* ***(v)*** *the government budget constraint (10) holds; and* ***(vi)*** *the open-economy aggregate resource constraint* (RC*) *holds.*

As is standard in the Ramsey literature, we use the conditions of a competitive equilibrium to substitute out prices (the primal approach). Namely,

Proposition 1. *An allocation* $\{c_t, n_t, k_t\}$ *is consistent with an open economy competitive equilibrium if and only if it satisfies the following implementability condition:*

$$\sum_{t=0}^{\infty} \beta^t \left(u_c(c_t, n_t)c_t + u_n(c_t, n_t)n_t\right) \geq u_c(c_0, n_0)a_0 \tag{IC*}$$

and the open economy aggregate resource constraint (RC*) *holds.*

2.5 Additional Constraints

The Ramsey problem for an open economy is to choose an allocation to maximize (1) subject to the implementability condition, (IC*), and the resource constraint, (RC*), under the assumption that the government can commit to a sequence of tax, transfers and debt repayment promises. We will nest that problem into a broader class of problems in which the government may face additional constraints. In particular, we consider constraints that take the form:

$$W_s(\{u_t\}_{t=s}^{\infty}, \{k_t\}_{t=s}^{\infty}) \geq 0, \text{ for } s \in \{0, 1, \dots\} \tag{PC}$$

where $u_t = u(c_t, n_t)$. We assume that the sequence of W_s are differentiable in all arguments.

We are interested in constraints that place a lower bound on utility, rather than an upper bound. As discussed below, these are the natural constraints in models of limited commitment. Specifically,

Assumption 2. *For all* $(s, i) \in \{0, 1, \ldots\} \times \{0, 1, \ldots\}$, $\partial W_s / \partial u_{s+i} \geq 0$.

We now give some examples that generate constraints of these form to motivate their interest. A straightforward interpretation of these constraints is arising in a limited commitment environment in which the government cannot commit to its promises. Suppose that deviating from a prescribed plan (either through default on the debt or changes in the tax allocation) at time t yields deviation utility $\underline{U}_t$ to the government in power at time t. Then, a constraint of the type above ensures that allocation is indeed an equilibrium. For example, Aguiar and Amador (forthcoming) introduce constraints on allocations of the form:

$$W_t = \sum_{j=0} \beta^j \left(\theta\delta^j + 1 - \delta^j\right) u(c_{t+j}, n_{t+j}) - \underline{U}_t \geq 0, \forall t \geq 0,$$

where θ and δ reflect the fact that political incumbents may face political turnover risk and prefer consumption to occur during incumbency. Setting $\theta = 1$ and $\delta = 0$ generates the limited commitment-benevolent government framework. In this environment, the deviation utility $\underline{U}_t$ may depend on the endogenous state variable k_t, and may depend on the sequence of g_t depending on whether these fiscal expenditures are hard wired or are promises that can be broken (for example, like pension promises to retirees). Similarly, Aguiar et al. (2009) model a limited-commitment government with a geometric discount factor $\tilde{\beta} \neq \beta$, with associated constraints:

$$W_t = \sum_{j=0} \tilde{\beta}^j u(c_{t+j}, n_{t+j}) - \underline{U}_t \geq 0, \forall t \geq 0. \tag{11}$$

Acemoglu et al. (2008) explore a similar constraint in a growth model with self-interested politicians.

The fact constraints of the type (PC) arise in models of endogenous default motivate the title of the paper. Nevertheless, limited commitment is not the only motivation for constraints such as (PC). In a full commitment environment, there may be reason to incorporate such constraints. Consider a dynastic model in which agents live for one period and bequeath assets to the next generation of the dynasty. Inter-generational altruism is governed by β. Now consider a Pareto problem in which the government maximizes the initial generation's utility (1), subject to giving generation t a utility level of at least $\underline{U}_t$. In this environment, the constraint set takes the form:

$$W_t = \sum_{s=0} \beta^s u(c_{t+s}, n_{t+s}) - \underline{U}_t \geq 0, \forall t \geq 1.$$

The role of these additional constraints is to motivate an environment in which the optimal allocation distorts inter-temporal tradeoffs from the perspective of the private agents whose decisions underly the implementability condition.

3 Efficient Allocations

In this section we characterize *efficient allocations*:

Definition 2. *An* **efficient allocation** *is a competitive equilibrium allocation that maximizes* (1) *subject to* (PC)*.*

By definition, an efficient allocation maximizes (1) subject to (IC*), (RC*) and (PC). Note that constraints (IC*) and (PC) are not necessarily convex. We assume that an interior efficient allocation exists and proceed to characterize it. To this end, let η denote the multiplier on (IC*), μ denote the multiplier on (RC*), and $\beta^s\lambda_s$, $s = 0, 1, ...$ denote the multipliers on constraints (PC). Necessary conditions for an interior optimum for $t \geq 1$ are (with a little re-arranging for later convenience):

$$\left(\frac{\beta^t}{q_t\mu}\right)\left[1+\eta+\eta\left(\frac{u_{cc}(c_t,n_t)}{u_c(c_t,n_t)}c_t+\frac{u_{cn}(c_t,n_t)}{u_c(c_t,n_t)}n_t\right)+\sum_{s=0}^{\infty}\beta^{s-t}\lambda_s\frac{\partial W_s}{\partial u_t}\right]=\frac{1}{u_c(c_t,n_t)} \tag{12}$$

$$\left(\frac{\beta^t}{q_t\mu}\right)\left[1+\eta+\eta\left(\frac{u_{cn}(c_t,n_t)}{u_n(c_t,n_t)}c_t+\frac{u_{nn}(c_t,n_t)}{u_n(c_t,n_t)}n_t\right)+\sum_{s=0}^{\infty}\beta^{s-t}\lambda_s\frac{\partial W_s}{\partial u_t}\right]=-\frac{F_n(k_t,n_t)}{u_n(c_t,n_t)} \tag{13}$$

$$F_k(k_t,n_t)+1-\delta\;+\left(\frac{\beta^t}{q_t\mu}\right)\sum_{s=0}^{\infty}\beta^{s-t}\lambda_s\frac{\partial W_s}{\partial k_t}=1+r_t^*, \tag{14}$$

with $q_t = \prod_{s=1}^{t}\frac{1}{1+r_s^*}$. For the initial period $t = 0$, we replace the parenthetical term after η in (12) and (13) with

$$\frac{u_{cc}(c_0,n_0)(c_0-a_0)}{u_c(c_0,n_0)}+\frac{u_{cn}(c_0,n_0)(n_0-a_0)}{u_c(c_0,n_0)}$$
$$\frac{u_{cn}(c_0,n_0)(c_0-a_0)}{u_n(c_0,n_0)}+\frac{u_{nn}(c_0,n_0)(n_0-a_0)}{u_n(c_0,n_0)},$$

respectively.

The following lemma follows from the first order conditions:

Lemma 1. *Suppose Assumptions 1 and 2 hold. In an interior efficient allocation, the resource constraint binds ($\mu > 0$).*

This lemma is intuitive in that the government seeks to maximize private agent utility and so will not leave resources unused. The only potential wrinkle in the present framework is if the constraint set imposed an upper bound on private agent consumption and the government was forced to waste resources. Such a pathological outcome is ruled out by assumption 2.

Our main result is now stated in the following proposition:

Proposition 2. *[Zero Labor Tax in the Long Run] Suppose Assumptions 1 and 2 hold. If an (interior) efficient allocation satisfies that:*

(a) $\liminf_{t\to\infty} c_t > 0$;

(b) if $u_c(u_{cn}n/u_c - u_{nn}n/u_n)$ *is unbounded for* $c > \epsilon_c > 0$*; then* $\limsup_{t\to\infty} n_t < \bar{n}$*; and*

(c) there exists $M > 0$ *and* T *such that* $1 > M > \beta(1 + r_t^\star)$ *for all* $t > T$*;*

then $\lim_{t\to\infty} \tau_t^n = 0$.

Condition (a) and (b) in the proposition place bounds on consumption and labor so that in the limit the allocation remains bounded away from the boundaries of the domain of u. The condition that $\beta(1 + r_t^*)$ is bounded away from one states that private agents are (in the limit) impatient relative to the world interest rate.

The standard Ramsey solution can be recovered by setting $\lambda_s = 0$ for all s. In this case, if $\eta \neq 0$, the labor tax wedge may be non-zero. This highlights the fact that the Ramsey full-commitment allocation without (PC) does not call for zero labor taxes in the long run. If $\beta(1 + r_t^*) < 1$, the Ramsey allocation calls for increasing marginal utility of consumption, which violates condition (a) in the proposition.

Proposition 2 does not tell us when an interior allocation will satisfy condition (a). The next proposition provides some sufficient conditions for this to be the case.

Proposition 3 (to be added)**.**

Proposition (2) states that labor is undistorted in the long run if agents are relatively impatient. That is, that labor tax distortions are *front loaded*. Perhaps one would think that distortions are backloaded due to impatience, or at the least distortions are smoothed to the extent possible. However, agents wish to front load consumption and leisure. This suggests borrowing to the greatest extent possible in the long run. Efficient allocation of labor is the counter-part to servicing a large aggregate debt.

In particular, suppose that the efficient allocation converges to a steady state $(c_\infty, n_\infty, k_\infty)$, with associated steady state interest rate $r^\star$ and fiscal expenditure g. Suppose as well that the constraints W_t are weakly decreasing in capital, so more capital tightens the constraints, although this is not strictly necessary. If W_t were increasing in k, the following proposition holds with the inequality of the last constraint reversed. Let B_∞ denote the steady state aggregate debt position. From the aggregate resource constraint,

$$B_\infty = \frac{1+r^\star}{r^\star}\left[F(k_\infty, n_\infty) - c_\infty - g - (r^\star + \delta)k_\infty\right].$$

The efficient allocation allocation maximizes steady state debt subject to the participation constraints:

Proposition 4. *[Maximal Steady State Debt] Suppose the efficient allocation converges to a steady state. Under the conditions of Proposition 2, the efficient allocation converges to the maximal steady state debt allocation, where the latter allocation is the solution to:*

$$\max_{c,n,k}\left\{\frac{1+r^\star}{r^\star}\left(F(k,n) - c - g - (r^\star+\delta)k\right)\right\},$$

subject to

$$u(c,n) \geq u(c_\infty, n_\infty),$$
$$k \leq k_\infty.$$

Note that the constraint set in the maximal debt problem is not a singleton, as there are many choices of consumption and leisure that deliver the same level of utility. What the proposition says is that the efficient allocation delivers the steady level of utility in a manner that maximizes net payments to foreign financial markets. The benefit of such an allocation to an impatient economy is that a large steady state debt finances front loading of consumption and leisure. The implication for fiscal policy is to leave labor decisions undistorted to maximize output.

We should emphasize that aggregate debt is maximized in the long run, not government debt. The debt referred to in the proposition is the aggregate debt of the country, which is the sum of private and public foreign liabilities. Even if the decentralization is such that the only foreign liabilities are government liabilities, these can be balanced by government claims against the domestic private sector, making the private sector the ultimate debtor. Indeed, the fact that the government does not tax labor income in the long run requires the government to hold enough claims against private agents or foreigners to fund government expenditures plus any net subsidy to capital income. This generates the somewhat surprising implication that an impatient economy may eventually run fiscal surpluses on the transition path, a point we discuss in detail in our example economy below.

From the perspective of the private agents, the efficient allocation also maximizes private steady state debt. In particular, the net payments a private agent makes to financial markets (including the capital rental market) at any point in time is $w_t n_t - c_t$. For a given level of utility, taxes on labor reduce the available income for net debt payments, and in this sense zero labor taxes allows private agents to maximize long run debt. In this regard, we stress that proposition 2 exploits the fact that private agents are impatient, and not just (or necessarily) the government. The proof of 2 relies on the fact that the compounded discount factor from the IC* constraint approaches zero faster than q_t, and (IC*) is derived from the private agents' problem independently of the government's objective and the additional constraints. In the absence of taxation, private agents' would like to pursue a path in which consumption and leisure are front loaded and long run debt is maximized, and the efficient fiscal policy faced with such a constraint ensures that this is accomplished to the extent allowable.

3.1 Example Economy: Sovereign Borrowing Constraints

We now study a specific environment to render the constraints (PC) concrete and fully characterize the efficient allocation. As previously mentioned above, a natural motivation for the additional constraints is that the country faces an external borrowing constraint. So let us focus on that case. Consider a small open economy without capital and with a linear production technology: $f(n) = n$. Assume also that g_t is constant and that the economy can borrow from abroad at a risk-free interest rate of $R^* \equiv 1 + r^* = 1/p^*_{t,t+1}$. We impose that $\beta R \leq 1$.

The per-period utility function of the domestic representative agent is:

$$u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{n^{1+\gamma}}{1+\gamma}. \tag{15}$$

At any point, the government of the economy can decide to cut all access from the international financial markets by defaulting on their external obligations. Such a move guarantees a utility level of $\underline{U}$ to the representative agent, where this payoff $\underline{U}$ can include the costs of default imposed to the country by the foreigners in a default event. For our analysis, we do not need to specify anything more about how $\underline{U}$ gets determined, except the following:

Assumption 3. *There exists $\tilde{c} > 0$ and $\tilde{n} < \bar{n}$ such that $\underline{U} > u(\tilde{c}, \tilde{n})/(1-\beta)$.*

In this example, constraint (PC) then takes the form:

$$W_t \equiv \sum_{j=t}^{\infty} \beta^{j-t} \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \psi \frac{n_t^{1+\gamma}}{1+\gamma} \right) \geq \underline{U}; \text{ for all } t. \tag{16}$$

The implementability condition takes the following form:

$$\sum_{t=0}^{\infty} \beta^t \left\{ c_t^{1-\sigma} - \psi n_t^{1+\gamma} \right\} \geq c_0^{-\sigma} a_0 \tag{17}$$

The efficient allocation for this economy can then be characterized as follows. Suppose that constraint (16) is not binding. Then the first order condition for consumption, equation (12), can be written in this environment as

$$(\beta R^*)^t \left(1 + \eta \left(1 - \sigma\right)\right) = \mu c_t^{\sigma}, \text{ while } W_t > \underline{U}, t > 0.$$

If $\beta R^* = 1$, then c_t is constant, while if $\beta R^* < 1$, c_t is strictly declining due to impatience.

Similarly, the first order condition for labor, equation (13), takes the form:

$$(\beta R^*)^t \left(1 + \eta \left(1 + \gamma\right)\right) = \psi^{-1} \mu n_t^{-\gamma}, \text{ while } W_t > \underline{U}, t > 0.$$

The right hand side is strictly declining in n_t. If $\beta R^* = 1$, then n_t is constant, while if $\beta R^* < 1$, n_t is strictly increasing due to impatience.

The labor tax wedge along this unconstrained path is:

$$\tau_t^n = \frac{\eta \left(\gamma + \sigma\right)}{1 + \eta(1-\sigma)}, \text{ while } W_t > \underline{U}, t > 0.$$

Regardless of relative impatience, while unconstrained by the borrowing constraint, labor tax is a constant. If $\beta R^* = 1$, we see that consumption, labor, and taxes are all constant, which accords with tax smoothing.

If $\beta R^* < 1$ taxes are constant while unconstrained, but the fact that c_t is falling and n_t is increasing over time implies that $u_t < u_{t-1}$ and this path is not sustainable in the long run. It must be then that at some point $W_t = \underline{U}$ by Assumption 3.

The following lemma states that $W_t = \underline{U}$ is an absorbing state for the utility level:

Lemma 2. *Suppose that $W_t = \underline{U}$ for $t > 0$, then $W_{t+s} = \underline{U}$ for all $s > 0$.*

We can also obtain an explicit solution for the behavior of the labor tax. In our current environment, $\partial W_s / \partial u_t = \beta^{t-s}$ for $s \leq t$, and zero otherwise. Substituting this into (12) and (13), we have

$$(\beta R^*)^t \left(1 + \eta\,(1-\sigma) + \sum_{j=0}^{t} \lambda_j\right) = \mu c_t^{\sigma}$$
$$(\beta R^*)^t \left(1 + \eta\,(1+\gamma) + \sum_{j=0}^{t} \lambda_j\right) = \psi^{-1} \mu n_t^{-\gamma}.$$

where we have used that $f'(n) = 1$. Taking the ratio of these two expressions to solve for the labor tax, we have:

$$\tau_t^n = \frac{\eta\,(\gamma + \sigma)}{1 + \eta(1-\sigma) + \sum_{j=0}^{t} \lambda_j}.$$

The fact that $\lambda_t > 0$ whenever the borrowing constraint binds implies that the summation $\sum_{j=0}^{t} \lambda_j$ is increasing over time along a constrained path, and the labor tax is falling. In the limit, $\tau_t^n = 0$, as implied by Proposition 2.

The efficient allocation is graphically depicted in Figure 1. The figure represents indifference curves in consumption-labor space. The curve $\underline{u} \equiv (1-\beta)\underline{U}$ denotes the flow utility that delivers $\underline{U}$. The tax wedge can be read off the slope of the indifference curve: $\frac{1}{1+\tau^n} = -\frac{u_n}{u_c} = \left.\frac{dc}{dn}\right|_{\underline{u}}$. We consider the initial allocation (after time 0) at point A, which is associated with some initial assets in private and public hands (a_0, A_0).

We overlay the dynamics of the efficient allocation on the indifference curves, using the figure as a phase diagram. For $u_t > \underline{u}$, the allocation is unconstrained by the borrowing limit. As noted above, the unconstrained allocation features falling consumption, increasing labor, and a constant tax wedge. The constant tax wedge implies parallel shifts across indifference curves as utility falls. Point B is the first point at which utility reaches $\underline{u}$. Once $u_t = \underline{u}$, utility is constant and we move along the indifference curve. As we move along, consumption and labor both increase and the tax wedge falls, eventually reaching zero in the limit. Point C represents the steady state of the economy.

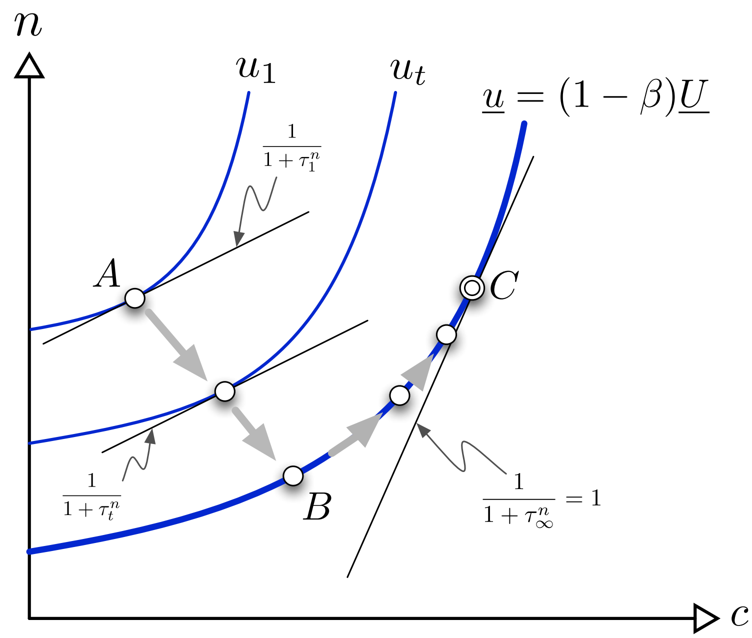


Figure 1: Path of Efficient Allocation when $\beta R < 1$. The blue lines represent indifference curves in the (c, n) space, and movements to the southeast represent reductions in utils.

The intuition while the economy is unconstrained by the borrowing limit is straightforward. Impatience relative to r^* makes a declining path of consumption and leisure optimal. Standard tax smoothing insights makes a constant tax rate efficient as well. The fact that the economy ultimately hits the constraint is also straightforward. However, the question remains why point B is not a steady state. That is, once the economy hits the borrowing constraint, why are there further dynamics, and why do these involve a declining labor tax and increasing consumption?

The intuition is related to Proposition 4. Output at point B is distorted downward by labor taxes, reducing the ability of the economy to service a large steady state debt. As the tax on labor is reduced, output increases. In particular, output increases more than consumption as long as the tax rate is strictly positive, raising the steady state net exports (i.e., net debt payments) of the economy. To see that consumption increases less than output (i.e., labor), note that along the indifference curve, $dc = -u_n/u_c dn = dn/(1+\tau^n)$. By moving along the indifference curve to the zero-tax steady state C, the government increases its capacity to service steady state debt. The benefit of this policy for impatient agents is that consumption and leisure can be front loaded when the debt is incurred.

4 Constraints on the Representative Agent

TO BE COMPLETED

5 A Closed Economy

In this section, we show how our results carry over to a closed economy. In a closed economy, all capital is owned by domestic agents, and the source tax on capital, τ_t^k, is equivalent to the residence based tax on capital, ϕ_t^k, so we can set the last one to zero without loss of generality. It follows then that $p_t = q_t$ and the aggregate resource constraint is the national income accounting identity:

$$c_t + g_t + k_{t+1} - (1-\delta)k_t \leq F(k_t, n_t). \tag{RC}$$

Market clearing requires that the private agents' initial wealth corresponds to their holdings of government debt plus the domestic capital stock: $a_t = b_t + p_{t-1}k_t/p_t$, where the fact that k_t is adjusted by the inter-temporal price p_{t-1}/p_t reflects the fact that a_t and b_t are in period-t (after tax) units, while k_t is the amount invested at the end of period $t-1$. We restrict $\tau_0^k = 0$ to eliminate the initial capital levy solution. The requirements of a closed economy competitive equilibrium are the same as their open economy counterparts, with the appropriate adjustment to the resource constraint and recognition that initial capital is a state variable in a closed economy:

Definition 3. *A* ***closed economy competitive equilibrium*** *from an initial position* (k_0, b_0) *consists of prices* $\{r_t, w_t, r_t^k\}$, *taxes* $\{\phi_t^k = 0, \tau_t^n, \tau_t^k\}$ *with* $\tau_0^k = 0$, *non-negative lump-sum transfers* $\{T_t\}$, *and quantities* $\{c_t, n_t, k_t\}$ *such that* ***(i)*** r_t, r_t^k, p_t, q_t *satisfy equations and (2), (3), (9);* ***(ii)*** *households optimize given prices and taxes subject to their budget constraint (4); that is, there exists* $\theta \geq 0$ *such that (4), (5) and (6) are satisfied* ***(iii)*** *firms maximize profits given prices and taxes; that is, equations (7) and (8) hold;* ***(iv)*** *the labor market clears;* ***(v)*** *the government budget constraint (10) holds; and* ***(vi)*** *each of the closed-economy aggregate resource constraints (RC) hold.*

As before, we use the primal approach. The closed economy version of lemma 1 is:

Proposition 1'. *An allocation is consistent with a closed economy competitive equilibrium if and* *only if it satisfies the implementability condition*

$$\sum_{t=0}^{\infty} \beta^t \left(u_c(c_t, n_t)c_t + u_n(c_t, n_t)n_t\right) \geq u_c(c_0, n_0)\Big(b_0 + \big(F_k(k_0, n_0) + 1 - \delta\big)k_0\Big) \tag{IC}$$

and each of the closed economy resource constraints (RC) *hold.*

Equation (PC) continues to define any additional constraints on the problem. The natural interpretation of (PC) in a closed economy is one of a lower bound on aggregate savings, rather than an upper bound on aggregate debt. In this regard, the dynastic model with insufficient private altruism (relative to the government's Pareto weights) is perhaps the most relevant interpretation. For the closed economy, we add an additional assumption on the functions W_t:

Assumption 4. $\partial W_s / \partial k_t \leq 0 \forall s, t$.

This assumption insures that additional capital (weakly) tightens the constraint. This is consistent, for example, with more capital raising the incentive of the government to renege on its tax promises.

The definition for an efficient allocation given in Definition 2 continues to hold with the relevant notion of equilibrium being a closed economy competitive equilibrium. To characterize (interior) efficient allocations, let $\beta^t \gamma_t$ denote the multiplier on the time t aggregate resource constraint. The first order conditions for $t \geq 1$ are:

$$\frac{1}{\gamma_t}\left[1 + \eta + \eta\left(\frac{u_{cc}(c_t, n_t)}{u_c(c_t, n_t)}c_t + \frac{u_{cn}(c_t, n_t)}{u_c(c_t, n_t)}n_t\right) + \sum_{s=0}^{\infty} \beta^{s-t}\lambda_s \frac{\partial W_s}{\partial u_t}\right] = \frac{1}{u_c(c_t, n_t)} \tag{18}$$

$$\frac{1}{\gamma_t}\left[1 + \eta + \eta\left(\frac{u_{cn}(c_t, n_t)}{u_n(c_t, n_t)}c_t + \frac{u_{nn}(c_t, n_t)}{u_n(c_t, n_t)}n_t\right) + \sum_{s=0}^{\infty} \beta^{s-t}\lambda_s \frac{\partial W_s}{\partial u_t}\right] = -\frac{F_n(k_t, n_t)}{u_n(c_t, n_t)} \tag{19}$$

$$F_k(k_t, n_t) + 1 - \delta + \frac{1}{\gamma_t}\sum_{s=0}^{\infty} \beta^{s-t}\lambda_s \frac{\partial W_s}{\partial k_t} = \frac{\gamma_{t-1}}{\beta\gamma_t}. \tag{20}$$

The initial period first order conditions ($t = 0$) are adjusted in the same way as they were in the open economy formulation. As in the case of the open economy, the resource multiplier is strictly positive in an efficient allocation after period 0:

Lemma 1'. *Suppose Assumptions 1 and 2 hold. If the efficient allocation is interior, then the resource constraint binds ($\gamma_t > 0$ for all $t \geq 1$).*

We can now state the closed economy version of Proposition 2:

Proposition 2′. *[Zero Labor Tax in the Long-run – Closed Economy] Suppose Assumptions 1, 2 and 4 hold. If an (interior) efficient allocation satisfies that:*

(a) $\liminf_{t\to\infty} c_t > 0$*;*

(b) if $u_c(u_{cn}n/u_c - u_{nn}n/u_n)$ *is unbounded for* $c > \epsilon_c > 0$*; then* $\limsup_{t\to\infty} n_t < \bar{n}$*; and*

(c) there exist $M > 0$ *and* T *such that* $1 > M > \beta(F_k(k_t, n_t) + 1 - \delta)$ *for all* $t > T$*;*

then $\lim_{t\to\infty} \tau_t^n = 0$*.*

Recall that in Proposition 2, the impatience condition (c) involved $\beta(1 + r_t^*)$, as r_t^* pins down the inter-temporal marginal rate of transformation in an open economy. In the closed economy case, we replace this with the condition $\beta\left(F_k(k_t, n_t) + 1 - \delta\right) << 1$, as the relevant marginal rate of transformation in a closed economy is the marginal product of capital. This impatience condition implies that capital above the modified golden rule from the perspective of the private agents. That is, capital is "over-accumulated" relative to the private agents' discount factor. In the closed economy, we impose that $\partial W_s/\partial k_t \leq 0$, which places some restrictions on the reasons why capital is above the modified golden rule. In particular, it implies that all else equal, reducing capital (weakly) relaxes the constraint, so the over-accumulation of capital is not hard-wired.[4] Rather, the over-accumulation of capital is required to ensure a certain level of future (generations') utility.

In the open economy, we proposed the intuition that the efficient allocation maximizes steady state aggregate debt (see proposition 4). In a closed economy, aggregate debt is zero by definition. Nevertheless, a similar intuition carries through to the closed economy case. The constraints in the closed economy ensure that sufficient resources are provided to future agents. From the perspective of the private agents, the efficient allocation in a closed economy is distorted by inducing savings, which in turn is necessary to sustain a relatively high level of future utility. The closed-economy equivalent of maximizing steady state aggregate debt is to minimizing the amount of capital left for future generations.

[4]This rules out such constraints as $k \geq \underline{k} > k^*$, where k^* is the modified golden rule capital. It allows the standard limited commitment constraints in which more capital makes deviation more profitable. Such a restriction on W was unnecessary in the open economy case as any over-accumulation of capital did not affect inter-temporal prices, which were pinned down by international financial markets.

Specifically, suppose that the efficient allocation converges to the steady state allocation $(c_\infty, n_\infty, k_\infty)$, with steady state fiscal expenditure g. Then the efficient allocation minimizes the steady state capital necessary to sustain steady state utility:

Proposition 4′. *[Minimal Steady State Capital] Suppose the efficient allocation converges to a steady state. Under the conditions of Proposition 2′, the efficient allocation converges to the minimal steady state capital allocation, where the latter allocation is the solution to:*

$$\min_{c,n,k} k,$$

subject to

$$\begin{aligned} u(c,n) &\geq u(c_\infty, n_\infty), \\ c &\leq F(k,n) - \delta k - c - g. \end{aligned}$$

This proposition highlights the link between the closed and open economy problems. In both cases, there is a floor on savings (or a ceiling on debt). Fiscal policy in the presence of impatience is designed to exhaust these constraints in order to maximize front loading. This requires efficient production in the steady state, or zero tax on labor inputs.

6 A Decentralization: Managing the portfolio of sovereign and domestic debt

In this section, we explore the time path of aggregates of the economy described in the section 3.1. From the analysis in that section, starting from a positive tax economy, we have that labor taxes are falling over time and both the private agent and the economy as a whole are running down assets. It is not clear, however, what is happening to government debt itself, which is the difference between private and aggregate wealth (recall from our accounting identities that $b + b^* = a - A$). In particular, the government may be borrowing from abroad to retire domestic debt. We can shed light on fiscal deficits through a numerical example.

Figure 2 depicts the path of a particular parameterization of the model discussed above.[5] Given that the model is quite simple, we present figure 2 as a guide to intuition

[5] Specifically, let $u(c,n) = \log c - \psi \frac{n^{1+\gamma}}{1+\gamma}$. We set $\gamma = 1$ and $g = 0.20$, and select ψ and $\underline{u}$ so that steady

rather than a quantitative exercise. Time is the horizontal axis in each panel, and period T denotes the first period in which the borrowing constraint binds. The first panel depicts consumption and labor, and the second panel depicts utility relative to $\underline{u}$. Consumption and leisure decline prior to period T, as the borrowing constraint is slack and agents are relatively impatient. At period T, utility has fallen to $\underline{u}$, as seen in panel (b), and remains there. Nevertheless, as discussed above, dynamics continue, with leisure continuing to fall but consumption now rising. This is supported by a declining tax on labor. In particular, as depicted in panel (c), labor taxes are constant prior to T, but then begin to fall. This is the front loading of taxes that is optimal when the economy is constrained. Eventually, the labor taxes converges to zero, as stated in Proposition 2.

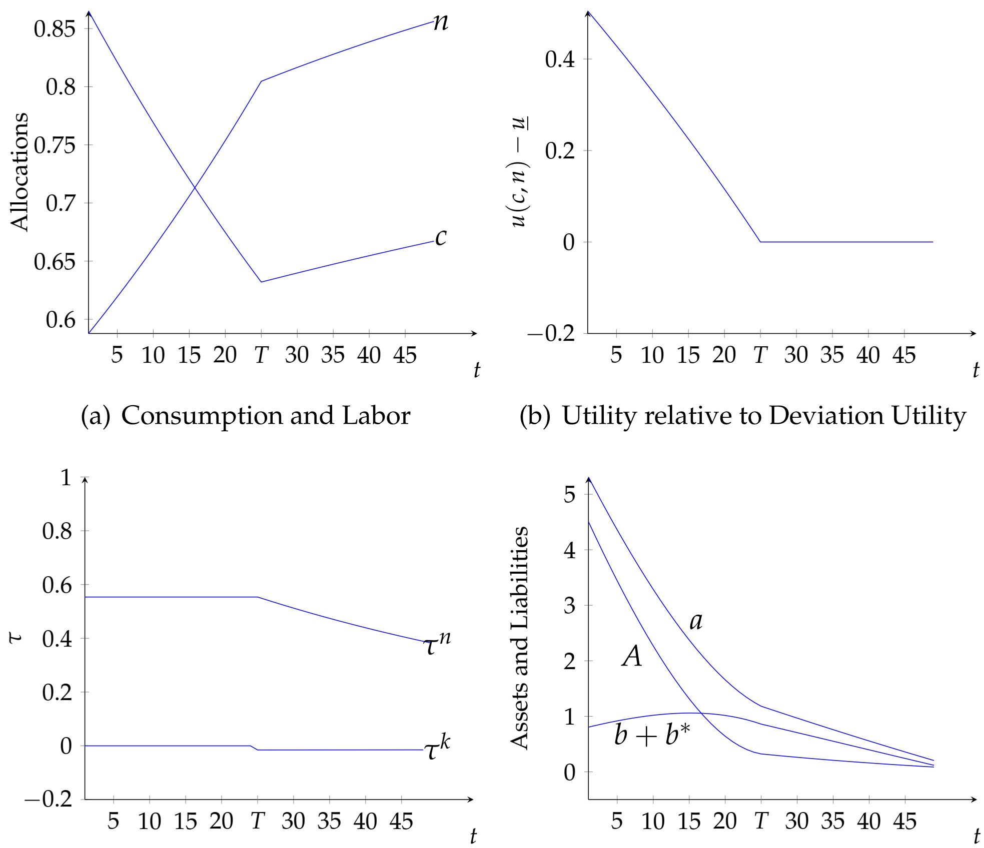
The tax on capital income is zero when the economy is unconstrained, and then slightly negative (a subsidy to savings) after period T. The subsidy to capital income is best interpreted as a tax on additional debt, which is necessary in this decentralization to keep the private agents from violating the aggregate borrowing limit. An alternative decentralization would be to constrain the private agents debt directly, and this wedge would reflect the multiplier on this constraint (verify this).

Panel (d) depicts the corresponding path of assets and liabilities. The country's aggregate net foreign asset position A falls rapidly while the borrowing constraint is slack. Once constrained, the economy continues to draw down assets and starts to accumulate foreign liabilities, although the process is slower after period T. Private assets a also fall, both before and after T. The "deceleration" at period T is less pronounced for private assets than it is for aggregate assets. This reflects that private assets need to be reduced in order to make labor efficiency consistent with the implementability constraint in the steady state. That is, the flip side of frontloading labor taxes is that the government pays down domestically held public debt. The government's total debt, $b + b^*$, is also depicted in panel (d), and is equal to $a - A$ by accounting identity. As might be expected, government debt is increasing while the economy is unconstrained. However, at some point *before* the economy becomes constrained, the government starts paying down its debt, and continues to do so after period T. In the limit, debt is reduced to the point that labor taxes are no longer necessary to fund government expenditures.

The dynamics depicted in figure 2 highlight the important role that government debt and net foreign assets play in supporting the convergence to first best labor. The model

state labor/income is 1 and steady state net foreign liabilities are 20 percent of aggregate income. The international interest rate is 0.05, and $\beta = 0.94$, so private agents discount at a higher rate than the world interest rate.

Figure 2: Time Path of Economic Aggregates



(a) Consumption and Labor

(b) Utility relative to Deviation Utility

(c) Labor and Capital Income Tax

(d) Country Assets, Private Assets, and Government Liabilities

does not make a clean prediction for the relative quantities of public debt held domestically and abroad without further restrictions. That is, if private agents can hold foreign assets directly, there is an indeterminacy in the following sense: The government can borrow from a domestic resident directly, or indirectly by having the domestic resident lend to foreign investors and then borrowing the same amount from foreign markets.

For many developing economies, private residents do not hold large net foreign asset positions, and the vast majority of international borrowing and lending is implemented by the government. If we set private agent foreign assets to zero, then we know that domestic assets a are equivalent to domestically held government debt.[6] Moreover, the country's net foreign assets equal government foreign reserves minus sovereign debt

[6]This follows from the absence of physical capital, but as we show below in an extension with capital, we continue to have a sharp prediction for domestically held debt as long as $a^* = 0$.

($-b^*$). Therefore, panel (d) of figure 2 indicates that domestically held government debt is falling, while net sovereign liabilities are increasing (or foreign assets are falling). Therefore, the path to efficient labor is one in which the public debt held abroad is increasing while the public debt held domestically is falling. As a normative prediction, the model states that the government should pay off domestically held debt first, while continuing to borrow from abroad. This is the optimal path to zero labor taxes for an impatient economy.

7 Conclusion

This paper characterized the optimal fiscal policy when agents are relatively impatient. A defining feature of the optimal policy is that labor taxes are front loaded. A consequence of such a policy is that the transition to the steady involves a conservative fiscal policy in which the government accumulates enough assets to finance expenditures in the absence of labor tax revenue. While the fiscal authority may run surpluses, the economy as a whole is accumulating foreign liabilities. As noted in the introduction, this normative description does not resemble the observed policy in many indebted economies. Such economies frequently run fiscal deficits and back load taxes. Fiscal authorities in practice often accumulate claims against the private sector, for example through state ownership of physical capital or international reserves. This could be considered a possible decentralization of the optimal policy. Nevertheless, the optimal policy of declining labor tax rates does not resemble observed responses to binding borrowing constraints.

Our model is a straightforward extension of the canonical model used to study optimal fiscal policy in unconstrained economies. It encompasses a large class of political economy frictions, particularly those that involve impatient politicians, as well as asset market frictions such as limited commitment. However, the model does not encompass other potential frictions in the asset markets, such as illiquidity of public or private debt, that may play a role in fiscal crises. Such additional complications do not a priori challenge the argument for front loading tax revenues, but we leave for future research a full analysis. Nevertheless, the intuition behind the result is likely to play a role in other environments. Namely, an economy that wishes to front load consumption and leisure should operate as efficiently as possible in the long run to maximize its debt servicing capacity; and, when taxes are distortionary, private indebtedness does not undermine efficiency to the same extent as public indebtedness.

References

Daron Acemoglu, Michael Golosov, and Aleh Tsyvinski. Political economy of mechanisms. *Econometrica*, 76(3):619–641, May 2008.

Mark Aguiar and Manuel Amador. Growth in the shadow of exproriation. *Quarterly Journal of Economics*, forthcoming.

Mark Aguiar, Gita Gopinath, and Manuel Amador. Investment cycles and sovereign debt overhang. *Review of Economic Studies*, 76:1–31, 2009.

Andrew Atkeson, Patric Kehoe, and V.V. Chari. Taxing capital income: A bad idea. *Federal Reserve Bank of Minneapolis Quarter Review*, 23:3–17, 1999.

Christopher Chamley. Optimal taxation of capital income in general equilibrium. *Econometrica*, 54(3):607–622, 1986.

Begona Dominguez. Public debt and optimal taxes without commitment. *Journal of Economic Theory*, 135:159–170, 2007.

Emmanuel Farhi and Iván Werning. Inequality and social discounting. *Journal of Political Economy*, 115:365–402, 2007.

Larry E. Jones, Rodolfo E. Manuelli, and Peter E. Rossi. On the optimal taxation of capital income. *Journal of Economic Theory*, 73:93–117, 1997.

Kenneth Judd. Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics*, 28:59–83, 1985.

Lars Ljungqvist and Thomas J. Sargent. *Recursive Macroeconomic Theory*. MIT Press, 2nd edition, 2004.

Robert E. Jr. Lucas and Nancy L. Stokey. Optimal monetary and fiscal policy in an economy without capital. *Journal of Monetary Economics*, 12:55–94, 1983.

Christopher Phelan. Opportunity and social mobility. *Review of Economic Studies*, 73:487–504, 2006.

Carmen Reinhart and Kenneth Rogoff. *This Time is Different: Eight Centuries of Financial Folly*. Princeton Univeristy Press, 2009.

Catrina Reis. Taxation without commitment. *working paper*, 2008.

A Proof of Proposition 1 and 1′

Proof. **The only if part:** In a competitive equilibrium, the budget constraint of the consumer most be holding with equality. Using this, substitute p_t by $\beta^t u_c(c_t, n_t)/u_c(c_0, n_0)$ and substitute w_t by $-u_n(c_t, n_t)/u_c(c_t, n_t)$. Then you get that:

$$\sum_{t=0}^{\infty} \beta^t \Big(u_c(c_t, n_t)c_t + u_n(c_t, n_t)n_t \Big) - u_c(c_0, n_0)a_0 = u_c(c_0, n_0)T \geq 0$$

where the last inequality follows from $T \geq 0$ and that $u_c \geq 0$.

The necessity that resource constraints hold follows from the definition of equilibria. This equation must hold for both the open and the closed economy. For the closed economy case, we note that $a_0 = b_0 + (1 + r_0)k_0$, and by the definition of equilibrium we have that $r_0 = F_k(k_0, n_0) - \delta$.

The if part: For the small open economy case, given an allocation, let us define the following objects:

$$\begin{aligned}
w_t &= -u_n(c_t, n_t)/u_c(c_t, n_t), \\
r_t^k &= r_t^* - \delta, \\
\tau_t^n &= -\frac{F_n(k_t, n_t)u_c(c_t, n_t)}{u_n(c_t, n_t)} - 1, \\
\tau_t^k &= F_k(k_t, n_t)/r_t^k - 1,
\end{aligned}$$

for all $t \geq 0$. Define as well:

$$\begin{aligned}
\phi_t^k &= 1 - \frac{1}{r_t^*} \left(\frac{u_c(c_{t-1}, n_{t-1})}{\beta u_c(c_t, n_t)} - 1 \right), \\
p_t &= \beta^t u_c(c_t, n_t)/u_c(c_0, n_0), \\
T_t &= 0,
\end{aligned}$$

for all $t \geq 1$; and

$$\begin{aligned}
T_0 &= \sum_{t=0}^{\infty} \beta^t \frac{u_c(c_t, n_t)c_t + u_n(c_t, n_t)n_t}{u_c(c_0, n_0)} - a_0 \geq 0, \\
\theta &= u_c(c_0, n_0).
\end{aligned}$$

Now note that p_t satisfies (3) by construction. So part (i) of the equilibrium definition is satisfied. Given the definition of $\theta \geq 0$, w_t and p_t, we get that conditions (5) and (6) are satisfied as well. Note that the budget constraint of the consumer is satisfied with T_t as defined above. So part (ii) is satisfied.

For part (iii), note that from the definition of r_t^k and τ_t^n and τ_t^k it follows that equations (7) and (8) hold. So part (iii) is satisfied. Part (v) holds as well, by hypothesis of the Proposition. We now show that part (iv) holds. We don't appeal directly to Walras' law because the budget constraint of the government is not necessarily holding with equality. However, rewriting equation (RC*), we get:

$$\sum_{t=0}^{\infty} q_t \left(g_t + T_t - \tau_t^n w_t n_t - \tau_t^k r_t^k k_t - \frac{\phi_t^k r_t}{1 + (1 - \phi_t^k) r_t} a_t \right) \\ + \sum_{t=0}^{\infty} q_t \left(c_t - w_t n_t - T_t + \frac{\phi_t^k r_t}{1 + (1 - \phi_t^k) r_t} a_t \right) \leq A_0 \quad (21)$$

where we used the first order conditions of the firm together witht $F = F_n n + F_k k$. Now, note that:

$$\begin{aligned} &\sum_{t=0}^{\infty} q_t \left(c_t - w_t n_t - T_t + \frac{\phi_t^k r_t}{1 + (1 - \phi_t^k) r_t} a_t \right) \\ &= \sum_{t=0}^{\infty} q_t \left(c_t - w_t n_t - T_t - a_t + \frac{1 + r_t}{1 + (1 - \phi_t^k) r_t} a_t \right) \\ &= \sum_{t=0}^{\infty} q_t \left(c_t - w_t n_t - T_t - a_t + \frac{1}{1 + (1 - \phi_t^k) r_t} a_{t+1} \right) + a_0 = a_0 \end{aligned}$$

where we have used the definition of a_t. Then, plugging back into (22):

$$\sum_{t=0}^{\infty} q_t \left(g_t + T_t - \tau_t^n w_t n_t - \tau_t^k r_t^k k_t - \frac{\phi_t^k r_t}{1 + (1 - \phi_t^k) r_t} a_t \right) \leq A_0 - a_0 = -b_0 \quad (22)$$

And thus part (iv) holds.

Taken together, the above implies that the sequences of prices, quantities and taxes we have constructed is a competitive equilibrium. So the allocation is consistent with an open economy competitive equilibrium.

The proof of the "if part" for the case of a closed economy is similar, so we omit it. □

B Proof of lemma 1 and 1′

Proof. **For the small open economy:** Suppose that $\mu = 0$. Take equations (12) and (13), interiority of the allocation implies that at any $t \geq 1$:

$$1 + \eta + \eta \left(\frac{u_{cc}(c_t, n_t)}{u_c(c_t, n_t)} c_t + \frac{u_{cn}(c_t, n_t)}{u_c(c_t, n_t)} n_t \right) + \sum_{s=0}^{\infty} \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial u_t} = 0$$
$$1 + \eta + \eta \left(\frac{u_{cn}(c_t, n_t)}{u_n(c_t, n_t)} c_t + \frac{u_{nn}(c_t, n_t)}{u_n(c_t, n_t)} n_t \right) + \sum_{s=0}^{\infty} \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial u_t} = 0$$

Subtracting the two equations above we get that:

$$\eta \frac{c_t}{u_c(t)} \left(u_{cc}(t) - \frac{u_{cn}(t) u_c(t)}{u_n(t)} \right) + \eta \frac{n_t}{u_c(t)} \left(u_{cn}(t) - \frac{u_{nn}(t) u_c(t)}{u_n(t)} \right) = 0$$

As long as consumption and leisure are normal goods, the terms inside the brackets are non-negative. But our strict concavity assumptions guarantee that at least one of them is strictly positive.[7] It follows then that, for the above equation to hold, $\eta = 0$ given that c_t and n_t are interior. But $\eta = 0$ implies that:

$$1 = -\sum_{s=0}^{\infty} \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial u_t}$$

which is a contradiction given that $\lambda_s \geq 0$ and $\frac{\partial W_s}{\partial u_t} \geq 0$.

For the closed economy: The proof is similar to the above for any $t \geq 1$. $\square$

C Proof of Propositions 2 and 2′

Proof. To recover the labor tax from the allocation, we use the agent's and firm's first order conditions to obtain:

$$\frac{-u_n(t)}{u_c(t)} = \left(\frac{1}{1 + \tau_t^n} \right) F_n(t),$$

[7] Suppose not, then we have that $u_c / u_n = u_{cn} / u_{nn}$ from the second term, and from the first we get $u_{cc} - \frac{u_{cn}^2}{u_{nn}} = 0$ which is a contradiction of our strict concavity assumption.

or

$$\tau_t^n = \frac{-F_n(t)u_c(t)}{u_n(t)} - 1. \tag{23}$$

For part (a): Consider the first order condition for consumption (12). Note that $\frac{\beta^t}{p_t^*} = \Pi_{s=0}^t \beta\left(1+r_s^*\right)$. Remember that $\beta(1+r_t^*) > 0$ for all t. And then, $\beta(1+r_t^*) < M_1 < 1$ for sufficiently large t implies that $\lim_{t\to\infty} \beta^t/p_t^* = 0$. Using (12) and (13), together with the interiority assumption and that $\mu > 0$ by Lemma 1, it follows that:

$$\eta\left(\frac{\beta^t}{p_t^*\mu}\right)\left[\left(u_{cc}(t) - \frac{u_{cn}(t)u_c(t)}{u_n(t)}\right)c_t + \left(u_{cn}(t) - \frac{u_{nn}(t)u_c(t)}{u_n(t)}\right)n_t\right] \\ = 1 + \frac{F_n(t)u_c(t)}{u_n(t)} = -\tau_t^n$$

Now note that for sufficiently large t, $c_t > c_0$ for some $c_0 > 0$ by condition (a). Condition (b) together with Assumption (3) guarantee that the terms inside the square brackets are bounded. Taking the limits as $t \to \infty$, it follows then that $\lim_{t\to\infty} \tau_t^n = 0$.

For part (b): The proof for the closed economy works in exactly the same way as the proof of the open economy, but replacing $\beta^t/(p_t^*\mu)$ by $1/\gamma_t$. Hence, we just need to show that $\gamma_t \to \infty$. From (20) we get the following difference equation:

$$\gamma_t = \frac{1}{\beta(F_k(k_t,n_t)+1-\delta)}\gamma_{t-1} - \frac{\sum_{s=0}^{\infty}\beta^{s-t}\lambda_s\frac{\partial W_s}{\partial k_t}}{F_k(k_t,n_t)+1-\delta}$$

Note that $\gamma_t > 0$ for all $t \geq 1$ (which holds by Lemma 1) implies that $F_k(k_t,n_t)+1-\delta > 0$ for all $t \geq 2$, given the assumption that $\partial W_s/\partial k_t \geq 0$. This implies that for sufficiently large $T_0 > 2$:

$$\gamma_t \geq \frac{1}{M_0}\gamma_{t-1} + A_t, \forall t > T_0 \tag{24}$$

for $A_t = -\frac{\beta\sum_{s=0}^{\infty}\beta^{s-t}\lambda_s\frac{\partial W_s}{\partial k_t}}{F_k(k_t,n_t)+1-\delta} \geq 0$. Using that $\gamma_t > 0$ for $t > T_0$, $0 < M_0 < 1$, it follows from (24) that $\lim_{t\to\infty}\gamma_t = \infty$. $\square$

D Proof of Proposition 4 and 4′

Proof. For the open economy:

The maximal steady state debt problem in the proposition is a convex programming problem. Let ψ_u and ψ_k denote the multipliers on the utility and capital constraints. The first order conditions are

$$\begin{aligned} \psi_u u_c(c,n) &= 1 \\ \psi_u u_n(c,n) &= -F_n(k,n) \\ \psi_k &= F_k(k,n) - r^\star - \delta \end{aligned}$$

where we have omitted the normalization term $(1+r^\star)/r^\star$ from the objective function as this term does not affect choices. These conditions, the constraints, plus the complementary slackness conditions are necessary and sufficient for an optimum. The conditions of proposition 2 ensure that $\psi_u > 0$ (that is, u_c and u_n are finite for any allocation that yields weakly greater utility than the steady state).

Let $x = (c, n, k, \psi_u, \psi_k) \in \Re^5_+$ represent an arbitrary allocation and multipliers, and x^* denote the x that satisfies the maximal debt first order conditions. Define the function $H(x) : \Re^5_+ \to \Re^5$ by:

$$H(x) = \begin{cases} \psi_u u_c(c,n) - 1, \\ \psi_u u_n(c,n) + F_n(k,n), \\ \psi_k - F_k(k,n) + r^\star + \delta, \\ \psi_u \left(u(c,n) - u(c_\infty, n_\infty)\right), \\ \psi_k \left(k - k_\infty\right). \end{cases}$$

Note that the strict concavity of u and F ensures that x^* is the unique zero of this function such that the constraints are satisfied.[8]

Now let $x^t = (c_t, n_t, k_t, \psi^t_u, \psi^t_k)$ denote the efficient allocation at time t, where $\psi^t_u = (q_t\mu)^{-1} \sum_{s=0}^{t} \beta^s \lambda_s \frac{\partial W_s}{\partial u_t}$ and $\psi^t_k = -\sum_{s=0}^{t} \beta^s \lambda_s \frac{\partial W_s}{\partial k_t}$ from the efficient allocation first order conditions (equations 12 - 14). Note that our assumptions on W ensure these are non-

[8] We omit the constraints themselves as elements of H, as the steady state efficient allocation will satisfy the constraints by definition.

negative. From the efficient allocation first order conditions, we have:

$$H(x^t) = \begin{cases} \left(\frac{\beta^t}{q_t \mu}\right) \left[1 + \eta + \eta \left(\frac{u_{cc}(c_t,n_t)}{u_c(c_t,n_t)} c_t + \frac{u_{cn}(c_t,n_t)}{u_c(c_t,n_t)} n_t\right)\right] u_c(c_t, n_t), \\ \left(\frac{\beta^t}{q_t \mu}\right) \left[1 + \eta + \eta \left(\frac{u_{cn}(c_t,n_t)}{u_n(c_t,n_t)} c_t + \frac{u_{nn}(c_t,n_t)}{u_n(c_t,n_t)} n_t\right)\right] u_n(c_t, n_t), \\ 0, \\ \psi_u^t \left(u(c_t, n_t) - u(c_\infty, n_\infty)\right), \\ \psi_k^t \left(k_t - k_\infty\right). \end{cases}$$

From the conditions of Proposition 2, the first two elements of $H(x^t)$ are converging to zero as t becomes large. The other terms converge to zero by definition as the allocation converges to the steady state. In particular, $H(x^\infty) = H(\lim_{t\to\infty} x^t) = 0$. As x^* is the unique zero of $H(x)$, we have that $x^\infty = x^*$.

For the closed economy:

Write the Lagrangian for the minimal capital problem:

$$\mathcal{L} = k + \psi \left(u(c_\infty, n_\infty) - u(c, n)\right) + \rho \left(c + g + \delta k - F(k, n)\right).$$

As in the open economy case, we have strictly convex programming problem. We can write the first order conditions as:

$$\begin{aligned} F_k(k, n) - \delta - \frac{1}{\rho} &= 0, \\ \frac{1}{u_c} - \frac{\psi}{\rho} &= 0, \\ \frac{-F_n}{u_n} - \frac{\psi}{\rho} &= 0, \end{aligned}$$

where the first condition ensures that $\rho > 0$. The conditions of proposition 2′ ensure that c is bounded away from zero and n away from $\bar{n}$. This ensures that $\psi > 0$ as well. Define $x = (c, n, k, \psi, \theta)$ and $H^c(x)$ as the left hand side of the above conditions plus the complementary slackness expressions. Let x^* denote the (unique) zero of $H^c(x)$ such that the constraints are satisfied.

Define x^t as the efficient closed economy allocation at time t, with

$$\rho^t \equiv \left[\frac{1}{\gamma_t}\sum_{s=0}^{\infty}\beta^{s-t}\lambda_s\frac{\partial W_s}{\partial k_t} - \frac{\gamma_{t-1}}{\beta\gamma_t}\right]^{-1}$$

$$\psi^t/\rho^t \equiv \frac{1}{\gamma_t}\sum_{s=0}^{\infty}\beta^{s-t}\lambda_s\frac{\partial W_s}{\partial u_t},$$

where we use the fact that the bracketed term in the first expression is equal to $F_k(k_t, n_t) - \delta$, which is strictly greater than zero in the efficient allocation. Then from (18)–(20), we have:

$$H^c(x^t) = \begin{cases} \frac{1}{\gamma_t}\left[1+\eta+\eta\left(\frac{u_{cc}(c_t,n_t)}{u_c(c_t,n_t)}c_t + \frac{u_{cn}(c_t,n_t)}{u_c(c_t,n_t)}n_t\right)\right], \\ \frac{1}{\gamma_t}\left[1+\eta+\eta\left(\frac{u_{cn}(c_t,n_t)}{u_n(c_t,n_t)}c_t + \frac{u_{nn}(c_t,n_t)}{u_n(c_t,n_t)}n_t\right)\right], \\ 0, \\ \psi^t\left(u(c_t,n_t) - u(c_\infty,n_\infty)\right), \\ \rho^t\left(c+g+\delta k_t - F(k_t,n_t)\right). \end{cases}$$

The first two terms converge to zero under the conditions for proposition 2′, and the remaining terms converge to zero by definition of the steady state and the closed economy aggregate resource constraint (RC). The constraints are satisfied by definition. Therefore, $H^c(x^\infty) = 0 = H^c(x^*)$, and the fact that H^c has a unique zero implies that x^∞ coincides with the minimal steady state capital allocation. □

E Proof of Lemma 2

Proof. Let's proceed by contradiction. Suppose that for some $t > 0$, $W_t = \underline{U}$ and $W_{t+1} > \underline{U}$. Then, from the first order conditions we know that $c_t \geq c_{t+1}$ and $n_t \leq n_{t+1}$. This implies that $u_t \equiv u(c_t, n_t) geu(c_{t+1}, n_{t+1}) \equiv u_{t+1}$. However note that $u_t + \beta(u_{t+1} + \beta u_{t+2} + ..) = \underline{U}$ and that $u_{t+1} + \beta u_{t+2} + \beta^2 u_{t+3} + .. > \underline{U}$ by the hypothesis of the Lemma. Then, it must be that $u_t + \beta\underline{U} < \underline{U}$, and thus $u_t < (1-\beta)\underline{U}$. From above, we know then that $u_{t+1} \leq u_t < (1-\beta)\underline{U}$. Using that $u_{t+1} + \beta u_{t+2} + ... > \underline{U}$, we have that $u_{t+2} + \beta u_{t+3} + .. > (\underline{U} - u_{t+1})/\beta > (\underline{U} - (1-\beta)\underline{U})/\beta = \underline{U}$. It follows then that at $t+2$ the borrowing constraint is not binding and thus, $u_{t+2} \leq u_{t+1}$. Proceeding in this fashion we can show that $u_{t+s} \leq (1-\beta)\underline{U}$ for all $s > 1$. But this violates the borrowing constraint as it implies that $\sum_{s=0}^{\infty}\beta^s u_{t+s+1} < \underline{U}$. □