Using Asset Prices to Measure the Cost of Business Cycles

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Abstract

We propose a method to measure the welfare cost of economic fluctuations that does not require full specification of consumer preferences, and instead uses asset prices. We measure *the marginal cost of consumption fluctuations,* the benefit of a small reduction in consumption fluctuations. We show that this measure is an upper bound to the benefits of reducing all consumption fluctuations. To measure the marginal cost of fluctuations we fit a variety of pricing kernels that reproduce key asset pricing statistics. We show that the pricing kernel has to be non-stationary. We find that consumers would be willing to pay a very high price for reducing overall consumption uncertainty. However, for consumption volatility corresponding to the business cycle frequencies, the benefits of eliminating high frequency variations equals the benefits of either a perment increase in consumption

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of about one half of a percent of consumption or a permanent an increase in the growth rate of consumption of 2 basis points. We also clarify the link between the cost of consumption uncertainty, the equity premium and the slope of the real term structure.

1. Introduction

In a seminal contribution, Lucas (1987) proposes a measure of the welfare cost of economic fluctuations. His measure is defined as the proportional increase of consumption that makes the representative agent indifferent with getting forever consumption at its deterministic trend level. He finds a small cost of business cycles. Subsequently, several studies have proposed estimates of this cost of business cycles under alternative assumptions on preferences and the consumption processes. Primarily as a function of the specification and parameterization of preferences, these estimates vary widely across studies.[1] In this paper, we propose to measure the welfare cost of business cycles without fully specifying consumer preferences, instead we directly use financial market data. We believe that by directly measuring the premia for aggregate risk we can circumvent some of the difficulties related to specifying a utility function.

We start by generalizing Lucas' cost of business cycles along two dimensions. First, we derive *the marginal cost of consumption fluctuations*. Lucas' cost of business cycles measures the welfare gain from removing all the business cycle risk, as such it can be thought of as a total cost. We define a cost that measures the welfare benefits from reduced fluctuations *at the margin*. We find two useful features to this definition: first, because it is a marginal cost we can use asset prices to estimate the cost of business cycles for a representative agent. Second, given that most economic policies would not intend to eliminate business cycle fluctuations entirely, knowing the potential benefits at the margin may be useful in itself. Our second dimension of generalization concerns the type of consumption fluctuations

[1] Lucas' estimates were below one percent of consumption. Obstfeld (1994) finds slightly higher costs than Lucas when allowing for stochastic growth trends and preferences that distinguish between risk aversion and intertemporal elasticity of substitution. Campbell and Cochrane (1998) find larger costs of business cycles by using habit-formation preferences that are also able to explain the equity premium. Atkeson and Phelan (1994) present an example of an incomplete markets economy that can generate the equity premium, but nevertheless has only small costs of business cycles. Other examples include: Dolmas (1998), Hansen, Sargent and Tallarini (1998), Krusell and Smith (1999), Otrok (1998), Tallarini (1998); and for the related literature about the welfare gains from international integration: Lewis (1996) and Van Wincoop (1999).

that one associates with business cycles. Lucas' consumption was subject to only temporary deviations from a deterministic trend. Several later studies specified consumption as a nonstationary process (for instance, Obstfeld (1994), Campbell and Cochrane (1995)), while the cost of business cycles continued to be measured as the gain from eliminating consumption uncertainty entirely, including the risks associated with the permanent stochastic components. Our definition is general enough to encompass this case, but also allows for business cycle fluctuations to be defined differently, for instance, as the cyclical consumption volatility excluding a possibly stochastic trend. We find this distinction useful because *a priori* one may think that some policies affect primarily economic behavior at business cycle frequencies without affecting the trend behavior of the economy. Even without this distinction about the source of economic fluctuations, it may be interesting to know the costs of the more short lived cyclical fluctuations relative to the costs of overall consumption uncertainty.

Before presenting our estimates, we derive some analytical results about the benefits from reducing consumption uncertainty. Under general specification we show that the marginal cost of reducing fluctuations is higher than the total cost of these fluctuations, hence our estimates are upper bounds. For more specific cases, such as expected discounted utility, we obtain a sharper bound where the marginal cost of consumption uncertainty is twice the total cost. The marginal cost is also an upper bound with consumption externalities of a general class we label as envy.

Several of the above cited studies have highlighted the relationship between Lucas' low cost of business cycles and the equity premium puzzle. In particular, studies with preference specifications that can solve the equity premium puzzle report usually costs of business cycles larger than Lucas' original estimates. We show that the marginal cost of consumption uncertainty is not equal to the (consumption) equity premium in general. The equity premium is the excess return of equity over a risk-less rate of return for a given holding period. In contrast, the marginal cost of consumption uncertainty is equal to the excess in the price of a perpetual bond with growing coupons over the price of a consumption equity claim. Two of the factors that make the marginal cost of consumption uncertainty different from the equity premium are the steepness of the term structure and the persistence of the shocks. We clarify the link between the two and provide examples where the quantitative differences are of first-order importance.

Motivated by our finding that the cost of consumption uncertainty can be expressed as the ratio between the prices of two assets. We derive a simple formula for the marginal cost. The marginal cost can be written as a function of long-term real yields, aggregate dividend-price ratios and an estimate of the economy's trend growth rate. If consumption were equal to dividends, this formula could be used to measure the cost of business cycles. Strictly speaking however, it is a measure of the cost associated with *all* the uncertainty about aggregate *dividends*. Measuring the cost of dividend uncertainty in this way, we find the costs to amount to about 500% of the level of consumption.

To relax the assumption of our simplified expression for the marginal cost, and to better measure the relevant prices we estimate a pricing kernel. We estimate a pricing kernel as a way to interpolate from the available asset prices the prices for the assets that measure the cost of consumption fluctuations. Our asset pricing kernel is consistent with US historical returns on equity, the term structure and the price/dividend ratios, all features that we show, on theoretical grounds, are important for measuring the cost of business cycles. In the state space for the pricing kernel we include variables that determine the market price of risk in different popular models. Since consumption is an element of the state space, our pricing kernel allows to differentiate consumption from dividends and to model its risk premia. We model consumption and the pricing kernel as non-stationary. This choice has quantitative implications for some of our measures. Our choice is motivated by a bound, in the spirit of the Hansen-Jagannathan bound, that implies that the pricing kernel cannot be stationary. We find the cost of business cycles to be between one half and two thirds of a percent for high frequency fluctuations of the type captured by the Hodrick-Prescott filter. These numbers are larger than those found by Lucas, but are not extremely large numbers.. However, the cost of all consumption uncertainty is similar to the marginal cost computed for dividends, potentially in the order of several hundred percents of the lifetime consumption. Finally we link these estimates with the benefits of increasing growth. We find that the benefits of eliminating business cycle fluctuations are smaller than the benefits of a permanent increase in the annual growth rate of consumption of 2 basis points.

The paper is organized as follows. Section 2 defines the marginal cost of consumption fluctuations. Section 3 compares marginal and total cost of consumption uncertainty. Section 4 studies the relationship between the marginal cost of consumption uncertainty and the equity premium. Section 5 presents our empirical estimates of the marginal cost of business cycles. Section 6 compares the cost and benefits of fluctuations with those of growth. Section 7 concludes.

2. Defining the Marginal Cost of Consumption Fluctuations

We start here by defining at a general level our measures of the cost of business cycles. We generalize Lucas' definition along two dimensions. First, allowing for partial reduction in consumption fluctuations we define a total cost function, and, as its derivative evaluated at the point where the consumers bear all the consumption fluctuations, we define the *marginal cost of economic fluctuations*. As a second dimension of generalization, we leave it open for the moment what type of consumption fluctuations are eliminated. As we discuss more in detail in the following sections, this could be, for instance, all consumption uncertainty or only cyclical consumption fluctuations for some frequencies.

The analysis is carried out under the assumptions of a representative agent economy. In each period t, the economy experiences one of finitely many events $z_t \in Z$. We denote by $z^t = (z_0, z_1 ... z_t)$ the history of events up through and including period t. We index consumption by histories, so we write $C : \mathbf{Z} \rightarrow R_+$, where $\mathbf{Z} \equiv \prod_{t \geq 1} Z^t$, or simply $\{C\} = \{C_t(z^t) : \forall t \geq 1, z^t \in Z^t\}$.

Definition 2.1. *We define the* ***total cost of consumption fluctuations function*** - (α) *as the solution of*

$$U\left(\left(1 + \text{-}\,(\alpha)\right)\{C\}\right) = U\left((1-\alpha)\{C\} + \alpha\{\mathfrak{C}\}\right), \tag{2.1}$$

where $\alpha \in [0,1]$, $C : \mathbf{Z} \rightarrow R_+$ *denotes the stochastic process of consumption and* $\mathfrak{C} : \mathbf{Z} \rightarrow R_+$ *denotes the process for the trend starting in period* 1. $U(.)$ *is a utility function, mapping consumption processes into* R.

The scalar α measures the fraction of risky consumption C that has been replaced by the less risky trend consumption $\mathfrak{C}$. The total cost function gives the total benefit from reducing consumption fluctuations as a function of the fraction of the reduction in fluctuations. It is straightforward to see that - $(0) = 0$, so that no reduction in fluctuations generates no benefit.

The next definition serves mainly notational convenience. We define the **total cost of consumption fluctuations** ω, as $\omega \equiv$ - (1), or equivalently $U((1+\omega)\{C\}) = U(\{\mathfrak{C}\})$. As a particular case, take $\alpha = 1$ and define the trend consumption to be $\{\mathfrak{C}\} = \{E_0(C)\}$, i.e. where $\mathfrak{C}(z^t) = E_0(C_t)$ for all t, and z^t. In this case, we have

$$U\left((1+\omega)\{C\}\right) = U\left(\{E_0(C)\}\right), \tag{2.2}$$

which is Lucas' definition of the cost of business cycles. Thus, Lucas' definition of the cost of business cycles can be seen as the total benefit associated with eliminating all the consumption fluctuations, that is $\alpha = 1$, and where consumption fluctuations are defined as consumption uncertainty, that is, resulting in the exchange of consumption for its expected path. When we use this specification for $\mathfrak{C}$ we refer to - as the cost of consumption *uncertainty*, since $\mathfrak{C}$ is deterministic, to distinguish it from the more general case, where we refer to - as the cost of consumption *fluctuations*.

Note, the specification in equation (2.2) differs slightly from Lucas' and the literature's standard specification because we choose to begin compensation as of $t = 1$, the standard has been to start compensation at $t = 0$. We choose this departure because our definition is more consistent with the idea of ex-dividend security prices, some of our qualitative results present themselves more tractably, and finally, the quantitative difference between the two will be insignificant.

For the next definition we assume that U is differentiable with respect to each $C_t(z^t)$ for all t and z^t. We denote the partial derivatives by $U_{z^t}(\{C\}) = \partial U(\{C\}) / \partial C_t(z^t)$.

Definition 2.2. *We define the* ***marginal cost of consumption fluctuations*** ω^m *as the derivative of the total cost function - evaluated at* $\alpha = 0$, *i.e.*

$$\omega^m \equiv -'(0) = \frac{\sum_{t=1}^{\infty} \sum_{z^t} U_{z^t}(\{C\}) \cdot (\mathfrak{C}(z^t) - C_t(z^t))}{\sum_{t=1}^{\infty} \sum_{z^t} U_{z^t}(\{C\}) \cdot C_t(z^t)}. \tag{2.3}$$

Thus, ω^m measures the percentage increase in consumption at which a marginal reduction in consumption fluctuations is valued per unit—it is the market price of consumption volatility. It is obtained by totally differentiating Eq (2.1) with respect to α.[2]

For any process $X : \mathbf{Z} \to R$, define $V_0[\{X\}]$ as follows,

$$V_0[\{X\}] \equiv \sum_{t=1}^{\infty} \sum_{z^t} U_{z^t}(\{C\}) \cdot X_t\left(z^t\right).$$

This is the shadow price, for the representative agent, of an asset with payouts given by $\{X\}$. Under this convention, it is immediate to see that

$$\omega^m = \frac{V_0[\{\mathfrak{C}\}]}{V_0[\{C\}]} - 1. \tag{2.4}$$

[2] For a discussion about the thought experiment underlying this definition we refer the reader to Section 3.3. about the costs of consumption fluctuations with consumption externalities.

Thus, one can interpret the marginal cost of consumption fluctuations as a ratio of the values of two securities: a claim to the consumption trend, $V_0[\{\mathfrak{C}\}]$, and a consumption equity claim, $V_0[\{C\}]$.

We think that these generalizations of Lucas' cost of business cycles have some attractive features. First, focusing on a *marginal* cost, we can hope to measure it by using information on the representative agent's marginal evaluation contained in security prices. Second, we can now think of the benefits of partial reduction in consumption fluctuations. Moreover, as our analysis in the next section shows, we can use our measure of the marginal cost to bound the standard total cost of business cycles for a large class of preference specifications, without the need to fully parameterize these preferences.

3. Comparing Marginal Cost and Total Cost of Consumption Fluctuations

To gain some insights into the definitions from the previous section we compare the marginal and total costs of consumption fluctuations. The main results of this section are the following: concavity of U implies that - is concave for small α, and hence the marginal cost ω^m is an upper bound for - (α) for small α. If U concave and homothetic, so that the total cost does not depend on the scale of the economy. In this case the total cost - (α) is concave, so that the marginal cost is an upper bound for the total cost, i.e. $\omega^m = -'(0) \geq \omega = -(1)$. We also examine the case where U is given the expected value of a concave utility, and where the "trend" is given by the expected value of consumption. In this case - is not only concave for α close to zero, but also for α close to one. If in addition, we have time separable utility, we show that, for small variance $\omega = \frac{1}{2}\omega^m$ and that the total cost is given by the insurance risk premium.

Our motivation for the study the relation between the marginal cost and the total cost is that the marginal cost can be measured using asset prices, while the total requires the knowledge of much more details of the utility function. Additionally the marginal cost may be interesting in itself, since some policies may only move in the direction of eliminating fluctuations. For instance, if most of the benefits from stabilization occur already for low values of α, then implementing a policy that moves in this direction, even if it does not makes consumption equal to the "trend" value, will be worthwhile. In a related comparison, Hansen, Sargent and Tallarini (1999) compute local and global mean-risk trade-off for consumers with risk-sensitive preferences, they find their local measure to be between 2 and

4 times higher than the global one.

We make the following initial assumptions. We assume that $U(\{C\})$ is increasing and concave in $\{C\}$. We also assume that the process $\{\mathfrak{C}\}$ is preferred to $\{C\}$, that is, $U(\{\mathfrak{C}\}) > U(\{C\})$. Under these conditions, it is straightforward to see that - $(\alpha) \geq 0$. Our first result is that - is concave for small α.

Proposition 3.1. *Let U be increasing, concave and twice differentiable. Then* - $''(0) \leq 0$.

This proposition implies that $\omega^m \equiv$ - $'(0) \geq$ - (α) for small α.

3.1. Homothetic preferences and scale-free cost functions

If we require that the cost of fluctuations - (α) be the same for the processes $\{C\}$ and $\{\mathfrak{C}\}$ as for the processes $\{\lambda C\}$ and $\{\lambda \mathfrak{C}\}$, where λ is any positive scalar, then we must impose some additional restrictions on the utility function U. This requirement implies that the cost of consumption fluctuations will not differ merely because economies are rich and poor. Specifically, we require U to be homothetic; that is, U is homogeneous of degree $1-\gamma$, *i.e.*, for any positive scalar $\lambda > 0$, and for $\lambda\{C\}$ defined as $\lambda\{C\}_t(z^t) = \lambda C_t(z^t)$ for each z^t we have

$$U(\lambda\{C\}) = \lambda^{1-\gamma} U(\{C\}).$$

Under this assumption, we obtain that the marginal cost is higher than the total cost.

Proposition 3.2. *Assume that U is increasing, concave and homothetic. Also assume that $\{\mathfrak{C}\}$ is preferred to $\{C\}$, i.e. $U(\{\mathfrak{C}\}) > U(\{C\})$ Then* - (α) *is concave, and thus*

$$\omega^m \equiv \text{-}\ '(0) \geq \text{-}\ (1) \equiv \omega.$$

Many preference specifications used in quantitative asset pricing studies satisfy this assumption, since many researchers impose the same form of scale invariance. Examples are the preferences used in Abel (1999), Campbell and Cochrane (1995), Constantinides (1990), Epstein and Zin (1991), Jermann (1998), Mehra and Prescott (1985), Tallarini (1998).

3.2. Expected utility

In this sub-section we explore some implications for the total and marginal cost $-$ and $-'$ when the utility U is characterized by expected utility. Throughout we assume that the trend $\{\mathfrak{C}\}$is given by the expected value of consumption, so we evaluate the elimination of all uncertainty.

Let U be given by expected value of a function $u : R_+^\infty \rightarrow R$. This specification allows time non-separabilities, including the ones emphasized by models where consumption display habit formation and durability. Notice that if u is concave, then U is concave and hence from our previous proposition $-$ is concave for α close to zero. Now we obtain a complementary result, showing that $-$ is concave for α close to 1.

Proposition 3.3. *Let $U(\{C\}) = E_0[u(C_1, C_2, ...C_t, ...)]$ and u be concave. Assume that $\mathfrak{C}_t(z^t) = E_0(C_t)$ for all z^t and t. Then $-'(1) = 0$ and $-(\alpha)$ is concave for α close to one.*

In the rest of this sub-section, we specialize this case further, by eliminating the time-non separability and by considering the case of small variance. We link the cost of fluctuation with the insurance premium and show that the total cost is about half of the marginal cost, i.e. $-(1) = 1/2 -'(0)$. The intuition for the second result is as follows, consider the equality $-(1) = -(0) + \int_0^1 -'(\alpha)\, d\alpha$. By definition $-(0) = 0$, thus we just have to compute an average across $-'(\alpha)$. By the previous proposition $-'(1) = 0$, then we show that an average with weights $1/2$ for $\alpha = 0$ and $\alpha = 1$, so that $-(1) = \frac{1}{2} -'(0)$ is appropriate if the variance of consumption is small.

Now we specialize the analysis to time separable utility and to the case of small variance. Consider a family of distributions over consumption indexed by their variance, $var(c) = \sigma^2$. Assume that these distributions have a common expected value $E(c) = \bar{c}$. We show that, for small σ^2, the marginal cost of consumption uncertainty is approximately twice the total cost, and that the total cost is approximately equal to the proportional insurance risk premium. We first consider the case for one period, and then the multiperiod case.

For the one period case, the total cost of consumption uncertainty $-(1, \sigma^2)$ is given by

$$E\left[u\left(c\left(1 + -\left(1, \sigma^2\right)\right)\right)\right] = u(\bar{c}), \tag{3.1}$$

and the marginal cost of consumption uncertainty $-'(0, \sigma^2)$ is given by

$$-'\left(0, \sigma^2\right) E\left[u'(c)\, c\right] = E\left[u'(c)(\bar{c} - c)\right]. \tag{3.2}$$

Proposition 3.4. *If* $\lim_{\sigma^2 \downarrow 0} \frac{\sigma\left[(c-\bar{c})^2\right]}{\sigma^2} < +\infty$ *then*

$$- \ \left(1, \sigma^2\right) = -\frac{1}{2}\frac{\bar{c}u''(\bar{c})}{u'(\bar{c})}\sigma^2(c/\bar{c}) + o\left(\sigma^2\right),$$

$$-'\left(0, \sigma^2\right) = -\frac{\bar{c}u''(\bar{c})}{u'(\bar{c})}\sigma^2(c/\bar{c}) + o\left(\sigma^2\right),$$

where $\sigma^2(x)$ *denotes the variance of* x*, and where* $f(x) = o(x)$ *means that* $\lim_{x\to 0} f(x)/x = 0$.

We illustrate this proposition with an example for CRRA utility and lognormal consumption.

Example 3.5. *Consider the case where the utility function has constant relative risk aversion* γ*, i.e.* $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$*, and where the log of consumption is distributed as* $N\left(\mu, \hat{\sigma}^2\right)$*. Recall that for log-normal consumption*

$$\sigma^2(c/\bar{c}) \equiv \frac{Var(c)}{[E(c)]^2} = \left(e^{\hat{\sigma}^2} - 1\right) = \hat{\sigma}^2 + o\left(\hat{\sigma}^2\right).$$

Simple computations show that,

$$- \ \left(1, \sigma^2\right) = e^{\frac{1}{2}\hat{\sigma}^2} - 1 = \frac{1}{2}\gamma\hat{\sigma}^2 + o\left(\hat{\sigma}^2\right),$$

$$-'\left(0, \sigma^2\right) = e^{\hat{\sigma}^2} - 1 = \gamma\hat{\sigma}^2 + o\left(\hat{\sigma}^2\right).$$

Now we consider the case of expected discounted utility for T periods, so that

$$U(\{C_t\}) = \sum_{t=1}^{T}\beta^t E_0\left[u(C_t)\right].$$

In this case, the total cost of consumption uncertainty $- \ (1, \sigma^2)$ solves

$$\sum_{t=1}^{T}\beta^t E_0\left[u\left(C_t\left(1 + - \ \left(1, \sigma^2\right)\right)\right)\right] = \sum_{t=1}^{T}\beta^t u(E_0 C_t),$$

and the marginal cost of consumption uncertainty $-'(0, \sigma^2)$ solves

$$-'\left(0, \sigma^2\right)\left[\sum_{t=1}^{T}\beta^t E_0\left[u'(C_t)C_t\right]\right] = \sum_{t=1}^{T}\beta^t E_0\left[u'(C_t)(E_0 C_t - C_t)\right].$$

We will assume that the conditional variances can grow at a deterministic rate, and that deviations from that growth path are small. Under such assumptions, we can show analogous results for the multiperiod case.

Lemma 3.6. *Assume that*

$$var_0\left(\frac{C_t}{E_0 C_t}\right) = \iota^t \sigma_t^2, \tag{3.3}$$

$$\sigma_t^2 = \sigma^2 + o_t\left(\sigma^2\right) \tag{3.4}$$

and that similar to the one-period case

$$\lim_{\sigma^2 \downarrow 0} \frac{\left(var_0\left[(C_t - E_0 C_t)^2\right]\right)^{1/2}}{var_0\left(C_t\right)} < \infty, \tag{3.5}$$

for all $t \leq T$. Then

$$- '\left(0, \sigma^2\right) = \frac{\sum_{t=1}^T \overline{\beta}^t \left[(E_0 C_t)^2 u''\left(E_0 C_t\right) \sigma^2 + o_t\left(\sigma^2\right)\right]}{\sum_{t=1}^T \overline{\beta}^t \left[(E_0 C_t) u'\left(E_0 C_t\right) + (E_0 C_t)^2 u''\left(E_0 C_t\right) \sigma^2 + o_t\left(\sigma^2\right)\right]}, \tag{3.6}$$

$$- \left(1, \sigma^2\right) = \frac{\sum_{t=1}^T \overline{\beta}^t \left[\frac{1}{2} (E_0 C_t)^2 u''\left(E_0 C_t\right) \sigma^2 + o_t\left(\sigma^2\right)\right]}{\sum_{t=1}^T \overline{\beta}^t \left[(E_0 C_t) u'\left(E_0 C_t\right) + o_t\left(\sigma^2\right)\right]}, \tag{3.7}$$

with $\overline{\beta} = \beta\iota$, and for all $T \geq 1$.

The following example applies this lemma to the CRRA utility function.

Example 3.7. *Let the period utility be $u(c) = c^{1-\gamma}/(1-\gamma)$, so that the coefficient of relative risk aversion is constant and equal to γ, and let consumption satisfy equation (3.5). In this case, we have*

$$\lim_{\sigma^2 \downarrow 0} \frac{- '\left(0, \sigma^2\right)}{\sigma^2} = \gamma,$$

$$\lim_{\sigma^2 \downarrow 0} \frac{- \left(1, \sigma^2\right)}{\sigma^2} = \frac{1}{2}\gamma,$$

for all $T \geq 1$.

From the previous lemma we have the main proposition for discounted, time separable, expected utility.

Proposition 3.8. *Under the conditions for the previous lemma,*

$$\lim_{\sigma^2 \downarrow 0} \frac{-\ (1, \sigma^2)}{-\ '(0, \sigma^2)} = \frac{1}{2}.$$

We end this section with a numerical example that verifies the accuracy of the approximation for small σ^2. In Figure 1 we trace out marginal and total costs as a function of the fraction of risk removed, α, for distributions with different skewness and a given variance. Consistent with our result above, marginal costs are basically linear, which implies $\frac{1}{2}\omega^m \approx \omega$. Figure 1 is based on a 2-point consumption distribution with standard deviation of 3.6%, a mean of 1 and CRRA preferences with risk aversion $\gamma = 10$. The three cases differ in the centered third moment. We introduce skewness by making down-moves 4 times as large a up-moves for negative skewness, while the probabilities are adjusted to keep the same mean and standard deviation; mutatis mutandis for positive skewness.

3.3. Cost of fluctuations with consumption externalities

Up to now, we have considered the case of a representative agent economy without consumption externalities. We define aggregate consumption externalities by including aggregate consumption $\{\bar{C}\}$ in the utility function as a separate argument, together with the agent's consumption $\{C\}$, so that utility is given by $U(\{C\}, \{\bar{C}\})$.

With externalities, the interpretation of the compensation - depends on the exact nature of the experiment. In particular, there are two ways of thinking about reducing fluctuations in consumption. One experiment is to stabilize consumption of only one agent, keeping aggregate consumption unchanged. Another experiment is to stabilize consumption of all the agents. In the first experiment, the total cost function is given by

$$U\left((1 + -\ (\alpha))\{C\}, \{C\}\right) = U\left(\alpha\{C\} + (1-\alpha)\{\mathfrak{C}\}, \{C\}\right). \tag{3.8}$$

Then, the benefit of a small reduction in fluctuations can be measured by the corresponding marginal cost. As explained above, the marginal cost $-\ '(0)$ equals the ratio of the prices of two securities, as in (2.4) where $U_{z^t}(\{C\})$ is replaced by $U_{z^t}(\{C\}, \{C\})$ defined as

$$U_{z^t}\left(\{C\}, \{\bar{C}\}\right) \equiv \frac{\partial U\left(\{C\}, \{\bar{C}\}\right)}{\partial C_t(z^t)}.$$

In the second experiment, the total cost function $\tilde{-}$ is defined as

$$U\left(\left(1+\tilde{-}\,(\alpha)\right)\{C\}\,,\{C\}\right) = U\left(\,\alpha\,\{C\}+(1-\alpha)\,\{\mathfrak{C}\}\,,\alpha\,\{C\}+(1-\alpha)\,\{\mathfrak{C}\}\,\right). \tag{3.9}$$

The function $\tilde{-}$ differs from $-$, since it includes the effect of the externality. However, because market prices do not internalize the effect of aggregate consumption, they do not equal $\tilde{-}\,'(0)$.

While consumption externalities can be of various forms, the consumption externalities used to account for the determinants of aggregate risk are of a particular type. We call "envy" a general type of externality that is related to the ones used in the literature, such as the "keeping up with the Joneses" in Abel (1999), and the "external habit" in Campbell and Cochrane (1995).

Definition 3.9. *We say that the representative agent exhibits envy if for any C and $\hat{C}$ such*

$$U\left(\left\{\hat{C}\right\},\{C\}\right) \geq U\left(\{C\}\,,\{C\}\right),$$

then

$$U\left(\left\{\hat{C}\right\},\{C\}\right) \geq U\left(\left\{\hat{C}\right\},\left\{\hat{C}\right\}\right).$$

The interpretation of this definition is that each agents uses the consumption of the representative agent as a benchmark for his own consumption. The next proposition shows that if the representative agent exhibits envy, the total benefit of stabilizing everybody'ss consumption are even smaller than the benefits from stabilizing one person's consumption.

Proposition 3.10. *Let's assume that the trend consumption $\{\mathfrak{C}\}$ is such that*

$$U\left(\{\mathfrak{C}\}\,,\{C\}\right) > U\left(\{C\}\,,\{C\}\right).$$

If U is increasing in its first argument, and if Definition 3.9 holds, then

$$-\,(\alpha) \geq \tilde{-}\,(\alpha)$$

for $\alpha \in [0,1]$.

As a corollary of the previous proposition, if $U\{\cdot, C\}$ has the properties of $U(\cdot)$ in Proposition 3.2, then

$$\omega^m \equiv -\,'(0) \geq -\,(1) \geq \tilde{-}\,(1)\,.$$

Thus, the ratio of the market prices of the securities in (2.4) is an upper bound for the total cost of fluctuations in both experiment, that is, whether consumption is stabilized for one agent or for all agents.

4. Comparing the Marginal Cost of Consumption Uncertainty with the Equity Premium

It seems widely recognized that there is a relationship between the equity premium and the cost of consumption uncertainty. Indeed, both are measures of the compensation required to bear aggregate risk. To our knowledge however no detailed comparison has been made in the literature. In this section, we compare the marginal cost of consumption uncertainty with the equity premium. To facilitate the comparison, we select an equity whose dividends are identical to consumption, which we call consumption-equity. It becomes very clear in this analysis that, in general, the marginal cost of consumption uncertainty is not equal to the equity premium, neither conceptually nor quantitatively.

We approach the comparison from two complementary angles. First, in Section (4.1), we use a decomposition into the fundamental risk components inspired by Campbell (1986) and Jermann (1998). This characterization has two important implications for estimating the marginal cost using asset price data. The first implication is that the slope of the term structure is an important determinant for the equity premium, but it is not a determinant for the marginal cost. The steeper the term structure is, the larger the equity premium can be, relative to the marginal cost of uncertainty. The second implication is that the degree of persistence of the pricing kernel and of the consumption processes have different impacts on the marginal cost and in the equity premium. The more persistent these processes are, the larger the marginal cost can be, relative to the equity premium. Second, in Section (4.2), we derive simple expressions for the equity premium and the marginal cost as functions of three elements: the dividend/price ratio, the real yield and the expected growth rate of the economy. We also present a preliminary estimate of the benefits of eliminating all dividend uncertainty where we find very high costs.

4.1. Decomposition into the fundamental risk components

Recall that in (2.4) we have defined $V_0\left[\{X\}\right]$ as the time zero implicit price of an asset that pays dividends $\{X\}$. Define $V_0\left[X_t\right]$ as the time zero price of an asset that pays a single dividend X_t at time t. In this subsection, we also consider these prices at times different from zero, for instance $V_1\left[X_t\right]$ denotes the price at time $t = 1$ of a security that pays X_t at time t. By a no-arbitrage argument the price of a consumption equity claim equals the value of a portfolio of claims to a single dividend equal to the consumption of each period, which we call, by analogy to the terms used for bonds, equity *strips*. Thus,

$$V_0\left[\{C\}_{t=1}^{\infty}\right] = \sum_{t=1}^{\infty} V_0\left[C_t\right].$$

To use the same notation for the prices of bonds, we denote by 1_t a dividend that is equal to one at t for all $z^t \in Z^t$ and zero otherwise. Define as $R_{0,t}\left[X_t\right]$ the time zero return until maturity of an asset with a single payment X_t at t, i.e.

$$R_{0,t}\left[X_t\right] \equiv \frac{V_t\left[X_t\right]}{V_0\left[X_t\right]} = \frac{X_t}{V_0\left[X_t\right]}.$$

Specializing the previous definition, denote the one-period holding return of an asset with single payment X_t at t as

$$R_{0,1}\left[X_t\right] \equiv \frac{V_1\left[X_t\right]}{V_0\left[X_t\right]}.$$

Using these definitions, denote by ν_0 the multiplicative excess expected equity premium,

$$1+\nu_0 \equiv \frac{E_0\left(R_{0,1}\left[\{C\}_{t=1}^{\infty}\right]\right)}{R_{0,1}\left[1_1\right]}.$$

Proposition 4.1. *Define the weights $w's$ as follows*

$$w_0\left[C_t\right] \equiv \frac{V_0\left[C_t\right]}{V_0\left[\{C\}_{t=1}^{\infty}\right]}, \tag{4.1}$$

the multiplicative consumption-equity premium can be written as

$$1+\nu_0 = \sum_{t=1}^{\infty} w_0\left[C_t\right]\left(\frac{E_0\left(R_{0,1}\left[C_t\right]\right)}{R_{0,1}\left[1_1\right]}\right), \tag{4.2}$$

and the marginal cost of consumption uncertainty can be written as

$$1+\omega_0^m = \sum_{t=1}^{\infty} w_0\left[C_t\right]\left(\frac{E_0\left(R_{0,t}\left[C_t\right]\right)}{R_{0,t}\left[1_t\right]}\right). \tag{4.3}$$

Equation (4.3) says that the marginal cost of consumption uncertainty equals **a weighted sum of dividend strip return premia with holding period until maturity**. In contrast, the multiplicative equity premium to a share that pays aggregate consumption, is equal to a **weighted sum of dividend strip premia for one-period holding returns.** Note, the weights, $w_0[C_t]$, are indeed the same for the marginal cost and for the equity premium!

When comparing the two expressions, (4.2) and (4.3), for the equity premium and the marginal cost, respectively, another fundamental difference becomes apparent. If the payouts are not random, then the holding returns until maturity, that determine the marginal cost, are not random either, that is,

$$R_{0,t}[X_t] = R_{0,t}[1_t] \equiv \frac{1_t}{V_0[1_t]},$$

and thus, the multiplicative premium is zero,

$$E_0(R_{0,t}[1_t]) = \frac{E_0(R_{0,t}[1_t])}{R_{0,t}[1_t]} = 1.$$

Therefore, the marginal cost is pure compensation for payout risk. This is in contrast to the one period holding returns that make up the equity premium. Realized returns depend on the valuation at time 1,

$$R_{0,1}[X_t] = R_{0,1}[1_t] \equiv \frac{V_1[1_t]}{V_0[1_t]},$$

so that premia are in general non-zero,

$$E_0(R_{0,1}[1_t]) = \frac{E_0(R_{0,1}[1_t])}{R_{0,1}[1_1]} \neq 1.$$

This comparison shows that the difference between equity premium and marginal cost will be a function of the slope of the term structure, as we discuss in more detail for the loglinear case in the next subsection.

4.1.1. Log-linear environment

We introduce a log-linear environment for two reasons. First, it allows us to sharpen the comparison made in the previous section between the equity premium and marginal cost. In particular, it helps to explain the different impact of the term structure risk and the payout uncertainty risk on the marginal cost and the equity premium. While the distinction between payout uncertainty risk and term structure risk is a general concept, in the log-linear environment these risk premia can be expressed in a separable way.[3] Additionally, we will use the log-linear environment later in the paper for estimating the cost of business cycles.

As it is well known, if there are no arbitrage opportunities, under technical regularity conditions, there must exists a non-negative process $\beta^t M_t$ such that all prices satisfy

$$V_0 [\{X\}] = \sum_{t=1}^{\infty} \beta^t E_0 \left[\frac{M_t}{M_0} X_t \right].$$

We call the process $\beta^t M_t$ a stochastic pricing kernel. We assume that the logarithm of kernel and the logarithm of the dividends of the assets of interest can be represented as linear functions of a linear VAR model. Specifically, let s_t be a state vector s_t following a multivariate, homoscedastic VAR

$$s_t = A s_{t-1} + \varepsilon_t, \tag{4.4}$$

where ε_t is a multivariate normal vector, i.i.d. through time. The square matrix A determines the dynamics of the system. We require the roots of A to be all smaller or equal to one, thus allowing for I(1) nonstationarity. Assume that

$$\begin{aligned} \ln(C_t) &= \left(\ln (1+g)^t\right) + l_c \cdot s_t, \\ \ln(M_t) &= l_m \cdot s_t, \end{aligned}$$

where the loadings, l_c and l_m, are row vectors and g is the trend growth rate.

Following the calculations in Jermann (1998), the multiplicative premium for one period holding return of a risky strip can be separated in two parts. One part is the term premium, i.e. the excess return of a long bond over the short rate, and the other is a compensation for the riskiness of the payout. Thus, the equity premium can be written as

$$1 + \nu_0 (s_0) = \sum_{t=1}^{\infty} w [C_t] (s_0) \left\{ \frac{E_0 (R_{0,1} [1_t])}{R_{0,1} [1_1]} \exp\left(-cov_0 (\log M_1, E_1 \log C_t)\right) \right\} \tag{4.5}$$

where we use the notation $\nu_0 (s_0)$, and $w [C_t] (s_0)$ to indicate that the equity premium and the weights depend on the current state vector s_0. Notice that the

[3] See Campbell (1986), Jermann (1998) and Abel (1999) for studies that focus on the distinctions between these two types of risk.

equity premium is a weighted average of two terms. The first term is the expected return of a t period real zero coupon bond over the short rate. This term is higher, when the real term structure is steeper. The second term, $-cov_0(\log M_1, E_1 \log C_t)$, measures how undiversifiable the risk about news on period t consumption is.[4]

By similar calculations the marginal cost equals

$$1 + \omega_0^m(s_0) = \sum_{t=1}^{\infty} w_0[C_t](s_0) \ \exp(-\operatorname{cov}_0(\log M_t, \log C_t)) \qquad (4.6)$$

where $-\operatorname{cov}_0(\log M_t, \log C_t)$ measures how undiversifiable consumption risk at horizon t is. Notice that for the marginal cost of consumption uncertainty, there is no term involving the real term structure.

In the case where $\log M_t$ and $\log C_t$ are very persistent, this covariance can be much larger in absolute value than the covariance in the expression for the equity premium.

Let us compare the covariances in the expressions for the marginal cost (4.5) and the equity premium (4.6). By definition of a the coefficient of correlation,

$$\begin{aligned} cov_0(\log M_t, \log C_t) &= \rho_0(\log M_t, \log C_t)\,\sigma_0(\log M_t)\,\sigma_0(\log C_t) \\ cov_0(\log M_1, E_1 \log C_t) &= \rho_0(\log M_1, E_1 \log C_t)\,\sigma_0(\log M_1)\,\sigma_0(E_1 \log C_t) \end{aligned}$$

The more persistent the processes for $\log M_t$, $\log C_t$ are the bigger the differences $\sigma_0(\log M_t) - \sigma_0(\log M_1)$ and $\sigma_0(\log C_t) - \sigma_0(E_1 \log C_t)$. Thus, the more persistent these processes are, the larger the marginal cost is, relative to the equity premium. For instance, if both $\log M_t$ and $\log C_t$ are $I(1)$, then the standard deviations in the marginal cost, $\sigma_0(\log M_t)$ and $\sigma_0(\log C_t)$, grow without bound, while the standard deviations for the equity premium, $\sigma_0(\log M_1)$ and $\sigma_0(E_1 \log C_t)$, are bounded.

Example: Separable CRRA utility. We illustrate the difference between the marginal cost of uncertainty and the equity premium using three specifications of a well understood economy. With time separable expected discounted CRRA utility, the kernel satisfies $\ln(M_t/M_0) = -\gamma \ln(C_t/C_0)$. The following results can be shown by direct calculations.

[4] The assumption that the system is log-linear further simplifies these formula so that this covariance is independent of the state s_0, and depends only on the maturity of the strip.

1. First, assume that consumption follows a random walk. In this case, interest rates are constant and thus there are no term premia. Payout uncertainty premia for k-period holding are related to the one-period return premia by

$$-cov_0 (\log M_t, \log C_t) = -\ t\ cov_0 (\log M_1, E_1 \log C_t) = t\ \gamma\sigma_\varepsilon^2$$

   Clearly, in this case, the marginal cost is bigger, likely substantially so, than the consumption-equity premium.

2. Second, assume that consumption growth rates follow an AR(1) with positive serial correlation. It can then easily be shown that the term structure is downward sloping, so that

$$\frac{E_0 (R_{0,1} [1_t])}{R_{0,1} [1_1]} < 1,$$

   and

$$-cov_0 (\log M_t, \log C_t) > -\ t\ cov_0 (\log M_1, E_1 \log C_t) > 0,$$

   so that there are even more reasons for which the marginal cost is bigger than the consumption-equity premium.

3. Third, the one case where the consumption-equity premium may actually be bigger than the cost of consumption uncertainty is with an AR(1) in consumption levels, with positive serial correlation, *i.e.* the growth rate of consumption is negatively autocorrelated . In this case, the term structure is upward slopping,

$$\frac{E_0 (R_{0,1} [1_t])}{R_{0,1} (1_1)} > 1,$$

   and

$$0 < -cov_0 (\log M_t, \log C_t) < -\ t\ cov_0 (\log M_1, E_1 \log C_t).$$

4.2. Decomposition into yields and growth rates

As shown in equation (2.4), the marginal cost of consumption uncertainty, can, by definition, be viewed as the ratio of the prices of two securities. For the particular case of comparing actual consumption to its expected growth path these two securities have intuitive interpretation and close real-world counterparts. Indeed, the appropriate version of equation (2.4) is

$$1 + \omega_0^m = \frac{V_0 (\{E_0 (C)\})}{V_0 (\{C\})}.$$

In the numerator, $V_0(\{E_0(C)\})$ is a perpetual bond that has a coupon that is growing at the consumption trend growth rate. In the denominator, $V_0(\{C\})$ is a consumption equity claim. Dividing both sides of the fraction by C_0 we have

$$1+\omega_0^m = \frac{V_0(\{E_0(C)\})/C_0}{V_0(\{C\})/C_0},$$

suggesting that the marginal cost is a function of two "price/dividend ratios," or, reciprocally, two yields. If we assume that aggregate consumption has a constant expected trend growth rate, g_0, this can be written as

$$1+\omega_0^m = \frac{\sum_{i=1}^{\infty}\left(\frac{1+g_0}{1+r_0}\right)^i}{\frac{1}{dp_0}} = (1+g_0)\left[\frac{dp_0}{r_0-g_0}\right], \tag{4.7}$$

where r_0 is the real yield of a bond with the duration of the growing perpetuity, and where dp_0 is the dividend yield of the consumption-equity.

In general, the expected excess return of equity cannot be written as a function of r_0, dp_0 and g_0. Nevertheless if we make some simplifying assumption we obtain an expression that depends on similar factors. If we assume that dividend yields are constant, we can write the expected return of equity as $(1+g_0)\cdot(1+dp_0)$. By definition, the equity premium is the expected excess return of equity over a risk-less investment, yielding a known return of y_0. Thus,

$$1+\nu_0 = \frac{(1+g_0)\cdot(1+dp_0)}{(1+y_0)}. \tag{4.8}$$

In general y_0 will be smaller than the yield of the long term bond r_0 used in equation (4.7). They will be equal only in special cases, for instance, if the term structure is flat. Clearly the formulae for the equity premium in (4.8) is different from the one for the marginal cost in (4.7). Notice that the equity premium ν_0, in (4.8) is approximately $\nu_0 \simeq g_0 + dp_0 - y_0$, and hence it cannot exceed a few percentage points. This is quite different for the marginal cost, ω_0^m, in equation (4.7), which is approximately, $\omega_0 \simeq dp_0/(r_0-g_0)$, and hence it could be much larger. In the next subsection, we provide quantitative content to equation (4.7) by measuring the marginal cost.

4.2.1. A measure of the marginal cost of dividend uncertainty

For a first quantitative cut, we plug historic averages into the formula for the marginal cost given by equation (4.7). Indeed, we can consider the dividend-price ratio, dp, to be the historical dividend-price ratio of the value weighted US. stock market. Of course, we are thus looking at how dividend uncertainty reduces the price of equity relative to a perpetual bond paying as coupons the expected dividends, we will focus on *consumption* fluctuations in the next section.

Combining the average historical dividend-price ratio with a historical long-term real yield and the dividend growth rate, we report in Table 1 a marginal cost for dividend uncertainty of 513.55%. That is, the buyer of an equity claim would have to be given a five times higher dividend to make him value equity as much as a perpetual bond paying coupons growing at the 1.29% trend growth rate of dividends. If dividends were equal to aggregate consumption, we would have had here our estimate of the marginal cost of consumption uncertainty. The additional statistics in Table 1 document some aspects of dividend and consumption behavior.

5. Measuring the Marginal Cost of Business Cycles

The approach used to measure the cost of business cycles in the previous section has the advantage of being simple and intuitive, but it has several shortcomings. First, it measures the elimination of all the uncertainty, as opposed to the elimination of business cycle fluctuations only. Second, the theory requires the use of an asset whose dividends are perfectly correlated with consumption. The measurement in the previous section uses a diversified equity portfolio, whose dividends display positive correlation with consumption, but that it is much smaller than one. Third, the calculation of the marginal cost can be very sensitive to small inconsistencies in the measurement of the different yields and growth rates. This is due to the non-linearities in (4.7), where dp_0 is divided by $r_0 - g_0$. For instance, the calculation of the duration adjusted yield, r_0, is not straightforward without specifying jointly the term structure and the expected growth rate of consumption.

In this section we address these three shortcomings. First, we will isolate the cost of consumption fluctuations corresponding to business cycle frequencies and compare it with the cost of all consumption uncertainty, as is usually done in the literature. Specifically, we isolate cyclical consumption movements corresponding to cycles of at most 8 years by using a band-pass filter approach. Second, we will price an asset that pays aggregate consumption, as opposed to aggregate dividends. Third, by explicitly modeling a pricing kernel, our estimates of dividend yields and growth rates will be consistent with each other.

Let us start here by overviewing the different steps involved in our estimation procedure, a more detailed step-by-step discussion follows in the subsequent subsections. Based on the log-linear environment defined in Section (4.1.1)

1. We select the state variables as variables that determine marginal utility of consumption in popular models, and estimate a linear autoregressive law of motion for the logarithm of these state variables, s_t.

2. We estimate the loading vector for the pricing kernel, l_m, and the effective time discount rate, β, from a set of asset pricing moment conditions determined by the theoretical considerations discussed above.

3. We specify the trend $\{\mathfrak{C}_\tau\}_{t=\tau}^{\infty}$, compute $V_t\left(\{\mathfrak{C}_\tau\}_{\tau=1}^{\infty}\right)(s_t)$ and $V_t\left(\{C_\tau\}_{\tau=1}^{\infty}\right)(s_t)$ for every s_t of our data-set, and report the sample mean of the marginal costs.

5.1. The state variables

We choose a multidimensional state vector to capture the dependence of marginal utility on non-contemporaneous consumption. These dependence has been emphasized in the literature in models with habit formation and in models with non-expected utility.

The state of the system consist of (1) aggregate consumption, (2) aggregate dividends, (3) the dividend-price ratio, (4) the long-short government bond yield spread and (5) the value-weighted realized stock return. This choice is motivated by empirical work on intertemporal asset pricing. For instance, Campbell (1996) has used dividend-price ratios, yield spreads, realized stock returns, and aggregate output as his state. We include consumption in the state space since we need to estimate the price of an asset with payouts equal to consumption.

We use annual time series covering 1889-1997, data sources and definitions are described in detail in the Data Appendix. We start by removing a linear deterministic growth trend from consumption and dividends. We then estimate by OLS a VAR for growth rates for consumption and dividends and for levels for the remaining variables. A system in levels, as in equation (4.4), is then recovered. The variance-covariance matrix of the innovations is obtained from the residuals of the estimated VAR.

By estimating the VAR in growth rates for consumption and dividends, we have introduced two unit roots into the matrix A. This decision to model consumption and the pricing kernel as non-stationary, has first-order quantitative implications for the measurement of the cost of consumption uncertainty. In the next subsection, we explain the reasons why we choose to model them as non-stationary.

5.1.1. Stationary versus nonstationary kernels

It is certainly possible to argue on statistical grounds that consumption and/or dividends can be modelled as stationary processes. However, we will select a nonstationary representation for consumption due to the fact that a stationary pricing kernel cannot *possibly* explain simultaneously the relatively flat historical term structure of interest rates and the relatively large historical equity premium, as we show below. A stationary pricing kernel that is volatile enough to generate high equity risk premia has too much predictable movements to be consistent with the relatively low long-short term spread. This result applies not only to the particular specification considered here, but to *any* stationary pricing kernel. Given the requirement of a nonstationary pricing kernel, we choose consumption and dividends to be nonstationary. We think that specifying consumption to be stationary and the pricing kernel to be non-stationary is an undesirable property of any representation.

The rest of this subsection makes the claims in the previous paragraph precise. First we set up some definitions for the term structure. We define **the continuously compounded term premium for a $k-$period discount bond** as

$$h_t^{cc}(k) = E_t\left\{\log\left(\frac{R_{t,t+1}[1_{t+k}]}{R_{t,t+1}[1_{t+1}]}\right)\right\}$$

where $R_{t,t+1}[1_{t+k}]$ measure the gross return from holding from time t to $t+1$ a claim to one unit of the consumption good to be delivered at time $t+k$. Define the **continuously compounded yield differential** between a k-period discount bond and a one-period, risk-less bond as

$$y_t^{cc}(k) = \log\left(\frac{V_t[1_{t+1}]}{(V_t[1_{t+k}])^{1/k}}\right).$$

We say that a kernel is stationary if $\{M_{t+1}\}$ is stationary and ergodic. We show here that with a stationary pricing kernel the excess returns for a discount bond that matures very far in the future takes a very simple form.

Proposition 5.1. *If the pricing kernel is stationary then*

$$h_t^{cc}(\infty) = \log E_t[M_{t+1}] - E_t \log M_{t+1},$$
$$y_t^{c}(\infty) = \log E_t[M_{t+1}] - \log M_t.$$

Now we relate our expression for the long-run term premia with some other observable quantities. We follow a similar argument as in Cochrane (1992) or Bansal and Lehmann (1997) to relate the size of the term premia with the return on the growth optimal portfolio.

Proposition 5.2. *If the pricing kernel is stationary, then*

$$E[h_t^{cc}(\infty)] = E[y_t^{cc}(\infty)] > E\left[\log\left(R_{t,t+1}^{j}\right)\right] - E[\log(R_{t,t+1}[1_{t+1}])]. \tag{5.1}$$

where $R_{t,t+1}^{j}$ *is the holding return on any asset.*

Equation (5.1) implies that the average long-short term spread is superior to the continuously compounded equity premium, which is rejected by historical averages, as we document below.

We finish this subsection with a simple quantitative check of our claim: a stationary pricing kernel will generate a term premium that is too high relative to the data. Consider the return on the US stock market as the asset $R_{t,t+1}^{j}$. Even in this case, the bound is clearly violated.

	1989-1997	1926-1997
(1) $E\left[\log\left(R_{t,t+1}^{Eq}\right)\right]$	0.0666	0.0710
(2) $E[\log(R_{t,t+1}(1_{t+1}))]$	0.0118	0.0058
(1)-(2)	0.0548	0.0652
$E[y_t^{cc}(\infty)]$	0.0059	0.0140
$E[h_t^{cc}(\infty)]$	0.0045	0.0136.

5.2. Estimating the loading vector l_m and β

Once the law of motion of the state is estimated we need to estimate the various loading coefficients on the states, l_m, plus the time discount factor, β. We will choose (l_m, β) in order to minimize $\Im = [\theta - f(l_m, \beta)]' \cdot [\theta - f(l_m, \beta)]$, where θ is a vector of moments to match and $f(l_m, \beta)$ contains the corresponding moments generated by our asset pricing model. In all cases for which we report results we use as many moments as parameters to estimate, and we were able to find parameter values such that $\Im = 0$.

Our analysis in Section 4 of the fundamental components of the equity premium and of the marginal cost of consumption uncertainty suggests we focus on a pricing kernel that is good at explaining historical stock price behavior and the real term structure. In the different estimates reported in Table 2, we choose the asset pricing kernel to replicate the US average dividend price ratio and the equity premium. We consider two ways of replicating the equity premium. First, by applying the pricing kernel to the estimated process of dividends, our model generates a series of conditional equity premia for which we compute the sample mean

$$f_j(l_m, \beta) = \frac{1}{T} \sum_{t=1}^{T} \left[1 + \nu_t^d\left(s_t; l_m, \beta\right)\right],$$

where $\nu_t^d(.)$ is defined as $\nu(.)$ in equation (4.5) except that the payout process represents aggregate dividends instead of aggregate consumption. Our second moment condition relative to the equity premium applies the pricing kernel to realized US. excess stock returns, $VWR_t - R_t^f$, so that

$$f_i(l_m, \beta) = \frac{1}{T} \sum_{t=1}^{T} \beta \exp\left(l_m\left(s_{t+1} - s_t\right)\right) \cdot \left(VWR_{t,t+1} - R_{t,t+1}^f\right),$$

with $\theta_i = 0$. In Table 2, we refer to these two conditions as $E\left(R^d/R^f\right)$ and $E\left(VWR - R^f\right)$ respectively. We also make the kernel fit the average real return on a risk-less short term bond and the average real yield on a 20 year bond.[5]

5.3. Computing the marginal cost of business cycles

We now describe how we specify the process corresponding to the consumption trend $\{\mathfrak{C}\}$. We choose two approaches. As our first approach, we specify the logarithm of the consumption trend to be a one-sided moving average of current and past consumption, $\ln(\mathfrak{c}_\tau) = \sum_{k=0}^{K} a(k) \cdot l_c \cdot s_{\tau-k}$, for appropriate moving average coefficients. As our second approach, we set trend consumption equal to its conditional expectation, $\mathfrak{C}_\tau = E_t(C_\tau)$, so that $\ln(\mathfrak{c}_\tau) = l_c \cdot A^{\tau-t} \cdot s_t$.

[5] Practically, we make the model fit the average yield of a zero coupon bond of 13 years maturity, given that this corresponds approximately to the duration of the 20 year government coupon bond in our historical data set.

We choose the moving-average coefficients so as to represent a band-pass filter that lets pass frequencies that correspond to cycles of Y years and more. Band-pass filters are represented in the time domain by infinite order two-sided moving averages. However, a requirement of our analysis is to have trend consumption at time t to be function of information available at time t, thus our choice of a one-sided moving average.

We chose the moving average coefficients of our trend consumption using the procedures presented by Baxter and King (1995,8). Let $\beta(\omega)$ be the frequency response function of the desired low-pass filter, which in our case is equal to one for frequencies lower than Y years and zero otherwise. Let $\alpha_K(\omega)$ be the frequency response function associated with a set of moving average coefficients $\{a_k\}_{k=1}^{K}$. We select the moving average coefficients $\{a_k\}$ so that α_K approximates β. In particular, our choice of $\{a_k\}$ minimizes

$$\int_{\pi}^{-\pi} |\beta(\omega) - \alpha_K(\omega)|^2 f(\omega)\, d\omega,$$

where $f(\omega)$ is a weighting function representing (an approximation to) the spectral density of the series to be filtered. In this minimization, we impose the condition $\alpha_K(0) = 1$, which implies that $\sum_{k=0}^{K} a_k = 1$.

We fix the length of the moving average coefficient to be fifteen (*i.e.*, $K = 15$) and consider filters that let pass cycles of length higher than eight, twelve, sixteen and twenty (i.e., $Y = 8, Y = 12, Y = 16$ and $Y = 20$). We use the spectral density of an AR(1) with autocorrelation .99 for the weighting function f. These one sided filters are far from a perfect low pass filter. Being one sided, these filters can not avoid to introduce a phase shift, i.e. the "trend" and its associated deviations will be lagging the series. The objective function displayed above is closely related to the variance of the difference of the desired series and the implied filter series. Thus this objective function trades-off the phase shift of the filtered series with the desired shape of the spectral density of it.[6] This can be seen by plotting the

[6] This can be seen by rewriting the objective function as follows:

$$\begin{aligned} & \int_{-\pi}^{\pi} |\beta(\omega) - \alpha_K(\omega)|^2 f(\omega)\, d\omega \\ = & \int_{-\pi}^{\pi} (|\beta(\omega)| - |\alpha_K(\omega)|)^2 f(\omega)\, d\omega \\ & + \int_{-\pi}^{\pi} 2\beta(\omega)\, |\alpha_K(\omega)| \left[1 - \cos(\psi_K(\omega))\right] f(\omega)\, d\omega \end{aligned}$$

square of the gain of the filter, as in Figure 10. The square gain should be one in between the desired frequencies and zero for higher frequencies. Instead, it tends to let pass up to 30% of the variance at higher frequencies, which makes the "trend" component too variable, or equivalently, it makes the deviations from trend too smooth. For instance, the filter designated as a 8 year low pass filter does let pass power at frequencies that are higher than 8 years. To obtain an idea of the effect on these filters on standard time series, we compare them with the Hodrick-Prescott filter, which many people use as a description of business cycle component. The HP filter itself is close to a band pass filter for quarterly data that suppresses the power in frequencies higher than a year and lower than 8 years. In Figures 2 and 3 we plot the deviations for the different filters applied to post II WW annual Consumption and GNP and the average annual deviations of these series obtained using the HP filter. The correlation of the HP filter deviation with the deviations obtained using any of these filters are about 0.7. The standard deviation of the Consumption (GNP) deviations are 0.84% (1%) , 1.09% (1.27%) , and 1.22% (1.43%) for the 8, 12 and 16 years approximate filters and 1.11 % (1.41%) for the HP filter. In this sense the HP filter deviations are in between those of the approximate 12 and 16 years filters. Figures 4, 5, 6, and 7 contains the time series for the deviations from trend using the 8, 12, 16 and 20 years approximate low pass filters.

We also report in Table 2 results based on some other popular moving-average filters. A geometric filter is specified so that $a_k = \delta_g^{k+1}$ and $\sum_{k=0}^{K} a_k = 1$, for $K = 5$, the same as for the frequency domain filters; thus $\delta_g = 0.5041$. A linear filter is specified so that $a_k = \delta_l \left(1 - \frac{k}{K+1}\right)$ and $\sum_{k=0}^{K} a_k = 1$, which gives a slope so that the next potential weight $a_{K+1} = 0$; thus $\delta_l = 0.2857$. Figures 8 and 9 have the deviations from trend using the filters with geometrically and linearly decaying weights.

5.4. Findings and discussion

In Table 2 we report estimates for various state space systems, various loading states and various moments to fit for the complete sample and for the postwar.

where $\psi_K(\omega)$ denotes the phase shift of the filter. This objective function can be written as the square of the difference of the desired frequency response with the square of the gain of the filter plus a terms that depends on the phase shift. This second terms is zero, if the filter has no phase shift. For instance a symmetric two sided filter has no phase shift, so it also fits the square of the difference of the frequency responses. Instead the one sided filters have to find the trade off between these two forces.

Overall, for our fluctuations defined by a 12 year filer we find a cost of about 0.7 of a percent of lifetime consumption a percent for the long period and about 0.5 of a percent for the post II world war period. Figure 4 to 9 show graphically the difference between the different definitions of trends for the US. consumption path. Results for the geometric filter are in-between the two frequency domain filters, the linear filter results in slightly larger costs.

The column labelled "inf" reports the cost of all consumption uncertainty, that is, including cycles of any length. We find very large costs, amounting to several hundred percents. While these results are quite striking, they are consistent with the orders of magnitudes we found for the cost of dividend uncertainty. These numbers are also consistent with the intuition provided by the log-linear random walk case in section (4.1.1). Fundamentally, consumers very much dislike the possibility that consumption can wander off very far from its expected path.[7]

Looking across the different specifications, our measures appear robust for the different moving average filters, even though the loading coefficients on some states differ sometimes substantially across cases. There is more variability for measures of consumption uncertainty. One reason for the variability in measuring the costs of consumption uncertainty is that in some cases, prices for the perpetual bonds with growing coupons become very large, *i.e.* long term yields are low relative to the trend growth rate.

We also present some estimates where in the VAR we include the ratio of dividends to consumption, assuming that this ration is stationary. The estimates do not vary a lot from the previous cases. If anything the cost of eliminating all uncertainty is estimated to be smaller than before.

5.5. The marginal cost of fluctuations in a simpler case

In this section we specialize the specification of the pricing kernel and state space to obtain a simpler relationship between the marginal cost of economic fluctuations and the excess return on a portfolio whose dividends equal consumption, which we denote "consumption equity". We specialize the kernel and the payoffs by assuming that they are given by random walks. This eliminates most of the dynamics: interest rates, dividend-price ratios, and expected excess returns are all constant. In this case we obtain that the marginal cost of fluctuations equals

[7] As explained in Section (5.2) we model consumption as non-stationary because a stationary consumption process is not consistent with having a kernel that can simultaneously explain a large equity risk premium and a relatively flat real term structure.

the expected excess return on "consumption equity" times a simple expression of $\{a_i\}$, the moving average coefficient that define $\{\mathfrak{C}_t\}$ as a function of $\{C_t\}$. For instance, for the 8, 12, 16 and 20 years filters used in the previous section the marginal cost of fluctuations equals 0.38, 0.72, 077 and 0.88 times the net expected excess return of consumption equity. We also show that, since the assumption precludes any dynamics, the expected excess return of the consumption equity equals the covariance between the pricing kernel and consumption growth rates. This covariance can be estimated by fitting a pricing kernel to average excess returns of assets. We estimate the marginal cost of economic fluctuations for different specifications of the state space. We found that the results are similar to the ones obtained fitting a more general version of the pricing kernel. Additionally we explore the consequences of introduction more assets in this simple framework.

We start by specializing the log-linear framework to the following:

$$s_{t+1} = g + s_t + \varepsilon_{t+1}$$

where ε_{t+1} is $N\left(0, \sum\right)$ and i.i.d trough time. In this case the pricing kernel can be written as:

$$\Delta m_{t+1} \equiv \log \frac{M_{t+1}}{M_t} = l \cdot g + l \cdot \varepsilon_{t+1}.$$

Notice that implicit in this specification of the state space is the assumption that consumption and dividends of any asset included in the state space follows a random walk, possibly with correlated innovations. Let D_t the dividend of any such asset and let $\Delta d_{t+1} \equiv \log \frac{D_{t+1}}{D_t}$. Under these assumption we have the following well known result.

Proposition 5.3. *If the log pricing kernel and the log dividend are random walks with homoskedastic innovations then interest rates, dividend-price ratios, expected returns of strips are equal to expected returns of equities, are all constant. Furthermore, if the innovations are normal, the ratio of the expected return of a risky strip paying D_{t+s} at date $t+s$ to the one period interest rate R_f and the excess return for equity paying dividends equal to the process $\{D\}$ are given by*

$$\frac{E_t\left(R_{t+1}\left[\{D\}\right]\right)}{R_f} = \frac{E_t\left(R_{t+1}\left[D_{t+s}\right]\right)}{R_f} = \exp\left(-cov\left(m_{t+1}, d_{t+1}\right)\right) \quad \textit{for all } s \geq 1, \tag{5.2}$$

and if $E\left(R_{t+1}\right)$ is a vector of excess expected returns then the loading vector is given by

$$\log E\left(R_{t+1}\right) = l \cdot \text{-} \tag{5.3}$$

where - $= \left[cov\left(\Delta s_{t+1,i}, \Delta d_{t+1,j}\right)\right]_{i,j}$ *is the variance-covariance matrix of innovations on the growth rate of state i with innovations on the growth rate of dividend of stock j.*

Under the assumptions used in the previous proposition dividend growth rates and total returns are perfectly correlated, since dividend-price ratios are constant. In our implementation we use consumption and/or returns of broad indices as states. With consumption in the state space, the previous proposition says that the multiplicative excess return of consumption equity equals minus the covariance of the kernel with consumption growth:

$$\frac{E\left(R_{t+1}\left[\{C_{t+s}\}_{s=1}^{\infty}\right]\right)}{R_f} = \exp\left(-cov\left(\Delta m_{t+1}, \Delta c_{t+1}\right)\right)$$

where R_f denotes the constant gross interest rate. Now we present a proposition that links the multiplicative excess return of consumption equity with the moving average coefficient of the "trend" $\{\mathfrak{C}_t\}$.

Proposition 5.4. *Let the "trend" $\mathfrak{C}_t$ be defined as*

$$\log \mathfrak{C}_t = \tau\left(t\right) + \sum_{i=0}^{K} a_i \log C_{t-i}$$

where $\tau\left(t\right)$ are constant chosen so that

$$E_0\left(\mathfrak{C}_t\right) = E_0\left(C_t\right).$$

Then the ratio between the price of a risky strip paying $\mathfrak{C}_t$ and one paying C_t is given by

$$\frac{V_0\left[\mathfrak{C}_t\right]}{V_0\left[C_t\right]} = \exp\left[cov_0\left(m_t, \sum_{j=0}^{J} a_j c_{t-j}\right) - cov_0\left(m_t, c_t\right)\right]$$

Furthermore if the log growth rate of pricing kernel and the state are i.i.d. normal,

$$\frac{V_0\left[\mathfrak{C}_t\right]}{V_0\left[C_t\right]} = \left(\frac{E\left(R_1\left[C_t\right]\right)}{R_f}\right)^{\left[1-a_0+1-(a_0+a_1)+\cdots 1-(a_0+a_1..+a_K)\right]} \tag{5.4}$$

where R_f is the gross interest rate and $R_1\left[C_t\right]$ is the gross return on a strip that pays C_t.

The role of the constants $\tau(t)$ is to ensure that $\mathfrak{C}_t$ has the same conditional expectation than C_t. This adjustment is a Jensen's inequality correction, due to our assumption that the logarithm of $\mathfrak{C}_t$,as opposed to just its level, is moving average of the logarithms of C_t. In the case where C_t follows a random walk with drift $\tau(t)$ is linear in t.

This proposition complements our earlier result. The first proposition explains how we identify the consumption equity premium by using excess return on securities, this proposition explains how we use that information and the values of the moving average coefficient to identify the marginal cost of fluctuations. In particular, recall that the marginal cost of economic fluctuations is given by the ratio of the prices of a claim with dividends equal to "trend" consumption to a claim with dividends equal to consumption. The ratio of the prices of these long lived securities can be written as

$$\frac{V_0[\{\mathfrak{C}\}]}{V_0[\{C\}]} = \sum_{t=1}^{\infty} w_t[C_t] \frac{V_0[\mathfrak{C}_t]}{V_0[C_t]}$$

where $w_t[C_t] = \frac{V_0[C_t]}{V_0[\{C\}]}$. Thus, the marginal cost of economic fluctuations is given by a weighted average of the ratio of strip prices. The ratio of these strip prices, with the exception of the strips for the first K securities, are independent of the date where they pay, and equal to the expression in (5.4). This expression only involves the consumption equity premium and a simple function of the values of the moving average coefficients. By using the approximation $\log(1+x) \cong x$, we can read (5.4) as

$$\omega^m \cong (r_{ce} - r_f)[1 - a_0 + 1 - (a_0 + a_1) + \cdots + 1 - (a_0 + a_1 + \cdots + a_K)]$$

where r_{ce} is the net expected return of the consumption equity and r_f is the net risk-free interest rate. This expression has the following interpretation: $(r_{ce} - r_f)$ captures the risk premium associated with consumption, and the factor $[1 - a_0 + \cdots + 1 - (a_0 + a_1$ captures the variability of the deviations from trend. To understand how this factor works, consider the case of moving average coefficients $\{a_i\}$ that are positive and decreasing in i and consider the following change: increase the values of $a_i's$ corresponding to long lags (large i) and decrease the ones corresponding to earlier lags (small i). This operation increases the value of the factor, and hence increases the marginal cost. The reason for this increase is that the trend corresponding to the new $\{a_i\}$ coefficients is smoother, since it distributes the weights more evenly. Consequently the deviations are more volatile, and thus the marginal cost is higher.

Now we present some examples of marginal cost computed using the result in the previous propositions. The first panel has the results for the long sample and the second for the post II world war sample. The first column displays the securities whose expected excess returns are use to fit the pricing kernel. We use the CRSP market portfolio or 10 CRSP portfolios of stocks sorted by the capitalization values of the firms. The second column contains the sates of the system, which are consumption growth and/or returns. The data set for the longer period is now shorter, starting at 1926 instead of 1890. The third column gives the values of the loadings coefficients for the first two cases. The forth two columns present the implied consumption equity premium. The next three columns gives the marginal cost for filters with weights corresponding to the approximate 8, 12 and 16 years frequencies, using the formula (5.4). The first two lines contains pricing kernel that are function of one state only: consumption growth and the excess return on the market portfolio respectively. The third has the excess return on the 10 CRSP size portfolios states. The fourth line has consumption growth, and the excess returns on the smallest and largest CRSP size portfolios. The last columns have summary statistics such as the standard deviation of consumption growth rate and of the market excess return and their coefficient of correlation.

Notice that the estimated consumption equity premium is in between 1.6% and 0.31%, a much smaller amount than the equity premium for the market portfolio.. The values for the factor $[1 - a_0 + \cdots + 1 - (a_0 + a_1 + \cdots + a_K)]$ corresponding to the approximate 8, 12 and 16 year filter are 0.38, 0.72, and 077 respectively . Thus, the combination of relatively small consumption equity premium with factors $[1 - a_0 + \cdots + 1 - (a_0 + a_1 + \cdots + a_K)]$ smaller than one, explains the relatively small values for the marginal cost of fluctuations. Notice that the estimated equity premium is smaller in the post II WW period, which explains the smaller marginal cost of fluctuations for the more recent period. Adding other equity portfolios, such as the CRSP size portfolios did not change the result much. This is due to the facts that, once the market portfolio has been included, the additional portfolios have a very small extra power explaining the variance of the growth rate of consumption. Finally we remark that the estimates of the marginal cost of fluctuations are of similar magnitudes of the one obtained using the more general model displayed in the previous section.

Overall the set up in this section is much simpler than the one used in the previous section, nevertheless it has some inconsistencies. The assumption that the pricing kernel follows a random walk has some counterfactual implications for features that are relevant. For instance, it implies that interest rates are constant.

Since the excess expected returns used to fit the kernel are computed using the short rate, which is smaller than the long rate, this result are likely to be biases to find larger cost. Also, the model implies constant price-dividend ratios, and the data dividend-price ratios have considerable low frequency variations.

This simple specification is useful to illustrate the effect of a "phase shift" on the filter that defines the "trend consumption". As explained above, we are interested in low pass filters to defined the "trend consumption" that let pass only low frequency power below some chosen frequency. Since the filter has to be one sided –i.e. the time t dividends can not depend on information not known at date t– then it will typically lag the ideal two sided band pass filter. The effect of this phase shift is to make the marginal cost larger. The intuition is that a phase shift makes the "trend consumption" more desirable since it becomes closer to the expected value. To make this more precise we show that if a trend given by moving average coefficients $\{a_i\}$ is lagged by one period, then the cost of fluctuations goes up by the value of the expected net consumption equity premium.

Proposition 5.5. *Let $a^K = \{a_i\}_{i=0}^K$ be the moving average coefficients that define the consumption trend, and let $m\left(a^K\right)$ be the factor that multiplies the consumption equity premium to determine the marginal cost of fluctuations with a trend given by the moving average coefficients a^K.T Consider the moving average coefficients a^{K+1} satisfying*

$$a_0^{K+1} = 0, \text{ and } a_{i+1}^{K+1} = a_i^K \text{ for } i = 1, 2, ..., K.$$

Then $m\left(a^{K+1}\right) = m\left(a^K\right) + 1$.

Denoting by $\omega^m(a)$ marginal cost for a trend corresponding to moving average coefficients a, using the previous two propositions, and the approximation $\log(1+x) \cong x$,we obtain

$$\omega^m\left(a^{K+1}\right) - \omega^m\left(a^K\right) \cong r_{ce} - r_f,$$

which means that with the phase shift of a year described above, the cost of business cycles by the magnitude of the net consumption equity premium.

6. Growth and Economic Fluctuations

In this section we use asset prices to estimate the benefits of increasing long term growth and to estimate the trade off between long term growth and fluctuations.

First we estimate the permanent constant increase in consumption that will leave agents indifferent with a 1% permanent increase in the growth rate of consumption. We estimate the benefits of an annual 1% permanent increase in the growth rate of consumption to be very large, well in excess of a permanent increase of 50% in consumption. Second we estimate the permanent increase in the growth rate that will be increase welfare as much as eliminating economic fluctuations. We estimate the benefits of reducing fluctuations to be smaller than the benefits of a permanent increase in the growth rate of 5 basis points per year.

We can use our definition of - to analyze the benefits of a permanent increase the growth rate of the economy, as in Lucas (1987). For that we let $\{\mathfrak{C}\}$ be defined as

$$\mathfrak{C}_t\left(z^t\right) = [1+\gamma]^t C_t\left(z^t\right).$$

In this case - (1) is the uniform compensation needed to make an agent indifferent between the current consumption and consumption growing at a rate γ higher, i.e.

$$U\left((1+ - (1))\{C_t\}\right) = U\left(\left\{[1+\gamma]^t C_t\right\}\right).$$

Let's denote the total cost of reducing the growth rate to γ as - $(1) \equiv \rho$. We can measure the marginal cost of reducing growth by

$$\rho^m \equiv -'(0) = \frac{V_0[\{\mathfrak{C}\}]}{V_0[\{C\}]} - 1.$$

Notice that this ratio can also be written as

$$\frac{V_0[\{\mathfrak{C}\}]}{V_0[\{C\}]} = \frac{\sum_{t=1}^{\infty} V_0\left[(1+\gamma)^t C_t\right]}{\sum_{t=1}^{\infty} V_0[C_t]} = \frac{\sum_{t=1}^{\infty} (1+\gamma)^t V_0[C_t]}{\sum_{t=1}^{\infty} V_0[C_t]} = \sum_{t=1}^{\infty} (1+\gamma)^t w_0[C_t],$$

where the weights w_0

$$w_0[C_t] = \frac{V_0[C_t]}{\sum_{j=1}^{\infty} V_0[C_j]},$$

are the same as in the equity premium and the cost of consumption uncertainty. Again, under the conditions for Proposition 3.2, we obtain that $\rho^m \geq \rho$. Using our estimated kernel Table 2, the benefits of increasing the rate of growth 1% –i.e. $\gamma = 0.01$– are higher than 20 % of consumption, and for many specifications on the order of 100% of consumption or higher. These number are much larger than the one computed by Lucas, which is 17%.

Now we find an expression for the increase in the growth rate of consumption that will leave agents indifferent with the elimination of economic fluctuations.

Unlike the cost of economic fluctuations - –where agents receive a compensation by a permanent increase in the level of consumption–, we now consider compensating agents by increasing the growth rate of consumption by λ. Let $\{\mathfrak{C}\}$ be the "trend" defined as before, and define $\lambda(\alpha)$ as the solution to

$$U\left(\left\{[1+\lambda(\alpha)]^t C_t\right\}\right) = U\left((1-\alpha)\{C\} + \alpha\{\mathfrak{C}\}\right) \tag{6.1}$$

where $\left\{[1+\lambda(\alpha)]^t C_t\right\}$ is the process with has value $[1+\lambda(\alpha)]^t C_t(z^t)$ at time t, event z^t. Notice that by definition $\lambda(0) = 0$. Assuming that $U(\cdot)$ is differentiable, differentiating both sides of (6.1) and evaluating them at $\alpha = 0$ we obtain

$$\begin{aligned} & \lambda'(0) \sum_{t\geq 1} \sum_{z^t \in Z^t} U_{z^t}(\{C\})\ C_t\left(z^t\right)\ t \\ = & \sum_{t\geq 1} \sum_{z^t \in Z^t} U_{z^t}(\{C\})\ \left[\mathfrak{C}_t\left(z^t\right) - C_t\left(z^t\right)\right] \end{aligned}$$

dividing both sides by $V_0[\{C\}]$ and rearranging,

$$\lambda'(0) = \frac{\frac{V_0[\{\mathfrak{C}\}] - V_0[\{C\}]}{V_0[\{C\}]}}{\sum_{t=1}^{\infty} t\ w_0[C_t]}.$$

where the weights w_0 are as in 3.2. Using the definition of the marginal cost of economic fluctuations we can rewrite this expression as

$$\lambda'(0) = \frac{\omega^m}{\sum_{t=1}^{\infty} t\ w_0[C_t]}. \tag{6.2}$$

We call $\sum_{t=1}^{\infty} t\ w_0[C_t]$ the duration of the consumption equity, by analogy with the definition of duration in the context of coupon bonds. Thus the trade-off between growth and fluctuations is given by the ratio of the marginal cost of fluctuations to the duration of the consumption equity. Before presenting estimates of $\lambda'(0)$, we notice that λ is concave for small α. In this case $\lambda'(0) \geq \lambda(\alpha)$ for small α.

Proposition 6.1. *Let U be increasing, concave and twice differentiable. Then $\lambda''(0) \leq 0$.*

Using our estimated pricing kernel of Table 2 we can see consumption equity duration of about 70 years, many times higher than 100 years, with considerable variations across specifications. These duration can be combined with the estimates of the benefits of reduction fluctuations to arrive at the trade off between growth and cyclical variations. Combining the estimates corresponding to the different specifications of Table 2, we obtain that the benefits of eliminating cyclical fluctuations is smaller than the benefit of a permanent increase of one basis point in the growth rate of consumption.

To obtain further insights in the magnitudes of ρ^m and λ, and about their determinants we specialize the analysis to the random walk case. In this case

$$\rho^m = \sum_{t=1}^{\infty} (1+\gamma)^t \, w_0 [C_t] - 1 = \frac{\gamma \ (1+r_{ce})}{1 + r_{ce} - (1+g_c)(1+\gamma)} \cong \frac{\gamma}{r_{ce} - g_c - \gamma}$$

where $g_c = E\left[\frac{C_{t+1}}{C_t}\right] - 1$ is the net rate of consumption growth and r_{ce} is the net expected gross return of the consumption equity. In the previous section we estimate $r_{ce} - r_f$ to be less than 1.5%. Thus if the risk free rate is $r_f = 2.5\%$ and the growth rate of consumption $g_c = 2\%$ then $\rho^m = 0.01/(0.015 + 0.025 - 0.02 - 0.01)$ is about 100%.[8]

Not surprisingly the duration of the consumption equity also has a particularly simple expression

$$\sum_{t=1}^{\infty} t \ w_0 [C_t] = \frac{1 + r_{ce}}{r_{ce} - g_c}.$$

Moreover, as we shown above, in the random walk case the marginal cost of economic fluctuations is given by

$$\omega^m \cong [r_{ce} - r_f] \, [1 - a_0 + \cdots + 1 - (a_0 + \cdots + a_K)]$$

where r_f is the net risk free and $\{a_i\}$ are the moving average coefficients that defines the trend $\{\mathfrak{C}\}$. Thus, replacing the expressions for the marginal cost of fluctuations and the duration of consumption equity, we obtain

$$\lambda'(0) = (r_{ce} - r_f)(r_{ce} - g_c) \frac{[1 - a_0 + \cdots + 1 - (a_0 + \cdots + a_K)]}{1 + r_{ce}}$$

Thus $\lambda'(0)$ is increasing in r_{ce}, roughly with its square. The reason is that the cost of fluctuations is linear in r_{ce} and the duration is decreasing in r_{ce}. Also, being the product of two net excess rates is a small number. In the previous section we estimate excess net consumption equity returns, $r_{ce} - r_f$, smaller than 1.5%. The

[8] We select this high value for the risk free rate r_f as a compromise between the short and end segments of the term structure, since in the random walk case the term structure is flat.

factor $[1 - a_0 + \cdots + 1 - (a_0 + \cdots + a_K)]$ varies between 0.38 and 0.88 for the 8 years and 20 filters we have considered. In that case, using the same values as in the numerical example for the benefit of growth, $\lambda'(0)$ lies in between 1.5 and 2.5 basis points. Hence, we conclude that the benefits of removing business cycles is smaller than a permanent increase in the growth rate of 2 basis points.

7. Summary and Conclusions

In this paper we have measured the cost of business cycle fluctuations using asset prices. We used a new approach that focuses on the *marginal* cost of consumption fluctuations, because asset prices correspond to marginal valuations of market participants. We have shown that the marginal cost of consumption fluctuations corresponds to the ratio of two asset prices. We established that the marginal cost of consumption fluctuations provides an upper bound to the benefits of reducing fluctuations completely. Our analysis showed that the equity premium and the cost of consumption uncertainty are related, but clearly distinct, conceptually and quantitatively. The steepness of the term structure and the persistence of the shocks are two of the features that make the equity premium differ from the marginal cost of consumption uncertainty.

We have estimated a pricing kernel, as a way to "interpolate" from existing asset prices the prices of the assets that measure the marginal cost of consumption fluctuations. We use a non-stationary pricing kernel. This is motivated by a bound that we have developed, similar in spirit to the Hansen-Jagannathan bound. This bound shows that, to be consistent with the term structure, the pricing kernel cannot be stationary. Our quantitative analysis highlights that addressing more specifically the issue of what business cycle fluctuations correspond to is of first-order importance. Specifically, we estimated the marginal cost of *all* consumption uncertainty to be extremely large. Nevertheless, when we define business cycles as fluctuations with frequencies less than or equal to 8 years we estimate the marginal cost of business cycles fluctuations to be below two thirds of a percent of lifetime consumption. Additionally the benefits of eliminating these type of fluctuations are smaller than the benefits of a permanent increase of the growth rate of consumption of 2 basis points per year.

Data Appendix

The data used in this paper is annual, covering the period 1889-1997 and comes mainly from two sources: Shiller (1998) and Ibbotson Associates. The breakdown by time series is given hereafter. For additional details we refer the reader to the original sources.

Consumption is real per capita consumption for nondurables and services from Shiller. We updated this series for the period after 1985 with NIPA data and population data from the US. Bureau of the Census.

Stock prices and corresponding dividends are the Standard and Poor Composite Stock Price Indexes from Shiller.

One-year returns are based on 6-month commercial paper returns from Shiller adjusted for a default premium for the period before 1926; after 1926, they are based on monthly holding periods for T-Bills from Ibbotson Associates.

The long-short yield spreads for government bonds are from Cambpell (1996) for the period before 1926; after 1926, from Ibbotson Associates.

All series that have not been originally deflated were deflated by the producer price index from Shiller for the period before 1926; after 1926 by the CPI from Ibbotson Associates.

Appendix

Proof of Proposition 3.2. If U is increasing and concave in $\{C\}$ there must exist a homogenous of degree one, positive and quasi-concave utility v that satisfies

$$U(\{C\}) = \frac{[v(\{C\})]^{1-\gamma}}{1-\gamma}.$$

First, we show that $-(\alpha)$ is concave in α. By homogeneity of U

$$(1 + -(\alpha))^{1-\gamma} \frac{[v(\{C\})]^{1-\gamma}}{1-\gamma} = \frac{[v((1-\alpha)\{C\} + \alpha\{\mathfrak{C}\})]^{1-\gamma}}{1-\gamma}$$

thus, after multiplying by $(1-\gamma)$, taking the $1/(1-\gamma)$ power and dividing by $v(\{C\})$ in both sides, we obtain

$$1 + -(\alpha) = \frac{v((1-\alpha)\{C\} + \alpha\{\mathfrak{C}\})}{v(\{C\})}.$$

Since $v(\cdot)$ is positive, quasi-concave and homogenous of degree one, it is concave. With $(1-\alpha)\{C\} + \alpha\{\mathfrak{C}\}$ linear in α, $v(.)$ it is also concave in α, thus $-(\alpha)$ is concave.

Now we use the concavity to obtain the desired inequalities,

$$-(1) = -(0) + \int_0^1 -'(\alpha)d\alpha \leq -'(0),$$

where the inequality uses $-(0) = 0$, the concavity $-$ and that $\alpha \leq 1$. ■

Proof of Proposition 3.1. Regarding $C = \{C\}$ as a vector on R^∞ let $\frac{\partial U(C)}{\partial C}$ and $\frac{\partial^2 U(C)}{\partial C \partial C'}$ be its first and second derivatives evaluated at C. Define $C_\alpha = (1-\alpha)\mathfrak{C} + \alpha C$. Then twice differentiating with respect to α the equation defining $-$,

$$\begin{aligned} & \frac{\partial^2 -(\alpha)}{\partial\alpha\partial\alpha} \frac{\partial U([1 + -(\alpha)]C)}{\partial C} C + \left(\frac{\partial -(\alpha)}{\partial\alpha} C\right)' \frac{\partial^2 U([1 + -(\alpha)]C)}{\partial C \partial C'} \left(\frac{\partial -(\alpha)}{\partial\alpha} C\right) \\ = & \ (\mathfrak{C} - C)' \frac{\partial^2 U(C_\alpha)}{\partial C \partial C'} (\mathfrak{C} - C) \end{aligned}$$

Taking $\alpha \downarrow 0$ since $C_0 = [1 + -(0)]C = C$,

$$\frac{\partial^2 -(0)}{\partial\alpha\partial\alpha} \frac{\partial U(C)}{\partial C} C = \left(\mathfrak{C} - [1 + \frac{\partial -(0)}{\partial\alpha}]C\right)' \frac{\partial^2 U(C)}{\partial C \partial C'} \left(\mathfrak{C} - [1 + \frac{\partial -(0)}{\partial\alpha}]C\right) \leq 0$$

where the inequality follows from the concavity of U . Since U is increasing $\frac{\partial^2 \text{-} (0)}{\partial\alpha\partial\alpha} \leq 0$. ■

Proof of Proposition 3.3. From the concavity of u and the definition of $\mathfrak{C}$

$$\begin{aligned} U\left((1-\alpha)\{C\}+\alpha\{\mathfrak{C}\}\right) &= E\left[u\left((1-\alpha)C_1+\alpha\mathfrak{C}_1,(1-\alpha)C_2+\alpha\mathfrak{C}_2,...(1-\alpha)C_t+\alpha\mathfrak{C}_t,...\right)\right] \\ &\leq u\left(\mathfrak{C}_1,\mathfrak{C}_2,...,\mathfrak{C}_1,...\right)=U\left(\{\mathfrak{C}\}\right) \end{aligned}$$

for any α. By the definition of - and the monotonicity of U, - has to reach a maximum on $\alpha = 1$ and thus $\text{-}'(1) = 0$ and $\text{-}(\alpha)$ is concave for α close to 1. ■

Proof of Proposition 3.4. First we consider the marginal cost $\text{-}'(0,\sigma^2)$. Using the mean value theorem to take a first order approximation of $u'(c)$ around $\bar{c}$, taking expectations in (3.2), and rearranging we have

$$\text{-}'\left(0,\sigma^2\right)=-\frac{E\left[u''\left(\tau\left(c\right)\right)\left(c-\bar{c}\right)^2\right]}{u'\left(\bar{c}\right)\bar{c}+E\left[u''\left(\tau\left(c\right)\right)\left(c-\bar{c}\right)c\right]},$$

where for each c, $\tau(c)$ is in $[\min\{\bar{c},c\},\max\{\bar{c},c\}]$. Notice that this implies

$$\begin{aligned} &\lim_{\sigma^2\downarrow 0} E\left[u''\left(\tau\left(c\right)\right)\left(c-\bar{c}\right)^2\right]/\sigma^2 \\ =\ &\lim_{\sigma^2\downarrow 0} E\left[u''\left(\tau\left(c\right)\right)\right]\lim_{\sigma^2\downarrow 0}\frac{E\left[\left(c-\bar{c}\right)^2\right]}{\sigma^2}+\lim_{\sigma^2\downarrow 0}\frac{cov\left[u''\left(\tau\left(c\right)\right),\left(c-\bar{c}\right)^2\right]}{\sigma^2} \\ =\ &\lim_{\sigma^2\downarrow 0} E\left[u''\left(\tau\left(c\right)\right)\right]\lim_{\sigma^2\downarrow 0}\frac{E\left[\left(c-\bar{c}\right)^2\right]}{\sigma^2} \\ &+\lim_{\sigma^2\downarrow 0}\rho\left[u''\left(\tau\left(c\right)\right),\left(c-\bar{c}\right)^2\right]\lim_{\sigma^2\downarrow 0}\sigma\left(u''\left(\tau\left(c\right)\right)\right)\lim_{\sigma^2\downarrow 0}\frac{\sigma\left[\left(c-\bar{c}\right)^2\right]}{\sigma^2} \\ =\ &u''\left(\bar{c}\right) \end{aligned}$$

where $\sigma(x)$ denotes the standard deviation of x and $\rho(x,y)$ denotes the correlation coefficient between x and y. Thus, by definition of $o(x)$ we have

$$E\left[u''\left(\tau\left(c\right)\right)\left(c-\bar{c}\right)^2\right]=u''\left(\bar{c}\right)\sigma^2+o\left(\sigma^2\right).$$

Also notice that

$$\lim_{\sigma^2\downarrow 0} E\left[u''\left(\tau\left(c\right)\right)\left(c-\bar{c}\right)c\right]=0,$$

and by calculation similar as the one above

$$E\left[u''\left(\tau\left(c\right)\right)\left(c-\bar{c}\right)c\right]=u''\left(\bar{c}\right)\sigma^2+o\left(\sigma^2\right).$$

Then, using the definition of (3.2)

$$\lim_{\sigma^2\downarrow 0}\frac{-\,'\left(0,\sigma^2\right)}{\sigma^2} = \lim_{\sigma^2\downarrow 0}\frac{-E\left[u''\left(\tau\left(c\right)\right)\left(c-\bar{c}\right)^2\right]/\sigma^2}{\left[u'\left(\bar{c}\right)\bar{c}+E\left[u''\left(\tau\left(c\right)\right)\left(c-\bar{c}\right)c\right]\right]} \tag{7.1}$$

$$= -\frac{u''\left(\bar{c}\right)}{\left[u'\left(\bar{c}\right)\bar{c}+\lim_{\sigma^2\downarrow 0}E\left[u''\left(\tau\left(c\right)\right)\left(c-\bar{c}\right)c\right]\right]}$$

$$= -\frac{u''\left(\bar{c}\right)}{u'\left(c\right)\bar{c}}.$$

Thus, by definition of $o\left(x\right)$ we have

$$-\,'\left(0,\sigma^2\right) = -\frac{u''\left(\bar{c}\right)}{u'\left(c\right)\bar{c}}\sigma^2+o\left(\sigma^2\right).$$

Now we consider the total cost of fluctuations $-\ \left(1,\sigma^2\right)$. Using a second order Taylor expansion on $u\left(c\left(1+-\ \right)\right)$ around $\bar{c}\left(1+-\ \right)$ and taking the expected value,

$$u(\bar{c}) = E\left[u\left(c\left(1+-\ \right)\right)\right] = u\left(\bar{c}\left(1+-\ \right)\right)+u'\left(\bar{c}\left(1+-\ \right)\right)\left(1+-\ \right)E\left[\left(c-\bar{c}\right)\right]$$
$$+\frac{1}{2}E\left[u''\left(\delta\left(c\right)\right)\left(c-\bar{c}\right)^2\left(1+-\ \right)^2\right]$$

where $\delta\left(c\right)$ is defined by the remainder of the Taylor approximation, and hence for each c, $\delta\left(c\right)\in\left[\min\left\{c,\bar{c}\left(1+-\ \right)\right\},\max\left\{c,\bar{c}\left(1+-\ \right)\right\}\right]$. Rearranging, we get

$$u(\bar{c})-u\left(\bar{c}\left(1+-\ \right)\right) = \frac{1}{2}E\left[u''\left(\delta\left(c\right)\right)\left(c-\bar{c}\right)^2\left(1+-\ \right)^2\right].$$

Then

$$\frac{1}{2}E\left[u''\left(\delta\left(c\right)\right)\left(c-\bar{c}\right)^2\left(1+-\ \right)^2\right]$$
$$= \frac{1}{2}\left(1+-\ \right)^2E\left[u''\left(\delta\left(c\right)\right)\right]E\left[\left(c-\bar{c}\right)^2\right]+\frac{1}{2}cov\left[u''\left(\delta\left(c\right)\right),\left(c-\bar{c}\right)^2\right]$$
$$= \frac{1}{2}\left(1+-\ \right)^2E\left[u''\left(\delta\left(c\right)\right)\right]\sigma^2+\frac{1}{2}\left(1+-\ \right)^2\rho\left[u''\left(\delta\left(c\right)\right),\left(c-\bar{c}\right)^2\right]\sigma\left(u''\left(\delta\left(c\right)\right)\right)\sigma\left(\left(c-\bar{c}\right)^2\right)$$

thus

$$\lim_{\sigma^2\downarrow 0}\frac{\frac{1}{2}\left(1+-\ \right)^2E\left[u''\left(\delta\left(c\right)\right)\right]\sigma^2+\frac{1}{2}\left(1+-\ \right)^2\rho\left[u''\left(\delta\left(\bar{c}\right)\right),\left(c-\bar{c}\right)^2\right]\sigma\left(u''\left(\delta\left(c\right)\right)\right)\sigma\left(\left(c-\bar{c}\right)^2\right)}{\sigma^2}$$

$$
\begin{aligned}
&= \frac{1}{2}\left[u''\left(\bar{c}\right)\right] \lim_{\sigma^2 \downarrow 0} \left(1 + -\ \right)^2 \\
&\quad + \lim_{\sigma^2 \downarrow 0} \left(1 + -\ \right)^2 \lim_{\sigma^2 \downarrow 0} \frac{1}{2}\rho\left[u''\left(\delta\left(c\right)\right), \left(c - \bar{c}\right)^2\right] \lim_{\sigma^2 \downarrow 0} \sigma\left(u''\left(\delta\left(c\right)\right)\right) \lim_{\sigma^2 \downarrow 0} \frac{\sigma\left(\left(c-\bar{c}\right)^2\right)}{\sigma^2} \\
&= \frac{1}{2}u''\left(\bar{c}\right).
\end{aligned}
$$

Since

$$
\lim_{\sigma^2 \downarrow 0} E\left[u''\left(\delta\left(c\right)\right)\right] = u''\left(\bar{c}\right),\ \lim_{\sigma^2 \downarrow 0} -\ = 0,\ \lim_{\sigma^2 \downarrow 0} \sigma\left(u''\left(\delta\left(c\right)\right)\right) = 0,\ \lim_{\sigma^2 \downarrow 0} \frac{\sigma\left(\left(c-\bar{c}\right)^2\right)}{\sigma^2} < \infty.
$$

So that

$$
\frac{1}{2}E\left[u''\left(\delta\left(c\right)\right)\left(c-\bar{c}\right)^2\left(1 + -\ \right)^2\right] = \frac{1}{2}u''\left(\bar{c}\right) + o\left(\sigma^2\right).
$$

Also by taking a first order Taylor expansion of $u(\bar{c})$ around $\bar{c}\left(1 + -\ \right)$,

$$
u\left(\bar{c}\right) = u\left(\bar{c}\left(1 + -\ \right)\right) - u'\left(\omega\left(-\ \right)\right) -\ \bar{c}
$$

for $\omega\left(-\ \right) \in \left[\bar{c}, \bar{c}\left(1 + -\ \right)\right]$. Since $\lim_{\sigma^2 \downarrow 0} -\ \left(1, \sigma^2\right) = 0$, then

$$
\bar{c} \leq \lim_{\sigma^2 \downarrow 0} \omega\left(-\ \left(1, \sigma^2\right)\right) \leq \bar{c}\left(1 + \lim_{\sigma^2 \downarrow 0} -\ \left(1, \sigma^2\right)\right)
$$

or

$$
\lim_{\sigma^2 \downarrow 0} \omega\left(-\ \left(1, \sigma^2\right)\right) = \bar{c}
$$

and thus

$$
\lim_{\sigma^2 \downarrow 0} \frac{u'\left(\omega\left(c\right)\right) -\ \bar{c}}{\sigma^2} = u'\left(\bar{c}\right)\bar{c} \lim_{\sigma^2 \downarrow 0} \frac{-}{\sigma^2},
$$

or equivalently

$$
u'\left(\omega\left(c\right)\right) -\ \bar{c} = u'\left(\bar{c}\right)\bar{c}-\ + o\left(\sigma^2\right).
$$

Then, using the definition of (3.1),

$$
\frac{1}{2}u''\left(\bar{c}\right) = -u'\left(\bar{c}\right)\bar{c} \lim_{\sigma^2 \downarrow 0} \frac{-}{\sigma^2}
$$

or

$$
\lim_{\sigma^2 \downarrow 0} \frac{-\ \left(1, \sigma^2\right)}{\sigma^2} = \frac{1}{2}\frac{u''\left(\bar{c}\right)}{u'\left(\bar{c}\right)\bar{c}}. \tag{7.2}
$$

Finally notice that

$$\sigma^2 (c/\bar{c}) = \frac{\sigma^2}{\bar{c}^2}$$

thus replacing this equality in (7.1) and (7.2) we obtain the desired results. ■

Proof of Lemma 3.6. It follows from a straightforward modification of the proof of the previous proposition. ■

Proof of Proposition 3.10. Notice that the left hand side of (3.8) and (3.9) are the same. Consider any arbitrary α. By definition of envy, making $\left\{\hat{C}\right\} = (1-\alpha)\{C\} + \alpha\{\mathfrak{C}\}$, we obtain the desired result. ■

Proof of Proposition (4.1). Using the definition of the $w's$ in equation (4.1), multiplying and dividing by $E_0(C_t)$ and using the definitions of $R_{0,t}[C_t]$ and $R_{0,t}[1_t]$ we have,

$$\begin{aligned} 1+\omega_0^m &= \frac{\sum_{t=1}^{\infty} V_0[E_0(C_t)]}{\sum_{t=1}^{\infty} V_0[C_t]} \\ &= \sum_{t=1}^{\infty} w_0[C_t]\left(\frac{V_0[E_0(C_t)]}{V_0[C_t]}\right) \\ &= \sum_{t=1}^{\infty} w_0[C_t]\left(\frac{E_0(C_t)}{V_0[C_t]}\Big/\frac{E_0(C_t)}{V_0[E_0(C_t)]}\right) \\ &= \sum_{t=1}^{\infty} w_0[C_t]\left(R_{0,t}[C_t]/R_{0,t}[1_t]\right). \end{aligned}$$

Using the definition of the equity premium, the decomposition of the price of equity as the sum of the strips, the definitions of $w_0[C_t]$, and the definition of excess one-period holding returns, we have:

$$\begin{aligned} 1+\nu_0 &= \frac{E_0(V_1[\{C_t\}])}{V_0[\{C_t\}]}\Big/\frac{1}{V_0(1_1)} \\ &= \frac{E_0\left(\sum_{t=1}^{\infty} V_1[C_t]\right)}{V_0[\{C_t\}]}\Big/\frac{1}{V_0(1_1)} \\ &= \frac{V_0[\{C_t\}]\,E_0\left(\sum_{t=1}^{\infty}\frac{V_0[C_t]}{V_0[\{C_t\}]}\frac{V_1[C_t]}{V_0[C_t]}\right)}{V_0[C_t]}\Big/\frac{1}{V_0(1_1)} \\ &= E_0\left(\sum_{t=1}^{\infty} w_0[C_t]\frac{V_1[C_t]}{V_0[C_t]}\right)\Big/\frac{1}{V_0(1_1)} \end{aligned}$$

$$
\begin{aligned}
&= \sum_{t=1}^{\infty} w_0 [C_t] \frac{E_0 (V_1 [C_t])}{V_0 [C_t]} / \frac{1}{V_0 (1_1)} \\
&= \sum_{t=1}^{\infty} w_0 [C_t] \left(\frac{E_0 (R_{0,1} [C_t])}{R_{0,1} [1_1]} \right). \blacksquare
\end{aligned}
$$

Proof of Proposition 5.1. By definition

$$
\begin{aligned}
h_t^{cc}(\infty) &= \lim_{k\to\infty} \left\{ E_t \left(\log \left[\frac{R_{t,t+1}[1_{t+k}]}{R_{t,t+1}[1_{t+1}]} \right] \right) \right\} \\
&= \lim_{k\to\infty} \left\{ E_t \log \left[\left(\frac{\frac{\beta^{k-1} \cdot E_{t+1}[M_{t+k}]}{M_{t+1}}}{\frac{\beta^k E_t[M_{t+k}]}{M_t}} \frac{\beta E_t [M_{t+1}]}{M_t} \right) \right] \right\} \\
&= \lim_{k\to\infty} \left\{ E_t \left(\log \left[\frac{E_{t+1}[M_{t+k}]}{M_{t+1}} \frac{E_t [M_{t+1}]}{E_t[M_{t+k}]} \right] \right) \right\} \\
&= \lim_{k\to\infty} \left\{ E_t \left(\log E_{t+1}[M_{t+k}] - \log M_{t+1} + \log E_t [M_{t+1}] - \log E_t[M_{t+k}] \right) \right\} \\
&= \lim_{k\to\infty} \left\{ E_t \log E_{t+1}[M_{t+k}] - E_t \log M_{t+1} + \log E_t [M_{t+1}] - \log E_t[M_{t+k}] \right\} \\
&= \log E_t [M_{t+1}] - E_t \log M_{t+1} + \lim_{k\to\infty} \left\{ E_t \log E_{t+1}[M_{t+k}] \right\} - \lim_{k\to\infty} \left\{ \log E_t[M_{t+k}] \right\} \\
&= \log E_t [M_{t+1}] - E_t \log M_{t+1},
\end{aligned}
$$

where the last step passes first the limit through the expectations operator—we assume enough regularity for this to be admissible (Lebesgue dominated convergence). Second, assuming that stationary is defined as requiring the existence of a finite unconditional mean, which implies that $\lim_{k\to\infty} \{\log E_{t+1}[M_{t+k}]\} = \lim_{k\to\infty} \{\log E_t[M_{t+k}]\} = \log E[M]$. This last equality shows how the price of a discount bond maturing in the distant future is deterministic in units of utility.

With yield differentials

$$
\begin{aligned}
y_t^{cc}(\infty) &= \lim_{k\to\infty} \left\{ \log \left[\frac{V_t [1_{t+1}]}{(V_t [1_{t+k}])^{1/k}} \right] \right\} \\
&= \lim_{k\to\infty} \left\{ \log \left[\frac{\beta E_t [M_{t+1}]}{M_t} \left(\frac{\beta^k \cdot E_t[M_{t+k}]}{M_t} \right)^{-1/k} \right] \right\} \\
&= \lim_{k\to\infty} \left\{ \log E_t [M_{t+1}] - \log M_t + (1/k) \log E_t[M_{t+k}] - (1/k) \log M_t \right\} \\
&= \log E_t [M_{t+1}] - \log M_t.
\end{aligned}
$$

This concludes the proof. ■

Proof of Proposition 5.2. First, that $E\left[h_t^{cc}(\infty)\right] = E\left[y_t^{cc}(\infty)\right]$ can be shown by taking unconditional expectations on the expression obtained in Proposition (5.1). The continuously compounded holding premium can be written as

$$\begin{aligned} h_t^{cc}(\infty) &= \log E_t\left[M_{t+1}\right] - \log M_t && (7.3)\\ &= \log \beta E_t\left[\frac{M_{t+1}}{M_t}\right] - \log\beta - E_t \log\frac{M_{t+1}}{M_t} && (7.4)\\ &= -\log R_{t,t+1}\left(1_{t+1}\right) - \log\beta - E_t\log\frac{M_{t+1}}{M_t}. \end{aligned}$$

For any risky gross asset return $R^j_{t,t+1}$:

$$1 = \beta E_t\left[R^j_{t,t+1}\frac{M_{t+1}}{M_t}\right]$$

so that,

$$\begin{aligned} 0 &= \log(1) = \log\left(\beta E_t\left[R^j_{t,t+1}\frac{M_{t+1}}{M_t}\right]\right)\\ &> E_t\left[\log\beta\left(R^j_{t,t+1}\frac{M_{t+1}}{M_t}\right)\right] = E_t\left[\log\left(\frac{M_{t+1}}{M_t}\right)\right] + \log\beta + E_t\left[\log\left(R^j_{t,t+1}\right)\right] \end{aligned}$$

where the strict inequality follows from the strict concavity of log. Then

$$-E_t\left[\log\left(\frac{M_{t+1}}{M_t}\right)\right] - \log\beta > E_t\left[\log\left(R^j_{t,t+1}\right)\right]. \tag{7.5}$$

Combining the two expressions we obtain

$$h_t^{cc}(\infty) > E_t\left[\log\left(R^j_{t,t+1}\right)\right] - \log R_{t,t+1}\left(1_t\right)$$

for any asset return $R^j_{t,t+1}$. Taking unconditional expectations we conclude the proof. ■

Proof of Proposition 5.3. First notice that the random walk assumption implies that interest rate are constant, since

$$E_t\left(\frac{M_{t+1}}{M_t}\right) = E\left(\frac{M_{t+1}}{M_t}\right)$$

and hence the term premium is zero. Then

$$\frac{E_t\left(R_{t+1}\left[D_{t+s}\right]\right)}{E_t R_{t+1}\left(1_{t+1}\right)} = \frac{E_t\left(R_{t+1}\left[D_{t+s}/D_t\right]\right)}{E_t R_{t+1}\left(1_{t+1}\right)} = E_t\left(\beta\frac{M_{t+1}}{M_t}\right)\frac{E_t\left(\frac{D_{t+1}}{D_t}E_{t+1}\left[\beta^{s-1}\frac{M_{t+s}}{M_{t+1}}\frac{D_{t+s}}{D_{t+1}}\right]\right)}{E_t\left[\beta^s\frac{M_{t+s}}{M_t}\frac{D_{t+1}}{D_t}\frac{D_{t+s}}{D_{t+1}}\right]}$$

$$= \exp\left(-cov_t\left(\log\frac{M_{t+1}}{M_t}, E_t\left(\log\frac{D_{t+s}}{D_t}\right)\right)\right) = \exp\left(-cov_t\left(m_{t+1}, d_{t+1}\right)\right) \text{ for all}$$

where the first inequality follows since D_t is known at t, the second from the definition of returns and pricing kernel, the third from the log-normality of $D's$ and $M's$ as in (4.5) since the term premium is zero, and the forth follows by the random walk assumptions. From this we obtain that the excess expected return of a stock paying $\{D_{t+s}\}$ is

$$\frac{E_t\left(R_{t+1}\left[\{D_{t+s}\}_{s=1}^{\infty}\right]\right)}{E_t R_{t+1}\left(1_{t+1}\right)} = \sum_{s=1}^{\infty} w\left(\left[D_{t+s}\right]\right)\frac{E_t\left(R_{t+1}\left[D_{t+s}\right]\right)}{E_t R_{t+1}\left(1_{t+1}\right)} = \frac{E_t\left(R_{t+1}\left[D_{t+s}\right]\right)}{E_t R_{t+1}\left(1_{t+1}\right)}$$

where the weights $w\left[D_{t+s}\right] = V_t\left[\{D_{t+s}\}\right]/V\left[\{D_{t+j}\}_{j=1}^{\infty}\right]$ add up to one. For this stock, the price-dividend ratio is given by

$$V_t\left[\{D_{t+s}\}_{s=1}^{\infty}\right]/D_t = \sum_{s=1}^{\infty}\beta^s E_t\left[\frac{M_{t+s}}{M_t}\frac{D_{t+s}}{D_t}\right] = \sum_{s=1}^{\infty}\prod_{i=1}^{s}E_t\left[\beta\frac{M_{t+i}}{M_{t+i-1}}\frac{D_{t+i}}{D_{t+i-1}}\right]$$

$$= \sum_{s=1}^{\infty}\left(E\left[\beta\frac{M_{t+1}}{M_t}\frac{D_{t+1}}{D_t}\right]\right)^s = \frac{E\left[\beta\frac{M_{t+1}}{M_t}\frac{D_{t+1}}{D_t}\right]}{1-E\left[\beta\frac{M_{t+1}}{M_t}\frac{D_{t+1}}{D_t}\right]}$$

where the third inequality uses the random walk assumption. Finally, notice for each asset i we have

$$\log E_t\left(R_{t+1,i}\right) = -cov_t\left(\Delta m_{t+1}, \Delta d_{t+1}^i\right) = -cov\left(l\cdot\varepsilon_{t+1}, \Delta d_{t+1}^i\right).$$

■

Proof of Proposition 5.4. First we find an expression for $\tau(t)$. Notice that using lognormality we get

$$E_0\left(\mathfrak{C}_t\right) = \exp(\tau) E_0 \exp\left(\sum_{i=1}^{K} a_i c_{t-j}\right) = \exp(\tau(t))\exp\left(\sum_{i=1}^{K} a_i E_0\left(c_{t-j}\right) + \frac{1}{2}var_0\left(\sum_{i=1}^{K} a_i c_{t-j}\right)\right)$$

and

$$E_0\left(C_t\right)=E_0\exp\left(c_t\right)=\exp\left(E_0c_t+\frac{1}{2}var_0\left(c_t\right)\right)$$

thus $\tau\left(t\right)$ solves $E_0\left(\mathfrak{C}_t\right)=E_0\left(C_t\right)$ so

$$\tau\left(t\right)=\exp\left(E_0c_t-\sum_{i=1}^{K}a_iE_0\left(c_{t-j}\right)+\frac{1}{2}var_0\left(c_t\right)-\frac{1}{2}var_0\left(\sum_{i=1}^{K}a_ic_{t-j}\right)\right).$$

Using the definition of the pricing kernel and lognormality

$$\frac{V_0\left[\mathfrak{C}_t\right]}{V_0\left[C_t\right]}=$$

$$\begin{aligned}
&= \frac{E_0\exp\left[\tau\left(t\right)+E_0m_t-m_0+\sum_{j=0}^{K}a_jc_{t-j}\right]}{E_0\exp\left[m_t-m_0+c_t\right]} \\
&= \frac{\exp\left[\tau\left(t\right)+E_0\left(m_t-m_0+\sum_{j=0}^{K}a_jc_{t-j}\right)+\frac{1}{2}var_0\left(m_t\right)\right]}{\exp\left[E_0\left(m_t-m_0+c_t\right)+\frac{1}{2}var_0\left(m_t\right)+\frac{1}{2}var_0\left(c_t\right)+cov_0\left(m_t,c_t\right)\right]} \\
&\quad \times\exp\left[\frac{1}{2}var_0\left(\sum_{j=0}^{K}a_jc_{t-j}\right)+cov_0\left(m_t,\sum_{j=0}^{K}a_jc_{t-j}\right)\right] \\
&= \exp\begin{bmatrix} E_0\left(\sum_{j=0}^{K}a_jc_{t-j}\right)-E_0\left(c_t\right)+ \\ \tau\left(t\right)+\frac{1}{2}var_0\left(\sum_{j=0}^{K}a_jc_{t-j}\right)-\frac{1}{2}var_0\left(c_t\right) \\ +cov_0\left(m_t,\sum_{j=0}^{K}a_jc_{t-j}\right)-cov_0\left(m_t,c_t\right)\end{bmatrix}.
\end{aligned}$$

and by definition of $\tau\left(t\right)$ we get

$$\frac{V_0\left[\mathfrak{C}_t\right]}{V_0\left[C_t\right]}=\exp\left[cov_0\left(m_t,\sum_{j=0}^{K}a_jc_{t-j}\right)-cov_0\left(m_t,c_t\right)\right].$$

By direct computation in the random walk case:

$$=\exp\left[cov_0\left(m_t,\sum_{j=0}^{K}a_jc_{t-j}\right)-cov_0\left(m_t,c_t\right)\right]$$

$$= \exp\left[cov_0\left(\sum_{k=(t-K)}^{t}\varepsilon_k^m, \sum_{k=(t-K)}^{t}\sum_{j=0}^{K}a_j\varepsilon_{k-j}^c\right) - cov_0\left(\sum_{k=(t-K)}^{t}\varepsilon_k^m, \sum_{k=(t-K)}^{t}\varepsilon_{k-j}^c\right)\right]$$

$$= \exp\left[\begin{array}{c} cov_0\left(\varepsilon_t^m, a_0\varepsilon_k^c\right) - cov_0\left(\varepsilon_t^m, \varepsilon_t^c\right) \\ cov_0\left(\varepsilon_{t-1}^m, (a_0+a_1)\,\varepsilon_{t-1}^c\right) - cov_0\left(\varepsilon_{t-K-1}^m, \varepsilon_{t-1}^c\right) + \cdots \\ cov_0\left(\varepsilon_{t-K}^m, (a_0+a_1+\cdots+a_K)\,\varepsilon_{t-K-1}^c\right) - cov_0\left(\varepsilon_{t-K}^m, \varepsilon_{t-K}^c\right) \end{array}\right]$$

$$= \exp\left[\ \sigma_{\Delta c\Delta m}\left[a_0 - 1 + (a_0+a_1) - 1 + \cdots + (a_0+a_1+\cdots+a_K) - 1\right]\ \right]$$

where ε^m and ε^c are the innovations of the log of the pricing kernel and the log of consumption. Where the first step uses the fact that the first $t-K$ shocks are common to both covariances; the second separates the covariances, and the third reorders and uses the result that for last term the sum of all the alphas equals 1. Finally, by the previous proposition we have that

$$\frac{E\left(R_1\left[C_t\right]\right)}{R_f} = \exp\left(-cov\left(\Delta m_t, \Delta c_t\right)\right) \equiv \exp\left(-\sigma_{\Delta c,\Delta m}\right)$$

which combined with the previous equality finishes the proof. ■

Proof of Proposition 5.5. Direct computation shows that

$$\begin{aligned} m\left(a^{k+1}\right) &= \left(a_0^{K+1} - 1\right) + \left(a_0^{K+1} + a_1^{K+1} - 1\right) + \left(a_0^{K+1} + a_1^{K+1} + a_2^K - 1\right) + \\ &\quad + \cdots + \left(a_0^{K+1} + a_1^{K+1} + \cdots + a_{K+1}^K - 1\right) \\ &= -1 + \left(a_0^K - 1\right) + \left(a_0^K + a_1^K - 1\right) + \cdots + \left(a_0^{K+1} + a_1^{K+1} + \cdots + a_K^K - 1\right) \\ &= m\left(a^K\right) + 1. \end{aligned}$$

■

Proof of Proposition 6.1. Regarding $C = \{C\}$ as a vector on R^∞ let $\frac{\partial U(C)}{\partial C}$ and $\frac{\partial^2 U(C)}{\partial C \partial C'}$ be its first and second derivatives evaluated at C. Define $C_\alpha = (1-\alpha)\,\mathfrak{C} + \alpha C$ and $\left(C_t\left[1+\lambda(\alpha)\right]^t\right)$ be the vector where each component consists of $C_t\left(z^t\right)\left[1+\lambda(\alpha)\right]^t$. Then twice differentiating with respect to α the equation defining λ,

$$\frac{\partial \lambda(\alpha)}{\partial \alpha}\,\frac{\partial U\left(\left(\left[1+\lambda(\alpha)\right]^t C_t\right)\right)}{\partial C}\ \left(C_t\left[1+\lambda(\alpha)\right]^{t-1} t\right) = \frac{\partial U\left(C_\alpha\right)}{\partial C}\ \left(\mathfrak{C} - C\right)$$

and

$$\frac{\partial^2 \lambda(\alpha)}{\partial \alpha \partial \alpha}\,\frac{\partial U\left(\left(\left[1+\lambda(\alpha)\right]^t C_t\right)\right)}{\partial C}\ \left(C_t\left[1+\lambda(\alpha)\right]^{t-1} t\right) +$$

$$
\begin{aligned}
& \left[\frac{\partial\lambda(\alpha)}{\partial\alpha}\right]^2 \frac{\partial U\left(\left([1+\lambda(\alpha)]^t C_t\right)\right)}{\partial C} \left(C_t\left[1+\lambda(\alpha)\right]^{t-2} t\,(t-1)\right) \\
& + \left(\frac{\partial\lambda(\alpha)}{\partial\alpha}\left(C_t\left[1+\lambda(\alpha)\right]^{t-1} t\right)\right)' \frac{\partial^2 U\left(\left([1+\lambda(\alpha)]^t C_t\right)\right)}{\partial C\partial C'} \left(\frac{\partial\lambda(\alpha)}{\partial\alpha}\left(C_t\left[1+\lambda(\alpha)\right]^{t-1} t\right)\right) \\
= \;& (\mathfrak{C}-C)' \frac{\partial^2 U(C_\alpha)}{\partial C\partial C'} (\mathfrak{C}-C)
\end{aligned}
$$

Taking $\alpha \downarrow 0$ since $C_\alpha = C$ and $\lambda(0) = 0$ then,

$$
\begin{aligned}
& \frac{\partial^2\lambda(0)}{\partial\alpha\partial\alpha} \frac{\partial U(C)}{\partial C} (C_t t) = -\left[\frac{\partial\lambda(\alpha)}{\partial\alpha}\right]^2 \frac{\partial U(C)}{\partial C} (C_t t\,(t-1)) \\
& + \left(\mathfrak{C} - C - \frac{\partial\lambda(\alpha)}{\partial\alpha}(C_t\; t)\right)' \frac{\partial^2 U(C)}{\partial C\partial C'} \left(\mathfrak{C} - C - \frac{\partial\lambda(\alpha)}{\partial\alpha}(C\; t)\right) \leq 0
\end{aligned}
$$

where the inequality follows from the concavity of U and the monotonicity of U . Since U is increasing , then $\frac{\partial^2\lambda(0)}{\partial\alpha\partial\alpha} \leq 0$ ■

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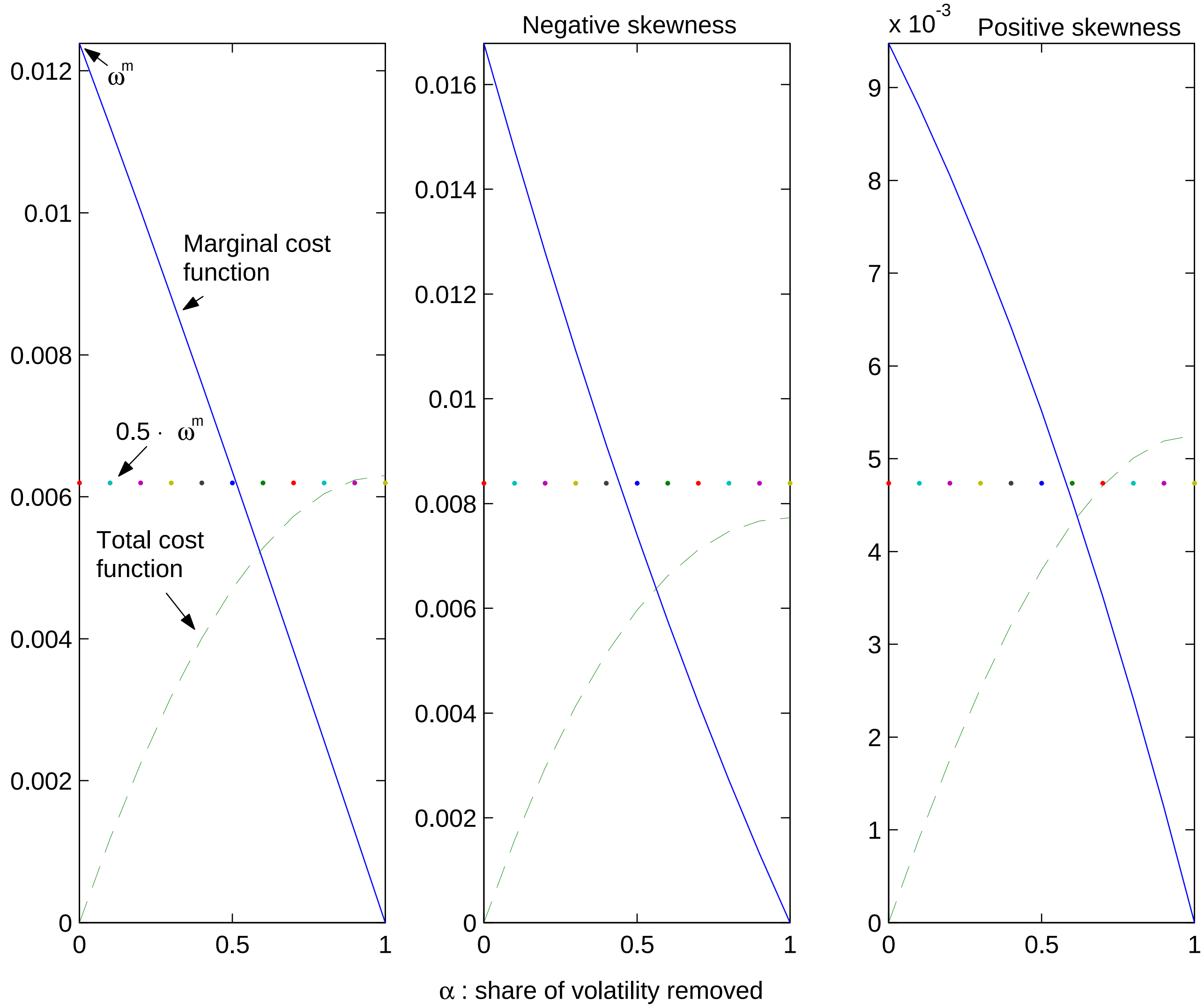
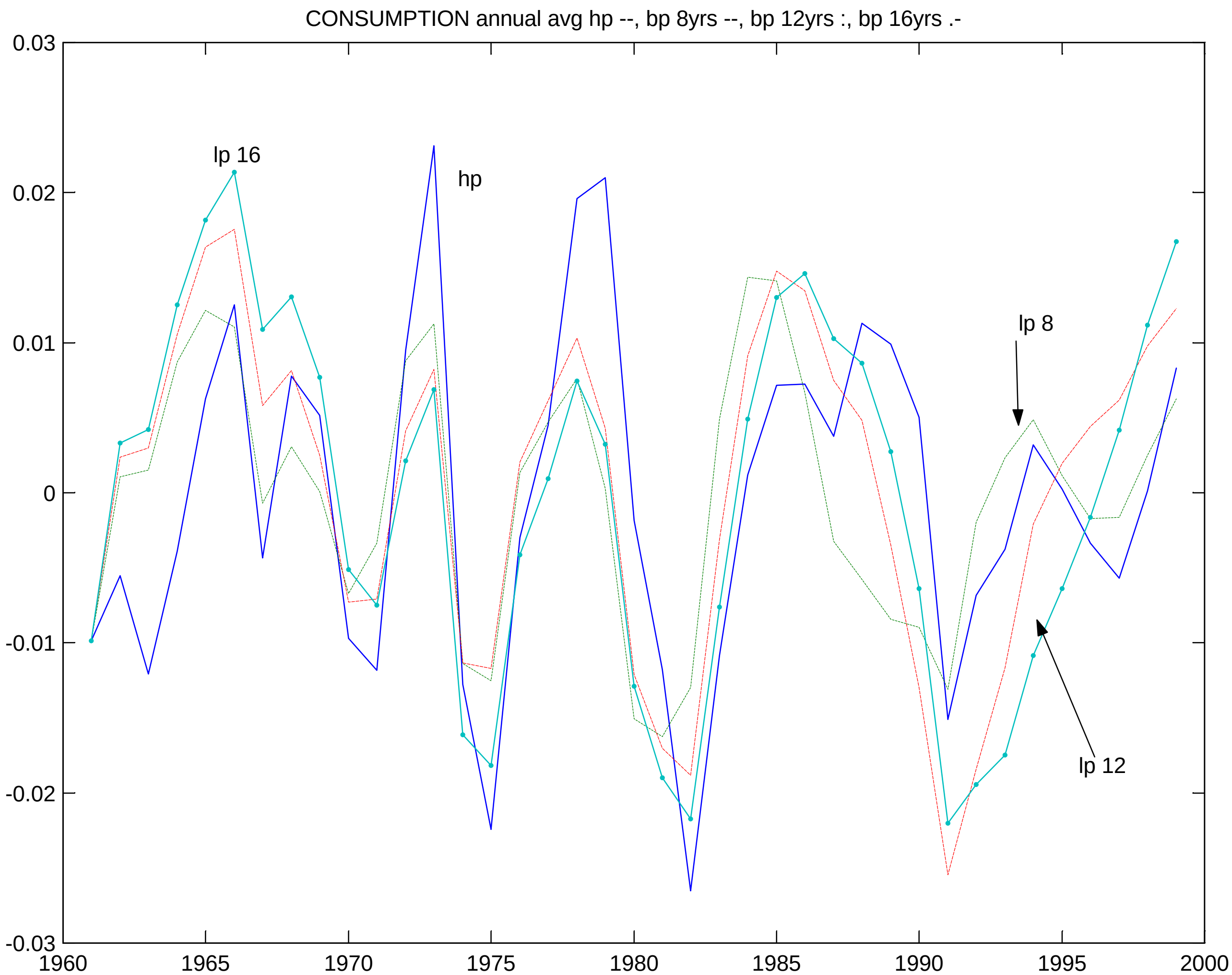
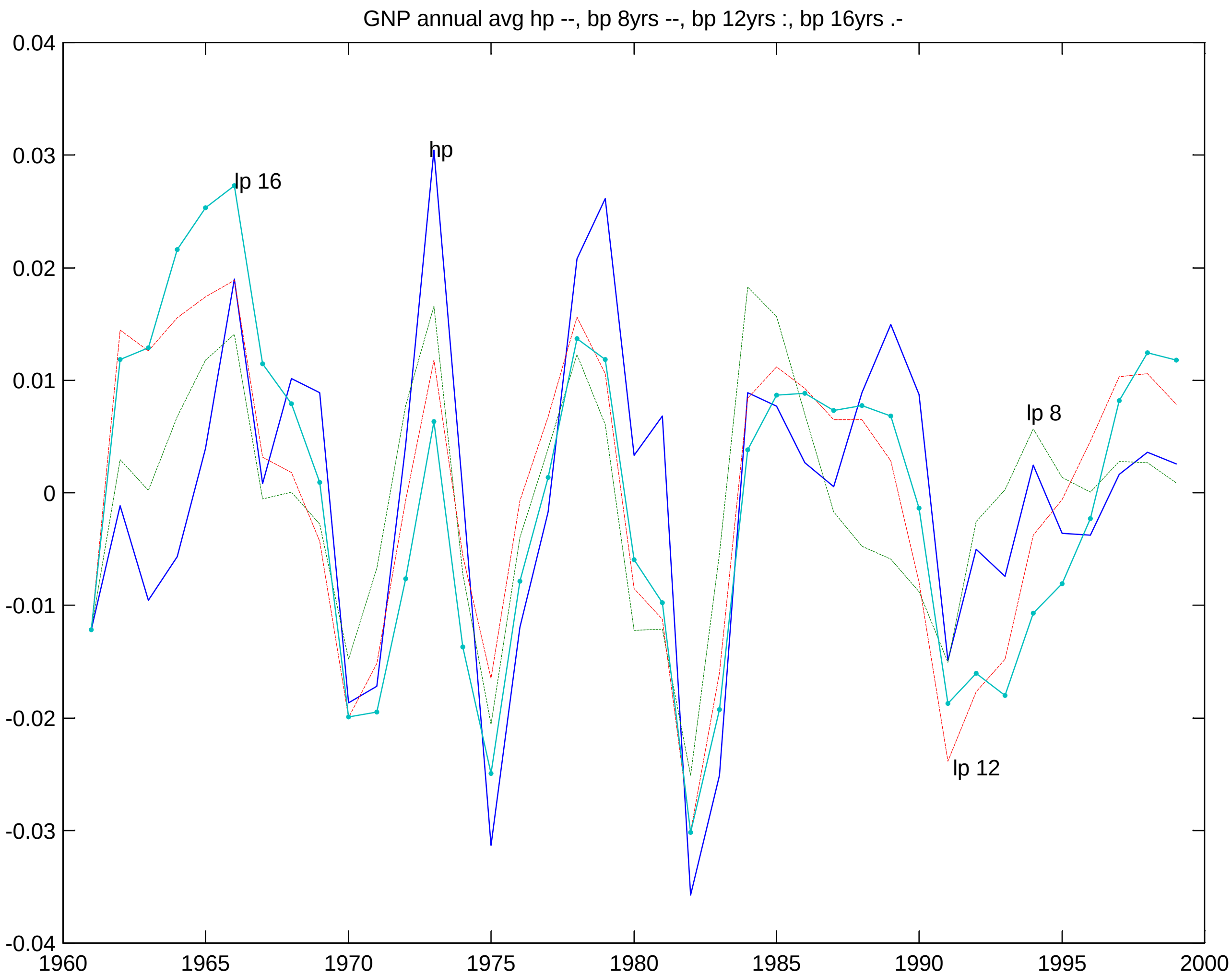
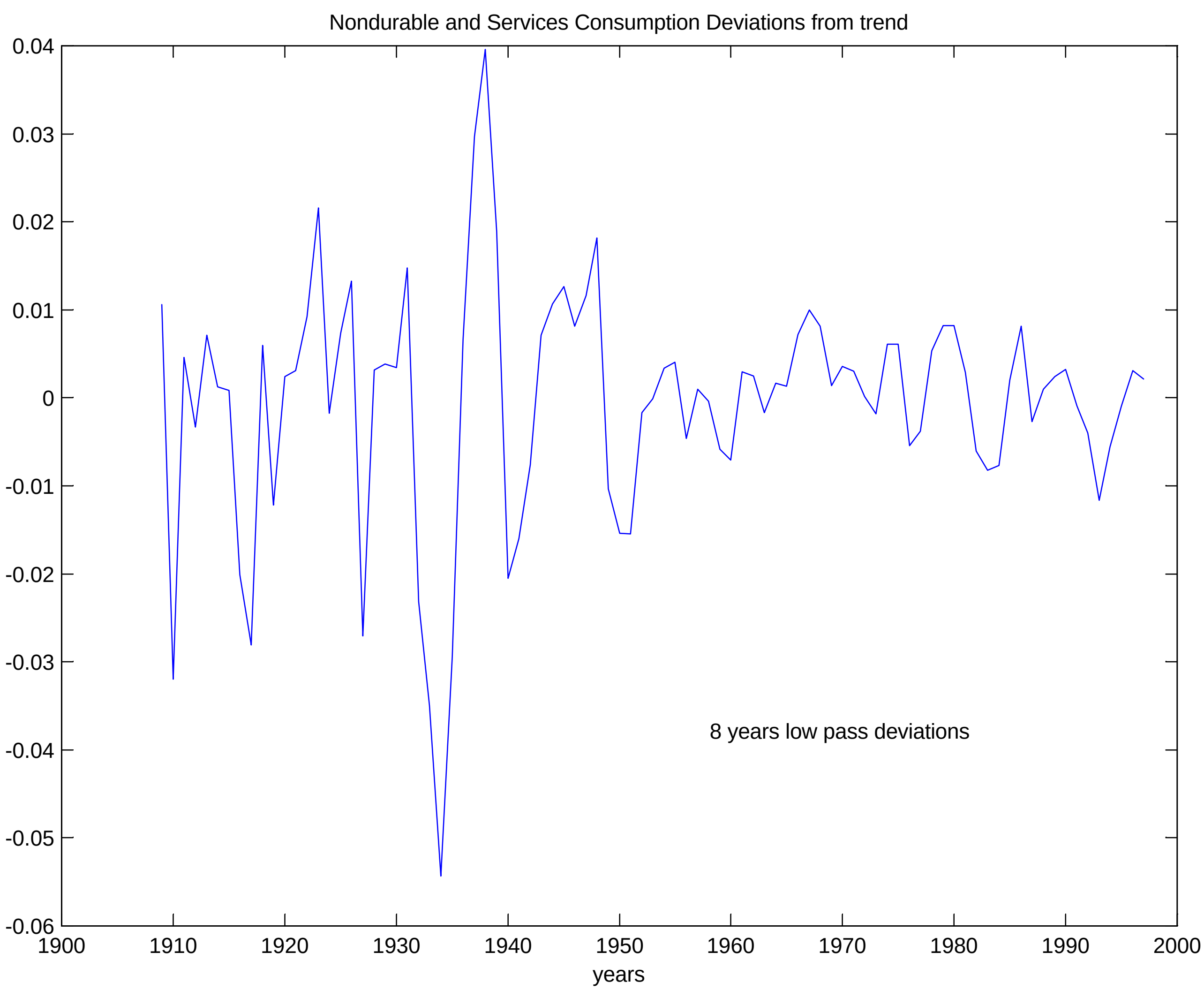
Figure 1: Marginal and Total Cost Functions

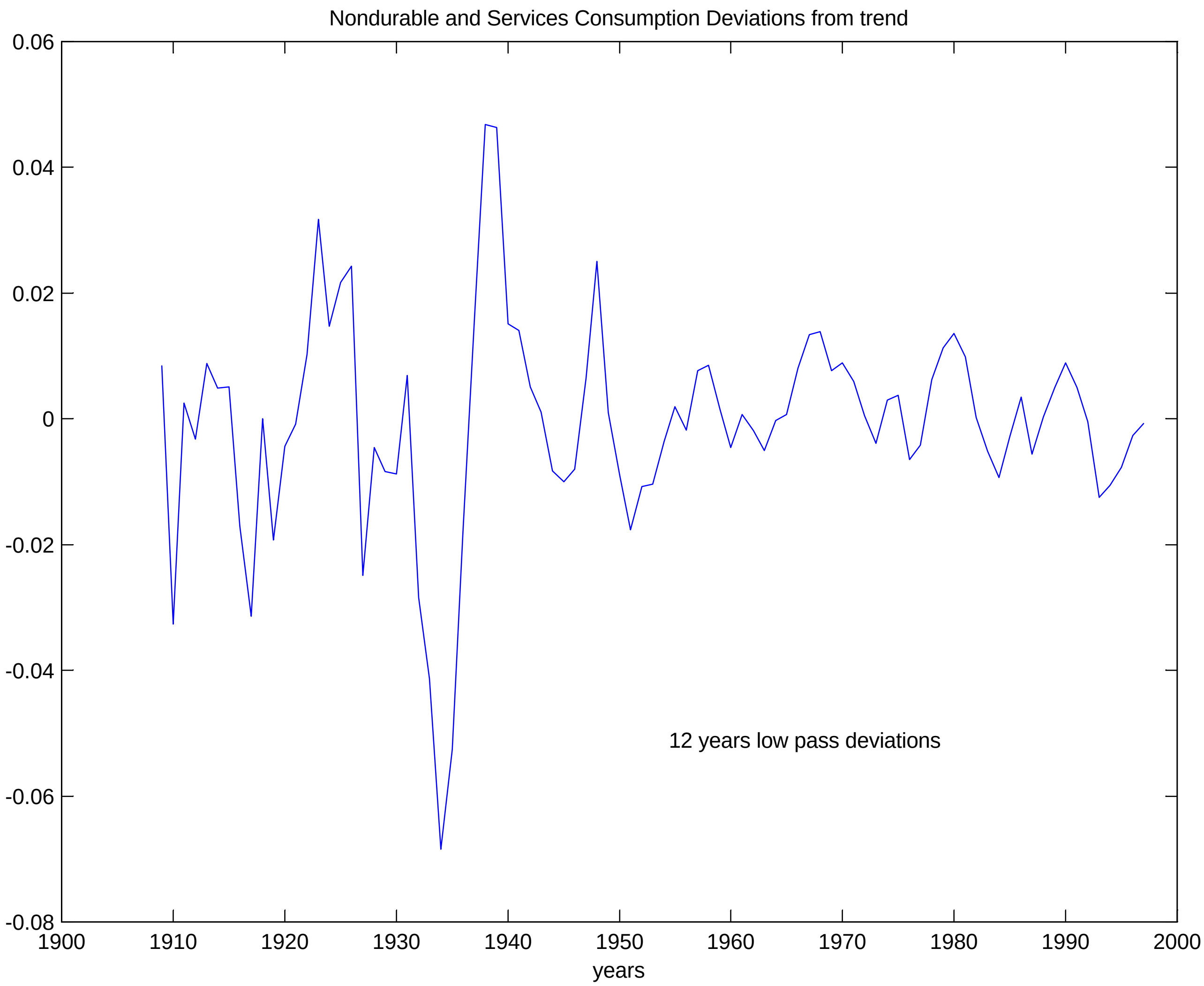



CONSUMPTION annual avg hp --, bp 8yrs --, bp 12yrs :, bp 16yrs .-
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
1960
1965
1970
1975
1980
1985
1990
1995
2000
lp 16
hp
lp 8
lp 12

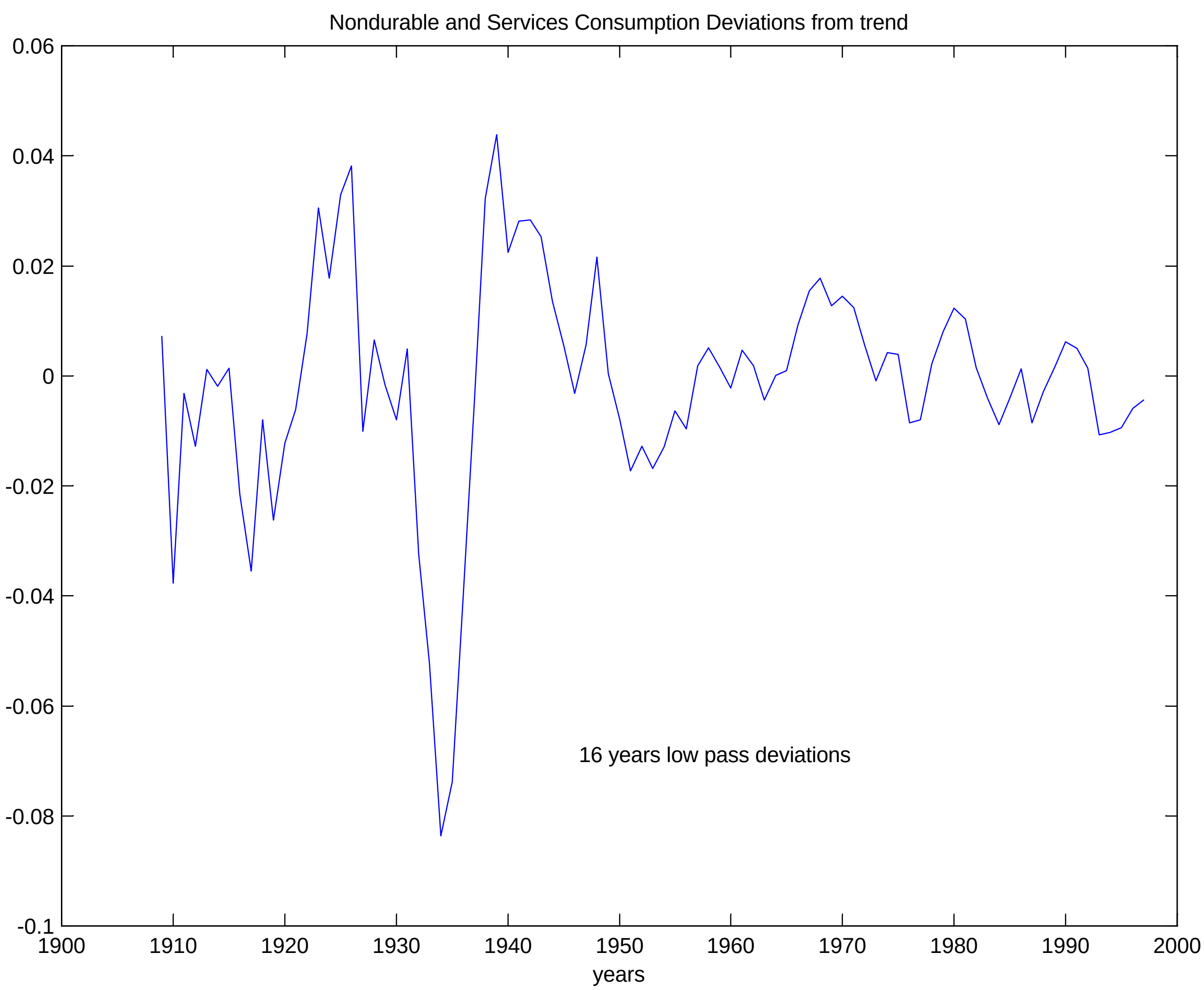


GNP annual avg hp --, bp 8yrs --, bp 12yrs :, bp 16yrs .-
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
1960
1965
1970
1975
1980
1985
1990
1995
2000
hp
lp 16
lp 8
lp 12

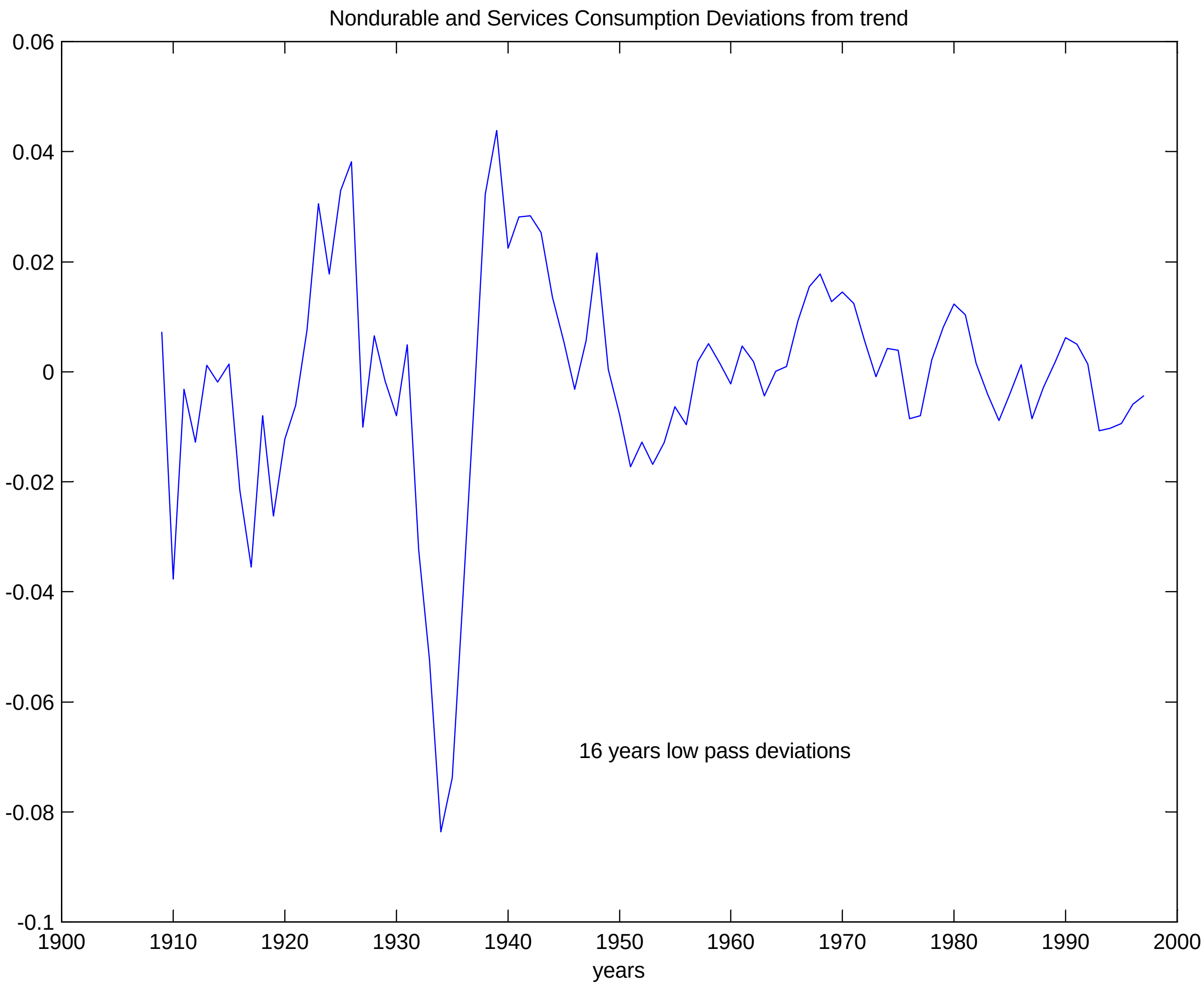

Nondurable and Services Consumption Deviations from trend
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
years
8 years low pass deviations



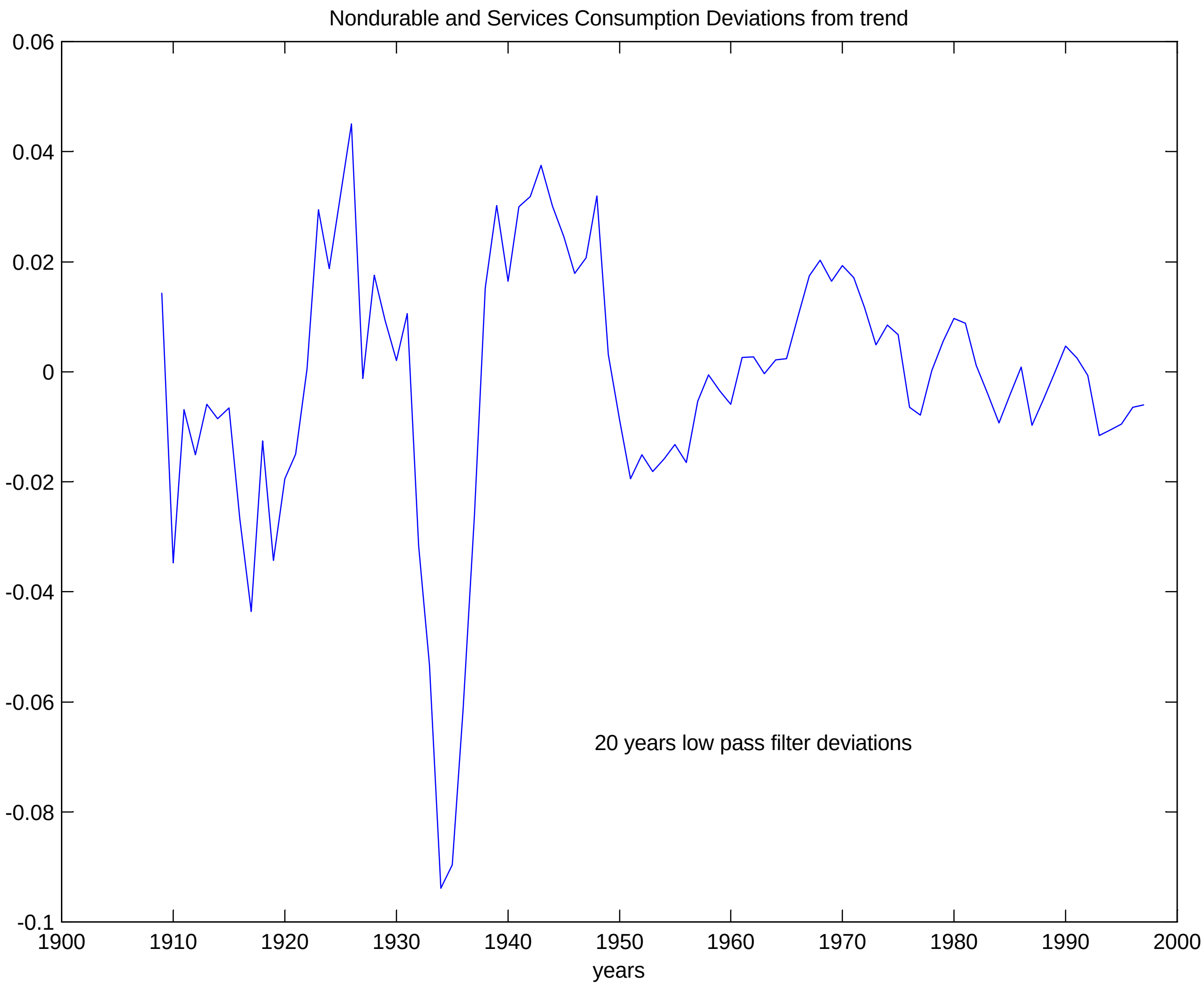
Nondurable and Services Consumption Deviations from trend
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
years
12 years low pass deviations

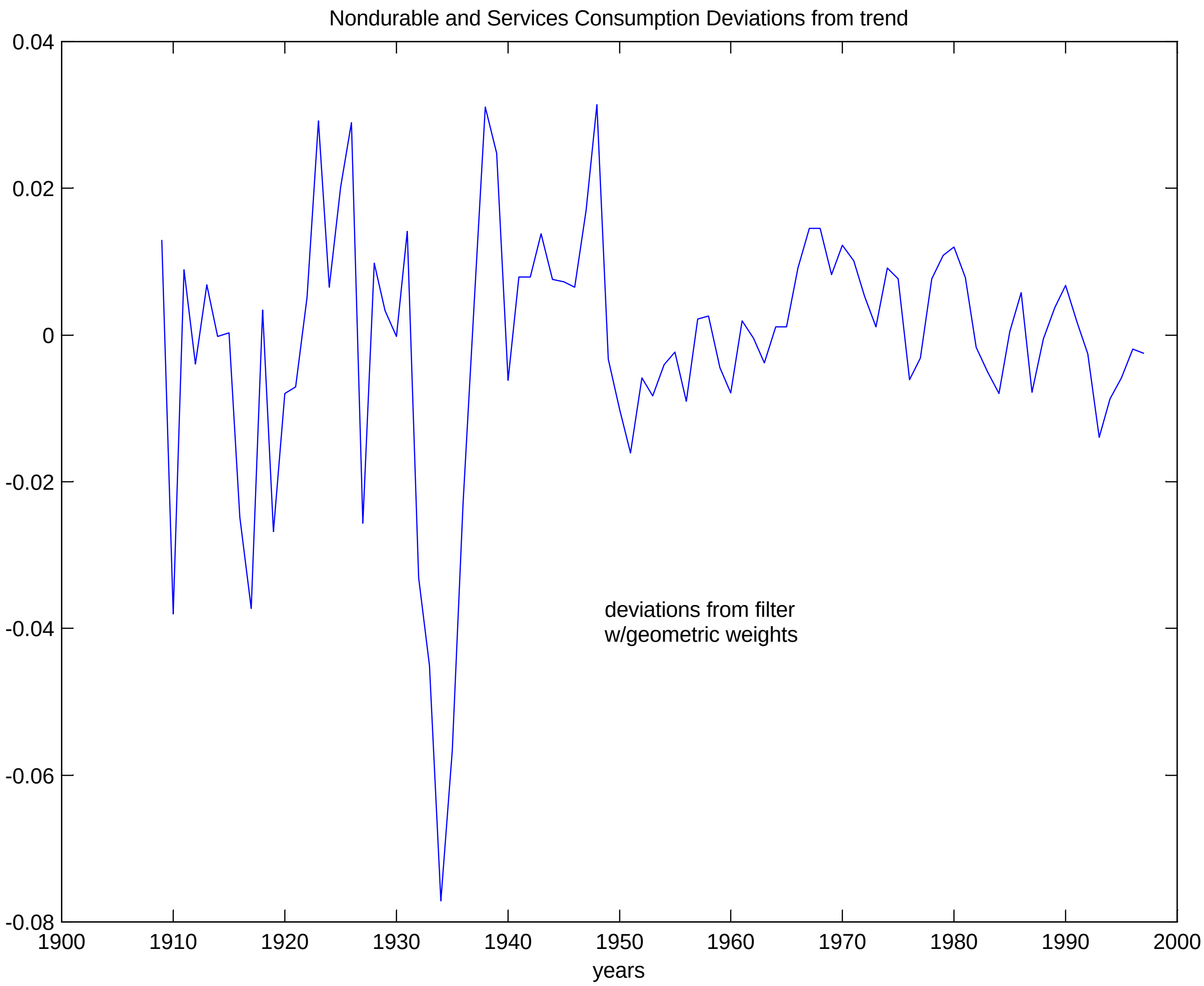


Nondurable and Services Consumption Deviations from trend
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
years
16 years low pass deviations

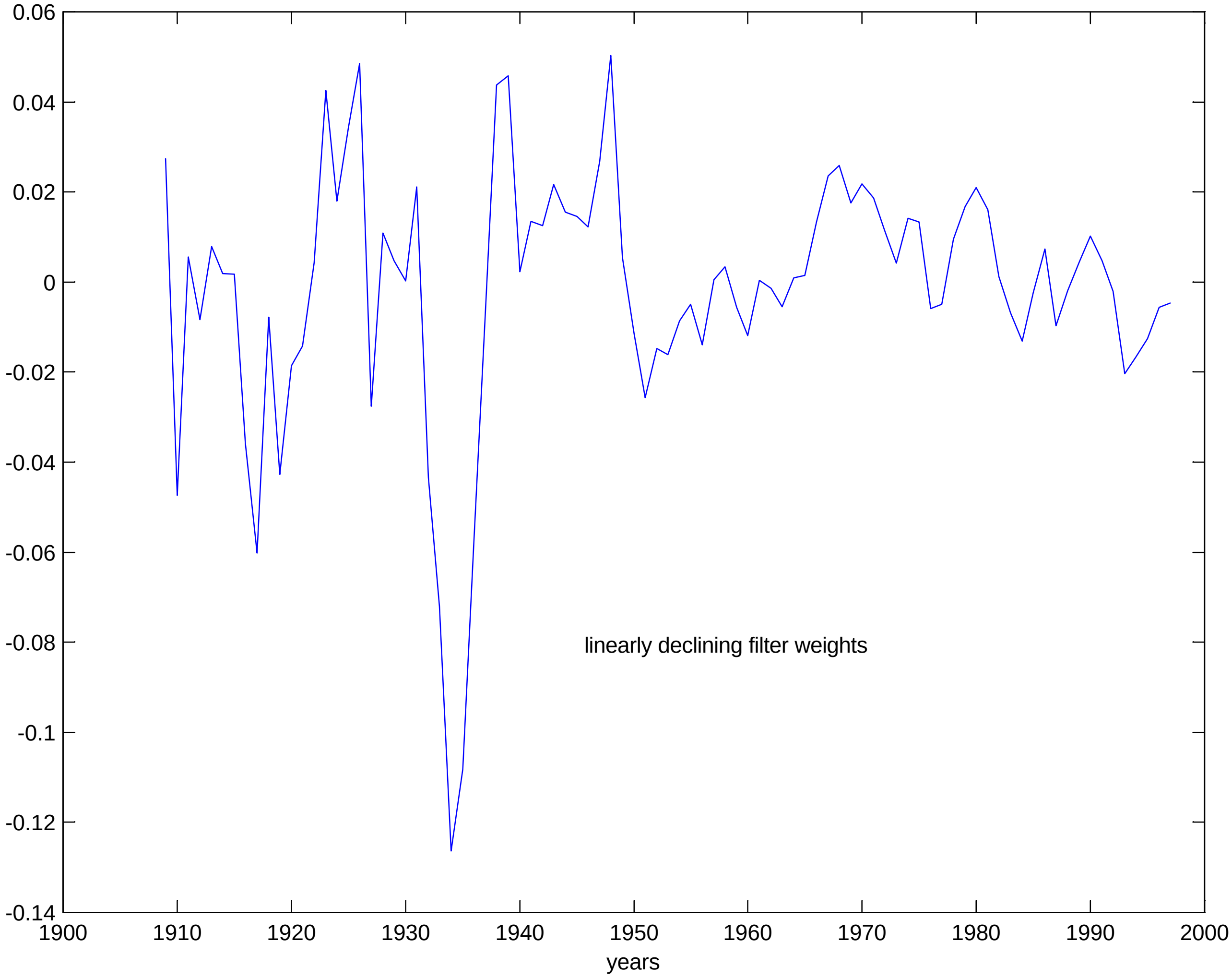


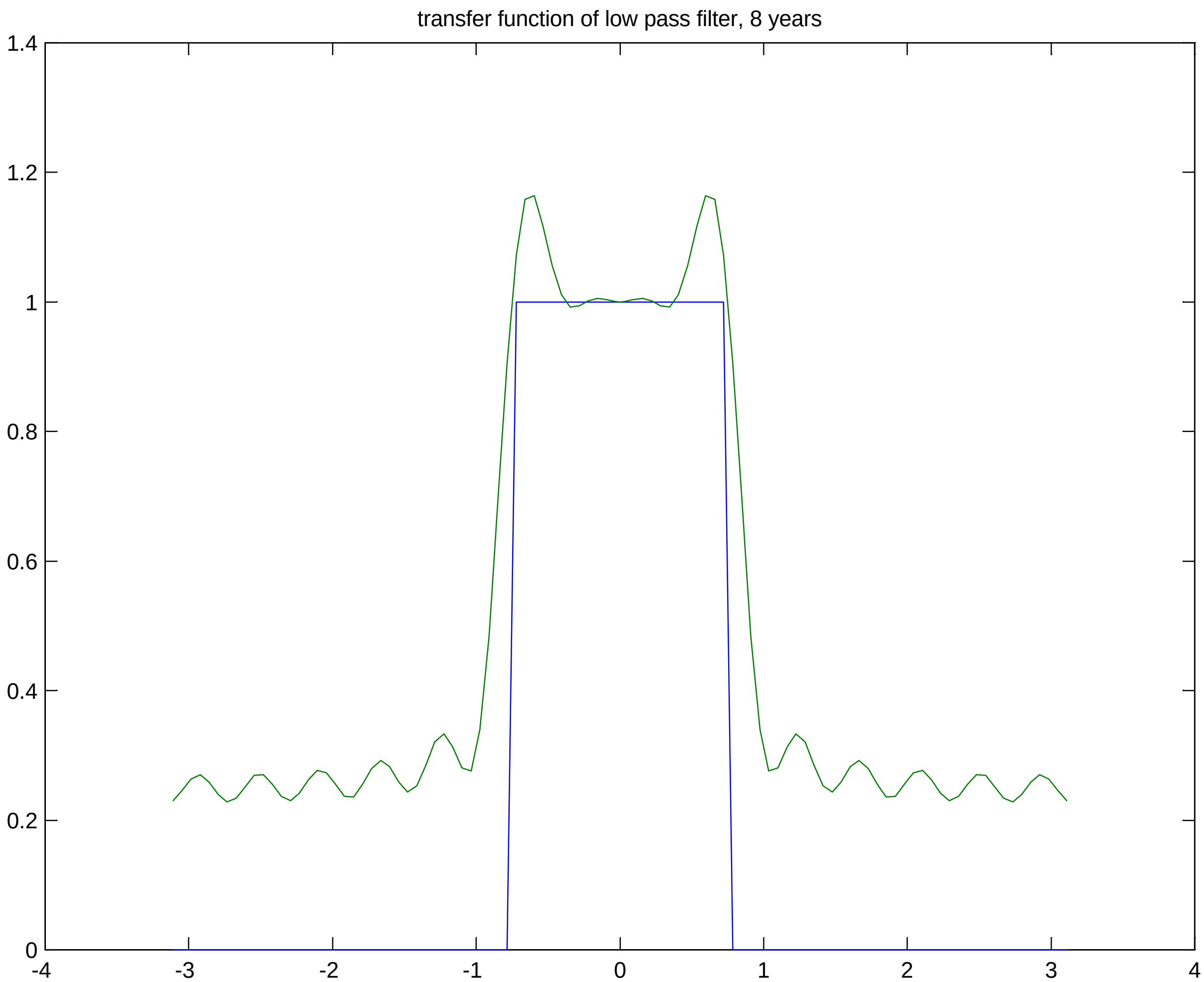
Nondurable and Services Consumption Deviations from trend
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
years
16 years low pass deviations


Nondurable and Services Consumption Deviations from trend
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
years
20 years low pass filter deviations


Nondurable and Services Consumption Deviations from trend
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
years
deviations from filter
w/geometric weights

Nondurable and Services Consumption Deviations from trend





transfer function of low pass filter, 8 years
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-4
-3
-2
-1
0
1
2
3
4

Table 1

Marginal Cost of Dividend Uncertainty

	1889-1997	1926-1997
Dividend-Price ratio	4.57%	4.36%
Longterm yield	2.05%	2.13%
Consumption trend growth	1.65%	1.80%
Dividend trend growth	1.29%	1.27%
((1+g)*dp) / (r - g)	6.1355	5.1219
Marginal Cost of Dividend Uncertainty	513.55%	412.19%
Std (Δ log C)	3.26%	2.46%
Std (Δ log D)	11.85%	11.33%
Corr(Δ log C,Δ log D)	0.32	0.47

See the Appendix for data sources

Table 2: 1889-1997

Marginal Cost of Consumption Fluctuations

Moments the kernel fits:	Coefficients on states: c	d	trm	d/p	vwr	β	Marginal cost: Low pass filters 8 years 15 lags	12years 15 lags	16 years 15 lags	20 years 15 lags	Other Filters infinite	geometric 5lags	linear 5lags	Benefits of 1% growth	Duration Consumptio Equity
1. E(D/P),E(Rd/Rf),Y(1),Y(13)	-9.0793	-1.5881	8.3099			0.898	0.42%	0.78%	0.88%	1.05%	341.72%	0.99%	1.83%	230.56%	70.47
2. E(D/P),E(VWR-Rf),Y(1),Y(13),	-9.3054	-1.5622	8.053			0.8969	0.43%	0.79%	0.89%	1.06%	353.26%	1.01%	1.85%	227.60%	70.19
3. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13),	-9.2107	-1.5793	8.192			0.897	0.43%	0.79%	0.89%	1.05%	347.07%	0.94%	1.73%	227.26%	70.16
4. E(D/P),E(VWR-Rf),Y(1),Y(13),	-4.0357	-2.3745		9.6365		0.9105	0.32%	0.60%	0.67%	0.69%	543.28%	0.72%	1.41%	1560.88%	98.90
5. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13),	-0.1929	-3.3088		10.0949		0.8951	0.24%	0.48%	0.54%	0.64%	604.27%	0.61%	1.15%	780.47%	115.55
6. E(D/P),E(Rd/Rf),Y(1),Y(13),	-4.3102 (not exact fit)	-2.5682		10.22		0.9008	0.35%	0.65%	0.73%	0.86%	1956.35%	0.82%	1.52%	4141.52%	109.09
7. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf)	-9.601	-1.5264	8.1928	-0.4905		0.8954	0.43%	0.80%	0.90%	1.07%	362.85%	1.02%	1.88%	221.43%	69.59
8. E(D/P),E(VWR-Rf),Y(1),Y(13),	-6.5264	-1.9255			-0.5325	0.9046	0.40%	0.73%	0.80%	0.93%	2862.13%	0.93%	1.68%	1344.64%	97.20
9. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13),	0.0624	-3.2975			-0.6974	0.8915	0.26%	0.49%	0.54%	0.63%	1263.46%	0.63%	1.16%	16561.54%	123.50
10. E(D/P),E(Rd/Rf),Y(1),Y(13),	-1.0029 (not exact fit)	-2.8789			-0.6934	0.9066	0.27%	0.51%	0.55%	0.64%	6151.07%	0.64%	1.19%	154985.17%	151.48
11. E(D/P),E(VWR-Rf),Y(1),Y(13),	-7.1807	-1.9775			-0.4007	0.9033	0.40%	0.71%	0.79%	0.93%	502.50%	0.90%	1.65%	571.11%	86.50
12. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13),	-0.6866	-3.238			-0.6284	0.8937	0.26%	0.49%	0.55%	0.65%	480.22%	0.63%	1.16%	8664.40%	116.65
13. E(D/P),E(Rd/Rf),Y(1),Y(13),	no fit														
14. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf)	0.0583	-2.1574		44.7437	1.657	0.9298	0.20%	0.43%	0.54%	0.68%	39.70%	0.49%	0.95%	109.37%	52.82
15. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf) (almost exact fit, less .2 basis point dif)	-13.4406	-1.0353	8.6103		0.4063	0.8761	0.50%	0.92%	1.06%	1.27%	206.03%	1.18%	2.17%	110.82%	53.17
Standard Deviation of Deviations from trend							1.35%	1.68%	2.02%	2.36%		1.74%	2.85%		

Moments
Dividend/price ratio: E(D/P) = 0.046
Equity premium: E(VWR-Rf) = 0.0677
Multiplicative equity premium: E(Rd/Rf-1) = 0.0698
Riskless rate: E(logY(1))= 0.0108
Yield spread: E(logY(13)-logY(1))= 0.0059

States
c = consumption
d = dividends
trm=long-short term spread
d/p = dividend-price ratio
vwr = realized value weighted stock return

Forecasting variables
1.-7. *c,d,trm,d/p*
8.-10.,14. *c,d,dp,vwr*
11.-13,15. *c,d,trm,vwr*

Table 2 Postwar data 54-97

Marginal Cost of Consumption Fluctuations

Moments the kernel fits:	Coefficients on states: c	d	trm	d/p	vwr	β	Marginal cost: 8 years	12 years	16 years	20 years	inf	geometric	linear	Benefits of 1% growth	Duration Consumpti Equity
							15 lags	15 lags	15 lags	5lags	5lags	5lags			
1. E(D/P),E(Rd/Rf),Y(1),Y(13)	-4.037	-2.0323	19.5			0.9547	0.05%	0.11%	0.15%	0.18%	3.66%	0.12%	0.24%	134.66	57.97
no fit, differences up to 50 basis points															
2. E(D/P),E(VWR-Rf),Y(1),Y(13),	-28.6395	-2.8152	23.945			0.8854	0.22%	0.42%	0.51%	0.62%	22.33%	0.51%	0.96%	63.41	39.28
3. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13)	-19.304	6.4914	41.6183			0.8471	0.15%	0.31%	0.35%	0.41%	302.10%	0.37%	0.70%	3125313.07	211.65
4. E(D/P),E(VWR-Rf),Y(1),Y(13),	-10.9436	-4.4006		53.3536		0.9215	0.14%	0.22%	0.24%	0.26%	17.86%	0.24%	0.41%	2039.49	102.06
5. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13)	no fit														
6. E(D/P),E(Rd/Rf),Y(1),Y(13),	-15.7315	-4.2218		37.4627		0.9169	0.16%	0.23%	0.27%	0.32%	5.61%	0.26%	0.45%	260.40	72.71
47-97, no fit, difference up to 70 basis point															
7. E(D/P),E(Rd/Rf),Y(1),Y(13), E(V	-6.8371	-1.9575	8.8942	2.0249		0.9061									
no fit	-13.0528	-1.1323	8.4302	-5.5749		0.8785									
8. E(D/P),E(VWR-Rf),Y(1),Y(13),	-11.8515	-4.2794			-2.0197	0.8903	0.18%	0.34%	0.36%	0.41%	115570.55%	0.44%	0.81%	2908024.26	508.09
9. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13)	-36.5338	-5.9338			-2.1304	0.7512	0.32%	0.57%	0.60%	0.69%	1156.26%	0.72%	1.29%	25.30	103.28
(1947-1997)															
10. E(D/P),E(Rd/Rf),Y(1),Y(13),	-23.7008	-5.1979			-1.8821	0.8371	0.21%	0.34%	0.37%	0.42%	54.54%	0.43%	0.75%	234.30	70.94
(1947-1997)	(almost exact fit)														
11. E(D/P),E(VWR-Rf),Y(1),Y(13),	-8.6172	-4.9868			-2.0417	0.9161	0.14%	0.21%	0.24%	0.28%	18.32%	0.25%	0.43%	29653.41	129.94
12. E(Rd/Rf),E(VWR-Rf),Y(1),Y(1:	no fit														
(no fit)															
13. E(D/P),E(Rd/Rf),Y(1),Y(13),	no fit														
14. E(D/P),E(Rd/Rf),Y(1),Y(13), E	-40.5724	5.403		94.4704	1.1613	0.7091	0.42%	0.73%	0.90%	1.01%	35.57%	0.97%	1.79%	31.49	24.44
15. E(D/P),E(Rd/Rf),Y(1),Y(13), E	-10.2481	-1.4752	10.2485		0.1942	0.8952									
(no fit)															

Moments *Dividend/price ratio: E(D/P) = 0.046*
Equity premium: E(VWR-Rf) = 0.0677
Multiplicative equity premium: E(Rd/Rf-1) = 0.0698
Riskless rate: E(logY(1))= 0.0108
Yield spread: E(logY(13)-logY(1))= 0.0059

States *c = consumption*
d = dividends
trm=long-short term spread
d/p = dividend-price ratio
vwr = realized value weighted stock return

Forecasting variables		
	1.-7.	*c,d,trm,d/p*
	8.-10.,14.	*c,d,dp,vwr*
	11.-13,15	*c,d,trm,vwr*

Table 2: 1889-1997

Marginal Cost of Consumption Fluctuations: Dividends and Consumption coingegrated

Moments the kernel fits:	Coefficients on states:							Marginal cost:				
								Low pass filters				
	c	d	trm	d/p	vwr	β		8 years	12years	16 years	20 years	infinite
								15 lags	15 lags	15 lags	15 lags	
1. E(D/P),E(Rd/Rf),Y(1),Y(13)	-13.0164	-0.8674	-8.2449			0.8975	o exact fit, but very clos	0.39%	0.69%	0.81%	0.99%	52.66%
2. E(D/P),E(VWR-Rf),Y(1),Y(13),	-12.9944	-1.4702	-8.0348			0.8898	no exact fit	0.39%	0.71%	0.84%	1.03%	49.10%
3. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13),	-13.0858	-0.7048	-3.2104			0.9046	no exact fit	0.41%	0.72%	0.84%	1.02%	81.39%
4. E(D/P),E(VWR-Rf),Y(1),Y(13),	-18.0239	-3.6034		-33.6666		0.8193	exact fit	0.37%	0.81%	0.98%	1.25%	181.17%
5. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13),	-15.7198	-0.5684		-7.2459		0.8841	no exact fit	0.45%	0.83%	0.96%	1.16%	157.04%
6. E(D/P),E(Rd/Rf),Y(1),Y(13),	-18.2968	-0.1533		-10.5133		0.8533	o exact fit, but very clos	0.52%	0.92%	1.08%	1.32%	89.07%
7. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf)	-16.5583	-0.4314	-3.4491	-4.617		0.8676	o exact fit, but very clos	0.49%	0.87%	1.02%	1.24%	70.41%
8. E(D/P),E(VWR-Rf),Y(1),Y(13),	-10.2308	-3.6605			1.1134	0.8856	exact fit	0.23%	0.56%	0.71%	0.93%	51.93%
9. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13),	-31.4179	1.7286			1.3809	0.6863	exact fit	0.91%	1.69%	1.90%	2.26%	infinite
10. E(D/P),E(Rd/Rf),Y(1),Y(13),							no fit					
11. E(D/P),E(VWR-Rf),Y(1),Y(13),	-23.1291	-2.8637			2.9842	0.7604	no exact fit	0.45%	1.06%	1.24%	1.52%	8.99E+06
12. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13),	-18.5015	-0.4133			0.8474	0.8585	no exact fit	0.52%	0.98%	1.10%	1.31%	2.21E+05
13. E(D/P),E(Rd/Rf),Y(1),Y(13),	no fit											
14. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf)	-9.0132	1.7678		80.1456	3.1688	0.916	exact fit	0.44%	0.76%	0.95%	1.16%	16.47%
15. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf)	-18.1404	-0.3375	-0.544		0.6212	0.8509	no exact fit	0.53%	0.95%	1.10%	1.32%	115.35%
Standard Deviation of Deviations from trend								1.35%	1.68%	2.02%	2.36%	

Moments
Dividend/price ratio: E(D/P) = 0.046
Equity premium: E(VWR-Rf) = 0.0677
Multiplicative equity premium: E(Rd/Rf-1) = 0.0698
Riskless rate: E(logY(1))= 0.0108
Yield spread: E(logY(13)-logY(1))= 0.0059

States
c = consumption
d = dividends
trm=long-short term spread
d/p = dividend-price ratio
vwr = realized value weighted stock return

Table 3: Random Walk Case: Consumption Equity Premium and Marginal Cost

	E(R/Rf)	State (Δs)	Loading (l_m)	Consumption Equity Premium	Marginal Cost Frequency domain filters 8 years	12 years	16 years		
1954-1997								std(Δc)	0.011
	Mkt	Δc	-83.9	1.09%	0.39%	0.85%	1.21%	std(Mkt)	0.169
	Mkt	Mkt	-2.6	0.22%	0.08%	0.17%	0.25%	corr(Δc,Mkt)	0.453
	10 CRSP size			0.37%	0.13%	0.29%	0.41%	E(Mkt-rf)	7.30%
	Dec.: 1,10	Δc, Mkt		0.31%	0.11%	0.24%	0.34%		
1926-1997								std(Δc)	0.025
	Mkt	Δc	-26.3	1.59%	0.57%	1.24%	1.77%	std(Mkt)	0.197
	Mkt	Mkt	-2.1	0.63%	0.22%	0.49%	0.70%	corr(Δc,Mkt)	0.628
	10 CRSP size			0.64%	0.23%	0.50%	0.71%	E(Mkt-rf)	8.01%
	Dec.: 1,10	Δc, Mkt		1.66%	0.59%	1.30%	1.85%		