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ABSTRACT

We exposit the link between money, velocity and prices in an inventory-theoretic model of the demand for money and explore the extent to which such a model can account for the short-run volatility of velocity, the negative correlation of velocity and the ratio of money to consumption, and the resulting "stickiness" of the aggregate price level relative to a benchmark model with constant velocity. We find that an inventory-theoretic model of the demand for money is a natural framework for understanding these aspects of the dynamics of money, velocity and prices in the short run.

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1. Introduction

In this paper, we examine the dynamics of money, velocity, and prices in an inventory-theoretic model of the demand for money. Data on the price level, the nominal money stock and real consumption are linked by an exchange equation of the form $P = vM/c$, where P is the price level, v is the velocity of money, and M/c is the ratio of the stock of money to real consumption expenditure. The long run behavior of these series differs markedly from their short run behavior. For example, over the last 40 years in the United States, the price level P has grown roughly in parallel with the ratio of a broad measure of money to consumption, while the velocity of money has moved very little in comparison to these other series. In contrast, during this same time period, the short run fluctuations of the ratio of money to consumption are strongly negatively correlated with the short run fluctuations in velocity. As a consequence of this negative correlation of money and velocity in the short-run, fluctuations in the price level P are not that highly correlated with fluctuations in M/c. We show that a simple inventory-theoretic model of money demand can account, at least qualitatively, for both the stability of velocity in the long run and the strong negative correlation of money and velocity in the short run. In this model, an exogenous increase in the money supply leads to an endogenous decline in the velocity of money, and as a result, the price level responds less than one-for-one to the change in the money supply. Hence, in comparison to a benchmark model in which the velocity of money is constant, prices in this model are sticky. We argue that this model of money demand offers a novel explanation for the short-run sluggishness of prices.

In Figure 1, we illustrate the long-run behavior of money, velocity, and prices. There we plot the log of the ratio of M2 to personal consumption expenditure for M/c, the log of the personal consumption expenditure deflator for P, and the log of the implied consumption velocity of M2 observed in monthly data from the United States over the last 40 years. In that figure, we see that that the price level P has risen substantially along with the ratio M/c while velocity has remained relatively stable. In Figure 2, we illustrate the short-run behavior of money and velocity. There we plot the deviations of M/c and v, measured as above, from their HP filtered trends. As is readily apparent, these two series are strongly negatively correlated.[1] In Figure 3, we plot the deviations of M/c and P from their HP-filtered trends. As one might expect given the strong negative correlation of money and velocity observed in Figure 2, the short-run fluctuations in prices are not that closely linked to the short-run fluctuations in M/c.

Our model is inspired by the analyses of money demand developed by Baumol (1952) and Tobin (1956). In their models, agents carry money (despite the fact that money is dominated in rate of return by interest bearing assets) because they face a fixed cost of trading money and these other assets. Our model is a simplified version of their framework. We study a cash-in-advance model in which the asset market and the goods market are physically separated. Households in the model have two financial accounts: a brokerage account in the asset market in which they hold a portfolio of interest bearing assets and a bank account in the goods market in which they hold money to pay for consumption. We modify this standard cash-in-advance model by assuming that households do not have the opportunity to exchange funds between their brokerage and bank accounts every period. Instead, we assume that they have the opportunity to transfer funds between these accounts only once every N periods.[2] Hence, households maintain an inventory of money in their bank account large enough to pay for consumption expenditures for several periods. They replenish this inventory with a transfer of funds from their brokerage account once every N periods. As households manage this inventory of money optimally, their money holdings follow a sawtooth pattern — rising rapidly with each periodic transfer of funds from their brokerage account and then falling slowly as these funds are spent smoothly over time — similar to the sawtooth pattern of money holdings derived by Baumol (1952) and Tobin (1956).

In this model, the velocity of money fluctuates around a steady-state value that is determined by the parameter N governing the frequency with which households have the opportunity to transfer funds between their brokerage and bank accounts. Hence, in our model, in the long-run, as the stock of money grows relative to consumption the price level grows by roughly the same amount and velocity remains relatively constant.

[1] In applying the HP-filter, we used a parameter of $3^2 * 1600 = 14400$ for monthly data. One obtains similar results using 12 month differenced data as opposed to HP filtered data to characterize the short-run fluctuations in money and velocity.

[2] Grossman and Weiss (1983) and Rotemberg (1984) solve similar models with $N = 2$.

Our inventory-theoretic model of money demand also has implications for the short run; it provides a natural accounting for the negative correlation of fluctuations in the ratio of money to consumption and velocity and hence for the corresponding sluggishness of prices. These short run implications of the model can be understood in two steps. First, consider how aggregate velocity is determined in this inventory-theoretic model of money demand. Households at different points in the cycle of depleting and replenishing their inventories of money in their bank accounts have different propensities to spend the money that they have on hand, or, equivalently, different individual velocities of money. Those households that have recently transferred funds from their brokerage account to their bank account will have a large stock of money in their bank account and will tend to spend this stock of money slowly to spread their spending smoothly over the interval of time that remains before they next have the opportunity to replenish their bank account. Hence, these households will have a relatively low individual velocity of money. In contrast, those households that have not had the opportunity to transfer funds from their brokerage account in the recent past and anticipate having the opportunity to make such a transfer soon will tend to spend the money that they have in the bank at a relatively rapid rate, and thus have a relatively high individual velocity of money. Aggregate velocity at any point in time is determined by the weighted average of these individual velocities of money of all of the households in the economy, with the weights determined by the distribution of money holdings across households.

Now consider the effects on aggregate velocity of an increase in the money supply brought about by an open market operation that occurs in some period t. In this open market operation, the government trades newly created money for interest bearing securities and households, on the opposite side of the transaction, trade interest bearing securities held in their brokerage accounts for newly created money. In any period in which the nominal interest rate is positive, this new money is purchased only by those households that currently have the opportunity to transfer funds from their brokerage account to their bank account since these are the only households that currently have the opportunity to begin spending this money. All other households choose not to participate in the open market operation since these households would have to leave this money sitting idle in their brokerage accounts where it would be dominated in rate of return by interest bearing securities. Hence, as a result of this open market operation, the fraction of the money stock held by those households currently able to transfer resources from their brokerage account to their bank account rises. Since these households have a lower-than-average propensity to spend this money, aggregate velocity falls. In this way, an exogenous increase in the supply of money leads to an endogenous reduction in the aggregate velocity of money and hence, a diminished, or sluggish, response of the price level.

We show that the response of velocity and the price level in our model to a one percent increase in the money supply depends on the frequency with which households replenish their bank accounts with transfers of funds from their brokerage accounts in the asset market as determined by the parameter N. With $N = 1$, as in a standard cash-in-advance model in which households can reallocate their wealth between interest bearing assets in their brokerage accounts in the asset market and money in their bank accounts in the goods market every period, a one percent increase in the money supply relative to the aggregate endowment has no impact on the velocity of money and, hence, leads immediately to a one percent increase in the price level. As N grows, the impact of a one percent increase in the money supply on velocity rapidly approaches $-1/2$ percent, so, on impact, this increase in the money supply leads to an increase in the price level of only $1/2$ percent. This sluggish response of the price level is persistent because it takes time for the original one percent increase in the stock of money to work its way through households' inventories. Specifically, for large N, velocity remains below its steady-state level, and hence prices adjust less than one percent, for roughly $N \log(2)$ time periods. After that time, velocity and prices overshoot their steady-state levels before converging in a series of dampened oscillations.

The parameter N governing the frequency with which households replenish their bank accounts also determines our model's implications for aggregate velocity in the steady-state — since this parameter determines the size of the inventory of money that households must hold to purchase their consumption. Thus, the empirical implications of our model for the sluggishness of prices are determined to a large extent by one's definition of money (since that definition determines one's measure of velocity and hence one's choice of N). In our simple model, defining money comes down to answering the question: What assets correspond to those that households in the model hold in their bank accounts and what assets do households hold and trade less frequently in their brokerage accounts?

We examine the empirical implications of our model using a broad measure of money: U.S. households' holdings of currency, demand deposits, savings deposits, and time deposits. In the data, U.S. households hold a large stock of such accounts, roughly 1/2 to 2/3 of annual personal consumption expenditure. They pay a large opportunity cost in terms of forgone interest to hold such accounts — on the order of 150-200 basis points. We choose to aggregate demand, savings, and time deposits because the opportunity cost to households does not appear to vary systematically across these three different types of accounts. To parameterize our model to match the observed ratio of US households' holdings of this broad measure of money relative to their personal consumption expenditure, we assume that households in our model transfer funds between their brokerage and bank accounts very infrequently — on the order of one every one and a half to three years. We argue that this assumption is not inconsistent with evidence summarized by Vissing-Jorgensen (2002) regarding the frequency with which US households trade assets held in their brokerage accounts.

We conduct two quantitative exercises with our model. In the first, we feed into the model the paths for the stock of M2 and for aggregate consumption observed in the U.S. economy in monthly data over the past 40 years and examine the model's predictions for velocity and the price level in the long and the short-run. In terms of its long-run implications, the model is similar to the data in that velocity is relatively stable and the price level increases in line with the growth of M2 relative to consumption. In terms of its short-run implications, the model produces fluctuations in velocity that have a surprisingly high correlation with the fluctuations in velocity observed in the data. This result stands in sharp contrast to the implications of a standard cash-in-advance model (this model with $N = 1$). In such a model, aggregate velocity is constant regardless of the pattern of money growth. We also find that the short run fluctuations in velocity in our model are not as large as those in the data. From the finding that the short run fluctuations in velocity in our model are highly correlated with those observed in the data, we conclude that a substantial portion of the unconditional negative correlation of the ratio of money to consumption and velocity might reasonably be attributed to the response of velocity to exogenous movements in money. From the finding that the short run fluctuations in velocity in our model are not as large as those in the data, however, we conclude that there may be other shocks to the demand for money which we have not modelled here. If this were the case, one would not expect this model to account for all of the variability of velocity observed in the data.

With this possibility in mind, in our second exercise, we consider the response of money, prices, and velocity in our model to an exogenous shock to monetary policy, modelled here as an exogenous, persistent shock to the short-term nominal interest rate similar to that estimated in the vector autoregression (VAR) literature as the response of the Federal Funds rate to a shock to monetary policy. Here we find that the corresponding paths for money and the price level are quite similar to the estimated responses of these variables in this VAR literature. With the increase in interest rates, the money stock initially declines for some time and the price level shows little or no response for a year or more. In this exercise, aggregate output is held constant by assumption and, hence, the sluggish response of the price level to this monetary policy shock is entirely due to the dynamics of money demand. We interpret this finding as a call for further work to identify the extent to which the sluggish response of prices to monetary policy shocks found in the VAR literature is a result of the dynamics of money demand or more conventional sources of price stickiness.

Grossman and Weiss (1983) and Rotemberg (1984) were the first to point out that open market operations could have effects on real interest rates and a delayed impact on the price level in inventory-theoretic models of money demand. The models that they present are similar to this model when the parameter $N = 2$. Those authors examine the impact of a surprise money injection on interest rates and prices in the context of otherwise deterministic models. Alvarez and Atkeson (1997) study the effects of open market operations on real interest rates and real exchange rates in a fully stochastic inventory-theoretic model of money demand. In that model, it is assumed that households have logarithmic utility and a constant probability of being able to transfer money between the asset market and the goods market. As a result of these assumptions, the individual velocity of money is constant across households and hence the aggregate velocity of money is constant both in the long and the short run. In the inventory-theoretic model of money demand that we present here, open market operations have effects on the real interest rate that are qualitatively similar to those in Alvarez and Atkeson (1997). We focus here on the implications of this model for the response of velocity and prices to money injections.

2. An inventory-theoretic model of money demand

Consider a cash-in-advance economy in which the asset market and the goods market are in physically separate locations. Time is discrete and denoted $t = 0, 1, 2, \ldots$. Agents in this economy are organized into households each comprised of a worker and a shopper. There are measure one households. We assume that each household has access to two financial intermediaries: one that manages its portfolio of assets and another that manages its money held in a transactions account in the goods market. We refer to the household's account with the financial intermediary in the asset market as its *brokerage account* and its account with the financial intermediary in the goods market as its *bank account*. There is a government that injects money into the asset market in this economy via open market operations. Households that participate in the open market operation purchase this money with assets held in their brokerage accounts. These households must transfer this money to their bank account before they can spend it on consumption.

The exogenous shocks in this economy are shocks to the money growth rate μ_t and shocks to the endowment of each household y_t. Since all households receive the same endowment, y_t is also the aggregate endowment of goods in the economy. Let $h_t = (\mu_t, y_t)$ denote the realized shocks in the current period. The history of shocks is denoted $h^t = (h_0, h_1, \ldots, h_t)$. From the perspective of time zero, the probability distribution over histories h^t has density $f_t(h^t)$.

As in a standard cash-in-advance model, each period is divided into two sub-periods. In the first sub-period, each household trades assets held in its brokerage account in the asset market. In the second sub-period, the shopper in each household purchases consumption in the goods market using money held in the household's bank account, while the worker sells the household's endowment in the goods market for money $P_t(h^t)y_t(h^t)$ where $P_t(h^t)$ denotes the price level in the current period. In the next period, a fraction $\gamma \in [0, 1]$ of the worker's earnings is deposited in the household's bank account in the goods market while the remaining $1 - \gamma$ of these earnings are deposited in the household's brokerage account in the asset market. We interpret γ as the fraction of total income that agents receive regularly deposited into their transactions accounts or as currency and we refer to γ as the *paycheck parameter* and to $\gamma P_{t-1}(h^{t-1})y_{t-1}(h^{t-1})$ as the household's *paycheck*. We interpret $(1-\gamma)$ as the fraction of total income that agents receive in the form of interest and dividends paid on assets held in their brokerage accounts.

Unlike a standard cash-in-advance model, we do not assume that households have the opportunity to transfer money between the asset market and the goods market every period. Instead, we assume that each household has the opportunity to transfer money between its brokerage account and its bank account only once every N periods. In other periods, a household can trade assets in it brokerage account and use money in its bank account to purchase goods, it simply cannot move money between these two accounts. We refer to those households that currently have the opportunity to transfer money between their brokerage and bank accounts as *active households* and those households that are currently unable to transfer money between these accounts as *inactive households*.

We assume that each period a fraction $1/N$ of the households are active. Each period, we index each household by the number time periods since it was last active, here denoted by $s = 0, 1, ..., N-1$. A household of type $s < N-1$ in the current period will be type $s+1$ in the next period. A household of type $s = N-1$ in the current period will be type $s = 0$ in the next period. Hence a household of type $s = 0$ is active in this period, a household of type $s = 1$ was active last period, and a household $s = N-1$ will be active next period.

In period 0, each household has an initial type s_0, with fraction $1/N$ of the households of each type $s_0 = 0, 1, ..., N-1$. Let $S(t, s_0)$ denote the type in period t of a household that was initially of type s_0. For all s_0, $S(0, s_0) = s_0$. For all periods t and s_0 such that $S(t, s_0) = 0, 1, ..., N-2$, in period $t+1$, $S(t+1, s_0) = S(t, s_0) + 1$. For the s_0 such that $S(t, s_0) = N-1$ in period t, $S(t+1, s_0) = 0$.

The households of type $s > 0$ are inactive in the current period. For an inactive household of type s, the quantity of money that it has on hand in its bank account at the beginning of goods market trading in the current period is denoted $M_t(s)$. The shopper in this household spends some of this money on goods, $P_t c_t(s)$, and the household carries the unspent balance in its bank account into next period, $Z_t(s) \geq 0$. The balance that this household has at the beginning of the period is equal to the quantity of money that it held over in its bank account last period $Z_{t-1}(s-1)$ plus its paycheck $\gamma P_{t-1}y_{t-1}$. Thus, the the evolution of money holdings and consumption for these households is given by:

$$M_t(s,h^t) = Z_{t-1}(s-1,h^{t-1}) + \gamma P_{t-1}(h^{t-1})y_{t-1}(h^{t-1}), \quad (1)$$

$$M_t(s,h^t) \geq P_t(h^t)c_t(s,h^t) + Z_t(s,h^t) \quad (2)$$

When a household is active, and hence of type $s=0$, it chooses a transfer of money $P_t x_t$ from its brokerage account in the asset market into its bank account in the goods market. Hence, the money holdings and consumption of active households satisfy:

$$M_t(0,h^t) = Z_{t-1}(N-1,h^{t-1}) + \gamma P_{t-1}(h^{t-1})y_{t-1}(h^{t-1}) + P_t(h^t)x_t(h^t), \quad (3)$$

$$M_t(0,h^t) \geq P_t(h^t)c_t(0,h^t) + Z_t(0,h^t). \quad (4)$$

In addition to the constraints on the household's bank account, equations (1)-(4) above, the household also faces a sequence of constraints on its brokerage account. We assume that in each period t, the household can trade in a complete set of one-period state contingent bonds, each of which pays one dollar into the household's brokerage account next period if the relevant contingency is realized. Let $B_{t-1}(s-1,h^t)$ denote the stock of bonds held by inactive households of type $s \geq 1$ at the beginning of period t following history h^t and $B_t(s,h^t,h')$ denote the stock of bonds purchased by that household that will pay off next period if history $h^{t+1} = (h^t,h')$ is realized next period. Let $A_t(s,h^t) \geq 0$ denote money held by the household in its brokerage account at the end of period t. Since an inactive household cannot transfer money between its brokerage account and its bank account, this household's bond and money holdings in its brokerage account must satisfy:

$$B_{t-1}(s-1,h^t) + A_{t-1}(s-1,h^{t-1}) + (1-\gamma)P_{t-1}(h^{t-1})y_{t-1}(h^{t-1}) - P_t(h^t)\tau_t(h^t) \quad (5)$$

$$\geq \int q_t(h^t,h')B_t(s,h^t,h')dh' + A_t(s,h^t)$$

where $q_t(h^t,h')$ is the price in period t given history h^t of a bond that will pay one dollar in period $t+1$ if shock h' is realized and $P_t(h^t)\tau_t(h^t)$ are nominal lump-sum taxes. We assume that each household's real bondholdings must remain within arbitrarily large bounds. The analogous constraint for active households is

$$B_{t-1}(N-1,h^t)+A_{t-1}(N-1,h^{t-1})+(1-\gamma)P_{t-1}(h^{t-1})y_{t-1}(h^{t-1})-P_t(h^t)\tau_t(h^t) \quad (6)$$

$$\geq \int q_t(h^t,h')B_t(0,h^t,h')dh'+P_t(h^t)x_t(h^t)+A_t(0,h^t),$$

where $P_t(h^t)x_t(h^t)$ is the transfer of money from brokerage to bank account chosen by the active households.

In the constraints (5) and (6) we have allowed each household the option of holding (noninterest bearing) money $A_t(s,h^t)\geq 0$ in their brokerage account in the asset market from period t to period $t+1$. Clearly, if nominal interest rates are always positive in equilibrium, no household would ever wish to do so since interest bearing bonds would dominate money in such equilibria. The nominal interest rate $i_t(h^t)$ is related to asset prices by:

$$\frac{1}{1+i_t(h^t)}=\int q_t(h^t,h')dh'. \quad (7)$$

At the beginning of period 0, all households of type $s_0\geq 1$ begin with balances $\bar{M}_0(s_0)$ in their bank accounts in the goods market. This quantity is the balance on the left side of (2) in period 0. For active households in period 0, the initial balance $\bar{M}_0(0)$ in (4) is composed of an initial given balance $\bar{Z}_0$ and a transfer P_0x_0 of their choosing. Each household of type s_0 also begins period 0 with initial balance $\bar{B}_0(s_0)$ in its brokerage account on the left side of constraints (5) and (6). The households initially have no money corresponding to A_{-1} in their brokerage accounts.

Let $B_t(h^t)$ be the total stock of government bonds in period t following history h^t. The government faces a sequence of budget constraints

$$B_{t-1}(h^t)=M_t(h^t)-M_{t-1}(h^{t-1})+P_t(h^t)\tau_t(h^t)+\int q_t(h^t,h')B_t(h^t,h')dh'$$

together with arbitrarily large bounds on the government's real bond issuance. We denote the government's policy for money injections as $\mu_t(h^t) = M_t(h^t)/M_{t-1}(h^{t-1})$. In period 0, the initial stock of government debt is $\bar{B}_0$ and $M_0 - M_{-1}$ is the initial monetary injection. This budget constraint implies that the government pays off its initial debt with a combination of lump-sum taxes and money injections achieved through open market operations.

For each date and state and taking as given the prices and aggregate variables, each household of type s_0 chooses transfers $x_t(h^t)$, cash and a bond portfolio to hold over in the asset market, $A_t(S(t, s_0), h^t)$ and $B_t(S(t, s_0), h^t, h')$, consumption , $c_t(S(t, s_0), h^t)$, and money holdings, $M_t(S(t, s_0), h^t)$ and $Z_t(S(t, s_0), h^t)$, to maximize expected utility:

$$\sum_{t=0}^{\infty} \beta^t \int u[c_t(S(t, s_0), h^t)] f_t(h^t) dh^t$$

subject to the constraints (1), (2), and (5) in those periods t in which $S(t, s_0) > 0$, and constraints (3), (2), and (6) in those periods t in which $S(t, s_0) = 0$.

An equilibrium of this economy is a collection of prices $\{q_t(h^t, h'), P_t(h^t)\}_{t=0}^{\infty}$, decision rules $\{c_t(s, h^t), x_t(h^t), A_t(s, h^t), B_t(s, h^t, h'), M_t(s, h^t), Z_t(s, h^t)\}_{t=0}^{\infty}$, and a government policy $\{\tau_t(h^t), \mu_t(h^t), B_t(h^t)\}_{t=0}^{\infty}$, such that the decision rules solve each household's problem when prices are taken as given and the goods market, the money market, and the bond market all clear for all t, h^t :

$$\frac{1}{N} \sum_{s=0}^{N-1} c_t(s, h^t) = y_t(h^t),$$

$$\frac{1}{N} \sum_{s=0}^{N-1} \left[M_t(s, h^t) + A_t(s, h^t) \right] = M_t(h^t), \text{ and}$$

$$\frac{1}{N} \sum_{s=0}^{N-1} B_{t-1}(s, h^t) = B_{t-1}(h^t).$$

To understand the determination of equilibrium asset prices, it is useful to examine the first order conditions of the household's problem. To do so, use (1) and (3) to substitute out for money holdings $M_t(s, h^t)$ in constraints (2) and (4). Let $\eta_t(S(t, s_0), h^t) \geq 0$ denote the Lagrange multipliers on the constraints (2) and (4) of household $s = S(t, s_0)$ at (t, h^t), and let $\lambda_t(S(t, s_0), h^t) \geq 0$ denote the Lagrange multipliers for the constraints (5) and (6).

Let $\delta_t^M(S(t,s_0),h^t) \geq 0$ denote the multipliers on the non-negativity constraints for money held in the bank, $Z_t(S(s_0,t),h^t)$, and let $\delta_t^A(S(t,s_0),h^t) \geq 0$ denote the multipliers on the non-negativity constraints for money held in the brokerage account, $A_t(S(t,s_0),h^t)$. The first order necessary conditions for the coalition's optimization problem include:

$$x_t(h^t): \qquad \eta_t(0,h^t) = \lambda_t(0,h^t) \tag{8}$$

$$c_t(s,h^t): \qquad \beta^t u'[c_t(s,h^t)] f_t(h^t) = P_t(h^t)\eta_t(s,h^t) \tag{9}$$

$$B_t(s,h^t,h'): \qquad \lambda_t(s,h^t) q_t(h^t,h') = \lambda_t(s+1,h^t,h') \tag{10}$$

$$Z_t(s,h^t): \qquad \delta_t^M(s,h^t) + \int \eta_{t+1}(s+1,h^t,h')dh' = \eta_t(s,h^t) \tag{11}$$

$$A_t(s,h^t): \qquad \delta_t^A(s,h^t) + \int \lambda_{t+1}(s+1,h^t,h')dh' = \lambda_t(s+1,h^t) \tag{12}$$

From (10), we have that the evolution over time of the marginal value of a dollar in a household's brokerage account (given by $\lambda_t(s,h^t)$) is the same for all households and is determined by bond prices $q_t(h^t,h')$. It is useful to define $Q_t(h^t) \equiv q_0(h^0,h_1)q(h^1,h_2)\ldots q(h^{t-1},h_t)$ as the price in period 0 of one dollar delivered in the asset market in period t following history h^t. From (10), we have that for all households

$$Q_t(h^t) = \frac{\lambda_t(S(t,s_0),h^t)}{\lambda_0(s_0,h^0)}.$$

From (8) and (9), we thus find that asset prices in period t are determined by the marginal utility of a dollar for the active households (of type s_0 such that $S(t,s_0)=0$):

$$Q_t(h^t) = \frac{1}{\lambda_0(s_0)}\beta^t \frac{u'[c_t(0,h^t)]}{P_t(h^t)} f_t(h^t). \tag{13}$$

State contingent bond prices are thus given by

$$q_t(h^t,h') = \frac{\lambda_0(s_0)}{\lambda_0(s_0+1)}\beta \frac{u'[c_{t+1}(0,h^t,h')]}{u'[c_t(0,h^t)]} \frac{P_t(h^t)}{P_{t+1}(h^t,h')} \frac{f_{t+1}(h^t,h')}{f_t(h^t)}. \tag{14}$$

In what follows, we will examine equilibria in which the initial endowments of bonds $\bar{B}_0(s_0)$ are such that $\lambda_0(s_0)$ is equal across all households. In such equilibria, the initial Lagrange multipliers $\lambda_0(s_0)$ drop out of all asset pricing formulas.

To this point, we have made explicit reference to uncertainty in the notation so as to give a clear characterization of state contingent asset prices. Having done this, we will suppress reference to histories h^t for the remainder of the paper to conserve on notation.

From (11), we obtain that for all inactive shoppers who hold money, $Z_t(s) > 0$, we have the familiar stochastic Euler equation for an agent who can save only with money:

$$1 = E_t \left\{ \beta \frac{u'[c_{t+1}(s+1)]}{u'[c_t(s)]} \frac{P_t}{P_{t+1}} \right\} \tag{15}$$

together with our pricing formula for bonds (14), which can be written as

$$\frac{1}{1+i_t} = E_t \left\{ \beta \frac{u'[c_{t+1}(0)]}{u'[c_t(0)]} \frac{P_t}{P_{t+1}} \right\}.$$

The asset pricing implications of our model are closely related to those obtained by Grossman and Weiss (1983), Rotemberg (1984), and Alvarez and Atkeson (1997). In particular, our model has predictions for the effects of money injections on real interest rates and real exchange rates arising from the segmentation of the asset market related to the predictions in those papers and those in Alvarez, Atkeson, and Kehoe (2002) and Alvarez, Lucas, and Weber (2002). We do not develop these implications here.

In what follows, we focus on equilibria in which two conditions are satisfied. The first condition is that the nominal interest rate i_t is positive in all periods t. This condition implies that households will hold not money in their brokerage accounts ($A_t(s) = 0$ for all t) since, in this case, interest bearing bonds always dominate money in the asset market. The second condition is that, for all periods t,

$$1 > E_t \left\{ \beta \frac{u'[c_{t+1}(0)]}{u'[c_t(N-1)]} \frac{P_t}{P_{t+1}} \right\}. \tag{16}$$

Since a shopper of type $s = N-1$ at date t become a shopper of types $s = 0$ at date $t+1$, from the first order conditions above, such a shopper will not hold money in the bank ($Z_t(N-1) = 0$) if this condition is satisfied.[3] In the remainder of the paper, we solve the

[3] More generally, however, there may exist situations where shoppers of type $s < N-1$ cease to hold

model under the assumption that $A_t(s) = 0$ and $Z_t(N-1) = 0$ in all periods t. After solving the model under these assumptions, one can use (14) and (7) to check that the implied interest rates are positive and check that (16) is satisfied.

3. How the model works

In this section, we first discuss the typical pattern of households' money holdings in this inventory-theoretic model of money demand. We then observe that one can analytically solve for the dynamic, stochastic equilibrium of our model in the case in which agents have utility $u(c) = \log(c)$ and the paycheck parameter is $\gamma = 0$. We use this special case to develop intuition for the dynamic relationship between money, velocity, and prices implied by the model in the long and the short run. In subsequent sections we approximate the solution to other parameterizations of the model numerically to study the quantitative implications of the model.

In our model, agents periodically withdraw money from the asset market and they spend that money only slowly in the goods market to ensure that it lasts until they have another opportunity to withdraw money from the asset market. As a result, households' equilibrium paths for money holdings have the familiar *saw-toothed* shape that is characteristic of inventory-theoretic models of money demand — declining steadily over time before jumping up once again when the next transfer of money from the asset market is in hand. In Figure 4, we illustrate this saw-tooth pattern in the steady-state path of real balances for an individual household.

This saw-toothed pattern of households' money holdings plays a key role in shaping our model's implications for the dynamics of money, velocity, and prices. This role can be seen most clearly in a specification of our model in which agents have utility $u(c) = \log(c)$ and the paycheck parameter $\gamma = 0$. In this specification of the model, households of type s spend a constant fraction $v(s)$ of their current money holdings and carry over the remaining

money in the bank from one period to another. The paycheck that shoppers receive makes it possible for them to consume even if they are not storing money; this may be optimal behavior if the return on money is sufficiently low.

fraction $(1 - v(s))$ into the next period.[4] Hence, each period

$$Z_t(s) = (1 - v(s))M_t(s) \text{ and} \tag{17}$$
$$P_t c_t(s) = v(s)M_t(s),$$

with

$$v(s) \equiv \frac{1-\beta}{1-\beta^{N-s}}. \tag{18}$$

We refer to the fraction $v(s)$ as the *individual velocity of money* for a household of type s. These individual velocities can be interpreted as average propensities to consume out of money holdings. Observe that these individual velocities $v(s)$ converge to $1/(N - s)$ as β approaches one and thus, in this limit, approach the individual velocities obtained if one simply assumed directly that households maintain constant nominal expenditure while inactive. As β/μ approaches one (where μ is the steady-state rate of money growth), the households' saw-toothed pattern of money holdings in steady-state is easy to compute — the money holdings in period $t + s$ of a household that was active in period t are given by

$$M_{t+s}(s) = M_t(0)\frac{N-s}{N}.$$

Given that the individual velocities of money for households of types s are constant over time in this specification of our model, aggregate velocity at any point in time is simply a function of the distribution of money holdings across these households with different individual velocities of money. To see this, recall that if the nominal interest rate is positive, so that households do not hold any money in the asset market ($A_t(s) = 0$), then money market

[4]The intuition that we present in this section is incomplete if we consider alternative parameterizations of utility or choose the paycheck parameter $\gamma > 0$. In these alternative parameterizations of the model, the individual velocities $v(s)$ are not constant over time but depend instead on expectations of future money growth and output.

clearing implies that

$$M_t = \frac{1}{N} \sum_{s=0}^{N-1} M_t(s).$$

Accordingly, we interpret $\{M_t(s)/M_t\}_{s=0}^{N-1}$ as the distribution of money holdings across households. Goods market clearing then implies that the aggregate velocity of money is determined by a weighted average of the individual velocities of money where the weights are given by the distribution of money holdings:

$$v_t \equiv \frac{P_t y_t}{M_t} = \frac{1}{N} \sum_{s=0}^{N-1} \frac{P_t c_t(s)}{M_t} = \frac{1}{N} \sum_{s=0}^{N-1} v(s) \left(\frac{M_t(s)}{M_t} \right). \tag{19}$$

Of course, in a steady-state, the distribution of money holdings across households of different types is constant over time, and hence, aggregate velocity is also constant. With constant aggregate velocity, the steady-state inflation rate is equal to the money growth rate. Hence, our model predicts that, in the long-run, the price level and the money supply grow together while the aggregate velocity of money stays roughly constant.

Out of steady-state, however, as shown in (19), the implications of this simple version of our model for the dynamics of prices, velocity, and money are determined by two factors: first, the differences in individual velocities $v(s)$ across households of different types s, and second, the effect of a money injection on the *distribution* of money holdings across households. These factors can be understood intuitively as follows.

First, consider the differences in individual velocities across households. These measures of individual velocity equal the flow of consumption obtained by that household relative to its money holdings at the beginning of the period. From Figure 4, we can immediately see that these individual velocities should increase with s since a household of type s close to zero holds a large stock of money relative to his consumption while a household of type s close to $N-1$ holds only a small stock of money relative to his consumption. In Figure 5, we illustrate the pattern of these individual velocities for households of type s as given in (18).

How does a one-time increase in the supply of money affect aggregate velocity in this economy? To answer this question, we solve for the evolution of the distribution of money holdings as a function of the money growth rate. From (17), the evolution of the distribution of money holdings for households of type $s = 1, \dots, N-1$ is given by:

$$\frac{M_t(s)}{M_t} = (1 - v(s-1)) \frac{M_{t-1}(s-1)}{M_{t-1}} \frac{1}{\mu_t}. \tag{20}$$

Since the distribution of money holdings must sum to one, the money holdings of active households are then given by:

$$\frac{1}{N} \frac{M_t(0)}{M_t} = 1 - \frac{1}{N} \sum_{s=1}^{N-1} (1 - v(s-1)) \frac{M_{t-1}(s-1)}{M_{t-1}} \frac{1}{\mu_t}. \tag{21}$$

These formulas show that an increase in the money growth rate μ_t shifts the distribution of money holdings towards the active households at the expense of the inactive households.[5]

Now we can see the effect of money growth on velocity. By redistributing money towards the active household, an increase in the supply of money tilts the distribution of money holdings towards agents with low individual velocities and away from agents with high individual velocities, lowering aggregate velocity. To derive this result analytically, from (19), (20), and (21) observe that

$$\frac{\partial (v_t \mu_t)}{\partial \mu_t} = v(0).$$

Hence the elasticity of velocity with respect to money growth is given by

$$\frac{\partial v_t}{\partial \mu_t} \frac{\mu_t}{v_t} = \left[\frac{\partial (v_t \mu_t)}{\partial \mu_t} - v_t \right] \frac{1}{v_t} = \left[\frac{v(0)}{v_t} - 1 \right].$$

[5] To check that these equations do, in fact, characterize the equilibrium allocations of money and consumption, one needs to check that the implied nominal interest rates are always positive and that shoppers of type $N-1$ do not hold money at the end of the period. Since $P_t c_t(0) = v(0) M_t(0)$ and $P_t c_t(N-1) = M_t(N-1)$ in our conjectured equilibrium allocation, one need only check that

$$E_t \left\{ \frac{\beta}{\mu_{t+1}} \frac{M_t(0)/M_t}{M_{t+1}(0)/M_t} \right\} < 1$$

and

$$\frac{M_t(N-1)}{M_t} \leq v(0) \frac{M_t(0)}{M_t}$$

at all dates and in all states of nature given the assumed stochastic process for μ_t.

Since the individual velocity of active households is less than aggregate velocity ($v(0) < v_t$), aggregate velocity declines when the supply of money increases. Given the exchange equation $M_t v_t = P_t y_t$, we see that the price level does not respond on impact one-for-one with an increase in the money supply since that increase in the money supply leads to an endogenous decrease in aggregate velocity.

To see how this elasticity of velocity with respect to money growth depends on N, consider the equilibrium of this model as the steady state value of $\beta/\mu \to 1$. In this limiting case, the nominal expenditure of each household is constant over time as is typically assumed in an inventory-theoretic model of money. In this limit, $v(0) = 1/N$ and velocity in steady-state is given by $2/(N+1)$ so that, under these assumptions

$$\frac{\partial v_t}{\partial \mu_t}\frac{\mu_t}{v_t} = -\frac{1}{2}\frac{N-1}{N} \qquad \text{and} \qquad \frac{\partial \pi_t}{\partial \mu_t}\frac{\mu_t}{\pi_t} = \frac{1}{2}\frac{N+1}{N},$$

where these derivatives are evaluated at the steady-state. We can see here that if $N = 1$, as in the standard cash-in-advance model, inflation responds one-for-one with the shock to money growth and velocity is constant. In contrast, for large N, prices respond only about 1/2 as much as money. This result follows from the geometry of money holdings implied by an inventory-theoretic model — a household that has just replenished its bank account will hold roughly twice as much money as an average household and hence have roughly half the velocity of the average household.

Following this effect on impact of money growth on velocity, the dynamics of velocity and prices that follow are determined by the subsequent evolution of the distribution of money holdings over time. It is easier to analyze the dynamics of velocity following a shock to money growth in a log-linearized version of the simple model. The log-linearized model has three sets of equations governing the evolution of velocity. First, there is an equation requiring that the sum of the log deviations of the fractions of money held by agents of type s is zero:

$$0 = \bar{m}(0)\hat{m}_t(0) + \sum_{s=1}^{N-1} \bar{m}(s)\hat{m}_t(s)$$

where $\bar{m}(s) \equiv M(s)/M$ in the steady-state and $\hat{m}_t(s) \equiv \log\left(M_t(s)/M_t\right) - \log \bar{m}(s)$. Second, there is a set of equations for $s = 1, \ldots, N-1$ governing the evolution of $\hat{m}_t(s)$

$$\hat{m}_t(s) = \hat{m}_{t-1}(s-1) - \hat{\mu}_t,$$

where these equations follow from the fact that individual velocities are constant. Third, there is the formula for velocity:

$$\bar{v}\hat{v}_t = v(0)\bar{m}(0)\hat{m}_t(0) + \sum_{s=1}^{N-1} v(s)\bar{m}(s)\hat{m}_t(s).$$

These equations imply that the deviations of aggregate velocity from its steady-state value follow an ARMA(N,N) process of the form:

$$\hat{v}_t = \sum_{s=0}^{N-1} v(s)\frac{\bar{m}(s)}{\bar{m}(0)}\hat{v}_{t-s-1} + \frac{1}{\bar{v}}\sum_{s=0}^{N-1}\left(v(s)\frac{\bar{m}(s)}{\bar{m}(0)} - \sum_{k=s}^{N-1} v(k)\bar{m}(k)\right)\hat{\mu}_{t-s}.$$

To get a sense of the coefficients of this process, observe that, with log utility and $\gamma = 0$, as β/μ gets close to one, we have:

$$v(s)\frac{\bar{m}(s)}{\bar{m}(0)} = v(0) = \frac{1}{N}.$$

Hence:

$$\hat{v}_t = \frac{1}{N}\sum_{s=1}^{N}\hat{v}_{t-s} + \sum_{s=0}^{N-1}\left(-\frac{1}{2} + \frac{s}{N}\right)\hat{\mu}_{t-s}.$$

Consider the response of the log of velocity to a one-time shock to money growth $\hat{\mu}_t$ and $\hat{\mu}_s = 0$ for all $s \neq t$. Using the approximation here, the impulse response of the log of velocity over the first $N-1$ periods is given by

$$\hat{v}_{t+k} = \frac{1}{2}\left(1 + \frac{1}{N}\right)^{k+1} - 1. \tag{22}$$

This impulse response starts with $\hat{v}_t = -1/2$, for large N it crosses zero at roughly $k = N\log 2$, and then rises above zero until $k = N$.

To illustrate these dynamics, in Figure 6, we show the responses of $\log(M_t)$, $\log(P_t)$ and $\log(v_t)$ to a one time unit shock to money growth when agents have log utility and $\gamma = 0$. As shown in Figure 6, at time $t = 0$, the money supply blips up by one unit and stays at its new level thereafter. In response to this injection, aggregate velocity falls, is negatively correlated with the money supply, and the price level responds less than one-for-one with the change in the money supply. Over time, aggregate velocity and prices rise, even overshooting their steady-state levels, and then gradually converge to steady-state with dampened oscillations.

The dynamics derived above and illustrated in Figure 6 can be understood as follows. Since the money growth rate is high for only one period, from (20) we see that the households who were active at the time of the money injection carry an abnormally large stock of money until they next have the opportunity to transfer funds from their brokerage account. As shown in Figure 5, their individual velocities rise each period until this next visit occurs. Thus, aggregate velocity remains below its steady-state level for a time initially as these agents have a low individual velocity and then rises past its steady-state level as the individual velocity for these agents rises. After N periods these agents have spent all of their money and they visit the asset market again. The periodic structure of the model (the pattern of shopping trips) introduces a sequence of dampened oscillations in velocity as the changes in the distribution of money holdings work their way through the system. After the first N periods, however, these effects of a money growth shock on velocity are quite small.

We have presented the solution of the model in this simple case to develop intuition for the qualitative effects of money injections on velocity and prices. Our key finding is that, in response to an increase in the money supply, aggregate velocity falls and thus the price level responds less than one-for-one with the money supply. Hence, prices in this model are sticky in the sense that they move substantially less than would be predicted by the simplest quantity theory. Specifically, the response of prices, on impact, is roughly half as large as the change in the supply of money with large N. Moreover, there is some persistence in this sluggish response of prices to changes in the quantity of money, and the extent of this persistence depends on the parameter N. A persistently sluggish response of prices to a change in money arises naturally from the dynamics of money holdings in this inventory-theoretic model of money demand.

In the next section, we study approximations to the solution of alternative specifications of our model by log-linearizing the equations of the model around the deterministic steady-state. We solve the resultant system of stochastic difference equations using the method of undetermined coefficients as described in Uhlig (1999).

4. Velocity and sluggish prices

We have shown how, qualitatively, the velocity of money declines in response to an increase in the supply of money and, as a result, prices respond sluggishly to an increase in the supply of money in our inventory theoretic model of money demand. The quantitative predictions of our model both for the velocity of money in the steady-state and for the short-run response of velocity to a money injection are determined by the parameters N and γ. In this section, we explore these quantitative implications. We first choose the parameters N and γ so that our model reproduces the average level of velocity for a broad monetary aggregate held by U.S. households. We then conduct two exercises with the model to illustrate its quantitative implications for the short-run dynamics of money, velocity, and prices with these parameter values.

In the first exercise, we feed into the model the sequences of money growth and aggregate consumption shocks observed in the data and compare the model's implications for the short-run fluctuations in velocity with those observed in the data. We find that velocity in the model is highly correlated with velocity in the data, but its fluctuations in the model are much smaller in magnitude than those observed in the data.

In our second exercise, we examine the responses of money, prices, and velocity in the model to a monetary shock that results in a persistent movement in the nominal interest rate similar to those estimated as the response of the Federal Funds rate to a monetary policy shock in the VAR literature. Here we find that the corresponding impulse responses of money and prices implied by our model are similar to those estimated in the VAR literature. In particular, prices in the model respond quite sluggishly to the change in monetary policy.

A. Steady-state velocity

The steady-state velocity implied by our model is a simple function of the parameters N and γ. In the example with $u(c) = \log(c)$ and $\gamma = 0$ that we used for intuition in the previous sections, we had individual velocities given by (18) which, for β close to 1, gives $v(s)$ close to $1/(N-s)$. With this approximation, in steady-state, aggregate velocity is given by:

$$\bar{v} = \frac{2}{N+1}.$$

Therefore, if we set the period length equal to one month and then seek to choose N so that the model produces aggregate annualized velocity equal to 2, we need to choose N to solve $2 = 24/(N+1)$ or $N = 11$ months. Obviously, to match a lower annualized figure for velocity, say 1.5, we would need to choose a larger N, here 15.

Holding N fixed, the model's implications for steady-state velocity are an increasing function of the paycheck parameter γ since the automatic deposit of paychecks into households' bank accounts allows for faster circulation of money. In this case, for β/μ close to one, aggregate velocity is well approximated by $\bar{v} = 2/(N+1)(1-\gamma)$. Here, for example, to produce annualized velocity close to 1.5 given $\gamma = 0.6$ would require $N = 38$.

B. Our choice of monetary aggregate

In this section, we choose the parameters of our the model to match the average velocity of a broad money aggregate — the sum of U.S. households' holdings of currency plus demand, savings, and time deposits.[6] In choosing this money aggregate, we consider currency and these bank accounts in the data as corresponding to funds held in households' bank accounts in the model, while stocks, bonds, money market and other mutual funds in the data as corresponding to assets held in households' brokerage accounts. We present evidence that households in the U.S. hold a large quantity of currency, demand, savings, and time deposits and pay a substantial cost to hold these assets in terms of foregone interest relative to the interest available on retail money market mutual funds or short-term Treasury securities. We match those observations in our model by assuming that households transfer money between their brokerage accounts and bank accounts very infrequently — on the order of only once every year to once every three years. While such an assumption may seem implausible, the microeconomic evidence summarized in Vissing-Jorgensen (2002) on

[6] This aggregate is essentially U.S. households' holdings of M2 less retail money market funds.

the frequency with which households trade financial assets held outside of their bank accounts so defined is consistent with these assumptions.

U.S. households hold substantial amounts of currency and low yielding bank accounts. In Figure 7, we report on US households' holdings of currency and demand deposits, time and savings deposits, and retail money market mutual funds. These data are from the Flow of Funds Accounts (2002). Figure 7 is a stacked line chart of these holdings relative to personal consumption expenditure. The height of the lowest line indicates holdings of currency and demand deposits relative to annualized personal consumption expenditure. The gap between that line and the next highest line indicates holdings of time and savings deposits. The gap between that second line and the third line indicates holdings of retail money market mutual funds.[7] These data give a measure of the velocity of money relative to personal consumption expenditure (at least for the money held by households) averaging roughly 1.5 and rising more recently towards 2. We use this average level of velocity of 1.5 to guide our choice of N and γ for the quantitative results that follow.

To document that these bank accounts have low yields, in Table 1 we summarize data on the rate of return paid on various types of bank deposits and other financial assets that are available from the web site of the Federal Reserve Bank of St. Louis.[8] In the top panel of Table 1, we report on the average user cost of holding currency, demand deposits, time and savings deposits, and retail money market mutual funds over the full time period for which the data are available as well as over the decade 1990-2001. The user costs reported in this table are equal to the difference between the rate of return on short-term Treasury securities (as reported in the spreadsheet from which the data are taken) less the rate of return on the asset in question. In panel b of Table 1, we show the average opportunity cost of M1, M2, and M2 less retail money market mutual fund shares. Here these average opportunity costs are measured as the weighted average of the opportunity cost of each type of deposit in the corresponding aggregate where the weights are given by the share of each type of deposit in

[7] Note that holdings of money market mutual funds were equal to zero before the middle of the 1970's.

[8] See the file of input data msinputs.zip available at http://www.stls.frb.org/research/msi/index.html. This file contains a spreadsheet that reports the data on the user cost of various types of bank deposits that has been collected by the Research Department at the Federal Reserve Bank of St. Louis as part of their project to construct Divisia monetary aggregates.

the corresponding monetary aggregate.

As is clear from Table 1, the average opportunity cost of holding time and savings deposits is roughly similar to that of holding demand deposits, both over the period 1959-2001 and more recently. In contrast, the opportunity cost of holding retail money market mutual fund shares has been essentially zero on average. Likewise, the opportunity cost of M2 less retail money market funds is on the order of 200 basis points (2 percentage points) and is not that substantially different than the opportunity cost of M1.

To parameterize our model to reproduce an average annual velocity of money of 1.5, we choose the length of a period to be one month and use two choices for the parameters N and γ. In the first of these, we set $N = 15$ months and $\gamma = 0$. We regard this parameterization of the model as a useful benchmark since, with log utility and these parameters, individual velocities are constant over time and aggregate velocity changes only because of changes in the distribution of money across agents. In our second choice of the parameters N and γ, we choose the paycheck parameter $\gamma = 0.6$ to match the fraction of personal income that is received as wage and salary disbursements observed in the data.[9] Here we are thinking that personal income not paid as wage and salary disbursements is paid directly into household's brokerage accounts rather than into their bank accounts. We then choose $N = 38$ so that with $\gamma = 0.6$, the model produces an average velocity of 1.5. We regard this second parameterization of the model as more interesting quantitatively. The interested reader can use the formulas for steady-state velocity presented in the previous subsection to find the parameter N implied by alternative choices of average velocity and the paycheck parameter γ.

The values $N = 15$ months and $N = 38$ months are the values that are required to account for the average level of low-yielding assets held by U.S. households given a range of assumptions about the paycheck parameter γ. These parameters imply, within the model, that households transfer funds between their brokerage accounts and bank accounts very infrequently. This assumption is not inconsistent with the available microeconomic evidence on the frequency with which agents trade financial assets held outside of their bank accounts.

The first set of such microeconomic data concerns the frequency with which households

[9] From Table 2.1 of the National Income and Product Accounts, we observe that this fraction has been equal to 60% on average over the period from 1959-2001.

trade equity. Such data are relevant since a household would have to trade equity to rebalance its portfolio between funds held in its bank account and equity held in its brokerage account. The Investment Company Institute (1999) conducted an extensive survey of households' holdings and trading of equity in 1998. They report on the frequency with which households traded stocks and stock mutual funds in 1998. They report that 48% of the households that held individual stocks outside of their retirement accounts neither bought nor sold any stock in 1998 and 63% of the households that held stock mutual funds outside of their retirement accounts neither bought nor sold mutual funds in 1998. Since a household would have to buy or sell some of these assets to transfer funds between these higher yielding assets held in a brokerage account and a lower yielding bank account, these data, interpreted in light of our model, would indicate choices of N ranging from roughly 24 (for roughly 1/2 of households trading these risky assets at least once within the year) to roughly 36 (for roughly 1/3 of households trading within the year).[10]

The second set of microeconomic data is that presented by Vissing-Jorgensen (2002). She studies micro data on the frequency of household trading of stocks, bonds, mutual funds and other risky assets obtained from the Consumer Expenditure Survey. In figure 6 in her paper, she shows the fraction of households who bought or sold one of these assets over the course of one year as a function of their financial wealth at the beginning of the year. She finds that the fraction of agents who traded one of these assets ranges from roughly 1/3 to 1/2 of the households owning these assets at the beginning of the year. Again, given our interpretation that households hold stocks, bonds, mutual funds and other risky assets in their brokerage accounts, these data would lead us to choose N between 24 and 36.

Traditionally, inventory-theoretic models of money demand have been used to study households' holdings of a narrow measure of money. As can be seen in Figure 7, households in the United States hold very little currency and demand deposits relative to their per-

[10] These data may also overstate the frequency with which households transfer funds between their equity accounts and their transactions accounts since some of the instances of equity trading are simply a reallocation of the equity portfolio. The Investment Company Institute reports that more than 2/3 of those households that sold individual shares of stock in 1998 reinvested all of the proceeds, while 57% of those households that sold stock mutual funds reinvested all of the proceeds. In the context of our model, reallocation of the household portfolio in the asset market is costless and does not generate cash that can be used to purchase goods.

sonal consumption expenditure. Their holdings of these assets have been trending downward steadily since 1952 and are now represent less than one month's worth of personal consumption expenditure. If we were to choose the parameters of our model to reproduce the observed velocity of a narrow definition of money such as M1 or currency alone, we would choose N to represent trading frequencies on the order of several weeks. This specification would require a period close to one day and N on the order of 15 and γ in the range of $1/2$. The variations in velocity that would occur in such a model would be at too high a frequency to be of interest relative to the data.

C. The response of velocity to U.S. money and output shocks

We now study the implications of our model for velocity in the short run when we feed in the money growth and endowment shocks observed in the U.S. data. We use monthly data on M2 as our measure of the monetary aggregate M_t, and we use monthly data on the deviation of the log of personal consumption expenditure from a linear trend as our measure of the shocks to the aggregate consumptio y_t. To solve for households' decision rules in the model, we estimate a VAR relating the current money growth rate and aggregate consumption to 12 lags of these variables and use this VAR as the stochastic process governing the exogenous shocks. We then generate the model's implications for velocity by feeding in the actual series for these shocks. To compare the implications of our model for the dynamics of money and velocity in the short-run to the data, we detrend the series implied by the model using the HP-filter.

Consider first the implications of our model with $N = 15$ months and $\gamma = 0$. In Figure 8, we show the HP-filtered series for velocity implied by our model together with the corresponding HP-filtered series for velocity from the data. The correlation between velocity in the model and the data is 0.4. In the figure, we have used different scales in plotting the series from the model and the data. These different scales reflect the fact that the standard deviation of velocity in the data is 3.6 times larger than the standard deviation of velocity in the model. The results are essentially identical when we compare the 12 month differences of the series from the model and the data: the correlation between the two again is 0.4 and the standard deviation of changes in the velocity in the data is again 3.6 times larger than the standard deviation of changes in velocity in the model.

In Figure 9, we make the same comparison between HP filtered velocity from the model and the data in the case in which $N = 38$ and $\gamma = 0.6$. Here, the correlation between the two series is higher at 0.6 and the standard deviation of velocity in the data is now only 2.6 times larger than that in the model. Again, the results for 12 month differenced data are similar, with a correlation of 0.5 and a relative standard deviation of 2.7.

Given that we have used nothing but steady-state information to choose the parameters of this model, we regard the high correlation between velocity from the model and the data as a remarkable success. Observe that if we had chosen $N = 1$, as in a standard cash-in-advance model, velocity as implied by the model would be constant at one regardless of the shock process and, hence, the correlation between velocity in the model and velocity in the data would be zero. We interpret this finding as offering support for the hypothesis that a substantial portion of the negative correlation between the short run movements of velocity and the ratio of money to consumption is due to the endogenous response of velocity to changes in the ratio of money to consumption.

While a promising first start, however, these specifications of the model clearly do not account for all of the variability of velocity observed in the data. Under both sets of parameter values, the short run variability of velocity in the data is substantially larger than that in the model. One possible explanation for this discrepancy between the model and the data may be that there are other shocks to the demand for money which we have not modelled here. If this were the case, one would not expect this model to account for all of the variability of velocity observed in the data. With this possibility in mind, in the next section we consider the response of money, prices, and velocity in our model to an exogenous shock to monetary policy, modelled here as an exogenous, persistent shock to the short-term nominal interest rate.

D. The response to a shock to monetary policy

There is a large literature that seeks to estimate the response of the macroeconomy to a monetary policy shock (see Christiano, Eichenbaum, and Evans (1999) for a survey of this literature). There appears to be a consensus in this literature that a shock to monetary policy, modelled as an exogenous, persistent increase in the short-term nominal interest rate, is associated with a persistent decrease in the supply of money and, at least initially, little or no response of the aggregate price level. (See Cochrane (1994) and Uhlig (2001) for additional examples of such estimates). As is evident from the exchange equation $Mv = Pc$, if the response of the economy to an exogenous shock to monetary policy is followed by a substantial movement in the quantity of money that is not matched by a corresponding movement in the price level, then that exogenous shock to monetary policy must also be followed by some combination of responses of consumption and the velocity of money. In this section, we examine the response of the velocity of money in our model to an exogenous, persistent increase in the short-term nominal interest rate to assess the extent to which the responses of money and prices following a monetary policy shock might be accounted for by the endogenous response of velocity to that shock.

To simulate the effects of a shock to monetary policy in our model, we solve for a path of money growth that is consistent with a predetermined, persistent movement in the short-term nominal interest rate. Before doing so, we first discuss two technical issues that arise when one solves our model under the assumption that the path for nominal interest rates is predetermined. We then show the impulse responses of money, prices, and velocity to an exogenous, persistent increase in the nominal interest rate.

The first technical issue has to do with the dynamics of equilibria in which the nominal interest rate follows an exogenously specified path. Under the assumption that the nominal interest rate follows a pre-specified path, one can show analytically that the matrix that describes the dynamics of the endogenous variables in this economy has eigenvalues that are all equal to zero. (This implies that, if the interest rate is set at its steady-state value but the initial distribution of money holdings is not, then the economy will reach steady-state in exactly N periods). Because these eigenvalues are repeated, this matrix is not diagonalizable, and hence, this variant of the model cannot be solved using standard methods such as those outlined by Blanchard and Kahn (1980) or Uhlig (1999). In a technical appendix to this paper, we develop a specific solution method for this model based on the use of the generalized Schur form that makes use of the information that the eigenvalues of the matrix that describes the equilibrium dynamics are all equal to zero.[11]

The second technical issue has to do with the invertibility of the equilibrium mapping between interest rates and money growth rates. In this model, there are many stochastic processes for money all consistent with the same exogenously specified path for nominal interest rates in equilibrium. In the experiments with the second variant of the model that we carry out below, we choose one of the many stochastic process for the gross growth rate of the money supply that result in an equilibrium in which the short-term nominal interest rate follows our prespecified stochastic process. The process for money growth that we choose is the unique one that has the property that a shock to the nominal interest rate, on impact, is associated with no movement in the current price level. This choice is consistent with the schemes used to identify shocks to monetary policy discussed in Christiano, Eichenbaum, and Evans (1999). We discuss these two issues in greater detail in the technical appendix to this paper.[12]

We now study the quantitative implications of our model having solved for a money growth process that results in equilibrium in which the log of the short-term gross interest rate follows a first-order autoregressive process with first order autocorrelation $\rho = 0.87$. This autocorrelation is produces a response of the nominal interest rate to a shock similar to that shown in Christiano, Eichenbaum, and Evans (1999) and Uhlig (2001). We focus on the parameterization of our model with $N = 38$ and $\gamma = 0.6$.

Figure 10 shows the impulse responses of the log of the money stock, velocity, and the aggregate price level in response to a shock to the short-term interest rate (also shown in the figure) for the specification of the model with $N = 38$ months and $\gamma = 0.6$. That this specification of the model generates large short-term movements in velocity that are strongly negatively correlated with the ratio of money to consumption can be seen clearly in these impulse responses. As a result of these negative comovements of money and velocity, the aggregate price level appears "sticky" in that it shows little or no response to the shock to

[11] We also found that direct methods based on use of the generalized Schur form, as suggested by Klein (2000) and others, did not correctly identify that the matrix describing the equilibrium dynamics of the variables had eigenvalues all equal to zero. This appears to be a numerical issue since this methodology should in theory work in cases with repeated eigenvalues.

[12] This appendix is available at www.atkeson.net\andy

interest rates for at least the first twelve months. It is only after 12 months have passed that the money stock and the price level begin to rise together in the manner that would be expected in a flexible price model following a persistent increase in the nominal interest rate. Recall that here, by assumption, there are no movements in aggregate output and consumption following this shock to the nominal interest rate.

Figure 11 shows the same impulse responses except that in this case the log of the growth rates of the money stock, velocity, and price level rather then the level of these variables is shown. This figure shows that there are persistent liquidity effects in this model both in the sense that a movement in the nominal interest rate is associated with a movement in the money growth rate in the opposite direction and also in the sense that a movement in the nominal interest rate is associated, at least at first, with a movement in the real interest rate (the difference between the nominal interest rate and the growth of the price level). The aggregate price level again appears "sticky" in the sense that inflation does not respond much to the movement in the nominal interest rate.[13]

In sum, these results indicate that our model can account for a substantial delay in the response of the price level to an exogenous shock to the nominal interest rate and it does so simply on the basis of the endogenous response of velocity to that interest rate shock.

5. Conclusion

In this paper, we have put forward a simple inventory-theoretic model of the demand for money and shown, in that model, that the price level does not respond immediately to an exogenous increase in the money supply. Instead, there is an extended period of price sluggishness that occurs because the exogenous increase in the money supply leads, at least initially, to an endogenous decrease in the velocity of money. We have argued that if this simple model is used to analyze the dynamics of money and velocity using a relatively broad measure of money, then it produces a sluggish response of the price level similar to that estimated in the VAR literature on the response of the economy to monetary policy shocks.

In keeping this model simple, we have abstracted from a number of issues that might

[13] As explained above, we have imposed that the reponse of prices to money be zero in the first period. We have imposed no constraint in the effect of money on price in any subsequent periods.

play an important role in the development of a more complete model. First, we have simply assumed that households have the opportunity to transfer funds between their brokerage and bank accounts only every N periods and have not allowed households to alter the timing of these transactions after paying some fixed cost. Perhaps it will be possible to extend this work to allow households to choose when to be active subject to a fixed cost using the techniques developed by Dotsey, King, and Wolman (1999).

Second, we have abstracted from any heterogeneity across households in their average money holdings and the corresponding frequency with which they transact between their brokerage and bank accounts. One might suspect that such heterogeneity would be important if wealthy households, those that hold the bulk of the financial assets in the economy, transfer funds between their brokerage and bank accounts more frequently than poor households. Adding such heterogeneity is relatively easy in this model since one can always include additional types of households with different assumed trading frequencies. The precise response of prices to monetary shocks in this kind of model will certainly depend on the details of the heterogeneity across households that one assumes. To date, we have not found clear results relating the details of such heterogeneity to implications for price sluggishness.

Third, we have abstracted from any differences between base money and our broader money aggregate and hence our model has no fluctuations in the money multiplier. Certainly, in the data, there is also sluggishness in the response of prices to changes in the quantity of base money that is due, at least in part, to fluctuations in the money multiplier. In this paper, we focus only on the sluggish response of prices to changes in the quantity of a broad measure of money and leave aside the study of the links between base money and broader measures of money.

Having abstracted from these and other potentially important questions, we cannot draw many specific quantitative conclusions from this analysis. We do, however, conclude with the broader point that the dynamics of money demand may play an important role in accounting for the sluggish response of prices to changes in monetary policy.

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Figure 1: In the long run M/c up, P up

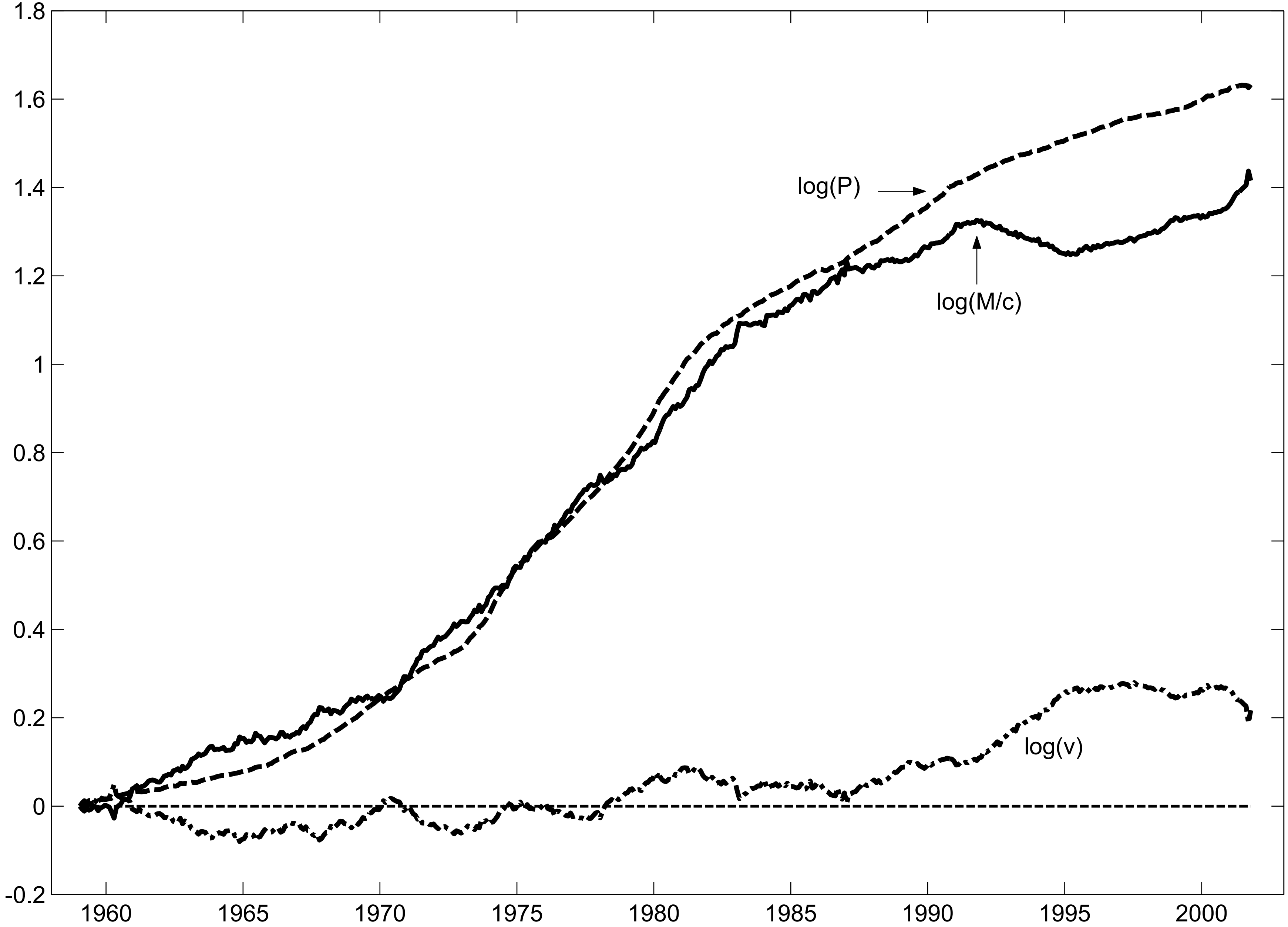


Figure 2: But in the short run M/c up, v down

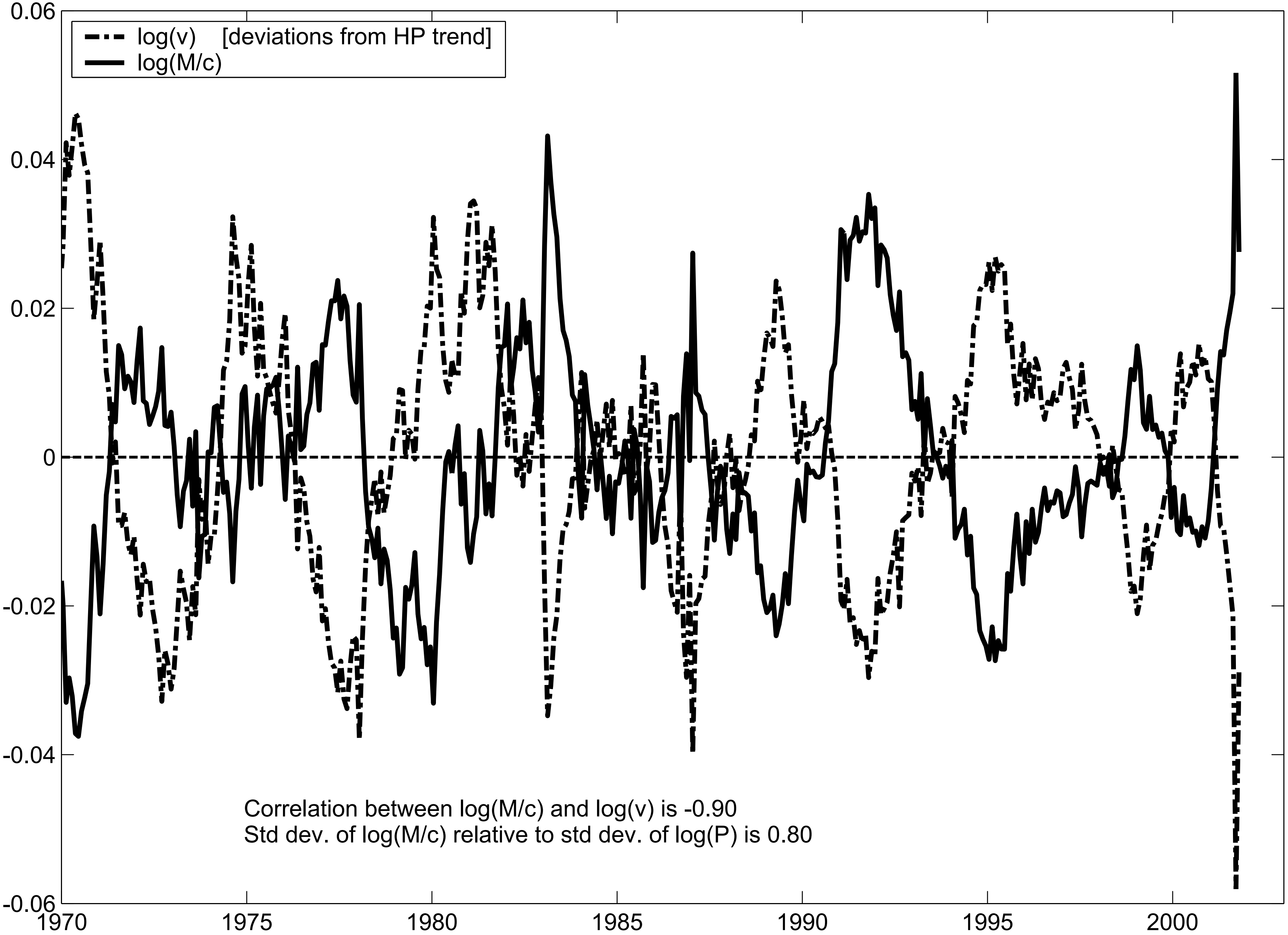


Figure 3: Money and prices in the short run

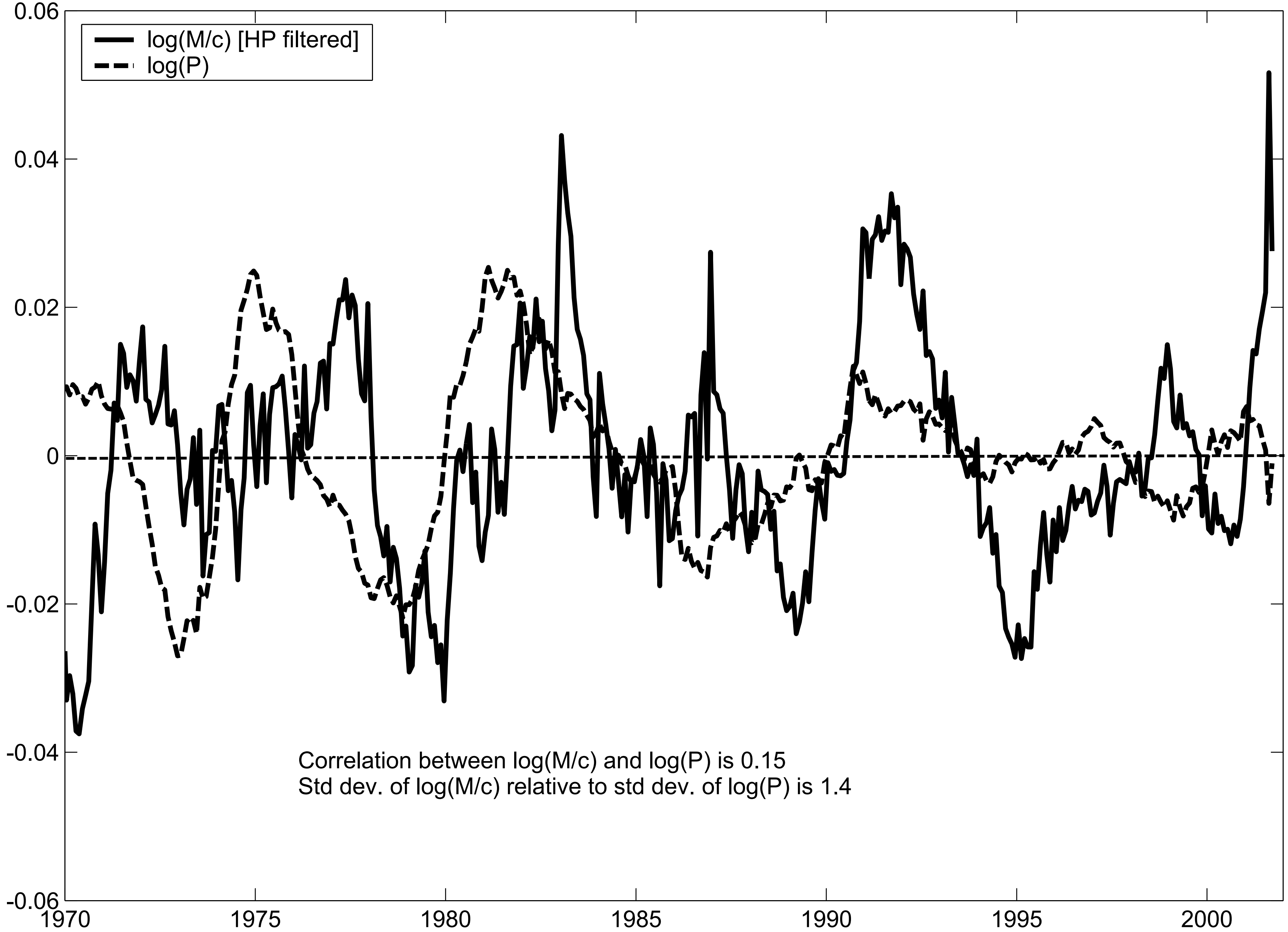


Figure 4: "Sawtoothed" money holdings ($N = 15$, $\gamma = 0$)

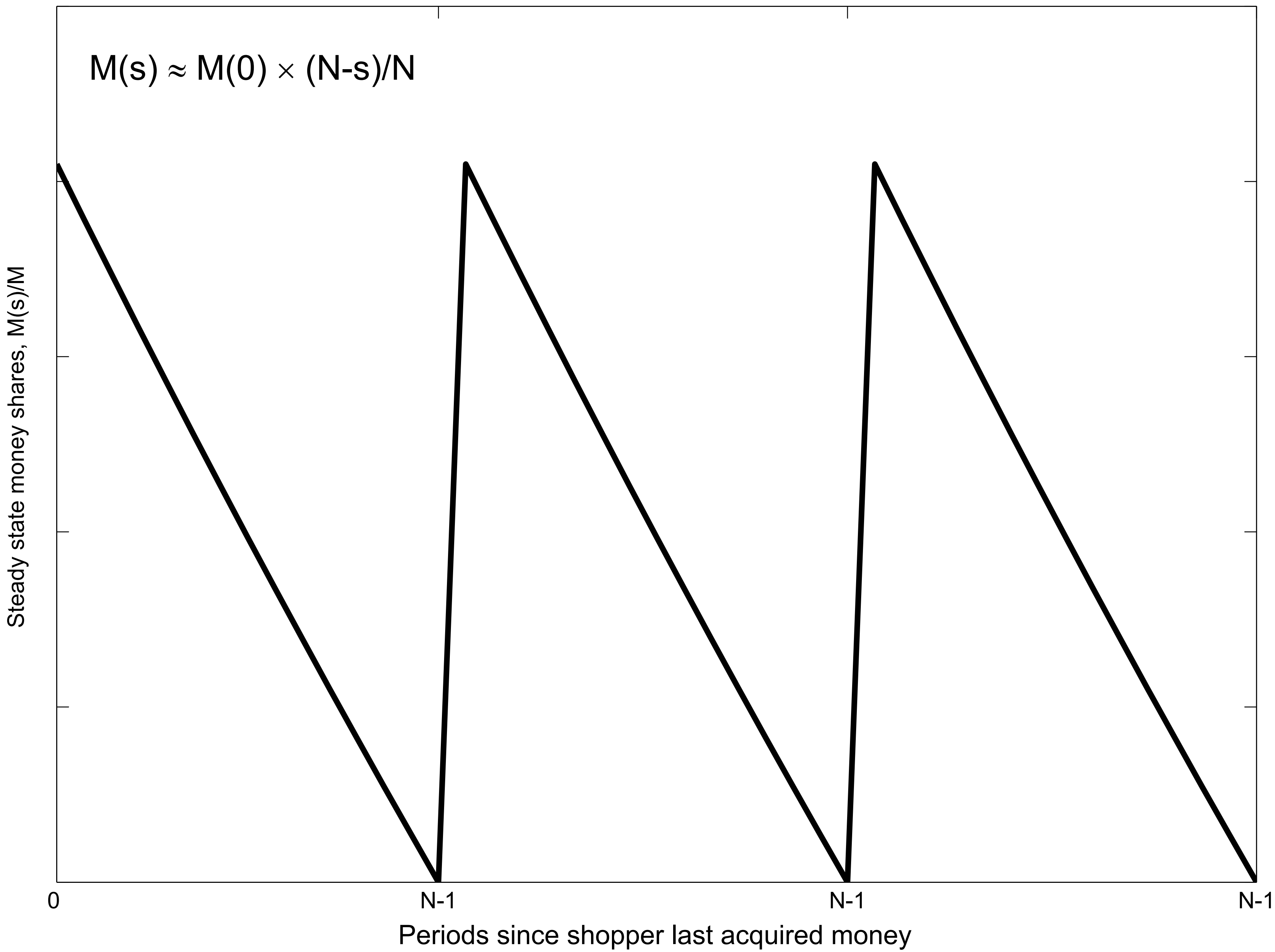


Figure 5: Individual velocities are increasing

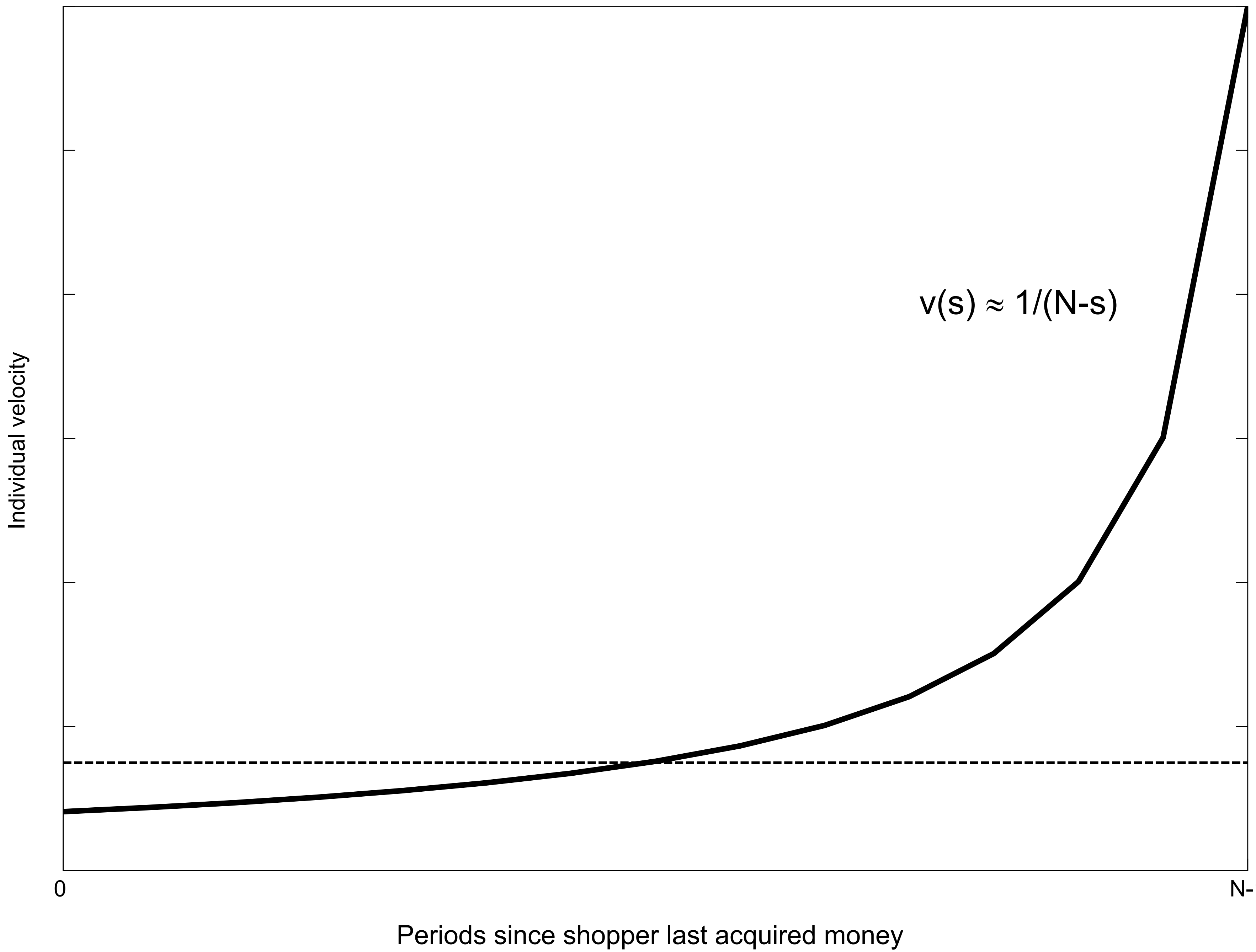


Figure 6: Money up, velocity down, prices sluggish (N = 15, γ = 0)

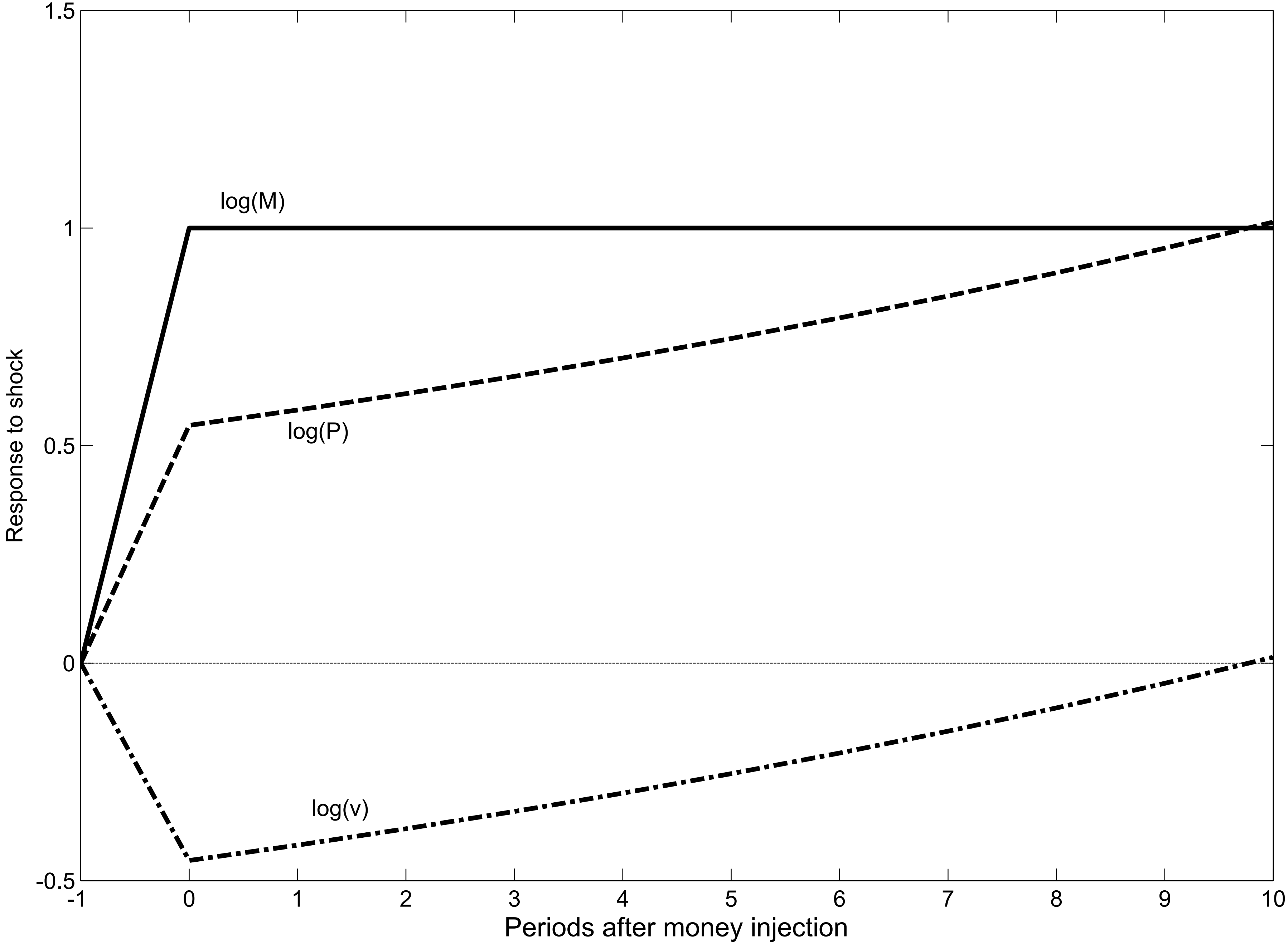


Figure 7: Household financial assets relative to personal consumption expenditure

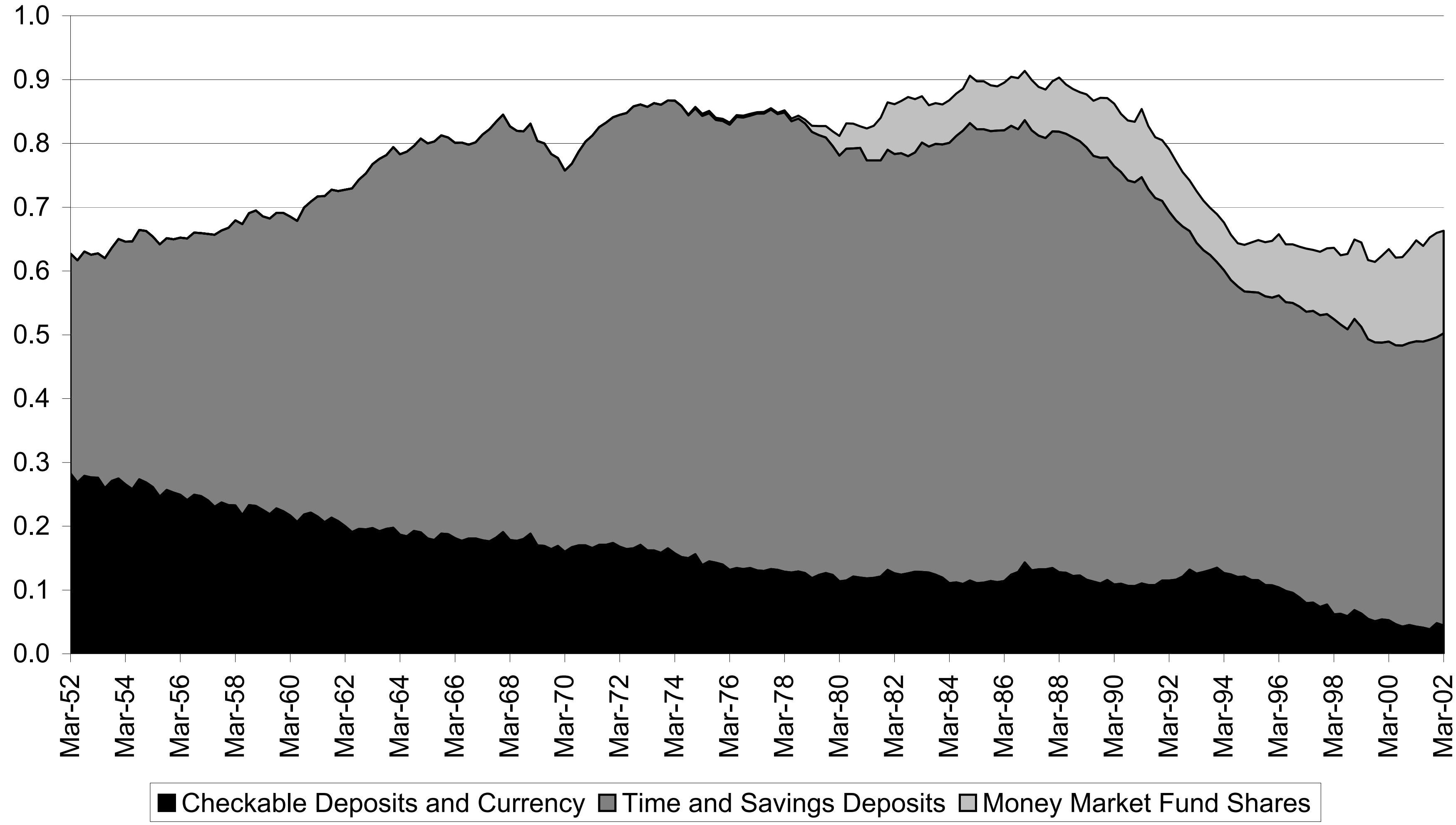


Figure 8: Model and data velocity (N = 15, γ = 0). HP filtered.

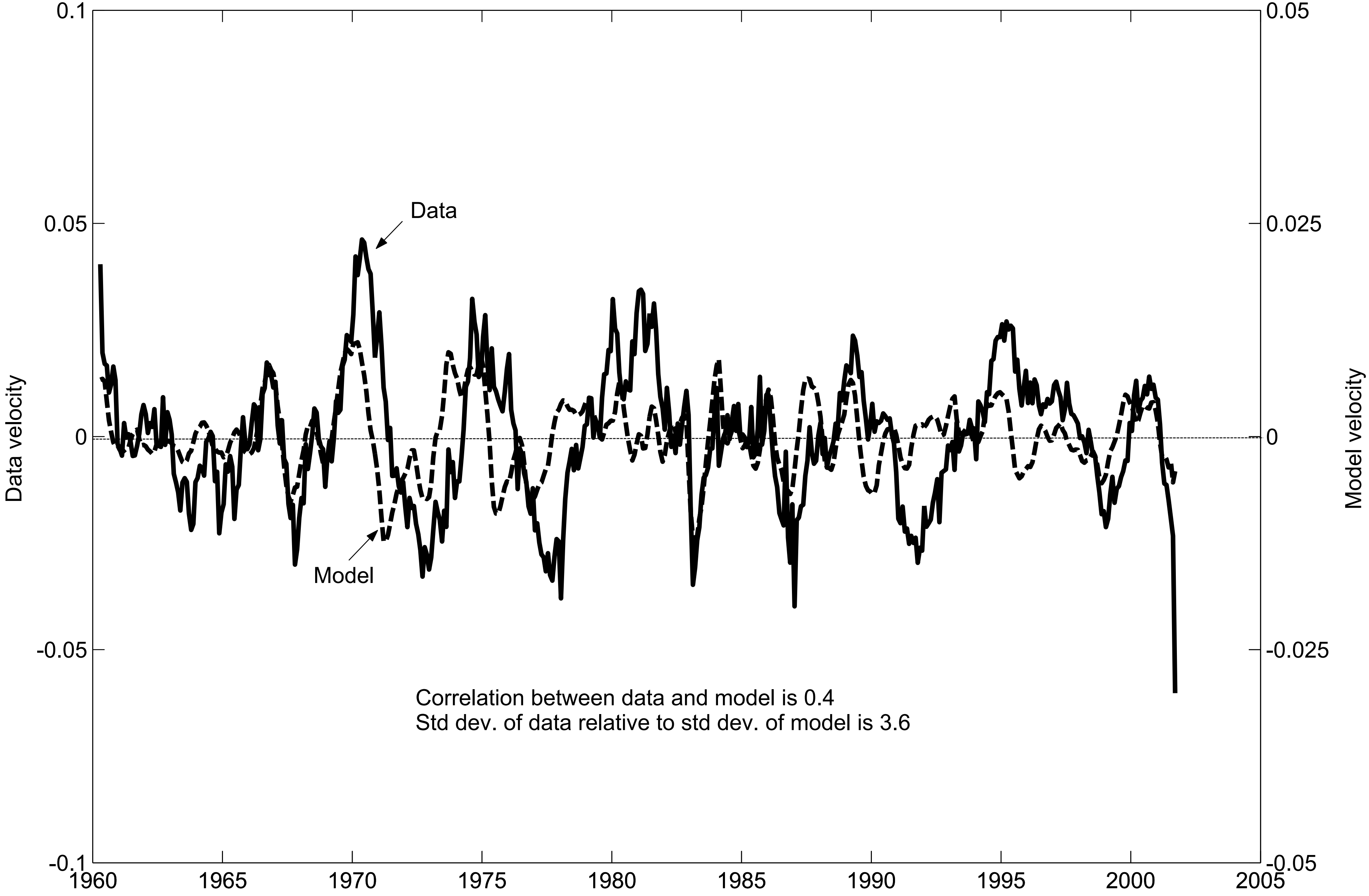


Figure 9: Model and data velocity (N = 38, γ = 0.6). HP filtered.

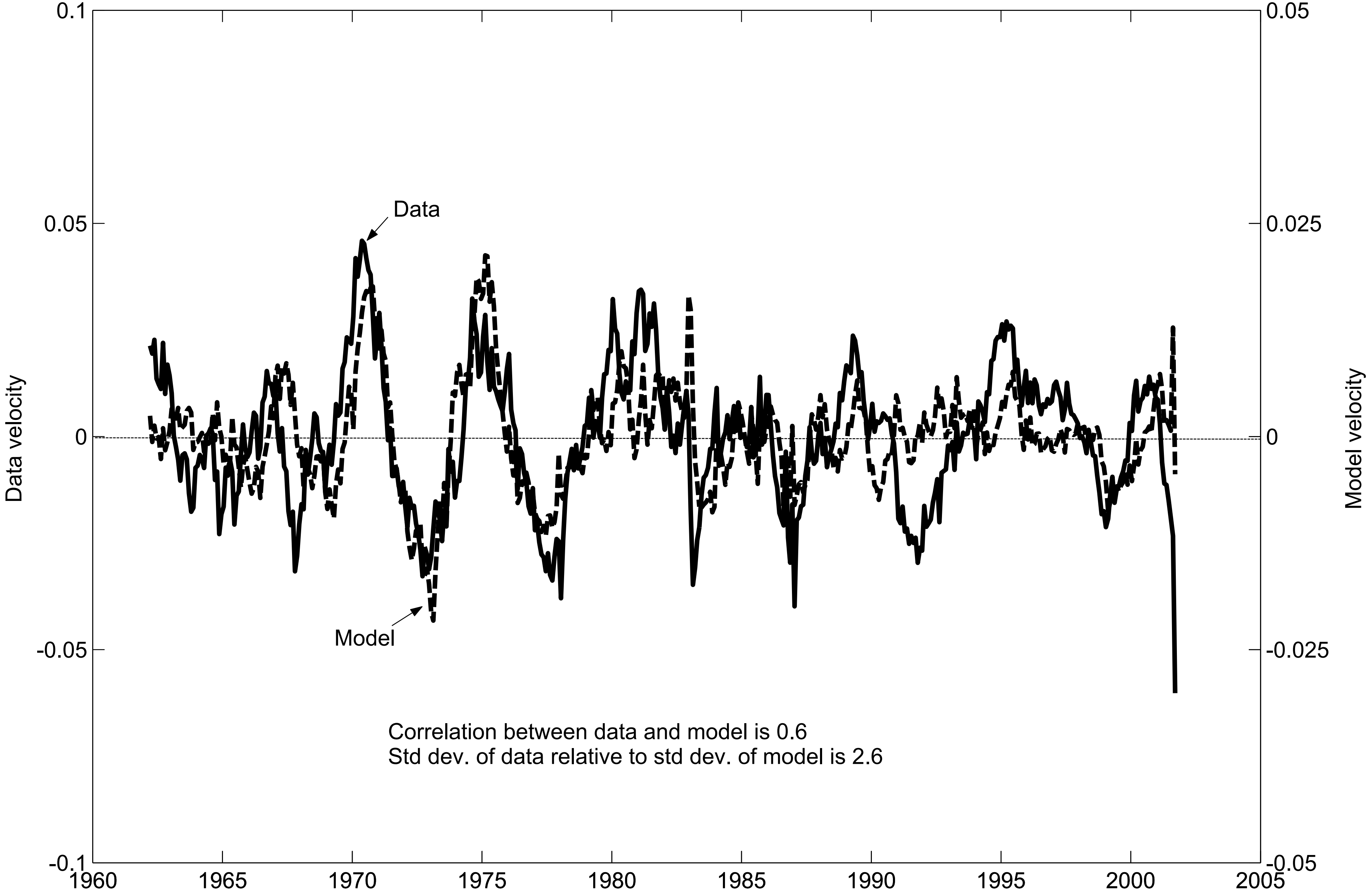
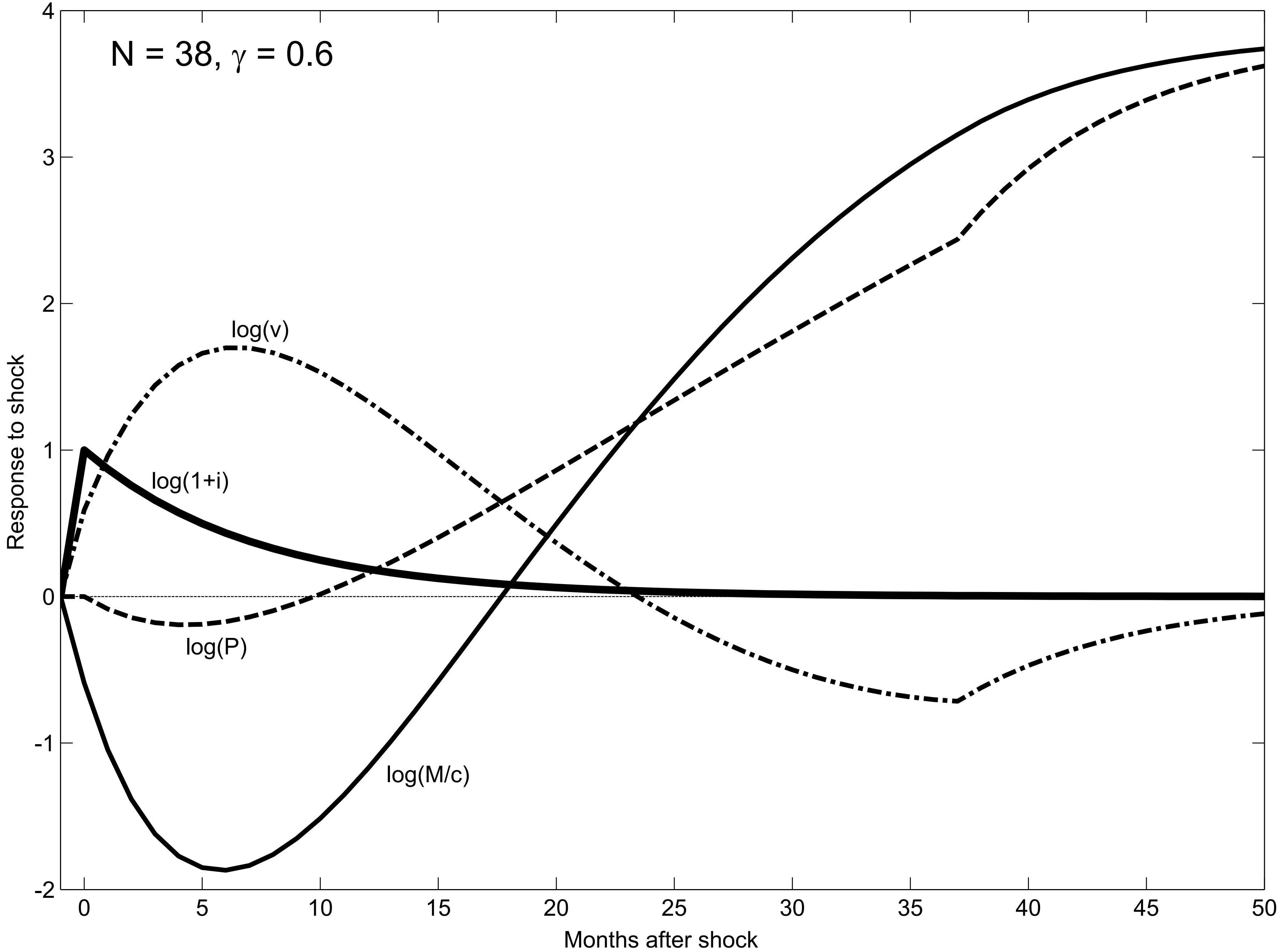


Figure 10: Sluggish price response to persistent interest rate shock



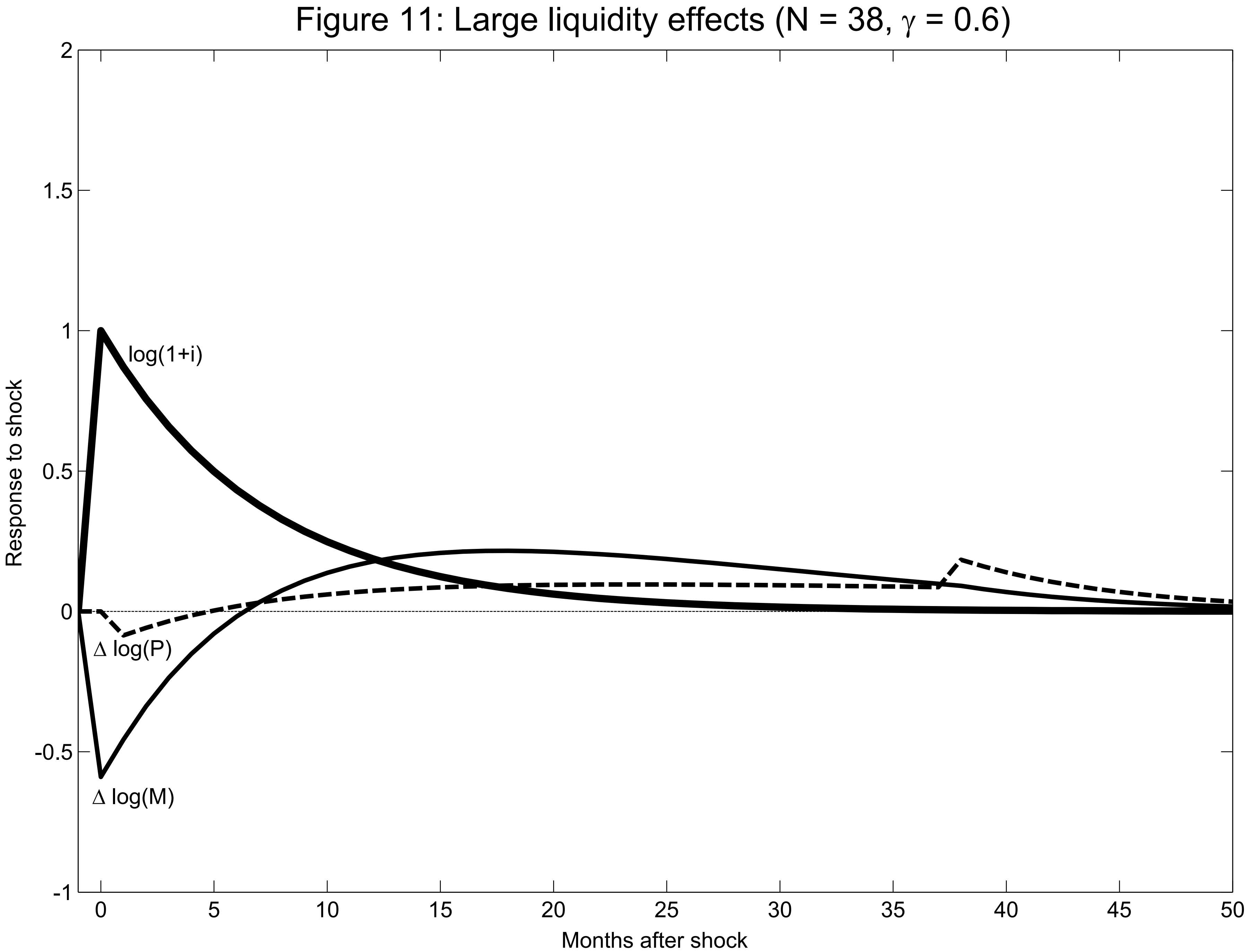


Figure 11: Large liquidity effects (N = 38, γ = 0.6)
Response to shock
Months after shock
log(1+i)
Δ log(P)
Δ log(M)
2
1.5
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
35
40
45
50

Table 1

Opportunity Cost of Various Monetary Assets

(a) *Short-Term Treasury Rate less own rate*

	average opportunity cost in percentage points	
Asset	1959-2001	1990-2001
Currency	5.22	4.32
Demand Deposits	1.98	1.33
Savings Deposits	1.50	1.71
Time Deposits	1.80	2.47
Retail Money Market Funds*	-0.33	-0.11

*1973-2001

(b) Short-Term Treasury Rate less own rate

	average opportunity cost in percentage points	
Aggregate	1959-2001	1990-2001
M1	2.97	2.71
M2	1.80	1.84
M2 less Retail Money Market Funds	1.95	2.17

Opportunity cost data constructed from the spreadsheets TB1ASAM.WKS
and ADJSAM.WKS available on the website of the Federal Reserve Bank of St. Louis
http://www.stls.frb.org/research/msi/index.html
These data are collected as part of the St. Louis Fed's project to construct
Divisia monetary services indices