New Trade Models, Same Old Gains?*

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Abstract

Micro-level data have had a profound influence on research in international trade over the last ten years. In many regards, this research agenda has been very successful. New stylized facts have been uncovered and new trade models have been developed to explain these facts. In this paper we investigate to which extent answers to new micro-level questions have affected answers to an old and central question in the field: How large are the gains from trade? A crude summary of our results is: "So far, not much."

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1 Introduction

What share of firms export? How large are exporters? How many products do they export? Over the last ten years, micro-level data have allowed trade economists to shed light on these and other micro-level questions. The objective of our paper is to look back at this research agenda and investigate to what extent answers to new micro-level questions have affected our answers to an old and central question in international trade: How large are the gains from trade? A crude summary of our results is: "So far, not much."

Motivated by the recent trade literature, our analysis focuses on models featuring five basic assumptions: Dixit-Stiglitz preferences, one factor of production, linear cost functions, complete specialization, and iceberg trade costs. Examples of trade models satisfying these restrictions include, among others, Krugman (1980), Eaton and Kortum (2002), Anderson and Van Wincoop (2003), and multiple variations and extensions of Melitz (2003).[1] Within that class of models, we identify two critical macro-level restrictions, a CES import demand system and a gravity equation,[2] and show that if these two restrictions hold, then under either perfect competition or monopolistic competition, there exists a common estimator of the gains from trade. This estimator only depends on the value of two aggregate statistics: (i) the share of expenditure on domestic goods, λ, which is equal to one minus the import penetration ratio; and (ii) a gravity-based estimator $\bar{\varepsilon}$ of the elasticity of imports with respect to variable trade costs, which we refer to as the "trade elasticity."

According to our results, whether gains from trade derive from reallocations across sectors, across firms within sectors, or across products within firms, the two previous aggregate statistics remain sufficient for welfare analysis. Put differently, within that particular, but important class of models, the mapping between trade data and welfare is independent of the micro-level details of the model we use.

In order to establish this stark equivalence result, we proceed as follows. We start by showing that the percentage change in the consumer price index associated with any small change in trade costs is equal to $-\widehat{\lambda}\big/\varepsilon$, where $\widehat{\lambda}$ is the percentage change in the share of expenditure devoted to domestic goods caused by the change in trade costs and ε is the true value of the trade elasticity. For $\varepsilon < 0$, which is the empirically relevant case, being more

[1] Notable extensions of Melitz (2003) satisfying the restrictions above include Chaney (2008), Arkolakis (2008), and Eaton, Kortum and Kramarz (2008).

[2] A CES import demand system is conceptually distinct from Dixit-Stiglitz preferences; it entails restrictions on the interplay between domestic demand and supply. The import demand system is CES if the elasticity of substitution of country j's import demand from country i (relative to the demand for domestic goods) with respect to the trade cost from i' to j is zero for $i' \neq i$ and is equal to a constant for $i' = i \neq j$.

open, $\widehat{\lambda} < 0$, implies a welfare gain. We then use our assumption that ε is constant across equilibria to integrate small changes in real income between the initial trade equilibrium and the autarky equilibrium. This allows us to establish that the total size of the gains from trade, i.e. the percentage change in real income necessary to compensate a representative consumer for going to autarky, is equal to $\lambda^{1/\varepsilon} - 1$. Finally, assuming that the true trade elasticity ε can be consistently estimated by $\overline{\varepsilon}$ using a gravity equation, we conclude that the gains from trade can be consistently estimated by $\lambda^{1/\overline{\varepsilon}} - 1$.

This last formula offers a very convenient way to measure gains from trade in practice. For example, the import penetration ratios for the U.S. and Belgium for the year 2000 were 7% and 27%, respectively.[3] This implies that $\lambda_{US} = 0.93$ and $\lambda_{BEL} = 0.73$. Anderson and Van Wincoop (2004) review studies that offer gravity-based estimates for the trade elasticity all within the range of -5 and -10. Thus, the total size of the gains from trade range from 0.7% to 1.5% for the U.S. and from 3.2% to 6.5% for Belgium, whatever the micro origins of these gains may be.

The common features of the trade models for which we derive these formulas are described in Section 2. As previously mentioned, these features consist of five basic assumptions and two critical macro-level restrictions: (*i*) a CES import demand system; and (*ii*) a gravity equation. In the rest of this paper, we simply refer to this class of models as "gravity" models.

Section 3 focuses on the case of gravity models with perfect competition, which will allow us to describe the logic behind our welfare formula in a very intuitive manner. In a neoclassical environment, a change in trade costs affects welfare through changes in terms-of-trade. Since there is only one factor of production, changes in terms-of-trade only depend on changes in relative wages and trade costs. Under complete specialization and a CES import demand system, these changes can be directly inferred from changes in the relative demand for domestic goods using an estimate of the trade elasticity, which the gravity equation provides.

A direct corollary of our analysis under perfect competition is that two very well-known gravity models, Anderson (1979) and Eaton and Kortum (2002), have identical welfare implications. In Anderson (1979), like in any other "Armington" model, there are only consumption gains from trade, whereas there are both consumption and production gains from

[3] Import penetration ratios are calculated from the *OECD Input-Output Database: 2006 Edition* as imports over gross output (rather than GDP), so that they can be interpreted as a share of (gross) total expenditures allocated to imports (see Norihiko and Ahmad (2006)).

trade in Eaton and Kortum (2002). Nevertheless, our results imply that the gains from trade in these two models are the same: as we go from Anderson (1979) to Eaton and Kortum (2002), the appearance of production gains must be exactly compensated by a decline in consumption gains from trade.

Section 4 turns to the case of gravity models with monopolistic competition. In this situation, the intuition behind our welfare formula is more subtle. In addition to their effects on relative wages, changes in trade costs now have implications for firms' entry decisions as well as their selection into exports. Both effects lead to changes in the set of goods available in each country, which must also be taken into account in our welfare analysis. A CES import demand system again greatly simplifies the analysis. On the one hand, it guarantees that the number of entrants must remain constant under free entry. On the other hand, it guarantees that any welfare change not caused by changes in the number of entrants—whether it affects relative wages or the set of goods available in a given country—can still be inferred from changes in the share of domestic expenditure using the trade elasticity offered by the gravity equation. Our welfare formula directly follows from these two observations.

Section 5 investigates the robustness of our results. We first explore how our simple welfare formula may extend to other gravity models. Following the recent literature on trade and firm heterogeneity, we consider models with restricted entry, as in Chaney (2008), endogenous marketing costs, as in Arkolakis (2008), and models with multi-product firms, in the spirit of Bernard, Redding and Schott (2009) and Arkolakis and Muendler (2007). Although some of these extensions are crucial to explain micro-level facts, e.g., the impact of trade liberalization on the distributions of firm size and firm productivity, we show that they leave our simple welfare formula unchanged.

Finally, we consider generalizations of gravity models, including models with multiple sectors, as in Costinot and Komunjer (2007) and Donaldson (2008), multiple factors, as in Bernard, Redding and Schott (2007) and Chor (2009), and tradable intermediate goods, as in Eaton and Kortum (2002), Alvarez and Lucas (2007), and Di Giovanni and Levchenko (2009). While our simple welfare formula no longer holds in these richer environments, we demonstrate that generalized versions can easily be derived using the same logic as in Sections 3 and 4. In particular, we show that conditional on a given market structure, either perfect or monopolistic competition, there still exists aggregate sufficient statistics for welfare analysis. Compared to our previous results, the main difference is that the equivalence between generalized gravity models with perfect and monopolistic competition may break down due to changes in the number of entrants.

Our paper is related to the recent literature in public finance trying to isolate robust insights for welfare analysis across different models; see e.g. Chetty (2009). As in that literature, using a "sufficient statistics approach" allows us to make welfare predictions without having to solve for all endogenous variables in our model. In a field such as international trade where general equilibrium effects are numerous, this represents a significant benefit.

In the international trade literature, there is now a large number of empirical papers focusing on the measurement of the gains from trade; see e.g. Feenstra (1994), Klenow and Rodríguez-Clare (1997), Broda and Weinstein (2006), Feenstra and Kee (2008), Goldberg, Khandelwal, Pavcnik and Topalova (2009), and Feenstra and Weinstein (2009). The purpose of such exercises is to quantify the contribution of particular margins, e.g., new goods or new products, to changes in the consumer price index. The goal of our paper is quite different: instead of establishing that a particular margin is small or large, we stress that in many new trade models, whatever the contribution of particular margins may be, the total size of the gains from trade can always be computed using the same aggregate statistics, λ and $\overline{\varepsilon}$.

From a theoretical standpoint, our paper builds on the seminal contribution of Eaton and Kortum (2002) who first computed real wages as a function of λ and ε in a Ricardian model with Dixit-Stiglitz preferences and productivity levels drawn from a Fréchet distribution. In recent work, Arkolakis, Demidova, Klenow and Rodríguez-Clare (2008) also used closed forms to compute the real wage as a function of λ and ε in a Melitz-type model with Dixit-Stiglitz preferences, monopolistic competition, free or restricted entry, and heterogenous firms with a Pareto distribution of productivity. Noting that the expression was similar to the one derived by Eaton and Kortum (2002)—and could have been derived by Krugman (1980)—Arkolakis, Demidova, Klenow and Rodríguez-Clare (2008) argued that the gains from trade in these models were the same.[4] The main difference between our paper and theirs is in terms of method and scope. Our analysis goes beyond their original claim by identifying two critical macro-level restrictions, a CES import demand system and gravity, such that for a particular, but important class of models, λ and $\overline{\varepsilon}$ are sufficient statistics for the estimation of the gains from trade. This general approach allows us to offer a unifying perspective on the welfare implications of gravity models under different market structures.

[4] In a recent paper, Feenstra (2009) uses duality theory to revisit, among other things, the results of Arkolakis, Demidova, Klenow and Rodríguez-Clare (2008). Under the same functional form assumptions, he shows how the gains from trade in the Melitz-type model computed by Arkolakis, Demidova, Klenow and Rodríguez-Clare (2008) can be interpreted as "production gains" from trade, whereas the gains from trade in Krugman (1980) can be interpreted as "consumption gains." However, he does not discuss the fact that conditional on trade data, the total size of the gains from trade predicted by these two models is the same. This is our main focus.

Another related paper is Atkeson and Burstein (2009), which focuses on the welfare gains from trade liberalization through its effects on entry and exit of firms and their incentives to innovate in a monopolistically competitive environment with symmetric countries as in Melitz (2003). At the theoretical level, they show that if small changes in trade costs are symmetric, then their impact on welfare must be the same as in Krugman (1980). While their analytical results and ours have the same flavor, the logic is quite different. Their results consist in showing that in this environment, the overall contribution of "new" margins, i.e. any margins not already present in Krugman (1980), must be offset, to a first-order approximation, by changes in entry. By contrast, this offsetting effect is (generically) absent from the gravity models with monopolistic competition considered in our paper. Our results simply state that whatever the welfare effects associated with these new margins are, the total size of the gains from trade can still be inferred from aggregate trade flows alone.

2 Gravity Models

The objective of this section is to clarify the scope of our analysis by describing some of the main features of the trade models considered in the next two sections.

The Basic Environment. Throughout this paper, we consider a world economy comprising $i = 1, ..., n$ countries; one factor of production, labor; and multiple goods indexed by $\omega \in \Omega$. The number of goods in Ω may be continuous or discrete. Each country is populated by a continuum of workers with identical Dixit-Stiglitz preferences

$$U_i = \left[\int_{\omega\in\Omega} q_i(\omega)^{\frac{\sigma}{\sigma-1}} d\omega\right]^{\frac{\sigma-1}{\sigma}},$$

where $q_i(\omega)$ is the quantity consumed of good ω in country i and $\sigma > 1$ is the elasticity of substitution between goods ω.[5] We denote by P_i the consumer price index in country i. We assume that workers are immobile across countries. L_i and w_i denote the total endowment of labor and the wage in country i, respectively. Finally, we assume that technology is such that all cost functions are linear in output.

Bilateral Imports. We denote by X_{ij} the value of country j's imports from country i, by $Y_j \equiv \sum_{i'=1}^{n} X_{i'j}$ the total expenditure in country j, and by $\lambda_{ij} \equiv X_{ij}/Y_j$ the share of country

[5] Since the number of goods in Ω may be continuous or discrete, one should think of U_i as a Lebesgue integral. Thus when Ω is a finite or countable set, $\int_{\omega\in\Omega} q_i(\omega)^{\frac{\sigma}{\sigma-1}} d\omega$ is equivalent to $\sum_{\omega\in\Omega} q_i(\omega)^{\frac{\sigma}{\sigma-1}}$.

$j's$ total expenditure that is devoted to goods from country i. In any trade equilibrium, we assume complete specialization in the sense that almost all goods are bought from only one source, though this source may vary across countries. Formally, if we denote by $\Omega_{ij} \subset \Omega$ the set of goods that country j buys from country i, complete specialization requires the measure of goods in $\Omega_{ij} \cap \Omega_{i'j}$ to be equal to zero for all i, $i' \neq i$, and j.[6] Accordingly, bilateral imports can be expressed as

$$X_{ij} = \int_{\omega \in \Omega_{ij}} p_j(\omega)\, q_j(\omega)\, d\omega, \tag{1}$$

where $p_j(\omega)$ denotes the price of good ω in country j.

Bilateral Trade Costs. Trade flows are subject to variable trade costs of the standard iceberg form: in order to sell one unit in country j, firms from country i must ship $\tau_{ij} \geq 1$ units. We assume that the matrix of variable trade costs $\boldsymbol{\tau} \equiv \{\tau_{ij}\}$ is such that $\tau_{ii} = 1$ for all i and $\tau_{il}\tau_{lj} \geq \tau_{ij}$ for all i, l, j. Depending on the market structure, trade flows may also be subject to fixed costs (Section 4).

The Import Demand System. Let $\mathbf{X} \equiv \{X_{ij}\}$ denote the $n \times n$ matrix of bilateral imports and $\mathbf{E}$ denote the vector of country-specific equilibrium variables in the economy. Under perfect competition (Section 3), $\mathbf{E}$ is equal to the vector of wages in each country, whereas under monopolistic competition $\mathbf{E}$ includes the vector of wages and number of entrants in each country (Section 4). We refer to the mapping from variable trade costs, $\boldsymbol{\tau}$, and equilibrium variables, $\mathbf{E}$, to bilateral imports, $\mathbf{X}$, as the *import demand system.* With a slight abuse of notation, we write $X_{ij} \equiv X_{ij}(\boldsymbol{\tau}, \mathbf{E})$. This mapping, of course, depends on the other primitives of the model: preferences, technology, and market structure. This simple formulation allows us to distinguish between the direct impact of variable trade costs and their indirect impact through general equilibrium effects.

Throughout this paper, we will restrict ourselves to a class of models where the import demand system satisfies two macro-level restrictions.

Macro-level Restriction (I): CES import demand system. Let $\varepsilon_j^{ii'} \equiv \partial \ln (X_{ij}/X_{jj}) / \partial \ln \tau_{i'j}$ denote the elasticity of relative imports with respect to variable trade costs and let $\boldsymbol{\varepsilon}_j \equiv \left\{\varepsilon_j^{ii'}\right\}_{i,i' \neq j}$ denote the associated $(n-1) \times (n-1)$ matrix. In any trade equilibrium, the

[6] This definition of complete specialization allows multiple countries to produce the same good. It only rules out equilibria such that multiple countries sell the same good in the same country.

import demand system is such that

$$\boldsymbol{\varepsilon}_j = \begin{pmatrix} \varepsilon & 0 & ... & 0 \\ 0 & ... & ... & ... \\ ... & ... & ... & 0 \\ 0 & ... & 0 & \varepsilon \end{pmatrix} \tag{2}$$

with $\varepsilon < 0$, which is the empirically relevant case. We refer to an import demand system such that Equation (2) is satisfied for all j as a "CES import demand system" and to ε as the "trade elasticity" of that system.[7]

It should be clear that this macro-level restriction imposes a lot of symmetry among countries. First, since all the diagonal terms are equal in Equation (2), any given change in bilateral trade costs, τ_{ij}, must have the same impact on relative demand, X_{ij}/X_{jj}, for all $i \neq j$. Second, since all the off-diagonal terms are equal to zero, any change in a third country trade costs, $\tau_{i'j}$, must have the same proportional impact on X_{ij} and X_{jj}. Put differently, changes in relative demand are "separable" across exporters: the relative demand for goods from country i, X_{ij}/X_{jj}, only depends on τ_{ij}.

Two additional comments are in order. First, the trade elasticity is a *partial derivative*: it captures the direct effect of changes in variable trade costs on $p_j(\omega)$ and Ω_{ij}, but not their indirect effect through changes in wages or the total number of entrants. Second, the trade elasticity is an *upper-level* elasticity: it tells us how changes in variable trade costs affect *aggregate* trade flows, whatever the particular margins of adjustment, $p_j(\omega)$ or Ω_{ij}, may be.

Macro-level restriction (II): Gravity. In any trade equilibrium, the import demand system satisfies "gravity" in the sense that bilateral imports can be decomposed into

$$\ln X_{ij}(\boldsymbol{\tau}, \mathbf{E}) = A_i(\boldsymbol{\tau}, \mathbf{E}) + B_j(\boldsymbol{\tau}, \mathbf{E}) + \varepsilon \ln \tau_{ij} + \nu_{ij}, \tag{3}$$

where $A_i(\cdot)$, $B_j(\cdot)$, ε and ν_{ij} all depend on preferences, technology, and market structure. Note that according to Equation (3), ν_{ij} is not a function of $\boldsymbol{\tau}$ and $\mathbf{E}$. This is an important restriction, which makes Equation (3) an assumption, rather than a mere definition of ν_{ij}.

[7] Our choice of terminology derives from the fact that in the case of a CES *demand* system, changes in relative demand, C_k/C_l, for two goods k and l are such that $\partial \ln(C_k/C_l)/\partial \ln p_{k'} = 0$ if $k' \neq k, l$ and $\partial \ln(C_k/C_l)/\partial \ln p_k = \partial \ln(C_{k'}/C_l)/\partial \ln p_{k'} \neq 0$ for all $k, k' \neq l$. Nevertheless, it should be clear that the assumption of a CES import demand system is conceptually distinct from the assumption of CES preferences. While the import demand obviously depends on preferences, it also takes into account the supply side as this affects the allocation of expenditures to domestic production.

Under standard orthogonality conditions, the previous gravity equation offers a simple way to obtain a consistent estimate, $\bar{\varepsilon}$, of the trade elasticity using data on bilateral imports, X_{ij}, and bilateral trade costs, τ_{ij}. For example, if the underlying distribution of τ_{ij} and ν_{ij} across countries satisfies $E\left(\nu_{ij}\ln\tau_{i'j'}\right)=0$, for any i, i', j, and $j'=1,...,N$, then $\bar{\varepsilon}$ can be computed as a simple difference-in-difference estimator. In the rest of this paper, we will remain agnostic about the exact form of the orthogonality condition associated with Equation (3), but assume that the same orthogonality condition can be invoked in all models. Without any risk of confusion, we can therefore refer to $\bar{\varepsilon}$ as the "gravity-based" estimate of the trade elasticity, whatever the particular details of the model may be.[8]

It is worth emphasizing that the two previous restrictions, CES and gravity, are different in nature and will play distinct roles in our analysis. CES imposes restrictions on how changes in variable trade costs affect relative import demands *across* trade equilibria. By contrast, gravity imposes restrictions on the cross-sectional variation of bilateral imports *within* a given trade equilibrium. The former property will allow us to express gains from trade as a function of the share of expenditure on domestic goods and the true trade elasticity, whereas the latter will be important to obtain an estimate of the trade elasticity from observable trade data. Finally, note that both properties are easy to check since they do not require to solve for the endogenous equilibrium variables included in $\mathbf{E}$.

Asymptotic Behavior. For technical reasons, we also assume that for any pair of countries $i\neq j$, $\lim_{\tau_{ij}\to+\infty}\left(w_i\tau_{ij}/w_j\right)=+\infty$. This mild regularity condition guarantees that trade equilibria converge to the autarky equilibrium as variable trade costs $\boldsymbol{\tau}$ go to infinity.

To summarize, the main features of the trade models analyzed in our paper include five basic assumptions: (*i*) Dixit-Stiglitz preferences; (*ii*) one factor of production; (*iii*) linear costs functions; (*iv*) complete specialization; and (*v*) iceberg trade costs; and two macro-level restrictions: (*i*) a CES import demand system; and (*ii*) gravity. Although these assumptions are admittedly restrictive, it is easy to check that they are satisfied in many existing trade models including Anderson (1979), Krugman (1980), Eaton and Kortum (2002), Anderson and Van Wincoop (2003), Bernard, Eaton, Jensen and Kortum (2003), and multiple variations and extensions of Melitz (2003), such as Chaney (2008), Arkolakis (2008), Eaton, Kortum and Kramarz (2008), and Arkolakis and Muendler (2007).[9]

[8]Of course, the exact value of $\bar{\varepsilon}$ as a function of trade data depends on the choice of the orthogonality condition. The crucial assumption for our purposes, however, is that conditional on the choice of the orthogonality condition, the exact value of $\bar{\varepsilon}$ is the same in all models.

[9]This being said, we wish to be very clear that our analysis does not apply to all variations and extensions

From now on, we refer to trade models satisfying the assumptions described in this section as "gravity" models. The rest of our paper explores the welfare implications of this class of models under two distinct market structures: perfect and monopolistic competition. The theoretical question that we are interested in is the following. Consider a hypothetical change in variable trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau}'$, while keeping labor productivity and labor endowments fixed around the world.[10] What is the percentage change in real income needed to bring a representative worker from some country j back to her original utility level?

3 Gains from Trade (I): Perfect Competition

We start by assuming perfect competition. Given our assumptions on technology and the number of factors of production, gravity models simplify into Ricardian models under perfect competition. In this case, the vector of country specific equilibrium variables is equal to the vector of wages, $\mathbf{E} = (w_1, ..., w_n)$. To simplify notations, we suppress the arguments $(\boldsymbol{\tau}, \mathbf{E})$ from our trade variables in the rest of this section.

3.1 Equilibrium conditions

Perfect competition requires goods to be priced at marginal costs:

$$p_j(\omega) = \frac{w_i \tau_{ij}}{z_i(\omega)}, \text{ for all } \omega \in \Omega_{ij}, \tag{4}$$

where $z_i(\omega) > 0$ is the labor productivity for the production of good ω in country i. In addition, perfect competition requires each good to be produced in the country that minimizes costs of production and delivery. Hence, we have

$$\Omega_{ij} = \left\{ \omega \in \Omega | \frac{w_i \tau_{ij}}{z_i(\omega)} = \min_{1 \leq i' \leq n} \frac{w_{i'} \tau_{i'j}}{z_{i'}(\omega)} \right\}. \tag{5}$$

of Melitz (2003). Helpman, Melitz and Rubinstein (2008), for example, falls outside the scope of our paper. In their model bilateral trade flows go to zero for sufficiently large bilateral trade costs. This corresponds to a situation in which ν_{ij} in Equation (3) is a function of trade costs, thereby violating our gravity property. For similar reasons, the trade elasticity is not constant in this model, contradicting our CES property.

[10] This is a non-trivial restriction. When measuring the gains from trade, we will implictly abstract from any direct channel through which changes in trade costs may affect labor productivity and labor endowments. See Grossman and Helpman (1991) for an overview of trade models allowing for such effects.

Finally, trade balance implies[11]

$$Y_j = w_j L_j, \tag{6}$$

Equipped with these three equilibrium conditions, we now investigate how changes in variable trade costs affects welfare in each country.

3.2 Welfare analysis

Without loss of generality, we focus on a representative worker from country j and use labor in country j as our numeraire, $w_j = 1$. We start by considering a small change in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau} + d\boldsymbol{\tau}$. Since the set of goods Ω is fixed under perfect competition, changes in the consumer price index satisfy

$$\widehat{P}_j = \int_{\Omega} \lambda_j (\omega) \widehat{p}_j(\omega) d\omega, \tag{7}$$

where $\widehat{x} \equiv dx/x$ denotes the relative change in a given variable x; and $\lambda_j (\omega)$ is the share of expenditure on good ω in country j.[12] Using Equations (4) and (5) and the fact that there is complete specialization, we can rearrange Equation (7) as

$$\widehat{P}_j = \textstyle\sum_{i=1}^{n} \lambda_{ij} \left(\widehat{w}_i + \widehat{\tau}_{ij}\right). \tag{8}$$

Equation (8) reminds us that in a neoclassical environment, all changes in welfare must be coming from changes in terms of trade. Since labor in country j is our numeraire, $\widehat{w}_j = 0$, these changes are exactly equal to $\widehat{w}_i + \widehat{\tau}_{ij}$. By the Envelope Theorem, changes in trade shares can only have a second order effect.

While the previous result is well-known, stronger welfare predictions can be derived in the case of a CES import demand system. The core of our analysis relies on the following lemma.

Lemma 1 *In any gravity model with perfect competition, percentage changes in the con-*

[11] Trade balance only plays a minor role in our analysis. All our results would hold under the weaker assumption that there are trade deficits, but that their relative magnitude is unaffected by changes in trade costs.

[12] Throughout this section, the assumption of Dixit-Stiglitz preferences is stronger than needed. We merely use the fact that all agents have identical homothetic preferences, which allows us to define *the* consumer price index in country j. Dixit-Stiglitz preferences will, of course, play a crucial role when we study monopolistic competition in Section 4.

sumer price index satisfy

$$\widehat{P}_j = -\widehat{\lambda}_{jj} \Big/ \varepsilon. \tag{9}$$

The formal proof as well as all subsequent proofs can be found in the Appendix. The logic can be sketched as follows. Under perfect competition, for any exporter i', a one percent increase in $w_{i'}$ has the same effect on country j and other exporters as a one percent increase in $\tau_{i'j}$. By definition of $\varepsilon_j^{ii'}$, changes in bilateral imports must therefore satisfy

$$\widehat{X}_{ij} - \widehat{X}_{jj} = \textstyle\sum_{i' \neq j} \varepsilon_j^{ii'} \left(\widehat{w}_{i'} + \widehat{\tau}_{i'j}\right). \tag{10}$$

A direct implication of Equation (10) is that if all elasticities $\varepsilon_j^{ii'}$ are known, changes in terms of trade can be inferred from changes in relative imports. To do so, we simply need to invert a system of $(n-1) \times (n-1)$ equations. Assuming that $\boldsymbol{\varepsilon}_j$ is invertible, which will always be true in the case of a CES import demand system, Equations (8) and (10) imply

$$\widehat{P}_j = \boldsymbol{\lambda}_j' \boldsymbol{\varepsilon}_j^{-1} \widehat{\mathbf{X}}_j, \tag{11}$$

where

$$\begin{aligned} \widehat{\mathbf{X}}_j' &= \left(\widehat{X}_{1j} - \widehat{X}_{jj}, ..., \widehat{X}_{(j-1)j} - \widehat{X}_{jj}, \widehat{X}_{(j+1)j} - \widehat{X}_{jj}, ...\widehat{X}_{nj} - \widehat{X}_{jj}\right); \\ \boldsymbol{\lambda}_j' &= \left(\lambda_{1j}, ...\lambda_{(j-1)j}, \lambda_{(j+1)j}, ...\lambda_{nj}\right). \end{aligned}$$

Equation (11) provides a general characterization of welfare changes as a function of initial trade shares, changes in trade flows, and upper level elasticities, whatever the particular characteristics of the import demand system may be. Lemma 1 then simply derives from the fact that in a gravity model, the import demand system is CES. The separability of changes in relative demand across exporters implies $\boldsymbol{\lambda}_j' \boldsymbol{\varepsilon}_j^{-1} \widehat{\mathbf{X}}_j = -\sum_{i=1}^n \lambda_{ij} \left(\widehat{X}_{ij} - \widehat{X}_{jj}\right) \Big/ \varepsilon_j^{ii}$, whereas the symmetry across exporters implies $-\sum_{i=1}^n \lambda_{ij} \left(\widehat{X}_{ij} - \widehat{X}_{jj}\right) \Big/ \varepsilon_j^{ii} = -\sum \widehat{\lambda}_{jj} \Big/ \varepsilon$. Equation (9) follows from these two observations.

It is worth emphasizing that Lemma 1 is a local result that does not depend on the assumption that ε is the same across all trade equilibria or countries. If we were to relax the specification of the import demand system so that we had $\varepsilon_j^{ii} = \varepsilon_j(\boldsymbol{\tau}, \mathbf{E})$ and $\varepsilon_j^{ii'} = 0$ for all j and $i \neq i'$, then Lemma 1 would still hold. By contrast, our global results will heavily rely on the fact that ε is invariant to changes in trade costs.

We now consider the welfare impact of large changes in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau}'$. Let P_j' denote the consumer price index in country j if trade costs are equal to $\boldsymbol{\tau}'$; and let $W_j \equiv 1 - \left(P_j' \big/ P_j\right)$ denote the (negative of) the percentage change in real income needed to bring a representative worker from country j back to her original utility level. Our first global result can be stated as follows.

Proposition 1 *In any gravity model with perfect competition, W_j can be consistently estimated by $1 - \left(\lambda_{jj}/\lambda'_{jj}\right)^{1/\bar{\varepsilon}}$, where λ'_{jj} and λ_{jj} are evaluated at the new and initial trade equilibrium, respectively.*

Proposition 1 derives from two observations. On the one hand, the fact that ε is constant implies that we can integrate Equation (9) between $\boldsymbol{\tau}$ and $\boldsymbol{\tau}'$ to get $W_j = 1 - \left(\lambda_{jj}/\lambda'_{jj}\right)^{1/\varepsilon}$. On the other hand, the fact that $\bar{\varepsilon}$ is a consistent estimator of ε implies, by a standard continuity argument, that $1 - \left(\lambda_{jj}/\lambda'_{jj}\right)^{1/\bar{\varepsilon}}$ is a consistent estimator of $1 - \left(\lambda_{jj}/\lambda'_{jj}\right)^{1/\varepsilon}$.

The implication of this proposition is that the welfare effect of a change in trade costs in a gravity model with perfect competition can be measured using only: (i) the initial and the new share of expenditure on domestic goods, λ_{jj} and λ'_{jj}; and (ii) the gravity-based estimate of the trade elasticity, $\bar{\varepsilon}$. This offers a parsimonious way to compute welfare changes resulting from changes in trade costs. In particular, one does not need to observe the way in which *all* prices change, as would be suggested by Equation (7); it is sufficient to have information about the trade elasticity, $\bar{\varepsilon}$, and the changes in trade flows as summarized by λ_{jj} and λ'_{jj}. Note also that since $\bar{\varepsilon}$ is negative in practice, see e.g. Anderson and Van Wincoop (2004), welfare increases, $W_j > 0$, whenever country j becomes more open, $\lambda'_{jj} < \lambda_{jj}$.

We define the *gains from trade in country j*, denoted by $\overline{W}_j$, as the percentage change in current income needed to bring country j's representative agent back to its original utility level after going to autarky, i.e. after increasing all τ_{ij}, $i \neq j$, to infinity.[13] Proposition 1 and the fact that $\lambda'_{jj} = 1$ under autarky immediately implies the following result.

Proposition 2 *In any gravity model with perfect competition, $\overline{W}_j$ can be consistently estimated by $(\lambda_{jj})^{1/\bar{\varepsilon}} - 1$, where λ_{jj} is evaluated at the initial equilibrium.*

This result implies that conditional on observed trade data, i.e. the values of λ_{jj} and $\bar{\varepsilon}$ in current trade equilibrium, the gains from trade predicted by all gravity models under perfect competition must be the same. Within that class of models, new sources of gains from trade may affect the composition of the gains from trade, but not their total size.[14]

[13] Formally, $\overline{W}_j$ is equal to $-W_j$ evaluated at the counterfactual equilibrium with $\boldsymbol{\tau} = +\infty$.

[14] Throughout this paper, we have chosen to interpret $\boldsymbol{\tau}$ as iceberg trade costs rather than tariffs. It should be clear that our analysis of how changes in $\boldsymbol{\tau}$ affect the price index does not depend on this particular interpretation. The only difference between the two interpretations is that changes in tariffs would also

3.3 Anderson (1979) vs. Eaton and Kortum (2002)

To get a better understanding of the equivalence result emphasized in Proposition 2, we now compare two well-known trade models, Anderson (1979) and Eaton and Kortum (2002).

On the demand side, both models assume Dixit-Stiglitz preferences. The main difference between the two models comes from the supply side. In Anderson (1979), countries cannot produce the goods produced by other countries: if $a_i(\omega) < +\infty$, then $a_{i'}(\omega) = +\infty$ for all $i' \neq i$.[15] By contrast, Eaton and Kortum (2002) assume that in each country, unit labor requirements are drawn from an extreme value distribution. From a qualitative standpoint, this is an important difference. It implies that there are both production and consumption gains from trade in Eaton and Kortum (2002), whereas there can only be consumption gains from trade in Anderson (1979).

Does that mean that the two models lead to different quantitative predictions about the size of the gains from trade? The answer is no. It is easy to check that both models fit the five basic assumptions of gravity models given in Section 2. Furthermore, the import demand system is such that

$$X_{ij} = \frac{T_i (w_i \tau_{ij})^{\varepsilon} w_j L_j}{\sum_{i'=1}^{I} T_{i'} (w_{i'} \tau_{i'j})^{\varepsilon}}, \tag{12}$$

where T_i and T_j are country-specific technology parameters, $\varepsilon \equiv$"$1-\sigma$" in Anderson (1979) and $\varepsilon \equiv$"$-\theta$" in Eaton and Kortum (2002). Because of alternative microtheoretical foundations, the (negative of the) trade elasticity is equal to the elasticity of substitution between goods (minus one) in Anderson (1979) and it is equal to the shape parameter of the productivity distribution in Eaton and Kortum (2002). In both models, however, Equation (12) implies that the import demand system satisfies our two macro-level restrictions: CES and gravity. We can therefore invoke Proposition 2 to conclude that conditional on two sufficient statistics, λ_{jj} and $\bar{\varepsilon}$, the gains from trade predicted by Anderson (1979) and Eaton and Kortum (2002) must be the same.[16]

This equivalence illustrates one of the main points of our paper in a very clear manner.

affect total income through changes in tariff revenues. Since tariff revenues are equal to zero under autarky, our estimator of the gains from trade would simply become $\overline{W}_j = (\lambda_{jj})^{1/\bar{\varepsilon}} (1 + T_j) - 1$, where T_j is the share of tariff revenues in country j's income in the initial equilibrium. Thus conditional on λ_{jj}, $\bar{\varepsilon}$, and T_j, the gains from trade predicted by all gravity models with perfect competition would remain the same.

[15] Under this assumption, endowment or "Armington" models such as Anderson (1979) can always be reinterpreted as particular Ricardian models; see Matsuyama (2007).

[16] Using Equation (12), one can actually show a stronger result. Conditional on trade data, $\mathbf{X}$ and $\bar{\varepsilon}$, the predicted changes in welfare associated with any counterfactual changes in trade costs, not just movements to autarky, are the same in Anderson (1979) and Eaton and Kortum (2002). See Appendix for details.

Since Eaton and Kortum (2002) allows countries to specialize according to comparative advantage whereas Anderson (1979) does not, one may think that the gains from trade predicted by Eaton and Kortum (2002) must be larger. Our analysis demonstrates that this is not the case. As we switch from Anderson (1979) to Eaton and Kortum (2002), the structural interpretation of the trade elasticity changes, reflecting the fact there is now one more margin, namely Ω_{ij}, for bilateral imports to adjust. However, conditional on the estimated value of the upper-level elasticity, $\bar{\varepsilon}$, more margins of adjustment can only affect the composition of the gains from trade.

4 Gains From Trade (II): Monopolistic Competition

We now turn to the case of monopolistic competition. In this environment, gains from trade may also derive from changes in the number of available varieties in each country, as in Krugman (1980), as well as from changes in aggregate productivity due to intra-industry reallocation, as in Melitz (2003).

In line with the previous literature, we now refer to goods as "varieties." We assume that there is an unbounded pool of potential entrants capable of producing differentiated varieties. In order to produce in country i, firms must incur a fixed entry cost, $f_e > 0$, in terms of domestic labor. M_i denotes the total measure of entrants in country i. Upon entry, these firms draw their productivity, $z(\omega)$, from a known distribution with density g_i. In order to sell their varieties to country j, firms from country i must then incur a fixed marketing cost, $f_{ij} \geq 0$, in terms of domestic labor. After marketing costs have been paid, trade flows are subject to iceberg trade costs $\tau_{ij} \geq 1$ as in the neoclassical case.

Throughout this section, we refer to gravity models satisfying the previous assumptions as "gravity models with monopolistic competition." In this case, the vector of country specific equilibrium variables is equal to the vector of wages and measures of entrants, $\mathbf{E} = (w_1, ..., w_n, M_1, ..., M_n)$. To simplify notation, we again suppress the arguments $(\boldsymbol{\tau}, \mathbf{E})$ from our trade variables in the rest of our analysis.

4.1 Equilibrium conditions

Because of Dixit-Stiglitz preferences, monopolists charge a constant markup over marginal cost. In any country j, the price of a variety ω from country i is given by

$$p_j(\omega) = \frac{\sigma \tau_{ij} w_i}{(\sigma - 1)\, z(\omega)} \text{ for all } \omega \in \Omega_{ij}. \tag{13}$$

The associated profits, $\pi_{ij}(\omega)$, of a firm with productivity $z(\omega)$ operating in country i and selling in country j can be written as

$$\pi_{ij}(\omega) = \left[\frac{\sigma \tau_{ij} w_i}{(\sigma - 1)\, z(\omega)\, P_j}\right]^{1-\sigma} \frac{Y_j}{\sigma} - w_i f_{ij}, \tag{14}$$

where $P_j = \left[\sum_{i=1}^{n} \int_{\omega \in \Omega_{ij}} p_j^{1-\sigma}(\omega) d\omega\right]^{\frac{1}{1-\sigma}}$ is the consumer price index in country j. The set of varieties from country i available in country j, Ω_{ij}, is determined by the following zero-profit condition

$$\Omega_{ij} = \{\omega \in \Omega | \pi_{ij}(\omega) \geq 0\}. \tag{15}$$

Equations (14) and (15) implicitly define a unique cut-off

$$z_{ij}^* = \left[\frac{\sigma \tau_{ij} w_i}{(\sigma - 1)\, P_j}\right] \left(\frac{w_i f_{ij} \sigma}{Y_j}\right)^{\frac{1}{\sigma-1}} \tag{16}$$

such that firms from country i sell variety ω in country j if and only if $z(\omega) \geq z_{ij}^*$. Finally, free entry implies that total expected profits are equal to fixed entry costs,

$$\sum_{i=1}^{n} E\left[\pi_{ji}(\omega)\right] = w_j f_e, \tag{17}$$

and trade balance implies that total income is equal to the total wage bill

$$Y_j = w_j L_j. \tag{18}$$

4.2 Welfare analysis

Without loss of generality, we again focus on a representative worker from country j and use labor in country j as our numeraire, $w_j = 1$. Like in Section 3, we first consider small changes in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau} + d\boldsymbol{\tau}$. Using Equations (13) and the definitions of P_j,

M_i, and z_{ij}^*, we can express changes in the consumer price index as

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij} \left[(\widehat{w}_i + \widehat{\tau}_{ij}) - \frac{\widehat{M}_i}{\sigma - 1} + \frac{\gamma_{ij} \widehat{z_{ij}^*}}{\sigma - 1} \right], \tag{19}$$

where

$$\gamma_{ij} \equiv \left(z_{ij}^* \right)^{\sigma} g_i \left(z_{ij}^* \right) / \int_{z_{ij}^*}^{+\infty} z^{\sigma-1} g_i (z) \, dz. \tag{20}$$

Compared to welfare changes in the neoclassical case, Equation (8), there are two extra terms, $\widehat{M}_i / (\sigma - 1)$ and $\gamma_{ij} \widehat{z_{ij}^*} / (\sigma - 1)$. Under monopolistic competition, these two terms reflect the fact that changes in trade costs may affect the set of varieties available in each country, thereby creating new potential sources of gains from trade.

First, trade costs may affect firms' entry decisions. If a change in trade costs raises the number of entrants in country i, $\widehat{M}_i > 0$, the total number of varieties in country j will increase and its consumer price index will decrease by $\lambda_{ij} \widehat{M}_i / (\sigma - 1) > 0$. Second, trade costs may affect firms' selection into exports. If a small change in trade costs lowers the cut-off productivity level, $\widehat{z_{ij}^*} < 0$, the total number of varieties in country j will also increase. But, unlike changes in M_i, changes in z_{ij}^* will affect the composition of varieties from country i in country j, as new exporters are less productive than existing ones. This argument explains why the consumer price index will decrease by $-\lambda_{ij} \gamma_{ij} \widehat{z_{ij}^*} / (\sigma - 1) > 0$ with the coefficient γ_{ij} adjusting for changes in the number and composition of varieties available in country j.

Again, the question that we want to ask is: Does the introduction of new sources of gains from trade lead to larger gains from trade? Using Lemma 2, we will demonstrate that in the case of a gravity model with monopolistic competition, the answer is still no.

Lemma 2 *In any gravity model with monopolistic competition, percentage changes in the consumer price index satisfy*

$$\widehat{P}_j = - \left. \widehat{\lambda}_{jj} \middle/ \varepsilon. \right. \tag{21}$$

Equation (21) shows that conditional on $\hat{\lambda}_{jj}$ and ε, the welfare impact of changes in trade costs is the same as under perfect competition. The proof of Lemma 2 can be sketched as follows. Using Equation (16) and the fact that $\widehat{Y}_j = \widehat{w}_j = 0$ by our choice of numeraire, we can express changes in the productivity cut-off, $\widehat{z_{ij}^*}$, as

$$\widehat{z_{ij}^*} = \widehat{w}_i + \widehat{\tau}_{ij} + \left(\frac{1}{\sigma - 1} \right) \widehat{w}_i - \widehat{P}_j. \tag{22}$$

Combining Equations (19) and (22), we then obtain

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij} \left[\frac{\left(\sigma - 1 + \gamma_{ij}\right)\left(\widehat{w}_i + \widehat{\tau}_{ij}\right)}{\sigma - 1 + \gamma_j} - \frac{\widehat{M}_i}{\sigma - 1 + \gamma_j} - \frac{\gamma_{ij}\widehat{w}_i}{\left(\sigma - 1 + \gamma_j\right)(1 - \sigma)} \right] \tag{23}$$

where $\gamma_j \equiv \sum_{i=1}^{n} \lambda_{ij}\gamma_{ij}$. Equation (23) illustrates two potential differences between perfect and monopolistic competition. The first one, which we have already mentioned, is that the set of varieties available in country j is not fixed. As a result, changes in terms of trade, $\widehat{w}_i + \widehat{\tau}_{ij}$, may not be sufficient to compute welfare changes; in principle, one may also need to keep track of changes in the number and composition of varieties, as captured by $-\widehat{M}_i \big/ \left(\sigma - 1 + \gamma_j\right) + \gamma_{ij}\widehat{w}_i \big/ \left(\sigma - 1 + \gamma_j\right)(1 - \sigma)$. The second difference, which is more subtle, is related to the impact of terms of trade changes, $\widehat{w}_i + \widehat{\tau}_{ij}$. Even in the absence of changes in the set of available varieties, Equation (23) shows that changes at the extensive margin, i.e. changes in Ω_{ij}, may directly affect the mapping between $\widehat{P}_j$ and $\widehat{w}_i + \widehat{\tau}_{ij}$. Because changes in terms of trade may lead to the creation and destruction of varieties with different prices in different countries, their impact may vary across countries, hence the correction term $\left(\sigma - 1 + \gamma_{ij}\right) \big/ \left(\sigma - 1 + \gamma_j\right)$. In contrast, under perfect competition new varieties never have a different impact since the price of a good no longer produced by one country is equal to its price in the new producing country.

The rest of our proof relies on the properties of a CES import demand system. We start by showing that under a CES import demand system, the second of these two differences necessarily is absent. Symmetry across countries implies $\gamma_{ij} = \gamma_j = 1 - \sigma - \varepsilon$ for all i and j, which means that the impact of changes in terms of trade must be the same for all exporters in any importing country. As a result, we can use the same strategy as under perfect competition to infer changes in terms of trade from changes in relative imports. After simple rearrangements, Equation (23) can be expressed as

$$\widehat{P}_j = -\frac{\widehat{\lambda}_{jj} - \widehat{M}_j}{\varepsilon}. \tag{24}$$

According to Equation (24), welfare changes in country j only depends on changes in two domestic variables: the share of expenditure on domestic goods, $\widehat{\lambda}_{jj}$; and the number of entrants, $\widehat{M}_j$.[17] To conclude the proof of Lemma 2, we show that, although the number of

[17] In the absence of a CES import demand system, one can show that changes in the price index still take a very simple form, namely $\widehat{P}_j = -\left(\hat{\lambda}_{jj} - \hat{M}_j\right) \big/ \left(1 - \sigma - \gamma_{jj}\right)$. In general, however, there is no simple mapping between trade elastisticities, $\boldsymbol{\varepsilon}_j$, and the relevant elasticity for welfare computations, $1 - \sigma - \gamma_{jj}$.

varieties consumed in country j may vary, we necessarily have $\widehat{M}_j = 0$. The formal argument uses the fact that under a CES import demand system, aggregate revenues are proportional to aggregate profits. As a result, the free entry condition completely pins down the number of entrants, independently of the value of variable trade costs.

Lemma 2 can again be used to analyze the impact of a change in variable trade costs. The exact same logic as in Section 3 leads to the two following propositions.

Proposition 3 *In any gravity model with monopolistic competition, W_j can be consistently estimated by $1 - \left(\lambda_{jj}/\lambda'_{jj}\right)^{1/\bar{\varepsilon}}$, where λ'_{jj} and λ_{jj} are evaluated at the new and initial trade equilibrium, respectively.*

Proposition 4 *In any gravity model with monopolistic competition, $\overline{W}_j$ can be consistently estimated by $(\lambda_{jj})^{1/\bar{\varepsilon}} - 1$, where λ_{jj} is evaluated at the initial equilibrium.*

A direct implication of Propositions 2 and 4 is that conditional on two sufficient trade statistics, λ_{jj} and $\bar{\varepsilon}$, the gains from trade predicted by gravity models with perfect and monopolistic competition are the same. Within the class of gravity models, as we switch from perfect to monopolistic competition, the composition of the gains from trade changes, but their total size does not.

Notwithstanding the importance of gravity models with monopolistic competition in the existing literature, it is obvious that the strong equivalence between these models and gravity models with perfect competition heavily relies on the fact that $\widehat{M}_j = 0$. With this in mind, we focus in the next subsection on the equivalence between two gravity models with monopolistic competition, Krugman (1980) and Melitz (2003). As we will see in Section 5, the equivalence between these two models can also be generalized to situations in which $\widehat{M}_j \neq 0$.

4.3 Krugman (1980) vs. Melitz (2003)

In line with our analysis under perfect competition, we conclude this section by comparing two well-known gravity models with monopolistic competition. The first one corresponds to the case in which g_i is a degenerate density function with all the mass at some single productivity level z_i that may differ across countries, and $f_{ij} = 0$ for all i, j. Under these assumptions, there is no firm heterogeneity and the model described in this section reduces to Krugman (1980). The second model corresponds to the case in which g_i is the density function associated with a Pareto distribution, $g_i(z) \equiv \theta b^{\theta} z^{-\theta-1}$ for $z \geq b$, as in most extensions and variations of Melitz (2003).[18] For expositional purposes, we will simply refer to this model as Melitz (2003), though it should be clear that we implicitly mean "Melitz (2003) with Pareto."[19]

In Krugman (1980), the absence of fixed exporting costs entails $z_{ij}^* = 0$ and $\gamma_{ij} = 0$ for all i, j. Since entry also is invariant to trade barriers, $\widehat{M_i} = 0$, this model therefore only features consumption gains from trade, just like the Armington model presented in Section 3. By contrast, in Melitz (2003), although entry remains invariant to trade barriers, $\widehat{M_i} = 0$, changes in trade costs affect the productivity cutoffs z_{ij}^*. These changes at the extensive margin may lead to changes in the number and composition of consumed varieties as well as changes in aggregate productivity.

Since trade leads to the exit of the least efficient firms in this richer model, it may be tempting to conclude that the gains from trade are larger. Our theoretical analysis contradicts this intuition. In both Krugman (1980) and Melitz (2003), it is easy to check that the import demand system is such that

$$X_{ij} = \frac{M_i T_i (w_i f_{ij})^{1+\frac{\varepsilon}{\sigma-1}} (w_i \tau_{ij})^{\varepsilon} w_j L_j}{\sum_{i'=1}^{I} M_{i'} T_{i'} (w_{i'} f_{i'j})^{1+\frac{\varepsilon}{\sigma-1}} (w_{i'} \tau_{i'j})^{\varepsilon}}. \tag{25}$$

where T_i and T_j are country-specific technology parameters; $\varepsilon \equiv 1 - \sigma$ in Krugman (1980) and $\varepsilon \equiv -\theta$ in Melitz (2003). Like in Section 3, the introduction of a new margin of adjustment changes the structural interpretation of the trade elasticity, from a preference to a technological parameter. Yet, because Equation (25) implies that the import demand system satisfies our two macro-level restrictions, CES and gravity, we can invoke Proposition 4 to conclude that conditional on $\bar{\varepsilon}$ and λ_{jj}, the gains from trade are the same in the two models.[20]

[18] A non-exhaustive list of examples includes Antras and Helpman (2004), Helpman, Melitz and Yeaple (2004), Ghironi and Melitz (2005), Bernard, Redding and Schott (2007), Chaney (2008), Arkolakis (2008), Eaton, Kortum and Kramarz (2008), and Helpman, Itskhoki and Redding (2008).

[19] Perhaps not surprisingly, one can show that in a gravity model with monopolistic competition, if g_i is differentiable for all z (which rules out Krugman (1980)), then g_i must be the density of a Pareto distribution with a common θ across all countries. Compared to the alternative strategy of assuming a Pareto distribution right away, our general approach presents two important advantages. First, as we already mentioned in Section 3, our local results still hold under the weaker assumption that trade elasticity may vary across trade equilibria and countries, i.e., ε_j^{ii} can vary with j, $\boldsymbol{\tau}$ and $\mathbf{E}$. Second, and more importantly, our strategy allows us to offer a unifying perspective on the welfare implications of gravity models under different market structures by identifying the crucial macro-level restrictions from which they derive.

[20] Unlike in the case of perfect competition, Equation (25) does not necessarily imply that Krugman (1980) and Melitz (2003) have the same welfare predictions for any counterfactual change in trade costs. Conditional on trade data, $\mathbf{X}$ and $\bar{\varepsilon}$, Melitz (2003) may predict different changes in trade flows than Krugman (1980) due

As a careful reader may have already noticed, our assumption that there exists a common orthogonality condition such that ε can be estimated using a gravity equation is somewhat stronger under monopolistic than perfect competition. In the case of Anderson (1979) and Eaton and Kortum (2002), we had $\nu_{ij} \equiv 0$ so that, for example, $E\left(\nu_{ij} \ln \tau_{i'j'}\right) = 0$ was trivially satisfied in both models. While the same is true in Krugman (1980), this is not the case in Melitz (2003) where $\nu_{ij} \equiv (f_{ij})^{1+\frac{\varepsilon}{\sigma-1}}$. For a difference-in-difference estimator to be a consistent estimator, we would therefore need additional assumptions about the joint distribution of fixed and variable trade costs. In our view, this issue is similar to the problem one would face under perfect competition if part of the variable trade costs were not observable. Again one would need observable and unobservable component of trade costs to be uncorrelated, or an instrumental variable, in order to avoid omitted variable bias. The fact that this unobserved component is fixed rather than variable does not change what we view primarily as an econometric issue, which we have little to contribute to.[21]

5 Extensions

The objective of this section is twofold. First, we wish to establish the robustness of our simple welfare formula by offering additional examples of gravity models, not considered in Sections 3 and 4, in which the gains from trade can be consistently estimated by $\lambda^{1/\bar{\varepsilon}} - 1$. Our choice of examples is motivated by recent developments in the literature on firm heterogeneity and trade. In line with this literature, we focus on variations and extensions of Melitz (2003) including: (*i*) restricted entry; (*ii*) endogenous marketing costs; and (*iii*) multi-product firms.[22] Although some of these extensions are crucial to explain micro-level facts, we show that they leave our simple formula unchanged. In a gravity model with restricted entry, endogenous marketing costs, or multi-product firms, the share of domestic expenditure and

to the different elasticity of trade flows with respect to wages in the two models, namely ε in Krugman (1980) versus $1 + \sigma\varepsilon/(\sigma - 1)$ in Melitz (2003). A notable exception is the case where all countries are symmetric. In this situation, there are no changes in wages, and so, the two models lead to the the same welfare predictions for any (symmetric) change in trade costs.

[21] As we pointed out in footnote 10, this issue is different from the economic issue raised by Helpman, Melitz and Rubinstein (2008): according to their model, ν_{ij} is a function of τ_{ij}.

[22] Another type of gravity model entails heterogeneous quality, as in Baldwin and Harrigan (2007) and Johnson (2009). While the introduction of quality considerations are crucial to explain the variation in the distribution of prices across firms and countries, it is isomorphic to a change in the units of account, which must again leave our welfare predictions unchanged. Quality considerations may, of course, have important distributional consequences in environments with multiple factors of production; see e.g. Verhoogen (2008) and Kugler and Verhoogen (2008). They may also matter in the presence of minimum quality requirements, which we are also abstracting from; see e.g. Hallak and Sivadasan (2009).

the trade elasticity remain sufficient statistics for welfare analysis.

Second, we wish to illustrate how our simple welfare formula may still hold in simple generalizations of gravity models. Motivated again by the existing trade literature, we consider generalizations of gravity models featuring: (*i*) multiple sectors; (*ii*) multiple factors; and (*iii*) tradable intermediate goods. While our simple welfare formula no longer holds in these richer environments, we demonstrate that generalized versions can easily be derived using the same logic as in Sections 3 and 4. For all extensions, formal proofs can be found in the Appendix.

5.1 Other gravity models

Throughout this subsection, we assume that all assumptions introduced in Section 2 hold and that we have monopolistic competition as in Section 4. Compared to Section 4, however, we relax some of our assumptions to allow for restricted entry, endogenous marketing costs, and multi-product firms.

Restricted entry. We start by considering the case in which entry is restricted, as in Chaney (2008). Instead of assuming that the total measure of entrants in country j is endogenously determined by a free entry condition, Equation (17), we assume that M_j is exogenously given. In this situation, the exact same logic as in Section 4 implies $\widehat{P}_j = -\widehat{\lambda}_{jj} \big/ \varepsilon$. Thus the only reason why our welfare formula may no longer hold is because changes in trade costs may now affect aggregate profits, and in turn, total income in country j. Under a CES import demand system, however, this effect is necessarily absent: aggregate profits are independent of the value of trade costs. As a result, our welfare formula must remain unchanged.

Endogenous marketing costs. We now turn to the case in which marketing costs are endogenous, as in Arkolakis (2008) and Eaton, Kortum and Kramarz (2008). In order to reach consumers with probability x in country j, a firm from country i must now pay a fixed cost equal to

$$f_{ij}(x) = f_{ij} \times \left[\frac{1 - (1 - x)^{1-\mu}}{1 - \mu} \right].$$

The model considered in Section 4 corresponds to the particular case in which $\mu = 0$. In this situation, the marginal cost of reaching an additional consumer is constant and firms find it optimal to reach every potential consumer or none at all.

Although the introduction of endogenous marketing costs is important to explain variations in the distribution of firm size, it is easy to show that it has no effect on our welfare formula. The introduction of a new margin, the share x of consumer that a firm wants to reach, again affects the structural interpretation of the trade elasticity, but nothing else. The share of domestic expenditure and the trade elasticity remain sufficient statistics for the computation of the gains form trade.

Multi-product firms. In Section 4, all firms can only produce one good. In the spirit of Bernard, Redding and Schott (2009) and Arkolakis and Muendler (2007) we assume here that each firm can produce up to N goods, which we will refer to as products. Since the is a continuum of firms and a discrete number of products per firm, there are no cannibalization effects: sales of one product do not affect the sales of other products sold by the same firm. We allow productivity levels to be correlated across different products within the same firm, and assume that firms incur in the same marketing costs for each product.

The introduction of multi-product firms has the same type of implications as the introduction of endogenous marketing costs. It matters crucially for micro-level phenomena, such as the impact of trade liberalization on firm-level productivity, but it has no effect on the magnitude of the gains from trade. As far as our welfare formula is concerned, the only thing that multi-product firms change is the structural interpretation of the trade elasticity, which now includes adjustments in the number of products within each firm.[23]

5.2 Generalized gravity models

This final subsection relaxes some of the assumptions of gravity models introduced in Section 2. We start by introducing multiple sectors and factors and conclude with tradable intermediate goods.

Multiple sectors. Suppose that goods $\omega \in \Omega$ are separated into $s = 1, ..., S$ groups of goods, Ω^s, which we refer to as sectors. Consumers in country i have preferences represented by the following utility function

$$U_j = \prod_{s=1}^{S} (Q_i^s)^{\eta_i^s},$$

where $0 \leq \eta_i^s \leq 1$ is the constant share of income on goods from sector s in country i; and

[23] This basic point would remain true if we did not have a CES import demand system. In that case, we would need to estimate more elasticities, i.e., the entire matrix $\boldsymbol{\varepsilon}_j$. But conditional on *upper-level* elasticities, predictions about the gains from trade would have to be the same with or without multi-product firms.

$Q_i^s \equiv \left[\int_{\omega\in\Omega^s} q_i(\omega)^{\frac{\sigma^s}{\sigma^s-1}} d\omega\right]^{\frac{\sigma^s-1}{\sigma^s}}$ is a Dixit-Stiglitz aggregator of goods in sector s with $\sigma^s > 1$ the elasticity of substitution between these goods. Compared to Section 2, the key difference is that CES and gravity now refer to properties of the import demand system at the *sector level*. Formally, CES now implies that bilateral imports from country i to country j in sector s, X_{ij}^s, satisfy

$$\partial \ln\left(X_{ij}^s / X_{jj}^s\right) / \partial \ln \tau_{i'j}^{s'} = \varepsilon^s \text{ if } s' = s \text{ and } i' = i;$$
$$\partial \ln\left(X_{ij}^s / X_{jj}^s\right) / \partial \ln \tau_{i'j}^{s'} = 0 \text{ otherwise.}$$

Similarly, gravity now implies that bilateral imports can be decomposed into

$$\ln X_{ij}^s(\boldsymbol{\tau}, \mathbf{E}) = A_i^s(\boldsymbol{\tau}, \mathbf{E}) + B_j^s(\boldsymbol{\tau}, \mathbf{E}) + \varepsilon^s \ln \tau_{ij}^s + \nu_{ij}^s,$$

with τ_{ij}^s the iceberg trade cost between i and j in sector s, as in Costinot and Komunjer (2007). All other properties of gravity models are unchanged.

Our results in the multi-sector case can be summarized as follows. Under perfect competition, our welfare formula generalizes to $\prod_{s=1}^S \left(\lambda_{jj}^s\right)^{\eta_j^s/\bar{\varepsilon}^s} - 1$, where λ_{jj}^s represents the share of expenditure in sector s that goes to domestic goods in the initial equilibrium. This formula is similar to the one derived by Donaldson (2008) using a multi-sector extension of Eaton and Kortum (2002). For $S = 1$, this formula reduces to the one derived in Section 3. For $S > 1$, however, we see that more aggregate statistics are necessary to estimate the gains from trade: elasticities and shares of expenditure at the sector level, but also data on the share of expenditure across sectors. This should not be too surprising: the less symmetry we assume across goods, the more information we need to estimate the gains from trade.

By contrast, our estimator of the gains from trade under monopolistic competition becomes $\prod_{s=1}^S \left(\lambda_{jj}^s \eta_j^s / \delta_j^s\right)^{\eta_j^s/\bar{\varepsilon}^s} - 1$, where δ_j^s is the share of employment in sector s in the initial equilibrium. Compared to the one-sector case, we see that the mapping between data and welfare is no longer the same under perfect and monopolistic competition. The reason is simple. The equivalence between these two market structures in Sectors 3 and 4 relied on the fact that there was no change in entry. In the multi-sector case, however, changes in employment across sectors lead to changes in entry, which must be reflected in the computation of the gains from trade. This explains the correction term, $\left(\eta_j^s / \delta_j^s\right)^{\left(\eta_j^s/\bar{\varepsilon}^s\right)}$, in our new formula.[24]

Although the equivalence between gravity models is admittedly weaker in the multi-sector case, it is worth emphasizing that the core insights of Propositions 2 and 4 still hold. In line with Section 3's results, multi-sector extensions of Eaton and Kortum (2002) and Anderson (1979) must therefore have the same welfare implications. The same is true about multi-sector extensions of Krugman (1980) and Melitz (2003), in line with Section 4's results. Conditional on a given market structure, either perfect or monopolistic competition, there still exists aggregate sufficient statistics for welfare analysis.

Multiple factors. Although the gravity models presented in Section 2 only feature one factor of production, labor, it is trivial to extend our results to situations in which there are $f = 1, ..., F$ factors, but all goods $\omega \in \Omega$ use these factors in the same proportions. In this situation, all our results go through with a "composite input" playing the same role as labor in Sections 3 and 4. The situation in which goods may vary in factor intensity is, of course, more complex.

One way to introduce differences in factor intensity is to assume that: (i) there are multiple sectors, as in the previous extension; (ii) all goods from the same sector have the same factor intensity; but (iii) factor intensity differs across sectors; see e.g. Bernard, Redding and Schott (2007) and Chor (2009). In the Appendix, we analyze the case where there are two factors of production, capital and labor, and production functions are Cobb-Douglas in all sectors. Under perfect competition, we show that our welfare formula generalizes to $\Psi_j \prod_{s=1}^{S} \left(\lambda_{jj}^s\right)^{\eta_j^s/\varepsilon^s} - 1$, where Ψ_j only depends on variables that can be evaluated in the initial equilibrium: shares of employment, δ_j^s, shares of expenditures, η_j^s, labor intensity, α_j^s, and the share of labor in country j' s total income, α_j. The correction term, Ψ_j, simply captures the fact that moving to autarky in a multi-factor world also has implications for relative factor demand, and therefore, relative factor prices.

Under monopolistic competition, a similar logic leads to a similar estimator. Formally, we show that the gains from trade can be estimated by $\widetilde{\Psi}_j \prod_{s=1}^{S} \left(\lambda_{jj}^s\right)^{\eta_j^s/\varepsilon^s} - 1$, where the correction term $\widetilde{\Psi}_j$ again depends on δ_j^s, η_j^s, α_j^s, and α_j. Like in the multi-sector-one-factor case, the difference between perfect and monopolistic competition derives from the fact that changes in employment across sectors lead to changes in entry under monopolistic

[24] In a related paper, Balistreri, Hillberry and Rutherford (2009) have developed variations of the Anderson (1979) and Melitz (2003) models with a non-tradeable sector to illustrate the same idea: if changes in trade costs lead to changes in entry, then models with perfect and monopolistic competition no longer have the same welfare implications.

competition, which has implications for the magnitude of the gains from trade.

To summarize, if labor is not the only factor of production, then one need more "local" data, α_j^s and α_j, in order to compute the magnitude of the gains from trade. But as in the previous extension, the core insights of Propositions 2 and 4 survive. We still have the equivalence between multi-factor extensions of Eaton and Kortum (2002) and Anderson (1979), as in Section 3, and the equivalence between multi-factor extensions of Krugman (1980) and Melitz (2003), as in Section 4.

Tradable intermediate goods. In Section 2, all goods were final goods. We now investigate how our welfare formula would generalize to environments in which goods $\omega \in \Omega$ are intermediate goods which can either be used to produce a unique non-tradeable final good or other intermediate goods, as in Eaton and Kortum (2002), Alvarez and Lucas (2007), Atkeson and Burstein (2009), and Di Giovanni and Levchenko (2009). Formally, we assume that after fixed costs have been paid (if any), the unit cost of production of good ω in country i, $c_i(\omega)$, can be written as

$$c_i(\omega) = \frac{w_i^{\beta_i} P_i^{1-\beta_i}}{z(\omega)}, \tag{26}$$

where $1 - \beta_i$ represents the share of other intermediate goods in the production of good ω. Similarly, we assume that fixed costs under monopolistic competition are such that firms from country i must incur: (i) a fixed entry cost equal to $w_i^{\alpha_i} P_i^{1-\alpha_i}\ f_e$ in order to produce in country i, where $1 - \alpha_i$ represents the share of intermediate goods in entry costs; and (ii) a fixed marketing cost equal to $w_i^{\beta_i} P_i^{1-\beta_i}\ f_{ij}$, in order to sell their varieties to country j. The models considered in Sections 3 and 4 correspond to the special case with $\alpha_i = \beta_i = 1$.

Under perfect competition, the introduction of intermediate goods amplifies the gains from trade as follows. Conditional on the observed values of the share of expenditure and the trade elasticity, the estimator of the gains from trade becomes $(\lambda_{jj})^{1/\left(\beta_j \bar{\varepsilon}\right)} - 1$. This expression is similar to the one derived in Eaton and Kortum (2002) and Alvarez and Lucas (2007). Jones (2009) convincingly argues that β_j is on average equal to $1/2$, hence a country like Belgium with $\lambda_{BEL} = 0.73$ experiences gains from trade (using $\bar{\varepsilon} = -5$) of 13% rather than 6%. Intuitively, a given decrease in λ_{jj} is now associated with bigger welfare gains in country j since it also captures the lower costs of intermediate goods. The larger the share β_j of intermediate goods in the production of other intermediate goods, the larger the amplification effect caused by this input-output loop.

Under monopolistic competition, we can use the same logic to show that the estimator of the gains from trade is $(\lambda_{jj})^{\frac{1}{\beta_j \bar{\varepsilon} + 1 - \alpha_j}} - 1$. For $\alpha_j = 1$, our welfare formula is therefore the same under both monopolistic and perfect competition. By contrast, for $\alpha_j \neq 1$, we see that conditional on trade data, λ_{jj} and $\bar{\varepsilon}$, the gains from trade predicted by models with monopolistic competition are larger, reflecting the increase in the number of entrants associated with the decrease in country j's consumer price index. If we assume that intermediate goods are just as important in entry costs as in marketing and production costs (i.e., $\alpha_j = \beta_j$), then we can use our modified formula to compute Belgium's gains from trade. Using again $\bar{\varepsilon} = -5$ and $\beta_j = 1/2$, these gains would now be 17% rather than 13%. Of course, if $\alpha_j > \beta_j$ then trade leads to a lower expansion of entry and lower gains from trade (relative to the case with $\alpha_j = \beta_j$).

The broad implications of this last extension are very similar to those we reached in the two previous ones: unless the introduction of intermediate goods does not lead to changes in the number of entrants, which is the case for $\alpha_j = 1$, the welfare implications of models with perfect and monopolistic competition are no longer the same. Nevertheless, within both classes of model, there still exist aggregate sufficient statistics for welfare analysis.

Although this section was not meant as an exhaustive analysis of all possible variations, combinations, and generalizations of gravity models, we wish to conclude by pointing out one class of extensions which we view as particularly important for future research. Throughout this section, we have relaxed various supply-side assumptions, but we have maintained the assumption of Dixit-Stiglitz preferences under monopolistic competition. Allowing for quasi-linear or translog preferences as in Melitz and Ottaviano (2008) and Feenstra and Weinstein (2009) would introduce variations in mark-ups, and hence, a new source of gains from trade. While the introduction of these pro-competitive effects would clearly affect the composition of the gains from trade, we strongly conjecture that the two sufficient statistics identified in our paper, λ and $\bar{\varepsilon}$, would still play a crucial role in determining their total size, albeit perhaps in a different way.[25]

6 Concluding Remarks

Micro-level data have had a profound influence on research in international trade over the last ten years. In many regards, this research agenda has been very successful. New stylized facts have been uncovered and new trade models have been developed to explain these facts. In this paper we have investigated to which extent answers to new micro-level questions have

[25] In Bernard, Eaton, Jensen and Kortum (2003), for instance, it is easy to check that although mark-ups vary at the firm-level, our simple welfare formula remains unchanged.

affected answers to an old and central question in the field: How large are the gains from trade? A crude summary of our results is: "So far, not much."

The first message of our paper is therefore a cautionary one. Although it may be tempting to think that new and richer trade models necessarily entail larger gains from trade, our analysis demonstrates that this is not the case. Within the class of trade models considered in this paper, the number of sources of gains from trade varies, but conditional on observed trade data, the total size of the gains from trade does not.

The second message of our paper is a positive one. The flip side of our strong equivalence results is that within a particular but important class of trade models, there exist two sufficient statistics for welfare analysis: (i) the share of expenditure on domestic goods; and (ii) a gravity-based estimate of the trade elasticity. Hence only a very limited amount of macro data may be necessary to make robust counterfactual predictions, whatever the micro level details of a particular trade model may be.

This last observation, however, leaves us with a puzzle. If many of our theoretical models predict the same gains from trade, how can these gains be so much smaller than the reduced-form estimates uncovered by empirical researchers? For instance, Feyrer (2009) concludes that an increase in trade volumes of 10% implies an increase in real income of 5%. While this elasticity lies below the previous estimates of Frankel and Romer (1999), it is an order of magnitude larger than the elasticity implied by gravity models.[26] How does one reconcile theory and empirics? If micro-level adjustments are not the answer, what sources of gains from trade can quantitatively account for the discrepancy between our models and these reduced-form estimates? These are exciting open questions waiting to be answered.[27]

[26] To see this, consider a hypothetical country with an import penetration ratio of 20%, halfway between the ratios of 27% and 7% for Belgium and the United States mentioned in the Introduction. A 10% increase in trade would increase the import penetration ratio to 22%, and so, lower the share of expenditure λ from 80% to 78%. Under the favorable assumption that $\bar{\varepsilon} = -5$, Propositions 1 and 3 would only predict an increase in real income of 0.5%. Even if we allow for the maximum amplification effects derived above with trade in intermediate goods under monopolistic competition, the welfare gains may only increase to 1.25%, still far from Feyrer's 5%.

[27] One interesting possibility is that trade may facilitate other ways through which countries gain from openness, e.g., multinational production or the diffusion of ideas. Ramondo and Rodríguez-Clare (2009) offer quantitative results consistent with that idea: in a model of trade and multinational production, they find that the gains from trade can be almost twice as large as those predicted by gravity models.

References

Alvarez, Fernando and Robert E. Lucas. 2007. "General Equilibrium Analysis of the Eaton-Kortum Model of International Trade." *Journal of Monetary Economics* 54(6):1726–1768.

Anderson, James E. 1979. "A Theoretical Foundation for the Gravity Equation." *The American Economic Review* 69(1):106–116.

Anderson, James E. and Eric Van Wincoop. 2003. "Gravity with Gravitas: A Solution to the Border Puzzle." *The American Economic Review* 93(1):170–192.

Anderson, James E. and Eric Van Wincoop. 2004. "Trade Costs." *Journal of Economic Literature* 42(3):691–751.

Antras, Pol and Elhanan Helpman. 2004. "2004." *Journal of Political Economy* 112(3):552–580.

Arkolakis, Costas. 2008. "Market Penetration Costs and the New Consumers Margin in International Trade." *NBER working paper 14214* .

Arkolakis, Costas and Marc-Andreas Muendler. 2007. "The Extensive Margin of Exporting Goods: A Firm-Level Analysis." *mimeo, University of California San Diego and Yale University* .

Arkolakis, Costas, Svetlana Demidova, Peter J Klenow and Andres Rodríguez-Clare. 2008. "Endogenous Variety and the Gains from Trade." *American Economic Review, Papers and Proceedings* 98(4):444–450.

Atkeson, Andrew and Ariel Burstein. 2009. "Innovation, Firm Dynamics, and International Trade." *mimeo, UCLA* .

Baldwin, Richard and James Harrigan. 2007. "Zeros, Quality and Space: Trade Theory and Trade Evidence." *NBER Working paper 13214* .

Balistreri, Edward J., Russell H. Hillberry and Thomas F. Rutherford. 2009. "Trade and Welfare: Does Industrial Organization Matter." *mimeo, University of Melbourne* .

Bernard, Andew B., Jonathan Eaton, J. Bradford Jensen and Samuel Kortum. 2003. "Plants and Productivity in International Trade." *The American Economic Review* 93(4):1268–1290.

Bernard, Andrew B., Stephen J. Redding and Peter Schott. 2007. "Comparative Advantage and Heterogeneous Firms." *Review of Economic Studies* 74(1):31–66.

Bernard, Andrew B., Stephen J. Redding and Peter Schott. 2009. "Multi-Product Firms and Trade Liberalization." *NBER Working Paper* 12782.

Broda, Cristian and David Weinstein. 2006. "Globalization and the Gains from Variety." *Quarterly Journal of Economics* 121(2):541–585.

Chaney, Thomas. 2008. "Distorted Gravity: The Intensive and Extensive Margins of International Trade." *The American Economic Review* 98(4):1707–1721.

Chetty, Raj. 2009. "Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods." *Annual Review of Economics* 1:451–488.

Chor, Davin. 2009. "Unpacking Sources of Comparative Advantage: A Quantitative Approach." *mimeo, Singapore Management University* .

Costinot, Arnaud and Ivana Komunjer. 2007. "What Goods Do Countries Trade? A Structural Ricardian Model." *NBER Working Papers 13691* .

Di Giovanni, Julian and Andrei A. Levchenko. 2009. "Firm Entry, Trade, and Welfare in Zipf's World." *mimeo, University of Michigan* .

Donaldson, Dave. 2008. "Railroads of the Raj: Estimating the Economic Impact of Transportation Infrastructure." *mimeo, London School of Economics* .

Eaton, Jonathan and Samuel Kortum. 2002. "Technology, Geography and Trade." *Econometrica* 70(5):1741–1779.

Eaton, Jonathan, Samuel Kortum and Francis Kramarz. 2008. "An Anatomy of International Trade: Evidence from French Firms." *NBER Working Paper 14610* .

Feenstra, Robert. 1994. "New Product Varieties and the Measurement of International Prices." *The American Economic Review* 84(1):157–177.

Feenstra, Robert. 2009. "Measuring the Gains from Trade under Monopolistic Competition." *mimeo, University of California, Davis* .

Feenstra, Robert and David Weinstein. 2009. "Globalization, Competition, and the U.S. Price Level." *mimeo, University of California, Davis and Columbia University* .

Feenstra, Robert and Hiau Looi Kee. 2008. "Export Variety and Country Productivity: Estimating the Monopolistic Competition Model with Endogenous Productivity." *Journal of International Economics* 74(2):500–518.

Feyrer, James. 2009. "Trade and Income – Exploiting Time Series in Geography." *NBER Working Paper* 14910.

Frankel, Jeffrey A. and David Romer. 1999. "Does Trade Cause Growth." *The American Economic Review* 89(3):379–399.

Ghironi, F. and M.J. Melitz. 2005. "International Trade and Macroeconomic Dynamics with Heterogeneous Firms." *Quarterly Journal of Economics* 120(3):865–915.

Goldberg, P., A. Khandelwal, Nina Pavcnik and P. Topalova. 2009. "Trade Liberalization and New Imported Inputs." *American Economic Review, Papers and Proceedings* 99(2):494–500.

Grossman, Gene and Elhanan Helpman. 1991. *Innovation and Growth in the Global Economy.* Cambridge, Massachussets: MIT press.

Hallak, Juan Carlos and Jagadeesh Sivadasan. 2009. "Firms' Exporting Behavior under Quality Constraints." *NBER Working Paper 14928* .

Helpman, Elhanan, Marc J. Melitz and Stephen R. Yeaple. 2004. "Export Versus FDI with Heterogeneous Firms." *The American Economic Review* 94(1):300–316.

Helpman, Elhanan, Marc Melitz and Yona Rubinstein. 2008. "Estimating Trade Flows: Trading Partners and Trading Volumes." *Quarterly Journal of Economics* 2(5):441–487.

Helpman, Elhanan, Oleg Itskhoki and Stephen J. Redding. 2008. "Inequality and Unemployment in a Global Economy." *NBER Working Paper* 14478.

Johnson, Robert C. 2009. "Trade and Prices with Heterogeneous Firms." *mimeo, Dartmouth College* .

Jones, Charles I. 2009. "Intermediate Goods and Weak Links: A Theory of Economic Development." *mimeo, Stanford University* .

Klenow, Peter J. and Andres Rodríguez-Clare. 1997. "Quantifying Variety Gains from Trade Liberalization." *mimeo, University of Chicago* .

Krugman, Paul. 1980. "Scale Economies, Product Differentiation, and the Pattern of Trade." *The American Economic Review* 70(5):950–959.

Kugler, Maurice and Eric Verhoogen. 2008. "The Quality-Complementarity Hypothesis: Theory and Evidence from Colombia." *mimeo, Columbia University* .

Matsuyama, Kiminori. 2007. Ricardian Trade Theory. In *The New Palgrave Dictionary of Economics*, ed. L. Blume and S. Durlauf. 2nd edition ed. Basingstoke Hampshire: Palgrave Macmillan.

Melitz, Marc J. 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71(6):1695–1725.

Melitz, Marc J. and Gianmarco I. P. Ottaviano. 2008. "Market Size, Trade, and Productivity." *The Review of Economic Studies* 75(1):295–316.

Norihiko, Yamano and Nadim Ahmad. 2006. "The OECD Input-Output Database: 2006 Edition." *Working Paper 2006/8, OECD* .

Ramondo, Natalia and Andres Rodríguez-Clare. 2009. "Trade, Multinational Production, and the Gains from Openness." *NBER working paper* 15604.

Verhoogen, Eric. 2008. "Trade, Quality Upgrading and Wage Inequality in the Mexican Manufacturing Sector." *Quarterly Journal of Economics* 123(2):489–530.

A Proofs (I): Perfect Competition

Proof of Lemma 1. In the main text, we have already established that

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij} \left(\widehat{w}_i + \widehat{\tau}_{ij} \right). \tag{27}$$

Under perfect competition we know that bilateral imports, X_{ij}, only depend on prices; and by Equation (4), we know that prices only depend on wages and variable trade costs through their product, $w_i \tau_{ij}$. Using these two observations and the definition of $\varepsilon_j^{ii'}$, we can express the percentage changes in relative imports as

$$\widehat{X}_{ij} - \widehat{X}_{jj} = \sum_{i' \neq j} \varepsilon_j^{ii'} \left(\widehat{w}_{i'} + \widehat{\tau}_{i'j} \right). \tag{28}$$

In the case of a CES import demand system, Equation (28) simplifies into

$$\widehat{X}_{ij} - \widehat{X}_{jj} = \varepsilon \left(\widehat{w}_{i'} + \widehat{\tau}_{i'j} \right). \tag{29}$$

Combining Equations (27) and (29), and noting that $\widehat{\lambda}_{ij} - \widehat{\lambda}_{jj} = \widehat{X}_{ij} - \widehat{X}_{jj}$, we obtain

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij} \left(\frac{\widehat{\lambda}_{ij} - \widehat{\lambda}_{jj}}{\varepsilon} \right). \tag{30}$$

To conclude the proof of Lemma 1, we note that $\sum_{i=1}^{n} \lambda_{ij} = 1$ implies $\sum_{i=1}^{n} \lambda_{ij} \widehat{\lambda}_{ij} = 0$. Combining this observation with Equation (30), we get Equation (9). **QED.** ■

Proof of Proposition 1. By Lemma 1, we know that

$$d \ln P_j = -\frac{d \ln \lambda_{jj}}{\varepsilon}. \tag{31}$$

Let λ_{jj} and λ'_{jj} denotes the share of expenditure on domestic goods in the trade equilibria associated with $\boldsymbol{\tau}$ and $\boldsymbol{\tau}'$, respectively. Similarly, let P_j and P'_j denote the consumer price in country j in the two equilibria. Since ε is constant across all trade equilibria, we can integrate Equation (31) between $\boldsymbol{\tau}$ and $\boldsymbol{\tau}'$ to get

$$\frac{P'_j}{P_j} = \left(\frac{\lambda_{jj}}{\lambda'_{jj}} \right)^{1/\varepsilon} \tag{32}$$

By definition, we know that $W_j \equiv 1 - \left(P_j' / P_j\right)$. Thus, Equation (32) implies

$$W_j = 1 - \left(\frac{\lambda_{jj}}{\lambda'_{jj}}\right)^{1/\varepsilon} \tag{33}$$

Since $\overline{\varepsilon}$ is a consistent estimator of ε, by assumption, and W_j is a continuous function of ε, by Equation (33), we can invoke the continuous mapping theorem to conclude that $1 - \left(\frac{\lambda_{jj}}{\lambda'_{jj}}\right)^{1/\overline{\varepsilon}}$ is a consistent estimator of W_j. **QED**. ■

Proof of Proposition 2. By assumption, we know that for any $i \neq j$, $\lim_{\tau_{ij} \to +\infty} (w_i \tau_{ij}) = +\infty$. Thus, we must have $\lambda'_{jj} = 1$ at $\boldsymbol{\tau} = +\infty$. Proposition 2 directly follows from this observation, Proposition 1, and the definition of $\overline{W}_j \equiv -\left(W_j\right)_{\boldsymbol{\tau}=+\infty}$. **QED**. ■

Counterfactual Changes in Trade Costs. In footnote 16, we have argued that conditional on trade data, $\mathbf{X}$ and $\overline{\varepsilon}$, the predicted changes in welfare associated with any counterfactual changes in trade costs are the same in Anderson (1979) and Eaton and Kortum (2002). We now demonstrate this result formally. By Equation (12), we know that the share of expenditures on goods from country i in country j is

$$\lambda_{ij} = \frac{T_i \left(w_i \tau_{ij}\right)^{\varepsilon}}{\sum_{i'=1}^{I} T_{i'} \left(w_{i'} \tau_{i'j}\right)^{\varepsilon}} \tag{34}$$

In both models, trade balance implies

$$w_i L_i = \sum_{j=1}^{I} \lambda_{ij} w_j L_j \tag{35}$$

Now consider a change in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau}'$. The share of expenditure on goods from country i in country j in the counterfactual equilibrium is given by

$$\lambda'_{ij} = \frac{T_i \left(w'_i \tau'_{ij}\right)^{\varepsilon}}{\sum_{i'=1}^{I} T_{i'} \left(w'_{i'} \tau'_{i'j}\right)^{\varepsilon}} \tag{36}$$

with trade balance given by

$$w'_i L_i = \sum_{j=1}^{I} \lambda'_{ij} w'_j L_j \tag{37}$$

For any variable x, let us denote by $\widetilde{x} = x'/x$. From equations 34 and 36 we obtain

$$\widetilde{\lambda}_{ij} = \frac{\left(\widetilde{w}_i \widetilde{\tau}_{ij}\right)^{\varepsilon}}{\sum_{i'=1}^{I} \lambda_{i'j} \left(\widetilde{w}_{i'} \widetilde{\tau}_{i'j}\right)^{\varepsilon}} \tag{38}$$

Similarly, from equations 37 and 38 we obtain

$$w_i' L_i = \sum_{j=1}^{I} \frac{(\widetilde{w}_i \widetilde{\tau}_{ij})^{\varepsilon}}{\sum_{i'=1}^{I} \lambda_{i'j} (\widetilde{w}_{i'} \widetilde{\tau}_{i'j})^{\varepsilon}} \widetilde{w}_j \lambda_{ij} w_j L_j$$

Using the fact that $Y_i = w_i L_i$, we can rearrange the previous expression as

$$\widetilde{w}_i Y_i = \sum_{j=1}^{I} \frac{\lambda_{ij} (\widetilde{w}_i \widetilde{\tau}_{ij})^{\varepsilon}}{\sum_{i'=1}^{I} \lambda_{i'j} (\widetilde{w}_{i'} \widetilde{\tau}_{i'j})^{\varepsilon}} \cdot \widetilde{w}_j Y_j \tag{39}$$

Since $Y_j \equiv \sum_{i'=1}^{n} X_{i'j}$ and $\lambda_{ij} \equiv X_{ij}/Y_j$, Equation 39 implies that conditional on trade data, $\mathbf{X}$, and the trade elasticity, ε, the proportional changes in wages predicted by Anderson (1979) and Eaton and Kortum (2002) are the same. By Equation 38, this further implies that the proportional changes in shares of expenditures predicted by the two models are the same as well. By Proposition 1, we know that the changes in welfare associated with a change in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau}'$ are given by

$$W_j = 1 - \left(\widetilde{\lambda}_{jj}\right)^{-1/\varepsilon}.$$

Since $\widetilde{\lambda}_{jj}$ is the same in Anderson (1979) and Eaton and Kortum (2002), W_j is the same as well. Our claim directly follows from this observation and the fact that $\overline{\varepsilon}$ is a consistent estimator of ε in both models. **QED**. ■

B Proofs (II): Monopolistic Competition

Proof of Lemma 2. In order to establish Equation (21), we proceed in 6 steps. For expositional purposes, we again suppress the arguments $(\boldsymbol{\tau}, \mathbf{E}) \equiv (\boldsymbol{\tau}, w_1, ..., w_n, M_1, ..., M_n)$, but it should be clear that, like in the main text, all endogenous variables, P_j, z_{ij}^*, and X_{ij} are functions of $(\boldsymbol{\tau}, w_1, ..., w_n, M_1, ..., M_n)$.

Step 1: *Percentage changes in the consumer price index are given by*

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij} \left[\frac{(\sigma - 1 + \gamma_{ij})(\widehat{w}_i + \widehat{\tau}_{ij})}{\sigma - 1 + \gamma_j} - \frac{\widehat{M}_i}{\sigma - 1 + \gamma_j} - \frac{\gamma_{ij} \widehat{w}_i}{(\sigma - 1 + \gamma_j)(1 - \sigma)} \right], \tag{40}$$

where $\gamma_j \equiv \sum_{i'=1}^{n} \lambda_{i'j} \gamma_{i'j}$.

In the main text, we have already established that

$$\widehat{P}_j = \sum_{i=1}^n \lambda_{ij} \left[\widehat{w}_i + \widehat{\tau}_{ij} - \frac{\widehat{M}_i}{\sigma - 1} + \frac{\gamma_{ij} \widehat{z_{ij}^*}}{\sigma - 1} \right], \tag{41}$$

By differentiating Equation (16) and using the fact that $w_j = 1$, we know that

$$\widehat{z_{ij}^*} = \widehat{w}_i + \widehat{\tau}_{ij} + \left(\frac{1}{\sigma - 1} \right) \widehat{w}_i - \widehat{P}_j. \tag{42}$$

Combining Equations (41) and (42), we obtain Equation (40).

Step 2: *Percentage changes in the cut-off productivity levels are given by*

$$\begin{aligned} \widehat{z_{ij}^*} \;&=\; \widehat{w}_i + \widehat{\tau}_{ij} + \left(\frac{1}{\sigma - 1} \right) \widehat{w}_i \\ &- \sum_{i'=1}^n \lambda_{i'j} \left[\left(\widehat{w}_{i'} + \widehat{\tau}_{i'j} \right) \left(\frac{1 - \sigma - \gamma_{i'j}}{1 - \sigma - \gamma_j} \right) + \frac{\gamma_{i'j} \widehat{w}_{i'}}{\left(1 - \sigma - \gamma_j \right) \left(1 - \sigma \right)} + \frac{\widehat{M}_{i'}}{1 - \sigma - \gamma_j} \right]. \end{aligned} \tag{43}$$

Equation (43) derives from Equations (40) and (42).

Step 3: *For any $i = 1, ..., n$, $j = 1, ..., n$, we must have $\gamma_{ij} = 1 - \sigma - \varepsilon$.*

Using Equations (1), (13), (15), (16), and the fact that $w_j = 1$, we can express bilateral imports by country j from country i as

$$X_{ij} = \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{P_j} \right)^{1-\sigma} M_i \left[\int_{z_{ij}^*}^{+\infty} z^{\sigma - 1} g_i(z) \, dz \right].$$

This implies

$$\widehat{X}_{ij} = (1 - \sigma) \left(\widehat{w}_i + \widehat{\tau}_{ij} \right) - (1 - \sigma) \widehat{P}_j + \widehat{M}_i - \gamma_{ij} \widehat{z_{ij}^*}. \tag{44}$$

Similarly, we have

$$\widehat{X}_{jj} = -(1 - \sigma) \widehat{P}_j + \widehat{M}_j - \gamma_{jj} \widehat{z_{jj}^*}. \tag{45}$$

Combining Equations (44) and (45), we obtain

$$\widehat{X}_{ij} - \widehat{X}_{jj} = (1 - \sigma) \left(\widehat{w}_i + \widehat{\tau}_{ij} \right) + \widehat{M}_i - \widehat{M}_j - \gamma_{ij} \widehat{z_{ij}^*} + \gamma_{jj} \widehat{z_{jj}^*},$$

which can be rearranged as

$$\begin{aligned} \widehat{X}_{ij} - \widehat{X}_{jj} &= (1-\sigma)\left(\widehat{w}_i + \widehat{\tau}_{ij}\right) + \widehat{M}_i - \widehat{M}_j \\ &\quad -\gamma_{ij} \textstyle\sum_{i'=1}^{n} \left[\left(\dfrac{\partial \ln z^*_{ij}}{\partial \ln w_{i'}}\right) \widehat{w}_{i'} + \left(\dfrac{\partial \ln z^*_{ij}}{\partial \ln \tau_{i'j}}\right) \widehat{\tau}_{i'j} + \left(\dfrac{\partial \ln z^*_{ij}}{\partial \ln M_{i'}}\right) \widehat{M}_{i'} \right] \\ &\quad +\gamma_{jj} \textstyle\sum_{i'=1}^{n} \left[\left(\dfrac{\partial \ln z^*_{jj}}{\partial \ln w_{i'}}\right) \widehat{w}_{i'} + \left(\dfrac{\partial \ln z^*_{jj}}{\partial \ln \tau_{i'j}}\right) \widehat{\tau}_{i'j} + \left(\dfrac{\partial \ln z^*_{jj}}{\partial \ln M_{i'}}\right) \widehat{M}_{i'} \right]. \end{aligned} \tag{46}$$

By definition, we know that $\varepsilon_j^{ii'} = \frac{\partial \ln(X_{ij}/X_{jj})}{\partial \ln \tau_{i'j}}$. Thus Equation (46) implies

$$\varepsilon_j^{ii'} = \begin{cases} 1-\sigma-\gamma_{ij}\left(\frac{\partial \ln z^*_{ij}}{\partial \ln \tau_{ij}}\right) + \gamma_{jj}\left(\frac{\partial \ln z^*_{jj}}{\partial \ln \tau_{ij}}\right), & \text{if } i' = i; \\ -\gamma_{ij}\left(\frac{\partial \ln z^*_{ij}}{\partial \ln \tau_{i'j}}\right) + \gamma_{jj}\left(\frac{\partial \ln z^*_{jj}}{\partial \ln \tau_{i'j}}\right), & \text{otherwise.} \end{cases}$$

Using Equation (43), we can simplify the previous expression to

$$\varepsilon_j^{ii'} = \begin{cases} 1-\sigma-\gamma_{ij} + \lambda_{ij}\left(\frac{1-\sigma-\gamma_{ij}}{1-\sigma-\gamma_j}\right)\left(\gamma_{ij}-\gamma_{jj}\right), & \text{if } i' = i; \\ \lambda_{i'j}\left(\frac{1-\sigma-\gamma_{i'j}}{1-\sigma-\gamma_j}\right)\left(\gamma_{ij}-\gamma_{jj}\right), & \text{otherwise.} \end{cases} \tag{47}$$

In a CES import demand system, we know that $\varepsilon_j^{ii'} = \varepsilon$ if $i' = i$ and $\varepsilon_j^{ii'} = 0$ otherwise. Combining this observation with Equation (47), we get $\gamma_{ij} = 1-\sigma-\varepsilon$ for all i,j.

Step 4: *Percentage changes in relative imports are given by*

$$\widehat{X}_{ij} - \widehat{X}_{jj} = \varepsilon\left[\widehat{\tau}_{ij} + \widehat{w}_i + \left(\frac{1-\sigma-\varepsilon}{\varepsilon(1-\sigma)}\right)\widehat{w}_i\right] + \widehat{M}_i - \widehat{M}_j. \tag{48}$$

Equation (48) derives from Equations (43) and (46) and the fact that $\gamma_{ij} = 1-\sigma-\varepsilon$ for all i,j.

Step 5: *Percentage changes in the consumer price index satisfy*

$$\widehat{P}_j = -\frac{\widehat{\lambda}_{jj} - \widehat{M}_j}{\varepsilon}. \tag{49}$$

Since $\gamma_{ij} = 1-\sigma-\varepsilon$ for all i,j, we can rearrange Equation (40) as

$$\widehat{P}_j = \textstyle\sum_{i=1}^{n} \lambda_{ij}\left[\left(\widehat{w}_i + \widehat{\tau}_{ij}\right) + \left(\dfrac{1-\sigma-\varepsilon}{\varepsilon(1-\sigma)}\right)\widehat{w}_i + \dfrac{\widehat{M}_i}{\varepsilon}\right],$$

Combining the previous expression with Equation (48), we get

$$\widehat{P}_j = \textstyle\sum_{i=1}^{n} \lambda_{ij} \left(\dfrac{\widehat{X}_{ij} - \widehat{X}_{jj}}{\varepsilon} \right) + \dfrac{\widehat{M}_j}{\varepsilon}.$$

Using the same logic as in Lemma 1, we then obtain Equation (49).

Step 6: *There are no changes in the measure of entrants,* $\widehat{M}_j = 0$.

Equations (14) and (17) imply

$$\textstyle\sum_{i=1}^{n} \dfrac{f_{ji}}{\left(z_{ji}^{*}\right)^{\sigma-1}} \displaystyle\int_{z_{ji}^{*}}^{+\infty} z^{\sigma-1} g_j(z)\, dz - \textstyle\sum_{i=1}^{n} f_{ji} \displaystyle\int_{z_{ji}^{*}}^{+\infty} g_j(z)\, dz = f_e$$

Differentiating the previous expression, we obtain

$$\textstyle\sum_{i=1}^{n} \theta_{ji} \widehat{z}_{ji}^{*} = 0, \tag{50}$$

where θ_{ji} is the share of total revenues in country j associated with sales in country i,

$$\theta_{ji} = \frac{\frac{f_{ji}}{\left(z_{ji}^{*}\right)^{\sigma-1}} \int_{z_{ji}^{*}}^{+\infty} z^{\sigma-1} g_j(z)\, dz}{\sum_{i'=1}^{n} \frac{f_{ji'}}{\left(z_{ji'}^{*}\right)^{\sigma-1}} \int_{z_{ji'}^{*}}^{+\infty} z^{\sigma-1} g_j(z)\, dz}. \tag{51}$$

Equations (17) and (18) further imply that

$$M_j \cdot \textstyle\sum_{i=1}^{n} \dfrac{\sigma f_{ji}}{\left(z_{ji}^{*}\right)^{\sigma-1}} \displaystyle\int_{z_{ji}^{*}}^{+\infty} z^{\sigma-1} g_j(z)\, dz = L_j.$$

Differentiating the previous expression, we obtain

$$\widehat{M}_j + \textstyle\sum_{i=1}^{n} \theta_{ji} \left(1 - \sigma - \gamma_{ij}\right) \widehat{z}_{ji}^{*} = 0. \tag{52}$$

Using the fact that $\gamma_{ij} = 1 - \sigma - \varepsilon$, Equations (50) and (52) imply

$$\widehat{M}_j = 0.$$

Combining the previous expression with Equation (49), we get Equation (21). **QED**. ■

C Proofs (III): Other Gravity Models

Restricted Entry. In the main text, we have argued that in a model with restricted entry, the gains from trade can still be consistently estimated by $(\lambda_{jj})^{1/\bar{\varepsilon}} - 1$. We now demonstrate this result formally. Since the equilibrium conditions (13)-(16) still hold under restricted entry, we can follow the exact same steps as in Lemma 2 to show that $\widehat{P}_j = -\left(\widehat{\lambda}_{jj} - \widehat{M}_j\right) \big/ \varepsilon$. Since $\widehat{M}_j = 0$ by assumption, we obtain

$$\widehat{P}_j = -\widehat{\lambda}_{jj} \big/ \varepsilon, \tag{53}$$

as stated in the main text. Let us now show that $\widehat{Y}_j = 0$. Under restricted entry, the trade balance condition, Equation (18), becomes

$$Y_j = w_j L_j + \Pi_j,$$

where Π_j are aggregate profits in country j. Differentiating the previous expression and using the fact that $\widehat{w}_j = 0$, we obtain

$$\widehat{Y}_j = \chi_j \widehat{\Pi}_j, \tag{54}$$

where $\chi_j = \Pi_j / Y_j$ is the share of aggregate profits in country j's total income. Using Equation (14), we can express aggregate profits as

$$\Pi_j = M_j \cdot \left[\sum_{i=1}^n \frac{f_{ji}}{\left(z_{ji}^*\right)^{\sigma-1}} \int_{z_{ji}^*}^{+\infty} z^{\sigma-1} g_j(z)\, dz - \sum_{i=1}^n f_{ji} \int_{z_{ji}^*}^{+\infty} g_j(z)\, dz \right]$$

Differentiating the previous expression and using $\widehat{M}_j = 0$, we get

$$\widehat{\Pi}_j = \sum_{i=1}^n \theta_{ji} \widehat{z}_{ji}^*, \tag{55}$$

where θ_{ji} is given by Equation (51). Similarly, we can express total income in country j as

$$Y_j = M_j \cdot \left[\sum_{i=1}^n \frac{\sigma f_{ji}}{\left(z_{ji}^*\right)^{\sigma-1}} \int_{z_{ji}^*}^{+\infty} z^{\sigma-1} g_j(z)\, dz \right]$$

Differentiating the previous expression and using $\widehat{M}_j = 0$, we then get

$$\widehat{Y}_j = \sum_{i=1}^{n} \theta_{ji} \left(1 - \sigma - \gamma_{ij}\right) \widehat{z}_{ji}^{*} \tag{56}$$

Combining Equations (54), (55), and (56), we obtain

$$\sum_{i=1}^{n} \theta_{ji} \left(1 - \sigma - \gamma_{ij}\right) \widehat{z}_{ji}^{*} = \chi_j \sum_{i=1}^{n} \theta_{ji} \widehat{z}_{ji}^{*}$$

Since $\gamma_{ij} = 1 - \sigma - \varepsilon$ under a CES import demand system, this implies

$$\left(\varepsilon - \chi_j\right) \left[\sum_{i=1}^{n} \theta_{ji} \widehat{z}_{ji}^{*}\right] = 0,$$

Using the fact that $\varepsilon < 0$, we obtain $\sum_{i=1}^{n} \theta_{ji} \widehat{z}_{ji}^{*} = 0$ and in turn

$$\widehat{Y}_j = 0. \tag{57}$$

Starting from Equations (53) and (57), we can use the same arguments as in Propositions 3 and 4 to conclude that our estimator of the gains from trade is still given by $\left(\lambda_{jj}\right)^{1/\bar{\varepsilon}} - 1$. **QED**.

Endogenous marketing costs. In the main text, we have argued that in a model with endogenous marketing costs, the gains from trade can still be consistently estimated by $\left(\lambda_{jj}\right)^{1/\bar{\varepsilon}} - 1$. To see this, note that the profit-maximization program of a firm with productivity z is now given by

$$\pi_{ij}\left(z\right) = \max_{x} \left\{ x \left[\frac{\sigma \tau_{ij} w_i}{\left(\sigma - 1\right) z P_j}\right]^{1-\sigma} \frac{Y_j}{\sigma} - w_i f_{ij} \left[\frac{1 - \left(1 - x\right)^{1-\mu}}{1 - \mu}\right] \right\} ,$$

where the optimal pricing rule is as in (13). The first-order condition of that program associated with x implies

$$x_{ij}\left(z\right) = 1 - \left(\frac{z_{ij}^{*}}{z}\right)^{\frac{\sigma - 1}{\mu}}, \text{ for all } z \geq z_{ij}^{*},$$

where $x_{ij}\left(z\right)$ represents the fraction of consumers from country j reached by a firm from country i with productivity z; and the productivity cut-off z_{ij}^{*} is still given by Equation (22). Since the price of a given good is infinite for a consumer who is not reached by a firm, the price index of a representative consumer in country j is now equal to

$$P_j^{1-\sigma} = \sum_{i=1}^{n} M_i \int_{z_{ij}^*}^{+\infty} \left[1 - \left(z_{ij}^* / z\right)^{\frac{\sigma-1}{\mu}}\right] \cdot \left(z w_i \tau_{ij}\right)^{1-\sigma} \cdot g_i(z)\, dz \ .$$

Differentiating the previous expression, we obtain

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij} \left[\left(\widehat{w}_i + \widehat{\tau}_{ij}\right) - \frac{\widehat{M}_i}{\sigma - 1} + \frac{\widetilde{\gamma}_{ij} \widehat{z_{ij}^*}}{\sigma - 1} \right], \tag{58}$$

where $\widetilde{\gamma}_{ij}$ is given by

$$\widetilde{\gamma}_{ij} \equiv \frac{(\sigma-1)}{\mu} \frac{\int_{z_{ij}^*}^{+\infty} z^{\sigma-1} \left(\frac{z_{ij}^*}{z}\right)^{\frac{\sigma-1}{\mu}} g(z)\, dz}{\int_{z_{ij}^*}^{+\infty} z^{\sigma-1} \left(1 - \left(\frac{z_{ij}^*}{z}\right)^{\frac{\sigma-1}{\mu}}\right) g(z)\, dz}.$$

Starting from Equation (58), we can then follow the exact same steps as in Lemma 2, Propositions 3 and 4. The only difference is that $\widetilde{\gamma}_{ij}$ now plays the role of γ_{ij}. **QED**.

Multi-product firms. In the main text, we have argued that in a model with multi-product firms, the gains from trade can still be consistently estimated by $(\lambda_{jj})^{1/\overline{\varepsilon}} - 1$. Before showing this formally, let us introduce the following notation. We denote by z_k the productivity of a firm in producing its k-th product for $k = 1, ..., K$. Without loss of generality, we order products for each firm such that $z_1 \geq ... \geq z_K$ and denote by $g_i(z_1, ..., z_K)$ the density function from which productivity levels are randomly drawn across firms. With a slight abuse of notation we denote by $g_i(z_1)$ the marginal density of the highest productivity level, and similarly, we denote by $g_i(z_k | z_{k-1}, ..., z_1)$ the associated conditional densities for $k = 2, ..., K$.

Using the above notation, we can now express the consumer price index of a representative agent in country j as

$$P_j^{1-\sigma} = \sum_{i=1}^{n} M_i \int_{z_{ij}^*}^{+\infty} S\left(z_1, z_{ij}^*\right) \cdot \left(w_i \tau_{ij}\right)^{1-\sigma} \cdot g_i(z_1)\, dz_1,$$

where the productivity cut-off z_{ij}^* is still given by Equation (22) and $S\left(z_1, z_{ij}^*\right)$ is constructed recursively as follows. For $k = K$, we set

$$S^K\left(z_1, ..., z_{K-1}, z_{ij}^*\right) \equiv \int_{z_{ij}^*}^{z_{K-1}} z_K^{\sigma-1} \cdot g_i\left(z_K | z_{K-1}, ..., z_1\right) dz_K \ .$$

Then for any $K > k \geq 2$, we set

$$S^k\left(z_1, ..., z_{k-1}, z_{ij}^*\right) \equiv \int_{z_{ij}^*}^{z_{k-1}} \left[z_k^{\sigma-1} + S^{k+1}(z_k, ..., z_1)\right] \cdot g_i\left(z_k | z_{k-1}, ..., z_1\right) dz_k,$$

Finally, we set

$$S\left(z_1, z_{ij}^*\right) \equiv z_1^{1-\sigma} + S^2\left(z_1, z_{ij}^*\right).$$

Differentiating the consumer price index we obtain

$$\widehat{P}_j = \textstyle\sum_{i=1}^n \lambda_{ij} \left[\left(\widehat{w}_i + \widehat{\tau}_{ij}\right) - \dfrac{\widehat{M}_i}{\sigma - 1} + \dfrac{\tilde{\tilde{\gamma}}_{ij} \widehat{z_{ij}^*}}{\sigma - 1} \right], \tag{59}$$

where $\tilde{\tilde{\gamma}}_{ij}$ is now given by

$$\tilde{\tilde{\gamma}}_{ij} \equiv \frac{\left(z_{ij}^*\right)^{\sigma-1} \frac{g_i\left(z_{ij}^*\right)}{z_{ij}^*} - \int_{z_{ij}^*}^{+\infty} \frac{S_2\left(z_1, z_{ij}^*\right)}{z_{ij}^*} g_i\left(z_1\right) dz_1}{\int_{z_{ij}^*}^{+\infty} S\left(z_1, z_{ij}^*\right) g_i\left(z_1\right) dz_1},$$

where S_2 refers to the derivative of S with respect to its second argument. Starting from Equation (58), we can then follow the exact same steps as in Lemma 2, Propositions 3 and 4. The only difference is that $\tilde{\tilde{\gamma}}_{ij}$ plays the role of γ_{ij}. **QED**.

D Proofs (IV): Generalized Gravity Models

Multiple sectors. In the main text, we have argued that in the multi-sector case, the gains from trade can be consistently estimated by $\prod_{s=1}^S \left(\lambda_{jj}^s\right)^{\eta_j^s / \bar{\varepsilon}^s} - 1$, under perfect competition, and $\prod_{s=1}^S \left(\lambda_{jj}^s \eta_j^s / \delta_j^s\right)^{\eta_j^s / \bar{\varepsilon}^s} - 1$, under monopolistic competition. We now demonstrate these two results formally.

Consider first the case of perfect competition. The same arguments as in Lemma 1 directly imply that

$$\widehat{X}_{ij}^s - \widehat{X}_{jj}^s = \varepsilon^s \left(\widehat{\tau}_{ij}^s + \widehat{w}_i\right), \tag{60}$$

and that

$$\widehat{P}_j = \sum_{s=1}^{S} \eta_j^s \sum_{i=1}^{n} \lambda_{ij}^s \left(\widehat{\tau}_{ij}^s + \widehat{w}_i\right), \tag{61}$$

where λ_{ij}^s is the share of expenditure on goods from country i in country j and sector s. Combining Equations (61) and (60) and simplifying yields

$$\widehat{P}_j = -\sum_{s=1}^{S} \eta_j^s \left(\frac{\widehat{\lambda}_{jj}^s}{\varepsilon^s}\right).$$

Integrating the previous expression as in the proof of Proposition 1 and using the definition of W_j, we get

$$W_j = 1 - \prod_{s=1}^{S} \left(\frac{\lambda_{jj}^s}{\lambda_{jj}^{s'}}\right)^{\eta_j^s/\varepsilon^s}. \tag{62}$$

Our estimator for the gains from trade under perfect competition derives from Equation (62) and the same argument as in the proof of Proposition 2.

Now consider the case of monopolistic competition. Using the same arguments as in Lemma 2, it is easy to show that

$$\widehat{P}_j = -\sum_{s=1}^{S} \eta_j^s \left(\frac{\widehat{\lambda}_{jj}^s - \widehat{M}_j^s}{\varepsilon^s}\right), \tag{63}$$

where M_j^s in the number of entrants in country j and sector s. In order to compute the changes in the number of entrants, we can adopt the same strategy as in Step 6 of the proof of Lemma 2. By free entry, for all $s = 1, ..., S$, we must have

$$\sum_{i=1}^{n} \frac{f_{ji}^s}{\left(z_{ji}^{s*}\right)^{\sigma^s-1}} \int_{z_{ji}^{s*}}^{+\infty} z^{\sigma^s-1} g_j^s(z)\, dz - \sum_{i=1}^{n} f_{ji}^s \int_{z_{ji}^{s*}}^{+\infty} g_j^s(z)\, dz = f_e^s,$$

where the s-superscripts reflect that all variables, parameters and functions may now vary at the sector level. Differentiating the previous expression, we obtain

$$\sum_{i=1}^{n} \theta_{ji}^s \widehat{z}_{ji}^{s*} = 0, \tag{64}$$

with θ_{ji}^s the share of total revenues in country j and sector s associated with sales in country

i. Free entry further implies that

$$M_j^s \cdot \sum_{i=1}^n \frac{\sigma^s f_{ji}^s}{\left(z_{ji}^{s*}\right)^{\sigma^s-1}} \int_{z_{ji}^{s*}}^{+\infty} z^{\sigma^s-1} g_j^s(z)\, dz = L_j^s,$$

where L_j^s is the endogenous amount of labor in sector s in country j. Differentiating the previous expression and using Equation (64), we obtain $\widehat{M}_j^s = \widehat{L}_j^s$. Together with Equation (63), this implies

$$\widehat{P}_j = -\sum_{s=1}^S \eta_j^s \left(\frac{\widehat{\lambda}_{jj}^s - \widehat{L}_j^s}{\varepsilon^s}\right)$$

Like in the case of perfect competition, we can then integrate the previous expression and use the definition of W_j to get

$$W_j = 1 - \prod_{s=1}^S \left(\frac{\lambda_{jj}^s L_j^{s'}}{\lambda_{jj}^{s'} L_j^s}\right)^{\eta_j^s/\varepsilon^s}. \tag{65}$$

Our estimator for the gains from trade under monopolistic competition derives from Equation (65) and the fact that with Cobb-Douglas preferences, the share of employment in sector s under autarky must be equal to the share of expenditure η_j^s. **QED**.

Multiple factors. As mentioned in the main text, we now generalize our multi-sector extension by assuming that there are two factors of production, capital and labor. We assume that production functions are Cobb-Douglas in all sectors. Formally, the variable cost of producing one unit of the variety ω of good s in country i is given by

$$c_i^s(\omega) = \frac{w_i^{\alpha_i^s} r_i^{1-\alpha_i^s}}{z_i^s(\omega)},$$

where w_i and r_i are the price of labor and capital in country i, respectively; and α_i^s is the labor intensity in sector s and country i. In the case of monopolistic competition, we further assume that labor intensity, α_j^s, is the same for fixed and variable costs. For future reference, we let $c_i^s \equiv w_i^{\alpha_i^s} r_i^{1-\alpha_i^s}$.

Consider first the case of perfect competition. Using the same logic as in the previous extension, it is easy to show that

$$\begin{aligned}\widehat{P}_j &= \textstyle\sum_{s=1}^S \eta_j^s \left[\widehat{c}_j^s + \sum_{i=1}^n \lambda_{ij}^s \left(\widehat{\lambda}_{ij}^s - \widehat{\lambda}_{jj}^s\right)/\varepsilon^s\right] \\ &= \textstyle\sum_{s=1}^S \eta_j^s \left[\widehat{c}_j^s - \widehat{\lambda}_{jj}^s/\varepsilon^s\right]\end{aligned}$$

Since labor in country j is our numeraire, $\widehat{w}_j = 0$, we can rearrange the previous expression as

$$\widehat{P}_j = \textstyle\sum_{s=1}^S \eta_j^s \left[\left(1-\alpha_j^s\right)\widehat{r}_j - \widehat{\lambda}_{jj}^s/\varepsilon^s\right]$$

Let us now compute $\widehat{r}_j$. First, note that because of Cobb-Douglas production functions, in each sector s, we must have

$$\frac{K_j^s}{L_j^s} = \left(\frac{1-\alpha_j^s}{\alpha_j^s}\right)\left(\frac{w_j}{r_j}\right),$$

where L_j^s and K_j^s are the amounts of labor and capital, respectively, in sector s in country j. By definition, we know that

$$\frac{K_j}{L_j} = \textstyle\sum_{s=1}^S \delta_j^s \left(\frac{K_j^s}{L_j^s}\right),$$

where $\delta_j^s \equiv L_j^s/L_j$ is the share of labor employed in sector j. Combining the two previous expressions we get

$$\frac{K_j}{L_j} = \left(\frac{w_j}{r_j}\right)\cdot \textstyle\sum_{s=1}^S \delta_j^s \left(\frac{1-\alpha_j^s}{\alpha_j^s}\right).$$

Differentiating the previous expression, we obtain

$$\widehat{r}_j = \left[\widehat{\textstyle\sum_{s=1}^S \delta_j^s \left(\frac{1-\alpha_j^s}{\alpha_j^s}\right)}\right],$$

which implies

$$\widehat{P}_j = \textstyle\sum_{s=1}^S \eta_j^s \left[\left(1-\alpha_j^s\right)\left[\widehat{\textstyle\sum_{s'=1}^S \delta_j^{s'} \left(\frac{1-\alpha_j^{s'}}{\alpha_j^{s'}}\right)}\right] - \widehat{\lambda}_{jj}^s/\varepsilon^s\right].$$

For an arbitrary change in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau}'$, the previous expression implies

$$P_j'/P_j = \Phi_j' \cdot \textstyle\prod_{s=1}^S \left[\lambda_{jj}^s/\left(\lambda_{jj}^s\right)'\right]^{\eta_j^s/\varepsilon^s}, \tag{66}$$

where

$$\Phi_j' \equiv \left[\sum_{s=1}^S \left(\delta_j^s\right)' \left(\frac{1-\alpha_j^s}{\alpha_j^s}\right) \Big/ \sum_{s=1}^S \delta_j^s \left(\frac{1-\alpha_j^s}{\alpha_j^s}\right)\right]^{\sum_{s=1}^S \left(1-\alpha_j^s\right)\eta_j^s}. \tag{67}$$

With multiple factors of production, trade balance implies

$$Y_j = w_j L_j + r_j K_j.$$

Thus for an arbitrary change in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau}'$, we have

$$\frac{Y_j'}{Y_j} = \frac{\alpha_j}{\alpha_j'} \tag{68}$$

where $\alpha_j \equiv w_j L_j / \left(w_j L_j + r_j K_j\right)$ and $\alpha_j' \equiv w_j L_j / \left(w_j L_j + r_j' K_j\right)$ are the share of labor in country j's income in the initial and counterfactual equilibrium, respectively. Using Equations (66) and (68), we can express the associated welfare change, $W_j \equiv 1 - \left(P_j' Y_j / P_j Y_j'\right)$, as

$$W_j = 1 - \left(\frac{\alpha_j'}{\alpha_j}\right) \Phi_j' \cdot \prod_{s=1}^S \left[\lambda_{jj}^s / \left(\lambda_{jj}^s\right)'\right]^{\eta_j^s / \varepsilon^s}$$

To complete the argument under perfect competition, we simply note that under autarky $\alpha_j' = \sum_{s=1}^S \alpha_j^s \eta_j^s$, $\left(\delta_j^s\right)' = \alpha_j^s \eta_j^s / \sum_{s'=1}^S \alpha_j^{s'} \eta_j^{s'}$, and $\left(\lambda_{jj}^s\right)' = 1$. Thus Equation (67) and the previous expression imply

$$\overline{W}_j = \Psi_j \cdot \prod_{s=1}^S \left(\lambda_{jj}^s\right)^{\eta_j^s / \varepsilon^s} - 1,$$

with

$$\Psi_j \equiv \left(\sum_{s=1}^S \frac{\alpha_j^s \eta_j^s}{\alpha_j}\right) \cdot \left[\sum_{s=1}^S \left(\frac{\eta_j^s \left(1-\alpha_j^s\right)}{\sum_{s'=1}^S \alpha_j^{s'} \eta_j^{s'}}\right) \Big/ \sum_{s=1}^S \delta_j^s \left(\frac{1-\alpha_j^s}{\alpha_j^s}\right)\right]^{\sum_{s=1}^S \left(1-\alpha_j^s\right)\eta_j^s}.$$

This completes our proof under perfect competition.

Now consider the case of monopolistic competition. In this situation, we assume that factor intensity is the same for fixed and variable costs in all sectors. The same argument as under perfect competition implies

$$\widehat{P}_j = \sum_{s=1}^S \eta_j^s \left[\left(1-\alpha_j^s\right) \left[\widehat{\sum_{s'=1}^S \delta_j^{s'} \left(\frac{1-\alpha_j^{s'}}{\alpha_j^{s'}}\right)}\right] - \frac{\widehat{\lambda}_{jj}^s - \widehat{M}_j^s}{\varepsilon^s}\right] \tag{69}$$

Let us now compute $\widehat{M}_j^s$. Like in the multi-sector-one-factor-case, free entry implies

$$\sum_{i=1}^{n} \theta_{ji}^s \widehat{z}_{ji}^{s*} = 0, \tag{70}$$

with θ_{ji}^s the share of total revenues in country j and sector s associated with sales in country i. Trade balance further implies that

$$M_j^s \cdot c_j^s \cdot \sum_{i=1}^{n} \frac{\sigma^s f_{ji}^s}{\left(z_{ji}^{s*}\right)^{\sigma^s-1}} \int_{z_{ji}^{s*}}^{+\infty} z^{\sigma^s-1} g_j^s\left(z\right) dz = w_j L_j^s + r_j K_j^s,$$

where L_j^s and K_j^s are the endogenous amounts of labor and capital, respectively, used in sector s in country j. Since production functions are Cobb-Douglas, we can rearrange the previous expression as

$$M_j^s \cdot c_j^s \cdot \sum_{i=1}^{n} \frac{\sigma^s f_{ji}^s}{\left(z_{ji}^{s*}\right)^{\sigma^s-1}} \int_{z_{ji}^{s*}}^{+\infty} z^{\sigma^s-1} g_j^s\left(z\right) dz = \frac{w_j L_j^s}{\alpha_j^s}$$

Differentiating the previous expression and using Equation (70), we obtain

$$\widehat{M}_j^s = \widehat{L}_j^s - \widehat{c}_j^s = \widehat{L}_j^s - \left(1-\alpha_j^s\right)\left[\sum_{s'=1}^{S} \delta_j^{s'} \widehat{\left(\frac{1-\alpha_j^{s'}}{\alpha_j^{s'}}\right)}\right], \tag{71}$$

where the second equality follows from the same logic as under perfect competition. Combining Equations (69) and (71), we obtain

$$\widehat{P}_j = \sum_{s=1}^{S} \eta_j^s \left[\left(1-\alpha_j^s\right)\left(\frac{\varepsilon^s-1}{\varepsilon^s}\right)\left[\sum_{s'=1}^{S} \delta_j^{s'} \widehat{\left(\frac{1-\alpha_j^{s'}}{\alpha_j^{s'}}\right)}\right] - \frac{\widehat{\lambda}_{jj}^s - \widehat{L}_j^s}{\varepsilon^s}\right]$$

The rest of the proof is similar to what we did under perfect competition. After some simple algebra, we obtain

$$\overline{W}_j = \widetilde{\Psi}_j \cdot \prod_{s=1}^{S} \left(\frac{\lambda_{jj}^s \eta_j^s}{\delta_j^s}\right)^{\eta_j^s/\varepsilon^s} - 1,$$

where

$$\widetilde{\Psi}_j \equiv \left(\sum_{s=1}^{S} \frac{\alpha_j^s \eta_j^s}{\alpha_j}\right) \cdot$$

$$\left[\sum_{s=1}^{S}\left(\frac{\eta_j^s\left(1-\alpha_j^s\right)}{\sum_{s'=1}^{S}\alpha_j^{s'}\eta_j^{s'}}\right)\Big/\sum_{s=1}^{S}\left(\frac{\delta_j^s\left(1-\alpha_j^s\right)}{\alpha_j^s}\right)\right]^{\sum_{s=1}^{S}\left(\frac{\varepsilon^s-1}{\varepsilon^s}\right)\left(1-\alpha_j^s\right)\eta_j^s} \cdot \prod_{s=1}^{S}\left(\frac{\alpha_j^s}{\sum_{s'=1}^{S}\alpha_j^{s'}\eta_j^{s'}}\right)^{\eta_j^s/\varepsilon^s}.$$

This completes the second part of our proof in the multi-factor case. **QED**.

Tradable intermediate goods. Consider first the case of perfect competition. We assume that intermediate goods are aggregated into a non-tradable good that can be either consumed or combined with labor to produce a "composite input" that will be used, in turn, in the production of intermediate goods. Formally, if we denote by K_i the quantity of the non-tradable aggregate good allocated to the production of the composite input in country i, then the quantity produced of this input is $Q_i = \beta_i^{\beta_i}(1-\beta_i)^{1-\beta_i} L_i^{\beta_i} K_i^{1-\beta_i}$. Since the price of the non-tradable good is equal to the consumer price index in country i, the unit cost of Q_i is given by $c_i = w_i^{\beta_i} P_i^{1-\beta_i}$, justifying Equation (26) in the main text.

Under these assumptions, the equilibrium conditions remain given by Equations (4)-(6), but with c_j and Q_j substituting for w_j and L_j. Using the composite input in country j as our numeraire, $c_j = 1$, we can therefore follow the same logic as in Lemma 1 to show that changes in the consumer price index satisfy $\widehat{P}_j = -\widehat{\lambda}_{jj}/\varepsilon$. Since w_j is no longer our numeraire, we however need to take changes in into account in our welfare computations. Formally, we have $W_j \equiv 1 - \left(w_j P_j' / w_j' P_j\right)$, where w_j and w_j' are the wages in the initial and the new equilibrium, respectively. This implies

$$\widehat{1-W_j} = \widehat{P}_j - \widehat{w}_j = -\widehat{\lambda}_{jj}/\varepsilon\beta_j, \tag{72}$$

where the second equality comes from the fact that

$$\beta_j \widehat{w}_j + (1-\beta_j)\widehat{P}_j = 0, \tag{73}$$

by our choice of numeraire. Starting from Equation (72), we can then use the same arguments as in Propositions 1 and 2 to conclude that our estimator of the gains from trade is now given by $(\lambda_{jj})^{1/\left(\beta_j\bar{\varepsilon}\right)} - 1$.

Consider now the case of monopolistic competition. We maintain the assumption that labor and the aggregate non-tradable good are used to produce a common input with unit cost $c_i = w_i^{\beta_i} P_i^{1-\beta_i}$. Compared to the case of perfect competition, we assume that this common input is used both for the production of intermediate goods and the payment of fixed marketing costs, now equal to $c_i f_{ij}$. In addition, we assume that labor and the non-tradable aggregate good can be combined for the payment of fixed entry costs. In order to produce in country i, a firm must pay $c_i^e \equiv w_i^{\alpha_i} P_i^{1-\alpha_i} f_e$.

Under these assumptions, Equations (13)-(16) and (18) still hold in equilibrium, but with c_j and Q_j substituting for w_j and L_j. Given that $c_j Q_j = w_j L_j + P_j K_j$, then we now have

$$Y_j = w_j L_j + P_j K_j. \tag{74}$$

By contrast, using the composite good in country j as our numeraire, $c_j = 1$, Equation (17) becomes

$$\sum_{i=1}^{n} \frac{f_{ji}}{\left(z_{ji}^*\right)^{\sigma-1}} \int_{z_{ji}^*}^{+\infty} z^{\sigma-1} g_j(z)\, dz - \sum_{i=1}^{n} f_{ji} \int_{z_{ji}^*}^{+\infty} g_j(z)\, dz = c_j^e. \tag{75}$$

Using Equations (13)-(16) and the same logic as in Lemma 2—Steps 1 through 5—it easy to show that changes in the consumer price index still satisfy

$$\widehat{P}_j = -\left(\widehat{\lambda}_{jj} - \widehat{M}_j\right)/\varepsilon. \tag{76}$$

We now build on Equation (75) and the logic of Step 6 in Lemma 2 to show that $\widehat{M}_j = -(\frac{1-\alpha_j}{\beta_j})\widehat{P}_j$. We use the following notations. We denote by L_j^Q and K_j^Q the amount of labor and the composite good used for production and marketing costs; and similarly, we denote by L_j^E and K_j^E the amounts of labor and the composite input used for entry. Our formal argument proceeds in three steps.

Step 1: *Percentage changes in the number of entrants satisfy*

$$\widehat{M}_j = \widehat{L}_j^E - (\frac{1-\alpha_j}{\beta_j})\widehat{P}_j. \tag{77}$$

Given our Cobb-Douglas aggregator, we know that

$$M_j f_e = \alpha_j^{-\alpha_j}(1-\alpha_j)^{\alpha_j-1}\left(L_j^E\right)^{\alpha_j}\left(K_j^E\right)^{1-\alpha_j}, \tag{78}$$

$$\frac{K_j^E}{L_j^E} = \left(\frac{1-\alpha_j}{\alpha_j}\right)\left(\frac{w_j}{P_j}\right), \tag{79}$$

Differentiating Equations (78) and (79), we obtain after rearrangements

$$\widehat{M}_j = \widehat{L}_j^E + (1-\alpha_j)\left(\widehat{w}_j - \widehat{P}_j\right)$$

Combining the previous expression with Equation (73), which still holds by our choice of numeraire, we obtain Equation (77).

Step 2: *Percentage changes in the amount of the composite good satisfy*

$$\widehat{K}_j = \psi_j \widehat{L}_j^E - \left(\frac{1}{\beta_j}\right)\widehat{P}_j, \tag{80}$$

where $\psi_j \equiv \left(\frac{\beta_j - \alpha_j}{\alpha_j \beta_j}\right)\left(\frac{w_j L_j^E}{P_j K_j}\right)$.

Given our CES aggregator, we know that

$$\frac{K_j^Q}{L_j^Q} = \left(\frac{1-\beta_j}{\beta_j}\right)\left(\frac{w_j}{P_j}\right). \tag{81}$$

By definition, we also know that $K_j = K_j^Q + K_j^E$ and $L_j = L_j^Q + L_j^E$. Using Equations (79) and (81), we can rearrange the previous expression as

$$K_j = \left[L_j\left(\frac{1-\beta_j}{\beta_j}\right) + L_j^E\left(\frac{\beta_j - \alpha_j}{\alpha_j\beta_j}\right)\right]\left(\frac{w_j}{P_j}\right).$$

Differentiating the previous expression, we obtain

$$\widehat{K}_j = \left(\frac{\beta_j - \alpha_j}{\alpha_j\beta_j}\right)\left(\frac{w_j L_j^E}{P_j K_j}\right)\widehat{L}_j^E + \left(\widehat{w}_j - \widehat{P}_j\right). \tag{82}$$

Equation (80) directly derives from Equations (73) and (82).

Step 3: *The amount of labor used for entry does not vary with trade costs:* $\widehat{L}_j^E = 0$.

Differentiating Equation (75), we get

$$(1-\sigma)\sum_{i=1}^n \theta_{ji}\widehat{z}_{ji}^* = \frac{\left[\alpha_j\widehat{w}_j + (1-\alpha_j)\widehat{P}_j\right]c_j^e}{\sum_{i=1}^n f_{ji}\left(z_{ji}^*\right)^{1-\sigma}\int_{z_{ji}^*}^{+\infty} z^{\sigma-1}g_j(z)\,dz}, \tag{83}$$

where θ_{ji} the share of total revenues in country j associated with sales in country i. Equations

(74) and (75) further imply that

$$M_j \cdot \sum_{i=1}^{n} \frac{\sigma f_{ji}}{\left(z_{ji}^*\right)^{\sigma-1}} \int_{z_{ji}^*}^{+\infty} z^{\sigma-1} g_j\left(z\right) dz = w_j L_j + P_j K_j .$$

Differentiating the previous expression and combining it with Equation (83), we obtain

$$\widehat{M}_j + \left(\frac{1-\sigma-\gamma}{1-\sigma}\right) \frac{\left[\alpha_j \widehat{w}_j + (1-\alpha_j)\widehat{P}_j\right] c_j^e}{\sum_{i=1}^{n} f_{ji}\left(z_{ji}^*\right)^{1-\sigma} \int_{z_{ji}^*}^{+\infty} z^{\sigma-1} g_j\left(z\right) dz} = \left(1-\kappa_j\right) \widehat{w}_j + \kappa_j \left(\widehat{P}_j + \widehat{K}_j\right),$$

where $\kappa_j \equiv P_j K_j / \left(w_j L_j + P_j K_j\right)$. Combining the previous expression with Equations (73), (77), and (80), we obtain

$$\widehat{L}_j^E = \frac{\widehat{w}_j\left(\alpha_j - \beta_j\right)}{\left(1-\beta_j\right)\left(1-\kappa_j \psi_j\right)} \left[1 - \frac{\left(1-\sigma-\gamma\right) c_j^e}{\left(1-\sigma\right) \sum_{i=1}^{n} f_{ji}\left(z_{ji}^*\right)^{1-\sigma} \int_{z_{ji}^*}^{+\infty} z^{\sigma-1} g_j\left(z\right) dz}\right]. \quad (84)$$

Equations (84) and (75) imply

$$\widehat{L}_j^E = \frac{\widehat{w}_j\left(\alpha_j - \beta_j\right)}{\left(1-\beta_j\right)\left(1-\kappa_j \psi_j\right)} \frac{\sum_{i=1}^{n} f_{ji} H_j\left(z_{ji}^*\right)}{\left(1-\sigma\right) \sum_{i=1}^{n} f_{ji}\left(z_{ji}^*\right)^{1-\sigma} \int_{z_{ji}^*}^{+\infty} z^{\sigma-1} g_j\left(z\right) dz}, \quad (85)$$

where $H_j\left(z_{ji}^*\right) \equiv \gamma \left(z_{ji}^*\right)^{1-\sigma} \int_{z_{ji}^*}^{+\infty} z^{\sigma-1} g_j\left(z\right) dz + \left(1-\sigma-\gamma\right) \int_{z_{ji}^*}^{+\infty} g_j\left(z\right) dz$. Notice that

$$H_j'\left(z_{ji}^*\right) = \left(1-\sigma\right) \left[\gamma \left(z_{ji}^*\right)^{-\sigma} \int_{z_{ji}^*}^{+\infty} z^{\sigma-1} g_j\left(z\right) dz - g_j\left(z_{ji}^*\right)\right] = 0, \quad (86)$$

where the second equality comes from the fact that $\gamma = \gamma_{ij} = \left(z_{ij}^*\right)^{\sigma} g_i\left(z_{ij}^*\right) / \int_{z_{ij}^*}^{+\infty} z^{\sigma-1} g_i\left(z\right) dz$. Since $\lim_{z_{ji}^* \to +\infty} H_j\left(z_{ji}^*\right) = 0$, Equation (86) implies $H_j\left(z_{ji}^*\right) = 0$ for all z_{ji}^*. Combining this observation with Equation (85), we obtain $\widehat{L}_j^E = 0$.

To conclude, note that Steps 1 and 3 imply $\widehat{M}_j = -(\frac{1-\alpha_j}{\beta_j})\widehat{P}_j$. Together with Equation (76), this implies $\widehat{P}_j = -\beta_j \widehat{\lambda}_{jj} / \left(\beta_j \varepsilon + 1 - \alpha_j\right)$. Using Equation (73) and the fact that $\widehat{1-W_j} = \widehat{P}_j - \widehat{w}_j$, we obtain $\widehat{1-W_j} = -\widehat{\lambda}_{jj} / \left(\beta_j \varepsilon + 1 - \alpha_j\right)$. The rest of the proof is the same as under perfect competition. **QED**.