# Aggregate Implications of Innovation Policy*

# Preliminary and Incomplete

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ABSTRACT

In this paper we present a model that integrates microeconomic and macroeconomic perspectives on the implications for aggregate output and welfare of changes in innovation policies. We develop a simple two-step algorithm to assess the long run impact of changes in innovation policies. We use this algorithm to show that in our model there is no special role for innovation policies distinct from a policy of subsidizing the profits of incumbent firms, even in the presence of spillovers from innovative activities. We also show that a wide range of policy changes have a long run impact in direct proportion to their aggregate impact on fiscal expenditures on those policies.

We use these results to compare the relative magnitudes of the impact on aggregates in the long-run of three innovation policies in the United States: the Research and Experimentation tax credit, Federal Expenditure on R&D, and the corporate profits tax. We argue that the corporate profits tax is a relatively important policy through its negative effects on innovation. We also use a calibrated version of our model to examine the absolute magnitude of the impact of these policies on aggregates. We show that, depending on the magnitude of spillovers, it is possible for changes in innovation policies to have very large impact on aggregates in the long run. However, over a fifteen year horizon, the impact of changes in innovation policies on aggregate output is not very sensitive to the magnitude of spillovers.

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## 1. Introduction

How do changes in economic policies that affect the costs and benefits to firms of innovative activity impact aggregate output, productivity, and welfare? In this paper, we address this question using a model that focuses on the role of innovative activities by heterogenous firms in contributing to aggregate productivity improvements in the economy. Our model is rich enough to capture the dynamic decisions of heterogeneous firms to both improve existing products (process innovation) and create new products (product innovation), and yet tractable enough to aggregate-up from firm-level decisions to obtain a deeper understanding of how aggregate output, productivity, and welfare should be expected to respond in general equilibrium to changes in innovation policies.

There is a very large empirical literature, following from the work of Zvi Griliches and many others, that uses detailed firm-level data to assess the impact of changes in innovation policies on firms' decisions to engage in innovative activities. At the same time, there is also a very large macroeconomic literature that aims to assess the aggregate implications of changes in innovation policy.[1] In this paper, we develop a model that integrates these two perspectives on the implications of innovation policy. As in Atkeson and Burstein (2010), our model extends Hopenhayn's (1992), Atkeson and Kehoe's (2005), and Luttmer's (2007) model of firm dynamics with entry of new firms or products to include a process innovation decision by incumbent firms following Griliches' (1979) model of knowledge capital. Our model features both physical capital and intangible capital accumulated by firms, and spillovers from firms' innovative activity.[2] We view our model as a tractable benchmark for examining the aggregate implications of innovation policy on aggregate output, productivity, and welfare.

We first show that one can use a two-step algorithm to assess the impact of changes in innovation policies on aggregate output, productivity, and welfare in the long run. In the first step, one uses a simple accounting procedure to measure the impact of changes in innovation policies on firm profitability. In this step, the analyst only needs to compute the impact at the margin of a change in innovation policy on the profitability of a typical firm. There is

[1] See Jones and Manuelli (2005) for a review of the impact of policies on growth in neoclassical endogenous growth models. See Howitt (2004) and Aghion and Howitt (2006) for a review of the impact of competition and innovation policy on growth. Also see Lee and Gordon (2005), OECD (2003, 2005) for reviews of the impact of innovation policy on aggregate outcomes.

[2] Our model extends McGrattan and Prescott (2005a,b) by modelling the accumulation of firm specific intangible capital. See Klenow and Rodriguez-Clare (2005) for a review of spillovers in models of economic growth.

no need in this first step for the analyst to take into account, at least locally, the dynamic response in the innovative activity of the typical firm to the change in innovation policy. For example, to compute the impact on aggregates in the long-run of a change in the Tax Credit for Research and Experimentation (R&E), in this first step, the analyst would simply need to compute the expected impact on the after-tax profitability of the typical firm of this change in the tax credit, holding fixed current levels of innovation expenditure and other decisions by firms. In the second step, the analyst uses the macroeconomic structure of the model to infer the long-run changes in aggregate output and productivity that must accompany, in general equilibrium, the change in firm profitability computed in the first step.

We use this algorithm to establish several analytical results on the long-run impact of changes in innovation policies. We show first that, globally, a uniform subsidy to innovative activity has an equivalent impact to a direct subsidy to firm profitability because subsidizing the cost of innovation is equivalent to subsidizing the returns to innovation. Second, we show that, locally, changes in the subsidy to any particular type of innovative activity all have an equivalent impact on aggregates as long as these policy changes have the same impact on firm profitability. Together, these results imply that in our model there is no special role for innovation policies distinct from a policy of subsidizing the profits of incumbent firms directly, even in the presence of spillovers from innovative activities.

We next use our model in two quantitative applications. First, we compare the relative magnitudes of the impact on aggregates in the long-run of three policies affecting innovative activity by firms in the U.S.: the R&E Tax Credit, Federal Expenditure on R&D, and the corporate profits tax. In this application, we use an analytical result from our model that, to a first-order approximation, one can measure the relative impact of a policy change on firm profitability, and hence on aggregates, from the impact of that policy change on government's fiscal expenditures. This result simplifies policy evaluation because there is no need to calculate changes in effective marginal tax or subsidy rates or changes in firms' decisions in response to these policy changes to evaluate the aggregate impact of a policy change. We use data on fiscal expenditures on these policies in the U.S. to argue that the corporate profits tax is a relatively important policy, in comparison to the R&E Tax Credit and Federal Expenditure on R&D, in terms of its aggregate effects on the long run accumulation of both tangible and intangible capital.

In our second quantitative application, we use a calibrated version of our model to examine the absolute magnitude of the impact on aggregates of a uniform subsidy to innovative activities leading to fiscal expenditures of 3% of GDP. This policy experiment has a similar impact on fiscal expenditures (and hence, to a first-order approximation, a similar long run impact on aggregates) as a policy of eliminating the corporate profits tax. We find that this policy change raises the research intensity of the economy by roughly 3 percentage points on a permanent basis, and has an impact on aggregate output over a 15 year horizon of between 1 and 4.5 percentage points, depending on the magnitude of the spillovers from innovative activity. These results imply that the change in the research intensity of the economy is roughly the same size as the fiscal expenditures on innovation subsidies, and that this change in the research intensity of the economy has a very small impact on the annualized growth rate of aggregate output over a 15 year horizon.

While our results about the impact of policy at a 15 year horizon are not very sensitive to a wide range of assumptions about the magnitude of spillovers from innovative activity, our model implies that there is a great deal of uncertainty about the impact of this policy change on aggregate output in the long run and welfare. We find that the level of aggregate output in the long run can change from anywhere between 2.8 percent in the absence of spillovers, to over $10,000$ percent with strong spillovers. The corresponding equivalent variation in consumption ranges from close to zero with no spillovers to roughly 50 percent with strong spillovers. This uncertainty about the long run impact of policy changes is not apparent at a 15 year horizon because the model's transition dynamics become extremely slow as the spillovers from innovative activities become large.

We conclude the paper with a discussion on the implications of our model for empirical work in the literature on the aggregate impact of change in innovation policies and on the social returns to innovation (see e.g. Kortum 1997, Jones and Williams 1998, Griffith, Redding, and Van Reenen 2001, Bloom, Griffith, and Van Reenen 2002, Bloom, Schankerman, Van Reenen 2010, and references therein). On the basis of the results of our model, we argue that existing regression evidence linking policy-induced changes in the research intensity of firms, industries, or aggregate economies to changes in output and productivity at the firm, industry, or aggregate levels, are not likely to shed much light on the magnitude of spillovers and hence on the long run aggregate implications of changes in innovation policies.

In deriving our main results, we assume that the parameters of our model lie in a region such that an increase in the productivity of incumbent firms in response to a change in policy crowds-out profits of entering firms. With this assumption, our model is a semi-endogenous growth model in which innovation policies affect the level of aggregate activity but not its growth rate, which is determined by the exogenous growth rate of general scientific knowledge and the population. There is a knife-edged set of parameter values for which the profits of entering firms are independent of other firms' productivities. With parameters set in this knife-edge case, our model is an endogenous growth model in which innovative activity by firms is the engine of growth. We provide a brief discussion of what results apply in the endogenous growth case. Note that the standard endogenous growth models such as the model of expanding varieties of Romer (1990), the quality-ladders model of Grossman and Helpman (1991) and Klette and Kortum (2004), and Schumpeterian models of Aghion and Howitt (1992), as well as many of the variants of these models discussed in Acemoglu (2009), chapters 13-14, satisfy this knife-edged property.[3]

The paper is organized as follows. Section 2 presents the model. Section 3 characterizes the BGP. Section 4 describes optimal policies. Section 5 presents analytic results on the aggregate impact, in the long run, of policy changes. Section 6 examines the relative magnitude of the impact of the three policies that affect innovative activity in the U.S. Section 7 presents the results from our calibrated model. Section 8 discusses the relationship between our results and existing empirical work on the aggregate implications of changes in the research intensity of the economy.

## 2. Model

In this section, we describe the physical environment, innovation policies, the equilibrium, and the implications of our model for firm dynamics.

### A. Physical Environment

Time is discrete and labeled $t = 0, 1, 2, \ldots$. There is two final goods, the first of which we call the *consumption good.* The representative household has preferences over this consumption good, $C_t$, given by

$$\sum_{t=0}^{\infty} \beta^t L_t \log(C_t/L_t)$$

[3] Our model of firm dynamics does not nest quality ladders or Schumpeterian models of firm dynamics in which entering firms directly displace incumbent firms in the production of existing products. In an appendix to be added, we show that our main results go through in a quality ladders version of the model

where $L_t$ is the population at date $t$, which grows at a steady rate of $g_L$ so $L_{t+1} = \exp(g_L)L_t$. We assume that $\beta \exp(g_L) < 1$.

Output of the consumption good, $Y_t$, is used for three purposes. First, as consumption by the representative household. Second, as investment in physical (tangible) capital, $K_{t+1} - (1-\delta_k) K_t$, where $K_t$ denotes the aggregate capital stock and $\delta_k$ denotes the depreciation rate of physical capital. Third, as an input into innovative activity, $X_t$. The resource constraint of the final consumption good is:

$$C_t + K_{t+1} - (1-\delta_k) K_t + X_t = Y_t. \tag{1}$$

At each date $t$, there is a continuum of size $N_t$ of incumbent firms each producing distinct intermediate goods. These intermediate goods producing firms are distinguished by an index $z$ of their current productivity, and we let $J_t(z)$ denote the distribution of $z$ across incumbent firms at date $t$ (so $\int_z dJ_t(z) = N_t$). The consumption good is produced as a CES aggregate of the output of this continuum of intermediate goods, according to

$$Y_t = \left( \int_z y_t(z)^{(\rho-1)/\rho} dJ_t(z) \right)^{\rho/(\rho-1)}, \tag{2}$$

where $y_t(z)$ is the output of an intermediate good producing firm with productivity index $z$ at date $t$. We assume that $\rho > 1$.

An incumbent intermediate good producing firm with productivity index $z$ produces its differentiated output with physical capital, $k$, and labor, $l$, according to

$$y_t(z) = \exp(z_t)^{1/(\rho-1)} k_t(z)^{\alpha} l_t(z)^{1-\alpha}, \tag{3}$$

where $0 < \alpha < 1$. Here, the productivity of the individual firm is given by $\exp(z_t)^{1/(\rho-1)}$.[4] As we show below, this normalization of productivity is convenient as the equilibrium size of the firm, measured in terms of labor $l_t(z)$, capital $k_t(z)$, revenues, or variable profits, is directly proportional to $\exp(z)$.

To innovate, firms must use a second final good, that we refer to as the *research good*,

[4] In an appendix to be added, we extend our model to allow for decreasing returns to scale in the production of intermediate goods, and a richer input-output structure (i.e.: final goods and intermediate goods are produced using the consumption good, capital and labor). Our main results remain unchanged.

as an input. Intermediate goods producing firms invest in two types of innovation in this economy: process and product innovation. Incumbent intermediate goods producing firms engage in *process innovation* to increase their productivity index $z$ from one period to the next. For an incumbent firm to increase its productivity index from $z$ in period $t$ to $z' = z + g_z$ in period $t+1$, it requires an expenditure of $c(\exp(g_z))\exp(z)$ units of the research good in period $t$, where $c(x)$ is a convex function. With this specification of process innovation costs, the cost for a firm of growing by a given percentage scales in direct proportion to the current size of the firm. We denote the curvature of the function $c(x)$ by $\eta_c = \frac{c''(x)x}{c'(x)}$.

Firms invest in *product innovation* as follows. A new firm producing a new variety can be created in period $t+1$ by an expenditure of $n_e$ units of the research good in period $t$. For simplicity, we assume that all newly created firms start with productivity index $\bar{z}$, which we normalize to 0.[5]

Letting $M_t \geq 0$ denote the mass of investment in new products in period $t$, then the resource constraint for the research good is given by

$$n_e M_t + \int_z c(\exp(g_{zt}(z)))\exp(z)dJ_t(z) = Y_{rt} \tag{4}$$

where $g_{zt}(z)$ is the growth rate of the productivity index $z$ chosen by intermediate goods producing firms with current index $z$ in period $t$ and $Y_{rt}$ is the output of the research good in period $t$.

We assume that a fraction $\delta_f$ of all incumbent firms and newly created firms exit exogenously at the beginning of each period. Thus, the evolution of the number of firms is given by

$$N_{t+1} = (1-\delta_f)(M_t + N_t) \tag{5}$$

The research good is produced as a Cobb-Douglas combination of the consumption good and labor according to

$$Y_{rt} = A_t H_t^{\gamma} L_{rt}^{\lambda} X_t^{1-\lambda} \quad , \ \text{where} \tag{6}$$

$$H_{t+1} = (1-\delta_r)H_t + Y_{rt} \tag{7}$$

[5] It is straightforward to allow for exogenous growth in $\bar{z}$ so that new goods improve over time. This is the approach taken in Atkeson and Kehoe (2005).

Here, $L_{rt}$ and $X_t$ denote the labor and consumption good, respectively, used in the production of the research good, and $A_t$ represents the stock of basic scientific knowledge which is assumed to evolve exogenously, growing at a steady rate of $g_A$ so $A_{t+1} = \exp(g_A)A_t$. Increases in this stock of knowledge improve the productivity of resources devoted to innovative activity. We interpret $A$ as a world-wide stock of scientific knowledge that is freely available for firms to use in innovative activities. The determination of $A$ is outside the scope of our analysis.

The variable $H_t$ is the spillover from cumulative innovative activity by firms, here modeled as an external learning effect — in innovating, researchers gain knowledge and experience useful for further innovation that is not captured as part of the private return either of the firm or the workers engaged in innovation. In what follows, we assume that $0 \leq \gamma < 1$.

The amount of labor used in current production, $L_{pt}$, is given by

$$L_{pt} = \int_z l_t(z) dJ_t(z) \ ,$$

The resource constraint for labor requires that labor used in current production plus labor used in the production of the research good must sum to a fixed total population $L_t$, that is $L_{pt} + L_{rt} = L_t$. Labor is mobile between intermediate goods production and innovation.

The amount of physical capital used in current production must satisfy the constraint

$$K_t = \int_z k_t(z) dJ_t(z) \ .$$

Finally, the law of motion of the distribution of $z$ across incumbent firms, $J_t(z)$, is determined by the exogenous exit rate, $\delta_f$, the process innovation decisions of incumbent firms, $g_{zt}(z)$, and the mass of investment in new products, $M_t$, in a standard manner.

## B. Policies and equilibrium

In this subsection, we describe the decentralization of this economy and define equilibrium with a collection of policies. In this decentralization, we assume that the representative household owns the physical capital stock and rents it to the intermediate good producing firms at rental rate $R_{kt}$. Each period, the household faces a budget constraint given by

$$C_t + K_{t+1} = \left[R_{kt} + (1 - \delta_k)\right] K_t + W_t L_t + D_t - E_t,$$

where $W_t$, $D_t$, and $E_t$ denote the economy-wide wage, aggregate dividends paid by intermediate good firms, and aggregate fiscal expenditures on policies (which are financed by lump-sum taxes collected from the representative household), respectively. We also define an interest rate for bonds denominated in the final consumption good, $\bar{r}_t$, which with log preferences is given by

$$1 + \bar{r}_t = \frac{1}{\beta} \frac{C_{t+1}/L_{t+1}}{C_t/L_t}. \tag{8}$$

The Euler equation for physical capital is given by

$$1 + \bar{r}_t = R_{kt+1} + 1 - \delta_k. \tag{9}$$

Intermediate good producing firms are offered three types of subsidies and are subject to one tax. These are abstract policies that are useful in deriving analytical results. In our quantitative work below we describe a mapping between actual policies and these abstract policies. The policies are as follows. First, firms are subject to a subsidy to variable profits from production (defined below), that we denote by $\tau_p$. Second, firms are subject to a subsidy to process innovation, that we denote by $\tau_g$. Third, firms are subject to a subsidy to product innovation that we denote by $\tau_e$. Finally, firms are subject to a tax on their use of physical capital, that we denote by $\tau_k$.

As we discuss below, the equilibrium of our model has the standard inefficiency arising from a monopoly markup in the production of intermediate goods. The policies that we consider above are not sufficient to undo these distortions from this markup. To have enough policies to implement the socially optimal allocation, we allow for a per-unit subsidy on production of the consumption good, $\tau_s$, that can be set to undo the distortions from the markup (note that subsidizing the production of the consumption good is equivalent to subsidizing the sales of intermediate good producers).

Competitive firms producing the final consumption good choose inputs and output to maximize profits each period, taking prices as given. In particular, they solve the problem

$$\max_{Y_t, \{y_t(z)\}} (1 + \tau_s) P_t Y_t - \int_z p_t(z) y_t(z) dJ_t(z),$$

subject to (2), where $p_t(z)$ is the price set by firms with productivity index $z$, and $P_t$ is the price of the final good paid by the representative household.

By standard arguments, in equilibrium prices must satisfy

$$(1+\tau_s)\,P_t = \int_z (p_t(z))^{1-\rho} dJ_t(z). \tag{10}$$

We normalize $P_t$ to 1. This profit maximization problem also gives input demands

$$y_t(z) = \left(\frac{p_t(z)}{1+\tau_s}\right)^{-\rho} Y_t\ .$$

The variable profits from production of a firm with productivity index $z$ in period $t$ are given by

$$(1+\tau_p)\left[p_t(z)y_t(z) - (1+\tau_k)\,R_{kt}k_t\,(z) - W_t l_t(z)\right],$$

This firm chooses price and quantity, $p_t\,(z)$ and $y_t\,(z)$, to maximize these variable profits subject to the demand above and the production function (3). Profit maximization implies that a firm with productivity index $z$ in period $t$ sets its price at

$$p_t(z) = \frac{\rho}{\rho-1}\frac{\left[(1+\tau_k)\,R_{kt}\right]^{\alpha} W_t^{1-\alpha}}{\exp(z)^{1/(\rho-1)}} \tag{11}$$

and its variable profits from production can be written as $(1+\tau_p)\,(1+\tau_k)^{\alpha(1-\rho)}\,\Pi_t \exp(z)$, with the constant in variable profits $\Pi_t$ defined by

$$\Pi_t = \kappa_0\,(1+\tau_s)^{\rho}\left(R_{kt}^{\alpha}W_t^{1-\alpha}\right)^{1-\rho} Y_t, \tag{12}$$

where $\kappa_0$ is a constant.[6] As these equations make clear, the variable profits earned by an incumbent firm scale with its productivity index $z$ in direct proportion to $\exp(z)$. We refer to $\Pi_t$ as the constant in variable profits of incumbent firms.

Firms' investments in process innovation are governed by the Bellman equation

$$V_t(z) = \max_{g_{zt}}\left[(1+\tau_p)\,(1+\tau_k)^{\alpha(1-\rho)}\,\Pi_t - (1-\tau_g)\,P_{rt}c\left(\exp(g_{zt})\right)\right]\exp(z) + \frac{(1-\delta_f)}{1+\bar{r}_t}V_{t+1}(z+g_z). \tag{13}$$

The value function $V_t(z)$ corresponds to the expected discounted present value of dividends paid by an incumbent firm with current productivity index $z$. The first term in the right-hand

[6] In particular, $\kappa_0 = \rho^{-\rho}(\rho-1)^{\rho-1}\left[\alpha^{\alpha}\,(1-\alpha)^{1-\alpha}\right]^{\rho-1}$.

side of (13) is the current dividend.

The zero-profit condition governing product innovation is given by

$$(1-\tau_e)\,P_{rt}n_e = \frac{(1-\delta_f)}{1+\bar{r}_t}V_{t+1}(\bar{z}) \tag{14}$$

as long as there is positive investment in product innovation, and an inequality with the cost of product innovation exceeding the profits otherwise. The dividend paid by an entering firm is $-(1-\tau_e)\,P_{rt}n_e$.

To aggregate the decisions of intermediate-good producers, we define an index of *total productivity* at time $t$, $N_t Z_t$, where $Z_t$ is an index of *average productivity* at time $t$ given by

$$Z_t = \frac{1}{N_t}\int_z \exp(z)dJ_t(z).$$

It is straightforward to show that in equilibrium, aggregate output of the consumption good is

$$Y_t = (N_t Z_t)^{1/(\rho-1)} K_t^{\alpha} L_{pt}^{1-\alpha}. \tag{15}$$

In parallel to physical capital, we refer to the index of total productivity $N_t Z_t$ as the aggregate stock of intangible capital in firms. In our applications, we use the sum of expenditures on consumption, $C_t$, and investment in physical capital, $K_{t+1}-(1-\delta_k)\,K_t$ as our measure of aggregate output (GDP) because in the National Income and Product Accounts, expenditures on innovative activities, $P_{rt}Y_{rt}$, are expensed rather than counted as a part of final output. Using (1), we have $GDP_t = Y_t - X_t$.[7]

With CES aggregators and Cobb-Douglas production functions, aggregate revenues of intermediate goods firms, $(1+\tau_s)\,Y_t$ are split into three components. A share $1/\rho$ accrues to variable profits from production (exclusive of the subsidy on variable profits), $(1+\tau_k)^{\alpha(1-\rho)}\,\Pi_t N_t Z_t = \frac{1}{\rho}(1+\tau_s)\,Y_t$, a share $\alpha(\rho-1)/\rho$ is paid to physical capital (inclusive of taxes on physical capital), $(1+\tau_k)\,R_{kt}K_t = \frac{\alpha(\rho-1)}{\rho}(1+\tau_s)\,Y_t$, and a share $(1-\alpha)(\rho-1)/\rho$ is paid as wages to production labor, $W_t L_{pt} = \frac{(1-\alpha)(\rho-1)}{\rho}(1+\tau_s)\,Y_t$. Hence, we have that payments to variable production labor represent a constant share of variable profits (exclusive of the subsidy on

[7] In using this measure of GDP, we are abstracting from measurement problems arising from the introduction of new products. Under the assumption that $Y$ from (15) is a physical good that is traded in the market, these measurement problems for GDP are less likely to arise.

variable profits)

$$W_t L_{pt} = (1-\alpha)(\rho-1)(1+\tau_k)^{\alpha(1-\rho)} \Pi_t N_t Z_t, \tag{16}$$

and that the ratio of physical capital to production labor is

$$\frac{K_t}{L_{pt}} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_{kt}(1+\tau_k)}. \tag{17}$$

Production of the research good is undertaken by competitive firms that take the spillover from innovation $H_t$ as given. Cost minimization in the production of the research good implies that the price of the research good, $P_{rt}$, is equal to:

$$P_{rt} = \frac{1}{H_t^{\gamma} A_t} \frac{1}{\lambda^{\lambda}(1-\lambda)^{(1-\lambda)}} W_t^{\lambda} \text{ , and} \tag{18}$$

$$\frac{\lambda}{1-\lambda} \frac{X_t}{L_{rt}} = W_t. \tag{19}$$

Cost minimization of the research good implies that

$$W_t L_{rt} = \lambda P_{rt} Y_{rt}. \tag{20}$$

So, using (16) and (20), the allocation of labor between production and research is:

$$\frac{L_{pt}}{L_{rt}} = \frac{(1-\alpha)(\rho-1)(1+\tau_k)^{\alpha(1-\rho)} \Pi_t N_t Z_t}{\lambda P_{rt} Y_{rt}}. \tag{21}$$

We define the research intensity of the economy, $s_r$, as the ratio of spending on innovative activities to GDP, $s_{rt} = P_{rt}Y_{rt}/(Y_t - X_t)$. Using $(i)$ $X_t = (1-\lambda) P_{rt} Y_{rt}$, $(ii)$ $Y_t / \left((1+\tau_k)^{\alpha(1-\rho)} \Pi_t N_t Z_t\right) = \rho/(1+\tau_s)$ together with equation (21), we can express the research intensity of the economy as

$$s_{rt} = \left[\frac{\rho\lambda}{(1-\alpha)(\rho-1)(1+\tau_s)} \frac{L_{pt}}{L_{rt}} - (1-\lambda)\right]^{-1}. \tag{22}$$

An equilibrium in this economy is a collection of sequences of aggregate prices $\{\bar{r}_t, P_{rt}, R_{kt}, W_t\}$, prices for intermediate goods $\{p_t(z)\}$, sequences of aggregate quantities $\{Y_t, K_t, X_t, C_t, L_{pt}, L_{rt}, H_t\}$, quantities of the intermediate goods and allocations of physical capital and labor $\{y_t(z), k_t(z), l_t(z)\}$, sequences of $\{\Pi_t\}$, and a sequences of firm value functions and process innovation decisions $\{V_t(z), g_{zt}\}$ together with distributions of firms, mass of incumbent firms, measures of product innovation, and aggregate productivities $\{J_t(z), N_t, M_t, Z_t\}$ such that, given a set of policies $\{\tau_p, \tau_e, \tau_g, \tau_k, \tau_s\}$ , initial stocks $\{A_0, L_0, H_0, K_0\}$, and an initial distribution of firms $J_0(z)$, households maximize their utility subject to their budget constraint, intermediate good firms maximize profits, all of the feasibility constraints are satisfied, and the distribution of firms evolve as described above.

**C. Firm profitability and firm dynamics**

Our model has two main implications for firm dynamics that we use frequently in our results below. First, the growth rate of incumbent firms is independent of firm size and is determined by what we term the level of firm profitability. Second, in any period with positive product innovation, the level of firm profitability is determined by the cost of product innovation. Putting these two implications together, we get a simple formula for determining how changes in innovation policy impact process innovation by incumbent firms.

We find it useful to denote the interest rate denominated in terms of the research good as $r_t$. This interest rate is defined by

$$(1 + r_t) = (1 + \bar{r}_t)\frac{P_{rt}}{P_{rt+1}} \tag{23}$$

To derive the two implications regarding firm dynamics, observe that the Bellman equation (13) governing process innovation for incumbent firms can be rewritten as

$$\frac{V_t(z)}{P_{rt}} = \max_{g_{zt}} \left[(1 + \tau_p)(1 + \tau_k)^{\alpha(1-\rho)} \frac{\Pi_t}{P_{rt}} - (1 - \tau_g)\, c\left(\exp(g_{zt})\right)\right] \exp(z) + \frac{(1 - \delta_f)}{1 + r_t}\frac{V_{t+1}(z + g_z)}{P_{rt+1}}. \tag{24}$$

It is straightforward to confirm that the value function in this Bellman equation takes the form $\frac{V_t(z)}{P_{rt}} = \bar{V}_t \exp(z)$, that is, that the discounted expected value of dividends for an incumbent firm denominated in units of the research good is directly proportional to the index of the firm's productivity $\exp(z)$, with the factor of proportionality given by $\bar{V}_t$. We refer to this factor of proportionality $\bar{V}_t$ as the *profitability* of firms in period $t$.

From this Bellman equation we see that all incumbent firms in period $t$ choose the same growth rate $g_{zt}$, given by the solution to

$$(1 - \tau_g)\, c'\left(\exp(g_{zt})\right) = \frac{(1 - \delta_f)}{1 + r_t}\bar{V}_{t+1}. \tag{25}$$

This result implies that the growth rate of incumbent firms in our model, $g_{zt}$ is always independent of firm size. With this result, we can write the law of motion for the index of total productivity as

$$N_{t+1}Z_{t+1} = (1-\delta_f)\exp(g_{zt})N_tZ_t + (1-\delta_f)\,M_t\exp(\bar{z})\,. \tag{26}$$

We now turn to our model's implications for firm profitability in any period in which there is positive product innovation. Given our normalization of the productivity of a new firm to $\bar{z}=0$, the zero-profit condition governing product innovation, equation (14), can be re-written as:

$$(1-\tau_e)\,n_e = \frac{(1-\delta_f)}{1+r_t}\bar{V}_{t+1}. \tag{27}$$

Combining these two expressions governing process and product innovation, (25) and (27), we get that in any period in which there is positive product innovation, the growth rate $g_z$ of incumbent firms (the level of process innovation) is simply determined by

$$(1-\tau_g)\,c'\left(\exp(g_{zt})\right) = (1-\tau_e)\,n_e. \tag{28}$$

To ensure the existence of equilibrium, we need parameter restrictions to ensure that profitability for incumbent firms $\bar{V}$ is well defined (finite) and that the index of average productivity, $Z$, is also well defined. Both of these conditions are restrictions on the growth rate of incumbent firms, $g_{zt}$. We present these restrictions in the next section for the case in which the economy is on a balanced growth path with positive product innovation every period.

## 3. Balanced Growth Path with Product Innovation

In our results on the aggregate implications of policy changes in the long-run, we analyze the change in our economy's balanced growth path corresponding to a given change in policy. We now characterize a balanced growth path (BGP) with product innovation in our model economy. On a BGP with product innovation, a subset of the variables are constant over time and others grow at a constant rate. In particular, on a BGP, the allocation of labor between production and innovation remains constant as does the profitability of firms, the interest rate in terms of both the final good and the research good, the rental rate of capital, and the index of average productivity of incumbent firms. Output of the consumption and research goods, consumption, the stock of physical capital, total productivity, wages, and cumulated experience with innovation, all grow at constant rates.

Our model has two types of BGPs with product innovation, one with semi-endogenous growth and one with endogenous growth, depending on parameter values. Specifically, we define a parameter $\theta$ as a function of the parameters $\gamma, \rho$, $\alpha$, and $\lambda$:

$$\theta = 1 - \frac{(1-\lambda)}{(\rho-1)(1-\gamma)(1-\alpha)}. \tag{29}$$

Our model is a semi-endogenous growth model with the growth rate along the BGP determined by the exogenous growth rates of scientific knowledge $g_A$ and of the population $g_L$ if $\theta > 0$. In this case, it is not possible to have endogenous growth because an increase in the index of total productivity $NZ$ results in general equilibrium in a decline in the profitability of firms and thus a decline of firms' investments in innovative activity. Growth can occur only to the extent that scientific progress reduces the cost of innovation and/or growth in population increases market size. We focus on equilibria that satisfy this parameter restriction in deriving our main results.

Our model is an endogenous growth model with the growth rate along the BGP determined by firms' investments in innovative activity only in the knife edged case in which $\theta = 0$. In this case, it is possible to have endogenous growth because an increase in the index of total productivity $NZ$ has no impact on the profitability of firms and thus firms find it optimal to continue to invest in innovation even as the productivity of their competitors grows. To the best of our knowledge, all endogenous growth models with innovation by firms satisfy some version of this knife-edge parameter restriction.[8]

We now characterize a BGP with semi-endogenous growth.

**A BGP in the Semi-Endogenous growth case.** When $\theta > 0$, the conditions characterizing a BGP have a block recursive structure that lies at the heart of our analytic results. We first characterize aggregate growth rates, the interest rate, and the rental rate of capital along a BGP with product innovation. Along such a BGP, these variables are independent of policies. We then characterize the growth rate of incumbent firms, the average productivity of firms, and the equilibrium level of firm profitability, from the zero-profit condition for product innovation. Finally, we solve for the levels along the BGP of the other

[8]Note that if $\theta < 0$, then our model does not have a BGP as in this case, the profitability of firms and their investments in innovation accelerate as the index of total productivity $NZ$ grows.

aggregate variables consistent with the equilibrium level of firm profitability.

We now characterize the aggregate growth rates, the interest rate, and the rental rate of capital. Conjecture that the average productivity of firms $Z$ is constant. Then, the growth rates and the interest rate in a BGP are determined from equations (1), (4), (6), (7), and (15). In particular, the growth rate of the measure of incumbent firms $g_N$ is given by

$$g_N = \frac{1}{\theta(1-\gamma)}(g_A + g_L), \tag{30}$$

the growth rate of total productivity $(NZ)$ is equal to the growth of the measure of incumbent firms, $g_N$, the growth rate of output of the consumption good (and hence consumption, physical capital, and the input into innovative activity $X$) is given by $g_Y = g_N / [(\rho - 1)(1-\alpha)] + g_L$ (from equations 1 and 15), the growth rate of output of the research good is given by $g_{Yr} = g_N$ (from 4), the growth rate of the wage is $g_Y - g_L$ (from equation 16), and the rental rate of capital is constant and given by $R_k = \beta^{-1} \exp(g_Y - g_L) - 1 + \delta_k$ (from equations 8 and 9). Finally, using (8), (18), and (23), the interest rate in terms of the research good is given by $r = \beta^{-1} \exp(g_N - g_L) - 1$.

We now present parameter restrictions needed to ensure the existence of equilibrium. With product innovation, the growth rate of incumbent firms, $g_z$ is constant and determined as the solution to (28). To ensure that average productivity $Z$ is finite in a BGP, we must have that the solution for the growth rate of incumbent firms $g_z$ not be too large. Specifically, we require the restriction that

$$(1-\delta_f)\exp(g_z) < \exp(g_N). \tag{31}$$

With this restriction, we have that average productivity on a BGP is given by

$$Z = \frac{\exp(g_N) - (1-\delta_f)}{\exp(g_N) - (1-\delta_f)\exp(g_z)}. \tag{32}$$

If condition (31) is violated, the BGP does not have positive product innovation.[9]

Given the interest rate, the equilibrium level of firm profitability $\bar{V}$ is solved from the zero profit condition for product innovation, (27). The constant in firm profits, $\Pi/P_r$, is

[9] Add a short description of BGP without product innovation.

determined from

$$\bar{V} = \frac{1+r}{1+r-(1-\delta_f)\exp(g_z)} \left[ (1+\tau_p)(1+\tau_k)^{\alpha(1-\rho)} \frac{\Pi}{P_r} - (1-\tau_g)\, c\,(\exp(g_z)) \right]. \tag{33}$$

To ensure that a solution for $\Pi/P_r$ exists, we must have $(1-\delta_f)\exp(g_z) < 1+r$, which is satisfied due to (31) and our assumption that $\beta \exp(g_L) < 1$.

With this parameter restriction, we solve for aggregates along the BGP as follows. Given the BGP growth rates, average productivity $Z$, the constant in firm profits $\Pi/P_r$, and a population level $L_t$, we solve for the allocation of labor between production and research from

$$\frac{L_p}{L_r} = \frac{(1-\alpha)(\rho-1)}{\lambda} \frac{(1+\tau_k)^{\alpha(1-\rho)}\, \Pi/P_r}{n_e \frac{\exp(g_N)-\left(1-\delta_f\right)\exp(g_z)}{1-\delta_f} + c\,(\exp(g_z))}, \tag{34}$$

and labor market clearing, $L_{pt} + L_{rt} = L_t$. Equation (34) is obtained from (4), (21) and (32). Next, for a given level of the stock of basic scientific knowledge $A_t$, we solve for the mass of firms $N_t$ from

$$\frac{\Pi}{P_r} = \kappa_1 (1+\tau_s)^{1+\frac{1}{1-\alpha}\frac{\gamma-\lambda}{1-\gamma}} (1+\tau_k)^{\alpha(\rho-1)\theta} (A_t)^{\frac{1}{1-\gamma}} (N_t Z)^{-\theta} (L_{rt})^{\frac{\gamma}{1-\gamma}} L_{pt}\ , \tag{35}$$

where $\kappa_1$ is a constant.[10] Equation (35) is obtained from (6), (7), (12), (15), (16), (17), (18), and (19), imposing that the spillover from cumulative innovative activity, $H$, is equal to its level along a BGP Finally, we solve for aggregate output, $Y_t$, using (15), the capital stock, $K_t$, using (17), the consumption good used in the production of the research good, $X_t$, using (16) and (19), and consumption, $C_t$, using (1).

There are two ways to see why the restriction that $\theta > 0$ ensures that the BGP of our model features semi-endogenous growth. The first is mechanically from equation (30): when $\theta > 0$, the growth rates are necessarily pinned down by resource constraints, production functions, and static pricing and production decisions by firms. The second is to see from equation (35) that $\theta > 0$ is required to obtain a unique solution for the total productivity of firms $N_t Z_t$, for given levels of $L_t$ and $A_t$. The intuition for why there is a unique level of total productivity $N_t Z_t$ for given levels of $L_t$ and $A_t$, when $\theta > 0$, is as follows. The parameter $\theta$ determines to what extent an increase in total productivity $NZ$ crowds out firm profitability

[10]In particular, $\kappa_1 = \left[(1-\alpha)/\lambda\right]^{\frac{\gamma-\lambda}{1-\gamma}} \left[\alpha^{\frac{\alpha}{1-\alpha}} (1-\lambda)\right]^{\frac{1-\lambda}{1-\gamma}} \rho^{-1} \left[\frac{\rho-1}{\rho}\right]^{\frac{\alpha}{1-\alpha}+\frac{\gamma-\lambda}{(1-\gamma)(1-\alpha)}} \left[\exp(g_N)+\delta_r-1\right]^{\frac{-\gamma}{1-\gamma}} (R_k)^{\alpha(\theta-1)(\rho-1)}$.

taking into account all the general equilibrium effects on aggregate prices and quantities. These general equilibrium effects are as follows:

1. An increase in total productivity $NZ$ of 1 percent increases output of the consumption good $Y$ by $1/[(\rho-1)(1-\alpha)]$ percent (taking into account the direct effect on productivity and the indirect effect through an increase in the capital stock), increasing the demand and hence the profitability of any individual firm by the same percentage.
2. An increase in the total productivity $NZ$ of 1 percent increases the wage by by $1/[(\rho-1)(1-\alpha)]$ percent and hence the marginal cost of production by $1/(\rho-1)$ percent, reducing the profitability of any individual firm measured in units of the final consumption good by 1 percent.
3. An increase in the total productivity $NZ$ of 1 percent increases the wage by $1/[(\rho-1)(1-\alpha)]$ percent and increases the spillover from cumulative innovative activity on the BGP by $(1-\lambda)/[(\rho-1)(1-\gamma)(1-\alpha)]$ percent. These two effects change the price of the research good by $(\lambda-\gamma)/[(\rho-1)(1-\gamma)(1-\alpha)]$ percent.

Adding these three effects together gives a combined effect of negative $\theta$ percent. When $\theta > 0$, the cumulative effect of these three general equilibrium effects is negative, meaning that an increase in the total productivity of incumbent firms crowds out the profitability of firms and hence crowds out the incentive for these firms to continue to innovate. Without an exogenous fall in the price of the research good brought about by exogenous scientific progress and/or an exogenous fall in wages brought about by an exogenous increase in population, productivity growth due to innovation by firms must cease in the long run.

In contrast, when $\theta = 0$, the general equilibrium effects discussed above exactly cancel and the crowding-out effect is not operative.[11] In this case it is possible to have a BGP with fully endogenous growth driven by firms' innovative activity. We now briefly describe a BGP with endogenous growth.

**A BGP in the Endogenous growth case.** To have a BGP with endogenous growth, we need $\theta = 0$ and $g_A + g_L = 0$.[12] In this case, we lose the recursive structure that

[11] This knife-edge condition is different from the one emphasized in Jones (1999) on the technology governing innovation. Our condition encompasses both the technology governing innovation and spillovers and the impact of product market competition on the profitability of creating new products.

[12] An alternative assumption for generating endogenous growth is to have $\lambda = \gamma = 1$ and $g_A = g_L = 0$. This is the case discussed in Acemoglu (2009), Chapter 13, to distinguish between semi-endogenous ($\gamma < 1$) and endogenous ($\gamma = 1$) growth. While we have assumed $\gamma < 1$, the case with $\lambda = \gamma = 1$ corresponds to the limit of cases subsumed in our analysis in which $\gamma$ and $\lambda$ both converge to 1 along the path that keeps $\theta = 0$.

we use in the semi-endogenous case because we lose equation (30). To solve for the BGP at a point in time, we must choose a value of $N_t$, and solve simultaneously for the rest of the variables using the remaining equations of the semi-endogenous growth case. Note that the level of $N_t$ is not pinned-down by the conditions for a BGP. Instead, the equilibrium level of $N_t$ must be determined along the transition path from a given initial condition $N_0$ (analogously to the level of capital in an $AK$ model).

## 4. Optimal Innovation Policies

In this section we present two results regarding the impact of innovation policy on aggregates in our economy. The first result is that a uniform subsidy to all innovative activity, $\tau_g = \tau_e$, has an equivalent impact on equilibrium allocations as a direct subsidy to firm's variable profits in the sense that the equilibrium allocations remains constant as long as the ratio $\frac{1+\tau_p}{1-\tau_g}$ remains constant. The second result characterizes optimal policy. Through both of these results, we show that in our model there is no special role for innovation policies distinct from a policy of subsidizing profits of incumbent firms, even in presence of spillovers from innovative activities.

*Proposition 1:* Let $\{\tau_p, \tau_g, \tau_e, \tau_k, \tau_s\}$ be a set of policies with $\tau_g = \tau_e$, and let there be an equilibrium allocation with product innovation every period corresponding to these policies. Let $\{\widetilde{\tau}_p, \widetilde{\tau}_g, \widetilde{\tau}_e, \widetilde{\tau}_k, \widetilde{\tau}_s\}$ be a set of alternative policies with $\tau_k = \widetilde{\tau}_k$, $\tau_s = \widetilde{\tau}_s$, $\widetilde{\tau}_g = \widetilde{\tau}_e$ and $\frac{1+\widetilde{\tau}_p}{1-\widetilde{\tau}_g} = \frac{1+\tau_p}{1-\tau_g}$. Then the equilibrium allocations corresponding to these two sets of policies are equal.

*Proof:* To prove this proposition, we guess and verify that the old allocation is an equilibrium allocation under the new policies. From equation (28) in an equilibrium with positive product innovation, the growth rate of incumbent firms $g_z$ is independent of innovation policies as long as $\tau_g = \tau_e$. From equation (27), with $r_t = \tilde{r}_t$, we must have $\widetilde{\bar{V}}_{t+1}/\left(1 - \widetilde{\tau}_e\right) = \bar{V}_{t+1}/\left(1 - \tau_e\right)$. From (24), we see that this condition holds if $\widetilde{\tau}_g = \widetilde{\tau}_e$ and $\frac{1+\widetilde{\tau}_p}{1-\widetilde{\tau}_g} = \frac{1+\tau_p}{1-\tau_g}$. Since $\tau_p$, $\tau_g$, and $\tau_e$ do not enter into any other equilibrium condition, our conjecture is verified. Q.E.D.

The intuition for this result is straightforward. A uniform subsidy to process and product innovation is equivalent to changing the equilibrium price of the research good. Since variable profits are the returns to innovative activities, a subsidy to these profits is a subsidy to the returns to innovation. One can achieve the same aggregate results by subsidizing the returns to or the costs of innovative activity.

We now characterize optimal policy on the BGP with product innovation in the semi-endogenous growth case of our model. The equilibrium of our model has two inefficiencies. The first is the standard inefficiency arising from a monopoly markup in the production of intermediate goods that distorts the mix of labor and consumption goods used in the production of the research good (when $\lambda < 1$) and the ratio of physical capital to labor used in the production of intermediate goods (when $\alpha > 0$). The second inefficiency arises because agents do not internalize the spillover from experience in innovative activities (when $\gamma > 0$). Hence, the equilibrium level of innovative activity is too low. A planner can undo these distortions with a subsidy to production of the consumption good and a subsidy to variable profits (or, equivalently from Proposition 1, with a subsidy to all innovative activities).

*Proposition 2:* If the social optimum allocation has a BGP with product innovation and semi-endogenous growth, then that optimal BGP corresponds to the equilibrium BGP with a subsidy to the production of the consumption good given by

$$1 + \tau_s^* = \frac{\rho}{\rho - 1}$$

and a subsidy to variable profits given by

$$\frac{1}{1 + \tau_p^*} = 1 - \gamma \frac{\beta \exp(g_L) [\exp(g_N) - (1 - \delta_r)]}{[\exp(g_N) - \beta \exp(g_L)(1 - \delta_r)]},$$

where $g_N$ is given by (30). The other policies are set to zero.

*Proof:* See Appendix.

To verify that the social optimum has product innovation, one simply must check that condition (31) is satisfied at the optimal value of $g_z$ given by $n_e = c'(\exp(g_z))$. With endogenous growth, Proposition 2 applies, except that the socially optimal growth rate of production of the research good $g_N$ must be solved for endogenously, as described in the Appendix.

As is clear from these two propositions, in a BGP there is no extra social benefit to subsidizing innovation directly even in the presence of spillovers from innovative activities. Note that the assumption that the spillovers from innovative activity impact equally the cost of both process and product innovation (without favoring either type of innovative activity) is important in deriving this result on the equivalence between innovation subsidies and subsidies to firm profits, as well as the result on the local equivalence of changes in the subsidy to any particular type of innovative activity that we present in the following Section.

## 5. Calculating Aggregate Effects of Changes in Policies

In this section we present a two-step algorithm for computing the implications for the change in aggregates from one BGP to another resulting from a change in policies in the case that our model has a BGP with semi-endogenous growth and positive product innovation. We then use this algorithm to derive analytical results on the effects of policies in several special cases. We conclude this section with a brief discussion of how to extend this algorithm in the case that our model has a BGP with endogenous growth.

Our algorithm takes advantage of the block recursive structure of the equations characterizing a BGP in our model with semi-endogenous growth and positive product innovation. When our model has semi-endogenous growth, aggregate growth rates and the interest rate in the BGP are invariant to policy changes and are as described in Section 3 above.

In the first step of our algorithm, we solve for the equilibrium level of firm profitability, $\bar{V}$, the growth rate of incumbent firms, $g_z$, the level of average productivity, $Z$, and the constant in firm profits, $\Pi/P_r$ from (27), (28), (32), and (33). In the second step, we solve for the allocation of labor between production and research from (34) and labor market clearing, and for the mass of firms $N_t$ from (35). All other aggregates in a BGP are pinned down by these variables.

### A. Changes in Subsidy to Variable Profits

We now use this algorithm to establish two propositions regarding the impact of a change in policies on BGP allocations.

*Proposition 3:* If our model has a BGP with semi-endogenous growth and positive product innovation, then a change in the subsidy to variable profits of size $\Delta \log(1+\tau_p)$ results in the following changes to the aggregates along the BGP:

1. The equilibrium level of firm profitability, $\bar{V}$, the growth rate of incumbent firms, $g_z$, and average productivity, $Z$, are unchanged;
2. The constant in firm profits, $\Pi/P_r$, changes by $\Delta \log(\Pi/P_r) = -\Delta \log(1+\tau_p)$;
3. The aggregate allocation of labor between production and research changes by $\Delta \log(L_p/L_r) = -\Delta \log(1+\tau_p)$;
4. To a first-order approximation, the change in the index of total productivity is $\Delta \log(NZ) = \frac{1}{(1-\gamma)\theta}\frac{L_p}{L}\Delta \log(1+\tau_p)$;

5. To a first-order approximation, the change in aggregate output, $GDP$, is

$$\Delta \log GDP = \left[\left(1 + \frac{1}{(1-\gamma)(\rho-1)(1-\alpha)\theta}\right)\frac{L_p}{L} - \frac{Y}{GDP}\right]\Delta \log(1+\tau_p) \ ;$$

6. To a first-order approximation, the change in the research intensity of the economy, $s_r$, is $\Delta \log s_r = \frac{Y}{GDP}\Delta \log(1+\tau_p)$.

*Proof:* Result 1 is based on equations (27), (28), and (32). Result 2 is based on equation (33), together with the fact that aggregate growth rates and the interest rate in the BGP are invariant to policy changes. Result 3 is based on equation (34), together with the fact that $g_z$ is unchanged. To calculate the log changes in $L_p$ and $L_r$ individually, we use a first-order approximation of the labor market clearing condition, $\frac{L_p}{L}\Delta \log(L_p/L) + \frac{L_r}{L}\Delta \log(L_r/L) = 0$, and the log change in $L_p/L_r$, to obtain $\Delta \log(L_p/L) = -(L_r/L)\Delta \log(1+\tau_p)$ and $\Delta \log(L_r/L) = (L_p/L)\Delta \log(1+\tau_p)$. To calculate the log change in $NZ$ in Result 4, we use equation (35). To calculate the log change in GDP in result 5, we use two intermediate calculations. First, $\Delta \log Y = \frac{1}{(\rho-1)(1-\alpha)}\Delta \log NZ + \Delta \log L_p$, where we are using equations (16), (17), and the fact that $R_k$ is constant to calculate the change in the physical capital stock, $\Delta \log K_t = \Delta \log Y_t$. Second, to a first-order approximation, $\Delta \log \frac{Y-X}{Y} = -\frac{X}{Y-X}\Delta \log\left(\frac{L_r}{L_p}\right)$, where we are using that $X$ is proportional to $WL_r$ and that $Y$ is proportional to $WL_p$. Combining these two calculations, $\Delta \log GDP = \Delta \log Y + \Delta \log \frac{Y-X}{Y}$, we obtain Result 5. To calculate the change in $s_r$ in Result 6, we use (22).Q.E.D.

Proposition 3 is the simplest application of our two-step algorithm for computing the aggregate implications of changes in innovation policies and it illustrates the logic of the algorithm in a straightforward manner. We start in result 1 with the observation that the equilibrium level of firm profitability $\bar{V}$ must remain constant in response to a change in the subsidy to variable profits to satisfy the equilibrium condition that there be zero profits associated with product innovation. In result 2, we see that to keep the level of firm profitability constant, the constant in variable profits, $\Pi/P_r$, must change to offset the direct impact of the subsidy to variable profits on firm profitability. This point follows immediately because a subsidy to variable profits has no impact on the level of process innovation, $g_z$ or on the average productivity of firms $Z$. In terms of firm dynamics, the only impact of a subsidy to variable profits is to encourage product innovation. Because the subsidy to variable profits has no impact on the life-cycle of a typical firm, the use of the research good for innovative activities per-firm is unchanged. This observation is the key to our calculation of the aggregate re-allocation of labor between production and research in result 3.

The resulting response of total productivity $NZ$ then follows from the downward sloping relationship between total productivity and the constant in variable profits that follows from our assumption that this is a semi-endogenous growth model. The response of total productivity to changes in innovation policy in this case is self-limiting because, as we have discussed above, an increase in the total productivity of incumbent firms crowds out the profitability of new firms and hence crowds out product innovation. As is evident in the expression in result 4, the slope of this relationship depends inversely on $\theta$ which measures the extent to which an increase in the productivity of all firms crowds out the profitability of a given firm. If $\theta$ is large, so that the crowding out is strong, then the response of total productivity associated with a given change in $\Pi/P_r$ is small. In this case, the response of innovative activity to a change in policy is crowded out before a large response of total productivity can occur. In contrast, as $\theta$ approaches zero, this crowding out effect is weak and the corresponding response of total productivity becomes large. The remaining aggregates follow immediately from the responses of the aggregate allocation of labor and total productivity.

In what follows, we consider the aggregate impact of alternative changes in innovation policies using extensions of this basic line of argument.

*Corollary 1:* If our model has a BGP with semi-endogenous growth and positive product innovation, then a uniform change to subsidies to process and product innovation of size $\Delta \log(1-\tau_g) = \Delta \log(1-\tau_e)$, result in changes in aggregates along the BGP listed in Proposition 3 with $\Delta \log(1+\tau_p)$ replaced by $-\Delta \log(1-\tau_g)$.

*Proof:* This follows immediately from Proposition 1. Q.E.D.

The application of our algorithm in this corollary to the case of a change in the uniform subsidy to both process and product innovation is a slight variation of the application above in the case of a subsidy to variable profits. Here, the equilibrium level of firm profitability $\bar{V}$ does respond to the change in the subsidy to innovation because the condition that process innovation earn zero profits inclusive of subsidies requires such a change. The uniform change in the subsidy to both process and product innovation results in a change in the constant in variable profits $\Pi/P_r$ of the same size as the response in proposition 3 because of the straightforward equivalence between a subsidy to the costs of innovation and a subsidy to the returns to innovation. Given the result that firm dynamics are again unchanged, the argument regarding changes in the aggregate allocation of labor between production and research and in total productivity are the same as before.

### B. Changes in Subsidies to Process or Product Innovation

In the next Proposition, we consider the aggregate impact of changes in subsidies to process and/or product innovation individually. In this case, changes in policies do affect the level of process innovation, the average productivity of firms, as well as the level of product innovation.

*Proposition 4:* If our model has a BGP with semi-endogenous growth and positive product innovation, and policies are initially set so that $\tau_g = \tau_e$, then a change in the subsidy to process innovation of size $\Delta \log (1-\tau_g)$ and/or a change in the subsidy to product innovation of size $\Delta \log (1-\tau_e)$ results in the following changes to the aggregates along the BGP,

1. The equilibrium level of firm profitability, $\bar{V}$, changes by $\Delta \log \bar{V} = \Delta \log (1-\tau_e)$;
2. To a first-order approximation, the growth rate of incumbent firms, $g_z$, and average productivity $Z$, change by

$$\Delta g_z = \frac{1}{\eta_c} \left[ \Delta \log (1-\tau_e) - \Delta \log (1-\tau_g) \right]$$

$$\Delta \log Z = \frac{(1-\delta_f)\exp(g_z)}{\exp(g_N) - (1-\delta_f)\exp(g_z)} \frac{1}{\eta_c} \left[ \Delta \log (1-\tau_e) - \Delta \log (1-\tau_g) \right]$$

3. To a first-order approximation, the constant in firm profits, $\Pi/P_r$, the aggregate allocation of labor between production and research, $L_p/L_r$, the index of total productivity, $NZ$, aggregate output, $GDP$, and the research intensity of the economy, $s_r$, change as in Proposition 3 with $-\left[s_g \Delta \log (1-\tau_g) + (1-s_g) \Delta \log (1-\tau_e)\right]$ replacing $\Delta \log (1+\tau_p)$, where $s_g$ is the ratio of process-innovation costs to variable profits, $s_g = \frac{(1-\tau_g)c(\exp(g_z))}{(1+\tau_p)(1+\tau_k)^{\alpha(1-\rho)}\Pi/P_r}$.

*Proof:* Result 1 is immediate from (27). In result 2, the change in $g_z$ and $Z$ is obtained by using a first-order approximation of equation (28) and equation (32), respectively. In result

3, the change in $\Pi/P_r$ is obtained by using a first-order approximation of (33),

$$\begin{aligned}\Delta\log(1-\tau_e) &= \frac{\Delta\log\Pi/P_r}{1-s_g} - \frac{s_g}{1-s_g}\Delta\log(1-\tau_g) \\ &- \left[\frac{(1-\tau_g)\,c'(\exp(g_z))}{(1+\tau_p)(1+\tau_k)^{\alpha(1-\rho)}\frac{\Pi}{P_r} - (1-\tau_g)\,c(\exp(g_z))} - \frac{(1-\delta_f)}{1+r-(1-\delta_f)\exp(g_z)}\right]\exp(g_z).\end{aligned}$$

and noting that the term in square brackets is equal to zero from the first-order condition (28) for $g_z$, and equations (27) and (33). To obtain the change in $L_p/L_r$, we use a first-order approximation of equation (34),

$$\Delta\log\frac{L_p}{L_r} = \Delta\log(\Pi/P_r) - \frac{c'(\exp(g_z)) - n_e}{n_e\frac{\exp(g_N)-(1-\delta_f)\exp(g_z)}{1-\delta_f} + c(\exp(g_z))}\exp(g_z)\,\Delta g_z$$

and note that the second term is zero from equation (28) and the fact that initial policies are set so that $\tau_g = \tau_e$ (note that this is the only step in which we use the assumption $\tau_g = \tau_e$). The remaining variables are calculated using the same steps as in the proof of Proposition 3. Note that if $\Delta\log(1-\tau_g) = \Delta\log(1-\tau_e)$, Proposition 4 is equivalent to Corollary 1. Q.E.D.

The application of our algorithm here in Proposition 4 to the case of changes to the individual subsidies to process and product innovation one at a time or in combination applies only to a first approximation because now firm dynamics, characterized by firms' investments in process innovation $g_z$ and average productivity $Z$ do change in response to the change in subsidies. In fact, in response to a change in innovation policy (in terms of a change in the subsidy to either process or product innovation) incumbent firms in our model can exhibit a wide range of different responses of their investments in process innovation depending on the curvature of the process innovation cost function given by $\eta_c$.

There are two key insights needed to establish that, to a first approximation, this response in incumbent firms' investments in process innovation does not matter for the aggregates. The first insight is that, to a first-order approximation, the change in the constant in firm profits $\Pi/P_r$ required to maintain zero profits to product innovation does not depend on the induced response of firms' process innovation $g_z$. This first insight follows directly from the envelope condition in firms' Bellman equation defining firm profitability $\bar{V}$ — since firms are choosing process innovation optimally in the original allocation, to a first-order approximation the change in firm profitability induced by a change in process innovation is zero. The second insight is that to a first-order approximation, the aggregate reallocation of labor between production and research also does not depend on the response of firms' investments in process innovation. This second insight is more subtle and depends both on the assumption that in the original allocation the subsidies to both types of innovation are equal ($\tau_e = \tau_g$) and on the result that all firms choose the same process innovation rate $g_z$. With these additional equalities, we have that, to a first-order approximation, the use of the research good for innovative activities per firm is again unchanged and hence the aggregate reallocation of labor is the same as in proposition 3.

Finally, consider the response of the index of total productivity $NZ$ to the change in policy. Here, with a change in the subsidy to process or product innovation separately, as noted in point 2, firms' investments in process innovation $g_z$ change and the resulting level of the average productivity of firms $Z$ also changes. The size of these responses depends on the curvature of firms' process innovation cost function $\eta_c$. If this cost function has low curvature, the response of process innovation and average productivity can be quite large. In equilibrium, however, it is the crowding out effect of an increase in total, not average, productivity on firm profits that determines the aggregate response of the economy. It is irrelevant for aggregates whether this response of total productivity is achieved through a large increase in average productivity due to a large increase in process innovation and a small change in product innovation or through a small change in average productivity and a large change in product innovation. If the average productivity of incumbent firms rises substantially in response to a change in policy, then the crowding out effect implies that there is less room for the creation of new products.

We next present a corollary of Proposition 4 which establishes that, to a first-order approximation, a change in the subsidy to process or product innovation individually has the same aggregate impact on the index of total productivity and output as a change in the subsidy to variable profits as long as these policy changes have the same impact on firm profitability holding fixed firms' process innovation decisions.

*Corollary 2:* If our model has a BGP with semi-endogenous growth and positive product innovation, and policies are initially set so that $\tau_g = \tau_e$, then a change in the subsidy to process innovation of size $\Delta \log(1 - \tau_g)$ and/or a change in the subsidy to product innovation of size $\Delta \log(1 - \tau_e)$ results in the same changes in the constant in firm profits,

$\Pi/P_r$, the aggregate allocation of labor between production and research, $L_p/L_r$, the index of total productivity, $NZ$, aggregate output, $GDP$, and the research intensity of the economy, $s_r$, to a first-order approximation, as a change in the subsidy to variable profits of size $\Delta\log(1+\tau_p) = -\left[s_g\Delta\log\left(1-\tau_g\right) + \left(1-s_g\right)\Delta\log\left(1-\tau_e\right)\right]$.

*Proof:* This follows immediately from Propositions 1 and Proposition 4. Q.E.D.

This second corollary reinforces the point that, to a first-order approximation, information on the response of firms' investments in process innovation to a change in innovation policy is not informative for the aggregate implications of that policy change: what is needed to evaluate policy, at least locally, is information on the impact of that policy change on the constant in firm variable profits holding fixed firms' decisions to invest in process innovation. As we have seen in proposition 4, changes in the individual subsidies to process or product innovation do have effects on firms' process innovation decisions. In contrast, as we have seen in proposition 3, changes in the subsidy to firms' variable profits have no effect on firms' process innovation decisions and hence no impact on average productivity. As this corollary establishes, despite this contrast, to a first-order approximation, these policy changes have the same impact on aggregates as long as they have the same impact on firm profitability holding fixed firms' process innovation decisions.

Note that the long-run change in aggregate consumption, $C$, corresponding to a change in these policies that leads to a given change in $\Pi/P_r$ is the same in Propositions 3, 4 and their Corollaries. This result follows from the fact that the change in the ratio of investment in physical capital to GDP is the same across all of these policies.

**C. Changes in Tax on Use of Physical Capital**

In the next Proposition we characterize the aggregate impact of a change in the tax $\tau_k$ on the firms' use of physical capital. A change in this tax has new effects that arise from the impact of this tax on intermediate goods firms' physical capital to labor ratio $(K/L_p)$.

*Proposition 5:* If our model has a BGP with semi-endogenous growth and positive product innovation, then a change in the tax to the use of physical capital of size $\Delta\log\left(1+\tau_k\right)$ results in the following changes to the aggregates along the BGP,

1. The equilibrium level of firm profitability, $\bar{V}$, the growth rate of incumbent firms, $g_z$, and average productivity, $Z$, are unchanged;
2. The constant in firm profits, $\Pi/P_r$, changes by $\Delta\log\left(\Pi/P_r\right) = \alpha\left(\rho-1\right)\Delta\log\left(1+\tau_k\right)$;

3. The aggregate allocation of labor between production and research, $L_p/L_r$, and the research intensity of the economy, $s_r$, are unchanged;
4. To a first-order approximation, aggregate output, $GDP$, changes by

$$\Delta \log GDP = -\frac{\alpha}{(1-\alpha)}\frac{1}{\theta}\Delta \log (1+\tau_k)$$

*Proof:* Results 1 and 2 are derived in the same way as in the proof of Proposition 3. Result 3 that $L_p/L_r$ and $s_r$ are unchanged is obtained from (34) combined with the fact that $(1+\tau_k)^{\alpha(1-\rho)}\,\Pi/P_r$ is constant. The change in $GDP$ in Result 3 is derived by first calculating $\Delta \log Y$ using the log-linear approximation of equations (15) and (35) together with the expression $\Delta \log K = \Delta \log Y - \Delta \log (1+\tau_k)$ from expressions (16) and (17). We then have $\Delta \log GDP = \Delta \log Y + \log\left(\frac{Y-X}{Y}\right)$, where $\Delta \log \frac{Y-X}{Y} = 0$ because $L_p/L_r$ is constant.

To understand the different aggregate implications of the policies considered in Proposition 3,4 and Proposition 5, it is helpful to express output of the final consumption per production worker $Y/L_p$, as

$$Y/L_p = (NZ)^{\frac{1}{(\rho-1)(1-\alpha)}}\,(K/Y)^{\frac{\alpha}{1-\alpha}}$$

The change in $Y/L_p$ is what is relevant for aggregates such as wages and GDP. The policies that we considered in Propositions 3 and 4, i.e. changes in subsidies to variable profits and innovation, affect the accumulation of total productivity $NZ$, but they do not affect the ratio of physical capital to production of the final consumption good $K/Y$. In contrast, the policy that we considered in Proposition 5, i.e. a change in the tax on the use of physical capital, affects both $NZ$ and $K/Y$. Note that the tax on the use of physical capital considered in Proposition 5 has an additional direct effect of changing the allocation of aggregate output between consumption and investment in physical capital. This direct effect is a standard implication of taxing physical capital.

**D. Using Fiscal Impact to Compare Aggregate Effects of Policy Changes**

In Propositions 3, 4, and 5 we computed the aggregate impact of a given change in the logarithm of subsidy or tax rates. To use these results in applications, one would have to measure changes in effective marginal tax or subsidy rates. In applying our algorithm to measure the aggregate impact of policies in the data, we find it more convenient to compute the aggregate impact of a change in subsidies or taxes measured in terms of the change in aggregate fiscal expenditures on these policies. We do this in the next Proposition.

Aggregate fiscal expenditures on policies $\tau_p$, $\tau_g$, $\tau_e$, $\tau_k$, $\tau_s$ at any point in time are given by

$$E = \tau_p \Pi N Z + P_{rt} \tau_g c\left(\exp\left(g_z\right)\right) N Z + P_r \tau_e n_e M - \tau_k R_k K + \tau_s Y$$

*Proposition 6:* Let our model have a BGP with semi-endogenous growth and positive product innovation, and suppose policies are initially set so that $\tau_p = \tau_g = \tau_e = 0$. Suppose that these policies change by $\Delta\tau_p$, $\Delta\tau_g$, $\Delta\tau_e$, and $\Delta\tau_s = 0$. Then, to a first-order approximation, the log change in the constant in firm profits, $\Pi/P_r$, is given by

$$\Delta \log\left(\Pi/P_r\right) = -\left[\Delta\tau_p + s_g \Delta\tau_g + \left(1 - s_g\right)\Delta\tau_e\right] \; , \tag{36}$$

and the change in aggregate expenditures on subsidies $E$ is given by

$$\Delta E = \Pi N Z \left[\Delta\tau_p + s_g \Delta\tau_g + \frac{\left(1 - s_g\right)}{\xi}\Delta\tau_e\right], \tag{37}$$

where

$$\xi = \frac{1}{\beta \exp\left(g_L\right)} \frac{1 - \beta \exp\left(g_L\right)\left(1 - \delta_f\right)\exp\left(g_z - g_N\right)}{1 - \left(1 - \delta_f\right)\exp(g_z - g_N)} \geq 1. \tag{38}$$

*Proof:* The change in $\Pi/P_r$ in equation (36) follows immediately from Propositions 3, 4, and 5, and Corollary 2. The change in $E$, starting at $\tau_p = \tau_g = \tau_e$, is

$$\begin{aligned} \Delta E &= \Pi N Z \Delta\tau_p + P_r c\left(\exp\left(g\right)\right) N Z \Delta\tau_g + P_r n_e M \Delta\tau_e \\ &= \Pi N Z \left[\Delta\tau_p + \frac{P_r c\left(\exp\left(g\right)\right) N Z}{\Pi N Z}\Delta\tau_g + \frac{P_r n_e M}{\Pi N Z}\Delta\tau_e\right]. \end{aligned}$$

Equation (37) is obtained from (i) the definition of $s_g$, and (ii) the free-entry condition with $\tau_p = \tau_g = \tau_e$ expressed as

$$P_r n_e M \xi = \Pi N Z - P_r c\left(\exp(g_z)\right) N Z, \tag{39}$$

which is derived using expressions (27), (33), and $M/\left(NZ\right) = \left[\exp\left(g_N\right) - \left(1 - \delta_f\right)\exp\left(g_z\right)\right]/\left(1 - \delta_f\right)$. Q.E.D.

Recall from Propositions 3 and 4 that, to a first-order approximation, changes in the aggregate allocation of labor between production and research, $L_p/L_r$, aggregate output, $GDP$, and the research intensity of the economy, $s_r$, are all proportional to the negative of the log change in the constant in variable profits, $\Pi/P_r$. Hence, what Proposition 6 implies is that if we start from a situation with no policies (but for $\tau_s$), a change in the subsidy to variable profits, and a change in the subsidy to process innovation all have aggregate impacts that are directly proportional to their aggregate fiscal impact. In contrast, a change in the subsidy to product innovation has an aggregate impact that is directly proportional to $\xi$ times its impact on expenditures in this subsidy.

The intuition for this result can be developed by understanding the impact of these policy changes on the level of the constant in firm profits. For changes in two of these policies, $\tau_p$ and $\tau_g$, in equilibrium, profitability $\bar{V}$ must remain unchanged and changes in the levels of subsidies received or taxes paid are proportional to size $\exp(z)$. Hence, differentiation of the firms' Bellman equation (33) together with the associated envelope conditions for firms' inputs and pricing decisions, gives that the equilibrium response in levels of the constant in variable profits, $\Delta(\Pi/P_r)$, is simply equal to the net-subsidy firms receive from these policies. Therefore, for these policies, the result in Proposition 6 is immediate.

The argument in the case of a change in the subsidy to product innovation, $\tau_e$, is slightly different. The relation between a change in the subsidy to product innovation and firm profitability differs by a factor of $\xi$ because the subsidy to product innovation is paid up-front when the product is created, while the variable profits are received in the future. The variable $\xi$ reflects the impact of discounting on these calculations. When $\beta \exp(g_L) < 1$, the BGP cross-section of firms' variable profits, capital use, and expenditures on process innovation are $\xi \geq 1$ times the equivalent sums entering into the discounted present value of these variables, while firms' expenditures on product innovation are not discounted and hence enter the same way into the cross section and discounted present value of these expenditures. Note that when $\beta \exp(g_L) = 1$, $\xi = 1$, and all of these policies have aggregate impact directly proportional to their impact on aggregate expenditures on policies $E$.

Proposition 6 makes clear the precise nature of the accounting exercise that is required in the first step of our algorithm. To a first-order approximation, a change in policies has three effects on the BGP expected discounted present value of dividends paid by an entering firm that must sum to zero in equilibrium (i.e. on the level of $\frac{(1-\delta_f)}{1+r}\bar{V} - (1-\tau_e)\, n_e$). The first effect is the change in the expected discounted value of dividends arising directly from a change in subsidies or taxes, holding fixed *all* firms' decisions, aggregate prices, and aggregate quantities. The second effect is the change in the expected discounted value of dividends arising from changes in firms' choices of prices, inputs, and process innovation. The third effect is the change in the expected discounted value of dividends arising from changes in aggregate prices and quantities ($W$, $Y$, and $P_r$). The second effect is equal to zero because of envelope condition of profit maximizing firms. The third effect is what is summarized by the expected discounted value of $\Delta\Pi/P_r$. Hence, in order to calculate the change in profitability $\Delta \log \Pi/P_r$ in our algorithm above, an accountant must simply calculate the first effect divided by $-\left(1+\tau_p\right)\left(1+\tau_k\right)^{\alpha(1-\rho)} \Pi/P_r$.[13]

As we have discussed, with $\theta = 0$, our model has a BGP with fully endogenous growth. Our simple two-step algorithm for computing the impact of a change in policy no longer applies because the BGP aggregate growth rates change with policies. As we show in an Appendix to be added, however, the change in the BGP growth rate $g_N$ induced by a change in subsidies to process or product innovation individually does not depend to a first-order approximation on the responsiveness of firms' investments in process innovation as determined by $\eta_c$. Moreover, to a first-order approximation, changes in the subsidy to process or product innovation individually have the same impact on the BGP growth rates as a change in the subsidy to variable profits as long as these policy changes have the same impact on firm profitability, holding fixed firms' process innovation decisions.

## 6. Applying our Algorithm

To these point, we have considered abstract policies. In this section we apply the results in Section 5 to assess the relative magnitudes of the impact on aggregates of three current policies affecting innovative activity by firms in the United States: the Research and Experimentation (R&E) Tax Credit, Federal spending on Research and Development (R&D), and the Corporate Profits Tax. As we discuss in greater detail below, we model changes in all three of these policies as combinations of changes in our abstract policies.

We consider the aggregate impact of eliminating these three policies. To measure the relative impact of these policies on aggregates in the United States, we use our results in Proposition 6 that, to a first-order approximation, the relative magnitudes of the aggregate effects of two different changes in policy are given by the relative magnitudes of the impact

[13] This procedure does not require that taxes and subsidies are initially set to zero.

of those policy changes on fiscal expenditures, holding all firms' decisions fixed. To use the results in Proposition 6, we apply our approximation to a baseline when none of these three policies are in place.

To compare the relative size of the impact of these three policies on aggregates in the United States, we use data from 2007 to measure fiscal expenditures on these three policies. According to the Office of Management and Budget (2009), the fiscal expenditure on the Research and Experimentation tax credit in 2007 was \$10 billion. The same data source lists Federal spending on R&D of \$139 billion.[14] Finally, the National Income and Product Accounts (Table 6.18) shows revenue of \$445 billion from corporate profits taxes.

We now describe in greater detail how we map the R&E Tax Credit, Federal spending on R&D, and the corporate profits tax into our framework. We model a change in the R&E Tax Credit, $\Delta\tau_{re}$, as a combination of changes in subsidies to process and product innovation of sizes $\Delta\tau_g$ and $\Delta\tau_e$, where the precise size of these changes for a given size of $\Delta\tau_{re}$ depends on the details of the rules on eligibility for the tax credit.[15] We abstract from these complications of the policy because, as we have shown in our model, all that we need to calculate the relative magnitude of these policies on aggregate in the BGP is the aggregate fiscal impact of the R&E tax credit. According to our model, in the end the R&E tax credit is simply a complicated and perhaps administratively expensive way of subsidizing firms.

We model a change in Federal spending on R&D as a change in a direct subsidy to the production of the research good, which is equivalent to a change in a uniform subsidy to process and product innovation of equal size, $\Delta\tau_g = \Delta\tau_e = \Delta\tau_{rd}$ and $\tau_p = 0$. The change in federal expenditure on R&D (starting in a baseline with $\tau_{rd} = 0$) corresponds to $P_r Y_r \Delta\tau_{rd}$.

Finally, we model a change in the corporate profits tax, $\Delta\tau_c$, as a combination of changes in taxes on variable profits and physical capital, and subsidies to innovative activities. As is standard, we assume that the change in the corporate profits tax implies a change in the

[14] Federal spending on R&D is often group into five categories: basic research, applied research, development, R&D equipment, and R&D facilities (see e.g. Office of Management Budget 2010). Over 50% of federal spending on basic research is in Health and Human services. The three leading agencies in applied research are Health ahd Human Services, Defense, and Energy. The Defense department and NASA account for the overwhelming majority of spending on development. Expenditures on equipment and facilities is spread across a variety of agencies. Spending in 2007 was divided among \$28 billion on basic research, \$27 billion on applied research, \$80 billion on development, and \$4 billion on equipment and facilities. For comparison, business spending in R&D in 2007 was measured at \$260 billion.

[15] In practice, the R&E tax credit in the US is a complicated policy that defines *qualified* research expenses and offers a credit only for those expenses that are incremental over a baseline amount that is also defined in the regulations (see e.g. Hall 2001).

tax on the use of physical capital depending on the rules for the deductibility of depreciation of physical capital. The impact of the corporate profits tax on innovation subsidies is more subtle. The corporate profits tax includes a subsidy to process innovation by incumbent firms if these firms partly expense this innovative activity. To the extent that expenditures on product innovation can also be partly expensed, the corporate profits tax can also include a subsidy to product innovation.[16] The overall effect of the corporate profits, however, is to divert a portion of the firms' dividends (payouts to both physical and intangible capital) to the government which, as we have shown above, discourages the accumulation of both types of capital.

To the extent that changes in these three policies amount to changes in the subsidy to variable profits, $\Delta\tau_p$, or changes in the subsidy to process innovation, $\Delta\tau_g$, then from Proposition 6, the relative aggregate impact, to a first-order approximation, of these policies can be measured directly from their relative fiscal impact — no parameters of the model enter into this calculation. To the extent that changes in these policies amount to changes in the subsidy to product innovation, $\Delta\tau_e$, the relative aggregate impact, to a first-order approximation, of these policies, can be measured from the fiscal impact of the component of the policy affecting the subsidy to product innovation, scaled-up by the parameter $\xi$ reflecting the difference in the discounted present value and cross-section of firm dividends. For example, in the case of the R&E tax credit, one needs to scale-up fiscal expenditure on that portion of the tax credit that is paid for the development of new products or firms. In the case of the corporate profits tax, one needs to scale-up fiscal expenditure arising from the expensing of product innovation costs.[17] **In this version of the paper, we have not yet considered the effect of these policies on the tax on the use of physical caital, $\tau_k$.**

The parameter $\xi$ defined in expression (38) is related to observables on the BGP as follows. Recall that if $\beta \exp(g_L) = 1$, then $\xi = 1$. In our model, $\beta \exp(g_L)$ is equal to the ratio of the growth of aggregate GDP to the consumption interest-rate, $\beta \exp(g_L) = \exp(g_Y) / (1 + \bar{r})$. In our calibration, we set $\beta \exp(g_L) = 0.99$. From equation (26) in the

[16] Gentry and Hubbard (2000), Cullen and Gordon (2007), and McGrattan and Prescott (2005a) discuss incomplete offsets of investment in intangible capital against corporate profits. Empirical work by Gentry and Hubbard (2000), Lee and Gordon (2005), Cullen and Gordon (2007), and Djankov et al (2010) find that corporate taxes reduce entry of new firms.

[17] The Office of Management and Budget (2009) provides an estimate for 2007 of the tax expenditures arising from the fact the corporate R&D expenditures are expensed rather than counted as investment, at $5 billion. This is a small portion of the revenue collected from the corporate profits tax. The portion of these tax expenditures corresponding to product innovation must be even smaller.

BGP, the term $1-(1-\delta_f)\exp(g_z-g_N)$ is equal to the share of production employment accounted for by new products. In our calibration, we set this employment share to 0.063, which is the share of employment in new establishments in the U.S. in 2007.[18] With these numbers, $\xi=1.16$. As a robustness check, consider data from Broda and Weinstein (2010) on the sales share of newly created consumer products obtained from ACNielsen's Homescan database. They report a sales share for newly created products of 9%, which leads to a value of $\xi=1.1$.

Hence, this scaling of fiscal expenditures on policies affecting the subsidy to product innovation does not substantially alter the relative ranking of the aggregate implications of the three policies that we consider from the ranking one would obtain from directly comparing fiscal expenditures.

On the basis of the data on fiscal expenditures and our value of the parameter $\xi$, we conclude that, to a first-order approximation, the long-term impact on aggregate output of the corporate profits tax is between 38 and 51 times larger larger (in terms of reducing aggregate output on the BGP) than the aggregate impact of the Federal Research and Experimentation tax credit (in terms of increasing aggregate output on the BGP) and between 2.8 and 3.7 as large as the aggregate impact of Federal spending on R&D.[19] We thus conclude that the corporate profits tax is a relatively important policy (in comparison to the R&E tax credit and federal expenditure on R&D) in terms of its aggregate effects on the long run accumulation of both tangible and intangible capital.

Note that our calculations do not apply directly to aggregate consumption if any of the policies that we consider affect the effective tax rate on the use of physical capital (in which case, these policies change the division of aggregate output between consumption and physical investment).

## 7. Quantitative Analysis on Aggregate Effects of Changes in Policies

We now perform a calculation of the absolute (as opposed to relative) magnitude of the aggregate impact of a change in innovation policy in a calibrated version of our model. We consider an economy that is on a BGP with subsidies and taxes set equal to zero, and ask

[18] The source of this figure is the 2007 Business Dynamics Statistics from the US Small Business Administration. Employment in entering establishments is the sum of employment in new firms (3.3%), as reported in Haltiwanger, Jarmin, and Miranda (2010), and employment in new establishments by existing firms (3%).

[19] These upper and lower are calculated by taking the ratio of fiscal expenditures in two policies, and either dividing or multiplying by the parameter $\xi=1.16$.

what is the aggregate impact of a uniform subsidy to innovative activities ($\tau_g = \tau_e$) that on the new BGP has fiscal expenditures of 3% of GDP. For the U.S. economy, this figure would have been approximately $420 bn in 2007 (i.e.: similar in magnitude to revenues collected from the corporate profits tax). We consider both the long run response and the transition dynamics from one BGP to another.

**A. Calibration**

To conduct this exercise, we must choose the parameters of our model. Table 1 lists the target moments and parameter values. A time period is defined to be a year. We normalize the level of population, $L$, at time zero to 1 and have it grow at rate $g_L = 0.01$. We normalize the level of scientific knowledge, $A_r$, at time zero to 1, and set its growth rate, $g_A$, so that, given the other parameter choices, growth rate of output per capita is 2%. We normalize the cost of product innovation, $n_e$, to 1. We parameterize the cost function for process innovation, $c(\exp(g_z)) = c_0\left[c_1 + \exp(g_z)^{1+\eta_c}\right]$. Given that the optimal choice of $g_z$ is not altered by the change in policy we consider both on the BGP and on the transition path, the choice of parameter $\eta_c$ does not affect our results and hence we normalize it.

We choose the parameters $\beta$, $\delta_k$, $\delta_f$, and $\lambda$ as follows. The discount factor $\beta$ corresponds to the ratio of the growth rate of per-capita GDP to the interest rate (in terms of the consumption good). We use a per-capita growth rate of GDP of 2% and an interest rate of 4% which implies $\beta = 0.98$. We set the depreciation rate on physical capital to $\delta_k = 0.10$. We choose the exogenous exit rate of incumbent firms to $\delta_f = 0.053$, which corresponds to the employment-weighted exit rate of U.S. establishments in 2007.[20] We set the share of labor in the production function of the research good to $\lambda = 0.01$. This choice is guided by the data from the NIPA satellite accounts on the price of inputs into R&D. In our model, the ratio of the relative price of research inputs to the price of final output is directly proportional to $W^{\lambda}$. Given that in the data, the relative price of research inputs to the GDP deflator shows no trend, we set $\lambda$ close to zero.

To calibrate the remaining parameters, we use as targets a standard share of labor in GDP of 66% and a share of dividends to owners of firms (payments to intangible capital) in GDP of 1% (obtained from McGrattan and Prescott 2005b). We denote this share of

[20] The source of this figure is the 2007 Business Dynamics Statistics from the US Small Business Administration. We include exit of all establishments, whether or not the firm that owns the establishment actually exits.

dividends by $\pi = (\Pi NZ - P_r Y_r)/GDP$. The share of rental payments to physical capital in GDP is calculated as a residual equal to 33%. Equation (17) together with $\pi = 0.01$ imply $\alpha = 1/3$. We use as a target an innovation intensity of the economy (share of intangible investment in GDP) of $s_r = 0.15$ (this estimate is taken from Corrado, Hulten, and Sichel 2009). Note that the parameter $\xi$ is related to $s_r$, $\pi$, and $s_g$, by[21]

$$\xi = \frac{1 - s_g}{\frac{s_r}{s_r + \pi} - s_g}.$$

Our calibration of $\xi = 1.16$ above implies $s_g = 0.54$. We choose the parameters $\rho$, $c_0$, $c_1$, $g_z$, and $g_N$ to hit our target values of $s_r$, $s_g$, $\pi$, $g_{Y/L}$, and the employment share of new products, using the equations describe above. This procedure results in values of parameters $\rho = 7.15$, $g_z = 0.07$, and $g_N = 0.08$.[22] Note that, with $\lambda$ close to zero, $L_p/L$ is essentially one and $GDP/Y = 1/(1 + s_r) = 0.86$.

This calibration procedure does not pin down the choice of parameters $\gamma$ and $\delta_r$ governing the spillover of the research good (that is, the values of all parameters and targets of our calibration are independent of $\gamma$ and $\delta_r$). Hence, the value of $\theta$ is not pinned down either. We report results below for a wide variety of values of $\gamma$ and briefly discuss how are results are not very sensitive to $\delta_r$.

**B. Long-Run Impact**

We now use our calibrated model to assess the long-run impact of a uniform subsidy to innovative activities that increase fiscal expenditures from 0% to 3% of GDP. The subsidy rates rise from $\tau_g = \tau_e = 0$ on the initial BGP to $\tau_g = \tau_e = 0.165$ for every period of the transition to the new BGP.

We first compute the change in GDP from the initial BGP to the new BGP, with spillover parameter $\gamma$ ranging from 0 to 0.74 (so that $\theta$ varies from $\theta = 0.75$ when $\gamma = 0$, to $\theta = 0.07$ when $\gamma = 0.73$). When $\gamma = 0$, the log change in GDP across BGPs is 0.028. When $\gamma = 0.74$, the log change in GDP across BGPs is 2.34 (GDP increases by a factor of $\exp(2.34) = 10.3$). If $\gamma$ is increased slightly further, so that $\theta$ is driven closer to 0, our

[21] We obtain this expression from equations (39), the definition of the research intensity of the economy as $s_r$ together with the resource constraint for the research good (4), and the definition of $\pi$.

[22] We do not report the values of $c_0$ and $c_1$ since they do not have any particular interpretation other than resulting in the target $g_z$ and $g_z$. We also note our calibration procedure results in a unique choice for all of our parameters.

model becomes an endogenous growth model and this change in GDP across BGPs rises towards infinity. We show the results for intermediate values of $\gamma$ in Figure 1. Clearly, our model's implications for the long-run impact of a given change in innovation subsidies varies tremendously depending on the assumed spillover parameter.

In Figure 1 we also show the log change in GDPs across BGP implied by our first-order approximation in Proposition 3. The changes in aggregate consumption are of similar magnitudes as shown in Figure 1.

The research intensity of the economy increases from $s_r = 0.15$ on the initial BGP to $s_r = 0.186$ on the new BGP. As discussed in Section 5, these changes in the research intensity of the economy across BGPs are independent of the spillover parameter $\gamma$. None of these results depend on the choice of $\delta_r$.

We now ask what our model's implications are for the behavior of aggregates during the transition from the initial to the new BGP.

**C. Transition Dynamics**

To compute the transition dynamics from one BGP to another, we solve the model numerically. We report on the behavior of aggregates over the first 15 years of the transition path. We focus on this 15 year horizon to show our model's implications for aggregates over a horizon that is relevant for applied work on the consequences of actual policy changes. We report in Figures 2-4 a number of statistics for aggregates.

Figure 2 displays the detrended log change in GDP 15 years after the policy change for the same range of values of $\gamma$ used in Figure 1. In this figure, we see that the range of responses of GDP over a fifteen years horizon is much smaller than the range of long-run responses shown in Figure 1. In particular, when $\gamma = 0$, the policy change raises GDP by 0.013 log percentage points relative to the initial BGP. When $\gamma = 0.74$, this policy-change raises GDP by roughly 0.045 log percentage points.

Note that in the case of $\gamma = 0$, the change in GDP over the first fifteen years is roughly half of the long-run change in GDP across BGPs, while in the case of $\gamma = 0.74$, the change in GDP over the first fifteen years is roughly 2% of the long-run change in GDP. This difference in responses over different horizons follows from the result that the transition dynamics of our model get significantly slower as the spillover parameter increases. In fact, when $\gamma = 0.74$, it takes more than 250 years for the cumulative change in GDP to reach half the long-run change (in contrast, when $\gamma = 0$, the half life is 16 years). If $\gamma$ is increased slightly further, so that $\theta$ is driven closer to 0, our model becomes an endogenous growth model, and the half life approaches infinity.[23]

Now consider the behavior of the research intensity of the economy during the transition. In Figure 3, we plot the change (in percentage points) of the research intensity of the economy, $s_r$, 15 years after the policy change. This change in the research intensity is roughly the same as the long-run change (0.035) across all the values of $\gamma$.

Finally, consider the impact of our policy change on welfare when transition dynamics are taken into account. Figure 4 displays the overall welfare gains (defined as the equivalent variation in consumption) taking into account the whole transition for the range of spillovers considered in Figures 1-3. Here we see that the welfare implications are close to zero when there are no spillovers[24] and they are very large — 0.47 log points of consumption in the initial BGP,[25] when the spillover is large. Thus, while the positive implications of our model for the behavior of aggregates over a 15 year horizon are not very sensitive to the assumed level of spillovers, the normative implications of our model are very sensitive to the choice of the spillover parameter.

It is clear that depending on the level of spillovers, innovation policies may have very large effects on long run GDP and welfare. Our results indicate that it is difficult, however, to use data on the aggregate response of the economy to a change in innovation policy to measure the size of these spillovers. In the next section, we discuss the implications of our results for the standard empirical approaches that have been used to address this question in the literature.

## 8. What Do We Know About The Long-Run Impact of Innovation Policy Changes?

With our calibrated model, we can answer the question of how permanent changes in innovation policy affect the research intensity of the economy and the levels of GDP and

[23] This slowdown of the transition dynamics as the spillover parameter increases is not likely to be an artifact of our specific calibration. For example, in a standard growth model extended to include spillovers to the accumulation of physical capital, it is straighforward to show that there is a trade-off between the size of the steady-state effects and the speed of transition from a policy change as the share of physical capital or the strength of spillovers varies. We note also that in our calibrated model, a lower value of the spillover depreciation parameter, $\delta_r$, reduces the half-life and slightly increases the increase in GDP over the first fifteen years of the transition.

[24] If the subsidy on the production of the final good, $\tau_s$, were optimally chosen to eliminate effective markups (as opposed to being set at $\tau_s = 0$) and $\gamma = 0$, then our policy change would reduce welfare.

[25] In particular, consumption in the initial BGP would have to be permanently multiplied by $\exp(0.47) = 1.6$ in order to provide the same level of utility as the one obtained in the transition after the policy change.

other aggregates over a fifteen year horizon even if we do not have precise information about the strength of spillovers from innovative activity. In particular, we found that a permanent subsidy to innovative activities with a fiscal impact of 3% of GDP per year raises the research intensity of the economy by roughly 3 to 4 percentage points both in the long run and at a 15 year horizon. This increase in research intensity is associated with an increase in GDP at a 15 year horizon ranging between 1 and 4 percentage points for a wide range of possible values of the spillover parameter $\gamma$.

What we cannot answer is the question of how aggregate output and welfare respond in the long run to a permanent policy change. We cannot answer this question because we do not know how aggregate output responds in the long run to a policy induced change in the research intensity of the economy. From Propositions 3 and 4, and Corollary 2, we have that changes in the aggregate research intensity driven by changes in three of our policies, are related to aggregate output in the BGP, to a first-order-approximation, by

$$\Delta \log GDP = \left[ \left( 1 + \frac{1}{(1-\gamma)(\rho-1)\theta} \right) \frac{L_p}{L} \frac{1}{1+(1-\lambda)s_r} - 1 \right] \Delta \log s_r.$$

What we have shown is that the term in square brackets is highly sensitive to the strength of spillovers, $\gamma$, and the implied value of $\theta$.

There are a number of papers in the literature that look to regression evidence to establish a connection between research intensity and output or productivity using data at the firm, industry, or aggregate level (see e.g. Kortum 1997, Jones and Williams 1998, Griffith, Redding, and Van Reenen 2001, Bloom, Griffith, and Van Reenen 2002, Bloom, Schankerman, Van Reenen 2010, and references therein). This evidence is often used to estimate the wedge between the private and social returns to innovation. We argue that this regression evidence linking research intensity and output or productivity is not likely to shed light on the long-run relationship between the research intensity in the aggregate and aggregate output or welfare for the following four reasons.

First, the relationship between research intensity and aggregate output depends on the particular policy being changed. If the change in the research intensity in our model arises instead from changes, for example, in exogenous scientific knowledge (captured in $A$), or changes in the subsidy to production of the consumption good $\tau_s$, then the model implies a different relationship between the changes in research intensity and aggregate output than the one that arises from changes in our policies. Hence, one needs to have a clear policy instrument to know which parameters are being estimated.

Second, it is not always clear that data used in regression results include the universe of process and product innovative activity. As we have seen in our model, the two types of innovation are substitutes: the larger is the rise in process innovation, the smaller is the response of product innovation. Moreover, for policies such as the corporate profits tax that distorts the use of physical capital, there is a further offset between the responses of physical and intangible capital. Regression evidence that captures only a subset of all innovative activity or capital accumulation may be highly misleading.

Third, the response of individual firms or industries to changes in policies tailored towards these firms or industries does not necessarily capture the general equilibrium effects that are central to our analysis. In particular, one would not expect the economy-wide wage level or the economy-wide level of output of the final consumption good $Y$ to respond to a change in firm or industry specific policies in the same way as when the change in policies is applied to the economy as a whole. Instead, in the case of industry or firm-specific policies, the elasticity of demand for that industry or firms' output plays a key role in determining the responsiveness of industry and firm output to a change in policy. It is straightforward to show that this partial equilibrium response of industry or firm-specific output to a change in policies tailored to those industries or firms can be either smaller or larger than the change in output of those industries and firms if that change in policies is applied to the economy as a whole. Hence, the implied relationship between the research intensity and output in an industry varies depending on whether the policy is applied to that specific industry or to all industries together.

Fourth, finally, our model implies very different relationships between research intensity and aggregate output depending on the time horizon. This problem is particularly severe when the spillovers are high. Given the slowdown in the transition dynamics as the spillover parameter increases, we suspect it would be difficult in practice to use data on the response of GDP to a policy change over a 15 year horizon to infer the degree of spillovers. This is because it would be difficult in practice to tell the difference between a 15 year cumulative, detrended, GDP change of 0.013 log percentage points (when $\gamma = 0$) and 0.045 log percentage points (when $\gamma = 0.74$). Using cumulative changes in GDP for shorter time horizons, these differences would be even smaller. Hence, we are skeptical that one could uncover useful measures of the aggregate implications of a policy induced change in the research intensity of the economy with no more than one or two decades of data.

## 9. Conclusion

In this paper, we have developed a model of the aggregate impact of changes in innovation policies and taxes and subsidies on firms more generally. This model integrates microeconomic perspectives using firm-level data and macroeconomic perspectives on this question. We see this work as offering three main contributions.

First, we have established the validity in our benchmark model of a very simple procedure for assessing the relative magnitudes, to a first-order approximation, of the long-run impact on aggregates of a wide range of policy changes. This procedure measures the relative magnitude of this long-run aggregate impact of a policy change by measuring the fiscal impact of this policy change holding fixed the endogenous decisions of firms. We have applied this procedure to measure the relative long-run impact on aggregates of three important policies in the U.S. that impact the decisions of firms to accumulate both intangible and tangible capital.

Second, in a calibrated version of our model, we have measured the impact on aggregates over a 15 year horizon of a policy change similar in magnitude in terms of its fiscal impact as eliminating the corporate profits tax in the U.S. We have found that this impact is moderate in size and not very sensitive to the assumed level of spillovers from innovative activities by firms. We have found, however, that the long-run impact of this policy change on aggregates and the welfare implications of this policy change are highly sensitive to the assumed level of spillovers from innovative activities by firms. If these spillovers are high, then a policy change of this magnitude would have a profound impact on welfare and aggregates in the long-run. Clearly, further research is needed to measure the extent of these spillovers.

Third, our model establishes a benchmark for evaluating the theoretically predicted impacts of policy changes on firm responses observed in micro data and the relationship between those firm level responses and the responses of macroeconomic aggregates. Our model has clear implications for the interpretation of much of the empirical work that has been done on this topic and our results cast doubt on the estimates of the magnitude of the long-run social return to innovation by firms that have been obtained in that work. Further research is needed to re-evaluate those measurements.

In our model, we have gained a great deal of tractability with a number of stark assumptions, such as a common growth rate for all continuing firms (Gibrat's Law), constant markups from CES demand, constant factor shares from Cobb-Douglas production functions, symmetric spillovers from both process and product innovation, and perhaps most importantly, completely elastic product innovation. Further research is needed to assess how sensitive are our results to empirically plausible deviations from these strong assumptions.

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# Appendices

*Proof of Proposition 2:* The problem of the planner is:

$$\max_{\{g_{zt},L_{pt},K_{t+1},X_t,N_{t+1}Z_{t+1},H_{t+1},\}} \sum_{t=0}^{\infty} \beta^t L_t \log\left(\frac{(N_tZ_t)^{\frac{1}{\rho-1}}(K_t)^{\alpha}(L_{pt})^{1-\alpha} - X_t + (1-\delta_k)K_t - K_{t+1}}{L_t}\right)$$

subject to the two following per-period constraints:

$$\mu_t\beta^t L_t: \; N_{t+1}Z_{t+1} = \left[(1-\delta_f)\exp(g_{zt}) - \frac{(1-\delta_f)}{n_e}c(\exp(g_{zt}))\right]N_tZ_t + \frac{(1-\delta_f)}{n_e}A_tH_t^{\gamma}(L_t-L_{pt})^{\lambda}X_t^{1-\lambda}$$

$$\nu_t\beta^t L_t: \; H_{t+1} = (1-\delta_r)H_t + A_tH_t^{\gamma}(L_t-L_{pt})^{\lambda}X_t^{1-\lambda}$$

with $K_0$, $N_0Z_0$, and $H_0$ are given, and where $\mu_t$ and $\nu_t$ denote the Lagrange multipliers of each constraint.

We first simplify the problem by showing that $g_{zt}$ is constant over time if there is positive product innovation. To see this, the FOC w.r.t to $g_{zt}$ is

$$\mu_t\exp(g_{zt})\left[(1-\delta_f) - \frac{(1-\delta_f)}{n_e}c'(\exp(g_{zt}))\right] = 0$$

If there is positive product innovation, we must have $\mu_t > 0$, so

$$n_e = c'(\exp(g_z)). \tag{40}$$

To simplify notation in this proof, we define $c = c(\exp(g_z))$.

The FOCs w.r.t to $L_{pt}$ and $X_t$ are, respectively,

$$(1-\alpha)\frac{1}{C_t}(N_tZ_t)^{\frac{1}{\rho-1}}\left(\frac{K_t}{L_{pt}}\right)^{\alpha} = \left(\mu_t\frac{(1-\delta_f)}{n_e} + \nu_t\right)\lambda A_tH_{rt}^{\gamma}(L_t-L_{pt})^{\lambda-1}X_t^{1-\lambda}$$

and

$$\frac{1}{C_t} = \left(\mu_t\frac{(1-\delta_f)}{n_e} + \nu_t\right)(1-\lambda)A_tH_{rt}^{\gamma}(L_t-L_{pt})^{\lambda}X_t^{-\lambda}.$$

Taking the ratio of these two expressions, we obtain

$$X_t = \frac{(1-\lambda)(1-\alpha)}{\lambda}(L_t-L_{pt})(N_tZ_t)^{\frac{1}{\rho-1}}\left(\frac{K_t}{L_{pt}}\right)^{\alpha}. \tag{41}$$

Plugging-in the solution for $X_t$ into the previous FOC w.r.t to $L_{pt}$, we re-write this constraint as:

$$(1-\alpha)^{\lambda}\frac{1}{C_t}(N_tZ_t)^{\frac{\lambda}{\rho-1}}\left(\frac{K_t}{L_{pt}}\right)^{\alpha\lambda}=\lambda^{\lambda}(1-\lambda)^{1-\lambda}\left(\mu_t\frac{(1-\delta_f)}{n_e}+\nu_t\right)A_tH_{rt}^{\gamma} \tag{42}$$

The FOCs w.r.t to $N_tZ_t$, $H_t$, and $K_t$ are, respectively,

$$\beta\exp(g_L)\frac{1}{\rho-1}\frac{(N_{t+1}Z_{t+1})^{\frac{2-\rho}{\rho-1}}\left(\frac{K_{t+1}}{L_{pt+1}}\right)^{\alpha}L_{pt+1}}{C_{t+1}}-\mu_t+\mu_{t+1}\beta\exp(g_L)(1-\delta_f)\left(\exp(g)-\frac{c}{n_e}\right)=0 \tag{43}$$

$$0=-\nu_t+\nu_{t+1}\beta\exp(g_L)(1-\delta_r)+\gamma\beta\exp(g_L)\left(\frac{(1-\lambda)(1-\alpha)}{\lambda}\right)^{1-\lambda}\times \tag{44}$$

$$\left(\mu_{t+1}\frac{(1-\delta_f)}{n_e}+\nu_{t+1}\right)A_{t+1}H_{t+1}^{\gamma-1}(L_{t+1}-L_{pt+1})(N_{t+1}Z_{t+1})^{\frac{1-\lambda}{\rho-1}}\left(\frac{K_{t+1}}{L_{pt+1}}\right)^{\alpha(1-\lambda)}$$

$$\frac{1}{C_t}=\frac{\beta\exp(g_L)}{C_{t+1}}\left(\alpha(N_{t+1}Z_{t+1})^{\frac{1}{\rho-1}}\left(\frac{K_{t+1}}{L_{pt+1}}\right)^{\alpha-1}+1-\delta_k\right). \tag{45}$$

We can further simplify (44) using (42):

$$-\nu_t+\nu_{t+1}\beta\exp(g_L)(1-\delta_r)+\beta\exp(g_L)\frac{\gamma(1-\alpha)}{\lambda}\frac{(L_{t+1}-L_{pt+1})}{H_{t+1}C_{t+1}}(N_{t+1}Z_{t+1})^{\frac{1}{\rho-1}}\left(\frac{K_{t+1}}{L_{pt+1}}\right)^{\alpha}=0. \tag{46}$$

We now impose that we are in a BGP with $(N_{t+1}Z_{t+1})/(N_tZ_t)=H_{t+1}/H_t=\exp(g_N)$. If $\theta>0$, the growth rate $g_N$ is equal to it level in the equilibrium, given by expression (30) (we discuss the case when $\theta=0$ below). In the BGP, from (42), $\mu_t$ and $\nu_t$ must each grow at the same rate $\exp(g_\mu)$ given by

$$\exp(g_\mu)=\exp(g_N)^{\frac{\lambda-1}{\rho-1}-\gamma}\exp(-g_A-g_L)\exp\left(g_{K/L}\right)^{\alpha(\lambda-1)}=\exp(-g_N).$$

where we used $g_{K/L}=\frac{g_N}{(\rho-1)(1-\alpha)}$ and (30). From (45), in the BGP,

$$\frac{K_t}{L_{pt}}=\left(\frac{\alpha}{R_k}(N_tZ_t)^{\frac{1}{\rho-1}}\right)^{\frac{1}{1-\alpha}}, \tag{47}$$

where $R_k=\beta^{-1}\exp(g_Y-g_L)-1+\delta_k$ as in the equilibrium. Combining (43), (46), (47), and the equation (from the law of motions of $NZ$ and $H$)

$$\frac{H_t}{N_t Z_t} = \frac{n_e}{1-\delta_f} \frac{\exp(g_N) - (1-\delta_f)\left(\exp(g_z) - \frac{c}{n_e}\right)}{\exp(g_N) - 1 + \delta_r}, \tag{48}$$

we obtain

$$\begin{aligned} \frac{L_{pt}}{L_{rt}} &= \frac{\mu_t}{\nu_t} \frac{(1-\delta_f)}{n_e} \frac{(\rho-1)\gamma(1-\alpha)}{\lambda} \times \\ & \frac{\left[\exp(g_N) - \beta \exp(g_L)(1-\delta_f)\left(\exp(g) - \frac{c}{n_e}\right)\right]}{\left[\exp(g_N) - (1-\delta_f)\left(\exp(g_z) - \frac{c}{n_e}\right)\right]} \frac{[\exp(g_N) - (1-\delta_r)]}{[\exp(g_N) - \beta \exp(g_L)(1-\delta_r)]}. \end{aligned} \tag{49}$$

In order to solve for $L_{pt}/L_{rt}$, we must solve for $\mu_t/\nu_t$. Using (42), (43), and the law of motion for $NZ$, we obtain

$$\frac{L_{pt}}{L_{rt}} \frac{\exp(g_L)\beta\lambda n_e}{(1-\alpha)(\rho-1)(1-\delta_f)} \left(\frac{(1-\delta_f)}{n_e} + \frac{\nu_t}{\mu_t}\right) = \frac{\left[\exp(g_N) - \beta \exp(g_L)(1-\delta_f)\left(\exp(g) - \frac{c}{n_e}\right)\right]}{\left[\exp(g_N) - (1-\delta_f)\left(\exp(g) - \frac{c}{n_e}\right)\right]}. \tag{50}$$

Combining (49) and (50) we obtain

$$\frac{\mu_t}{\nu_t} = \frac{n_e}{(1-\delta_f)\gamma\beta\exp(g_L)} \frac{\exp(g_N)(1-\gamma\beta\exp(g_L)) - \beta\exp(g_L)(1-\delta_r)(1-\gamma)}{\exp(g_N) - 1 + \delta_r}$$

and

$$\begin{aligned} \frac{L_{pt}}{L_{rt}} &= \frac{(\rho-1)(1-\alpha)}{\beta \exp(g_L)\lambda} \left[\frac{\exp(g_N) - \beta\exp(g_L)(1-\delta_f)\left(\exp(g_z) - \frac{c}{n_e}\right)}{\exp(g_N) - (1-\delta_f)\left(\exp(g_z) - \frac{c}{n_e}\right)}\right] \\ & \times \left[\frac{\exp(g_N)(1-\gamma\beta\exp(g_L)) - \beta\exp(g_L)(1-\delta_r)(1-\gamma)}{[\exp(g_N) - \beta\exp(g_L)(1-\delta_r)]}\right]. \end{aligned} \tag{51}$$

We now derive the equilibrium level of $L_{pt}/L_{rt}$ assuming that $\tau_g = \tau_e$. Using (34), (38), and two equations used in the proof of Proposition 6,

$$(1-\tau_g) P_r n_e M\xi = (1+\tau_p)\Pi NZ - (1-\tau_g) P_r c(\exp(g_z)) NZ,$$

and $M/(NZ) = [\exp(g_N) - (1-\delta_f)\exp(g_z)]/(1-\delta_f)$, we obtain

$$\frac{L_{pt}}{L_{rt}} = \frac{(\rho-1)(1-\alpha)}{\beta\exp(g_L)\lambda}\left(\frac{1-\tau_g}{1+\tau_p}\right)\left[\frac{\exp(g_N) - \beta\exp(g_L)(1-\delta_f)\left(\exp(g_z) - \frac{c}{n_e}\right)}{\exp(g_N) - (1-\delta_f)\left(\exp(g_z) - \frac{c}{n_e}\right)}\right]. \quad (52)$$

We are now ready to calculate the policies such that the equilibrium allocations coincide with the optimal allocations in a BGP. To set equation (28) equal to (40), we need $\tau_g^* = \tau_p^*$. To set equation (51) equal to (52), we need

$$\frac{1-\tau_g^*}{1+\tau_p^*} = 1 - \gamma\frac{\beta\exp(g_L)[\exp(g_N) - (1-\delta_r)]}{[\exp(g_N) - \beta\exp(g_L)(1-\delta_r)]}.$$

To set equation (41) equal to (19), using the equilibrium wage expression,

$$W_t = \frac{(1-\alpha)(\rho-1)}{\rho}(1+\tau_s)(N_t Z_t)^{\frac{1}{\rho-1}}(K_t/L_{pt})^a,$$

we need $(1+\tau_s^*) = \rho/(\rho-1)$. Finally, to set equation (17) equal to (47), we set $\tau_k^* = 0$.

Note that in the specification of our model with $\theta = 0$, we cannot use (30) to solve for $g_N$. In this case, for any given policies, $g_N$ solves equations (27) and (33). The expressions for optimal policies are the same as those under $\theta > 0$, but $g_N$ needs to be endogenously determined.

Other appendices mentioned in the text to be added.

# Table 1: Model Calibration

| Calibrated parameters | | Targets |
|---|---|---|
| Growth rate of population, $g_L$ | 0.01 | Growth rate of population = 0.01 |
| Depreciation rate of physical capita, $\delta_k$ | 0.1 | Depreciation rate of physical capital = 0.10 |
| | | |
| Growth rate of number of varieties, $g_N$ | 0.082 | Growth rate of per-capita GDP = 0.02 |
| Discount factor, $\beta$ | 0.98 | Difference between interest rate and growth rate = 0.01 |
| Exit rate of incumbent firms, $\delta_f$ | 0.053 | Employment-weighted exit rate of U.S. establishments in 2007 |
| Share of labor in production function of research good, $\lambda$ | 0.01 | Trend difference, U.S. price of inputs into R&D and GDP deflator |
| Share of phsycal capital, $\alpha$ | 1/3 | Share of labor in GDP = 0.66 |
| Elasticity of substitution across products, $\rho$ | 7.15 | Research intensity (share of intangible investment in GDP), $s_r$=0.15 |
| Parameters in innovation cost function, $c_0$ and $c_1$ | 0.0147 and 0.0445 | Share of dividends (payments to intangible capital) in GDP = 0.01 |
| | | Share of production employment by new establishments = 0.063 |

Imply ratio of process-innovation costs to variable profits, $s_g$=0.54, growth rate of z, $g_z$=0.07 , and $\xi = 1.16$

## Parameters governing spillover from cumulative innovative activity

| | | |
|---|---|---|
| Share, $\gamma$ | 0 to 0.74 | Implies $\theta$ ranging between 0.76 (when $\gamma$=0) and 0.07 (when $\gamma$=0.74) |
| Depreciation rate, $\delta_r$ | 0.1 | Sensitivity, $\delta_r$=0.05 and $\delta_r$=0.15 |

## Other parameters that do not affect results

| | |
|---|---|
| Population at time zero, L | 1 |
| Level of scientific knowledge at time 0, Ar | 1 |
| Cost of product innovation, $n_e$ | 1 |
| Curvature of process innovation cost function, $\eta_c$ | 10 |

## Policies

| | |
|---|---|
| Taxes and subsidies on initial balanced growth path (BGP): $\tau_j$=0 , j=p,s,g,e,k | |
| Taxes and subsidies on new BGP: $\tau_g$=$\tau_e$=0.165 and $\tau_j$=0 , j=p,s,k | Fiscal expenditures = 3% of GDP on new BGP |

Figure 1: GDP, log change across balanced growth paths

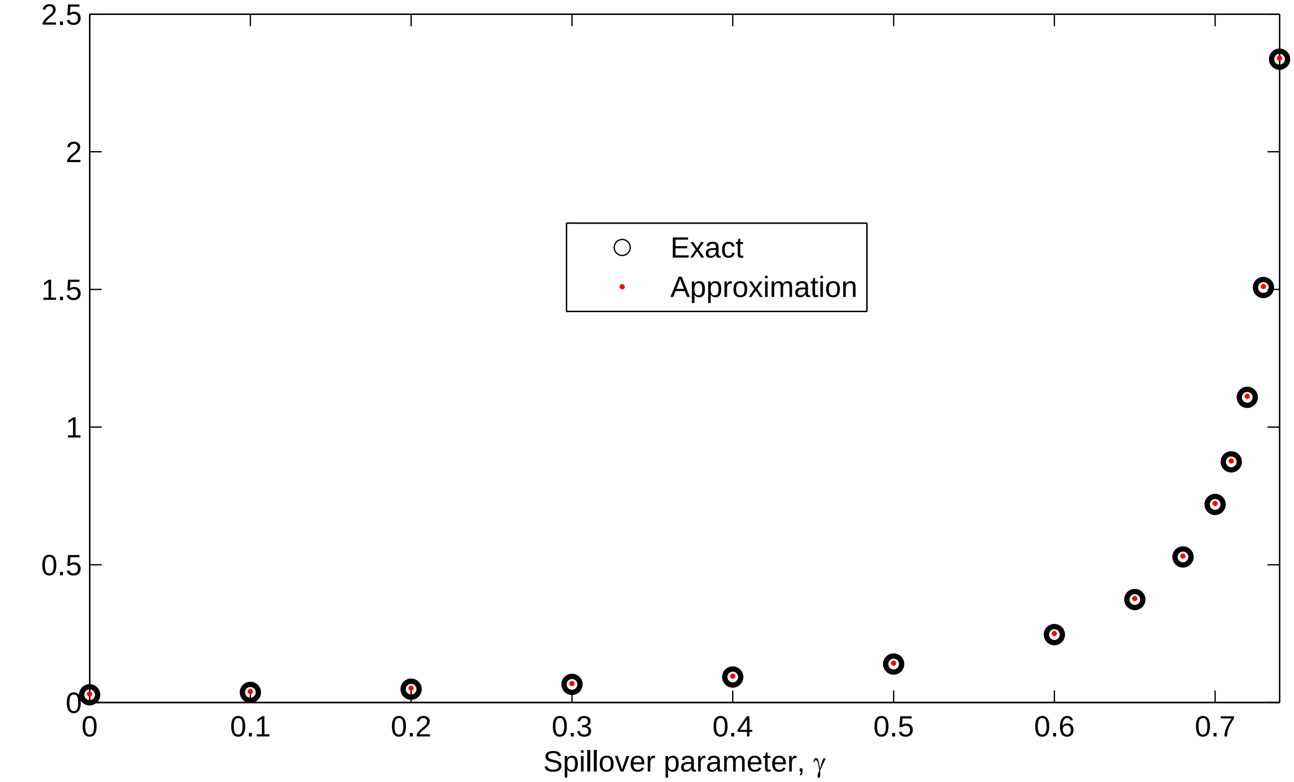


Figure 2: GDP, log change 15 years after policy change

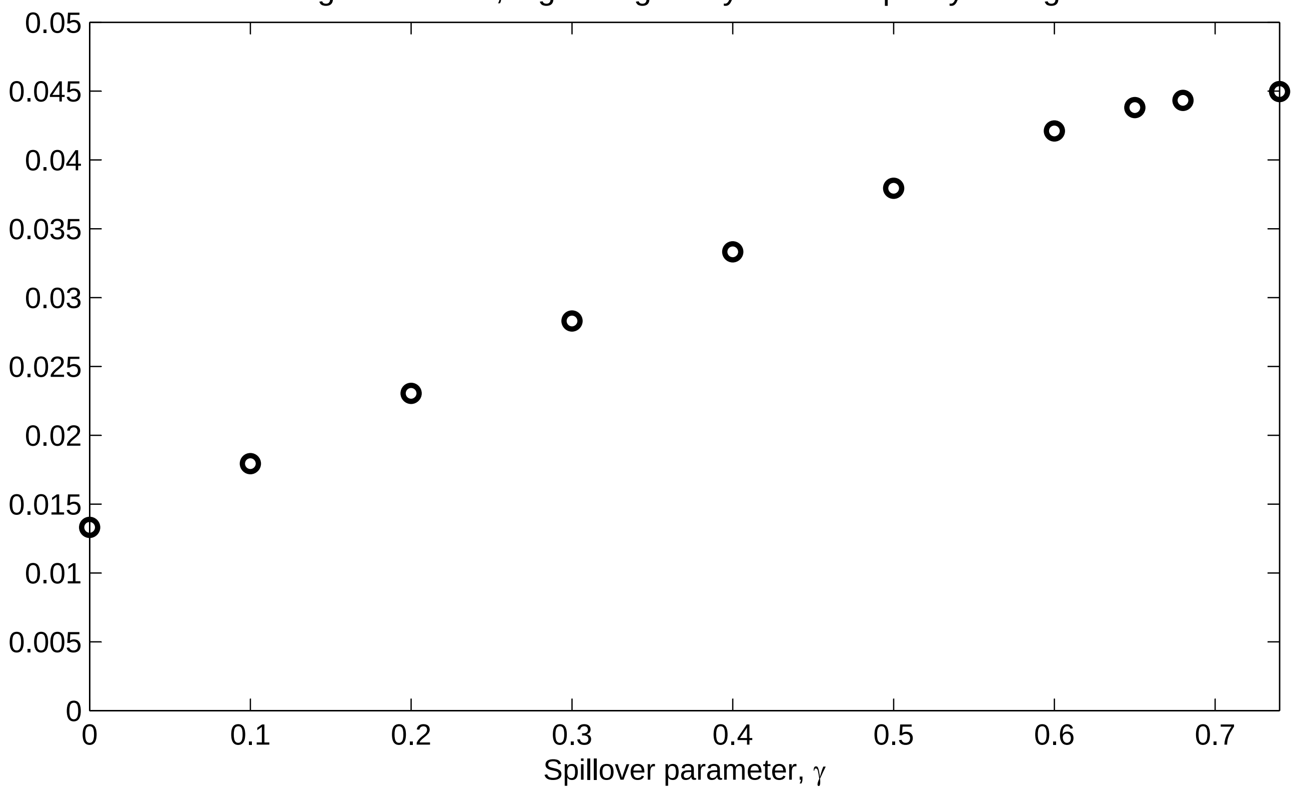

Figure 3: Research intensity of the economy, change 15 years after policy change

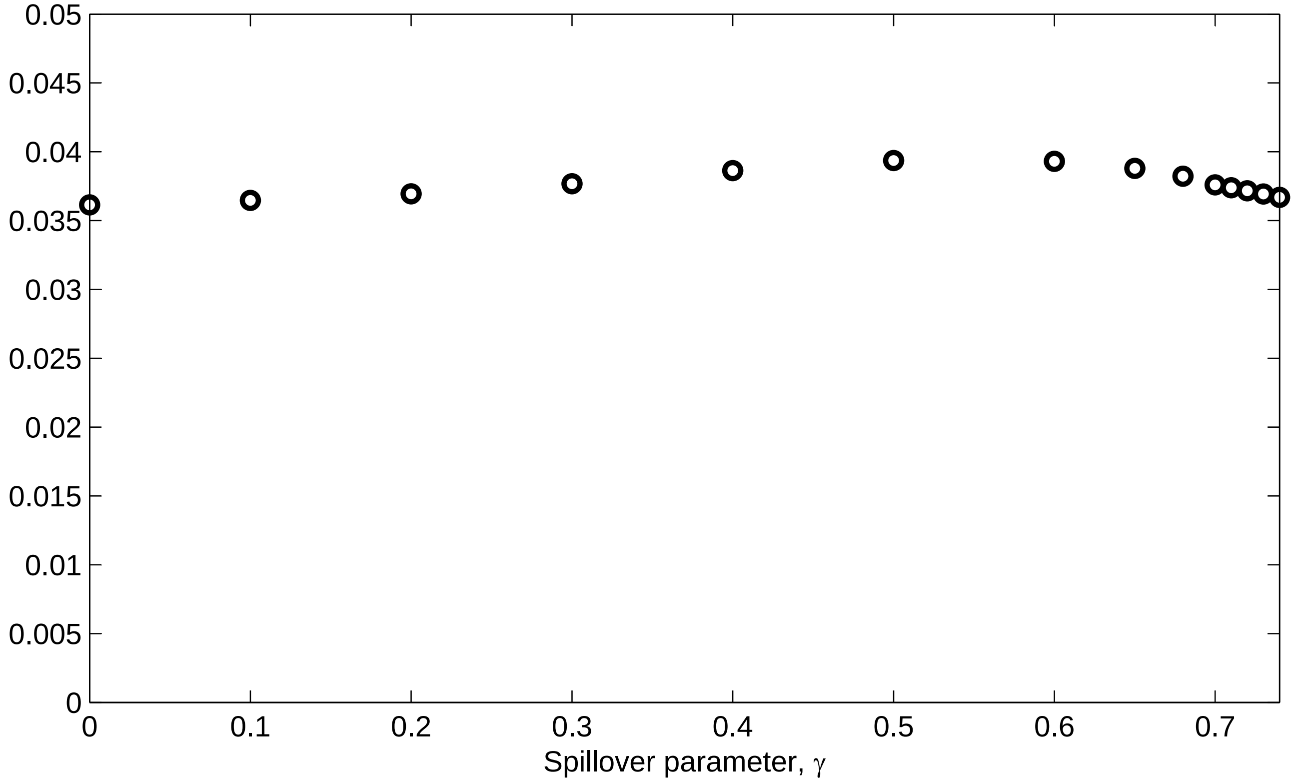


Figure 4: Welfare (equivalent variation, log change in initial consumption)