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Financial Markets and Fluctuations in Uncertainty*

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ABSTRACT

Researchers have documented that in the recent financial crisis the large decline in economic activity and credit has been accompanied by a large increase in the dispersion of growth rates across firms. We build a quantitative general equilibrium model in which financial frictions interact with increases in uncertainty at the firm level to generate a contraction in economic activity and a large increase in the dispersion of growth rates across firms. We find that our model can generate about 75% of the decline in output of the Great Recession of 2007. A promising feature of our model is that it generates large labor wedges, a feature of the recent data on business cycles.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1. Introduction

The recent financial crisis has been accompanied by severe contractions in economic activity and credit as well as large increases in the cross section dispersion of firm growth rates (Bloom, Floetotto and Jaimovich 2009). Motivated by these observations we build a quantitative general equilibrium model with heterogeneous firms and financial frictions in which increases in uncertainty at the firm level lead to an increase in the cross section dispersion of firm growth rates and a contraction in economic activity.

The basic mechanism is that in the presence of imperfect financial markets an increase in firm level uncertainty leads firms to contract the size of their projects to avoid default. The increase in uncertainty also has an amplification effect on firms' output through tighter credit. Firm level credit is more restricted because of the rise in default risk that comes with higher firm level uncertainty.

We quantify our model and ask, Can an increase in firm level uncertainty that generates the observed increase in the cross section dispersion in the recent recession lead to a sizable contraction in economic activity? We find that our model can generate about 75% of the decline in output seen in the data. More generally, we compare our model's implications to the data post 1970 for the average peak to trough changes and business cycle statistics. We find that the model generates quite volatile business cycles and labor fluctuations that are large relative to output.

Recently, Chari, Kehoe and McGrattan (2007) have decomposed business cycles into the components due to various wedges and have argued that labor wedges can account for 2/3 of the fluctuations in output. A striking feature of the Great Recession that started in 2007 is that it was associated mainly with a worsening of the labor wedge. A promising feature of our model is that it can generate a substantial worsening of the labor wedge during the episode. More generally, in our model with financial frictions changes in the dispersion of firm level shocks manifest themselves in aggregate data as movements in the labor wedge.

Our model has three key ingredients. First, firms must choose the scale of their projects with a decreasing returns to scale technology in advance. Second, the loans to firms cannot depend on their shocks and firms default if they have insufficient funds to pay for their debt. Third, since firms must pay a fixed cost to enter, in equilibrium they make positive expected profits in each period that they do not default. The cost of default is the loss of future expected profits.

Given these ingredients, when firms choose their project size they face a trade off between expected return and risk. As firms increase their size they increase the expected return conditional on not defaulting but they also increase the probability of default. For a given variance of idiosyncratic shocks, they choose their optimal size to balance off the increase in expected return against the losses from default. At a given project size, when the variance of the idiosyncratic shocks increases the probability of default increases. Thus, in the face of such an increase in variance, firms become more cautious and decrease the size of their projects. At the aggregate level, these firm level responses imply that when the dispersion of idiosyncratic shocks increases aggregate output and employment fall.

The result that firms decrease the size of their projects when the variance of idiosyncratic shocks increases depends critically on the lack of complete markets. If firms had access to complete markets then (with i.i.d. idiosyncratic shocks) an increase in the variance of these shocks would lead to no change in their optimal size. With such complete markets they would face no tradeoff between expected return and default risk. When the variance of shocks increases, firms keep their preferred size and then restructure the pattern of payments across states so that they never default.

In our model output declines not only because the size of each incumbent declines but also because the number of entering firms declines. High dispersion makes potential new entrants less willing to enter because conditional on entering a smaller scale is optimal and at this smaller scale the value of operating a firm is lower. For a given fixed cost of entry as the value a firm decreases the number of operating firms also decreases.

We consider a quantitative version of the model in which idiosyncratic firm productivity shocks have stochastic volatility. We choose the parameters of this shock process so that the model produces the time variation in cross section dispersion of the growth rate of sales observed in a panel of Compustat firms. We find that generates volatile business cycles and can explain 77% of the fall in output of the current recession. The model also produces volatility of labor and the labor wedge relative to output similar to that in the data.

The algorithm we use to compute the equilibrium extends the solution methods of Kahn and Thomas (2008) that handle idiosyncratic and aggregate shocks to allow for free entry and endogenous debt contracts with default risk. Allowing for free entry in a decentralized model of heterogenous firms adds a substantial complication in the solution method as the entry decision has to be solved for in the simulation part of the algorithm.

In terms of the literature, this work is related to a growing literature that studies time varying volatility. Bloom, Floetotto, and Jaimovich (2009) and Bloom (2009) show that in the presence of adjustment costs, firms drop their investment and hiring when hit by a high uncertainty shock. [1] A key difference between our approach and that of Bloom et al. is that

[1] Other papers that study the effects of uncertainty on invesment in the presence of adjustment costs

in our work the financial frictions manifest themselves as labor wedges and the fixed cost frictions in the Bloom et al. paper manifest themselves as TFP shocks. Christiano, Motto, and Rostagno (2009) also explore the business cycle implications of uncertainty shocks. They show that in a DSGE model with nominal rigidities and financial frictions uncertainty shocks to investment account for a significant portion of the fluctuations in output.

Our work is also related to the work on heterogenous firms and financial frictions. For example, Cooley and Quadrini (2001) develop a model of heterogenous firms with incomplete financial markets and default risk and explore its implications for the dynamics of firms growth and exit. It is also related to the work of Cooley, Marimon, and Quadrini (2004) who find in a general equilibrium setting that limited enforceability of financial contracts amplifies the effects on output of technology shocks.

2. Model

Consider a dynamic model of a continuum of heterogeneous firms and a continuum of identical households. Firms use a decreasing returns technology with labor as an input to produce consumption goods. This technology is subject to both aggregate shocks and idiosyncratic productivity shocks. Households provide labor services to firms and trade assets that are contingent on the aggregate shock. They own all firms and receive lump sum transfers.

As we have noted, our model has three key ingredients. First, firms must choose the scale of their projects with a decreasing returns to scale technology in advance of the realization of shocks that affect the variance of idiosyncratic productivity. Second, the loans

include Bernanke (1983), Abel and Eberly (1996), and Caballero and Engel (1999).

to firms cannot depend on their idiosyncratic shocks and firms default if they have insufficient funds to pay for their debt. Third, since firms must pay a fixed cost to enter, in equilibrium they make positive expected profits in each period that they do not default. The cost of default is the loss of future expected profits.

The timing of decisions is as follows. In the beginning of each period idiosyncratic and aggregate shocks are realized. After observing shocks, incumbent firms choose to repay their debts or default and new entrants decide on entry. Households then consume, work and decide on new asset holdings. Firms at this stage decide whether to default or not. Continuing firms produce, pay their workers and their debts, decide on new project sizes and new debt holdings, pay taxes and distribute dividends. Defaulting firms exit.

A. Firms

A continuum of firms produce output y_t with a stochastic decreasing returns technology using labor ℓ_t as an input

$$y_t = z_t \ell_t^{\theta}$$

where z_t is an exogenous idiosyncratic productivity shock. Firms commit to the scale of the project before the realization of their shocks. Specifically, they commit to an employment level ℓ_t in period $t-1$ that is not contingent on the realization of either the aggregate or the idiosyncratic shocks at t. The idiosyncratic productivity shock z_t follows a Markov process with transition function $\pi_z(z_t|z_{t-1}, \sigma_{t-1})$ where σ_{t-1} is an aggregate shock that controls the dispersion (or standard deviation) of firm productivity. The aggregate shock follows a Markov process with transition function $\pi_\sigma(\sigma_t|\sigma_{t-1})$.

Firms have access to one period debt contracts which are not contingent on shocks. Firms begin period t with some debt due b_t. Hence at the beginning of period t, each firm is indexed by $v_t = (\ell_t, b_t, z_t)$ which records their labor commitments, their debt due, and their current idiosyncratic shock. We let Υ_t denote the measure of v_t across firms. The aggregate state is denoted $S_t = (\Upsilon_t, \sigma_t)$.

In each period t, each firm chooses a debt contract in which, conditional on not defaulting, promises to pay b_{t+1} at $t+1$. The price of such claim is given by a price schedule $q(S_t, z_t, \ell_{t+1}, b_{t+1})$ that depends on the current aggregate state S_t, the firms' current idiosyncratic shock z_t and current decisions of the firm, namely, its labor commitment ℓ_{t+1} and its borrowing level b_{t+1}. Note that b_{t+1} can be negative so that firms can save.

After shocks are realized existing firms decide whether to repay or default, denoted $\phi = 1$ or $\phi = 0$ respectively. Firms that repay continue while firms that default exit. The dividends for a continuing firm are

$$(1) \quad d_t = (1-\tau^c)(z_t \ell_t^\theta - w_t \ell_t) - b_t + q(S_t, z_t, \ell_{t+1}, b_{t+1}) b_{t+1}$$

where τ^c denotes corporate taxes. Dividends are restricted to be nonnegative, $d_t \geq 0$.

The state of a firm at t is (v_t, S_t). Given the state, the budget set is defined as

$$\Gamma(v_t, S_t) = \{d_t | \ d_t \geq 0\}.$$

where d_t is given by (1). Note that firms with large enough debt have an empty budget set which forces them to default. That is, given the bond price schedules $q(S_t, z_t, \ell_{t+1}, b_{t+1})$ for borrowing for new debt b_{t+1}, there is a large enough inherited debt b_t at t such that no new debt contract can deliver non-negative dividends. For such a configuration the only option for the firm is to default. Such a firm then exits. We capture this formally by requiring that if $\Gamma(v_t, S_t) = \varnothing$ then firms default by setting $\phi(v_t, S_t) = 0$. In our model this is the only time a firm will default.

Let $V(v_t, S_t)$ denote the value of firm. For any individual state (v_t, S_t) such that the budget set is empty let $V(v_t, S_t)$ equal zero. For all other states in which the budget set is nonempty, $V(v_t, S_t)$ equals the value conditional on repaying today and acting optimally from now on. In each period, continuing firms choose new project sizes ℓ_{t+1}, new loans b_{t+1}, and dividends d_t. For states in which the budget set is nonempty, the value of $V(v_t, S_t)$ equals

$$\max_{\{d_t, b_{t+1}, \ell_{t+1}\}} d_t + \sum_{z_{t+1}, \sigma_{t+1}} \gamma \left\{ Q\left(\sigma_{t+1} | S_t\right) V(v_{t+1}, S_{t+1}) \pi_z(z_{t+1} | z_t, \sigma_t) \right\}$$

subject to the budget constraint

$$d_t = (1 - \tau^c)(z_t \ell_t^\theta - w_t \ell_t) - b_t + q(S_t, z_t, \ell_{t+1}, b_{t+1}) b_{t+1}$$

a non-negative dividend condition $d_t \geq 0$, and the law of motion for aggregate states. $Q\left(\sigma_{t+1} | S_t\right)$ is the price in state S_t for one unit of goods in period $t+1$ when the aggregate shock is σ_{t+1} and the parameter γ makes it attractive for firms to borrow so that it is not optimal for them to build up a large amount of savings in order to completely avoid the possibility of default. The law of motion has two parts. The aggregate shock σ_{t+1} evolves according to $\pi_\sigma(\sigma_{t+1}|\sigma_t)$ while the measure over firms idiosyncratic states evolves according to

$$(2) \quad \Upsilon_{t+1} = H(S_t)$$

This problem gives the decision rules for new sizes of projects $\ell(v_t, S_t)$, borrowing $b(v_t, S_t)$, repayment $\phi(v_t, S_t)$, and dividends $d(v_t, S_t)$.

When a firm defaults, it is liquidated, so society loses the built up knowledge inherent in z_t. However, defaulting firms produce and pay their workers whenever operating profits are positive $z_t\ell_t^\theta - w_t\ell_t \geq 0$. Bond holders receive any residual goods from such firms. If $z_t\ell_t^\theta - w_t\ell_t < 0$ for defaulting firms, so that at their chosen scale in $t-1$ they make negative operating profits, they hire no labor and are liquidated immediately. Let $\kappa_t(v_t, S_t)$ be an indicator function for the firm that equals 1 if $z_t\ell_t^\theta - w_t\ell_t \geq 0$ and 0 otherwise.

Consider next firm entry. There is a continuum of identical potential projects every period. To enter, firms have to pay an entry cost ξ in period t and decide on the size of the project ℓ_{t+1}^e for the following period. The idiosyncratic shocks of new entrants z_{t+1} are drawn from a distribution with transition function $\pi_e(z_{t+1}|\sigma_t)$. The value function of entrants is given by

$$V^e(S_t) = \max_{\{\ell_{t+1}^e\}} -\xi + \sum_{z_{t+1},\sigma_{t+1}} \left[Q\left(\sigma_{t+1}|S_t\right) V(\ell_{t+1}, 0, z_{t+1}, S_{t+1})\pi_e(z_{t+1}|\sigma_t)\right]$$

subject to the evolution of the aggregate states. This problem gives project sizes for new entrants $\ell_{t+1}^e(S_t)$. Let $M(S_t)$ denote the measure of new entrants.

Consider next the bond price schedules faced by firms. Each firm borrows from a financial intermediary. The intermediary offers firms bond price schedules such that each debt contract compensates for the expected loss in the case of default. To develop the expression for the value of a contingent loan to a firm, suppose the current aggregate state is S_t and that a firm with current idiosyncratic shock z_t and new scale ℓ_{t+1} buys a bond b_{t+1}. At $t+1$ when the aggregate shock is σ_{t+1} and the idiosyncratic shock is z_{t+1}, that firm repays completely whenever the repayment indicator $\phi(\ell_{t+1}, b_{t+1}, z_{t+1}, S_{t+1})$ is one, and when this indicator is zero it repays $\max\{z_{t+1}\ell_{t+1}^{\theta} - w_{t+1}\ell_{t+1}, 0\}$, which is all of its operating profits . The intermediary values these repayments using the price for contingent claims $Q(\sigma_{t+1}|S_t)$. Hence, the value for such a contingent loan is given by

$$
\begin{aligned}
& q(S_t, z_t, \ell_{t+1}, b_{t+1})b_{t+1} = \\
& \sum_{z_{t+1},\sigma_{t+1}} Q\left(\sigma_{t+1}|S_t\right) \pi_z(z_{t+1}|z_t, \sigma_t)\phi(\ell_{t+1}, b_{t+1}, z_{t+1}, S_{t+1})b_{t+1} \\
+ & \sum_{z_{t+1},\sigma_{t+1}} Q\left(\sigma_{t+1}|S_t\right) \pi_z(z_{t+1}|z_t, \sigma_t)(1 - \phi(\ell_{t+1}, b_{t+1}, z_{t+1}, S_{t+1})) \max\{z_{t+1}\ell_{t+1}^{\theta} - w_{t+1}\ell_{t+1}, 0\}
\end{aligned}
$$

B. Households

Households are identical, discount the future at rate β, and have a period utility function $u(c_t, h_t)$ where c_t and h_t are consumption and labor in period t. Households provide labor h_t to firms at t for wage w_t, and receive aggregate dividends D_t and a lump sum transfer T_t. Households have access to one period securities $B_{t+1}(\sigma_{t+1})$ that are contingent on the aggregate shock σ_{t+1} at $t+1$, with prices $Q(\sigma_{t+1}|S_t)$.

The recursive problem for households, taking as given the wage, state contingent prices, aggregate dividends, and lump sum transfers, is the following

$$V^H(B_t(\sigma_t), S_t) = \max_{\{c_t, h_t, B_{t+1}(\sigma_{t+1})\}} u(c_t, h_t) + \beta \sum_{\sigma_{t+1}} \pi_\sigma(\sigma_{t+1}|\sigma_t) V^H(B_{t+1}(\sigma_{t+1}), S_{t+1})$$

subject to the budget constraint

$$c_t + \sum_{\sigma_{t+1}} Q(\sigma_{t+1}|S_t) B_{t+1}(\sigma_{t+1}) = w_t h_t + D_t + T_t + B_t(\sigma_t),$$

the no Ponzi scheme condition $B_{t+1}(\sigma_{t+1}) \geq -\bar{B}$, and the law of motion for aggregate states. Note that the transfers to consumers in period t are simply the taxes collected from firms in that period.

C. Equilibrium

Equilibrium in the goods market require that the total consumption of households equals the total output produced by non-defaulting firms and defaulting firms net of the cost of new entrants

$$(3) \quad c(S_t) = y(S_t) - M(S_t)\xi.$$

where total output is defined as

$$(4) \quad y(S_t) = \int [\phi(v_t, S_t) + (1 - \phi(v_t, S_t))\kappa(v_t, S_t)] z_t \ell_t^\theta d\Upsilon_t(v_t; \sigma_t)$$

Labor market clearing implies that households labor supply in $t+1$ equals the period t labor commitments of operating firms in $t+1$, so that for each σ_{t+1}

$$\int [\phi(v_t, S_t) + (1-\phi(v_t, S_t))\kappa(v_t, S_t)]\ell(v_t, S_t) d\Upsilon_t(v_t; \sigma_t) + M(S_t)\ell^e(S_t) = h(B(\sigma_{t+1}), S_{t+1}).$$

Finally, the total borrowing by firms equals the total savings by consumers

$$\int \phi(v_t, S_t) q(S_t, z_t, \ell_{t+1}, b_{t+1}) b_{t+1} d\Upsilon_t(v_t; \sigma_t) = \sum_{\sigma_{t+1}} Q(\sigma_{t+1}|S_t) B_{t+1}(\sigma_{t+1})$$

The measure of firms evolves over time as firms enter, exit, and change their labor and borrowing choices. Given the state $S_t = (\Upsilon_t, \sigma_t)$, the transition function is given by

$$H(v_{t+1}; S_t) = \int \Lambda(v_{t+1}, v_t|S_t)\Upsilon_t(v_t) dv_t + M(S_t)\Lambda^e(v_{t+1}|S_t). \tag{5}$$

where, given $v_{t+1} = (\ell_{t+1}, b_{t+1}, z_{t+1})$, the function $\Lambda(v_{t+1}, v_t|S_t)$ equals

$$\begin{cases} \pi_z(z_{t+1}|z_t, \sigma_t) & \text{if } \ell_{t+1} = \ell(v_t, S_t),\ b_{t+1} = b(v_t, S_t),\ \text{and } \phi(v_t, S_t) = 1 \\ 0 & \text{otherwise} \end{cases}$$

and the function $\Lambda^e(v_{t+1}|S_t)$ equals

$$\begin{cases} \pi_e(z_{t+1}|\sigma_t) & \text{if } \ell_{t+1} = \ell^e(S_t) \\ 0 & \text{otherwise} \end{cases}$$

To interpret these transition functions note that $\Lambda(v_{t+1}, v_t|S_t)$ specifies that the probability that a firm with some $v_t = (\ell_t, b_t, z_t)$ transits to $v_{t+1} = (\ell_{t+1}, b_{t+1}, z_{t+1})$ in aggregate state S_t equals $\pi_z(z_{t+1}|z_t, \sigma_t)$ when such firm chooses the particular ℓ_{t+1} and b_{t+1} in v_{t+1}.

We now define the equilibrium of this economy. Given the initial distribution Υ_0 and an initial aggregate shock σ_0, a recursive equilibrium consists of policy and value functions of firms $\{d(v_t, S_t),\ b(v_t, S_t),\ \ell(v_t, S_t),\ \phi(v_t, S_t),\ V(v_t, S_t)\}$, households policy functions for consumption $c(B_t, S_t)$, hours $h(B_t, S_t)$ and savings $B(\sigma_{t+1}, S_t)$, the wage rate $w(S_t)$ and discount bond price $Q(\sigma_{t+1}, S_t)$, bond price schedules $q(S_t, z_t, \ell_{t+1}, b_{t+1})$, the mass of new entrants $M(S_t)$, and (vi) the evolution of aggregate states Υ_t governed by the transition function $H(S_t)$, such that for all t: (i) The policy and value functions of firms satisfy their optimization problem, (ii) households decisions are optimal, (iii) loan contracts reflect the expected default losses such that every contract breaks even in expected value, (iv) domestic good, labor, and credit markets clear, (v) the free entry condition holds

$$V^e(S_t)M(S_t) = 0,$$

and (vi) the evolution of the measure of firms is consistent with the policy functions of firms, households and shocks.

D. Sketch of the Computation Algorithm

Solving the model requires keeping track of the measure of firms $\Upsilon_t(v_t)$ as it changes with the idiosyncratic and aggregate shocks. We first combine the problem of households, firms, and the financial intermediary. In recursive notation, optimization from households imply that wages and intertemporal prices equal

$$Q(\sigma'|\sigma,\Upsilon) = \beta\pi_\sigma(\sigma'|\sigma)\frac{u_c(c',h')}{u_c(c,h)}$$

$$w(\sigma,\Upsilon) = \frac{u_h(c,h)}{u_c(c,h)}$$

The firms problem can be simplified further by noting that the only relevant individual states for the firm decisions are the idiosyncratic shock and the total after tax profits net of total debt $x = (1-\tau^c)(z\ell^\theta - w\ell) - b$. Using households' first order condition we can then write in recursive form the combined program of firms and the financial intermediary as follows.

First for a given state (x, z, σ, Υ) the firm default if its budget set is empty: If

$$\{x + q(\sigma,\Upsilon,z,\ell',b')b' < 0 \ \forall \ \{\ell',b'\}\}$$

then $\phi(z,x,\sigma,\Upsilon) = 0$ and $V(x,z,\sigma,\Upsilon) = 0$.

Conditional on having a non-empty budget set then the firms' problem is

$$(6) \quad V(x,z,\sigma,\Upsilon) = \max_{\ell',b'} d + \sum_{\sigma',z'} Q(\sigma'|\sigma,\Upsilon)\pi_z(z'|z,\sigma)[V(x'(.),z',\sigma',\Upsilon')]$$

where the choices of ℓ', b' map into the future firm state x' as follows

$$x' = (1-\tau^c)(z'\ell'^\theta - w(\sigma',\Upsilon')\ell') - b'$$

subject to the budget constraint

$$d = x + q(\sigma, \Upsilon, z, \ell', b')b',$$

non-negative dividend condition.

$$(7) \quad x + q(\sigma, \Upsilon, z, \ell', b')b' \geq 0$$

the break even conditions for the financial intermediary

$$q(\sigma, \Upsilon, z, \ell', b')b' =$$

$$\sum_{\sigma', z'} Q(\sigma'|\sigma, \Upsilon)\, \pi_z(z'|z, \sigma)\phi(x'(.), z', \sigma', \Upsilon')b'$$
$$+ \sum_{\sigma', z'} Q(\sigma'|\sigma, \Upsilon)\, \pi_z(z'|z, \sigma)(1 - \phi(x'(.), z', \sigma', \Upsilon')) \max\{z\prime\ell'^{\theta} - w'\ell', 0\}$$

and the evolution of the aggregate states where σ' evolves according to $\pi_\sigma(\sigma'|\sigma)$ while the measure over firms idiosyncratic states evolves according to

$$(8) \quad \Upsilon' = H(\Upsilon, \sigma)$$

Following the algorithms of Krusell and Smith (1998) and Kahn and Thomas (2008) we parameterize the measure of firms with a small number of moments and solve the firm's problem. In particular we approximate the measure of firms with the number of operating firms N, and the last period aggregate shock, σ_{-1}. Given these 2 states, we construct forecasts functions for the next period's number of firms and aggregate consumption and labor

$$(9) \quad N' = f_N(N, \sigma_{-1}, \sigma), \ c = f_c(N, \sigma_{-1}, \sigma), \ h' = f_h(N, \sigma_{-1}, \sigma)$$

To solve the equilibrium of the model, we start with candidate forecasts functions (9) and solve the program (6). Solving such problem requires finding fixed points for both the value function $V(x, z, \sigma, N, \sigma_{-1})$ and the repayment decision $\phi(z, x, \sigma, N, \sigma_{-1})$. The solution gives policy rules for firms $\ell(x, z, N, \sigma_{-1}, \sigma)$, borrowing $b(x, z, N, \sigma_{-1}, \sigma)$, repayment $\phi(x, z, N, \sigma_{-1}, \sigma)$, and dividends $d(x, z, N, \sigma_{-1}, \sigma)$. Using these policy rules we simulate the model. Given initial N in every period of the simulation, we find the number of firms the next period N' such the free entry condition holds:

$$\xi = \sum_{z',\sigma'} \left[Q\left(\sigma'|\sigma, N, \sigma_{-1}\right) V(\ell^e(N, \sigma, \sigma_{-1}), 0, z', N', \sigma, \sigma')\pi_e(z'|z, \sigma)\right]$$

During each period we also compute aggregate consumption and labor such that the market clearing conditions hold. After the simulation we compute new forecasting functions (9) using the equilibrium sequences fo N, c, and h for given the sequence of σ shocks. We repeat this procedure until the forecasting functions converge. In the final stage, all forecasting functions have an $R^2 > 0.98$.

3. Uncertainty, default and the labor wedge

In this section we construct a simple example to illustrate how fluctuations in uncertainty gives rise to fluctuations in the labor wedge in the presence of default risk.

Consider a two period model where a firm produces in period 1 with a decreasing returns technology that uses labor as its input $z\ell^\theta$ where z is drawn from a distribution $f(z)$. The firm pays in period 1 its wage bill $w\ell$ and conditional on not liquidating receives in period 2 a constant value V. The firm decides the scale of its operation in period 0 and has to liquidate if its cash flows in period 1 are negative.

Consider first the case when financial markets are complete. With perfect financial markets the firm can guarantee positive cash flows in every state in period 1 by using state contingent assets. Hence, the choice of labor solves the following problem

$$\max_{\ell} \int_0^\infty [z\ell^\theta - w\ell] f(z) dz + V$$

The optimal scale of operation ℓ^* is such that the expected marginal product of labor equal the wage

$$\theta \ell^{*\theta-1} E(z) = w.$$

With perfect financial markets fluctuations in the volatility of the idiosyncratic shock z that does not affect its mean will have no impact on firms' labor choice.

Consider now the case of financial autarky as an example of extreme financial market imperfections. In this case, firms with large employment have to liquidate when they experience low productivity due to insufficient cash flow to cover the wage bill. Effectively, the firm chooses its labor input ℓ as well as a cutoff productivity $\hat{z}$ below which it liquidates. The firm solves the following problem

$$\max_{\ell,\hat{z}} \int_{\hat{z}}^{\infty} [z\ell^{\alpha} - w\ell]\phi(z)dz + \int_{\hat{z}}^{\infty} V\phi(z)dz$$

subject to

$$\hat{z}\ell^{\alpha} - w\ell = 0.$$

This last condition defines the cutoff productivity $\hat{z}$ below which the firm liquidates because for any $z < \hat{z}$ the firm would have negative cash flow. The larger the scale ℓ the larger is the probability of liquidation for the firm because of the loss in future value of V.

In this environment, the optimal choice of labor maximizes not only period 1 profits (as in the case of perfect financial markets), but also future profits by preventing liquidation. The choice of ℓ^* satisfies:

$$(10) \quad \theta\ell^{*\theta-1}\frac{E(z|z \geq \hat{z})}{1-\Phi(\hat{z})} = w + V\frac{\phi(\hat{z})}{1-\Phi(\hat{z})}\frac{d\hat{z}}{d\ell^*}$$

where $\hat{z}\ell^{*\alpha} - w\ell^* = 0$.

When financial markets are imperfect and firms face liquidation risk, the choice of ℓ equates the effective marginal product of labor in the states in which the firm is operative to the marginal cost of labor which includes the wage and the loss in future value. Condition (10) illustrates the labor wedge arising from default risk that makes the marginal product of labor larger than the wage.

In contrast to the case of perfect financial markets, fluctuations in the volatility of idiosyncratic shocks affect the choice of labor in this case. Increases in volatility generally rise the hazard rate $\frac{\phi(\hat{z})}{1-\Phi(\hat{z})}$ which in turn leads to a smaller choice of labor and and a larger labor wedge. Intuitively, a rise in volatility increases liquidation risk; firms then have incentives to lower this risk by reducing their scale.

4. Quantitative Analysis

A. Parametrization

We use features of the time variation in the cross section distribution of firms in the United States to help inform the choice of some key parameters. Many of the parameters of preferences and technology are fairly standard and are chosen to reflect commonly used values.

Consider first the setting of some standard parameters. The utility function is assumed to take the form $u(c,h) = \log(c) - \xi\frac{h^{1+\nu}}{1+v}$. We set $\nu = 0.25$ which implies a labor elasticity of 4. This estimates are in the range of elasticities used in macroeconomics as reported by Rogerson and Wallenius (2009). The exponent of the production function θ is set to the labor share 0.7 times the decreasing returns to scale parameter 0.975. The decreasing returns parameter is taken from Basu and Fernald (1997). The corporate tax τ_c is set to 25% which is the average effective corporate tax from 2002 to 2006 from the Monthly Treasury Statement published by the Treasury Department.

Consider next the parameterization of the Markov processes over idiosyncratic productivity shocks and aggregate shocks. The Markov process over firms' idiosyncratic productivity shock is parameterized as follows. In each period a fraction δ of firms receive a productivity shock $z_t = 0$ which is absorbing. The productivity process of the remaining firms is assumed to follow the process

$$\log z_t = \rho_z \log z_{t-1} + \sigma_{t-1}\varepsilon_t$$

where $\varepsilon_t \sim N(0,1)$. The innovations ε_t are independent across firms. The aggregate shock follows the process

$$\log \sigma_t = (1-\rho_\sigma)\log \mu_\sigma + \rho_\sigma \log \sigma_{t-1} + \upsilon_t$$

where $\upsilon_t \sim N(0,\varphi^2)$.

Our data source for firm data is Compustat. We set the parameters governing the aggregate and idiosyncratic shocks $\mu_\sigma, \varphi, \rho_\sigma,\ \rho_z$, to match the time variation of both the distribution and average annual sales growth in the data. The four moments we target are the mean, standard deviation and autocorrelation of the cross section interquartile range of annual sales growth, and the mean firm autocorrelation of sales growth across firms from 1970 to 2009, which equals 0.65. Using quarterly data, annual sales growth is computed as $(sales_t - sales_{t-1})/0.5(sales_{t-4} + sales_t)$ with sales deflated by CPI for firms in Compustat with at least 160 quarters of observations. The parameter γ is chosen to match the average debt to sales for firms in Compustat of 3.6. The fraction δ is parameterized to the average U.S. failure rates and the entry cost ξ is calibrated so that the number of firms relative to output 5. The resulting parameters are $\mu_\sigma = 0.16$, $\varphi = 0.16$, $\rho_\sigma = 0.84$, $\rho_z = 0.7$, $\delta = 0.025$, $\gamma = 0.7$ and $\xi = 0.74$.

Table (1) presents the calibration results and some additional moments of the model and the data. Both in the data and the model the average growth dispersion across firms is large and volatile although in the model both statistics are a somewhat larger. The average interquartile range of sales growth are 0.15 in the data and 0.23 in the model with a standard deviation over time of 3.3 and 6 respectively.

As the table shows, underlying the dispersion and time variation of sales growth across firms is the dispersion of the firm's leverage ratios both in the data and the model. The mean leverage ratio across firms are 3.6 in both the data and the model. The mean leverage ratio is also very volatile over time with a standard deviation of 49 in the data and 91 in the model. Moreover, firms differ substantially in their leverage ratios at any point in time; the average interquartile range of leverage equal 2.5 in the data while it is equal to 1.2 in the model. This heterogeneity in the cross section of leverage is also time varying with a standard deviation of 83 in the data and 38 in the model. Overall, the model is consistent with the data in terms of the large variability in firms' leverage ratios across time and across firms.

B. Impulse Response to a Dispersion Shock

Here we discuss the impulse response of both firm level variables and aggregate variables to an increase in dispersion. We use these responses to help provide intuition for the model's mechanism.

Specifically, to set the initial conditions we consider a long enough sequence of realizations in which the low dispersion shock is realized so that all aggregates do not change from one period to the next. We then use as an initial condition the resulting measure over individual states. Starting from this distribution we suppose that in time period 4 the dispersion shock changes from low to high and then stays there for then onwards.

The shock in the impulse response is an increase in σ_t from 12% to 16% which corresponds to one standard deviation of the σ_t process. To help interpret the magnitude of the shock, the IQR of sales growth increases from 0.17 to 0.25 with this shock.

We start with the responses of an individual firm with some particular individual state (z, ℓ, b). We choose the level of z to be the mean level. Note that this level of z has the property that when the dispersion shock increases the conditional mean of future idiosyncratic shocks in unaffected. We choose the levels of ℓ and b for an average firm in the distribution.

In Figure 1 we plot the labor ℓ' and debt choices qb' of this firm for 10 quarters. We see that when dispersion increases the firm decreases the scale of production by about 7% and decreases the value of its borrowing by about 1.5%. The intuitive idea is that at the original scale of production, when dispersion increases firms would be forced into default more often. When firms default they lose the future stream of positive expected profits. To avoid losing this stream the firm decreases the size of the project. After period 6, the firm starts increasing its employment because of the general equilibrium effect of lower wages which we discuss more in detail below.

The intuition why firms contract their scale after high uncertainty shocks is similar to our example in section 3. Since the firm has decreasing returns to scale ($\theta < 1$), the scale of operation controls the tradeoff between expected return and risk. A smaller scale lowers expected profits for the next period but lowers the risk, namely the probability that the firm will have to default. Now consider our firm with the mean idiosyncratic productivity. When the dispersion shock increases, the conditional mean of the idiosyncratic productivity shock is unchanged but the conditional variance of this shock increases. This change increases the risk more than the expected return and hence the firm finds it optimal to choose a lower scale for the project. For a similar reason the firm decreases it outstanding debt.

To get a feel for the importance of financial frictions at the firm level we consider what a firm with no financial frictions would do when faced with an increase in dispersion. Specifically, we consider the choices of a single firm with access to a complete set of financial markets with complete enforcement of debt contracts that faces the same prices as in our economy and with idiosyncratic productivity equal to the mean level. In particular, this firm has access to debt which allows it to pay back different amounts depending on the level of its shocks. This firm chooses the scale of labor ℓ_{CM} so that the expected marginal product of labor equals the expected wage rate weighted by the stochastic discount factor

$$\sum_{z_{t+1},\sigma_{t+1}} Q\left(\sigma_{t+1}|S_t\right)\pi_z(z_{t+1}|z_t,\sigma_t)\theta z_{t+1}\ell_{CM}^{\theta-1} = \sum_{\sigma_{t+1}} Q\left(\sigma_{t+1}|S_t\right)w_{t+1}.$$

Figure 2 plots the labor choices for such a firm as well as the choices of labor for a firm in our economy. The scale of firms with no financial frictions are always higher than the scale of firms in our model. In fact, the distance between these two choices of labor is a measure of the financial distortions at the level of the individual firm. When dispersion is low, the size of the firm with no financial frictions is about 20% larger than a similar firm in our model. When dispersion increases, financial distortions are amplified and the difference in size between these two firms reaches more than 40%.

In the model, a firm's choice of its scale depends on that firm's debt, as well as on the aggregate and idiosyncratic shocks. All else equal, highly indebted firms choose smaller scales. The left panel of Figure 3 illustrates this generic negative relationship between project size and debt for two aggregate shocks while holding constant operating profits at the mean level. Firms with large debt choose smaller scales because debt increases the risk of default and hence these firms find it optimal to reduce such risk with a more conservative scale. Specifically, given the debt schedules, large debt due shrinks firms' budget sets especially for low idiosyncratic shocks. Firms prefer to become smaller and expand the budget set in those idiosyncratic shocks to avoid default. As the figure shows, when debt is large enough, firms' budget set is empty and they default and exit.[2] As the figure also shows, high debt is disproportionately disruptive in times of high dispersion as the level of debt for which the firm shrinks its scale and defaults is lower with high dispersion.

Default in the model happens when firms cannot roll-over their debt even though the firm has a positive value. Hence, default happens due to *liquidity* as opposed to *solvency* problems. The right panel of Figure 3 shows the value of the firm as a function of debt for the two aggregate shocks. Clearly, the higher the debt of a firm is the lower its value and once the debt gets high enough, the firm's value is zero. The interesting part of the relation is that at a critical value of the debt the value of the firm discretely jumps down to zero. At this critical value the firm is just able to borrow enough to pay off its existing debt. Hence, for slightly higher values of debt the firm cannot borrow enough and must default. The reason the value function jumps at this critical value is that by defaulting the firm loses a strictly positive discounted stream of expected future profits. This leads to the question, Why can the firm with a positive value not borrow more? The reason is that the firm cannot borrow freely at the contingent prices used in the valuation of the future dividends. In particular, the firm cannot borrow contingently on its shocks. Because of this restriction the firm cannot

[2] Many model of firm dynamics and financial frictions such Hopenhayn and Albuquerque (2003), Cooley and Quadrini (2001), and Arellano, Bai, and Zhang (2010) share this feature with our model.

pledge resources based only on the expected stream of profits. Hence, it is possible for a firm to be illiquid, in that it cannot borrow, even though it is solvent, in that it has positive value.

We now turn to the impulse responses at the aggregate level. In Figure 4a we plot the impulse response of the main aggregates, output, labor, and consumption, for 10 quarters. In the period when the shock hits, the *impact* period, output and labor do not change because the size of the projects have been set in the previous period. In the period after the shock hits, output falls about 3% and labor falls more than 4.5%. The reason why aggregate output and labor fall is that incumbent firms shrink their scale with higher dispersion and there is less entry of new firms. The dynamics of consumption differ from those of output and labor. On impact, consumption rises about 1% and then falls. Consumption rises on impact because investment in new firms fall more than output.

To get a longer perspective in Figure 4b, we plot these impulse responses for 100 quarters. We see that consumption and output continue to fall and eventually settle down to be about 3.5% lower than their starting points. After its initial drop, labor slowly rises and eventually settles down to be about 2.3% lower than their starting points.

In Figure 5a, we plot uncontingent interest rates (in levels relative to the initial one) and wage rates[3]. Except for the impact period, interest rates barely change. The wage rate rises by 1% on impact and then falls. As we see in Figure 5b, the wage rate continues to decline until it is about 3.8% below its original level. The decline in wages after shock hits is the main reason why aggregate labor slowly rises after the initial decline.

In Figure 6a we see that the measure of firms in the economy decreases substantially

[3]The uncontingent interest rate is the rate on claim in period t that delivers 1 unit of consumption in all states in period $t+1$.

after the high uncertainty shock. The long impulse response in Figure 6b shows that this measure eventually settles down to about 8% below its original level. High dispersion is associated with a smaller measure of firms which is driven by the incentives to enter. As seen above, high dispersion exacerbates the financial frictions and leads firms to choose small scales. Hence during high dispersion times, it becomes less attractive to pay the fixed cost and enter. Since the resources spent on fixed costs to set up firms represent a type of investment we have that investment decreases in high dispersion times. After the increase in dispersion the measure of firms declines slowly as consumers find it optimal to smooth their consumption.

In Figures 7a and 7b we graph the labor wedge and TFP. These are two commonly used measures in business cycle analysis. We define the labor wedge as

$$(11)\quad 1-\tau_l=-\frac{U_{ht}}{U_{ct}}/(\theta y_t/h_t).$$

and define TFP as output y_t divided by $h_t^{\theta}N_t^{1-\theta}$ where N_t is the measure of firms[4]. Interestingly, we see that the labor wedge falls over 2% after the dispersion shock is realized while TFP (and, clearly labor productivity y_t/l_t) increases a modest amount, about .5%.

Note that even with complete financial markets the labor wedge would not be zero. That is true because firms must commit to their scale before the realization of shocks. Hence, we think of (11) as a simple statistic that is often computed in the data and is used to evaluate models. Nonetheless, this labor wedge moves around substantially more in our model than it would in the complete markets version of our model. The reason it does so is that in

[4] Here N_t is the measure of firms. We note that in the economy with complete markets the aggregate production function would be $h_t^{\theta}N_t^{1-\theta}$.

our model the financial frictions make it not optimal for firms to equate the value marginal product of labor to the value of the wage as seen in Figure 2.

In the model, measured TFP rises a small amount because high dispersion in idiosyncratic productivity increases the reallocation possibilities across firms given the decreasing returns to scale in production.

C. Business Cycle Implications

So far we have investigated the implications for our model following a one time shock to dispersion. Next, we consider the type of business cycle statistics that the model generates. In examining these statistics it is important to keep in mind that in our benchmark results we have purposefully abstracted from other shocks to highlight the quantitative importance of dispersion shocks.

The data is quarterly from 1970:1 to 2009:4 and logged and detrended with separate linear trends. We consider both changes in aggregates from peak to trough, business cycle statistics and explore in detail the episode of the 2007 Great Recession.

Peak Trough Analysis

Table (2) shows that dispersion shocks interacting with financial frictions are powerful in generating volatile business cycles. Consider first the changes in aggregates from peak to trough. In the data, the average contraction of GDP from peak to trough is 5% while in the model the contraction is 5.6%. In the data and in the model the contraction in aggregate output is largely due to a decline in labor, although in the model labor falls by more. In the data, on average labor falls by 5.1% from peak to trough while in the model it declines by 7.1%. The model thus generates a recession mostly from movements in labor and hence predicts that labor declines more than output declines.

Probably the most important feature of the model is that it generates an average decline in the labor wedge in recessions of 5.2% which is close to the observed in the data of 5.8%. Thus, our model with financial frictions and dispersion shocks generate movements in aggregate variables which during downturns show up as a collapse in the labor wedge.

During this period TFP falls modestly from peak to trough. Specifically TFP falls on average about 1.3% while output falls about 5%. In the model, we have a moderate decline in TFP of 0.1%.

Business Cycle Statistics in the Benchmark Model

Turning now to second moments statistics our model with only dispersion shocks generates highly volatile business cycles. In fact the model overstates the volatility of GDP. The model also generates, as in the data, that the volatility of labor and the labor wedge relative to output are quite high, although in the data these are higher. The relative volatility of labor in the model is 1.0 whereas in the data it is 1.3; the relative volatility of the labor wedge in the model is 0.5 whereas in the data it is 1.7. The labor and the labor wedge are also positively correlated with GDP in the model and data, though in the model those correlations are a bit higher than in the data.

Classic business cycle models with only TFP shocks do not generate the high volatility of labor relative to output observed in the data because in those models the labor wedge is constant. As noted above, Chari, Kehoe and McGrattan (2007) show that fluctuations in the labor wedge modeled as an exogenous stochastic process can account for about 2/3 of the fluctuations in output. In our model, financial frictions at the firm level and dispersion shocks increase the volatility of aggregate labor which results in a volatile labor wedge.

The variation of TFP in the model is modest and contrary to the data its fluctuations are negatively correlated with GDP. Adding TFP shocks in the model would bring the volatility and correlation of TFP closer to the data.

The fluctuations in TFP also show that our mechanism stands in contrast with the results in Bloom, Floetotto and Jaimovich (2009). They show that in a model with adjustment costs for capital and labor, high dispersion generates a TFP decline. In our model, high uncertainty delivers a modest rise in TFP. The reason is that high dispersion provides better reallocation opportunities for the economy. In fact, in a model with complete asset markets high dispersion would generate an increase in aggregate output because better possibilities for reallocation rise aggregate productivity. Nonetheless, both models generate a contraction in aggregate output in response to high uncertainty but through different margins. In their environment, the contraction in output is accounted by an endogenous decline in TFP, while in our model the contraction is accounted mainly by a decline in the labor wedge.

The Great Recession of 2007

The next experiment we do in this section is analyze how much of the movement in aggregates in the current recession can be accounted for by our model.

In this experiment we let the initial number of firms be the one that arises in the limit after a long sequence of relatively low uncertainty shocks. We then choose the sequence of shocks so that the IQR of sales growth that the model produces is similar to that in the data. In Figure 8 we show the IQR of sales growth in the model and the data. We think of this procedure as backing out the dispersion shocks from data on dispersion of sales growth.

Given our initial condition and this sequence of shocks we then simulate the model.

Figure 9 and 10 show the resulting movements in output and labor. From Figure 9 we see that over the period 2007:4 to 2009:1 the model generates about the same overall decline in output as in the data. However after 2009:1, the model generates a slow recovery whereas in the data output remained repressed. The dynamics for labor are similar to those for output. From Figure 10 we that the model produces a similar decline in labor as in the data, from 2007:4 to 2009:1, and then produces a slow recovery, whereas in the data employment remained low. Mechanically the reason that the model produces a slow recovery towards the end of the recession is that by that time the uncertainty shock stabilizes and then declines. At this point, firms are able adjust their debt holdings accordingly and capable of expanding somewhat their operations. Overall, the model can explain 77 percent of the contraction of output during the recent recession.

Figures 11 and 12 show the resulting series for labor wedges and aggregate TFP. From Figure 11 we see that the labor wedge in the model is similar to that in the data up to 2009:1, stabilizing thereafter. The model can explain about 51 percent of the worsening of the labor wedge of the recent recession. From Figure 12 we see that the model produces a modest increase in aggregate TFP up to 2008:4 and then a decline. In the data the TFP falls and then rises. Note that both in the model and in the data TFP fluctuations are modest and for the overall event TFP is unchanged.

5. Conclusion

We have built a quantitative general equilibrium model in which financial frictions interact with increases in uncertainty at the firm level to generate a contraction in economic activity and a large increase in the dispersion of growth rates across firms. Firms in our model scale down on their employment in more uncertain times to reduce the risk of default. The reduction in firm level employment without changes in productivity generates at the aggregate level large and volatile labor wedges, which is a feature of recent data on business cycles. We find that in our model can generate substantial volatility in business cycles and for the current recession can account for about 77 percent of the output collapse and all of the labor decline.

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