A Game-Theoretic View of the Fiscal Theory of the Price Level

Marco Bassetto*

March 8, 2000

Abstract

The goal of this paper is to probe the validity of the fiscal theory of the price level by modeling explicitly the market structure in which households and the governments make their decisions. I describe the economy as a game, and I am thus able to state precisely the consequences of actions that are out of the equilibrium path. I show that there exist government strategies that lead to a version of the fiscal theory, in which the price level is determined by fiscal variables alone. However, these strategies are more complex than the simple budgetary rules usually associated with the fiscal theory, and the government budget constraint cannot be merely viewed as an equilibrium condition.

1 Introduction

This paper stems from a recent heated debate on the relationship between the price level and fiscal policy. This relationship has a long tradition in macroeconomics. Milton Friedman stressed extensively that inflation is chiefly a monetary phenomenon and that price stability can be achieved by stabilizing the money supply.[1] Sargent and Wallace [16] showed that monetary and fiscal policy are intertwined through the government budget constraint; the objective of a stable money supply is inconsistent with a persistent fiscal deficit. Sargent [15] studied several inflationary episodes and argued that fiscal deficits were primarily responsible for the ultimate recourse of policymakers to the printing press.

A recent string of papers[2] have pushed the link between the price level and fiscal policy further, developing a "fiscal theory of the price level". These papers observe that in low and

*Preliminary and incomplete. Comments welcome. I am indebted to Lawrence Christiano and Larry Jones for helpful comments and discussions. Address: Department of Economics, Northwestern University, 2003 Sheridan Road, Evanston, IL 60208; phone (847) 491-8233; email m-bassetto@nwu.edu

[1] See e.g. Friedman and Schwartz [7].

[2] To my knowledge, Leeper [11] started this line of research, and Sims [19] and Woodford [21] are the seminal contributions. Woodford has developed the idea further in [22, 23, 25, 24]. Cochrane [4] has extended the analysis to long-term debt, and Dupor [6] to the exchange-rate determination in an open-economy framework. Loyo [12] has applied the theory to study inflation episodes in Brazil.

moderate-inflation countries governments borrow mainly by issuing nominal bonds. The presence of nominal bonds introduces an additional link between fiscal and monetary policy, and the revenues that the government can achieve by implicitly defaulting on its debt through inflation are much larger than the seigniorage revenues emphasized by Sargent and Wallace. However, the key distinction between the "traditional" view of inflation and the fiscal theory of the price level is much deeper than the mere presence of nominal debt. According to the fiscal theory, money is completely secondary in determining the price level, which is instead driven by the sequence of primary surpluses and deficits. The price level is simply the instrument through which the real value of debt stays in line with the present value of future government surpluses.

The key difference between the fiscal theory and the traditional view lies in the interpretation of the government budget constraint, which links the real value of debt to the present value of primary surpluses the government will run in the future. The advocates of the theory view this link as an equilibrium condition: an imbalance between the real value of debt and the surpluses would trigger changes in the price level that would lead back towards an equilibrium, either by reducing or by increasing the value of the nominal debt. The traditional view interprets the link as a constraint on policy, which forces government action, either through a fiscal adjustment or through a default on debt or through money-induced inflation, whenever the real value of debt and the present value of primary surpluses tend not to be equal. It is this difference that has spurred the major controversy.[3]

The goal of this paper is to reach a clearer and less controversial understanding of the constraints imposed on monetary and fiscal policy by their interdependence. I describe the entire economy as a game, and I provide a market microstructure that shows how prices arise from the actions of the players in the economy. Specifically, prices are formed by the bidding process of households and the government on specialized trading posts where goods and assets are traded pairwise. While the market structure I describe is highly stylized, it is able to clearly set apart constraints on the set of actions that the government can take from relations that hold only in equilibrium, thereby shedding light on the key source of controversy.

In a companion paper [1], I show that the standard definition of a competitive equilibrium and of a commitment equilibrium fail to describe out-of-equilibrium paths, and I provide a more complete definition of an equilibrium for an economy with a large player (the government) and many atomistic players. In this paper, I apply the definition to a specific game which is well suited to address the validity of the fiscal theory of the price level.

I show that, in the environment I describe, there exist government strategies that lead to a version of the fiscal theory, in which the price level is determined by fiscal variables alone. However, these strategies are more complex than the simple budgetary rules usually associated with the fiscal theory, and the government budget constraint cannot be merely viewed as an equilibrium condition.

Section 2 illustrates the fiscal theory of the price level and the theoretical criticism against it. Section 3 describes the market structure I assume. Section 4 contains the main the results of

[3]Among the authors that have attacked the view that the government budget constraint is purely an equilibrium condition are Buiter [2]. Other papers that express similar views are by McCallum [13] and Kocherlakota and Phelan [10].

the paper, section 5 talks about extensions that are in progress and section 6 concludes.

2 Ricardian and non-Ricardian Policy Rules

In this version of the paper, I study a cashless economy, in which money is purely a unit of account. This specification is often pursued by the papers that adopt the fiscal theory of the price level, consistently with their idea that money as a medium of exchange is secondary in determining the price level.

I choose a cashless specification because it is simpler and still captures the main insights of the debate. In section 5, I discuss how I plan to introduce a transaction role for money in a future extension and what aspects of the debate on the fiscal theory can only be addressed by this introduction.

Let us consider an economy with a continuum of identical households that live for two periods (1 and 2) and a government. Households receive a constant exogenous endowment of a single homogeneous good in each period, which we normalize to 1. A nonconstant endowment and production could be easily introduced without altering the results, but they would make the notation more cumbersome and would introduce many more markets to keep track of in the game-theoretic version. Each household starts the first period with B_1 units of government bonds. A government bond is a claim to 1 "dollar", which is just a unit of account. All debt is assumed to mature in one period; once again, this is not an important assumption, but saves on notation considerably. The government has access to lump-sum taxes in both periods; with the tax revenues T_1 and T_2, it finances some exogenous government spending in either period (G_1 and G_2), as well as repayment of its original debt. We assume no uncertainty.

Households have preferences given by

$$u(c_1) + u(c_2) \tag{1}$$

where u is a strictly increasing and concave function satisfying Inada conditions and c_j is consumption in period j. We use lower-case letters for variables that refer to a single household, and upper-case letters for the corresponding aggregates. We only look for symmetric equilibria, in which each household is taking the same action; therefore, lower-case and upper-case variables will always coincide *in equilibrium.*

Government spending does not enter in the households' utility; as usual, it could be added in a strongly separable way without affecting the results.

The household's flow budget constraints are

$$\begin{aligned} P_1 c_1 &\leq P_1(1 - T_1) + B_1 - \frac{b_2^d}{R_1} \\ P_2 c_2 &\leq P_2(1 - T_2) + b_2^d \end{aligned} \tag{2}$$

P_j is the price level, i.e., the inverse of the value of a dollar; R_1 is the nominal interest rate in the economy and b_2^d is the amount of newly-issued government bonds with period-2 maturity that the household demands in period 1.

The government budget constraint for this economy is[4]

$$\begin{aligned} P_1 G_1 &= P_1 T_1 + \frac{B_2}{R_1} - B_1 \\ P_2 G_2 &= P_2 T_2 - B_2 \end{aligned} \tag{3}$$

where B_2 is the supply of bonds in period 2.

A **competitive equilibrium** is an allocation (C_1, C_2, B_2^D), a price system (P_1, P_2, R_1) and a government policy (T_1, T_2, B_2) such that:

(i) given the price system and the government policy, the allocation maximizes the households' utility subject to the budget constraint 2;

(ii) the government budget constraint (3) is satisfied;

(iii) Markets clear, i.e. $B_2^D = B_2$.

The definition of a competitive equilibrium describes the actions taken by the households and the government at the equilibrium; it does not specify what would happen if the government took a different policy, or if the price system were different from the equilibrium one.

We define a **fiscal policy rule** as a mapping from the price P_1 into T_1, and from the vector (P_1, P_2) to T_2. While this economy does not have money, we still define a **monetary policy rule** as a mapping from the price P_1 to an interest rate R_1. The rationale behind this definition is the perception that the cashless economy is only a limiting concept and that the central bank retains the ability to peg the nominal interest rate as we drive the economy to the cashless limit. In the game we describe below, the ability of the government to peg the interest rate will explicitly come out of the model. The definition of a fiscal and monetary policy rule here is more limited than the one in Woodford [21, 22] or in Kocherlakota and Phelan [10], as I specify which variables the government is targeting in its rule. This is only done for simplicity of exposition.

We define a policy rule to be the combination of a fiscal and monetary policy rule.

The literature distinguishes two types of rules, which I will call Ricardian and non-Ricardian, following Woodford [22]. A policy rule is **Ricardian** if it satisfies the government budget constraint for any price vector; it is non-Ricardian otherwise. This definition is justified by the fact that, in any Ricardian rule, the present value of taxes (payments from the households to the government) less the value of debt (present value of payments from the government to the households) is identically equal to the present value of government spending, a constant that does not depend on the price levels (P_1, P_2). With a Ricardian rule, an increase in P_1 that decreases the value of nominal government debt held by the households is matched by a reduction in the present value of taxes, and does not affect the households' choices, provided the real interest rate remains constant.

While the previous argument justifies the name "Ricardian", the key distinction from our perspective is that a non-Ricardian policy rule allows the government to violate its budget

[4] In what follows, I do not allow the government to waste any resources (other than spending itself...). The analysis would be similar if the government had access to free disposal; in that case, violations of (3) would only be a problem when taxes are too small.

constraint out of equilibrium, whereas a Ricardian rule meets the budget constraint both in and out of an equilibrium. If government spending were allowed to vary, instead of being exogenous, the name Ricardian vs. non-Ricardian would no longer capture the key difference.

Proponents of the fiscal theory of the price level assume that the government can commit to non-Ricardian policies. While their arguments are not cast in a model that properly specifies out of equilibrium behavior, their reasoning is (a variation of) the following. For any price P_1, tax T_1 and interest rate $R_2 > 0$, it is possible to find a supply of government debt B_2 such that the flow budget constraint is satisfied in period 1. If the policy rule is non-Ricardian, then there are some price vectors (P_1, P_2) for which the budget constraint in period 2 is not satisfied; at this price vector, the government would "offer" bonds B_3 that mature after the end of the economy to meet its flow budget constraint. Since nobody is willing to buy these bonds, there is excess supply and prices will have to adjust.

The opponents of the fiscal theory[5] insist that any rule that is non-Ricardian is simply a misspecification: no matter what the prices are, the government should always choose a policy that satisfies its intertemporal budget constraint, which includes the transversality condition $B_3 = 0$.

In order to deem non-Ricardian rules admissible, it is necessary to interpret the intertemporal budget constraints differently: the households' budget constraints are viewed as binding both in and out of equilibrium, whereas the government budget constraint is interpreted as a "government valuation equation" that only holds at the equilibrium price (see e.g. Cochrane [5]). Woodford [25] justifies this asymmetry with two arguments:

(i) if the households were not subject to budget constraints, they would demand an infinite amount of goods, so there would be no equilibrium; the same is not true for the government, which (for exogenous reasons) has an interior satiation point;

(ii) households are price takers, whereas the government is a big player capable of moving prices.

Neither of these arguments is compelling. The possibility or impossibility of violating the budget constraint out of equilibrium should not have anything to do with preferences. Having the ability to affect prices is not the same as having the ability of violating a budget constraint for any given price vector.

The admissibility of non-Ricardian rules has dramatic implications on the determinacy of the price level, which we now turn to.

Proposition 1 *If the government adopts a Ricardian policy rule, P_1 is indeterminate; more precisely, given any strictly positive value, there exists a competitive equilibrium in which P_1 attains that value.*

Proof. Under a given policy rule, a (symmetric) competitive equilibrium is characterized by the following equations:

[5] See e.g. Buiter [2], Kocherlakota and Phelan [10].

(i) first-order conditions for the household's problem

$$u'(C_1) = \frac{P_1 R_1}{P_2} u'(C_2) \tag{4}$$

(ii) household budget constraints at equality

$$\begin{aligned} P_1 C_1 &= P_1(1 - T_1) + B_1 - \frac{B_2^d}{R_1} \\ P_2 C_2 &= P_2(1 - T_2) + B_2^d \end{aligned} \tag{5}$$

(iii) government budget constraints (at equality) (3)

(iv) market clearing conditions

$$\begin{aligned} C_1 &= 1 - G_1 \\ C_2 &= 1 - G_2 \\ B_2^d &= B_2 \end{aligned} \tag{6}$$

(v) policy rule specification: $T_1 = T_1(P_1)$, $R_1 = R_1(P_1)$ and $T_2 = T_2(P_1, P_2)$.

Let $\bar{P}_1$ be a strictly positive value. We show that, if the policy rule is Ricardian, there exists a (unique) competitive equilibrium in which $P_1 = \bar{P}_1$. Given $\bar{P}_1$, the policy rule specifies a unique value for T_1 and R_1. We can substitute these values to obtain a unique value for the supply of bonds B_2 from the government budget constraint. Consumption and the demand for bonds can be uniquely determined by the market clearing conditions; these choices satisfy the household budget constraint in period 1 by Walras' law, as can be verified by substitution. The price level in the second period is determined by (4): even though the government cannot set the initial price level, it controls inflation through the choice of the nominal interest rate. If the policy rule is Ricardian, $T_2(P_1, P_2)$ is consistent with the period-2 budget constraint of the government; finally, the household budget constraint in period 2 is redundant because of Walras' law. QED.

Proposition 1 is the cashless counterpart to the well-known result that, in many monetary models, nominal interest-rate targeting leads to price indeterminacy.

While in a Ricardian regime the fiscal policy cannot help in determining the initial price level, the result obviously changes when we no longer require $T_2(P_1, P_2)$ to be such that the government budget constraint is met at all prices. The fiscal theory of the price level is most often derived by assuming that the government sets the real value of taxes T_1 and T_2 and the nominal interest rate R_1 *independently* of the prices.

Proposition 2 *Assume that the policy rule specifies unconditional values for T_1, T_2 and R_1. There exists at most one competitive equilibrium that is consistent with such a rule; the equilibrium exists provided T_1 or T_2 are sufficiently large.*

Proof. A competitive equilibrium must satisfy the same equations we listed in proposition (1). As before, we can uniquely determine consumption from the market clearing conditions. We can solve (3) and (4) as a system of 3 equations in P_1, P_2 and B_2, which yields the following unique result:

$$\begin{aligned} P_1 &= \frac{B_1}{(T_1 - G_1) + (T_2 - G_2)\frac{u'(C_2)}{u'(C_1)}} \\ P_2 &= \frac{B_1 R_1}{(T_1 - G_1)\frac{u'(C_1)}{u'(C_2)} + (T_2 - G_2)} \\ B_2 &= \frac{B_1 R_1 (T_2 - G_2)}{(T_1 - G_1)\frac{u'(C_1)}{u'(C_2)} + (T_2 - G_2)} \end{aligned} \tag{7}$$

This system yields positive prices P_1 and P_2 if T_1 or T_2 are large enough. Finally, market clearing implies that $B_2^d = B_2$, and the household's budget constraints are satisfied by Walras' law. QED.

The policy rule described in proposition 2 is consistent with a competitive equilibrium only if the initial real value of debt takes a particular value. This is the source of the fiscal theory of the price level: if taxes do not respond to meet the government budget constraint, then the price level must do so to guarantee that the real value of debt acts as the residual variable. Taxes must not be too low, for otherwise they would require a negative real value of debt, which is ruled out (assuming $B_1 > 0$) as prices must be positive.

The fiscal theory of the price level follows from the assumption that the policy rule in proposition 2 (or variants of it, as in Loyo [12], where the interest rate reacts to inflation) is a good description of the actual policy rule followed in many countries. Accordingly, the papers that advocate the fiscal theory view the price level as being primarily determined by the dynamics of government deficits (surpluses) and debt.

Both the papers that advocate the fiscal theory and those who deny its possibility or plausibility contain discussions of policy rules and often vague descriptions of out-of-equilibrium dynamics and adjustment to the equilibrium. However, all of these papers define an equilibrium as a competitive equilibrium, which is not a good concept to address the consequences of deviations from the equilibrium path.

To my knowledge, no paper has attempted to cast the problem in an environment in which it is possible to explicitly discuss the household and government behavior out of the equilibrium path. By writing the economy as a game, I am able to answer explicitly the following questions: is it possible for the government to commit to non-Ricardian policy rules? Can price determinacy be achieved through the fiscal policy when the monetary policy is characterized by interest-rate targeting? What actions lead to out-of-equilibrium prices, and what is the evolution of the economy out of equilibrium?

3 A Game-Theoretic Version of the Economy

In order to model the economy we described above as a game, we need to be explicit about the way prices are formed from the actions by the households and the government. In what follows, I model the market structure as a version of trading posts that is similar to Shubik [18].[6] While I make a number of assumptions on the details of how trading takes place, it is straightforward to show that these details could be changed without affecting the results. What can potentially make a difference is the main assumption that trading takes place simultaneously and through trading posts.[7]

The players of the game are households and the government. Every time a player wishes to trade, it has to submit a bid to a specialized trading post, which I will equivalently call a "market". Each market deals with pairs of goods or assets, and there is a market for any exchange that the government and the households may wish to entertain. Accordingly, in period 1 there are 3 trading posts: in the first, goods are exchanged for maturing bonds; in the second, goods are exchanged for newly issued bonds that mature in period 2; in the third, maturing bonds can be exchanged for newly issued bonds that mature in period 2. In period 2, the only trading post is one where goods are exchanged for maturing bonds.

As in Shubik [18], each household that wants to trade must submit an unconditional bid for the amount it wishes to sell on a given market. The bid must represent a quantity of the good (or bond) *sold*, rather than bought, because only in this way households can meet their binding obligation at any price. In equilibrium, households have perfect foresight about the relative price in each market, and a single household cannot alter any price through its actions. For this reason, households would be strictly indifferent between using unconditional bids or more-sophisticated bid schemes.

I assume that the government submits unconditional sale bids in all markets except the one where maturing bonds are exchanged for newly-issued bonds; to keep the analogy with the previous section in which the government targeted interest rates, I assume that the government sets the relative price of maturing vs. newly-issued nominal bonds (i.e., the nominal interest rate), and offers to trade any quantity the households desire. This bidding scheme is feasible for the government, which can produce nominal bonds in unlimited supply. Being a large player, the government could potentially have an interest in submitting more-complex bids. However, even for the government a bid is interpreted as a binding commitment, in and out of equilibrium; whatever bidding mechanism the government uses, I thus require it to be able to meet its obligations even when *many* households make an unexpected bid. That would still leave room for potentially complicated bids, in which perhaps rationing is sometimes involved; however, I show that the government can attain price determinacy even by using the simple bidding scheme proposed here.

Each trading post (except the one that determines the nominal interest rate) clears simply by setting the price equal to the ratio of the supply of the two objects to be exchanged; at that price, market clearing is achieved as an identity, independently of the bids, and exchange takes

[6] I assume enough symmetry that these trading rules yield the Walrasian outcome. As Shubik [18] points out, this is far from guaranteed in general. A more-complicated version with multilateral trading posts could overcome this problem.

[7] An alternative model of the microstructure of the determination of prices in a competitive equilibrium is provided by the search-theoretic approach developed by Rubinstein and Wolinsky [14] and Gale [8, 9]. However, this approach is considerably more cumbersome to deal with, and introducing a government in their environment would require significant adaptations that are currently beyond the scope of this project.

place.

As in the previous section, lower-case variables refer to single households and upper-case variables refer to aggregates.

The timing of the economy is as follows.

(i) Households start with 1 unit of the period-1 good and B_1 units of government debt maturing in period 1. The government levies a first installment of period-1 taxes, $T_1^1 \in [0, 1]$ and sets a price $P_{B_1 B_2}$ at which it stands ready to exchange maturing bonds for new bonds. From here on, I index prices by the objects that are being exchanged at each trading post. The government submits a sale bid for $C_1^{B_1}$ units of goods in the market for maturing bonds, subject to $C_1^{B_1} \leq T_1^1$. It also submits a sale bid for $B_2^{C_1}$ units of new bonds in exchange for goods. While we assume here that the government submits its bids first, nothing would change if we assumed that the bids are submitted jointly by the government and the households; this is true because we only look at commitment equilibria in which the government specifies its strategy ex ante.

(ii) Trading opens. There are bilateral trading posts for each possible exchange; in our case 3 exchanges are possible: goods for maturing government bonds, goods for new bonds issued by the government and maturing bonds for new bonds. Each household may submit a sale bid for $b_1^{C_1}$ units of bonds in the market for goods, and another sale bid for $b_1^{B_2}$ units of bonds in the market for new bonds maturing next period, subject to the constraint that $b_1^{C_1} + b_1^{B_2} \leq b_1 \equiv B_1$, i.e., the sale bids cannot exceed the total amount of bonds the household starts with. I use superscripts to indicate the object the player wishes to buy in each market: e.g., C_1 represents period-1 goods, B_2 represents bonds maturing in period 2. There is no point in distinguishing between lower- and upper-case on the superscript, as it only refers to the type of good, not the quantity; for this reason, I always use upper-case letters. Each household may also submit a sale bid of $c_1^{B_2}$ units of goods in exchange for new bonds, subject to the constraint that $c_1^{B_2} \leq 1 - T_1^1$.

(iii) For the markets in which the price is not set by the government, the ratio of the quantities of the unconditional bids sets the price and exchange takes place.[8] The government meets the demand of new bonds in the market in which it sets the price. We thus have

$$
\begin{aligned}
P_{C_1 B_1} &= \frac{B_1^{C_1}}{C_1^{B_1}} \\
P_{C_1 B_2} &= \frac{B_2^{C_1}}{C_1^{B_2}} \\
B_2^{B_1} &= B_1^{B_2} P_{B_1 B_2}
\end{aligned}
\tag{8}
$$

The relative price of goods and maturing bonds $P_{C_1 B_1}$ determines the value of the unit of account (the "dollar") for the cashless economy. For this reason, I interpret $P_{C_1 B_1}$ as the general level of prices; it thus corresponds to P_1 as defined in section 2. I explain below that

[8] Prices are not defined in markets to which no bid is submitted.

this may be different in a model in which there is money and I explain how the analysis will be generalized. $P_{B_1B_2}$ is the relative price of the unit of account in the two periods, i.e., it is the nominal interest rate in the economy, which we called R_1 in the previous section.

(iv) The government levies a second installment of taxes (or transfers) $T_1^2 \in [-T_1^1 + C_1^{B_1} - B_2^{C_1}P_{C_1B_2}, 1-T_1^1+C_1^{B_1}-C_1^{B_2}]$. The bounds ensure that the government has enough resources to carry out the transfer or the households have enough resources *in the aggregate* to meet the tax obligation. If an individual household bid more than the others, it might not have enough resources to meet the tax obligation at this stage. We assume that in this case the government taxes all of its available endowment, so the household will not be able to consume in period 1. Since we assumed Inada conditions on the utility function, we do not need to worry about households bidding their endowment strategically to escape taxation. The reason we allow a second installment in the first period is to give the government a way of reacting to an unexpected shortfall in resources raised through borrowing.

(v) Consumption and government spending take place. Each household consumes

$$c_1 = \max\{0, 1 - T_1 - c_1^{B_2} + \frac{b_1^{C_1}}{P_{C_1B_1}}\} \tag{9}$$

where $T_1 = T_1^1 + T_1^2$ and starts period 2 with $b_2 = b_1^{B_2}P_{B_1B_2} + c_1^{B_2}P_{C_1B_2}$ units of nominal bonds. The government spends

$$G_1 = T_1 + B_2^{C_1}P_{C_1B_2} - C_1^{B_1} \tag{10}$$

units in the first period.

(vi) Households start with 1 unit of the period-2 good. The government levies a lump-sum tax $T_2 \in [0, 1]$. In the second period, we do not distinguish between a first and a second installment in taxes, although we could do so. In the last period, the government cannot raise any resources by borrowing and hence cannot face an unexpected shortfall in its resources; as a consequence, distinguishing between a first and second installment is superfluous. The only market open in period 2 is the one where maturing bonds are traded for goods. The government submits a bid $C_2^{B_2} \leq T_2 - G_2$.

(vii) Each household submits a bid $b_2^{C_2} \leq b_2$.

(viii) The price is determined as before by the ratio of bids, i.e.

$$P_{C_2B_2} = \frac{B_2^{C_2}}{C_2^{B_2}} \tag{11}$$

(ix) Each household consumes

$$c_2 = 1 - T_2 + \frac{b_2^{C_2}}{P_{C_2B_2}} \tag{12}$$

The government spends

$$G_2 = T_2 - C_2^{B_2} \tag{13}$$

The household's preferences over the outcomes are described by (1). As for the government, the papers that address the fiscal theory of the price level do not model its preferences explicitly. In line with the exogenous policy they take, I look for strategies that let the government achieve an exogenous "target" level of taxes, which I normalize to $\bar{T}$ in both periods.[9]

Definition 1 A **competitive equilibrium** is an allocation

$$(C_1, C_2, T_1, T_2, B_2, B_1^{C_1}, B_1^{B_2}, C_1^{B_2}, B_2^{C_2}, C_1^{B_1}, B_2^{B_1}, B_2^{C_1}, C_2^{B_2})$$

and a price system

$$(P_{C_1B_1}, P_{B_1B_2}, P_{C_1B_2}, P_{C_2B_2})$$

such that:

(i) Given the price system and taxes (T_1, T_2), $(C_1, C_2, B_2, B_1^{C_1}, B_1^{B_2}, C_1^{B_2}, B_2^{C_2})$ solves the household maximization problem:

$$\begin{aligned} \max_{c_1,c_2,b_2,b_1^{C_1},b_1^{B_2},c_1^{B_2},b_2^{C_2}\in\mathbb{R}_+^7} \quad & u(c_1) + u(c_2) \text{ s.t.} \\ c_1 = 1 - T_1 + & \frac{b_1^{C_1}}{P_{C_1B_1}} - c_1^{B_1} \\ c_2 = 1 - T_2 + & \frac{b_2^{C_2}}{P_{C_2B_2}} \\ b_1^{C_1} + & b_1^{B_2} \le b_1 \\ b_2 = b_1^{B_2}P_{B_1B_2} + & c_1^{B_2}P_{C_1B_2} \\ & b_2^{C_2} \le b_2 \\ & c_1^{B_1} \le 1 - T_1 \end{aligned} \tag{14}$$

(ii) Markets clear and the government budget constraints hold, i.e. equations (8), (11), (10) and (13) are satisfied.

As usual, the definition of a competitive equilibrium only involves only the outcome of the game. The information a competitive equilibrium gives us is that each household would optimally choose the prescribed allocation if it expects everybody else to choose the same allocation, the government to follow the specified policy and the price system to be the one included in the definition. A competitive equilibrium does not convey any information on how the households or the government would react if people behaved differently. Compared with the definition of a

[9]The assumption of a constant target can be easily relaxed without affecting any of the results.

competitive equilibrium in section 2, the only difference is that we need here to specify the trade volume and the relative price in each market. The set of consumption levels (C_1, C_2), prices $(P_1 = P_{C_1B_1}, P_2 = P_{C_2B_2}, R_1 = P_{B_1B_2})$, government taxes (T_1, T_2) and period-2 bond holdings $B_2 = B_2^d$ compatible with a competitive equilibrium is the same under both definitions; the latter definition only specifies more details of how trading actually takes place within the market structure assumed here.

A household strategy is the following:

1. bids $(b_1^{C_1}, b_1^{B_2}, c_1^{B_2})$ as functions of the actions taken by the government up to that node, i.e. $(T_1^1, P_{B_1B_2}, C_1^{B_1}, B_2^{C_1})$;
2. a bid $b_2^{C_2}$ as a function of the government choices $(T_1^1, T_1^2, T_2, P_{B_1B_2}, C_1^{B_1}, B_2^{C_1}, C_2^{B_2})$, of the aggregate bids by the households in period 1 $(B_1^{C_1}, B_1^{B_2}, C_1^{B_2})$ and of its previous bids $(b_1^{C_1}, b_1^{B_2}, c_1^{B_2})$.

Consumption was not included, as it can be deducted mechanically from (9) and (12).

A government strategy is the following.

1. A tax T_1^1, bids $C_1^{B_1}$, $B_2^{C_1}$ and a price $P_{B_1B_2}$.
2. A tax T_2 as a function of the previous actions taken by the government $(T_1^1, P_{B_1B_2}, C_1^{B_1}, B_2^{C_1})$ and by households $(B_1^{C_1}, B_1^{B_2}, C_1^{B_2})$. The actions taken by each individual household are unobservable (except to the household itself); only their aggregates are common knowledge.

I dropped T_1^2 and $C_2^{B_2}$ from the definition of a government strategy: they are determined as a residual by (10) and (13).

I assume that the government can commit to a strategy before the game begins; time inconsistency is not an issue I am interested in, since government preferences are not explicitly modeled. In this paper, I am only studying whether there exists a strategy in the game that corresponds to the fiscal theory of the price level. Establishing whether such a strategy is part of a plausible equilibrium would require to model more completely the government preferences and is beyond the scope of this work.

In this setup, commitment means that there is an additional stage at the beginning of the game in which the government picks (commits to) the strategy it will follow throughout the game I described. This definition corresponds to the one in Schelling [17]. In a companion paper (Bassetto [1]), I discuss more in detail some issues relating to the existence of a subgame perfect equilibrium in the game with commitment, and I contrast the definition of a commitment equilibrium given here with that contained in Chari and Kehoe [3] and Stokey [20].

4 Ricardian and non-Ricardian Strategies in the Game

It is interesting to study two different cases. In the first case, government spending is identically zero; in this case, the target level of taxes always exceeds spending and there is never a need for the government to raise additional resources through borrowing.[10] Government debt exists

[10] This analysis could easily be extended to cases in which government spending is positive but below the target level of taxes in both periods.

in this case only as an initial condition, and is repaid using the revenues in excess of spending. In the second case, we maintain the assumption that $G_2 = 0$, but we assume that $G_1 > \bar{T}$: in the first period, the target level of taxes in insufficient to finance government spending, and the government needs to raise additional resources by borrowing. We do not consider the case in which $G_2 > \bar{T}$: this would only be possible if the government started with negative debt B_2, which we rule out.

I am interested in knowing when and whether the government can adhere to its target level of taxes both in and out of the equilibrium, and what are the "minimal" deviations that are needed if it is impossible to keep faith to the target.

4.1 No Government Spending

Proposition 3 *If $G_1 = G_2 = 0$, there exist government strategies in which taxes are $\bar{T}$ both in and out of equilibrium. If the government adopts any such strategy, there is a unique (symmetric) subgame perfect equilibrium in the subgame that follows the government commitment; the price level is thus uniquely determined. Furthermore, any such strategy achieves the same initial price level $P_{C_1B_1}$, whereas inflation and hence the price level $P_{C_2B_2}$ depends on the particular strategy.*

The complete proof is in the appendix; I describe here the outline and the intuition. The government strategy sets $T_1^1 = \bar{T}$, and the nominal interest rate $P_{B_1B_2}$ at any (strictly positive) level. The government bids the entire amount $C_1^{B_1} = \bar{T}$ in exchange for maturing bonds while it does not submit any bid on the market between goods and new bonds. In period 2, the government levies a tax $T_2 = \bar{T}$ and uses the revenues to bid $C_2^{B_2} = \bar{T}$ in exchange for bonds maturing in period 2. It can be immediately verified from the description of the game that these actions can be taken independently of the choices by the households, and that they deliver the target level of taxes independently of the household actions and hence both in and out of equilibrium.

With the given government strategy, there is a unique equilibrium, in which the unit of account (the "dollar") has a well-defined value. As in Cochrane [5], government debt in this example is essentially an entitlement to a future payoff and a "dollar" simply represents a share of the debt; in equilibrium, households will submit bids such that these shares are correctly priced as if they were any other asset.

We want next to establish whether the suggested government strategy is Ricardian. If we write the government budget constraint adapted from (3), we obtain

$$B_1 = \bar{T} P_{C_1B_1} + \frac{\bar{T} P_{C_2B_2}}{P_{B_1B_2}} \tag{15}$$

which only holds at the equilibrium price level. For prices that are out of equilibrium, (15) is violated, so the strategy is non-Ricardian according to the definition in section 2.

However, prices only deviate from the equilibrium values when households fail to make their equilibrium bids. There are two types of deviations: in the first type, households fail to redeem part of the debt. As an example, they bid less than B_2 in the second period, in which case $P_{C_2B_2}$ decreases and the present value of taxes seems to exceed the value of debt. This excess is only apparent, for it is the result of many households failing to claim their parts of repayments: if we only count debt that is presented for redemption, the government budget constraint holds. In the second type of deviation, households do not waste any of their debt, but they misallocate B_1 across the two markets, redeeming too many bonds and rolling over too few or vice versa. Substituting (8), it can be easily verified that (15) always holds for prices that follow this type of deviation; the strategy is "Ricardian" with respect to this type of deviations.

By studying the market structure behind a competitive equilibrium, we are able to see that the government is subject to budget constraints that must hold in and out of an equilibrium: equations (10) and (13). Equation (15) is instead not a true government budget constraint, because it assumes that all of the debt will be redeemed: this is a correct assumption on the equilibrium path, but may be violated out of equilibrium.

In section 2, we argued that a policy rule that satisfies (3) in and out of equilibrium is called Ricardian because the present value of taxes net of debt repayment is independent of the price level, which did not happen for non-Ricardian rules. However, in this example the validity of (15) out of equilibrium is not connected to the present value of taxes net of debt repayment; in fact, this present value *is* independent of the price level for the government strategy we analyze, but (15) may be violated because it assumes *all* debt has to be repaid.

4.2 Variable Government Spending

In the case discussed above, all of the debt is inherited from the past, and the government is only setting terms to repay it. We now look at the case in which $G_1 > \bar{T}$. In this case, the government would like to run a primary deficit in the first period. In the previous example, the government participated in the markets only by buying government debt, which would have otherwise been worthless to the households; in this example, the government needs to buy goods in the first period, and must thus persuade the households to trade resources that are intrinsically valuable to them. For the sake of simplicity, we retain the assumption that $G_2 = 0$.

While the government was able to meet its target in and out of equilibrium when spending was less than taxes in both periods, it is trivial to see that this is not possible when target spending exceeds the target level of taxes. No matter what the government strategy is, households have the option of not participating in the markets where goods are traded for future bonds. If households do not participate in this market, equation (10) implies $G_1 \leq T_1$. In this case, there is thus no government strategy that includes $T_1 = \bar{T}$ independently of the history of play. In the environment we study, any rule that unconditionally requires the government to set spending above taxes in any given period is meaningless.

The previous observation seems to defeat the fiscal theory of the price level. In all of the papers that I am aware of, an unconditional path for taxes and spending is assumed. Nonetheless, the following proposition rescues the fiscal theory by showing that the government can adopt a strategy that leads to a unique equilibrium in the game; in such an equilibrium, taxes are at the target level and the price level is uniquely determined by spending and taxes.

Proposition 4 *Assume that there exists a competitive equilibrium in which $T_1 = T_2 = \bar{T}$. Then the government can commit to a strategy such that the outcome of the unique equilibrium in the* *subgame following the commitment coincides with such a competitive equilibrium.*

The complete proof is contained in the appendix. I present here the outline and the intuition behind the result. Let

$$(\tilde{C}_1, \tilde{C}_2, \bar{T}, \bar{T}, \tilde{B}_2, \tilde{B}_1^{C_1}, \tilde{B}_1^{B_2}, \tilde{C}_1^{B_2}, \tilde{B}_2^{C_2}, \tilde{C}_1^{B_1}, \tilde{B}_2^{B_1}, \tilde{B}_2^{C_1}, \tilde{C}_2^{B_2}) \tag{16}$$

be the competitive equilibrium allocation and let the associated price system be

$$(\tilde{P}_{C_1B_1}, \tilde{P}_{B_1B_2}, \tilde{P}_{C_1B_2}, \tilde{P}_{C_2B_2}) \tag{17}$$

A government strategy that achieves the desired result is the following. In period 1, the government sets $T_1^1 = \bar{T}$. It bids $\tilde{C}_1^{B_1}$ units of goods in exchange for maturing bonds and $\tilde{B}_2^{C_1}$ units of new bonds in exchange for goods, and sets the nominal interest rate at $\tilde{P}_{B_1B_2}$. The second installment of taxes T_1^2 is set so that (10) holds; this installment depends thus on the household bid $C_1^{B_2}$. Independently of what happened in period 1, the government sets taxes at $\bar{T}$ and bids $\tilde{C}_2^{B_2} = \bar{T}$ in exchange for bonds maturing in period 2; it follows that $G_2 \equiv 0$.

The intuition behind this strategy is simple. The government cannot guarantee that borrowing will raise enough resources to cover the target level of spending. However, the government strategy offers new lenders a fixed amount of period-2 goods. As a consequence, when households lend less (more) than the desired amount to the government, the rate of return on the debt becomes automatically very (un)attractive, which rules out a second equilibrium with lower (higher) lending. The key aspect of this strategy is the ability of the government to separate the resources promised to new lenders from those reserved to previous lenders that roll over their debt. In period 1, the government is selling claims to its future surplus on two markets. By choosing $P_{B_1B_2}$ and $B_2^{C_1}$, it is controlling the share of that surplus that goes to either group of creditors. Ceteris paribus, a higher $P_{B_1B_2}$ and/or a lower $B_2^{C_1}$ implies a smaller share for new lenders and a larger share for previous debt holders, which leads people to lend fewer new resources to the government. In order to raise exactly the target level of revenues, the government must attain the appropriate mix of debt on the two markets. The initial price level is determined by the households bids in redeeming debt for goods in period 1. These bids in turn depend on the amount of goods the government is offering in period 1 and on the share of the future surplus that is offered to them through new bonds.

Once again, Cochrane's [5] analogy between the price of a government debt and the price of stock is very well suited for the microstructure I am introducing. However, this does not imply that the government budget constraint can be viewed simply as a "government valuation equation": out of equilibrium, the government is forced to raise taxes above its target level and it is the ability of adjusting its use of resources in a very specific way that leads to uniqueness of the equilibrium. Cochrane is correct in claiming that "no budget constraint forces Microsoft (or Amazon.com!) to adjust future earnings *to match current valuations*" (emphasis added), but overlooks the fact that Amazon.com would indeed have to adjust their earnings if their valuation and the households' willingness to subscribe their capital changed.[11] To the extent that the adjustment in their earnings *would not* match any alternative valuation, the uniqueness of the equilibrium valuation holds.

[11] Microsoft may be able to promise the same earnings independently of its valuation because it is quite possible

5 Extensions

5.1 Many periods

The extension of the results derived above to a multiperiod economy is straightforward. It is particularly interesting to extend the analysis to infinite-horizon economies.

The flow budget constraint of the government in an infinite-horizon economy becomes

$$P_t G_t = P_t T_t + \frac{B_{t+1}}{R_t} - B_t, \quad t = 1, 2, ... \tag{18}$$

Unlike in the finite-horizon case, the sequence of flow budget constraints does not imply that the intertemporal budget constraint is satisfied; for this to happen, the sequence of taxes and debt that is offered must also satisfy the transversality condition

$$\lim_{t\to\infty} B_t \prod_{s=1}^{t-1} \frac{1}{R_s} = 0 \tag{19}$$

Given a sequence of taxes and spending, it is now always possible to find a sequence of government debt that satisfies the flow budget constraint; it is formally no longer necessary for the government to offer bonds that mature after the end of the economy. Nonetheless, a generic sequence of taxes and spending and interest rates will imply a sequence of debt that violates the transversality condition, which is exactly the analogous of the condition $B_3 = 0$ in our two-period economy.

In an infinite-horizon economy, a policy rule is thus called Ricardian if it satisfies the transversality condition independently of the sequence of prices, and non-Ricardian otherwise.

Our game can be generalized to this case, provided preferences are adjusted to introduce a discount factor. The key step in generalizing the results we obtained so far is proving that the one-step principle can be applied to obtain a subgame-perfect equilibrium.

5.2 Money

The introduction of money is very important to compare the economy I present here to a standard monetarist model in which the price level is essentially determined by the quantity of nominal balances in the economy. However, the fiscal theory of the price level stems precisely from the failure of such models to deliver price determinacy in many instances. In particular, I have assumed throughout this paper that the "monetary authority" follows an interest rate peg. Such a policy typically leads to indeterminacy in both the nominal money supply and the price level.

Money plays an important role also in Buiter's [2] criticism of the fiscal theory. In his framework, a non-Ricardian policy is interpreted as a policy that defaults on part of the debt; as a consequence, debt trades at a discount over its nominal value. In our cashless economy, it

that all of its periods of negative cash-flow pertain to the past. In other words, Microsoft may just be similar to our first example, in which the government does not spend and is only repaying old debt. The budget constraint hits the company (and the government) when it needs to raise fresh resources, not while it is able to finance internally any investment.

is impossible for the debt to trade at a discount, as we defined the value of a dollar precisely in terms of debt; in order for this to be a possibility, it is necessary to introduce a second nominal asset (money) whose price relative to debt may not be fixed.

Money can be introduced in the game described above through a "cash-in-advance" technology that prevents some barter trading posts from opening. Household are divided into n symmetric groups, with n even, that lie on a circle. Each group i produces a good that cannot be bartered with the good at the "opposite extreme", i.e. $i \pm n/2$, so these trades require money. Because each group is still formed by a continuum of households, each household behaves as price taker. I now assume that each household likes to consume all n types of goods. Trading posts are open for all pairwise combinations of goods (except for the opposite extreme goods), for all goods vs. money, for goods vs. bonds and money vs. bonds. In this case, a "dollar" is the price of money, not national debt.

While this is work in progress, I conjecture that the price of a dollar of money and a dollar of debt will coincide identically only if the government explicitly pursues a policy that pegs the relative price; such a policy implies a commitment to monetization of the debt should households wish to get rid of it by selling it on the market rather than rolling it over.

The fiscal theory of the price level is unlikely to survive if accompanied by a money-supply rule, which is inconsistent with any monetization. However, an interest-rate peg is consistent with a peg of the relative price of money and debt, as the government can freely adjust the supply both of money and bonds; this suggests that a strategy similar to that described in section 4 may achieve price determinacy through appropriate management of debt.

6 Conclusion

While this research is unlikely to lay to rest the dispute on the validity of the fiscal theory of the price level, it shows how the question can at least be cast in a more complete model in which the definition of an equilibrium is not controversial.

In this paper, I show that the usual version of the government budget constraint is not adequate to describe the restrictions on the government policy out of equilibrium. Nonetheless, the government does face budget constraints on its actions even out of equilibrium; the policy rules postulated by proponents of the fiscal theory violate these constraints and are thus misspecified.

I rescue the fiscal theory by displaying a strategy in which the fiscal side of the economy determines the price level in an environment in which the traditional monetarist analysis would imply indeterminacy. This strategy is very much in the spirit of the fiscal theory of the price level: the government guarantees a stream of real payments to the current holders of debt independently of the current or future price level.

A Proof of proposition 3

I solve the household's problem backwards.

When submitting its bid in period 2, each household inherits as a given its previous consumption c_1 and its level of nominal bonds b_2. At this stage, the household can only choose how much of b_2 to bid in exchange for additional period-2 goods; the price it expects on that market is given by (11), which is a strictly positive number and is independent of its bid (assuming $B_2 > 0$). The household will thus bid all of its b_2 bonds and consume $c_2 = 1 - T_2 + \frac{b_2^{C_2}}{P_{C_2B_2}}$.

In period 1, the household has to submit 3 bids. Given that the government does not offer new bonds in exchange for goods, the household expects a price $P_{C_1B_2} = 0$ if it submits a bid on that market, so no bids will be submitted. The household is thus left with the problem to allocate the initial amount of bonds b_1 between the bid for new bonds and that for goods. From the perspective of an individual household, each unit bid for goods yields $1/P_{C_1B_1}$ units of the consumption good, and each unit bid for new bonds yields $P_{B_1B_2}$ units of new bonds. While $P_{C_1B_1}$ is not known to the household ex ante, in equilibrium the household has perfect foresight about it.[12] The household also knows that each unit of new bonds will fetch $1/P_{C_2B_2}$ units of period-2 goods. Its problem becomes thus exactly (14). The mechanism I designed corresponds to a Walrasian economy from the perspective of each household: each household is simply taking prices as given and maximizing by allocating its resources.[13] While mathematically the problem is identical, conceptually a household faces a more-complex problem in the economy I consider: it has to form beliefs not only about future prices, as in a dynamic Walrasian equilibrium, but also about current prices, which are determined only after the bid has been submitted.

The first-order condition for household bids at an interior yields:[14]

$$u'(c_1) = \frac{P_{B_1B_2} P_{C_1B_1} u'(c_2)}{P_{C_2B_2}} \tag{20}$$

which is the standard Euler equation, together with $B_1^{C_1} + B_1^{B_2} = B_1$.

An equilibrium in the subgame in which the government strategy is specified, as above, by $T_1 = \bar{T}$, $C_1^{B_1} = \bar{T}$, $B_2^{C_1} = 0$, $B_{B_1}^2 = \bar{B}$, $T_2 \equiv \bar{T}$, $C_2^{B_2} \equiv \bar{T}$ is characterized as follows. From the government strategy, $\frac{B_2^{C_2}}{P_{C_2B_2}} = \bar{T}$ after any history. From the government strategy, (11) and (12) we obtain $C_2 = 1$ independently of the household bids. Notice that this is a result on C_2, which is average consumption; in principle, each household could consume more or less than 1. Similarly, the government strategy, (8) and (9) imply $C_1 = 1$ independently of the history. Using $C_1 = C_2 = 1$, we see from (20) that inflation is equal to the nominal interest rate chosen by the government. This is because consumption is constant and there is no discount factor, so the real interest rate must be 0.

We can solve for the bids and the initial price using (8), (11), $B_1^{C_1} + B_1^{B_2} = B_1$ and $B_2 = B_1^{B_2} P_{B_1B_2}$, from which we obtain $B_1^{C_1} = B_1^{B_2} = 1/2$. The initial equilibrium price is $P_{C_1B_1} =$

[12] There is no uncertainty because the government is not playing mixed strategies, and the households' choices are uncorrelated (even if we assumed they were playing mixed strategies, which I do not).

[13] There is no market for private debt, which makes households borrowing constrained; this is irrelevant in my setup with identical households.

[14] In equilibrium, households must be choosing an interior point when allocating maturing bonds to the 2 markets. If this were not the case, there would be one period in which goods are offered in exchange for maturing bonds, but no bonds are redeemed; it would then be enough to bid an arbitrarily small amount to obtain the goods essentially for free.

$\frac{B_1}{2\bar{T}}$: it is uniquely determined and is independent of the nominal interest rate chosen by the government. QED.

B Proof of proposition 4

Let

$$(\tilde{C}_1, \tilde{C}_2, \bar{T}, \bar{T}, \tilde{B}_2, \tilde{B}_1^{C_1}, \tilde{B}_1^{B_2}, \tilde{C}_1^{B_2}, \tilde{B}_2^{C_2}, \tilde{C}_1^{B_1}, \tilde{B}_2^{B_1}, \tilde{B}_2^{C_1}, \tilde{C}_2^{B_2})$$

be the competitive equilibrium allocation and let the associated price system be

$$(\tilde{P}_{C_1B_1}, \tilde{P}_{B_1B_2}, \tilde{P}_{C_1B_2}, \tilde{P}_{C_2B_2})$$

We prove the proposition for the case in which the government participates in all markets: $(\tilde{C}_1^{B_1}, \tilde{B}_2^{C_1}) \gg 0$. The proof of the other cases is analogous, except that prices are not defined in the markets in which the government does not participate; in those markets, household (correctly) expect any bid they submit to be wasted, and hence in equilibrium they would not submit bids.

Consider the following government strategy. In period 1, the government sets $T_1^1 = \bar{T}$. It bids $\tilde{C}_1^{B_1}$ units of goods in exchange for maturing bonds and $\tilde{B}_2^{C_1}$ units of new bonds in exchange for goods, and sets the nominal interest rate at $\tilde{P}_{B_1B_2}$. The second installment of taxes T_1^2 is set so that (10) holds; this installment depends thus on the household bid $C_1^{B_2}$. Independently of what happened in period 1, the government sets taxes at $\bar{T}$ and bids $\tilde{C}_2^{B_2} = \bar{T}$ in exchange for bonds maturing in period 2; it follows that $G_2 \equiv 0$.

We now look at the household response if the government commits to the strategy above. In period 2, households will bid all of their maturing bonds against goods, independently of the previous history, so for each household $b_2^{C_2} = b_2$ and in the aggregate $B_2^{C_2} = B_2$, independently of the previous history. In a competitive equilibrium in which $G_1 > T_1$ it is necessarily the case that $B_2 > 0$ and $T_2 > 0$, so we know $\tilde{B}_2 > 0$ and hence $\tilde{P}_{C_2B_2} \in (0, +\infty)$.

Each household has beliefs about the bids that will be submitted by the others, and uses (8) and (11) to get a belief about the prices that will arise in each trading post. Given its beliefs about prices, the household solves (14). In a symmetric equilibrium, the solution to (14) must coincide with the belief that the household has about the behavior of other households.

In a symmetric equilibrium, the bids submitted by the households can be derived from the following requirements.

(i) First-order conditions for (14):

$$\frac{u'(C_1)}{P_{C_1B_1}} = u'\Big(1 - \bar{T} + B_2 P_{C_2B_2}\Big)\frac{P_{B_1B_2}}{P_{C_2B_2}} + \mu, \qquad (21)$$
$$\mu \geq 0 \text{ if } B_1^{C_1} > 0, \quad \mu \leq 0 \text{ if } B_1^{C_1} < B_1$$

$$u'(C_1) = u'\Big(1 - \bar{T} + B_2 P_{C_2B_2}\Big)\frac{P_{C_1B_2}}{P_{C_2B_2}} + \nu, \qquad (22)$$
$$\nu \geq 0 \text{ if } C_1^{B_2} < 1 - T_1, \quad \nu \leq 0 \text{ if } C_1^{B_2} > 0$$

$$
\begin{aligned}
C_1 = 1 - T_1 + \frac{B_1^{C_1}}{P_{C_1 B_1}} - C_1^{B_2} \\
B_1^{C_1} + B_1^{B_2} = B_1 \\
B_2 = B_1^{B_2} P_{B_1 B_2} + C_1^{B_2} P_{C_1 B_2}
\end{aligned}
\tag{23}
$$

where μ and ν are Kuhn-Tucker multipliers;

(ii) Equations (8) and (11), which describe the price formation at the trading posts.

(iii) The decisions to which the government is committed:

$$
\begin{aligned}
&P_{B_1 B_2} = \tilde{P}_{B_1 B_2} \\
&B_2^{C_1} = \tilde{B}_2^{C_1} \\
&C_1^{B_2} = \tilde{C}_1^{B_2} \\
&C_2^{B_2} = \tilde{C}_2^{B_2} = \bar{T}
\end{aligned}
\tag{24}
$$

(iv) The government budget constraint

$$
T_1 = T_1^1 + T_1^2 = G_1 - C_1^{B_1} + C_1^{B_2} \tag{25}
$$

The allocation and price system in (16) and (17) form a competitive equilibrium, which implies that equations (21), (22), (23), (8), (11), (24) and (25) must hold. The competitive equilibrium we are considering is thus an equilibrium outcome of the subgame in which the government committed to the strategy above. The household strategy in this equilibrium calls for bidding $\tilde{B}_1^{B_2}$, $\tilde{B}_1^{C_1}$ and $\tilde{C}_1^{B_2}$ in the first period, and bidding all of the period 2 bonds in the second period independently of the previous history.

We next need to prove that this is the unique symmetric equilibrium.

Notice that, in an equilibrium, we must have $\mu \leq 0$ and $\nu \leq 0$. If μ were greater than 0, equation (21) implies that households would not be bidding maturing bonds in exchange for goods. In this case, a single household could capture the entire government bid of goods by submitting an arbitrarily small bid on the market shunned by all others: it would face an arbitrarily favorable price on that market, which would contradict the optimality of not submitting a bid. Similarly, we have $\nu \leq 0$: since the government is offering new bonds in exchange for goods, households must be submitting strictly positive bids on that market.

There are thus four cases, depending on whether either constraint is binding. In all four cases, repeated substitution shows that there exists a unique solution to the system of equations (21), (22), (23), (8), (11), (24) and (25), which yields the desired result. QED.

It is worth noticing that, in the more natural case in which $\mu = 0$ and $\nu = 0$, (21) and (22) imply

$$
P_{B_1 B_2} = P_{C_1 B_2} / P_{C_1 B_1} \tag{26}
$$

This relationship stems from the fact that, from the perspective of a single household, this economy has redundant markets. The same consumption vector can be achieved either by rolling some debt over or by redeeming it for goods while at the same time purchasing new bonds with goods. In equilibrium, a household must be indifferent between the two strategies in order to participate in all markets, and this links the prices on the 3 markets that are open in period 1.[15]

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