Investment dynamics with fixed capital adjustment cost and capital market imperfections

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ABSTRACT

This paper analyzes a model of investment with fixed investment costs and capital market imperfections. In this model finance influences the level of capital firms hold, as well as the frequency at which they invest. In consequence investment reacts nonlinearly with respect to shocks to productivity and liquidity. Liquidity and productivity shocks are complements and the influence of finance is strongest if a firm wishes to significantly adjust capital for fundamental reasons.

This theoretical model is confronted with UK company data in a two-step estimation that first identifies the long-run relationship of productivity, capital and finance. Here we find no significant influence of finance on the capital decision of a firm. However, when the short-run investment function is estimated, finance has a significant impact, which is also strongest for strong fundamental investment incentives. Moreover, the investment function is strongly convex in the fundamentals themselves, indicating fixed costs of capital adjustment.

JEL classification: E22; E44; G31; C32; C33

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1 Introduction

Economists' knowledge of micro-level and aggregate investment is still far from being conclusive. Seemingly well established however is the view that the workhorse of the neoclassical investment model, the q-model of investment, has a hard time explaining empirically observed patterns of investment.[1] The question of which assumption of the neoclassical model leads to its failure to what extent is yet to be answered.

Beginning with Fazzari et al. (1988) the empirical literature has emphasized the role of financial factors in firm-level investment. More recently attention has been drawn to the role of non-convexities in the investment technology. This paper aims at merging both of these strands and shows that financial factors and non-convexities are both simultaneously important since each significantly influences the effect of the other.

This interaction has not been analyzed much, but very recently a few contributions have drawn attention to the issue: Holt (2003) provides a theoretical real options model of irreversible investment that shows how financial frictions and irreversibility of investment interact as complements, Whited (2004) provides evidence that firms which are identified as financially constrained exhibit investment spikes much less frequently, and Caggese (2003) develops a formal test for financial constraints based on the irreversibility of fixed investment.

Our approach differs in both methodology and focus from these studies: With non-convex adjustment costs, firms invest infrequently and lump their investment projects. This opens two ways for finance to affect investment: First, finance can alter the target level of capital to which a company adjusts and secondly, it can influence the frequency at which investment projects are carried out.

When we asses empirically which of the two channels is more important, finance shows at best a minor influence on the capital levels that companies hold. By contrast however, finance has a significant influence on investment. In consequence, finance has only an intertemporal substitution effect, that is more liquidity speeds up investment.

This result itself is already informative in finding out about the actual form of frictions involved. For example, one would expect the relative strength of both effects (level vs. frequency) to be just the reverse in a model in which the main influence of finance comes via the cost of capital and convex adjustment cost lead to partial adjustment of the current stock of capital to its target level every period. In such model a change in finance translates into a change in the target level of capital to which actual capital smoothly adjusts over time, which marks a clear difference to our model of interacting

[1] See Caballero (2000).

frictions.

While this interaction between finance and non-convex adjustment costs has not been much analyzed, non-convexities themselves–such as irreversibility or fixed costs of investment and other economies of scale–have been discussed widely and have been theoretically analyzed in a very general framework by Abel and Eberly (1994). Empirical evidence for non-convexities is mostly drawn from the Longitudinal Research Database (LRD).

Doms and Dunne (1998), for example, report that–at the plant-level–a small fraction of investment *activities* is associated with an overwhelmingly large fraction of changes in the capital stock. Cooper et al. (1999) use the LRD to estimate a hazard model of investment. They find a time-increasing investment hazard and thus evidence for non-convexities. A more direct approach has been taken by Caballero et al. (1995). They estimate "mandated investment" by imposing a long-run relation between earnings, capital employed and the cost of capital. Explaining actual investment by mandated investment in a second step, they empirically find the convex relationship predicted by the non-convex adjustment cost model of investment.

Additional evidence for non-convexities has also been drawn from other data than the LRD.[2] For instance, Caballero and Engel (1999) estimate a model of aggregate investment dynamics that rests on a microeconomic model which features stochastic non-convex adjustment costs. On the basis of 2-digit industry level panel data they find significant fixed costs of investment and obtain a better fit with their structural model than with partial adjustment (convex cost) alternative models.

Besides this strong emphasis that the investment literature has put on the role of non-convexities in investment decisions, it also has questioned some of the earlier papers on financial factors in investment.[3] A whole series of papers has elaborated problems of measurement errors and biased estimators that arise in q-theoretical regressions and result in spuriously positive estimates of financial variables.[4] Most disappointing for the financial factors in investment literature are the results of Gomes (2001): In a pecking order of finance framework he shows that the presence of financial frictions is neither

[2]Abel and Eberly (2002) employ the Compustat data in a q-theoretic framework - their (1994) augmented adjustment cost model - and find evidence for non-convexities in the capital adjustment technology, especially for fixed costs. Goolsbee and Gross (2000) use micro-level airline industry data within the framework of Caballero et al. (1995) and find evidence for a convex investment-function. However, this evidence vanishes by aggregation - even on the airline (company) level there is nearly no evidence for non-convex adjustment costs.

[3]For a very exhaustive survey over the literature on capital market imperfections and investment see Hubbard (1998).

[4]See for instance Kaplan and Zingales (1997), Cummins et al. (1999), Erickson and Whited (2000) or Gomes (2001)

sufficient nor necessary to obtain a (seemingly) significant *positive coefficient* on *cash flow* in a q-theoretic investment regression.[5]

Despite Gomes' (2001) strong scepticism against including *cash-flow in q-regressions* of investment to pick up financial frictions, Gomes' (2001, p. 1279) contribution itself arguably calls for the inclusion of other measures of the financial status of a company in an investment regression with non-convex adjustment costs. He points out in particular that the investment behavior corresponding to a pecking order of finance model "[...] is somewhat similar to those used in the investment with fixed cost literature" when not controlling for the financial status.

Obviously this makes a departure from the standard "Tobin's q plus cash-flow" model of investment necessary. Because of that, we modify the Caballero and Engel (1999) model, such that it captures most prominent specifications of financial frictions: wealth dependent cost of capital, credit rationing and the absence of external equity finance. From this model, we infer that the (expected) investment of a company is a function of only two variables: the firm's mandated investment and the firm's equity ratio, which is the book value of a firm's equity over the book value of its assets.[6]

In particular, our merged model predicts that finance influences the frequency of investment; that this influence is strong especially when there are strong fundamental incentives to adjust the stock of capital; and that investment is a convex function in fundamental investment incentives–like in the pure non-convex adjustment cost model.

The departure from the q plus cash-flow framework is also partly motivated by the fact that other non-q models, which are thus not subject to Gomes' criticism, have been able to provide evidence for financial frictions.[7]

Moreover, with the direct effect of finance on the frequency of investment in the combined model, it is also able to tackle another puzzle apparent in the investment literature: As a fact also evident in the sample of UK firms used in the present paper,

[5] The explanation for this can be summarized in the following way: On the one hand, average q will measure marginal q with errors, which generates a significant cash flow coefficient even without any financial frictions. On the other, true marginal q already (partly) measures the impact of financial frictions–if such impact exist indeed.

[6] This result reflects that from a general theoretical angle a measure for the line-of-credit is more likely to be the appropriate indicator of financial constraints to investment than cash-flow is. A fact that was pointed out early in the investment literature by Blinder (1988, p. 199). In particular if agency or other informational market imperfections link discount factors to the availability of internal funds and hence generate an investment liquidity correlation, the correct measure should be a stock- and not a flow-item. The result also has an empirical advantage: the pre-determined equity ratio (taken from the opening balance of a company) is by contrast to cash-flow unlikely to strongly correlate with current productivity shocks that influence the investment decision.

[7] See for instance Whited (1992), Whited (2004), Bond and Meghir (1994), or Gilchrist and Himmelberg (1998).

liquidity affects investment in the short run but it affects capital much less in the long run. In particular when investment is estimated as an error-correction-process, evidence for this has been reported but at the same time this has hardly attracted attention.[8] An extreme example is Guariglia (1999, pp. 47) who reports that firm size and stock based liquidity proxies are empirically independent while simultaneously liquidity influences investment significantly. The investment-frequency effect of liquidity in our model is able to explain such observations.

Having analyzed the model theoretically, we then asses it empirically in an analysis that draws on the ideas developed in Caballero et al. (1995). Infrequent investment establishes a cointegration relation between the static optimal target level of capital and the actual capital a company employs. This allows to recover the gap between actual and desired level of capital from an estimation of a cointegrating vector between capital, total factor productivity and the equity ratio. However, to minimize the influence of measurement errors in this estimation, we deviate from Caballero et al.'s (1995) procedure and combine their direct method with the idea of Cooper and Haltiwanger (2002) to measure productivity indirectly. With our sample of UK companies, the Cambridge DTI database, this allows to generate three preliminary measures of productivity from which we then infer a final estimate of productivity as the common factor imbedded in all three of these.

This common factor is non-stationary and cointegrated with the level of capital a company employs. By contrast, finance has no influence on the level of capital employed in the long-run capital. The cointegration error of this long-run regression identifies the amount of mandated investment, which is the fundamental investment incentive. Having generated an estimate of mandated investment, the investment function is estimated non-parametricly as a function of the equity ratio and mandated investment. This second estimation shows that investment is a non-linear function of both finance and fundamentals. It is convex in fundamentals, it is significantly influenced by finance, and–in line with the theoretical prediction–this influence is the stronger, the stronger fundamental incentives are.

The remainder of this paper is organized as follows: Section 2 develops a model that describes firm level investment under the assumption of capital market imperfections and fixed costs of investment. Within this section, Section 2.1 describes the company's choice problem whereas 2.2 discusses the properties of the investment function. Section 3 presents empirical evidence for the model and draws on firm level investment data

[8] See e.g. Hubbard (1998) or Mairesse et al. (1999) for an overview of the empirical literature on firm-level investment and the time structure of liquidity effects.

from the Cambridge DTI database. Section 4 concludes and an appendix follows.

2 A theoretical model

2.1 Firm-level investment

We begin by presenting and discussing the representative problem of a firm which is at the same time subject to financial constraints and fixed adjustment costs. For simplicity and as in Caballero and Engel (1999), this firm is a monopolisticly competitive single-plant and single-product firm. The investment decision is modelled in discrete time and a firm faces an infinitely elastic supply of all factors. At the beginning of each period all uncertainty about that period is resolved and is common knowledge from then on. Thereafter, each firm decides upon investment.

2.1.1 Adjustment technology and financial constraint

If firms want to change their capital stock they have to pay some fixed costs; all other factors may be adjusted without cost. At the end of every period each firm has to pay back its last period's debt plus interest, has to pay for any new purchased capital goods and for all other factors. Moreover, firms can issue new debt and pay out dividends.

Besides a non-convexity in the adjustment cost, firms face a capital market imperfection: As in Gilchrist and Himmelberg (1998) and as an extremely simplified version of the pecking order theory of finance, there will be a no-new-equity constraint first of all, i.e. firms cannot issue new shares or to have negative dividends.

> Assumption 1: Once founded, firms are unable to issue new equity. In particular, dividend payments D_t must be non-negative at any point in time: $\forall t : D_t \geq 0$.

This simplified version of the pecking order theory is necessary to keep the model tractable.[9] Nevertheless, the general results should not change if this assumption were replaced by a more complex version of financial transaction costs as in Gomes (2001).

Moreover, this assumption does not contradict empirical findings. For example Friedman (1982) shows empirically that firms hardly use any external equity finance at all. Additionally, this assumption is theoretically supported by Fries et al. (1997), who show that full collataralization- and "no new equity"-constraints may theoretically arise as an industry equilibrium.

Secondly, we assume a collateral constraint, i.e. the amount of debt a firm may issue is limited by the actual stock of capital.

[9] Although it greatly simplifies, this assumption is actually taken for the sake of tractability by many papers with a structrural model for financing constraints, see for examples Whited (1992), Gilchrist and Himmelberg (1998), or Bayraktar et al. (2003).

Assumption 2: The maximum fraction of capital that can be pledged for debt shall be $\hat{b}$. Moreover, $\hat{b} \leq \sup \left\{ b \in \mathbb{R}_{++} | \, b \leq \frac{1-\delta}{1+r(b)} \right\}$, where δ is the depreciation rate and r is the interest rate that might depend on b.

This assumption can be viewed as a version of Hart and Moore's (1994) debt capacity model. The qualification ensures that the company has at no time more outstanding debt obligations than it holds assets. This ensures that the firm always has a positive book value of equity and will not have to declare bankruptcy.

The third, last, and weakest assumption regarding capital market imperfections is that the interest rate on debt r only depends on the financial leverage $\frac{B}{K}$. As Gilchrist and Himmelberg (1998), we assume r to be homogenous of degree zero in B and K, and to be weakly increasing in B. This does not rule out r to be independent of B and K.

Assumption 3: The interest rate is a differentiable function of $\frac{B_{t+1}}{K_t}$ with $r = r\left(\frac{B_{t+1}}{K_t}\right) \geq 0$ and $r'\left(\frac{B_{t+1}}{K_t}\right) \geq 0$.

2.1.2 Periodic sales

Now let K^* denote the frictionless stock of capital of a firm, which is the stock of capital that would be chosen in the absence of fixed costs of investment and capital market imperfections. Let K be the actual capital employed. The firm generates earnings by employing capital and the perfectly flexible adjustable factors. Optimizing over these factors yields a semi-reduced function of earnings per-period (EBIT), Π,

This function is linear homogenous in the frictionless stock of capital K^* and can be written as:[10]

$$\Pi(z, K^*) = \pi(z)K^*. \tag{1}$$

In this function z denotes the capital imbalance, which is the ratio of actual capital employed to frictionless capital: $z := \frac{K}{K^*}$. Henceforth, z typically refers to the capital imbalance at the beginning of each period and before investment takes place, while z^o refers to the target level of the capital imbalance, which is the imbalance after investment. The earnings function π itself is strictly concave and fulfills the (Inada) conditions $\pi(0) = 0$ and $\lim_{z\to 0}\pi'(z) = +\infty$. To ensure that profits are bounded, we denote the discount factor by ψ and the depreciation rate by δ and assume $\lim_{z\to+\infty} \pi'(z) < \psi\delta$.

To model the non-convex costs we assume that when a firm invests, production halts. The duration of this stop is determined by the random variable w which represents the

[10] This assumption of linear homogeneity just as the other assumptions with respect to π are for example fulfilled if demand is iso-elastic and the production function is Cobb-Douglas. See Caballero and Engel (1999) for details.

time used for the installation of the new capital, which is analogous to the adjustment costs assumption in Caballero and Engel (1999). Therefore, the costs, A, of adjusting the capital stock are given by:

$$A(z^o, K^*, w) := w\pi(z^o)K^*, \tag{2}$$

where z^o denotes the capital-imbalance after adjustment, such that $z_t^o := \frac{K_t + Investment_t}{K_t^*}$. Note that in the presence of depreciation or a positive trend in productivity the firm will typically invest up to a larger stock of capital than the frictionless optimal one, K_t^*, to keep the distance between desired and actual stock of capital small on average.

2.1.3 Dynamics of the stochastic variables

So far we have put no restrictions on the stochastic dynamics of the random variables K_t^* and w_t. Both variables together completely determine firm heterogeneity and investment dynamics, so that any assumptions on these variables are crucial. A minimal assumption for keeping the model tractable is that both variables exhibit the Markov property. Furthermore, we shall assume that the stop in production can be at most for one entire period, so that $w_t \in]0;1[$. Additionally w_t shall be i.i.d., and K_t^* follows a geometric random walk (with drift), whose innovations ξ_t are normally distributed and serially uncorrelated (besides a possible drift), yet they can be correlated across firms:

$$\frac{K_t^*}{K_{t-1}^*} = \exp(\xi_t). \tag{3}$$

2.1.4 Capital market and the firm's objective

Although firms are hence situated in a risky environment, they are assumed to be risk-neutral. They seek to maximize the expected, discounted dividend stream and do so by choosing some capital imbalance z^o (the amount of capital employed, $z^o K^*$) and the amount of debt used to finance production, B_{t+1}.

In order to finance investment, a firm can either cut back dividend payments D_t or raise debt B_{t+1}. As assumed, a firm is unable to sell any new shares or raise equity by negative dividends (assumption 1). Moreover, the amount of debt a firm can issue is limited by the actual stock of capital employed (assumption 2). Additionally, the interest rate is a function of the debt ratio $b_t := \frac{B_t}{K_{t-1}}$ and is weakly increasing in b_t

(assumption 3).[11] Therefore, dividend payments, D_t, are given by

$$\begin{aligned} D_t &= D(z_t^o, B_{t+1}, K_t^*, w_t, z_t, B_t) \\ &:= \Pi(z_t^o, K_t^*) - A(z_t^o, K_t^*, w_t)\mathsf{I}_{\{z_t^o \neq z_t\}} - K_t^*(z_t^o - z_t) + B_{t+1} - (1 + r(b_t))B_t \end{aligned} \tag{4}$$

in which I is an indicator function for investment activity. If the time invariant discount factor is denoted by ψ and the value of the firm is denoted by V, then the following Bellman equation determines both firm value V and the optimal investment policy:

$$\begin{aligned} &V(K_t^*, w_t, z_t, B_t) = \\ &\max_{(z_t^o, B_{t+1}) \in X} \left\{ D(z_t^o, B_{t+1}, K_t^*, w_t, z_t, B_t) + \psi \mathsf{E}_t V\left(K_{t+1}^*, w_{t+1}, z_{t+1}\left(z_t^o, \xi_{t+1}\right), B_{t+1}\right) \right\}. \end{aligned} \tag{5}$$

In this expression $X := X\left(K_t^*, w_t, z_t, B_t\right)$ is the correspondence of financially feasible capital-imbalance and debt pairs. E_t denotes the expectations operator, conditional on information available at time t.

To simplify notation, let the ratio of the book value of equity over the book value of capital be denoted by e_t. This "equity ratio" is a function of last period's debt ratio $b_t = \frac{B_t}{K_{t-1}}$, and is given by $e_t := e(b_t) = 1 - \frac{(1+r(b_t))}{1-\delta} b_t$. To reduce the number of state variables and to obtain a more convenient formulation of the problem at hand, we subtract the book value of equity from firm value V and divide by K_t^*. This defines a new value function $v := \frac{V}{K_t^*} - e_t z_t$. This function represents the difference between the economic value of a firm's equity and the equity's book value (relative to its optimal stock of capital K^*). As both the equity ratio e_t and frictionless capital K_t^* are determined before the optimal policy decision is taken, maximizing v and maximizing V yield the same optimal policy.

To also replace the set of alternatives X by a set that only includes relative variables, we define the correspondence Y that gives financially feasible plans in terms of z^o and

[11] The interst rate for bonds increases by assumption in our model, and is not derived endogenously. In fact, as we have explicitely ruled out bankruptcies, debt is even risk-free. Hence, this assumption is strictly speaking inconsistent with the model. Yet, to rule out risky debt is only to simplify and focus the analysis. Introducing another risk term that enters after investment decisions are made and adding bankruptcy costs for the debt holders would generate an upward sloping interest function. However, this would also complicate the analysis substantially.

b^o for capital-imbalance and debt pairs ("plans") with strictly positive capital: [12]

$$Y(z,e,w) := \left\{ (z^o, b^o) \in \mathsf{R}_{++} \times \mathsf{R}_+ | 1 - e\frac{z}{z^o} - \frac{\pi(z^o)}{z^o}[1 - w\mathsf{I}_{\{z^o \neq z\}}] \leq b^o \leq \hat{b} \right\}. \quad (6)$$

The first inequality in this definition of Y represents the positive-dividend constraint and the second inequality reflects the debt-ceiling $\hat{b}$, the maximum debt-to-capital ratio. After some tedious calculations,[13] we obtain for v

$$\begin{aligned} v(z,e,w) := \max_{(z^o,b^o)\in Y\cup(0,0)} & \pi\left(z^o\right) + \left[(1-\delta)\psi e\left(b^o\right) - (1-b^o)\right] z^o - \pi(z^o) w \mathsf{I}_{\{z^o \neq z\}} \\ & \psi \mathsf{E}_t \left\{ v\left(z^o \frac{1-\delta}{\exp(\xi_{t+1})}, e\left(b^o\right), w_{t+1}\right) \exp(\xi_{t+1}) \right\}. \end{aligned} \quad (7)$$

From this equation, we see that value v is composed of four elements, all normalized by K^* : current cash-flow $\pi\left(z^o\right)$, the difference between the discounted book value of equity at the beginning of the next period $(1-\delta)\psi z_t^o e\left(b^o\right)$ and its current value $\left(1-b^o\right) z^o$, adjustment cost and finally the expected value of v discounted for one period.

To simplify notation further, we now define an auxiliary value function $\tilde{v}$. This function equals the expression in (7) without adjustment costs. Hence, $\tilde{v}(z,b)$ is defined by

$$\begin{aligned} \tilde{v}(z,b) := & \pi\left(z\right) + \left[(1-\delta)\psi e\left(b\right) - (1-b)\right] z \\ & + \psi \mathsf{E}_t \left\{ v\left(\frac{z(1-\delta)}{\exp(\xi_{t+1})}, e\left(b\right), w_{t+1}\right) \exp(\xi_{t+1}) \right\}. \end{aligned} \quad (8)$$

In turn, firm value v can now be expressed relatively simple in terms of $\tilde{v}$ as

$$v\left(z,e,w\right) = \max_{(z^o,b^o)\in Y(z,e,w)\cup(0,0)} \tilde{v}(z^o,b^o) - \pi(z^o) w \mathsf{I}_{\{z^o \neq z\}}. \quad (9)$$

However, the inclusion of state $(0,0)$ complicates the analysis of the firm's choice problem somewhat. Choosing $(0,0)$ means that the firm will leave the market. Yet, the following Lemma proves that it is always possible and profitable for a firm to avoid bankruptcy in our model and hence no firm chooses $(0,0)$ to leave the market. Intuitively default is not profitable because of the monopoly rents that the firm would forgo by defaulting.

Lemma 1 *(a) Y is non-empty and*

[12] We also restrict the firms to be unable to hold financial assets, i.e. $b \geq 0$. This is done for technical convinience. Alternatively, we may assume that the firm's discountfactor is larger than the market rate for financial assets, so that firms prefer to pay out dividends over accumulating financial assets.

[13] See appendix.

(b) employing zero capital is suboptimal, i.e.

$$\max_{(z^o,b^o)\in Y(z,e,w)} \tilde{v}(z^o,b^o) - \pi(z^o) w \mathsf{I}_{\{z^o\neq z\}} > \psi \mathsf{E}_t[v(0,0,w_{t+1}) > 0$$

Proof. *See appendix.* ■

Because of the above Lemma, firms never stop production completely, and they never declare bankruptcy. Because of the above Lemma, the optimal policy is always an element of Y and thus the Bellman equation defining v simplifies to:

$$v(z,e,w) = \max_{(z^o,b^o)\in Y(z,e,w)} \tilde{v}(z^o,b^o) - \pi(z^o) w \mathsf{I}_{\{z^o\neq z\}} \ . \tag{10}$$

2.1.5 Adjustment

In general, the correspondence Y of financially feasible plans is only upper-hemicontinuous, thus v might not be continuous everywhere. The lack of continuity arises because of the fixed adjustment costs. If a firm employs a large stock of capital and is heavily indebted, it may find itself unable to cover the debt obligations without selling capital if the capital imbalance or the debt level rises marginally. In this case the firm may be forced to substantially decrease its stock of capital to repay debt since it also has to cover the additional cost of disinvestment. Because of this it is necessary to distinguish two cases when describing firm level investment:

(a) The firm is in danger of becoming insolvent: this happens, if

$$1 - e_t - \frac{\pi(z_t)}{z_t} > \bar{b} \ . \tag{11}$$

In this case the firm has a negative cash flow and cannot sustain the actual level of capital employed by issuing new debt. Expressed formally, this means that the set of debt-ratios that can be attained without adjustment of capital,

$$Z(z,e) := \left\{ b^o \in \mathsf{R}_{++} | 1 - e - \tfrac{\pi(z)}{z} \leq b^o \leq \bar{b} \right\},$$

is empty. In this situation, the firm must (heavily) cut back production and so increases its average productivity. In consequence, a firm always disinvests if in financial distress.

(b) The firm is not in danger of becoming insolvent:

Then, a firm adjusts its stock of capital (i.e. it invests) in period t if the expected increase in value outweighs adjustment costs. Denote the (optimal) capital-imbalance after adjustment with $z^* = z^*(z, e, w)$ and the ratio of debt to capital after adjustment by $b^* = b^*(z, e, w)$. Additionally, denote the optimal debt level without capital adjustment by $b^+ := \arg\max_{b^o \in Z(z_t, e_t)} \tilde{v}(z_t, b^o)$. Here, again $Z(z, e)$ is the set of feasible debt-ratios with only financial readjustment. Then a firm invests if

$$\underbrace{\tilde{v}(z^*, b^*) - \tilde{v}\left(z_t, b^+\right)}_{\text{gain in value}} \geq \underbrace{\pi(z^*) w_t}_{\text{adjusment cost}},$$

which can be equivalently posed as

$$w_t \leq \frac{\tilde{v}(z^*, b^*) - \tilde{v}(z_t, b^+)}{\pi(z^*)}. \tag{12}$$

As shown in the appendix, the value of a firm that adjusts is monotonically decreasing in w_t, so that for every (e_t, z_t) there exists an unique trigger $\bar{w}$ such that

$$\underbrace{\tilde{v}\left(z^*, b^*\right) - w\pi\left(z^*\right)}_{\text{Value of a firm adjusting the stock of capital}} \gtreqless \underbrace{\tilde{v}(z_t, b^+)}_{\text{Value when only readjusting finance}} \quad \text{if } w \lesseqgtr \bar{w}\ . \tag{13}$$

Since $\bar{w}$ is implicitly defined by the maximum values $\tilde{v} - w\pi\mathbb{1}$ attains over Z and Y, the reaction of $\bar{w}$ induced by changes in e or z are directly related to the Lagrangian multipliers associated to the positive-dividend constraint

$$z^o - ez - \pi(z^o)[1 - w\mathbb{1}_{\{z^o \neq z\}}] - b^o z^o \leq 0. \tag{14}$$

Denoting the multipliers for the full optimization by λ^* and for the financial readjustment by λ^+. As shown in the appendix (Proposition 4), we can express $\frac{\partial \bar{w}}{\partial e}$ and $\frac{\partial \bar{w}}{\partial z}$ as

$$\frac{\partial \bar{w}}{\partial e} = \frac{1}{\pi\left(z^*\right)} \left(\lambda^* - \lambda^+\right) z \tag{15}$$

$$\frac{\partial \bar{w}}{\partial z} = \frac{1}{\pi\left(z^*\right)} \left[\left(\lambda^* - \lambda^+\right) e - \frac{\partial \tilde{v}(z, b^+)}{\partial z} + \lambda^+ \left(1 - b^+\right) - \lambda^+ \pi'\left(z\right)\right]. \tag{16}$$

This means the influence of finance is directly related to the difference between λ^* and λ^+. The maximal costs at which the company is willing to adjust its stock of capital increases in e if λ^* is larger than λ^+ and decreases otherwise.

The multiplier λ^* is for example larger than λ^+ if investment cannot be paid from current cash flow, while the optimal debt level b^+ can be reached with still paying out dividends. In this case $\lambda^* > 0 = \lambda^+$ and an increase in the equity ratio allows the company to invest at larger costs, $\frac{\partial \bar{w}}{\partial e} > 0$. Although $\lambda^+ = 0$ might be an extreme case, with non-convex adjustment costs investment projects are often sufficiently large, so that they exceed current cash flow which results in $\lambda^* > 0$.

However, for firms making losses from a suboptimal large capital stock, the situation is just reverse. Since selling capital generates a cash-flow that can be use to pay off debt, λ^* will be small, potentially $\lambda^* = 0$, while λ^+ will be large, as the company's debt burden increases to finance losses, moving b further away from the unconstrained optimal level. Consequently, we can expect $\frac{\partial \bar{w}}{\partial e} \geq 0$ for those firms that invest substantially and for those firms which sell substantial amounts of capital, we expect $\frac{\partial \bar{w}}{\partial e} \leq 0$.

2.2 Cross-sectional investment

2.2.1 Aggregation

Since the firm's investment decision only depends on the comparison of two values w_t and $\bar{w}(z_t, e_t)$, $\bar{w}(z_t, e_t)$ defines a critical value Ω which is the largest value of the stoppage duration w_t for which the firm chooses to invest.

$$\Omega(z,e) := \begin{cases} \overline{w}(z,e) & \text{if } Z(z,e) \neq \emptyset \\ 1 & \text{if } Z(z,e) = \emptyset^{14} \end{cases} \tag{17}$$

As only contemporary state variables matter for the aggregation, the time indices of state variables are suppressed henceforth for e and z. Let $G(w)$ be the distribution of w. Then the investment hazard can be defined as $\Lambda(z,e) := G(\Omega(z,e))$ and we can define the average capital imbalance $\bar{z}^*$ obtained after investment by firms whose equity ratio has been e and capital imbalance has been z before investment. This average capital imbalance is the conditional expectation of $z^*(z,e,w)$, conditional on $w \leq \Omega(z,e)$. This means $\bar{z}^*$ is given by

$$\bar{z}^*(z,e) := \Lambda(z,e)^{-1} \int^{\Omega(z,e)} z^*(z,e,w)dG(w), \tag{18}$$

Since adjustment-cost shocks w are i.i.d., the cross-sectional average investment rate equals the expected investment rate $i(z,e)$ unconditional on the firm's adjustment cost parameter w. The expected investment rate i in turn is the product of the probability of investment $\Lambda(z,e)$ and the typical investment rate of investing firms $\left[\frac{\bar{z}^*(z,e)}{z} - 1\right]$ and we obtain

$$i(z,e) := \left(\frac{\bar{z}^*(z,e)}{z} - 1\right) \Lambda(z,e). \tag{19}$$

[14]Since $w \in]0;1[$, $\Pr(w \leq 1) = 1$.

Now, differentiating this expression for the investment rate i with respect to e yields an interesting decomposition of the effect of a change in finance:

$$\frac{\partial i(z,e)}{\partial e} = \underbrace{\frac{\Lambda(z,e)}{z}\left(\frac{\partial \bar{z}^*(z,e)}{\partial e}\right)}_{\text{level-effect}} + \underbrace{\frac{(\bar{z}^*(z,e)-z)}{z}\frac{\partial \Lambda(z,e)}{\partial e}}_{\text{frequency-effect}} \tag{20}$$

As one can see from this equation, in our fixed capital adjustment cost model, finance has two ways to influence investment decisions. First, there is an effect coming from a change in the target level of capital, to which the firm adjusts by investment. This effect is a long-run or level effect, since it alters the stock of capital a firm likes to hold. By contrast, the second way finance influences investment has only short run impact on the firm's level of capital. Due to the direct effect of finance on the critical adjustment cost $\bar{w}$ and hence on the frequency of investment, financial healthier firms invest more often. In consequence, their capital imbalance will be smaller, in turn decreasing the probability of further investment. It is because of this frequency effect that investment can be more sensitive to the financial situation than the optimal stock of capital is.

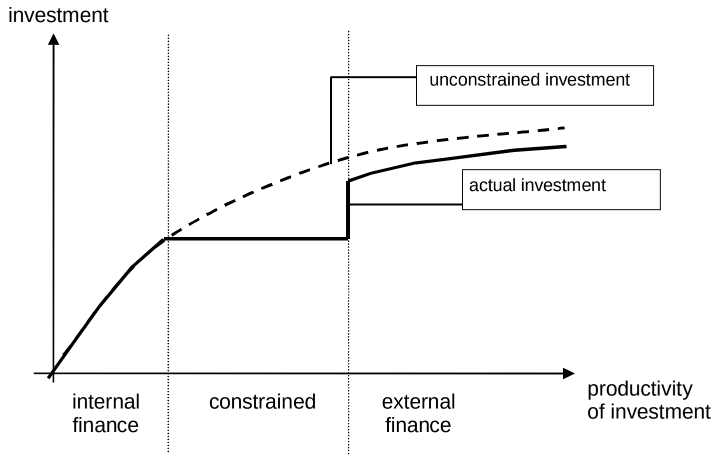
Moreover, this frequency effect makes fundamental investment incentives, which are captured by the capital imbalance, and finance complements in the investment decision in two senses. Firstly, only if the company wants to carry out large investment projects, i.e. $\frac{(\bar{z}^*(z,e)-z)}{z}$ is large, an increase in the adjustment frequency Λ can have a large effect. Secondly, from the discussion of $\frac{\partial \bar{w}}{\partial e}$ we know that the effect of finance on the critical level of adjustment cost is only large if z takes on very small or very large values.

2.3 Discriminating between our model and alternatives

This frequency effect is central in discriminating between the model of this paper and the two most prominent models of financial frictions with convex adjustment cost: pecking-order of finance models à la Myers and Majluf (1984) and financial accelerator models with liquidity- or wealth-dependent cost of capital à la Bernanke and Gertler (1989) or Bernanke et al. (1998). In the latter models, when adjustment costs are convex, the influence of finance comes via the target level of capital.[15] An effect of liquidity on the speed of adjustment in these models can only be of second-order: Slower adjustment marginally saves internal funds, such that the marginal gains of faster adjustment and

[15] However, note that in a model with convex costs z^o has to be defined somewhat differently. In this case it is the capital imbalance at which a firm would not actively change the capital imbalance by investing. Yet and although firms adjust their capital imbalance in the short-run towards this level, outside its long run equilibrium level (where e is endogenuous) z^o is never actually reached by active investment.

Figure 1: A stylized version of the pecking order model



the marginal-costs of internal funds are equalized. More liquidity hence only influences adjustment speed by altering its own *marginal* cost. By contrast, we have seen that in the fixed adjustment cost model, with investment being an extramarginal decision, a change in liquidity renders some projects unprofitable at given adjustment costs. This gives liquidity a first-order influence on adjustment.

In contrast, a pecking-order model of finance features a short run effect of liquidity. In this model, three regimes of firm finance typically emerge as stylized in figure 1.[16]

To the left are those firms with a high value of z (whose difference between K and K^* is small). They are financially unconstrained. Since they do not wish to invest much, they can rely on internal finance, and their investment decision is independent of their liquidity constraint. When the gap between K and K^* widens and firms obtain an intermediate value of z, they become strictly constrained by liquidity and a change in liquidity changes investment. Only firms with a low value of z rely on external finance, because their investment is highly profitable. Their investment revenues are large enough to cover the extra cost of external finance. Essentially this means that internal finance and fundamental incentives are *substitutes* in the pecking-order model with convex adjustment costs, in case the firm likes to invest for fundamental reasons. By contrast, our model predicts them to be *complements*, larger liquidity increases the likelihood of investment especially when there is much investment mandated. Since $\frac{\partial i}{\partial z} < 0$, this expressed formally is $\frac{\partial^2 i}{\partial z \partial e} < 0$ in our model and $\frac{\partial^2 i}{\partial z \partial e} \geq 0$ in the pecking-order model.

[16] See Gomes (2001), Bond and Meghir (1994), or Whited (1992) for details.

Within our model, both, the debt-ceiling and the liquidity dependence of the cost of capital shape investment decisions. Which of the two frictions is more important in the data can be evaluated by comparing $\frac{\partial i}{\partial z} \times \frac{\partial \bar{z}^*}{\partial e}$ and $\frac{\partial i}{\partial e}$. If only the liquidity dependence were of importance, $\frac{\partial i}{\partial z} \times \frac{\partial \bar{z}^*}{\partial e}$ and $\frac{\partial i}{\partial e}$ should be close to equal. In this case a drop in the equity ratio for example only raises the managerial discount factor. This is similar to an increase in z, and implies that the typical investment project becomes smaller. Although this also influences the investment frequency, this effect is just the effect a marginal change in z would have: smaller investment projects only pay at lower investment costs.

If the debt-ceiling is the important financial friction, $\frac{\partial i}{\partial e}$ can be expected to exceed $\frac{\partial i}{\partial z} \times \frac{\partial \bar{z}^*}{\partial e}$ substantially. Here, liquidity corresponds to a number of investment options a firm can expect to have at most over some given period of time. The smaller the number of options is, the larger the value of each option will be. This option value adds another factor to the fixed cost of adjusting the stock of capital.

All this means that we can discriminate between the various investment models using first- and higher-order derivatives. Thus, even if we later estimate the investment function non-parametrically and even if the estimated derivatives have no structural interpretation in the form of coefficients of an adjustment-cost function, the estimates allow to draw structural conclusions.

3 Empirical evidence

3.1 Measuring the capital imbalance

To test our model of interacting frictions, we need an empirical approach that also nests alternative models. This precludes structural estimation, and forces us to rely on a reduced form representation of our model on the lines of the two-step model of Caballero et al. (1995). In this two-step approach, first the capital imbalance z is estimated as a proxy of fundamental investment incentives. Thereafter, investment is regressed on this proxy and on the equity ratio to obtain the (short-run) expected investment function. This function is estimated employing non-parametric estimation techniques and we primarily base our inference on non-parametric average derivative estimates. Although this strategy allows no direct inference about the parameters of the economic primitives (e.g. the adjustment cost function), it still allows to discriminate as we have just argued before.

Hence, the first goal is to construct an estimator for the capital imbalance z. In contrast to Caballero et al. (1995), it cannot be assumed that the desired capital is proportional to the stock of capital K^* that a plant would hold in the absence of *adjustment* *costs,* since the capital market imperfection complicate the firm's choice. The following Lemma helps to simplify the problem.

Lemma 2 *For all* $\lambda > 0$, $z^*\left(w, \lambda e, \lambda^{-1} z\right) = z^*(z, e, w)$, *i.e. in determining the optimal capital imbalance* e *and* z *enter multiplicatively.*

Proof. *Note that for those firms that invest* ($z^o \neq z$) *in the right-hand side of* (10) z *and* e *only appear in the restriction* $(z^o, b^o) \in Y(z, e, w)$. *In* Y *however,* e_t *and* z_t *enter only multiplicatively.* ■

Hence, z and e enter only multiplicatively in z^*. Taking logs of z and e (without changing notation) and denoting all other logs as small letters, we then can rewrite the optimal capital imbalance z^*, defined in the previous chapter, as a function in two arguments, $(z + e)$ and w. Abusing notation slightly, we replace $z^*(z, e, w)$ by $z^*(z + e, w)$.

Moreover, if we neglect the different effect of e and z on $\Omega(e, z)$ and hence on the composition of $\bar{z}^*$, then the average target capital imbalance $\bar{z}^*$ can also be approximated by a function of the sum of $z + e$. We linearize $\bar{z}^*(z + e)$ with a first order approximation around $\bar{z}^*(0)$.[17] This linearization yields, for the dynamically desired stock of capital $\tilde{k}$ (in logs):[18]

$$\bar{z}_i^*(z_{it} + e_{it}) = \tilde{k}_{it} - k_{it}^* = \alpha_{i0} + \beta_i(z_{it} + e_{it-1}). \tag{21}$$

In this equation, the equity ratio e_{it-1} of the end-of-period $t - 1$ was substituted for e, since it reflects the equity ratio in the opening balance of the company, which is e in our model.

Still, z_{it} has to be replaced by an observable. For an isoelastic production/sales function (in logs: $y_{it} = \psi_{it}^0 + \psi_i^k k_{it}$) we obtain from the first order condition for the statically optimal stock of capital k_{it}^*:

$$k_{it}^* = \xi_{it} := \frac{\psi_{it}^0 + \ln \psi_i^k - c}{1 - \psi_i^k}, \tag{22}$$

In this equation c denotes log cost-of-capital and ψ_i^k is the reduced form elasticity of sales to capital–when all other flexibly adjusted factors have been partialed out. Now combining (21) and (22) yields (allowing for a non-unit elasticity of k^* with respect to

[17] This means firms hold the static optimal stock of capital and hence $K/K^* = 1$ and $z = \ln(K/K^*) = 0$. Additionally this assumes that firms hold no debt. Thus $e = \ln(1) = 0$ and $e + z = 0$.

[18] In Caballero and Engel (1999) for comparison, the optimal capital imbalance is a constant.

ξ, the right hand side of (22)):

$$\frac{\bar{z}_{it}^{*} - z_{it}}{1-\beta_i} = \tfrac{\alpha_{i0}}{1-\beta_i} + \theta_i \xi_{it} - k_{it} + \tfrac{\beta_i}{1-\beta_i} e_{it-1} \tag{23}$$

In this equation $\bar{z}_{it}^{*} - z_{it}$ gives the log difference between dynamically optimal capital and the capital currently employed, which is approximately the investment rate i upon adjustment. Hence, we might term $\bar{z}_{it}^{*} - z_{it}$ mandated investment. If firms adjust their stock of capital over time, we can expect the difference $\bar{z}_{it}^{*} - z_{it}$ to be stationary, while ξ_{it} and k_{it} are most likely to be non-stationary. Additionally, the equity ratio e_{it} in the opening balance is predetermined. Consequently, there must be a cointegration relation and θ and β can be estimated from a panel-cointegration regression (Caballero et al., 1995, p. 15).

3.2 Estimation procedure

3.2.1 Inferring productivity

Although, we do not directly observe ψ_{it}^{0} and ψ_{i}^{k}, there are various ways to estimate these parameters, and this estimation builds the first step of the total estimation procedure. The most direct way was to infer the parameters from an estimation of the production function

$$y_{it} = \psi_{it}^{0} + \psi_{i}^{k} k_{it}. \tag{24}$$

However, estimating this production function is problematic since productivity is non stationary, capital is endogenous, adjustment is lumpy, and time-to-build makes a specific dynamic model necessary. With a panel of medium length in the time dimension this approach hence becomes infeasible.

As an alternative, we calculate ψ_{i}^{k} from average expenditure shares on machinery α_M, land α_K and labor α_L and then obtain ψ_{it}^{0} as the direct residual from (24) . Moreover, on the basis of average expenditure shares we can replace sales y_{it} by $\frac{M_{it}}{\alpha_M}$ or $\frac{w_{it}L_{it}}{\alpha_w}$, which are the ratios of the current expenditure for one production factor over its average expenditure share. Then we obtain $\hat{\psi}_{it}^{0} = \frac{M_{it}}{\alpha_M} - \hat{\psi}_{i}^{k} k_{it}$ for example. This indirect procedure has been proposed by Cooper and Haltiwanger (2002). The appendix provides more details on both, the direct and the indirect procedures.

Since all three alternatives give valid estimates of ξ, but since also every single procedure on its own has also its disadvantages, we generate the actual estimate of ξ_{it} as the common factor of all three of them. Essentially, this means we assume every single of the three values $\xi_{it}^{(j)}$ measures ξ with a noise or measurement error $\eta_{it}^{(j)}$. Hence we specify

$$\xi_{it}^{(j)} = b_j \xi_{it} + \eta_{it}^{(j)}, \tag{25}$$

and recover ξ_{it} by factor analysis. It turns out that there is indeed only one factor common to all measures. Details are again available in the appendix.

3.2.2 Estimation of the long run relation

The second step of the analysis is the estimation of the cointegration relation that follows from (23). Here we employ Phillips and Moon's (1999) "full-modified panel cointegration estimator" (henceforth PFM-OLS). This estimator is $\sqrt{n}T$-consistent, asymptotic normal and corrects for possible endogeneity of the regressors.

A drawback of the PFM-OLS estimator is that it is formulated for balanced panels with integrated regressors only. The data we have is an unbalanced panel and at least for e we would rather assume it to be a predetermined I(0) process. However, the PFM-OLS estimator is a generalization of the full-modified OLS estimator of Phillips and Hansen (1990) and Phillips (1995) and hence we expect the results of Phillips (1995) to carry over to the panel case as well, i.e. the estimator is $\sqrt{nT}$-consistent and asymptotic normal for parameters of stationary regressors. The standard errors for an $I(0)$ regressor are calculated in analogy to the time-series case.

The PFM-OLS estimator involves the estimation of an average (long-run) covariance matrix. This is obtained by generating covariance-matrix estimates for each single firm following a Newey-West (1987) type approach and then averaging over firms. For the actual estimation, we use a Bartlett-kernel with a bandwidth of $\sqrt{T_i}+1$, where T_i denotes the number of observations per firm. However, we restrict the sample of firms to be used for covariance estimation to firms with at least 8 consecutive observations.

The cointegration relation that we estimate follows directly from (23):

$$k_{it} = \alpha_i + \gamma_t + \theta_i \xi_{it} + \kappa_i e_{it-1} + u_{it}, \tag{26}$$
$$\kappa := \tfrac{\beta_i}{1-\beta_i} \text{ and } u_{it} = \tfrac{\bar{z}_{it}^{*} - z_{it}}{1-\beta_i}.$$

The error term u reflects the stationary cointegration error. The time-dummies γ_t have been included since the data that will be used covers a period of large shocks to inflation, so that measuring the cost of capital is problematic. However, assume that the cost of capital differ across firms only due to different finance, κe_{it-1}, and time invariant, firm specific factors, α_i, or time specific factors, γ_t. Then our estimation will reflect the cost of capital correctly, since γ_t and α_i are removed by two-way within transformations.[19]

[19]Note that this also controls for different baseline access to capital markets.

With the estimate of θ at hand, we can use (22) to calculate end-of-period t capital imbalances $\hat{z}_{it} = u_{it}$. This capital imbalance, evaluated at the end of the previous period $\hat{z}_{it-1}$, can serve as a proxy for the capital imbalance z_t at the beginning of the period t.

3.2.3 Estimation of the investment function

In the third estimation step, we express the expected investment rate i_{jt} as a function of $(\hat{z}_{jt-1}, e_{jt-1})$, which has to be estimated non-parametrically. To compare the short- and long-run behavior, average derivatives of $i(z,e)$ are estimated. These are the counterparts to the coefficients in linear models.

The employed nonparametric estimation procedure is local linear kernel estimation. However, with non-parametric estimators it is not as straightforward to account for firm-specific effects as it is in the parametric case (Ullah and Roy, 1998). To account for fixed effects the investment function shall meet the following assumption:

Assumption 5: For each firm j at time t investment i is given by

$$i^+_{jt}(\hat{z}_{jt-1}, e_{jt-1}) = i(\hat{z}^+_{jt-1}, e^+_{jt-1}) + v_{jt} \ . \tag{27}$$

Moreover, we assume $\mathsf{E}(i(\cdot,\cdot)) = 0$ to identify i and define generally x^+_{jt} as the two-way within transformed variable x to remove fixed time and individual effects.

Under this assumption function i can be directly estimated from the within-transformed data employing standard non-parametric local linear estimation. This technique is best understood as a weighted least squares (WLS) estimation whose weights are generated by a kernel function that measures the distance of observations to the point at which the function is locally estimated.

3.2.4 Estimation of average derivatives of the investment function

A major drawback of the nonparametric estimators are their wide confidence bounds. Because of this, we also estimate average derivatives of $i(z,e)$, which allow to draw more reliable conclusions as their confidence bounds are relatively close. This is because non-parametric average derivative estimators converge with parametric rates of convergence (Rilstone, 1991).

Several nonparametric estimators for average derivatives are available in our panel data setting (Ullah and Roy, 1998). Again, we concentrate on derivative estimates from a local linear estimation. Since the local regression is a WLS on

$$i^+_{jt} = i(z,e) + \left(z^+_{jt-1} - z\right) b_z(z,e) + \left(e^+_{jt-1} - e\right) b_e(z,e) + \nu_{jt}, \tag{28}$$

it produces two candidates for derivative estimates. We can either numerically generate $b^* := \partial_{(z,e)}\hat{\imath}(z,e)$ or alternatively take $b^{**} := \hat{b}_z, \hat{b}_e$ as a direct estimates for the derivatives.

The pointwise derivative estimators, b^{**} are asymptotically normally distributed. Its small samples bias differs from that of b^* in such way that in most cases the numerical estimator b^* is preferable (Ullah and Roy, 1998). However, this estimator has a not yet known variance (Pagan and Ullah, 1999). Therefore, we use both estimators, but have to rely on the direct estimator b^{**} for inference.[20]

Average derivative estimates are generated as the mean of the pointwise estimates. When we calculate average derivatives over a subset of observations we calculate them as the mean of those observations that fall into the subsample. We do not re-estimate with only the observations from the subsample. The cross- and higher-order-derivatives are computed as numerical estimates. Just as for the numerical estimator of the first-order derivative the asymptotic variance is not yet known for average higher-order-derivative estimates.

3.3 Data

The data we employ comes from the BSO-dataset of the Cambridge/DTI Database.[21] This database contains annual accounting data from UK companies from 1976 to 1990. Observations of 50494 company-years are included in the data. About half of them are of manufacturing firms. For the subsequent analysis the dataset has been restricted to companies of the manufacturing sector with positive fixed capital and positive equity. Out of these, we only keep those firms that report a complete series of employee remuneration, since we are otherwise unable to infer capital productivity from the data. Moreover, the sample is restricted to firms with 7 or more consecutive observations to make the cointegration analysis sensible. Additionally we drop those firms whose equity ratio (in logs) in any period of time differed from the sample mean by 5.5 times the standard

[20] Additionally, for panel data there is the fixed effects estimator of Ullah and Roy (1998) available. Just as b^* and b^{**} under assumption 5, this estimator b^{FE} uses the within transformed data for the regression. However, it calculates the weights with the kernel on the basis of the original data. By contrast, the estimators b^* and b^{**} use also the within transformed data to evaluate the kernel. The advantage of the fixed-effects estimator is that assumption 5 could be replaced. Instead the estimator assumes that the investment function exhibits fixed idiosyncratic effects is otherwise homogenous among the firms in the not transformed data. The downside of this estimator is that bias behavior and variance are unknown. Experimenting with this estimator in a previous version of paper produced no qualitatively ddifferent results.

[21] See Goudi et al., 1985. The database is freely available by HMSO after registration. It covers a representative sample of UK company accounts. The same database has for example recently been used by Geroski et al. (2003) to test Gibrat's law.

deviation or from the firm specific mean by 3.5 times the standard deviation. Also we drop those firms whose investment rate differed by 7 times the standard deviation from the mean. The former selection removes in particular firms with extremely low values of equity and firms with extreme changes in the financial structure. The latter removes, in particular, firms that exhibit investment spikes that are too large to be neither outlier nor measurement error. Together this two selections remove 274 observations.

We choose relatively large bounds to remove outliers to avoid to much of a selection bias. In particular, we are conservative in removing outliers on the basis of the investment rate since investment rates will by nature be heavily dispersed with fixed adjustment cost. Hence, large investment spikes do not necessarily mean outliers. Altogether the selection reduces the sample size substantially to 6272 observations from 631 different firms, but it should be noted that most of the removed observations are removed due to missing data or due to a too small number of observations.

The BSO dataset contains capital and investment data for land and buildings as well as for other tangible assets (e.g. tools and machinery). Since reported depreciation rates for machinery are about 50%, we restrict the investment analysis to land and buildings.[22] Besides the more modest depreciation rates of land and buildings the greater ex ante appropriateness of the fixed adjustment cost model also favors land and buildings as the investment good to be analyzed. The data on machinery is only used to infer factor productivity.

All data that we use have been inflated to 1996 prices with the price index for producer output taken from the International Financial Statistics database of the IMF. The capital series have been generated using the perpetual inventory method, taking the reported capital in the first period of observation as a starting value. Details on how the data items are constructed are available in the appendix.

Table 1 reports descriptive statistics for those variables of the sample that we use. For the data on investment in land and building we find the following. When a cut-off value of a *net*-investment rate of 30% is used to indicate an investment spike,[23] 19% of all firm years exhibit such spike. On average, these spikes account for 39% of each firm's investment activity over the total observation period. On the other hand 15% of all firm years show a *gross*-investment rate of less than 5% in absolute terms, so that there also is substantial evidence for inactivity. Active disinvestment (less than -5% *gross*-

[22] Although our model might still hold true for fast depreciating capital goods, we would need data at a higher frequency to sensibly analyse the data. At a depreciation rate of 50% capital goods are replaced on average every second year on a regular basis, if the stock of capital stays constant. Hence, we can expect to hardly find any influence of fixed costs in yearly data.

[23] Net-investment rate here means investment rate over the firm average depreciation rate.

Table 1: Descriptive Statistics, Unit Root Tests and Spike and Inactivity Indicators

Variable	Obs	Mean	Std. Dev.	Min	Max
capital productivity, ξ, common factor	6272	-0.000	1.029	-4.921	3.564
land and buildings, k (logs)	6272	9.777	1.633	1.608	15.579
equity ratio, e (logs)	6272	-0.242	0.334	-2.404	1.272
investment rate, i	6272	0.336	0.539	-0.837	8.177
sales, y (log)	6272	11.571	1.508	4.106	16.640
remuneration, wL (log)	6272	9.949	1.517	0.639	14.971
machinery M (log)	6272	10.274	1.614	3.637	16.069

Results of a Breitung Meyer Test				Activities in %	
variable	coefficient (ρ-1)	std. error	lags	average depreciation rate	15.0
k, (a)	0.0604	0.008	1		
k, (b)	0.0188	0.009	1	percentage of spikes	18.5
ξ, (a)	0.0548	0.008	1		
ξ, (b)	0.0421	0.009	1	percentage of inactivity	14.7
e, (a)	-0.0431***	0.017	2		
e, (b)	-0.0800***	0.017	2	percentage of disinvestment	4.7
cointegration error				fraction of total investment	
z	-0.0223**	0.010	1	accounted for by spikes	39.3

***,** significant rejection of the unit root
on the 1% / 2% level, one-sided test $(\rho - 1) < 0$
(a) without / (b) with time dummy,
augmentation lags: sequential t-test max=4, min=1

investment rate) is observed only rarely (in less than 5% of all cases). Therefore on a priori grounds, the assumption of fixed adjustment costs or irreversibility of investment seems to be relatively plausible for the DTI data on land and buildings.

For the equity ratio, we see a substantial number of firms holding more liquid assets than having outstanding debt (appr. 16% of all observations), so that they are effectively lenders and their log equity ratio is positive. Hence, these companies seem to be more "patient" than the market or gain some additional value from retaining earnings, for example greater flexibility as in our model.

Since Cooper and Willis (2004) have highlighted the importance of the unit-root assumption for productivity in fixed adjustment cost models, we pretest this assumption. The results of a Breitung-Meyer (1994) test for unit roots in panel data are also reported in table 1.[24] The analyzed series are capital (land and buildings) productivity ξ^*, capital

[24] The Breitung Meyer test has been chosen, since the panel is much larger along the cross sectional dimension than along the time dimension and is also unbalanced. The test accounts for both fixed individual effects and cross-sectional correlation and is an "N-asymptotic" test.

Table 2: Estimates from the cointegration regression: (PFM-OLS, two-way within)

Full Sample			By Industries							
	κ	θ	Industry		κ^b	θ	Industry		κ^b	θ
PFM-OLS	0.009	0.913^{***}	21	PFM	-0.12	1.03^{***}	33	PFM	0.02	1.18^{***}
st. err. I(1)	0.032	0.036		OLS	-0.03	0.96^{***}		OLS	-0.06	1.19^{***}
st. err. I(0)	0.030^a	0.050^a	23	PFM	0.26^{**}_{***}	0.91^{***}	36	PFM	0.09	1.28^{***}
				OLS	0.25^{***}	0.91^{***}		OLS	0.13^{**}	1.18^{***}
OLS	0.012^{*}	0.911^{***}	26	PFM	-0.07	0.51^{***}	48	PFM	-0.05	0.54^{***}
st.err.^{c}	0.022	0.022		OLS	0.01	0.54^{***}		OLS	-0.04	0.67^{***}

^{a}The standard errors are obtain as panel analogues to Phillips (1995, p. 1038, eq. (14)).

^{b} $^{**}_{***}$ superscript refers to I(1) and subscript to I(0) standard errors for the PFM estimator for κ,

^{c} OLS: Usual panel corrected SE, **,*** denote significance on the 5% and 1% level respectively.

k, the equity ratio e and the gap z. While productivity and capital both seem to be integrated of order 1, both e and the gap z (cointegration error) are stationary, at least once we control for time-effects.

3.4 Estimation results for the target (long run) capital imbalance

Table 2 reports estimates of the cointegration equation (26) for the whole dataset as well as for those industries for which more than 450 observations are available. These industries are food [21], drink [23], chemicals [26], mechanical engineering [33], electrical engineering [36] and paper, printing & publishing [48].

The estimated long-run elasticity of capital with respect to the ξ is 0.913. Since ξ has, due to normalization, only 2/3 of the variance of the original measures from which it is constructed, the theoretical prediction for θ would be approximately 1.5, from which the estimate significantly deviates. However, the result that is more striking is that finance measured by the equity ratio has not only *no statistically significant* influence on the level of the capital stock, but also that the estimated coefficient $\hat{\kappa}$ is virtually zero.[25]

When we look at the sample stratified by industries the result does not substantially change. In almost all cases finance has no significant influence. Only for industry 23 (and 36 based on OLS results) would we conclude that more financial means increase the stock of capital a company wishes to hold. Strangely the I(0) standard errors calculated as panel analogies to Phillips (1995, p. 1038) are smaller than the I(1) errors calculated under the assumption of superconsistency.

Going back to the results that we obtained under the assumption of a homogeneous panel, the finding that κ is zero has an important statistical implication for the analysis

[25]Note that this is not a result of e being stationary. Since the cointegration error z is stationary κ is identified also asymptotically, see Phillips (1995).

of investment behavior that follows. For q-models of investment it has been argued by a number of authors that a statistically significant influence of cash-flow on investment might result from the inability to measure future profitability of investment correctly. As a result cash flow actually predicts profitability but its statistically significant influence in investment does not identify financial market imperfections. A series of papers have followed that idea and replaced the stock market value of a company by analysts' forecasts of company's earnings in the q regressions. These papers then find no additional role of cash flow in investment, see Cummins et al. (1999) and Bond and Cummins (2001) and Bond et al. (2004).[26] Since in our approach the equity ratio does not explain the capital choice of a company in the long run once we control for productivity ξ, in turn e can also not contain any information on the long run profitability of investment which is not included in ξ. Consequently, if we find an influence of finance e in investment later on, we cannot attribute it to picking up long run investment prospects.

3.5 Investment behavior

3.5.1 Density and conditional expectations estimates

With the cointegration estimates it is now possible to construct the time series $\hat{z}_{it}$ for each firm. We standardize i, e and z for the non-parametric analysis. To obtain a first graphical idea of the investment data, the densities of i, e and z and the conditional density of the investment rate $f(i|z)$ are estimated. The results are displayed in figures 2 and 3 (a). In these estimations and in all following estimations a normal product kernel is used. For the density estimation, a variable bandwidth is used, which we generate using an adaptive two-stage estimator (e.g. Pagan and Ullah, 1999, p. 13), where the preliminary density estimate is parametric normal. The overall bandwidth for variable j, h_j, is selected by Silverman's (1986) interquartile rule of thumb, with interquartile range R_j, the number of observations $n = 5674$ (we loose one year because we take the lag of z) and $h_j = 0.9 \min\left(\frac{R_j}{1.34}, 1\right) n^{-1/(4+\dim)}$.

The investment rate and the equity ratio distributions show substantial excessive kurtosis, while the capital imbalances z are closer to normally distributed. When we look at the distribution of i conditional on z displayed in figure 3(a), we find both for firms with large and with small mandated investment two peaks in the distribution, one associated with adjustment, the other with inactivity. However, these two peaks are only clearly marked on the boundary of the support, where the estimator becomes less reliable.

[26] However, similar to our results on investment later on , Bond et al. (2004) still find a role for the cash holdings of a company in determining the firms investment activity.

Figure 2: Non-parametric density estimates, i, e, and z

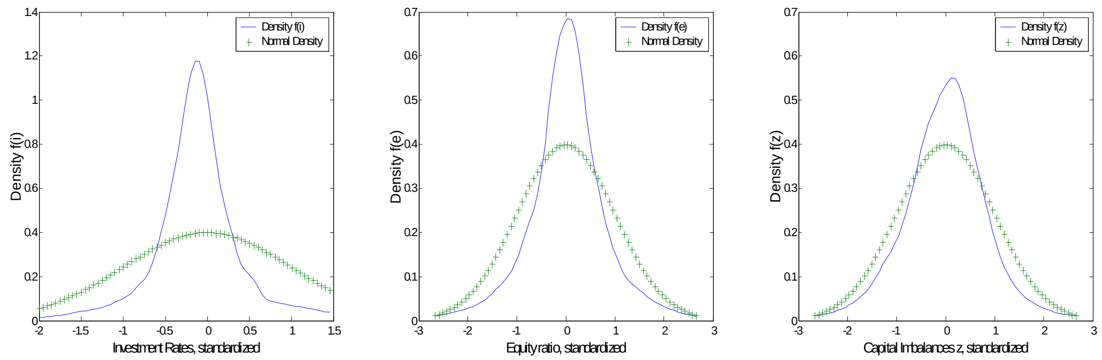
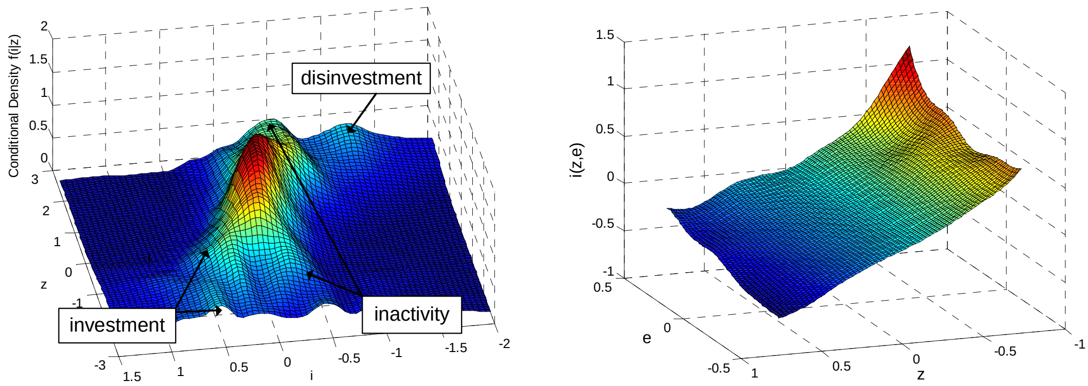


Figure 3: Non-parametric estimates

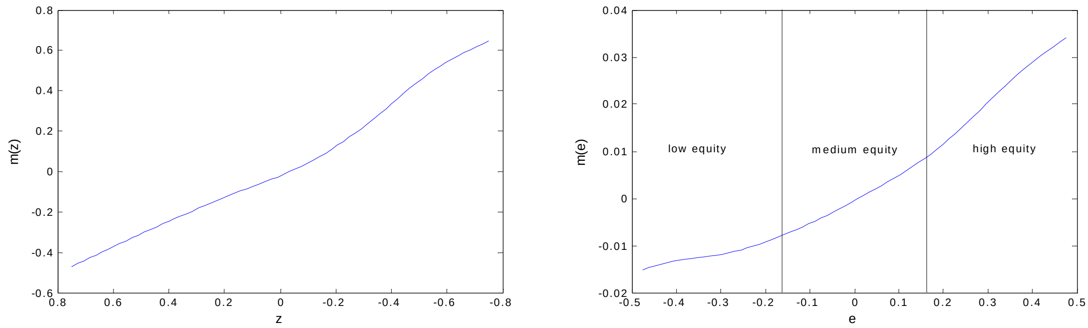


(a) conditional density f(i|z) (b) expected investment rate i(z,e)

Now, we turn towards the estimation of the expected investment function. As explained, we use a local linear estimation technique. Again the bandwidth chosen is a flexible bandwidth, here following Fan and Gijbels (1992), where bandwidth $h(x_i) = hf(x_i)^{-1/5}$ and f is the density of the standard normal distribution. The common constant h is determined such as to minimize the mean integrated squared error. This results for variable j in a bandwidth $h_j = a_j^{1/6} n^{-1/6}$ with $a_j = 0.29\hat{\sigma}_u^2 \left(\sum_{i=1}^{n} \frac{(\hat{m}_j''(x_i)^2)}{n} \right)^{-1}$, where $\hat{m}_j''(x_i)$ is the second-order derivative of investment with respect to variable j. We estimate this derivative on the basis of a fourth order polynomial that is estimated with OLS; $\hat{\sigma}_u^2$ is the variance of the OLS residual. Using different bandwidth within a certain range, produced no substantially different estimation results.

Figure 3(b) displays the estimated investment function. For the display the argu-

Figure 4: Local linear estimate of an additive seperable investment function, i(z,e) = m(z) + m(e)



ments and the investment rate have been transformed back to the non-standardized scale. Moreover note, that the scale for z runs from positive values (low mandated investment) on the left to negative values (high mandated investment) on the right.

The investment function is nonlinear in a twofold sense. First, there is some interaction of fundamental investment incentives z and finance e. If mandated investment is large (negative z), more equity increases the slope of the investment function, so that it reacts more strongly to changes in z, the fundamental investment incentive. On the other hand, if the firm fundamentally should decrease the stock of capital, more equity makes the firm more reluctant to adjust, and the firm sells less than it does with low equity. Recall that this corresponds to our analysis of the influence of finance on the critical size of adjustment costs (15) and hence to a frequency effect as of (20).

Secondly, the investment function appears convex for negative z and somewhat concave for positive z, which corresponds to incentives to invest or to sell capital respectively. This curvature is a central prediction of non-convex adjustment cost models. So far, the non-linearity we find only for the point-estimates whose confidence bounds are wide–as typical for non-parametric estimations.

One way to increase the accuracy of the regressions is to impose more structure, for example a certain functional form. However, this will result in biased estimates if the functional form is misspecified. A potential form of such misspecification bias can be seen in figure 4, note that the scale for z has also been reversed. There we estimate the generalized additive model (see Pagan and Ullah, 1999, pp.137)

$$\mathsf{E}[i|(z,e)] = m_1(z) + m_2(e). \tag{29}$$

In this model there is no interaction of equity and capital-imbalance and thus this source

Table 3: Average gradient of the investment rate $i(e, z)$

Number of observations: $N = 5641$			std. deviation
derivative	$\hat{b}^{*}$	$\hat{b}^{**}$	for $\hat{b}^{**}$
b_z	-0.6841	-0.6697	0.0065
b_e	0.1584	0.1414	0.0128

of non-linearity that we found in figure 3 (b) is removed. Two results are apparent effects of this specification. First, the investment rate seems more or less linear in the capital imbalance and secondly when we look at equity, those firms with low equity that might be considered most constrained are least reactive to changes in finance, a "fact" firstly emphasized by Kaplan and Zingales (1997). However, when we look at average derivatives, which is an alternative way to alleviate the problem of wide confidence bounds of non-parametric estimation, these findings appear to be an artifact generated by misspecification.

3.5.2 Average derivative estimates

These average derivative estimates for both estimators are reported in table 3. Note, that all variables have been standardized (divided by their standard deviation s_x) for the estimation. The reported derivatives are transformed back to the original scale by multiplying the derivative with respect to z or e with $\frac{s_i}{s_z}$ and $\frac{s_i}{s_e}$ respectively (ratios of standard deviations of i and z or e). Although both estimators yield quantitatively slightly different estimates, they qualitatively do not differ: Both the equity-ratio and the capital imbalance z have a significant effect. If the true model were a partial adjustment model with linear error-correction $i = \beta_z (k - k^*) + \beta_e e$, then we should have obtained a long run relation of $\frac{\partial k^*}{\partial e} = -\frac{\beta_e}{\beta_z} \approx 0.23$, since the cointegrating vector corresponds to the space of zero adjustment.

Table 4 displays the average derivatives for subsamples stratified according to in which terzile e or z fall.[27] Since the non-parametric average derivative estimators converge with a parametric speed of convergence, the standard errors increase due to the stratification only by factor $\sqrt{3}$.

The first part of the table stratifies the sample according to z, going from low mandated investment (high z) to high mandated investment (low z). The first row of the table displays $\frac{\partial i}{\partial z}$ and provides strong evidence that non-convex adjustment cost matter in the investment decision. It shows that investment reacts most strongly to z when mandated

[27] We do not include top and bottom 0.5% observations in the first and third terzile to avoid potential problems at the boundary of their support.

Table 4: Average derivatives for single-dimensionally stratified subsamples

	stratified according to z				stratified according to e		
	z high	z medium	z low		e low	e medium	e high
b_z	-0.5821	-0.5207	-0.9077	b_z	-0.6321	-0.6426	-0.7226
b_e	0.1620	0.0889	0.1748	b_e	0.1074	0.1691	0.1442

standard errors equal approximately 1.74 times the full-sample errors
Derivatives are direct estimates $\bar{b}^{**}$

investment is large (low z) and that it reacts least strongly to mandated investment if z takes intermediate values.

The second row of the table displays $\frac{\partial i}{\partial e}$. Here it shows that the influence of finance on investment is pronounced if z takes either very high or very low values. In the light of our model, this makes a lot of sense: If the main impact of finance comes via investment frequency, the absolute size of this impact can only be large when adjustments are also large.

That for both, low and high z, $\frac{\partial i}{\partial e}$ is positive relates to the reaction of $\bar{w}$ to changes in e. When z is low, more liquidity increases the threshold $\bar{w}$ for adjustment costs up to which a firm adjusts. By contrast, more liquidity will lower $\bar{w}$ when z is large. The former effect leads to more upward adjustment, whereas the latter leads to less downward adjustment, so that both increase the investment rate.

Overall however, larger financial means increase the adjustment speed, $-\frac{\partial i}{\partial z}$, as we can see from the first row of the second part of table. Moreover, in contrast to the results we had obtained from the generalized additive model as in figure 4 there is no significant difference in the influence of finance between low equity and high equity firms.

The impression from the differences in first order derivatives is also validated by looking at the average estimated Hessian displayed in table 5. The estimates are generated as numerical estimates and again we stratify the sample along z. The table reports the Hessian for z coming from the first and third terzile. When mandated investment is large (low z), investment i is convex in z and i is concave for low mandated investment. Moreover, the cross derivative is negative in the former and positive in the latter case. Taking into account that $-z$ measures mandated investment, this means finance and fundamentals are complements for those firms with large mandated investments. An increase in equity has the same effect on the first-order derivative $\frac{\partial i}{\partial z}$ that an increase in mandated investment has.

Table 5: Average Hessian

	low z (high incentive)		high z (low incentive)	
	z	e	z	e
z	0.5767	-0.0325	-0.1473	0.1310
e	-0.0352	0.2947	0.1310	0.0613

Derivatives are numerical estimates $\bar{b}^*$

4 Conclusion

In this paper a model of investment that incorporates an imperfect capital market and fixed investment costs was presented. Even though no closed form was deduced some new results were obtained by analyzing that model. The major result is to identify the difference between a short-run effect of liquidity on the frequency of investment and a long-run effect on the optimal stock of capital. While models that generate a liquidity finance correlation via agency or strategic motives–for example Myers (1977) or Brander and Lewis (1986)–predict the influence of finance through the stock of capital, the model in this paper rather emphasizes an alternative effect via the frequency of investment projects. This suggests a stronger short-run than long-run influence of finance in investment, and empirically we find exactly this.

In particular, the investment rate is found to be a nonlinear function, where investment opportunities (capital imbalance) and finance (equity ratio) strongly interact. Imposing only additive separability in estimating this function already leads to a severe error. This error could well be the cause of the puzzling finding reported in the literature that "*a priori* unconstrained" firms react more strongly to changes in their financial variables than constrained ones. For further empirical research, this result suggest a need for flexible functional approaches, for example to estimate investment equations in a generalized error correction framework as done in this paper.[28]

Within this wide class of generalized error correction methods, our approach was completely non-parametric for the short-run dynamics (or error correction function). We generated accuracy of the estimates by relying only on average derivative estimates. An alternative were to take a flexible but parametric approach to generalize the error-correction process. This might provide further insights. In particular our estimation is unable to recover the economic primitives of the investment decision. Though our empirical method can rule out some model alternatives, still a whole class of investment models might generate the observed dynamics. Yet, indirect inference methods may help

[28]See de Jong (2001) for an overview on generalized error-correction methods.

to identify the economic primitives on the basis of our reduced form estimations.[29]

From a policy perspective, our findings have some implications again without recovering economic primitives. Suppose there are shocks to the balance sheet positions of firms–for example through exchange rates as in Céspedes et al. (2004), Aghion et al. (2001) or Devereux and Lane (2001)–then this paper's model predicts the real impact of such shocks to have a different timing and impact than the usual financial accelerator model of Bernanke and Gertler (1989) and Bernanke et al. (1998). Productive firms will delay investments, unproductive ones are more likely to sell capital to repay debt. The total effect comes much more up front than over time.

Moreover, the total impact will depend on the distribution of capital imbalance in the economy. If there is no aggregate capital imbalance but some firms need to increase their stock of capital, while others need to sell capital, a deterioration of balance sheets will lead to a drop in investment. By contrast, in the same aggregate situation the same deterioration will lead to no strong reaction if also no single firm has a marked capital imbalance. Thus, if fixed adjustment costs are present, policies that influence the balance sheet will be rated differently depending on the distribution of capital imbalances. Therefore, with financial frictions, policy makers, for example central banks, need to observe the distribution of capital-imbalances even more than with only fixed adjustment in order to predict policy implications. More specifically the effects of finance and fundamentals cannot be considered separately as the magnitude of each effect depends on the state of the other variable.

Another example for policy implications would be a (corporate) tax reform. There the implications on the costs of retaining earnings have to be taken into account,too. Since a rise in the average equity ratio, not only raises investment in the short run, but also increases the propensity to adjust it may increase efficiency, as capital imbalances become smaller.

[29] See for example Cooper and Willis (2003). However the recent contribution of Bayer (2004) to the debate on gap regressions initiated by Cooper and Willis (2004) raises some concern about the feasibility of such approach, in particular for macro-data.

5 Appendix

In this appendix, we first derive the Bellman equation which is central to our model. Thereafter, we show the existence and uniqueness of a solution to this equation. Then, some properties of the induced optimal-policy function are discussed. Finally a data appendix follows.

5.1 Deriving the Bellman Equation

5.1.1 Constraint set

All variables, functions etc. are defined as in the main text, unless stated differently. The correspondence, X, of financial feasible capital-imbalance and debt pairs is given by

$$X\left(K_t^*, w_t, z_t, B_t\right) = \left\{ z^o, B_{t+1} \in \mathbb{R}_+^2 | D(z^o, B_{t+1}, K_t^*, w_t, z_t, B_t) \geq 0 \wedge B_{t+1} \leq \hat{b} z^o K_t^* \right\}. \tag{30}$$

The first constraint is the positive dividend constraint the latter reflects the debt ceiling.

Dividends are given as in the main text by

$$D = \Pi(z_t^o, K_t^*) - A(z_t^o, K_t^*, w_t)\mathbb{I}_{\{z_t^o \neq z_t\}} - K_t^*(z_t^o - z_t) + B_{t+1} - (1 + r\,(b_t))B_t. \tag{31}$$

For notational convenience, we sometimes suppress the arguments of a function. Dividing D by K_t^* yields

$$\frac{D}{K_t^*} = \frac{\Pi(z_t^o, K_t^*)}{K_t^*} - \frac{A(z_t^o, K_t^*, w_t)\mathbb{I}_{\{z_t^o \neq z_t\}}}{K_t^*} - (z_t^o - z_t) + \frac{B_{t+1}}{K_t^*} - (1 + r\,(b_t))\frac{B_t}{K_t^*}. \tag{32}$$

Using $b_t := \frac{B_t}{K_{t-1}}$, we obtain $\frac{B_{t+1}}{K_t^*} = b^o z^o$. And for $\frac{B_t}{K_t^*}$ we obtain $\frac{B_t}{K_t^*} = \frac{B_t}{K_{t-1}} \frac{K_{t-1}}{K_t} \frac{K_t}{K_t^*} = b_t z_t \left(1-\delta\right)^{-1}$, because the stock of capital before investment is given by $K_{t-1} = \frac{K_t}{1-\delta}$. Hence,

$$\begin{aligned}\frac{D}{K_t^*} &= \pi\left(z^o\right) - \pi(z^o) w_t \mathbb{I}_{\{z^o \neq z_t\}} - \left(z^o - z_t\right) + b^o z^o - \frac{\left(1 + r\,(b_t)\right)}{1-\delta} b_t z_t && (33)\\ &= \pi(z^o)[1 - w_t \mathbb{I}_{\{z^o \neq z_t\}}] + \left(1 - \frac{(1 + r(b_t))}{1-\delta} b_t\right) z_t - \left(1 - b^o\right) z^o && (34)\end{aligned}$$

Now we introduce our short hand notation for the equity ratio

$$e_t := e(b_t) = 1 - \frac{(1 + r(b_t))}{1-\delta} b_t.$$

and obtain

$$\frac{D}{K_t^*} = \pi\left(z^o\right) - \pi(z^o)w_t \mathsf{I}_{\{z^o \neq z_t\}} - \left(z^o - z_t\right) + b^o z^o - \frac{\left(1 + r\left(b_t\right)\right)}{1-\delta} b_t z_t \tag{35}$$

$$= \pi(z^o)[1 - w_t \mathsf{I}_{\{z^o \neq z_t\}}] + e\left(b_t\right) z_t - \left(1 - b^o\right) z^o \tag{36}$$

This allows us to define X in relative terms as

$$\hat{X}\left(K_t^*, w_t, z_t, b_t\right) := \left\{ z^o, b^o \in \mathsf{R}_{++} \times \mathsf{R}_+ \,\middle|\, \begin{array}{c} \pi(z^o)[1 - w_t \mathsf{I}_{\{z^o \neq z_t\}}] + e\left(b_t\right) z_t \\ - \left(1 - b^o\right) z^o \geq 0 \wedge b^o \leq \hat{b} \end{array} \right\} \cup \{(0,0)\} \tag{37}$$

Since r is a continuous increasing function, e is a one-to-one mapping. Since $\hat{X}$ is independent of K_t^*, we define the positive part of $\hat{X}$ as a correspondence only in $\left(z_t, C, w_t\right)$:

$$Y\left(z_t, C, w_t\right) := \left\{ z^o, b^o \in \mathsf{R}_{++} \times \mathsf{R}_+ \,\middle|\, \pi(z^o)[1 - w_t \mathsf{I}_{\{z^o \neq z_t\}}] + C - \left(1 - b^o\right) z^o \geq 0 \geq b^o - \hat{b} \right\}. \tag{38}$$

For $e_t = e(b_t)$ and $C = e_t z_t$ we have $\hat{X} = Y \cup \{(0,0)\}$. Note that in this appendix we slightly differ in notation from the main text, when denoting the equity argument of Y and v later. Here we use only the compound $C = ez$, whereas in the main text we only have the equity ratio. We can do this replacement because of Lemma 2. This 'trick' will help to show that $\mathsf{E}_t\left[v(\cdot)\exp(\xi_{t+1})\right]$ is differentiable with respect to b.

Lemma 3 (Lemma 1 main text) *(a) Y is non-empty and*

(b) employing zero capital is suboptimal, i.e.

$$\max_{(z^o, b_t) \in Y} \tilde{v}(z^o, b_t) - \pi(z^o) w_t \mathsf{I}_{\{z^o \neq z_t\}} > \psi \mathsf{E}_t[v(w_{t+1}, 0, 0) > 0$$

Proof. *(a) Because of Assumption 2 $e_t \geq 0$, By definition $z \geq 0$ so that also $e_t z_t \geq 0$ holds. Thus to prove (a), it is sufficient to show, that the company can repay all debt, b_t, by selling all its capital. Than it reinstalls a small amount of capital $\underline{z}(w_t)$ for which it pays only from current earnings although it suffers from the stop in production:*

$$\exists \underline{z}(w_t) : \underline{z} - \pi(\underline{z})[1 - w_t \mathsf{I}_{\{\underline{z} \neq z_t\}}] \leq 0.$$

Because $\lim_{x \to 0} \pi'(x) = +\infty$ by assumption and $\frac{\pi(x)}{x} \geq \pi'(x)$ since π is concave , this "self-financing" $\underline{z}$ always exists.

(b) If the firm opts to stop production, it sells all capital, pays back debt and pays the

rest–its equity–out as dividend. Using $\underline{z}$ from part (a) a firm can always pay out all equity and still set $b_{t+1} = b^o = 0$ as well. Moreover, this means the company will start with a positive value of equity in the next period. Now it can again sell $\underline{z}$ and pays this out as a dividend and then is in exactly the same situation as the company that had chosen $(0, 0)$ in the previous period, but it can pay a dividend that is $\underline{z}$ units larger. This also implies that, $v(\cdot)$ must be bounded from below by a positive real number, so that $\psi E_t[v(w_{t+1}, 0, 0)] > 0$. ■

Moreover, define

$$Z(z, C) := \left\{ b^o \in \mathbb{R}_+ \,\middle|\, \pi(z) + C - (1 - b^o) z \geq 0 \geq b^o - \hat{b} \right\} \tag{39}$$

$$\text{and } Y^*(C, w) := \left\{ z^o, b^o \in \mathbb{R}_{++} \times \mathbb{R}_+ \,\middle|\, \pi(z^o)(1 - w) + C - (1 - b^o) z^o \geq 0 \geq b^o - \hat{b} \right\}. \tag{40}$$

One can easily verify that for $C = ez$, $Y = Y^* \cup (\{z\} \times Z)$. Moreover, due to the concavity of π both Y^* and Z are convex-valued, the union Y however may be not.

5.1.2 Value function

Now denote the value of a firm by V. For notational convenience define $Y := Y(z_t, C(z_t, b_t), w_t)$, $C = z_t e(b_t)$ and drop the arguments of $\hat{X}$ and Y.

V is determined by the following Bellman equation.

$$V(K_t^*, z_t, b_t, w_t) := \max_{(z^o, b^o) \in \hat{X}} \left\{ D(z^o, b^o, K_t^*, w_t, z_t, b_t) + \psi \mathsf{E}_t \left[V(K_{t+1}^*, w_{t+1}, z_{t+1}, b^o) \right] \right\} \tag{41}$$

Now, divide both sides by K_t^* and use (36). Replacing $\hat{X}$ by Y this yields because of Lemma 3

$$\frac{V(K_t^*, z_t, b_t, w_t)}{K_t^*} = \max_{(z^o, b^o) \in Y} \left\{ .\pi(z^o)[1 - w_t \mathsf{I}_{\{z^o \neq z_t\}}] + e(b_t) z_t - (1 - b^o) z^o + \psi \mathsf{E}_t \left[\frac{V(K_{t+1}^*, z_{t+1}, b^o, w_{t+1})}{K_t^*} \right] \right\} \tag{42}$$

$$= e(b_t) z_t + \max_{(z^o, b^o) \in Y} \left\{ \pi(z^o)[1 - w_t \mathsf{I}_{\{z^o \neq z_t\}}] - (1 - b^o) z^o + \psi \mathsf{E}_t \left[\frac{V_{t+1}}{K_{t+1}^*} \frac{K_{t+1}^*}{K_t^*} \right] \right\} \tag{43}$$

The second equality stems from the fact, that $e(b_t) z_t$ is not a function of (z^o, b^o) and is thus not affected by the maximization.

Finally, subtract $e(b_t) z_t$ from both sides and add $e(b^o) z_{t+1} - e(b^o) z_{t+1}$ in the expectations operator. Using $v = \frac{V}{K^*} - e(b_t) z_t$ then yields

$$
\begin{aligned}
v(z_t, e_t z_t, w_t) &:= \tfrac{V(K_t^*, z_t, b_t, w_t)}{K_t^*} - e_t z_t \\
&= \max_{(z^o, b^o) \in Y} \left\{ \begin{array}{c} \pi(z^o)[1 - w_t \mathbf{1}_{\{z^o \neq z_t\}}] - (1 - b^o) z^o, \\ +\psi \mathsf{E}_t \left[\left(\frac{V(K_{t+1}^*, z_{t+1}, b^o, w_{t+1})}{K_{t+1}^*} + e(b^o) z_{t+1} - e(b^o) z_{t+1} \right) \frac{K_{t+1}^*}{K_t^*} \right] \end{array} \right\}
\end{aligned}
\tag{44}
$$

$$
v(z_t, C, w_t) = \max_{(z^o, b^o) \in Y} \left\{ \begin{array}{c} \pi(z^o)[1 - w_t \mathbf{1}_{\{z^o \neq z_t\}}] - (1 - b^o) z^o + \psi(1 - \delta) e(b^o) z^o \\ +\psi \mathsf{E}_t \left[v\left(\frac{z^o(1-\delta)}{\exp(\xi_{t+1})}, e(b^o) \frac{z^o(1-\delta)}{\exp(\xi_{t+1})}, w_{t+1}\right) \exp(\xi_{t+1}) \right] \end{array} \right\}
\tag{45}
$$

The third equality is obtained by some term replacements. Equation (3) yields $\frac{K_{t+1}^*}{K_t^*} = \exp(\xi_{t+1})$ and

$$
z^o(1 - \delta) = \frac{(K_t + I_t)(1 - \delta)}{K_t^*} = \frac{K_{t+1}}{K_{t+1}^*} \frac{K_{t+1}^*}{K_t^*} = z_{t+1} \frac{K_{t+1}^*}{K_t^*}.
$$

That maximizing v indeed leads to an equivalent policy to maximizing V has been intuitively explained in the main text.

5.2 Existence and uniqueness

From now on time-indices will be suppressed. To bring out the discrete choice nature, we denote

$$
v(z, C, w) = \left\{ \begin{array}{ll} \max\{v_{no\ \text{adj}}(z, C), v_{\text{adj}}(C, w)\} & \text{for } Z \neq \emptyset \\ v_{\text{adj}}(C, w) & \text{for } Z = \emptyset \end{array} \right.
\tag{46}
$$

with

$$
\begin{aligned}
v_{no\ \text{adj}}(z, C) &:= \max_{b^o \in Z(z, C)} \left\{ \begin{array}{c} \pi(z) - (1 - b^o) z + \psi(1 - \delta) e(b^o) z \\ +\psi \int\int \left[v\left(z \frac{1-\delta}{\exp(\xi)}, e(b^o) z \frac{1-\delta}{\exp(\xi)}, \epsilon \right) \exp(\xi) \right] dF(\xi) dG(\epsilon) \end{array} \right\} \\
v_{\text{adj}}(C, w) &:= \max_{(z^o, b^o) \in Y^*(C, w)} \left\{ \begin{array}{c} \pi(z^o)(1 - w) - (1 - b^o) z^o + \psi(1 - \delta) e(b^o) z^o \\ +\psi \int\int \left[v\left(z^o \frac{1-\delta}{\exp(\xi)}, e(b^o) z \frac{1-\delta}{\exp(\xi)}, \epsilon \right) \exp(\xi) \right] dF(\xi) dG(\epsilon) \end{array} \right\}.
\end{aligned}
$$

Assumption 6: $\mu_\xi := \psi \mathsf{E}_t[\exp(\xi_{t+1})] < 1$.[30]

[30] This assumption is equivalent to assumption A.6 in Caballero and Engel (199, p. 811). Assume ξ is normally distributed with variance σ^2. Then this assumption is equivalent to $\exp\left(d + \frac{\sigma^2}{2}\right) < 1 + r$. Approximately, this is $r - d > \frac{\sigma^2}{2}$. Economically this means that productivity and hence value of a given stock of capital grows at a smaller rate than the market rate of return.

Suppose this assumption did not hold and neglect adjustment costs for the moment. It is easy to see that a firm could obtain infinite expected value by choosing a stock of capital that is small enough to

Definition 1 *Let T be defined by posing $(Tv)(z, C, w)$ equal to the right hand side of (45). This operator is defined on the set $\mathcal{B}$ of all real-valued, almost everywhere (a.e.) continuous and bounded functions with domain $D = R_{++} \times R_{+} \times [0, 1]$*

Lemma 4 *The mapping T preserves boundedness,*

Proof. *To show that T preserves boundedness, one has to show that for any bounded function u $(Tu)(\cdot)$ is bounded. Consider $u \in \mathcal{B}$, that is bounded from above by $\overline{u}$ and bounded from below by $\underline{u}$, then $(Tu)(\cdot)$ is bounded from above because*

$$
\begin{aligned}
(Tu)(z, C, w) &\leq \mu_\xi \overline{u} + \sup_{(z^o, b^o) \in Y} \left\{ \pi(z^o)[1 - w_t \mathbb{1}_{\{z^o \neq z_t\}}] - (1 - b^o) z^o + \psi (1 - \delta) e(b^o) z^o \right\} \\
&\leq \mu_\xi \overline{u} + \sup_{0 \leq z^o, 0 \leq b^o \leq \widehat{b}} \{\pi(z^o) - (1 - b^o) z^o + \psi (1 - \delta) e(b^o) z^o\} \\
&= \mu_\xi \overline{u} + \sup_{0 \leq z^o, 0 \leq b^o \leq \widehat{b}} \{[1 - \psi (1 + r(b^o))] b^o z^o + \pi(z^o) - (1 - \psi(1 - \delta)) z^o\} \\
&\leq \mu_\xi \overline{u} + \sup_{0 \leq z^o, 0 \leq b^o \leq \widehat{b}} \{(1 - \psi) z^o + \pi(z^o) - (1 - \psi(1 - \delta)) z^o\} \\
&= \mu_\xi \overline{u} + \sup_{0 \leq z^o} \{\pi(z^o) - \psi \delta z^o\} \qquad (47)
\end{aligned}
$$

The first inequality reflects the boundedness of u, which implies $\mathsf{E}_t \left[u \exp(\xi_{t+1}) \right] \leq \mu_\xi \overline{u}$. The second inequality results from dropping adjustment costs.
With the definition of $e(b)$ we obtain the first equality. The third inequality now follows from dropping r and $r(b^o) \geq 0$, and thereafter replacing b^o by 1 since $1 \leq \widehat{b} < 1$. The last supremum is bounded, because $\pi(z^o) - \psi \delta z^o$ obtains its maximum. This follows from our concavity assumption on π and our assumption on the first derivative of π that leads to

$$
\lim_{z^o \to 0} \pi'(z^o) - \psi \delta > 0 > \lim_{z^o \to \infty} \pi'(z^o) - \psi \delta.
$$

In fact inequality (47) states that the value of a company is always less than the maximum value the company can obtain in the next period plus the static maximum profit.
That $(Tu)(\cdot)$ is bounded from below follows from

$$
\begin{aligned}
(Tu)(z, ze, w) &\geq \mu_\xi \underline{u} + \sup_{(z^o, b^o) \in Y} \left\{ \pi(z^o)[1 - w_t \mathbb{1}_{\{z^o \neq z_t\}}] - (1 - b^o) z^o + \psi (1 - \delta) e(b^o) z^o \right\} \\
&> \mu_\xi \underline{u}.
\end{aligned}
$$

reproduce its depreciation plus the interest rate in the first period. In the next period it can be expected, that this stock of capital (depreciated capital replaced) generates a positive profit, which grows at a larger rate than the interest rate. In this sense, assumption 6 is an equilibrium condition for the capital-market.

The last inequality follows directly from Lemma 3.1–the optimality of no-bankruptcy. ■

Lemma 5 *T preserves almost everywhere (a.e.) continuity, in particular for any a.e. continuous function u $T(u)$ is continuous everywhere outside $A := \{(z, C)|C = (1 - \theta)z - \pi(z)\}$.*

Proof. *For every u that is bounded and continuous a.e. parameter integrals in (45) are continuous in z^o, b^o. Together with the other parts being continuous, the whole maximized function. Since, both Y and Z are continuous correspondences except for the (z, C)-pairs in $A := \{(z, C)|C = (1 - \theta)z - \pi(z)\}$, the maximization fulfills the assumptions of Berge's theorem at all points outside A. A is the set of points at which Z switches to being empty, it is those (z, e) pairs at which a marginal decrease in equity will force the firm to adjust capital to avoid bankruptcy.*
With Berge's theorem, $(Tu)(z, C, w)$ must be continuous for all points $(z, C) \notin A$. Now, A is a curve in R^2 and so has measure 0. Thus, (Tu) is continuous a.e. ■

Lemma 6 *T satisfies Blackwell's condition.*

Proof. *First notes that if $f_1, f_2 \in B$ and if $\forall (z, C, w) \in D : f_1(z, C, w) \leq f_2(z, C, w)$, then (because $\exp(d + \xi) > 0$) the expected value in (46) preserves the inequality, and so does the max-function. Thus*

$$(Tf_1)(z, C, w) \leq (Tf_2)(z, C, w).$$

Straightforward algebra shows that

$$(Tf + a)(z, C, w) = (Tf)(z, C, w) + \mu_\xi a$$

Assumption A.6 now yields the second Blackwell condition. ■

Proposition 1 *Equation (45) has exactly one solution (which belongs to* B*).*

Proof. *Lemmas 4 to 6 yields that T defines a contraction mapping on the metric space* B *with a modulus strictly smaller than one. The existence and uniqueness now follows from the contraction mapping theorem (See Theorem 3.2 in Stockey, Lucas and Prescott, 1989)* ■

5.3 Optimal policy

Now define the following function related to the solution of the Bellman equation of $v(z, ze, w)$:

$$I(z,C) := \psi \int\int v\left(z\frac{1-\delta}{\exp(\xi)}, C\frac{1-\delta}{\exp(\xi)}, \epsilon\right) \exp(\xi) \, dF(\xi)dG(\epsilon) \tag{48}$$

Lemma 7 *$I(z,b)$ is analytic in z and b and is hence differentiable of all order, if $e(b)$ is analytic*

Proof. *Note that I can be written as a convolution of an continuous function K and a normal density:*

$$\begin{aligned} I(z,C) &:= \psi \int K(\ln z - \xi, \ln C - \xi) \exp(\xi) dF(\xi) \\ K(s,u) &:= \int v\left(\exp\left(s + \ln\left(1-\delta\right)\right), \exp\left(u + \ln\left(1-\delta\right)\right), \epsilon\right) dG(\epsilon) \end{aligned}$$

K is continuous as it is a parameter integral, with a bounded and a.e. continuous kernel. Since the convolution of a normal density and a continuous function is analytic (see e.g. Theorem 9 on p. 59 in Lehmann (1986)), I is analytic in both arguments and hence it is analytic in z, and also in b if $e(b)$ is analytic. ■

Note that we can now express $v_{no \text{ adj.}}$ and $v_{adj.}$ in terms of I as

$$v_{no \text{ adj.}}(z,C) = \pi(z) + \max_{b^o \in Z(z,C)} \left(\psi(1-\delta)e(b^o) - (1-b^o)\right)z + I(z,b^o) \tag{49}$$

$$v_{\text{adj}}(C,w) := \max_{(z^o,b^o)\in Y^*(C,w)} \left\{\pi(z^o)(1-w) - (1-b^o)z^o + \psi(1-\delta)e(b^o)z^o + I(z^o,b^o)\right\} \tag{50}$$

Lemma 8 *There exist Lagrangian multipliers $\left(\lambda^+, \mu^+\right) \geq 0$ and $(\lambda^*, \mu^*) \geq 0$ to the two optimization problems above.*

Proof. *Since I is differentiable, all functions involved on the right hand side of the above two equations are differentiable. There must be a set of Lagrangian multipliers $\left(\lambda^+, \mu^+\right) \geq 0$ and $(\lambda^*, \mu^*) \geq 0$ associated with the constrained optimization problem when the two constraints that build Z*

$$g_n^1(b^o) = (1-b^o)z - \pi(z) \leq C \tag{51}$$

$$g_n^2(b^o) = b^o \leq \hat{b}, \tag{52}$$

and the two that build Y^*

$$g_a^1(z^o, b^o) = (1 - b^o) z^o - \pi(z^o)(1 - w) \leq C \tag{53}$$
$$g_a^2(z^o, b^o) = b^o \leq \hat{b}. \tag{54}$$

are differentiable with a non-zero first order derivative in either z^o *or* b^o. *Since* $z^o > 0$, *this obviously holds true.* ∎

Lemma 9 *Suppose that* $v_{no\ adj.}$ *and* $v_{adj.}$ *are differentiable. Note that this does not hinge on the differentiability of* v, *but hinges on the Hessian to the optimization problem* (49). *Differentiability follows from a non-singular Hessian. Then*

$$\frac{\partial v_{no\ adj.}}{\partial e} = z\lambda^+, \ \frac{\partial v_{no\ adj.}}{\partial z} = \frac{\partial \mathbf{e}(z, b^+)}{\partial z} + \lambda^+ e - \lambda^+ \left(1 - b^+\right) + \lambda^+ \pi'(z) \tag{55}$$
$$\frac{\partial v_{adj..}}{\partial e} = z\lambda^*, \ \frac{\partial v_{adj.}}{\partial z} = \lambda^* e \tag{56}$$

Proof. *Denote* b^+ *and* (z^*, b^*) *some maximizers of the no adjustment and the adjustment case respectively. Under the assumed differentiability the Envelope Theorem generates the result. Recall that* $\mathbf{e}(z, b^+) := \pi(z) + (\psi(1 - \delta) e(b^+) - (1 - b^+)) z + I(z, b^+)$.
∎

Next we define an auxiliary function, which is defined for arbitrary weakly positive adjustment costs w not restricted to $[0, 1]$:

$$J(z, b, w) := \mathbf{e}(z, b) - w\pi(z) \tag{57}$$
$$= \pi(z)(1 - w) - (1 - b) z + \psi(1 - \delta) e(b) z + I(z, ze(b)) \tag{58}$$

We first show that critical adjustment costs exist, so that the firm becomes indifferent between adjusting and not adjusting.

Lemma 10 *The function* $J(z, b, w)$ *is bounded from above for* (z, b), *so that* $\sup_{(z^o, b^o) \in Y} J(z, b^o, w)$ *is finite.*

Proof. *As* v *satisfies the Bellman-equation, it must be bounded. However, since*

$$v(z, ze, 0) = \sup_{(z^o, b^o) \in Y^*(ze, 0)} J(z^o, b^o, 0) \geq \sup_{(z^o, b^o) \in Y^*(ze, w)} J(z^o, b^o, w)$$

always hold, J *must be bounded, too.* ∎

Corollary 1 *$v_{no\ adj.}$ and $J_{\max}(z,e,w) := \sup_{(z^o,b^o)\in Y^*(ze,w)} J(z^o,b^o,w)$ are both bounded.*

Lemma 11 *(a) J and $J_{\max}$ are strictly monotonously decreasing in w without bound. (b) $v_{no\ adj.}$ is independent of w.*

Proof. *(a) For any $w_1, w_2 \in [0;1] : w_1 < w_2$ we have:*

$$\begin{aligned} J(z,b,w_2) &= \mathbf{e}(z,b) - w_2\pi(z) \\ &< \mathbf{e}(z,b) - w_1\pi(z) = J(z,b,w_1) \end{aligned}$$

And since $Y_2 := Y(z,e,w_2) \subset Y(z,e,w_1) =: Y_1$, we get:

$$\begin{aligned} J_{\max}(z,e,w_2) &= \max_{(z^o,b^o)\in Y_2} J(z^o,b^o,w_2) \leq \max_{(z^o,b^o)\in Y_1} J(z^o,b^o,w_2) \\ &< \max_{(z^o,b^o)\in Y_1} J(z^o,b^o,w_1) = J_{\max}(w_1,z,e) \end{aligned}$$

(b) follows directly from the definition of $v_{no\ adj.}$. ■

Proposition 2 *Define for $Z \neq \emptyset$ as an implicit function $\overline{w}(z,e)$ by*

$$J_{\max}(ez,\overline{w}) - v_{no\ adj.}(z,ez) = 0 \tag{59}$$

Then firms adjust if their current adjustment cost factor w is smaller than $\overline{w}(z,e)$ or if $Z = \emptyset$.

Proof. *That a unique $\overline{w}(z,e)$ equating $J_{\max}$ and $v_{no\ adj.}(z,ez)$ exists, follows from $J_{\max}(ez,w) \geq v_{no\ adj.}(z,ez)\ \forall(z,e)$ together with the monotonicity of $J_{\max}$ in w. As argued in the main text firms adjust if $Z = \emptyset$ or $J_{\max}(ze,w) > v_{no\ adj.}(z,ze)$. Since $J_{\max}$ is monotonously decreasing in w this inequality holds if and only if $w < \overline{w}$.* ■

Next, we show that the set of maximizers must at least be finite, if there is no unique maximizing (z,b)-pair

Proposition 3 *Suppose that π and r are analytic, that $J(z,b,w)$ is analytic and thus the set Q of $(z^o,b^o) \in Y^*(C,w)$ such that $J(z^o,b^o,w) = J_{\max}(C,w)$ is a non-empty set with a finite number of points.*

Proof. *Since I is analytic, so is J. As J is analytic it must be continuous, too. Since $Y^*(C,w)$ is compact this ensures that J attains its maximum on a non-empty compact set Q. Since J is analytic the maxima are isolated, so that Q contains a finite number of elements.* ■

Finally, we derive the reaction of critical costs $\bar{w}$ to changes in e and z.

Proposition 4 *Suppose that the assumptions of 9 hold, then $\overline{w}(z,e)$ is differentiable on the open and convex set $\bar{A} := \left\{(e,z) \in R_{+}^{2} | e > (1-\hat{\theta}) - \frac{\pi(z)}{z}\right\}$, of which A is the border. Then also $\Omega(z,e)$ is differentiable on $R_{+}^{2}\backslash A$. Moreover, if $\overline{w}(z,e) \leq 1$*

$$\frac{\partial \bar{w}}{\partial e} = \frac{1}{\pi\left(z^{*}\right)}\left(\lambda^{*}-\lambda^{+}\right) z \tag{60}$$

$$\frac{\partial \bar{w}}{\partial z} = \frac{1}{\pi\left(z^{*}\right)}\left[\left(\lambda^{*}-\lambda^{+}\right) e - \frac{\partial \tilde{v}(z,b^{+})}{\partial z} + \lambda^{+}\left(1-b^{+}\right) - \lambda^{+}\pi'\left(z\right)\right] \tag{61}$$

Proof. *As (59) implicitly defines $\bar{w}$, $\bar{w}$ is differentiable when $J_{\max}$ and $v_{no\ adj.}$ are differentiable, this ensures the assumption. We can alternatively (see main text) write the equation which implicitly defines $\bar{w}$ as:*

$$\bar{w}\left(z,e\right)\pi\left(z^{*}\left(z,e,\bar{w}\right)\right) = \tilde{v}(z^{*}\left(z,e,\bar{w}\right), b^{*}\left(z,e,\bar{w}\right)) - \tilde{v}(z, b^{+}\left(z,e\right)).$$

Differentiating this equation with respect to e, yields

$$\frac{\partial \bar{w}}{\partial e}\pi\left(z^{*}\right) + \bar{w}\frac{\partial \pi}{\partial z}\frac{\partial z^{*}}{\partial e} = \frac{\partial}{\partial e}\tilde{v}(z^{*}\left(z,e,\bar{w}\right), b^{*}\left(z,e,\bar{w}\right)) - \frac{\partial}{\partial e}\tilde{v}(z, b^{+}\left(z,e\right)).$$

Now subtract $\bar{w}\frac{\partial \pi}{\partial z}\frac{\partial z^{*}}{\partial e}$ from both sides, and observe that $v_{adj}(z, ze, \bar{w}) = \tilde{v}\left(z^{*}\left(z,e,\bar{w}\right), b^{*}\left(z,e,\bar{w}\right)\right) - w\pi\left(z^{*}\left(z,e,\bar{w}\right)\right)$. This yields

$$\frac{\partial \bar{w}}{\partial e}\pi\left(z^{*}\right) = \frac{\partial}{\partial e}v_{adj}(z, ze, \bar{w}) - \frac{\partial}{\partial e}v_{no\ adj}(z,e).$$

Analogously we obtain

$$\frac{\partial \bar{w}}{\partial z}\pi\left(z^{*}\right) = \frac{\partial}{\partial z}v_{adj}(z, ze, \bar{w}) - \frac{\partial}{\partial z}v_{no\ adj}(z,e).$$

Lemma 9 now yields the stated result. ■

5.4 Data Appendix

5.4.1 Construction of the database and definition of the variables

The Cambridge DTI data covers in total 50494 company-year observations with companies coming from a variety of industries. 20 of these industries are classified as manufacturing. Our finally selected sample–as discussed in the main text–consists of 6272 observations from 631 firms, the number of observations for each industry are shown in

Table 6: Number of observations per industry

Industry		Observations	Industry		Observations
5	mixed activities, manuf.	190	38	vehicles	226
18	mineral and ore extract.	74	39	metal goods	285
21	food	493	41	textiles	416
23	drink	549	43	leather	45
24	tobacco	19	44	footwear & clothing	236
26	chemicals	771	46	non-metallic mineral	256
31	metals	278	47	timber	84
33	mechanical engineering	950	48	paper, printing, publ.	494
36	electrical engineering	491	49	other manufacturing	306
37	shipbuilding	14	91	manufacturing	95

Table 7: Definitions of variables in terms of basic sample items

Item	Variable No.	Item	Variable No.
Land & buildings, $K^{\#}$ (gross)	var114	Sales, Y	var127
Depreciation on $K^{\#}$., DK	var117	Liabilities, D	var8+var9+var10
Machinery, etc. , $M^{\#}$ (gross)	var115		+var11+var12-var5
Depreciation on $M^{\#}$, DM	var118	Liquid assets,	var19+var20+var21
Employee remuneration, wL	var135	LA	+0.5*var18

table 6. Each company year in the DTI data includes 150 variables extracted from the published company account.

These variables are numbered var1-var150 and in what follows reference is to these numbers. Besides this data, the database also includes identification variables. Table 7 gives the basic definitions of the variables that we use.

The preliminary values for buildings $K^{\#}$ and machinery $M^{\#}$ just as the values for depreciation, sales and remuneration are directly taken from the data. Both actual capital series, K and M, are generated by the perpetual inventory method, taking the first reported value for a company (in period $T_i^{\min}$) as a starting value, i.e.

$$K_{it} = \frac{K_{it}^{\#} - \left(K_{it-1}^{\#} - DK_{it-1} \right)}{P_t} + K_{it-1} \text{ for } t > T_i^{\min}$$

$$K_{it} = \frac{K_{it}^{\#}}{P_t}, \; t = T_i^{\min}$$

The annual investment hence is $I_{it} := \frac{K_{it}^{\#} - \left(K_{it-1}^{\#} - DK_{it-1}\right)}{P_t}$. The series for machinery M is constructed analogously to K.

All other items are directly inflated using the price index. To calculate the equity ratio–which is based on book values–we first add up all long-term liabilities (var8), bank loans (var9), trade liabilities (var10), interest liabilities (var11), and tax liabilities minus tax reserves (var12-var5). From this number we then subtract the value of liquid assets, which are composed of marketable securities (var19), tax reserve certificates (var20), cash (var21) and 50% of the trade claims the company holds (var18).[31] This sum we now divide by the book value of assets, $M + K$, and subtract the result (the debt ratio) from 1 to obtain the equity ratio:

$$e_{it} = \frac{M_{it} + K_{it} - (D_{it} - LA_{it})}{M_{it} + K_{it}} = 1 - \frac{D_{it} - LA_{it}}{M_{it} + K_{it}}.$$

Depreciation rates are calculated as the book value of depreciation over the actual book value of the asset, e.g. $\delta_{it}^K = \frac{DK_{it}}{K_{it}^{\#}}$. Firm average depreciation rates $\bar{\delta}_{iK}$ are then calculated as the simple arithmetic average over a firm's depreciation rates. Finally investment rates are calculated as

$$i_{jt} = \frac{I_{jt}}{K_{jt-1}\left(1 - \delta_{jt}^K\right)}.$$

5.4.2 Constructing the capital productivity measure

To construct the measure ξ_{it} of the desired stock of capital, we start from the optimization problem of a firm that employs three factors: Land K, Machinery, M, and Labor L. It produces according to a Cobb-Douglas production function with decreasing returns to scale (e.g. due to monopoly power)

$$\ln(Y_{it}) = \alpha_{it}^0 + \alpha_{iK} \ln(K_{it}) + \alpha_{iL} \ln(L_{it}) + \alpha_{iM} \ln(M_{it}), \ \alpha_{iK} + \alpha_{iL} + \alpha_{iM} < 1 \quad (62)$$

Assuming M and L can be adjusted without adjustment costs yields as first order optimality conditions

$$\alpha_{iL} Y_{it} = w_{it} L_{it} \quad (63)$$

$$\alpha_{iM} Y_{it} = (r + \delta_{iM}) M_{it}. \quad (64)$$

Assuming this also holds for K in the long-run, we can estimate α_{iX} as the industry

[31] This means we use a relatively conservative valuation rule for trade claims. From the balance it is not clear how liquid these claims are.

average

$$
\begin{aligned}
\hat{\alpha}_{iK} &= \frac{1}{T_i} \sum_{t=1}^{T_i} \frac{\left(r + \bar{\delta}_{iK}\right) K_{it}}{Y_{it}} \\
\hat{\alpha}_{iM} &= \frac{1}{T_i} \sum_{t=1}^{T_i} \frac{\left(r + \bar{\delta}_{iM}\right) M_{it}}{Y_{it}} \\
\hat{\alpha}_{iL} &= \frac{1}{T_i} \sum_{t=1}^{T_i} \frac{w_{it} L_{it}}{Y_{it}}.
\end{aligned}
$$

Moreover, substituting (63) and (64) into (62), we obtain

$$
\ln\left(Y_{it}\right)\left(1 - \alpha_{iL} - \alpha_{iM}\right) = \alpha_{it}^{0} + \alpha_{iK} \ln\left(K_{it}\right) + \alpha_{iL} \ln\left(\frac{\alpha_{iL}}{w_{it}}\right) + \alpha_{iM} \ln\left(\frac{\alpha_{iM}}{r + \delta_M}\right).
$$

Substituting in $\psi_i^K := \frac{\alpha_{iK}}{(1-\alpha_{iL}-\alpha_{iM})}$ and

$$
\psi_{it}^{0} := \alpha_{it}^{0} + \alpha_{iL} \ln\left(\frac{\alpha_{iL}}{w_{it}}\right) + \alpha_{iM} \ln\left(\frac{\alpha_{iM}}{r + \bar{\delta}_{iM}}\right), \tag{65}
$$

we obtain the semi-reduced production function mentioned in the main text

$$
y_{it} = \psi_{it}^{0} + \psi_i^K k_{it}.
$$

and ξ defined as

$$
\xi_{it} := \frac{\psi_{it}^{0} + \ln \psi_i^k - c}{1 - \psi_i^k}. \tag{66}
$$

Now, there are three ways to estimate ξ_{it}. All three methods estimate ψ_i^K by using the expenditure shares as estimates for the alphas and hence yield $\hat{\psi}_i^K = \frac{\hat{\alpha}_{iK}}{(1-\hat{\alpha}_{iL}-\hat{\alpha}_{iM})}$. However, they differ in the way ψ_{it}^{0} is estimated. Basically they all result from inverting (62) to obtain total factor productivity α_{it}^{0} and then subsequently substituting in (66). However, they differ in such way that the indirect calculations replace y_{it} by the expenditure on other factors. For the actual calculations, these three options sometimes ignore constant factors that will be cancelled out by the within transformation anyway.

Table 8: Factor analysis and descriptive statistics
for the three measures of capital productivity

Factor	Eigenvalue	Difference	Proportion	Cumulative
$1(\xi_{it})$	2.6140	2.650	1.035	1.035
2	-0.0361	0.016	-0.014	1.021
3	-0.0524	.	-0.021	1.000

Factor Loadings			Scoring Coefficients	
Variable	ξ_{it}	Uniqueness	Variable	ξ_{it}
$\xi_{it}^{(1)}$	0.8989	0.1920	$\xi_{it}^{(1)}$	0.3459
$\xi_{it}^{(2)}$	0.9511	0.0953	$\xi_{it}^{(2)}$	0.3608
$\xi_{it}^{(3)}$	0.9552	0.0876	$\xi_{it}^{(3)}$	0.3621

Variable	Obs	Mean	Std. Dev.	Min	Max
$\xi^{(1)}$, direct	6272	12.675	1.532	5.389	17.449
$\xi^{(2)}$, machinery	6272	12.152	1.655	5.021	17.986
$\xi^{(3)}$, labor	6272	11.578	1.539	1.893	16.890

These three preliminary productivity estimates are given by

$$\xi_{it}^{(1)} = \frac{y_{it} - \hat{\alpha}_{iK} k_{it} - \hat{\alpha}_{iM} \ln(M_{it}) - \hat{\alpha}_{iL} \ln(w_{it} L_{it})}{1 - \hat{\alpha}_{iK} - \hat{\alpha}_{iL} - \hat{\alpha}_{iM}} \tag{67}$$

$$\xi_{it}^{(2)} = \frac{1 - \hat{\alpha}_{iL} - \hat{\alpha}_{iM}}{1 - \hat{\alpha}_{iK} - \hat{\alpha}_{iL} - \hat{\alpha}_{iM}} \ln\left(\frac{M_{it}}{\alpha_{iM}}\right) - \frac{\hat{\alpha}_{iK}}{1 - \hat{\alpha}_{iK} - \hat{\alpha}_{iL} - \hat{\alpha}_{iM}} k_{it} \tag{68}$$

$$\xi_{it}^{(3)} = \frac{1 - \hat{\alpha}_{iL} - \hat{\alpha}_{iM}}{1 - \hat{\alpha}_{iK} - \hat{\alpha}_{iL} - \hat{\alpha}_{iM}} \ln\left(\frac{w_{it} L_{it}}{\alpha_{iL}}\right) - \frac{\hat{\alpha}_{iK}}{1 - \hat{\alpha}_{iK} - \hat{\alpha}_{iL} - \hat{\alpha}_{iM}} k_{it}. \tag{69}$$

All three measures are highly correlated, but differ somewhat. If we assume that all three of them measure the true capital productivity ξ_{it} with an error, i.e.

$$\xi_{it}^{(j)} = b_j \xi_{it} + \eta_{it}^{(j)} \tag{70}$$

we can recover ξ_{it} by factor analysis as the common factor in all three measures. Indeed, we find only one significant common factor. Interestingly, in line with Cooper and Haltiwanger's (2002) finding, the indirect measure based on the labor employment decision, $\xi_{it}^{(3)}$, is most informative as a stand alone measure, which can be seen from the factor loadings (uniqueness).

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