Innovation, Firm Dynamics, and International Trade*

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Abstract

We present a general equilibrium model of the decisions of firms to innovate and to engage in international trade. We use the model to study the changes in aggregate productivity and welfare that arise as firms' exit, export, process- and product innovation decisions respond to a change in the marginal cost of international trade.

We first consider three important special cases of our model that we can solve analytically. In the first special case, all firms export. In the second special case, as in the model of Melitz (2003), only the most productive firms export but firms have no productivity dynamics after entry. In the third special case, firms have endogenous productivity dynamics but exit and export decisions are independent of size. We then extend our results to parameterized specifications of the model we must solve numerically.

Our central finding is that, despite the fact that a change in trade costs can have a substantial impact on individual firms' exit, export, and process innovation decisions, the firms' free-entry condition places a constraint on the overall response of aggregate productivity to the change in trade costs. In particular, we show that the steady-state response of product innovation largely offsets the impact of changes in firms' exit, export, and process innovation decisions on aggregate productivity. We also find that the dynamic welfare gains from a reduction in trade costs are very similar to the welfare gains that arise directly from the reduction in trade costs.

Our results suggest that micro evidence on individual firms' responses to changes in international trade costs may not be informative about the macroeconomic implications of changes in these trade costs for aggregate productivity and welfare.

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1. Introduction

There is a large and rapidly growing empirical literature that has documented that a reduction in international trade costs can have a substantial impact on individual firms' decisions to produce, export, and engage in research and development to improve the cost or quality of existing products.[1] Motivated by these observations, we build a simple general equilibrium model of these decisions, and we use this model to examine the question: do considerations of the impact of a reduction in trade costs on heterogeneous firms' decisions to produce, export, and innovate, lead to new answers to the macroeconomic question of the impact of a reduction in trade costs on aggregate productivity and welfare? Our answer is largely, no.

For the last several decades, research in international trade has modeled comparative advantage as an attribute of the firm.[2] We follow this approach, and model firms as producing differentiated products that are traded subject to both a fixed and a marginal cost of exporting. Our model of innovation builds on Griliches' (1979) knowledge capital model of firm productivity. Each firm has a stock of a firm specific factor that determines its current profit opportunities. Our model includes two forms of innovation: innovation to increase the stock of this firm specific factor in an existing firm — *process innovation*, and innovation to create new firms with a new initial stock of the firm specific factor — *product innovation*.

We use the model to study the changes in aggregate productivity and welfare that arise as firms' exit, export, process and product innovation decisions respond to a change in the marginal cost of international trade.[3] In our analysis, we find it useful to decompose the change in aggregate productivity that arises from a change in the marginal costs of trade into two components. The first component is the *direct effect* of a change of trade costs on productivity, holding fixed firms' exit, export, process, and product innovation decisions. The magnitude of this effect is simply determined by the share of exports in production, and hence is independent of the details of our model of heterogeneous firms' decisions. The second effect is the *indirect effect* that arises from changes in firms' exit, export, process,

[1] Bernard, Jensen, Schott and Redding (2007) survey this literature. In addition, see the work of Aw, Roberts and Xu (2009), Bloom, Draca, and Van Reenen (2009), Bustos (2007), De Locker (2007), and Lileeva and Trefler (2007).

[2] See Bernard, Eaton, Jensen, and Kortum (2003), Krugman (1979), Melitz (2003), and the review in Helpman (2008) for theoretical models of the role of firms in international trade.

[3] Throughout this paper, we consider an ideal measure of aggregate productivity that takes into account the introduction of new varieties. This concept of aggregate productivity is not necessarilly what is measured in the data (see, for example, Bajona, Gibson, Kehoe, and Ruhl 2008). We focus on the ideal measure of productivity because it is this that matters for welfare in our model.

and product innovation decisions.

What determines the magnitude of the indirect effect? An earlier theoretical literature stemming from the work of Krugman (1979), Grossman and Helpman (1991), and Rivera-Batiz and Romer (1991) looked at this question focusing only on the impact of a reduction in international trade costs on firms' decisions to create new product varieties — that is to engage in product innovation. Our main finding is that the more complex models of the heterogeneous responses of firms' exit, export, process and product innovation decisions that have followed this earlier work, lead to largely similar implications for the magnitude of the indirect effect of a reduction in trade costs on aggregate productivity.

To establish this finding, we first present analytical results regarding the steady-state impact of a change in marginal trade costs on aggregate productivity for three important special cases of our model. In the first special case, we assume that *all firms export.* This specification of our model extends the work of Krugman (1979) in considering firms' exit and process innovation decisions. In the second special case, only the most productive firms export, but firms have *no productivity dynamics* after entry, and hence this special case of our model corresponds to the model in Melitz (2003). In the third special case, which we refer to as the *exogenous selection* version of our model, firms have productivity dynamics due to endogenous process innovation, but exit and export decisions are independent of size. In the second and third special cases, we also assume that the real interest rate is zero. We find analytically that the indirect effect on aggregate productivity of a change in the marginal costs of trade is, to a first-order approximation, the same in all three of these special cases of our model, and equal to the indirect effect found in the earlier models with only product innovation. Hence, for these special cases, the details of how a change in trade costs affects firms' exit, export, and process innovation decisions do not affect at all our model's implications for aggregate productivity in the steady state.

Our model does imply that when firms are heterogeneous, a reduction in trade costs leads to a reallocation of production, export status, and investments in process innovation from smaller, less-productive, non-exporting firms to larger, more-productive, exporting firms and this reallocation does lead to a change in the productivity of the average firm. Why is it that this reallocation does not matter for our model's implications for aggregate productivity? The logic of our argument depends critically on firms' free-entry condition: the profits associated with creating a new product must be zero in equilibrium. Ceteris-paribus, a reduction in international trade costs raises the profits associated with creating a new product in direct proportion to the share of exports in the expected present value of profits of entering firms, independent of the details of firms' exit, export, and process innovation decisions. This is because an envelope condition implies that, at the margin, changes in these decisions lead to no additional effects of a change in trade costs on expected profits. In equilibrium, to satisfy the free-entry condition, this increase in expected profits must be offset by an increase in the real wage and a change in aggregate output. Our result that the details of firms' exit, export, and process innovation decisions do not matter for aggregate productivity thus follows from the fact that changes in the real wage and aggregate output required to satisfy the free-entry condition are also independent of the details of these decisions. To establish the link between the real wage, aggregate output, and aggregate productivity in our model, we show that, with CES demand functions and when either all firms export or the real interest rate is zero, the real wage and aggregate output are functions only of aggregate productivity. Therefore, the free-entry condition requires that whatever change in the productivity of the average firm that arises from changes in firms' exit, export, and process innovation decisions must be offset by a change in product innovation so as to ensure that the responses of the real wage and output are independent of these decisions.

In these analytical results, we make strong assumptions and consider only marginal changes in trade costs. To calculate the indirect effect of a change in trade costs on aggregate productivity when we relax those assumptions, we must solve the model numerically. To do so, we consider a parameterized version of our model that accounts for some salient features on the size dynamics and distribution of large firms, and the share of exporters in output and employment in the U.S. economy. Our numerical results confirm our analytical findings both when the real interest rate is low or when firms' investments in process innovation are inelastic to changes in the incentives to innovate.

We find, however, that in a specification of our model with both elastic process innovation and positive real interest rates, the counter-balancing changes in process and product innovation, while still substantial, are not exactly offsetting. In this case, our model with heterogeneous firms does give a new answer for the impact of a decline in marginal trade costs on aggregate productivity, output, and consumption in steady-state, relative to a model that considers only product innovation. It is for this reason that we qualified our answer to the question that motivates this paper. While we cannot say that there is no additional indirect effect on aggregate productivity in this specification of our model, we can say that this effect is small relative to the responses of the productivity of the average firm (two orders of magnitude smaller). Moreover, this indirect effect can be negative. With regards to welfare, in this specification of our model we find that the gains associated with the indirect effect of a reduction in trade costs on aggregate productivity are negligible because the transition dynamics to the new steady-state are very slow.

Our model is closely related to several papers in the literature. If we assume that firms' process innovation choices are inelastic, then our model is an open economy version of the models in Hopenhayn (1992) and Luttmer (2007) in which firms experience exogenous random shocks to their productivity.[4] Our model of process innovation is similar to the one in Ericson and Pakes (1995), in which the fruits of innovative activity are stochastic, which implies that our model can account for simultaneous growth and decline, and entry and exit of firms in steady-state.[5] Our model also relates to the models in Yeaple (2005) and Bustos (2007), which study the adoption of technology improvements by exporters and non-exporters in response to a change in trade costs. Costantini and Melitz (2007) use a model to study how the dynamics of trade liberalizations shapes the pace of these technology upgrades.[6] Our result that a change in international trade costs has no impact on innovative effort if all firms export echoes the result in Eaton and Kortum (2001) in a model of quality ladders embedded in a multi-country Ricardian model of international trade. Our paper also complements the models of firm-level innovation of Klette and Kortum (2004) and Lentz and Mortensen (2006).

Our paper is also related to a large body of work on the aggregate implications of trade liberalizations. Baldwin and Robert-Nicoud (2008) study a variant of Melitz's (2003) model that features endogenous growth through spillovers. They show that a reduction in international trade costs can increase or reduce growth via changes in product innovation, depending on the nature of the spillovers and the form of the production function of new goods. Our model abstracts from such spillovers. Arkolakis et. al. (2008) show that the welfare gains from a reduction in trade costs in models with heterogeneous firms and endogenous exit

[4]Such a model is considered in Irarrazabal and Opromolla (2006). Furthermore, Arkolakis (2007) extends this model of firm dynamics to account for other salient features of the data on firm dynamics by domestic and exporting firms.

[5]Doraszelski and Jaumandreu (2006) estimate a Griliches' knowledge capital model in which innovative investments within the firm also lead to stochastic productivity improvements.

[6]See also the related work of Navas-Ruiz and Sala (2007), and Long, Raff and Stähler (2008).

and export decisions are equal to the welfare gains in simpler models of trade that abstract from endogenous exit and export decisions, if these models are calibrated to match the same share of trade in GDP and the same elasticity of trade flows with respect to the marginal trade costs. We extend their results by showing that, even if the elasticity of trade differs across these models, the endogenous choices of exit and exporting have no first-order effects on aggregate productivity when the real interest rate is zero or the distribution of initial productivities is Pareto.[7]

The paper is organized as follows. Section 2 presents our model and Section 3 characterizes the symmetric steady-state equilibrium. Section 4 characterizes the steady-state impact of a change in trade costs in specifications of our model that we can solve for analytically. Section 5 extends the results of Section 4 to specifications of our model that we must solve for numerically. Section 6 concludes. The Appendix provides proofs and other details for our analytic results.

2. The Model

Time is discrete and labeled $t = 0, 1, 2, \ldots$. There are two countries: home and foreign. Variables pertaining to the foreign country are denoted with a star. Households in each country are endowed with L units of time. Production in each country is structured as follows. There is a single final nontraded good that can be consumed or used in innovative activities, a continuum of differentiated intermediate goods that are produced and can be internationally traded subject to a fixed and a variable trade cost, and a nontraded intermediate good that we call the *research good*. This research good is produced using a combination of final output and labor, and is used to pay the costs associated with both process and product innovation, as well as the fixed costs of exporting and production. The productivities of the firms producing the differentiated intermediate goods are determined endogenously through equilibrium process innovation, and the measure of differentiated intermediate goods produced in each country is determined endogenously through product innovation.

Intermediate goods are differentiated products each produced by heterogeneous firms indexed by two firm specific state variables, z and n_x, indexing the firm's productivity and its fixed cost of exporting, respectively. In what follows, we index the firm's production,

[7] In related work, Alessandria and Choi (2007) and Baldwin and Forslid (2006) study the welfare gains of trade liberalizations in a dynamic version of Melitz' model that abstracts from process innovation.

pricing, and export decisions by these state variables. We assume that the fixed cost of exporting, n_x, evolves exogenously for each firm according to a Markov process in which the distribution of this cost next period given cost n_x this period, is given by $\Gamma(n_x'|n_x)$.

A firm in the home country with state variables $s = (z, n_x)$ has productivity equal to $\exp(z)^{1/(\rho-1)}$ and produces output $y_t(s)$ with labor $l_t(s)$ according to the CRS production technology[8]

$$y = \exp(z)^{1/(\rho-1)} l. \tag{2.1}$$

In addition, to continue, the firm requires a fixed cost of n_f units of the research good every period. We rescale firm productivity using the exponent $1/(\rho - 1)$ for expositional convenience, where $\rho > 1$. As we explain below, with this rescaling, each firm's equilibrium labor and variable profits are proportional to $\exp(z)$.

The output of this firm can be used in the production of the home final good, with the quantity of this domestic absorption denoted $a_t(s)$. Alternatively, some of this output can be exported to the foreign country to be used in the production of the foreign final good. The quantity of the output of this firm used in the foreign country is denoted $a_t^*(s)$.

International trade is subject to both fixed and iceberg type costs of exporting. The iceberg type marginal cost of exporting is denominated in terms of the intermediate good being exported. The firm must export Da^* units of output, with $D \geq 1$, to have a^* units of output arrive in the foreign country for use in the production of the foreign final good.

Let $x_t(s) \in \{0, 1\}$ be an indicator of the export decision of home firms with state variables s (it is 1 if the firm exports and 0 otherwise). Then, feasibility requires that

$$a_t(s) + x_t(s) D a_t^*(s) = y_t(s) \tag{2.2}$$

and that $x_t(s) n_x$ units of the research good be used to pay fixed costs of exporting.

A firm in the foreign country with state variables s has the same production technology, with output denoted $y_t^*(s)$, labor $l_t^*(s)$, and domestic absorption $b_t^*(s)$. Exports to the home country, $b_t(s)$, are subject to both fixed and marginal costs and hence feasibility requires that $x_t^*(s) D b_t(s) + b_t^*(s) = y_t^*(s)$, and that $x_t^*(s) n_x$ units of the foreign research good be used to pay the fixed costs of exporting.

[8] Our model can be easily extended to include other forms of physical and human capital. Consideration of these forms of capital will lead to the standard amplification of the impact of changes in productivity on output.

The home final good is produced from home and foreign intermediate goods with a constant returns production technology of the form

$$Y_t = \left[\int a_t(s)^{1-1/\rho} \, \mathrm{d}M_t + \int x_t^*(s) b_t(s)^{1-1/\rho} \, \mathrm{d}M_t^* \right]^{\rho/(\rho-1)} , \tag{2.3}$$

where M_t is the measure of operating firms in the home country over the state s, and M_t^* the corresponding measure in the foreign country. Production of the final good in the foreign country is defined analogously.

The final good in the home country is produced by competitive firms that choose output Y_t and inputs $a_t(s)$ and $b_t(s)$ subject to (2.3) to maximize profits taking prices P_t, $p_{at}(s)$, $p_{bt}(s)$, export decisions $x_t(s)$, $x_t^*(s)$, and measures of operating intermediate goods firms M_t and M_t^* as given. All prices in the home country in period t are stated relative to the price of the research good in the home country in the same period, which is normalized to one. Standard arguments give that equilibrium prices must satisfy

$$P_t = \left[\int p_{at}(s)^{1-\rho} \, \mathrm{d}M_t + \int x_t^*(s) p_{bt}(s)^{1-\rho} \, \mathrm{d}M_t^* \right]^{1/(1-\rho)} , \tag{2.4}$$

and are related to quantities by

$$\frac{a_t(s)}{Y_t} = \left(\frac{p_{at}(s)}{P_t} \right)^{-\rho} \text{ and } \frac{b_t(s)}{Y_t} = \left(\frac{p_{bt}(s)}{P_t} \right)^{-\rho} . \tag{2.5}$$

Analogous equations hold for prices and quantities in the foreign country.

The research good in the home country is produced with a constant returns to scale production technology that uses Y_{rt} units of the home final good and L_{rt} units of labor to produce $L_{rt}^{\lambda} Y_{rt}^{1-\lambda}$ units of the research good, with $\lambda \in [0,1]$. The foreign research good is produced symmetrically. We denote the relative price of the research good across countries by W_{rt}^*. In each country, the research good is produced by competitive firms. Standard cost minimization requires that

$$\frac{\lambda}{1-\lambda} \frac{Y_{rt}}{L_{rt}} = \frac{W_t}{P_t} , \; \frac{\lambda}{1-\lambda} \frac{Y_{rt}^*}{L_{rt}^*} = \frac{W_t^*}{P_t^*}, \tag{2.6}$$

and that given our choice of numeraire,

$$1 = \lambda^{-\lambda} (1-\lambda)^{-(1-\lambda)} (W_t)^{\lambda} (P_t)^{1-\lambda} , \text{ and } W_{rt}^* = \lambda^{-\lambda} (1-\lambda)^{-(1-\lambda)} (W_t^*)^{\lambda} (P_t^*)^{1-\lambda} . \tag{2.7}$$

Intermediate goods firms in each country are monopolistically competitive. A home firm with state variables s faces a static profit maximization problem of choosing labor input $l_t(s)$, prices $p_{at}(s)$, $p_{at}^*(s)$, quantities $a_t(s)$, $a_t^*(s)$, and whether or not to export $x_t(s)$, to maximize current period profits taking as given wages for workers W_t, and prices and output of the final good in both countries P_t, P_t^*, Y_t, and Y_t^*. This problem is written

$$\Pi_t(s) = \max_{y,l,p_a,p_a^*,a,a^*,x\in\{0,1\}} p_a a + x p_a^* a^* - W_t l - x n_x \tag{2.8}$$

subject to (2.1), (2.2), and (2.5).

Productivity at the firm level evolves over time depending both on idiosyncratic productivity shocks hitting the firm and on the level of investment in productivity improvements undertaken within the firm. We model the evolution of firm productivity as follows. At the beginning of each period t, every existing firm has probability δ of exiting exogenously and corresponding probability $1-\delta$ of surviving to produce. Surviving firms can choose to operate and pay a fixed cost of operation n_f in terms of the research good, or to exit. A continuing firm with state s that invests $\exp(z)\, c(q)$ units of the research good in improving its productivity in the current period t, has probability q of having productivity $\exp(z+\Delta_z)^{1/(\rho-1)}$ and probability $1-q$ of having productivity $\exp(z-\Delta_z)^{1/(\rho-1)}$ in the next period $t+1$. We refer to the firm's choice of q as the *process innovation decision* of the firm, and to the firm's expenditure of $\exp(z)c(q)$ units of the research good as its investment in process innovation. We assume that $c(q)$ is increasing and convex in q.[9]

With this evolution of firm productivity, the expected discounted present value of profits for a firm with state variables s satisfies a Bellman equation

$$V_t(z, n_x) = \max\left[0, V_t^o(z, n_x)\right] , \tag{2.9}$$

$$V_t^o(z, n_x) = \max_{q\in[0,1]} \Pi_t(z, n_x) - \exp(z)c(q) - n_f + \tag{2.10}$$

$$(1-\delta)\frac{1}{R_t}\int_{n_x'} \left[qV_{t+1}(z+\Delta_z, n_x') + (1-q)V_{t+1}(z-\Delta_z, n_x')\right] \mathrm{d}\Gamma(n_x'|n_x),$$

where $\Pi_t(s)$ is given by (2.8), and R_t is the world interest rate in period t (in units of the home research good). Note that here we express this Bellman equation for the expected discounted present value of profits for the firm $V_t(s)$ in units of the research good. We find

[9] With this scaling of the innovation cost function, $\exp(z)$, we are assuming that the process innovation cost required to increase the size of the firm by a fixed percentage scales with the size of the firm. This will imply that, for sufficiently large firms, their growth rate is independent of size, consistent with Gibrat's law. Note also that if the time period is small, our binomial productivity process approximates a geometric Brownian motion in continuous time, as in Luttmer (2007). Our model differs from his in that firms control the drift of this process through investments of the research good.

this convention useful in characterizing equilibrium. We let $q_t(s)$ denote the optimal process innovation decision of the firm in the problem (2.10).

Since for each value of n_x the value function $V_t^0(z, n_x)$ is strictly increasing in z, it is clear that at each date t, the decision of firms to operate (2.9) follows a cutoff rule with firms with productivity above a cutoff $\bar{z}_t(n_x)$ choosing to operate and firms with productivity below that cutoff exiting. Note that if $n_f = 0$, then $V_t^o(s) = V_t(s)$ and $\bar{z}_t(n_x) = -\infty$, and hence there is no endogenous exit.

New firms are created with an investment of the research good. Investment of n_e units of the research good in period t yields a new firm in period $t+1$ with initial state variables s drawn from a distribution G. In any period in which there is entry of new firms, free entry requires that

$$n_e = \frac{1}{R_t} \int V_{t+1}(s) \mathrm{d}G. \tag{2.11}$$

Note that both sides of this equation are expressed in units of the research good. Let M_{et} denote the measure of new firms entering in period t that start producing in period $t+1$. The analogous Bellman equation holds for the foreign firms as well. We refer to M_{et} as the *product innovation* decision as this is the mechanism through which new differentiated products are produced.

Households in the home country have preferences of the form $\sum_{t=0}^{\infty} \beta^t \log(C_t)$, where C_t is the consumption of the home final good at date t. Households in the foreign country have preferences of the same form over consumption of the foreign final good C_t^*. Each household in the home country faces an intertemporal budget constraint of the form

$$P_0 C_0 - W_0 L + \sum_{t=1}^{\infty} \left(\prod_{j=1}^{t} \frac{1}{R_j} \right) (P_t C_t - W_t L) \leq \bar{W} \ , \tag{2.12}$$

where $\bar{W}$ is value of the initial stock of assets held by the household. Households in the foreign country face similar budget constraints with wages, prices, and assets all labelled with stars.

Feasibility requires that for the final good,

$$C_t + Y_{rt} = Y_t \tag{2.13}$$

in the home country, and the analogous constraint holds in the foreign country. The feasibility constraint on labor in the home country is given by

$$\int l_t(s) \mathrm{d}M_t + L_{rt} = L \tag{2.14}$$

where $\int l_t(s)dM_t$ denotes total employment in production of intermediate goods, and L_{rt} denotes total research employment, and likewise in the foreign country.

The feasibility constraint on the research good in the home country is

$$M_{et}n_e + \int \left[n_f + x_t\left(s\right)n_x + \exp(z)c(q_t(s))\right]\mathrm{d}M_t = L_{rt}^{\lambda}Y_{rt}^{1-\lambda}, \tag{2.15}$$

and likewise in the foreign country.

The evolution of M_t over time is given by the exogenous probability of exit δ, the decisions of operating firms to invest in their productivity $q_t(s)$, and the measure of entering firms in period t, M_{et}. The measure of operating firms in the home country with state variables less than or equal to $s' = (z', n'_x)$, denoted by $M_{t+1}(z', n'_x)$, is equal to the sum of three inflows of firms that decide to operate in period $t+1$: new firms founded in period t, firms continuing from period t that draw positive productivity shocks (and hence had productivities lower than $z' - \Delta_z$ in period t), and firms continuing from period t that draw negative productivity shocks (and hence had productivities lower than $z' + \Delta_s$ in period t). We write this as follows.

For $z' \geq \bar{z}'_{t+1}(\tilde{n}'_x)$,

$$\begin{aligned} M_{t+1}(z', n'_x) = M_{et} \cdot \left[G(z', n'_x) - G(\bar{z}'_{t+1}\left(n'_x\right), n'_x)\right] + \\ +(1-\delta)\int_0^{n'_x}\left[\int_{-\infty}^{z'-\Delta_z}\int_{\{n_x\}} q_t(z, n_x)\mathrm{d}M_t(z, n_x)\right]\mathrm{d}\Gamma(\tilde{n}'_x|n_x) \\ +(1-\delta)\int_0^{n'_x}\left[\int_{\bar{z}'_{t+1}(\tilde{n}'_x)}^{z'+\Delta_z}\int_{\{n_x\}}\big(1 - q_t(z, n_x)\big)\mathrm{d}M_t(z, n_x)\right]\mathrm{d}\Gamma(\tilde{n}'_x|n_x). \end{aligned} \tag{2.16}$$

For $z' < \bar{z}'_{t+1}(\tilde{n}'_x)$, $M_{t+1}(z', n'_x) = 0$. The evolution of $M^*_t(z)$ for foreign firms is defined analogously.

We assume that the households in each country own those firms that initially exist at date 0. Thus we require that the initial assets of the households in both countries adds up to the total value of these firms

$$\bar{W} + \bar{W}^* = \int V_0(s)\mathrm{d}M_0 + \int V_0^*(s)\mathrm{d}M_0^* \tag{2.17}$$

An equilibrium in this economy is a collection of sequences of aggregate prices and wages $\{R_t, P_t, P^*_t,\ W_t, W^*_t, W^*_{rt}\}$, and prices for intermediate goods $\{p_{at}(s), p^*_{at}(s),\ p_{bt}(s), p^*_{bt}(s)\}$, a collection of sequences of aggregate quantities $\{Y_t, Y^*_t,\ C_t, C^*_t, Y_{rt},\ Y^*_{rt}, L_{rt}, L^*_{rt}\}$, and quantities of the intermediate goods $\{a_t(s), a^*_t(s), b_t(s),\ b^*_t(s), l_t(s), l^*_t(s)\}$, initial assets $\bar{W}, \bar{W}^*$, and a collection of sequences of firm value functions and profit, exit, export, and process innovation decisions $\{V_t(s), V_t^*(s),\ V_t^o(s), V_t^{o*}(s), \Pi_t(s), \Pi_t^*(s), \bar{z}\left(n_x\right), \bar{z}^*\left(n_x\right), x_t(s), x_t^*(s),\ q_t(s), q_t^*(s)\}$ together with measures of operating and entering firms $\{M_t, M_{et}, M_t^*, M_{et}^*\}$ such that household in each country maximize their utility subject to their budget constraints, intermediate goods firms in each country maximize within period profits, final goods firms in each country maximize profits, all of the feasibility constraints are satisfied, and the measures of operating firms evolve as described above.

In most of our analysis, we focus our attention on equilibria that are *symmetric* in the following sense. First, we assume that the distribution of initial assets is such that expenditure is equal across countries at date 0 and hence in every period. Second, we assume that each country starts with the same distribution of operating firms by productivity and hence, because prices and wages are equal across countries, continue to have the same distribution of operating firms by productivity in each subsequent period. In such a symmetric equilibrium, we have $Y_t = Y_t^*$, $P_t = P_t^*$, $W_t/P_t = W_t^*/P_t^*$, and $W_{rt}^* = 1$.

A *steady-state* of our model is an equilibrium in which all of the variables are constant. A *symmetric steady-state* is an equilibrium that is both symmetric and a steady-state. In what follows, we omit time subscripts when discussing steady-states.

3. Characterizing Symmetric Steady-State

In this section, we present the equations that characterize a symmetric steady-state of our model. We first characterize the firms' pricing, exit, export, and process innovation decisions. We show that these decisions are the solution to a one-dimensional fixed-point problem. We then characterize the aggregate quantities and prices, taking as given the firms' exit, export, and process innovation decisions. We then present one of the central results in the paper: in steady-state, the combined impact of firm's exit, export, process- and product innovation decisions on aggregate productivity must offset each other so as to ensure that firms' profits are consistent with free-entry.

Consider the static profit maximization problem (2.8) for an operating firm in the home country. All operating firms choose a constant markup over marginal cost, so equilibrium prices are given by

$$p_a(s) = \frac{\rho}{\rho - 1}\frac{W}{\exp(z)^{1/(\rho-1)}},\ \text{and}\ p_a^*(s) = \frac{\rho}{\rho - 1}\frac{DW}{\exp(z)^{1/(\rho-1)}}.$$

Given the demand of final goods firms for intermediate inputs (2.5), home intermediate firms with state variables s have variable profits on their home sales in terms of the numeraire, $\Pi_d \exp(z)$, with the constant in variable profits Π_d given by

$$\Pi_d = \frac{(W/P)^{1-\rho} PY}{\rho^\rho (\rho - 1)^{1-\rho}} \ , \tag{3.1}$$

and variable profits $\Pi_x \exp(z)$ on their foreign sales, with $\Pi_x = \Pi_d D^{1-\rho}$. As is standard, domestic variable profits are decreasing in the real wage W/P, increasing in the price charged by other firms P, and increasing in the scale of final goods production Y.

Total static profits are

$$\Pi(s) = \Pi_d \exp(z) + \max(\Pi_x \exp(z) - n_x, 0) \tag{3.2}$$

We now characterize the firms' exit, export, and process innovation decisions, as the unique solution of a one-dimensional fixed point problem. We solve for a fixed-point over the constant Π_d in firm's variable profits, defined in (3.1).

Consider first firms' export decisions, $x(s)$. Given a value of Π_d, firms' exporting decisions are determined by the static condition that variable profits from exports must exceed fixed costs of exporting, or

$$x(z, n_x) = 1 \text{ if and only if } \Pi_d D^{1-\rho} \exp(z) \geq n_x. \tag{3.3}$$

To solve for firms' steady-state exit and process innovation decisions, we must solve the firms' Bellman equation, (2.9), removing the time subscripts from all variables, and letting $R_t = 1/\beta$. Standard arguments give that this Bellman equation has a unique solution $V(s)$, corresponding to any given value of Π_d under appropriate parameter restrictions.[10] In addition, the solution for $V(s)$ is weakly increasing in Π_d, while the value function of operating firms, $V^o(s)$, is strictly increasing in Π_d.

We use the free-entry condition (2.11) to solve for the equilibrium value of Π_d. To see that a unique solution for Π_d exists, first observe that the right side of the free-entry condition (2.11) is weakly increasing in Π_d and that if it is strictly positive (when a positive mass of

[10] The parameter restrictions required ensure that the net present value of firms' profits remain bounded for any choice of process innovation. A strong sufficient condition is that $\beta(1-\delta)\exp(\Delta_z) < 1$. When numerically solving our model, we check the following weaker sufficient conditions: For all $q \in [0,1]$ such that $\beta(1-\delta)[q\exp(\Delta_z) + (1-q)\exp(\Delta_z)] \geq 1$, then $\Pi_d\left(1 + D^{1-\rho}\right) - c(q) < 0$. The interpretation of this condition is that if it possible for a firm to choose process innovation so that variable profits grow faster than the interest rate, then the variable profits associated to this process innovation decision are negative.

newly entering firms choose to operate), then it is also strictly increasing in Π_d. Second, note that the right hand side of (2.11) is equal to zero when $\Pi_d = 0$, and becomes arbitrarily large as Π_d gets large. Since the fixed cost of entry is strictly positive, there is a unique solution for Π_d.

The solution to this problem now gives us firm's exit decisions $\bar{z}(n_x)$, export decisions $x(s)$, and process innovation decisions $q(s)$.[11] These decisions, under certain parameter restrictions, imply from (2.16) a steady-state distribution of state variables across firms scaled by the mass of entering firms, $\tilde{M}(s) = M(s)/M_e$. The parameter restrictions required imply that the equilibrium process innovation decision of large firms leads them to shrink in expectation.[12]

Now assume that the firms' exit, export, and process innovation decisions are given and lead to a steady-state scaled distribution across states, $\tilde{M}(s)$. To solve for aggregate quantities and prices, it is convenient to define two indices of aggregate productivity across firms implied by firm's decisions,

$$\begin{aligned} Z_d &= \int (1 - x(z, n_x)) \exp(z) \mathrm{d}\tilde{M}(z, n_x) \ \text{, and} \\ Z_x &= \int x(z, n_x) \exp(z) \mathrm{d}\tilde{M}(z, n_x) \, . \end{aligned} \tag{3.4}$$

The first of these is an index of productivity aggregated across all operating, non-exporting home firms, and the second is an index of productivity aggregated across all home firms that export, both scaled by the mass of entering firms. In a symmetric steady-state, the second index, Z_x, is also an index of productivity aggregated across all foreign firms that export to the home country.

From the firm's static profit maximization problem (2.8), we have that the production employment of home firms in a symmetric steady-state is given by

$$l(s) = \left(\frac{\rho - 1}{\rho}\right)^{\rho} \left(\frac{W}{P}\right)^{-\rho} Y \exp(z) \left(1 + x(s) D^{1-\rho}\right) \ , \tag{3.5}$$

[11] This result relies on our assumption that all innovation activities use the same research good. If different inputs were required for product and process innovation, then a change in trade costs might affect the relative price of the inputs into these activities and thus affect equilibrium process innovation. In this case, the full model must be solved simultaneously.

[12] To check that a stationary distribution exists, one must check that the equilibrium process innovation decisions satisfy

$$\lim_{z \to \infty} (1 - \delta) \left[q(z, n_x) \exp(\Delta_z) + (1 - q(z, n_x)) \exp(-\Delta_z) \right] < 1$$

for all values of n_x.

Given that firm revenues are proportional to firm employment, the share of exports in the value of production of intermediate inputs is given by

$$s_x = \frac{Z_x D^{1-\rho}}{Z_d + \left(1 + D^{1-\rho}\right) Z_x}. \tag{3.6}$$

Note that the share of total production employment accounted for by exporters is $s_x \left(1 + D^{1-\rho}\right) / D^{1-\rho}$.

It is also convenient to compute the average expenditure on the research good per entering firm, which we denote by Υ, with

$$\Upsilon = n_e + \int \left[n_f + x\left(s\right) n_x + \exp(z) c(q(s))\right] \mathrm{d}\tilde{M}(z, n_x). \tag{3.7}$$

Given Π_d, Z_d, Z_x, and Υ, the symmetric steady-state values of W/P, Y, L_r, Y_r, M_e, and C solve the following six equations: (2.6),

$$\frac{W}{P} = \frac{\rho - 1}{\rho} \left[M_e \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right)\right]^{1/(\rho-1)}, \tag{3.8}$$

$$Y = \left[M_e \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right)\right]^{1/(\rho-1)} \left(L - L_r\right) , \tag{3.9}$$

$$L_r = \frac{\lambda}{\lambda + \zeta\left(\rho - 1\right)} L, \tag{3.10}$$

$$\Pi_d = \frac{\lambda^\lambda \left(1 - \lambda\right)^{1-\lambda}}{\rho^\rho \left(\rho - 1\right)^{1-\rho}} \left(W/P\right)^{1-\rho-\lambda} Y \text{ , and} \tag{3.11}$$

$$C = Y \left[1 - \frac{1}{\zeta} \frac{1 - \lambda}{\rho}\right], \tag{3.12}$$

where $\zeta = \Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] / \Upsilon$ is the ratio of total variable profits to total expenditure on the research good. We derive these equations in the Appendix.

Since labor is the only variable factor of production, aggregate productivity from equation (3.9) is given by

$$Z = \left[M_e \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right)\right]^{1/(\rho-1)}. \tag{3.13}$$

In solving our model, we make use of the following two Lemmas regarding the aggregate allocation of employment, L_r, and the ratio of consumption to final output, C/Y. Lemma 1 states that L_r and C/Y change with a change in marginal trade costs D only if the ratio of total variable profits to total expenditure on the research good also changes. We show in Lemma 2 that if the real interest is zero ($\beta = 1$), the aggregate allocation of labor and the ratio of consumption to final output are independent of marginal trade costs.

Lemma 1: The steady-state allocation of labor in the research good, L_r, and the steady-state ratio of consumption to output, C/Y, are functions only of the ratio of total variable profits to total expenditure on the research good, ζ, and the parameters λ, ρ and L.

Proof: See expressions (3.10) and (3.12).

Lemma 2: With $\beta = 1$, in steady state, average variable profits across firms equals average expenditure across firms on the research good, so $\zeta = \Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] / \Upsilon = 1$. Hence, by (3.10), L_r is a constant fraction of the labor force given by $L_r = \lambda / \left(\lambda + \rho - 1\right) L$ independent of the trade cost D.

Proof: Free entry requires that for an entering firm, the expected present value of variable profits equals the expected present value of expenditures on the research good. In a steady-state in which the real interest rate is zero, these expected present values are equal to their cross-sectional averages across firms. More details are given in the Appendix.

In our model, households' utility is not defined when $\beta = 1$. We interpret Lemma 2 as a useful limiting result as the discount factor approaches 1.

These Lemmas give us the following algorithm to solve for a symmetric steady-state of the model as a function of the marginal trade cost D. First, use the free-entry condition (2.11) to solve for the equilibrium value of Π_d. Associated with the equilibrium value of Π_d, are firms' exit, export, and process innovation decisions determining the aggregate productivity indices Z_x, and Z_d, as well as the expenditure per entering firm on the research good Υ. Use (3.10) to compute L_r and use (3.8), (3.9), and (3.11) to solve for equilibrium product innovation M_e. Expressions (3.9) and (3.12) then determine output and consumption. With this algorithm, we see that our model has a certain recursive structure. In equilibrium, the free-entry condition pins down firms' exit, export, and process innovation decisions as well as the aggregate allocation of labor between production employment and research. Product innovation then adjusts to satisfy the remaining equilibrium conditions.

We use this recursive structure of our model to analyze the impact of a change in the marginal trade cost on the steady-state equilibrium levels of aggregate productivity, output and welfare. From (3.13), it is clear that aggregate productivity is determined by the exit, export, process- and product innovation decisions of firms. A central result of the paper is that, in steady-state, the combined impact of these decisions on aggregate productivity must offset each other so as to ensure that firms' profits are consistent with free-entry. In particular, from the steady-state equilibrium conditions, (3.8), (3.9) and (3.11), we have

$$\Delta \log \Pi_d = (2 - \rho - \lambda) \Delta \log Z + \Delta \log (L - L_r), \tag{3.14}$$

where Δ denotes the total derivative of a variable.

The intuition for (3.14) is as follows. The free-entry condition, as captured by our Bellman equation, pins down how the variable profits earned by a firm with a given productivity level must change in response to a change in marginal trade costs. From (3.1), this change in variable profits also pins down the change in the real wage and aggregate output that must occur in the new steady-state. Since the real-wage and aggregate output are determined by aggregate productivity and the aggregate allocation of labor, we have that the free-entry condition for firms pins down how aggregate productivity and the aggregate allocation of labor must respond to a change in marginal trade costs. We use this result in the next two sections to derive analytical and numerical results regarding the impact of changes in trade costs on aggregate productivity, output, and welfare.

The economics of the coefficient on aggregate productivity in (3.14) is as follows. An increase in aggregate productivity raises both the real wage and output one-for-one, and decreases the price of the final good in terms of the research good at the rate λ. From (3.1), the combined effect of an increase in aggregate productivity on the constant on variable profits is given by $(2 - \rho - \lambda)$.

In what follows, we impose the parameter restriction $\rho + \lambda > 2$ so that an increase in aggregate productivity lowers the constant on variable profits. When this restriction is violated, it is socially optimal to choose an unbounded level of entry M_e and consumption C in the steady-state. To see this, consider a planner seeking to choose Y_r and M_e to maximize $C = Y - Y_r$, holding fixed the levels of Z_x, Z_d, Υ, and L_r. Using (2.15) to solve for Y_r in terms of M_e, the objective in this problem can be stated as $\kappa M_e^{\frac{1}{\rho-1}} - M_e^{\frac{1}{1-\lambda}}$, with $\kappa > 0$. This function is concave in M_e and hence has an interior maximum if and only if $\rho + \lambda > 2$. Therefore, when this condition is violated, the planner would find it optimal to set $M_e = \infty$. We rule-out this parameter configuration because we do not find it interesting to consider an economy in which it is feasible to have unbounded consumption in the steady-state.[13]

[13]Given the parameter assumption that $\rho + \lambda > 2$, one can show that the social planner chooses exit, export, and process innovation decisions in steady-state equal to those chosen in equilibrium. Moreover, the optimal and equilibrium steady-state allocations are identical if $\lambda = 1$, and the optimal levels of output, consumption and product-innovation are higher than the equilibrium level of these variables when $\lambda < 1$. The intuition for this result is that the equilibrium monopoly distortion alters the value of entry relative to the cost of entry. This proof is available upon request.

4. Trade costs and aggregate productivity and output: Analytical results

In this section, we present analytic results regarding the impact of a change in marginal trade costs on aggregate productivity and output for three important special cases of our model. In the first special case, we assume that *all firms export*. In the second special case, only the most productive firms export, but firms have *no productivity dynamics* after entry, and hence this special case of our model corresponds to the model in Melitz (2003). In the third special case, that we refer to as the *exogenous selection* version of our model, firms have endogenous productivity dynamics from process innovation, but firms' exit and export decisions are independent of size. In the second and third special cases, we also assume that the real interest rate is zero. We now show that a change in the marginal costs of trade has the same impact on steady-state productivity, to a first-order approximation, in all three of these special cases of our model.

To a first-order approximation, a change in the marginal trade cost D has two effects on aggregate productivity. The first effect is a *direct effect* of a change of trade costs on productivity, holding fixed firms' exit, export, process, and product innovation decisions. The second effect is an *indirect effect* that arises from changes in firms' exit, export, process, and product innovation decisions. More formally, from equation (3.13), this decomposition is given by

$$\Delta \log Z = \underbrace{-s_x \Delta \log D}_{\textbf{Direct effect}} + \tag{4.1}$$

$$\underbrace{\frac{1}{\rho-1}\left[s_x \frac{1+D^{1-\rho}}{D^{1-\rho}} \Delta \log Z_x + \left(1 - s_x \frac{1+D^{1-\rho}}{D^{1-\rho}}\right) \Delta \log Z_d + \Delta \log M_e\right]}_{\textbf{Indirect effect}}.$$

The indirect effect of a change in trade costs on aggregate productivity consists of two components. The first component (i.e., the sum of the first two terms in the square bracket) is the indirect effect of a change in trade costs on the productivity of the average firm. The second component, given by $\Delta \log M_e / (\rho - 1)$, is the indirect effect that arises from product innovation.[14]

[14] Note that the line of argument we use here to analyze the direct and indirect effects arising from a change in trade costs does not extend naturally to the analysis of a change in tariffs that are rebated to a household. A change in tariffs does not entail the same direct effect as a change in trade costs because it does not change the resources consumed in international trade.

To calculate the indirect effect, we proceed as follows. The expression (3.14) can be written as

$$\Delta \log \Pi_d = (2 - \rho - \lambda) * (\text{Direct Effect} + \text{Indirect Effect}) + \Delta \log (L - L_r). \tag{4.2}$$

In our three special cases, we show below that from the Bellman equation, the steady-state change in the constant in variable profits that is consistent with free-entry is given by

$$\Delta \log \Pi_d = (\rho - 1)\, s_x \Delta \log D = (1 - \rho) * \text{Direct Effect}. \tag{4.3}$$

When all firms export, or when the real interest rate is zero, the steady-state aggregate allocation of labor is unchanged with D, so that $\Delta \log (L - L_r) = 0$ (see Lemmas 1 and 2 for the case in which $\beta = 1$). Plugging these results into (4.2) gives that the ratio of the indirect effect to the direct effect of a change in trade costs on aggregate productivity is given by

$$\frac{\text{Indirect Effect}}{\text{Direct Effect}} = \frac{1 - \lambda}{\rho + \lambda - 2}. \tag{4.4}$$

This expression (4.4) regarding the relative size of the indirect and direct effects is a straightforward implication of a standard model of trade with homogeneous firms and monopolistic competition, no productivity dynamics, no fixed costs of production or exporting, and no spillovers, such as the model described in Krugman (1979).

Our main result is that (4.4) characterizes the relative size of the indirect and direct effects in all three special cases of our model. This result has two important implications. First, if $\lambda = 1$, so that the research good is produced entirely with labor, then there is no indirect effect. Hence, the steady-state change in productivity, to a first-order approximation, is simply given by the direct effect. This means that in equilibrium, the changes in productivity induced by changes in firms' exit, export, process, and product innovation decisions (i.e. the indirect effect) must entirely offset each other, to a first-order approximation, in the new steady-state. Second, under the more general assumption that $\lambda < 1$, the indirect effect on productivity has the same magnitude, to a first-order approximation, whether or not one considers endogenous process innovation, and endogenous or exogenous choices by firms to export and exit. In the quantitative section of the paper, we explore the extent to which this analytical result holds in more general cases of our model.

When computing the welfare effects from a change in the marginal trade cost D, we must consider both the impact of a change in marginal trade cost on consumption in steady-state, as well as the transition dynamics for consumption. In Lemma 1, we prove that the change in the ratio of consumption to output in steady-state is determined by the same factor ζ that determines the aggregate allocation of labor, L_r. Since in all three special cases of our model, ζ remains constant, we have that the steady-state consumption moves one for one with steady-state output, and that the steady-state change in aggregate output is equal to the change in aggregate productivity. The transition dynamics are computed numerically. At the end of this section, we discuss why if the steady-state effects of a change in marginal trade costs are large, then the transition dynamics are slow.

4.1. All firms export

In this subsection, we first show in Proposition 1 that in an economy with no fixed costs of international trade, changes in the marginal costs of trade have no impact at all on the incentives of firms in the steady-state to engage in process innovation. We use this proposition to show that $\Delta \log Z_x = \Delta \log Z_d = 0$ in response to a change in marginal trade costs, and that the change in the constant in variable profits is given by (4.3). We then show in Proposition 2 that the aggregate allocation of labor is unchanged, and the ratio of indirect to direct effects of a change in the marginal trade cost on aggregate productivity is given by (4.4).

Proposition 1: Consider a world-economy with no fixed costs of trade ($n_x = 0$). A change in the marginal cost of trade, D, has no impact on the steady-state process innovation decisions of firms, $q\left(s\right)$.

Proof: We first prove this proposition under the assumption that the economy is in a symmetric steady state equilibrium. With $n_x = 0$ for all firms, (3.3) implies that all firms export, and the variable profits of a firm with productivity z are $\Pi_d \left(1 + D^{1-\rho}\right) \exp\left(z\right)$. Hence, under the assumption that all firms export, the Bellman equation in steady-state, (2.9), can be written with $\tilde{\Pi} \exp\left(z\right)$ replacing $\Pi_t\left(s\right)$, where $\tilde{\Pi} = \Pi_d \left(1 + D^{1-\rho}\right)$. Our arguments in the previous section imply that there is a unique level of $\tilde{\Pi}$ that satisfies the free-entry condition (2.11), independent of the parameter D. The corresponding process innovation decisions that solve the Bellman equation at this level of $\tilde{\Pi}$ are the equilibrium exit and process innovation decisions. These are also independent of D.

In a steady-state that is not symmetric, the appropriate definition of $\tilde{\Pi}$ is $\Pi_d + \Pi_x D^{1-\rho}$, and the same logic applies. Clearly, the analogous results hold for foreign firms. Q.E.D.

This result holds because, in an economy in which every firm exports, the increased incentives to innovate resulting from the increase in profits that come from a reduction in marginal trade costs affect all firms proportionally. The free-entry condition then requires that the increase in profits is exactly offset by an increase in the cost of the research good necessary for innovation. Recalling that we have normalized the price of the research good to one, this is the intuition for the result that $\tilde{\Pi} = \Pi_d \left(1 + D^{1-\rho}\right)$ remains unchanged. As a result, the optimal process innovation decision of all firms is unchanged.[15]

Proposition 2: Consider a world-economy with no fixed costs of trade ($n_x = 0$). In response to a change in the marginal cost of trade D, the aggregate labor allocation L_r is unchanged, and the ratio of the indirect effect to the direct effect is given by (4.4). This indirect effect corresponds entirely to a change in product innovation.

Proof: We prove the proposition by calculating the terms in (4.2). From Proposition 1, $\Delta \log \Pi_d = -\Delta \log \left(1 + D^{1-\rho}\right)$. Since all firms export, the share of exports in intermediate goods' output is equal to the export intensity of each firm, which is given by $D^{1-\rho} / \left(1 + D^{1-\rho}\right)$. This gives (4.3). It is an immediate corollary of Proposition 1 that the firms' exit decision are also unchanged. Hence, the scaled distribution of firms across states, $\tilde{M}(s)$, the productivity indices, Z_d and Z_x, and the ratio of total variable profits to total expenditures on research goods, $\zeta = \Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] / \Upsilon$ remain unchanged. From Lemma 1, L_r is also unchanged. Our result follows from expression (4.2). Even though our proof used (4.2), which is a first-order approximation of the change in steady-state profits (3.14), one can extend this result to the full non-linear model. Q.E.D.

4.2. No productivity dynamics (Melitz 2003)

Consider a version of our model with fixed operating and export costs, that assumes $\Delta_z = 0$ and a time-invariant value of n_x so that there are no dynamics of firm productivity and export decisions of active firms. In this version of our model, firms choose not to engage in process innovation, and hence this model corresponds to the one in Melitz (2003). Proposition 3 states that the ratio of indirect to direct effects on aggregate productivity from a change in the marginal trade cost in this version of our model is given by (4.4).

Proposition 3: In a symmetric steady-state of our model with $\Delta_z = 0$, a time-invariant value of n_x, and $\beta = 1$, to a first-order approximation the ratio of the indirect effect to the

[15] Given this intuition, it is clear that in our model firm level process innovation decisions are also unaffected if a country moves from autarky to free trade, or by changes in tariffs or tax rates on firm profits, revenues, or factor use that alter the variable profit function in the same weakly separable manner with z. Proposition 1 would also hold in a two-sector model in which the aggregate outputs of each sector are imperfect substitutes, and firms face separate entry condition of the form (2.11).

direct effect of a change in the marginal trade cost D on aggregate productivity is given by (4.4).

Proof: Because $\beta = 1$, Lemma 1 applies in this version of our model, so L_r remains unchanged with a change in the marginal trade cost. Because there are no dynamics in productivity or export decisions, active firms' value functions in steady-state are given by

$$V(z, n_x) = \frac{1}{\delta} \max \left\{0, \Pi_d \exp(z) - n_f + \max \left\{0, \Pi_d \exp(z) D^{1-\rho} - n_x\right\}\right\}. \quad (4.5)$$

The free-entry condition is still (2.11). Because there is no option value to continuing with an unprofitable firm, firms exit if they draw initial productivity z such that firm static profits in the domestic market are less than zero, $\Pi_d \exp(z) < n_f$. Likewise, firms choose to export only if the static profits associated with doing so are positive, $\Pi_d \exp(z) D^{1-\rho} > n_x$. Using these results, differentiating the free-entry condition (2.11), gives (4.3). The details of this derivation are provided in the Appendix. Our result is obtained from plugging this last expression into (4.2). Q.E.D.

Again, note that if $\lambda = 1$, so that the research good is produced entirely with labor, there is no indirect effect on aggregate productivity from a change in marginal trade costs arising from endogenous changes in exit and export decisions. Any increase in aggregate productivity that results from changes in firms' exit and export decisions is exactly offset by a decline in product innovation.

The key intuition for this proposition is that, because with no productivity dynamics there are no option values associated with the decisions of exiting and exporting, the marginal firms earn zero profits from these two activities. Hence, at the margin, changes in the exit and export decisions have no first order effects on an entering firm's expected profits in steady-state. With $\beta = 1$, the aggregate allocation of labor remains unchanged. All this implies that the ratio of indirect to direct effects on aggregate productivity are the same as in the model in which all firms export. Hence, as long as the fixed and marginal trade costs are chosen to match the same share of exports in output of intermediate goods, the response of aggregate productivity, output, and consumption in steady-state to a given percentage change in marginal trade costs are the same regardless of whether all firms export or not.

Note that in Proposition 3, we relied on the assumption that $\beta = 1$ to use Lemma 2 to show that L_r is independent of D. This Lemma does not apply when $\beta < 1$ and not all firms export. We can extend Proposition 3 to allow for $\beta < 1$, as follows. Consider this version of the model with $\Delta_z = 0$ and a time-invariant level of n_x. Suppose, in addition, that the productivity distribution of entering firms G is such that $\exp(z)$ is Pareto (as in Baldwin and Robert-Nicoud 2008, and Chaney 2008). We show in the Appendix that L_r is also unchanged with D, and the ratio of the indirect effect on productivity of a change in the marginal trade cost D to the direct effect of this change in trade costs is also given by (4.4).

Proposition 3 implies that in this model with no process innovation, a change in marginal trade costs has the same impact on productivity, output, and consumption, to a first-order approximation, as one would find in a model in which firms exit and export decisions are given by exogenous random processes as long as both models are parameterized to target a common share of exports in output of intermediate goods. We next study whether the results in Proposition 3 extend to a model with such exogenous selection but also with endogenous process innovation and productivity dynamics.

4.3. Exogenous selection

Here we consider the responses of process- and product innovation, aggregate productivity, and output to a reduction in the costs of international trade with productivity dynamics when not all firms export. We do so in the *exogenous-selection* version of our model, in which firms' exit and export decisions are exogenous. In this version of our model, a change in marginal trade costs results in a reallocation of process innovation across firms. This reallocation of process innovation is a portion of the indirect effect of a change in marginal trade costs on productivity that is not present when all firms export or when there are no productivity dynamics. Despite this reallocation of process innovation, we show that (4.4) still applies.

In this version of our model, we assume that the fixed costs of operating n_f equal zero, and that the fixed cost of exporting, n_x, follows a two-state Markov in which $n_x \in \{l, h\}$, with $l = 0$ and $h = \infty$, with a Markov transition matrix

$$\Gamma = \begin{pmatrix} \gamma_l & 1-\gamma_l \\ 1-\gamma_h & \gamma_h \end{pmatrix},$$

with $\gamma_l > 1/2$ and $\gamma_h > 1/2$. All entering firms start with $z = 0$, and with probability g_i they have $n_x = i$ for $i = l, h$. With these assumptions, firms exit and export decisions are exogenous and independent of current productivity z. It is this feature of the equilibrium of this version of our model that makes it analytically tractable. We refer to our model with these parameters as the *exogenous-selection* version of our model.

Lemma 3: In a symmetric steady-state in the exogenous-selection version of our model, the firms' value function $V(z, n_x)$ have the form $V_i \exp(z)$ for $i = l, h$, and the process innovation decisions $q(z, n_x)$ have the form q_i for $i = l, h$, where V_i and q_i solve

$$V_l = \Pi_d \left(1 + D^{1-\rho}\right) - c(q_l) + \beta(1-\delta)\alpha_l \left[\gamma_l V_l + (1-\gamma_l) V_h\right] \ ,$$

$$V_h = \Pi_d - c(q_n) + \beta(1-\delta)\alpha_h \left[\gamma_h V_h + (1-\gamma_h) V_l\right] \ ,$$

$$q_i \in \arg\max_{q \in [0,1]} \ -c(q) + \beta(1-\delta)\alpha_i \left[\gamma_i V_i + (1-\gamma_i) V_{-i}\right] \text{ for } i = l, h \ , \tag{4.6}$$

with α_i denoting the expected growth rate of productivity for continuing firms, given by

$$\alpha_i = \left[q_i \exp(\Delta_z) + (1-q_i)\exp(-\Delta_z)\right].$$

In this symmetric steady-state, we have $q_l \geq q_h$.

The value of Π_d is determined by the free-entry condition

$$n_e = \beta(g_l V_l + g_h V_h) \ , \tag{4.7}$$

and the indices of aggregate productivity Z_d and Z_x solve

$$\begin{pmatrix} Z_x \\ Z_d \end{pmatrix} = (1-\delta) A \begin{pmatrix} Z_x \\ Z_d \end{pmatrix} + \begin{pmatrix} g_l \\ g_h \end{pmatrix} \text{, with} \tag{4.8}$$

$$A = \begin{pmatrix} \alpha_l \gamma_l & \alpha_h (1-\gamma_h) \\ \alpha_l (1-\gamma_l) & \alpha_h \gamma_h \end{pmatrix}.$$

The aggregates values of W/P, Y, L_r, M_e, Y_r, and C are the solution to (2.6), (3.8), (3.9), (3.10), (3.11), and (3.12), with

$$\Upsilon = n_e + c(q_l) Z_x + c(q_h) Z_d.$$

Proof: The characterization of the value functions follows because firms never pay a fixed cost of operating or exporting, so they drop out from the Bellman equation (2.9). It follows immediately that the value functions and process innovation decisions that we put forward, solve that Bellman equation. Observe that since $\gamma_l \geq 1/2$, $\gamma_h \geq 1/2$, and $1 + D^{1-\rho} > 0$, then $V_l > V_h$. Then, since $c(\cdot)$ is convex, from (4.6) we have that $q_l \geq q_h$, with this inequality strict if $q_i \in (0, 1)$. The intuition for this result is as follows. Exporters have a bigger market. Given that exporting status is persistent, they also expect to have a bigger market in the future. Hence, they have a greater incentive to innovate.

Equation (4.8) can be understood as follows. A fraction δ of firms exit exogenously every period. All continuing exporters have expected productivity growth rate α_l. A fraction γ_l of these remain exporters, and a fraction $(1-\gamma_l)$ become non-exporters. Likewise, all continuing non-exporters have expected productivity growth rate α_h, and transition of export status determined by γ_h. All entering firms have productivity index $z=0$, and hence productivity 1. A fraction g_l of these entrants are exporters, and the remainder are non-exporters. Q.E.D.

We now study the impact of a reduction in trade costs on this economy. From the free-entry condition (4.7), we see that a reduction in trade costs must raise the value of exporting firms, V_l, and lower the value of non-exporting firms, V_h. If export status is sufficiently persistent, then the incentives for process innovation, captured in (4.6), increase for exporters and decrease for non-exporters, leading to a reallocation of process innovation across firms.

We can obtain analytical results regarding the impact of the reduction in trade costs on aggregate productivity and output in this special case of our model if we also set $\beta=1$, as follows.

Proposition 4: In a symmetric steady-state in the exogenous-selection version of our model with $\beta=1$, to a first-order approximation, the ratio of the indirect effect to the direct effect on aggregate productivity of a change in the marginal trade cost D is given by (4.4).

Proof: We obtain this result regarding a change in the margin of the trade cost by differentiating the Bellman equation and the free-entry condition to obtain the steady-state change in profits, and then we obtain the result from (4.2). In particular, differentiating the Bellman equation, with $\beta=1$, gives

$$\Delta V_l = \left(1+D^{1-\rho}\right)\Delta\Pi_d + \Pi_d\Delta\left(1+D^{1-\rho}\right) + (1-\delta)\,\alpha_l\left[\gamma_l\Delta V_l + (1-\gamma_l)\,\Delta V_h\right] \text{ and}$$

$$\Delta V_h = \Delta\Pi_d + (1-\delta)\,\alpha_h\left[\gamma_h\Delta V_h + (1-\gamma_h)\,\Delta V_l\right]\ ,$$

where we have used an envelope condition to cancel out the terms that arise from marginal changes in process innovation. Writing these in vector form, we obtain

$$\begin{pmatrix} \Delta V_l \\ \Delta V_h \end{pmatrix} = \left(1-(1-\delta)\,A'\right)^{-1}\begin{pmatrix} \left(1+D^{1-\rho}\right)\Delta\Pi_d + \Pi_d\Delta\left(1+D^{1-\rho}\right) \\ \Delta\Pi_d \end{pmatrix}. \qquad (4.9)$$

Free-entry requires that

$$g_l\Delta V_l + g_h\Delta V_h = 0.$$

Using the last two expressions and the fact that $(1-(1-\delta)A')^{-1} = \left([1-(1-\delta)A']^{-1}\right)'$ implies

$$
\begin{aligned}
0 &= \begin{pmatrix} g_l & g_h \end{pmatrix} \left([1-(1-\delta)A']^{-1}\right)' \begin{pmatrix} (1+D^{1-\rho})\,\Delta\Pi_d + \Pi_d \Delta (1+D^{1-\rho}) \\ \Delta\Pi_d \end{pmatrix} \\
&= \begin{pmatrix} Z_x & Z_d \end{pmatrix} \begin{pmatrix} (1+D^{1-\rho})\,\Delta\Pi_d + \Pi_d \Delta (1+D^{1-\rho}) \\ \Delta\Pi_d \end{pmatrix},
\end{aligned}
\tag{4.10}
$$

where the last equality follows from (4.8). This then implies (4.3). Our result is obtained from plugging (4.3) into (4.2) and taking into account that L_r is independent of D. Q.E.D.

From Proposition 4, observe first that if $\lambda = 1$, there is no indirect effect of a reduction in trade costs on aggregate productivity in the steady-state. Hence, in this case, the impact of the change in process innovation on the productivity of the average firm must be exactly offset by the change in product innovation. More generally, recall that the impact of a change in trade costs on process innovation is independent of the parameter λ. In equilibrium, it is product innovation that must adjust differently depending on the parameter λ.

We now discuss how the results in Proposition 4 vary if we allow for $\beta < 1$ in our model with exogenous selection. In order to do so, it is useful to define *hybrid* indices of aggregate productivity, $\tilde{Z}_x$ and $\tilde{Z}_d$, as

$$
\begin{pmatrix} \tilde{Z}_x \\ \tilde{Z}_d \end{pmatrix} = (1-\delta)\,\beta A \begin{pmatrix} \tilde{Z}_x \\ \tilde{Z}_d \end{pmatrix} + \begin{pmatrix} g_l \\ g_h \end{pmatrix}. \tag{4.11}
$$

Note that in defining these hybrid indices of aggregate productivity, we used expression (4.8) where the effective survival rate is $\beta(1-\delta)$ instead of $(1-\delta)$. The hybrid share of exports in intermediate goods' output, $\tilde{s}_x$, is defined by expression (3.6), with $\tilde{Z}_x$ and $\tilde{Z}_d$ in place of Z_x and Z_x. This hybrid share of exports in intermediate goods' output corresponds to the share of exports in the discounted present value of revenues for an entering firm. If $\beta = 1$, we have $\tilde{s}_x = s_x$. If $\beta < 1$, and if entering firms are less (more) likely to be exporters relative to old surviving firms, then $s_x > \tilde{s}_x$ ($s_x < \tilde{s}_x$).

Following the same logic as in Proposition 4, one can show that, with $\beta \leq 1$, we have

$$
\Delta \log \Pi_d = (1-\rho) * \frac{\tilde{s}_x}{s_x} * \text{Direct Effect}. \tag{4.12}
$$

Observe that if entering firms are very likely to be non-exporters (low g_l), and if export status is persistent, then $\tilde{s}_x$ is close to zero, and aggregate variable profits Π_d are roughly unchanged with D. Then, (4.6) implies that there there will be a large increase in process innovation by exporters relative to non-exporters. In contrast, if entering firms are very likely to be exporters (high g_h), then $\tilde{s}_x$ is high, and Π_d falls by more with D. This larger decline in aggregate variable profits leads to a smaller increase in process innovation by exporting firms. Hence, the average export status of entering firms is an important determinant of the reallocation of process innovation in response to a change in trade costs.

The result (4.12) raises the possibility that the indirect effect on aggregate productivity of a change in trade costs could offset, rather than amplify, the direct effect. In particular, if one assumes that process innovation is highly inelastic, then $\Delta \log Z_x = \Delta \log Z_d = 0$, and using Lemma 1 and (4.2) one can show that the ratio of the indirect effect to the direct effect is

$$\frac{\text{Indirect Effect}}{\text{Direct Effect}} = -1 + \frac{\rho - 1}{\rho + \lambda - 2} \left[\frac{L_r}{L} + \frac{\tilde{s}_x}{s_x} \left(1 - \frac{L_r}{L} \right) \right]. \tag{4.13}$$

This expression is negative when $\tilde{s}_x / s_x$ is small and λ is large. For example, if $\lambda = 1$, then the indirect effect partly offsets the direct effect if and only if $\tilde{s}_x < s_x$.

4.4. Transition Dynamics

So far, we have focused on steady-state comparisons. One can also compute transitions in our model out of steady-state, although to take all the general equilibrium effects into account this must be done numerically. In our quantitative analysis below, we find that this model can have very slow transition dynamics despite the fact that the only state variable is the distribution of productivities across firms. One can gain some intuition for this result if one considers equation (4.8) in the exogenous selection version of the model, interpreted as a first-order difference equation for the aggregate productivity indices Z_{xt} and Z_{dt}. That equation implies that if q_{lt}, q_{ht} and M_{et} change once and for all following a one-time change in trade costs in period 0, then the transition dynamics of these aggregate productivity indices are given by

$$\begin{pmatrix} Z_{xt} - \bar{Z}_x \\ Z_{dt} - \bar{Z}_d \end{pmatrix} = (1 - \delta)^t A^t \begin{pmatrix} Z_{x0} - \bar{Z}_x \\ Z_{d0} - \bar{Z}_d \end{pmatrix}, \tag{4.14}$$

where $\bar{Z}_x$ and $\bar{Z}_d$ denote the new steady-state values of these indices. Note that A is a matrix with all non-negative elements and that $(1 - \delta)^t A^t$ must converge to zero to have a steady-state. If $(1 - \delta)^t A^t$ converges to zero rapidly, then the transition dynamics are fast. If $(1 - \delta)^t A^t$ dies out slowly, then the transition dynamics are slow.

This matrix $(1 - \delta) A$ also determines in our model the productivity of the average firm relative to that of the average entering firms. On average, entering firms have productivity $[(1 + D^{1-\rho}) \;\; 1] \, [g_l \;\, g_h]'$, and that the average firm has productivity$[(1 + D^{1-\rho}) \;\; 1] *$

$\sum_{t=0}^{\infty} (1-\delta)^t A^t [g_l \; g_h]'$. Hence, if $(1-\delta)^t A^t$ dies out rapidly, then the productivity of the average firm is similar to the average productivity of an entering firm. In this case, process innovation is not playing a big role in determining firms' productivities and transition dynamics are fast. In contrast, if $(1-\delta)^t A^t$ dies out slowly, so that the productivity of the average firm is substantially larger than the average productivity of an entering firm, then process innovation is playing a big role in determining firms' productivities, but the transition dynamics are slow. Therefore, there is a trade-off in our between the importance of process innovation for firms' productivities and the speed of transition to steady state.

5. Quantitative Analysis

We now present a quantitative version of our model that is consistent with some salient features of the data on firm size dynamics (both in terms of employment and export status), and the firm size distribution in the U.S. economy. We use this quantitative version of our model to extend our results from the previous section on the impact of a change in the marginal costs of international trade on aggregate productivity, output, and welfare for specifications of the model that we cannot solve for analytically. In particular, in our quantitative model we simultaneously have endogenous selection in firms' exit and export decisions, and endogenous process innovation. We consider how the quantitative implications of the model change as we vary the real interest rate and the elasticity of process innovation to changes in the incentive to innovate.

We do so in four experiments. In the first experiment, we set the real interest rate to zero and allow the elasticity of process innovation to vary. The results from this experiment conform closely to our analytical results in the previous section. The ratio of the indirect to the direct effects of a change in trade costs on aggregate productivity is very close to (4.4). In the second experiment, we consider a positive real interest rate and inelastic process innovation. Here we find, again, that the results from this experiment conform closely to our analytical results, summarized in (4.13). In the third experiment, we consider a positive real interest rate and elastic process innovation. Here we find that it is possible to have a substantially larger steady-state response of aggregate productivity than we have previously found, but this effect is small relative to the responses of the productivity of the average firm. Moreover, the welfare gains are very similar to those that are obtained from consideration of the direct effect alone. In the fourth experiment, we redo experiment three with the only change that we consider the implications of a larger change in trade costs. We find roughly the same results.

Calibration

We choose time periods equal to two months so there are six time periods per year.[16] We parameterize the distribution G of productivity draws and export cost of entrants so that all firms enter with a common productivity index $z_0 = 0$ and all firms share a common fixed cost of exporting n_x that is constant throughout the firm's active life.

In our quantitative exercises we assume that the process innovation cost function has the form $c(q) = h\exp(bq)$ so that the curvature of this function is indexed by the parameter b. If this parameter b is high (low), so that this curvature is high (low), then we have that process innovation is very inelastic (elastic) to changes in the incentives to innovate. We consider alternative values of b ranging from very large values ($b = 3000$), in which the process innovation decisions of firms are highly inelastic and hence effectively constant as in the model of Luttmer (2007), to lower values of b ($b = 30$ and $b = 10$), in which process innovation decisions are elastic so that the reallocation of process innovation following a change in trade costs is quite large.

We now discuss how the remaining parameters of the model are chosen to reproduce a number of salient features of U.S. data on firms dynamics, the firm size distribution, and international trade. The parameters of the model that we must choose are the steady-state interest rate given by $1/\beta$, the total number of workers L, the parameters governing the variance of employment growth for surviving firms Δ_z, the exogenous exit rate of firms δ, the marginal trade cost D, the fixed costs of operation n_f, and entry n_e, the fixed costs of exporting n_x, and the parameters of the innovation cost function h and b, We also need to choose the elasticity of substitution among intermediates in final output ρ, and the share of labor in the production of research goods λ. In our model, the distribution of employment across firms in a symmetric steady-state depends on the elasticity parameter ρ only through the trade intensity for firms that do export given by $D^{1-\rho}/(1 + D^{1-\rho})$. Since our calibration procedure is based on employment data, we choose the trade intensity $D^{1-\rho}/(1 + D^{1-\rho})$ as a parameter, and hence our steady-state calibration is invariant to the choice of ρ. For similar reasons, our steady-state calibration is also invariant to the choice of λ.

These parameters are set as follows. We consider two values of β: $\beta = 1$ so that the real

[16] As we reduce the period length, we keep the entry period of new firms at one year. Otherwise, the allocations would change significantly as the cost of waiting for a new draw would decline.

interest rate is zero, and β set so that the steady-state interest rate (annualized) is 5%. We normalize $L = 1$. Now consider the parameters shaping the law of motion of firm productivity z (Δ_z, δ, n_e, n_f, n_x, $D^{1-\rho}$, h and b). We choose Δ_z so that the standard deviation of the growth rate of employment of large firms in the model is 25% on an annualized basis. This figure is in the range of those for US publicly-traded firms reported in Davis et. al. (2006).[17] We choose the exogenous death rate δ so that the model's annual employment-weighted death rate of large firms is 0.55%, consistent with the corresponding one for large firms in the US data.[18] Note that in our model, over the course of one year, large firms do not choose to exit endogenously because they have productivity far away from the threshold productivity for exit. Hence δ determines the annual exit rate of these firms directly. We normalize $n_e = 1$, and set $n_f = 0.1$.[19]

Corresponding to each value of b (3000, 30 and 10), we choose the parameters n_x, $D^{1-\rho}$, and h to match the following three observations. First, the fraction of exports in gross output of intermediate goods is $s_x = 7.5\%$. Second, the fraction of total production employment accounted for by exporting firms is $s_x \left(1 + D^{1-\rho}\right) / D^{1-\rho} = 40\%$.[20] Third, we match the shape of the right tail of the firm size distribution in the U.S. Here, our calibration procedure is similar to that in Luttmer (2007). Specifically, consider representing the right tail of the distribution of employment across firms in the U.S. data with a function that maps the logarithm of the number of employees $\log(l)$ into the logarithm of the fraction of total employment employed in firms with this employment or larger. It has been commonly observed that this function is close to linear for large firms. In calibrating the model with inelastic process innovation (fixed q for all firms) we set the model parameters so that the model matches the slope coefficient of this function for firms within a certain size range.[21] Concretely, we target a slope of -0.2 for firms ranging between 1000 and 5000 employees.

[17] We abstract from the trend in employment growth rate volatility discussed in Davis et. al. (2006) and pick a number that roughly matches the average for the period 1980-2001.

[18] This is the 1997-2002 average employment-based failure rate of US firms larger than 500 employees, computed using data reported by the Statistics of U.S. Businesses and Nonemployer Statistics.

[19] The statistics that we report are invariant to proportional changes in all three fixed costs and h.

[20] Bernard et. al. (2005) report that the fraction of total US empoyment (excluding a few sectors such as agriculture, education, and public services) accounted for by exporters is 36.3% in 1993 and 39.4% in 2000. The average of exports and imports to gross output for the comparable set of sectors is roughly 7.5% in the U.S. in 2000. The steady state of our model abstracts from trends in trade costs that would lead to changes in trade volumes over time.

[21] The slope coefficient for sufficiently large firms can be solved for analytically in our model. In particular, given the choice of process-innovation q for large firms, then the slope coefficient is $1 + \log(y) / \Delta_z$, where y is the root of $y = (1-\delta) q + (1-\delta)(1-q) y^2$ which is less than one in absolute value.

Note that firm sizes in terms of number of employees in the model are simply a normalization. We choose this normalization so that the median firm in the employment-based size distribution is of size 500. In other words, 50% of total employment in the model is accounted for by firms of size under 500.[22] The calibrated model then implies a value of process innovation q for large firms. As we lower b, we adjust the model parameters to keep the value of q for large firms constant and thus keep the dynamics of large firms unchanged.

Table 1 summarizes the observations used in the calibration, as well as the resulting parameter values, for each level of b. Recall that by calibrating the model to data on firm size, we did not need to take a stand on the values of ρ and λ. The aggregate implications of changes in trade costs are, however, affected by the values of ρ and λ. In our benchmark parameterization, we set $\rho = 5$, and λ equal to either 1 or 0.5.[23]

Experiment 1

In our first experiment, we consider the calibration of our model in which the real interest rate is 0% ($\beta = 1$). This calibration of our model combines the endogenous selection of firms' exit and export decisions of the Melitz (2003) model, together with productivity dynamics driven by endogenous process innovation. From Lemma 1, since the real interest rare is zero, we have that there is no change in the aggregate allocation of labor. In this experiment, we evaluate the accuracy of (4.3) and (4.4) derived in our analytical results of Section 3. In this experiment, as well as in Experiments 2 and 3, we set $\Delta \log D$ to a small negative number number (i.e., a decline in the marginal trade costs) and compute the change in the symmetric steady state of the model. We report all changes as elasticities (ratios of changes in the log of the variables to $\Delta \log D$) with a minus sign so that these elasticities can be interpreted as the increase in aggregate productivity, output, etc. in response to a decline in trade costs. We repeat these experiments for our three values of b ($b = 3000$, $b = 30$, and $b = 10$), and our two values of λ ($\lambda = 1$ and $\lambda = 0.5$) for a total of 6 parameter configurations of the model. Results are reported in Table 2.

Since the share of exports in intermediate goods' output is $s_x = 0.075$ and $\rho = 5$, it is clear that for all six of these cases, that our analytical formula (4.3) is very accurate. When $\lambda = 1$, our formula (4.4) for the ratio of the indirect to direct effect is also quite accurate. In this case, the indirect effect is roughly zero because product innovation adjusts to offset

[22] This is the size of the median firm in the US firm employment-based size distribution on average in the period 1999-2003, as reported by the Statistics of U.S. Businesses and Nonemployer Statistics.

[23] Our choice of $\rho = 5$ roughly coincides with the average elasticity of substitution for US imports of differentiated 4-digit goods estimated in Broda and Weinstein (2006) for the period 1990-2001.

the changes in exit, export, and process innovation. This implies that the aggregate change of aggregate productivity and output in these three cases is very close to what one would obtain from the direct effect alone.

Note that when process innovation is elastic, there is a large reallocation of process innovation from non-exporters to exporters. This reallocation of process innovation leads to a large change in the share of exports in output. In particular, the elasticity of the export share s_x to a change in D is 3.7 with $b = 3000$, 9.9 with $b = 30$, and 25.9 with $b = 10$ (we do not report these numbers in Table 2). This reallocation leads to a large increase in the productivity of the average firm. However, in each case, there is a large offsetting movement in product innovation that leaves the indirect effect of a reduction in marginal trade costs on aggregate productivity roughly unchanged.

For those cases in Table 2 with $\lambda = 0.5$, the conclusions are similar in that the numerical results are close to the analytical predictions. Here the change in aggregate productivity and output is larger (0.086 instead of 0.075) because the indirect effect is larger, as predicted by (4.4).

From Lemma 1, we have that when the real interest is zero, the steady-state change ratio of consumption to output is unchanged with a change in trade costs. This results is confirmed in that the response of aggregate consumption reported in Table 2 is the same as the response of aggregate output.

Experiment 2

In Experiment 2, we consider the parameterization of our model in which the annualized real interest rate is 5% ($\beta < 1$), but process innovation is assumed to be inelastic ($b = 3000$). This version of the model is the one we discussed at the end of the analytic section, extended with endogenous selection in exit and export decisions. We perform the same aggregate experiments as in Experiment 1 using values of λ equal to 1 and 0.5, and report the results in Table 3.

We find that the formulas for the change in the constant in variable profits, (4.12), and the ratio of the indirect to the indirect effects, (4.13), are very accurate. Our main finding of this experiment is that, with a small value of $\tilde{s}_x$, the indirect effect of a change in marginal trade costs is negative. That is, the decline in product innovation more than offsets the changes in the productivity of the average firm. Hence, the resulting changes in aggregate productivity and output are *smaller* than those that arise from the direct effect alone. In particular, the direct effect on aggregate productivity is 0.075, which is larger than the resulting change in output (0.01 with $\lambda = 1$ and 0.008 with $\lambda = 0.5$).

This result that the indirect effect is negative is largely driven by the results that the change in variable profits is so small, as given by $(1 - \rho)\,\tilde{s}_x$. The intuition for this result is that entering firms start small and it takes many periods for them to become exporters. Hence, with a positive real interest rate, changes in marginal trade costs do not have a significant impact on the variable profits of entering firms. To illustrate the importance of firm dynamics for this result, consider an alternative parameterization of our model in which the constant h in the process innovation cost function is set to a higher level so that entering firms on average do not grow substantially. In this alternative parameterization of our model, s_x and $\tilde{s}_x$ are both roughly equal to 0.075. This alternative parameterization might be relevant for capturing productivity dynamics at the product level rather than at the firm level if one thinks that new products enter at a relatively larger scale. In this parameterization, entering products are roughly the same size as the average firm, and hence have a relatively high probability of being exported shortly after entry. When we repeat Experiment 2 in this parameterization of our model, reported in Columns 3 and 4 of Table 3, the change in variable profits is larger in absolute terms, and the indirect effect is not negative. In terms of the impact on output, these results are similar to those we obtained in Columns 1 and 4 of Table 2. This result suggests that, quantitatively, the hybrid export share $\tilde{s}_x$ is important in determining the effects of a change in marginal trade costs on productivity and output in steady-state.

Experiment 3

In our third experiment, we examine the aggregate impact of a change in trade costs in a parameterization of our model that is not close to one that we solved analytically. In particular, we consider a specification of our model with a positive real interest rate, endogenous exit and export decisions, and elastic process innovation. We consider the parameterization of our model in which the annualized real interest rate is 5% ($\beta < 1$), values of b governing the elasticity of process innovation equal to 30 and 10, and $\lambda = 1$ and $\lambda = 0.5$. We report the results in Table 4.

We see in Columns 2 and 4, with $b = 10$, that there can be a large reallocation of labor (the elasticity of production labor is 0.29) and less of an offset of product innovation to the change in the productivity of the average firm (the ratio of the indirect effect to the direct effect on productivity is 0.26). Both of these effects can contribute to a substantial amplification of the direct effect of a reduction in trade costs on output. The response of aggregate output is also large compared to that found in Table 3 in Columns 1 and 2, which assumes inelastic process innovation. In particular, if $\lambda = 1$, the elasticity of aggregate output to a reduction in D is 0.03 with $b = 3000$, 0.15 with $b = 30$, and 0.38 with $b = 10$. Thus, with $b = 10$, the response of output is five times bigger than what would arise from the direct effect alone. Note, however, that there is still a substantial offsetting effect between process and product innovation. The elasticity of the indirect effect on the productivity of the average firm and the elasticity of product innovation are both two orders of magnitude larger than their combined effect on aggregate productivity.

Welfare

Our results so far concern the impact of a change in the marginal trade cost on steady-state levels of productivity, output and consumption. We now ask whether considerations of firms' exit, export, process- and product innovation decisions have a substantial effect on the model's implications of a change in trade costs on welfare over and above the direct effect. To compute the welfare implications of such a change in trade costs, we must compute the transition dynamics from one steady-state to another, which in general must be done numerically.

Our welfare metric is the equivalent variation in consumption from a change in marginal trade costs, defined as the change in consumption at the old steady-state that leaves households indifferent between the old steady-state and the transition to the new steady-state.

To put these welfare gains in perspective, we compare them to the magnitude of the direct effect of a change in the marginal trade cost on aggregate productivity. This direct effect is the equivalent variation in consumption if there are no changes in firms' exit, export, process, and product innovation decisions, no reallocation of aggregate labor, and hence no transition dynamics.

For very low interest rates, transition dynamics are not important for welfare and the equivalent variation in consumption is very close to the change in consumption from the old steady state to the new one. Hence, in Experiment 1, which assumes an real interest rate equal to zero, the welfare gains from a reduction in trade costs is given by the steady-state change in consumption. As presented in Table 2 and discussed above, the steady-state indirect effects on aggregate productivity stemming from changes in firms' exit, export, process- and product innovation decisions are very small (and close to zero when $\lambda = 1$). Hence, consideration of these decisions do not have a substantial effects on the welfare implications of a change in trade costs relative to the direct effect alone.

Consider now the more interesting welfare implications of our model with positive real interest rates as specified in Experiments 2 and 3. In Tables 3 and 4 we report the ratio of the elasticity of the equivalent variation in consumption with respect to $\Delta \log D$ to the elasticity of consumption due to the direct effect alone (given by s_x), minus one. A value of zero in this statistic indicates that there are no effects on welfare arising from the indirect effect and the reallocation of aggregate labor in the transition to a new steady-state. A value of one in this statistic indicates that the welfare effects arising from the indirect effect and reallocation of aggregate labor in the transition to a new steady-state are as large as the elasticity of output due to the direct effect. Note that the value of this statistic can be negative if the welfare implications of the indirect effect and the aggregate reallocation of labor are negative.

In both Experiments 2 and 3, as reported in columns 1-4 of Tables 3 and 4, we see that our welfare statistic is close to zero. Hence, there are almost no effects on welfare arising from the indirect effect and the reallocation of aggregate labor in the transition to a new steady-state, despite the fact that in the long run the indirect effect and the aggregate reallocation of labor both can contribute to a large change aggregate output and consumption.

These results follow from the fact that in those cases in which there is a large steady-state response of aggregate productivity and output to a change in marginal trade costs, the transition dynamics are very slow, and hence contribute little to welfare. To illustrate these slow transition dynamics, Figure 1 plots the elasticity of the ratio of exports to output of intermediate good firms during the transition. As is evident in the figure, when entering firms are small, these transition dynamics take over 100 years to play out. In our analytical section, we argued that our model's aggregate transition dynamics are connected to its firm dynamics. When entering firms are small relative to the average firm, aggregate transition dynamics are slow. When entering firms are larger, these aggregate transition dynamics are faster. To illustrate this point, we also show in Figure 1 the transition dynamics for exports relative to output of intermediate good firms for the specifications of our model in which entering firms are large relative to the average firm, as described in columns 6 and 7 of Table 4. In particular, in these specifications, entering firms on average do not grow substantially, so the actual and hybrid shares of employment in exporters are similar. We see that, for this specification of our model, the aggregate transition dynamics are substantially faster. Note, however, that even though the transition dynamics are faster under these alternative specifications, our welfare statistics are still roughly zero because, in the long run, the indirect effect and the aggregate reallocation of labor both contribute to only a small change in aggregate output and consumption.[24]

Experiment 4

In Experiment 3, we computed the welfare gains arising from the indirect effects and reallocation of aggregate labor in the transition to a new steady-state following a small change in trade costs. We now consider a large change in trade costs. In particular, using the same parameter values as in Experiment 3, we compute the welfare gains that arise from a 28% change in $D^{1-\rho}$. We report our findings in Table 5.

Depending on the elasticity of process innovation, this change in trade costs results in the long-run in an increase in the export share from 7.5% to 9.3% ($b = 3000$), or to 20.5% ($b = 10$).[25] In spite of the large change in trade patterns resulting from a reallocation of process innovation from non-exporters to exporters, there is a large offsetting response of product innovation. As reported in Table 5, the change in aggregate productivity is again two orders of magnitude smaller than the change in the productivity of the average firm. Overall, the welfare gains that arise from the indirect effects and reallocation of aggregate labor in the transition to a new steady state are still small. They are less than 20% of the welfare gains from the direct effect for all of the combinations of λ and b that we considered in Table 4.

[24] The result that consideration of firm's exit, export, process- and product innovation decisions have a very small impact on the welfare implications of a change in marginal trade costs can also be understood in the lens of the planning solution of our model. As discussed above, the equilibrium allocations of our model coincide with the planning solution under $\lambda = 1$, or if $\lambda < 1$ in the presence of a per-unit subsidy that eliminates distortionary monopoly markups. In the planning problem, with firms' exit, export and process innovation decisions optimally chosen, the envelope condition implies that, to a first-order approximation, the increase in the discounted flow of utility from a change in marginal trade costs is equal to the discounted present value of the direct effect of this change on aggregate productivity. By the envelope condition, changes in firms' exit, export and process innovation decisions are of second order importance for welfare.

[25] Choosing a larger reduction in D with $b = 10$ leads to an even larger increase in the growth rate of exporting firms and a non-stationary firm size distribution. Throughout the paper we only focus on parameterizations that give rise to stationary steady-states.

6. Concluding remarks

In this paper we build a model of the endogenous change in aggregate productivity that arises in general equilibrium as firms' exit, export, process- and product innovation decisions respond to a change in trade costs. Our central finding is that, despite the fact that a change in trade costs can have a substantial impact on individual firms' exit, export, and process innovation decisions, firms' free-entry condition places a constraint on the overall response of aggregate productivity to the change in trade costs. In particular, we show that the steady-state response of product innovation largely offsets the impact of changes in firms' exit, export, and process innovation decisions on aggregate productivity. We also find that the dynamic welfare gains from a reduction in trade costs are not substantially larger than those welfare gains from the direct effect alone, despite the fact that consideration of firms' exit, export, and process innovation lead to very large dynamic responses of exports and the firm size distribution. Our results suggest that micro evidence on individual firms' responses to changes in international trade costs is not informative about the macroeconomic implications of change in these trade costs for aggregate productivity and welfare unless this evidence were to also shed light on the dynamics of product innovation.

Our model has abstracted from three important considerations. First, we have assumed constant elasticity of demand. This assumption implies that changes in trade cost have no impact on firms' markups and that there is no strategic interaction in firms' process innovation decisions. Our model could be extended to allow for variable markups (see Melitz and Ottaviano 2008 for a model of trade and heterogeneous firms with non-constant elasticity of demand, or Aghion et. al. 2003 for a model of process innovation with strategic interactions between firms). Second, we have assumed that all firms are single-product firms. In doing so, we have abstracted from the effects that a reduction in trade costs might have on product innovation by incumbent firms. Consideration of process and product innovation in models with multi-product firms would be an important extension of this paper (see Klette and Kortum 2004, Luttmer 2007, and Bernard, Redding and Schott 2007 for models of multi-product firms). Third, we have also abstracted from spillover effects that might lead to endogenous growth. Given the work of Baldwin and Robert-Nicoud (2008) on the role of spillovers in the Melitz model, we anticipate that it would be possible in our model to generate a wide variety of results depending on the details of the spillovers.

In this paper we have focused on the implications of a reduction in trade costs on innovation at the firm level. More generally, one might consider a broader array of economic changes and policies that would impact firms' innovation decision. We conjecture that our main result regarding the role of the free-entry condition in constraining the response of aggregate productivity would extend to these situations as well. In particular, one might find similar results regarding the aggregate implications of various innovation policies designed to stimulate innovation at the firm level: the response of process innovation conducted by existing firms might be offset in equilibrium by a change in product innovation. We leave consideration of this extension for future work.

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Appendix

Determination of aggregate variables in symmetric steady-state

The definition of the price index of the final good in the home country (2.4) implies that the real wage is given by (3.8). Using (3.5), the labor market clearing condition (2.14) can be expressed as

$$L = \left(\frac{\rho-1}{\rho}\right)^{\rho}\left(\frac{W}{P}\right)^{-\rho} Y M_e\left[Z_d + \left(1+D^{1-\rho}\right)Z_x\right] + L_r. \tag{6.1}$$

Using (3.8) and (6.1), aggregate output is given by (3.9).

Using (2.6) and (3.8), the resource constraint on the research good, (2.15), can be expressed as

$$\Upsilon M_e = \left(\frac{1-\lambda}{\lambda}\frac{\rho-1}{\rho}\right)^{1-\lambda}\left[M_e\left(Z_d + \left(1+D^{1-\rho}\right)Z_x\right)\right]^{\frac{1-\lambda}{\rho-1}} L_r. \tag{6.2}$$

Using (2.7), the constant on variable profits (3.1) in a symmetric steady state is given by (3.11). Using (3.8) and (3.9), the constant on variable profits can be written as (3.14). Pre-multiplying (3.14) by $M_e\left(Z_d + \left(1+D^{1-\rho}\right)Z_x\right)$, dividing this expression by (6.2), and re-arranging terms, we obtain (3.10).

We also have, from (2.13), that $C = Y - Y_r$, or using (2.6), $C = Y - L_r\frac{1-\lambda}{\lambda}\frac{W}{P}$. Using (3.8), (3.9), and (3.10), we obtain (3.12). Note that $0 \leq C \leq Y$, because $\zeta \geq 1$ and $\rho > 1$. Q.E.D.

Proof of Lemma 2

The last two lines in the equation (2.16) define an operator that maps distributions of firms across states into new distributions of firms across states. We denote this operator by

T, and re-write (2.16) as

$$M_{t+1} = TM_t + GM_{et}.$$

Hence, the steady-state distribution of firms across states, scaled by the measure of entering firms, is given by

$$\tilde{M} = \sum_{n=0}^{\infty} T^n G \ .$$

This distribution is the sum of the distribution of firms across firms for firms that are $n = 0$ to $n = \infty$ periods old.

Note that if one integrates our Bellman equation (2.9) with $\beta = 1$, with respect to any arbitrary distribution of firms across states $H(s)$, one obtains

$$\int V(s) \, \mathrm{d}H(s) =$$

$$\int \left[\Pi_d \left(1 + x(s) D^{1-\rho}\right) \exp(z) - x(s) n_x - n_f - c(q(s) \exp(z))\right] \mathrm{d}H(s) + \int V(s) \, \mathrm{d}TH(s)$$

Iterating on this expression, using G as the initial distribution in place of H gives that

$$\int V(s) \, \mathrm{d}G(s) = \sum_{n=0}^{\infty} \int \left[\Pi_d \left(1 + x(s) D^{1-\rho}\right) \exp(z) - x(s) n_x - n_f - c(q(s) \exp(z))\right] \mathrm{d}T^n G.$$

Using the free-entry condition gives

$$n_e = \sum_{n=0}^{\infty} \int \left[\Pi_d \left(1 + x(s) D^{1-\rho}\right) \exp(z) - x(s) n_x - n_f - c(q(s) \exp(z))\right] \mathrm{d}T^n G.$$

Reversing the order of summation and integration gives the result. Q.E.D.

Additional details of Proposition 3

Here we provide additional details for Proposition 3 in the version of our model with $\Delta_z = 0$ and a time-invariant fixed export cost n_x so that there are no dynamics in productivity and export decisions. We assume that there is a single (as well as time-invariant) value of n_x to simplify our presentation, but our results carry through if we assume that there are multiple levels of n_x. We allow for $\beta < 1$.

The steady-state value of a firm with productivity z is given by

$$V(z) = \frac{1}{1 - \beta(1 - \delta)} \max\left\{0, \Pi_d \exp(z) - n_f + \max\left\{0, \Pi_d \exp(z) D^{1-\rho} - n_x\right\}\right\}. \quad (6.3)$$

The free-entry condition is

$$\beta \int V(z) \, \mathrm{d}G(z) = n_e \ ,$$

where $G(z)$ is the distribution of productivity of entering firms.

The exit cutoff $\bar{z}$ is defined by $\Pi_d \exp(\bar{z}) = n_f$, and the export cut-off $\bar{z}_x$ is defined by $\Pi_d D^{1-\rho} \exp(\bar{z}_x) = n_x$. We assume, without loss of generality, that $n_f < n_x D^{\rho-1}$ so that the export cutoff is strictly higher than the exit cutoff.

Using the value functions (6.3), the free-entry condition can be written as

$$\Pi_d \left(\delta Z_d + \left(1 + D^{1-\rho}\right) \delta Z_x\right) - (1 - G(\bar{z})) n_f - (1 - G(\bar{z}_x)) n_x = \frac{(1 - \beta(1-\delta))}{\beta} n_e , \quad (6.4)$$

where the indices of aggregate productivity scaled by the measure of entering firms are

$$Z_d = \frac{1}{\delta} \int_{\bar{z}}^{\bar{z}_x} \exp(z) \, \mathrm{d}G(z) \text{ and } Z_x = \frac{1}{\delta} \int_{\bar{z}_x}^{\infty} \exp(z) \, \mathrm{d}G(z) .$$

Differentiating (6.4), we obtain

$$\Delta \Pi_d \delta \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right) + \Pi_d \delta Z_x \Delta \left(1 + D^{1-\rho}\right) +$$

$$(n_f - \Pi_d \exp(\bar{z})) \, dG(\bar{z}) \, \Delta \bar{z} + \left(n_x - \Pi_d D^{1-\rho}\right) \exp(\bar{z}_x) \, dG(\bar{z}_x) \, \Delta \bar{z}_x = 0$$

Using the cutoff definitions, the last two terms drop-out, so

$$\Delta \Pi_d \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right) + \Pi_d Z_x \Delta \left(1 + D^{1-\rho}\right) = 0 ,$$

which results in (4.3).

The average expenditure on the research good per entering firm, Υ, from (3.7) is

$$\Upsilon = n_e + \frac{1 - G(\bar{z})}{\delta} n_f + \frac{1 - G(\bar{z}_x)}{\delta} n_x. \quad (6.5)$$

The free-entry condition (6.4) can be expressed, using (6.5), as

$$\Pi_d \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right) - \Upsilon = \frac{(1 - \beta)}{\delta \beta} n_e. \quad (6.6)$$

If $\beta = 1$, then $\zeta = \Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] / \Upsilon = 1$ (confirming the result in Lemma 1), which implies from Lemma 2 that L_r is unchanged with D. Hence, the ratio of the indirect to direct effect of changes in trade costs on aggregate productivity is given by (4.4).

We now show that, if we allow for $\beta < 1$, and assume that G is such that $\exp(z)$ is distributed Pareto, we obtain that $\zeta = \Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] / \Upsilon$ is invariant to D, so Lemma 1 applies and hence L_r is unchanged with D. Therefore, the ratio of the indirect to direct effect of changes in trade costs on productivity is again given by (4.4).

In particular, we assume that the cdf of $\exp(z)$ is

$$G(\exp(z)) = 1 - \left(\frac{\exp(z_0)}{\exp(z)}\right)^{\sigma}, \text{ for } \exp(z) > \exp(\bar{z}_0).$$

Under this assumption, we have

$$Z_d = \int_{\exp(\bar{z})}^{\exp(\bar{z}_x)} \frac{\sigma}{\delta} \frac{\exp(\bar{z}_0)^{\sigma}}{\exp(z)^{\sigma}} \mathrm{d}\exp(z) = \frac{\sigma \exp(\bar{z}_0)^{\sigma}}{\delta(\sigma - 1)} \left[\exp(\bar{z})^{1-\sigma} - \exp(\bar{z}_x)^{1-\sigma}\right] \text{ , and}$$

$$Z_x = \int_{\exp(\bar{z}_x)}^{\infty} \frac{\sigma}{\delta} \frac{\exp(\bar{z}_0)^{\sigma}}{\exp(z)^{\sigma}} \mathrm{d}\exp(z) = \frac{\sigma \exp(\bar{z}_0)^{\sigma}}{\delta(\sigma - 1)} \exp(\bar{z}_x)^{1-\sigma},$$

or using the definitions of $\bar{z}$ and $\bar{z}_x$,

$$Z_d = \frac{\sigma \exp(\bar{z}_0)^{\sigma}}{\delta(\sigma - 1)} (\Pi_d)^{\sigma-1} \left[n_f^{1-\sigma} - \left(\frac{n_x}{D^{1-\rho}}\right)^{1-\sigma}\right], \text{ and}$$

$$Z_x = \frac{\sigma \exp(\bar{z}_0)^{\sigma}}{\delta(\sigma - 1)} (\Pi_d)^{\sigma-1} \left(\frac{n_x}{D^{1-\rho}}\right)^{1-\sigma}.$$

Therefore, we have

$$\Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] = \frac{\sigma}{\delta(\sigma - 1)} (\exp(\bar{z}_0) \Pi_d)^{\sigma} \left[n_f^{1-\sigma} + n_x^{1-\sigma} \left(D^{1-\rho}\right)^{\sigma}\right] \tag{6.7}$$

Using the cutoff definitions, we can express Υ as

$$\Upsilon = n_e + \frac{1}{\delta} (\exp(\bar{z}_0) \Pi_d)^{\sigma} \left[n_f^{1-\sigma} + n_x^{1-\sigma} \left(D^{1-\rho}\right)^{\sigma}\right]. \tag{6.8}$$

Combining (6.7) and (6.8) we obtain

$$\Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] = \frac{\sigma}{\sigma - 1} (\Upsilon - n_e)$$

Combined with (6.6), this implies that both $\Pi_d [Z_d + Z_x (1 + D^{1-\rho})]$ and Υ are independent of D. Therefore, Lemma 1 applies.

Figure 1 : Transition Dynamics of Exports/GDP from a Decline in Marginal Trade Costs

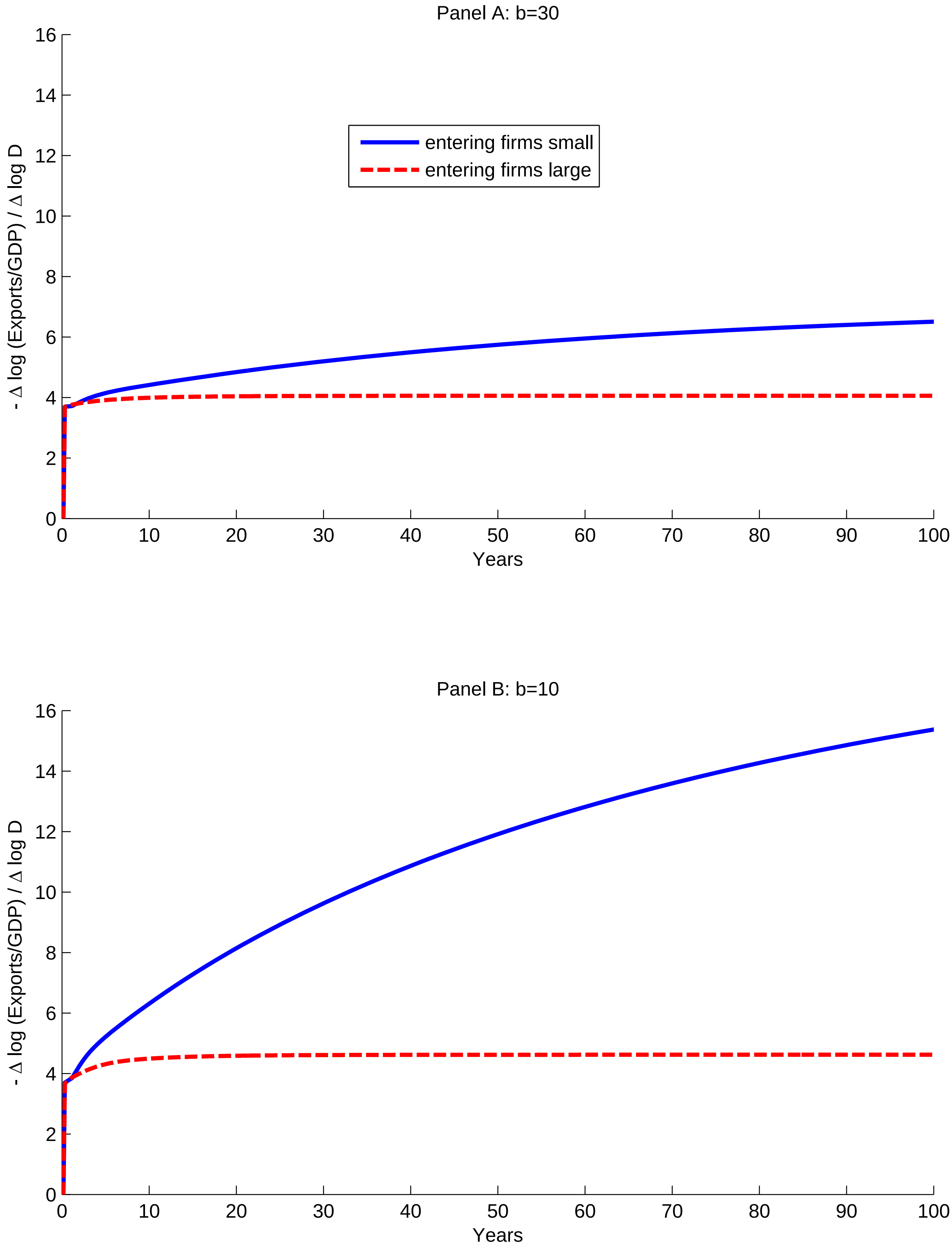

TABLE 1: Model Parameterization

Calibrated Parameters		1	2	3
Curvature process innovation cost function, b		3000	30	10
Exogenous death rate, δ (annualized)		0.005	0.005	0.005
Process innovation step size, Δz (annualized)		0.25	0.25	0.25
Level process innovation cost function, h (or employment-based right-tail coefficient of large firms)		1.3E+42 (-0.25)	0.31 (-0.25)	0.49 (-0.25)
Marginal trade cost parameter , D^(1-ρ)		0.231	0.231	0.231
Fixed cost of international trade, nx		1.4	0.7	0.285
Targets	**US Data**			
Employment growth rate of large firms, annual standard deviation	0.25	0.25	0.25	0.25
Annual employment-based exit rate, firms larger 500 employees	0.0055	0.0055	0.0055	0.0055
Employment-based right tail coefficient, firms of size 1000 to 5000	-0.2	-0.21		
Exports / GDP (of intermediate goods in model)	0.08	0.08	0.08	0.08
Employment share of exporters (production employment in model)	0.40	0.41	0.41	0.40

Other parameters

Annualized interest rate, 1/β annualized: 0 and 0.05
Share of labor in production of research good, λ : 1 and 0.5
Elasticity of substitution across intermediate goods, ρ : 5
Fixed entry cost, ne : 1
Fixed operation cost, nf : 0.1

TABLE 2

Experiment 1: Reduction in Marginal Trade Costs, Zero Real Interest Rate

	Research good produced with labor only λ=1			Research good produced with labor + goods λ=0.5		
Parameterization	1	2	3	4	5	6
Real interest rate, r	0	0	0	0	0	0
Curvature of process innovation cost function, b	3000	30	10	3000	30	10
Share of labor in research good production, λ	1	1	1	0.5	0.5	0.5
Export share, sx	0.075	0.075	0.076	0.075	0.075	0.076
Hybryd export share,	0.075	0.075	0.076	0.075	0.075	0.076
Elasticity of aggregate variables across steady-states negative of log change in variable / log change in D						
Constant on variable profits, Πd	-0.302	-0.301	-0.304	-0.302	-0.301	-0.304
Aggregate productivity, Z	0.076	0.075	0.076	0.086	0.086	0.087
Direct effect	0.075	0.075	0.076	0.075	0.075	0.076
Productivity of the average firm	0.000	1.170	3.780	0.000	1.170	3.780
Product Innovation	0.000	-1.170	-3.781	0.011	-1.160	-3.771
Aggregate Production Labor, L-Lr	0.000	0.000	0.000	0.000	0.000	0.000
Output, Y	0.075	0.075	0.076	0.086	0.086	0.087
Consumption, C	0.075	0.075	0.076	0.086	0.086	0.087
Ratio indirect / direct effect, numerical	0.00	0.00	0.00	0.15	0.14	0.15
Ratio indirect / direct effect, theoretical	0.00	0.00	0.00	0.14	0.14	0.14

TABLE 3

Experiment 2: Reduction in Marginal Trade Costs, Positive Real Interest Rate, Inelastic Process Innovation

	Small entering firms		Large entering firms	
	λ=1	λ=0.5	λ=1	λ=0.5
Parameterization	1	2	3	4
Real interest rate, r	0.05	0.05	0.05	0.05
Curvature of process innovation cost function, b	3000	3000	3000	3000
Share of labor in research good production, λ	1	0.5	1	0.5
Export share, sx	0.076	0.076	0.078	0.078
Hybryd export share,	0.004	0.004	0.075	0.075
Elasticity of aggregate variables across steady-states				
negative of log change in variable / log change in D				
Constant on variable profits, Πd	-0.019	-0.019	-0.301	-0.301
Aggregate productivity, Z	0.010	0.008	0.076	0.086
Direct effect	0.076	0.076	0.078	0.078
Productivity of the average firm	0.000	0.000	0.000	0.000
Product Innovation	-0.066	-0.068	-0.002	0.008
Aggregate Production Labor, L-Lr	0.020	0.010	0.002	0.001
Output, Y	0.030	0.019	0.077	0.087
Consumption, C	0.030	0.028	0.077	0.088
Ratio indirect / direct effect, numerical	-0.87	-0.89	-0.03	0.11
Ratio indirect / direct effect, theoretical	-0.88	-0.90	-0.03	0.11
Welfare / direct effect - 1	0.00	-0.04	0.00	0.11

TABLE 4

Experiment 3: Reduction in Marginal Trade Costs, Positive Real Interest Rate, Elastic Process Innovation

	Entering firms small				Entering firms large	
	λ=1		λ=0.5		λ=1	
Parameterization	1	2	3	4	5	6
Real interest rate, r	0.05	0.05	0.05	0.05	0.05	0.05
Curvature of process innovation cost function, b	30	10	30	10	30	10
Share of labor in research good production, λ	1	1	0.5	0.5	1	0.5
Export share, sx	0.076	0.075	0.076	0.075	0.075	0.085
Hybryd export share,	0.009	0.022	0.009	0.022	0.073	0.086
Elasticity of aggregate variables across steady-states						
negative of log change in variable / log change in D						
Constant on variable profits, Πd	-0.034	-0.090	-0.034	-0.090	-0.291	-0.344
Aggregate productivity, Z	0.036	0.095	0.027	0.071	0.074	0.099
Direct effect	0.076	0.075	0.076	0.075	0.075	0.085
Productivity of the average firm	0.626	2.644	0.626	2.644	0.045	0.145
Product Innovation	-0.666	-2.626	-0.675	-2.650	-0.047	-0.132
Aggregate Production Labor, L-Lr	0.112	0.289	0.060	0.158	0.005	0.003
Output, Y	0.148	0.384	0.087	0.229	0.079	0.102
Consumption, C	0.148	0.384	0.142	0.381	0.079	0.105
Ratio indirect / direct effect, numerical	-0.52	0.26	-0.65	-0.06	-0.02	0.16
Welfare / direct effect - 1	0.000	0.010	-0.004	0.036	0.000	0.000

TABLE 5

Experiment 4: Large Reduction in Marginal Trade Costs

	Research good produced with labor only λ=1			Research good produced with labor + goods λ=0.5		
Parameterization	1	2	3	4	5	6
Real interest rate, r	0.05	0.05	0.05	0.05	0.05	0.05
Curvature of process innovation cost function, b	3000	30	10	3000	30	10
Share of labor in research good production, λ	1	1	1	0.5	0.5	0.5
Export share, initial steady state	0.076	0.076	0.075	0.076	0.076	0.075
Export share, new steady state	0.093	0.109	0.205	0.093	0.109	0.205
Elasticity of aggregate variables across steady-states negative of log change in variable / log change in D						
Constant on variable profits, Πd	-0.020	-0.043	-0.122	-0.020	-0.043	-0.122
Aggregate productivity, Z	0.006	0.042	0.195	0.007	0.031	0.137
Direct effect + productivity of the average firm (*)	0.109	0.922	14.176	0.109	0.922	14.176
Product Innovation	-0.103	-0.880	-13.981	-0.102	-0.891	-14.039
Aggregate Production Labor, L-Lr	0.006	0.126	0.657	0.003	0.068	0.357
Output, Y	0.013	0.169	0.852	0.010	0.099	0.494
Consumption, C	0.013	0.169	0.852	0.012	0.161	0.832
Welfare / direct effect - 1	0.034	0.054	0.133	0.001	0.051	0.178

(*): We do not separately report the direct and indirect effects on average productivity because equation 4.1 is not very precise with a large change in D.