IDIOSYNCRATIC PRODUCTION RISK, GROWTH AND THE BUSINESS CYCLE

George-Marios Angeletos and Laurent E. Calvet*
MIT and Harvard University

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We introduce a neoclassical growth economy with idiosyncratic production risk and incomplete markets. The general equilibrium is characterized in closed form. Uninsurable production shocks introduce a risk premium on private equity and typically result in a *lower* steady-state level of capital than under complete markets. In the presence of such risks, the anticipation of low investment and high interest rates in the future discourages risk-taking and feeds back into low investment in the present. An endogenous macroeconomic complementarity thus arises, which slows down convergence and amplifies the magnitude and persistence of the business cycle. These results – contrasting sharply with those of Aiyagari (1994) and Krusell and Smith (1998) – highlight that idiosyncratic *production* or *capital-income* risk can have significant adverse effects on capital accumulation and aggregate volatility.

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1. Introduction

This paper investigates the impact of idiosyncratic production and capital-income risk on the level and volatility of macroeconomic activity. We introduce a neoclassical economy with decentralized production and incomplete insurance markets, in which the equilibrium is characterized in closed form. Even though agents face no borrowing constraints and wealth heterogeneity does not impact aggregate dynamics, incomplete risk sharing leads to substantial underaccumulation of capital, slows down convergence to the steady state, and generates a powerful amplification and propagation mechanism over the business cycle.

The standard neoclassical growth model of Cass, Koopmans and Brock-Mirman assumes complete markets, implying that private agents can fully diversify idiosyncratic risk in their labor and capital income. Following Bewley (1977), previous research introduces incomplete markets in the form of uninsurable idiosyncratic risk in *labor* income, while production and investment take place in a common aggregate technology.[1] Financial incompleteness leads to overaccumulation of capital in the steady state (Aiyagari, 1994) and has no quantitatively important effect on business-cycle dynamics (Krusell and Smith, 1998).

This paper departs from the Bewley class of models by considering uninsurable idiosyncratic risk in *production* activities and *capital* income. Each agent is a private producer operating her own neoclassical technology with her own capital stock. Production is subject to individual-specific uncertainty, which generates idiosyncratic risk in capital income. Incomplete risk sharing has strikingly different implications than in Bewley-type models. It leads to substantial underaccumulation of capital in the steady state and generates strong amplification and persistence over the business cycle.

Our focus on idiosyncratic production and capital-income risk is motivated by a number of empirical observations. Large undiversifiable entrepreneurial and investment risks are paramount not only in the developing world, but also in the most advanced economies. In a recent study of US private equity, Moskowitz and Vissing-Jørgensen (2001) document that entrepreneurs and private investors face a "dramatic lack of diversification" and an extreme dispersion in returns. In addition, these agents control a large fraction of savings and investment in the economy.[2] For agency and moral-hazard

[1] See Ríos-Rull (1995) and Ljungqvist and Sargent (2000, ch. 14) for a review of this literature.

[2] In the United States, private companies accounted for about half of production and corporate equity in 1998 and for more than two-thirds in the 70's and 80's. Moskowitz and Vissing-Jørgensen (2001) also observe: "About 77 percent of all private equity is owned by households for whom private equity constitutes at least half of their total net worth. Furthermore, households with private equity ownership invest on average more than 70 percent of their private holding in a single private company, in which the household has an active management interest. [...] Survival rates of private firms are around 34 percent over the first 10 years of the firm's life. Furthermore, even conditional on survival, entrepreneurial

reasons, executives of publicly traded firms are also very exposed to firm-specific risks in the production and investment decisions they make on behalf of shareholders. Furthermore, labor income often includes returns to entrepreneurial activity, education, or some form of human or intangible capital.[3] Controlling for these effects produces even larger estimates of capital-income risk.

The presence of undiversifiable production shocks does not impact the average return but increases the uncertainty of individual investment. As a result, agents discount future income by a risk premium on private equity. This decreases investment demand and leads to *under*accumulation of capital in the steady state as compared to complete markets.[4] We note that this result originates in risk aversion, and thus contrasts with the *over*accumulation of capital obtained in Bewley-type models from precautionary or buffer-stock savings.

Perhaps more surprisingly, undiversifiable production risk has novel implications for business-cycle dynamics. The private equity premium characterizes the willingness to engage in risky projects at a given point in time. This quantity depends on the ability to self-insure through borrowing and lending in later periods, and is thus sensitive to future credit conditions. In our model, the anticipation of low savings and high interest rates in the future feeds back into high risk premia and low investment in the present. At the same time, intertemporal consumption smoothing implies that low investment in the present propagates to low savings and high interest rates in the future. As a result, low savings and low investment can be self-sustaining for long periods of time.

Undiversifiable production risk thus gives rise to an endogenous dynamic macroeconomic complementarity, which generates amplification and persistence over the business cycle. Figure 1 illustrates this mechanism by considering an economy hit at date $t = 0$ by an unanticipated negative wealth shock. The solid lines represent the transmission of the shock over time in the standard neoclassical growth model. The immediate impact is to reduce savings, increase interest rates and reduce investment at $t = 0$. The shock then propagates to lower wealth, higher interest rates, and lower investment in later periods. This transmission channel originates in intertemporal consumption smoothing and is the fundamental propagation mechanism of the complete-markets RBC paradigm. In the presence of uninsurable idiosyncratic production risk, it is complemented by a risk-taking effect. As agents anticipate higher interest rates at $t = 1$, they become less willing to engage in risky projects and further reduce investment at $t = 0$. Similarly, the anticipation of higher interest rates at $t = 2$ feeds back to even higher risk premia

investment appears extremely risky, generating a wide distribution of returns."

[3] Hall (2001) emphasizes the role of intangible capital in productivity growth and stock returns. Entrepreneurial activity and capital income also have important empirical implications for the wealth distribution (Castaneda *et al.,* 1998; Quadrini, 2000) and portfolio choices (Heaton and Lucas, 2000).

[4] Our model thus complements earlier research investigating the impact of *aggregate* production risk on investment and diversification (e.g. Acemoglu and Zilibotti, 1997).

and even lower investment at $t = 0$ and $t = 1$. This second channel is represented by the dashed arrows in the figure. The combination of the intertemporal-smoothing and risk-taking effects thus amplifies the impact of the exogenous shock and slows down the recovery of the economy.

Note that when private agents plan how much to save and invest in one period, they do not internalize the impact that equilibrium interest rates have on optimal investment and equilibrium risk premia in earlier periods. The macroeconomic complementarity thus originates in a pecuniary externality in risk-taking; it is a genuine general-equilibrium implication of missing markets. Moreover, the complementarity arises only when agents face idiosyncratic production risk, and is thus absent from the Bewley class of models.

We emphasize that the proposed transmission mechanism originates in the endogenous countercyclicality of risk premia. This is an important observation. Any model in which risk premia increase in anticipation of an economic slowdown is likely to produce similar business-cycle effects. Furthermore, although there is no available empirical information on private premia, the market price of risk is strongly countercyclical,[5] which suggests that our theoretical arguments may have substantial empirical content.

Our model belongs to the class of general-equilibrium economies with incomplete markets and heterogeneous infinitely-lived agents.[6] Such models generally suffer from the "curse of dimensionality" because the wealth distribution – an infinite-dimensional object – is a relevant state variable. Banerjee and Newman (1993), Galor and Zeira (1993) and others have stressed the potential importance of wealth heterogeneity for fluctuations and growth in a variety of settings. In a calibrated Bewley-type economy, however, Krusell and Smith (1998) show that wealth heterogeneity has *approximately* no impact on aggregate dynamics. By adopting a CARA-normal specification (exponential preferences and Gaussian risks), we render equilibrium prices and macro aggregates *exactly* independent of the wealth distribution. This allows the characterization of general equilibrium in closed form, which, to the best of our knowledge, is new to the incomplete-market growth literature. Moreover, this research highlights that incomplete markets can have important implications for capital accumulation, medium-run growth and business cycles, *even* when wealth heterogeneity does not influence aggregate dynamics.

The absence of borrowing constraints is another important feature of our model. The transmission mechanism we identify is thus different from – and in fact comple-

[5] See for instance Campbell (1999) for a review of the empirical relation between asset prices and business cycles.

[6] Examples include Aiyagari (1994, 1995), Calvet (2001), Constantinides and Duffie (1996), Heaton and Lucas (1996), Huggett (1993, 1997), Krusell and Smith (1998), Ríos-Rull (1996), and Scheinkman and Weiss (1986).

mentary to – the effect of credit-market imperfections (Bernanke and Gertler, 1989, 1990; Kiyotaki and Moore, 1997; Aghion, Banerjee and Piketty, 1999). This earlier research focuses on borrowing constraints that affect the *ability* to invest. Some empirical research, however, has questioned the impact of borrowing constraints on the cyclical behavior of investment (e.g. Kaplan and Zingales, 1997). In this paper, we abstract from credit-market imperfections and focus on incomplete insurance, which only affects the *willingness* to invest.[7] We show that a countercyclical private equity premium is sufficient to generate amplification and persistence in aggregate fluctuations, even in the absence of financial constraints or balance-sheet effects.[8] Furthermore while this earlier research does not address the large cyclical variation in the market price of risk, our paper highlights the important interactions between business cycles and risk premia.

The rest of the paper is organized as follows. Section 2 introduces the economy and Section 3 analyzes the individual decision problem. In Section 4 we characterize the general equilibrium in closed form, analyze the steady state, and describe the propagation and amplification mechanism arising in the presence of idiosyncratic production risk. Section 5 presents numerical simulations and Section 6 concludes. All proofs are in the Appendix.

2. A Ramsey Economy with Incomplete Risk Sharing

This section introduces a neoclassical growth economy with decentralized production, CARA preferences, Gaussian idiosyncratic uncertainty, and an exogenous incomplete asset span.

2.1. Technology and Idiosyncratic Risks

Time, indexed by $t \in \mathbb{N} \equiv \{0, 1, ...\}$, is discrete and infinite. The economy is stochastic and all random variables are defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Individuals are indexed by $j \in \mathbb{J} = \{1, ..., J\}$.[9] They are all born at date 0, live forever, and consume a single consumption good in every date.

Each individual is also a producer, or entrepreneur, who operates his own production scheme using his own labor and capital stock. The technology is standard neoclassical. It exhibits constant returns to scale with respect to capital and labor, has diminishing marginal returns with respect to each input, and satisfies the Inada conditions. There

[7] In contrast to the Bernanke-Gertler class of models, our approach considers infinitely-lived utility-maximizing agents. This permits direct comparison with standard RBC frameworks.

[8] Our analysis highlights that, even if a borrowing constraint is not currently binding, the risk of adverse financial conditions in the future affects incentives to invest in the present. This type of dynamic feedback can have important implications for the cyclical behavior of both asset prices and aggregate investment.

[9] The model directly extends to economies with a continuum of agents.

are no adjustment costs and no indivisibilities in investment. The individual can invest in a single type of capital. We denote by k_t^j the stock of capital that individual j holds at the beginning of period t, and by l_t^j his effective labor endowment in period t. The individual output in period t is $A_t^j F(k_t^j, l_t^j)$. The production function F is deterministic and identical in the population. We assume that all individuals have the same labor supply $l_t^j = \bar{l}$, and thus conveniently consider the function $f(k) \equiv F(k, \bar{l})$.[10] The total factor productivity A_t^j is a random shock specific to agent j. The individual controls k_t^j through his investment at date $t-1$, while A_t^j is observed only at date t. Production is thus subject to idiosyncratic uncertainty, which we also call *technological, entrepreneurial,* or *investment* risk.

For comparison with production shocks, it is useful to also introduce endowment risks. We let e_t^j denote the exogenous stochastic endowment of the consumption good that individual j receives in period t. These shocks model risks that are outside the control of individuals and do not affect production or investment opportunities. The overall non-financial income of individual j in period t is

$$y_t^j = A_t^j f(k_t^j) + e_t^j. \tag{2.1}$$

The random shock A_t^j is multiplicative, while the endowment risk e_t^j is additive.

2.2. Asset Structure

Idiosyncratic risks can be partially hedged by trading a limited set of short-lived securities indexed by $m \in \{0, 1, .., M\}$. Purchasing one unit of security m at date t yields a random amount of consumption $d_{m,t+1}$ at date $t+1$. The price of the security at date t is denoted by $\pi_{m,t}$. The vectors $\pi_t = (\pi_{m,t})_{m=0}^M$ and $d_t = (d_{m,t})_{m=0}^M$ are respectively the price vector and the payoff vector at date t. Security $m = 0$ is a riskless bond, and assets $m \in \{1, ..., M\}$ are risky. The bond delivers $d_{0,t} = 1$ with certainty in every t, and $R_t \equiv 1/\pi_{0,t}$ and $r_t \equiv R_t - 1$ denote respectively the gross and the net interest rate between t and $t+1$.

The asset span is exogenous and generally incomplete. We rule out default, short-sales constraints and any other credit-market imperfections. Finally, for simplicity, we assume that all assets are in zero net supply.[11] At the outset of every period t, individuals are informed of the contemporaneous realization of asset payoffs d_t and idiosyncratic shocks $\{(A_t^j, e_t^j)\}_{j \in \mathbb{J}}$. Information is thus homogeneous across agents and

[10] f satisfies $f'(k) > 0 > f''(k)\ \forall k \in (0, \infty)$, $\lim_{k \to 0} f'(k) = \infty$, and $\lim_{k \to \infty} f'(k) = 0$.

[11] Ricardian equivalence holds in our model because agents have infinite horizons and can freely trade the riskless bond. Therefore as long as public debt is financed by lump-sum taxation, there is no loss of generality in assuming that the riskless bond is in zero net supply.

generates a filtration $\{\mathcal{F}_t\}_{t=0}^{\infty}$.[12] We denote by $\mathbb{E}_t$ the expectation operator conditional on $\mathcal{F}_t$.

2.3. Preferences

To distinguish between intertemporal substitution and risk aversion, we adopt a preference specification that belongs to the Kreps-Porteus/Epstein-Zin non-expected utility class. Consider two concave Bernoulli utilities U and Υ. A stochastic consumption stream $\{c_t\}_{t=0}^{\infty}$ generates a stochastic utility stream $\{u_t\}_{t=0}^{\infty}$ defined by the recursion

$$U(u_t) = U(c_t) + \beta U[\mathbb{CE}_t(u_{t+1})] \ \ \forall t \geq 0, \tag{2.2}$$

where $\mathbb{CE}_t(u) \equiv \Upsilon^{-1}[\mathbb{E}_t\Upsilon(u)]$ denotes the certainty equivalent of u. The curvature of Υ thus governs risk aversion, while the curvature of U governs intertemporal substitution. When $\Upsilon = U$, these preferences reduce to standard expected utility, $U(u_0) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$. Finally, note that u_t is measured in consumption units.

2.4. CARA-Normal Specification

Closed-form certainty equivalents are not possible in general, but can be obtained in the CARA-normal case.

Assumption 1 (Exponential Preferences) Agents have identical recursive utility (2.2) with

$$U(c) = -\Psi \exp(-c/\Psi), \qquad \Upsilon(c) = -(1/\Gamma)\exp(-\Gamma c). \tag{2.3}$$

Assumption 2 (Gaussian Risks) The idiosyncratic risks $\{(A_t^j, e_t^j)\}_{j\in\mathbb{J}}$ and asset returns d_t are jointly normal.

Note that a high Ψ corresponds to a strong willingness to substitute consumption through time, while a high Γ implies a high risk aversion. The two assumptions are motivated by analytical tractability but are not critical for the main arguments of the paper. Section 5 will propose a calibration method that assumes a constant *relative* risk aversion and a constant elasticity of intertemporal substitution at the steady state.

In addition, we consider

Assumption 3 (No Persistence) The idiosyncratic shocks $\{(A_t^j, e_t^j)\}_{j\in\mathbb{J}}$ and the asset payoffs d_t are i.i.d. across time.[13]

[12] The results of this paper are not modifed when income shocks are privately observed and the structure of the economy is common knowledge.

[13] More generally, the results of the paper remain unchanged when: (1) idiosyncratic shocks and dividends follow linear processes, and (2) the residuals of the projection of $\{(A_t^j, e_t^j)\}_{j\in\mathbb{J}}$ on d_t are serially uncorrelated.

Individual investment choices are then independent of contemporaneous idiosyncratic income and productivity shocks, which greatly simplifies aggregation. In the quantitative analysis of Section 5, we will mimic persistent idiosyncratic shocks by increasing the length of a time period.

Assumption 4 (No Aggregate Uncertainty) In all dates and events, $\sum_j A_t^j/J = \mathbb{E}_{t-1}A_t^j = \overline{A}$ and $\sum_j e_t^j/J = \mathbb{E}_{t-1}e_t^j = 0$.[14]

This restriction, too, only serves the tractability of the model; it will imply that asset prices and all macro variables are deterministic in equilibrium. In Section 4, we discuss the implications of idiosyncratic productivity risk for an economy with aggregate uncertainty.

3. Decision Theory

This section examines the decision problem of an individual agent. We show that the portfolio choice reduces to a mean-variance problem and then derive the optimal saving and investment rules.

3.1. The Individual Problem

Consider an individual j in period t. Denote his consumption by c_t^j, physical-capital investment by i_t^j, non-financial income by y_t^j, and portfolio of the bond and the risky assets by $\theta_t^j = (\theta_{m,t}^j)_{m=0}^M$. The agent's budget constraint in period t is

$$c_t^j + i_t^j + \pi_t \cdot \theta_t^j = y_t^j + d_t \cdot \theta_t^j, \tag{3.1}$$

where y_t^j is given by (2.1). The agent accumulates capital according to $k_{t+1}^j = (1-\delta)k_t^j + i_t^j$, where $\delta \in [0,1]$ is the fixed depreciation rate of capital. To simplify notation, we conveniently rewrite the decision problem in terms of stock variables. We let

$$w_t^j \equiv A_t^j f(k_t^j) + (1-\delta)k_t^j + e_t^j + d_t \cdot \theta_t^j$$

represent the agent's total *wealth* (or *cash-in-hand*) at date t. We then restate the budget constraint (3.1) as

$$c_t^j + k_{t+1}^j + \pi_t \cdot \theta_t^j = w_t^j. \tag{3.2}$$

Given a price sequence $\{\pi_t\}_{t=0}^\infty$, agent j chooses an adapted plan $\{c_t^j, k_{t+1}^j, \theta_t^j, w_t^j\}_{t=0}^\infty$ that maximizes utility and satisfies the budget constraint (3.2).

[14] This assumption follows naturally from the Law of Large Numbers when the economy contains an infinity of agents facing independent shocks.

The indirect utility of wealth $V_t^j(w)$ satisfies the Bellman equation

$$U\left(V_t^j(w_t^j)\right) = \max_{(c_t^j, k_{t+1}^j, \theta_t^j)} U(c_t^j) + \beta\, U\left(\mathbb{CE}_t[V_{t+1}^j(w_{t+1}^j)]\right), \tag{3.3}$$

subject to (3.2) and the transversality condition $\lim_{t\to\infty} \beta^t U\left(\mathbb{CE}_t[V_{t+1}^j(w_{t+1}^j)]\right) = 0$. We denote the corresponding optimal consumption rule by $c_t^j(w)$.

3.2. Consumption-Investment Choice

CARA preferences and the absence of short-sales constraints imply that the demand for risky assets and productive capital are independent of wealth. We can thus anticipate that the wealth distribution will not affect aggregate dynamics. This property, together with Assumption 4, will ensure that all macro variables are deterministic. This subsection thus develops individual decision theory when interest rates are deterministic and traded assets have risk premia equal to zero.[15]

Along the equilibrium price path, an educated guess is that the value function and the optimal consumption rule are linear in wealth:

$$V_t^j(w) = a_t^j w + b_t^j, \qquad c_t^j(w) = \widehat{a}_t^j w + \widehat{b}_t^j, \tag{3.4}$$

where $a_t^j, \widehat{a}_t^j > 0$ and $b_t^j, \widehat{b}_t^j \in \mathbb{R}$ are non-random coefficients to be determined. In every period t, the future Gaussian wealth w_{t+1}^j generates a certainty-equivalent utility $\mathbb{CE}_t[V_{t+1}^j(w_{t+1}^j)]$. We infer from (2.3) and (3.4) that

$$\mathbb{CE}_t\left[V_{t+1}^j(w_{t+1}^j)\right] = V_{t+1}^j\left[\mathbb{E}_t w_{t+1}^j - \Gamma_t^j \mathrm{Var}_t(w_{t+1}^j)/2\right], \tag{3.5}$$

where $\Gamma_t^j \equiv \Gamma a_{t+1}^j$ measures absolute risk aversion in period t with respect to wealth variation in period $t+1$. We henceforth call Γ_t^j the *effective* degree of risk aversion at date t.

Without loss of generality, we normalize all risky securities ($m \geq 1$) to have zero expected payoffs. Since there is no risk premium, the assets have zero prices and are thus only used for hedging purposes. For any $(c_t^j, k_{t+1}^j, \theta_{0,t}^j)$, the optimal portfolio $(\theta_{m,t}^j)_{m=1}^M$ is chosen to minimize the conditional variance of wealth. This has a simple geometric interpretation. We project A_{t+1}^j and e_{t+1}^j on the asset span:

$$A_{t+1}^j = \kappa^j \cdot d_{t+1} + \eta_{t+1}^j, \qquad e_{t+1}^j = \xi^j \cdot d_{t+1} + \varepsilon_{t+1}^j.$$

The projections $\kappa^j \cdot d_{t+1}$ and $\xi^j \cdot d_{t+1}$ represent the *diversifiable* components of the idiosyncratic production and endowment risks. The residuals η_{t+1}^j and ε_{t+1}^j, which

[15] See Angeletos and Calvet (2000) for a more general analysis.

are orthogonal to d_{t+1}, correspond to the *undiversifiable* shocks and alone determine individual choices. Assumption 3 implies that these variables are i.i.d. through time. To maintain the symmetry of the model, we also assume that residuals η^j_{t+1} and ε^j_{t+1} are identically distributed across agents. Their variances

$$\sigma_A^2 \equiv \mathrm{Var}(\eta^j_{t+1}|\mathcal{F}_t) = \mathrm{Var}(\eta^j_{t+1}) \qquad \sigma_e^2 \equiv \mathrm{Var}(\varepsilon^j_{t+1}|\mathcal{F}_t) = \mathrm{Var}(\varepsilon^j_{t+1})$$

are useful measures of financial incompleteness.[16] We observe that $\sigma_A = 0$ and $\sigma_e = 0$ under complete markets. Aiyagari (1994) and Krusell and Smith (1998) considered economies with additive idiosyncratic risks but no idiosyncratic production risks, which in our model corresponds to $\sigma_e > 0$ but $\sigma_A = 0$. This paper focuses on the presence of uninsurable idiosyncratic production risks, and thus $\sigma_A > 0$.

After optimal hedging, individual wealth reduces to $w^j_{t+1} = (\overline{A} + \eta^j_{t+1})f(k^j_t) + (1-\delta)k^j_t + \varepsilon^j_{t+1} + \theta^j_{0,t}$. It has conditional variance $\mathrm{Var}_t(w^j_{t+1}) = \sigma_e^2 + f(k^j_{t+1})^2\sigma_A^2$. We define $\Phi(k) \equiv \overline{A}f(k) + (1-\delta)k$ as the *expected production function*; and

$$G(k,\Gamma) \equiv \Phi(k) - \Gamma\left[\sigma_e^2 + f(k)^2\sigma_A^2\right]/2$$

as the *risk-adjusted output.* By (3.5), the plan $(c^j_t, k^j_{t+1}, \theta^j_{0,t})$ maximizes

$$U(c^j_t) + \beta U\left\{V^j_{t+1}\left[G(k^j_{t+1},\Gamma^j_t) + \theta^j_{0,t}\right]\right\}$$

subject to the budget constraint, $c^j_t + k^j_{t+1} + \theta^j_{0,t}/R_t = w^j_t$.

The first-order conditions (FOCs) with respect to k^j_t and $\theta^j_{0,t}$ imply the key condition for investment demand:

$$R_t = \frac{\partial G}{\partial k}(k^j_{t+1},\Gamma_t) = \Phi'(k^j_{t+1}) - \Gamma^j_t f(k^j_{t+1})f'(k^j_{t+1})\sigma_A^2. \tag{3.6}$$

Under complete markets ($\sigma_A = 0$), the agent equates the marginal product of capital with the interest rate: $R_t = \Phi'(k^j_{t+1})$. In the presence of uninsurable production shocks ($\sigma_A > 0$), however, the return on investment is adjusted for risk. The difference between the expected marginal product of capital and the risk-free rate, $\Phi'(k^j_{t+1}) - R_t$, represents the *risk premium on private equity.* Note that it is proportional to the uninsurable production risk σ_A^2 and the effective risk aversion Γ^j_t.

The FOC with respect to the riskless rate implies the Euler equation:

$$\mathbb{E}_t c^j_{t+1} - c^j_t = \Psi\ln(\beta R_t) + \Gamma\mathrm{Var}_t(c^j_{t+1})/2. \tag{3.7}$$

Expected consumption growth thus increases with the variance of consumption. This reflects the standard precautionary motive for savings (Leland, 1968; Sandmo, 1970;

[16] The assumption that σ_A and σ_e are independent of t can easily be relaxed. In addition, we could let σ_A and σ_e depend on *aggregate* wealth to capture that risk sharing worsens during recessions.

Caballero, 1990; Kimball, 1990). The sensitivity of the growth rate to consumption risk is governed by the risk aversion Γ, while its sensitivity to the interest rate is governed by the intertemporal substitution Ψ.

The envelope and Euler conditions imply after simple manipulation that $\widehat{a}_t^j = a_t^j$ and $a_t^j = 1/[1 + (a_{t+1}^j R_t)^{-1}]$. Forward iteration yields

$$a_t^j = \frac{1}{1 + \sum_{s=0}^{+\infty} (R_t R_{t+1} ... R_{t+s})^{-1}}. \tag{3.8}$$

The marginal utility of wealth is thus the inverse of the price of a perpetuity delivering one unit of the consumption good in each period $s \geq t$. We also infer from (3.8) that *effective* risk aversion $\Gamma_t^j \equiv \Gamma a_{t+1}^j$ is an increasing function of *future* interest rates.

The solution to the individual choice problem is summarized below:

Proposition 1 (Individual Choice) For any path $\{R_t\}_{t=0}^{\infty}$, the value function and consumption rule are linear in wealth, as in (3.4), and the coefficients a_t^j and $\widehat{a}_t^j$ are equal and satisfy (3.8). The demand for investment is given by

$$R_t = \Phi'(k_{t+1}^j) - \Gamma_t^j f(k_{t+1}^j) f'(k_{t+1}^j) \sigma_A^2. \tag{3.9}$$

Consumption and savings are characterized by the Euler equation,

$$\mathbb{E}_t c_{t+1}^j - c_t^j = \Psi \ln(\beta R_t) + \frac{\Gamma}{2} \mathrm{Var}_t(c_{t+1}^j), \tag{3.10}$$

where $\mathrm{Var}_t(c_{t+1}^j) = (a_{t+1}^j)^2 [\sigma_e^2 + f(k_{t+1}^j)^2 \sigma_A^2]$. Finally, effective risk aversion $\Gamma_t^j \equiv \Gamma a_{t+1}^j$ increases with future interest rates.

3.3. Comparative Statics

Consider the impact of incomplete markets on capital accumulation. The optimality condition (3.9) defines optimal investment as a function of the contemporaneous interest rate, the effective risk aversion, and the production risk: $k_{t+1}^j = k(R_t, \sigma_A, \Gamma_t^j)$. This function decreases with the interest rate R_t and the production risk σ_A, but is independent of the endowment risk σ_e. When $\sigma_A > 0$, the optimal investment k_{t+1}^j also decreases with the effective risk aversion Γ_t^j and thus, by (3.8), with future interest rates.

Proposition 2 (Investment) The demand for investment decreases with uninsurable production risk: $\partial k_{t+1}^j / \partial \sigma_A < 0$. When $\sigma_A > 0$, investment is also discouraged by high future interest rates: $\partial k_{t+1}^j / \partial R_s < 0$ for all $s > t$.

Higher interest rates in the future increase the effective risk aversion and thus raise the risk premium on private equity in the present. We expect that the feedback between future credit conditions and current risk-taking is much more general than our model, and would be strengthened by the presence of borrowing constraints. In Section 4.3, we will demonstrate that this feedback generates a dynamic macroeconomic complementarity, which can be the source of amplification and persistence over the business cycle.

We note that high idiosyncratic production risk and high interest rates tend to *reinforce* each other's negative impact on capital accumulation.[17] Because downturns are associated with large risks and bad credit conditions, the amplification and persistence effects documented in this paper are likely to be empirically stronger during recessions.

Consider next the impact of incomplete markets on savings. Although endowment risk unambiguously increases precautionary savings, the effect of technological shocks is small or ambiguous because production risk is endogenous. It is indeed possible that when σ_A goes up, the individual scales back investment k_{t+1}^j so much that output risk $f(k_{t+1}^j)\sigma_A$ actually *decreases.* From the Euler equation (3.10), we conclude:

Proposition 3 (Savings) An increase in endowment risk σ_e raises both wealth risk $\mathrm{Var}_t(w_{t+1}^j)$ and consumption growth $\mathbb{E}_t c_{t+1}^j - c_t^j$. The impact of production risk σ_A is generally ambiguous. For example, in the case of a Cobb-Douglas technology with capital share $\alpha = 1/2$, there is a threshold $\overline{\sigma}_A$ such that $\partial Var_t(w_{t+1}^j)/\partial\sigma_A < 0$ if and only if $\sigma_A > \overline{\sigma}_A$.

While Bewley-type models focus on the effect of incomplete markets on precautionary savings, we observe that this channel is weak or even ambiguous in the presence of production risks. In contrast, the effect on the private equity premium originates in risk aversion, unambiguously reduces investment, and seems likely to dominate in equilibrium.

4. General Equilibrium and Steady State

We now characterize in closed form the general equilibrium and the steady state of the economy.

[17] In the proof of Proposition 2 (Appendix A), we show that $\partial^2 k_{t+1}^j/(\partial\sigma_A^2 \partial R_s) < 0 \ \forall s > t$, at least in the neighborhood of complete markets.

4.1. Definitions

By Assumptions 2-4, undiversifiable idiosyncratic risks cancel out in aggregate and are normally distributed in every period t:

$$\sum_j \eta_t^j = \sum_j \varepsilon_t^j = 0, \qquad \begin{pmatrix} \eta_t^j \\ \varepsilon_t^j \end{pmatrix} \sim \mathcal{N}\left[0, \begin{pmatrix} \sigma_A^2 & 0 \\ 0 & \sigma_e^2 \end{pmatrix}\right] \ \forall j.$$

The variances σ_A^2 and σ_e^2 parsimoniously quantify the structure of risks, and the economy is fully specified by the parameters $\mathcal{E} = (\beta, \Gamma, \Psi, F, A, \delta, \sigma_A, \sigma_e)$.

Condition (3.8) implies that, in every period, agents share the same marginal utility of wealth and effective risk aversion: $a_t^j = a_t$ and $\Gamma_t^j = \Gamma a_{t+1}$. We then infer two important properties from condition (3.9). First, because agents have CARA preferences and face no borrowing constraints, the optimal investment k_{t+1}^j is independent of contemporaneous individual wealth w_t^j. Second, because technology $(f, \delta, \overline{A})$, investment risk σ_A, and effective risk aversion Γa_{t+1} are identical in the population, all agents choose the same level of investment: $k_{t+1}^j = K_{t+1}\ \forall j$. Similarly, all agents face identical consumption risk: $\mathrm{Var}_t(c_{t+1}^j) = (a_{t+1})^2[\sigma_e^2 + f(K_{t+1})^2\sigma_A^2]$. The CARA-normal specification thus implies that wealth heterogeneity is irrelevant for aggregate dynamics.

Since there is no exogenous aggregate uncertainty, we focus on equilibria in which the interest rate is deterministic and there is no risk premium on financial assets. Financial securities thus play only one role in the model – the definition of the uninsurable components of idiosyncratic production and endowment risks.

Definition An *incomplete-market equilibrium* is a deterministic price sequence $\{\pi_t\}_{t=0}^{\infty}$ and a collection of state-contingent plans $(\{c_t^j, k_{t+1}^j, \theta_t^j, w_t^j\}_{t=0}^{\infty})_{j=1}^J$ such that: (1) the plan $\{c_t^j, k_{t+1}^j, \theta_t^j, w_t^j\}_{t=0}^{\infty}$ maximizes the utility of each agent j; and (2) asset markets clear in every date and event: $\sum_{j=1}^J \theta_t^j = 0$.

4.2. Equilibrium Characterization

Let C_t, W_t and K_t respectively denote the population averages of consumption, wealth and capital in period t. Note that the initial mean wealth, $W_0 = \sum_{j=1}^J w_0^j/J$, is an exogenous parameter for the economy. Since a_t^j, Γ_t^j, k_{t+1}^j, and $\mathrm{Var}_t(c_{t+1}^j)$ are identical across agents, aggregation is straightforward and we conclude:

Theorem 1 (General Equilibrium) There exists an incomplete-market equilibrium in which the macro path $\{C_t, K_{t+1}, W_t, R_t\}_{t=0}^{\infty}$ is deterministic and all agents choose identical levels of productive investment. For all $t \geq 0$, the equilibrium path satisfies

$$R_t = \Phi'(K_{t+1}) - \Gamma a_{t+1} f(K_{t+1}) f'(K_{t+1}) \sigma_A^2 \tag{4.1}$$

$$C_{t+1}-C_t = \Psi \ln(\beta R_t) + \Gamma a_{t+1}^2 \left[\sigma_e^2 + f(K_{t+1})^2 \sigma_A^2\right]/2 \quad (4.2)$$

$$a_t = \left[1 + \sum_{s=0}^{+\infty} (R_t R_{t+1} ... R_{t+s})^{-1}\right]^{-1} \quad (4.3)$$

$$C_t + K_{t+1} = W_t \quad (4.4)$$

$$W_{t+1} = \Phi(K_{t+1}). \quad (4.5)$$

Conditions (4.1) and (4.3) follow directly from the individual decision problem. Equation (4.2) is obtained by aggregating the individual Euler equations. If there were no undiversifiable idiosyncratic production risks, (4.1) would reduce to the familiar complete-markets condition $R_t = \Phi'(K_{t+1})$. If in addition there were no undiversifiable endowment risks, then (4.2) would reduce to the complete-markets Euler equation $U'(C_t) = \beta R_t U'(C_{t+1})$. Finally, conditions (4.4) and (4.5) express the resource constraint and the production frontier of the economy.

Under incomplete markets, aggregate consumption growth increases with the variance of individual consumption, reflecting the standard precautionary motive. More interestingly, idiosyncratic production shocks introduce a risk premium on private equity, which reduces aggregate investment for a given risk-free rate. When each agent invests an additional unit of capital, aggregate output increases deterministically by $\Phi'(K_{t+1})$, but individual risk-adjusted returns are only $\Phi'(K_{t+1}) - \Gamma a_{t+1} f(K_{t+1}) f'(K_{t+1}) \sigma_A^2 = R_t$. The premium $\rho_t \equiv \Phi'(K_{t+1}) - R_t = (\Gamma a_{t+1}) f(K_{t+1}) f'(K_{t+1}) \sigma_A^2$ thus quantifies the gap between the social and private return on investment.

We finally note from (3.8) that the effective risk aversion Γa_{t+1} and thus the risk premium ρ_t increase with all future interest rates $\{R_s\}_{s=t+1}^{\infty}$. An anticipated increase in future rates raises the premium on private equity and thereby decreases the demand for investment. In the next subsection, we show that this feedback generates a dynamic macroeconomic complementarity, which induces persistence and amplification in the transitional dynamics.

4.3. Propagation and Amplification: An Endogenous Dynamic Macroeconomic Complementarity

Consider an economy in steady state, which is hit at date t by an unanticipated negative wealth shock. The impact of such a shock in a *complete-market* Ramsey economy is well-known. Consumption and investment fall, interest rates rise, and the economy converges monotonically and asymptotically back to the steady state. The transition takes some time under complete markets only because agents seek to smooth consumption. But, when markets are *incomplete*, the intertemporal substitution effect is complemented by a risk-taking effect. Anticipating high interest rates in the near future, private agents are less willing to invest in risky production. This effect amplifies the fall in initial investment and slows down convergence to the steady state, as compared to complete markets.

This mechanism is illustrated in a simplified version of the model.

Example. For expositional simplicity, we consider that markets are incomplete at date $t = 0$ but complete in all subsequent periods $t \in \{1, .., \infty\}$. Assume in addition that the aggregate endowment $e_t \equiv \sum_j e_t^j / J$ equals zero when $t \neq 1$, and let $I_t = K_{t+1}$ denote the gross investment in period t. We characterize equilibrium and then analyze the response of the economy to an aggregate shock.

Since markets are complete at every $t \geq 1$, we can easily solve for the Ramsey path generated by a wealth level W_1. Since agents smooth consumption through time, productive investment is an increasing function of contemporaneous wealth: $I_t = I^*(W_t)$. The wealth level next period is then $W_{t+1} = \Phi(I_t)$. Iterating these two equations, we infer that I_t can be expressed as an increasing function of I_1 alone. By (3.8), effective risk aversion Γ_0 is determined by future interest rates $\{R_t = \Phi'(I_t)\}_{t=1}^{\infty}$, and is thus a decreasing function of period-1 investment alone: $\Gamma_0 = \Gamma^*(I_1)$, $\Gamma^{*\prime} < 0$. A lower I_1 reduces I_t and increases R_t in all periods $t \geq 1$, implying a high effective risk aversion in period 0.

We now characterize equilibrium at $t = 1$. The wealth level W_1 is determined by previous investment and contemporaneous endowment: $W_1 = \Phi(I_0) + e_1$. Since markets are complete at $t = 1$, we infer that $I_1 = I^*(W_1)$ and thus

$$I_1 = I^*(\Phi(I_0) + e_1). \tag{4.6}$$

More investment in period 0 (or more endowment in period 1) generates more wealth and therefore more investment in period 1. This reflects intertemporal consumption-smoothing under complete markets.

Consider now equilibrium in the initial period $t = 0$. Since markets are incomplete, I_0 satisfies

$$\Phi'(I_0) = R_0 + \rho(I_0, I_1), \tag{4.7}$$

where $\rho(I_0, I_1) \equiv \Gamma^*(I_1) f(I_0) f'(I_0) \sigma_A^2$ is the risk premium on private equity. This relation defines I_0 as a function of the contemporaneous interest rate R_0 and future investment I_1.[18] If production risks are fully insurable ($\sigma_A = 0$), the risk premium on private equity is zero and condition (4.7) reduces to $\Phi'(I_0) = R_0$. The chosen I_0 is then independent of I_1: $\partial I_0 / \partial I_1 = 0$. On the other hand if $\sigma_A > 0$, the risk premium $\rho(I_0, I_1)$ is a decreasing function of I_1, and thus: $\partial I_0 / \partial I_1 > 0$. The anticipation of low investment in the future leads investors to expect low savings and high interest rates in later periods,

[18] The second-order condition of the individual decision problem implies $\partial[\Phi'(I_0) - \rho(I_0, I_1)]/\partial I_0 \equiv \partial^2 G / \partial I_0^2 < 0$. It follows that $\partial I_0 / \partial R_0 = 1/(\partial^2 G / \partial I_0^2) < 0$ and $\partial I_0 / \partial I_1 = (\Gamma^{*\prime})(f f')\sigma_A^2 / (\partial^2 G / \partial I_0^2) \geq 0$, since $\Gamma^{*\prime} < 0$.

which in turn discourages risk-taking in the present. This *complementarity* is at the heart of our propagation and amplification mechanism. It arises only if $\sigma_A > 0$, and hinges on the property that the anticipation of a recession in the near future is associated with high risk premia on private equity in the present.

We now examine the impact at date 0 of an *anticipated* recession or investment slump at date 1. To be specific, we assume that the slump originates in an exogenous decrease in the aggregate endowment e_1. For simplicity, we also treat the initial interest rate R_0 as exogenously fixed.[19] When $\sigma_A = 0$, by (4.7) the optimal investment I_0 is independent of the expected decline in I_1. For fixed R_0, we conclude that $dI_0/de_1 = 0$, $dW_1/de_1 = 1$, and $dI_1/de_1 = I^{*\prime}$. Under complete markets, the anticipation of a recession or an investment slump in period 1 does not affect investment in period 0; and the impact of the exogenous wealth shock on contemporaneous income and investment is not amplified.

On the other hand, when $\sigma_A > 0$, the investment levels I_0 and I_1 are complementary by (4.7). The anticipation of an exogenous negative wealth shock in period 1 signals high future interest rates and induces private agents to scale down their risky investment in period 0. The reduction in I_0 implies a further reduction in $W_1 = \Phi(I_0) + e_1$, which in turn further lowers I_1 by the wealth effect (4.6). The anticipation of the endogenous reduction in W_1 and I_1 leads to an even lower I_0, and another feedback between I_1 and I_0 is initiated. The overall impact of the initial exogenous shock can be quite large, and in particular $dI_0/de_1 > 0$, $dW_1/de_1 > 1$, and $dI_1/de_1 > I^{*\prime}$.

In the presence of undiversifiable idiosyncratic production risks, an aggregate shock propagates from one period to another via the dynamic complementarity of investment. This propagation amplifies the contemporaneous impact of the shock and leads to increased persistence. ■

We now make several remarks on this mechanism. First, incomplete markets generate a particular type of *pecuniary externality.* In the presence of uninsurable production shocks, risk-taking depends on future interest rates. When private agents decide how much to save and invest in a future period, they do not internalize the impact of their choices on future interest rate and therefore on current investment.

Second, the pecuniary externality generates a *dynamic macroeconomic complementarity.* Because interest rates are endogenous and influence risk-taking, the anticipation

[19] This is equivalent to assuming an infinitely elastic supply for savings. In our model, the supply for savings at $t = 0$ is derived from the contemporaneous Euler equation. The anticipation of a recession in period 1 (lower e_1) increases the supply of savings at $t = 0$ under either complete or incomplete markets, but decreases the demand for investment only under incomplete markets. The endogeneity of the interest rate thus tends to dampen but not to offset the amplification and propagation mechanism we are proposing. The calibrations of Section 5 will confirm that the mechanism is quite powerful in general equilibrium.

of low aggregate investment in the future feeds back into low aggregate investment in the present. Low levels of investment can thus be self-sustaining for long periods of time. This dynamic macroeconomic complementarity is the basis of amplification and persistence over the business cycle. The reader may be familiar with a standard example of macroeconomic complementarity – the production externalities considered by Bryant (1983) and Benhabib and Farmer (1994).[20] In this literature, an individual's marginal productivity is assumed to increase in the aggregate stock of capital, which generates a complementarity in investment. Note that this type of production externality is exogenous and *ad hoc*. In contrast, the complementarity in our model is endogenously generated by a pecuniary externality, and is thus a genuine general-equilibrium implication of a market imperfection.

Third, our mechanism hinges on the fact that idiosyncratic uncertainty affects production and investment. It is thus not present in Bewley-type economies (e.g., Aiyagari, 1994; Krusell and Smith, 1998) that only consider endowment risks. This explains why this earlier research did not find the propagation and amplification mechanism identified in this paper.

We finally note that our results stem from two basic premises. First, the risk premium on private equity, and thus the willingness to invest in risky production, depend on the ability to self-insure against future variations in returns. Second, this ability is lower during recessions and risk premia are higher at the onset of an economic downturn. These two premises are obviously much more general than our specific model.[21] They alone imply that private investment is lower when a recession is anticipated to persist in the near future, which can slow down recovery and make the recession partially self-fulfilling. Moreover, our results are consistent with the evidence that risk premia on public equity are highly countercyclical. The proposed transmission mechanism has thus strong empirical content and is probably robust to alternative specifications.

4.4. Steady State

We now analyze how the presence of undiversifiable idiosyncratic production risks affects the capital stock in the long run. A *steady state* is a fixed point $(C_\infty, W_\infty, K_\infty, R_\infty)$ of the dynamic system (4.1)-(4.4). We easily show:

Theorem 2 (Steady State) The consumption level is $C_\infty = \Phi(K_\infty) - K_\infty$, while the

[20] Cooper (1999) provides an overview of macroeconomic complementarities.

[21] Consider for instance an economy with incomplete insurance *and* credit markets imperfections. We expect that a risk-averse agent will take less risk and thus invest less in the present when he anticipates a higher borrowing rate, a higher probability to use credit, or a higher probability to face a binding borrowing constraint at a *future* date. Note that this effect occurs whether or not *current* investment is financially constrained.

interest rate and the aggregate capital stock satisfy

$$R_\infty = \Phi'(K_\infty) - \rho_\infty, \tag{4.8}$$

$$\ln(\beta R_\infty) = -\frac{\Gamma}{2\Psi}\sigma_c^2, \tag{4.9}$$

where $\rho_\infty \equiv \Gamma(1-R_\infty^{-1})f(K_\infty)f'(K_\infty)\sigma_A^2$ and $\sigma_c^2 \equiv (1-R_\infty^{-1})^2\left[\sigma_e^2 + f(K_\infty)^2\sigma_A^2\right]$.

The first equation corresponds to the aggregate demand for productive investment, and the second to the aggregate supply of savings. The coefficient ρ_∞ is the risk premium on private equity, and σ_c is the standard deviation of individual consumption. We note that $R_\infty = 1/\beta$ when markets are complete ($\sigma_A = \sigma_e = 0$), but $R_\infty < 1/\beta$ in the presence of undiversifiable idiosyncratic risks ($\sigma_A > 0$ and/or $\sigma_e > 0$). The property that the risk-free rate is below the discount rate under incomplete markets has been proposed as a possible solution to the low risk-free rate puzzle (e.g., Weil, 1992; Huggett, 1993; Constantinides and Duffie, 1996; Heaton and Lucas, 1996).

The steady state is unique when markets are complete, and by continuity when σ_A and σ_e are sufficiently small.[22] The comparative statics are then easily derived from (4.8)-(4.9):

Proposition 4 (Comparative Statics) The capital stock K_∞ increases with the endowment risk σ_e, the discount factor β and the mean productivity $\overline{A}$. On the other hand, the variation of K_∞ with the production risk σ_A is generally ambiguous.

Endowment and production risks have thus very different effects on the steady state. Consider first the case when $\sigma_e > 0$ but $\sigma_A = 0$. There is no risk premium on private investment and the steady state equations reduce to

$$R_\infty = \Phi'(K_\infty) \quad \text{and} \quad \frac{\ln(\beta R_\infty)}{(1-1/R_\infty)^2} = -\frac{\Gamma}{2\Psi}\sigma_e^2.$$

A higher σ_e implies a higher consumption risk, increases the precautionary supply of savings, and reduces the interest rate. Since $R_\infty = \Phi'(K_\infty)$, the capital stock necessarily increases. This is precisely the effect considered by Aiyagari (1994).

We now consider the case when $\sigma_A > 0$. Production shocks introduce a risk premium on private investment and the steady state is determined by the system (4.8)-(4.9). The production risk σ_A affects both the savings supply (like σ_e) and the investment demand (unlike σ_e). An increase in σ_A tends to encourage the precautionary supply of savings, reduce the interest rate, and thereby stimulate investment. On the other hand, a higher σ_A raises the private risk premium and reduces the demand for investment at any level

[22] See the Appendix for a discussion of uniqueness.

of the interest rate. There is thus a conflict between the savings and the investment effect.

Intuition suggests that the investment channel can dominate in two cases. First, when agents have a weak precautionary motive, a higher production risk increases the variance of consumption but has little effect on savings. Second, when real returns have a strong impact on long-run savings (the steady-state supply of savings is very elastic with respect to R_∞), an increase in precautionary savings will lead to only a small reduction in the equilibrium interest rate. In either case, the new steady state is mainly determined by the reduction in investment demand. We observe that these arguments hinge on the sensitivities of savings to consumption risk and interest rates. In our framework, they are both valid when the coefficient Ψ is sufficiently high. The numerical simulations of the next Section will demonstrate that K_∞ decreases with σ_A unless the elasticity of intertemporal substitution is implausibly low. Therefore, while Aiyagari (1994) suggests that the economy *over*invests under incomplete insurance, we instead conclude that *under*investment is the most likely scenario.

5. Calibration and Numerical Results

We now calibrate our infinite-horizon economy, and numerically examine how uninsurable risks affect the steady state and the convergence rate.

5.1. Calibrated Economies

We begin by specifying technology and risks. The production function is assumed to be Cobb-Douglas, $f(K) = K^\alpha$ for $\alpha \in (0, 1)$, and the mean productivity is $\bar{A} = 1$. We calibrate the standard deviations of the uninsurable risks as percentages of GDP. For instance when $\sigma_A = 0.25$, the standard deviation of gross output, $\sigma_A f(K_\infty)$, represents 25% of mean production, $f(K_\infty)$. Similarly, we assume that the standard deviation of the endowment risk is proportional to steady-state output.[23]

We next consider the specification of preferences. A difficulty with exponential utilities is that the *relative* risk aversion and the elasticity of intertemporal substitution (EIS) vary across consumption levels. It is to show that at a given point C these coefficients are respectively equal to ΓC and Ψ/C. We can remedy this problem by choosing parameters Γ and Ψ that match a given relative risk aversion γ and a given EIS ψ at the steady state. For example when markets are complete, long run consumption is $C^*_\infty(\alpha,\beta,\delta) \equiv [(\beta^{-1}+\delta-1)/\alpha - \delta][\alpha/(\beta^{-1}+\delta-1)]^{1/(1-\alpha)}$.[24] We thus set $\Gamma =$

[23] More specifically, we impose $[Var(\varepsilon^i_{t+1})]^{1/2} = \sigma_e f(K_\infty)$. This renormalization does not affect individual decision-making, and simply leads us to replace σ_e by $\sigma_e f(K_\infty)$ in equations (4.2) and (4.9).

[24] This follows from a simple calculation. Since $\Phi'(K^*_\infty) = R^*_\infty = 1/\beta$, the steady capital stock is $K^*_\infty = [\alpha/(\beta^{-1}+\delta-1)]^{1/(1-\alpha)}$ and average consumption is then given by $C^*_\infty = \Phi(K^*_\infty) - K^*_\infty$.

$\gamma/C^*_\infty(\alpha,\beta,\delta)$ and $\Psi = \psi\, C^*_\infty(\alpha,\beta,\delta)$ for every (α,β,δ). The calibration of Γ and Ψ is broadly similar under incomplete markets, as discussed in Appendix B.

Overall, a *calibrated economy* is parameterized by $\mathcal{E}^{cal} = (\beta,\gamma,\psi,\alpha,\delta,\sigma_A,\sigma_e)$, where β is the discount factor, γ the relative risk aversion, ψ the EIS, α the income share of capital, δ the depreciation rate, and σ_A and σ_e the idiosyncratic production and endowment risks as percentages of GDP.

5.2. Calibrated Steady State

We now characterize the comparative statics of the calibrated steady state around $\sigma_A = \sigma_e = 0$.

Proposition 5 (Comparative Statics) As we move away from complete markets, the interest rate R_∞ decreases with σ_e and σ_A. The capital stock K_∞ increases with the endowment risk σ_e; it decreases with the production risk σ_A if and only if $\psi > \underline{\psi}$, where $\underline{\psi} \equiv (\beta^{-1}-1)\left[(\beta^{-1}-1)+(1-\alpha)\delta\right]/(2\alpha^2)$.

As discussed in Section 4.4, the endowment risk σ_e stimulates precautionary savings but does not affect investment demand. As a result, a higher σ_e unambiguously reduces R_∞ and increases K_∞. In contrast, the productivity risk σ_A has the conflicting effects of increasing precautionary savings and reducing investment demand. When the EIS ψ is high, variations in consumption risk have little impact on the interest rate, and the investment effect dominates.

We note that the lower bound $\underline{\psi}$ is typically smaller than 0.20 for plausible values of (β,α,δ). For instance, $\underline{\psi} = 0.20$ when each time period lasts a year and the technology only uses physical forms of capital: $(\beta,\alpha,\delta) = (0.95, 0.35, 0.05)$. We similarly obtain $\underline{\psi} = 0.14$ with a longer time interval and a broader definition of capital: $(\beta,\alpha,\delta) = (0.75, 0.70, 0.25)$. Since empirical evidence suggests an EIS close to 1 and certainly well above 0.20, the most plausible scenario is that uninsurable technological risks reduce the long run capital stock as we move away from complete markets.

5.3. Numerical Simulations of the Steady State

Numerical simulations were performed for many values of $\mathcal{E}^{cal} = (\beta,\gamma,\psi,\alpha,\delta,\sigma_A,\sigma_e)$. On one hand, when ψ is *very* small (typically less than 0.20), the capital stock K_∞ is a single-peaked function of σ_A. The introduction of a new asset increases the long run capital stock if markets are very incomplete (high σ_A); but if markets are nearly complete (low σ_A), the savings effect dominates and financial innovation decreases K_∞. On the other hand, for moderate or high ψ (typically larger than 0.20), the investment effect always dominates. Better sharing of production risks then unambiguously increases the long-run capital stock.

Figure 2 illustrates the monotonicities of the capital stock K_∞ and the net interest rate $r_\infty \equiv R_\infty - 1$ for a typical RBC specification in annual frequency. We set $\beta = 0.95$, $\gamma = 4$, $\psi = 1$, $\delta = 0.05$, and consider either $\alpha = 0.70$ (Panel A) or $\alpha = 0.35$ (Panel B). Panel A considers both physical and human forms of capital, and Panel B physical capital only. In each graph, the solid line corresponds to $\sigma_e = 0$ and the dashed one to $\sigma_e = 50\%$. In both cases, K_∞ monotonically decreases as σ_A varies from 0 to 100%, and the decline is more pronounced for the larger value of the capital share α. In particular for $\sigma_A = 100\%$, the capital stock is about 25% lower than its complete-market value if $\alpha = 0.35$ (Panel B), and 40% lower if $\alpha = 0.70$ (Panel A).

The simulations in Figure 2 assume that an idiosyncratic shock lasts only one year. Since infinitely-lived agents can easily self-insure against short transitory shocks, the impact of missing insurance markets on the steady state is relatively modest. Financial incompleteness is expected to have stronger effects when idiosyncratic shocks are highly persistent, and investment is subject to long irreversibilities or adjustment costs. Persistent production shocks and long irreversibilities are empirically valid assumptions, especially for human capital formation, large R&D projects, and investments involving specialization, indivisibilities, and long horizons. Unfortunately, we cannot explicitly introduce these features in the model without losing tractability. Persistence can however be captured in our simulations by increasing the length of the time period. The interval between t and $t+1$ then corresponds to the horizon of an investment project and the average life of an idiosyncratic productivity shock.

Figure 3 illustrates the case of a 5-year investment horizon. We choose $\beta = 0.75$ and $\delta = 0.25$ over the 5-year period, which correspond to discount and depreciation rates of about 5% per year. We also set $\gamma = 4$, $\psi = 1$, and $\alpha \in \{0.35, 0.70\}$. The effect of σ_A on K_∞ is now very strong. At $\sigma_A = 100\%$, the capital stock is 30% of its complete-market value if $\alpha = 0.35$ (Panel B); it is only 15% of the complete-market level when $\alpha = 0.70$ (Panel A).

In contrast to Aiyagari (1994), Figures 2 and 3 show that incomplete markets can imply both a low risk free rate and a low capital stock. Furthermore, in an incomplete-market economy, the risk-free rate can be a very poor proxy for the marginal productivity of capital. In Panel B of Figure 3, the marginal productivity of capital is 18% per year when $\sigma_A = 100\%$, as compared to a yearly interest rate of 4%.

The simulations provide useful insights on the interaction between endowment and production risks. In Figures 2 and 3, the dashed lines correspond to $\sigma_e = 50\%$ and the solid ones to $\sigma_e = 0$. We observe that the steady state becomes less sensitive to σ_e as σ_A increases. This is because when σ_A is large, individuals are already holding a buffer stock that can be used to self-insure against both investment and endowment risks. The precautionary effect of σ_A similarly diminishes with σ_e, implying that the investment effect dominates more easily when there are already large precautionary savings in the economy.

We note that when σ_e and σ_A are approximately equal, the impact of production risk on capital stock is prevalent. For instance in Figures 2 and 3, the capital stock K_∞ is well below its complete-markets value for $\sigma_A = \sigma_e = 50\%$. When production and endowment risks contribute equally to idiosyncratic income variation, the adverse investment effect tends to be stronger than the favorable precautionary impact induced by *both* types of income risk.

We conclude that the *quantitative* impact of σ_A on K_∞ can be large.[25] Although empirical estimates of σ_A or σ_e are not readily available, we know that idiosyncratic production, entrepreneurial, and investment risks are very substantial in reality. For example, the survival rate of a new private firm is only 34% after 10 years. The distribution of returns to entrepreneurial activity is also extremely wide even conditionally on survival.[26] In addition, private savings are very low in the United States. These facts are consistent with our model and suggest that substantial underinvestment due to incomplete risk-sharing is a very likely empirical scenario.

5.4. Persistence

We showed in Section 4.3 that uninsurable production risks generate a dynamic macroeconomic complementarity in investment, which can be a source of amplification and propagation over the business cycle. We now quantify this effect by examining how σ_A alters convergence to the steady state.

In Appendix C, we linearize the dynamic system (4.1)-(4.5) around the steady state and calculate the stable eigenvalue λ. The local dynamics can then be approximated by $\log(K_{t+1}/K_\infty) = \lambda \log(K_t/K_\infty)$. The quantity $1-\lambda$ is called the *convergence rate*. Incomplete insurance slows down convergence if $1-\lambda$ decreases with σ_A. Numerical simulations show that this is indeed the case for a wide range of plausible parameter values. Consider our earlier example of a 5-year investment project (Figure 3). We calibrate the model using $\beta = 0.75$ (discount rate $\approx 5\%$ per year), $\delta = 0.25$ (depreciation rate $\approx 5\%$ per year), $\gamma = 4$, $\psi = 1$, and $\alpha \in \{0.35, 0.70\}$. In Figure 4 we illustrate the convergence rate and the half-life of an aggregate wealth shock[27] as σ_A varies from 0 to 100%. The convergence rate decreases rapidly with σ_A. With a narrow definition of capital ($\alpha = 0.35$, Panel B), the half-life of a shock almost doubles as σ_A increases from 0 to 100%. The effect is even stronger when incompleteness affects both physical and

[25] The numerical results of Figures 1 and 2 are not very sensitive to changes in ψ and γ. A higher ψ weakens the effect of σ_A on R_∞ and strengthens its impact on K_∞, because it increases the interest elasticity of savings. On the other hand, γ tends to have a small ambiguous effect, since a higher γ increases both the precautionary motive and the risk premium on investment.

[26] See Moskowitz and Vissing-Jorgensen (2001).

[27] The half-life T of a deviation from the steady state is defined by $\lambda^T = 1/2$, or $T = -\log_2 \lambda$.

human capital ($\alpha = 0.7$, Panel A).[28]

Undiversifiable productivity shocks thus substantially slow down convergence to the steady state, as implied by the dynamic macroeconomic complementarity discussed in Section 4. We anticipate that convergence would be even slower in the presence of borrowing constraints.

While this paper considers a one-sector economy, it is straightforward to extend our framework to multiple sectors. For instance, we can introduce two production technologies, one with high mean return and high risk and another with low mean return and low risk. We can also include several forms of investment, such as physical and human or intangible capital. In such an environment, incomplete risk sharing distorts not only the aggregate levels of savings and investment, but also the cross-sectoral allocation of capital and labor. This cross-sectoral distortion reduces aggregate productivity, implying a further reduction in steady-state capital and income.[29] Moreover, the anticipation of stringent future credit conditions induces not only a lower level of overall risk taking, but also a substitution away from high-risk high-return investment opportunities. As the economy shifts to safer but less productive technologies during downturns, the persistence and the amplitude of the business cycle are further increased.

These results suggest that production risk could generate additional persistence over the business cycle in standard RBC models with aggregate uncertainty (e.g. Kydland and Prescott, 1982). Furthermore, the magnitude of uninsurable productivity shocks appears as a potential determinant of both the steady state and conditional convergence. Cross-country variation in the degree of risk sharing may thus help explain the large diversity of productivity levels and growth rates around the world (e.g. Barro, 1997; Jones, 1997).

6. Concluding Remarks

This paper examines a standard neoclassical growth economy with heterogeneous agents, decentralized production, and uninsurable production and endowment risks. Under a CARA-normal specification for preferences and risks, we obtain closed-form solutions for individual choices and aggregate dynamics. Uninsurable production shocks introduce a

[28] Figure 3 also demonstrates the asymmetry between production and endowment risk. While endowment risk does not introduce a dynamic macroeconomic complementarity, the precautionary motive tends to boost savings above the steady state and thus speed up convergence in an initially poor economy. The convergence rate thus tends to *decrease* with σ_A but *increase* with σ_e. When σ_A and σ_e are equal, the investment effect dominates and the convergence rate is lower than under complete markets.

[29] A multi-sector extension of our paper would thus complement the endogenous-growth literature examining the effect of uninsurable investment risks on the allocation of savings across different investment opportunities, such as liquid and illiquid assets (e.g., Bencivenga and Smith, 1991) and storage and risky production (e.g. Greenwood and Jovanovic, 1990; Obstfeld, 1994).

risk premium on private equity and reduce the aggregate demand for investment. As a result, the steady-state capital stock tends to be lower under incomplete markets, despite the low risk-free rate induced by the precautionary motive. Undiversifiable idiosyncratic production risks generate a powerful dynamic macroeconomic complementarity between future and current investment. Based on the endogenous countercyclicality of the risk premium, this mechanism amplifies the impact of an exogenous aggregate shock on output and investment, slows down convergence to the steady state, and increases the persistence of the business cycle.

Credit-market imperfections and non-convexities in production have been viewed by many authors as a source of persistence in the business cycle. Although these departures from the neoclassical growth model are not considered here, we find that incomplete risk sharing alone is sufficient to generate underinvestment and introduce a powerful propagation mechanism. The presence of uninsurable production risks reduces the individual's *willingness* to invest. Introducing borrowing constraints would in addition restrict the *ability* to undertake risky projects and increase the sensitivity of risk premia and investment demand to future credit conditions. While CARA preferences rule out wealth effects on risk taking, the private equity premium is even more countercyclical when wealth encourages risk taking. During a recession, agents are unwilling to take risk not only because they anticipate stringent future credit conditions but also because they are poorer. The impact of credit constraints and wealth on risk premia could thus reinforce the steady-state and business-cycle effects documented in the paper. Furthermore, we anticipate that the impact of risk premia on investment represents a source of amplification and persistence that is much more general than our specific model.

The next step is to construct a full-fledged RBC model with isoelastic preferences, decentralized production, borrowing constraints, and both aggregate and idiosyncratic production uncertainty. This can be accomplished by combining our framework with the numerical analysis of Krusell and Smith (1998). This extension would permit a quantitative evaluation of the interaction between risk premia and business cycles, which is the heart of our argument. It would also permit a reassessment of the impact of wealth heterogeneity on aggregate dynamics. In addition, idiosyncratic capital-income risk may help match the skewness of the wealth distribution in the data,[30] and have important asset pricing implications.[31] We leave these questions open for future research.

[30] Krusell and Smith (1998) show that a calibrated Bewley-type model fails to match the large dispersion of wealth observed in the US economy unless persistent idiosyncratic shocks are introduced in the discount factor. We conjecture that persistent capital income risk may have similar effects than these utility shocks, and could thus help match the empirical wealth distribution.

[31] Constantinides and Duffie (1996) show that Bewley-type models can explain the magnitude and countercyclicality of the public equity premium by assuming that labor-income risk is itself highly countercyclical. Our findings suggest that idiosyncratic production uncertainty can generate large countercyclicality in risk premia even when exogenous shocks are acyclical. Moreover, Heaton and Lucas

Finally, like Aiyagari (1994) and Krusell and Smith (1998), this paper treats the financial structure as exogenous. There is an important literature on how incomplete risk sharing can originate in private information[32] or lack of commitment.[33] Our results suggest that a promising extension to this research may consider production economies with idiosyncratic capital-income risk and endogenous asset markets. This would provide a more detailed picture of the relation between economic growth, the process of financial innovation, and the business cycle.

(2000) show empirically that proprietary income risk has a strong impact on portfolio holdings, which is suggestive of the potential importance of capital-income risk for asset prices.

[32] See for example Townsend (1982), Green (1987), Banerjee and Newman (1991), Atkeson and Lucas (1992), and Cole and Kocherlakota (2001).

[33] See for example Kehoe and Levine (1993), Kocherlakota (1996), and Alvarez and Jermann (2000).

Appendix A: Proofs

Proof of Proposition 1 (Individual Choice)

A sketch of the portfolio-decision choice is provided in the text. We present here a more detailed derivation under the assumption that there is no risk premium on financial securities.

Since the asset structure includes a riskless bond, there is no loss of generality in assuming that $\mathbb{E}_t d_{m,t+1} = 0$ for all $m \geq 1$. In the absence of a risk premium, it follows that $\pi_{m,t} = 0, \forall m \geq 1$. We thus rewrite the optimization problem (3.3) as

$$\max_{(c_t^j, k_t^j, \theta_{0t}^j)} U(c_t^j) + \beta U \left\{ V_{t+1}^j \left[\mathbb{E}_t w_{t+1}^j - \frac{\Gamma a_{t+1}^j}{2} \min_{(\theta_{1t}^j, .., \theta_{Jt}^j)} \mathrm{Var}_t(w_{t+1}^j) \right] \right\}. \tag{6.1}$$

This suggests a two-step solution. We successively solve for the optimal portfolio of risky assets, and then for the consumption-investment choice.

Given any $(c_t^j, k_t^j, \theta_{0,t}^j)$, the optimal portfolio $(\theta_{j,t}^j)_{j=1}^J$ minimizes the conditional variance of wealth, $\mathrm{Var}_t(w_{t+1}^j) = \mathrm{Var}_t \left[A_{t+1}^j f(k_{t+1}^j) + e_{t+1}^j + \sum_{j=1}^J d_{j,t+1}^j \theta_{j,t}^j \right]$. Without loss of generality, we normalize $\mathrm{Var}(d_{m,t+1}) = 1$ and $\mathrm{Cov}(d_{m,t+1}; d_{n,t+1}) = 0$ for $m \neq n$. The FOCs then imply

$$\theta_{m,t}^j = -\mathrm{Cov}_t \left[d_{m,t+1} \, ; \, A_{t+1}^j f(k_{t+1}^j) + e_{t+1}^j \right], \ \forall m \geq 1.$$

This result has a natural geometric interpretation. For all t, we can project (or regress) A_{t+1}^j and e_{t+1}^j on the asset span. This yields $A_{t+1}^j = \kappa^j \cdot d_{t+1} + \eta_{t+1}^j$ and $e_{t+1}^j = \xi^j \cdot d_{t+1} + \varepsilon_{t+1}^j$, where κ^j, ξ^j are deterministic constants and $\eta_{n,t+1}^j, \varepsilon_{t+1}^j$ are random variables orthogonal to d_{t+1}. The optimal portfolio fully hedges the diversifiable component of idiosyncratic risks:

$$\theta_t^j \cdot d_{t+1} = - \left[f(k_{t+1}^j) \kappa^j + \xi^j \right] \cdot d_{t+1}. \tag{6.2}$$

Individual wealth then reduces to $w_{t+1}^j = (\overline{A} + \eta_{t+1}^j) f(k_{t+1}^j) + (1-\delta) k_{t+1}^j + \varepsilon_{t+1}^j + \theta_{0,t}^j$. Thus, $\mathbb{E}_t w_{t+1}^j = \overline{A} f(k_{t+1}^j) + (1-\delta) k_{t+1}^j + \theta_{0,t}^j$ and $\mathrm{Var}_t(w_{t+1}^j) = \sigma_e^2 + f(k_{t+1}^j)^2 \sigma_A^2$.

We now turn to the optimal consumption, saving and investment decision. We define $\Phi(k) \equiv \overline{A} f(k) + (1-\delta) k$ and $G(k, \Gamma a) \equiv \Phi(k) - \Gamma a \left[\sigma_e^2 + f(k)^2 \sigma_A^2 \right]/2$. It follows that $\mathbb{E}_t w_{t+1}^j = \Phi(k_{t+1}^j) + \theta_{0,t}^j$ and $\mathbb{E}_t w_{t+1}^j - \Gamma a_{t+1}^j \mathrm{Var}_t(w_{t+1}^j)/2 = G(k_{t+1}^j, \Gamma a_{t+1}^j) + \theta_{0,t}^j$. Combining with (6.1), we conclude that the optimal $(c_t^j, k_{t+1}^j, \theta_{0,t}^j)$ maximizes

$$U(c_t^j) + \beta U \left\{ V_{t+1}^j \left[G(k_{t+1}^j, \Gamma a_{t+1}^j) + \theta_{0,t}^j \right] \right\}, \tag{6.3}$$

subject to $c_t^j + k_{t+1}^j + \theta_{0,t}^j / R_t = w_t^j$. The FOCs with respect to k_t^j and $\theta_{0,t}^j$ give

$$\begin{aligned} U'(c_t^j) &= \beta U' \left\{ V_{t+1}^j \left[G(k_{t+1}^j, \Gamma a_{t+1}^j) + \theta_{0,t}^j \right] \right\} a_{t+1}^j \frac{\partial G}{\partial k}(k_{t+1}^j, \Gamma a_{t+1}^j), \\ U'(c_t^j) &= \beta U' \left\{ V_{t+1}^j \left[G(k_{t+1}^j, \Gamma a_{t+1}^j) + \theta_{0,t}^j \right] \right\} a_{t+1}^j R_t. \end{aligned}$$

Dividing these equalities yields $R_t = \partial G/\partial k$.

We now write the envelope condition: $U'[V_t^j(w_t^j)]a_t^j = U'(c_t^j)$. Using (2.3) and (3.4), this reduces to $c_t^j = a_t^j w_t^j + b_t^j - \Psi \ln a_t^j$. We infer that $\widehat{a}_t^j = a_t^j$ and $\widehat{b}_t^j = b_t^j - \Psi \ln a_t^j$. Using (2.3) and (3.4), we rewrite the FOC with respect to $\theta_{0,t}^j$ as

$$\begin{aligned} U'(c_t^j) &= \beta R_t U' \left\{ V_{t+1}^j \left[\mathbb{E}_t w_{t+1}^j - \Gamma a_{t+1}^j \mathrm{Var}_t(w_{t+1}^j)/2 \right] \right\} a_{t+1}^j \\ &= \beta R_t U' \left\{ a_{t+1}^j \mathbb{E}_t w_{t+1}^j - \Gamma (a_{t+1}^j)^2 \mathrm{Var}_t(w_{t+1}^j)/2 + b_{t+1}^j - \Psi \ln a_t^j \right\}. \end{aligned}$$

Using $a_{t+1}^j = \widehat{a}_{t+1}^j,\ b_{t+1}^j = \widehat{b}_t^j + \Psi \ln a_{t+1}^j$ and the consumption rule, the above reduces to

$$U'(c_t^j) = \beta R_t U' \left[\mathbb{E}_t c_{t+1}^j - \Gamma \mathrm{Var}_t(c_{t+1}^j)/2 \right].$$

This gives the Euler condition (3.10).

Combining the envelope condition with the FOC for $\theta_{0,t}^j$ gives

$$U'[V_t^j(w_t^j)]a_t^j = \beta U' \left\{ V_{t+1}^j \left[G(k_{t+1}^j, \Gamma a_{t+1}^j) + \theta_{0,t}^j \right] \right\} a_{t+1}^j R_t,$$

or equivalently $V_t^j(w_t^j) = V_{t+1}^j \left[G(k_{t+1}^j, \Gamma a_{t+1}^j) + \theta_{0,t}^j \right] - \Psi \ln(\beta a_{t+1}^j R_t / a_t^j)$. The budget constraint and the consumption rule imply that $\theta_{0,t}^j = R_t(1-\widehat{a}_t^j) w_t^j - R_t k_{t+1}^j - R_t \widehat{b}_t^j$, where $\widehat{a}_t^j = a_t^j$. We thus infer

$$\begin{aligned} a_t^j \cdot w_t^j \ + \ b_t^j \quad &= \quad a_{t+1}^j R_t (1-a_t^j) \cdot w_t^j \ + \\ &\quad + a_{t+1}^j \left[G(k_{t+1}^j, \Gamma a_{t+1}^j) - R_t k_{t+1}^j - R_t \widehat{b}_t^j \right] + b_{t+1}^j - \Psi \ln(\beta a_{t+1}^j R_t / a_t^j). \end{aligned}$$

Since this linear relation holds for every w_t^j, we conclude that $a_t^j = a_{t+1}^j R_t (1-a_t^j)$ or equivalently $a_t^j = 1/[1 + (a_{t+1}^j R_t)^{-1}]$. Iterating forward yields (3.8). ***QED***

Proof of Proposition 2 (Investment)

By the implicit function theorem, the first-order condition (3.6) implies $\partial k_{t+1}^j / \partial(\sigma_A^2) = -\mu \Gamma_t$ and $\partial k_{t+1}^j / \partial \Gamma_t = -\mu \sigma_A^2$, where $\mu \equiv f(k_{t+1}^j) f'(k_{t+1}^j) / (-\partial^2 G/\partial k^2)$. We infer from the second-order condition $\partial^2 G / \partial k^2 < 0$ that $\mu > 0$. Finally, condition (3.8) implies that $\partial \Gamma_t / \partial R_s > 0$ and thus $\partial k_{t+1}^j / \partial R_s = -\mu \sigma_A^2 \partial \Gamma_t / \partial R_s < 0$ for all $s > t$. Differentiating this relation at $\sigma_A = 0$, we obtain $\partial \left| \partial k_{t+1}^j / \partial R_{t+s} \right| / \partial(\sigma_A^2) = \mu \partial \Gamma_t / \partial R_s > 0$. ***QED***

Proof of Proposition 3 (Savings)

Since $\mathrm{Var}_t(w_{t+1}^j) = \sigma_e^2 + \sigma_A^2 f(k_{t+1}^j)^2$, we infer that $\partial \mathrm{Var}_t(w_{t+1}^j)/\partial \sigma_e^2 > 0$. On the other hand, $\partial \mathrm{Var}_t(w_{t+1}^j)/\partial \sigma_A^2 = f(k_{t+1}^j)^2 + [2\sigma_A^2 f(k_{t+1}^j) f'(k_{t+1}^j)](\partial k_{t+1}^j / \partial \sigma_A^2)$ has an ambiguous sign. Consider the special case $f(k) = \sqrt{k}$ and $\delta = 1$. Then $G(k, \Gamma) = (2\sqrt{k})^{-1}(\overline{A} - \Gamma \sigma_A^2 \sqrt{k})$ and $R_t = \partial G(k_{t+1}^j, \Gamma_t^j)/\partial k$ imply $k_{t+1}^j = \overline{A}^2 / (2R_t + \Gamma_t \sigma_A^2)^2$. Hence,

$\mathrm{Var}_t(w^j_{t+1}) = \sigma_e^2 + \sigma_A^2 \overline{A}^2/(2R_t + \Gamma_t \sigma_A^2)^2$. We conclude that $\partial \mathrm{Var}_t(w^j_{t+1})/\partial \sigma_A^2 < 0$ if and only if $\sigma_A^2 > 2R_t/\Gamma_t$. ***QED***

Proof of Theorem 1 (General Equilibrium)

We now derive the equations characterizing general equilibrium. First, note that (3.8) implies $a^j_t = a_t$ for all j, t. We infer from the optimality condition (3.9) that $k^j_{t+1} = K_{t+1}$ for all j. Equation (3.9) then reduces to (4.1) and the Euler equation (3.10) can be rewritten as

$$\mathbb{E}_t c^j_{t+1} - c^j_t = \Psi \ln(\beta R_t) + \frac{\Gamma a^2_{t+1}}{2}[\sigma_e^2 + f(K_{t+1})\sigma_A^2].$$

We aggregate these equalities across agents and infer (4.2). Finally, (4.4) and (4.5) follow from aggregating the budget constraints and Assumption 4 (absence of aggregate uncertainty). ***QED***

Proof of Theorem 2 (Steady State)

The steady state is defined by the system $(4.8)-(4.9)$. The second equation implies $R_\infty \leq 1/\beta$. The transversality condition imposes that $R_\infty > 1$ and $a_\infty > 0$. The interest rate R_∞ is therefore bounded between 1 and $1/\beta$. Since $R_\infty > 1$, the first equation implies $\overline{A}F'(K_\infty) + 1 - \delta > 1$, or equivalently $K_\infty < \widehat{K} \equiv (F')^{-1}(\delta/\overline{A})$. The capital stock K_∞ is therefore contained between 0 and $\widehat{K}$.

Each steady-state equation implicitly defines the interest rate as a function of the capital stock. Consider for instance equation (4.9). It is useful to define the functions $m_1 : (1, \beta^{-1}] \to [0, +\infty)$, $m_1(R) \equiv (2\Psi/\Gamma)(1 - R^{-1})^{-2}\ln[1/(R\beta)]$, and $m_2 : [0, \widehat{K}) \to [\sigma_e^2, \sigma_e^2 + f(\widehat{K})^2\sigma_A^2)$, $m_2(K) \equiv \sigma_e^2 + f(K)^2\sigma_A^2$. We observe that m_1 is decreasing in R and m_2 is increasing in K. The steady state equation (4.9) is equivalent to $m_1(R) = m_2(K)$. For each $K \in [0, \widehat{K})$, the equation $m_1(R) = m_2(K)$ has a unique solution, $R_2(K) \equiv m_1^{-1}[m_2(K)]$, which maps $[0, +\infty)$ onto $(1, m_1^{-1}(\sigma_e^2)] \subseteq (1, \beta^{-1}]$. Similarly, the steady state equation (4.8) implicitly defines a decreasing function $R_1(K)$, which maps $(0, \widehat{K}]$ onto $[1, +\infty)$. The intersection of R_1 and R_2 gives K_∞.

Consider the function $\Delta(K) \equiv R_2(K) - R_1(K)$. When $K \to 0$, we observe that $R_2(K)$ is bounded and $R_1(K) \to +\infty$, implying $\Delta(K) \to -\infty$. We also note that $\Delta(\widehat{K}) = R_2(\widehat{K}) - 1 > 0$. Hence, there exists at least one steady state for any (σ_A, σ_e). Under complete markets, the steady state is unique since the function R_2 is constant and R_1 is decreasing. By continuity, the steady state is also unique when σ_A and σ_e are sufficiently small. ***QED***

Proof of Proposition 4 (Comparative Statics)

Consider the functions R_1 and R_2 defined in the proof of Theorem 2. Observe that $R_1(K)$ and $R_2(K)$ are both decreasing. We know that $|R_1'(K_\infty)| > |R_2'(K_\infty)|$ when the steady state is unique. An increase in σ_e or β leaves the function $R_1(K)$ unchanged and pushes down the function $R_2(K)$. The steady state is therefore characterized by a lower interest rate and a higher capital stock. Similarly, an increase in $1-\delta$ and $\overline{A}$ pushes up $R_1(K)$, also leading to a lower interest rate and a higher capital stock. An increase in Γ or σ_A reduces both $R_1(K)$ and $R_2(K)$, reflecting the fact that $\Gamma\sigma_A$ enters in both the investment demand and the savings supply. Γ and σ_A can therefore have ambiguous effects, as verified in simulations. ***QED***

The proof of **Proposition 5** is presented in Appendix B after the discussion of the calibration method.

Appendix B: Calibrated Economies

We now present the calibration of Γ and Ψ, which allows the comparison between our CARA economy and the standard isoelastic setup used in RBC models. Relative risk aversion at the steady-state consumption level is ΓC_∞. We restrict the incomplete-market economy $\mathcal{E}$ so that ΓC_∞ remains invariant at a fixed level γ.

We next consider Ψ. The elasticity of intertemporal substitution (EIS) is equal to Ψ/C_∞ at the steady state consumption level. Similar to the calibration of risk aversion, we could restrict Ψ/C_∞ to remain constant at a fixed level ψ. In Angeletos and Calvet (2000), we adopted this method and the additional restriction $\Psi = 1/\Gamma$ (expected utility). We found that idiosyncratic production risks strongly reduce the convergence rate to the steady state, confirming the predictions contained in Section 4.

In this paper, however, we propose a more elaborate calibration method that stems from the following observation. Consider a *complete-market* Ramsey economy with intertemporal utility $\sum_{t=0}^{+\infty}\beta^t U(c_t)$, where U is a smooth strictly concave function. Gross output is $\Phi(K) = f(K) + (1-\delta)K$. The local dynamics around the steady state are approximated by $\ln(K_{t+1}/K_\infty) \approx \lambda \ln(K_t/K_\infty)$, where λ is the stable eigenvalue of the linearized system. It is easy to show that[34]

$$\lambda = \frac{1}{2}\left\{1+\beta(\beta^{-1}-1+\delta)M_\infty + \frac{1}{\beta} - \sqrt{[1+\beta(\beta^{-1}-1+\delta)M_\infty]^2 - \frac{4}{\beta}}\right\},$$

where

$$M_\infty = \frac{f''(K_\infty)/f'(K_\infty)}{U''(C_\infty)/U'(C_\infty)}.$$

The ratio M_∞ quantifies the relative curvatures of the production and utility functions. The eigenvalue λ is thus fully determined by (β,δ) and M_∞. The Cobb-Douglas specification $f(K) = K^\alpha$ implies that $f''(K)/f'(K) = -(1-\alpha)/K$. With a CARA utility

[34] Cass (1965) derives a similar result for continuous time economies.

$U(C) = \Psi \exp(-C/\Psi)$, we also know that $U''(C)/U'(C) = -1/\Psi$. The ratio M_∞ then reduces to $(1-\alpha)\Psi/K_\infty$, and

$$\lambda = \frac{1}{2}\left\{1+\beta(\beta^{-1}-1+\delta)(1-\alpha)\frac{\Psi}{K_\infty}+\frac{1}{\beta}-\sqrt{\left[1+\beta(\beta^{-1}-1+\delta)(1-\alpha)\frac{\Psi}{K_\infty}\right]^2-\frac{4}{\beta}}\right\}. \tag{6.4}$$

Under complete markets, the convergence rate $g = 1-\lambda$ is thus fully determined by the parameters (α, β, δ) and the ratio Ψ/K_∞, which quantifies the relative curvature of the production and utility functions at the steady state.

When we move from complete to incomplete markets, two phenomena affect the eigenvalue λ and thus the convergence rate $g = 1-\lambda$. First, the transitional dynamics are affected by new terms in (4.1)-(4.5): the risk premium in the investment-demand equation and the consumption variance in the Euler equation. Second, changes in the steady state affect the relative curvature Ψ/K_∞ and thereby the eigenvalue λ. This second effect reflects the shift of the steady-state to different points on the production and utility functions. It is thus purely mechanical and sheds little light on the impact of incomplete risk sharing on the transitional dynamics. For this reason, we prefer to neutralize this effect by keeping Ψ/K_∞ (or equivalently M_∞) invariant at a prespecified level as we vary σ_A and σ_e.[35] This in turn requires an appropriate calibration of Ψ/K_∞. When markets are complete, we impose that the intertemporal elasticity Ψ/C_∞ be equal to a given coefficient ψ. This allows us to choose a value of ψ that matches empirical estimates of the EIS. A simple calculation also implies $C_\infty/K_\infty = q^*-\delta$, where $q^* \equiv (\beta^{-1}-1+\delta)/\alpha$. The ratio Ψ/K_∞ is therefore equal to $\psi C_\infty/K_\infty = \psi(q^*-\delta)$ under complete markets. When markets are incomplete, we keep Ψ/K_∞ invariant at its complete-market level $\psi(q^*-\delta)$. Our calibration thus disentangles the dynamic effect of financial incompleteness from purely mechanical changes in the relative curvatures of the production and utility functions.

Following this methodology, we define[36]

[35] The alternative calibration method, which keeps constant the EIS Ψ/C_∞ at ψ but lets Ψ/K_∞ vary, also implies a very substantial increase in persistence when σ_A increases from zero. But, because K_∞ typically decreases with σ_A, the change in Ψ/K_∞ tends to reduce persistence. For large production risks, the convergence rate $g = 1-\lambda$ is then slightly non-monotonic in σ_A in some simulations (while staying far below the complete market value). It is then interesting to consider the *shadow complete-market* convergence rate obtained by substituting the incomplete market capital stock in (6.4). The difference between the actual convergence rate and its shadow value is then monotonically increasing in σ_A. The dynamic effect of σ_A thus unambiguously slows down convergence.

[36] Our calibration method also has the following alternative interpretation. Instead of adjusting the EIS around the incomplete-markets steady state, we can set it at a predetermined level $\Psi/C_\infty = \psi$, but assume that the production function is exponential rather than Cobb-Douglas: $f(K) = 1-\exp(-\phi K)$. We calibrate the coefficient ϕ by setting the income share of capital equal to α in the complete-market steady state. This specification implies $M_\infty = \phi/\Psi = \beta(\beta^{-1}-1+\delta)(1-\alpha)\psi(q^*-\delta)$ and generates

Definition A *calibrated economy* $\mathcal{E}^{cal} = (\beta, \gamma, \psi, \alpha, \delta, \sigma_A, \sigma_e)$ is an incomplete market economy $\mathcal{E} = (\beta, \Gamma, \Psi, f, \delta, \overline{A}, \sigma_A, \sigma_e')$ such that $\Gamma C_\infty = \gamma$, $\Psi/K_\infty = \psi(q^* - \delta)$, $q^* \equiv (\beta^{-1} - 1 + \delta)/\alpha$, $\overline{A} = 1$ and $\sigma_e' = \sigma_e f(K_\infty)$.

We then easily show

Proof of Proposition 5 (Calibrated Steady State)

Given $\mathcal{E}^{cal} = (\beta, \gamma, \psi, \alpha, \delta, \sigma_A, \sigma_e)$, let $q_\infty \equiv f(K_\infty)/K_\infty = K_\infty^{\alpha-1}$ denote the output-capital ratio in the steady state, implying $f'(K_\infty) = \alpha q_\infty$ and $C_\infty/K_\infty = q_\infty - \delta$. The calibration of Γ and Ψ implies $\Gamma C_\infty = \gamma$ and $\Psi/K_\infty = \psi(q^* - \delta)$. The steady-state system (4.8) – (4.9) thus reduces to

$$R_\infty = 1 - \delta + \alpha q_\infty(1 - \gamma\Delta\sigma_A^2), \quad \ln(\beta R_\infty) = -\frac{\gamma\Delta^2}{\psi\nu}(\sigma_A^2 + \sigma_e^2). \tag{6.5}$$

where $\Delta \equiv (1 - R_\infty^{-1})q_\infty/(q_\infty - \delta)$ and $\nu \equiv 2(q^* - \delta)/(q_\infty - \delta)$. When $\sigma_A = \sigma_e = 0$ (complete markets), $R_\infty = 1/\beta$, $q_\infty = q^* \equiv (\beta^{-1} - 1 + \delta)/\alpha$, $\Delta = (1-\beta)q^*/(q^* - \delta)$, and $\nu = 2$ like in the standard Ramsey model. When σ_A^2 and σ_e^2 are positive but close to 0, the first-order variations in R_∞ and q_∞ are obtained by keeping Δ and ν constant in (6.5). We thus get $d(\ln R_\infty) = -(\gamma\Delta^2)/(\psi\nu) \cdot d(\sigma_A^2)$ and $dq_\infty = \gamma\Delta q_\infty(1 - \underline{\psi}/\psi)d(\sigma_A^2)$, where $\underline{\psi} = (q^* - \delta)(\beta^{-1} - 1)/(2\alpha)$. It follows that $dq_\infty/d(\sigma_A^2) > 0$ and thus $dK_\infty/d\sigma_A < 0$ if and only if $\psi > \underline{\psi}$. ***QED***

Appendix C: Local Dynamics

We now derive the local dynamics of (4.1)-(4.5) around the steady state. An equilibrium path can be calculated by a backward recursion of the *state vector* $z_t = (a_t, C_t, W_t)$.

Lemma (Equilibrium Recursion) For any vector $z_{t+1} = (a_{t+1}, C_{t+1}, W_{t+1}) \in (0, 1] \times \mathbb{R} \times [0, +\infty)$, there exists a unique $(a_t, C_t, W_t, K_{t+1}, R_t) \in (0, 1) \times \mathbb{R}^2 \times \mathbb{R}_+^2$ satisfying the equilibrium recursion (4.1) – (4.4).

Proof. Given $z_{t+1} = (a_{t+1}, C_{t+1}, W_{t+1})$, the feasibility condition implies that $K_{t+1} = \Phi^{-1}(W_{t+1})$. The interest rate is then given by $R_t = \partial G(K_{t+1}, \Gamma a_{t+1})/\partial K$. Finally, equations (4.3) – (4.4) assign unique values to a_t, C_t, and W_t. *QED*

The lemma implies the existence of a unique recursion mapping $\mathbf{H}$ such that $z_t = \mathbf{H}(z_{t+1})$ for all t. Note that this mapping is implicitly defined by

$$\begin{aligned} a_t &= 1/[1 + (a_{t+1}R_t)^{-1}], \\ C_t &= C_{t+1} - \Psi \ln(\beta R_t) - \Gamma a_{t+1}^2 \left[f(K_{t+1})^2 \sigma_A^2 + \sigma_e^2\right]/2, \\ W_t &= C_t + K_{t+1}, \end{aligned}$$

exactly the same calibrated steady state.

where $K_{t+1} = \Phi^{-1}(W_{t+1})$ and $R_t = \partial G(K_{t+1}, \Gamma a_{t+1})/\partial K$. The Jacobian of $\mathbf{H}$ is

$$\mathbf{D}_z\mathbf{H} = \begin{bmatrix} \frac{\partial a_t}{\partial a_{t+1}} & 0 & \frac{\partial a_t}{\partial W_{t+1}} \\ \frac{\partial C_t}{\partial a_{t+1}} & 1 & \frac{\partial C_t}{\partial W_{t+1}} \\ \frac{\partial C_t}{\partial a_{t+1}} & 1 & \frac{\partial (C_t + K_{t+1})}{\partial W_{t+1}} \end{bmatrix}.$$

We observe that $\partial K_{t+1}/\partial W_{t+1} = 1/\Phi'(K_{t+1}) > 0$, and

$$\frac{\partial R_t}{\partial a_{t+1}} = -\Gamma f(K_{t+1}) f'(K_{t+1}) \sigma_A^2 \leq 0,$$
$$\frac{\partial R_t}{\partial W_{t+1}} = \frac{1}{\Phi'(K_{t+1})} \left\{ f''(K_{t+1}) \left[\overline{A} - \Gamma a_{t+1} f(K_{t+1}) \sigma_A^2 \right] - a_{t+1} [f'(K_{t+1})]^2 \Gamma \sigma_A^2 \right\} < 0.$$

Consider the function $\chi(v) \equiv 1/(1+v^{-1}) = v/(1+v)$, which has derivative $\chi'(v) = 1/(1+v)^2 = [\chi(v)/v]^2$. Since $a_t = \chi(a_{t+1}R_t)$, we infer that

$$\frac{\partial a_t}{\partial a_{t+1}} = \left(\frac{a_t}{a_{t+1}R_t} \right)^2 \left(R_t - a_{t+1} \left| \frac{\partial R_t}{\partial a_{t+1}} \right| \right),$$
$$\frac{\partial a_t}{\partial W_{t+1}} = -\left(\frac{a_t}{a_{t+1}R_t} \right)^2 a_{t+1} \left| \frac{\partial R_t}{\partial W_{t+1}} \right| < 0.$$

The Euler equation $C_t = C_{t+1} - \Psi \ln(\beta R_t) - \Gamma a_{t+1}^2 \left[f(K_{t+1})^2 \sigma_A^2 + \sigma_e^2 \right]/2$ then implies

$$\frac{\partial C_t}{\partial a_{t+1}} = \frac{\Psi}{R_t} \left| \frac{\partial R_t}{\partial a_{t+1}} \right| - \Gamma a_{t+1} [f(K_{t+1})^2 \sigma_A^2 + \sigma_e^2]$$
$$\frac{\partial C_t}{\partial W_{t+1}} = \frac{\Psi}{R_t} \left| \frac{\partial R_t}{\partial W_{t+1}} \right| - \Gamma a_{t+1}^2 \sigma_A^2 \frac{f(K_{t+1}) f'(K_{t+1})}{\Phi'(K_{t+1})}$$

Let $\mathbf{I}$ denote the identity matrix. The characteristic polynomial $P(x) \equiv \det(\mathbf{D}_z\mathbf{H} - x\mathbf{I})$ can be rewritten as

$$P(x) = \left(\frac{\partial a_t}{\partial a_{t+1}} - x \right) \left\{ x^2 - \left[1 + \frac{\partial (C_t + K_{t+1})}{\partial W_{t+1}} \right] x + \frac{\partial K_{t+1}}{\partial W_{t+1}} \right\} + x \frac{\partial C_t}{\partial a_{t+1}} \frac{\partial a_t}{\partial W_{t+1}}.$$

The roots of P are the eigenvalues of the *backward* dynamical system. (The eigenvalue λ considered in Section 5 and Appendix B thus satisfies $P(1/\lambda) = 0$ and $1/\lambda > 1$.) Since $P(-\infty) = +\infty$ and $P(+\infty) = -\infty$, there always exists a real eigenvalue. Simple calculation shows that $P(1) > 0$ if and only if $|R_2'(K_\infty)| > |R_1'(K_\infty)|$, where R_1 and R_2 are defined in the proof of Theorem 2. When the steady state is unique, this inequality is satisfied, the characteristic polynomial has at least one root in $(1, +\infty)$ and the stable manifold has a dimension no smaller than 1.

When markets are complete ($\sigma_A = \sigma_e = 0$), we know that $R_\infty = \Phi'(K_\infty) = 1/\beta$. We then infer that around the steady state,

$$\frac{\partial K_{t+1}}{\partial W_{t+1}} = \beta, \quad \frac{\partial R_t}{\partial a_{t+1}} = 0, \quad \frac{\partial R_t}{\partial W_{t+1}} = -\overline{A} \left| f''(K_\infty) \right|,$$

$$\frac{\partial a_t}{\partial a_{t+1}} = \beta, \quad \frac{\partial C_t}{\partial a_{t+1}} = 0, \quad \frac{\partial C_t}{\partial W_{t+1}} = \Psi \beta^2 \overline{A} \left| f''(K_\infty) \right|.$$

The characteristic polynomial reduces to $P(x) = (\beta - x)\, Q(x)$, where

$$Q(x) = x^2 - \left[1 + \beta + \Psi \beta^2 \overline{A} \left| f''(K_\infty) \right|\right] x + \beta.$$

The characteristic polynomial P has the obvious root $x = \beta$, which belongs to $(0,1)$ and thus corresponds to an unstable solution of the forward dynamical system. We next observe that $Q(0) > 0$ and $Q(1) < 0$. This implies that the quadratic polynomial Q has one root in the interval $(0,1)$ and one root in $(1,+\infty)$. Overall, the Jacobian matrix $\mathbf{D}_z\mathbf{H}$ has two eigenvalues in the interval $(0,1)$ and one eigenvalue $x = 1/\lambda$ larger than 1. When the production function is Cobb-Douglas $f(K) = K^\alpha$, it is easy to check that the stable root λ is given by (6.4).

When markets are incomplete, the dynamical system is still locally determined around the steady state when σ_A and σ_e are not very large. This follows by continuity from our finding that the cubic polynomial $P(x)$ has only one root outside $(0,1)$ when $\sigma_A = \sigma_e = 0$. We also numerically check that all the economies considered in Section 5 are locally determined. Since $P(x)$ is a cubic, there is a closed-form solution for the incomplete-markets convergence rate, which is omitted for expositional simplicity. Using Mathematica, we easily derive the analytical expression of λ and run the numerical experiments of Section 5.

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Figure 1. Amplification and Propagation Mechanism

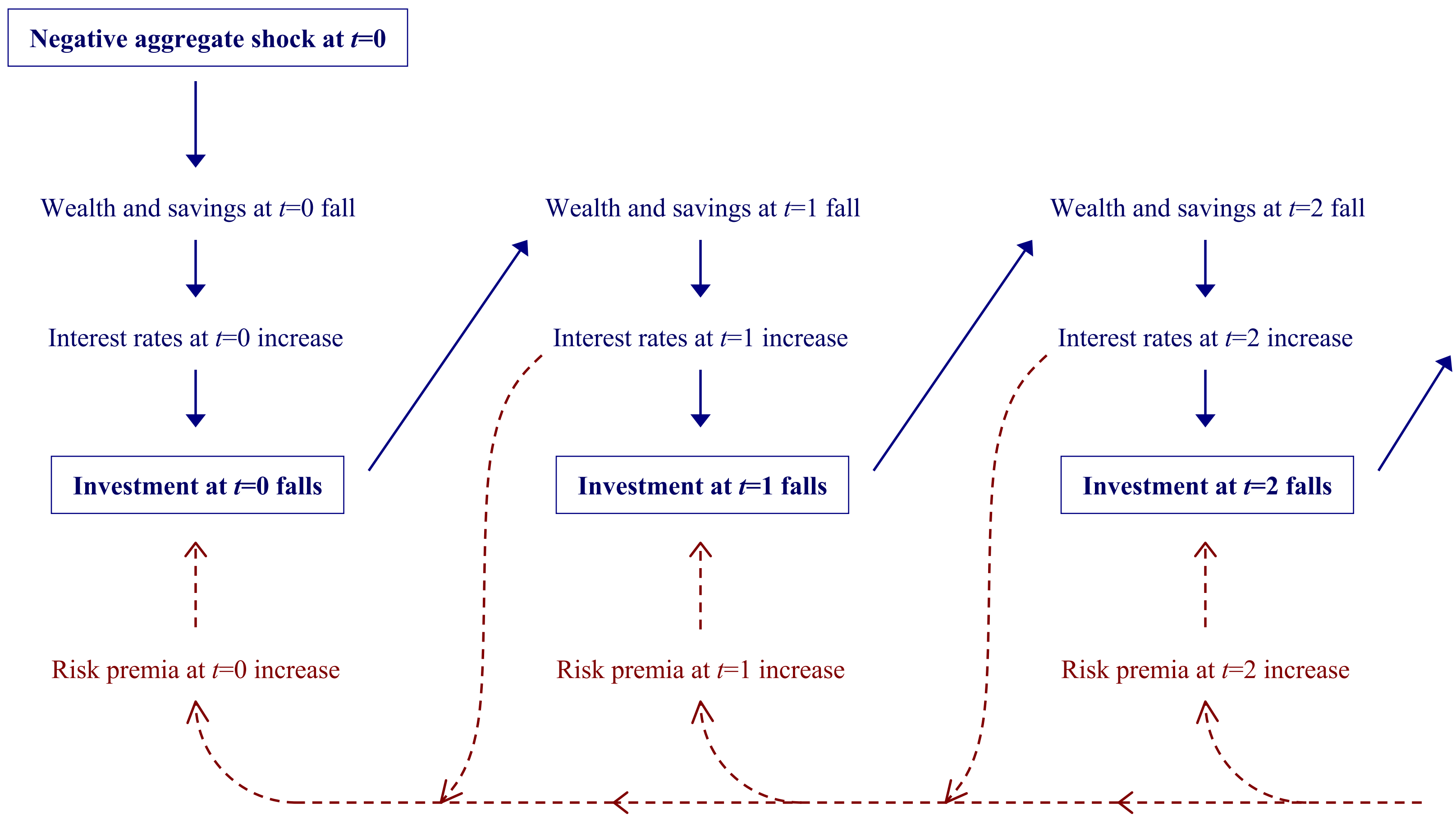

Figure 2.A (α = 0.70)

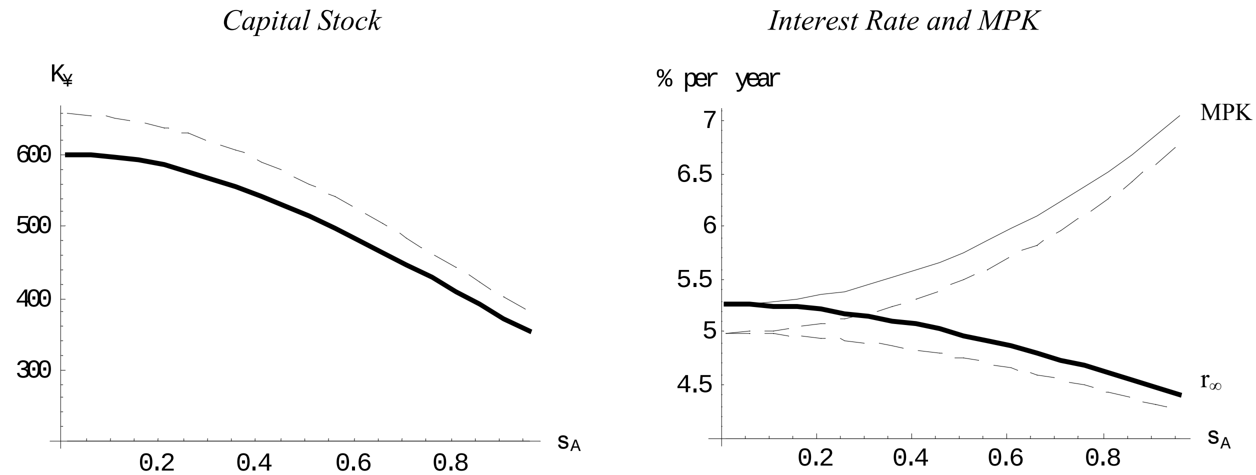

Figure 2.B (α = 0.35)

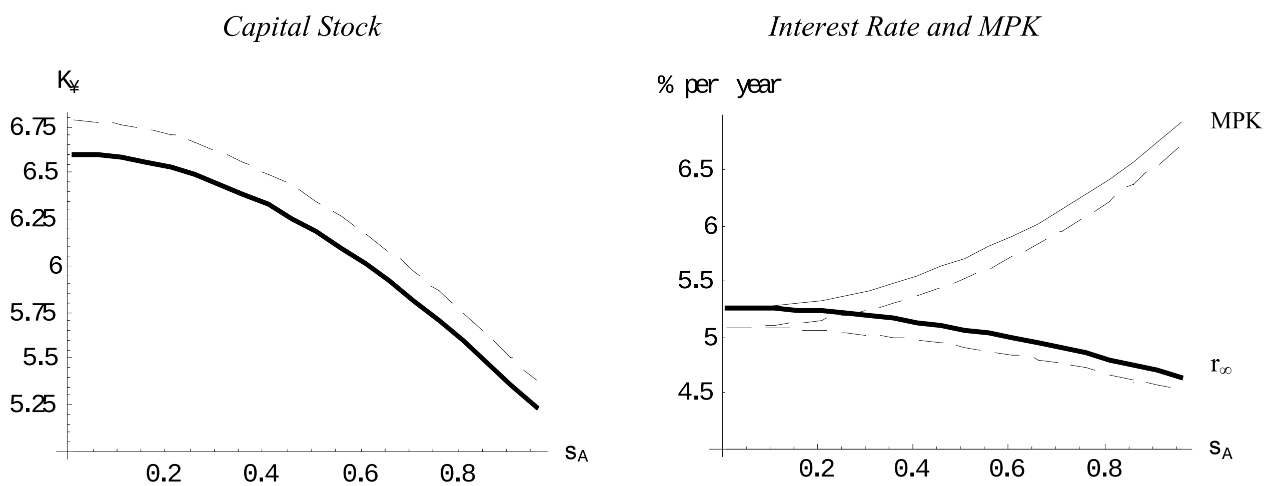

FIGURE 2. We perform an RBC calibration of the model with a time period of one year. The discount rate is 5% per year, the depreciation rate is 5% per year, the degree of relative risk aversion is 4, and the elasticity of intertemporal substitution is 1. The income share of capital is 70% in Panel A and 35% in Panel B. The solid lines correspond to $\sigma_e = 0$ (no idiosyncratic endowment risk) and the dashed ones to $\sigma_e = 50\%$ (of steady-state GDP). The plots show the steady-state level of the capital stock, the interest rate, and the marginal product of capital (MPK), as idiosyncratic production risk σ_A varies between zero and 100% of steady-state GDP.

Figure 3.A (α = 0.70)

Capital Stock *Interest Rate and MPK*

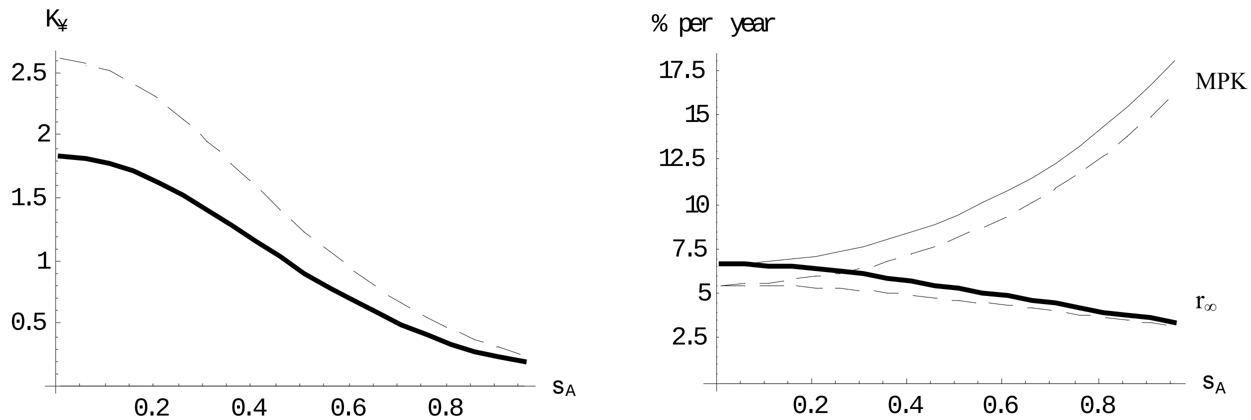

Figure 3.B (α = 0.35)

Capital Stock *Interest Rate and MPK*

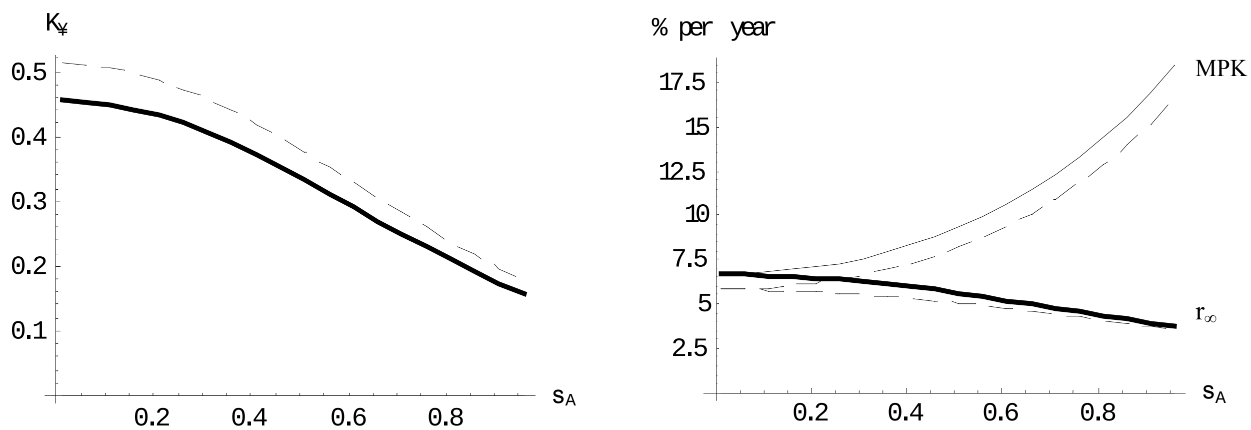

FIGURE 3. We assume the same parameter values as in Figure 1, but now use a five year time period (for both the length of an investment project and the duration of an idiosyncratic production shock). The solid lines correspond to $\sigma_e = 0$ and the dashed ones to $\sigma_e = 50\%$. The plots show the steady-state level of the capital stock, the interest rate, and the marginal product of capital (MPK), as idiosyncratic production risk σ_A varies between zero and 100%.

Figure 4.A (α = 0.70)

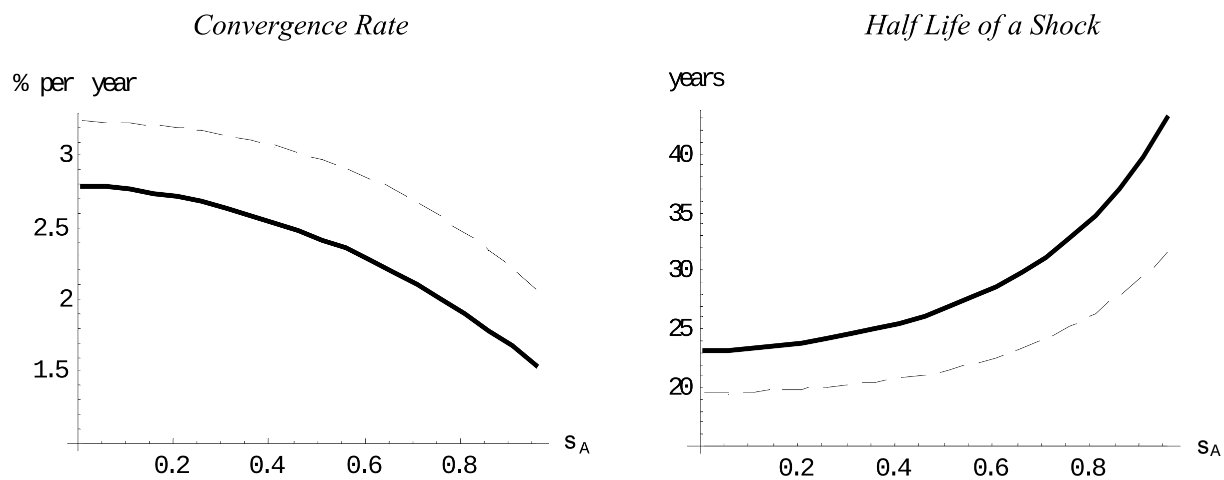

Figure 4.B (α = 0.35)

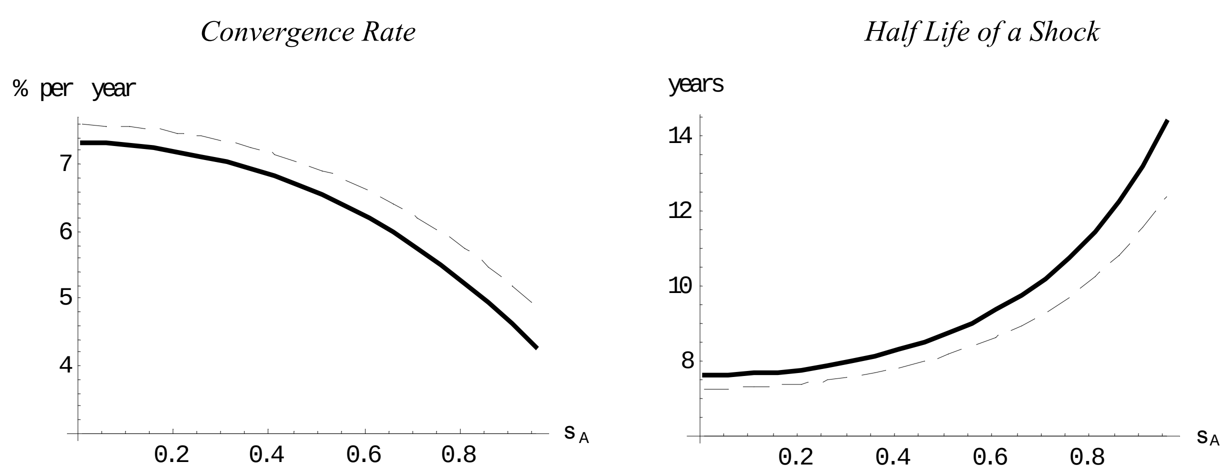

FIGURE 4. Assuming the same parameters as in Figure 2, we plot the convergence rate and the half-life of the deviation from the steady state as idiosyncratic production risk σ_A varies between zero and 100%. The solid lines correspond to $\sigma_e = 0$ and the dashed ones to $\sigma_e = 50\%$.