A Theory of Growth and Volatility at the Aggregate and Firm level*

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Abstract

This paper presents an endogenous growth model that explains the evolution of the first and second moments of productivity growth at the aggregate and firm level during the post-war period. Growth is driven by the development of both (i) idiosyncratic R&D innovations and (ii) general innovations that can be freely adopted by many firms. Firm-level volatility is affected primarily by the Schumpeterian dynamics associated with the development of R&D innovations. On the other hand, the variance of aggregate productivity growth is determined mainly by the arrival rate of general innovations. Ceteris paribus, the share of resources spent on development of general innovations increases with the stability of the market share of the industry leader. As market shares become less persistent, the model predicts an endogenous shift in the allocation of resources from the development of general innovations to the development of R&D innovations. This results in an increase in R&D, an increase in firm-level volatility, and a decline in aggregate volatility. The effect on productivity growth is ambiguous.

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On the empirical side, this paper documents an upward trend in the instability of market shares. It shows that firm volatility is positively associated with R&D spending, and that R&D is negatively associated with the correlation of growth between sectors which leads to a decline in aggregate volatility.

Keywords: Endogenous growth, aggregate volatility, firm-level volatility, innovation.
JEL: D9, E3, L1

1 Motivation

The literature on endogenous growth has made substantial progress in the past 15 years. In spite of these advances, however, there remains much to be learnt about the determinants of long-run productivity growth. In our opinion, the existing literature suffers from two main limitations.

First, state-of-the-art models (Aghion and Howitt [1998, Ch. 12], Dinopoulos and Thompson [1998], Jones [1995], Kortum [1997], Peretto [1998], Segerstrom [1998] and Young [1998]) predict a positive relationship between the growth rate of productivity and the share of Research & Development (R&D) in GDP. However, this prediction does not seem to be true of data for the United States (US) during the post-war period. Figure 1 illustrates the smoothed growth rate of productivity as well as the evolution of the share of private R&D in GDP as measured by the NSF. No clear relationship seems to exist between the two variables.[1] In fact, Comin [2004] shows that R&D expenditures, as defined by the NSF, can account for only a small fraction of productivity growth in the US during the post-war period. This finding suggests that there are other (probably) purposeful investments that lead to important improvements in productivity, investments that are not embodied in new products and, as a result, are not included in the NSF's definition of R&D.[2]

Second, in addition to having trouble explaining the first moments of growth processes, the existing theories have left the second moments out of their scope, as though their determinants were orthogonal to the determinants of the first moments.[3] This presumption, however, is debatable in light of the interesting dynamics of volatility during the post-war period. Two strands of the literature have characterized the evolution of volatility at the aggregate and firm level. McConnell and Perez-Quiros [1999] and Stock and Watson [2003] have shown that the volatility of aggregate variables such as output, hours worked and labor productivity growth has declined during the post-war period. At the firm level, however, these same variables have become more volatile (Comin and Mulani [2004], Chenney et al. [2003] and this paper). Perhaps most importantly, these diverging

[1] Examination of TFP growth or output growth results in similar conclusions. Similarly, the upward trend in R&D also holds for total R&D expenses in the US and in the OECD.

[2] These alternative innovations should not be confused with what the NSF classifies as process innovation. Process innovation applies to the development of new industrial processes such as those that lead to the production of steel or chemical products. In our context, this is the same as standard R&D that leads to a new product or an improved version of an existing product.

[3] There exists literature that has attempted to explore the effects of exogenous increases in aggregate volatility on growth (Ramey and Ramey [1995], Barlevy [2003]). A key difference between that literature and this paper is that here volatility (both aggregate and firm-level) is endogenous to analysis.

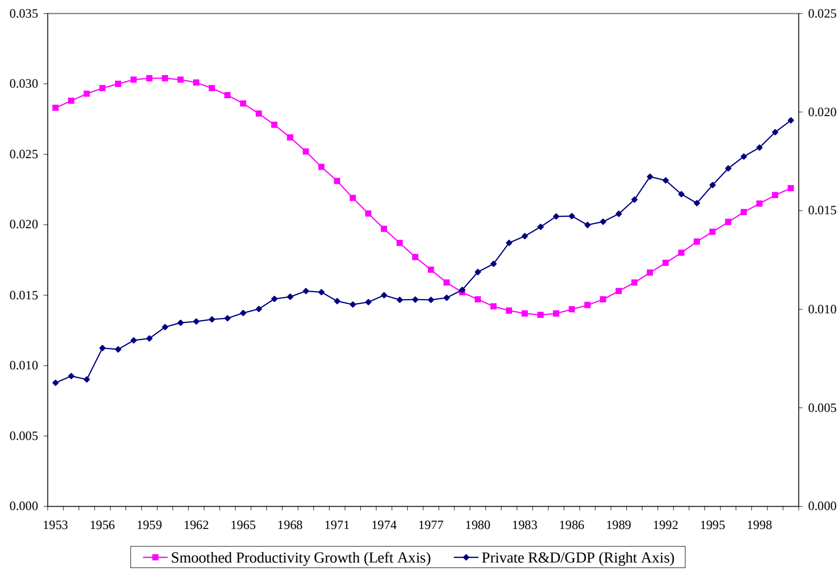

Figure 1: Evolution of (Smoothed) Productivity Growth and Private R&D share in GDP.

trends are also true for the volatility of productivity growth. Figure 2 illustrates the time series of the volatility of productivity growth at the aggregate and firm level. The left axis plots the standard deviation of 10-year centered rolling windows of annual productivity growth. The right axis plots the evolution of the same variable averaged for firms in the COMPUSTAT data base.[4,5] The opposite trends are evident.[6]

Standard macroeconomic models are not equipped to explain these diverging trends in volatility. Existing representative-agent models cannot account for the divergence since they predict that the second moments of the aggregate and individual variables are identical.[7] In principle, models with firm heterogeneity, such as Bertola and Caballero [1990], can accommodate different trends in aggregate and firm-level volatility by assuming different trends in the variance of the exogenous

[4]For each firm in COMPUSTAT, we compute its volatility as the centered standard deviation of 10 consecutive annual growth rates of the sales per worker. The firm volatility measure plotted in Figure 2 is the average across firms of their volatility. volatility

[5]Comin and Mulani [2003] shows that the upward trend in firm-level volatility is not the result of a compositional bias in the COMPUSTAT sample. See also Comin and Philippon [2005] for a more detailed discussion.

[6]As mentioned earlier, Comin and Mulani [2003] and Chenney et al. [2003] have documented upward trends in the volatility of growth rate of firm-level sales and employment. This paper is, to the best of our knowledge, the first to document the upward trend in the firm-level volatility of the growth rate of sales per worker.

[7]Gabaix [2005] presents a model where shocks to large firms generate aggregate fluctuations. As in representative agent models, Gabaix's model predicts that the evolution of firm-level and aggregate volatility is similar.

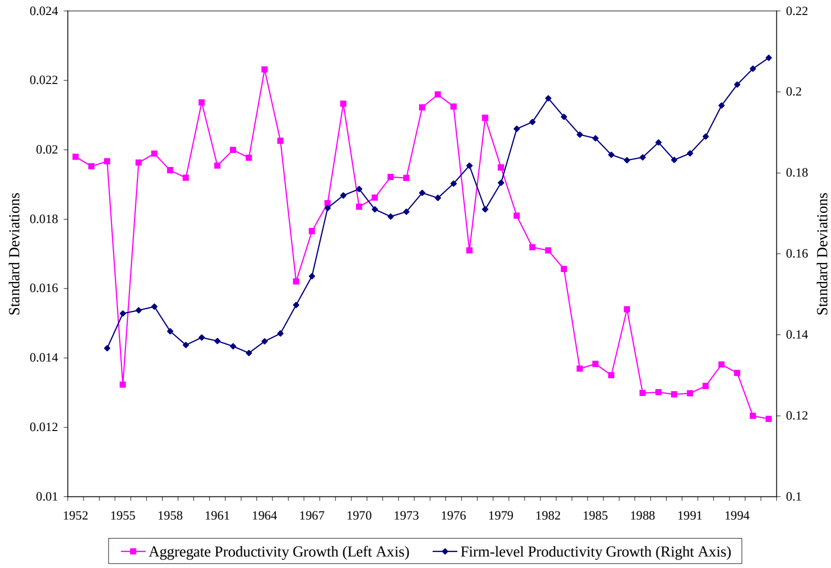

Figure 2: Evolution of the Aggregate and Firm-level Volatility of productivity

aggregate and firm-specific shocks. However, Comin and Philippon [2005] provide evidence that these diverging trends are not simply a coincidence: A common component can account for an important part of both trends. The goal should then be to build a model where, in response to a shock, firm-level and aggregate second moments respond in opposite ways. This is not the case in current models of firm heterogeneity because the interactions between firms embedded in these models are not adequate: most of them are partial equilibrium models and treat firms as independent entities. Even though more recent versions of these models have incorporated general equilibrium interactions, they seem insufficient to generate the co-movement patterns that drive the diverging trends in volatility.

Addressing the above concerns, this paper builds an endogenous growth model that attempts to enhance our understanding of the determinants of productivity growth and that has implications for firm and aggregate volatility that are consistent with the evidence.

To this end, this paper takes on a fresh route that, we believe, is more promising. It builds on the quality-ladder models of Aghion and Howitt [1992] and Grossman and Helpman [1991, Ch. 4]. In this context, standard R&D investments lead firms to develop new products that replace the current leading products. Such improvements in productivity lead to substantial firm-level volatility since incumbents incur losses while entrants enjoy capital gains. However, these innovations are to a large extent sector specific and have only a minor effect on volatility at the aggregate level.

To explain the movements in aggregate growth and volatility, it is necessary to consider a second type of innovation - general innovations.[8] General innovations have two properties. First, they are applicable to several firms and sectors.[9] Second, a firm that develops a general innovation (for the most part) cannot appropriate the benefits enjoyed by other firms when adopting it. This is the case because general innovations - such as the mass production system, mass customization, flow manufacturing and other organizational innovations, improved process controls, product development, testing practices and pre-production planning, new personnel and accounting practices, financial innovations and new credit instruments such as the credit card, general programming languages such as Basic, Hypertext or Fortran, the use of electricity as the source of energy in a plant,... - are hard to patent and relatively easy to reverse-engineer.[10][11]

These two properties have interesting implications. First, general innovations have a large effect on aggregate growth because they affect many firms symmetrically. Furthermore, since general innovations do not arrive smoothly over time, investments in the development of general innovations may lead to substantial volatility in aggregate productivity growth. Second, the inability of innovators to appropriate the social value of general innovations means that their incentives to develop them depend on their firm's productivity gain from implementing the innovation. These productivity gains are larger for more valuable firms. As a result, general innovations are typically conducted by (large) leading firms.

In equilibrium, there is a negative relationship between the resources spent on R&D and resources spent on the development of general innovations. Since (1) R&D leads to turnover in market leaders and to a decline in the value of leading firms and (2) the private return to a general innovation increases in the value of the firm, a force that leads the economy to invest more in R&D may induce a decline in the rate of development of general innovations.

[8] A large majority (though surely not all) of the innovations that fit our notion of general innovations are explicitly excluded by the NSF from their definition of R&D. See Comin [2004] for more on NSF's definition of R&D.

[9] This feature links our general innovations to the General Purpose Technologies (GPT's) of Bresnahan and Trajtenberg [1995] and Helpman and Trajtenberg [1998]. Unlike GPT's, general innovations do not have to be revolutionary. An interesting difference from a modelling point of view is that the development of GPT's and their components is usually modelled as either being exogenous or supported by patents. Our general innovations are developed endogenously and are not patentable.

[10] See Table 1 for an incomplete list of general innovations together with a small description of the innovation. For a longer description see Appendix 1.

[11] Hellwig and Irmen [2001] and Boldrin and Levine [2000] have also highlighted the importance of innovations that are not patentable.

This trade off between R&D and general innovations accounts for the trends observed in growth and volatility at the aggregate level. first, productivity growth increases with the development of both R&D and general innovations. But since an increase in R&D intensity leads to a decline in the arrival rate of general innovations, the relationship between R&D and productivity growth is ambiguous. Second, aggregate volatility is primarily affected by the arrival rate of general innovations because this determines the co-movement of growth across sectors by causing simultaneous fluctuations in them. Hence, a decline in investments in general innovations, leads to a decline in aggregate volatility. Finally, firm-level volatility is primarily driven by market turnover. An increase in R&D intensity leads to turnover in the market leader and an increase in firm-level volatility.

In addition to developing a new model of growth and volatility, this paper also provides empirical evidence of the forces and mechanisms emphasized by the model. First, it documents two new facts: (i) a very significant increase in the market turnover rate and (ii) a substantial decline in the correlation of productivity growth across sectors. Second, it shows that turnover and firm volatility have increased to a greater extent in sectors that have experienced higher increases in R&D intensity. Finally, this paper establishes that R&D is negatively associated with aggregate volatility by showing that sectors that experienced higher increases in R&D, also experienced greater declines in the correlation between their own growth and the rest of the economy.[12]

While there must be other forces that have contributed to the trends in firm-level and, specially, aggregate volatility, the mechanisms emphasized by our model are quantitatively significant. A calibration of the model shows that it can account for (1) the lack of a relationship between R&D and productivity growth, (2) 75 percent of the increase in firm volatility and, (3) over 40 percent of the decline in aggregate volatility.

The rest of the paper is organized as follows. Section 2 presents the formal model and undertakes the comparative statics exercises. Section 3 discusses and evaluates predictions of the model in both qualitative and quantitative terms. Section 4 concludes.

[12] Comin and Philippon [2005] decompose the variance of aggregate growth into 2 components - (a) a sectoral variance component and (b) a correlation-between-sectors component and show that the decline in aggregate variance is mostly due to the decline in the correlation of sectoral growth.

2 Model

The following describes an endogenous technological change model that delivers endogenous growth and endogenous volatility at the aggregate and firm-level. Most of the insights hold in a one-sector version of the model. However, a multisector formulation is necessary to understand the determinants of the co-movement of growth across sectors, which is essential for the evolution of aggregate volatility.

2.1 Set up

Preferences

The representative consumer enjoys a utility flow that is linear on the units of final output consumed (c_t). The present discounted value of utility is represented as:

$$U = \int_0^\infty c_t e^{-rt} dt, \tag{1}$$

where r denotes the instantaneous discount rate. Consumers supply, inelastically, a mass of L units of labor. They also pay some lump sum of taxes T_t.

Production

Final output (y) is produced by combining the outputs produced in the N sectors ($\{y_s\}_{s=1}^N$), as specified in the following production function:

$$y = \prod_{s=1}^N y_s^{1/N}. \tag{2}$$

Sectoral output is produced competitively by combining $m+1$ sector-specific intermediate goods, where m is a fixed number. Each intermediate good is produced by one and only one producer. In each sector, there are two types of intermediate goods. The good with highest quality (q_{ls}) is the leading intermediate good. Consumers perceive this as a differentiated intermediate good because of its superior technical properties. The rest of producers cannot compete with the leading intermediate good and must produce standard intermediate goods. The standard intermediate goods produced by the various followers are not differentiated.

Let x_s^l denote the number of units of leading intermediate good employed to produce sector $s\prime$ output. Similarly, let x_{si}^f denote the number of intermediate goods from the i^{th} standard producer employed in the production of sector s' output. Then the production function for sector s' output can be expressed as:

$$y_s = q_{ls} X_s, \tag{3}$$

where q_{ls} is the quality of the leading intermediate good and X_s is the intermediate good composite, both in sector s. X_s takes the following functional form:

$$X_s = \left(\beta \left(x_s^l \right)^{\alpha} + (1-\beta) \left(\sum_{i=1}^{m} x_{si}^f \right)^{\alpha} \right)^{1/\alpha}, \tag{4}$$

where $\beta < 1$ is the market share of the leading intermediate good, and $\alpha \in (0,1)$ is the elasticity of substitution between the leading good and the composite of standard intermediate goods.

The production of a unit of intermediate good in sector s requires a_s units of labor. a_s declines with the efficiency of the production process, h_s, such that:

$$a_s = h_s^{-\psi_h}, \text{with } \psi_h > 0 \tag{5}$$

Innovation

Intermediate good producers can undertake two types of innovations. First, they can try and develop an intermediate good with quality higher than q_{ls}. In particular, after spending $(1-s_{R\&D})n_{si}^q$ units of aggregate output, they face a probability $\lambda_i^q = \lambda_0^q n_{si}^q / y$ over an instantaneous time-interval dt of developing a new leading good with quality $\delta_q q_{ls}$ $(\delta_q > 1)$.[13,14] In this formulation, λ_0^q measures the probability of succeeding in the development of a superior intermediate good per fraction of GDP spent on R&D. $s_{R\&D}$ denotes an R&D subsidy that is financed by the lump sum taxes paid by consumers.

Second, intermediate goods producers can also invest in the improvement of the production process of their intermediate good (i.e. reducing the cost of production, a_s). Specifically, each intermediate goods firm can invest n^h units of aggregate output and face an instantaneous probability $\lambda^h = \lambda_0^h \left(n^h / y \right)^{\rho_h}$, with $0 < \rho_h < 1$, of successfully increasing h up to $\delta_h h$, with $\delta_h > 1$.

These two types of innovations differ in their appropriability. Firms that invent a new product or improve the quality of an existing product can patent the innovation and extract a substantial fraction of the surplus enjoyed by other firms from such an innovation. On the other hand, firms that

[13] Griliches [1984] finds evidence in favor of the linearity of the R&D production technology using firm-level data.

[14] This formulation has several interesting features. First, the lower demand elasticity of the leading intermediate good is instrumental to generate cross-sectional variation in sales per worker. Second, by not having to carry around the distribution of qualities for intermediate goods, we make substantial progress towards an analytical solution of the model. Third, the absence of entry and exit simplifies the computation of firm-level moments.

develop general innovations, such as improvements in management practices, cannot appropriate the benefits experienced by other firms when they adopt the same practices. This is the case because such general innovations are easy to reverse engineer and because most of them are not embodied in a good and therefore are hard to patent. These characteristics are reflected in the assumption that general innovations are immediately (and costlessly) adopted by all producers.

A second difference between the two types of innovations is their applicability. The impact of new or improved goods is often restricted to a small number of sectors. General innovations, such as improvements in management or in the organization of production mentioned above, can be applied to many different economic activities that cover a wide array of sectors. For concreteness, we assume that R&D innovations are sector specific while, for the time being, general innovations diffuse to all the sectors in the economy.

Government

The government collects lump sum taxes from the consumers to finance the exogenous R&D subsidy at every instant.

2.2 Optimality

Let p_s^x denote the cost of acquiring one unit of the composite of intermediate goods in sector s. Let X_s^f be the total quantity of standard intermediate goods used in the production of sector s' output. That is $X_s^f \equiv \left(\sum_{i=1}^m x_{si}^f\right)$.

Demand of sectoral output by producers of final output is given by

$$p_s y_s = y/N. \tag{6}$$

Similarly, demand for the leading and total standard intermediate goods from producers of sectoral output in the sector are:

$$x_s^l = X_s \left(\frac{\beta p_s^x}{p_s^l}\right)^{\frac{1}{1-\alpha}} \tag{7}$$

$$X_s^f = X_s \left(\frac{(1-\beta) p_s^x}{p_s^f}\right)^{\frac{1}{1-\alpha}}, \tag{8}$$

where p_s^f is the price of the standard intermediate goods composite.

Given these demand functions, the producers of leading and standard intermediate goods in sector s charge prices:[15]

$$p_s^l = \frac{a_s w}{\alpha} \quad (9)$$
$$p_s^f = a_s w \quad (10)$$

The resulting cost of acquiring one unit of the composite of intermediate goods, p_s^x, is equal to:

$$p_s^x = \frac{a_s w}{\xi}, \text{ where } \xi \equiv \left[(\beta \alpha^{\alpha})^{\frac{1}{1-\alpha}} + (1-\beta)^{\frac{1}{1-\alpha}} \right]^{\frac{1-\alpha}{\alpha}}. \quad (11)$$

Combining expressions (6) through (11), we can solve for the nominal sales of leading and standard intermediate good producers:

$$p_s^l x_s^l = \frac{y\xi}{\alpha N} \left(\frac{\alpha\beta}{\xi} \right)^{\frac{1}{1-\alpha}} \equiv \varkappa^l y \quad (12)$$
$$p_s^f x_{si}^f = \frac{y\xi}{mN} \left(\frac{1-\beta}{\xi} \right)^{\frac{1}{1-\alpha}} \equiv \varkappa^f y \quad (13)$$

Let $c(\lambda) \equiv \left(\lambda / \lambda_0^h \right)^{\frac{1}{\rho_h}}$ be the cost of undertaking the investments in general innovations needed to face a probability λ of developing a general innovation. Profits net of innovation expenses for leading and standard intermediate good producers are expressed as:

$$\pi_s^l y = (1-\alpha)\varkappa^l y - \left(c(\lambda_{ls}^h) + \frac{\lambda_{ls}^q (1 - s_{R\&D})}{\lambda_0^q} \right) y \quad (14)$$
$$\pi_{si}^f y = -\left(c(\lambda_{si}^h) + \frac{\lambda_{si}^q (1 - s_{R\&D})}{\lambda_0^q} \right) y, \quad (15)$$

where $\varkappa^l \equiv \frac{\xi}{\alpha N} \left(\frac{\alpha\beta}{\xi} \right)^{\frac{1}{1-\alpha}}$ and $\varkappa^f \equiv \frac{\xi}{mN} \left(\frac{1-\beta}{\xi} \right)^{\frac{1}{1-\alpha}}$ are the ratios of nominal sales to GDP for the leading and standard intermediate goods and λ_l^x and λ_{si}^x denote the hazard rates for innovations of type x faced by the leading and the i^{th} standard intermediate good firms, for $x = h$, q. Note that followers have zero operating profits since the price they charge for standard intermediate goods is equal to the constant marginal cost of production.

After having solved the static problem, we set up the Bellman equations for the problem of the firm. In general, the state variables for this problem would be the distribution of quality levels across sectors ($\{q_{sl}\}_{s=1}^N$) and the efficiency in the production of intermediate goods (h). However,

[15] As usual in the literature, we assume that the share of leading intermediate goods, β, is small enough for the leading intermediate good producer to find negligible the effect of its price on the price of the intermediate good composite, p_s^x.

the Cobb-Douglas specification for the production of final output implies that the quality of the leading intermediate good in a sector (q_{ls}) matters only to the extent that it affects the average quality of the leading intermediate goods, q_l, defined as:

$$q_l = \prod_{s=1}^{N} q_{ls}^{1/N}. \tag{16}$$

Further, q_l and h affect the value of the firm only through aggregate output, y. Therefore, the value of a firm divided by aggregate output is independent of q_l and h. Let v_s^l and v_{si}^f denote the values of an intermediate good producer of the leading and i^{th} standard good in sector s divided by aggregate output. To define these values, it proves useful to introduce additional notation. Let $\triangle q$ and $\triangle h$ be defined, respectively, as $\delta_q^{1/N}$ and $\delta_h^{\psi_h}$. Let $\overrightarrow{\lambda_{-j}^{z}}$ denote the vector that contains the hazard rates for innovations on z (for $z = q, h$) for all the intermediate good producers other than j (for $j = sl$, si, and s, where the last refers to all producers in sector s). Finally, G^z denotes the law of motion for $\overrightarrow{\lambda_{-sj}^{z}}$ which is taken as given by the firm. Using this notation, v_s^l and v_{si}^f are defined by the following Bellman equations:

$$\begin{aligned}
v_s^l\left(.;\overrightarrow{\lambda_{-sl}^{q}},\overrightarrow{\lambda_{-sl}^{h}}\right) &= \max_{\lambda_{sl}^{h},\lambda_{sl}^{q}} \pi_s^l dt + (1+rdt)^{-1}[\left(\sum_{z=1}^{N}\lambda_{lz}^{q} + \sum_{z\neq s}\sum_{i=1}^{m}\lambda_{zi}^{q}\right) dt\triangle q v_s^l(.;\overrightarrow{\lambda_{-sl}^{q\prime}},\overrightarrow{\lambda_{-sl}^{h\prime}}) \quad \text{(Vl)}\\
&+\sum_{j=1}^{m}\lambda_{sj}^{q} dt\triangle q v_{si}^f(.;\overrightarrow{\lambda_{-si}^{q\prime}},\overrightarrow{\lambda_{-si}^{h\prime}})\\
&+\triangle h\sum_{z=1}^{N}(\lambda_{zl}^{h} + \sum_{i=1}^{m}\lambda_{zi}^{h}) dt v_s^l(.;\overrightarrow{\lambda_{-l}^{q\prime}},\overrightarrow{\lambda_{-l}^{h\prime}})\\
&+\left(1-\sum_{z=1}^{N}\left(\lambda_{zl}^{q} + \sum_{i=1}^{m}\lambda_{zi}^{q} + \lambda_{zl}^{h} + \sum_{i=1}^{m}\lambda_{zi}^{h}\right) dt\right) v_s^l(.;\overrightarrow{\lambda_{-sl}^{q\prime}},\overrightarrow{\lambda_{-sl}^{h\prime}})]\\
&s.t.\\
\overrightarrow{\lambda_{-ls}^{q\prime}} &= G^q(\overrightarrow{\lambda_{-ls}^{q}},\overrightarrow{\lambda_{-ls}^{h}});\ \overrightarrow{\lambda_{-ls}^{h\prime}} = G^h(\overrightarrow{\lambda_{-ls}^{q}},\overrightarrow{\lambda_{-ls}^{h}})
\end{aligned}$$

$$
\begin{aligned}
v^f_{si}\left(.;\overrightarrow{\lambda^q_{-si}},\overrightarrow{\lambda^h_{-si}}\right) &= \max_{\lambda^h_{si},\lambda^q_{si}} \pi^f_{si}dt + (1+rdt)^{-1}[\left(\sum_{z=1}^{N}\lambda^q_{zl} + \sum_{z\neq s}\left(\sum_{j=1}^{m}\lambda^q_{sj}\right) + \sum_{j\neq i}\lambda^q_{sj}\right) dt\triangle q v^f_{si}(.;\overrightarrow{\lambda^{q\prime}_{-si}},\overrightarrow{\lambda^{h\prime}_{-si}}) \\
&\quad +\lambda^q_{si}dt\triangle q v^l(.;\overrightarrow{\lambda^{q\prime}_{-si}},\overrightarrow{\lambda^{h\prime}_{-si}}) + \triangle h\sum_{z=1}^{N}(\lambda^h_{zl} + \sum_{j=1}^{m}\lambda^h_{zj})dtv^{si}(.;\overrightarrow{\lambda^{q\prime}_{-si}},\overrightarrow{\lambda^{h\prime}_{-si}}) \qquad \text{(Vf)} \\
&\quad + \left(1 - \sum_{z=1}^{N}\left(\lambda^q_{zl} + \sum_{j=1}^{m}\lambda^q_{zj} + \lambda^h_{zl} + \sum_{j=1}^{m}\lambda^h_{zj}\right)dt\right) v^{si}(.;\overrightarrow{\lambda^{q\prime}_{-si}},\overrightarrow{\lambda^{h\prime}_{-si}})] \\
&\quad s.t. \\
\overrightarrow{\lambda^{q\prime}_{-si}} &= G^q(\overrightarrow{\lambda^q_{-si}},\overrightarrow{\lambda^h_{-si}});\ \overrightarrow{\lambda^{h\prime}_{-si}} = G^h(\overrightarrow{\lambda^q_{-si}},\overrightarrow{\lambda^h_{-si}}).
\end{aligned}
$$

These functional equations are self-explanatory. They simply capture the capital gains enjoyed and losses suffered by each type of firm when an innovation, R&D-driven or general, arrives.

Optimal Investments

Producers of standard intermediate goods have the option of introducing a good of higher quality. Optimal investment in developing this superior intermediate good induces followers to equalize the marginal cost of the R&D investment to its expected marginal benefit:

$$
\overbrace{(1 - s_{R\&D})}^{\text{Marginal Cost}} = \overbrace{\lambda^q_0(v^l_s(q_l\delta_q^{1/N}, h) - v^f_{si}(q_l, h))}^{\text{Expected Mg. Benefit from Embodied Innovations}} \tag{17}
$$

In principle, current leaders can also increase the quality of their intermediate good. They face the same marginal cost as followers, but the expected marginal benefit is now equal to $\lambda^q_0(V^l_s(q_l\delta_q, h) - V^l_s(q_l, h))$. This implies that, if in equilibrium $V^l > V^{si}$, only followers will invest in increasing the quality of the leading intermediate good, as is the case in standard quality-ladder models.[16]

[16] Comin and Ludvigson [2006] provide empirical evidence that supports this prediction. Comin and Ludvigson follow the finance literature and rank firms by the book-to-market value of their assets. Since producers of standard intermediate goods have high growth prospects, they identify the producers of standard goods with the firms in the lowest book-to-market portfolio. Conversely, producers of leading intermediate goods have low growth prospects and correspond to the firms in the highest book-to-market portfolio (i.e. the value portfolio). Comin and Ludvigson [2006] show that growth firms are very much like our standard intermediate good producers in that they are younger firms (about five years younger than value firms) and much smaller (about 35 percent less capital and 10 percent less employment). In addition, Hansen, Heaton and Li [2005], and Comin and Ludvigson [2006] show that the growth firms undertake much more R&D than value firms. That is not only as a share of their sales (four times more in the five portfolio classification of Hansen et al. and twice more in the three portfolio classification of Comin and

Leaders have incentives to come out with general innovations that reduce the marginal cost of producing intermediate goods for all producers. In an interior solution, the optimal investment in general innovations by the leader results in the following equality:

$$\overbrace{\frac{1}{\rho_h}\left(\frac{\lambda_{sl}^h}{\lambda_0^h}\right)^{\frac{1-\rho_h}{\rho_h}}}^{\text{Marginal Cost}} = \overbrace{\lambda_0^h(v_s^l\left(q_l, h\delta_h\right) - v_s^l\left(q_l, h\right))}^{\text{Expected Mg. Benefit from general Innovations}} \quad . \tag{18}$$

Followers, in principle, can also come out with general improvements in productivity. The intensity of their investments in general innovations equalizes the marginal cost with the expected marginal benefit.

$$\overbrace{\frac{1}{\rho_h}\left(\frac{\lambda_{sf}^h}{\lambda_0^h}\right)^{\frac{1-\rho_h}{\rho_h}}}^{\text{Marginal Cost}} = \overbrace{\lambda_0^h(v_{si}^f\left(q_l, h\delta_h\right) - v_{si}^f\left(q_l, h\right))}^{\text{Expected Mg. Benefit from general Innovations}} \quad . \tag{19}$$

In equilibrium, however, since the private value of these innovations is proportional to the value of the firm, the followers' incentive to undertake general innovations is lower than the leader's.

2.3 Equilibrium

We close the model by clearing the labor market. This implies the following condition.

$$L = \sum_{s=1}^{N} a_s \left(x_s^l + \sum_{i=1}^{m} x_{si}^f\right) \tag{20}$$

Now we can formally define a equilibrium of this economy. That is a tuple $\left(\{\lambda_{si}^q\}_{i=1}^m, \lambda_{sl}^h, \{\lambda_{si}^h\}_{i=1}^m, \left\{x_{si}^f\right\}_{i=1}^m, x_s^l, v_s^l, \left\{v_{si}^l\right\}, w\right)_{s=1}^N$ that satisfies the firms' optimization conditions (7) and (8), the firms' Bellman equations (Vl) and (Vf), the optimal investment in innovation conditions (17), (18) and (19) and the labor market clearing condition (20). Note that this equilibrium is stationary since the problem of the firm has no state variable.

The analysis presented here is restricted to the Stationary Symmetric Equilibrium (SSE) of this economy. That is, an equilibrium of this economy where:

- $\lambda_{sl}^h = \lambda_l^h$, $x_s^l = x^l$, $v_s^l = v^l$ $\forall s$.

Ludvigson), but also in absolute terms (about 20 percent more in the three portfolio decomposition of Comin and Ludvigson).

- $\lambda_{si}^q = \lambda_s^q/m$, $\lambda_{si}^h = \lambda_{f.}^h$, and $x_{si}^f = x_.^f$ $v_{si}^f = v^f$ $\forall s, i$,

where λ_s^q is the probability that a R&D innovation is developed in a sector, and v^f and v^l are, respectively, the value of following and leading firms divided by aggregate output in the symmetric equilibrium.

Next we solve for the equilibrium investment intensities in the development of R&D and general innovations. First, we note that the value of standard intermediate goods firms, v^f, is zero. This follows from the fact that standard intermediate goods producers make zero operating profits i.e. they incur loses equal to the cost of undertaking innovations (i.e. $\pi_{si} = -\frac{\lambda_s^q(1-s_{R\&D})}{\lambda_0^q m} - c(\lambda_{f.}^h)$). In equilibrium, these static loses precisely compensate the expected capital gains from becoming market leaders, making the net value of a standard intermediate good producer zero.

To see this formally, note that the equilibrium condition that determines the R&D intensity can be rewritten as:

$$0 = \frac{\lambda_s^q}{m}\left(\frac{-(1-s_{R\&D})}{\lambda_0^q} + \left(\triangle q v^l - v^f\right)\right). \tag{21}$$

Using the expression for π^f and the linearity of the R&D technology, expression (21) can be rewritten as:

$$0 = \pi^f + c(\lambda_{f.}^h) + \frac{\lambda_s^q}{m}\left(\triangle q v^l - v^f\right).$$

In the symmetric equilibrium, the value of a standard intermediate good producer can be expressed as:

$$rv^f = \pi^f + \frac{\lambda_s^q}{m}(\triangle q v^l - v^f) + \lambda_s^q(\frac{m-1}{m})(\triangle q - 1)v^f + \lambda_s^q(N-1)(\triangle q - 1)v^f + \lambda^h(\triangle h - 1)v^f, \tag{22}$$

where λ^h is the probability that some firm in the economy develops a general innovation.

Plugging expression (21) into (22) yields:

$$rv^f = -c(\lambda_{f.}^h) + \lambda_s^q(\frac{m-1}{m})(\triangle q - 1)v^f + \lambda_s^q(N-1)(\triangle q - 1)v^f + \lambda^h(\triangle h - 1)v^f.$$

This equation implies that $v^f \leq 0$ and that $v^f = 0$ if and only if $\lambda_{f.}^h = 0$. But note that the optimal investment in general innovations by producers of standard intermediate goods is given by

$$c'(\lambda_{f.}^h) = \lambda_0^h(\triangle h - 1)v^f.$$

Since the marginal private value of general innovations (RHS) is smaller or equal to zero, following firms will not invest in the development of general innovations. That is, $\lambda_{f.}^h = 0$ and therefore

$v^f = 0$.[17]

Plugging this into the Bellman equation for leading intermediate good producers, it follows that

$$v^l = \frac{(1-\alpha)\varkappa^l - (\frac{\lambda^h/N}{\lambda_0^h})^{\frac{1}{\rho_h}}}{r + \lambda_s^q(1-(\triangle q-1)(N-1)) - \lambda^h(\triangle h-1)}. \tag{23}$$

The following system of equations determines the optimal investment intensities in R&D and general innovations:

$$\begin{aligned} 1 - s_{R\&D} &= \lambda_0^q \triangle q v^l && \text{(Lq)} \\ c'(\lambda^h/N) &= \lambda_0^h(\triangle h - 1)v^l && \text{(Lh)} \end{aligned}$$

Isolating v^l from condition (Lq), it follows that, in the SSE, $v^l = (1 - s_{R\&D})/(\lambda_0^q \triangle q)$. Plugging this back into condition (Lh) results in:

$$c'(\lambda^h/N) = \frac{(1-s_{R\&D})\lambda_0^h(\triangle h-1)}{\lambda_0^q \triangle q}.$$

Plugging in the functional form specified for $c(.)$, results in the expression for the rate of arrival of general innovations:

$$\lambda^h = N\lambda_0^h \left(\frac{(1-s_{R\&D})\rho_h\lambda_0^h(\triangle h-1)}{\lambda_0^q \triangle q}\right)^{\frac{\rho_h}{1-\rho_h}} \tag{24}$$

Proposition 1 characterizes the effects of the various parameters on the arrival rate of general innovations (λ^h). In particular, it describes the effects of the two most relevant parameters for our analysis, $s_{R\&D}$ and λ_0^q.

Proposition 1 *: λ^h increases with λ_0^h, ρ^h and $\triangle h$, declines with $s_{R\&D}$, λ_0^q and $\triangle q$ and is unaffected by α, β and r.*

Investments in general innovations are optimal when the leading firm equalizes the marginal cost to the expected marginal benefit from them. The rate of general innovations in the SSE is increasing in λ_0^h and ρ^h since both these parameters reduce the marginal cost of developing general innovations. λ^h also increases with $\triangle h$ since it increases the marginal benefit from these innovations.

The rest of the comparative static exercises described in proposition 1 operate through the value of leading firms, v^l. In the SSE, v^l is equal to $(1-s_{R\&D})/(\lambda_0^q \triangle q)$. This is the case since, in equilibrium,

[17] The lack of investments in general innovations by followers is a robust feature of the equilibrium. This may be the case even when the intermediate goods produced by followers are differentiated, if there are some small fixed costs of undertaking general innovations.

standard intermediate good firms undertake R&D investments until the expected marginal cost of developing an embodied innovation ($(1-s_{R\&D})/\lambda_0^q$) equals the capital gain experienced when developing a superior leading product ($\triangle q v^l$). As a result, the rate of arrival of R&D-driven innovations (λ^q) adjusts until v^l equals $(1-s_{R\&D})/(\lambda_0^q \triangle q)$. This is why changes in the demand elasticity (α), the market share of leading intermediate goods (β) or the discount rate (r) have no effect on v^l or λ^h.[18]

Changes in the R&D subsidy ($s_{R\&D}$), the efficiency of R&D investments (λ_0^q) or the quality of improvements associated with them ($\triangle q$), however, do have an effect on λ^h. Increases in $s_{R\&D}$, λ_0^q or $\triangle q$ make the development of an embodied innovation more attractive and lead to increases in the arrival rate of R&D-driven innovations. This mechanism reduces the value of leading firms and hence the marginal value of general innovations for leading firms. As a result, λ^h is decreasing in $s_{R\&D}$, λ_0^q and $\triangle q$ in the SSE.

Plugging back the expression for λ^h (24) into equation (Lq) allows us to solve for the arrival rate of embodied innovations at the sector level, λ_s^q:

$$\lambda_s^q = \frac{\left[\lambda_0^q \triangle q/(1-s_{R\&D})\right](1-\alpha)\varkappa^l + (N-\rho_h)\left[\rho_h \lambda_0^h (\triangle h - 1)\left[(1-s_{R\&D})/(\lambda_0^q \triangle q)\right]^{\rho_h}\right]^{1/(1-\rho_h)} - r}{1-(\triangle q - 1)(N-1)}$$

The following conditions are useful to characterize the effects of the parameters of interest on λ_s^q and on the share of private R&D expenses in GDP ($\lambda_s^q(1-s_{R\&D})/\lambda_0^q * N$).

Condition 1: $(1-\alpha)\varkappa^l - \left(\frac{\rho_h \lambda_0^h (\triangle h - 1)(1-s_{R\&D})}{\lambda_0^q \triangle q}\right)^{1/(1-\rho_h)} \left(\frac{N-\rho_h}{1-\rho_h}\right) > 0$

Condition 2: $r > (\triangle h - 1)\left[\lambda^h (1 + \frac{(N-\lambda_0^h)}{N}\frac{\rho_h}{1-\rho_h})\right]$, where λ^h is defined in expression (24).

Proposition 2 *: Suppose that $1-(\triangle q-1)(N-1) > 0$. Then, (i) λ_s^q increases with $s_{R\&D}$, λ_0^q and $\triangle q$ if and only if condition 1 holds; (ii) the share of R&D in GDP increases with $\triangle q$ if condition 1 holds; (iii) the share of R&D in GDP increases with $s_{R\&D}$ and λ_0^q if conditions 1 and 2 hold.*

These results are also intuitive. For a given v^l, $s_{R\&D}$, λ_0^q and $\triangle q$ raise the expected capital gain per unit of output spent on developing R&D innovations. Therefore, the value of the leader must decline for the marginal benefit to equal the marginal cost of these innovations in equilibrium. In principle, one may think that v^l can adjust both by increasing λ_s^q and by reducing λ^h. In a one sector economy (*i.e.* $N=1$), however, the envelope theorem ensures that $\partial v^l/\partial \lambda^h = 0$ at the optimum.

[18] This result contrasts with standard models of R&D, where there is only one form of innovation. In these models, when the market size increases, more resources are allocated to the innovation activity in equilibrium.

Therefore, a higher arrival rate of R&D innovations λ_s^q is required to equalize marginal cost with the benefit of embodied innovations. In a multisector economy, some general innovations are developed in other sectors. As a result, $\partial v^l / \partial \lambda^h > 0$ at the optimum. Condition 1 is a regularity condition that ensures that the decline in λ^h is large enough to necessitate a decline in λ_s^q to equalize the marginal cost with the expected marginal benefit from R&D innovations. In this environment, a higher arrival rate of embodied innovations, λ_s^q brings about the decline in v^l in response to an increase in $s_{R\&D}$, λ_0^q or $\triangle q$.

To guarantee that the increase in λ_s^q associated with the increase in λ_0^q or $s_{R\&D}$ is in part driven by an increase in the share of private R&D in GDP, we have to ensure that, ceteris paribus, followers experience an increase in the expected capital gain when λ_0^q or $s_{R\&D}$ increase. This might not be the case because, in addition to increasing the probability of success per private unit of output invested in R&D, λ_0^q also raises the leader's effective discount rate, thus reducing v^l. This second force is dominated by the effect of λ_0^q on the probability of taking over the market leader if the interest rate, r, is sufficiently large. Similarly for $s_{R\&D}$, an increase in the subsidy also raises the effective discount rate by affecting the level of λ_s^q for given share of private R&D in GDP. This effect is dominated by the effect of $s_{R\&D}$ on the marginal cost of undertaking R&D innovations if the interest rate is sufficiently large for condition 2 to hold.[19]

Combining propositions 1 and 2, it follows that increases in λ_0^q or $s_{R\&D}$ lead to increases in λ_s^q and declines in λ^h. In other words, λ_0^q and $s_{R\&D}$ cause the rate of R&D-driven and general innovations to move in opposite directions. This negative co-movement between R&D-driven and general innovations is the key theoretical result to understand the post-war dynamics of growth and volatility at the aggregate and firm level.

2.4 Moments

After solving for the equilibrium of the economy, we turn our attention to the implications that this has for the first and second moments of the growth rates of output and productivity at the aggregate and firm level.

[19] These two forces are also present in standard Schumpeterian models (i.e. Aghion and Howitt [1992]) but there the productivity of R&D unambiguously dominates the effect on the discount rate. The difference in our framework stems from the effect of general innovations on the effective discount rate.

2.4.1 Aggregate moments

Growth is the result of both embodied and general innovations. For any given sector s, the growth rate of the sector's output (or productivity) (γ_{y_s}) is equal to the number of embodied innovations in the sector times the log of their effect on sectoral output plus the number of general innovations developed in the whole economy times their effect on sectoral output. Formally,

$$\gamma_{y_s} = \gamma_{y/l_s} = \#q_s * \ln(\delta_q) + \#h * \ln(\triangle h),$$

where $\#q_s$ is the number of new embodied innovations developed in the sector during the period, and $\#h$ is the number of new general innovations developed in the economy.

The growth rate of the economy (γ_y) is the average of the sectoral growth rates:

$$\gamma_y = \gamma_{y/l} = \frac{\sum_{n=1}^{N} \gamma_{y_n}}{N} = \frac{\sum_{n=1}^{N} \#q_n}{N} * \ln(\delta_q) + \#h * \ln(\triangle h).$$

Since new technologies arrive with a Poisson rate, it is straightforward to compute the instantaneous expectation and variance of the growth rate of productivity at the sector and aggregate level:

$$\begin{aligned} E\gamma_{y_s} &= \lambda_s^q \ln(\delta_q) + \lambda^h \ln(\triangle h) \\ E\gamma_y &= \lambda_s^q \ln(\delta_q) + \lambda^h \ln(\triangle h) \end{aligned}$$

These expressions show that aggregation does not have any effect on the expected growth rate of productivity since aggregate and sectoral expected growth rates coincide.

Propositions 1 and 2 have established a negative co-movement between R&D and general innovations in response to changes in R&D subsidies ($s_{R\&D}$) and the efficiency of R&D (λ_0^q). Therefore, the effect of increases in λ_0^q or $s_{R\&D}$ on expected growth is ambiguous. In particular, these parameter changes reduce expected productivity growth if and only if the relative productivity gain associated with an embodied innovation is relatively small: $\ln(\triangle q)/\ln(\triangle h) \leq -\frac{\partial \lambda^h}{\partial x}/\frac{\partial \lambda^q}{\partial x}$, for $x = s_{R\&D}$, λ_0^q.

Equation (26) provides the formula for the variance of the aggregate growth rate of the economy. Since R&D-driven and general innovations follow independent Poisson processes, the variance of the growth rate of aggregate output (and productivity) is linear in the hazard rates, where the coefficients of λ_s^q and λ^h are the squared contribution to productivity growth from each type of innovation.

$$V\gamma_{y_s} = \lambda_s^q(\ln(\delta_q))^2 + \lambda^h(\ln(\triangle h))^2 \tag{26}$$

$$V\gamma_y = \frac{\lambda_s^q}{N}(\ln(\delta_q))^2 + \lambda^h(\ln(\triangle h))^2 \tag{27}$$

Comparing expressions (26) and (27) shows that aggregation does affect the second moments of the growth rate of productivity. R&D-driven innovations are sector specific and are averaged away at the aggregate level. Hence, R&D-driven innovations have a larger effect on the volatility of the sectoral growth rate than on the volatility of the aggregate growth. General innovations, on the other hand, are adopted across the economy. Thus, their impacts are the same at the aggregate and sectoral level.

Another way to illustrate this point is by conducting a variance-covariance decomposition of the variance of aggregate growth. Recall that $\gamma_y \equiv \frac{\sum_{n=1}^{N}\gamma_{y_n}}{N}$. Hence,

$$\begin{aligned}
V(\gamma_y) &= E\left(\frac{\sum_{n=1}^{N}\gamma_{y_n}}{N} - E\gamma_y\right)^2 \\
&= E\left(\frac{\sum_{n=1}^{N}\gamma_{y_n} - E\gamma_{y_s}}{N}\right)\left(\frac{\sum_{n'=1}^{N}\gamma_{yn'} - E\gamma_{y_s}}{N}\right) \\
&= \frac{V(\gamma_{y_s})}{N} + E\frac{\sum_{n=1}^{N}\sum_{n'\neq n}\left(\gamma_{y_n} - E\gamma_{y_s}\right)\left(\gamma_{yn'} - E\gamma_{ys}\right)}{N^2} \\
&= \frac{V(\gamma_{y_s})}{N} + \frac{N(N-1)}{N^2}Cov(\gamma_{y_n}, \gamma_{y_{n'}}),
\end{aligned} \tag{28}$$

where $cov(\gamma_{y_n}, \gamma_{y_{n'}})$ denotes the covariance between the growth rates of two generic sectors n and n'.

In expression (28), as the number of sectors (N) increases, the importance of the sectoral variance in aggregate variance declines and aggregate volatility increasingly depends on the covariance of growth across sectors. Sectoral variance $V(\gamma_{y_s})$ depends on the arrival rate of embodied innovations developed in the sector (λ_s^q) and the arrival rate of general innovations developed in the economy (λ^h). The sectoral covariance, on the other hand, is equal to $(\ln(\triangle h))^2\ \lambda^h$ and depends solely on the hazard rate for general innovations. Therefore, as the number of sectors increases, the variance on aggregate growth increasingly depends of the intensity of general innovations while the arrival rate of R&D-driven innovations becomes less relevant.

This observation has important implications for the comparative statics associated with λ_0^q and $s_{R\&D}$. Propositions 1 and 2 showed that an increase in λ_0^q leads to an increase in λ_s^q and a decline in λ^h. Since sectoral volatility positively depends on both of these, λ_0^q has an ambiguous effect on the variance of sectoral growth.[20] However, since the sectoral covariance depends only on the frequency of arrival of general innovations, an increase in λ_0^q unambiguously leads to a decline in

[20] In particular, sectoral volatility declines with an increase in λ_0^q if and only if $(\ln(\triangle q)/\ln(\triangle h))^2 \leq -\frac{\partial\lambda^h}{\partial\lambda_0^q}/\frac{\partial\lambda^q}{\partial\lambda_0^q}$.

the covariance of sectoral growth. Furthermore, since the covariance component dominates sectoral variance in economies with a relatively large number of sectors, the variance of aggregate growth declines unambiguously when λ_0^q increases. As we shall see in section 3, these predictions for the evolution of aggregate variance, sectoral variance and sectoral covariance are consistent with post-war data in the US.

The covariance of sectoral growth can trivially be decomposed into the product of standard deviations and the correlation of sectoral growth:

$$cov(\gamma_{y_s}, \gamma_{y'_s}) = \sqrt{V(\gamma_{y_s})V(\gamma_{y_{s'}})} * corr(\gamma_{y_s}, \gamma_{y_{s'}})$$

When looking at actual data, the variance of growth in a sector typically depends on other factors such as the sector size and age. As a result, it is useful to examine instead the implications of the model for the correlation of growth across sectors. The correlation of growth between sectors s and s' depends on λ^h and λ_s^q as follows:

$$corr(\gamma_{y_s}, \gamma_{y_{s'}}) = \frac{(\triangle h)^2 \lambda^h}{(\triangle q)^2 \lambda_s^q + (\triangle h)^2 \lambda^h}. \tag{29}$$

Note that the sectoral correlation is increasing in λ^h and decreasing in λ_s^q. It follows from propositions 1 and 2 that increases in $s_{R\&D}$ and λ_0^q lead to declines in the correlation of sectoral growth.

2.4.2 Firm-level moments

Expected firm-level sales growth - denoted by $E\gamma_{salesi}-$ is affected by the rates of arrival of general innovations and R&D innovations in the economy through the effects that these have on aggregate demand growth ($E\gamma_y$). In addition, the firm's R&D intensity is positively associated with sales growth because it increases the probability of taking over the market leader through an increase in the sales of the followers combined with a reduction in sales of the leader. Hence, at the firm level, there is a positive relationship between R&D intensity and expected growth.

In the symmetric equilibrium, expected growth rate of sales for leaders and followers are given by the following expressions:

$$E\gamma_{salesi} = \begin{cases} E\gamma_y - \lambda_s^q \ln(\beta m/((1-\beta))) \text{ for } i = l \\ E\gamma_y + \lambda_s^q/m \ln(\beta m/((1-\beta))) \text{ for } i = f \end{cases}$$

These same considerations help us understand the determinants of the expected growth rate of sales per worker. Here, market turnover affects the firm' sales per worker because market leaders charge higher markups than producers of standard intermediate goods. The possibility of a change in the market position creates an expected gain (loss) in the sales per worker for standard (leading) intermediate good producers, as is clear in the next expression:

$$E\gamma_{salesi/L_i} = \begin{cases} \gamma_y - \lambda_s^q \ln(1/\alpha) \text{ for } i = l \\ \gamma_y + \lambda_s^q/m \ln(1/\alpha) \text{ for } i = f \end{cases} .$$

The firm-level volatility of the growth rates of sales and sales per worker depends on the variance of the aggregate growth rate of the economy and the risk of turnover in the market leader. Expressions (30) and (31) present the average variances of the growth rate of sales and sales per worker.[21]

$$var(\gamma_{sales_i}) = var(\gamma_y) + \lambda_s^q \left(\frac{1+\beta(m-1)}{m}\right)\left(\ln(\frac{\beta m}{(1-\beta)})\right)^2 \quad (30)$$

$$var(\gamma_{sales_i/L})) = var(\gamma_y) + \lambda_s^q \left(\frac{1+\beta(m-1)}{m}\right)(\ln(1/\alpha))^2 \quad (31)$$

The variance of aggregate output in the US data is approximately two orders of magnitude smaller than the variance of firm-level volatility. Hence, the quantitatively important term is the latter which is driven by the turnover rate, λ_s^q. An increase in $s_{R\&D}$ or λ_0^q leads to higher turnover (λ_s^q) both directly and through the higher investments in the development of R&D-driven innovations that it triggers. In this way, $s_{R\&D}$ and λ_0^q raise firm-level volatility.

Expressions (30) and (31) also imply an effect of average firm-volatility on R&D intensity. Specifically, λ_0^q, which determines firm volatility through λ_s^q, raises R&D expenses as shown in proposition 2. Therefore, an increase in the exogenous component of firm volatility triggers R&D expenses.

2.5 Imperfect Diffusion of General Innovations

We have shown above the importance of covariance of growth across sectors in the evolution of aggregate volatility. It is also illuminating to explore the model' predictions for the cross-section variation in the covariance of sectoral growth. Note that, in the current version, the model predicts no cross-sectional variation in the correlation of growth between sectors. This follows from the instantaneous adoption of general innovations in all sectors.

[21] Firm-level variances are weighted by the share of firm sales.

In this section, we enrich the model by relaxing the assumption that general innovations are applicable to all the sectors in the economy. Specifically, we introduce two new assumptions: (i) the intermediate good producers of a given sector can freely adopt all the general innovations developed in the sector and (ii) the random variable, that determines whether a general innovation is suitable to be adopted in a sector other than the one in which it was developed, follows a Bernoulli distribution that is independent across sectors and innovations.

Let ξ denote the probability that a general innovation is adopted in a sector other than the one in which it was developed. The previous assumptions imply that the arrival rate of general innovations in sector n is equal to $\lambda_{sn}^h + \xi(N-1)\overline{\lambda}_{s(-n)}^h$, where λ_{sn}^h denotes the rate of development of general innovations in sector n and $\overline{\lambda}_{s(-n)}^h$ denotes the average rate of development of general innovations in the sectors other than n. The covariance of growth in two sectors, n and n', depends on how frequently they adopt the same general innovations. Clearly, the probability of such a coincidence is higher for the technologies developed in either of the sector than for technologies developed in other sectors. Specifically, the probability that a technology developed in n (or n') is suitable for adoption in n' (n) is ξ. The probability that a technology develop in a sector other than n and n' is suitable for adoption in n and n' is $\xi^2 < \xi$. Thus, the covariance between the growth in sectors n and n' is:

$$cov(\gamma_{y_n}, \gamma_{y_{n'}}) = \left[\xi(\lambda_n^h + \lambda_{n'}^h) + \xi^2(N-2)\overline{\lambda}_{-(n,n')}^h\right](\ln(\triangle h))^2 ,$$

where $\overline{\lambda}_{-(n,n')}^h$ denotes the average rate of development of general innovations in the sectors other than n and n'. Averaging over all the sectors n', the average covariance of the growth of sector n with the growth rate in the other sectors is

$$cov_n = \left[\xi(\lambda_n^h + \overline{\lambda}_{(-n)}^h) + \xi^2(N-2)\overline{\lambda}_{(-n)}^h\right](\ln(\triangle h))^2 .\text{[22]} \tag{32}$$

To explore the cross-section variation in this covariance, suppose that the efficiency of investments in the development of embodied innovations (λ_0^q) varies across sectors. From proposition 1, we know that in sectors with higher values of λ_0^q, leading firms will have fewer incentives to develop general innovations. As a result, we should observe a negative correlation between λ_0^q and λ_n^h in the cross-section. Moreover, from expression (32), there should also be a negative cross-sectoral relationship between λ_0^q and the average covariance of a sector. Proposition 2, predicts a positive

[22] As in the previous subsection, it is useful to remember that the comparative statics for the correlation of sectoral growth are qualitatively identical to those for the covariance.

relationship between λ_0^q and the R&D intensity. Therefore, the model implies a negative cross-sectional relationship between R&D intensity in the sector and the average covariance of growth in the sector.

Using the same logic as in section 2.4, it follows that the variance of growth in the sector n is

$$var_n = \lambda_n^q (\ln(\delta_q))^2 + (\lambda_n^h + \xi\bar{\lambda}_{-n}^h) (\ln(\triangle h))^2 \tag{33}$$

The average correlation of growth between sector n and the rest of the sectors then is:

$$corr_n = \frac{cov_n}{\sqrt{var_n \overline{var_{-n}}}}, \tag{34}$$

where $\overline{var_{-n}}$ is the average variance across sectors other than n. Given the negative effect of λ_0^q on cov_n and the positive effect it has on var_n, the model implies a negative cross-sectional relationship between R&D intensity in the sector and the sectoral correlation of growth.

2.6 Predictions

In summary, following are the predictions resulting from the model:

- Expected growth:
  - (E1) At the firm level, there is a positive relationship between firms' R&D expenses and growth in sales per worker.
  - (E2) At the sector and aggregate level, the relationship between R&D expenses and productivity growth is ambiguous.
- Volatility:
  - (V1) An increase in λ_0^q leads to higher turnover (λ^q) and higher sectoral R&D expenses.
  - (V2) R&D has a positive effect on average firm-level volatility of the growth rate of sales and sales per worker.
  - (V3) At the sector level, the relationship between average R&D intensity and volatility is ambiguous.
  - (V4) At the aggregate level, an increase in λ_0^q leads to a decline in the volatility of the growth rate of output and labor productivity.

- Co-movement:
  - (C1) An increase in λ_0^q leads to a decline in the covariance and correlation of the growth rate of productivity across sectors.
  - (C2) Since λ_0^q has a positive effect on firm-level volatility (and on R&D), the model predicts a negative relationship between the average firm-level volatility in a sector (or the R&D to sales ratio in a sector) and the correlation between the sector's growth and the growth rate in the other sectors of the economy.

3 Evidence and Discussion

This section addresses three questions. Is there any evidence that λ_0^q and $s_{R\&D}$ have increased in the US during the post-war period? Is there any indication that the evolution of the level, volatility and co-movement of productivity growth at the aggregate and firm level are associated with the mechanisms described in the model? Finally, in a quantitative sense, are the mechanisms presented above able to generate the dynamics of volatility and growth observed in the US economy?

3.1 Driving Forces

As we shall see below, market turnover has increased significantly since the 1950's. This upward trend may have been triggered in the by the increase in R&D subsidies, $s_{R\&D}$, and by increases in the probability of developing an R&D innovation per unit of output invested, λ_0^q.

Over the past two decades, tax credits for R&D have become widespread and increasingly generous. Currently, over half (29) of all US states offer an R&D tax credit. There is a literature that has shown that these R&D tax credits have lead to a substantial increase in the share of private R&D in GDP (for example, Hall [1993], Mamuneas and Nadiri [1996], or Bloom, Griffith, and Van Reenen [2002]). This is the case even after controlling for the displacement of R&D from low to high subsidy states (Wilson [2005]).

Though harder to quantify, the growing trend of outsourcing services or the production of certain components has made it easier for followers to figure out ways to improve on the products and services provided by market leaders. This diffusion of knowledge beyond the boundaries of market leaders has increased the productivity of private R&D expenses, λ_0^q.[23] Similarly, firm employees often

[23] These changes in the flow of knowledge are unlikely to have a significant impact on the productivity of investments

learn to make a product or deliver a service to the point that they find it advantageous to create their own company and compete with the market leader. These business dynamics require that workers obtain a holistic understanding of the process of production. The gradual disappearance of Taylorism and improved analytic abilities of workers due to the spread of college education has meant that workers now acquire such a holistic understanding. As a result, these business dynamics have probably become a more relevant source of turnover.[24]

Figure 3 plots a measure of the inverse of the turnover rate for the sample of firms in the COMPUSTAT data base. Specifically, for each 2 digit sector and year, firms are ranked by the level of sales per worker. After creating a vector of percentiles for every year in the post-war period, persistence in rankings is measured by computing the correlation between the vectors of rankings in two years, five and ten years apart (i.e. 1950 with 1955, and 1950 with 1960, respectively). Repeating the same exercise for all the years in the post-war period results in a time series for the turnover in market leadership.[25]

Both of these statistics indicate that there has been an increase in market turnover. In the early 50's, the correlation of rankings was 0.9 for the 5-year-apart measure and 0.8 for the 10-year-apart measure. These correlations have declined in a fairly monotonic manner reaching 0.71 and 0.66, respectively, at the end of the sample in 2002. Comin and Philippon [2005] document similar trends for rankings based on market capitalization and profit rate.

This increase in turnover, however, is not only the result of exogenous factors but also the result of endogenous choices made by firms. An important path to market leadership is technological superiority.[26] Baumol [2002] emphasizes the role of research and development of superior goods as

in developing general innovations, λ_0^h. This is the case because those that now acquire more easily the knowledge are followers and followers develop a small share of all the general innovations developed in the economy.

[24]One example that illustrates this view is Mountain Hardwear, an outdoors gear company founded in 1993 by workers that left North Face and Sierra Designs. They justify their success as follows: "we decided to take a fresh approach to making great gear. Figuring that if we made innovative, technologically advanced tents, outdoor clothing and sleeping bags, consumers would buy them. We were right. [...] But it wasn't just about making great gear. From all those years of working in the outdoor industry, we knew what we liked about the business, and we also knew what we wanted to change."

[25]This measure of turnover is unlikely to be affected by entry into the COMPUSTAT sample. This is the case because when there are more firms in sample, it is more likely that a firm is taken over by some other firm, but the decline in the percentile associated with this decline in the ranking will be smaller if there were fewer firms in sample.

[26]R&D, however, is by no means the only investment that leads firms to leadership. In the case of Mountain Hardware, the founders had acquired product-specific knowledge from working for the previous market leaders, North Face and Sierra Designs, that gave them a competitive edge.

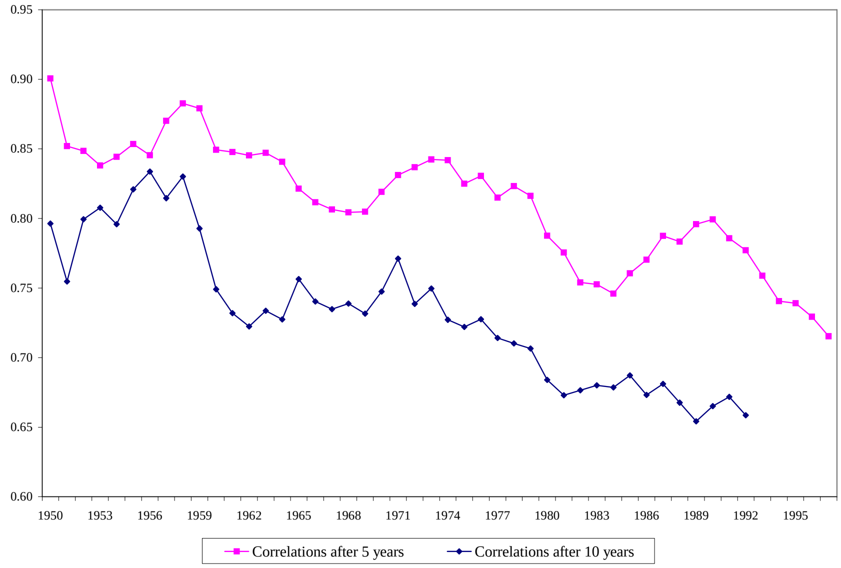

Figure 3: Correlations of firm percentiles by sales per worker.

a competitive mechanism far more important than competition in prices. Figure 1 has showed that one measure of these efforts in developing superior products (the NSF measures of non-federally financed R&D over GDP) has almost tripled since the early 50's. Similarly, firms in other G-7 countries were conducting much less R&D than the US in 1970 but had matched R&D intensity in the US by the turn of the century.

3.2 Evaluation of the Model's Predictions

The model's predictions are consistent with the facts described in the introduction. It predicts the lack of a clear relationship between R&D intensity and productivity growth at the aggregate level, the upward trend in firm-level volatility and the downward trend in aggregate volatility. Next, we provide evidence of other trends predicted by the model and that these are in fact driven by the mechanisms of the model.

Expected Growth

Firms that make greater investments in R&D are more likely to experience the improvements in productivity associated with becoming the new market leader. This is why the model predicts a positive relationship between R&D spending and expected growth in sales per worker at the firm level (prediction E1). Griliches [1980, 1986] and Griliches and Mairesse [1984] have examined panels of firm-level data covering the post-war period and observed a strong, significant relationship between R&D intensity and productivity or TFP growth at the firm level, even after including firm-level fixed effects.

As we move to the sector or aggregate levels, the model's predictions about the relationship between R&D and productivity growth become ambiguous (E2). This ambiguity follows from the negative relationship that exists between R&D and the development of general innovations. If, in addition, general innovations do not diffuse perfectly across sectors, sectors that develop fewer general innovations will also implement fewer innovations. As a result, the model predicts that in sectors with more R&D investments, the contribution to growth from general innovations will be lower and the resulting relationship between R&D and growth will be ambiguous.

Many studies have estimated the relationship between R&D intensity and TFP growth using sector level data.[27] These studies, typically find a significant positive relationship when examining the cross-section. However, once sector-level fixed effects are introduced as regressors, the coefficient of the R&D intensity becomes insignificant (Jones and Williams [1998]). This may be the case for two reasons. First, noise in the data may make it difficult to identify the relationship between R&D and TFP growth in the time series.[28] Second, it may be the case that after allowing for a sector-specific average growth rate, R&D has no effect on sectoral TFP growth. In any case, the fact that it is possible to identify the effect of R&D on TFP growth in the time dimension using firm-level data but not using sector-level data seems to indicate that the relationship becomes less clear as we move to more aggregate levels. As discussed above, one reason for the disappearance of this relationship may be the negative effect that R&D has on the development of general technologies in the sector.[29]

The above econometric exercises provide indirect evidence in favor of the joint hypothesis that general innovations are an important source of productivity growth and that R&D dampens the development of general innovations. We are not the first ones to highlight the importance of general

[27] See Jones and Williams [1998] for references.

[28] However, this attenuation bias should be stronger when using firm-level data which is probably noisier than sector-level data. Yet, Griliches and Mairesse [1984] and others have no problem identifying a strong and significant effect of R&D on productivity growth at the firm level.

[29] Abdih and Joutz [2005] reach similar conclusions using a different methodology. They estimate cointegration relationships between R&D labor, patent applications (i.e. R&D output), the stock of patents and TFP. Their results provide evidence of the ambiguous effect of R&D on TFP because they show that, while there is a very strong positive relationship between R&D labor and patent applications, there is no statistical relationship between patents and TFP.

innovations. Mokyr [2002] has emphasized the role of some general innovations for growth. In particular, he claims that "much of the productivity increase in the twentieth century was the result of the perfection of production techniques and process innovation. [...] These led to a continuous transformation in organizational methods, most obviously in mass production in manufacturing techniques but eventually in services and agriculture as well."

Unfortunately, direct measures on the intensity of investment in general innovations are not available. This makes it difficult to directly test the negative effect of R&D on the development of general innovations. A very imperfect substitute is to create a (very incomplete) list of general innovations and show that most of them were introduced either before WWII or between the 50's and early 60's when firm turnover was low. Table 1 provides our list of general innovations, most of which were developed before 1970. A brief description of each technology and why they qualify as a general innovation is relegated to Appendix 1.

Turnover, Volatility and R&D

The model predicts that the interaction between R&D and turnover/volatility generates an interesting lead-lag relationship (predictions V1 and V2). Proposition 2 implies that λ_0^q, a component of current firm volatility, leads to subsequent R&D. In addition, R&D should lead to more turnover and to higher average firm volatility in the sector. This second mechanism should operate with a lag since it takes some time to develop and market an innovation and to become the market leader.[30]

To investigate this relationship, we build a panel of annual R&D intensities, turnover rates and average firm volatility in 35 2-digit sectors that cover the US economy between 1950 and 1996. For each sector, we compute the ratio of R&D expenses to total sales, the median standard deviation of a 10 year rolling window of growth in sales per worker and the persistence in the rankings of sales per worker as in figure 3. Then, we estimate the regressions:

$$\begin{aligned}
\lambda_{it}^q &= \alpha_{0i} + \alpha_1 t + \beta(j) * (R\&D/Sales)_{it-j} + \epsilon_{it} \\
\sigma_{it} &= \alpha_{0i} + \alpha_1 t + \beta(j) * (R\&D/Sales)_{it-j} + \epsilon_{it}
\end{aligned}$$

In this specification, we introduce both a sector-level fixed effect and a time trend, to reduce the possibility of spurious correlations between R&D and volatility. Figures 4 and 5 report the estimates of $\beta(j)$ for various lags (j) of R&D together with the 95 percent confidence interval that result from the Newey-West standard errors.

[30] Shakerman and Pakes [1984] calibrate the time it takes to develop and market an innovation to be between 2 and 4 years.

The lead-lag relationship between R&D and volatility/turnover is evident from these figures. As suspected, current volatility has a significant impact on future R&D that peaks at approximately $t+3$. In addition, there is an evident effect of past R&D on current volatility/turnover that peaks at $t-5/t-4$. This effect is always positive, statistically significant and at least as large as the contemporaneous correlation between R&D and firm volatility. Also noteworthy is the symmetrical nature of the lead-lag relationships between R&D and volatility and between R&D and turnover.

These estimates are checked for robustness using two variations. First, the time trend is replaced by year dummies. In addition, other measures of volatility such as mean sales growth, median sales growth or mean sales per worker growth, are used. The use of these variations continues to result in the same bimodal cross-correlogram between R&D and firm-volatility.

Of course, there is always the possibility that the estimated relationship between R&D and volatility is driven by some third variable omitted from the regression. However, we consider such a scenario unlikely. For this to be the case, the omitted variable would have to be positively and significantly correlated with (i) current volatility, (ii) R&D at t-4, t-5, t and (iii) the leads of R&D from t+1 to t+4. It would also have to be uncorrelated with R&D at t-3, t-2 and t-1. These restrictions are, on the other hand, naturally satisfied by our model.

As we move to higher levels of aggregation, the trend in volatility, as well as its relationship with R&D, changes. At the sector level, the model predicts an ambiguous relationship between average R&D intensity and volatility (V3). In the model, aggregate volatility depends by-and-large on the arrival rate of general innovations. As turnover increases, the market leaders' private value of developing general innovations declines, as does aggregate volatility (prediction V4). This is how the model accounts for the observed downward trend in aggregate volatility.

Co-movement

Since general innovations are widely applicable in the economy, a decline in the intensity of their development should lead to a decline in the correlation of growth across sectors (prediction C1).

To explore the evolution of the correlation of growth across sectors, we proceed as follows. First, $corr([\gamma_{s,\tau}]_{t-4}^{t+5}, [\gamma_{j,\tau}]_{t-4}^{t+5})$ is defined as the correlation between the annual growth rate in sectors s and j during the 10-year period centered at t. Then, for every sector s, the average correlation with the rest of the sectors is computed as follows:

$$corr_{s,t}^{\text{sec}} = \sum_{j \neq s} \frac{\omega_j^{\text{sec}}}{\sum_{h \neq s} \omega_h^{\text{sec}}} corr([\gamma_{s,\tau}]_{t-4}^{t+5}, [\gamma_{j,\tau}]_{t-4}^{t+5}) \, , \tag{35}$$

where ω_j^{sec} denotes the average share of sector $j's$ sale in the total sales of the economy. Finally, aggregate correlation is defined as a weighted average of the sectoral correlations:

$$corr_t^a = \sum_s \omega_s^{\text{sec}} corr_{s,t}^{\text{sec}}.$$

Figures 7 and 8 show a clear downward trend in the average correlation ($corr_t^a$) of productivity and TFP growth across sectors during the post-war period.[31] Comin and Philippon [2005] show that the decline in the correlation of sectoral growth is mostly driven by the decline in the covariance of growth across sectors (as opposed to a decline in the variance of sectoral growth).

To gain insight into the importance of R&D in the decline of the co-movement of growth across sectors, we exploit the cross-sectional implications of the model. Recall that once we recognize that general innovations are more likely to diffuse in the sector where they are developed, it follows that sectors with higher turnover should develop and adopt less general innovations, and should have a lower correlation, on average, with the other sectors (prediction C2). To test this prediction, we estimate the following specification:

$$corr_{s,t}^{\text{sec}} = \alpha_s + \beta t + \gamma RD_{s,t} + \epsilon_{st}, \tag{36}$$

where $corr_{s,t}^{\text{sec}}$ is defined in expression (35) and $RD_{s,t}$ denotes the R&D intensity in sector s at time t. The first and seventh columns in Table 2 report the estimate of γ when $corr_{s,t}^{\text{sec}}$ is measured by the correlations of productivity and TFP growth, respectively. In both cases, R&D is associated with a significant decline in correlation. Specifically, the estimates of γ are -3.3 for productivity and -2.5 for TFP growth, with $p - values$ of 2 percent. This implies that the increase in R&D is associated with a decline of between 7.5 and 10 percentage points of the 10 and 25 percentage point decline observed in the sectoral correlation of TFP or productivity growth. These estimates are robust to replacing the time trend with year dummies.

Columns 2 and 8 of Table 2 replace the R&D intensity as explanatory variable with the firm-level volatility in the sector. Consistent with the model, higher firm-level volatility in a sector is also associated with lower correlation of sectoral growth with other sectors.[32]

[31] See Comin and Philippon [2005] for more on this.

[32] These results are robust to restricting the sample to the private sectors, using other variables to measure firm volatility, using the median instead of the average to measure the firm volatility in the sector, using a measure of turnover in the sector as independent variable instead of a measure of firm volatility and including a time trend or no trend at all instead of the year fixed effects.

In principle, the estimated effect of R&D on sectoral correlation can be driven by omitted variable bias. For example, it could be argued that R&D intensity may be related to the sensitivity of sectors to aggregate shocks. However, to the extent that this sensitivity has not changed significantly over time, this effect should be captured by the sector fixed effect. One kind of aggregate shock that have been related to the decline in aggregate volatility are oil price shocks. To test whether the omission of the sensitivity of the sector to oil prices is biasing our estimates of γ towards significance we run regression (36) controlling for the share of energy in the sector. Columns 3,4, 9 and 10 show that including the share of energy in the control set does not at all affect the estimates of the effect of R&D or firm volatility on sectoral correlation. Further, in columns 5, 6, 11 and 12, we show that these results hold when we restrict our sample to the sectors other than energy.

Another explanation for the decline in aggregate volatility is proposed by Thesmar and Thoenig [2004]. Building on Arrow [1971], they claim that financial innovation can lead to greater risk taking by firms on the one hand but fewer aggregate credit crunches on the other. Their analysis implies that sectors that benefit more from financial innovation are going to experience larger declines in their correlation with the rest of the economy because of the lower exposure to credit crunches and binding collateral constraints (Bernanke, Gertler and Gilchrist [1996]). Lower exposure to financial stress will lead to lower aggregate volatility. Comin and Philippon [2005] empirically explore this hypothesis by including in regression (36) two additional controls that proxy for the degree of financial dependence in the sector: the amount of debt and equity issued in the sector, each divided by the total sales in the sector. In contrast to R&D, both measures of financial market dependence are positively associated with the correlation of sectoral growth (albeit this relationship is statistically insignificant). Therefore, improvements in financial markets do not seem to be a major force in the decline in aggregate volatility. More importantly for our purposes, the negative effect of R&D on the correlation of sectoral growth is not driven by the omission of measures of external financial dependence.

In summary, the existing theories proposed to explain the decline in aggregate volatility do not seem to be driving the negative relationship between R&D and the correlation of sectoral growth. This reinforces the view that, as suggested by our model, this relationship is causal.[33]

[33] Philippon [2003] argues that an increase in competition in the goods market leads firms to adjust their prices faster, which reduces the impact of aggregate demand shocks. While intuitively appealing, Philippon [2003]'s is a within-sector explanation with no implication for the evolution of sectoral co-movement.

3.3 Calibration

The above showed econometric evidence in favor of the model's mechanisms. In what follows, we undertake two calibration exercises to assess the model's quantitative ability to generate the observed evolutions of aggregate growth, and aggregate and firm volatility.

Firm-level volatility

Recall that the variance of the growth rates of sales and sales per worker at the firm level are given by the expressions:

$$var(\gamma_{sales_i}) = var(\gamma_y) + \lambda_s^q \left(\frac{1+\beta(m-1)}{m}\right) \left(\ln(\frac{\beta m}{(1-\beta)})\right)^2 \quad (37)$$

$$var(\gamma_{sales_i/L})) = var(\gamma_y) + \lambda_s^q \left(\frac{1+\beta(m-1)}{m}\right) (\ln(1/\alpha))^2 \quad (38)$$

The first term in both expressions represents the variance of growth in aggregate output. In the US, this term is approximately two orders of magnitude smaller than the variance of firm-level growth and hence irrelevant to the evolution of firm-level volatility. The quantitatively relevant effect of an increase in λ_0^q or $s_{R\&D}$ comes from the second term. The turnover rate, λ_s^q, increases both due to the exogenous increase in λ_0^q and the endogenous increase in R&D intensity. In the post-war period, R&D has increased by a factor of 3 in the US (figure 1). If to this we add the exogenous increase in λ_0^q, the linearity of the production function for new R&D-driven innovations implies that λ_s^q has also increased by at least a factor of three in the post-war period.

Independent estimates of λ_s^q can be computed from the evolution of the persistence of rankings in sales per worker in figure 3.[34] These calculations indicate that in the mid 50's, λ_s^q was approximately 2 percent while in the mid 90's, it was 2.5 to 3 times higher. Comin and Philippon [2005] conduct similar exercises using other measures of market leadership such as profit rates and market value. Specifically, they compute the probability that a firm currently ranked in the top 20^{th} percentile of its sector by profits rate or market value, is not in the top 20th percentile in 5 years. These exercises imply that the turnover rate has increased by a factor of 5-6 during the post-war period.

With these estimates, it is simple to understand the power of the model to induce a very significant increase in firm volatility. In expressions (37) and (38), the first term is quantitatively irrelevant. The second term depends on fixed parameters and λ_s^q. Our estimates indicate that λ_s^q has increased by at least a factor of 2.5 to 3. In the data, firm variance has increased by a factor

[34] See Appendix 3 for the formal derivation.

of approximately 4 in the post-war period. Therefore, the model can account for, at least, 62 to 75 percent of the increase in the variance of firm-level growth.

Aggregate Volatility and Productivity Growth

One way to assess the model's ability to generate the observed evolution of aggregate growth and volatility would be to calibrate all the parameters of the technology to develop general innovations and use them in the model along with the evolution of R&D-style innovations to pin down the evolution of λ^h. However, the lack of independent information to calibrate f_0^h, ρ_h and λ_0^h makes this route unfeasible.

Alternatively, we assume that the post-war decline in the correlation of productivity growth across sectors is driven by the decline in the development of general innovations. We then use this information to pin down the evolution of λ^h and explore the model's implications for the evolution of productivity growth and aggregate volatility in 1950 and 2000.

Specifically, we use the following 6-step procedure:[35]

(i) Calibrate the initial turnover rate (λ^q_{s1950}) to match the initial correlation of rankings in figure 3. As shown in Appendix 3, this yields an estimate for λ^q_{s1950} of 2 percent.

(ii) Using the value of λ^q_{s1950} and the initial correlation and variance of sectoral growth (0.5 and 0.0005 respectively), pin down the values for $\lambda^h_{1950} * (\ln(\triangle h))^2$, $\lambda^q_{s1950} * (\ln(\triangle q))^2$ and $\ln(\triangle q)$.

(iii) Using the average initial growth rate of productivity (0.025), calibrate $\ln(\triangle h)$ and λ^h_{1950}.

(iv) Calibrate the final turnover rate, λ^q_{s2000}, to 2.5 times the initial turnover rate (i.e. 5 percent).

(v) Using the final correlation of sectoral growth (0.25) and the calibrated value of $\ln(\triangle h)$, compute the final rate of arrival for general innovations (λ^h_{2000}).

(vi) With this information and the number of sectors (35), compute the final expected growth rate of productivity ($E\gamma_{y2000}$) and the initial and final variance of aggregate productivity growth ($V\gamma_{y1950}, V\gamma_{y2000}$).

Table 3 shows the actual as well as the model's predictions for the final expected growth rate of labor productivity and the initial and final standard deviations of aggregate productivity growth.

[35] A more detailed explanation of this calibration is presented in Appendix 2.

Moment	Data	Model
$E\gamma_{y2000}$	0.02	0.017
$V\gamma_{y1950}$	$4*10^{-4}$	$2.56*10^{-4}$
$V\gamma_{y2000}$	$1.44*10^{-4}$	$1.44*10^{-4}$
Increment in $V\gamma_y$	$-2.56*10^{-4}$	$-1.12*10^{-4}$

Table 3

This simple calibration illustrates two things. First, the model can easily explain the lack of a relation between R&D and productivity growth at the aggregate level. Despite the substantial increase in R&D expenses, the model predicts a small decline in expected productivity growth for the year 2000.[36] Second, the mechanisms emphasized by the model can account for a significant fraction of the decline in aggregate volatility. The model underpredicts the initial level of aggregate volatility. This is not surprising given that the only type of aggregate disturbances are technology shocks, a scenario that is clearly unrealistic. However, the predicted decline in the variance of aggregate productivity growth represents over 40 percent of the observed decline in aggregate volatility. This estimate must be taken with caution because of the identification assumption that the decline in the co-movement of sectoral growth is entirely driven by the decline in the development of general innovations. However, this assumption may not be far from reality given the important negative effects of R&D on sectoral correlation that we have estimated above. Moreover, this rough estimate of the contribution of our endogenous technological change mechanisms to the decline in aggregate volatility are consistent with Stock and Watson [2003]'s conclusion: after considering the effects of a more active monetary policy and lower commodity price shocks, 50 percent of the decline in aggregate volatility must be due to less volatile technology shocks.

4 Conclusion

A thorough understanding of the forces that drive growth in the US is an essential prerequisite for undertaking informed policy recommendations. This paper has presented a new growth theory for the US that is superior to current models because it overcomes two hurdles that we believe any valid theory should pass. First, it explains the relationship between R&D and productivity growth

[36]This calibration implies that about 90 percent of aggregate productivity growth was driven by general innovations in 1950. this fraction declined to 67 percent by 2000.

at the firm-level as well as the lack of a relationship between the two at the sector and aggregate level. Second, it explains the evolution of the second moments of productivity growth at the firm and aggregate level. In particular, it explains the diverging trends in firm and aggregate volatility and the fact that the decline in aggregate volatility is in a large fraction due to a decline in the correlation of sectoral growth.

In addition to being consistent with these facts, this paper has also provided evidence on the importance of the mechanisms emphasized by the model. In particular, it has showed that firm volatility and market turnover are associated with past R&D and that current market turnover is associated with subsequent R&D. Perhaps most importantly, it has showed that sectors that have experienced higher increases in R&D have also experienced greater declines in the correlation of their growth with the rest of the economy. This indicates that there is a strong connection between aggregate and firm volatility. Furthermore, it supports the view that this connection operates mainly through the effect of R&D on the decline in the co-movement of growth across sectors. By no means does this imply that all of the decline in aggregate volatility (or increase in firm volatility) is driven by this common component associated with R&D. But it does show that this component is an important piece of the puzzle.

Finally, our model suggests that sectoral co-movement is driven by the development of general innovations, and the decline of their importance in growth is at the root of the observed dynamics for the first and (to some extent) second moments of aggregate productivity growth. Since general innovations are, by-and-large, not included in the NSF measure of R&D and there is no measure of the investments made to develop them or the number of general innovations developed, we are unable to directly explore the determinants of general innovations. In this paper, we have evaluated our theory of general innovations by exploring the validity of its implications for the second moments of growth. Let's hope that the current lack of data around general innovations does not keep us from searching for the keys to growth in the right place, like in the old economist' joke.

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Figure 4
Effect of R&D on Volatility at Various Lags and 95 Percent Confidence Intervals

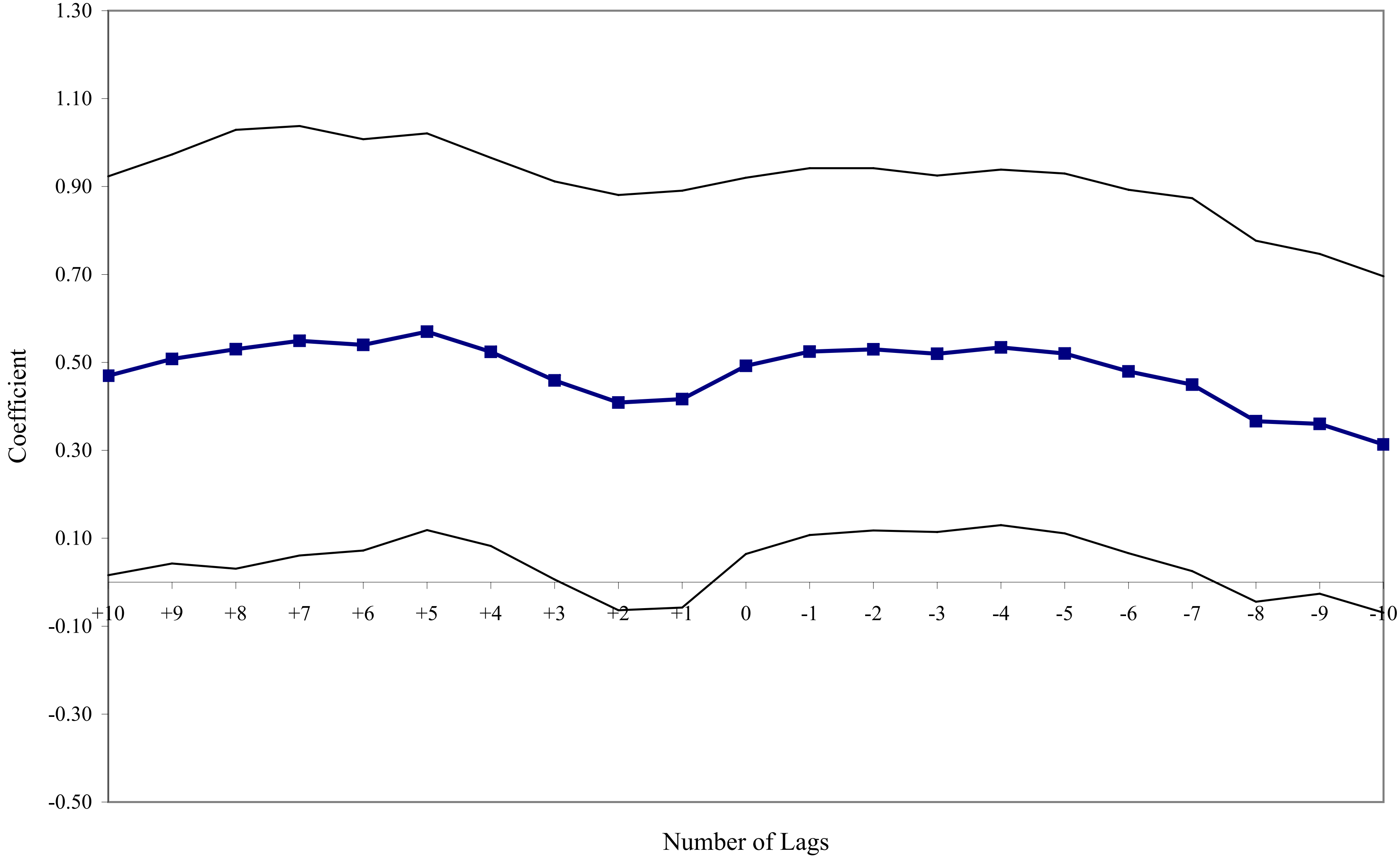

Figure 5
Effect of R&D on Turnover at Various Lags with 95 Percent Confidence Interval

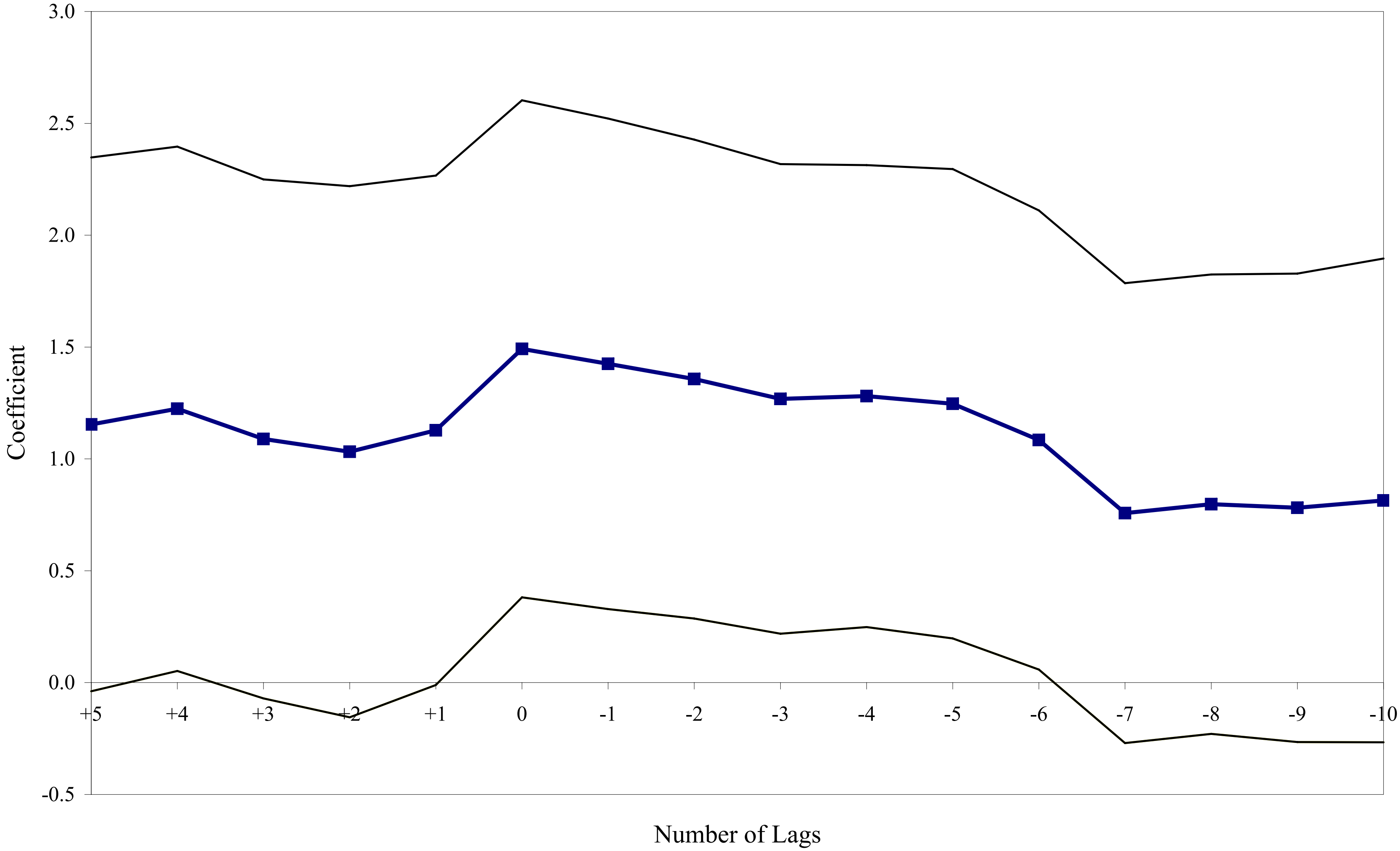

Figure 6
Evolution of Sectoral Correlation of Productivity Growth

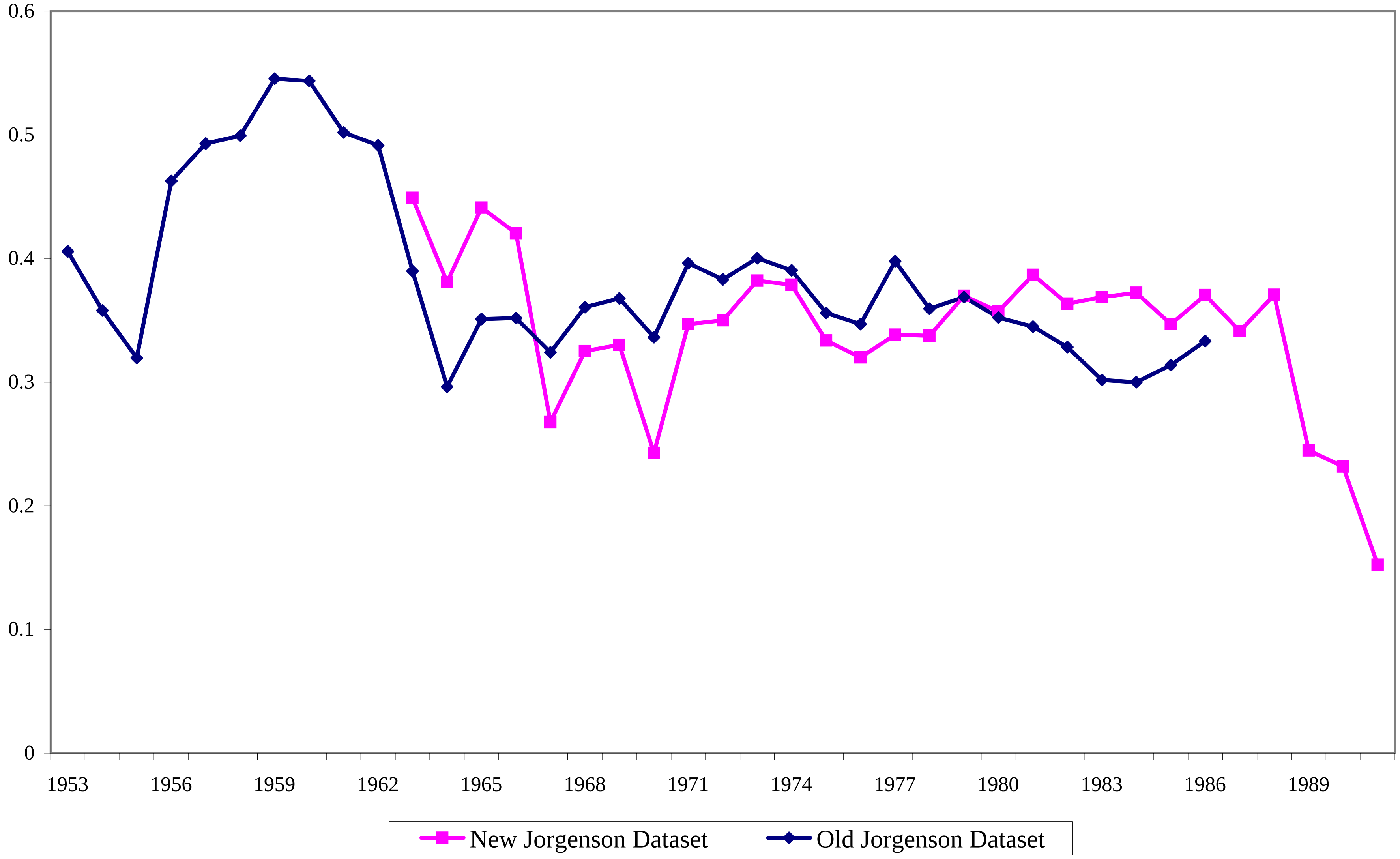

Figure 7
Correlation of Sectoral TFP Growth

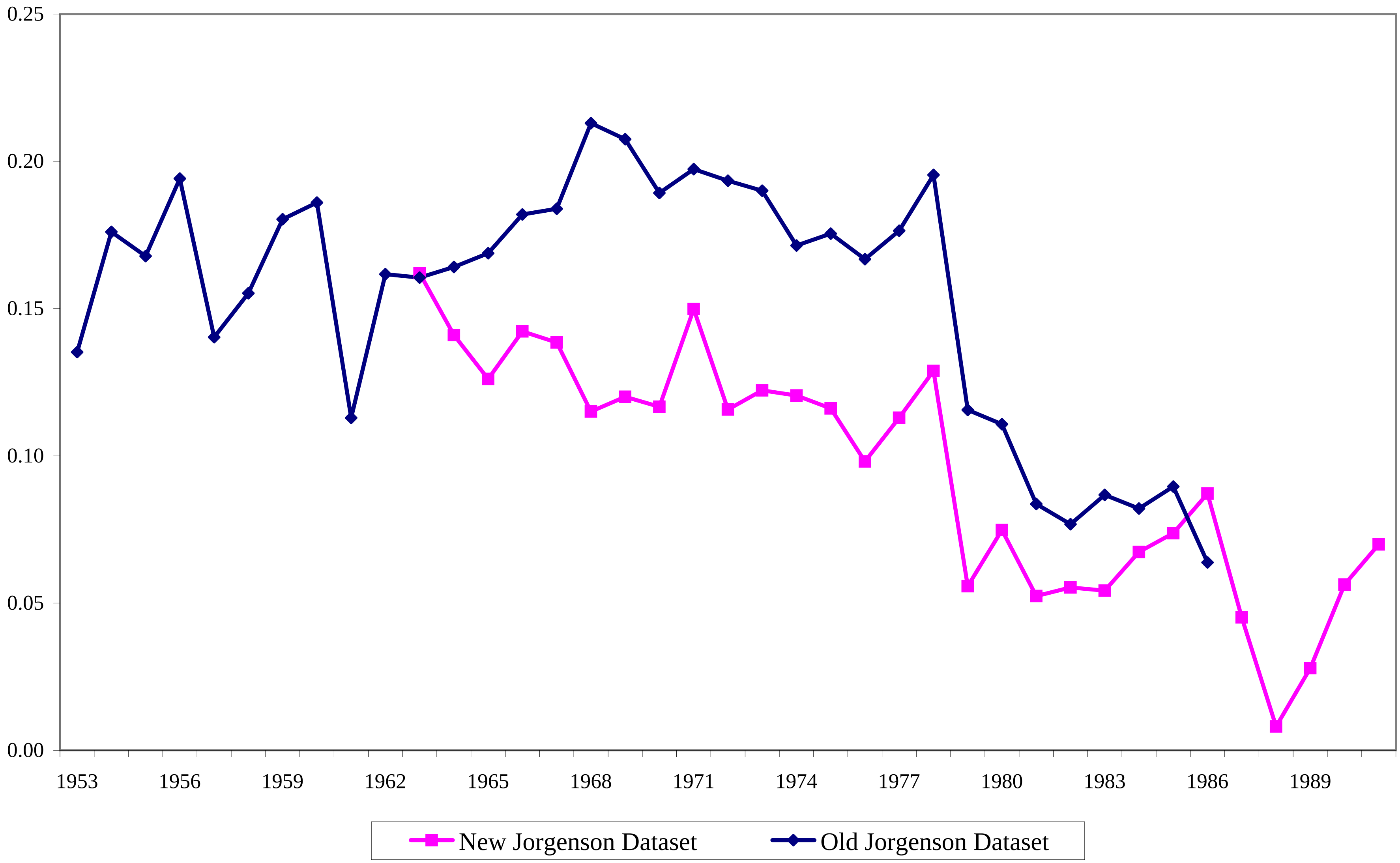

Table 1
Examples of General Innovations

Innovation	Date	Importance
Management and Production Design		
Mass Production	1900	Fixed costs spread out over larger volumes meant lower costs.
Ford Assembly Line	1913	Shorter assembly time resulted in lower production costs.
Scientific Management	1911	Used a scientific approach to production processes to improve productivity.
McKinsey Management Consulting	1923	Introduced a streamlined approach to consulting services.
Multi-Divisional Structure	1920's	Introduced the idea of autonomous divisions responsible for pursuing goals, independent of each other.
Just-in-Time Manufacturing	1950's	Improved synergies between adjacent production processes to minimize inventories.
Human Resource Management		
Hawthorne Studies	1924-1933	Highlighted the importance of the relationship between the employee morale and productivity.
Industrial Psychology	1940's-50's	Emphasized contextual variables for purposes of training and positive organization change.
Survey Feedback	1940's	Highlighted the importance of sharing feedback with employees.
Sensitivity Training	1946	Focused on the importance of open discussion in small groups.
Credit/Banking		
Credit card	1950	Helped businesses and consumers undertake credit transactions in a more extensive and systematic manner.
Electronic Recording Method of Accounting	1950's	Helped computerize the banking industry.
Magnetic Ink Character Recognition	1950's	Allowed computerized tracking and accounting of check transactions.
Electronic Money	1972	Introduced an electronic alternative to check processing.
Computer / Software / Internet		
Hypertext	1945	Basis of the eventual World Wide Web.
Arpanet	1969	Enabled the exchange of information over large geographic distances.
Fortran	1957	High-level programming language that made for improved scientific, engineering and mathematical applications.
Computers	1936	Enabled the automation of an assortment of functions.
Internet Search Engines	1990	Greatly reduced cost of gathering of information.
Trade		
Mall	1922	Started the modern-day one-stop shop for all consumers.
Department Store	1877	Improved the efficiency of retail and distribution.
Internet Shopping	1990's	Provided firms with a new avenue to sell and buy goods and services.
Marketing		
Coupons	1895	Effective promotion/marketing tool.
Mail order catalog	1872	Enabled businesses to target consumers that did not access to retail outlets.
Chemical Engineering		
Chemical Engineering	1920's	Improved the design and control of similar operations at plants in several different industries.

Table 2: R&D, Firm-level Volatility and Sectoral Co-movement

	Dependent variable											
	correlation in sectoral productivity growth						correlation in sectoral TFP growth					
R&D	-3.28		-3.11		-3.39		-2.49		-3.11		-3.1	
	(1.42)		(1.44)		(1.45)		(1.09)		(1.06)		(1.07)	
Firm level volatility		-0.297		-0.287		-0.27		-0.237		-0.239		-0.25
		(0.122)		(0.121)		(0.121)		(0.083)		(0.084)		(0.085)
Energy share			-0.49	-0.35	-0.077	0.076			-0.057	0.065	-0.18	-0.038
			(0.37)	(0.36)	(0.36)	(0.36)			(0.19)	(0.18)	(0.24)	(0.22)
N	1011	1011	1011	1011	982	982	1011	1011	1011	1011	982	982
Sectors	All	All	All	All	Non-energy		All	All	All	All	Non-energy	

Notes:
Newey-West standard errors are reflected in parentheses.
Firm volatility is measured by the sectoral average of the firm-level variance of the growth rate of sales.
All regressions include sector and year fixed effects.

Appendix I: Discussion of General Technologies

We present here several examples of inventions that meet the two criteria that characterize our notion of general technologies. First, while these innovations originated in a particular context, the general nature of the idea underlying them meant they were applied to many economic activities across industries and sectors. Second, the disembodied nature of these innovations meant that they could not be patented. As a result, firms could not appropriate the benefit from these innovations when competitors, whether within or across industries, adopted them.

I. Production Design

A. Mass production of cars and Ford's assembly line

Mass production first originated in the automobile industry in the United States in 1901. American car manufacturer Ransome Eli Olds (1864-1950) invented the basic concept of the assembly line and mass produced the first automobile, the Curved Dash Oldsmobile. Henry Ford (1863-1947) invented an improved version of the assembly line by installing the first conveyor belt-based assembly line in his car factory in Ford's Highland Park, Michigan plant, around 1913-14. The assembly line reduced production costs for cars by reducing assembly time.

The philosophy of mass production was simple. Fixed overhead costs were spread out over larger and larger volumes of production, thus lower and lower prices became possible. This strategy that characterized mass production was to become the defining characteristic of American industry throughout the twentieth century.

B. Scientific Management

Scientific management is the study of relationships between workers and machines. Frederick Taylor, regarded as the Father of Scientific Management, published Principles of Scientific Management in 1911, in which he proposed work methods designed to increase worker productivity. Taylor realized that organization productivity could be increased by enhancing the efficiency of production processes. This involved breaking down each task to its smallest unit and to figure out the one best way to do each job. Emphasis was laid on ensuring the worker indulged in only those motions essential to the task. Taylor looked at interaction of human characteristics, social environment, task, and physical environment, capacity, speed, durability, and cost. The overall goal was to remove human variability.

The results were profound. Productivity under Taylorism went up dramatically. In a famous experiment on the output of a worker loading pig iron to a rail car, Taylor increased the worker's output from 12 to 47 tons per day. New departments arose such as industrial engineering, personnel, and quality control. There was also growth in middle management as there evolved a separation of planning from operations. Rational rules replaced trial and error; management became formalized and efficiency increased. This model, based on merit and unquestioned authority, was a dramatic improvement over earlier models of organization.

C. Management Consulting

McKinsey and Co. was one of the first management consulting firms established in 1923 in Chicago. While the consulting industry had originated before then, the introduction of McKinsey's innovative approach to analyzing and solving problems constituted an important general technology. The McKinsey way of consulting can be decomposed in the following three steps. First the consultant gathers as much factual information about the client's organization as possible. Second, after a thorough analysis of the facts, an initial hypothesis is determined, to be tested with the client. Finally, a set of recommendations are presented to the client. These recommendations are limited to what can be realistically done given the resources of the client, the consulting firm and the amount of time required. Further, the recommendations are proposed along with milestones to be achieved as intermediate steps towards the ultimate target.

D. Multi-Divisional Structure

Faced by stiff competition from Ford Motors, General Motors helped pioneer the Multi-divisional organizational structure in the 1920's. The organization was divided into several divisions, each responsible for the production of the car and its marketing to the assigned market segment. Each was to have its own managerial team with complete autonomy over its operational decisions. The central office's role would be restricted to evaluate each divisions performance and coordinate overall strategy. The system helped General Motors transition from a chaotic organization into a streamlined and efficient competitor in the automobile industry. As a result of the organizational change, GM's market share grew to 45 percent in 1940 from 11 percent in 1921. The multi-divisional structure has since become a standard organizational feature of the corporate world, enabling many companies to efficiently produce a wide array of products.

E. Just-in-Time Manufacturing

Toyota introduced the 'Just-in-Time' system of manufacturing in the 1950's. Traditional manufacturing setups meant inventory systems were effective only under large economies of scale. Toyota's tiny scale of operations and lack of capital meant that it could not compete with the lower prices of its competitors under such models. Elimination of the inventories meant that Toyota had to tighten coordination between successive stages of production. The lack of inventories to buffer disruptions between adjacent stages of production meant improvements in the reliability of every step of the process. The new system meant fewer interruptions in the production process, faster identification of flaws in the cars and better communication with suppliers. The success of its manufacturing system has helped it and other corporations achieve world success in their respective industries.

II. Human Resources Management

A. The Hawthorne Studies

Beginning in 1924 and continuing until 1933, the Western Electric Company sponsored a series of experiments for studying worker productivity and morale at its Hawthorne Works near Chicago. The intent of these studies was to determine the effect of working conditions on productivity.

The studies collectively highlighted the importance of positive worker attitude and provided information about factors other than physical working conditions that contribute to productivity. In particular, researchers found that a group norm regarding the rate of productivity significantly affects individual performance, and that informal authority from influential group members often overrode formal authority from the supervisor. A major outcome of the interviews was to teach supervisors how to handle employee complaints. Smaller work groups and greater freedom were found to be the greatest drivers of the observed increase in productivity. These findings on the relationship between improvements in productivity and better employee morale were applied to a wide ranging group of employment settings.

B. Industrial Psychology

Industrial psychology involved the testing of morale and efficiency at businesses, industrial and military organizations. Edwin A. Fleishman (1953) undertook what was a typical project of its time at the International Harvester Company. Fleishman studied the relationship of training programs on the leadership of supervisors and their sensitivity to and consideration of the needs and feelings of subordinates. While supervisors showed an initial response to the training program by being more considerate towards their subordinates, in due course, they reverted back to their original behavior. The reversal of the behavior was attributed to the culture or climate of the department the subjects came from. In what came to be known as a critical point in organizational change, the study highlighted the difference between focusing on the individual and focusing on contextual variables (such as group norms and organizational culture).

C. Survey Feedback

The organizational survey feedback method first showed up in the late 1940's. Questionnaires were being used to systematically assess employee morale and attitudes in organizations. Floyd

Mann's study in 1957, guided by Rensis Likert, went a long way in developing what we now know as the Survey Feedback method. The method involved data collection by questionnaire to determine employee's perceptions of the management of the organization. The second aspect of the method was reporting the results back to the employees who answered the questionnaire. Once the results of the survey had been conveyed, managers, using the help of the subordinates, would chart out a plan to undertake positive changes in areas of concern as reflected in the survey results. The study emphasized that the effectiveness of the method relied on what the manager did with the information from the survey. Positive changes occurred when the manager discussed the results with his subordinates

D. Sensitivity Training

Sensitivity training refers to small group discussions where the primary, almost exclusive source of learning is the behavior of the group members themselves. Participants receive feedback from one another regarding their behavior in the group. Sensitivity training, also known as T-groups, became the earliest tool of what came to be known as organizational development. Kurt Lewin discovered the concept when undertaking a training workshop in Connecticut in 1946. He was asked to conduct a workshop that would help improve community leadership in general and interracial relationships in particular. Lewin brought in trainers and researchers and along with the participants engaged in lectures, role play and general group discussions. In the evenings, the trainers and researchers would evaluate the events of the day. The workshop acquired its significance however when participants happened to observe and participate in the evaluations as well. Participants began to object to the interpretation of their behavior on several occasions. The observation by the participants resulted in the three-way discussion among the researchers, trainers and participants. The participants in turn became more sensitive to their own behavior in terms of how they were being perceived by others and the impact their behavior was having on others. Carl Rogers labeled this mode of learning as "perhaps the most significant social invention of the century".

III. Credit/banking

Improvements in the credit and banking sector have, both directly and indirectly, resulted in improvements in businesses across all sectors of the economy.

A. Credit card

The credit card industry began in the United States in the 1930s when oil companies and hotel chains began issuing credit cards to customers for purchases made at their own gas stations and hotels. The bank credit card was introduced in the 1950s. While store or book credit allowed irregular repayment and installment loans required regular repayment, the credit cards of the early 1950s combined both types of credit. In 1951, Franklin National Bank released the first revolving charge card. Using the revolving card a customer could borrow money, repay it and borrow again as long as the borrower remained under their credit limit. The organizations that are now called Visa and MasterCard sprang up to create interchange, a nation-wide system designed to settle credit card transactions between banks, merchants and customers.

Today, with help from Visa and MasterCard, financial institutions are marketing credit cards to people all over the world. Credit cards have allowed consumers to carry debt, something that previously required a bank loan – a much more intensive process than a credit-card approval. Credit cards have been the primary instrument that fueled international consumerism and high consumer debt, each of which has spurred multiple trickle-down industries.

B. Credit Reporting

In Manhattan during the 1830s, Lewis Tappan developed extensive credit records while handling credit in his brother's wholesale silk business. He then extended this aspect of the business to other suppliers who needed information. He contracted with agents and correspondents throughout the country to "gossip" about the solvency, prospects, and character of local businesses. He established R. G. Dun & Co., an information hub that could rapidly service new inquiries and add new information and in the process helped found the business of credit reporting in the United States. The credit reporting system and improvements in the same have helped firms minimize risk. With access to the credit history of their customers, firms could target only consumers meeting their criteria of their acceptable levels of risk. It has helped institutions reduce bad debts and streamline their bottom lines.

C. ERMA and MRCI

During the 1950s, ERMA, the Electronic Recording Method of Accounting computer processing system, began as a project to computerize the banking industry. ERMA computerized the manual processing of checks and account management and automatically updated and posted checking accounts. MICR, the magnetic ink character recognition, was also part of ERMA. MICR allowed computers to read special numbers at the bottom of checks that allowed computerized tracking and accounting of check transactions. These inventions led to a more efficient banking system.

D. Electronic money

The widespread use of electronic currency began with the automated clearinghouse (ACH), set up by the US Federal Reserve in 1972 to provide the US Treasury and commercial banks with an electronic alternative to check processing. Payments made today in nearly all of the deposit currencies in the world's banking systems are handled electronically through a series of inter-bank computer networks.

Although banks have been able to move currency electronically for decades, only recently has the average consumer had the capability to use electronic transfers in any meaningful way. The increasing power and decreasing cost of computers — coupled with advancements in communication technology that make global interaction available at vastly reduced costs — have together made the digital transfer of funds a reality for millions of individuals around the world.

IV. Computer / Software / Internet

While innovations in this category clearly exhibit the characteristics of general technologies, they are included in the NSF's definition of Basic Research. In this sense, they are exceptions to the rule: general technologies are not R&D.

A. Arpanet

Arpanet was created during the Cold War to meet the need for large powerful computers in the country that were networked with each other to overcome geographic differences. Four computers were the first connected in the original ARPAnet. As the network expanded, different models of computers were connected, creating compatibility problems. The solution rested in a better set of protocols called Transmission Control Protocol/Internet Protocol (TCP/IP) designed in 1982. To send a message on the network, a computer broke down its data into IP (Internet Protocol) packets, like individually addressed digital envelopes. TCP (Transmission Control Protocol) ensured the packets were delivered from client to server and reassembled in the right order. Several other innovations occurred under ARPAnet - email (or electronic mail), the ability to send simple messages to another person across the network (1971); telnet, a remote connection service for controlling a computer (1972); and file transfer protocol (FTP), which allowed information to be sent from one computer to another in bulk (1973). Each of these inventions has made it significantly easier for businesses to communicate and share information both across and within each other.

B. Fortran

At IBM in 1954, John Backus and a group started to design the FORmula TRANslator System, or FORTRAN0. At the time, computers were slow and unreliable and all programming was done in machine or assembly code. The authors of FORTRAN claimed that the resulting code would be as efficient as handcrafted machine code. Work on FORTRAN was completed in 1957 and for many years after, FORTRAN dominated programming, and was the common tongue for computer programmers.

C. Computers

Conrad Zuze invented the first freely programmable computer, the Z1 Computer, in 1936. However, the computers that are an integral part of all commercial activity today are the result numerous related innovations since then. From the creation of the transistor in 1947, the first commercial computer in 1951 to the introduction of the integrated circuit in 1958 and the microprocessor in 1971, several innovations have come together to integrate the use of computers in our lives. This general technology has had an unparalleled impact on all commercial activity – from the organization of businesses, to record keeping, to communication and the speedy automation of otherwise time consuming tedious tasks. Every business regardless of industry has adopted the use of computers in order to improve production and increase efficiency.

D. Internet Search Engines

The first Internet search engine, called 'Archie', was created in 1990 by Alan Emtage, a student at McGill University. Since then numerous search engines have enabled people to search for and gather information in a more inexpensive and convenient manner than ever before. Information is used to produce virtually any good and service. Search engines increase the efficiency in the process of gathering information. Thus, search engines increase productivity in a wide range of sectors. Whether innovations in search engines are appropriable is more debatable. Clearly, they are not embodied and non-patentable. However, the effectiveness of the search engine and the advertising revenues depend in part on the number of users. Since users may respond to innovations in the search engine a part of the revenues created by these innovations will be appropriable. Having said that, we still believe that, the lack of patents makes the concept of search engines a general innovation.

V. Trade

The introduction of malls and department stores constitute a general technology because improvements in the distribution of goods and services benefited a variety of industries in the economy.

A. The Mall

The first shopping mall was the Country Club Plaza, founded by the J.C. Nichols Company and opened near Kansas City, MO, in 1922. The first enclosed mall called Southdale opened in Edina, Minnesota in 1956. In the 1980s, giant mega malls were developed. Mega malls revolutionized the retail industry. The geographical concentration of hundreds of stores offering goods and services catering to every walk of life meant consumers could now indulge in a one-stop shopping experience.

Since their inception, mega malls have helped all retail outlets, independent of their industry, cater to a much larger population of consumers.

B. Department Stores

In 1877, John Wanamaker opened "The Grand Depot", a six story round department store in Philadelphia. He is credited with creating the first White Sale, modern price tags, and the first in-store restaurant. He also pioneered the use of money-back guarantees and newspaper ads to advertise his retail goods. Along with the retail giants of the day including, Marshall Field in Chicago, Alexander T. Steward in New York, Wanamaker was one of the first to discover the vast power of buying wholesale and how it could cut costs to reduce retail prices.

C. Internet Shopping

Shopping on the internet has opened a new portal for doing business for virtually every type of business in every industry. Every day, millions of dollars are transacted in exchange for every imaginable product or service through the internet. The wide applicability of this invention is evident. Similar to internet search engines, shopping on the internet is also not perfectly non-appropriable. Specific websites that create a brand image in creating a market for purchase and sale of goods and services (e.g. ebay, shopping.com) are able to extract a revenue stream from the transactions. However, the concept of a website used to create a virtual marketplace for transactions is a general innovation because it is not patentable and any individual or business is free to create such a website.

VI. Marketing

A. Coupons

Asa Candler, a Philadelphia pharmacist, invented the coupon in 1895. Candler, who purchased the Coca-Cola Company, placed coupons in newspaper for a free Coke from any fountain - to help promote the new soft drink. Today coupons are an integral part of promotion campaigns for every business. Cut-out coupons are included in newspapers as an advertising tool. They are also embedded in products so as to encourage repeat purchases. Over the years, coupons have been adopted as marketing tool across industries to help businesses build a brand image and target their customers in a more efficient manner.

B. Mail Order Catalog

Aaron Montgomery Ward invented the idea of a mail order catalog. As a traveling salesman, he realized that his rural customers could be better served by mail-order, a revolutionary idea at the time. The first catalog consisted of a single sheet of paper with a price list, 8 by 12 inches, showing the merchandise for sale with ordering instructions. Today, mail-order catalogs are an integral part of major retail businesses. They have helped businesses across sectors to tap into the market of consumers who are unwilling or unable to access the retail outlets. Serving as an effective marketing medium, mail order catalogs have opened up new segments of consumers previously unavailable to these businesses.

VII. Chemical Engineering

Arthur D. Little introduced the concept of the 'unit operations' in 1915. It referred to activities such as mixing, heating, filtering, verizing among others that featured in any chemical process. Chemical engineering research was directed towards the improvement of such processes. The concept of unit operations was instrumental to the success of Pre-production Planning. Pre-production made possible the transition from the confines of the laboratory to large scale production and was critical to the development of chemical engineering. In its stages of infancy, chemical engineering research was applied to the paper and pulp industry and contributed to the at the time new sulfite process of converting wood pulp into paper. In more recent times, advances in the field have had a substantial impact across several sectors, perhaps most noticeably on the petrochemical industry.

Appendix II: Discussion of Calibration

In this appendix, we discuss in greater detail the calibration conducted in section 3.3 to explore the model predictions for aggregate volatility and growth. In particular, we explain each of the 6 steps.

(i) and (iv) calibrate the turnover rates (λ^q_{s1950} and λ^q_{2000}) to match the initial correlation of rankings in figure 3.

We proceed in two steps. First, we use the model to compute the productivity percentiles of the leader and the followers in a sector. Second, we use the model to compute the expected correlation of the percentiles over time as a function of λ^q.

At any given moment in time, the market leader has higher productivity than the m followers. These in turn have the same level of sales per worker. The percentile of the leader $p_l = 1/(2(m+1))$, while the percentile of the followers $p_f = (m+2)/(2(m+1))$. Let's denote by $\overrightarrow{p_t}$ the $(m+1)$ x 1 vector that contains the percentile of each firm at year t. The mean and variance of $\overrightarrow{p_t}$ are constant and given by $\mu_p = 0.5$ and $Var_p = m/(2(m+1))^2$, respectively.

The correlation of percentiles between years t and $t+1$ is given by the following expression:

$$\begin{aligned} Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+1}}) &= \frac{Cov(\overrightarrow{p_t}, \overrightarrow{p_{t+1}})}{Var_P} \\ &= \frac{E\left[\sum_{i=1}^{m+1}(p_{it}-\mu_p)(p_{it+1}-\mu_p)/(m+1)\right]}{Var_p}, \end{aligned} \tag{39}$$

where E denotes the expectation of $\overrightarrow{p_{t+1}}$ conditional on $\overrightarrow{p_t}$.

With probability $1-\lambda^q_s$, no firm will take over the market leader and $\overrightarrow{p_{t+1}}$ will be the same as $\overrightarrow{p_t}$. In this event, $\sum_{i=1}^{m+1}(p_{it}-\mu_p)(p_{it+1}-\mu_p)/(m+1) = \sum_{i=1}^{m+1}(p_{it}-\mu_p)^2/(m+1) = Var_p$. With probability λ^q_s, one firm will take over the market leader and they will swap their percentiles at year $t+1$. For the market leader, $(p_{it}-\mu_p) = -m/(2(m+1))$, while for the followers, $(p_{it}-\mu_p) = 1/(2(m+1))$. Hence,

$$\begin{aligned} Cov(\overrightarrow{p_t}, \overrightarrow{p_{t+1}}) &= (1-\lambda^q)Var_p + \lambda^q_s\left[\frac{m-1}{m+1}\frac{1}{(2(m+1))^2} - \frac{2}{m+1}\frac{m}{(2(m+1))^2}\right] \\ &= (1-\lambda^q)Var_p - \frac{2\lambda^q_s Var_p}{m(m+1)} \\ &\simeq (1-\lambda^q)Var_p, \end{aligned}$$

where the last approximation holds when m is sufficiently large. Substituting into (39), it follows that

$$Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+1}}) \simeq (1-\lambda^q_s)$$

It also follows that for small λ_s^q,

$$Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+5}}) \simeq (1 - 5\lambda_s^q).$$

Since in 1950 $Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+5}}) \simeq 0.9$, we calibrate λ_{s1950}^q to 0.02. Similarly, since in 2000 $Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+5}}) \in (0.7, 0.75)$, we calibrate λ_{s2000}^q to (0.05, 0.06).

(ii) Using the value of λ_{s1950}^q and the initial correlation and variance of sectoral growth, pin down the values for $\lambda_{1950}^h * (\ln(\triangle h))^2$, $\lambda_{s1950}^q * (\ln(\triangle q))^2$ and $\ln(\triangle q)$.

In the multisector version of the model, we have seen that the variance of sectoral growth and the correlation of sectoral growth are given by the following expressions:

$$\begin{aligned} V\gamma_{y_s} &= \lambda_s^q (\ln(\triangle q))^2 + \lambda^h (\ln(\triangle h))^2 & (40) \\ corr(\gamma_{y_s}, \gamma_{y_{s'}}) &= \frac{(\triangle h)^2 \lambda^h}{(\triangle q)^2 \lambda_s^q + (\triangle h)^2 \lambda^h} & (41) \end{aligned}$$

It follows that:

$$\lambda_s^q (\ln(\triangle q))^2 = V\gamma_{y_s}/(1+\Phi),$$

where

$$\Phi \equiv \frac{corr(\gamma_{y_s}, \gamma_{y_{s'}})}{1 - corr(\gamma_{y_s}, \gamma_{y_{s'}})}.$$

It also follows from (40) and (41) that $\lambda^h (\ln(\triangle h))^2 = \Phi V\gamma_{y_s}/(1+\Phi)$ and (trivially) $\ln(\triangle q) = \sqrt{\lambda_s^q (\ln(\triangle q))^2 / \lambda_s^q}$.

We calibrate $corr(\gamma_{y_s}, \gamma_{y_{s'}})_{1950}$ to 0.5 (figure 6) and $V_{\gamma_{ys}}(1950)$ to 0.0005 both computed using the Jorgenson and Stiroh 35-KLEM dataset. That pins down $\lambda_{s1950}^q (\ln(\triangle q))^2$, $\lambda_{1950}^h (\ln(\triangle h))^2$ and $\ln(\triangle q)$, which is assumed to be constant.

(iii) **Using the average initial growth rate of productivity, calibrate $\ln(\triangle h)$ and λ_{1950}^h.**

The expected growth rate of the economy is given by the following expression:

$$E\gamma_y = \lambda_s^q \ln(\triangle q) + \lambda^h \ln(\triangle h) \tag{42}$$

It follows that:

$$\ln(\triangle h) = \frac{\lambda^h (\ln(\triangle h))^2}{E\gamma_y - \lambda_s^q \ln(\triangle q)}. \tag{43}$$

Further, once $\ln(\triangle h)$ is known, $\lambda^h = \lambda^h (\ln(\triangle h))^2 / (\ln(\triangle h))^2$. We use BLS data reported in figure 1 to calibrate $E\gamma_{y1950}$ to 0.025 and then use expression (43) to pin down $\ln(\triangle h)$ and λ_{1950}^h.

(v) Using the final correlation of sectoral growth and the calibrated value of $\ln(\triangle h)$, compute the final rate of arrival of general innovations (λ^h_{2000}).

From expression (41), it follows that

$$\lambda^h = \Phi \lambda^q_s (\ln(\triangle q))^2 / (\ln(\triangle h))^2.$$

Substituting in (i) Φ_{2000} , which we set to 0.25 based on figure 6, (ii) λ^q_{s2000},which we have set to 0.05 based on the discussion above and (iii) the calibrated values of $\ln(\triangle q)$ and $\ln(\triangle h)$, we can pin down λ^h_{2000}.

(vi) With this information and the number of sectors (35), compute the final expected growth rate of productivity ($E\gamma_{y2000}$), the initial and final variance of aggregate productivity growth ($V\gamma_{y1950}, V\gamma_{y2000}$).

This follows by evaluating the following two expressions at λ^q_{s1950}, λ^h_{1950}, λ^q_{s2000}, λ^h_{2000}.

$$\begin{aligned} E\gamma_y &= \lambda^q_s \ln(\triangle q) + \lambda^h \ln(\triangle h) \\ V\gamma_y &= \frac{\lambda^q_s}{N} (\ln(\triangle q))^2 + \lambda^h (\ln(\triangle h))^2 \end{aligned}$$