Dynamic Poaching in Credit Card Lending*

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ABSTRACT

The paper develops a positive theory of dynamic competition between credit card lenders, featuring balance transfers and default. Based on our theory and our quantitative results, we argue that the observed outcomes in the US credit card market are consistent with reduced effectiveness of personal bankruptcy protection, and inefficiently elevated costs of unsecured credit for intertemporal smoothing purposes. The model also delivers balance transfers as an equilibrium phenomenon.

JEL: D1,D8,G2
Keywords: credit cards, personal bankruptcy, credit lines, non-exclusivity, unsecured credit, balance transfer, credit score

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1 Introduction

A statistical US household holds an option to draw as much as $40k\footnote{Data for 2007. Source: Page 226 of H.R. 5244, the Credit Cardholder’s Bills of Rights: Providing New Protections for Consumers, Hearing Before the Subcommittee on Financial Institutions and Consumers Credit, House Committee on Financial Services, 110 Cong. 109, 226 (April 17, 2008) (testimony of Travis Plunkett, Consumer Federation of America), available at \url{http://www.house.gov/apps/list/hearing/financialsvcs_dem/hr041708.shtml}.} in credit card funds. Perhaps not all, but a significant fraction of these funds can be used to finance day-to-day consumption, and can be defaulted on under Chapter 7 or 13 of the US personal bankruptcy law. By historic standards, this is an unprecedented level of insurance provided by the private credit markets, raising an important question whether the provision this type of insurance is efficient.

By design, the institution of personal bankruptcy in the unsecured credit market is meant to provide state contingency when private contracts cannot be made state contingent. According to the theory of incomplete contracts, bankruptcy protection can result in significant welfare gains, potentially leading to an allocation that exhibits some notion of constrained efficiency or second best. The typical conditions to achieve second best are: two-sided commitment contracts that can be signed by lenders and borrowers, and default that is sufficiently costly for borrowers in the absence of any shocks that bankruptcy is intended to insure against. In practice, it has been shown that a simple form of punishment for default can come a long way in insuring particularly severe shocks like persistent job loss, or health and serious family problems (e.g. divorce, unwanted pregnancy)\footnote{See, for example, Livshits, McGee & Tertilt (2006)}. In terms of policy, this research has provided a powerful argument in support of some form of personal bankruptcy regulations.

In the case of the credit card market, due to the two-sided commitment requirement, the above arguments are somewhat questionable. In fact, the credit card market features the most extreme form of lack of commitment on the borrower side: full non-exclusivity of contracts with increasingly popular options of balance transfers attached to more than 60% of credit card offers. In this context, a characterization of the possible distortions caused by non-exclusivity is very much needed to better understand the economic impact of personal bankruptcy protection laws. This should allow to assess potential solutions, welfare costs, and give policy makers some guidance about what the observable symptoms of the underlying inefficiencies might be.

To study these questions both theoretically and quantitatively, we develop a positive
theory of competition in a lending market featuring credit lines, balance transfers and default. Specifically, in our model borrowers use unsecured credit lines to smooth consumption intertemporally, and can default on their debts by maxing out on available credit limits. Consumers can accept multiple credit lines to optimize on the interest payments to their existing debt. Lenders observe time-varying credit-worthiness of borrowers and choose whether to extend credit and what kind of contract to offer in order to undercut incumbent lenders.

![Credit Card Revenue and Cost Structure, 2003 (Daly (2004)).](image)

Figure 1: Credit Card Revenue and Cost Structure, 2003 (Daly (2004)).

The above features are generally informed by what we see in the data. First, the income statement of the credit card industry illustrated in Figure 1 reveals that default is, in fact, an important feature of this market, and lenders must secure significant revenue sources to offset losses associated with it. Second, the same figure shows lenders actually rely on interest to generate the bulk of their revenue, consistent with the view that credit cards are primarily credit lines. Third, a centralized credit reporting system effectively removes any informational barriers between entrants and incumbent. The fact that 17% of balances are transferred per annum (see Evans & Schmalensee (2005)) suggests that entry frictions are potentially small in this market.

Using our model, we show that non-exclusivity of contracts creates a strategic entrant-incumbent relation between lenders, resulting in a mis-allocation of both credit and implicit insurance. Both underinsurance and overinsurance are possible. The source of underinsurance is straightforward. In general, undercutting precludes initial lenders from offering low interest rate contracts. Since in our model these lenders are the primary providers of insurance across most states of the world, this results in too little insurance and excessive marginal rates. The
source of over-insurance is more subtle, but it turns out a potent force. It is brought about by the possibility that borrowers may engage in a strategic default, i.e. they may default on their debts in the absence of negative shocks. The threat of strategic default limits how much total credit lenders can extend, giving rise to a notion of credit limit carrying capacity. From the perspective of initial (ex-ante) lenders, this situation creates an incentive to crowd out future lenders by committing early on to overly generous credit limits, thereby inducing endogenous borrower commitment. Such commitment is desirable and allows to implement a great deal of insurance in some cases, which is, however, excessive relative to the second best. This force dominates in our quantitative model, and results in both excessive credit limits and interest rate on insurance that is provided in equilibrium.

Quantitatively, [to be completed].

Our results yield two key policy implications. First, as other studies have suggested, relaxing non-exclusivity requirements may be welfare improving by allowing bankruptcy protection to be more effective in insuring households against adverse shocks. Second, recent reforms in bankruptcy law, such as introducing means testing, may have the unintended consequence of increasing charge-off rates by exacerbating crowding out and overinsurance.

Related literature—we are not the first ones to study non-exclusivity of credit contracts exposed to default risk. The effect of non-exclusivity with loan contracts has been studied by DeMarzo & Bizer (1992), and in a related context also by Kahn & Mookherjee (1998), Parlour & Rajan (2001), Bisin & Guaitoli (2004), Petersen & Rajan (1995), and Hatchondo & Martinez (2007). DeMarzo & Bizer (1992) study lender competition in a moral hazard environment. In their model, higher debt levels (loan size) increase exogenous default probability and, unlike in our model, there is no new information arrival between rounds of competition. In their second best allocation, borrower’s credit should not be fully satiated at the marginal interest rate or, equivalently, the borrower should be credit constrained in equilibrium. This arises because too much credit elevates default probability beyond optimal levels. This is not the case in equilibrium: because non-exclusivity allows the borrower to accept multiple loans, she always fulfills her credit needs. Since later entrants do not have any informational advantage, it does not matter who actually satiates borrower’s credit needs in equilibrium.

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Footnote 3: Means testing was introduced by the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005. It limits eligibility to file under Chapter 7 (liquidation) by requiring the filer’s income to be below the median in the individual’s domiciled state. Debtors with income above the median can only file under Chapter 13 (restructuring). These changes were intended to reduce default rates and limit fraudulent behavior.
In contrast, in our paper competitors take advantage of the fact that, due to a partial resolution of uncertainty, borrowers may no longer need insurance at the later stages of the competition—late lenders engage in poaching by exploiting the change in borrower preferences. Distortion then arises due to a conceptually different mechanism: credit lines provide ex-ante lenders with an effective tool to deter entry (or commit the borrower) and preempt ex-post ‘satiation’ by future competitors. The mechanism we analyze is not only different but, most importantly, does not rely on moral hazard considerations, which could in principle be added to our framework.\textsuperscript{4}

A second strand of the literature we build on deals with quantitative modelling of unsecured credit. Our quantitative approach is closest to the OLG framework proposed by Livshits, McGee & Tertilt (2006), and enriches it with the credit card market features described above. Other important contributions in this area include Chatterjee, Corbae, Nakajima & Rios-Rull (2007) and Athreya (2002), and also notable in this context is the work by Athreya, Tam & Young (2008), Sanches (2008), Rios-Rull & Mateos-Planas (2007), Nara-jabad (2007), Drozd & Nosal (2007), and Mateos-Planas (2011).

2 Analytic Model

We begin by developing a one-period analytic setup, which we later extend to a multi-period life-cycle environment that we calibrate and solve numerically. While several aspects of the analytic setup are simplified to streamline the intuition, it incorporates all the relevant features present in our quantitative model.

The economy is populated by a large number of two types of ex-ante identical agents, consumers and lenders. Consumers have deterministic income $y > 0$ and face an i.i.d. binary expense shock $\kappa \in \{0, x > 0\}$ that occurs with probability $p$ (e.g. medical bills). They start the period with some pre-existing debt $B > 0$—debt levels are endogenous in our life-cycle model, and smooth consumption within the period by resorting to unsecured credit and the option to default.

Lenders have deep pockets, and maximize profits by extending credit to consumers. They face constant cost of funds normalized to 0%—which is also the saving rate for consumers.

\textsuperscript{4}Note that the possibility of a strategic default in normal times is a form of moral hazard in our model. However, as long as the sum of credit limits is below the highest feasible aggregate credit limit, the probability of default is independent of the size of the credit lines.
2.1 Credit Markets

Markets are incomplete (state non-contingent) and stylized to capture the salient features of the U.S. credit card market. In particular, credit takes the form of non-exclusive credit lines that provide a free option to borrow up to a specified limit $L$ at a fixed interest rate $R$. Lenders sequentially compete in two rounds of Bertrand competition, separated by the arrival of information about the expense shock —this is meant to capture the presence of a highly developed credit reporting system in the U.S. Consumers can only accept one contract per round.[5] Lenders can commit not to change pre-authorized credit limits after the arrival of information.[6]

2.2 Timing of Events

The timing of events for any given consumer is as follows: (i) in the beginning of the period, markets open and lenders compete a la Bertrand to extend credit to the consumer; (ii) a signal $s = \{0, x\}$ revealing the shock is publicly observed —we generalize the analytic results to the case of noisy signals in the supplementary online appendix; (iii) after the signal is observed, the credit market reopens with probability $\zeta > 0$, and lenders compete to extend an additional credit line to the consumer; (iv) the shock $\kappa$ is realized and the consumer chooses how much to borrow to smooth consumption between two consumption subperiods, and whether to default —the consumer is allowed to max out on all credit lines before defaulting.

2.3 Consumer Problem

After the lending market closes, the consumer enters the consumption stage with, at most, two contracts on hand: one from the first round, $C = (R, L)$, and, if the market reopened

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[5] A simple game can decentralize this outcome: within each round of competition, borrowers shop sequentially for lines while lenders observe all contracts being accepted in the process and can change the terms of unaccepted offers. Lenders can commit to not change terms, unless the borrower applies for more credit within the same round. In this context, under reasonable assumptions, merging two same-round lines into one is always better, as it reduces the marginal interest rate while providing the same aggregate limit.

[6] They are two ways of motivating this assumption. First, U.S. law imposes full commitment to the utilized portion of the line, and borrowers could cash credit limits before lenders retract the non-utilized portion of the line. Specifically, the 2009 law requires lenders to offer a 5-year repayment plan (or longer) at an interest rate no higher than the one at which borrowing took place. Second, commitment could be enforced through reputational considerations, in case pre-authorization is associated with commitment by borrowers. Anecdotal evidence suggest this is the case. See the Congressional Hearings for the Credit CARD Act of 2009 (March 24, 2009 - Senate Hearing 111-323, e.g. page 48.).
after the signal realization, one from the second round, \( C' = (R', L') \). We assume without loss of generality that interest rates satisfy \( R > R' \). For simplicity, we also assume that consumers cannot cancel the initial credit line \( C \) after observing the signal. (Our quantitative model allows consumers to cancel the first round contracts after observing the signal but before knowing whether the market re-opens or not.)

### 2.3.1 Consumption stage

The *ex-post* utility function of the consumer depends on consumption level in the two sub-periods, denoted \( c_1 \) and \( c_2 \), respectively. It is given by the composition of the intertemporal CES aggregator \( G(c_1, c_2) = \left( c_1^{1-\sigma} + c_2^{1-\sigma} \right)^{-\frac{1}{1-\sigma}} \), and a CES utility function over aggregated consumption \( u(\cdot) \). A special case of this formulation is the usual CES utility exhibiting the same elasticity across dates and states of the world.

The consumer maximizes her utility by borrowing across consumption sub-periods and choosing whether to default or not. Borrowing cannot be higher than \( L + L' \) and, upon default, all credit card debt as well as the expense shock \( \kappa \) can be discharged at a pecuniary penalty proportional to income. As we mention below, similar results would apply to the case of non-pecuniary (utility) penalties. Prior to defaulting, the consumer can max out on both lines, implying that the implicit insurance against the shock is determined by the aggregate credit limit that is available in state \( \kappa = x \).

Formally, the consumer chooses a binary default decision \( \delta(\kappa, C, C') \in \{0, 1\} \) and borrowing level \( b(\kappa, C, C') \), to maximize her indirect utility function

\[
V(\kappa, C, C') \equiv \max_{\delta \in \{0, 1\}} V^\delta(\kappa, C, C').
\]

where \( V^\delta(\kappa, C, C') = u(G(c_1, c_2)) \) stands for the indirect utility function *conditional* on default decision. In case of no default \( (\delta = 0) \), the budget constraint in the first subperiod is determined by consumption \( (c_1) \), flow of income \( (y) \) *plus* new borrowing \( (b) \) *less* pre-existing debt \( (B) \) and current interest payments \( (\rho(\cdot)/2) \):

\[
c_1 + B + \rho(b, C, C')/2 = y + b, \text{ with } b \leq L + L'.
\]

Second subperiod budget constraint is given by consumption \( (c_2) \), income \( (y) \) *less* payment

\[\text{Specifically, if } u(c) = \frac{c^{1-\sigma}}{1-\sigma} \text{ then } u(c_1) + u(c_2) = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma} = \frac{G(c_1, c_2)^{1-\sigma}}{1-\sigma} = u(G(c_1, c_2)).\]
of the principal \((b)\), interest payments and expense shock \((\kappa)\):

\[
c_2 + \rho(b, C, C')/2 + \kappa + b = y.
\]

The interest payment function is given by

\[
\rho(b, C, C') = \begin{cases} 
R \max(b - L', 0) + R' \min(L', b), & b > 0 \\
0, & b \leq 0,
\end{cases}
\]

and it reflects the fact that the consumer uses the cheaper line \((C')\) first.

In case of default, the first subperiod budget constraint allows the agent to default on interest due as well, i.e.,

\[
c_1 + B = y + b,
\]

while the second subperiod period budget constraint reflects the fact that the agent maxes out and discharges both her credit card debt and the expense shock:

\[
c_2 + b = \theta \kappa y + L + L',
\]

where \(\theta \kappa\) is the fraction of income net of default penalties, which can differ across shock realizations. We want to point out that, although we follow the literature in making the expense shock fully defaultable, our results still apply to the case in which the consumer can only discharge a fraction \(\phi < 1\) of the shock. This is because the budget constraint under partial discharge is equivalent to a budget constraint under full discharge and a higher default penalty, i.e. a lower \(\theta_x\).\(^8\)

### 2.3.2 Default Decision and Credit Limit Carrying Capacity

The default decision is a function of equilibrium contracts, and hence cannot be fully characterized without solving the model. However, the fact that the consumer is allowed to max out before defaulting places a bound on how much credit she can get. Specifically, given any fixed initial credit limit and interest rates, there is a well-defined aggregate credit limit or credit limit carrying capacity, above which the consumer decides to default. This notion of capacity

\(^8\)The equivalent penalty is given by \(\theta'_{x} = \theta_{x} y - (1 - \phi)y\).
plays a fundamental role in the provision of insurance, since it limits the second round entry due to the possibility of strategic default, i.e., default in the absence of an expense shock. The existence of such capacity is irrespective of whether default penalties are pecuniary (our focus on pecuniary penalties is just for analytical convenience and could be generalized to any form of punishment).

**Definition 1.** (Credit Limit Carrying Capacity) Given \((L, R, R') \geq 0\), \(L_{\text{max}}(\kappa; L, R, R') \) represents the total credit limit such that \( V^0(\kappa, \cdot) < V^1(\kappa, \cdot) \) for all \( L' > L_{\text{max}}(\kappa; L, R, R') - L \) and \( V^0(\kappa, \cdot) \geq V^1(\kappa, \cdot) \) otherwise.

To ease notation, we write \( L_{\text{max}}(L, R) \) whenever \( \kappa = 0 \) and \( R' = 0 \). Notice that the consumer always defaults after the shock (\( L_{\text{max}}(x; \cdot) = 0 \)) when the shock is bigger than the default penalty \((x > (1 - \theta_x)y)\).

### 2.4 Lender Problem

Lenders offer credit lines to consumers and maximize expected profits in equilibrium by best responding to competitors’ offers and consumers’ optimal behavior in a Bertrand environment. Equilibrium in the lending market is required to be subgame perfect Nash, which we find by applying backward induction.

The profit functions of first round lenders incorporate both the potential default losses and the revenue loss caused by a balance transfer on the repayment path. Formally, for any set of contracts \( C \) and \( C' \) obeying \( R' < R \) the profit function of initial lenders is given by

\[
\pi(\kappa, C, C') = (1 - \delta(\kappa, C, C')) R \max(b - L', 0) - \delta(\kappa, C, C') L, \tag{4}
\]

while the profit function of second round lenders is

\[
\pi'(\kappa, C, C') = (1 - \delta(\kappa, C, C')) R' \min(L', b) - \delta(\kappa, C, C') L'. \tag{5}
\]

Given profits on each path, determined by the shock (signal) realization and the state of the market (open/close), we can compute expected profits of initial lenders. Of particular interest is the case in which \( C \) provides insurance, i.e., the agent defaults when hit by the shock and repays otherwise, and second round lenders extend credit all the way up to \( L_{\text{max}}(\cdot) - L \).
after observing $s = 0$ —future lenders set $L' = 0$ following $s = x$ if default is expected. In this context, expected profits of initial lenders are given by

$$(1 - p)R [\zeta (b_0 - (L_{\max}(.) - L)) + (1 - \zeta)b_1] - pL,$$

(6)

where $b_0$ and $b_1$ respectively denote borrowing under entry and under no entry when $\kappa = 0$. This expression shows how increasing $L$ could generate additional revenue by reducing the size of the balance transfer $L_{\max}(.) - L$ given that, as we show below, $L_{\max}$ is decreasing in $L$. This gives rise to the crowding out motive mentioned in the introduction —at $L = L_{\max}(.)$ future lenders are completely crowded out. This motive is present when initial lenders heavily rely on revenue under entry, i.e., when $\zeta$ and $b_0$ are not very low. Otherwise, initial lenders may find optimal to accommodate entry and rely on the interest revenue raised when the friction of probabilistic entry is strong enough.9

2.4.1 Equilibrium Contracts

Due to Bertrand competition, equilibrium contracts must maximize consumer’s indirect utility, subject to a zero profit condition (in expectation). By backward induction, given any first round contract $C$, second round lenders choose $C'(C, s)$ to maximize consumer’s indirect utility conditional on the signal realization, i.e.,

$$(C', s) = \arg \max_{C'} V(s, C, C'), \text{ subject to } \pi'(\kappa, \cdot) \geq 0.$$ 

Since signals perfectly reveal the shock (see the online Appendix for the case of noisy signals), it is clear that second round lenders will best respond to $C$ under Bertrand competition by charging the risk free interest rate and extending credit up to the repayment capacity.10

Lemma 1. Given any first round contract $C = (R, L)$, $C'(C, s)$ is given by $R' = 0$ and $L' = \max\{0, L_{\max}(s; L, R, 0) - L\}$.

Proof. Omitted.

9As far as profits are concerned it does not matter whether the friction is introduced in a probabilistic or deterministic way (e.g. loyalty programs).

10Note that this best response may not be unique. If the agent only borrows from $C'$ but does not fully utilize the line, any line with a limit between consumer’s borrowing level and $L_{\max} - L$ is also a best response. All these contracts are fully equivalent and so it is without loss to focus on this particular best response, which is unique when the line is fully utilized.
Anticipating $C'(\cdot, s)$, first round lenders choose the contract $C^*$ that satisfies

$$C^* = \arg\max_C EV(\kappa, C, C'(C, s)), \text{ subject to } E\pi \geq 0.$$ 

By Bertrand competition, any equilibrium contract must yield zero expected profits. Thus, it is convenient to define, for any $L$, the zero profit interest rate associated to the relevant case in which the consumer defaults when hit by the shock but repays in the absence of it.

**Definition 2.** $\mathcal{R}(L)$ is the lowest $R$ satisfying $E\pi = 0$ when $L' = L_{\max}(L, R) - L$ and $R' = 0$.

**Definition 3.** $L$ is profit feasible if $\mathcal{R}(L)$ is well-defined. $L$ is said to be strictly feasible if $E\pi > 0$ for some $R$ given $C'$.

In addition, we define the limit $L$ that assures complete crowding out.

**Definition 4.** $L_{\max}$ is the credit limit that satisfies $L_{\max} = L_{\max}(L_{\max}, \mathcal{R}(L_{\max}))$.

### 2.5 Equilibrium and Constrained Efficiency

Equilibrium in our model is the collection of policy functions $\delta, b, B', C^*, C'$ jointly satisfying the consumer and lender problems laid out above.

We next define, for comparison purposes, a notion of constrained efficient or second best allocation (CEA). To this end, we keep the same restrictions on the types of contracts that can be offered (credit lines yielding non-negative profits, one per round) and let a benevolent planner with access to the same information as lenders, choose the utility maximizing allocation that can be implemented with such contracts.

**Definition 5.** Constrained efficient contracts satisfy

$$\left(C, C'\right) = \arg\max_{C, C'} EV(\kappa, C, C'), \text{ subject to } E\pi \geq 0 \text{ and } \pi' \geq 0. \quad (7)$$

### 3 Results

We now present the analytic results for the one period model. All the proofs are in the Appendix. Throughout the section, we evaluate the main assumptions and illustrate differences between equilibrium and second best allocations with the following numerical example.
Leading Example  Consider the following parameter values: the consumer has CRRA utility with $\sigma = 2$, income is normalized to 1; pre-existing debt amounts to 25% of income ($B = 0.25$); and default costs are 25% of income across shock realizations ($\theta_0 = \theta_x = 0.75$). In addition, the expense shock is at least 25% of income ($x \geq 0.25$), implying that she always default after being hit by the shock ($\theta_x y \geq y - x$); and shock probability is $p = 0.047$ — similar to the delinquency rate on credit cards during 2010/11.\footnote{Source: Federal Financial Institutions Examination Council (FFIEC) Consolidated Reports of Condition and Income (1985-2000: FFIEC 031 through 034; 2001: FFIEC 031 & 041).}

3.1 Assumptions

Before stating the main results, we introduce a set of assumptions that allow us to simplify proofs and focus attention on the interesting case in which lenders make a non-trivial choice of credit limits to balance implicit insurance against the shock and interest rate distortion of intertemporal smoothing. Specifically, we analyze the case in which the consumer finds default optimal when hit by the shock and it is feasible to provide some insurance. If the agent does not default the solution is trivial: all lenders will charge the risk free rate and no transfer of resources across shock realizations takes place. On the other hand, if the agent always defaults no contracts will be offered.

The first assumption is somewhat technical and assures that $L_{max}$ is continuous and differentiable. This assumption is not needed at all, but greatly simplifies the proofs.\footnote{Results without this assumption are available upon a request.} Let $G_i$ denote the partial derivative of $G$ w.r.t. its $i$th argument.

**Assumption 1.** $G_1(y - B, y) - G_2(y - B, y) < G_1(y, y)$. This condition is typically quite slack. Given the income level and preferences in our leading example, it is satisfied when pre-existing debt is no larger than 40% of income.

**Lemma 2.** $L_{max}(L, R)$ is differentiable and decreasing in $L$ for all $R > 0$.

The next assumption states that initial lenders can feasibly extend credit. Again, the problem becomes trivial otherwise, since the set of potential equilibrium contracts would only include risk free second round lines offered when $\kappa = 0$. We state it in an endogenous form, but the lemma below assures that for sufficiently small $p$ the assumption always holds.
Assumption 2. For $\zeta = 1$, there exists $L > 0$ that is strictly profit feasible.

Lemma 3. If $B > 0$ and $\theta_0 < 1$ there exists $\bar{p}$ such that Assumption 2 holds for all $p < \bar{p}$.

Given the parameters of the example, the bound on $p$ implied by this lemma is $\bar{p} = 0.075$ when $\zeta = 1$, and higher when competition is less intense ($\zeta < 1$).\footnote{In comparison, the highest recorded annual delinquency rate on credit cards is 6.5% (2009).}

Our last assumption restricts attention to the case in which lenders can give contracts that do not constrain borrowing in state $\kappa = 0$. This case captures the motivation behind the paper: when credit limits are not binding, credit lines are effectively distinct from standard loan contracts, since the free option to borrow is used differently across states of the world.\footnote{We could also dispense with this assumption at the expense of complicating the proofs by dealing with a number of additional subcases.}

Assumption 3. $B/2 < L_{\text{max}}$.

The above assumption is a sufficient condition for non-binding borrowing constraints when the aggregate limit is at the repayment capacity, which must be the case in equilibrium (Lemma 1). Intuitively, borrowing $B/2$ would arise at zero interest rates, assuring perfect consumption smoothing. Thus, if $L_{\text{max}} \geq B/2$ the consumer is not constrained at any $R \geq 0$ when $L = L_{\text{max}}$. It turns out that $L_{\text{max}}(L, R(L))$ is decreasing in $L$, which implies that the consumer is not constrained as long as $L + L' = L_{\text{max}}(L, R(L))$.\footnote{The intuition is simple. When comparing two potential equilibrium contracts, we must have that the one with higher $L$ implies lower utility when $\kappa = 0$ since utility clearly goes up with $L$ when $\kappa = x$. Otherwise, if utility goes up on both states the lower $L$ contract is Pareto dominated and would never arise in equilibrium. Hence, higher $L$ must lead to a lower $L_{\text{max}}(L, R(L))$ since the latter is the aggregate limit at which utility under default when $\kappa = 0$ equals utility under repayment (which is lower at the higher $L$).}

3.2 Equilibrium Characterization

We begin by stating the key feature of the planning solution: unless the planner uses the initial credit line to relax the intertemporal credit constraint, a higher credit limit $L$ must necessarily correspond to a strictly higher interest rate $R$. The intuition is straightforward: increasing a non-binding $L$ leaves borrowing levels unaffected, and so the interest rate must be raised to compensate any additional default losses implied by a more generous credit limit (recall that the consumer can max out and default on limits). We will show that, in equilibrium, the opposite is true whenever $L$ crowds out $L'$.

Lemma 4. If $L$ is not binding then $\Delta R/\Delta L > 0$ in the planner’s problem.
Next, we characterize equilibrium for two distinct cases, depending on whether the entry friction is present.

3.2.1 No Entry Impediments ($\zeta = 1$).

Here we analyze the special case of sure entry following a good signal ($s = 0$). In such a case, the aforementioned crowding out motive always exists, since initial lenders trivially must rely on revenue under entry. Thus, whenever initial lender profits are non-negative (which requires $b_0 > L'$), higher $L$ leads to lower $L'$, thereby increasing borrowing from the first round line.

The next lemma establishes that the opposite result to Lemma 4 holds under non-exclusivity: the relation between the first round credit limit and the zero profit interest rate is negative. These lemmas formalize the central message of our paper, which we generalize below: the threat of a balance transfer changes the ex-ante lender’s incentives to provide implicit insurance.

**Lemma 5.** If $\zeta = 1$ then $\Delta R/\Delta L \leq 0$ for all feasible $L$, and the set of feasible $L$ is an interval with $L_{\text{max}}$ being its upper bound.

The intuition behind this result is as follows. First, starting from a feasible contract $C = (L, R)$, if lenders increase $L$ by $\Delta$ then $L_{\text{max}}(L, R)$ goes down (Lemma 2), crowding out $L'$ by more than $\Delta$. Second, borrowing must be higher than $L'$ for $C$ to be profit feasible when $\zeta = 1$. Given this, it turns out that if the marginal interest rate remains fixed, the increase in borrowing from $C'$ to $C$ exceeds $\Delta$. Hence, the increase in initial lenders’ revenue under entry is at least $R\Delta$, which yields extra expected revenue of at least $(1 - p)R\Delta$, while the increase in expected default losses is given by $p\Delta$. Now, since the lowest feasible $R$ is the one associated with full utilization of the line ($E\pi = (1 - p)RL - pL$), we must have $R \geq p/(1 - p)$. As a result, the increase in expected revenue is higher than the increase in default losses, which implies $L$ can be increased and, simultaneously, $R$ can be reduced.

Our next goal is to characterize the implications of the above results on borrower consumption allocations, which is a non-trivial task. This is because an allocation is defined by consumption levels on multiple paths, one per state-of-the-market/shock combination, and thus a deadweight loss function cannot be easily derived. To overcome this issue, we resort to certainty equivalent consumption along each shock path, which are denoted by $c_H$ in case of $\kappa = 0$, and $c_L$ when $\kappa x$. Certainty equivalents have two additional advantages: it allows to measure implicit insurance in equilibrium (higher $c_L/c_H$ implies more insurance),
and all our results can then be conveniently illustrated in the \((c_H, c_L)\) space. In particular, we can represent three important objects of the underlying equilibrium selection process: indifference curves, feasible consumption frontiers, and actuarially fair transformation lines (AFTL), i.e., lines with (negative) slope equal to the fair insurance rate \((1-p)/p\). Indifference curves (IC) have the usual properties. They are decreasing and convex-shaped, flatter than AFTL below the 45° line, i.e., at allocations exhibiting underinsurance \((c_L/c_H < 1)\), and steeper than AFTL at allocations with overinsurance (above the 45° line). The profit feasible consumption frontier (PFCF) is defined as the set of \((c_H, c_L)\) such that there is no other profit feasible \((c'_H, c'_L) > (c_H, c_L)\). Finally, the resource feasible consumption frontier (RFCF) depicts the set of consumption combinations satisfying the planner’s constraints that are not Pareto dominated.

Figures 2 and 3 illustrate our main results, which are formally stated below: the PFCF is globally steeper than the AFTL whenever \(\Delta R/\Delta L < 0\) (Proposition 1). In other words, increasing insurance while lowering the marginal interest rate causes the marginal rate of transformation of resources from normal to distressed times to be better than fair. This is because both insurance and consumption smoothing go up. In contrast, the RFCF is flatter than the AFTL whenever the first round credit limit is not binding, that is, whenever the planner does not use \(L'\) to relax borrowing constraints (Proposition 2). In this context, there is a sharp separation between the equilibrium and the second best allocation (CEA). While CEA exhibits underinsurance \((c_L < c_H)\), the equilibrium allocation exhibits more insurance than CEA (except when the borrower prefers no insurance at all), and may even lead to overinsurance \((c_L > c_H)\). The latter happens when the penalty for defaulting is higher following \(\kappa = 0\) than \(\kappa = x\) (see Corollary 1).

In conclusion, equilibrium contracts can take on two forms in the absence of entry frictions: \(L\) close or at \(L_{\max}\); and \(L = 0\) (no insurance) if the consumer prefers no distortion of intertemporal smoothing over having too much insurance. Consequently, non-exclusivity results in a polarization of implicit insurance levels across individuals. Specifically, some consumers may get too much insurance, while others will be underinsured.

We next state the formal results.

**Definition 6.** Let \(c_{\max}\) be the pair \((c_H, c_L)\) associated to \(L = L_{\max}\) and \(R = R(L_{\max})\). That is, \(c_{\max}\) is the point associated with the highest possible \(c_L\).
Figure 2: Equilibrium Above Full Insurance ($\theta_0 < \theta_x$)

**Proposition 1.** *(Global steepness in the limit)* If $\zeta = 1$ the PFCF is globally steeper than AFTL and contains $c_{\text{max}}$.

**Corollary 1.** If the equilibrium allocation exhibits positive insurance then it is given by $c_{\text{max}}$ or it exhibits $c_L > c_H$.

The corollary directly follows from the fact that ICs are flatter than the AFTL below the 45-degree line. This implies that when $c_{\text{max}}$ is below the 45° line the only candidates for equilibrium are $W$ and $c_{\text{max}}$ (Figure 3). If, on the other hand, $c_{\text{max}}$ is strictly above the 45° line, the equilibrium may be interior (Figure 2), but it will always exhibit overinsurance.

To complete our analysis, we next provide an analogue of all of the above results for the planner. This applies to consumption combinations at which $L$ is not binding on any path.

**Proposition 2.** *(Planner)* The RFCF is flatter than AFTL at all points at which $L$ is not binding and $L' = 0$.

The flatness of the RFCF, combined with ICs being flatter than the AFTL below the 45° line implies that, while equilibrium may exhibit overinsurance w.r.t. first best, the second best
allocation always exhibits underinsurance as long as borrowing constraints are not binding.

**Corollary 2.** *CEA exhibits* $c_L < c_H$ *or binding* $L$.

Our leading example shows that the difference in insurance provision between CEA and the equilibrium allocation can be quite big. The optimal equilibrium contract in the example is given by $L = L_{\text{max}} = 0.237$ and $R = 0.0597$, which implies an utilization rate $b/L = 42\%$ and full insurance, i.e., $c_L/c_H = 100\%$. In contrast, CEA is given by $L = 0.137$ and $R = 0.306$, with an utilization rate of $b/L = 81\%$, and substantial underinsurance: $c_L/c_H = 94\%$. The charge-off rate $(pL/(1 - p)b)$ in equilibrium is 11.7% versus 6.1% in CEA — which emphasizes our earlier point that at the individual level we typically observe higher levels of debt discharged through default.

### 3.2.2 Entry impediments $\zeta < 1$

We next show that the negative relationship between credit limits and interest rates, and thus the steepness of the PFCF, hold at points close to the top of the frontier ($c_{\text{max}}$) as long as $\zeta$ is above some cutoff. Note that entry impediments in the most extreme version turn the
equilibrium problem into the planning problem (at non-binding L), and so such cutoff is to be expected. Somewhat surprisingly, however, we find that the cutoff, given by $B/(2L_{\text{max}})$, is typically quite low. For instance, in our leading example it is equal to 0.53.\[16\]

**Lemma 6.** If $\zeta \geq \frac{1}{2}B/L_{\text{max}}$ then $\Delta R/\Delta L \leq 0$ for all $L$ close to $L_{\text{max}}$.

**Proposition 3.** (Steepness at the top) If $\zeta \geq \frac{1}{2}B/L_{\text{max}}$ then the PFCF is steeper than AFTL at all points close to $c_{\text{max}}$.

The condition for the proposition to hold is stated in endogenous form since it depends on $L_{\text{max}}$. The next lemma provides an exogenous sufficient condition, which is typically quite slack. For instance, it implies $\zeta > 0.582$ for the parameters of the example.

**Lemma 7.** A sufficient condition for $\zeta \geq \frac{1}{2}B/L_{\text{max}}$ to hold is

$$\zeta > \frac{1}{2} \frac{B}{G(\frac{y-B}{y})} + B - (1 + \theta_0)y.$$  

Generally, for moderate levels of $\zeta$, equilibrium will still exhibit a substantial degree of polarization, with $L$ typically being close to $L_{\text{max}}$ or, alternatively, being very low if non-crowding out contracts are preferred by the consumer.

### 3.2.3 Relation to exclusivity

Our results suggest that exclusivity is potentially a better regime, as it also may involve a great of ex-ante competition without the distortionary effect of ex-post entry.

The next lemma states that, as long as $L$ is not binding, contract exclusivity delivers the second best allocation. The intuition is fairly straightforward. As long as $L$ is not binding under no entry, then it must be non-binding under entry also. Thus, increasing $L'$ does not relax borrowing constraints and, since doing so undercuts revenue from $L$, it would cause the marginal rate to go up and $c_H$ to go down.\[17\] Thus, $L'$ must be zero when $L$ is not binding in the planning solution, implying that exclusivity implements this allocation.

**Lemma 8.** If CEA exhibits non-binding $L$ then it coincides with the equilibrium allocation under contract exclusivity.

\[16\]The data suggests that competition is very intense in the U.S. credit card market: just in the third quarter of 2010 consumers received over 1.3 billion unsolicited offers, 64% of them with an introductory low interest rate and the option to transfer existing balances.

\[17\]Since default losses do not change with $L'$, $E\pi = 0$ requires that interest payments when $\kappa = 0$ do not change either. Thus, increasing $L'$ while keeping $c_L$ unchanged necessarily lowers $c_H$.  

17
3.2.4 Relation to first-best

Last but not least, we derive a condition under which the consumer will be overinsured also relative to first best (complete markets). This condition assures that \( c_{max} \) is above the 45° line, implying overinsurance in equilibrium.

**Proposition 4.** *(Relation to first best)* At \( c_{max} \), \( c_L > c_H \) if and only if \( \theta_0 < \theta_x \).

This last result also shows that the effect of changes in bankruptcy law may have unintended consequences. This is because under non-exclusivity the credit limit carrying capacity is an additional and important determinant of the level of insurance, while otherwise it is not. A relevant example is the recent introduction in 2005 of income means testing for bankruptcy eligibility, which forces households with incomes above the median to file through the more costly Chapter 13. In our model, we could interpret such policy as an increase in the default penalty in normal times. Such a policy would always reduce strategic default under exclusive contracts without affecting the default options after being hit by the shock. However, under non-exclusivity, it may inefficiently increase the amount discharged through default in bad times by inducing more generous credit limits.

3.3 Noisy Signals

As we mention above, we study the noisy signal case in the online supplementary appendix and obtain similar results as long as the precision of signals is not too low.

4 Quantitative Model

Here we extend the one-period model to a multi-period OLG environment. The setup builds on Livshits, McGee & Tertilt (2006) and, with some modifications, incorporates into this model the within period sequential competition laid out earlier in the paper. Contracts are assumed to have one-period maturity. The crucial difference is that debt is endogenous. We calibrate this model match some features of the U.S. unsecured credit market data.

To integrate the analytic model into a life-cycle framework, we simplify the consumer problem by assuming only one consumption sub-period. Two sub-periods are excessive in an environment with life-cycle smoothing needs, and only complicate the analysis. We then extend the basic analytic setup by incorporating the following two additional features: (i) decision to cancel the initial credit line after observing the signal and before observing whether
the market reopens, and (ii) noisy signals. To evaluate our quantitative results, we compare each period equilibrium contracts to the counterfactual contracts that a constrained planner would offer instead (assuming the allocation reverts to equilibrium after each period, i.e., the planner uses the equilibrium value function to pin down continuation values).

4.1 Setup

Agents live for 18 periods in the model (each representing three calendar years), with the first 15 being working age periods and the last 3 representing retirement. Income is stochastic and follows a three state Markov process. The consumer problem is fully dynamic and implies that the consumer maximizes intertemporal utility. The state at the beginning of period is given by \((t, B, y, \delta)\), where \(t\) denotes age and \(\delta\) is the default status. Temporal utility is given by

\[
u(c, t) = - \left( \frac{c}{s_t} \right)^{1-\sigma},
\]

where \(c\) stands for consumption in the period and \(s_t\) is the consumer (household) size adjustment factor (it is age dependent). This feature follows closely Livshits, McGee & Tertilt (2006), and other life-cycle setups.

Timing within each period is as follows: (1) the consumer accepts initial contract \(C\); (2) a signal \(s\) of precision \(\pi\) about the shock is publicly observed\^{18} (3) the consumer decides whether to cancel the initial contract; (4) the market reopens with probability \(\zeta\) and the consumer accepts a second contract \(C'\); (5) the expense shock \(\kappa\) is revealed to the consumer; (6) the consumer chooses \(c\) and whether to file for bankruptcy at the end of the period, which determines next period \(\delta\). Punishment for default given by a pecuniary cost \(1 - \theta\) paid during the period of filing (at the beginning of the next period). During the filing period the agent is assumed to be excluded from the credit market and, while all debt is discharged, she consumes \(\theta y_t\), where \(y_t\) denotes income in period \(t\). As in Livshits, McGee & Tertilt (2006), the consumer is not allowed to default in two consecutive periods, and instead she holds the option to roll-over any expense shock debt to the next period at a penalty interest rate \(r_p = 72\%\), and default later. We relegate the details of the model to the Appendix.

\^{18}Specifically, with probability \(\pi\) the signal reveals the shock realization while it is non-revealing with probability \(1 - \pi\).
4.2 Calibration

The model is calibrated to match bankruptcy filing rates under Chapter 7 of 5.72 per thousand (year 2002), independent evidence on the size of income shocks and medical bills, replacement rate of 50%, and a credit card debt to income ratio of 9% (year 2002). We set the base risk free rate and the annual return on savings at 3.5% (Gourinchas & Parker (2002)). The cost of funds includes a wedge (proportional to borrowing) to account for the difference between saving and credit card rates of 13.4% (annually) less the annual charge-off rate of 6.2% from the data, as reported by the FRB (year 2002, average). We then calibrate the intertemporal discount factor $\beta$ and punishment for defaulting $\theta$ to match the filing rate and level of unsecured debt.

Income is calibrated to the same process as the one used by Livshits, McGee & Tertilt (2006) (persistent AR1 plus a transitory component discretely affecting 10% of population) and mapped onto a three state Markov process with the same age profile. The value of the expense shock is taken from Livshits, McGee & Tertilt (2006) to match medical bills in the data, which yields a shock that hits with probability 6.7% (in a three year period) of magnitude equal to 23% of average income. There is no uncertainty about retirement income, which is the sum of 20% of the average income in the economy and 30% of personal income during the 15-period working age. The lenders’ problem is similar to the one in the analytical model, except that they face positive cost of funds.

In terms of specific parameters to the lending market, we arbitrarily set $\zeta = 0.75$ and $\pi = 0.5$ (signal precision), and experiment with alternative values. See more details in the supplementary appendix.

4.3 Findings: VERY PRELIMINARY

The model matches the aggregate bankruptcy statistics quite well. Debt, filing rates and charge-offs in the model are matched. Since our calibration target is the filing rate rather than overall default or delinquency rates, we use and adjusted charge-off rate (2.5% in 2002) that only accounts for credit card losses attributable to bankruptcy filings, rather than the overall charge-off rate (6.2%) — the latter include any debt deemed unrecoverable. Interest

\footnote{We use year 2002 data whenever possible since this is the last year we have data for credit card losses directly attributable to bankruptcy filings. See Table 2 for data sources.}
Table 1: Parameter values and targets.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor $\beta$</td>
<td>0.85</td>
<td>unsecured debt to income (joint)</td>
</tr>
<tr>
<td>Punishment $\theta$</td>
<td>0.68</td>
<td>filing rate (joint)</td>
</tr>
<tr>
<td>Saving rate* $r$</td>
<td>0.10</td>
<td>saving rate + credit card premium</td>
</tr>
<tr>
<td>Cost of funds $rcf$</td>
<td>0.25</td>
<td>saving rate + credit card premium</td>
</tr>
<tr>
<td>Signal precision $\pi$</td>
<td>0.5</td>
<td>arbitrary</td>
</tr>
<tr>
<td>Competition intensity $\zeta$</td>
<td>0.8</td>
<td>arbitrary</td>
</tr>
</tbody>
</table>

* Triennial.

rates on credit cards are close (calibration target). The model is also roughly consistent with the statistics pertaining to the population of bankrupts: (i) bankrupts are slightly poorer in the model than the median in the population, and (ii) the amount defaulted on (net of recoveries) by a statistical bankrupt is of the same order of magnitude as in the survey data reported by Livshits, McGee & Tertilt (2006). Nevertheless, there are known shortcomings. For example, almost all defaults are triggered by fully dischargeable expense shocks (e.g., medical bills). This is a known problem of consumer bankruptcy models: without such large shocks it is difficult to generate any meaningful amount of defaults under reasonable calibrations.

The results in the benchmark case are illustrated in Figure 4 and in Table 2. As we can see, the key difference between the equilibrium allocation and the counterfactual (per-period) second best is large. The figure depicts debt discharged, with the diagonal representing second best. There are many cases in which overinsurance arises (points below the diagonal). Quantitatively, the model implies that 72% of agents who discharge debt are overinsured by about 45% (the aggregate credit limit under default being 45% higher than second best), and those underinsured (28%) are so by about 18%. At the same time, the average annual interest rate paid by those who pose some default risk is about 1 percentage point higher than the second best rate. This dynamic competition premium is quite big: it is equivalent to 25% of the total credit card annual premium over cost of funds (year 2002). The marginal rate is 1.25 percentage points higher (a third of the average annual premium). It also generates a substantial amount of balance transfers, equal to about 13% of all balances that pose some

\[20\] In the data, this is not a significant aspect, as less than 5% of debt defaulted on under Chapter 7 is recovered.
default risk. In the data, balance transfers constitute about 17% of existing balances per annum. To the best of our knowledge, this model is the first one to endogenously give rise to balance transfers in equilibrium.

Figure 4: Quantitative model: Second best versus equilibrium discharge of debt.

5 Conclusions
We have provided a detailed study of the effects of non-exclusivity of unsecured credit card contracts on the provision of insurance through the institution of personal bankruptcy. We have derived conditions under which non-exclusivity may lead to a distorted provision of insurance in the presence of dynamic competition with balance transfers. Our results suggest that the current regime may result in a suboptimal allocation of credit and insurance. While we abstract from all normative considerations, our results can loosely be interpreted as stemming from the desirability of a standing option to borrow on demand and having access to

21 The amount of balance transfers drops to 4% when we include risk free contracts.
Table 2: Quantitative Results

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Benchmark Model</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CEA</td>
<td>EA</td>
</tr>
<tr>
<td><strong>Key Aggregate Statistics</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC Debt to Income $^a$</td>
<td>9.10</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>Adjusted Net Charge-off Rate $^b$</td>
<td>2.48</td>
<td>2.44</td>
<td>2.67</td>
</tr>
<tr>
<td>Filing rate $^c$</td>
<td>5.50</td>
<td>5.51</td>
<td>5.32</td>
</tr>
<tr>
<td><strong>Insurance Contracts: Comparison to Second Best</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. CC-Discharge to Median Income of Bankrupts $^d$</td>
<td>50.50</td>
<td>0.63</td>
<td>0.71</td>
</tr>
<tr>
<td>Balances Transferred (per period) $^e$</td>
<td>17.00</td>
<td>0.00</td>
<td>9.96</td>
</tr>
<tr>
<td>Median Insurance (in percent of CEA)</td>
<td>-</td>
<td>100</td>
<td>117.1</td>
</tr>
<tr>
<td>Frequency of Over-insurance (in percent)</td>
<td>-</td>
<td>-</td>
<td>72</td>
</tr>
<tr>
<td>Median Over-insurance (in percent of CEA)</td>
<td>-</td>
<td>-</td>
<td>45.4</td>
</tr>
<tr>
<td>Frequency of Under-insurance</td>
<td>-</td>
<td>-</td>
<td>28</td>
</tr>
<tr>
<td>Median Under-insurance (in percent of CEA)</td>
<td>-</td>
<td>-</td>
<td>18</td>
</tr>
<tr>
<td>Avg. Excess Marginal $R$</td>
<td>-</td>
<td>-</td>
<td>1.35</td>
</tr>
<tr>
<td>Avg. Excess $R$</td>
<td>-</td>
<td>-</td>
<td>0.95</td>
</tr>
<tr>
<td>Entry with Crowding Out Motive (in percent)</td>
<td>-</td>
<td>-</td>
<td>85.4</td>
</tr>
</tbody>
</table>

Notes: Sources listed at the end of the tables.

$^a$Aggregate income in the economy. We assume that 95% revolving debt is in credit cards. Data for 2002, source: FRB and BEA.

$^b$Since not all discharged debt corresponds to formal filings, we should require our model to match a net charge-off rate lower than the one reported in the data (annual average of 6.2 percent. Data for 2002. Source: FRB). Specifically, we calculate the adjusted charge-off rate by using CC bankruptcy losses rather than net charge-offs ($18.19 billion. Data for 2002. Source: The Nilson Report, 2003). The adjusted charge-off rate is computed by dividing these losses by 0.95 × (aggregate revolving debt) (Fourth quarter 2002. Source: FRB).


$^d$This is for chapter 7 filers. Average CC discharge is calculated by (i) estimating the fraction of CC bankruptcy losses attributed to chapter 7 (see footnote 21); and (ii) dividing the estimated chapter 7 losses by the number of non-business Chapter 7 filings (Data for year 2002. Source: US Courts). To attribute bankruptcy losses to chapter 7 filings, we compute the ratio of (unsecured non-priority) debt discharged through chapter 7 to debt discharged through chapters 7 and 13 (84 percent. Data for 1997. Source: WEFA report, 1998). Median income of chapter 7 filers. Data for 2001. Source: Lawless, Littwin, Porter, Pottow, Thorne & Warren (2008).

$^e$This number does not readily map onto the data, and should not be compared. For data we report balance transfers per annum to provide some reference point.
implicit insurance, which cannot be *efficiently* provided under non-exclusivity. Our analysis suggests that the recent trends in the U.S. credit card market of attracting consumers through balance transfer offers (over 3 billion in 2010) reduces the benefit of a higher flexibility to borrow by impairing the effectiveness of bankruptcy protection. In a broader context, our paper implies that ex-post competition comes at the cost of distorting implicit contracting and that relaxing non-exclusivity requirements can improve welfare. In light of this we think that a more careful analysis is needed to learn under which circumstances ex-ante competition alone may be counterproductive.
Appendix

A1. Quantitative Model: Details

As mentioned above, the state at the beginning of the period is \((t, B, y, \delta)\) where \(\delta\) is the default flag and can take on three different values. If \(\delta = 0\) the agent was allowed but decided not to file for bankruptcy at the end of last period. When \(\delta = 1\) the agent was allowed to and filed for bankruptcy. Finally, \(\delta = 2\) when the agent is hit by the shock after having filed for bankruptcy in the previous period and rolls the shock over at the penalty interest rate \(r_p\).

A1. Initial State: No Bankruptcy Flag on Record \((\delta = 0)\)

The timing of events is as follows:

1. The consumer accepts initial contract offered by banks in this state \(\mathcal{C} = (R, L)\).
2. The realization of signal \(s\) takes place — the state is now \((B, y, \delta, s, \mathcal{C})\).
3. The consumer chooses whether to cancel contract \(\mathcal{C}\) \((\xi = 0, 1)\); if she cancels \(\mathcal{C} = \emptyset\).
4. If the market reopens, the consumer accepts second round contract offered by banks in this state \(\mathcal{C'} = (R', L')\). If the market does not reopen \(\mathcal{C'} = \emptyset\).
5. The expense shock is revealed \(\kappa\) — the state is now \((B, y, \delta, \kappa, \mathcal{C}, \mathcal{C'})\).
6. The consumer chooses consumption filing decision at the end of the period, which determines whether she starts next period with or without bankruptcy flag \(\delta\).

Given this timing, the implied budget constraint is given by

\[
c = y + B - b, \quad b \leq L + L',
\]

and the debt transition equation is

\[
B' = b + R' \min\{b, L'\} + R \max\{b - L', 0\} + \kappa.
\]

If the consumer decides to file, \(\delta = 1\) in the next period. Finally, the value function is

\[
V^{\delta=0}(B, y) = \max_{b,\xi} \left\{ u(c, t) + \beta \max\{V^{\delta=0}(B', y), V^{\delta=1}(B', y)\} \right\},
\]

where expectation is taken w.r.t. \(\kappa\).
A2. Initial State: First Bankruptcy Flag on Record ($\delta = 1$)

When in default for the first time ($\delta = 1$), all debt $B$ is discharged, and the consumer consumes her endowment less pecuniary cost of defaulting. In case of any expense shock, she rolls it over to the next period at a penalty interest rate $r_p$, implying, respectively, the following budget constraint and debt transition equation:

$$c = \theta y - b, \text{ and } B' = \kappa - b + r_p(\kappa - b).$$

At the end of the period the consumer has a choice of exiting default state, or continuing in default. The consumer can not discharge her debt again. The value function for this state is

$$V^{\delta=1}(B, y) = E_y \max_b \{u(c, t) + \beta \max \{V^{\delta=0}(B', y), V^{\delta=2}(B', y)\}\}.$$  

A3. Initial State: Second Bankruptcy Flag on Record ($\delta = 2$)

The consumer rolls over her debt at a penalty interest rate and has an option to file again or come out of default starting from her accumulated debt. The budget constraint and debt transition equations are

$$c = y - b, \text{ and } B' = B + \kappa - b + r_p(B + \kappa - b)$$

The value function for this state is

$$V^{\delta=2}(B, y) = E_y \max_b \{u(c, t) + \beta \max \{V^{\delta=0}(B', y), V^{\delta=1}(B', y)\}\}.$$  

A2. Proofs

This appendix contains the proofs of the results presented in the paper. For an extension of the model to imperfect signals we refer the reader to the online supplementary appendix. All results hold with minor qualifications.

A1. Preliminaries

In this section, we introduce some definitions and expressions that we use repeatedly in the proofs. First, we use some properties of the CES aggregator to rewrite aggregated consump-
tion and first order conditions for borrowing in a convenient way. After that, we introduce some additional definitions and notation. Let $\lambda$ denote the state of the market in the second round: open when $\lambda = 0$ and closed when $\lambda = 1$.

Notice that $G$ is symmetric, increasing and concave in each of its arguments, and homogeneous of degree one. This allows to express aggregated consumption given $\lambda$ and $\kappa$, denoted by $c_{\lambda|\kappa}$, as

$$c_{\lambda|\kappa} = G(c_{1,\lambda|\kappa}, c_{2,\lambda|\kappa}) = E_{\lambda|\kappa}G(\lambda|\kappa, 1 - \lambda|\kappa),$$

(A1)

where $c_{1,\lambda|\kappa}$ and $c_{2,\lambda|\kappa}$ respectively denote first and second period consumption, $E_{\lambda|\kappa} = c_{1,i|j} + c_{2,\lambda|\kappa}$ and $q_{\lambda|\kappa} = \frac{c_{1,\lambda|\kappa}}{c_{1,\lambda|\kappa} + c_{2,\lambda|\kappa}}$. Furthermore, it implies that the marginal interest rate on any repayment path ($\kappa = 0$) fully pins down the share of total consumption consumed in the first period ($q_{\lambda|\kappa}$), as long as borrowing constraints are not binding. Of particular interest is the entry case in which both lines are utilized when $R' = 0$ and $L' = L_{\max}(R, L) - L$. In this case, consumption levels when $\kappa = 0$ are given by

$$c_{1,\lambda|0} = y - B + (1 - R/2)b_{\lambda|0} + (1 - \lambda)(L_{\max}(R, L) - L),$$

(A2)

and

$$c_{2,\lambda|0} = y - (1 + R/2)b_{\lambda|0} - (1 - \lambda)(L_{\max}(R, L) - L),$$

(A3)

where $b_{\lambda|0}$ denotes borrowing from $C$. If $L$ is not binding, $b_{\lambda|0}$ satisfies the first order condition (FOC)

$$(1 - R/2)G_1(c_{1,\lambda|0}, c_{2,\lambda|0}) - (1 + R/2)G_2(c_{1,\lambda|0}, c_{2,\lambda|0}) = 0,$$

where $G_j$ is the partial derivative of $G$ w.r.t. the $j$th argument. Since $G_1$ and $G_2$ are homogeneous of degree zero, the FOC can be expressed as

$$(1 - R/2)G_1(q_{\lambda|0}, 1 - q_{\lambda|0}) - (1 + R/2)G_2(q_{\lambda|0}, 1 - q_{\lambda|0}) = 0,$$

(A4)

implying that $q_{\lambda|0}$ it is uniquely pinned down by $R$, less than half for $R > 0$ and, by the concavity of $G$, strictly decreasing in $R$.

In addition, $L_{\max}(L, R)$ satisfies

$$(2y - B - Rb_{0|0})G_1(q_{0|0}, 1 - q_{0|0}) = ((1 + \theta_0)y - B + L + L') G(q_{0|0}, q_{0|0}),$$

(A5)
where \( q_{D,i}^D = \min \left\{ 0.5, \frac{y-B+L+L'}{(1+b_0)y-B+L+L'} \right\} \). That is, when defaulting, the consumer can fully smooth consumption across periods unless maxing out on \( L \) and \( L' \) in period 1 is not enough to equalize consumption levels under default. This is because the consumer defaults on both principal and interest. When both lines are utilized \( L_{\text{max}} \) is obtained by finding the credit limit \( L' \) that simultaneously satisfies (A4) and (A5).

Finally, \( R(L) \) satisfies the zero profit condition of first round lender:

\[
(1 - p)R(L) \left( \zeta b_{0|0} + (1 - \zeta)b_{1|0} \right) = pL. \tag{A6}
\]

**A2. Proof of Propositions 1 and 3**

The two propositions are a direct consequence of three lemmas. The first two, Lemmas 5 and 6 above, establish that it is feasible to increase \( L \) and reduce \( R \) when entry is sufficiently likely and \( L \) is relatively close to \( L_{\text{max}} \). Moreover this is true for all feasible \( L \) when the probability of entry is close to one. Given this, the next lemma establishes that the slope of the PFCF is steeper than the AFTL whenever the relationship between \( L \) and \( R(0) \) is negative. This result relies on the fact that, by Lemma 1 (i) we must have \( L' = L_{\text{max}}(L, R(L)) - L, R' = 0 \) and \( R = R(L) \) on the frontier; and (ii) borrowing constraints are not binding when \( \lambda = \kappa = 0 \) and \( L + L' = L_{\text{max}}(L, R(L)) \) by Assumption 3.

**Lemma 9.** If \( \frac{\Delta R}{\Delta L} \leq 0 \) and \( \zeta \geq \frac{B}{2L_{\text{max}}} \) then \( \frac{\Delta c_L}{\Delta c_M} < -\frac{1-p}{p} \) on the PFCF.

**Proof of Lemmas 5 and 6.** The proof logic is based on three key features. First, by Assumption 2 there exists an interval of feasible \( L \): if it is possible to make strictly positive profits at some \( L > 0 \) it is also possible to break even in a neighborhood of \( L \), given that the continuity of \( L_{\text{max}} \) and \( G \) guarantees that profits are continuous in \( L \) and \( R \). In particular, this means that there is some feasible \( L > 0 \) strictly lower than \( L_{\text{max}} \). Second, if \( L \) is feasible and it is possible to raise \( L \) by \( \Delta L \) without lowering profits, while keeping \( R \) fixed, then \( L + \Delta L \) must be feasible too. Thus, if we prove that \( \frac{\Delta R}{\Delta L} \leq 0 \) for some feasible \( L < L_{\text{max}} \) for all \( \Delta L < L_{\text{max}} - L \) we are automatically proving feasibility of all credit limits between \( L \) and \( L_{\text{max}} \). Third, a necessary condition for \( \frac{\Delta R}{\Delta L} \leq 0 \) is that both credit lines are utilized when \( \lambda = 0 \). Otherwise, increasing \( L \) would not raise revenue while it would increase default losses, pushing profits down. Notice that this must be the case for all feasible \( L \) when \( \zeta = 1 \), since a feasible contract with \( L > 0 \) must be utilized to cover default losses. In addition, if a contract
is feasible under $\zeta = 1$ it must be feasible for any $\zeta < 1$.

Given these three facts, we prove the lemmas by finding a necessary and sufficient condition for $\frac{\Delta R}{\Delta L} \leq 0$. We then show that it holds for all feasible $L$ when $\zeta = 1$, which, by the argument above, implies that the set of feasible $L$ must be an interval containing $L_{max}$. Finally, we show that if $\zeta > \frac{B}{2L_{max}}$ the condition holds with slack at $L = L_{max}$, implying that it holds at $L$ close to $L_{max}$—which are feasible for all $\zeta$ since they are feasible at $\zeta = 1$.

To ease the exposition, we work out the case of differentiable $R$ and argue at the end that the same logic applies when $R$ exhibits kinks or discontinuities. Let $\frac{\partial}{\partial L}$ denote the derivative w.r.t. $L$ holding $R$ fixed, but not policy functions— a similar definition applies to $\frac{\partial}{\partial R}$.

In order to find a restriction on $\zeta$ we differentiate the zero profit condition (A6) w.r.t. $L$, which yields

$$\frac{\partial EIR}{\partial R} \frac{dR}{dL} + R(L)\zeta \frac{\partial b_{00}}{\partial L} \geq \frac{p}{1-p},$$

where $EIR = R(L)\left(\zeta b_{00} + (1-\zeta)b_{10}\right)$, i.e., it is the expected interest revenue of the first round contract. The inequality comes from the fact that increasing $L$ may increase revenue when $\lambda = 1$ by relaxing the borrowing constraint. In addition, we must have $\frac{\partial EIR}{\partial R} \geq 0$ on the PFCF. Otherwise, the first round lender could decrease the interest rate, thus making the consumer better off, and increase profits. Thus, a necessary and sufficient condition for $\frac{dR}{dL} \leq 0$ is

$$R(L)\zeta \frac{\partial b_{00}}{\partial L} \geq \frac{p}{1-p}. \quad (A7)$$

To obtain $\frac{\partial b_{00}}{\partial L}$ we use the fact that $\frac{\partial q_{00}}{\partial L} = 0$. From (A2) and (A3) we have that

$$q_{00} = c_{1,00} = \frac{y - B + (1 - R/2)b_{00} + (L_{max} - L)}{2y - B - Rb_{00}}. \quad (A8)$$

Differentiating the RHS w.r.t. $L$ (keeping $R(L)$ fixed) and setting it to zero yields

$$(1 - (1/2 - q_{00}) R(L)) \frac{\partial b_{00}}{\partial L} + \frac{\partial L_{max}}{\partial L} - 1 = 0. \quad (A9)$$

$^{22}$Note that, although borrowing constraints are not binding when $\lambda = \kappa = 0$ by Assumption $L$ may still bind when $\lambda = 1$ or $\kappa = 1$. 

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Next we obtain $\frac{\partial \mathcal{L}_{\text{max}}}{\partial L}$ by differentiating (A5):

$$\frac{\partial \mathcal{L}_{\text{max}}}{\partial L} = -R(L) \frac{\partial b_{0|0}}{\partial L} G(q_{0|0}, 1 - q_{0|0}) G(0.5, 0.5).$$

Plugging $\frac{\partial \mathcal{L}_{\text{max}}}{\partial L}$ into (A9) yields

$$\frac{\partial b_{0|0}}{\partial L} = \frac{1}{1 - (1/2 - q_{0|0}) R(L) - R(L) \frac{G(q_{0|0}, 1 - q_{0|0})}{G(0.5, 0.5)},}$$

which combined with (A7) yields the following condition for $\frac{\partial R}{\partial L} \leq 0$:

$$\zeta \geq \frac{p}{1 - p} \left( 1 - (1/2 - q_{0|0}) R(L) - \frac{G(q_{0|0}, 1 - q_{0|0})}{G(0.5, 0.5)} \right). \quad (A10)$$

Notice that the lowest interest rate at which a first period contract can break even is the rate associated with full utilization of $L$ both under entry and non-entry, i.e., $b_{0|0} = b_{1|0} = L$, which yields a zero profit interest rate of $\frac{p}{1 - p}$. This implies that the RHS of (A10) is strictly less than one. Hence, it is always satisfied for $\zeta$ close to one. That is, the lemma holds for all feasible $L$ when $\zeta$ is sufficiently close to one. This proves Lemma 4.

The next step is to show that when $\zeta \geq \frac{B}{2L_{\text{max}}} \quad (A10)$ holds with slack at $L = L_{\text{max}}$. When $L = L_{\text{max}}$ we have that $b_{0|0} = b_{1|0}$ and thus, by the zero profit condition, $R b_{0|0} = \frac{p}{1 - p} L_{\text{max}}$.

In addition, from (A8) we have that

$$(1 - (1/2 - q_{0|0}) R)b_{0|0} = q_{0|0}(2y - B) - (y - B).$$

Using these expressions we can write (A10) for $L = L_{\text{max}}$ as

$$\zeta \geq \frac{q_{0|0}(2y - B) - (y - B)}{L_{\text{max}}} - \frac{p}{1 - p} \frac{G(q_{0|0}, 1 - q_{0|0})}{G(0.5, 0.5)}.$$

Given that $q_{0|0} \leq 1/2$, the RHS of this expression is bounded above by $\frac{B}{2L_{\text{max}}}$.

We finish the proof by arguing that the above argument continues to hold when $R$ is not differentiable or even discontinuous. This is because $EIR$ is initially increasing in $R$. Thus, as argued above, since $R$ is the lowest rate satisfying zero-profit, $EIR$ must be increasing at $R$, even if it is not quasi-concave in $R$. Given this, if increasing $L$ by $dL$ at some $R = R$ raises $EIR$ more than the increase in default losses then the new zero-profit rate can only
drop, either continuously or discontinuously if $EIR$ happens to be decreasing at the root of the new zero-profit condition closest to $R$.

**Proof of Lemma 9.** The proof shows that raising $L$ without increasing the rate on the PFCF does not hurt intertemporal smoothing on repayment paths while providing insurance and (possibly) improving smoothing on the default path. We do so by differentiating certainty equivalents on each path w.r.t. $L$.

To ease the exposition we assume differentiability of the frontier but the proof arguments would follow through without imposing this assumption.

We compute the slope of the PFCF by first calculating $\frac{dc_i}{dL}$ for $i = L, H$ and then computing

$$\frac{dc_L}{dc_H} = \frac{dc_L}{dL} \cdot \frac{dc_H}{dL}.$$  

Notice that $L' = 0$ when $\kappa = x$ and thus $c_L = c_{0|x} = c_{x|x}$. The certainty equivalent when $\kappa = 0$ is given by

$$u(c_H) = \zeta u(c_{0|0}) + (1 - \zeta) u(c_{1|0}).$$  

Differentiating (A11) w.r.t. $L$ we get

$$\frac{dc_H}{dL} = \zeta \frac{u'(c_{0|0})}{u'(c_H)} \frac{dc_{0|0}}{dL} + (1 - \zeta) \frac{u'(c_{1|0})}{u'(c_H)} \frac{dc_{1|0}}{dL},$$

where $u'$ is the marginal utility of consumption. Notice that, when $\frac{dR}{dL} \leq 0$, we must have $\frac{dc_{1|0}}{dL} \geq 0$ since interest goes down and borrowing constraints are relaxed as long as there is no second round line. Also, since higher $L$ leads to higher $c_L$, it must be that $\frac{dc_{0|0}}{dL} < 0$ on the PFCF, otherwise we could increase both $c_L$ and $c_H$. Finally, the concavity of $v$ implies that $\frac{u'(c_{0|0})}{u'(c_H)} \leq 1$ and $\frac{u'(c_{1|0})}{u'(c_H)} \geq 1$ given that $c_{0|0} \geq c_H \geq c_{1|0}$. Accordingly,

$$\frac{dc_H}{dL} \geq \zeta \frac{dc_{0|0}}{dL} + (1 - \zeta) \frac{dc_{1|0}}{dL} \geq \zeta \frac{dc_{0|0}}{dL}. \quad (A12)$$

Given this, we need to show that

$$\frac{\partial c_{0|x}}{\partial L} < -\frac{1 - p}{p}. \quad (A13)$$
First we compute \( \frac{\partial c_{0|x}}{\partial L} \). If \( L \) is not binding when \( \kappa = x \) the consumer fully smooths consumption across periods, i.e., \( q_{\lambda|x} = 1/2 \). In addition, we have that \( E_{0|x} = (1 + \theta_x)y - B + L \), which leads to

\[
\frac{\partial c_{0|x}}{\partial L} = G(0.5, 0.5) . \tag{A14}
\]

If \( L \) is binding then \( c_{0|x} = G(y - B + L, \theta_x y) \) and thus \( \frac{\partial c_{0|x}}{\partial L} = G_1(y - B + L, \theta_x y) > G(0.5, 0.5) \) by the homogeneity and symmetry of \( G \) implying that the LHS of (A14) is greater than the RHS and the results below still follow through.

In order to calculate \( \frac{dc}{dL} \) we use the zero profit condition of the first round lender, given by (A6), to express \( E_{0|0} \) as a function of \( L \) and \( L_{\text{max}} \) and then differentiate \( E_{0|0}G(q_{0|0}, 1 - q_{0|0}) \) w.r.t. \( L \). Adding (A2) and (A3) and using (A6) we get

\[
E_{0|0} = 2y - B - \frac{p}{(1 - p)} L + \frac{1 - \zeta}{\zeta} R(L)b_{x|0} .
\]

Notice that \( \frac{\partial b_{x|0}}{\partial L} \geq 0 \). In addition, recall that \( q_{\lambda|0} \) is pinned down by \( R \) and does not depend on \( L \). Thus, when we differentiate \( E_{0|0}G(q_{0|0}, 1 - q_{0|0}) \) w.r.t. \( L \) we obtain

\[
\zeta \frac{\partial c_H}{\partial L} \geq - \frac{p}{1 - p} G(q_{0|0}, 1 - q_{0|0}) . \tag{A15}
\]

Dividing (A14) by (A15) we get that

\[
\frac{dc_L}{dc_H} = \frac{dc_L}{dc_H} \leq - \frac{1 - p}{p} \frac{G(0.5, 0.5)}{G(q_{0|0}, 1 - q_{0|0})} < - \frac{1 - p}{p} ,
\]

given that \( G(q_{0|0}, 1 - q_{0|0}) < G(0.5, 0.5) \) since \( q_{0|0} < 1/2 \) for \( R > 0 \). \( \square \)

A3. Remaining Proofs

Proof of Lemma 2. We prove the result by developing a sufficient condition for the LHS and RHS of (A5) to satisfy single crossing w.r.t. \( L' \), with the LHS crossing the RHS from above.

---

\( ^{23} \)Recall that the consumer does not pay any interest when she defaults.

\( ^{24} \)To see why, notice that

\[
G(0.5, 0.5) = 0.5(G_1(0.5, 0.5) + G_2(0.5, 0.5)) = G_1(0.5, 0.5) < G_1((y - B + L)/(2\theta_x y), 0.5),
\]

where the last inequality is due to the concavity of \( G \) w.r.t. each of its arguments and the fact that for \( L \) to be binding we must have \((y - B + L)/(2\theta_x y) < 1/2 \).
We do so by differentiating both sides w.r.t. $L'$. We need to consider two cases: for low $L'$ both lines may be utilized, while as $L'$ grows eventually only the second round line will be used. When both lines are used the LHS is given by

$$c_{0|0} = G(y - B + (1 - R/2)\hat{b}_{0|0} - L'R/2, y - (1 + R/2)\hat{b}_{0|0} - L'R/2),$$

where $\hat{b}_{0|0}$ denotes total borrowing. If $L$ is not binding, by the envelope theorem we have that

$$\frac{\partial c_{0|0}}{\partial L'} = \frac{R}{2} \left( G_1(c_{1,0|0}, c_{2,0|0}) + G_2(c_{1,0|0}, c_{2,0|0}) \right).$$

Since $R(G_1 + G_2) = 2(G_1 - G_2)$ by the FOC for borrowing we must have that

$$\frac{\partial c_{0|0}}{\partial L'} \leq G_1(c_{1,0|0}, c_{2,0|0}) - G_2(c_{1,0|0}, c_{2,0|0}) < G_1(y - B, y) - G_2(y - B, y),$$

where the last inequality comes from the concavity of $G$.

When $L$ is binding, $L'$ must be binding too so the effect of increasing $L'$ is similar to the next case.

When only the second round line is used by the consumer, increasing $L$ only affects $c_{0|0}$ whenever $L'$ is binding. Accordingly,

$$\frac{\partial c_{0|0}}{\partial L'} = G_1(c_{1,0|0}, c_{2,0|0}) - G_2(c_{1,0|0}, c_{2,0|0}) < G_1(y - B, y) - G_2(y - B, y).$$

Thus, we just need to show that the rate at which consumption under default grows with $L'$ is higher than $G_1(y - B, y) - G_2(y - B, y)$. This is guaranteed by Assumption [1] since, as shown in the above proofs, this rate is at least $G(0.5, 0.5) = G_1(y, y)$. Continuity and differentiability are a direct consequence of the differentiability of $G$, the continuity of $c_{0|0}$ and the fact that $\frac{\partial c_{0|0}}{\partial L'}$ has the same expression regardless whether the borrower is constrained or not.

Finally, it is easy to see why $L_{\text{max}}$ is decreasing in $L$. As long as $L + L'$ remains fixed, raising $L$ lowers $c_{0|0}$ whenever $L'$ is fully utilized while the RHS of (A5) does not change. Thus, for (A5) to hold $L + L'$ must go down, since increasing $L + L'$ would increase the RHS more than the LHS by the above argument. If $L'$ is not fully utilized after increasing $L$ while keeping $L + L'$ fixed then both sides of (A5) remain constant and $L_{\text{max}}$ does not change. □
Proof of Lemma 3. First, notice that $L_{\text{max}}$ is bounded above by $(1 - \theta_0)y$. To see why notice that the highest $c_{0|0}$ is associated with full smoothing at $R = 0$. Thus, from (A5) we have that

$$(2y - B)G(0.5, 0.5) = ((1 + \theta_0)y - B + L_{\text{max}}(0, L, 0))G(q_{0|0}^D, 1 - q_{0|0}^D),$$

which yields, for all $L$,

$$L_{\text{max}}(0, L, 0) = (1 - \theta_0)y - \frac{G(0.5, 0.5)}{G(q_{0|0}^D, 1 - q_{0|0}^D)} \leq (1 - \theta)y.$$

Given this, default losses when $\kappa = x$ are bounded above by $p(1 - \theta_0)y$. In addition, at $R = 0$ and $L = L_{\text{max}}(0, L, 0)$ we have that $b_{0|0} = \min\{B/2, L_{\text{max}}(0, L, 0)\}$, which is greater than zero as long as $B > 0$ and $\theta_0 < 1$. But then, since $L_{\text{max}}$ and $b_{0|0}$ are continuous we can find $R > 0$ such that interest revenue, given by $Rb_{0|0}$ is strictly positive. This, in turn, implies that we can find $\bar{p} > 0$ such that $\bar{p}L \leq \bar{p}(1 - \theta_0)y \leq Rb_{0|0}$, i.e. $L$ is strictly feasible at all $p < \bar{p}$.

Proof of Lemma 4. We obtain the sufficient condition by finding a lower bound on $L_{\text{max}}$. Notice that $L_{\text{max}}$ satisfies

$$G(y - B - (1 - R/2)b_{0|0}, y - (1 + R/2)b_{0|0}) = ((1 + \theta_0)y - B + L_{\text{max}})G(q_{0|0}^D, 1 - q_{0|0}^D).$$

The LHS is minimized by setting $b_{0|0} = 0$ and $G(q_{0|0}^D, 1 - q_{0|0}^D) = G(0.5, 0.5)$, which yields

$$L_{\text{max}} \geq \frac{G(y - B, y)}{G(0.5, 0.5)} - (1 + \theta_0)y + B.$$

Proof of Proposition 2. We proceed by computing the slope of the RFCF using

$$\frac{dc_L}{dc_H} = \frac{dc_L}{dl} \frac{dl}{dc_H}$$

By the argument in the text, if $L < L_{\text{max}}$ is not binding we have that $L' = 0$. Accordingly, $c_H$ and $c_L$ now equal aggregated consumption on the repayment and default paths, respectively.

Continuity of $b_{0|0}$ is straightforward when it is given by the FOC (A4). If borrowing constraints are binding then $b_{0|0}$ equals $L_{\text{max}}(0, L, 0)$.  

25 Continuity of $b_{0|0}$ is straightforward when it is given by the FOC (A4). If borrowing constraints are binding then $b_{0|0}$ equals $L_{\text{max}}(0, L, 0)$. 

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Given this, for all \( c_L \) at which \( L \) is not binding, we have that

\[
\frac{dc_L}{dL} = G(0.5, 0.5).
\]

In order to calculate \( \frac{dc_H}{dL} \) we use the zero-profit condition of the first round line, given by \( EIR = (1 - p)Rb_H = pL \), where \( b_H \) denotes the borrowing level when \( \kappa = 0 \). In addition, since \( L \) is not binding the first order condition pinning down \( b_H \) is satisfied with equality. Therefore, we can apply the envelope theorem and obtain

\[
\frac{dc_H}{dL} = \frac{p}{dEIR} \frac{\partial G(y - B + (1 - R/2)b_H, y - (1 + R/2)b_H)}{\partial R} \frac{dR}{dEIR} = -pb_H \frac{G_1(q_H, 1 - q_H) + G_2(q_H, 1 - q_H)}{2} \frac{dR}{dEIR},
\]

where \( q_H \) denotes the fraction of first period consumption over total consumption under \( \kappa = 0 \) and the last equality uses the fact that \( G_j \) is homogeneous of degree zero for \( j = 1, 2 \). Now, since \( b_H \) is decreasing in \( R \) and \( EIR \) is increasing in \( R \) on the frontier\(^{26}\) we must have \( 0 \leq \frac{dEIR}{dR} < (1 - p)b_H \) and thus

\[
\frac{dc_H}{dL} < -\frac{p}{1 - p} \frac{G_1(q_H, 1 - q_H) + G_2(q_H, 1 - q_H)}{2}.
\]

Notice that \( \frac{G_1(q_H, 1 - q_H) + G_2(q_H, 1 - q_H)}{2} \) is decreasing in \( q_H \) for \( q_H \leq 1/2 \)\(^{27}\) Thus,

\[
\frac{G_1(q_H, 1 - q_H) + G_2(q_H, 1 - q_H)}{2} > \frac{G_1(0.5, 0.5) + G_2(0.5, 0.5)}{2} = G(0.5, 0.5),
\]

where the last equality comes from applying Euler’s theorem. But this implies that the slope of the frontier is flatter than the AFTL:

\[
\frac{dc_L}{dc_H} = \frac{dc_L}{dL} \frac{dc_L}{dc_H} > -\frac{1 - p}{p} \frac{2G(0.5, 0.5)}{G_1(q_H, 1 - q_H) + G_2(q_H, 1 - q_H)} > -\frac{1 - p}{p}.
\]

\(^{26}\)Otherwise, the planner could lower the interest rate and increase revenues, making the consumer strictly better off.

\(^{27}\)Differentiating this expression w.r.t. \( q_H \) we get \( G_{11} - G_{22} \), where \( G_{ii} \) denotes the second partial derivative w.r.t to \( G \)’s \( i \)th argument. It is straightforward to check that \( G_{11}(c_1, c_2) < G_{22}(c_1, c_2) < 0 \) if \( c_2 > c_1 \geq 0 \), implying that \( G_1(q_H, 1 - q_H) + G_2(q_H, 1 - q_H) \) is decreasing for \( q_H < 1/2 \).
Proof of Proposition 4. Notice that at $L = L_{\text{max}}$ consumption under entry and no-entry ($\lambda = 0, 1$) coincide, and so certainty equivalents equal aggregated consumption given $\kappa$. Also, since $L + L' = L_{\text{max}}$ on the PFCF, from the definition of $L_{\text{max}}$ we have that consumption under repayment equals consumption under default for $\kappa = 0$. Hence, to check whether $c_L > c_H$, we just need to compare consumption under default for different values of $\kappa$, which boils down to comparing $\theta_0$ and $\theta_x$.

Proof of Lemma 8. I directly follows from the argument in the text.

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