Power Laws and the Mega-Idiosyncratic Origins of Aggregate Fluctuations

Xavier Gabaix*
MIT, Department of Economics and NBER

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Abstract

If firm sizes have a small dispersion, idiosyncratic firm-level shocks lead to negligible aggregate fluctuations. This has led economists to appeal to macroeconomic (sectoral or aggregate shocks) shocks to explain aggregate fluctuations. However, the empirical distribution of firms is fat-tailed. This paper shows how, in a world with fat-tailed firm size distribution, idiosyncratic firm-level fluctuations aggregate up to non-trivial aggregate fluctuations. We illustrate why and how this happens, and contend that aggregate fluctuations come in large part from idiosyncratic shocks to firms. We show empirically that idiosyncratic volatility is indeed large enough to account for GDP volatility. The idiosyncratic movements of the largest 100 firms in the US appear to explain about 40% of variations in output and the Solow residual. This "granular" hypothesis suggests new directions for macroeconomic research, in particular that macroeconomic questions will be clarified by looking at the behavior of large firms. This mechanism might be useful for understanding the fluctuations of many aggregate quantities, such as business cycle fluctuations, inventories, inflation, short or long run movements in productivity, and the current account.

1 Introduction

This paper proposes a simple origin for aggregate shocks. It develops the view that a large part of aggregate shocks comes from idiosyncratic shocks to individual firms. This approach sheds light on a number of issues that are difficult to address in models that postulate aggregate shocks. Though economy-wide shocks (inflation, wars, policy shocks) are no doubt important, they have a difficulty explaining most fluctuations (Cochrane 1994). Often, the explanation for year to year jumps of aggregate quantities is elusive. On the other hand, there is a host of anecdotal evidence for important idiosyncratic shocks. For instance, the McKinsey Institute (2001) estimates that in 1995-1999, 1/6 of the increase in productivity growth of the whole U.S.

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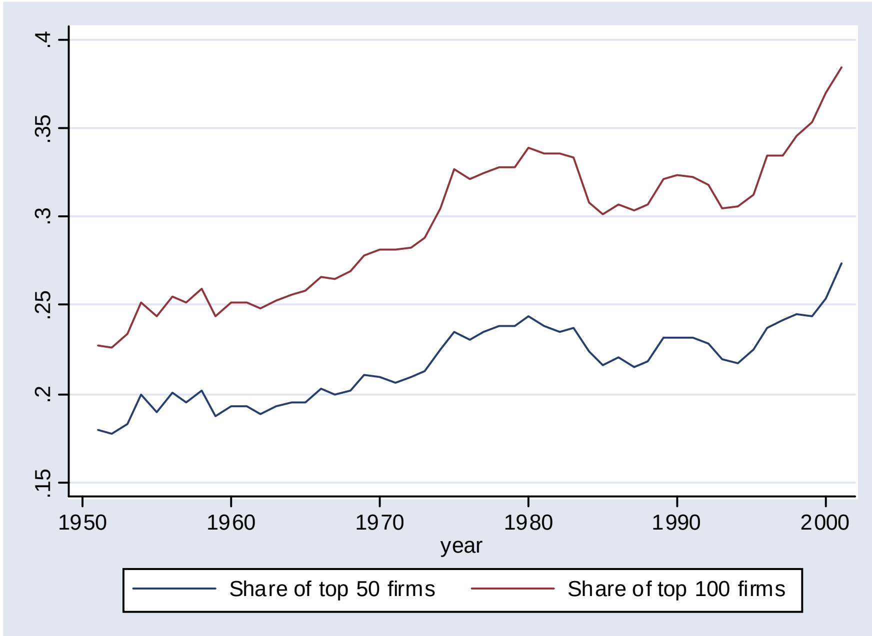

Figure 1: Share of the top 50 and 100 non-oil firms in Compustat. The share is the sum of the sales of the firms, divided by GDP. Hulten's theorem (Appendix E) motivates the use of sales rather than value added.

economy was due to one firm, Wal-Mart[1]. Likewise, shocks to GDP may stem from a variety of events such as a success by Nokia, the difficulties of a Japanese bank, new sales by Boeing, a new chip by Intel, and a downsizing at Nestlé.

Idiosyncratic shocks aggregate to non-trivial shocks, because modern economies have many large firms. For instance, in Japan, the top 10 firms account for 35% of the exports (Canals et al. 2004). For the U.S., Figure 1 reports the total sales of the of the top 50 and 100 firmsas a fraction of GDP. On average, the sales of the top 50 firms are 23% of GDP, while the sales of the top 100 firms are 30% of GDP. The top 100 firms hence represent a large part of the macroeconomic activity. It is perhaps not surprising that understanding their actions give a good insight into aggregate economy. Indeed, by looking at their respective actions, one can take a peek into the inner workings of the economy, indeed of about 1/4 of the economy as a whole.

This hypothesis, that idiosyncratic shocks generate aggregate shocks, offers a microfoundation for the "aggregate shocks" of real business cycle models. Hence real business cycle shocks are not, at heart, mysterious "aggregate productivity shocks". Rather they are well-defined shocks to individual firms[2]. This view sheds lights on a number of issues, such as the dependence of the amplitude of GDP fluctuations with GDP level, the microeconomic composition of GDP, the distribution of GDP and firm-level fluctuation.

Some of the mathematics will be involved, so it is useful to highlight the main argument. First, a result based on Hulten (1978) shows that, if firm i has a productivity shock $d\pi_i$, those

[1] In their interesting study, McKinsey (2001) seek to understand why U.S. productivity growth increased from 1.5% to 2.8% per year in the second half of the 1990s.

[2] These shocks can propagate to the rest of the economy. There is a very large literature on these "propagation mechanisms". This paper focuses on the original shocks, not their propagation.

shocks are i.i.d., then the standard deviation of GDP growth is:

$$\sigma_{GDP} = h_S \sigma_\pi \tag{1}$$

where h_S is the Sales herfindahl of the economy:

$$h_S = \left(\sum_{i=1}^{N} \left(\frac{\text{Sales of firm } i}{\text{GDP}} \right)^2 \right)^{1/2}$$

and σ_π is the standard deviation of the i.i.d. productivity shocks. Second, microeconomic volatility is very large. We find that, even for large firms, the volatility of productivity is $\sigma_\pi = 20\%$/year. Third, as countries have large firms the sales herfindahl h_S is high. For instance, for the U.S. in 2002, it is $h_S = 6.2\%$. Using (1), we predict a GDP volatility equal to: $\sigma_{GDP} = 20\% \cdot 6.2\% = 1.2\%$. This is the order of magnitude of business cycle fluctuations. Using non-US data leads to even larger business cycle fluctuations.

We will also show how demand linkages such as Long and Plosser (1982)'s generate an amount of comovement among firms that resembles the one of business cycles. Hence, firm level shocks create both non-trivial aggregate fluctuations, but also comovement. We have all the ingredients we need for a business cycle.

The main theoretical contribution is to break the curse of $1/\sqrt{N}$ diversification. A simple diversification argument shows that, in an economy with N firms with independent shocks, aggregate fluctuations should have a size proportional to $1/\sqrt{N}$. Given modern economies can have millions of firms, this suggests that those idiosyncratic fluctuations will be negligible. Horvath (1998,2000) and Dupor (1999) discuss ways out of this problem based on the sparsity of the input output matrix. We offer a simple alternative solution. When firm size is power law distributed, then conditions under which one derives the central limit theorem break down, and other mathematics (due to Paul Lévy) apply. In the central case of Zipf's law, aggregate volatility scales like $1/\ln N$, rather than $1/\sqrt{N}$. The draconian $1/\sqrt{N}$ diversification is replaced by a much milder one that goes in $1/\ln N$. In an economy with fat tailed distribution of firms, diversification effects due to country size are quite small. Section 6 provide gathers the empirical evidence on this, and is very congruent with the model.

We will present the argument with several degrees of sophistication. Section 2 develops a simple model that can be calibrated. Section 3 provides a calibration that shows that our effects of the right order of magnitude to account for macroeconomic fluctuations. Section 4 shows directly that the idiosyncratic movements of firms appear to explain, year by year, a good fraction (40%) of actual fluctuations in GDP and the Solow residual. Section 5 revisits how demand linkages can in turn create comovements. Section 6 discusses some extensions.

1.1 Related literature

1.1.1 Macroeconomics

A few papers have proposed way to generate macro shocks from purely micro shocks. A pioneering paper is Jovanovic (1987), which we discuss in section 2.2. It relies on an extremely large multiplier M that has an order of magnitude of 1000 – the square root of the number of firms in the economy. This high multiplier has proved an obstacle of the Jovanovic model by macroeconomists. Different routes were explored by very innovative papers, Durlauf (1993) and Bak et al. (1993). Durlauf (1993) generates macroeconomic uncertainty with idiosyncratic shocks and local interactions between firms. The action comes from the non-linear interactions between firms, while in our paper the core comes from the skewed distribution of firms. Durlauf's model is analytically difficult, and we suspect that embedding our power law distributed firm in his models could be quite interesting. This is difficult to do at this point. Bak et al. (1993) explore self-organizing criticality [3]. While we have much sympathy for their approach (which is very different from ours), their model generates fluctuations that are probably "too fat tailed": they have a power law exponent of 1/3, so that fluctuations don't even have a mean, much less a variance. Nirei (2003) proposes an elaborate model whose spirit is related to Bak et al. 1993, and finds fluctuations with a power law exponent 1/2.

Long and Plosser (1983) worked out the view that sectoral (rather than firm) shocks might account for GDP fluctuations. As their model has a small number of sectors, those shocks can be viewed as mini aggregate shocks. Horvath (1998, 2000) and Conley and Dupor (2003) explore this hypothesis further. They find that sector-specific shock are an important source of aggregate volatility. Studies disagree somewhat on the share of sector specific shocks, aggregate shocks, and complementarities. Shea (2002) quantifies that complementarities play a major role in aggregate business cycle fluctuations. Caballero, Engel and Haltiwanger find that aggregate shocks are important (1997), while Horvath (1998) find that sector-specific shocks go a long way to explain aggregate disturbances. Finally Horvath (1998,2000) and Dupor (1999) debate about whether N sectors can have a volatility that does not decay in $N^{-1/2}$. We find an alternative solution to their debate. This solution, formalized in Proposition 2, is that firm size distribution is very skewed[4], that a few large firms dominate the economy. Also, we propose that thinking about firms might be a useful way to think about the world. Many "industry shocks" originate in the decision of one large firms (Toyota, WalMart, IBM) to introduce a radical innovation. The shocks are also easier to explain: they are the fruit of R&D efforts, and bets on the organization of production.

1.1.2 Power laws in economics

A growing number of economic variables appear to follow power laws. The earliest is the distribution of incomes (Pareto, 1896). Many power laws have an exponent 1, i.e. they follow Zipf's law. A number of economic systems appear to follow Zipf's law: cities (Zipf 1949, Gabaix and Ioannides 2004), firms (Axtell 2001, Okuyama et al. 2003), mutual funds (Gabaix, Reuter and Ramalho 2003), web sites (Barabasi and Albert 1999). Gabaix (1999) provides an explanation and a survey of the literature. Stock market fluctuations also follow power laws. Intriguingly, the exponent is typically either 3 or 3/2. Gabaix et al. (2003, 2004) survey and propose an explanation for a series of puzzling facts on the distribution of stock market returns. They base their explanation on the power law distribution of large traders. This is analogous to the way this paper bases GDP fluctuations on a power law distribution of large firms.

2 The essence of the idea

2.1 A simple "islands" economy

To illustrate the idea, we consider a very simple economy, composed of N firms that are independent islands with no feedback[5]. In this economy there are only idiosyncratic shocks to

[3] Also see the pedagogical version in Scheinkman and Woodford (1994).

[4] Canals et al. (2004) find that this is particularly true for the exports, whose distribution are extremely skewed. For instance, they find that the root-Herfindahl of exports is about 50%.

[5] Appendix A fleshes out such a model.

firms. We study its aggregate volatility. We call this volatility the GDP volatility coming from idiosyncratic shocks, σ_{GDP}. Say that firm i produces S_{it}. In a year t, it has a growth rate:

$$\frac{\Delta S_{i,t+1}}{S_{i,t}} = \frac{S_{i,t+1} - S_{it}}{S_{it}} = \sigma_i \varepsilon_{i,t+1} \tag{2}$$

where σ_i is firm i's volatility and the $\varepsilon_{i,t+1}$ are independent random variables with mean 0 and variance 1. Total GDP is:

$$Y_t = \sum_{i=1}^{N} S_{it} \tag{3}$$

and GDP growth is:

$$\frac{\Delta Y_{t+1}}{Y_t} = \frac{1}{Y_t} \sum_{i=1}^{N} \Delta S_{i,t+1} = \sum_{i=1}^{N} \sigma_i \frac{S_{it}}{Y_t} \varepsilon_{i,t+1}.$$

As the shocks $\varepsilon_{i,t+1}$ are uncorrelated, the variance of GDP growth is:

$$\sigma_{GDP}^2 = var \frac{\Delta Y_{t+1}}{Y_t} = \sum_{i=1}^{N} \sigma_i^2 \left(\frac{S_{it}}{Y_t} \right)^2.$$

The volatility of GDP fluctuations coming from the idiosyncratic micro shocks are

$$\sigma_{GDP} = \left(\sum_{i=1}^{N} \sigma_i^2 \cdot \left(\frac{S_{it}}{Y_t} \right)^2 \right)^{1/2}. \tag{4}$$

Hence the variance of GDP, σ_{GDP}^2, is the weighted sum of the variance σ_i^2 of idiosyncratic shocks with weights equal to $\left(\frac{S_{it}}{Y_t} \right)^2$, the squared share of output that firm i accounts for. We shall use equation (4) throughout the paper.

If the firms all have the same volatility $\sigma_i = \sigma$, we get the following simple identity:

$$\sigma_{GDP} = \sigma h \tag{5}$$

with

$$h = \left[\sum_{i=1}^{N} \left(\frac{S_{it}}{Y_t} \right)^2 \right]^{1/2}. \tag{6}$$

h is the square root of the Herfindahl of the economy. For simplicity, we call it the "herfindahl" of the economy.

In the body of this paper, we work with the "bare-bones" model (2)-(3). This can be viewed as the linearization of a host of richer models. We present such a model in Appendix A. Our arguments apply if feedback mechanisms are added, as we do in section 24.1. We take advantage of the high tractability and portability of the simple model.

2.2 The $1/\sqrt{N}$ argument for the appeal to aggregate shocks

First, we briefly recall the reason why macroeconomics usually appeals to common (or at least sector-wide) aggregate shocks. With a large number of firms N, one could expect the sum of their σ_{GDP} shocks to be vanishingly small. Indeed, take firms of initially identical size equal to $1/N$ of GDP, and identical standard deviation $\sigma_i = \sigma$. Then (5)-(6) gives:

$$\sigma_{GDP} = \frac{\sigma}{\sqrt{N}}.$$

To get an idea of the order of magnitude delivered by this view, we take an estimate of firm volatility $\sigma = 20\%$ from Appendix B, and consider an economy with $N = 10^6$ firms[6]. We get

$$\sigma_{GDP} = \frac{\sigma}{\sqrt{N}} = \frac{20\%}{10^3} = 0.02\% \text{ per year.}$$

This theoretical annual GDP volatility of 0.02% is just too small to account for the empirically measured size of macroeconomic fluctuations. This is why economists typically appeal to aggregate shocks.[7] We will see that in fact this argument will fail, because large firms in modern economies have a size much bigger than $1/N$. Before we do that, we show that more general modeling assumptions predict a $1/\sqrt{N}$ scaling.

Proposition 1 *Consider an islands economy with N firms whose sizes are drawn from a distribution with finite variance. Suppose that they all have the same volatility σ. Then the economy's GDP volatility is:*

$$\sigma_{GDP} = \frac{E\left[S^2\right]^{1/2}}{E\left[S\right]} \frac{\sigma}{\sqrt{N}}.$$

Proposition 1 should be contrasted to Proposition 2 below. Its proof is in Appendix D.

We now show how a different model of the size distribution of firms leads to dramatically different results.

2.3 The $1/\sqrt{N}$ argument breaks down with power law firms

2.3.1 Empirical evidence shows that the distribution of firms has fat tails

A long literature establishes that the distribution of firm sizes (sales, assets, or number of employees give the same results) is very skewed. A good model parametrization is a power law distribution:

$$P\left(S > x\right) = a x^{-\zeta}. \tag{7}$$

for $x > a^{1/\zeta}$. To estimate this, it is useful to take the density:

$$f\left(x\right) = \frac{\zeta a}{x^{\zeta+1}}$$

and its logarithm:

$$\ln f\left(x\right) = -\left(\zeta + 1\right) \ln x + C \tag{8}$$

where C is a constant. A long literature has estimated the size distribution of firms, but typically the sample would include only firms listed in the stock market. Axtell (2001) breaks new ground by using the Census, which lists all the U.S. firms.

[6] Axtell (2001) reports that in 1997 there were 5.5 million firms in the United States.

[7] One way around this has been proposed by Jovanovic (1987), who observes that when the multiplier is very large ($1/\left(1-\lambda\right) = M \sim \sqrt{N}$, so $1 - \lambda \sim 1/\sqrt{N}$), we get non-vanishing aggregate fluctuations. The problem is that empirically, such a large multiplier (of order of magnitude $\sqrt{N} \simeq 1000$) is very implausible. The impact of government purchases or trade shocks, for instance, would be much higher than we observe. Hence most economists do not think plausible the "extremely large multiplier" route.

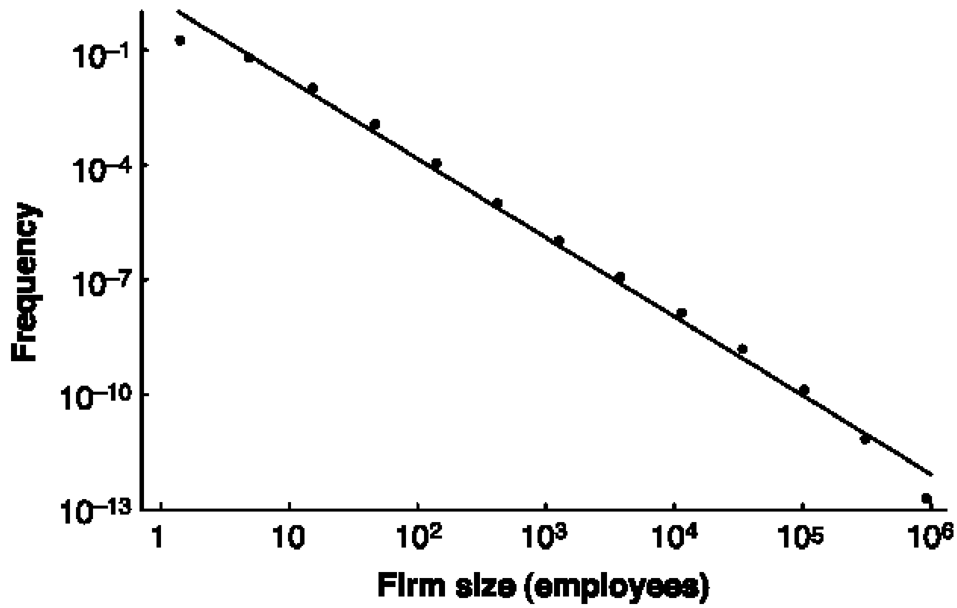

Figure 2: Log frequency $\ln f(S)$ vs log size $\ln S$ of U.S. firm sizes (by number of employees) for 1997. OLS fit gives a slope of 2.059 (s.e.= 0.054; R^2 =0.992). This corresponds to a frequency $f(S) \sim S^{-2.059}$, i.e. a power law distribution with exponent $\zeta = 1.059$. This is very close to Zipf's law, which says that $\zeta = 1$. Source: Axtell (2001).

We reproduce his[8] plot of (8) in Figure 2. The horizontal axis shows $\ln x$, where x is the size of a firm in number of employees. The vertical axis shows the log of the fraction of firms with size x, $\ln f(x)$. One expects to see a straight line in the region where (8) holds, and indeed the Figure shows a very nice fit. An OLS fit of (8) yields an $R^2 = 0.992$, and a slope $= -2.059$, with a standard error of 0.054. This yields an estimate of $\zeta = 1.059 \pm 0.054$.

In the rest of the paper we will often take the approximation $\zeta = 1$, the "Zipf" value. This value ($\zeta \simeq 1$) is often found in the social sciences, for instance in the size of cities (Zipf 1949), and the in the amount of assets under management of mutual funds (Gabaix, Ramalho and Reuter 2003). The origins of this distribution are becoming better understood (see Gabaix (1999), and Gabaix and Ioannides (2004) for a survey of various candidate explanations).

The power law distribution (7) has fat tails, and thus produces some very large firms. We look at the implications for GDP fluctuations in the next section.

2.3.2 GDP volatility when the volatility of a firm does not depend on of its size

Proposition 1 does not address what happens when the variance of sizes is infinite. More precisely, the empirical distributions we find, with power laws $\zeta < 2$, have infinite variance. The next Proposition examines what happens in that case of a "fat tailed" distribution of firms. Its proof is in Appendix D.

Proposition 2 *Consider an islands economy with N firms that have power law distributions*

[8] Okuyama et al. (1999) also find that $\zeta \simeq 1$ for Japanese firms.

(7) with exponent $\zeta \in [1,2)$ and volatility σ. Then its GDP volatility is:

$$\sigma_{GDP} \sim \frac{v_\zeta}{\ln N}\sigma \text{ for } \zeta = 1$$
$$\sigma_{GDP} \sim \frac{v_\zeta}{N^{1-\frac{1}{\zeta}}}\sigma \text{ for } 1 < \zeta < 2$$

where v_ζ is a random variable that is independent of N and σ.

The main conclusion is that if firms have fat tails, σ_{GDP} decreases as $N^{-\beta}$ for $0 \leq \beta < 1/2$, and thus decays much more slowly than $N^{-1/2}$. In the Zipf limit $\zeta = 1$, we get $\beta = 0$, and the decay is barely perceptible[9].

2.3.3 GDP volatility when the volatility of a firm depends on its size

This section completes the theoretical picture, but in the first reading we recommend the reader skip to section 3.

We just understood the benchmark case where all firms have the same volatility σ. We now turn to the case where the volatility decreases with size, which seems to be the case empirically. We examine the functional form suggested by the empirical discussion in section 6.1

$$\sigma^{\text{Firm}}(S) = kS^{-\alpha} \tag{9}$$

for $\alpha \geq 0$.

Proposition 3 *Take an islands economy with N firms that have power law distributions $P(S > x) = x^{-\zeta}$ for $\zeta \in [1, \infty]$. Assume that the volatility of a firm of size S is*

$$\sigma^{Firm}(S) = kS^{-\alpha} \tag{10}$$

for some $\alpha \geq 0$. Then, GDP fluctuations have the form:

$$\frac{\Delta Y_t}{Y_t} = kN^{-\alpha'} g_t \tag{11}$$

with

$$\alpha' = \min\left(\frac{1}{2}, \frac{\alpha + \zeta - 1}{\zeta}\right) \tag{12}$$

and g_t is a symmetrical Lévy stable distribution with exponent min $\{\zeta/(1-\alpha), 2\}$.

In particular, the volatility $\sigma(S)$ of GDP decreases in a power law fashion as a function of its size S [10]:

$$\sigma^{\text{GDP}}(S) \sim S^{-\alpha'}. \tag{13}$$

[9] If there are N identical firms, $1/h_N^2 = N$. So $1/h_N^2$ reveals the "effective" number of firms in the economy, for diversification purposes. So, in a Zipfian world (where $\zeta = 1$), the effective number of firms is not N but $(\ln N)^2$. For $1 < \zeta < 2$, the effective number of firms scales as $N^{2-2/\zeta}$. This notion of the "effective" number of firms is important as long as diversification plays a role, as is the case in Caballero and Engel (2004) and the present paper.

[10] In this paper, $f(S) \sim g(S)$ for some functions f, g, means that the ratio $f(S)/g(S)$ tends, for large S, to a positive real number. So f and g have the same scaling "up to a constant real factor".

Corollary 4 *(Similar scaling of firms and countries). For $\zeta = 1$ and $\alpha \leq 1/2$, we have $\alpha' \simeq \alpha$, i.e. firms and countries should see their volatility scale with a similar exponent:*

$$\sigma^{Firms}(S) \sim \sigma^{GDP}(S) \sim S^{-\alpha}$$

In section 6.1, we will present some evidence that the above prediction holds. The above Propositions indicate that the volatility could decay very slowly with size. In the next section we examine whether these effects are large enough.

3 Empirical evidence on concentration and firm-level volatility

3.1 Firm-level volatility

Most estimations of plant-level or firm-level volatility find very large volatilities σ, with an order of magnitude $\sigma = 30\%$ to $\sigma = 50\%$ per year. Appendix B reviews the evidence. For instance, the volatility of firm size in Compustat is a very large 40% per year. Much of the work has been done on the median firm, rather than on large firms. Hence in this section we look at the volatility of large firms, namely the top 100 non-oil firms each year.

Measuring the volatility of a firm is difficult, because various frictions make imperfect the link between changes in core productivity and short term changes in observables fundamental. So one simple measure of the volatility of a firm is the volatility of the stock market returns. If a firm produces a_{it} per year, of which a fraction f is paid in dividens, and the dividend grows at a rate μ, then the Gordon formula predicts a stock price $p_t = a_t f / (R - \mu)$, where R is the discount rate. In particular, the variance of returns is equal to the variance of productive capacity a. We find an average annualized volatility of idiosyncratic returns $\sigma = 27\%$.[11]

A next measure is the volatility of two measures of growth rates: $\Delta \ln (\text{Sales}_{it}/\text{Employees}_{it})$, and $\Delta \ln\text{Sales}_{it}$. For both, we calculate the cross-sectional variance for each year, among the top 100 firms of the previous year, and take the average.[12] We find a standard deviation of 12% for both measures of growth rate.

Finally, the correlation between growth rates is small. Amongst the top 100 firms, we find a sample correlation of 0.04 for growth rate of the sales per employee, and a correlation of 0.15 for the growth rate of sales.[13]

In conclusion, we find, for the top 100 firms, a volatility of 27% based on the stock price, and a volatility of 12% based on sales and sales / employees. All measures have pros and cons, so in what follows we use a simple average of stock market and same year volatility, which yields $\sigma = 20\%$ per year for firm level volatility.

3.2 Herfindahls and induced volatility

We now examine the theoretically appropriate measure of the size of firms. The key is given by a theory of Hulten (1978), which shows that the sales, rather than value added, is the appropriate measure. Indeed, suppose a competitive economy with several competitive firms or sectors, and

[11] The volatility of returns without removing the market component is 31%. The volatility of a stock price is mostly idiosyncratic.

[12] For each year, we measure the cross-sectional variance of growth rates, $\sigma_t^2 = K^{-1} \sum_{i=1}^{K} g_{it}^2 - \left(K^{-1} \sum_{i=1}^{K} g_{it}\right)^2$, with $K = 100$.

[13] For each year, we measure the sample correlation $\rho_t = \left[\frac{1}{K(K-1)} \sum_{i \neq j} g_{it} g_{jt}\right] / \left[\frac{1}{K} \sum_i g_{it}^2\right]$, with $K = 100$.

that firm i has a Hicks-neutral productivity growth $d\pi_i$. Hulten (1978) shows that the increase in GDP is:

$$\frac{d\text{GDP}}{\text{GDP}} = \sum_i \frac{\text{Sales of firm } i}{\text{GDP}} d\pi_i \tag{14}$$

The weights add up to more than 1. This reflects the fact that productivity growth in a firm generate an increase in the social value of all the inputs it uses. The firms' sales are the proper statistics for that social value. For clarity, Appendix E offers a simple derivation of Hulten's theorem, and shows that the result holds under weaker conditions that Hulten's.

We now draw the implications for the volatility of GDP. Suppose productivity shocks $d\pi_i$ are i.i.d. with standard deviation σ_π. Then, the variance of productivity growth is:

$$var \frac{d\text{GDP}}{\text{GDP}} = \sum_i \left(\frac{\text{Sales of firm } i}{\text{GDP}} \right)^2 var\,(d\pi_i)$$

so

$$\sigma_{GDP} = h_S \sigma_\pi \tag{15}$$

where h_S is the Sales herfindahl:

$$h_S = \left(\sum_{i=1}^{N} \left(\frac{\text{Sales}_{it}}{\text{GDP}_t} \right)^2 \right)^{1/2}. \tag{16}$$

Hulten's theorem allows us to simplify a lot the analysis. For the total volatility, one does not need to know the details of the input-output matrix. The sales herfindahl is the sufficient statistics.

We also report the "workforce herfindahl" h_W:

$$h_W = \left(\sum_{i=1}^{N} \left(\frac{\text{Workforce}_{it}}{\text{Total workforce}_t} \right)^2 \right)^{1/2} \tag{17}$$

It is less well motivated theoretically, but may be useful.

We get our herfindahls from Acemoglu, Johnson and Mitton (2004), who analyze the Dun and Bradstreet data. This data has a good coverage of the major firms for many countries. It is not without problems, but at least it provides an order of magnitude for the empirical values of the herfindahls.

		All Countries	Rich Countries	USA
Sales herfindahl	h_S	22.0	26.6	6.1
Workforce herfindahl	h_W	3.8	4.0	1.2
GDP volatility induced by idiosyncratic firm-level shocks	$\sigma_{GDP} = \sigma h_S$	4.4	5.2	1.2

Table 1: Sales herfindahl h_S and Workforce herfindahl h_W (Eqs.16–17) in 2002. Units are %. Rich countries are the countries with GDP per capita greater than \$13,000. For the induced GDP volatility, we use take $\sigma_{GDP} = \sigma h_S$, with a firm-level volatility $\sigma = 20\%$. See Eq. 15. Source: Acemoglu, Johnson and Mitton (2004) for the international data, and Compustat for the USA data.

As seen above, a good estimate for the firm-level volatility is $\sigma = 20\%$. Table 1 displays the results. We see that the sales herfindahl h_S is quite large: $h_S = 22\%$ for all countries, and $h_S = 6.1\%$ for the USA. By Eq. 15 this corresponds to a GDP volatility

$$\sigma_{GDP} = 20\% \times 6.1\% = 1.2\%$$

for the USA, and $\sigma_{GDP} = 20\% \times 22\% = 4.4\%$ for a typical country. This is very much in the order of magnitude of GDP fluctuations. As shown in Section 24.1, feedback mechanisms can increase this estimate. We conclude that *idiosyncratic volatility is quantitatively large enough to explain macroeconomic volatility.*

4 Using idiosyncratic movements to explain aggregate fluctuations

4.1 The granular residual

We now present tentative evidence that the idiosyncratic movements of the top 100 firms affect explain a large fraction – 40 % – of the movements of total factor productivity (TFP). Doing this exactly right requires vastly better data, and a different focus than this paper. However, we wish to present some suggestive evidence.

We choose a simple proxy for the labor productivity of firm i, sales per worker:

$$z_{it} := \ln \frac{\text{Sales of firm } i \text{ in year } t}{\text{Number of employees in firm } i \text{ and year } t} \tag{18}$$

This proxy is crude, but has the merit of demanding only simple data. It is also more likely to be available with non-U.S. data. than less frugal measures, such that a firm-level Solow residual. In any case, it has good predictive power, so it is helpful to show granular effects in a clear way.

Firm-level productivity growth Δz is affected by a common factor f and an idiosyncratic factor ε

$$\Delta z_{it} = z_{it} - z_{i,t-1} = f_t + \varepsilon_{it} \tag{19}$$

We choose to focus on the $K = 100$ firms that had the largest sales in year $t-1$. The number 100 is largely arbitrary. The size-weighted average productivity growth of large firms is:

$$A_t^S := \left(\sum_{i=1}^{K} \frac{\text{Sales}_{i,t-1}}{\text{GDP}_{t-1}} \right)^{-1} \left(\sum_{i=1}^{K} \frac{\text{Sales}_{i,t-1}}{\text{GDP}_{t-1}} \Delta z_{it} \right) \tag{20}$$

and their equal–weighted average productivity is:

$$A_t^e = K^{-1} \sum_{i=1}^{K} \Delta z_{it} \tag{21}$$

We call their difference the "granular residual" A_t^Δ

$$A_t^\Delta \quad : \quad = A_t^S - A_t^e = \left(\sum_{i=1}^{K} \frac{\text{Sales}_{i,t-1}}{\text{GDP}_{t-1}} \right)^{-1} \left(\sum_{i=1}^{K} \frac{\text{Sales}_{i,t-1}}{\text{GDP}_{t-1}} \left(\Delta z_{it} - \overline{\Delta z_t} \right) \right) \tag{22}$$

$$\overline{\Delta z_t} \quad = \quad K^{-1} \sum_{i=1}^{K} \Delta z_{it} \tag{23}$$

In words, the granular residual is the difference between the size-weighted and the equal-weighted averages of the productivity growth or the top $K = 100$ firms. Thus, it is free of a common component f_t, and reflects how much the very large firms grew, compared to the typical firm. To see this, we observe that under representation (19),

$$\begin{aligned} A_t^{\Delta} &= \left(\sum_{i=1}^{K} \frac{\text{Sales}_{i,t-1}}{\text{GDP}_{t-1}}\right)^{-1} \left(\sum_{i=1}^{K} \frac{\text{Sales}_{i,t-1}}{\text{GDP}_{t-1}} \left(\varepsilon_{it} - \overline{\varepsilon_t}\right)\right) \\ \overline{\varepsilon}_t &= K^{-1} \sum_{i=1}^{K} \varepsilon_{it} \end{aligned} \tag{24}$$

The common shock term disappears. The granular residual A_t^{Δ} reflects only the idiosyncratic movements of large firms. Hence the granular residual is a residual in two senses. It is the residual productivity of large firms, and under the granular hypothesis it is a empirical proxy for the Solow residual of large firms, purged of the aggregate shocks captured by A_t^e.

4.2 The granular residual seems to explain much of the fluctuations in GDP and the Solow residual

As is now clear to the reader, under the granular hypothesis, the idiosyncratic movements of large firms do affect aggregate productivity, and their granular residual affects TFP. To examine the , we use annual U.S. Compustat data from 1951 to 2001. We take the Solow residual from Hall (2004), which itself comes from the Bureau of Labor Statistics. For the granular residual, we take for each year $t-1$ the $K = 100$ largest non-oil firms in Compustat.[14] We regress GDP growth and the Solow residual on the granular residual.

$$\begin{aligned} \text{GDP Growth}_t &= 0.028 + 0.71 A_t^{\Delta} + 0.40 A_{t-1}^{\Delta} \\ & \quad\ (5.0) \qquad (5.6) \qquad\quad (5.0) \\ R^2 &= 37\%. \\ \text{Solow Residual}_t &= 0.0085 + 0.70 A_t^{\Delta} + 0.31 A_{t-1}^{\Delta} \\ & \quad\ (3.5) \qquad (5.5) \qquad\quad (2.7) \\ R^2 &= 40\%. \end{aligned}$$

Table 2: For the year $t = 1952$ to 2001, we regress the Solow Residual, and GDP growth, on the granular residual A_t^{Δ} of the top 100 non-oil firms. The firms are the largest by sales of the previous year. Robust $t-$statistics are in parentheses.

These regressions are heartening support of the granular hypothesis. Again, if the important shocks were only aggregate (f_t in Eq.19), then the fit of the regressions in Table 2 should be nil. This is because common shocks are purged from the granular residual. In particular, it is impossible to explain those results with a representative firm framework, or in a framework in which idiosyncratic shocks cancel out in the aggregate.

We conclude that the idiosyncratic movements of the top 100 firms in the economy, explain a large (40%) fraction of the Solow residual and a large fraction (37%) of GDP fluctuations.

[14] We exclude firms in the oil industry, because the wild swings in oil prices make (18) a too poor proxy of the productivity of oil firms.

4.3 Dissecting the results a bit further

After this main empirical punchline, we look further at the detail of the mechanism. In the "strong version" of the granular hypothesis, equal weighted movements A_t^e in productivity should not even predict the Solow residual – only size weighted movements do. This prediction is confirmed in Table 3. On the other hand, equal weighted movements A_t^e do predict about 15% of GDP, which confirms that indeed they capture some of the economy-wide movements, such as demand factors.

$$
\begin{aligned}
\text{GDP Growth}_t &= 0.021 + 0.18 A_t^e - 0.25 A_{t-1}^e \\
& \quad\ (4.7) \qquad (0.99) \qquad (\text{-}1.95) \\
R^2 &= 15\%. \\
\text{Solow Residual}_t &= 0.014 + 0.014 A_t^e - 0.22 A_{t-1}^e \\
& \quad\ (4.5) \qquad (0.09) \qquad (\text{-}2.3) \\
R^2 &= 10\%.
\end{aligned}
$$

Table3: For the year $t = 1952$ to 2001, we regress the Solow Residual, and GDP growth, on the equal-weighted productivity growth rate A_t^e of the top 100 non-oil firms. The firms are the largest by sales in year $t-1$. Robust $t-$statistics are in parentheses.

5 Enriching the model with demand linkages

5.1 Demand linkages create plausibly strong output comovement

The above calibration showed that idiosyncratic shocks can account to a large aggregate volatility. We provide here some detail about the comovement they imply. Shea (2002) present a series of models that generate comovement. We take his "instantaneous" version of the Long Plosser (1982) model. There are N firms. The representative consumer has utility: $U = \exp \sum_{i=1}^{N} \theta_i \ln C_i$. Firm i produces Q_i with L_i units of labor, and X_{ik} inputs from firm k. The production function is Cobb-Douglas:

$$
Q_i = \lambda_i \exp \left(b \left[(1-\alpha) \ln L_i + \alpha \ln K_i \right] + \sum_k \phi_k \ln X_{ik} \right)
$$

with $1 = b + \sum_k \phi_k$. The clearing constraints are $Q_i = C_i + \sum_k X_{ki}$ and $K = \sum_i K_i, L = \sum_i L_i$, where L is the fixed labor supply. We assume that firms behave competitively[15].

The analysis is standard. The economic importance of firms is captured by

$$
\gamma_i = \frac{\text{Sales of firm } i}{\text{GDP}} = \frac{p_i Q_i}{\text{GDP}} = \frac{\phi_i}{b} + \theta_i
$$

while its share of value added is $L_i / L = b\gamma_i$.

[15] This is to simplify the analysis. Firms could be competitive because markets are contestable (Baumol 1982). Otherwise, our "firms" can be interpreted as "sectors". There is some debate about the size of markups. Basu and Fernald (1997) find markups less than 10%, while other studies find higher markups, and much of macroeconomics uses on zero markups.

Let hats mean proportional changes, i.e. $\widehat{Z} = dZ/Z$. If firm i has a productivity shock $\widehat{\lambda}_i$, then Eq. 14 indicates that GDP increases by:

$$\widehat{Y} = \sum_i \gamma_i \widehat{\lambda}_i. \tag{25}$$

while the production of firm i increases by:

$$\widehat{Q}_i = \widehat{C}_i = \widehat{\lambda}_i + (1-b)\,\widehat{Y} \tag{26}$$

The term $\widehat{Y}$ generates a comovement of between firms. To quantify it, we first interpret b, then quantify it.

To interpret b, imagine that all firms have productivity growth $\widehat{\lambda}_i = \widehat{\Lambda}\%$. Then GDP growth is

$$\widehat{Y} = \widehat{\Lambda} \sum_i \gamma_i = \widehat{\Lambda} \sum_i \frac{\phi_i}{b} + \theta_i = \widehat{\Lambda}\left(\frac{1-b}{b} + 1\right) = \frac{1}{b}\widehat{\Lambda}.$$

So $1/b$ is the "multiplier" of productivity shocks: a uniform 1% increase in productivity translates into a $1/b\%$ increase in GDP.

How big is b empirically? b is 1 minus the share of intermediate inputs ("materials") in the production function. For the U.S., the Jorgensen, Gollop and Fraumeni (1987) data, updated in 1996, gives $b = 0.50$.[16] So we conclude that b is between 0.15 and 0.5. Alternatively, $b = h_W/h_S$ is the ratio of value added to sales of a typical firm, which yields $b = 0.15$. In what follows, we take the conservative estimate of Jorgensen, Gollop and Fraumeni (1987), which gives $b = 0.5$. This translates into a "productivity multiplier" $1/b = 2$.

This allows us to quantify better intensity of the comovement. Shea (2002) proposes a useful measure of comovement. If, by a statistical or mental procedure, we removed the common component of firms firm level volatility would be $\widehat{Q}_i = \widehat{\lambda}_i$ rather than (26). GDP increase would be:

$$\widehat{Y}_{\text{No Cov}} = \sum_i \frac{\text{Value added of firm } i}{\text{GDP}} \widehat{\lambda}_i$$

But with the Long Plosser demand linkages, GDP increase is: $\widehat{Y} = \sum_i \frac{\text{Sales of firm } i}{\text{GDP}} \widehat{Q_i}$. We have $\widehat{Y}_{\text{No Cov}} = b\widehat{Y}$. So the ratio of GDP variance attributed to comovements is:

$$1 - \frac{var\left(\widehat{Y}_{\text{No Cov}}\right)}{var\widehat{Y}} = 1 - b^2 \tag{27}$$

This is the type of ratio that Shea calculates. He finds that 80% to 96% of the variance is due to complementarities. We compare this to what the model predicts. If $b = 0.2$ (resp. if $b = 0.5$), then $1 - b^2 = 96\%$ (resp. 75%) of comovements are attributable to complementarities. We conclude that Long Plosser demand linkages generate enough realistically high comovements between firms.

This section shows that, to analyze the size of complementarities, it is enough (under some conditions) to work with the herfindahls of the economy. One does not need to know the details of the input output matrix. When we use the empirical values for the value added to sales ratio b, we find that the complementarities generated by demand linkages indeed generate a large enough comovement across firms. We conclude that our "granular" hypothesis, when augmented by the Long Plosser model, generates both plausible aggregate fluctuations and comovements between firms.

[16] Susanto Basu kindly provided this number.

5.2 Long Plosser demand linkages create perfect correlation in employment

A prominent feature of business cycles is the large comovement of employment across sector. In a granular world, productivity shocks are specific to a firm. Indeed, so one could think that in such a world, a successful firm gains employment at the expense of unsuccessful firms. We show that, in an instantaneous Long Plosser world at least, this is not the case. In fact, even though the primitive productivity shocks are perfectly *uncorrelated* across firms, the employment shocks are perfectly *correlated*[17].

Proposition 5 *Consider the above Long-Plosser economy, and make labor supply elastic. Call ψ the elasticity of labor supply, and say that the utility is:*

$$U = \exp\left[\sum_{i=1}^{N} \theta_i \ln C_i\right] - L^{1+1/\psi} \tag{28}$$

Then, if firm i has productivity shock $\widehat{\lambda}_i$, TFP growth is:

$$\widehat{\lambda} = \sum_i \gamma_i \widehat{\lambda}_i \tag{29}$$

where γ_i =Sales of firms i / GDP. GDP growth is:

$$\widehat{Y} = \frac{1+\psi}{1+\alpha\psi}\widehat{\lambda}. \tag{30}$$

Employment growth in firm i is:

$$\widehat{L}_i = \frac{1}{\alpha + 1/\psi}\widehat{\lambda} \tag{31}$$

where $1-\alpha$ is the labor share of value added. So even though the primitive productivity shocks $\widehat{\lambda}_i$ are perfectly uncorrelated across firms, the employment shocks $\widehat{L}_i$ are perfectly correlated across firms.

Hats mean proportional changes, e.g.. $\widehat{\lambda} = d\lambda/\lambda$.

Proof. The proof is very simple. Because of the Cobb-Douglas consumption and production structure, labor shares are constant. If total labor supply is L_t, labor employed by sector i is $L_{it} = b\gamma_i L_t$. There is perfect comovement of employment across firms We have $\widehat{L}_{it} = \widehat{L}_t$.

We next derive the expression of fluctuations in aggregate employment L. The production function is CRS in (K, L), and so create an amount of the final consumption good equal to: $BK^{\alpha}L^{1-\alpha}$. When firm i has productivity $\lambda_i = e^{\pi_i}$, the above derivations show that GDP is:

$$Y = A\lambda K^{\alpha} L^{1-\alpha} \tag{32}$$

for some constant A and $\lambda = \exp\left(\sum_i \gamma_i \ln \lambda_i\right)$. Optimal labor supply L maximizes $A\lambda K^{\alpha}L^{1-\alpha} - L^{1+1/\psi}$, and so is: $L = \left[(1-\alpha) A\lambda K^{\alpha}\right]^{1/(\alpha+1/\psi)}$. Taking logs and differentiating, this yields $\widehat{L} = \frac{1}{\alpha+1/\psi}\widehat{\lambda}$. Finally, taking the log of (32) and differentiating gives:

$$\widehat{Y} = \widehat{\lambda} + (1-\alpha)\widehat{L} = \left(1 + \frac{1-\alpha}{\alpha+1/\psi}\right)\widehat{\lambda} = \frac{1+\psi}{1+\alpha\psi}\widehat{\pi} \tag{33}$$

[17] I thank Robert Hall for prompting me to address this question.

As is known, a non-zero elasticity of labor supply ψ increases the impact of productivity growth on GDP to more than more for one. ■

Proposition 5 shows that that Long Plosser demand linkages generate perfect comovement of employment across firms. This is due to the Cobb-Douglas nature of the utility and production functions. The lesson for more general models is that demand linkages easily generate large comovements in employment.

6 Evidence on scalings and distributions

This section examines Proposition 2's predictions for the scaling of country level and firm level quantities. The reader may skip this section in the first reading.

6.1 Scaling of firm-level volatility

Here we summarize some evidence for the scaling of the growth rate of firms (9) and the scaling of GDP growth (13). It has been discussed in a series of papers by Stanley et al. (1996), Amaral et al. (1997), Canning et al. (1998) and Lee et al. (1998). In a nutshell, firms and countries have identical, non-trivial, scaling of growth rates. Stanley et al. (1996) and Amaral et al. (1997) study how the volatility of the growth rate of firms changes with size[18] S . To do this, one divides the firms in a number of bins of sizes S, calculate the standard deviation of the growth rate of their sales $\sigma(S)$, and plots $\ln \sigma(S)$ vs $\ln S$. One finds a roughly affine shape, displayed in Figure 3:

$$\ln \sigma^{\text{firms}}(S) = -\alpha \ln S + \beta. \tag{34}$$

Exponentiation gives (9). A firm of size S has volatility proportional to $S^{-\alpha}$ with $\alpha = 0.15$. This means that large firms have a smaller proportional standard deviation than small firms, but this diversification effect is weaker than would happen if a firm of size S was composed of S independent units of size 1, which would predict $\alpha = 1/2$.

Canning et al. (1999) do the same analyses for country growth rates and find[19] that countries with a GDP of size S also have a volatility of size $S^{-\alpha'}$, with $\alpha' = 0.15$. The two graphs are plotted in Figure 3. The slopes are indeed very similar, and statistical tests reported in Canning et al. (1998) say that one cannot reject the null that $\alpha = \alpha'$. This is particularly interesting in light of Proposition 3 and Corollary 4, which say that this should be the case if Zipf's law holds[20].

One important caveat is in order. The above estimate of α, the scaling exponent of firms, is likely biased upwards. The reason is that it is estimated only with firms in Compustat, i.e. listed in the stock market. For a given size, a firm that is highly volatile is more likely to be in Compustat than a less volatile firm. This effect is weaker for big firms. This implies that the value of α measured in a sample composed only of firms in Compustat is likely to be larger than the true empirical value. So, the empirical value we find is more likely to be an upper bound on

[18] The measure of size can be assets, sales, or number of employees. Those three measures give similar results.

[19] Another way to see their result is to regress:

$$\begin{aligned} \ln \sigma_i &= -\alpha \ln Y_i + \beta \ln \text{GDP/Capita} + \gamma \text{Openness} \\ &\quad + \delta \text{Gvt share of GDP} + \text{constant} \end{aligned}$$

where σ_i is the standard deviation of $\ln Y_{it}/Y_{it-1}$ and Y_i the mean of the Y_{it}. We run this over the top 90% of the countries to avoid the tiniest countries, and find that $\alpha = .15$ with a standard deviation of .015.

[20] Acemoglu and Zilibotti (1997) propose a different mechanism by which large countries are more diversified and have a smaller volatility.

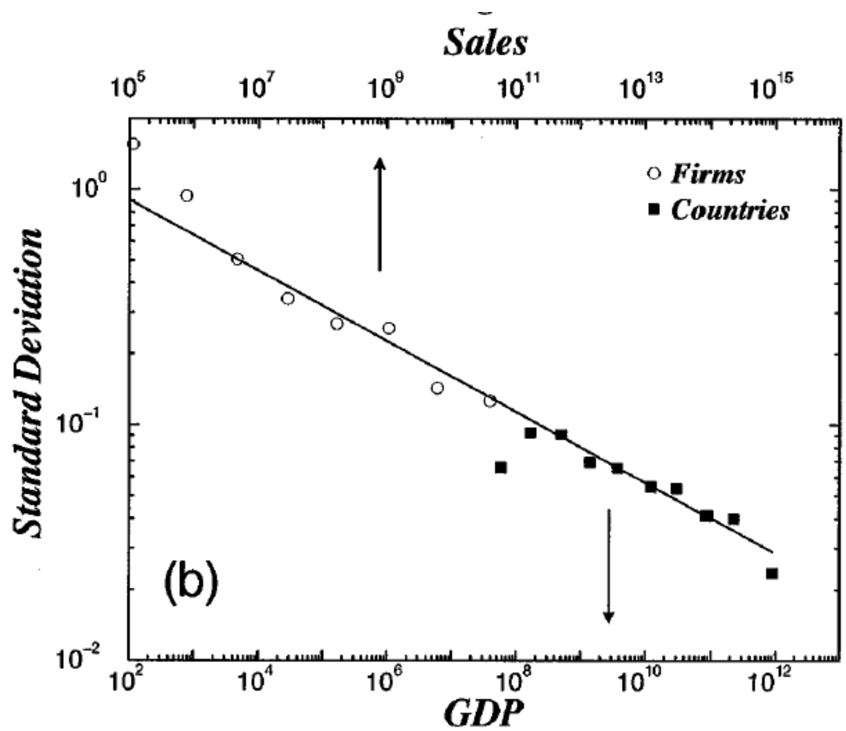

Figure 3: Standard deviation of the distribution of annual growth rates (log log axes). Note that $\sigma(S)$ decays with size S with the same exponent for both countries and firms, as $\sigma(S) \sim S^{-\alpha}$, with $\alpha = .15$. The size is measured in sales for the companies (top axis) and in GDP for the countries (bottom axis). The firm data are taken from the Compustat from 1974, the GDP data from Summers and Heston (1991). Source: Lee et al. (1998).

the true α rather than the true value. The best way to estimate the true value of α would be to run a regression (34) on a sample that includes all firms, not just firms listed in Compustat (Census data, for example lists more firms). It is possible, indeed, that the best value is $\alpha = 0$, as random growth models have long postulated. More research is needed to assess this.

6.2 The distribution of fluctuations in firms and GDP growth

This section examines the prediction of Proposition 3 for the distribution of fluctuations in firms and GDP.

6.2.1 Fluctuations without border effects: Empirical evidence on a Lévy distribution of firms' fluctuations

One can reinterpret Proposition 3 by interpreting a large "firm" as a "country" made up of smaller entities. If those entities follow a power law distribution, then Proposition 3 applies and predicts that the fluctuations of the growth rate $\Delta \ln S_{it}$, once rescaled by $S_{it}^{-\alpha}$, follows a Levy distribution with exponent $min\{\zeta/(1-\alpha), 2\}$. Amaral et al. (1997) and Canning et al. (1998) plot this empirical distribution, and we reproduce their finding in Figure 4. We next compare this graph to Proposition 3's prediction – a symmetrical Lévy distribution with exponent $1/(1-\alpha)$ and $\alpha = 0.15$. Figure 5 draws this distribution ($\ln p(x)$ vs x).We see that the shapes are both much fatter than a Gaussian.

We now investigate the best fist, assuming that the growth rate follows a symmetrical Lévy distribution with exponent β. The Gaussian benchmark corresponds to $\beta = 2$. Calling g_{it} the growth rate of firm i in year t, we transform $\gamma_{it} = A_t g_{it} + B_t$ such that for all t's, $E[\gamma_{it}] = 0$ and Median$(|\gamma_{it}|) = 1$. We plot the distribution of γ_{it}, which is strikingly close to a Lévy with exponent $1/(1-\alpha)$. There are some deviations, for very large $|\gamma|$. Hypothesizing that for $|\gamma_{it}| \leq \overline{\gamma}$, γ_{it} follows a Lévy with exponent β, we estimate β by maximum likelihood. We take $\overline{\gamma} = 10$. As $P(|\gamma_{it}| \leq \overline{\gamma}) = 0.99$ empirically, this means that we fit the 99% of the points. We do this for each year separately, which give us a series of β's. We find:

$$\begin{aligned}
\text{Mean of } \beta &= 1.28 \\
\text{Standard deviation of } \beta_t &= 0.11 \\
\sigma(\beta)/(\text{Number of years})^{1/2} &= 0.016.
\end{aligned}$$

Empirically, we conclude that $\beta = 1.28$ with a standard deviation of 0.016. The prediction is $1/(1-\alpha) = 1.18$ for $\alpha = 0.15$. Thus, the empirical data is fairly close to the theoretical prediction.

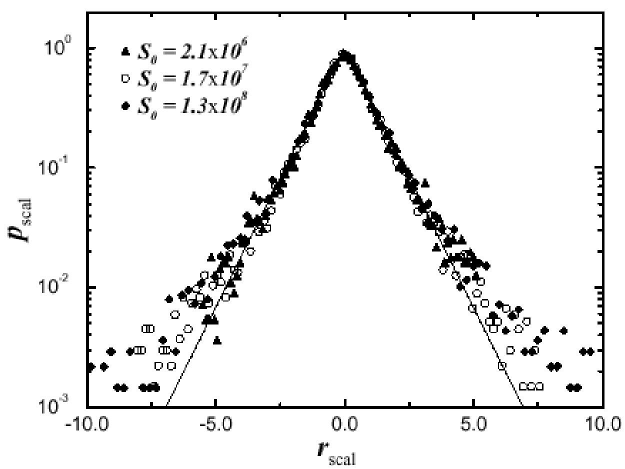

Figure 4: Empirical distribution of the fluctuation of firm sizes. The shape is very similar to that of the Levy distribution predicted by the model (see Figure 5 below). Source: Amaral et al. (1997).

6.2.2 Fluctuations with border effects: Distribution of GDP growth

The above theory needs to be amended slightly for GDP, because typically the largest firm in a country only accounts for a small fraction (say couple of percentage points) of a country's GDP. We speculate that this is because of antitrust concerns.

We now modify the analysis to incorporate this fact. The payoff will be a better prediction of the shape of GDP fluctuations. We adopt the following representation. If we have a country with N firms, the size of firms S_i are drawn from a power law with exponent $\zeta = 1 + \varepsilon$, but with bounded support $[1, mN]$. The density is assumed to be a power law with an exponent ζ

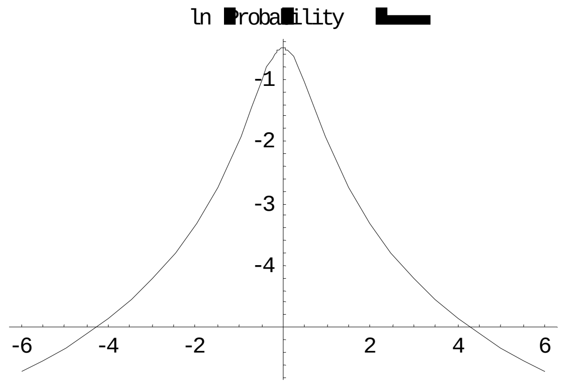

Figure 5: Log of a symmetrical Levy distribution with an exponent of $1/(1-\alpha)$, with $\alpha = 0.15$.

in $[1, mN]$, i.e.:

$$f(S) = \frac{\zeta}{1-(mN)^{-\zeta}} S^{-\zeta-1}.$$

The total size is $Y = \sum_{i=1}^{N} S_i$. We can also establish the distribution of the fluctuations in Y.

Proposition 6 *If the subcomponents cannot have a size bigger than mN, for some finite m, we have, given the standard deviation σ_i of a country, that the fluctuations are normal*

$$\frac{\Delta Y}{Y} =^d Y^{-\alpha} u$$

where u is a normal variable. In particular, if $m < \infty$, all moments are finite. Given only the size Y of the country, the fluctuations have the density:

$$f_{m,\alpha}(g) = \int_0^\infty e^{-\psi_{m,2-2\alpha}\left(k^2/2\right)} \cos(kg) \frac{dk}{\pi} \tag{35}$$

and all the moments are finite. We call this distribution a "modified Levy distribution". In the limit $m \to \infty$, this distribution tends to a symmetrical Lévy distribution with an exponent of $1/(1-\alpha)$. In the limit $m \to 0$, this distribution tends to a Gaussian.

The proof is in Appendix D.

We find a new "universal" distribution that does not depend on the details of the shocks to the individual firms. This is analogous to the fact that in the central limit theorem the limiting distribution does not depend on the details of the distribution of the initial shocks.

We make a few observations on our modified Levy distribution. When $m \to \infty$, there are no restrictions on the support of the subunits, and we get the the Lévy $1/(1-\alpha)$ distribution predicted by Proposition 3. When $m \to 0$, even the largest firms are small (they are bounded above by $mY/E[S]$). Since the total variance is the sum of lots of small variances, the central limit theorem applies, and hence the fluctuations are Gaussian. The proof shows that their order of magnitude is $m^{1/2-\alpha}$.

The empirical distribution is plotted in Figure 6. Figure 7 shows the corresponding theoretical plot for the distribution of growth rates. We see that the two distributions are quite close.

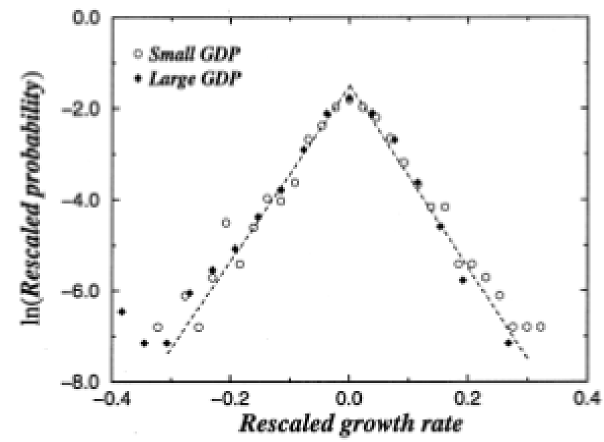

Figure 6: Empirical distribution of GDP fluctuations. Source: Canning et al. (1998)

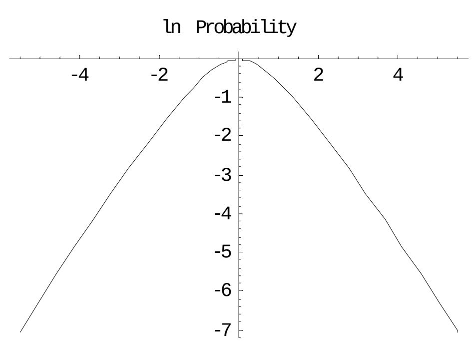

Figure 7: ln(Probability of a growth rate g) vs g under the null of the modified Lévy distribution predicted by the model (with parameters $2-2\alpha=1.7$ and $m=1$).

7 Conclusion

There are clearly "macroeconomic" shocks: monetary policy shocks, policy shocks, trade (e.g. exchange rate) shocks, and possibly aggregate productivity shocks. However, is it possible that, though they are the most visible ones, they are not the major contributors to GDP fluctuations. The present paper lays down the theoretical possibility that idiosyncratic shocks are an important, and possibly the major, part of the origins of business cycle fluctuations.

It may be worthwhile to contemplate the possible consequences of the hypothesis that idiosyncratic shocks to large firms are an important determinant of the volatility of aggregate quantities.

First, one may understand the origins of fluctuations better: they do not come from mysterious "aggregate productivity shocks," but from concretely observable shocks to the large players, such as Wal-Mart, Intel, and Nokia.

Second, these shocks to large firms, initially independent of the rest of the economy, offer a rich source of shocks for VARs and impulse response studies – the real-side equivalent of the "Romer and Romer" shocks for monetary economics. For instance, a strike, or the tenure of a new CEO could be a source of for a macroeconomic shocks plausibly independent from the rest of the economy.

Third, this gives a new theoretical angle for the propagation of fluctuations: For instance, if Wal-Mart innovates, its competitors may suffer in the short term, but then scramble to catch-up. This creates rich industry-level dynamics (that are already actively studied in IO) should be very useful for studying macroeconomic fluctuations, since they allow to trace the dynamics of productivity shocks.

Fourth, this could explain the reason why people, in practice, do not know "the state of the economy" – i.e. the level of productivity, in the RBC language. In our view, this is because "the state of the economy" depends on the behavior (productivity and investment behavior, among others) of many large firms. So the integration is not easy, and no readily accessible single number can summarize this state. This could offer a new and relevant mechanism for the dynamics of "animal spirits".

Finally this mechanism might explain a large part of the volatility of many aggregate quantities such as inventories, inflation, short or long run movements in productivity, and the current account. The latter is explored in Canals et al. (2004).

8 Appendix A: A simple model illustrating the "islands" economy

The paper presents a mechanism that emerges from a variety of economic structures. Here we present one possible type of model that generates the mechanism. Markets are competitive. Firm i has a capital K_{it}. It invests in a technology with random productivity A_{it} such that $E[A_{it}]$ is constant across i's and

$$\sigma(A_{it}) = bK_{ti}^{-\alpha}. \tag{36}$$

A variety of mechanisms (e.g. Amaral et al. (1998), Sutton (2001)) can generate the microeconomic scaling presented in equation (36). These mechanisms typically assume that firms of size S are made up of N smaller units, with $N \sim S^{\alpha/2}$, which generates (9) and (36). Capital is fully reinvested, so that:

$$K_{i,t+1} = A_{i,t+1}K_t. \tag{37}$$

GDP is simply $Y_t = \sum_i A_{i,t}K_{t-1}$.

Adding labor does not change the conclusion of this paper. Suppose that the production function is $A_{i,t}F(K_{ti}, L_{ti})$, with constant returns to scale. Risk neutral firms maximize

$$\max_{L_{it}} E[A_{it}]\, F(K_{ti}, L_{ti}) - w_t L_{ti}.$$

The quantity of labor chosen L_{ti} is $L_{it} = \lambda_t K_{it}$, for a factor of proportionality λ, so that:

$$K_{i,t+1} = A_{i,t+1}F(K_{it}, \lambda_t K_{it}) - w_t\lambda_t K_{it} = (A_{i,t+1}F(1, \lambda_t) - w_t\lambda_t)\, K_{it}.$$

The equation of motion follows the same structure as (37), with random productivity:

$$A'_{it} = A_{i,t+1}F(1, \lambda_t) - w_t\lambda_t.$$

GDP is $Y_t = \sum_i A_{i,t}K_{t-1}F(1, \lambda_{t-1})$ and evolves as the stochastic sum in the paper.

Random growth theories (see Gabaix 1999, Luttmer 2004 and the survey in Gabaix and Ioannides 2004) propose further microfoundations for such a process, derive how it converges to a power law distribution, and work out the conditions under which it leads to Zipf's law.

9 Appendix B: Evidence on firm-level volatility

9.1 Idiosyncratic volatility is very large

Our data on idiosyncratic volatility come from Compustat. For large firms it is likely that Compustat is very representative, as it includes most of these firms. For small firms, Compustat may not be fully representative, just like the stock market may not represent all firms[21]. It is still the best dataset we have so far. Future studies using Census data will give us much better estimates.

As firms can die, some choices have to be made on which firms are included in the sample. Luckily, various specifications give very similar results. The simplest exercise is to follow a set a firms for an extended amount of time. To have good statistics, we need many firms, and thus

[21] Indeed this "selection bias" creates an upward bias in the measurement of α in the microeconomic scaling law (9).

fairly recent data as the Compustat coverage has been growing. Thus, we use all firms for all years from 1980 to 2002. The results do not depend at all on the starting year,1980. Yet, a much earlier starting date would yield too few firms while a much later one would yield too few years. We remove foreign firms and we use reports on sales (data12: sales(net) in MM$). Alternative measures give similar results, as indeed they are proportional in the medium run. We deflate sales using BEA Implicit Price Deflators for Gross Domestic Product (year 2000=100). Thus we have observations from 6155 firms (21016 if we don't remove firms absent in 1980) from 1980 to 2002. This adds up to 76926 (186075 if we don't remove firms absent in 1980) data-points (year-firm) on sales, and 69743 (159660 if we don't remove firms absent in 1980) data-points (year-firm) on the growth rate $g_{it} = \ln(S_{it}/S_{it-1})$.

The raw standard deviation of the g_{it} is 0.442. This means that the standard deviation of the sales of firms in Compustat is 44.2% a year – a very high number. This number is a bit smaller for large firms, according to (9). The average standard deviation is a very similar number, 0.462.

Simple standard deviation	$stddev(g_{it}) = 0.442$
Average standard deviation	$\sqrt{E[\sigma_i^2]} = 0.462$
Absolute deviation	$absdev_{tot} = E[\lvert g_{it} - E[g_{it}]\rvert] = 0.204$
Interquartile range	$IQR_{tot}(g_{it}) = 0.193$

Table: Statistics on the dispersion of growth rates $g_{it} = \ln(S_{it}/S_{it-1})$, where S_{it} are the sales of firm i at time t.

As firms' growth rates have fat tails, the variance might not be a very robust estimator. We look at two robust estimators of deviation. The interquartile range is the value of the 75% percentile minus the 25% percentile of growth rates. It is equal to 0.193. For a Gaussian with standard deviation σ, the interquartile range is 0.675σ, so if the distribution was Gaussian we would infer a standard deviation 0.193/0.675= 0.285. Again, this is a very high dispersion. Our last measure of dispersion is the absolute deviation, which gives 0.204. If the distribution is Gaussian, we would infer a standard deviation of $0.203\sqrt{\pi/2} = 0.254$. As the distribution has tails fatter than a Gaussian, the true standard deviation is higher than those last two values.

We conclude from this analysis that indeed, the typical standard deviation of the growth rate of firm in Compustat is very high, with a point estimate of 0.44% per year, which is robust to a variety of other measures of dispersion.

It is clear that this must be accounted for by idiosyncratic shocks, as the standard deviation of macroeconomic quantities such as GDP growth is much lower. To verify this formally, we run the following regression with fixed effects and AR(1) noise:

$$\begin{aligned} g_{it} &= \alpha_i + f_t + \varepsilon_{it} \\ \varepsilon_{it} &= \gamma \varepsilon_{it-1} + u_{it} \end{aligned}$$

where u_{it} is i.i.d. with mean zero. We find a standard deviation

$$\begin{aligned} \sigma(f_t) &= 0.044 \\ \sigma(\varepsilon_{it}) &= 0.400 \\ \sigma(g_{it} - \alpha_i) &= 0.402. \end{aligned}$$

Hence aggregate shocks account for only 1.25% $(= \sigma(f_t)^2 / \sigma(g_{it} - \alpha_i)^2)$ of the variance of firm growth rate. Likewise, the correlation of the growth rate between two random firms is only 0.012.

9.2 Firm-level scaling

The scaling law says that the growth rate of a firm of size S, in a year t, has a standard deviation:

$$\sigma(S,t) = \text{standard deviation}\left(\ln S_{t+1} - \ln S_t \mid S_t = S\right) = b_t S^{-\alpha_t}. \tag{38}$$

Amaral et al. (1997) present evidence for the scaling law for a particular year t. Here we extend their empirical analysis.

We first proceed with size as a measure of sales. We estimate α_t for each year and plot in the resulting values of α_t in Figure 8. We show here that α_t has remained fairly constant throughout the years. Its mean value is 0.188.

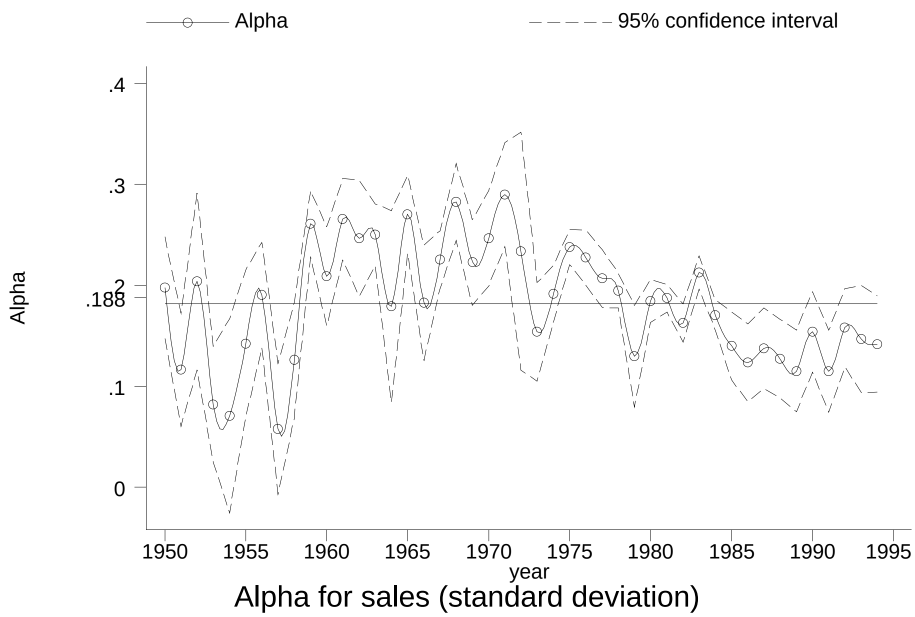

Figure 8: Time series of the scaling exponent α_t for the growth of sales. For each year t we estimate the scaling exponent α_t such that $\sigma(g_t \mid S_t = S) \sim S^{-\alpha_t}$.

Interestingly, the coefficient b_t has increased over the time.

10 Appendix C: Lévy's theorem

The basic theorem can be found in most probability textbooks, e.g. Durrett (1996, p.153).

Theorem 7 *Suppose that $x_1, x_2, \ldots$ are i.i.d. with a distribution that satisfies:*
(i) $\lim_{x\to\infty} P(x_1 > x)/P(|x_1| > x) = \theta \in [0,1]$
(ii) $P(|x_1| > x) = x^{-\zeta} L(x)$
with $\zeta \in (0,2)$ *and* $L(x)$ *slowly varying*[22]*. Let* $s_n = \sum_{i=1}^n x_i$*, and*

$$a_n = \inf\{x : P(|x_1| > x) \leq 1/n\} \text{ and } b_n = nE\left[x_1 1_{|x_1| \leq a_n}\right]$$

[22] $L(x)$ is said to be slowly varying (e.g. Embrechts *et al.* 1997, p.564) if

$$\lim_{x\to\infty} L(tx)/L(x) = 1 \text{ for all } t > 0.$$

Prototypical examples are $L = a$ and $L(x) = a \ln x$ for a non-zero constant a.

As $n \to \infty$, $(s_n - b_n)/a_n \to^d Y$ where Y is a Lévy distribution with exponent ζ.

In practice, for a power law distribution $P(x_1 > x) = (x/x_0)^{-\zeta}$, $a_n = x_0 n^{-1/\zeta}$.

A symmetrical Lévy distribution with exponent $\zeta \in (0,2]$ has the distribution $\lambda(x,\zeta) = \frac{1}{\pi}\int_0^\infty e^{-k^\zeta}\cos(kx)\,dk$ and the cumulative $\Lambda(x,\zeta) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty e^{-k^\zeta}\frac{\sin(kx)}{k}dk$.

For $\zeta = 2$, a Levy distribution is a Gaussian. For $\zeta < 2$, the distribution has power law tail with exponent ζ. Unfortunately, there are no closed form formulae for λ and Λ except in the case $\zeta = 1$ (Cauchy distribution) and $\zeta = 2$ (Gaussian distribution).

11 Appendix D: Longer derivations

11.1 Proof of Proposition 1

Because of $\sigma_{GDP} = \sigma h$, we examine h.

$$N^{1/2}h = \frac{\left(N^{-1}\sum_{i=1}^N S_i^2\right)^{1/2}}{N^{-1}\sum_{i=1}^N S_i}$$

The law of large numbers ensures that $N^{-1}\sum_{i=1}^N S_i^2 \to^{a.s.} E\left[S^2\right]$, and $N^{-1}\sum_{i=1}^N S_i \to^{a.s.} E[S]$. This yields: $N^{1/2}h \to^{a.s.} E\left[S^2\right]^{1/2}/E[S]$.

11.2 Proof of Proposition 2

Because of $\sigma_{GDP} = \sigma h$, we examine h.

$$h = \frac{\left(\sum_{i=1}^N S_i^2\right)^{1/2}}{\sum_{i=1}^N S_i} \tag{39}$$

We treat the cases where $\zeta > 1$ and $\zeta = 1$ separately.

When $1 < \zeta \le 2$. By the law of large numbers,

$$N^{-1}\sum_{i=1}^N S_i \to E[S].$$

However, S_i^2 has power law exponent $\zeta/2 \le 1$, as shown by:

$$P\left(S^2 > x\right) = P\left(S > x^{1/2}\right) = a\left(x^{1/2}\right)^{-\zeta} = ax^{-\zeta/2}.$$

So to handle the numerator of (39), we use Lévy's Theorem from Appendix C. This implies:

$$N^{-2/\zeta}\sum_{i=1}^N S_i^2 \to u$$

where u is a Levy distributed random variable with exponent $\zeta/2$. So

$$N^{1-1/\zeta}h = \frac{\left(N^{-2/\zeta}\sum_{i=1}^N S_i^2\right)^{1/2}}{N^{-1}\sum_{i=1}^N S_i} \to^d \frac{u^{1/2}}{E[S]}.$$

When $\zeta = 1$. Some more care is required, because $E[S] = \infty$. We use Theorem 7, which gives $b_n = n \ln n$, hence:

$$N^{-1}\left(\sum_{i=1}^{N} S_i - N \ln N\right) \rightarrow^d g$$

where g is a Levy with exponent 1. We conclude $\ln N \cdot h \rightarrow^d u^{1/2}/g$.

11.3 Proof of Proposition 3

As $\Delta S_i/S_i = S_i^{-\alpha} u_i$:

$$\frac{\Delta Y_{t+1}}{Y_t} = \frac{\sum_{i=1}^{N} \Delta S_{it}}{Y_t} = \frac{\sum_{i=1}^{N} S_i^{1-\alpha} u_{it}}{\sum_{i=1}^{N} S_i}. \tag{40}$$

By the law of large numbers:

$$N^{-1} Y_t = N^{-1} \sum_{i=1}^{N} S_i \rightarrow \overline{S}.$$

To tackle the numerator, we observe that $S_i^{1-\alpha}$ has power law tails with exponent $\zeta' = \zeta/(1-\alpha)$. We need to consider two cases.

If $\zeta' < 2$, $x_i = S_i^{1-\alpha} u_i$, which has power law tails with exponent ζ', and by Levy's theorem:

$$N^{-1/\zeta'} \Delta Y_t = N^{-1/\zeta'} \sum_{i=1}^{N} S_i^{1-\alpha} u_{it} \rightarrow^d g$$

where g is a Levy with exponent ζ'.

If $\zeta' \geq 2$, $S_i^{1-\alpha} u_i$ has finite variance, and $N^{-1/2} \Delta Y_t \rightarrow^d g$, where g is a Gaussian.

We conclude that in both cases:

$$N^{-\max(1/2, 1/\zeta')} \Delta Y_t \rightarrow^d g$$

for a distribution g. So

$$N^{1-\max(1/2,1/\zeta')} \frac{\Delta Y_{t+1}}{Y_t} \rightarrow^d \frac{g}{\overline{S}} g.$$

We conclude that the Proposition holds, with

$$\begin{aligned} \alpha' &= 1 - \max\left(1/2, 1/\zeta'\right) = 1 + \min\left(-1/2, -1/\zeta'\right) \\ &= \min\left(1/2, 1 - 1/\zeta'\right) = \min\left(1/2, 1 - \frac{1-\alpha}{\zeta}\right). \end{aligned}$$

11.4 Proof of Proposition 6

We start by stating:

Proposition 8 *If the subcomponents cannot have a size bigger than* mN*, for some finite* m*, the variance of* Y *scales as:*

$$\sigma_Y^2 \sim Y^{-2\alpha} V$$

where V *is a random variable whose Laplace transform is:*

$$L^V(k) := E\left[e^{-kV}\right] = e^{-\psi_{m,2-2\alpha}(k)}$$

where $\psi(k)$ *is defined in (44). In the limit* $m \to \infty$*,* V *is a totally positive Lévy distribution with exponent* $1/(2-2\alpha)$.

In particular, all the moments are finite. Indeed, one can easily calculate the cumulants of V (the κ_i such that $-\ln L^V(k) = \sum \kappa_i k^i / i!$) and find:

$$\kappa_i(V) = \frac{m^{\gamma i-1}}{\gamma i - 1}.$$

Recall that the 4 first cumulants $(\kappa_i)_{i=1\ldots4}$ are respectively $\langle V\rangle, varV$, $\left\langle (V-\langle V\rangle)^3\right\rangle$, and $\left\langle (V-\langle V\rangle)^4\right\rangle - 3varV$; i.e. the mean, variance, skewness and excess kurtosis. We proceed to the proof. We define:

$$V_N := \frac{1}{N^{2-2\alpha}} \sum_{i=1}^{N} S_i^{2-2\alpha}$$

where S_i is drawn from the above distribution. We study V_N in the limit of large $N's$. We know from the analysis above, that for $m=\infty$, V_N tends to a Lévy distribution with exponent $1/(2-2\alpha)$.We study its behavior for $m<\infty$. The tool of choice is the Laplace transform (using $\zeta = 1+\varepsilon \simeq 1$)

$$\begin{aligned} L^{V_N}(k) \quad &: = E\left[e^{-kV_N}\right] = E\left[\exp \frac{-k}{N^{2-2\alpha}} \sum_{i=1}^{N} S_i^{2-2\alpha}\right] \\ &= E\left[\exp \frac{-k}{N^{\gamma}} S_i^{\gamma}\right]^N \text{ with } \gamma = 2-2\alpha. \end{aligned} \tag{41}$$

Now

$$\begin{aligned} H \quad &: = E\left[\exp\frac{-k}{N^\gamma}S_i^\gamma\right] = \int_1^{mN} \frac{\zeta}{1-(mN)^{-\zeta}} S^{-\zeta-1} \exp\left(\frac{-k}{N^\gamma}S_i^\gamma\right) dS \\ &= \frac{1}{1-(mN)^{-1}} \int_1^{mN} \exp\left(\frac{-k}{N^\gamma}S_i^\gamma\right)\frac{dS}{S^2} \\ &= \frac{1}{1-(mN)^{-1}} N^{-1} \int_{N^{-\gamma}}^{m^\gamma} \frac{\exp(-kt)}{\gamma t^{1+1/\gamma}} dt \text{ by the change in variables } S = Nt^{1/\gamma}. \end{aligned}$$

As $N\to\infty$, $H \sim N^{-1}\int_{N^{-\gamma}}^{m^\gamma} \frac{dt}{\gamma t^{1+1/\gamma}} \sim 1$. So we use (verifying that $H(k=0)=1$)

$$\begin{aligned} H-1 &= N^{-1}\int_{N^{-\gamma}}^{m^\gamma} \frac{\exp(-kt)-1}{\gamma t^{1+1/\gamma}} dt + o\left(\frac{1}{N}\right) \\ &= -\frac{1}{N}\psi(k) + o\left(\frac{1}{N}\right) \end{aligned} \tag{42}$$

with the new function:

$$\psi_{m,\gamma}(k) := \int_0^{m^\gamma} \frac{1-\exp(-kt)}{\gamma t^{1+1/\gamma}} dt \tag{43}$$

which has a closed form in terms of the incomplete Gamma function $\Gamma(a,z) = \int_0^z e^{-t} t^{a-1} dt$:

$$\psi_{m,\gamma}(k) = -\frac{k^{1/\gamma}}{\gamma}\Gamma\left(-\frac{1}{\gamma}, k\, m^\gamma\right) - m. \tag{44}$$

Finally, Eqs (41) and (42) give, in the limit of large N's:

$$\ln L^{V_N}(k) = N \ln H = N \ln \left(1 - \frac{1}{N}\psi(k) + o\left(\frac{1}{N}\right)\right) = -\psi(k) + o(1).$$

Thus V_N converges in distribution to a well-defined random variable V, whose Laplace transform is: $L^V(k) = e^{-\psi(k)}$.

We can also establish the distribution of the fluctuations in Y. $\frac{\Delta Y}{Y} =^d Y^{-\alpha} V^{1/2} u$ from above. Thus the Fourier transform of the GDP growth is:

$$F(k) = E\left[e^{-ikV^{1/2}u}\right] = E\left[e^{-k^2 V}/2\right] = e^{-\psi\left(k^2/2\right)}$$

so taking the inverse Fourier transform we get (35).

When $m \to \infty$,

$$\psi_{m,\gamma=2-2\alpha}\left(k^2/2\right) \to \int_0^\infty \frac{1-\exp\left(-k^2/2t\right)}{\gamma t^{1+1/\gamma}} dt = \frac{k^{2/\gamma}\Gamma(-1/\gamma)}{2^{1/\gamma}\gamma} = bk^{1/(1-\alpha)}$$

for some b. The characteristic function is that of a symmetric Lévy distribution.

When $m \to 0$,

$$\psi_{m,\gamma}(k) = \int_0^{m^\gamma} \frac{1-\exp(-kt)}{\gamma t^{1+1/\gamma}} dt \sim \int_0^{m^\gamma} \frac{kt}{\gamma t^{1+1/\gamma}} dt = \frac{m^{\gamma-1}}{\gamma-1}k = \frac{m^{1-2\alpha}}{1-2\alpha}k$$

so that $\psi_{m,\gamma}\left(k^2/2\right) \sim \frac{m^{1-2\alpha}}{1-2\alpha}k^2/2$, which shows that $\Delta Y/Y \left(\frac{m^{1-2\alpha}}{1-2\alpha}\right)^{-1/2}$ tends to a standard Gaussian distribution.

12 Appendix E: Hulten's theorem with and without instantaneous reallocation of factors

For clarity, we rederive and extend Hulten (1978)'s result, which says that a Hicks-neutral productivity shock $d\pi_i$ to firm i causes an increase in GDP equal to:

$$\text{GDP growth} = \sum_i \frac{\text{Sales of firm } i}{\text{GDP}} d\pi_i.$$

There are N firms. Firm i produces good i, and uses a quantity X_{ij} is intermediary inputs from firm j. It also uses L_i units of labor, K_i units of capital. It has productivity π_i. If production is: $Q_i = e^{\pi_i} F^i(X_{i1}, ..., X_{iN}, L_i, K_i)$. The representative agent consumer C_i of good i, and has a utility function is $U(C_1, ..., C_N)$. Production of firm i serves as consumption, and intermediary inputs, so: $Q_i = C_i + \sum_k X_{ki}$. The optimum in this economy reads:

$$\max_{C_i, X_{ik}, L_i, K_i} U(C_1, ..., C_N) \text{ s.t.}$$

$$C_i + \sum_k X_{ki} = e^{\pi_i} F^i(X_{i1}, ..., X_{iN}, L_i, K_i); \sum_i L_i = L; \sum_i K_i = K$$

The Lagrangian is:

$$\begin{aligned} W &= U(C_1, ..., C_N) + \sum_i p_i \left[e^{\pi_i} F^i(X_{i1}, ..., X_{iN}, L_i, K_i) - C_i - \sum_k X_{ki}\right] \\ &\quad + w\left[L - \sum_i L_i\right] + r\left[K - \sum_i K_i\right]. \end{aligned}$$

Assume marginal cost pricing[23]. GDP is this economy is $Y = wL + rK = \sum_i p_i C_i$. The value added of firm i is $wL_i + rK_i$, and its sales are $p_i Q_i$.

If each firm i has a shock $d\pi_i$ to productivity, we differentiate the expression of W to find GDP growth:

$$\frac{dW}{W} = \frac{1}{W} \sum_i p_i \left[e^{\pi_i} G^i \left(X_{i1}, ..., X_{iN}, L_i, K_i \right) d\pi_i \right] = \sum_i \frac{\text{Sales of firm } i}{\text{GDP}} d\pi_i,$$

which is Eq. 14.

The above analysis shows that Hulten's theorem holds even if, after the shock, the capital, labor, and material inputs are not reallocated. This is a simple consequence of the envelope's theorem. Hence Hulten's result holds also if there are frictions to adjust labor, capital, or intermediate inputs.

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