

International Trade and Macroeconomic Dynamics with Heterogeneous Firms*

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Abstract

We develop a stochastic, general equilibrium, two-country model of trade and macroeconomic dynamics. Productivity differs across individual, monopolistically competitive firms in each country. Firms face some initial uncertainty concerning their future productivity when making an irreversible investment to enter the domestic market. In addition to the sunk entry cost, firms face both fixed and per-unit export costs. Only a subset of relatively more productive firms export, while the remaining, less productive firms only serve their domestic market. This microeconomic structure endogenously determines the extent of the traded sector and the composition of consumption baskets in both countries. Exogenous shocks to aggregate productivity, sunk entry costs, and trade costs induce firms to enter and exit both their domestic and export markets, thus altering the composition of consumption baskets across countries over time. The microeconomic features have important consequences for macroeconomic variables. Macroeconomic dynamics, in turn, feed back into firm level decisions, further altering the pattern of trade over time. Our model generates deviations from purchasing power parity that would not exist absent our microeconomic structure with heterogeneous firms. It provides an endogenous, microfounded explanation for a Harrod-Balassa-Samuelson effect in response to aggregate productivity differentials and deregulation. In addition, the deviations from purchasing power parity triggered by aggregate shocks display substantial endogenous persistence for very plausible parameter values, even when prices are fully flexible.

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1 Introduction

The study of international macroeconomic dynamics in the context of formal models is usually separated from that of the determinants and evolution of trade patterns. The reason is that the vast majority of macroeconomic models take the pattern of international trade and the structure of markets for goods and factors of production as given.¹ The determinants of trade patterns are the subject of extensive study in trade theory. However, the latter focuses on the microeconomics of trade patterns. Analysis is generally limited to comparison of long-run positions or growth dynamics after changes in one or more determinants of the trade pattern. There is little or no consideration of short- to medium-run business cycle dynamics and how these may trigger changes in trade patterns over such cycles.²

The separation that exists between modern international macroeconomics and trade theory is hard to accept on general equilibrium grounds. Modern international macroeconomics prides itself with being microfounded. Yet, it neglects to analyze how macro phenomena can affect the microeconomic underpinnings of the macro structure. Similarly, much of trade theory fails to recognize that micro dynamics have aggregate effects that can feed back into further micro adjustments over time.

This paper makes a start at bridging the gap between international macroeconomics and trade theory by using Melitz's (2003) model of trade with monopolistic competition and heterogeneous firms as microeconomic underpinning of a two-country, dynamic, stochastic, general equilibrium (DSGE) model of international trade and macroeconomics.³

International macro and trade models often assume monopolistic competition. It is usually the case that firms make profits from selling output in macro models. Nevertheless, there is usually no attention to entry and exit dynamics, and how these can affect the transmission of shocks to the macroeconomy. Once one allows for entry and exit of firms into and from markets over the economic cycle in an international macro model, the trade pattern becomes endogenous, and it is no longer possible to separate macro and micro phenomena in a general equilibrium framework.

We adopt a simplified version of Melitz's (2003) setup, although one that is rich enough to deliver

¹See Lane (2001) for a survey of the recent literature. We discuss the relation between our work and some exceptions to this trend in international macroeconomics below.

²Baldwin and Krugman (1989) is an exception here. They analyze the effects of cycles in exchange rates on trade patterns. Krugman (1994) is a standard reference for modern trade theory.

³Melitz (2003) extends Hopenhayn's (1992*a*, 1992*b*) work to explain the endogenous selection of heterogeneous firms in an industry in a general equilibrium setting under monopolistic competition. His analysis focuses on steady-state effects of trade on intra-industry reallocations and aggregate productivity.

a variety of novel results. We assume that productivity differs across individual, monopolistically competitive firms in each country. Firms face some initial uncertainty concerning their future productivity when making an irreversible investment to enter the domestic market. In addition to the sunk entry cost, firms face both fixed and per-unit export costs. Forward-looking firms formulate entry and export decisions based on expectations of future market conditions. Only a subset of relatively more productive firms export, while the remaining, less productive firms only serve their domestic market. This microeconomic structure endogenously determines the extent of the traded sector and the composition of consumption baskets in both countries. Exogenous shocks to aggregate productivity or entry and trade costs induce firms to enter and exit both their domestic and export markets, thus altering the composition of consumption baskets across countries over time.

We start by introducing this microeconomic structure in a flexible-price model with no international trade in financial assets. This allows us to focus on the role of goods market dynamics in a relatively simple setup. We show that the microeconomic features of our model have important consequences for macroeconomic variables. Macroeconomic dynamics, in turn, feed back into firm level decisions, further altering the pattern of trade over time. Our model generates deviations from purchasing power parity (PPP) that would not exist absent our microeconomic structure with heterogeneous firms. It provides an endogenous, microfounded explanation for a Harrod-Balassa-Samuelson (HBS) effect in response to aggregate productivity differentials and deregulation, which we model as a decrease in the size of sunk entry costs.⁴

Textbook analysis of the HBS effect assumes the exogenous existence of a non-traded sector and a favorable productivity shock to the traded sector alone (Obstfeld and Rogoff, 1996, pp. 210-212). The shock causes the relative price of non-traded goods to increase and the real exchange rate to appreciate as a consequence. An equal increase in productivity in the two sectors has no effect on the real exchange rate. Balassa (1964), Harrod (1933), and Samuelson (1964) first pointed out the tendency for countries with higher productivity in the traded relative to the non-traded sector to have higher prices.

All goods are *tradeable* in our model; some are *non-traded* in equilibrium. More productive firms self-select endogenously into the traded sector in each country, and shocks, such as changes in productivity, are aggregate. But the intuition for an endogenous HBS effect is straightforward.

⁴Blanchard and Giavazzi (2003) consider three proxies for deregulation in a closed economy model: higher substitutability across goods, lower entry costs, lower bargaining power of workers. We compare some of our results for the case of lower entry costs to theirs below.

Our model predicts that more productive economies, or less regulated ones, will exhibit higher average prices relative to their trading partners. This real exchange rate appreciation is driven by the two key new features of our setup: We show that effective labor units must relatively appreciate in the economy providing the more attractive environment for firms. This is a key consequence of introducing firm entry – along with entry costs for the deregulation scenario. (Krugman, 1980, described this effect in his seminal, first new trade theory paper.) Given the existence of a non-traded sector, implied by the fixed export costs, the relative increase in labor costs must induce an appreciation. In addition, changes in relative labor costs further induce changes in the composition of the traded sector in both countries. Import prices in the economy with higher labor costs (say, home) rise as lower productivity foreign firms can now export to it. Conversely, import prices in the economy with lower labor costs decrease as lower productivity home exporters are forced to drop out of the export market. These effects necessarily reinforce the real exchange rate appreciation. Here, the endogenous determination of the traded sector plays a key role.

In addition to endogenizing the HBS effect, we show that the deviations from PPP triggered by aggregate shocks display substantial endogenous persistence for very plausible parameter values, even if prices are fully flexible in our model. More generally, the introduction of micro dynamics motivated by heterogeneity and entry and trade costs significantly improves the ability of the model to generate endogenously persistent dynamics – a stumbling bloc for many well-known DSGE macro models.

When we remove the assumption of financial autarky and allow for international trade in bonds, the model predicts that more productive economies, or less regulated ones, will run persistent foreign debt positions to finance faster entry of new firms into a relatively more favorable environment. This combines with an endogenous HBS effect to deliver dynamics of foreign debt and the real exchange rate that are consistent with stylized facts for the United States in the 1990s.

A stochastic exercise shows that the model matches several important moments of the U.S. and international business cycle quite well for reasonable assumptions about parameters and productivity. Standard international real business cycle (RBC) models, such as Backus, Kehoe, and Kydland’s (1992), fail to generate the observed, positive GDP correlation across countries. Backus, Kehoe, and Kydland’s model also generates higher cross-country correlation of consumption than aggregate output, while the opposite is true in the data (the consumption-output anomaly). Risk-sharing under complete asset markets automatically yields perfect correlation between the consumption differential across countries and the real exchange rate in Backus and Smith (1993), who

document evidence that there is no systematic relation between the real exchange rate and relative consumption in the data (the consumption-real exchange rate anomaly). Our model generates positive GDP correlation across countries, it does not automatically produce high correlation between relative consumption and the real exchange rate, and it improves substantially on the traditional RBC model as far as the consumption-output anomaly is concerned. These results confirm Obstfeld and Rogoff's (2001) finding that trade costs help explain international macroeconomic puzzles.

The assumption that firms pay fixed export costs that do not vary with export volume is clearly central to our results. Recent micro-level empirical studies have documented the importance and relevance of sunk market entry costs in explaining firm export behavior. Recent empirical studies have also overwhelmingly substantiated the intra-industry selection mechanism of firms into export markets that is at the heart of micro-level dynamics in our model. Bernard and Jensen (2001) (for the U.S.), Bernard and Wagner (2001) (for Germany), Das, Roberts, and Tybout (2001) (for Colombia), and Roberts and Tybout (1997*a*) (for Colombia) all estimate substantial magnitude of sunk export market entry costs, important enough to generate large hysteresis effects associated with a firm's export market participation. At the same time, there is evidence that endogenous changes in the composition of the traded sector take place over the normal length of a business cycle. Bernard and Jensen (2001) study a panel of U.S. manufacturing plants between 1987 and 1997. They find that, on average, 13.9 percent of non-exporters begin to export in any given year during the sample, and 12.6 percent of exporters stop.

Interviews with managers making export decisions confirm that firms in differentiated product markets face significant fixed costs associated with entry in export markets (Roberts and Tybout, 1997*b*): A firm must find and inform foreign buyers about its product and learn about the foreign market. It must research the foreign regulatory environment and adapt its product to ensure conformity to foreign standards (including testing, packaging, and labeling requirements). An exporting firm must also set up new distribution channels in the foreign country and conform to all the shipping rules specified by the foreign customs agency. Governments often manipulate some of these costs to erect non-tariff barriers to trade. Regardless of their origin, these costs are most appropriately modeled as independent of the firm's export volume decision. We assume fixed, per-period export costs rather than sunk export market entry costs to keep our model simple. This implies that there is no hysteresis in entry and exit decisions in export markets.⁵ Yet, fixed, per-period export costs are sufficient to deliver endogenous selection of the most productive firms into

⁵In Melitz (2003), entry costs in the export markets are sunk after a firm has observed its productivity.

export markets.⁶ The sunk nature of entry costs into the industry in our model then contributes to the persistence of dynamics.

As far as bringing together trade theory and macroeconomics, Dornbusch, Fischer, and Samuelson (DFS, 1977) is probably the best known antecedent of our work.⁷ Our model differs from DFS in several respects. Essentially, DFS is a model of inter-sectoral specialization based on technology differences. Melitz's (2003) is a model of intra-sectoral specialization. Even if we think of DFS as a model of intra-sectoral specialization, there is no entry cost and product variety is fixed in that model (there is no entry or exit of "firms" – where now a "firm" is a sector in DFS). The measure of technology is relative in DFS, in the sense that a "good firm" in one country is a "bad firm" in another. In contrast, a good technology draw is not relative in our setup. Finally, in DFS, "firms" in both countries compete to produce the same good (hence, only one survives). In our model, firms never compete to produce the same good.

Baldwin and Krugman's (1989) model is closer to ours in spirit. They analyze how cycles in nominal exchange rates can have long-lasting effects through their impact on trade patterns. Their model features entry and exit decisions as a key mechanism of transmission. However, theirs is a partial equilibrium, stylized framework. We highlight important trade and macro consequences of general equilibrium dynamics – and leave the introduction of price stickiness for future work.⁸

More recent contributions to the international macroeconomic literature have emphasized the role of trade costs and composition effects in the propagation of shocks in open economies. Backus, Kehoe, and Kydland (1992) show that the inclusion of trade frictions in the form of a quadratic resource cost of movements in the trade balance improves the quantitative performance of a two-country, international, RBC model. Obstfeld and Rogoff (2001) present simple models in which the addition of per-unit trade costs and the potentially endogenous nature of tradedness help explain

⁶Aw, Chung, and Roberts (2000) (for Taiwan), Bernard and Jensen (1999) (for the U.S.), and Clerides, Lach, and Tybout (1998) (for Colombia, Mexico, and Morocco) all find evidence that more productive firms self-select into export markets. Bernard and Jensen (1999), Clerides, Lach, and Tybout (1998), Roberts and Tybout (1997a), and Roberts, Sullivan, and Tybout (1995) all introduce a fixed export cost into the theoretical sections of their work to explain the self-selection of firms into the export market. However, they do so in partial equilibrium models with fixed distribution of firm productivity levels.

⁷See also Obstfeld and Rogoff (1996, pp. 235-257) on the DFS model. Kehoe and Ruhl (2002) develop a version of the model suitable for calibration to match observed post-trade-liberalization export growth on the intensive and extensive margins. Kraay and Ventura (2002) extend the DFS model to allow for international trade in a complete set of Arrow-Debreu securities and study the consequences of trade integration on the dynamics of the trade balance.

⁸Ricci (1997) develops a two-country, monopolistic competition model with nominal rigidity in which firms choose where to locate *ex ante* based on expected profits. In his model, countries are more specialized under flexible than under fixed exchange rates because the exchange rate performs a favorable adjustment role for firms that are located in the country that is more specialized in the goods produced by those firms, whereas it is a source of disturbance for other firms.

a number of puzzles in international macroeconomics. Burstein, Eichenbaum, and Rebelo (2002) focus on the role of composition effects in the transmission of the inflationary consequences of exchange rate devaluations. Cuñat and Maffezzoli (2002) introduce Heckscher-Ohlin trade features in an international real business cycle model and show that this improves its empirical performance. Kraay and Ventura (2000 and 2001) develop models that explain features of international business cycles by focusing on specialization and the factor content of trade. Corsetti, Martin, and Pesenti (2003) put forth a two-country, sticky-wage model with entry subject to a fixed cost that increases in the number of existing firms and “melting-iceberg” costs of international trade. All goods are traded in their model, and they explore the implications of entry for the transmission of monetary shocks. Bergin and Glick (2003*a, b*) introduce heterogeneous, good-specific, iceberg costs in perfect-foresight, two-period, small, open, endowment economy models. Betts and Kehoe (2001) also consider heterogeneous, per-unit trade costs in a multi-country, trade and macro model with complete asset markets and differentiated goods. As in our paper, endogenous non-tradedness is central to macro dynamics in Bergin and Glick’s work and in Betts and Kehoe’s. The focus on fixed costs, heterogeneous productivity, and entry and exit decisions of monopolistically competitive firms over the business cycle distinguishes our approach from those of these contributions.

The rest of the paper is organized as follows. Section 2 describes the benchmark model with financial autarky. Section 3 presents results on the determinants of the real exchange rate in our setup. These results guide our interpretation of the impulse responses in Section 4, which analyzes the dynamics of the model in response to shocks to aggregate productivity, sunk entry costs (interpreted as changes in domestic goods market regulation facing firms in each country), and trade costs (interpreted as changes in trade policy). Section 5 discusses how allowing for international bond trading affects dynamics and expounds on the implications of our model for current account dynamics. It also presents the results of a stochastic simulation of the model. Section 6 concludes.

2 The Model

We begin by developing a version of our model in which there is no international trade in financial assets.

Household Preferences and Intratemporal Choices

The world consists of two countries, home and foreign. We denote foreign variables with a superscript star. Each country is populated by a unit mass of atomistic households. Households and firms in each country use cash currency for transactions in goods, financial, and labor markets. All contracts and prices in the world economy are written in nominal terms. Prices are flexible. Thus, we can focus on real variables in the solution. However, since the composition of consumption baskets in the two countries changes over time, so does the range of good-prices over which home and foreign consumption-based price indexes are defined. It follows that the unit of measure actually changes over time when we define real variables in units of consumption by deflating nominal ones with the corresponding consumption-based price index. The presence of money in the economy ensures existence of a convenient, underlying, constant unit of measure for all variables. As it is common in the recent literature in international macroeconomics, we introduce money in the economy by assuming that agents derive utility from money holdings.⁹

The representative home household supplies L units of labor inelastically in each period at the nominal wage rate W_t , denominated in units of home currency. The household maximizes expected intertemporal utility from consumption (C) and holdings of real money balances (M/P , where M is nominal money and P is the price index):

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s^{1-\gamma}}{1-\gamma} + \chi \log \frac{M_s}{P_s} \right), \quad (1)$$

where $\beta \in (0, 1)$ is the subjective discount factor, $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution, and $\chi > 0$ is the weight of utility from money balances in overall utility.¹⁰

At time t , the household consumes the basket of goods C_t , defined over a continuum of goods Ω :

$$C_t = \left(\int_{\omega \in \Omega} c_t(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}, \quad (2)$$

where $\theta > 1$ is the symmetric elasticity of substitution across goods. At any given time t , only a subset of goods $\Omega_t \subset \Omega$ are available. Let $p_t(\omega)$ denote the home currency price of a good $\omega \in \Omega_t$.

⁹Betts and Kehoe (2001) also introduce money in their flexible-price model of trade and macroeconomics to look at the implications of their model for the nominal exchange rate.

¹⁰Since price flexibility allows us to disregard nominal variables and focus on real variables in the solution, the functional form of the (separable) utility from money balances is inconsequential in this paper. We follow Obstfeld and Rogoff (1995) and assume that χ is small enough that money has a negligible effect on welfare.

The consumption-based price index for the home economy is then

$$P_t = \left(\int_{\omega \in \Omega_t} p_t(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}, \quad (3)$$

and home's demand for each individual good ω is

$$c_t(\omega) = \left(\frac{p_t(\omega)}{P_t} \right)^{-\theta} C_t. \quad (4)$$

The foreign household maximizes a similar utility function, with identical parameters. It supplies L^* units of labor inelastically in each period at the nominal wage rate W_t^* per unit, denominated in units of foreign currency. The foreign consumption basket is

$$C_t^* = \left(\int_{\omega \in \Omega} c_t^*(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}.$$

The subset of goods available for consumption in the foreign economy during period t is $\Omega_t^* \subset \Omega$ and can differ from the subset of goods that are available in the home economy. Letting $p_t^*(\omega)$ denote the foreign currency price of good $\omega \in \Omega_t^*$, the foreign consumption-based price index is

$$P_t^* = \left(\int_{\omega \in \Omega_t^*} p_t^*(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}, \quad (5)$$

and foreign demand for individual good ω is

$$c_t^*(\omega) = \left(\frac{p_t^*(\omega)}{P_t^*} \right)^{-\theta} C_t^*. \quad (6)$$

Firms

There is a continuum of firms in each country, each producing a different variety $\omega \in \Omega$. Production requires only one factor, labor. Aggregate labor productivity is indexed by Z_t (Z_t^*), which represents the effective units of labor per home (foreign) worker. Firms are heterogeneous as they produce with different technologies indexed by relative productivity z . The output of a home firm with relative productivity z is

$$y_t(z) = Z_t z l_t(z), \quad (7)$$

where $l_t(z)$ is the firm's labor demand. Productivity differences across firms therefore translate into differences in the unit cost of production. This cost, measured in units of the consumption good, is $w_t/(Z_t z)$, where $w_t \equiv W_t/P_t$ is the real wage. Similarly, foreign firms are indexed by their productivity z and unit costs (measured in units of the foreign consumption good) $w_t^*/(Z_t^* z)$, where $w_t^* \equiv W_t^*/P_t^*$ is the real wage of foreign workers.¹¹

Prior to entry, firms are identical and face a sunk entry cost of $f_{E,t}$ ($f_{E,t}^*$) effective labor units, equal to $w_t f_{E,t}/Z_t$ ($w_t^* f_{E,t}^*/Z_t^*$) units of the home (foreign) consumption good. Upon entry, home firms draw their productivity level z from a common distribution $G(z)$ with support on $[z_{\min}, \infty)$. Foreign firms draw their productivity level from an identical distribution. This relative productivity level remains fixed thereafter. Since there are no fixed production costs, all firms produce in every period, until they are hit with a “death” shock, which occurs with probability $\delta \in (0, 1)$ in every period. This exit inducing shock is independent of the firm's productivity level, so $G(z)$ also represents the productivity distribution of all producing firms. Home and foreign firms can serve both their domestic market as well as the export market. Exporting is costly, and involves both a “melting-iceberg” trade cost $\tau_t \geq 1$ ($\tau_t^* \geq 1$) as well as a fixed cost $f_{X,t}$ ($f_{X,t}^*$) (measured in units of effective labor). We assume that firms hire workers from their respective domestic labor markets to cover these fixed costs. These costs, in real terms, are then $w_t f_{X,t}/Z_t$ for home firms (in units of the home consumption good) and $w_t^* f_{X,t}^*/Z_t^*$ for foreign firms (in units of the foreign consumption good). The fixed export costs are paid on a period-by-period basis rather than sunk upon entry in the export market.¹²

All firms face a residual demand curve with constant elasticity θ in both markets, and set fully flexible prices that reflect the same proportional markup $\theta/(\theta - 1)$ over marginal cost. Let $p_{D,t}(z)$ and $p_{X,t}(z)$ ($p_{D,t}^*(z)$ and $p_{X,t}^*(z)$) denote the nominal domestic and export prices of a home (foreign) firm. We assume that export prices are denominated in the currency of the export market. Prices, in real terms relative to the price index in the destination market, are then given by:

$$\begin{aligned} \rho_{D,t}(z) &\equiv \frac{p_{D,t}(z)}{P_t} = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z}, & \rho_{D,t}^*(z) &\equiv \frac{p_{D,t}^*(z)}{P_t^*} = \frac{\theta}{\theta - 1} \frac{w_t^*}{Z_t^* z}, \\ \rho_{X,t}(z) &\equiv \frac{p_{X,t}(z)}{P_t} = Q_t^{-1} \tau_t \rho_{D,t}(z), & \rho_{X,t}^*(z) &\equiv \frac{p_{X,t}^*(z)}{P_t} = Q_t \tau_t^* \rho_{D,t}^*(z), \end{aligned} \tag{8}$$

¹¹We use the same index z for both home and foreign firms as this variable only captures firm productivity relative to the distribution of firms in that country.

¹²Even if there is substantial evidence of the importance of sunk export costs, introducing them in our model implies complications that we leave for future work. We could have also modeled an overhead fixed cost for selling in the domestic market – so long as this cost is low enough that firms with productivity z_{\min} earn a non-negative profit from domestic sales. We ignore this cost for simplicity.

where $Q_t \equiv \varepsilon_t P_t^*/P_t$ is the consumption-based real exchange rate (units of home consumption per unit of foreign consumption; ε_t is the nominal exchange rate, units of home currency per unit of foreign). However, due to the fixed export cost, firms with low productivity levels z may decide not to export in any given period. When making this decision, a firm decomposes its total profit $d_t(z)$ ($d_t^*(z)$) (returned to households as dividends) into portions earned from domestic sales $d_{D,t}(z)$ ($d_{D,t}^*(z)$) and from potential export sales $d_{X,t}(z)$ ($d_{X,t}^*(z)$). All these profit levels (dividends) are expressed in real terms in units of the consumption basket in the firm's location.¹³ They are given by:

$$d_t(z) = d_{D,t}(z) + d_{X,t}(z), \quad d_t^*(z) = d_{D,t}^*(z) + d_{X,t}^*(z),$$

where

$$d_{D,t}(z) = \frac{1}{\theta} [\rho_{D,t}(z)]^{1-\theta} C_t,$$

$$d_{X,t}(z) = \begin{cases} \frac{Q_t}{\theta} [\rho_{X,t}(z)]^{1-\theta} C_t - \frac{w_t f_{X,t}}{Z_t} & \text{if firm } z \text{ exports,} \\ 0 & \text{otherwise,} \end{cases}$$

and

$$d_{D,t}^*(z) = \frac{1}{\theta} [\rho_{D,t}^*(z)]^{1-\theta} C_t^*,$$

$$d_{X,t}^*(z) = \begin{cases} \frac{Q_t^{-1}}{\theta} [\rho_{X,t}^*(z)]^{1-\theta} C_t - \frac{w_t^* f_{X,t}^*}{Z_t^*} & \text{if firm } z \text{ exports,} \\ 0 & \text{otherwise.} \end{cases}$$

As expected, a firm's total profit increases with its productivity level z (even though these firms set relatively lower prices – see (8)). A firm will export if and only if it would earn non-negative profit from doing so. For home firms, this will be the case so long as productivity z is above a cutoff level $z_{X,t} = \inf \{z : d_{X,t}(z) > 0\}$. A similar cutoff level $z_{X,t}^* = \inf \{z : d_{X,t}^*(z) > 0\}$ holds for foreign exporters. We assume that the lower bound cost z_{\min} is low enough relative to the export costs that $z_{X,t}$ and $z_{X,t}^*$ are both above z_{\min} . This ensures the existence of an endogenously determined non-traded sector: the set of firms who *could* export, but decide not to. These firms, with productivity levels between z_{\min} and the export cutoff level, only produce for their domestic market.¹⁴ This set of firms fluctuates over time with changes in the profitability of the export

¹³Note that an exporter's relative price $\rho_{X,t}(z)$ ($\rho_{X,t}^*(z)$) is expressed in units of C_t^* (C_t) (the consumption good at the location of sales) but the profits from export sales $d_{X,t}(z)$ ($d_{X,t}^*(z)$) are expressed in units of C_t (C_t^*) (the consumption basket in the firm's location).

¹⁴All firms that pay the sunk entry cost will produce for their domestic market as they all earn positive profits

market, inducing changes in the cutoff levels $z_{X,t}$ and $z_{X,t}^*$. Since we do not have sunk export costs, firms “freely” enter and exit the export market as soon as export conditions change.

Firm Averages

In every period, there is a mass $N_{D,t}$ ($N_{D,t}^*$) of firms producing in the home (foreign) country. These firms have a distribution of productivity levels over $[z_{\min}, \infty)$ given by $G(z)$. Among these firms, there are $N_{X,t} = [1 - G(z_{X,t})] N_{D,t}$ and $N_{X,t}^* = [1 - G(z_{X,t}^*)] N_{D,t}^*$ exporters. These exporters have a distribution of productivity levels over $[z_{X,t}, \infty)$ and $[z_{X,t}^*, \infty)$ given by the conditional distribution of $G(z)$ on these intervals. Following Melitz (2003), we define two special “average” productivity levels – an average \tilde{z}_D for all producing firms in each country, and an average $\tilde{z}_{X,t}$ ($\tilde{z}_{X,t}^*$) for all home (foreign) exporters – as:

$$\tilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\theta-1} dG(z) \right]^{\frac{1}{\theta-1}},$$

$$\tilde{z}_{X,t} \equiv \left[\frac{1}{1 - G(z_{X,t})} \int_{z_{X,t}}^{\infty} z^{\theta-1} dG(z) \right]^{\frac{1}{\theta-1}}, \quad \tilde{z}_{X,t}^* \equiv \left[\frac{1}{1 - G(z_{X,t}^*)} \int_{z_{X,t}^*}^{\infty} z^{\theta-1} dG(z) \right]^{\frac{1}{\theta-1}}.$$

As shown in Melitz (2003), these productivity averages summarize all the information on the productivity distributions relevant for all macroeconomic variables.¹⁵ In essence, our model is isomorphic to one where $N_{D,t}$ ($N_{D,t}^*$) firms with productivity level \tilde{z}_D produce in the home (foreign) country and $N_{X,t}$ ($N_{X,t}^*$) firms with productivity level $\tilde{z}_{X,t}$ ($\tilde{z}_{X,t}^*$) export to the foreign (home) market.

In particular, $p_{D,t}(\tilde{z}_D)$ ($p_{D,t}^*(\tilde{z}_D)$) represents the average nominal price of home (foreign) firms in their domestic market, and $p_{X,t}(\tilde{z}_{X,t})$ ($p_{X,t}^*(\tilde{z}_{X,t}^*)$) represents the average nominal price of home (foreign) exporters in the export market. The price index at home therefore reflects the prices of the $N_{D,t}$ home firms (with average price $p_{D,t}(\tilde{z}_D)$) and the $N_{X,t}^*$ foreign exporters to the home market (with average price $p_{X,t}^*(\tilde{z}_{X,t}^*)$). The home price index can thus be written¹⁶

$$P_t = \left\{ N_{D,t} [p_{D,t}(\tilde{z}_D)]^{1-\theta} + N_{X,t}^* [p_{X,t}^*(\tilde{z}_{X,t}^*)]^{1-\theta} \right\}^{\frac{1}{1-\theta}},$$

from doing so (including the firm with the lowest productivity level $z = z_{\min}$).

¹⁵The averages are based on weights that are proportional to relative output shares of firms.

¹⁶See Melitz (2003) for proofs.

which implies

$$N_{D,t} (\tilde{\rho}_{D,t})^{1-\theta} + N_{X,t}^* (\tilde{\rho}_{X,t}^*)^{1-\theta} = 1, \quad (9)$$

where $\tilde{\rho}_{D,t} \equiv \rho_{D,t}(\tilde{z}_D)$ and $\tilde{\rho}_{X,t}^* \equiv \rho_{X,t}^*(\tilde{z}_{X,t}^*)$ represent the average relative prices of home producers and foreign exporters in the home market. Similarly, the price index in foreign reflects the domestic prices of the $N_{D,t}^*$ foreign producers and the export prices of the $N_{X,t}$ home exporters:

$$P_t^* = \left\{ N_{D,t}^* [p_{D,t}^*(\tilde{z}_D)]^{1-\theta} + N_{X,t} [p_{X,t}(\tilde{z}_{X,t})]^{1-\theta} \right\}^{\frac{1}{1-\theta}},$$

which implies

$$N_{D,t}^* (\tilde{\rho}_{D,t}^*)^{1-\theta} + N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} = 1, \quad (10)$$

where $\tilde{\rho}_{D,t}^* \equiv \rho_{D,t}^*(\tilde{z}_D)$ and $\tilde{\rho}_{X,t} \equiv \rho_{X,t}(\tilde{z}_{X,t})$ represent the average relative prices of foreign producers and home exporters in the foreign market.

The productivity averages \tilde{z}_D , $\tilde{z}_{X,t}$, and $\tilde{z}_{X,t}^*$ are constructed in such a way that $\tilde{d}_{D,t} \equiv d_{D,t}(\tilde{z}_D)$ ($\tilde{d}_{D,t}^* \equiv d_{D,t}^*(\tilde{z}_D)$) represents the average firm profit earned from domestic sales for all home (foreign) producers; and $\tilde{d}_{X,t} \equiv d_{X,t}(\tilde{z}_{X,t})$ ($\tilde{d}_{X,t}^* \equiv d_{X,t}^*(\tilde{z}_{X,t}^*)$) represents the average firm export profits for all home (foreign) exporters.¹⁷ Thus, $\tilde{d}_t = \tilde{d}_{D,t} + [1 - G(z_{X,t})] \tilde{d}_{X,t}$ and $\tilde{d}_t^* = \tilde{d}_{D,t}^* + [1 - G(z_{X,t}^*)] \tilde{d}_{X,t}^*$ represent the average *total* profits of home and foreign firms, since $1 - G(z_{X,t})$ and $1 - G(z_{X,t}^*)$ represent the proportion of home and foreign firms that export and earn export profits.¹⁸

Firm Entry and Exit

In every period, there is an unbounded mass of prospective entrants in both countries. These entrants are forward looking, and correctly anticipate their future expected profits \tilde{d}_t (\tilde{d}_t^*) in every period (the pre-entry expected profit is equal to post-entry average profit) as well as the probability δ (in every period) of being hit with the exit-inducing shock. We assume that entrants at time t only start producing at time $t + 1$, which introduces a one-period time-to-build lag in the model.¹⁹ The exogenous exit shock occurs at the very end of the time period (after production and entry). Some entrants may therefore never produce as they can incur the exit shock immediately upon

¹⁷This implies $d_{D,t}(\tilde{z}_D) = \int_{z_{\min}}^{\infty} d_{D,t}(z) dG(z)$ and $d_{X,t}(\tilde{z}_{X,t}) = \{1/[1 - G(z_{X,t})]\} \int_{z_{X,t}}^{\infty} d_{X,t}(z) dG(z)$. Similar results hold for the average profits of foreign firms. See Melitz (2003) for proofs.

¹⁸Again, \tilde{d}_t and \tilde{d}_t^* represent the average firm profit levels in the sense that $\tilde{d}_t = \int_{z_{\min}}^{\infty} d_t(z) dG(z)$ and $\tilde{d}_t^* = \int_{z_{\min}^*}^{\infty} d_t^*(z) dG(z)$.

¹⁹Kydland and Prescott (1982) first introduced time-to-build lags in business cycle analysis. See Backus, Kehoe, and Kydland (1992) for an international RBC model that adopts Kydland and Prescott's setup.

entry. Prospective home entrants in period t therefore compute their expected post-entry value given by the present discounted value of their expected stream of profits $\{\tilde{d}_s\}_{s=t+1}^\infty$:

$$\tilde{v}_t = E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_{t+s}}{C_t} \right)^{-\gamma} \tilde{d}_s. \quad (11)$$

This will also represent the average value of incumbent firms *after* production has occurred (since both the new entrants and the incumbents then face the same probability $1 - \delta$ of survival and production in the subsequent period). The firm discounts future profit with the household's stochastic discount factor, adjusted for the probability of firm survival $1 - \delta$. Entry will incur until the average firm value \tilde{v}_t is equalized with the entry cost $w_t f_{E,t}/Z_t$. The average value of foreign firms \tilde{v}_t^* is defined in an analogous way, leading to the following free entry conditions for firms in both countries:

$$\tilde{v}_t = \frac{w_t f_{E,t}}{Z_t}, \quad \tilde{v}_t^* = \frac{w_t^* f_{E,t}^*}{Z_t^*}. \quad (12)$$

These conditions will hold so long as there is a positive mass $N_{E,t}$ and $N_{E,t}^*$ of entrants in both countries. We assume that the macroeconomic shocks are small enough that this holds in every period. Finally, the timing of entry and production we have assumed leads to the following equation of motion for the number of producing firms in both countries:

$$N_{D,t} = (1 - \delta)(N_{D,t-1} + N_{E,t-1}), \quad N_{D,t}^* = (1 - \delta)(N_{D,t-1}^* + N_{E,t-1}^*). \quad (13)$$

Parametrization of Productivity Draws

In order to solve our model, we parametrize the distribution of firm productivity draws $G(z)$. We assume that productivity z is distributed Pareto with lower bound z_{\min} and shape parameter $k > \theta - 1$. The assumption of Pareto distribution induces a size distribution of firms that is also Pareto, which fits firm-level data quite well. k indexes the dispersion of productivity draws: Dispersion decreases as k increases and the firm productivity levels are increasingly concentrated toward their lower bound z_{\min} .²⁰ These assumptions imply that

$$G(z) = 1 - \left(\frac{z_{\min}}{z} \right)^k$$

²⁰The standard deviation of log productivity is equal to $1/k$. The condition that $k > \theta - 1$ ensures that the variance of firm size is finite.

and that the average productivities \tilde{z}_D , $\tilde{z}_{X,t}$, and $\tilde{z}_{X,t}^*$ are given by

$$\begin{aligned}\tilde{z}_D &= \left[\frac{k}{k - (\theta - 1)} \right]^{\frac{1}{\theta-1}} z_{\min}, \\ \tilde{z}_{X,t} &= \left[\frac{k}{k - (\theta - 1)} \right]^{\frac{1}{\theta-1}} z_{X,t}, \quad \tilde{z}_{X,t}^* = \left[\frac{k}{k - (\theta - 1)} \right]^{\frac{1}{\theta-1}} z_{X,t}^*.\end{aligned}\tag{14}$$

The share of exporting firms in both countries are then:

$$\frac{N_{X,t}}{N_{D,t}} = 1 - G(z_{X,t}) = \left(\frac{z_{\min}}{\tilde{z}_{X,t}} \right)^k \left[\frac{k}{k - (\theta - 1)} \right]^{\frac{k}{\theta-1}},\tag{15}$$

$$\frac{N_{X,t}^*}{N_{D,t}^*} = 1 - G(z_{X,t}^*) = \left(\frac{z_{\min}}{\tilde{z}_{X,t}^*} \right)^k \left[\frac{k}{k - (\theta - 1)} \right]^{\frac{k}{\theta-1}}.\tag{16}$$

Finally, the zero export profit conditions for the cutoff firms with productivity $z_{X,t}$ and $z_{X,t}^*$ imply that average export profit levels must satisfy:

$$\tilde{d}_{X,t} = w_t \frac{f_{X,t}}{Z_t} \frac{\theta - 1}{k - (\theta - 1)}, \quad \tilde{d}_{X,t}^* = w_t^* \frac{f_{X,t}^*}{Z_t^*} \frac{\theta - 1}{k - (\theta - 1)}.\tag{17}$$

Household Budget Constraint and Intertemporal Choices

Households in each country hold three types of assets: domestic money, shares in a mutual fund of domestic firms, and domestic, risk-free bonds.

Focus on the home economy. Let x_t be the share in the mutual fund of home firms held by the representative home household *entering* period t . The mutual fund pays a total profit in each period (in units of home currency) that is equal to the average total profit of all home firms that produce in that period, $\tilde{D}_t N_{D,t}$, where $\tilde{D}_t \equiv P_t \tilde{d}_t$. During period t , the representative home household buys x_{t+1} shares in a mutual fund of $N_{H,t} \equiv N_{D,t} + N_{E,t}$ home firms (those already operating at time t and the new entrants). Only $N_{D,t+1} = (1 - \delta) N_{H,t}$ firms will produce and pay dividends at time $t + 1$. Since the household does not know which firms will be hit by the exogenous exit shock δ at the *very end* of period t , it finances continuing operation of all pre-existing home firms and of all new entrants during period t . The date t price (in units of home currency) of a claim to the future profit stream of the mutual fund of $N_{H,t}$ firms is equal to the average nominal price of claims to future profits of home firms, $\tilde{V}_t \equiv P_t \tilde{v}_t$.²¹

²¹Given our assumptions, there is no loss of generality from writing the portfolio problem of the representative household in terms of a mutual fund that pays dividends equal to total average profits. Writing the problem in terms of share holdings in individual firms would complicate the notation and ultimately result in identical equilibrium

The household enters period t with bond holdings B_t in units of consumption, mutual fund share holdings x_t , and nominal money holdings M_{t-1} . It receives gross interest income on bond holdings, dividend income on mutual fund share holdings and the value of selling its initial share position, and labor income. The household allocates these resources between money balances and purchases of bonds and shares to be carried into next period, consumption, and lump-sum tax payments (T_t). The period budget constraint in units of domestic currency is:

$$P_t B_{t+1} + \tilde{V}_t N_{H,t} x_{t+1} + M_t + P_t C_t + P_t T_t = P_t (1 + r_t) B_t + \left(\tilde{D}_t + \tilde{V}_t \right) N_{D,t} x_t + M_{t-1} + W_t L,$$

where r_t is the consumption-based interest rate on holdings of bonds between $t - 1$ and t (known with certainty as of $t - 1$).²² In real terms:

$$B_{t+1} + \tilde{v}_t N_{H,t} x_{t+1} + \frac{M_t}{P_t} + C_t + T_t = (1 + r_t) B_t + \left(\tilde{d}_t + \tilde{v}_t \right) N_{D,t} x_t + \frac{M_{t-1}}{P_t} + w_t L. \quad (18)$$

The home household maximizes (1) subject to (18). The foreign household maximizes its utility function subject to a similar budget constraint:

$$B_{*,t+1}^* + \tilde{v}_t^* N_{F,t}^* x_{t+1}^* + \frac{M_t^*}{P_t^*} + C_t^* + T_t^* = (1 + r_t^*) B_{*,t}^* + \left(\tilde{d}_t^* + \tilde{v}_t^* \right) N_{D,t}^* x_t + \frac{M_{t-1}^*}{P_t^*} + w_t^* L^*, \quad (19)$$

where $N_{F,t}^* \equiv N_{D,t}^* + N_{E,t}^*$ is the number of foreign firms before the exit shock at the end of period t that will imply $N_{D,t+1}^* = (1 - \delta) N_{H,t}^*$.²³

The Euler equations for bond holdings at home and abroad are:

$$(C_t)^{-\gamma} = \beta (1 + r_{t+1}) E_t \left[(C_{t+1})^{-\gamma} \right], \quad (20)$$

$$(C_t^*)^{-\gamma} = \beta (1 + r_{t+1}^*) E_t \left[(C_{t+1}^*)^{-\gamma} \right]. \quad (21)$$

Using $N_{H,t} = N_{D,t+1}/(1 - \delta)$ and $N_{F,t}^* = N_{D,t+1}^*/(1 - \delta)$, the Euler equations for share holdings

conditions.

²²Underlying this constraint is the assumption that households actually trade bonds denominated in units of currency that pay a risk-free nominal interest rate. Given nominal bond holdings B_t^n entering period t , $B_t \equiv B_t^n / P_{t-1}$. Nominal bond holdings B_t^n produce a gross nominal return $(1 + i_t) B_t^n$ at time t , where i_t is the nominal interest rate between $t - 1$ and t . The nominal interest rate is indexed to inflation to ensure a risk-free, gross real return in units of the consumption basket $1 + r_t \equiv (1 + i_t) P_{t-1} / P_t$. Therefore, nominal bond holdings B_t^n entering period t produce a nominal return $P_t (1 + r_t) B_t$ during period t , or a risk-free real return $(1 + r_t) B_t$.

²³We denote the foreign household's bond holdings entering period $t + 1$ with $B_{*,t+1}^*$. We use B_{t+1}^* ($B_{*,t+1}$) to denote the foreign (home) household's holdings of home (foreign) bonds when we extend the model to allow for international borrowing and lending.

in the two countries are:

$$\tilde{v}_t = \beta(1 - \delta) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (\tilde{v}_{t+1} + \tilde{d}_{t+1}) \right], \quad (22)$$

$$\tilde{v}_t^* = \beta(1 - \delta) E_t \left[\left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} (\tilde{v}_{t+1}^* + \tilde{d}_{t+1}^*) \right]. \quad (23)$$

As it should be expected, forward iteration of equation (22) and absence of speculative bubbles yield the asset price solution in equation (11).²⁴

Aggregate Accounting and Balanced Trade

We assume that money is injected in the economy through lump-sum transfers of seignorage revenue. The home government budget constraint is $P_t T_t = -(M_t - M_{t-1})$. Similarly for the foreign government.

Aggregating the budget constraints (18) and (19) across (symmetric) home and foreign households, imposing the equilibrium conditions under financial autarky ($B_t = B_{*,t}^* = 0$ and $x_{t+1} = x_t = x_{t+1}^* = x_t^* = 1$), and using the government budget constraints yields the aggregate accounting equations for the home and foreign economies:

$$C_t = w_t L + N_{D,t} \tilde{d}_t - N_{E,t} \tilde{v}_t, \quad (24)$$

$$C_t^* = w_t^* L^* + N_{D,t}^* \tilde{d}_t^* - N_{E,t}^* \tilde{v}_t^*. \quad (25)$$

Consumption in each period must equal labor income plus investment income net of the cost of investing in new firms. Net investment income is the sum of profits from domestic sales and export profits, minus entry costs.

To close the model, observe that financial autarky implies balanced trade: The value of home exports must equal the value of foreign exports:

$$Q_t N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} C_t^* = N_{X,t}^* (\tilde{\rho}_{X,t}^*)^{1-\theta} C_t. \quad (26)$$

²⁴We omit money demand equations given that money is neutral in our model. Similarly omitted transversality conditions for bonds and shares must be satisfied to ensure optimality.

Summary

Table 1 summarizes the main equilibrium conditions of the model. The equations in the table constitute a system of 25 equations in 25 endogenous variables determined during period t : $\tilde{\rho}_{D,t}$, $\tilde{\rho}_{X,t}^*$, $\tilde{\rho}_{X,t}$, w_t , w_t^* , $\tilde{d}_{D,t}$, $\tilde{d}_{D,t}^*$, $\tilde{d}_{X,t}$, $\tilde{d}_{X,t}^*$, $N_{E,t}$, $N_{E,t}^*$, $\tilde{z}_{X,t}$, $\tilde{z}_{X,t}^*$, $N_{D,t+1}$, $N_{D,t+1}^*$, $N_{X,t}$, $N_{X,t}^*$, r_{t+1} , r_{t+1}^* , \tilde{v}_t , \tilde{v}_t^* , C_t , C_t^* , Q_t . There are 4 endogenous state variables that are predetermined as of time t : the risk-free interest rates, r_t and r_t^* , and the total numbers of firms at home and abroad, $N_{D,t}$ and $N_{D,t}^*$. Finally, the model features 8 exogenous variables: the aggregate productivities Z_t and Z_t^* , and the policy variables $f_{E,t}$, $f_{E,t}^*$, $f_{X,t}$, $f_{X,t}^*$, τ_t , τ_t^* . We interpret changes in $f_{E,t}$ and $f_{E,t}^*$ as changes in market regulation facing a country's firms in the respective domestic markets and changes in $f_{X,t}$, $f_{X,t}^*$, τ_t , and τ_t^* as changes in trade policy. Since $f_{X,t}$ and τ_t are trade costs facing home firms, they are best interpreted as the foreign government's trade policy instruments. Similarly, $f_{X,t}^*$ and τ_t^* describe the home government's trade policy.

Given the solutions for the average variables in the system of Table 1, we can back out the dynamics of individual firm variables, which are central to understand intuitions. Because \tilde{z}_D is simply proportional to z_{\min} , the dynamics of the average variables $\tilde{\rho}_{D,t}$, $\tilde{\rho}_{D,t}^*$, $\tilde{d}_{D,t}$, $\tilde{d}_{D,t}^*$, $\tilde{c}_{D,t} = \theta \tilde{d}_{D,t} / \tilde{\rho}_{D,t}$, and $\tilde{c}_{D,t}^* = \theta \tilde{d}_{D,t}^* / \tilde{\rho}_{D,t}^*$ are also informative of what happens at the level of individual firms operating only in the domestic market in the home and foreign economies. Instead, to understand dynamics at the level of *individual* exporting firms, one cannot focus on the behavior of the average variables $\tilde{\rho}_{X,t}$, $\tilde{\rho}_{X,t}^*$, $\tilde{d}_{X,t}$, and $\tilde{d}_{X,t}^*$, because these reflect the endogenous dynamics of the export cutoff productivities $\tilde{z}_{X,t}$ and $\tilde{z}_{X,t}^*$. To understand the effect of exogenous shocks on individual home and foreign exporters, we must remove the effect of changes in the cutoffs $\tilde{z}_{X,t}$ and $\tilde{z}_{X,t}^*$ from the average variables. Consider home exporters. The response of the price of an individual exporter ($\rho_{X,t}(z)$) to a shock is given by the response of the product $\tilde{z}_{X,t} \tilde{\rho}_{X,t}$. The component of average export dividends (in units of home consumption) that does not depend on the fixed export cost is $(Q_t/\theta) (\tilde{\rho}_{X,t})^{1-\theta} C_t^*$. Therefore, the response of $(Q_t/\theta) (\tilde{z}_{X,t} \tilde{\rho}_{X,t})^{1-\theta} C_t^*$ tells us the response of the component of profits that do not depend on the fixed cost for an individual exporter (we denote this variable with $\hat{d}_{X,t}(z)$). Finally, the response of $c_{X,t}(z) = (\tilde{z}_{X,t} \tilde{\rho}_{X,t})^{-\theta} C_t^*$ informs us on the response of the quantity of output sold by an individual home exporter in the foreign market. In what follows, $y_{D,t}(z)$ and $y_{X,t}(z)$ denote the output quantities produced by a home firm with productivity z for the domestic and export market (similar notation is used for foreign firms).

When interpreting the results of the model, it is also useful to introduce the following additional variables:

- $\tilde{T}_t \equiv \varepsilon_t \tilde{p}_{X,t} / \tilde{p}_{X,t}^*$ denotes the average terms of trade of the home economy: the ratio of the average price of home exports to the average price of home imports (both in units of home currency), or the average quantity of foreign exports per one unit of home exports. An increase in \tilde{T}_t is an improvement in home's average terms of trade.
- $\mu_t \equiv \varepsilon_t W_t^* / W_t = Q_t w_t^* / w_t$ is the relative wage (units of home labor per unit of foreign labor). An increase in μ_t signals that labor is becoming relatively more costly in the foreign country.
- $TOL_t \equiv \mu_t Z_t / Z_t^*$ is the *terms of labor*: the relative cost of *effective* labor at home and abroad. If TOL_t rises, effective labor is becoming more costly in the foreign country.
- The average price of home traded goods in the home market is:

$$\tilde{\rho}_{T,t} = \left\{ \frac{1}{1 - G(z_{X,t})} \int_{z_{X,t}}^{\infty} [\rho_{D,t}(z)]^{1-\theta} dG(z) \right\}^{\frac{1}{1-\theta}} = \frac{Q_t}{\tau_t} \tilde{\rho}_{X,t}.$$

The average price of home non-traded goods is:

$$\begin{aligned} \tilde{\rho}_{NT,t} &= \left\{ \frac{1}{G(z_{X,t})} \int_{z_{\min}}^{z_{X,t}} [\rho_{D,t}(z)]^{1-\theta} dG(z) \right\}^{\frac{1}{1-\theta}} \\ &= \left\{ \frac{N_{D,t}}{N_{D,t} - N_{X,t}} \left[(\tilde{\rho}_{D,t})^{1-\theta} - \frac{N_{X,t}}{N_{D,t}} (\tilde{\rho}_{T,t})^{1-\theta} \right] \right\}^{\frac{1}{1-\theta}}. \end{aligned}$$

Hence, the average relative price of non-traded in terms of traded goods is:

$$\begin{aligned} \widetilde{TNT}_t \equiv \frac{\tilde{\rho}_{NT,t}}{\tilde{\rho}_{T,t}} &= \left\{ \frac{N_{D,t}}{N_{D,t} - N_{X,t}} \left[\left(\frac{\tilde{\rho}_{D,t}}{\tilde{\rho}_{T,t}} \right)^{1-\theta} - \frac{N_{X,t}}{N_{D,t}} \right] \right\}^{\frac{1}{1-\theta}} \\ &= \left\{ \frac{N_{D,t}}{N_{D,t} - N_{X,t}} \left[\left(\frac{\tilde{z}_{X,t}}{\tilde{z}_D} \right)^{1-\theta} - \frac{N_{X,t}}{N_{D,t}} \right] \right\}^{\frac{1}{1-\theta}} \end{aligned}$$

Note also that $\widetilde{TNT}_t = \frac{\tilde{z}_{X,t}}{\tilde{z}_{NT,t}}$, where $\tilde{z}_{NT,t}$ is the average productivity of firms that do not export. Therefore, the average relative price of non-traded home goods in terms of traded is also the average percent productivity advantage of exporters over non exporters. A similar expression defines the

average relative price of foreign non-traded goods in terms of foreign traded ones.

- $s_{D,t} \equiv N_{D,t} (\tilde{p}_{D,t})^{1-\theta} \left(s_{D,t}^* \equiv N_{D,t}^* (\tilde{p}_{D,t}^*)^{1-\theta} \right)$ is the share of home (foreign) expenditure on domestic goods.
- $N_{X,t} (\tilde{p}_{X,t})^{1-\theta}$ and $N_{X,t}^* (\tilde{p}_{X,t}^*)^{1-\theta}$ are the shares of expenditure on export goods in home and foreign, respectively. (The price index equations (9) and (10) ensure that the shares spent on domestic and export goods add to 1.)
- $N_{D,t} (\tilde{p}_{D,t})^{1-\theta} - N_{X,t} (Q_t \tau_t^{-1} \tilde{p}_{X,t})^{1-\theta}$ and $N_{D,t}^* (\tilde{p}_{D,t}^*)^{1-\theta} - N_{X,t}^* [Q_t^{-1} (\tau_t^*)^{-1} \tilde{p}_{X,t}^*]^{1-\theta}$ are the shares of home and foreign expenditure, respectively, on non-traded (non-exported) goods.
- $y_t \equiv w_t L + N_{D,t} \tilde{d}_t$ ($y_t^* \equiv w_t^* L^* + N_{D,t}^* \tilde{d}_t^*$) is home (foreign) GDP.

3 Entry, Endogenous Non-Tradedness, and the Real Exchange Rate

Since we focus on real exchange rate dynamics in much of the analysis below, this section illustrates some results and intuitions on real exchange rate determination in our model. We begin by proving that PPP would hold in all periods if there were no export costs.

Manipulating the price index equations and using the (average) pricing equations and the definition of the terms of labor yields:

$$\begin{aligned} \left(\frac{P_t}{\tilde{p}_{D,t}} \right)^{1-\theta} &= N_{D,t} + N_{X,t}^* \left(TOL_t \tau_t^* \frac{\tilde{z}_D}{\tilde{z}_{X,t}^*} \right)^{1-\theta}, \\ \left(\frac{\varepsilon_t P_t^*}{\tilde{p}_{D,t}^*} \right)^{1-\theta} &= N_{D,t}^* (TOL_t)^{1-\theta} + N_{X,t} \left(\tau_t \frac{\tilde{z}_D}{\tilde{z}_{X,t}} \right)^{1-\theta}, \end{aligned}$$

where $\tilde{p}_{D,t}$ is shorter notation for $p_{D,t}(\tilde{z}_D)$. Therefore, the consumption-based real exchange rate Q_t must be such that:

$$Q_t^{1-\theta} = \frac{N_{D,t}^* (TOL_t)^{1-\theta} + N_{X,t} \left(\tau_t \frac{\tilde{z}_D}{\tilde{z}_{X,t}} \right)^{1-\theta}}{N_{D,t} + N_{X,t}^* \left(TOL_t \tau_t^* \frac{\tilde{z}_D}{\tilde{z}_{X,t}^*} \right)^{1-\theta}}. \quad (27)$$

Equation (27) holds regardless of whether there is financial autarky or not. Suppose $\tau_t = \tau_t^* = 1$ and $f_{X,t} = f_{X,t}^* = 0$. Because there is no fixed export cost, all firms export and there are no non-traded goods: $\tilde{z}_{X,t} = \tilde{z}_{X,t}^* = \tilde{z}_D$, $N_{X,t} = N_{D,t}$, $N_{X,t}^* = N_{D,t}^*$. Then, equation (27) immediately implies that $Q_t = 1$ in all periods. Absent trade costs, our model is isomorphic to Obstfeld and

Rogoff's (1995) as far as the real exchange rate is concerned: PPP holds since all goods are traded, the law of one price holds, and preferences for consumption are identical across countries.

The definition of the real exchange rate $Q_t \equiv \varepsilon_t P_t^*/P_t$ uses the consumption-based price indexes in the two economies, P_t and P_t^* . These price indexes change over time for the variety effect implied by entry of new firms and availability of new goods in the economy. Arguably, this is not how the Bureau of Labor Statistics calculates the CPI for the U.S. economy: CPI data are based on average prices rather than constantly adjusted for availability of new varieties. Denote the consumer price level based on average prices with \tilde{P}_t . The average price level \tilde{P}_t satisfies

$$\left(\tilde{P}_t\right)^{1-\theta} = \frac{N_{D,t}}{N_{D,t} + N_{X,t}^*} (\tilde{p}_{D,t})^{1-\theta} + \frac{N_{X,t}^*}{N_{D,t} + N_{X,t}^*} (\tilde{p}_{X,t}^*)^{1-\theta},$$

where $\tilde{p}_{X,t}^*$ is shorter notation for $p_{X,t}^*(\tilde{z}_{X,t}^*)$. Therefore, P_t and \tilde{P}_t are such that $(P_t)^{1-\theta} = \left(N_{D,t} + N_{X,t}^*\right) \left(\tilde{P}_t\right)^{1-\theta}$. A similar relation holds abroad: $(P_t^*)^{1-\theta} = \left(N_{D,t}^* + N_{X,t}\right) \left(\tilde{P}_t^*\right)^{1-\theta}$. Let us now define a measure of the real exchange rate based on average consumer price levels: $\tilde{Q}_t \equiv \varepsilon_t \tilde{P}_t^*/\tilde{P}_t$. Using the results above, it is immediate to verify that

$$\tilde{Q}_t = \left(\frac{N_{D,t}^* + N_{X,t}}{N_{D,t} + N_{X,t}^*}\right)^{\frac{1}{\theta-1}} Q_t. \quad (28)$$

Note that, if $\tau_t = \tau_t^* = 1$ and $f_{X,t} = f_{X,t}^* = 0$, then $\tilde{Q}_t = 1$, because $Q_t = 1$, $N_{X,t} = N_{D,t}$, and $N_{X,t}^* = N_{D,t}^*$. As expected, PPP holds for both measures of the real exchange rate in the absence of trade costs. The ‘‘average’’ real exchange rate \tilde{Q}_t provides a measure of the real exchange rate that is empirically more appealing than Q_t , as it is in line with what we would obtain from using consumer price data constructed by the Bureau of Labor Statistics.

Appendix A presents the solution for the symmetric steady state of the model. Denote steady-state levels of variables by dropping the time subscript. We assume $f_E = f_E^*$, $f_X = f_X^*$, $\tau = \tau^*$, $L = L^*$, and $Z = Z^* = 1$. The model has a unique, symmetric steady state with $\tilde{Q} = Q = TOL = 1$ under these assumptions. We then log-linearize the system in Table 1 around the steady state under assumptions of log-normality and homoskedasticity of exogenous stochastic shocks, and we solve for the dynamics in response to exogenous shocks numerically with the method of undetermined coefficients as in Uhlig's (1999) implementation of Campbell (1994). We denote percentage deviations from the steady state with sans serif fonts. Log-linearizing (27) and (28), using the definition of $s_{D,t}$ and the expressions for average prices in Table 1, and combining the

resulting equations yields:²⁵

$$\begin{aligned} \tilde{Q}_t &= (2s_D - 1) \text{TOL}_t - (1 - s_D) [(\tilde{z}_{X,t} - \tilde{z}_{X,t}^*) - (\mathbf{t}_t - \mathbf{t}_t^*)] \\ &\quad - \frac{1}{\theta - 1} \left(s_D - \frac{N_D}{N_D + N_X} \right) [(\mathbf{N}_{D,t}^* - \mathbf{N}_{X,t}) - (\mathbf{N}_{D,t} - \mathbf{N}_{X,t}^*)], \end{aligned} \quad (29)$$

where \mathbf{t}_t (\mathbf{t}_t^*) denotes the percentage deviation of τ_t (τ_t^*) from the steady state. Under our assumptions on parameter values below, $s_D > 1/2$. Hence, *ceteris paribus*, when $\text{TOL}_t < 0$ (when home labor in effective units becomes more expensive relative to foreign), the real exchange rate \tilde{Q}_t appreciates. The intuition is familiar. When home effective labor becomes relatively more expensive, this increases the price of non-traded goods at home relative to foreign and leads to appreciation. This is a channel for real exchange rate appreciation that depends on existence of a non-traded goods sector, but not on the fact that the latter is determined endogenously and can change over time. Abstracting from the fact that a non-traded sector exists in our specific model if and only if there are strictly positive fixed export costs (which ensure the endogenous determination of what is traded and what is not), this is the traditional channel for real appreciation in the textbook analysis of the HBS effect. The direct contribution of endogenous non-tradedness to real exchange rate dynamics plays a key role through the term that depends on the export cutoff differential $\tilde{z}_{X,t} - \tilde{z}_{X,t}^*$ and the term that reflects variation in the relative numbers of home exporters into the foreign economy and foreign exporters into home. Focus on the export cutoffs first. Changes in relative labor costs further induce changes in the composition of the traded sector in both countries. Import prices in the economy with higher labor costs (say, home) rise as lower productivity foreign firms can now export to it. Conversely, import prices in the economy with lower labor costs decrease as lower productivity home exporters are forced to drop out of the export market. These effects necessarily reinforce the real exchange rate appreciation caused by an increase in the effective cost of home labor. As for the last term in equation (29), $s_D - N_D/(N_D + N_X)$ is the difference between the market (expenditure) share of home firms competing in the home market and the number share of these firms. It reflects the average productivity difference between home firms (\tilde{z}_D) and foreign exporters ($\tau \tilde{z}_X$, adjusted for the iceberg export cost). In steady state, this difference is negative under our parametrization below, as the average productivity advantage of exporters (58 percent) is larger than the transport cost (30 percent). Since exporters have lower

²⁵If $\tau = \tau^* = 1$ and $f_X = f_X^* = 0$, then $s_D = N_D/(N_D + N_X) = 1/2$ and $\tilde{z}_{X,t} = \tilde{z}_{X,t}^* = 0$. Of course, absent changes in trade costs, $\tilde{Q}_t = 0$.

prices relative to domestic firms, a decrease in the relative number of home exporters into the foreign economy ($N_{D,t}^* - N_{X,t} > 0$) leads, *ceteris paribus*, to an increase in the average price of consumption abroad and depreciation of \tilde{Q}_t . Conversely, a decrease in the relative number of foreign exporters into home ($N_{D,t} - N_{X,t}^* > 0$) causes appreciation. If $N_{D,t} - N_{X,t}^* > N_{D,t}^* - N_{X,t}$, these effects reinforce the real appreciation generated by $\text{TOL}_t < 0$.

We can combine (29) with the log-linear version of (28) to understand the difference in the dynamics of Q_t relative to \tilde{Q}_t . It is:

$$Q_t = (2s_D - 1)\text{TOL}_t - (1 - s_D) [(\tilde{z}_{X,t} - \tilde{z}_{X,t}^*) - (\tau_t - \tau_t^*)] - \frac{s_D}{\theta - 1} (N_{D,t}^* - N_{D,t}) + \frac{1 - s_D}{\theta - 1} (N_{X,t}^* - N_{X,t}). \quad (30)$$

Just like the price indexes P_t and P_t^* can change either because average prices change or because product variety does, Q_t is affected both by changes in average relative prices – \tilde{Q}_t – and changes in the relative number of varieties consumed – $(N_{D,t}^* + N_{X,t}) / (N_{D,t} + N_{X,t}^*)$. The terms of labor, export cutoffs, and iceberg trade costs affect Q_t directly through their effects on average prices and \tilde{Q}_t . Hence, the corresponding terms in the log-linear equation for Q_t are unchanged. The effects of changes in the number of firms in the domestic and foreign market are dominated by the effect of product variety on prices. *Ceteris paribus*, increases in the number of varieties available for consumption push the price indexes down.²⁶ An increase in the number of foreign exporters relative to home ($N_{X,t}^* - N_{X,t} > 0$) leads to depreciation of Q_t because, *ceteris paribus*, it pushes the home price index down relative to the foreign one. An increase in the total number of home firms relative to foreign ($N_{D,t}^* - N_{D,t} < 0$) generates depreciation for the same reason. In particular, the sign of the effect of $N_{D,t}^* - N_{D,t}$ on Q_t changes relative to the effect on \tilde{Q}_t , exactly because product variety becomes dominant over average price effects in the determination of Q_t .

Consumption Smoothing and the Real Exchange Rate under Financial Autarky

The results on the real exchange rate that we discussed above hold regardless of whether there is international trade in financial assets or not. We can develop additional intuition on the determinants of real exchange rate dynamics under financial autarky by observing that the balanced trade

²⁶This is true both in nominal and real terms. Take for example the home price index. It is $(P_t)^{1-\theta} = (N_{D,t} + N_{X,t}^*) (\tilde{P}_t)^{1-\theta}$. Multiplying both sides by $(W_t)^{1-\theta}$ shows that the consumption-based price index in units of labor (P_t/W_t) falls – the real wage rises – if product variety increases even if there is no change in the average price of consumption in units of labor (\tilde{P}_t/W_t).

condition (26) implies:

$$Q_t = \frac{N_{X,t}^*}{N_{X,t}} \left(\frac{\tilde{\rho}_{X,t}^*}{\tilde{\rho}_{X,t}} \right)^{1-\theta} \frac{C_t}{C_t^*}. \quad (31)$$

Recall the definition of the average terms of trade of the home economy: $\tilde{T}_t \equiv \varepsilon_t \tilde{p}_{X,t} / \tilde{p}_{X,t}^*$. It is easy to verify that $\tilde{\rho}_{X,t} / \tilde{\rho}_{X,t}^* = \tilde{T}_t / Q_t$. Therefore, equation (31) can be rewritten as:

$$(Q_t)^\theta = \frac{N_{X,t}^*}{N_{X,t}} \tilde{T}_t^{\theta-1} \frac{C_t}{C_t^*}. \quad (32)$$

Ceteris paribus, increases in C_t/C_t^* , $N_{X,t}^*/N_{X,t}$, and \tilde{T}_t must be accompanied by depreciation of the consumption-based real exchange rate to preserve balanced trade.

Assume temporarily that $f_{X,t} = f_{X,t}^* = 0$ and $\tau_t = \tau_t^* = 1$. Suppose further that we reduce our model to Obstfeld and Rogoff's (1995) setup by assuming constant numbers of firms at home and abroad, so that the ratio $N_{X,t}^*/N_{X,t} = N_{D,t}^*/N_{D,t}$ is constant. In this case, C_t/C_t^* is proportional to $\tilde{T}_t^{1-\theta}$. In log-linear terms,

$$C_t - C_t^* = -(\theta - 1) \tilde{T}_t.$$

Adjusting for the fact that the definition of the terms of trade in Obstfeld and Rogoff's model is the reciprocal of ours, this is the relation between the consumption differential and the terms of trade in that model if one removes international borrowing or lending. In other words, holding the ratio $N_{X,t}^*/N_{X,t}$ constant, the standard model with PPP implies that the terms of trade adjust exactly to offset movements in the consumption differential to preserve balanced trade.

As equation (32) shows, the consumption differential is not tied to the average terms of trade in our setup. Suppose that a favorable relative productivity shock (an increase in Z_t for given Z_t^*) causes home consumption to rise above foreign and the average terms of trade to deteriorate. Terms of trade deterioration implies that (on average) imports are more expensive for home consumers, which, *ceteris paribus*, has a negative effect on welfare. In our model, agents can react to this by reallocating resources away from the export sector and letting Q_t absorb part of the movement in the consumption differential. This incentive – and the fact that variety rather than average price effects dominate the dynamics of Q_t relative to \tilde{Q}_t – will explain our result that Q_t depreciates in response to a favorable productivity shock, whereas \tilde{Q}_t appreciates according to an endogenous, microfounded HBS effect.

4 International Trade and Macroeconomic Dynamics

Calibration

We calibrate parameters as follows. We interpret periods as quarters and set $\beta = .99$ and $\gamma = 2$ – both standard choices for quarterly business cycle models. We normalize the endowment of labor of each household to 1 and set the size of the exogenous firm exit shock $\delta = .025$ to mimic the evidence of destruction of one in ten jobs in the course of a year. We borrow the value of θ from Bernard, Eaton, Jensen, and Kortum (BEJK, 2003) and set $\theta = 3.8$, which was calibrated to fit U.S. plant and macro trade data. BEJK also reports that the standard deviation of log U.S. plant sales is 1.67. In our theoretical model, this standard deviation is equal to $1/(k - \theta + 1)$. Given $\theta = 3.8$, we deduce $k = 3.4$, which satisfies the requirement $k > \theta - 1$. We postulate $\tau = 1.3$, roughly in line with Obstfeld and Rogoff (2001), and set the steady-state fixed export cost f_X such that the proportion of exporting plants matches the number reported in BEJK (21 percent). This leads to a fixed export cost f_X equal to 23.5 percent of the per-period, amortized flow value of the entry cost, $[1 - \beta(1 - \delta)] / [\beta(1 - \delta)] f_E$.²⁷ Changing the entry cost f_E while maintaining the same ratio f_X/f_E does not affect any of the impulse responses. The total number of firms in steady state is inversely proportional to f_E – and the size and value of all firms are similarly proportional to f_E . Basically, changing f_E for given ratio f_X/f_E amounts to changing the unit of measure for output and number of firms. But it does not matter whether output is measured in units, tens of units, etc., and the same is true for the number of firms. Hence, we set f_E to 1 for simplicity. For the same reason, we normalize z_{\min} to 1 without loss of generality. Our calibration implies that exporters are on average 58.2 percent more productive than non-exporters. The steady-state share of expenditure on domestic goods is .733, and the share of expenditure on non-traded domestic goods is .176. The relative size differential of exporters relative to non-exporters in the domestic market is 3.61.

It may be argued that the value of θ results in a steady-state markup that is too high relative to the evidence. A standard choice in the macro literature is $\theta = 6$ to deliver a 20 percent markup of price over marginal cost (Rotemberg and Woodford, 1992). However, it is important to observe that, in models without any fixed cost, $\theta/(\theta - 1)$ is a measure of both markup over marginal cost and average cost. In our model with entry costs, free entry ensures that, *on average*, firms earn zero

²⁷We tried using different values of τ (1.1, 1.2, 1.25) and recalculated f_X relative to f_E to match the 21 percent of exporting plants. The impulse responses were very similar in all cases.

profits *net* of the entry cost. This means that, on average, firms price at average cost (inclusive of the entry cost). More productive firms price above their average cost (since the latter is lower as they spread the entry cost over larger amounts of output produced), and less productive firms price below their average cost. The firm with productivity z_{\min} must always price below its average cost – which means that the net present value of its profits never end up covering the entry cost. Thus, although $\theta = 3.8$ implies a fairly high markup over marginal cost, our model delivers reasonable markups over average costs.

Productivity Shocks

Following Backus, Kehoe, and Kydland (1992) and Baxter (1995), we assume the following bivariate process for the percentage deviations of home and foreign productivities Z_t and Z_t^* from the steady state:

$$\begin{bmatrix} Z_t \\ Z_t^* \end{bmatrix} = \begin{bmatrix} \phi_Z & \phi_{ZZ^*} \\ \phi_{Z^*Z} & \phi_{Z^*} \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_t^Z \\ \xi_t^{Z^*} \end{bmatrix}, \quad (33)$$

where the persistence parameters ϕ_Z and ϕ_{Z^*} are in the interval $[0, 1]$, the spillover parameters ϕ_{ZZ^*} and ϕ_{Z^*Z} are non-negative, and ξ_t^Z and $\xi_t^{Z^*}$ are zero-mean innovations. In Section 5, we calibrate the matrix of persistence and spillover parameters and the variance-covariance matrix of the innovations to match empirical evidence. Here, we assume $\phi_{ZZ^*} = \phi_{Z^*Z} = 0$ and focus on the consequence of a 1 percent increase in Z_t above the steady state with persistence $\phi_Z = 1$ or $\phi_Z = .9$.

A Permanent Increase in Home Productivity

Figure 1 presents the responses to a permanent 1 percent increase in home productivity. Consider first the long-run effects, in the new steady state. (A period corresponds to a quarter in our exercise, but the numbers 10 and 20 below the horizontal axis in each panel of the figures denote 10 and 20 years after the shock, respectively.)

Home is now endowed with more units of effective labor. At equal relative cost per unit of effective labor (holding the terms of labor – *TOL* – constant), whenever there are positive transport costs, entering firms will want to locate in the bigger market. Here, home is the bigger market (even when *TOL* does not change), as it has more units of effective labor. Thus, labor costs in foreign must relatively decrease (the terms of labor must fall) in order to keep firms in its market, and its labor force employed. This effect is described by Krugman (1980, pp. 954-955) in his seminal, first

new trade theory paper. (If all prospective entrants decided to locate into home, firms would stop entering the foreign economy altogether, and pre-existing foreign firms would be eventually wiped out by the “death” shock δ .) Note that the free entry condition for firms in home and foreign plays a key role in determining this comparative static.

If the new effective labor units at home were all used by existing firms, then the increase in production would induce positive profits for all firms in the long run. Instead, the free entry condition implies entry of new home firms, and N_D increases.

Home consumption (C) increases owing to higher labor and dividend income (w and \tilde{d} , respectively). The increase in C (and in w and \tilde{d}) reflects also the availability of more varieties. For instance, since the number of available varieties increases, C would increase even if consumption of each individual variety remained constant. We can separate the variety effect from the other mechanisms at work in our model by decomposing changes in demand facing an individual producer among three channels: aggregate changes in demand (or changes in total nominal expenditure deflated by the average price of all varieties), changes in a firm’s price relative to the average price, and changes in the number of competing firms. Consider variety z . Recall the relation between the consumption-based price index P and the average price level \tilde{P} : $(P_t)^{1-\theta} = (N_{D,t} + N_{X,t}^*) (\tilde{P}_t)^{1-\theta}$. Home demand for variety z can then be written as $(p_t(z)/\tilde{P}_t)^{-\theta} (N_{D,t} + N_{X,t}^*)^{-1} (P_t C_t / \tilde{P}_t)$. The term $P_t C_t / \tilde{P}_t$ captures the first channel, the term $(p_t(z)/\tilde{P}_t)^{-\theta}$ captures the second channel, and the term $(N_{D,t} + N_{X,t}^*)^{-1}$ captures the third channel. The latter disappears when the number of firms is fixed. In our model, holding overall demand and relative prices constant, increases in the number of firms competing in the home market decrease the demand level for each firm. (Alternatively, one can look at this effect through the consumption-based price index by recalling that, holding the average price level constant, an increase in the number of firms reduces the overall price index.)

In Figure 1, import demand at home for all foreign firms ($y_X^*(z)$) increases. This is induced by the overall increase in home demand (PC/\tilde{P}) and the decrease in the prices of foreign exports relative to domestic goods ($p_X^*(z)/\tilde{p}_D = \rho_X^*(z)/\tilde{\rho}_D$ falls) caused by lower TOL .²⁸ These two forces dominate the opposite effect of the increase in the number of firms selling at home. Increased demand for imports induces entry of new foreign exporters into the home market. Consequently, the foreign export productivity cutoff (z_X^*) falls, as less productive foreign firms are now exporting.

²⁸It is $P_t C_t / \tilde{P}_t = (N_{D,t} + N_{X,t}^*)^{-1/(\theta-1)} C_t$. Log-linearizing this expression and using $\theta = 3.8$ and $N_X/N_D = .21$ in the initial steady state, it is easy to verify that PC/\tilde{P} increases in Figure 1.

Export demand for home firms ($y_X(z)$) decreases, induced mainly by the increase in relative export prices: $p_X(z)/\tilde{p}_D^* = \rho_X(z)/\tilde{\rho}_D^*$ rises since TOL falls. A higher number of home exporters (N_X) than in the initial steady state contributes to this effect. Changes in foreign demand and in the overall number of foreign firms (N_D^*) are small compared to this relative price change. (Foreign income and consumption increase by little.) Hence, the relative price effect dominates, and $y_X(z)$ is lower than in the initial steady state. The number of home exporters is higher than in the initial steady state since all home firms are permanently more productive. However, *ceteris paribus*, the decrease in export demand and the increase in the effective cost of home labor (w/Z) induce exit of home exporters: The cutoff productivity for home exporters (z_X) rises, forcing relatively less productive home firms to abandon the foreign market.²⁹

Now consider changes along the transition.

Absent sunk entry costs, there would be an immediate adjustment to the new steady state number of firms. However, the sunk costs – along with the household saving/investment decision – induce a slow response in the number of firms. The sunk entry costs make the N_D variable behave very much like a capital stock, and the N_E variable very much like investment in a more standard macro model.

Abstract temporarily from the increase in N_X^* on impact. The immediate increase in home overall demand for given number of firms supplying goods in the home market induces higher initial demand for existing goods sold in the domestic market. Absent a change in TOL , the demand increase is spread evenly over home produced and imported varieties. This would represent an excess demand for foreign effective labor relative to the new increased level of effective labor available at home. This results in an initial increase in TOL . The increase in N_X^* on impact does not reverse this immediate effect. From that point on, the number of home firms steadily increases, but by less than PC/\tilde{P} , which shifts home demand toward home produced goods and reverses the initial excess demand for foreign effective labor. The steady increase in the number of varieties available in the home economy over time contributes to the increasing paths of home consumption (C) and the real wage (w).

Import demand at home ($y_X^*(z)$) increases monotonically, as the overall increase in demand at home outweighs the initial reversal in the direction of the relative price of foreign exports ($p_X^*(z)/\tilde{p}_D$)

²⁹Recall that the fixed export cost in units of consumption is wf_X/Z . Since f_X is unchanged, *ceteris paribus*, increases in w/Z increase the burden of the fixed cost on the profitability of exporting. Among other sources of real wage variation, upward pressure on w originates in the variety effect and in increased labor demand to cover the sunk entry cost for new home entrants.

initially rises). Thus, there is no change in the direction of the foreign export productivity cutoff z_X^* during the transition.

Export demand for home firms initially increases, as the relative price is the dominant effect. Initially, $y_X(z)$ increases as $p_X(z)/\tilde{p}_D^*$ falls in response to the immediate upward movement of TOL . Thus the home export cutoff productivity level z_X initially falls, as temporarily stronger demand allows relatively less productive firms to export.

The average real exchange rate \tilde{Q} depreciates in the short run, owing to the initial increase in TOL , but it subsequently appreciates. Its path resembles that of TOL but, as we observed above, the effect of the terms of labor is reinforced by the dynamics of the export cutoffs and the implied adjustment in the composition of the traded sector.

The consumption-based real exchange rate Q depreciates in the short and in the long run, as its dynamics are dominated by the variety effect. Its qualitative dynamics track those of the consumption differential C/C^* . The average terms of trade deteriorate, since, *ceteris paribus*, higher productivity makes home exports cheaper relative to foreign exports as in the standard model with PPP. Absent changes in Q and in the relative number of exporters in the two countries, terms of trade deterioration would fully offset the consumption differential to preserve balanced trade.³⁰ Terms of trade deterioration is dampened by a relative reallocation of resources away from the export sector in the home economy (except in the initial period) and into the export sector abroad. This leaves room for consumption-based real depreciation to perform part of the adjustment necessary to preserve balanced trade.

A Transitory Increase in Home Productivity

Figure 2 presents the responses to a 1 percent increase in home productivity with persistence .9 – a value of the persistence parameter at the lower end of the range usually considered in the RBC literature. The shock has no permanent effect, as all endogenous variables we focus on are stationary in response to stationary exogenous shocks. Our interest here is also in the persistence properties of the model’s endogenous variables relative to the persistence of a transitory shock.

Home consumption rises on impact, in response to higher income. As in the case of the short-run response to a permanent shock, absent a change in TOL , this would result in excess demand for effective foreign labor. Therefore, TOL rises temporarily, which pushes the relative price of

³⁰We report the response of $\tilde{T}_t^{\theta-1}$ in the figures to facilitate comparison with the consumption differential in equation (32).

home exports ($p_X(z)/\tilde{p}_D^*$) down and the relative price of foreign exports ($p_X^*(z)/\tilde{p}_D$) up. The effect of higher home demand (PC/\tilde{P}) prevails on the higher relative price of foreign exports and the increase in the number of firms selling at home, and $y_X^*(z)$ increases. As a consequence, the number of foreign exporters rises, and the cutoff productivity level z_X^* falls as less productive foreign firms now find it profitable to export.

Financial autarky implies that foreign consumers cannot share the favorable consequences of the transitory increase in home productivity through international asset trading motivated by the desire of home households to smooth the dynamics of C . Foreign consumption increases by little in response to increased income from exporting to home.

The initial decline in the relative price of home exports $p_X(z)/\tilde{p}_D^*$ temporarily boosts output of exporters, so that $y_X(z)$ increases initially. In the short run, the number of home exporters increases, both because all firms are temporarily more productive for given fixed export cost and because the expansion in demand makes it profitable to export for relatively less productive firms. *Ceteris paribus*, the increase in N_X contributes to higher C^* through availability of more varieties. But this effect is short lived. Households use investment in new firms to smooth the consequences of the shock on consumption. The number of entrants in the home economy increases and so does the total number of home firms. In turn, entry and the increase in the total number of home firms push the relative cost of home effective labor up (TOL falls), which reverses the movement in the relative price of home exports and quickly brings $y_X(z)$ below the steady state. The number of home exporters decreases as productivity returns to the steady state and output for each exporter has fallen. The decrease in $y_X(z)$ is mirrored by an increase in the export productivity cutoff for home firms (z_X), which causes relatively less productive home firms to leave the export market.

The path of the average real exchange rate \tilde{Q} is qualitatively similar to that of TOL . The favorable productivity shock results in real appreciation throughout most of the transition for the same reasons we discussed above. Similarly, the consumption-based real exchange rate Q depreciates tracking the consumption differential C/C^* , and the terms of trade deteriorate.

The dynamics of key endogenous variables in Figure 2 display substantial persistence, well beyond the exogenous .9 persistence of the productivity shock. Approximately 84 percent of the initial increase in productivity has been reabsorbed 10 years after the shock. At that point in time, the average real exchange rate \tilde{Q} still needs to cover roughly half the distance between the peak appreciation (which happens approximately 4 years after the shock) and the steady state.³¹

³¹This result does not depend on the presence of a steady-state iceberg cost $\tau = \tau^* \neq 1$. A similarly persistent

A large literature has developed in the past few years trying to explain real exchange rate movements in terms of nominal rigidity and local currency pricing. (See Chari, Kehoe, and McGrattan, 2002, and references therein.) The success has been, at best, mixed. Plausible degrees of nominal rigidity and local currency pricing (supported by the assumption of market segmentation) succeed in generating volatile real exchange rates, but only special assumptions deliver persistence in line with the data. Benigno (2003) highlights inertia in endogenous interest rate setting by central banks and differences in nominal rigidity as sources of real exchange rate persistence. Burstein, Eichenbaum, and Rebelo (2002) and Corsetti and Dedola (2002) recently put forth models that allow for the presence of distribution sectors and are a promising extension of the sticky-price literature in which structural features of the economy beyond nominal rigidity matter for real exchange rate dynamics. We propose a different mechanism that delivers substantial real exchange rate persistence in response to transitory shocks: firm entry and reallocation in and out of markets in a world of flexible prices.³²

Deregulation

We model deregulation in the domestic market as a permanent decrease in the sunk entry cost f_E relative to its initial steady-state level. This is consistent with Blanchard and Giavazzi (2003). Figure 3 presents the responses to a permanent 1 percent decrease in the home sunk entry cost.

As for the case of a permanent change in productivity, consider long-run effects first.

Absent changes in TOL , all entering firms will want to locate in home to take advantage of the lower entry costs. In equilibrium, home labor costs must rise relative to foreign to keep foreign labor employed, *i.e.*, TOL must fall. The lower entry costs must also induce entry of new firms, so that N_D rises. As in Blanchard and Giavazzi (2003), lower entry costs result in a larger number of firms and a higher real wage (w).

Along the transition, the major difference with the case of a permanent productivity increase

deviation from PPP happens if $\tau = \tau^* = 1$.

³²Imbs, Mumtaz, Ravn, and Rey (2002) argue that the “consensus” half-life of PPP deviations estimated by Rogoff (1996) (three to five years) is the outcome of aggregation bias. They show that the half life of deviations from PPP for disaggregated prices is much shorter (little longer than a year). In our view, there is no inconsistency in the observations that disaggregated price indexes converge to PPP in two years and aggregate CPIs complete their convergence in six years. The observations need not be explained in terms of aggregation “bias.” (Indeed, Caballero and Engel, 2003, argue that using disaggregated data biases estimates of the speed of macroeconomic adjustment upward.) Benigno (2003) points out that differences in nominal rigidity across sectors within a country can generate additional persistence in the dynamics of relative aggregate CPIs through their effect on sectoral relative prices. In our model, in the absence of per-unit trade costs, the law of one price holds for all traded goods. But the mechanism we discussed generates persistent changes in relative aggregate price indexes.

is that home no longer also enjoys an increase in the number of effective labor units relative to foreign. There is therefore no need for an initial increase in TOL to keep the new units of labor at home employed. In fact, there is an initial increase in demand for home labor relative to foreign, driven by the labor demand of new entrants in the home market. Another major difference is that home consumers no longer enjoy an immediate boost in demand/consumption linked to higher productivity. In fact, consumption decreases on impact as consumers increase their savings to finance the entry of new firms.

There are two competing effects on import demand at home ($y_X^*(z)$). The decrease in consumption demand has a negative effect, while the relative decrease in foreign labor costs (lower TOL) has a positive effect through its impact on relative prices ($p_X^*(z)/\tilde{p}_D$ falls). An initial decrease in N_X^* reinforces the effect of TOL , since it implies that the number of firms competing in the home market falls on impact. Over time, the entry of new home firms and an increasing number of foreign exporters have a negative effect on $y_X^*(z)$. Initially, the decrease in consumption demand dominates, and $y_X^*(z)$ falls. Over time, changes in both consumption and relative prices induce an increase in $y_X^*(z)$. Changes in the foreign export cutoff level z_X^* mirror the change in export demand: z_X^* increases initially as $y_X^*(z)$ falls, and z_X^* falls thereafter as $y_X^*(z)$ increases.

Changes in home export demand ($y_X(z)$) are dominated by the change in the relative price $p_X(z)/\tilde{p}_D^*$ – both the change in foreign consumption and the effect of changes in the number of firms selling in the foreign market are small relative to the changes in this variable. $p_X(z)/\tilde{p}_D^*$ rises as TOL falls. Hence, $y_X(z)$ falls over the entire transition. As usual, the change in the export cutoff z_X mirrors this change: z_X increases over the entire transition.

The Endogenous Harrod-Balassa-Samuelson Effect

We pause to comment on the real exchange rate and the HBS effect before analyzing the consequences of changes in trade policy. We focus on the average real exchange rate \tilde{Q} , since the latter is consistent with the empirical measures of the real exchange rate on which the HBS evidence is based.

Consistent with the HBS effect, our model predicts that more productive economies, or less regulated ones, will exhibit higher average prices relative to their trading partners. This real exchange rate appreciation (based on average prices) is driven by two key new features of our model:

1. *Entry*: Effective labor units must relatively appreciate in the economy providing the more attractive environment for firms. This is a key consequence of introducing firm entry (along with entry costs for the deregulation scenario). Given the existence of a non-traded sector, the relative increase in labor costs must induce an appreciation.³³
2. *Endogenous Non-Tradedness*: Changes in relative labor costs further induce changes in the composition of the traded sector in both countries:
 - Import prices in the economy with higher labor costs (home) rise as lower productivity firms can now export to it. Conversely, import prices in the economy with lower labor costs decrease as exporters with relatively lower productivity are forced to drop out of the export market. These effects necessarily reinforce the real exchange rate appreciation.
 - In the long run, the relative number of foreign exporters in the home market decreases in response to a permanent home productivity shock ($N'_D - N'^*_X > 0$, where a prime denotes the long-run movement from the initial steady state). This pushes the average price of consumption at home upward. Instead, the relative number of home exporters in the foreign market increases ($N'^*_D - N'_X < 0$), pushing the average price of consumption abroad downward. Both changes strengthen the appreciation of \tilde{Q} . In the case of deregulation at home, $N'_D - N'^*_X > N^*_D - N'_X$ contributes to the real appreciation.

Changes in expenditure shares also contribute to the real appreciation: As the relative labor costs increase in the more productive economy, the expenditure share on non-traded goods increases. Conversely, the expenditure share on non-traded goods in the other economy decreases. As non-traded goods have higher prices (relative to traded goods), the shift in expenditures further increases the real exchange rate appreciation.

Endogenous determination of the traded sector over time plays a key role in our microfounded, endogenous HBS effect. All goods are tradeable in our model; some are non-traded in equilibrium, and the margin moves in response to shocks. Thus, the non-tradedness margin *within* tradeable sectors is central in our theory. How relevant is it on empirical grounds? As noted by Bergin and Glick (2003*a, b*), there is evidence in the empirical trade literature that entry and exit into/from export markets do happen at frequencies that are relevant for business cycle analysis (Bernard and Jensen, 2001). However, such entry and exit do not occur across sectors but within sectors: We

³³As expected, the relative price of home non-traded goods in units of traded ones (\widetilde{TNT}) increases in the long run in figures 1 and 3.

do not observe some sectors switching between traded and non-traded. We observe firms/plants moving in and out of the export sectors within tradeable sectors. A model based on a traded/non-traded classification across sectors will therefore not capture the relevant margin for this process. As mentioned above, only 21 percent of U.S. plants export. There is thus a very substantial non-traded margin of firms within the tradeable manufacturing sector.

Entry and exit into the export market is also important in explaining aggregate trade flows (and hence other macro variables): Roberts and Tybout (1997*b*) report that the large export increases associated with devaluations in Mexico and Colombia were in the most part driven by the entry of new plants into the export market. That is, over 50 percent of the growth in exports is accounted for by new exporters (and not expansion of existing exporters). For the U.S., Bernard and Jensen (2003) report that 40 percent of the growth in U.S. exports over the period 1987-1992 was driven by new exporters. The changing non-tradedness margin within tradeable sectors is therefore empirically important for aggregate changes.³⁴

Trade Policy

We focus on the consequences of trade liberalization: a symmetric, worldwide decrease in the iceberg costs τ and τ^* or a symmetric, worldwide decrease in the fixed export costs f_X and f_X^* .

Lower Iceberg Costs

Figure 4 illustrates the response of the economy to a worldwide, permanent, 1 percent decrease in the iceberg trade costs τ and τ^* . Since the shock is symmetric, there is no movement in relative, cross-country variables such as the terms of labor, the terms of trade, and the real exchange rate. We report only changes in home variables as they are identical to those in foreign variables. Lower iceberg costs result in lower export prices and higher output for each individual exporter ($y_X(z)$). There is an immediate, substantial increase in the number of exporting firms, mirrored by a decrease in the export productivity cutoff. The increase in export output demand and the fact that more exporters are demanding labor to cover fixed export costs contribute to upward pressure on the real wage. Consumption rises in response to higher household wage and dividend income. However, *ceteris paribus*, a higher real wage increases the burden of the sunk entry cost in the domestic market. For this reason, the number of entrants drops, and the total number of firms that produce

³⁴Klein, Schuh, and Triest (2003) find evidence that real exchange rate movements lead to significant reallocations of labor in U.S. manufacturing. Our model provides a theoretical foundation for the effect of firm reallocation across sectors on the aggregate economy.

in each economy falls gradually. It is this gradual fall in N_D that causes the long-run increase in w and C to be smaller than the short-run movement through the variety effect. A higher real wage also drives the price of output for domestic sale up, so that domestic sales actually fall, though by less than the increase in output of each individual exporter, and aggregate GDP (y) rises.

Lower Fixed Costs

Figure 5 shows the responses to a worldwide, permanent, 1 percent decrease in the fixed export costs f_X and f_X^* . Qualitatively, most effects are similar to those of a reduction in the size of the iceberg costs, although the size of the responses is significantly smaller in most cases. The main qualitative difference relative to Figure 4 is that export prices rise and export output falls. As in Figure 4, trade liberalization results in a sizable increase in the number of exporters. *Ceteris paribus*, this puts upward pressure on the real wage. Since there is no change in iceberg costs and the real exchange rate, both prices for domestic and export sales must rise. Higher export prices, a small change in overall demand, and the fact that a larger number of exporters implies that each of them is facing a lower demand combine to generate a decrease in output for each exporter.

The dynamics of consumption in figures 4 and 5 imply that the welfare gains from trade liberalization are much larger when liberalization is accomplished by lowering per-unit trade costs than by reducing the fixed export cost. The changes in individual exporter output and the number of exporters in each country following trade liberalization are consistent with Kehoe and Ruhl's (2002) evidence that a substantial portion of the increase in trade after liberalization takes place on the extensive margin.

5 International Trade in Bonds

We now extend the model of Section 2 to allow for international trade in bonds. This allows us to study both how international bond trading affects the results we obtained so far and how the microeconomic dynamics in our model affect current account movements relative to more standard setups without the features of ours. Since the extension to international borrowing and lending does not contain especially innovative features relative to the autarky setup, we limit ourselves to describing its main ingredients in words here and present the relevant model equations in an appendix.

We assume that agents can trade bonds domestically and internationally. Home bonds, issued by

home households, are denominated in home currency. Foreign bonds, issued by foreign households, are denominated in foreign currency. We keep the assumption that nominal returns are indexed to inflation in each country, so that the bonds issued by each country provide a risk-free, real return in units of that country's consumption basket.

International asset markets are incomplete, as only risk-free bonds are traded across countries. In the absence of any other change in our model, this would imply indeterminacy of steady-state net foreign assets and non-stationarity of the model. The choice of the initial position of the economy would be a matter of convenience, and all shocks would have permanent consequences via wealth reallocation across countries regardless of the nature of the disturbances, thus undermining the reliability of log-linear approximation and the validity of stochastic analysis. (Ghironi, 2000, discusses the issue in detail.) To solve this problem, we assume that agents must pay fees to domestic financial intermediaries when adjusting their bond holdings. We specify these fees as quadratic in the stock of bonds. This convenient specification is sufficient to pin down the steady state uniquely and deliver stationary model dynamics in response to temporary shocks.³⁵ Schmitt-Grohé and Uribe (2003) demonstrate that different approaches to the issue of steady-state determinacy and model stationarity under incomplete markets generate similar dynamics when the models are calibrated to match a given economy. We choose the cost of adjusting bond holdings over the alternatives for its analytical convenience and ease of interpretation. (One can think of the fees that households must pay as capturing a form of limited participation in financial markets as in Cooley and Quadrini, 1999.) Realistic choices of parameter values imply that the cost of adjusting bond holdings has a very small impact on model dynamics, other than pinning down the steady state and ensuring mean reversion in the long run.³⁶

We assume that financial intermediaries rebate the revenues from bond-adjustment fees to domestic households. In equilibrium, the markets for home and foreign bonds clear, and each country's net foreign assets entering period $t + 1$ depend on interest income from asset holdings entering period t , labor income, net investment income, and consumption during period t . The change in asset holdings between t and $t + 1$ is the country's current account. Home and foreign current accounts add to zero when expressed in units of the same consumption basket. There are now three Euler equations in each country: the Euler equation for share holdings, which is

³⁵Turnovsky (1985) first used this approach, later adopted by Benigno, P. (2001) and Laxton and Pesenti (2003), among others.

³⁶We could have introduced costs of adjusting bond holdings in the autarky case above, but the costs would have had no role in equilibrium, as holdings of bonds are always zero under autarky, leaving the system of equations to be solved unchanged.

unchanged, and Euler equations for holdings of domestic and foreign bonds. The fees for adjusting bond holdings imply that the Euler equations for bond holdings feature a term that depends on the stock of bonds, and which is central to steady-state determinacy and model stationarity. Euler equations for bond holdings in each country imply a no-arbitrage condition between bonds. In the log-linear model, this no-arbitrage condition relates the interest rate differential across countries to expected real exchange rate depreciation in a standard fashion.

The balanced trade condition closed the model in the case of financial autarky. We show that balanced trade implies labor market clearing in each country in Appendix B. Since the balanced trade condition no longer holds once we allow for international bond trading, we must impose labor market clearing conditions at home and abroad to close the model. These conditions state that the amount of labor used in production and to cover sunk entry costs and fixed export costs in each country must equal labor supply in that country in each period.

Since the costs of adjusting bond holdings pin down zero holdings of both domestic and foreign bonds in both countries as the unique steady state, the model with international bond trading of Appendix C has exactly the same steady state as the model under autarky (under the same assumptions about exogenous variables). We linearize the system and solve it using the same log-linearization technique and the method of undetermined coefficients.³⁷

Impulse Responses

We consider the same productivity and deregulation shocks as in the case of autarky. The responses to trade liberalization are identical to those under financial autarky. Since the trade liberalization scenarios involve identical changes in trade policy instruments in the two countries and the initial steady state is symmetric, there is no incentive for agents to trade assets across countries in equilibrium even if they are allowed to do so.

We set the scale parameter for the cost of adjusting bond holdings to .0025 – sufficient to generate stationarity in response to transitory shocks but small enough to ensure that we are not attributing too large a role to this friction in the dynamics of our model.

³⁷Since steady-state holdings of bonds are zero, the percentage deviations of bond stocks from the steady state are normalized by the steady-state level of consumption when linearizing the model (for instance, $B_{t+1} \equiv dB_{t+1}/C$). Similarly for the current account and the trade balance.

A Permanent Increase in Home Productivity

Figure 6 shows impulse responses to a permanent 1 percent increase in home productivity. The response of several key variables to the shock is qualitatively similar to that under financial autarky. The permanent nature of the shock implies that home households do not have an incentive to accumulate or decumulate net foreign assets to smooth the effect of a *transitory* fluctuation in income on consumption. The path of C is very similar to that in Figure 1.

As in the case of financial autarky, TOL must decrease in the long run (home effective labor must relatively appreciate) to prevent all entering firms from locating to the home economy. The free entry condition implies entry of new firms, and N_D increases.

The home economy runs a current account deficit in response to the shock and accumulates net foreign debt.³⁸ Home households borrow from abroad to finance more investment in new, more productive home firms than under financial autarky in the first years after the shock, as one can see by comparing the responses of N_E in figures 1 and 6. The ability to borrow from abroad allows home agents to front-load the increase in the number of home firms that is triggered by the shock, so that the profile of N_D is steeper than in Figure 1. The home household's incentive to front-load entry of more productive firms is mirrored by the foreign household's desire to invest savings in the more attractive economy. Although foreign households cannot hold shares in the mutual portfolio of home firms (since we allow only bonds to be traded across countries), the return on bond holdings is tied to the return on holdings of shares in home firms by no-arbitrage between bonds and shares within the home economy. Therefore, foreign households can share the benefits of higher home productivity by lending to home.

Faster entry of new home firms puts upward pressure on the home real wage, so that TOL no longer depreciates on impact and home exporters no longer enjoy an initial boost in demand: $y_X(z)$ is below the steady state throughout the transition to the new, lower long-run position. In turn, this is mirrored by an increase in the export productivity cutoff z_X .

As under financial autarky, more overall demand for consumption at home results in more demand for imports: $y_X^*(z)$ rises and relatively less productive foreign firms enter the export market (z_X^* falls). Foreign consumption increases by less than under financial autarky on impact, and it takes a few years to achieve the new steady-state level, as foreign agents save in the form

³⁸The impulse response of \mathbf{a} in Figure 6 cumulates holdings of domestic and foreign bonds. It shows the level of \mathbf{a} at the *end* of each period. (The same is true for \mathbf{a}^* .) The result that home runs a current account deficit does not change if one looks at the response of $P_t C A_t / \tilde{P}_t$ to adjust for the effect of changes in the number of available varieties on the current account in units of the consumption basket.

of lending to the home economy in the initial periods. In fact, the decision of foreign households to save in the form of foreign lending to the more productive country implies that less resources are available to fund entry of new firms in the foreign economy, so that N_E^* falls substantially in the first few years and there is a more pronounced decrease in the total number of foreign firms (N_D^*) than in Figure 1, resulting in a downward movement in foreign GDP at the time of the peak negative response in N_D^* .

The average real exchange rate \tilde{Q} appreciates in response to the appreciation of the terms of labor and the endogenous adjustment of the traded sectors in both countries. As the results of Section 3 suggested, opening the economy to international asset trading does not change the functioning of the endogenous HBS mechanism at work in our model.

A Transitory Increase in Home Productivity

Figure 7 shows impulse responses to an increase in home productivity with persistence .9. Dynamics are qualitatively similar to those under financial autarky. As in the case of a permanent shock, an important difference is the absence of an initial depreciation of the terms of labor, again motivated by faster entry of new firms into the home economy than under autarky. Home households borrow initially to finance faster firm entry. However, borrowing is quickly reversed, and home runs current account surpluses for approximately seven years after the shock. The path of the current account is such that home's net foreign assets are actually above the steady state throughout the transition, except in the initial periods. When the shock is not permanent, lending abroad to smooth the consequences of a temporary, favorable shock on consumption becomes the main determinant of net foreign asset dynamics.³⁹

Deregulation

Figure 8 shows impulse responses to deregulation of the home market. The comparison with Figure 3 can be understood along the same lines as that between figures 1 and 6. Home households borrow from abroad to front-load entry of new firms in the more favorable home market. The home country runs a current account (and trade) deficit and accumulates foreign debt. Home consumption initially declines and is permanently higher in the long run. Foreign consumption moves by more than in Figure 3 as foreign households initially save in the form of foreign lending

³⁹Foreign households initially invest in the more productive home economy and then borrow to share the consumption benefits of temporarily higher home productivity.

and then receive income from their positive asset position. The terms of labor and the average real exchange rate \tilde{Q} appreciate.

Once we allow for international borrowing and lending, our model predicts that (permanently) more productive, or less regulated, economies will have a stronger real exchange rate (the HBS effect) and will run persistent (though not permanent) foreign debt positions to finance faster entry of new firms in the economy. These predictions appear in line with the experience of the United States in the 1990s.

International Business Cycles

Backus, Kehoe, and Kydland (BKK, 1992) show that introducing trade costs in an international RBC model improves its ability to replicate second moments of U.S. and international data. They specify a resource cost of trade as a quadratic function of net exports. But even the introduction of these trade costs is not sufficient to remove the result that consumption is more strongly correlated across countries than aggregate output in their model, contrary to the data (the consumption-output anomaly). Backus and Smith (1993) show that international RBC models with internationally complete asset markets tie the cross-country consumption differential to the real exchange rate through international risk-sharing. Contrary to the prediction of perfect positive correlation between relative consumption and the real exchange rate, they document evidence of no clear pattern in the data (the consumption-real exchange rate anomaly). Chari, Kehoe, and McGrattan (CKM, 2002) report evidence of negative correlation between relative consumption and the real exchange rate for the U.S. relative to Europe. Their sticky-price model does better than BKK as far as the consumption-output anomaly is concerned, but it too fails when it comes to relative consumption and the real exchange rate – and to generating sufficient persistence in the latter.

Obstfeld and Rogoff (2001) argue that introducing iceberg trade costs helps explain a variety of puzzles in international comovements, including the BKK consumption-output anomaly. They observe that trade costs and incomplete asset markets can explain the consumption-real exchange rate anomaly.⁴⁰ Our model features trade costs and incomplete asset markets. Here, we investigate its ability to mimic key features of international business cycles and its performance in relation to the puzzles highlighted above.

The model includes only one source of fluctuations at business cycle frequencies, the shocks to

⁴⁰Benigno and Thoenissen (2003) show that international asset market incompleteness plays a central role in dealing with the Backus-Smith puzzle in a model in which intermediate goods are traded and households consume only non-traded goods.

aggregate productivities Z_t and Z_t^* . In this section, we calibrate the bivariate productivity process (33) based on U.S. and international data, holding other exogenous variables at their steady-state levels. For purposes of comparison, we use the symmetrized estimate of the bivariate productivity process for the U.S. and an aggregate of European economies in BKK and set

$$\begin{bmatrix} \phi_Z & \phi_{ZZ^*} \\ \phi_{Z^*Z} & \phi_{Z^*} \end{bmatrix} = \begin{bmatrix} .906 & .088 \\ .088 & .906 \end{bmatrix}.$$

This matrix implies a small, positive productivity spillover across countries, such that, if home productivity rises during period t , foreign productivity will also increase at $t + 1$. We set the standard deviation of the productivity innovations to .00852 (a .73 percent variance) and the correlation to .258 (corresponding to a .19 percent covariance), as estimated by BKK. We calculate the implied second moments of endogenous variables using the frequency domain technique described in Uhlig (1999) and compare the model-generated moments with those of U.S. and international data computed in BKK and reported in Table 2 for the reader's convenience.⁴¹ (Benigno, 2003, and CKM are the sources on the empirical properties of the real exchange rate. Benigno reports averages of data in Bergin and Feenstra – BF –, 2001, and CKM.) For consistency with BKK and CKM, who focus on the high-frequency properties of business cycles in the U.S. and abroad, we report second moments of Hodrick-Prescott (HP)-filtered variables. The productivity process has eigenvalues .994 and .818. A stationary process for productivity and model stationarity imply that all endogenous variables of interest are stationary. However, productivity and key endogenous state variables – such as the number of firms and asset stocks – are persistent enough that model-generated moments calculated without HP filtering pick up low frequency fluctuations that are not featured in the HP-filtered data in BKK.

Households invest in the entry of new firms in the economy in our model. The number of new entrants behaves very much like investment in capital, with the latter proxied by the number of firms that produce in each period. For comparison with BKK's results on investment and capital, we compute second moments for $N_{E,t}$ and $N_{D,t}$ as well as for their overall values in terms of average firm valuation ($\tilde{v}_t N_{E,t}$ and $\tilde{v}_t N_{D,t}$, respectively). Consistent with BKK, we define saving as the difference between GDP and consumption ($y_t - C_t$) and investigate the correlation between the saving rate ($1 - C_t/y_t$) and the investment rate ($\tilde{v}_t N_{E,t}/y_t$) relative to the data. To control for the role of the variety effect on measurement, we consider two measures of real variables, in units of

⁴¹Results based on model simulation are similar.

consumption (for instance, y_t) and deflated by the average price level \tilde{P}_t ($P_t y_t / \tilde{P}_t$). In most cases, results are very similar across measures.

Table 3 reports our results for the benchmark calibration. The model underpredicts the standard deviation of aggregate output (measured by GNP in BKK) and overpredicts that of consumption, although it is successful at generating less volatile consumption than GDP. The ratio of the standard deviation of $P_t C_t / \tilde{P}_t$ to $P_t y_t / \tilde{P}_t$ is roughly .59, ten percent higher than the consumption-output volatility ratio in BKK’s data. Like fixed investment in the data, investment in new firms is substantially more volatile than GDP. The trade balance/GDP ratio is much more volatile than the net exports/output ratio in the data. The trade costs in our model do not prevent the trade balance/GDP ratio from being quite volatile as the BKK specification does by penalizing trade balance movements directly.⁴² Instead, the real exchange rate is clearly less volatile than in the data.

The model is quite successful at reproducing the autocorrelation function of key U.S. aggregate variables with output: The autocorrelation functions for output itself, consumption, investment in new firms, the stock of productive firms, and the trade balance in Table 3 all reproduce the qualitative pattern in Table 2. In the cases of output, consumption, and investment success is also reasonable, if not striking, on quantitative grounds.

The BKK model fails to deliver positive correlation between home and foreign output. A puzzling negative cross-country correlation of aggregate outputs is a standard result of the international RBC literature. Our model successfully generates positive correlation between foreign and domestic GDPs. However, as the BKK setup, ours fails to generate cross-country consumption correlation that is smaller than the correlation across GDPs for the same parametrization of productivity. The contemporaneous correlation between saving and investment rates is positive, but stronger than in the data. The autocorrelation function for the real exchange rate \tilde{Q}_t displays substantial persistence, with a first-order coefficient equal to .89 roughly in line with the BF and CKM evidence. The correlation between relative consumption spending (the ratio of home to foreign nominal consumption spending deflated by the respective average price levels) and the real exchange rate \tilde{Q}_t is negative – as in the CKM data – but too large in absolute value. The correlation between relative consumption and the consumption based interest rate – which is 1 in Backus and Smith’s (1993) complete markets world – is .71.

Reynolds (1993) and Baxter and Crucini (1995) find that estimated spillover coefficients in the

⁴²Note, however, that the volatility of the trade balance itself is much smaller.

process (33) are insignificantly different from zero. Baxter and Crucini (1995) argue in favor of a near-unit-root process for productivity. Accordingly, Baxter (1995) and Baxter and Farr (2001) set the coefficients ϕ_{ZZ^*} and ϕ_{Z^*Z} to zero and consider values of $\phi_{ZZ} = \phi_{Z^*Z^*}$ ranging between .995 and .999 in their exercises. Table 4 reports results for persistence .999. (As Baxter, 1995, we keep the same variance-covariance matrix of the productivity innovations as in BKK.) The majority of qualitative patterns are unchanged. As before, the model overpredicts the standard deviation of consumption and the trade balance-GDP ratio. It underpredicts the standard deviation of GDP and the real exchange rate. It does quite well in terms of the standard deviations of investment in new firms and the total stock of firms.

The model generates substantial GDP persistence in Table 4. The autocorrelation function for investment in new firms is very close to the empirical autocorrelation of fixed investment. The contemporaneous correlation of home consumption to home GDP is too strong relative to the data. Most importantly, however, the consumption-output anomaly of BKK weakens substantially: The correlation of consumption across countries (C and C^*) is smaller than that of GDP (y and y^*). When productivity spillovers are present, the response of foreign consumption to a home shock is larger, as foreign households anticipate that foreign productivity will rise too. When we remove the spillovers, the increase in foreign consumption following a home shock is muted, resulting in lower consumption correlation than GDP correlation across countries. The consumption-output anomaly persists if we consider variables deflated by the respective average prices, but the correlation between PC/\tilde{P} and P^*C^*/\tilde{P}^* is only slightly larger than that between Py/\tilde{P} and P^*y^*/\tilde{P}^* . The correlation between saving and investment rates is now in the same range as observed in the data. As in the case of the BKK productivity process, the correlation between relative consumption spending deflated by average prices and the real exchange rate is negative, but too large in absolute value. Importantly, the correlation between C/C^* and Q now drops to .13.

Overall, we interpret the results of the stochastic exercise of this section as supportive of the novel features of our model as a mechanism for the propagation of business cycles across countries and over time. Even after HP filtering (and thus removing low-frequency fluctuations that are arguably important for medium- to long-run transmission), the model is very successful at generating persistent dynamics of endogenous variables and matching several key moments of the data for reasonable parameter values and a standard productivity process. Consistent with Obstfeld and Rogoff's (2001) arguments, the introduction of trade costs and market incompleteness pushes results in the right direction with respect to important puzzles in international macroeconomics,

although assumptions about the exogenous shock process also play an important role.

6 Conclusions

We developed a two-country, stochastic, general equilibrium model of international trade and macroeconomic dynamics. Relative to existing international macro models, ours has the advantage of matching several features of empirical evidence in the micro, trade literature. It does so while preserving substantial tractability and the ability to provide intuitions for the main results.

We assumed that firms face some uncertainty about their future productivity when they make the decision whether or not to sink the resources necessary to enter the domestic market. Consistent with overwhelming empirical evidence, we assumed that firms face fixed costs as well as per-unit costs when they export. As a consequence of the fixed export cost, only the relatively more productive firms self-select into the export market. Aggregate productivity shocks, changes in domestic market regulation, and changes in trade policy cause firms to enter and exit markets and generate deviations from PPP that would not exist absent our microeconomic structure.

Our model provides a fully microfounded, endogenous explanation for the HBS effect, according to which more productive economies have relatively stronger real exchange rates. All goods are tradeable in our setup, some are non-traded in equilibrium, and the non-tradedness margin changes over time. In contrast to the textbook treatment of the HBS effect, shocks are aggregate rather than sector-specific. Our model predicts that more productive economies, or less regulated ones, will exhibit higher average prices relative to their trading partners. This real exchange rate appreciation is driven by entry and endogenous non-tradedness, the two key new features of our setup.

The same new features result in substantial persistence of key endogenous variables in response to transitory exogenous shocks. In particular, our model results in persistent PPP deviations in a world of flexible prices.

When we allow for international borrowing and lending, the model predicts that more productive, or less regulated, economies will experience HBS real appreciation and run persistent foreign debt positions to finance faster entry of new firms in the economy. Thus, our framework provides a novel perspective on recent stylized facts for the U.S. economy that are broadly in line with these predictions. In addition, the model matches several important moments of the U.S. and international business cycle quite well for reasonable assumptions about parameters and productivity. In particular, it generates positive GDP correlation across countries, it does not constrain

the correlation between relative consumption and the real exchange rate to being perfect, and it improves on the standard international RBC setup as far as cross-country correlations of consumption and GDP are concerned, confirming Obstfeld and Rogoff's (2001) argument that trade costs help explain international macroeconomic puzzles.

Baldwin and Krugman (1989) used a mechanism similar to ours in a sticky-price, partial equilibrium model to explain persistent effects of nominal exchange rate movements. Although it does well on persistence grounds, the flexible-price setup of this paper underpredicts the volatility of GDP and the real exchange rate. We conjecture that including nominal rigidity will amplify the response of the real exchange rate to shocks and subject it to the consequences of monetary shocks, complementing our persistence-inducing mechanism with increased real volatility. We will extend our model to incorporate nominal rigidity in future work.

Appendix

A. The Steady State

We denote constant, steady-state levels of variables by dropping the time subscript and assume: $f_E = f_E^*$, $f_X = f_X^*$, $\tau = \tau^*$, $L = L^*$, and $Z = Z^* = 1$. Under these assumption, the steady state of the model is symmetric: $\tilde{Q} = Q = \mu = 1$ and the levels of all other endogenous variables are equal across countries.

Solving for \tilde{z}_X

Given the solution for the average export productivity \tilde{z}_X , we can obtain the cutoff level z_X from (14). We can solve for \tilde{z}_X as follows. The Euler equation for share holdings yields:

$$\tilde{v} = \frac{\beta(1-\delta)}{1-\beta(1-\delta)} \left(\tilde{d}_D + \frac{N_X}{N_D} \tilde{d}_X \right). \quad (34)$$

Combining this equation with the free entry condition $\tilde{v} = f_E w$ implies:

$$\tilde{d}_D + \frac{N_X}{N_D} \tilde{d}_X = \frac{[1 - (1-\delta)\beta]}{(1-\delta)\beta} f_E w. \quad (35)$$

The steady-state zero profit export cutoff equation is:

$$\tilde{d}_X = w f_X \frac{\theta - 1}{k - (\theta - 1)}. \quad (36)$$

Also, steady-state profits from selling at home and abroad are:

$$\tilde{d}_D = \frac{1}{\theta} (\tilde{\rho}_D)^{1-\theta} C, \quad (37)$$

$$\tilde{d}_X = \frac{1}{\theta} (\tilde{\rho}_X)^{1-\theta} C - wf_X. \quad (38)$$

These two equations imply:

$$\tilde{d}_D = \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_D} \right)^{\theta-1} (\tilde{d}_X + wf_X). \quad (39)$$

Optimal pricing yields:

$$\tilde{\rho}_D = \frac{\theta}{\theta-1} \tilde{z}_D^{-1} w. \quad (40)$$

$$\tilde{\rho}_X = \frac{\theta}{\theta-1} \tau \tilde{z}_X^{-1} w. \quad (41)$$

Hence:

$$\frac{\tilde{\rho}_X}{\tilde{\rho}_D} = \frac{\tau \tilde{z}_D}{\tilde{z}_X}. \quad (42)$$

Substituting this equation into (39), we have:

$$\tilde{d}_D = \left(\frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} (\tilde{d}_X + wf_X), \quad (43)$$

or, taking (36) into account,

$$\tilde{d}_D = \left(\frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} \left(wf_X \frac{\theta-1}{k-(\theta-1)} + wf_X \right). \quad (44)$$

The steady-state share of exporting firms in the total number of domestic firms is:

$$\frac{N_X}{N_D} = (z_{\min})^k (\tilde{z}_X)^{-k} \left[\frac{k}{k-(\theta-1)} \right]^{\frac{k}{\theta-1}}. \quad (45)$$

Substituting equations (36), (44), and (45) into (35), using $\tilde{z}_D = \{k/[k-(\theta-1)]\}^{\frac{1}{\theta-1}} z_{\min}$, and rearranging yields:

$$(\tilde{z}_X)^{1-\theta} (\tau z_{\min})^{\theta-1} \left[\frac{k}{k-(\theta-1)} \right]^2 + (\tilde{z}_X)^{-k} (z_{\min})^k \left[\frac{k}{k-(\theta-1)} \right]^{\frac{k}{\theta-1}} \frac{\theta-1}{k-(\theta-1)} = \frac{[1-(1-\delta)\beta] f_E}{(1-\delta)\beta f_X}.$$

This equation can be rewritten as:

$$\xi_1 (\tilde{z}_X)^{1-\theta} + \xi_2 (\tilde{z}_X)^{-k} = \xi_3, \quad (46)$$

where

$$\begin{aligned} \xi_1 &\equiv (\tau z_{\min})^{\theta-1} \left[\frac{k}{k - (\theta - 1)} \right]^2 > 0, \\ \xi_2 &\equiv (z_{\min})^k \left[\frac{k}{k - (\theta - 1)} \right]^{\frac{k}{\theta-1}} \frac{\theta - 1}{k - (\theta - 1)} > 0, \\ \xi_3 &\equiv \frac{[1 - (1 - \delta)\beta] f_E}{(1 - \delta)\beta} \frac{f_E}{f_X} > 0. \end{aligned}$$

The left-hand side of equation (46) is a hyperbola. This guarantees existence and uniqueness of $\tilde{z}_X > 0$, the exact value of which we obtain numerically.

Solving for $\tilde{\rho}_X$

The law of motion for the total number of domestic firms implies:

$$N_E = \frac{\delta}{1 - \delta} N_D. \quad (47)$$

The steady-state aggregate accounting equation is:

$$C = wL + N_D \tilde{d}_D + N_X \tilde{d}_X - N_E w f_E.$$

Using (35) and (47), this can be rewritten as:

$$\frac{C}{w} = L + N_D f_E \frac{1 - \beta}{(1 - \delta)\beta}. \quad (48)$$

Equations (36) and (38) imply:

$$\frac{C}{w} = \tilde{\rho}_X^{\theta-1} \frac{\theta k}{k - (\theta - 1)} f_X \quad (49)$$

The price index equation $N_D \tilde{\rho}_D^{1-\theta} + N_X \tilde{\rho}_X^{1-\theta} = 1$ yields:

$$\frac{\tilde{\rho}_X^{\theta-1}}{N_D} = \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_D} \right)^{\theta-1} + \frac{N_X}{N_D},$$

or, using equations (42) and (45),

$$\frac{\tilde{\rho}_X^{\theta-1}}{N_D} = \left(\frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} + \left(\frac{z_{\min}}{\tilde{z}_X} \right)^k \left[\frac{k}{k - (\theta - 1)} \right]^{\frac{k}{\theta-1}}. \quad (50)$$

Together, equations (48), (49), and (50) yield the following equation for $\tilde{\rho}_X$:

$$\tilde{\rho}_X^{1-\theta} = \left[\frac{\theta k}{k - (\theta - 1)} f_X - K^{-1} f_E \frac{1 - \beta}{(1 - \delta) \beta} \right] L^{-1}.$$

where K is the right-hand side of equation (50).

Special Case: All Firms Export In this case, equation(49) no longer holds since the zero cutoff profit condition (36) no longer holds. Using equations (37) and (38), equation (35) can be written as:

$$\tilde{\rho}_X^{1-\theta} \frac{C}{\theta} \left(\tau^{\theta-1} + 1 \right) - w f_X = \frac{[1 - (1 - \delta) \beta]}{(1 - \delta) \beta} f_E w,$$

which implies:

$$\frac{C}{w} = \tilde{\rho}_X^{\theta-1} \frac{\theta}{\tau^{\theta-1} + 1} \left\{ f_X + \frac{[1 - (1 - \delta) \beta]}{(1 - \delta) \beta} f_E \right\}. \quad (51)$$

Equation (51) now replaces equation (49) when solving for $\tilde{\rho}_X$. This yields the following expression for $\tilde{\rho}_X$:

$$\tilde{\rho}_X^{1-\theta} = \left[\frac{\theta}{\tau^{\theta-1} + 1} \left\{ f_X + \frac{[1 - (1 - \delta) \beta]}{(1 - \delta) \beta} f_E \right\} - K^{-1} f_E \frac{1 - \beta}{(1 - \delta) \beta} \right] L^{-1}.$$

Solving for the Remaining Variables

The solutions for other endogenous variables are straightforward

- $N_D = K^{-1} \tilde{\rho}_X^{\theta-1}$;
- $\tilde{\rho}_D = \frac{\tilde{z}_X}{\tau \tilde{z}_D} \tilde{\rho}_X$ using (42);
- $w = \tilde{\rho}_X \frac{\theta-1}{\theta \tau} \tilde{z}_X$ using (41);
- $C = w \left[L + N_D f_E \frac{1-\beta}{(1-\delta)\beta} \right]$ using (48);
- $N_E = \frac{\delta}{1-\delta} N_D$ using (47);
- $N_X = N_D (z_{\min})^k (\tilde{z}_X)^{-k} \left[\frac{k}{k - (\theta - 1)} \right]^{\frac{k}{\theta-1}}$ using (45);
- $\tilde{d}_D = \frac{1}{\theta} (\tilde{\rho}_D)^{1-\theta} C$ using (37);

- $\tilde{d}_X = \frac{1}{\theta} (\tilde{\rho}_X)^{1-\theta} C - wf_X$ using (38);
- $\tilde{v} = wf_E$ (using the free entry condition);
- $1 + r = 1/\beta$ (using the Euler equation for bond holdings).

Symmetry of the steady state ensures $\tilde{z}_X^* = \tilde{z}_X$, $\tilde{\rho}_X^* = \tilde{\rho}_X$, $N_D^* = N_D$, $N_E^* = N_E$, $N_X^* = N_X$, $\tilde{d}_D^* = \tilde{d}_D$, $\tilde{d}_X^* = \tilde{d}_X$, $\tilde{v}^* = \tilde{v}$, in addition to $C^* = C$, $w^* = w$, and $r^* = r$.

B. Labor Market Clearing

Recall that a firm with productivity z produces $Z_t z$ units of output per worker. Consider separately the labor used to produce goods for the domestic and export markets: let $l_{D,t}(z)$ and $l_{X,t}(z)$ represent the number of workers hired to produce goods for each market. These only represent production workers; in addition, each new entrant hires $f_{E,t}/Z_t$ workers to cover the entry cost, and each exporter hires $f_{X,t}/Z_t$ workers to cover the fixed export cost in every period. The profits earned from domestic sales for a firm with productivity z are then given by:

$$\begin{aligned} d_{D,t}(z) &= \rho_{D,t}(z) Z_t z l_{D,t}(z) - w_t l_{D,t}(z) \\ &= \frac{1}{\theta - 1} w_t l_{D,t}(z), \end{aligned} \tag{52}$$

using $\rho_{D,t}(z) = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z}$ from (8). This relationship holds for a firm with average productivity \tilde{z}_D , and also for averages across all domestic firms. This implies that the average number of production workers hired to cover domestic sales is $(\theta - 1) \tilde{d}_{D,t}/w_t$. The total number of such workers hired at home is thus $N_{D,t}(\theta - 1) \tilde{d}_{D,t}/w_t$.

The profits earned from export sales for an exporting firm with productivity z are given by:

$$\begin{aligned} d_{X,t}(z) &= Q_t \rho_{X,t}(z) \frac{Z_t z l_{X,t}(z)}{\tau_t} - w_t \left[l_{X,t}(z) + \frac{f_{X,t}}{Z_t} \right] \\ &= \frac{1}{\theta - 1} w_t l_{X,t}(z) - w_t \frac{f_{X,t}}{Z_t}, \end{aligned} \tag{53}$$

using $\rho_{X,t}(z) = Q_t^{-1} \tau_t \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z}$ from (8). Note that only $Z_t z l_{X,t}(z)/\tau_t$ export units are sold, although $Z_t z l_{X,t}(z)$ are produced (the remaining fraction having “melted” away in an iceberg fashion while crossing the border). Again, this relationship holds for a firm with average export productivity $\tilde{z}_{X,t}$, and also for averages across all exporters. The average number of production workers hired

to cover export sales is thus $(\theta - 1)\tilde{d}_{X,t}/w_t + (\theta - 1)f_{X,t}/Z_t$. Multiplying by $N_{X,t}$ yields the total number of such workers for the home economy.

The total number of production workers hired in the home economy is then

$$\frac{\theta - 1}{w_t}N_{D,t}d_{D,t}(z) + \frac{\theta - 1}{w_t}N_{X,t}\tilde{d}_{X,t} + \frac{\theta - 1}{Z_t}N_{X,t}f_{X,t}.$$

Adding the total number of workers hired by new entrants, $N_{E,t}f_{E,t}/Z_t$, and those hired by exporters to cover the fixed costs, $N_{X,t}f_{X,t}/Z_t$, yields the aggregate labor demand for the home economy:

$$L_t = \frac{\theta - 1}{w_t}N_{D,t}\tilde{d}_{D,t} + \frac{\theta - 1}{w_t}N_{X,t}\tilde{d}_{X,t} + \frac{\theta}{Z_t}N_{X,t}f_{X,t} + \frac{1}{Z_t}N_{E,t}f_{E,t}. \quad (54)$$

Equating L_t to labor supply (L) yields the equilibrium condition (69) below. The derivation of (70) is analogous.

Balanced Trade Implies Labor Market Clearing

We now demonstrate that balanced trade under financial autarky implies labor market clearing.

The balanced trade condition

$$Q_t N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} C_t^* = N_{X,t}^* (\tilde{\rho}_{X,t}^*)^{1-\theta} C_t$$

can be written (see the home price index identity (9)):

$$Q_t N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} C_t^* = \left[1 - N_{D,t} (\tilde{\rho}_{D,t})^{1-\theta} \right] C_t.$$

This condition can be re-written as

$$C_t = \theta N_{X,t} \left(\tilde{d}_{X,t} + w_t \frac{f_{X,t}}{Z_t} \right) + \theta N_{D,t} \tilde{d}_{D,t},$$

since

$$\begin{aligned} \tilde{d}_{D,t} &= \frac{1}{\theta} (\tilde{\rho}_{D,t})^{1-\theta} C_t, \\ \tilde{d}_{X,t} &= \frac{Q_t}{\theta} (\tilde{\rho}_{X,t})^{1-\theta} C_t^* - \frac{w_t f_{X,t}}{Z_t}. \end{aligned}$$

Combining this with the aggregate accounting identity (see (24))

$$C_t = w_t L + N_{D,t} \tilde{d}_{D,t} + N_{X,t} \tilde{d}_{X,t} - N_{E,t} w_t f_{E,t} / Z_t$$

yields the labor market clearing condition for the home economy:

$$L = \frac{\theta - 1}{w_t} N_{D,t} \tilde{d}_{D,t} + \frac{\theta - 1}{w_t} N_{X,t} \tilde{d}_{X,t} + \frac{\theta}{Z_t} N_{X,t} f_{X,t} + \frac{1}{Z_t} N_{E,t} f_{E,t}.$$

The proof for the foreign economy follows the same steps.

C. International Bond Trading

The budget constraint of the representative home household, in units of the home consumption basket, is now:

$$\begin{aligned} & B_{t+1} + Q_t B_{*,t+1} + \frac{\eta}{2} (B_{t+1})^2 + \frac{\eta}{2} Q_t (B_{*,t+1})^2 + \tilde{v}_t N_{H,t} x_{t+1} + \frac{M_t}{P_t} + C_t + T_t \\ = & (1 + r_t) B_t + Q_t (1 + r_t^*) B_{*,t} + \left(\tilde{d}_t + \tilde{v}_t \right) N_{D,t} x_t + T_t^{fi} + \frac{M_{t-1}}{P_t} + w_t L, \end{aligned} \quad (55)$$

where B_{t+1} denotes holdings of home bonds, $B_{*,t+1}$ denotes holdings of foreign bonds, $(\eta/2) (B_{t+1})^2$ is the cost of adjusting holdings of home bonds, $(\eta/2) (B_{*,t+1})^2$ is the cost of adjusting holdings of foreign bonds (in units of foreign consumption), T_t^{fi} is the fee rebate, taken as given by the household, and equal to $(\eta/2) \left[(B_{t+1})^2 + Q_t (B_{*,t+1})^2 \right]$ in equilibrium. For simplicity, we assume that the scaling parameter $\eta > 0$ is identical across costs of adjusting holdings of home and foreign bonds. Also, there is no cost of adjusting equity holdings. The justification for this is that, in equilibrium, bond holdings differ from zero only because of transactions with a foreign counterpart. As a consequence, in equilibrium, bond-adjustment fees actually capture fees on international transactions, which we assume absent from domestic transactions such as those involving equity to avoid adding unnecessary complication.⁴³

⁴³We also experimented with a specification of the cost of adjusting bond holdings as a function of overall assets: $(\eta'/2) (A_{t+1})^2$, where $A_{t+1} \equiv B_{t+1} + Q_t B_{*,t+1}$. The specification we use has the advantage of pinning down uniquely the steady-state levels and dynamics of B_{t+1} and $B_{*,t+1}$ as well as of their aggregate. The alternative specification only pins down the latter. It is possible to verify that the two specifications yield identical log-linear dynamics under the assumptions that the steady-state levels of B_{t+1} and $B_{*,t+1}$ are zero when the cost $(\eta'/2) (A_{t+1})^2$ is used and $\eta' = (1/2) \eta$.

The representative foreign household faces a similar constraint, in units of foreign consumption:

$$\begin{aligned} & \frac{B_{t+1}^*}{Q_t} + B_{*,t+1}^* + \frac{\eta (B_{t+1}^*)^2}{2 Q_t} + \frac{\eta}{2} (B_{*,t+1}^*)^2 + \tilde{v}_t^* N_{F,t}^* x_{t+1}^* + \frac{M_t^*}{P_t^*} + C_t^* + T_t^* \\ = & \frac{(1+r_t)}{Q_t} B_t^* + (1+r_t^*) B_{*,t}^* + \left(\tilde{d}_t^* + \tilde{v}_t^* \right) N_{D,t}^* x_t^* + T_t^{fi*} + \frac{M_{t-1}^*}{P_t^*} + w_t^* L^*, \end{aligned} \quad (56)$$

where B_{t+1}^* denotes holdings of the home bond, $B_{*,t+1}^*$ denotes holdings of the foreign bond, and $T_t^{fi*} = (\eta/2) \left[(B_{t+1}^*)^2 / Q_t + (B_{*,t+1}^*)^2 \right]$ in equilibrium.

Home and foreign households maximize the respective intertemporal utility functions subject to the respective constraints. The first-order conditions for the choices of share holdings in mutual portfolios of domestic firms at home and abroad are unchanged relative to the case of financial autarky. Instead, we have the following Euler equations for bond holdings. At home:

$$(C_t)^{-\gamma} (1 + \eta B_{t+1}) = \beta (1 + r_{t+1}) E_t [(C_{t+1})^{-\gamma}], \quad (57)$$

$$(C_t)^{-\gamma} (1 + \eta B_{*,t+1}) = \beta (1 + r_{t+1}^*) E_t \left[\frac{Q_{t+1}}{Q_t} (C_{t+1})^{-\gamma} \right]. \quad (58)$$

Abroad:

$$(C_t^*)^{-\gamma} (1 + \eta B_{t+1}^*) = \beta (1 + r_{t+1}) E_t \left[\frac{Q_t}{Q_{t+1}} (C_{t+1}^*)^{-\gamma} \right], \quad (59)$$

$$(C_t^*)^{-\gamma} (1 + \eta B_{*,t+1}^*) = \beta (1 + r_{t+1}^*) E_t [(C_{t+1}^*)^{-\gamma}]. \quad (60)$$

The presence of the terms that depend on the stock of bonds on the left-hand side of these equations is crucial for steady-state determinacy and model stationarity. It ensures that zero holdings of bonds is the *unique* steady state in which the product of β times the gross interest rate equals 1 in each country, so that economies return to this initial position after temporary shocks. If we had perfect foresight and $\eta = 0$, Euler equations for bond holdings at home and abroad would imply the common no-arbitrage condition $(1 + r_{t+1}) / (1 + r_{t+1}^*) = Q_{t+1} / Q_t$. This is the familiar condition that says that the real interest rate differential must be equal to expected real depreciation for agents to be indifferent between home and foreign bonds. With perfect foresight and $\eta > 0$, no-arbitrage conditions for home and foreign households imply:

$$\frac{1 + r_{t+1}}{1 + r_{t+1}^*} = \frac{Q_{t+1}}{Q_t} \frac{1 + \eta B_{t+1}}{1 + \eta B_{*,t+1}} = \frac{Q_{t+1}}{Q_t} \frac{1 + \eta B_{t+1}^*}{1 + \eta B_{*,t+1}^*}. \quad (61)$$

Equilibrium requires that home and foreign bonds be in zero net supply worldwide:

$$B_{t+1} + B_{t+1}^* = 0, \quad (62)$$

$$B_{*,t+1} + B_{*,t+1}^* = 0. \quad (63)$$

Home and foreign holdings of each individual bond must add up to zero because each country is populated by a unitary mass of identical households that make identical equilibrium choices and only the home (foreign) country issues home (foreign) currency bonds. Using (62) and (63) in conjunction with the second equality in (61) makes it possible to show that $B_{t+1} = B_{*,t+1}$ and $B_{t+1}^* = B_{*,t+1}^*$ at an optimum. Since households face quadratic costs of adjusting bond holdings with identical scale parameters across bonds, it is optimal to adjust holdings of different bonds equally so as to spread the cost evenly. The same result holds in the log-linear version of the stochastic model.

Aggregate accounting implies the following laws of motion for net foreign assets at home and abroad:

$$B_{t+1} + Q_t B_{*,t+1} = (1 + r_t)B_t + Q_t (1 + r_t^*) B_{*,t} + w_t L + N_{D,t} \tilde{d}_t - N_{E,t} \tilde{v}_t - C_t, \quad (64)$$

$$\frac{B_{t+1}^*}{Q_t} + B_{*,t+1}^* = \frac{(1 + r_t)}{Q_t} B_t^* + (1 + r_t^*) B_{*,t}^* + w_t^* L^* + N_{D,t}^* \tilde{d}_t^* - N_{E,t}^* \tilde{v}_t^* - C_t^*, \quad (65)$$

where holdings of individual bonds across countries are tied by the equilibrium conditions (62) and (63). Given these conditions, multiplying (65) times Q_t and subtracting the resulting equation from (64) yields an expression for home net foreign asset accumulation as a function of interest income and of the cross-country differentials between labor income, net investment income, and consumption:

$$\begin{aligned} B_{t+1} + Q_t B_{*,t+1} &= (1 + r_t)B_t + Q_t (1 + r_t^*) B_{*,t} + \frac{1}{2} (w_t L - Q_t w_t^* L^*) \\ &\quad + \frac{1}{2} \left(N_{D,t} \tilde{d}_t - N_{D,t}^* Q_t \tilde{d}_t^* \right) - \frac{1}{2} \left(N_{E,t} \tilde{v}_t - N_{E,t}^* Q_t \tilde{v}_t^* \right) - \frac{1}{2} (C_t - Q_t C_t^*). \end{aligned} \quad (66)$$

Current accounts are by definition equal to the changes in aggregate bond holdings in the two

countries:

$$CA_t \equiv B_{t+1} - B_t + Q_t (B_{*,t+1} - B_{*,t}), \quad (67)$$

$$CA_t^* \equiv \frac{B_{t+1}^* - B_t^*}{Q_t} + B_{*,t+1}^* - B_{*,t}^*. \quad (68)$$

It is straightforward to verify that the bond-market clearing conditions (62) and (63) imply $CA_t + Q_t CA_t^* = 0$ (a country's borrowing must equal the other country's lending) and $C_t + Q_t C_t^* = w_t L + Q_t w_t^* L^* + N_{D,t} \tilde{d}_t + N_{D,t}^* Q_t \tilde{d}_t^* - (N_{E,t} \tilde{v}_t + N_{E,t}^* Q_t \tilde{v}_t^*)$ (since the world as a whole is a closed economy, world consumption must equal world labor income plus world net investment income).

Labor market clearing conditions at home and abroad require:

$$L = \frac{\theta - 1}{w_t} (N_{D,t} \tilde{d}_{D,t} + N_{X,t} \tilde{d}_{X,t}) + \frac{1}{Z_t} (\theta N_{X,t} f_{X,t} + N_{E,t} f_{E,t}), \quad (69)$$

$$L^* = \frac{\theta - 1}{w_t^*} (N_{D,t}^* \tilde{d}_{D,t}^* + N_{X,t}^* \tilde{d}_{X,t}^*) + \frac{1}{Z_t^*} (\theta N_{X,t}^* f_{X,t}^* + N_{E,t}^* f_{E,t}^*). \quad (70)$$

We thus have 29 endogenous variables determined during period t : $\tilde{\rho}_{D,t}, \tilde{\rho}_{D,t}^*, \tilde{\rho}_{X,t}, \tilde{\rho}_{X,t}^*, w_t, w_t^*, \tilde{d}_{D,t}, \tilde{d}_{D,t}^*, \tilde{d}_{X,t}, \tilde{d}_{X,t}^*, N_{E,t}, N_{E,t}^*, \tilde{z}_{X,t}, \tilde{z}_{X,t}^*, N_{D,t+1}, N_{D,t+1}^*, N_{X,t}, N_{X,t}^*, r_{t+1}, r_{t+1}^*, \tilde{v}_t, \tilde{v}_t^*, C_t, C_t^*, Q_t, B_{t+1}, B_{*,t+1}, B_{t+1}^*, B_{*,t+1}^*$. There are 8 endogenous state variables that are predetermined as of time t : the risk-free interest rates, r_t and r_t^* , the total numbers of firms at home and abroad, $N_{D,t}$ and $N_{D,t}^*$, and the stocks of bonds, $B_t, B_{*,t}, B_t^*$, and $B_{*,t}^*$. Finally, the model features the same 8 exogenous variables as in the financial autarky case. The 29 endogenous variables above are determined by a system of 29 equations that is identical to the system in Table 1 in the following blocs: Prices, Price Indexes, Profits, Free entry, Zero-profit export cutoffs, Share of exporting firms, Number of firms, Euler equation (shares). The 5 equations in Euler equation (bonds), Aggregate accounting, and Balanced trade are replaced by the 9 equations in Table C.1.

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Table 1. Model summary, financial autarky

Prices	$\tilde{\rho}_{D,t} = \frac{\theta}{\theta-1} (\tilde{z}_D Z_t)^{-1} w_t$ $\tilde{\rho}_{D,t}^* = \frac{\theta}{\theta-1} (\tilde{z}_D Z_t^*)^{-1} w_t^*$ $\tilde{\rho}_{X,t} = \frac{\theta}{\theta-1} \tau_t (\tilde{z}_{X,t} Z_t)^{-1} Q_t^{-1} w_t$ $\tilde{\rho}_{X,t}^* = \frac{\theta}{\theta-1} \tau_t^* (\tilde{z}_{X,t}^* Z_t^*)^{-1} Q_t w_t^*$
Price indexes	$N_{D,t} (\tilde{\rho}_{D,t})^{1-\theta} + N_{X,t}^* (\tilde{\rho}_{X,t}^*)^{1-\theta} = 1$ $N_{D,t}^* (\tilde{\rho}_{D,t}^*)^{1-\theta} + N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} = 1$
Profits	$\tilde{d}_{D,t} = \frac{1}{\theta} (\tilde{\rho}_{D,t})^{1-\theta} C_t$ $\tilde{d}_{D,t}^* = \frac{1}{\theta} (\tilde{\rho}_{D,t}^*)^{1-\theta} C_t^*$ $\tilde{d}_{X,t} = \frac{Q_t}{\theta} (\tilde{\rho}_{X,t})^{1-\theta} C_t^* - w_t \frac{f_{X,t}}{Z_t}$ $\tilde{d}_{X,t}^* = \frac{Q_t^{-1}}{\theta} (\tilde{\rho}_{X,t}^*)^{1-\theta} C_t - w_t^* \frac{f_{X,t}^*}{Z_t^*}$
Free entry	$\tilde{v}_t = w_t \frac{f_{E,t}}{Z_t}$ $\tilde{v}_t^* = w_t^* \frac{f_{E,t}^*}{Z_t^*}$
Zero-profit export cutoffs ⁴⁴	$\tilde{d}_{X,t} = w_t \frac{f_{X,t}}{Z_t} \frac{\theta-1}{k-(\theta-1)}$ $\tilde{d}_{X,t}^* = w_t^* \frac{f_{X,t}^*}{Z_t^*} \frac{\theta-1}{k-(\theta-1)}$
Share of exporting firms	$\frac{N_{X,t}}{N_{D,t}} = (z_{\min})^k (\tilde{z}_{X,t})^{-k} \left[\frac{k}{k-(\theta-1)} \right]^{\frac{k}{\theta-1}}$ $\frac{N_{X,t}^*}{N_{D,t}^*} = (z_{\min})^k (\tilde{z}_{X,t}^*)^{-k} \left[\frac{k}{k-(\theta-1)} \right]^{\frac{k}{\theta-1}}$
Number of firms	$N_{D,t} = (1-\delta) (N_{D,t-1} + N_{E,t-1})$ $N_{D,t}^* = (1-\delta) (N_{D,t-1}^* + N_{E,t-1}^*)$
Euler equation (bonds)	$(C_t)^{-\gamma} = \beta (1+r_{t+1}) E_t [(C_{t+1})^{-\gamma}]$ $(C_t^*)^{-\gamma} = \beta (1+r_{t+1}^*) E_t [(C_{t+1}^*)^{-\gamma}]$
Euler equation (shares)	$\tilde{v}_t = \beta (1-\delta) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\tilde{v}_{t+1} + \tilde{d}_{D,t+1} + \frac{N_{X,t+1}}{N_{D,t+1}} \tilde{d}_{X,t+1} \right) \right]$ $\tilde{v}_t^* = \beta (1-\delta) E_t \left[\left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} \left(\tilde{v}_{t+1}^* + \tilde{d}_{D,t+1}^* + \frac{N_{X,t+1}^*}{N_{D,t+1}^*} \tilde{d}_{X,t+1}^* \right) \right]$
Aggregate accounting	$C_t = w_t L + N_{D,t} \tilde{d}_{D,t} + N_{X,t} \tilde{d}_{X,t} - N_{E,t} \tilde{v}_t$ $C_t^* = w_t^* L^* + N_{D,t}^* \tilde{d}_{D,t}^* + N_{X,t}^* \tilde{d}_{X,t}^* - N_{E,t}^* \tilde{v}_t^*$
Balanced trade	$Q_t N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} C_t^* = N_{X,t}^* (\tilde{\rho}_{X,t}^*)^{1-\theta} C_t$

⁴⁴These conditions hold only when $f_{X,t}$ and $f_{X,t}^*$ are strictly positive. If all firms export (*i.e.*, if $f_{X,t} = f_{X,t}^* = 0$), then these conditions must be replaced with $\tilde{z}_{X,t} = \tilde{z}_{D,t}$ and $\tilde{z}_{X,t}^* = \tilde{z}_{D,t}^*$. The same is true in the bond trading case.

Table 2. Business cycle data⁴⁵

<i>U.S.</i>											
Variable	Standard deviation										
	Percentage						Relative to output				
Output	1.71						1.00				
Consumption	.84						.49				
Investment	5.38						3.15				
Capital stock	.63						.37				
Net exports/output	.45										
<i>Corr</i> (Variable _{t+j} , Output _t), $j = -5, \dots, 5$											
Variable ↓	-5	-4	-3	-2	-1	0	1	2	3	4	5
Output	-.03	.15	.38	.63	.85	1.00	.85	.63	.38	.15	-.03
Consumption	.20	.38	.53	.67	.77	.76	.63	.46	.27	.06	-.12
Investment	.09	.25	.44	.64	.83	.90	.81	.60	.35	.08	-.14
Capital stock	-.60	-.60	-.54	-.43	-.24	.01	.24	.46	.62	.71	.72
Net exports/output	-.51	-.51	-.48	-.43	-.37	-.28	-.17	.00	.17	.30	.38
<i>International</i>											
Contemporaneous cross correlation											
Country	<i>with U.S.</i>			<i>within country</i>							
	Output	Consumption		Saving-investment rate				Net exports/output-output			
Austria	.31	.07		.29				-.42			
Finland	.02	-.01		.09				-.36			
France	.22	-.18		-.04				-.17			
Germany	.42	.39		.42				-.27			
Italy	.39	.25		.06				-.62			
Switzerland	.27	.25		.38				-.66			
United Kingdom	.48	.43		.07				-.21			
Europe	.70	.46									
U.S.	1.00	1.00		.68				-.36			
Real exchange rate											
Source	First-order autocorr.	Std. dev. (ratio to output)				Corr. with relative consumption					
CKM	.83	5.50				-.35					
BF	.80	4.81									

⁴⁵Source: Backus, Kehoe, and Kydland (1992) unless otherwise noted.

Table 3. Model-generated moments, BKK productivity process

Standard deviation											
Variable	Percentage		Relative to y (Py/\tilde{P})								
y (Py/\tilde{P})	.9961 (.7950)		1.00 (1.00)								
C (PC/\tilde{P})	.6929 (.4681)		.6956 (.5888)								
$\tilde{v}N_E$ ($P\tilde{v}N_E/\tilde{P}$)	3.6356 (3.5002)		3.6498 (4.5754)								
N_E	3.6314		3.6456 (relative to y)								
$\tilde{v}N_D$ ($P\tilde{v}N_D/\tilde{P}$)	.3316 (.3795)		.3329 (.4773)								
N_D	.2697		.2707 (relative to y)								
TB/y	1.0178		1.0218								
TB (PTB/\tilde{P})	.1571 (.2968)		.1577 (.3733)								
\tilde{Q}	.0278		.0279 (relative to y)								
$Corr(\text{Variable}_{t+j}, y_t), j = -5, \dots, 5$ (corr. with $P_t y_t / \tilde{P}_t$ if variable deflated with \tilde{P})											
Variable ↓	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	-.02	.11	.27	.47	.71	1.00	.71	.47	.27	.11	-.02
Py/\tilde{P}	-.02	.10	.27	.47	.71	1.00	.71	.47	.27	.10	-.02
C	-.05	.07	.22	.41	.64	.92	.69	.49	.32	.17	.05
PC/\tilde{P}	-.08	.03	.18	.37	.59	.87	.66	.48	.32	.19	.08
$\tilde{v}N_E$.06	.16	.29	.44	.62	.84	.50	.24	.05	-.08	-.17
$P\tilde{v}N_E/\tilde{P}$.08	.18	.31	.46	.65	.86	.49	.22	.02	-.12	-.21
N_E	.07	.17	.29	.44	.62	.84	.49	.23	.04	-.10	-.19
$\tilde{v}N_D$	-.42	-.38	-.31	-.20	-.03	.19	.46	.61	.67	.67	.61
$P\tilde{v}N_D/\tilde{P}$	-.34	-.37	-.39	-.39	-.37	-.33	.06	.30	.45	.51	.52
N_D	-.41	-.38	-.31	-.21	-.05	.16	.44	.59	.65	.65	.60
TB/y	-.01	-.13	-.29	-.48	-.71	-.99	-.66	-.41	-.20	-.05	.07
TB	-.17	-.17	-.16	-.14	-.11	-.06	.20	.34	.40	.39	.34
PTB/\tilde{P}	-.05	-.13	-.23	-.36	-.50	-.67	-.33	-.10	.05	.14	.18

Table 3, continued

$Corr(\tilde{Q}_{t+j}, \tilde{Q}_t), j = -5, \dots, 5$											
-5	-4	-3	-2	-1	0	1	2	3	4	5	
.19	.37	.55	.73	.89	1.00	.89	.73	.55	.37	.19	
Contemporaneous cross correlation											
$y^*, y (P^*y^*/\tilde{P}^*, Py/\tilde{P})$				$C^*, C (P^*C^*/\tilde{P}^*, PC/\tilde{P})$				Saving rate, "Investment" rate			
.44 (.21)				.92 (.86)				.95			
$(PC/\tilde{P})/(P^*C^*/\tilde{P}^*), \tilde{Q}$						$C/C^*, Q$					
-.99						.71					

Table 4. Model-generated moments, Baxter productivity process

Standard deviation											
Variable	Percentage		Relative to y (Py/\tilde{P})								
y (Py/\tilde{P})	1.0521 (.7195)		1.00 (1.00)								
C (PC/\tilde{P})	.9136 (.5749)		.8683 (.7990)								
$\tilde{v}N_E$ ($P\tilde{v}N_E/\tilde{P}$)	3.6873 (3.3538)		3.5047 (4.6613)								
N_E	3.6489		3.4682 (relative to y)								
$\tilde{v}N_D$ ($P\tilde{v}N_D/\tilde{P}$)	.3697 (.4693)		.3514 (.6522)								
N_D	.3033		.2883 (relative to y)								
TB/y	1.3288		1.2630								
TB (PTB/\tilde{P})	.4662 (.7583)		.4431 (1.0539)								
\tilde{Q}	.1297		.1233 (relative to y)								
$Corr(\text{Variable}_{t+j}, y_t), j = -5, \dots, 5$ (corr. with $P_t y_t / \tilde{P}_t$ if variable deflated with \tilde{P})											
Variable ↓	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	.01	.14	.31	.50	.73	1.00	.73	.50	.31	.14	.01
Py/\tilde{P}	.03	.17	.33	.53	.75	1.00	.75	.53	.33	.17	.03
C	.01	.14	.30	.50	.73	1.00	.73	.49	.30	.14	.01
PC/\tilde{P}	.02	.16	.32	.52	.74	1.00	.74	.52	.33	.17	.04
$\tilde{v}N_E$.12	.22	.34	.48	.64	.82	.51	.27	.08	-.07	-.17
$P\tilde{v}N_E/\tilde{P}$.18	.27	.37	.48	.59	.70	.41	.18	.01	-.12	-.21
N_E	.13	.23	.35	.49	.64	.81	.50	.25	.06	-.08	-.18
$\tilde{v}N_D$	-.36	-.30	-.21	-.09	.08	.30	.51	.63	.67	.66	.61
$P\tilde{v}N_D/\tilde{P}$	-.29	-.33	-.36	-.38	-.40	-.40	-.04	.21	.38	.47	.51
N_D	-.36	-.32	-.24	-.13	.02	.21	.45	.59	.65	.66	.62
TB/y	-.07	-.19	-.34	-.51	-.72	-.95	-.65	-.40	-.20	-.04	.08
TB	-.16	-.21	-.26	-.32	-.39	-.45	-.19	-.01	.13	.21	.26
PTB/\tilde{P}	-.15	-.23	-.32	-.41	-.51	-.62	-.34	-.13	.03	.14	.21

Table 4, continued

$Corr(\tilde{Q}_{t+j}, \tilde{Q}_t), j = -5, \dots, 5$											
-5	-4	-3	-2	-1	0	1	2	3	4	5	
.01	.14	.30	.49	.73	1.00	.73	.49	.30	.14	.01	
Contemporaneous cross correlation											
$y^*, y (P^*y^*/\tilde{P}^*, Py/\tilde{P})$				$C^*, C (P^*C^*/\tilde{P}^*, PC/\tilde{P})$				Saving rate, "Investment" rate			
.32 (.44)				.30 (.46)				.47			
$(PC/\tilde{P})/(P^*C^*/\tilde{P}^*), \tilde{Q}$						$C/C^*, Q$					
-.98						.13					

Table C.1. Model summary, bond trading

Euler equations (bonds)	$(C_t)^{-\gamma} (1 + \eta B_{t+1}) = \beta (1 + r_{t+1}) E_t [(C_{t+1})^{-\gamma}]$
	$(C_t)^{-\gamma} (1 + \eta B_{*,t+1}) = \beta (1 + r_{t+1}^*) E_t \left[\frac{Q_{t+1}}{Q_t} (C_{t+1})^{-\gamma} \right]$
	$(C_t^*)^{-\gamma} (1 + \eta B_{t+1}^*) = \beta (1 + r_{t+1}) E_t \left[\frac{Q_t}{Q_{t+1}} (C_{t+1}^*)^{-\gamma} \right]$
	$(C_t^*)^{-\gamma} (1 + \eta B_{*,t+1}^*) = \beta (1 + r_{t+1}^*) E_t [(C_{t+1}^*)^{-\gamma}]$
Net foreign assets	$B_{t+1} + Q_t B_{*,t+1} = (1 + r_t) B_t + Q_t (1 + r_t^*) B_{*,t}$
	$+ \frac{1}{2} (w_t L - Q_t w_t^* L^*) + \frac{1}{2} (N_{D,t} \tilde{d}_{D,t} - N_{D,t}^* Q_t \tilde{d}_{D,t}^*)$
	$+ \frac{1}{2} (N_{X,t} \tilde{d}_{X,t} - N_{X,t}^* Q_t \tilde{d}_{X,t}^*)$
	$- \frac{1}{2} (N_{E,t} \tilde{v}_t - N_{E,t}^* Q_t \tilde{v}_t^*) - \frac{1}{2} (C_t - Q_t C_t^*)$
Bond market equilibrium	$B_{t+1} + B_{t+1}^* = 0$
	$B_{*,t+1} + B_{*,t+1}^* = 0$
Labor market equilibrium	$L = \frac{\theta-1}{w_t} (N_{D,t} \tilde{d}_{D,t} + N_{X,t} \tilde{d}_{X,t}) + \frac{1}{Z_t} (\theta N_{X,t} f_{X,t} + N_{E,t} f_{E,t})$
	$L^* = \frac{\theta-1}{w_t^*} (N_{D,t}^* \tilde{d}_{D,t}^* + N_{X,t}^* \tilde{d}_{X,t}^*) + \frac{1}{Z_t^*} (\theta N_{X,t}^* f_{X,t}^* + N_{E,t}^* f_{E,t}^*)$

Figure 1

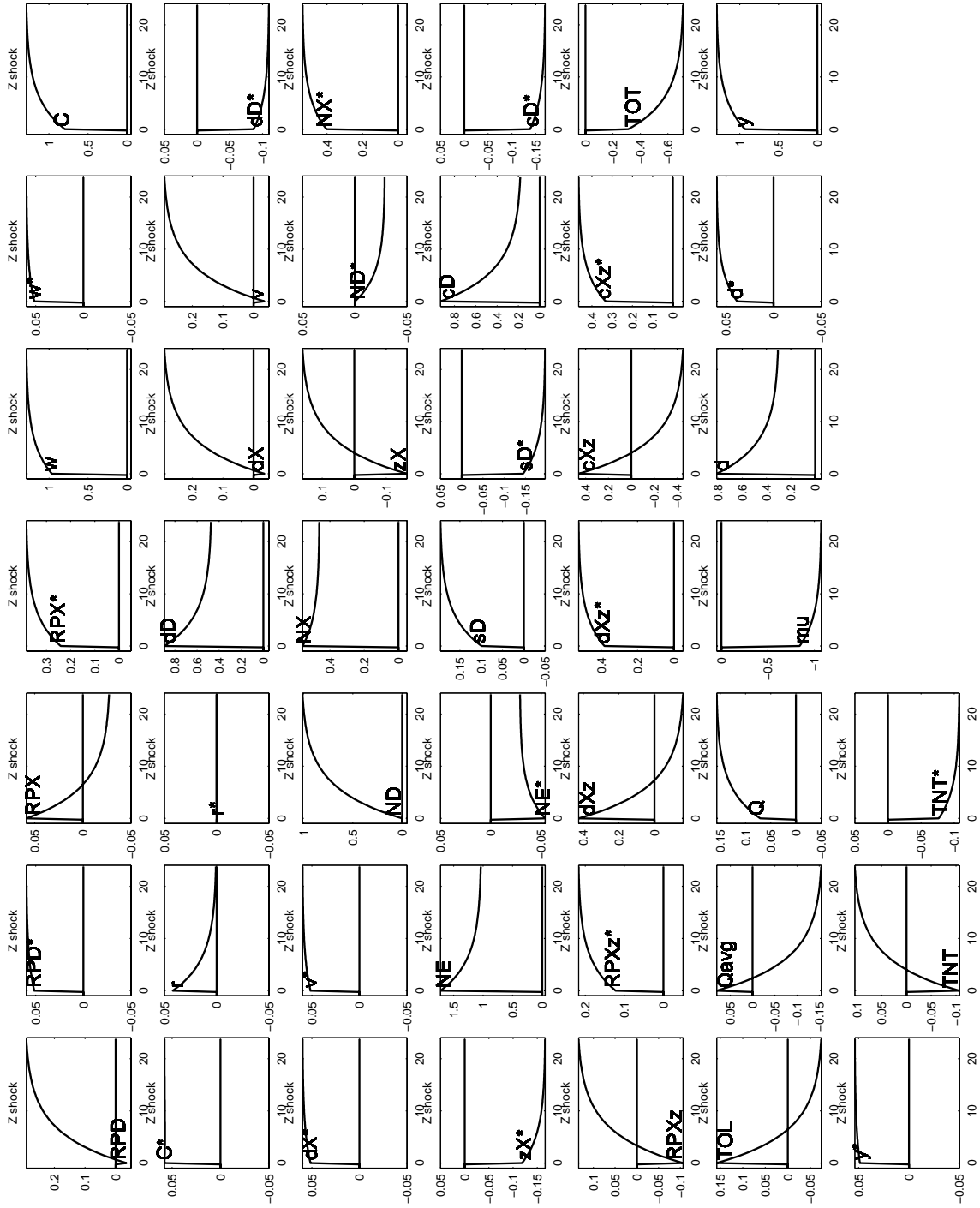


Figure 2

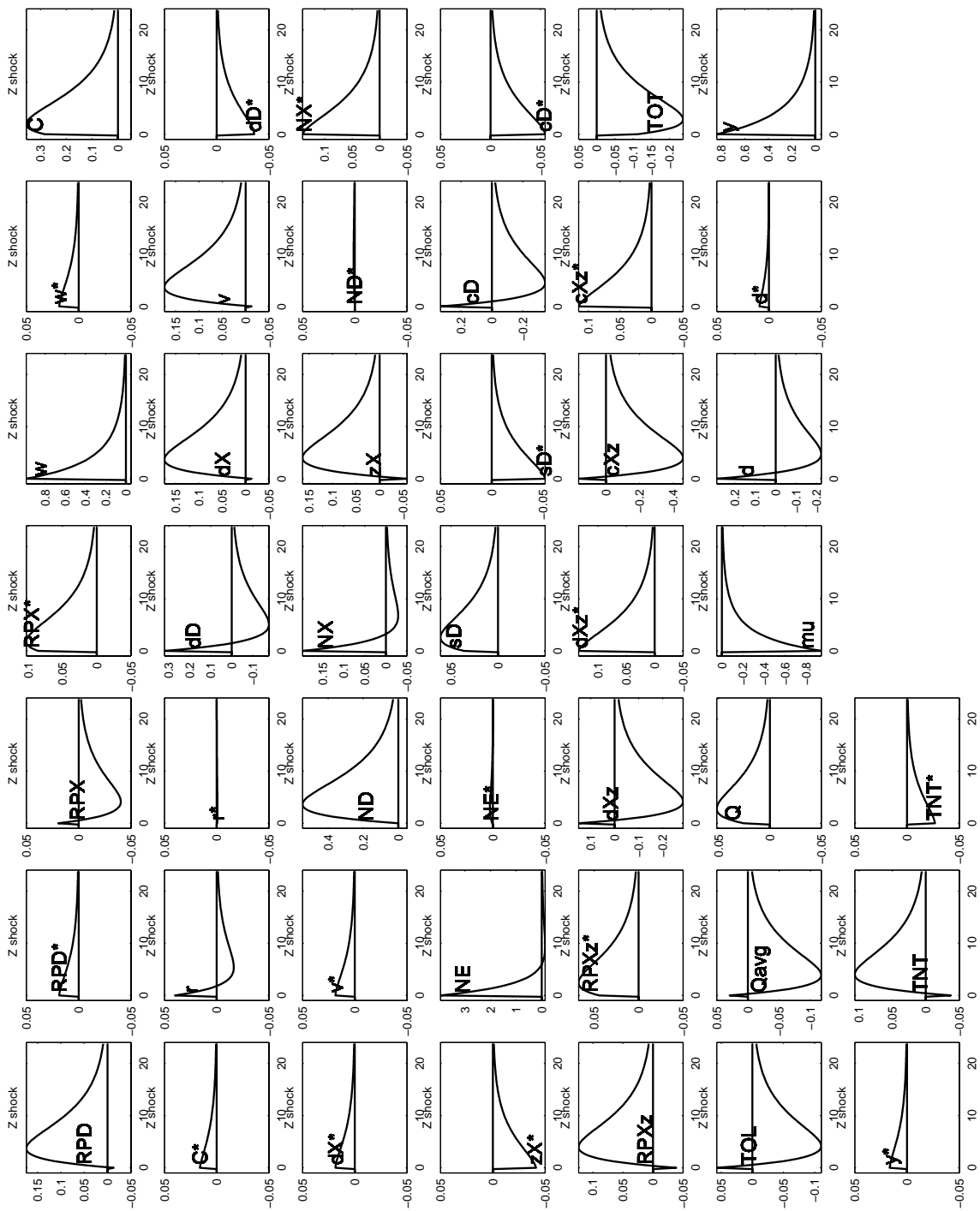


Figure 3

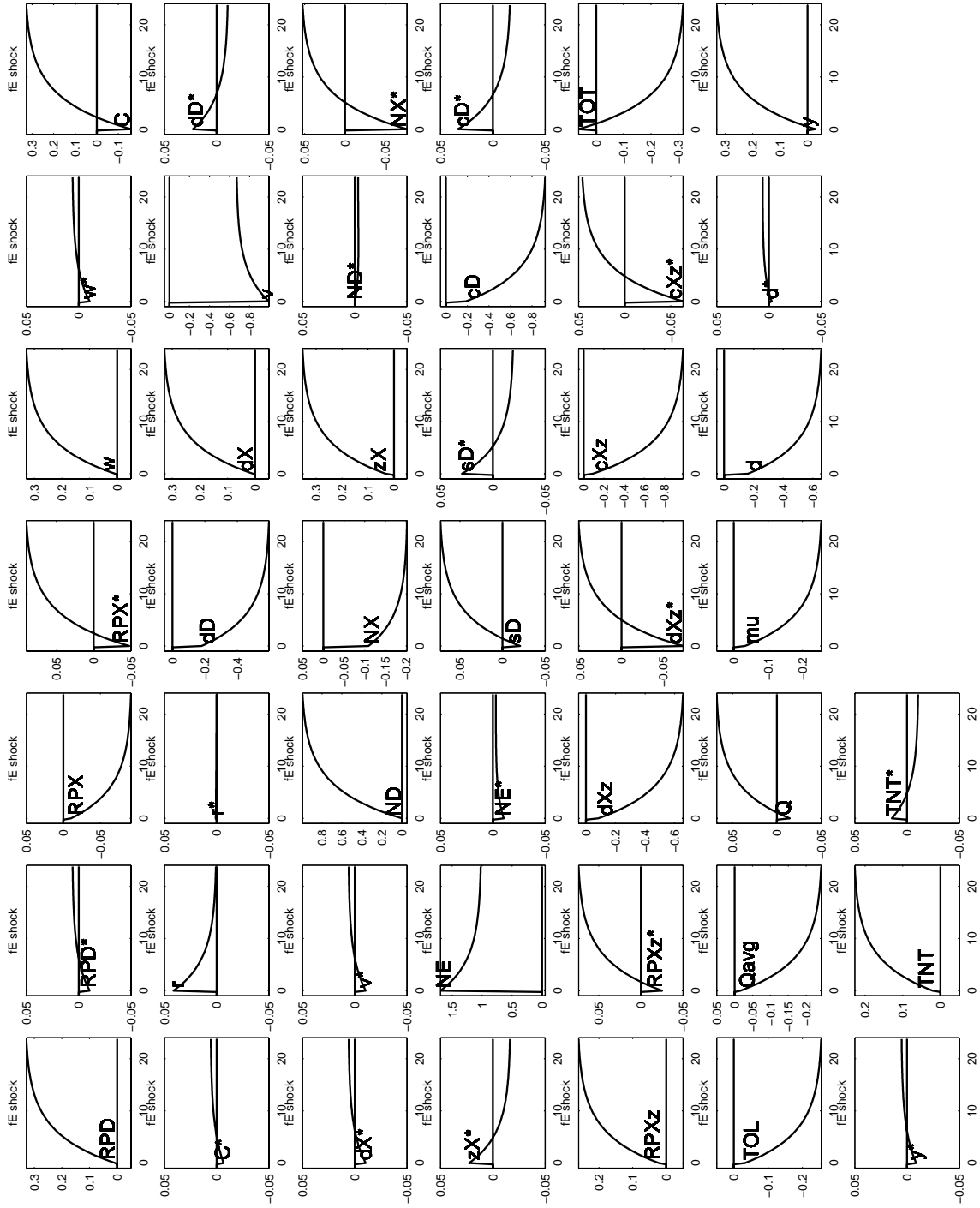


Figure 4

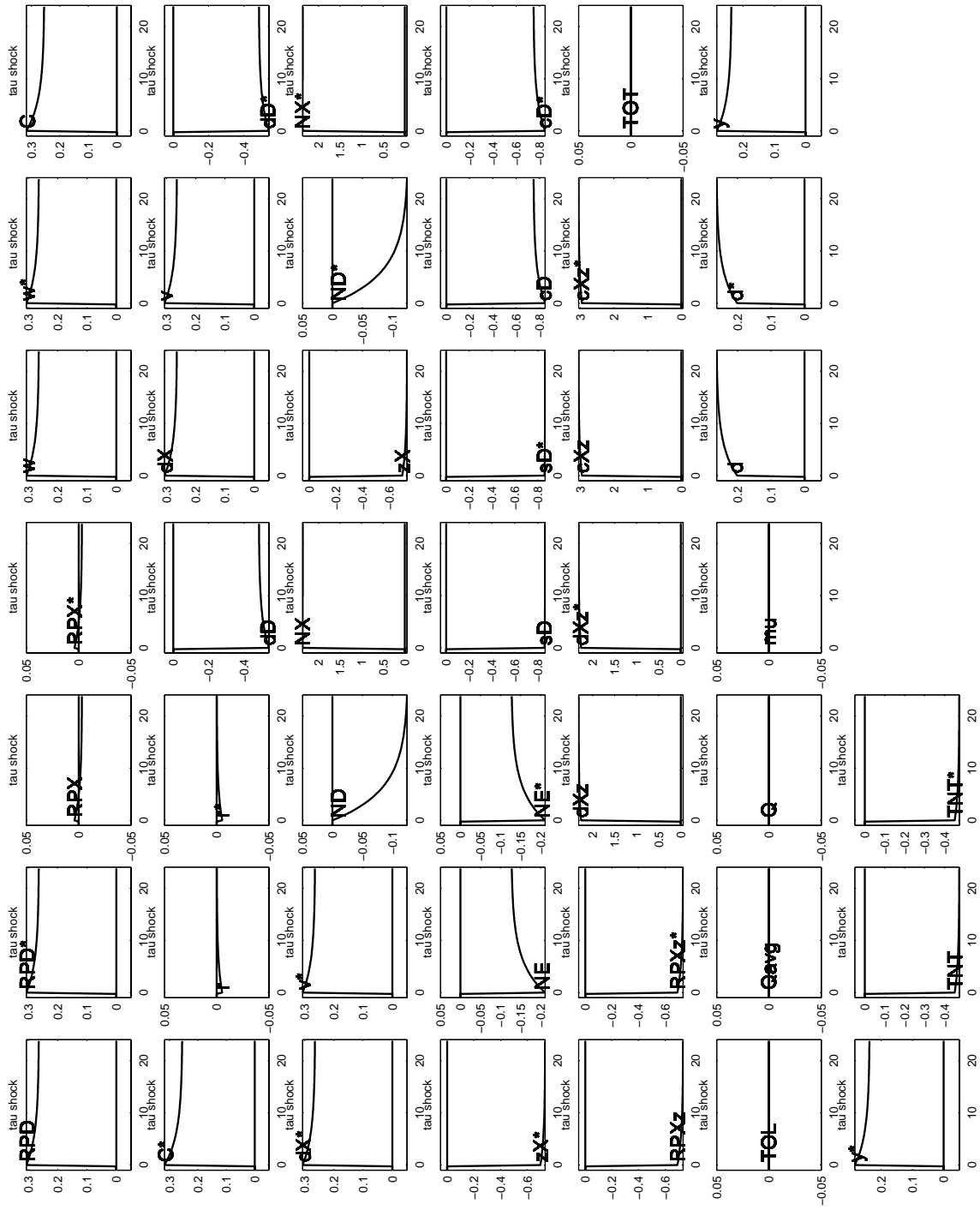


Figure 5

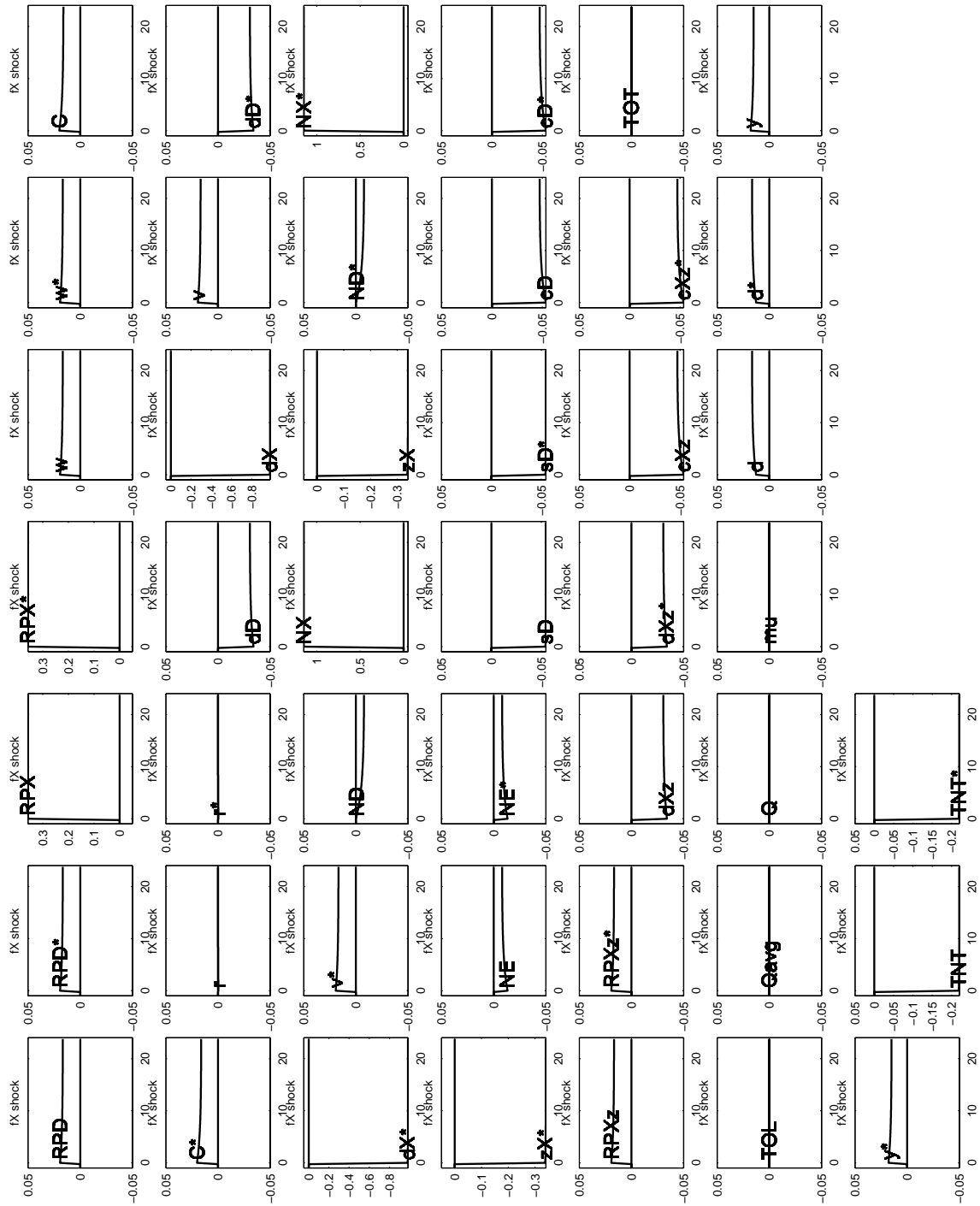


Figure 6

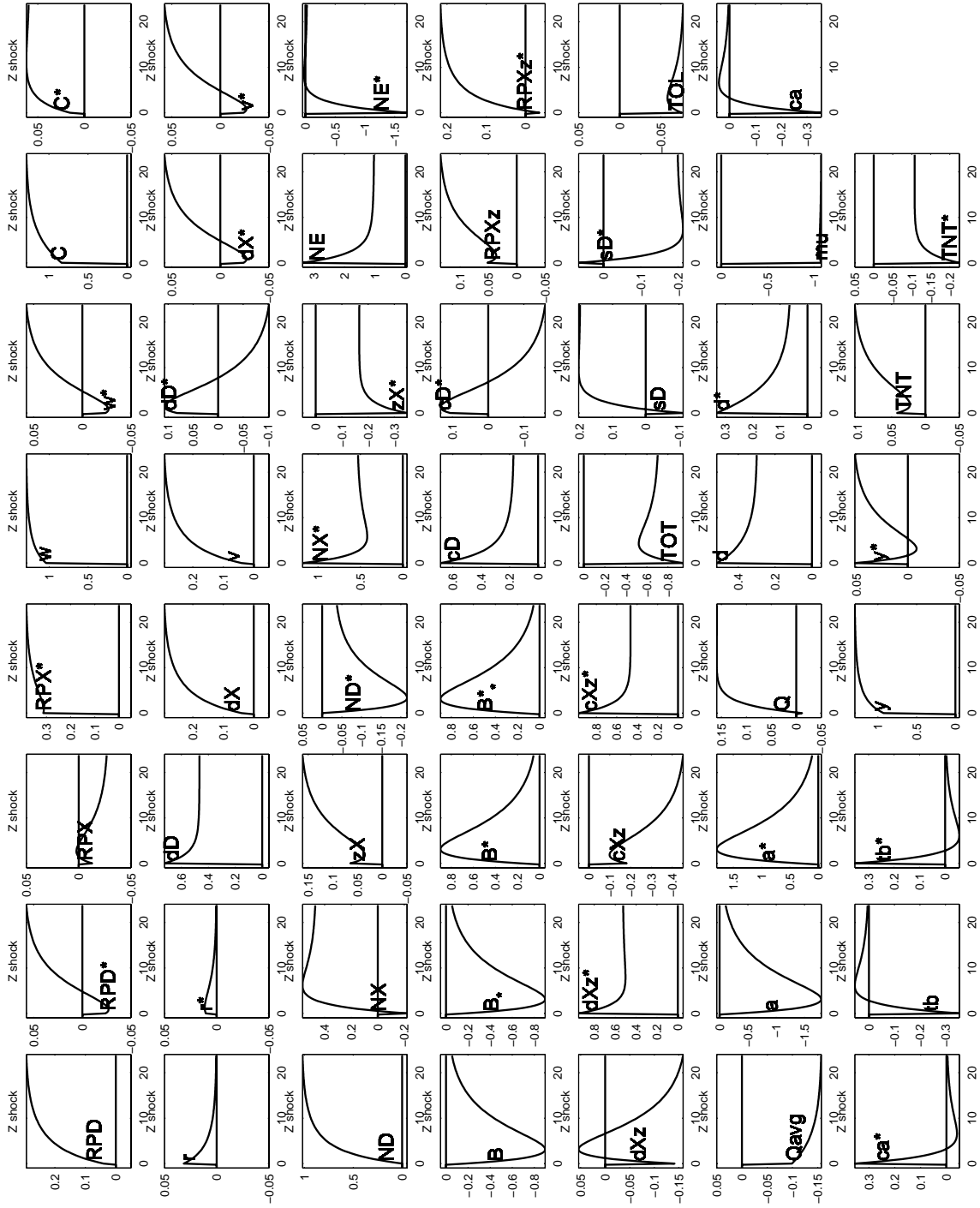


Figure 7

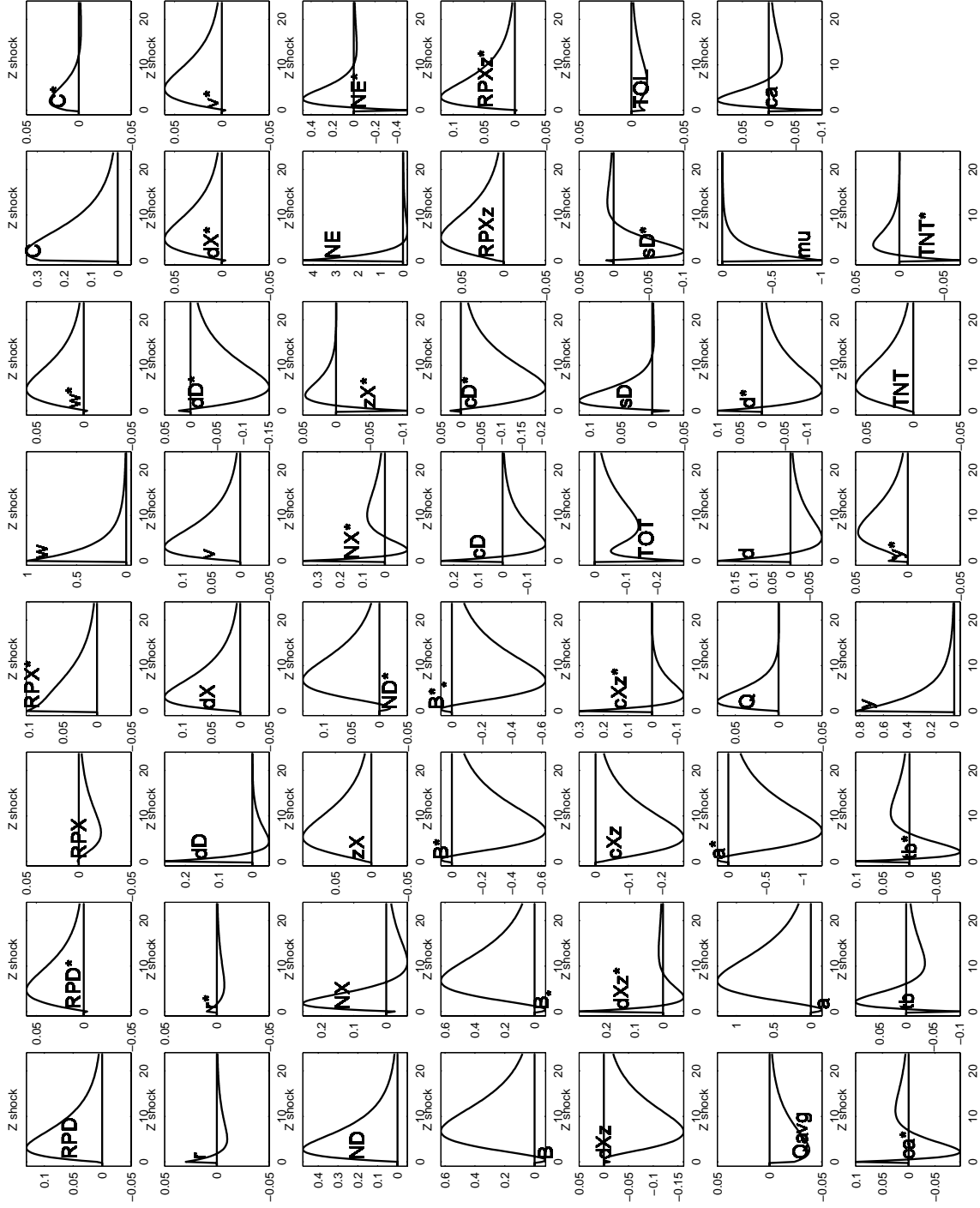


Figure 8

