Trading Frenzies and Their Impact on Real Investment

Itay Goldstein, Emre Ozdenoren, and Kathy Yuan*

October 16, 2010

Abstract

We study a model where a capital provider learns from the price of a firm's security in deciding how much capital to provide for new investment. This feedback effect from the financial market to the investment decision gives rise to trading frenzies, where speculators all wish to trade like others, generating large pressure on prices. Coordination among speculators is sometimes desirable for price informativeness and investment efficiency, but speculators' incentives push in the opposite direction, so that they coordinate exactly when it is undesirable. We analyze the effect of various market parameters on the likelihood of trading frenzies to arise.

*This version is still work in progress. We thank the editor, three anonymous referees, Roland Benabou, Josh Coval, Alex Edmans, Joao Gomes, Johan Hombert, Peter Kondor, Miklos Koren, Alessandro Pavan, Haresh Sapra, Anjan Thakor, Dimitri Vayanos, Liyan Yang, and seminar and conference participants at AFA Meeting, CEU, LSE, MBS, NBER Corporate Finance Meeting, NYU, Paris School of Economics, Tel Aviv University, UCL, University of Toronto, Utah Winter Finance Conference, and Wharton for their comments.
Goldstein is from the University of Pennsylvania; e-mail: `itayg@wharton.upenn.edu`. Ozdenoren is from the London Business School and CEPR; e-mail: `eozdenoren@london.edu` and `emreo@umich.edu`. Yuan is from the London School of Economics and CEPR; e-mail: `K.Yuan@lse.ac.uk`.

1 Introduction

Trading frenzies in financial markets occur when many speculators rush to trade in the same direction leading to large pressure on prices. Financial economists have long been searching for the sources of trading frenzies, asking what causes strategic complementarities in speculators' behavior. This phenomenon is particularly puzzling given that the price mechanism in financial markets naturally leads to strategic substitutes, whereby the expected change in price caused by speculators' trades makes others want to trade in the opposite direction.

We argue in this paper that the potential effect that financial-market trading has on the real economy, i.e., on firms' cash flows, may provide the mechanism for trading frenzies to arise. Intuitively, suppose that speculators in the financial market short sell a stock, leading to a decrease in its price. Since the stock price provides information about the firm's profitability, it affects decisions by various agents, such as capital providers. Seeing the decrease in price, capital providers update downwards their expectation of the firm's profitability. This weakens the firm's access to capital and thus hurts its performance.[1] As a result, the firm's value decreases, and short sellers are able to make a profit. This creates a source for complementarities, whereby the expected change in value caused by speculators' trades makes others want to trade in the same direction, and generates a trading frenzy.

Let us describe the model in more detail. We study an environment where a capital provider decides how much capital to provide to a firm for the purpose of making new real investment. The decision of the capital provider depends on his assessment of the productivity of the proposed investment. In his decision, the capital provider uses two sources of information: his private information and the information aggregated by the price of the firm's security which is traded in the financial market. The reliance of capital provision on financial-market prices establishes the effect that the financial market has on the real economy. We refer to this effect as the 'feedback effect'.[2]

The financial market in our model contains many small speculators trading a security, whose payoff is correlated with the cash flow obtained from the firm's investment. Speculators trade on the basis of information they have about the productivity of the investment. They have access to two signals: the first signal is independent across speculators (conditional on

[1] Other agents that may be affected by the information in the price are managers, employees, customers, etc.

[2] Note that the financial market is a secondary market, and hence the only feedback from it to the firm's cash flow is informational; there is no transfer of cash from the market to the firm.

the realization of the productivity), while the second one is correlated among them.[3] The correlated signals introduce common noise in information into the model, which can be due to a rumor, for example. A trading frenzy occurs when speculators put large weight on the correlated signal relative to the idiosyncratic signal, and so they tend to trade similarly to each other.

To close the model, we introduce noisy price-elastic supply in the financial market. The market is cleared at a price for which the demand from speculators equals the exogenous supply. The endogenous price, in turn, reflects information about the productivity of the investment, as aggregated from speculators' trades. But, given the structure of information and trading, the information in the price contains noise from two sources – the noisy supply and the common noise in speculators' information. The information in the price is then used by the capital provider, together with his private information, when making the decision about capital provision and investment

Analyzing the weight speculators put on the correlated signal relative to the idiosyncratic signal, we shed light on the determinants of trading frenzies. In a world with no strategic effects, this weight is naturally given by the ratio of precisions between the correlated and the idiosyncratic signals. But, in the equilibrium of our model, there are two strategic effects that shift the weight away from this precisions ratio. The first effect is the usual outcome of a price mechanism. When speculators put weight on the correlated information, this information gets more strongly reflected in the price, and then the incentive of each individual speculator to put weight on the correlated information decreases. This generates strategic substitutes and pushes the weight that speculators put on the correlated information below the ratio of precisions.[4] The second effect arises due to the feedback effect from the price to the capital provision decision. When speculators put weight on the correlated information, this information gets to have a stronger effect on the capital provision to the firm and hence on the real value of its traded security. Then, the incentive of each speculator to put weight on this information increases. This leads to strategic complementarities that make speculators put a larger weight on the correlated signal.

This second effect is what causes a trading frenzy, leading speculators to put large weight on their correlated information, and to trade in a coordinated fashion. When this effect dominates, our model generates a pattern that looks like a 'run' on a stock by many speculators, who are driven by common noise in their correlated signals (e.g. rumor), leading to a

[3]In our model, the correlation is perfect, but this is not essential.

[4]Strategic substitutes due to the price mechanism appear in various forms in the literature on financial markets. See, for example, Grossman and Stiglitz (1980).

price decline, lack of provision of new capital, and collapse of real value. This echoes some highly publicized events such as the bear raid on Overstock.com in 2005 or the bear raids on Bear Stearns and Lehman Brothers in 2008.

We investigate what circumstances increase the tendency for a trading frenzy in our model. First, we show that speculators are more likely to trade in a coordinated fashion when the supply in the financial market is more elastic with respect to the price. This can be interpreted as a more liquid market. In such a market, the strategic substitutes due to the price mechanism are weak, as informed demand is easily absorbed by the elastic supply without having much of a price impact. Hence, speculators tend to put more weight on correlated information and trade more similarly to each other. Second, we find that when there is small variance in the supply function, i.e., when there is small variance in noise/liquidity trading in the financial market, speculators tend to put large weights on their correlated signals and thus to act in a coordinated fashion. This is because in these situations, the capital provider relies more on the information in the price since the price is less noisy, and so the feedback effect from the market to the firm's cash flows strengthens, increasing the scope of strategic complementarities. Third, the precision of various sources of information also plays an important role in shaping the incentive to rely on correlated vs. uncorrelated information. Intuitively, there will be more coordination when speculators' correlated signals are sharper and when their uncorrelated signals are noisier. Interestingly, there will be more coordination when the capital provider has less precise information of his own, as then the feedback from the market to his decision is stronger.

Another question we ask is whether trading frenzies are good or bad for the efficiency of the capital provision decision. We find that they are sometimes good and sometimes bad, and that there is a conflict between the level of coordination in equilibrium and the one that maximizes the efficiency of the capital provision decision. The efficiency of the capital provision decision is maximized when the informativeness of the price is highest. It turns out that when there is high variance of noise/liquidity trading in the market, higher degree of coordination among speculators increases price informativeness. This is because, in noisy markets, coordination among speculators is beneficial in suppressing the noise in liquidity trading that reduces the informativeness of the price. In such markets, trading frenzies among speculators are actually desirable because they enable decision makers to detect some trace of informed trading in a market subject to large volume of liquidity trading and noise. On the other hand, when the market is less noisy, the importance of coordination among speculators declines, and the additional noise that coordination adds via the excess weight that speculators put on their correlated information (which translates into weight on common noise) makes coordination undesirable. Hence, the conflict arises because high levels of coordination are desirable in noisy markets, but in equilibrium, speculators coordinate more in less noisy markets.

Our paper builds on a small, but growing, branch of models in financial economics that consider the feedback effect from trading in financial markets to corporate decisions. The basic motivation for this literature goes back to Hayek (1945), who posited that market prices provide an important source of information for various decision makers. Empirical evidence for this link is provided by Luo (2005) and Chen, Goldstein, and Jiang (2007). On the theoretical side, earlier contributions to this literature include Fishman and Hagerty (1992), Leland (1992), Khanna, Slezak, and Bradley (1994), Boot and Thakor (1997), Dow and Gorton (1997), Subrahmanyam and Titman (1999), and Fulghieri and Lukin (2001).

Several recent papers in this literature are more closely related to the mechanism in our paper. Ozdenoren and Yuan (2008) show that the feedback effect from asset prices to the real value of a firm generates strategic complementarities. In their paper, however, the feedback effect is modeled exogenously and is not based on learning. As a result, their paper does not deliver the implications that our paper delivers on the effect of liquidity and various information variables on coordination and efficiency. Khanna and Sonti (2004) also model feedback exogenously and show how a single trader can increase the value of his existing inventory in the stock by trading to affect the value of the firm. Goldstein and Guembel (2008) do analyze learning by a decision maker, and show that this might lead to manipulation of the price by a single potentially informed trader. Hence, the manipulation equilibrium in their paper is not a result of strategic complementarities among heterogeneously informed traders. Dow, Goldstein, and Guembel (2007) show that the feedback effect generates complementarities in the decision to produce information, but not in the trading decision.[5]

Our paper is most closely related to Goldstein, Ozdenoren, and Yuan (2009) and Angeletos, Lorenzoni, and Pavan (2010). Both of these papers derive endogenous complementarities as a result of learning from the aggregate action of agents. To analyze trading frenzies and their impact on real investments, our model embeds this mechanism in a model where a capital provider learns from the price in the financial market to make an investment decision. There are a few technical advancements in our analysis due to the nature of the problem that we study. For example, having a price mechanism introduces strategic substitutes into

[5]Complementarities in the decision to produce information also arise due to other reasons in several other papers. For example see, Froot, Scharfstein, and Stein (1992); Hirshleifer, Subrahmanyam, and Titman (1994); Bru and Vives (2002); and Veldkamp (2006a and 2006b).

the model.[6] Our model also generates new insights that are important for our application, such as the effect of supply elasticity and noise trading on coordination in financial markets.

The remainder of this paper is organized as follows. In Section 2, we present the model setup and characterize the equilibrium of the model. In Section 3, we solve the model. Section 4 analyzes the determinants of coordination among speculators in our model. In Section 5, we discuss the implications for the efficiency of investments and the volatility of prices and investments. Section 6 concludes.

2 Model

The model has one firm and a traded asset. There is a capital provider who has to decide how much capital to provide to the firm for the purpose of making an investment. There are three dates, $t = 0, 1, 2$. At date 0, speculators trade in the asset market based on their information about the fundamentals of the firm. At date 1, after observing the asset price and receiving private information, the capital provider of the firm decides how much capital the firm can have and the firm undertakes investment accordingly. Finally, at date 2, the cash flow is realized and agents get paid.

2.1 Investment

The firm in this economy has access to a production technology, which at time $t = 2$ generates cash flow $\tilde{F}I$. Here, I is the amount of investment financed by the capital provider, and $\tilde{F} \geq 0$ is the level of productivity. Let $\tilde{f}$ denote the natural log of productivity, $\tilde{f} = \ln \tilde{F}$. We assume that $\tilde{f}$ is unobservable and drawn from a normal distribution with mean $\bar{f}$ and variance σ_f^2. We use τ_f to denote $1/\sigma_f^2$. As will become clear later, assuming a log-normal distribution for the productivity shock $\tilde{F}$ is important for the tractability of our model.

At time $t = 1$ the capital provider chooses the level of capital I. Providing capital is costly and the capital provider must incur a private cost of: $C(I) = \frac{1}{2}cI^2$, where $c > 0$. This cost can be thought of as the cost of raising the capital, which is increasing in the amount of capital provided, or as effort incurred in monitoring the investment (which is also increasing in the size of the investment). The capital provider's benefit increases in the cash flow generated by the investment. To ease the exposition, we say that he captures the full amount $\tilde{F}I$. But, as we discuss below, the model will generate similar results if we assume

[6] Another technical detail that we highlight in the description of the model is the use of log-normal distributions, which is necessary in a setting of feedback from financial-market prices to investment decisions.

that the capital provider gets a portion of the cash flow: $\beta\tilde{F}I$ (but then we will need to carry another parameter, β). The capital provider chooses I to maximize the value of the cash flow from investing in the firm's production technology minus his cost of raising capital $C(I)$, conditional on his information set, $\mathcal{F}_l$, at $t = 1$:

$$I = \arg\max_I E[\tilde{F}I - C(I)|\mathcal{F}_l]. \tag{1}$$

The solution to this maximization problem is:

$$I = \frac{E[\tilde{F}|\mathcal{F}_l]}{c}. \tag{2}$$

The capital provider's information set, denoted by $\mathcal{F}_l$, consists of a private signal $\tilde{s}_l$ and the asset price P observed at date 0 (we will elaborate on the formation of P next). That is, $\mathcal{F}_l = \{\tilde{s}_l, P\}$. The private signal $\tilde{s}_l$ is a noisy signal about $\tilde{f}$ with precision τ_l: $\tilde{s}_l = \tilde{f} + \sigma_l\tilde{\epsilon}_l$, where $\tilde{\epsilon}_l$ is distributed normally with mean zero and standard deviation one and $\tau_l = 1/\sigma_l^2$. Later, we will conduct comparative statics with respect to the precision of the capital provider's private signal. It is important to emphasize that even though our capital provider learns from the information in the price, he still may have good sources of private information. In fact, his signal can be more precise than other signals in the economy. Despite this, he still attempts to learn from the market, as agents in the market have other signals that are aggregated by the price.

2.2 Speculative Trading

The traded asset is a claim on the payoff from the firm's investment $\tilde{F}I$, which is realized at the final date $t = 2$. The price of this risky asset at $t = 0$ is denoted by P. One way to think about the traded asset is as a derivative, whose payoff is tied to the return from the investment. It can also be viewed as equity, to the extent that the value of the firm is $\tilde{F}I$ (and so the cost of the investment $C(I)$ is privately incurred by the capital provider). We could analyze our model assuming that this value is shared between the capital provider and shareholders, such that the former receives $\beta\tilde{F}I$ and the latter receive $(1-\beta)\tilde{F}I$. This would not change our results, but will add complexity due to the additional parameter β. Hence, we omit β in the paper. It should be noted that, no matter what the nature of the asset is, our market is a secondary market with no cash transfers to the firm. The only effect of the market on the firm will be via the information revealed in the trading process.

In the market, there is a measure-one continuum of heterogeneously informed risk-neutral speculators indexed by $i \in [0, 1]$. Each speculator is endowed with two signals about $\tilde{f}$ at time 0. The first signal, $\tilde{s}_i = \tilde{f} + \sigma_s \tilde{\epsilon}_i$, is privately observed where $\tilde{\epsilon}_i$ is independently normally distributed across speculators with mean zero and unit variance. The precision of this signal is denoted as $\tau_s = 1/\sigma_s^2$. The second signal is $\tilde{s}_c = \tilde{f} + \sigma_c \tilde{\epsilon}_c$. This signal is observed by all speculators and $\tilde{\epsilon}_c$ is independently and normally distributed with mean zero and unit variance and $\tau_c = 1/\sigma_c^2$.[7]

Each speculator can buy or sell up to a unit of the risky asset. The size of speculator i's position is denoted by $x(i) \in [-1, 1]$. This position limit can be justified by limited capital and/or borrowing constraints faced by speculators.[8] Due to risk neutrality, speculators choose their positions to maximize expected profits. A speculator's profit from shorting one unit of the asset is given by $P - \tilde{FI}$, where $\tilde{FI}$ is the asset payoff and P is the price of the asset. Similarly, a speculator's profit from buying one unit of the asset is given by $\tilde{FI} - P$.

Formally, speculator i chooses $x(i)$ to solve:

$$\max_{x(i)\in[-1,1]} x(i)\, E\left[\tilde{FI} - P | \mathcal{F}_i\right], \tag{3}$$

where $\mathcal{F}_i$ denotes the information set of speculator i and consists of $\tilde{s}_i$ and $\tilde{s}_c$. Since each speculator has measure zero and is risk neutral, an informed speculator optimally chooses to either short up to the position limit, or buy up to the position limit. We denote the aggregate demand by speculators as $X = \int_0^1 x(i)\, di$, which is given by the fraction of speculators who buy the asset minus the fraction of those who short the asset.

2.3 Market Clearing

At date 0, conditional on his information, each speculator submits a market order to buy or sell a unit of the asset to a Walrasian auctioneer. The Walrasian auctioneer then obtains the aggregate demand by speculators X and also a noisy supply curve from uninformed traders, and sets a price to clear the market. The noisy supply of the risky asset is

[7]The assumption that the second signal is a common signal greatly simplifies the analysis. However, it is not necessary. The necessary element is that the noise in the information observed by speculators has a common component that cannot be fully teased out by the capital provider. In Goldstein, Ozdenoren, and Yuan (2010), we analyzed an alternative setup, where the second signal is specified as a heterogenous private signal with a common noise component $\tilde{\epsilon}_c$ and an agent-specific noise component $\tilde{\epsilon}_{2i}$. That is, $\tilde{s}_{ci} = \tilde{f} + \sigma_c \tilde{\epsilon}_c + \sigma_{\epsilon 2} \tilde{\epsilon}_{2i}$, where $\tilde{\epsilon}_c$ and $\tilde{\epsilon}_{2i}$ are independently normally distributed variables with mean zero and variance one. That paper, however, was simpler on other dimensions, as there was no price formation for the traded asset.

[8]The specific size of this position limit on asset holdings is not crucial for our results. What is crucial is that informed speculators cannot take unlimited positions; if they do, strategic interaction among informed speculators will become immaterial.

exogenously given by $Q(\tilde{\xi}, P)$, a continuous function of an exogenous demand shock $\tilde{\xi}$ and the price P. The supply curve $Q(\tilde{\xi}, P)$ is strictly decreasing in $\tilde{\xi}$, and increasing in P, that is, it is upward slopping in price. The demand shock $\tilde{\xi} \in \mathbb{R}$ is independent of other shocks in the economy, and $\tilde{\xi} \sim N(0, \sigma_\xi^2)$. As always, we denote $\tau_\xi = 1/\sigma_\xi^2$. The usual interpretation of noisy supply/demand is that there are agents who trade for exogenous reasons, such as liquidity or hedging needs. They are usually referred to as "noise traders".

To solve the model in closed form, we assume that $Q(\tilde{\xi}, P)$ takes the following functional form:

$$Q(\xi, P) = 1 - 2\Phi\left(\tilde{\xi} - \alpha \ln P\right), \tag{4}$$

where $\Phi(\cdot)$ denotes the cumulative standard normal distribution function. The parameter α captures the elasticity of the supply curve with respect to the price. It can be interpreted as the liquidity of the market: when α is high, the supply is very elastic with respect to the price, and so large informed demand is easily absorbed in the price without having much of a price impact. This notion of liquidity is similar to that in Kyle (1985), where liquidity is considered high when the informed trader has a low price impact. The basic features assumed in (4), i.e., that the supply is increasing in price and also has a noisy component, are standard in the literature. It is also common in the literature to assume particular functional forms to obtain tractability. The specific functional form assumed here is close to that in Dasguta (2007) Hellwig, Mukherji, and Tsyvinski (2006) .

2.4 Equilibrium

We now turn to the definition of equilibrium.

DEFINITION 1: [Equilibrium with Market Orders] An equilibrium consists of a price function, $P(\tilde{f}, \tilde{\epsilon}_c, \tilde{\xi}) : \mathbb{R}^3 \to \mathbb{R}$, an investment policy for the capital provider $I(\tilde{s}_l, P) : \mathbb{R}^2 \to \mathbb{R}$, strategies for speculators, $x(\tilde{s}_i, \tilde{s}_c) : \mathbb{R}^2 \to [-1, 1]$, and the corresponding aggregate demand $X(\tilde{f}, \tilde{\epsilon}_c)$, such that:

- For speculator i, $x(\tilde{s}_i, \tilde{s}_c) \in \arg\max_{x(i)\in[-1,1]} x(i) E\left[\tilde{F}I - P | \tilde{s}_i, \tilde{s}_c\right]$;
- The capital provider's investment is $I(\tilde{s}_l, P) = E\left[\tilde{F} | \tilde{s}_l, P\right]/c$.
- The market clearing condition for the risky asset is satisfied:

$$Q(\tilde{\xi}, P) = X(\tilde{f}, \tilde{\epsilon}_c) \equiv \int x(\tilde{f} + \sigma_s \tilde{\epsilon}_i, \tilde{f} + \sigma_c \tilde{\epsilon}_c) d\Phi(\tilde{\epsilon}_i). \tag{5}$$

Definition 2: A *linear monotone equilibrium* is an equilibrium where $x(\tilde{s}_i, \tilde{s}_c) = 1$ if $\tilde{s}_i + k\tilde{s}_c \geq g$ for constants k and g, and $x(\tilde{s}_i, \tilde{s}_c) = -1$ otherwise. In words: in a monotone linear equilibrium, a speculator buys the asset if and only if a linear combination of his signals is above a cutoff g, and sells it otherwise.

In the rest of the paper we focus on linear monotone equilibria.

3 Solving the Model

In this section, we explain the main steps that are required to solve our model. Restricting attention to a linear monotone equilibrium, we first use the market clearing condition to determine the asset price. We then characterize the information content of the asset price to derive the capital provider's belief on $\tilde{f}$ based on $\{P, \tilde{s}_l\}$ and solve for the optimal investment problem. Finally, given the capital provider's investment rule and the asset pricing rule, we solve for individual speculators' optimal trading decision.

In a linear monotone equilibrium, speculators short the asset whenever $\tilde{s}_i + k\tilde{s}_c \leq g$ or, equivalently, $\sigma_s\tilde{\epsilon}_i \leq g - (1+k)\,\tilde{f} - k\sigma_c\tilde{\epsilon}_c$. Hence, their aggregate selling can be characterized by: $\Phi\left(\left(g - (1+k)\,\tilde{f} - k\sigma_c\tilde{\epsilon}_c\right)/\sigma_s\right)$. Conversely, they purchase the asset whenever $\tilde{s}_i + k\tilde{s}_c \geq g$ or, equivalently, $\sigma_s\tilde{\epsilon}_i \geq g - (1+k)\,\tilde{f} - k\sigma_c\tilde{\epsilon}_c$. Hence, their aggregate purchase can be characterized by $1 - \Phi\left(\left(g - (1+k)\,\tilde{f} - k\sigma_c\tilde{\epsilon}_c\right)/\sigma_s\right)$. The net holding from speculators is then:

$$X\left(\tilde{f}, \tilde{\epsilon}_c\right) = 1 - 2\Phi\left(\frac{g - (1+k)\,\tilde{f} - k\sigma_c\tilde{\epsilon}_c}{\sigma_s}\right). \tag{6}$$

The market clearing condition together with Equation (4) indicate that

$$1 - 2\Phi\left(\frac{g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c}{\sigma_s}\right) = 1 - 2\Phi\left(\tilde{\xi} - \alpha \ln P\right). \tag{7}$$

Therefore the equilibrium price is given by

$$P = \exp\left(\frac{(1+k)\tilde{f} + k\sigma_c\tilde{\epsilon}_c - g + \sigma_s\tilde{\xi}}{\alpha\sigma_s}\right) = \exp\left(\frac{\tilde{f} + k\tilde{s}_c - g + \sigma_s\tilde{\xi}}{\alpha\sigma_s}\right), \tag{8}$$

which can be rewritten as

$$z(P) \equiv \frac{g + \alpha\sigma_s \ln P}{1+k} = \tilde{f} + \frac{k}{1+k}\sigma_c\tilde{\epsilon}_c + \frac{1}{1+k}\sigma_s\tilde{\xi} = \left(\frac{1}{1+k}\right)\tilde{f} + \frac{k}{1+k}\tilde{s}_c + \frac{1}{1+k}\sigma_s\tilde{\xi}. \tag{9}$$

From the above equation, we can see that $z(P)$, which is a sufficient statistic for the information in P, provides some information about the realization of the productivity shock

$\tilde{f}$. Yet, the signal $z(P)$ is not fully revealing of $\tilde{f}$, as it is also affected by the noise in the common signal $\tilde{\epsilon}_c$ and by the noisy demand $\tilde{\xi}$. Since the capital provider observes $z(P)$, he will use it to update his belief about the productivity. Note that $z(P)$ is distributed normally with a mean of $\bar{f}$ and a variance of $\sigma_p^2 = (k/(1+k))^2\sigma_c^2 + (1/(1+k))^2\sigma_s^2\sigma_\xi^2$. We denote the precision of $z(P)$ as a signal for $\tilde{f}$ as:

$$\tau_p = \frac{1}{\sigma_p^2} = \frac{(1+k)^2\tau_c\tau_\xi\tau_s}{k^2\tau_\xi\tau_s + \tau_c}. \tag{10}$$

After characterizing the information content of the price, we can derive the capital provider's belief on $\tilde{f}$. That is, conditional on observing $\tilde{s}_l$ and $z(P)$, the capital provider believes that $\tilde{f}$ is distributed normally with mean

$$\frac{\tau_f\bar{f} + \tau_l\tilde{s}_l + \tau_p z(P)}{\tau_f + \tau_l + \tau_p} \tag{11}$$

and variance $1/(\tau_f + \tau_l + \tau_p)$. Then, using the capital provider's investment rule in Equation (1) and taking expectations, we can express the level of investment as:

$$\begin{aligned} I &= \frac{1}{c}E[\tilde{F}|\tilde{s}_l = s_l, P] = \frac{1}{c}E[\exp\left(\tilde{f}\right)|\tilde{s}_l = s_l, P] \\ &= \frac{1}{c}\exp\left(\frac{\tau_f\bar{f} + \tau_l s_l + \tau_p z(P)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)}\right). \end{aligned} \tag{12}$$

Given the capital provider's investment policy in (12) and the price in (8), we can now write speculator i's expected profit from buying the asset given the information that is available to him (shorting the asset would give the negative of this):

$$\begin{aligned} & E\left[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c\right] \\ = \; & \frac{1}{c}E\left[\exp\left(\frac{\tau_f\bar{f} + \tau_l s_l + \tau_p z(P)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)} + \tilde{f}\right)|\tilde{s}_i, \tilde{s}_c\right] \\ & -E\left[\exp\left(\frac{\tilde{f} + k\tilde{s}_c - g + \sigma_s\tilde{\xi}}{\alpha\sigma_s}\right)|\tilde{s}_i, \tilde{s}_c\right]. \end{aligned} \tag{13}$$

Note that we made use here of the fact that $\tilde{F} = \exp\left(\tilde{f}\right)$. This is where using the natural log of the productivity parameter plays a key role. Using the properties of the exponential function, we can express the value of the firm $\tilde{F}I$ as $\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z(P)}{\tau_f+\tau_l+\tau_p} + \frac{1}{2(\tau_f+\tau_l+\tau_p)} + \tilde{f}\right)$, where the expression in parentheses is linear in $\tilde{f}$. This enables us to get a linear closed-form solution, which would otherwise be impossible in a model of feedback.

Conditional on observing $\tilde{s}_i$ and $\tilde{s}_c$, speculator i believes that $\tilde{f}$ is distributed normally with mean

$$\frac{\tau_f\bar{f} + \tau_s\tilde{s}_i + \tau_c\tilde{s}_c}{\tau_f + \tau_s + \tau_c} \tag{14}$$

and variance $1/(\tau_f + \tau_s + \tau_c)$. Hence, substituting for $z(P)$ (from (9)) and taking expectations, Equation (13) can be rewritten as:

$$E\left[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c\right] = \frac{1}{c}\exp(a_0 + a_1\tilde{s}_i + a_2\tilde{s}_c) - \exp(b_0 + b_1\tilde{s}_i + b_2\tilde{s}_c), \tag{15}$$

where the coefficients a_0, a_1, a_2, b_0, b_1, and b_2 are functions of k and of the model's parameters. Explicit expressions for these coefficients are provided in the proof of Proposition 1 in the appendix.

In equilibrium, a speculator who receives a private signal $\tilde{s}_i = g - k\tilde{s}_c$ must be indifferent between buying the asset or shorting it. That is,

$$E\left[P - \tilde{F}I|\tilde{s}_i = g - k\tilde{s}_c, \tilde{s}_c\right] = 0. \tag{16}$$

Substituting $\tilde{s}_i = g - k\tilde{s}_c$ into (15), and taking logs, the indifference condition of (16) becomes:

$$\ln\frac{1}{c} + a_0 + a_1(g - k\tilde{s}_c) + a_2\tilde{s}_c = b_0 + b_1(g - k\tilde{s}_c) + b_2\tilde{s}_c. \tag{17}$$

In a linear monotone equilibrium, this indifference condition must hold for all $\tilde{s}_c$. Hence, the coefficient for $\tilde{s}_c$ must be zero:

$$a_2 - a_1k = b_2 - b_1k. \tag{18}$$

Using this, we solve for the weight k that a speculator puts on the common signal in his trading strategy. Recall that a_1, a_2, b_1, and b_2 are also functions of k, and hence (18) becomes a high-order polynomial. Analyzing this polynomial, we obtain the result in the following proposition. The proof of this proposition, as well as all other proofs, is in the Appendix.

PROPOSITION 1: For α large enough, there exists a monotone linear equilibrium with $k^* > 0$. This equilibrium is unique when τ_f is sufficiently small.

The weight k^* that speculators put on the common signal in equilibrium captures the degree of coordination in their trading decisions. When k^* is high, speculators put a large weight on the common information when deciding whether to sell or buy the asset. This leads to large coordination among them and gives rise to a trading frenzy. In the upcoming sections, we develop a series of results on the determinants of coordination and its implications for the efficiency of the investment decision and for the volatility of prices. We focus on the case of large supply elasticity (large α) and imprecise prior (small τ_f), for which we know that there exists a unique equilibrium.

4 The Determinants of Speculators' Coordination

The weight that speculators put on the common signal in this model is affected by the degree to which there are strategic complementarities or strategic substitutes among them. Strategic substitutes are generated by the usual price mechanism. Since the aggregation of speculators' orders affects the price, when others put large weight on the common signal, the common signal gets to have a large effect on the price, and then the incentive of each speculator to put weight on the common signal decreases. Strategic complementarities, on the other hand, are generated here by the feedback effect that prices have on the investment decision and thus on the real value of the firm. When others put more weight on the common signal, this signal gets to have a large effect on the information available to the capital provider, and hence on the real value of the security. Then, the incentive of each speculator to put weight on the common signal increases.

In a world without these strategic effects, the weight that speculators put on the common signal relative to the private signal would be equal to the ratio of precisions between the signals: τ_c/τ_s. But, with strategic effects, the equilibrium weight on the common signal k^* reflects the sum of the strategic effects on top of the precisions ratio; where the strategic substitutes due to the price mechanism push k down and the strategic complementarities due to the feedback effect push it up. In the rest of this section, we formally isolate the various determinants of coordination to understand the impact of each factor on the equilibrium level of coordination.

4.1 Impact of Learning by the Capital Provider

Suppose that there is no feedback effect from prices to real values, because the capital provider does not learn from the price. In this case, the capital provider's decision on how much capital to provide becomes (this equation is analogous to Equation (12) in the full model):

$$\begin{aligned} I &= \frac{1}{c}E[\tilde{F}|\tilde{s}_l = s_l] \\ &= \frac{1}{c}\exp\left(\frac{\tau_f}{\tau_f+\tau_l}\bar{f} + \frac{\tau_l}{\tau_f+\tau_l}s_l + \frac{1}{2(\tau_f+\tau_l)}\right). \end{aligned} \tag{19}$$

We again solve for the linear monotone equilibrium where speculators short sell the asset if and only if $\tilde{s}_i + k_{BM}\tilde{s}_c \leq g_{BM}$ (the subscript BM stands for 'benchmark'), and purchase the asset otherwise. Given the investment rule in (19), the expected profit for speculator i from buying the asset, given the information available to him, becomes (this equation is analogous to Equation (13) in the full model):

$$E[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c] = E\left[\frac{1}{c}\exp\left(\frac{\tau_f \bar{f} + \tau_l s_l}{\tau_f + \tau_l} + \frac{1}{2(\tau_f + \tau_l)}\right)\tilde{F}|\tilde{s}_i, \tilde{s}_c\right] \\ -E\left[\exp\left(\frac{1}{\alpha\sigma_s}\left(\tilde{f} + k_{BM}\tilde{s}_c - g_{BM} + \sigma_s\tilde{\xi}\right)\right)|\tilde{s}_i, \tilde{s}_c\right]. \tag{20}$$

We know that a speculator observing $\tilde{s}_i = g_{BM} - k_{BM}\tilde{s}_c$ is indifferent between buying and shorting the asset. Following similar steps to those in the full model, we obtain (see more details in the proof of Proposition 2) the weight that speculators put on the common signal in the case of no feedback effect from price to real investment:

$$k_{BM} = \frac{\left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)\tau_c\tau_f + \left(2 - \frac{\sqrt{\tau_s}}{\alpha}\right)\tau_c\tau_l}{\left(\frac{\sqrt{\tau_s}}{\alpha}\right)(\tau_f + \tau_l)(\tau_c + \tau_f + \tau_s) + \left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)\tau_f\tau_s + \left(2 - \frac{\sqrt{\tau_s}}{\alpha}\right)\tau_l\tau_s}. \tag{21}$$

Inspecting (21), we can see that k_{BM} is lower than τ_c/τ_s, and that it approaches τ_c/τ_s as α gets very large. The intuition is as follows: τ_c/τ_s represents the ratio of precisions between the common signal and the idiosyncratic signal. This is the relative weight that speculators would put on the common signal if there were no strategic interactions. In a world without feedback effect, the only strategic interaction between the speculators comes from the price mechanism, which generates strategic substitutes: as speculators put more weight on the common signal, the price reflects the common signal more strongly, inducing other speculators to reduce the weight they put on the common signal. As α gets very large, this effect weakens, and speculators converge to the weight of τ_c/τ_s. The following proposition summarizes the properties of k_{BM} and its relation to the equilibrium weight k^* on the common signal.

PROPOSITION 2: For α large enough, if the capital provider does not learn from the price when making lending decisions, the weight speculators put on the common signal k_{BM} is strictly below the equilibrium weight k^* they put in the full model (with a feedback effect).

We can see that when we shut down the feedback effect from the price to real investment, the weight that speculators put on the common signal decreases. This is in line with our discussion above, according to which the feedback effect from prices to real investment is the source of complementarity in speculators' strategies, making them want to put more weight on the common signal. Hence, the feedback effect is the cause of trading frenzies in our model.

4.2 Impact of Supply Elasticity

The parameter α captures the elasticity of supply with respect to price in our model. When α is high, the supply of shares is very sensitive to the price, meaning that an increase in demand by informed traders is quickly absorbed in the market, so that informed trading does not have a large price impact. As mentioned above, α can then be interpreted as a measure of liquidity, and our model can be used to tell what is the effect of liquidity on trading frenzies. The following proposition tells us that the extent to which speculators coordinate on the common signal increases in the level of liquidity α.

Proposition 3: The equilibrium level of coordination k^* is increasing in α, and for α large enough $k^* > \tau_c/\tau_s$.

In illiquid markets, order flows have a large effect on the price. Then, when speculators put more weight on the common signal, this signal has a substantial effect on the price, and so other speculators want to put less weight on the common signal. This effect decreases as α goes up and liquidity improves. Hence, in liquid markets there is a greater tendency for coordination and trading frenzies. As the proposition shows, when α is large enough, the weight on the common signal increases even beyond the ratio of precisions τ_c/τ_s.

4.3 Impact of Noise Trading

Noise trading is captured in our model by the variable $\tilde{\xi} \sim N(0, \sigma_\xi^2)$. A high level of σ_ξ^2 implies that the market is exposed to large levels of noise trading. In the literature on financial markets, this introduces noise to the price, and in the presence of a feedback effect, it makes it harder to base investment decisions on the price. In our model, we examine the effect of noise trading on speculators' coordination. As we will see later, this will have further implications for the informativeness of the price.

Proposition 4: For α large enough, the equilibrium weight k^* that speculators put on the common signal is decreasing in the variance of noise demand σ_ξ^2.

The intuition here goes as follows: With high variance in the noise demand, there is high variance in the market price for reasons that are not related to speculators' trades. As a result, the reliance of the capital provider on the information in the price decreases. This weakens the feedback effect and hence the strategic complementarities among speculators, leading to a lower level of k^*.

Finally, it is worth noting that changes in the position limits of speculators will have similar effects to changes in the variance of noise trading. For example, if speculators could choose positions in the range $[-2, 2]$ (instead of $[-1, 1]$, assumed in the paper), they would have more impact on the capital provider's decision for a given level of σ_ξ^2 and thus would put a larger weight on the common signal in equilibrium. Hence, the effect of loosening speculators' trading constraints is similar to that of reducing the variance of noise trading.

4.4 Impact of the Information Structure

We now establish comparative statics results on the effect of the informativeness of various signals on the equilibrium level of coordination. The results are summarized in the next proposition.

PROPOSITION 5: For α large enough and τ_f small enough: $\partial k^*/\partial\tau_s < 0$, $\partial k^*/\partial\tau_l < 0$, and $\partial k^*/\partial\tau_c > 0$.

These results are intuitive. Speculators put more weight on the common signal relative to the private signal when the common signal is more precise (τ_c is higher) and the private signal is less precise (τ_s is lower). Hence, trading frenzies are more likely when the common information becomes more precise relative to speculators' idiosyncratic sources of information. Less obvious is the result that the tendency for coordination among speculators decreases when the capital provider has more precise information (τ_l is higher). The reason is that when the capital provider has more precise information, he relies less on the price, and so the feedback effect from markets to real decisions weakens, and there is less scope for strategic complementarities.

5 Coordination, Investment Efficiency, and Non-Fundamental Volatility

In this section, we explore the effect that coordination has on the efficiency of investment decisions and on market volatility. To analyze investment efficiency, we look at the ex ante expected net benefit of investment (i.e. expected net benefit before any of the signals are realized given the prior belief that $\tilde{f}$ is normally distributed with mean $\bar{f}$ and precision τ_f) from the perspective of the capital provider. We keep the information structure the same as before, and in particular, in the interim stage we allow the capital provider to obtain information only from his private signal and the price. So our efficiency criterion is given by:

$$E_0\left[\max_I E\left[\tilde{F}I - \frac{1}{2}cI^2 \,|\tilde{s}_l = s_l, P\right]\right], \tag{22}$$

where a speculator purchases the asset if $\tilde{s}_i + k\tilde{s}_c \geq g$ and shorts it otherwise (for constant k and g) and P is the market clearing price. We denote the optimal level of coordination k_{OP} to be the one that maximizes investment efficiency as in (22).

The following proposition characterizes k_{OP}, and how it is linked to the accuracy of the information inferred from the market price, τ_p:

PROPOSITION 6: The level of coordination that maximizes investment efficiency is $k_{OP} = \tau_c/\tau_s\tau_\xi$, which also maximizes τ_p.

The capital provider cares about the events in the security market only to the extent that they affect the quality of the information he has when making the investment decision. Hence, the level of coordination that maximizes investment efficiency is the one that maximizes the accuracy of the information in the market price. Examining the expression for the price signal in (9), we can see that there is a tradeoff in setting the level of coordination. The tradeoff arises because there are two sources of noise in the price, one coming from the noise demand $\widetilde{\xi}$ and the other one from the noise in the common signal $\tilde{\epsilon}_c$. The first source of noise becomes more prominent when speculators' private information is noisy (τ_s is low) because then noise trading becomes more important. A high level of coordination reduces the effect of the first source of noise – as coordinated speculative trading helps overcoming the large volume of noise trading – and increases the effect of the second source of noise – as coordinated speculative trading increases the weight on the common signal. Therefore, the optimal level of coordination will be high when the potential damage from noise demand is high (τ_ξ and τ_s are low) or when the potential damage from noise in the common signal is low (τ_c is high). Then, $k_{OP} = \tau_c/\tau_s\tau_\xi$.

It is interesting to compare the optimal level of coordination characterized here with the level of coordination that is obtained in equilibrium. From Proposition 4 we know that in equilibrium speculators coordinate more when the variance in the noise demand is low (τ_ξ is high). A high τ_ξ implies that speculators' trades have more effect on the capital provider's decision, increasing the scope of strategic complementarities. Yet, this is exactly when coordination is not desirable for the efficiency of the investment. Hence, there is a sharp contrast between the profit incentives of speculators and the efficiency of the investment. Speculators coordinate more exactly when it is inefficient to do so. The following proposition summarizes the comparison between the optimal level of coordination and the equilibrium level of coordination.

PROPOSITION 7: For α large enough, there exists $\bar{\tau}_\xi$ such that $k_{OP} > k^*$ for $\tau_\xi < \bar{\tau}_\xi$ and $k_{OP} < k^*$ for $\tau_\xi > \bar{\tau}_\xi$.

The proposition says that speculators coordinate too much in less noisy markets and coordinate too little in noisy markets. Interestingly, this implies that trading frenzies are only sometimes undesirable. When there is high variation in noise demand, price informativeness would improve if speculators coordinated their trades more to provide a signal that overcomes the effect of noise demand. Yet, it is exactly in this case that they find coordination less profitable in equilibrium.

We close this section by noting some of the implications of inefficient coordination levels. Deviations from the optimal level of coordination k_{OP} are manifested in our model by higher levels of non-fundamental volatility. The following proposition establishes the link between the level of coordination and non-fundamental volatility – volatility that does not come from the variability in fundamental – of price and investment.

PROPOSITION 8: (a) Non-fundamental volatility of asset price is minimized at $k = k_{OP}$ (where its value is $1/(\tau_c + \tau_s\tau_\xi)$), decreases in k when $k < k_{OP}$ and increases in k when $k > k_{OP}$. In particular, when $k > k_{OP}$, non-fundamental volatility of asset prices is higher because prices are more sensitive to the noise component in speculators' common signal $\tilde{\epsilon}_c$. When $k < k_{OP}$, non-fundamental volatility of asset prices is higher because prices are more sensitive to the noise demand $\tilde{\xi}$.

(b) Similarly, non-fundamental volatility of investment is minimized at $k = k_{OP}$ (where its value is $1/(\tau_l + \tau_c + \tau_s\tau_\xi)$), decreases in k when $k < k_{OP}$ and increases in k when $k > k_{OP}$.

This proposition indicates that the strategic interactions among speculators in the financial markets often lead to non-fundamental volatility in prices as well as real activities. The source of this non-fundamental volatility could come from either too low coordination (that is, when the market is characterized by a high amount of trading by noise investors) or too high coordination (that is, when the market has low noise trading and the noise in the correlated signals among speculators is high).

6 Conclusion

We study strategic interactions among speculators in financial markets and their real effects. Two opposite strategic effects exist. On the one hand, speculators wish to act differently from each other as a certain action by other speculators changes the price in a way that reduces the profit for other speculators from this action. On the other hand, due to the feedback effect from the price to the real investment, a certain action by speculators changes the real value of the firm in a way that increases the incentive of other speculators to take this action. This creates a basis for trading frenzies, where speculators rush to trade in the same direction, putting pressure on the price and on the firm's value. We characterize which effect dominates when and analyze the resulting level of coordination in speculators' actions.

The interaction among speculators affects the informational content of the price. Since prices affect real investment in our model, we can ask what level of coordination is most efficient for real investment. In general, speculators' incentives to coordinate go in opposite direction to the optimal level of coordination. Speculators want to coordinate more when there is low amount of noise trading, but this is when coordination is less desirable from an efficiency point of view. Hence, our model shows that there is always either too much or too little coordination, and this reduces the efficiency of investment and creates excess volatility in the price.

Interestingly, our paper is also related to an old debate on whether speculators stabilize prices. The traditional view is that by buying low and selling dear, rational speculators stabilize prices. Hart and Kreps (1986) argue that when speculators can hold inventories and there is uncertainty about preferences, speculative activity may cause excess price movement. Our paper contributes to this literature by pointing out that when speculative activity has an effect on real investments, speculators might coordinate on correlated sources of information, and create excess volatility in prices. In our model, this reduces efficiency of real investments.

References

Angeletos, George-Marios, Guido Lorenzoni, and Alessandro Pavan, 2010, Beauty contests and irrational exuberance: A neoclassical approach, MIT working paper.

Boot, Arnoud, and Anjan Thakor, 1997, Financial system architecture, *Review of Financial Studies* 10, 693–733.

Bru, Lluis, and Xavier Vives, 2002, Informational externalities, herding and incentives, *Journal of Institutional and Theoretical Economics* 158, 91–105.

Chen, Qi, Itay Goldstein, and Wei Jiang, 2007, Price informativeness and investment sensitivity to stock price, *Review of Finacial Studies* 20, 619–650.

Dasgupta, Amil, 2007, Coordination and delay in global games, *Journal of Economic Theory* 134, 195–225.

Dow, James, Itay Goldstein, and Alexander Guembel, 2007, Incentives for information production in markets where prices affect real investment, Working Paper.

Dow, James, and Gary Gorton, 1997, Stock market efficiency and economic efficiency: Is there a connection?, *Journal of Finance* 52, 1087–1129.

Fishman, Mike J., and Kathleen M. Hagerty, 1992, Insider trading and the efficiency of stock prices, *RAND Journal of Economics* 23, 106–122.

Froot, Kenneth, David Scharfstein, and Jeremy Stein, 1992, Herd on the street: informational inefficiencies in a market with short-term speculation, *Journal of Finance* 47, 1461–1484.

Fulghieri, Paolo, and Dmitry Lukin, 2001, Information production, dilution costs, and optimal security design, *Journal of Financial Economics* 61, 3–42.

Goldstein, Itay, and Alexander Guembel, 2008, Manipulation and the allocational role of prices, *Review of Economic Studies* 75, 133–164.

Goldstein, Itay, Emre Ozdenoren, and Kathy Yuan, 2009, Learning and complementarities in speculative attacks, *Review of Economic Studies* forthcoming.

Grossman, Sanford, and Joseph Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review* 70, 393–408.

Hayek, Friedrich, 1945, The use of knowledge in society, *American Economic Review* 35, 519–530.

Hellwig, Christian, Arijit Mukherji, and Aleh Tsyvinski, 2006, Self-fulfilling currency crises: The role of interest rates, *American Economic Review* 96, 1769–87.

Hirshleifer, David, Avanidhar Subrahmanyam, and Sheridan Titman, 1994, Security analysis and trading patterns when some investors receive information before others, *Journal of Finance* 49, 1665–1698.

Khanna, Naveen, Steve L. Slezak, and Michael H. Bradley, 1994, Insider trading, outside search and resource allocation: Why firms and society may disagree on insider trading restrictions, *Review of Financial Studies* 7, 575–608.

Khanna, Naveen, and Ramana Sonti, 2004, Value creating stock manipulation: Feedback effect of stock prices on firm value, *Journal of Financial Markets* 7, 237–270.

Kyle, Albert S., 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315–1335.

Leland, Hayne, 1992, Insider trading: Should it be prohibited?, *Journal of Political Economy* 100, 859–887.

Luo, Yuanzhi, 2005, Do insiders learn from outsiders? evidence from mergers and acquisitions, *Journal of Finance* 60, 1951–1972.

Ozdenoren, Emre, and Kathy Yuan, 2008, Feedback effects and asset prices, *Journal of Finance* 63, 1939–1975.

Subrahmanyam, Avanidhar, and Sheridan Titman, 1999, The going-public decision and the development of financial markets, *Journal of Finance* 54, 1045–1082.

Veldkamp, Laura, 2006a, Information markets and the comovement of asset prices, *Review of Economic Studies* 73, 823–845.

———, 2006b, Media frenzies in markets for financial information, *American Economic Review* 96, 577–601.

Appendix

Proof of Proposition 1: Based on (13) and the updating done by the speculator based on his information, the coefficients in (15) are given as follows:

$$
\begin{aligned}
a_0 = \; & \frac{\tau_f \bar{f} + \frac{1}{2}}{\tau_f+\tau_l+\tau_p} + \frac{\left(\tau_f+2\tau_l+\tau_p\left(1+\frac{1}{1+k}\right)\right)}{\tau_f+\tau_l+\tau_p}\left(\frac{\tau_f \bar{f}}{\tau_f+\tau_s+\tau_c}\right) \\
& + \left(\frac{\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p}{\tau_f+\tau_l+\tau_p}\right)^2 \frac{1}{2(\tau_f+\tau_s+\tau_c)} + \frac{1}{2}\left(\frac{\tau_l}{\tau_f+\tau_l+\tau_p}\right)^2 \sigma_l^2 \\
& + \frac{1}{2}\left(\frac{\tau_p}{\tau_f+\tau_l+\tau_p}\right)^2 \left(\frac{1}{1+k}\right)^2 \sigma_s^2 \sigma_\xi^2
\end{aligned}
$$

$$
a_1 = \frac{\left(\tau_f + 2\tau_l + \tau_p\left(1+\frac{1}{1+k}\right)\right)}{\tau_f+\tau_l+\tau_p} \frac{\tau_s}{\tau_f+\tau_s+\tau_c}
$$

$$
a_2 = \frac{\tau_p \frac{k}{1+k}}{\tau_f+\tau_l+\tau_p} + \frac{\left(\tau_f + 2\tau_l + \tau_p\left(1+\frac{1}{1+k}\right)\right)}{\tau_f+\tau_l+\tau_p} \frac{\tau_c}{\tau_f+\tau_s+\tau_c}
$$

$$
b_0 = \frac{1}{\alpha\sigma_s}\left(\frac{\tau_f \bar{f} + \frac{1}{2}\frac{1}{\alpha\sigma_s}}{\tau_f+\tau_s+\tau_c} - g\right) + \frac{1}{2\alpha^2}\sigma_\xi^2
$$

$$
b_1 = \frac{1}{\alpha\sigma_s}\frac{\tau_s}{\tau_f+\tau_s+\tau_c}
$$

$$
b_2 = \frac{1}{\alpha\sigma_s}\left(\frac{\tau_c}{\tau_f+\tau_s+\tau_c} + k\right)
$$

Then, the condition in (18) becomes:

$$
\frac{\tau_p}{\tau_f+\tau_l+\tau_p}\frac{k}{1+k} + \left(\frac{\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p}{\tau_f+\tau_l+\tau_p}\right)\left(\frac{-\tau_s k + \tau_c}{\tau_f+\tau_s+\tau_c}\right) - \frac{1}{\alpha\sigma_s}\left(\left(\frac{-\tau_s k+\tau_c}{\tau_f+\tau_s+\tau_c}\right) + k\right) = 0.
$$

This equation can be rewritten as:

$$
\begin{aligned}
0 = \; & \frac{\tau_p}{\tau_f+\tau_l+\tau_p}\frac{k}{1+k} + \left(\frac{\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p}{\tau_f+\tau_l+\tau_p}\right)\left(\frac{-\tau_s k + \tau_c}{\tau_f+\tau_s+\tau_c}\right) - \left(\frac{-\tau_s k+\tau_c}{\tau_f+\tau_s+\tau_c}\right) - k \\
& + \left(1 - \frac{1}{\alpha\sigma_s}\right)\left(\frac{-\tau_s k+\tau_c}{\tau_f+\tau_s+\tau_c} + k\right),
\end{aligned}
$$

or equivalently,

$$
\begin{aligned}
0 \;=\; & \frac{\tau_p}{\tau_f+\tau_l+\tau_p}\frac{k}{1+k} + \left(\frac{\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p}{\tau_f+\tau_l+\tau_p}\right)\left(\frac{-\tau_s k + \tau_c}{\tau_f+\tau_s+\tau_c}\right) - \left(\frac{-\tau_s k+\tau_c}{\tau_f+\tau_s+\tau_c}\right) - k \\
& + \left(1 - \frac{1}{\alpha\sigma_s}\right)\left(\frac{\tau_f+\tau_c}{\tau_f+\tau_s+\tau_c}k + \frac{\tau_c}{\tau_f+\tau_s+\tau_c}\right).
\end{aligned}
$$

We focus our attention on the following parts of the left hand side:

$$
F(k) = \frac{\tau_p}{\tau_f+\tau_l+\tau_p}\frac{k}{1+k} + \left(\frac{\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p}{\tau_f+\tau_l+\tau_p}\right)\left(\frac{-\tau_s k + \tau_c}{\tau_f+\tau_s+\tau_c}\right) - \left(\frac{-\tau_s k+\tau_c}{\tau_f+\tau_s+\tau_c}\right) - k.
$$

and

$$G(k) = \left(1 - \frac{1}{\alpha\sigma_s}\right)\left(\frac{\tau_f + \tau_c}{\tau_f + \tau_s + \tau_c}k + \frac{\tau_c}{\tau_f + \tau_s + \tau_c}\right)$$

where

$$\tau_p = \frac{(1+k)^2\tau_c\tau_\xi\tau_s}{k^2\tau_\xi\tau_s + \tau_c}.$$

We denote $r \equiv \tau_\xi\tau_s$. Simplifying the above two equations (by multiplying through with $(\tau_c + \tau_f + \tau_s)(\tau_c\tau_f + \tau_c\tau_l + \tau_c r + \tau_c k^2 r + \tau_f k^2 r + k^2\tau_l r + 2\tau_c k r)$ and dividing by r) we obtain:

$$\begin{aligned} H(k) = &-k^3\left((\tau_c + \tau_f + \tau_s)(\tau_c + \tau_f + \tau_l) + \tau_l\tau_s\right) - \tau_c k^2(\tau_c + \tau_f - \tau_l + 2\tau_s) \\ &- \tau_c k(\tau_s - \tau_c) + \tau_c^2 \\ &- \frac{1}{r}\left(\tau_c k\left(\tau_c\tau_f + \tau_c\tau_l + \tau_f\tau_l + \tau_f\tau_s + 2\tau_l\tau_s + \tau_f^2\right) - \tau_c^2\tau_l\right) \end{aligned} \tag{23}$$

$$\begin{aligned} J(k) = &\left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)\left((\tau_c + \tau_f)(\tau_c + \tau_f + \tau_l)k^3 + \tau_c(3\tau_c + 3\tau_f + \tau_l)k^2\right. \\ &\left. + \tau_c(\tau_f + 3\tau_c)k + \tau_c^2\right) \\ &+ \left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)\frac{\tau_c}{r}\left((\tau_f + \tau_l)(\tau_c + \tau_f)k + \tau_c(\tau_f + \tau_l)\right) \end{aligned} \tag{24}$$

For an equilibrium, we need $H(k) + J(k) = 0$.

First, we focus on existence of an equilibrium with $k > 0$. $H(k) + J(k)$ has a positive root if and only if

$$\alpha > \sqrt{\tau_s}\frac{\tau_f + \tau_l + r}{\tau_f + 2\tau_l + 2r}.$$

To see this, note that the coefficient for k^3 is always negative, implying that the value of $H(k) + J(k)$ becomes negative as k becomes large. So, there exists a strictly positive root for the polynomial if its value is strictly positive at $k = 0$, which is given by the above inequality. If the inequality is violated than the value of the polynomial is negative at $k = 0$. Its derivative at $k = 0$ is given by

$$\begin{aligned} &-\tau_c(\tau_s - \tau_c) - \frac{1}{r}\left(\tau_c\left(\tau_c\tau_f + \tau_c\tau_l + \tau_f\tau_l + \tau_f\tau_s + 2\tau_l\tau_s + \tau_f^2\right)\right) \\ &+ \left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)\tau_c(\tau_f + 3\tau_c) + \left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)\frac{\tau_c}{r}\left((\tau_f + \tau_l)(\tau_c + \tau_f)\right). \end{aligned}$$

At $\frac{\sqrt{\tau_s}}{\alpha} \geq \frac{\tau_f+2\tau_l+2r}{\tau_f+\tau_l+r}$ the derivative is negative. This means that $H(k)+J(k)$ is decreasing at $k=0$ for $\frac{\sqrt{\tau_s}}{\alpha} \geq \frac{\tau_f+2\tau_l+2r}{\tau_f+\tau_l+r}$. Moreover, the second derivative is negative when $\frac{\sqrt{\tau_s}}{\alpha} \geq \frac{\tau_f+2\tau_l+2r}{\tau_f+\tau_l+r}$, and thus the expression will keep decreasing. Therefore the polynomial cannot have a positive root.

While the above analysis ensures that the speculator is indifferent at the threshold signal, another equilibrium condition is needed to ensure that the speculator buys the asset when his private signal exceeds the threshold and sells when his private signal falls below the threshold. For this to hold, we need that $E\left[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c\right]$ will be increasing in s_i:

$$\frac{\tau_f + 2\tau_l + \tau_p\left(1+\frac{1}{1+k}\right)}{\tau_f+\tau_l+\tau_p}\left(\frac{\tau_s}{\tau_f+\tau_s+\tau_c}\right) > \frac{\sqrt{\tau_s}}{\alpha}\frac{\tau_s}{\tau_f+\tau_s+\tau_c}$$

$$\frac{\sqrt{\tau_s}}{\alpha} < \frac{\tau_c r(k+2)(k+1) + (\tau_f+2\tau_l)(\tau_c+k^2 r)}{\tau_c r(k+1)^2 + (\tau_f+\tau_l)(\tau_c+k^2 r)}$$

$$\frac{\sqrt{\tau_s}}{\alpha} < \frac{(\tau_c+\tau_f+2\tau_l)k^2 + 3\tau_c k + \frac{\tau_c}{r}(\tau_f+2\tau_l+2r)}{(\tau_c+\tau_f+\tau_l)k^2 + 2\tau_c k + \frac{\tau_c}{r}(\tau_f+\tau_l+r)}$$

A sufficient condition for existence is that $\alpha \geq \sqrt{\tau_s}$.

For uniqueness, we need to check the sign of the discriminant of $H(k)+J(k)$ for α large and τ_f small. Letting α go to infinity and τ_f go to zero we obtain the discriminant as $-\frac{\tau_c^3}{r^3}$ times

$$\begin{aligned}
&\left(\begin{array}{c} 64\tau_c^4\tau_l + 128\tau_c^3\tau_l^2 + 160\tau_c^3\tau_l\tau_s + 64\tau_c^2\tau_l^3 + 416\tau_c^2\tau_l^2\tau_s + 144\tau_c^2\tau_l\tau_s^2 \\ +284\tau_c\tau_l^2\tau_s^2 + 56\tau_c\tau_l\tau_s^3 + 8\tau_l\tau_s^4 \end{array}\right) r^3 \\
&+\left(\begin{array}{c} 64\tau_c^5\tau_l + 192\tau_c^4\tau_l^2 + 160\tau_c^4\tau_l\tau_s + 192\tau_c^3\tau_l^3 + 608\tau_c^3\tau_l^2\tau_s + 144\tau_c^3\tau_l\tau_s^2 + 64\tau_c^2\tau_l^4 \\ +448\tau_c^2\tau_l^3\tau_s + 440\tau_c^2\tau_l^2\tau_s^2 + 56\tau_c^2\tau_l\tau_s^3 + 416\tau_c\tau_l^3\tau_s^2 + 80\tau_c\tau_l^2\tau_s^3 + 8\tau_c\tau_l\tau_s^4 + 48\tau_l^2\tau_s^4 \end{array}\right) r^2 \\
&+\left(128\tau_c\tau_l^4\tau_s^2 - 32\tau_c^2\tau_l^3\tau_s^2 - 16\tau_c^2\tau_l^2\tau_s^3 - 52\tau_c^3\tau_l^2\tau_s^2 - 64\tau_c\tau_l^3\tau_s^3 + 32\tau_c\tau_l^2\tau_s^4 + 96\tau_l^3\tau_s^4\right) r \\
&+\left(64\tau_l^4\tau_s^4 + 32\tau_c\tau_l^3\tau_s^4\right)
\end{aligned} \tag{25}$$

The coefficient of r^2 in (25) is strictly positive so the quadratic part of (25) is minimized at:

$$r = -\frac{128\tau_c\tau_l^4\tau_s^2 - 32\tau_c^2\tau_l^3\tau_s^2 - 16\tau_c^2\tau_l^2\tau_s^3 - 52\tau_c^3\tau_l^2\tau_s^2 - 64\tau_c\tau_l^3\tau_s^3 + 32\tau_c\tau_l^2\tau_s^4 + 96\tau_l^3\tau_s^4}{2\left(\begin{array}{c} 64\tau_c^5\tau_l + 192\tau_c^4\tau_l^2 + 160\tau_c^4\tau_l\tau_s + 192\tau_c^3\tau_l^3 + 608\tau_c^3\tau_l^2\tau_s + 144\tau_c^3\tau_l\tau_s^2 + 64\tau_c^2\tau_l^4 \\ +448\tau_c^2\tau_l^3\tau_s + 440\tau_c^2\tau_l^2\tau_s^2 + 56\tau_c^2\tau_l\tau_s^3 + 416\tau_c\tau_l^3\tau_s^2 + 80\tau_c\tau_l^2\tau_s^3 + 8\tau_c\tau_l\tau_s^4 + 48\tau_l^2\tau_s^4 \end{array}\right)}.$$

Substituting this back to the quadratic above we find that the minimized value is:

$$\frac{1}{2}\frac{\tau_l^3\tau_s^4}{\begin{pmatrix} 8\tau_c^5+24\tau_c^4\tau_l+20\tau_c^4\tau_s+24\tau_c^3\tau_l^2+76\tau_c^3\tau_l\tau_s+18\tau_c^3\tau_s^2+8\tau_c^2\tau_l^3+56\tau_c^2\tau_l^2\tau_s \\ +55\tau_c^2\tau_l\tau_s^2+7\tau_c^2\tau_s^3+52\tau_c\tau_l^2\tau_s^2+10\tau_c\tau_l\tau_s^3+\tau_c\tau_s^4+6\tau_l\tau_s^4 \end{pmatrix}}$$
$$\times\begin{pmatrix} 343\tau_c^6+2352\tau_c^5\tau_l+1176\tau_c^5\tau_s+5376\tau_c^4\tau_l^2+6944\tau_c^4\tau_l\tau_s+1344\tau_c^4\tau_s^2+4096\tau_c^3\tau_l^3 \\ +13312\tau_c^3\tau_l^2\tau_s+6448\tau_c^3\tau_l\tau_s^2+512\tau_c^3\tau_s^3+8192\tau_c^2\tau_l^3\tau_s+9984\tau_c^2\tau_l^2\tau_s^2+1984\tau_c^2\tau_l\tau_s^3 \\ +5120\tau_c\tau_l^3\tau_s^2+2048\tau_c\tau_l^2\tau s^3+128\tau_c\tau_l\tau_s^4+192\tau_l^2\tau_s^4 \end{pmatrix}$$

which is strictly positive. Since the quadratic term is strictly positive at its minimum, it is positive for all r. Since r^3 term is positive for $r>0$ as well, (25) is strictly positive for all $r>0$. That is, the discriminant is strictly negative for large enough α and small enough τ_f, and hence $H(k)+J(k)=0$ has a unique root. QED.

Proof of Propositions 2 and 3:

First, we derive k_{BM}. Based on (20) and following steps similar to those in the proof of Proposition 1, we know that the indifference condition for a speculator observing $\tilde{s}_i = g_{BM} - k_{BM}\tilde{s}_c$ is:

$$\begin{aligned} &\ln\left(\frac{1}{c}\right)+\frac{\tau_f\bar{f}+\frac{1}{2}}{\tau_f+\tau_l}+\left(\frac{\tau_f+2\tau_l}{\tau_f+\tau_l}\right)\left(\frac{\tau_f\bar{f}+\tau_s\left(g_{BM}-k_{BM}\tilde{s}_c\right)+\tau_c\tilde{s}_c}{\tau_f+\tau_s+\tau_c}\right) \\ &+\left(\frac{\tau_f+2\tau_l}{\tau_f+\tau_l}\right)^2\frac{1}{2(\tau_f+\tau_s+\tau_c)}+\frac{1}{2}\left(\frac{\tau_l}{\tau_f+\tau_l}\right)^2\sigma_l^2 \\ =\ &\frac{1}{\alpha\sigma_s}\left(\frac{\tau_f\bar{f}+\tau_s\left(g_{BM}-k_{BM}\tilde{s}_c\right)+\tau_c\tilde{s}_c+\frac{1}{2}\frac{1}{\alpha\sigma_s}}{\tau_f+\tau_s+\tau_c}+k_{BM}\tilde{s}_c-g_{BM}\right)+\frac{1}{2\alpha^2}\sigma_\xi^2. \end{aligned} \tag{26}$$

Since the above equality must be satisfied for all $\tilde{s}_c$, we set the coefficient of $\tilde{s}_c$ to zero:

$$\left(\frac{\tau_f+2\tau_l}{\tau_f+\tau_l}\right)\left(\frac{1}{\tau_f+\tau_s+\tau_c}\right)(-k_{BM}\tau_s+\tau_c)=\left(\frac{1}{\alpha\sigma_s}\right)\left(\left(\frac{1}{\tau_f+\tau_s+\tau_c}\right)(-k_{BM}\tau_s+\tau_c)+k_{BM}\right). \tag{27}$$

This leads to the expression for k_{BM} in (21).

Now, we show that in the full model (with feedback effect) $k^*>\tau_c/\tau_s$ for α large enough. To see this note that $H(\tau_c/\tau_s)+J(\tau_c/\tau_s)>0$ for α large enough. Since $H(k)+J(k)$ has a unique root and crosses the axis from above the conclusion follows. Next, note that $k_{BM}<\tau_c/\tau_s$ and thus $k_{BM}<k^*$ for α large enough proving Proposition **2.** Finally, observe by inspecting (23) and (24) that $H(k)+J(k)$ shifts up as α increases, so its unique root k^* increases in α.

Proof of Proposition 4:

Consider the following terms involving $1/r$ in $H(k) + J(k)$:

$$-\frac{1}{r}\left(\tau_c k(\tau_c\tau_f + \tau_c\tau_l + \tau_f\tau_l + \tau_f\tau_s + 2\tau_l\tau_s + \tau_f^2) - \tau_c^2\tau_l\right)$$
$$+\left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)\frac{\tau_c}{r}\left((\tau_f + \tau_l)(\tau_c + \tau_f)k + \tau_c(\tau_f + \tau_l)\right)$$

For α large enough, these terms are negative iff k exceeds τ_c/τ_s. So for $k > \tau_c/\tau_s$, $H(k)+J(k)$ shifts up as r goes up. By Proposition 3, for α large enough, k^* which implicitly depends on r exceeds τ_c/τ_s for all r. Since $H(k) + J(k)$ crosses the axis once from above at k^*, we see that k^* must be increasing in r. Since increasing σ_ξ and r are inversely related, an increase in σ_ξ leads to a decrease in k^*. QED.

Proof of Proposition 5

Let

$$\begin{aligned}
D(k) = &-3\left((\tau_f + \tau_c + \tau_l)(\tau_f + \tau_c + \tau_s) + \tau_l\tau_s\right)k^2 - 2\tau_c(\tau_f + \tau_c - \tau_l + 2\tau_s)k \\
&+ \tau_c(\tau_c - \tau_s) - \frac{1}{r}\left(\tau_c(\tau_f\tau_c + \tau_f\tau_l + \tau_f\tau_s + \tau_c\tau_l + 2\tau_l\tau_s + \tau_f^2)\right) \\
&+ \left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)\left(3(\tau_f + \tau_c)(\tau_f + \tau_c + \tau_l)k^2 + 2\tau_c(3\tau_f + 3\tau_c + \tau_l)k + \tau_c(\tau_f + 3\tau_c)\right) \\
&+ \frac{\tau_c}{r}\left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)(\tau_f + \tau_c)(\tau_f + \tau_l)
\end{aligned}$$

Note $\partial(H+J)/\partial k = D(k)$. When the equilibrium is unique $D(k^*) < 0$ since the $H(k)+J(k)$ crosses zero from above.

Next we show that for α large $\frac{\partial k^*}{\partial \tau_s} < 0$. For α large,

$$\frac{\partial k^*}{\partial \tau_s} \approx \frac{\frac{1}{\tau_s^2\tau_\xi}\left(\tau_c^2\tau_f + 2\tau_c^2\tau_l + \tau_c\tau_s^2\tau_\xi k^* + 2\tau_c\tau_s^2\tau_\xi\,(k^*)^2 + (\tau_c\tau_s^2\tau_\xi + \tau_f\tau_s^2\tau_\xi + 2\tau_l\tau_s^2\tau_\xi)\,(k^*)^3\right)}{D(k^*)} < 0.$$

Next we show that for α large $\frac{\partial k^*}{\partial \tau_c} > 0$. For α large,

$$\frac{\partial k^*}{\partial \tau_c} \approx \frac{\begin{pmatrix} \tau_s\,(k^*)^3 - 2(2\tau_c + \tau_f + \tau_l - \tau_s)\,(k^*)^2 - (8\tau_c + \tau_f - \tau_s)k^* - 4\tau_c \\ -\frac{1}{r}\left(-\tau_s(\tau_f + 2\tau_l)k^* + 2\tau_c\tau_f + 4\tau_c\tau_l\right)\end{pmatrix}}{D\,(k^*)}.$$

Note that

$$\begin{aligned}
&\tau_c\Big(-\tau_s\,(k^*)^3 + 2\,(2\tau_c + \tau_f + \tau_l - \tau_s)\,(k^*)^2 + (8\tau_c + \tau_f - \tau_s)\,k^* + 4\tau_c \\
&+ \frac{1}{r}\left(-\tau_s\,(\tau_f + 2\tau_l)\,k^* + 2\tau_c\tau_f + 4\tau_c\tau_l\right)\Big) > -\tau_s\,(\tau_c + \tau_f + 2\tau_l)\,(k^*)^3 \\
&+ 2\tau_c\,(\tau_f + \tau_l - \tau_s + \tau_c)\,(k^*)^2 + \tau_c\,(\tau_f - \tau_s + 4\tau_c)\,k^* + 2\tau_c^2 \\
&+ \frac{\tau_c}{r}\left(\tau_c\tau_f + 2\tau_c\tau_l - \tau_s\,(\tau_f + 2\tau_l)\,k^*\right) \approx H(k^*) + J(k^*) = 0.
\end{aligned}$$

Next we show that for α large $\frac{\partial k^*}{\partial \tau_l} < 0$. For α large,

$$\frac{\partial k^*}{\partial \tau_l} \approx \frac{\frac{2}{r}(k^*\tau_s - \tau_c)(r\left(k^*\right)^2 + \tau_c)}{D\left(k^*\right)} < 0$$

since $k^* > \tau_c/\tau_s$ for α large. QED

Proof of Proposition 6:

We substitute optimal I into Equation (22) and compute the expectations:

$$\begin{aligned}
& \frac{1}{c}E\left[\exp\left(\tilde{f}\right)\exp\left(\frac{\tau_f\bar{f} + \tau_l s_l + \tau_p z\left(P\right)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)}\right)\right] \\
& -\frac{1}{2}\frac{1}{c}E\left[\exp 2\left(\frac{\tau_f\bar{f} + \tau_l s_l + \tau_p z\left(P\right)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)}\right)\right] \\
= \; & \frac{1}{c}E\left[\exp\left(2\tilde{f} + \frac{\tau_f\left(\bar{f} - \tilde{f}\right) + \tau_l\sigma_l\tilde{\epsilon}_l + \tau_p\left(z\left(P\right) - \tilde{f}\right)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)}\right)\right] \\
& -\frac{1}{2}\frac{1}{c}E\left[\exp 2\left(\tilde{f} + \frac{\tau_f\left(\bar{f} - \tilde{f}\right) + \tau_l\sigma_l\tilde{\epsilon}_l + \tau_p\left(z\left(P\right) - \tilde{f}\right)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)}\right)\right] \\
= \; & E\frac{1}{c}\left[\exp\left(\frac{\tau_f + 2\tau_l + 2\tau_p}{\tau_f + \tau_l + \tau_p}\tilde{f} + \frac{\tau_f\bar{f}}{\tau_f + \tau_l + \tau_p} + \frac{\tau_l\sigma_l\tilde{\epsilon}_l + \tau_p\left(z\left(P\right) - \tilde{f}\right)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)}\right)\right] \\
& -\frac{1}{2}\frac{1}{c}E\left[\exp 2\left(\tilde{f} + \frac{\tau_f\left(\bar{f} - \tilde{f}\right) + \tau_l\sigma_l\tilde{\epsilon}_l + \tau_p\left(z\left(P\right) - \tilde{f}\right)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)}\right)\right] \\
= \; & \frac{1}{c}\exp\left(2\bar{f} + \frac{1}{2(\tau_f + \tau_l + \tau_p)} + \frac{1}{2}\frac{\left(\tau_f + 2\tau_l + 2\tau_p\right)^2\frac{1}{\tau_f}}{\left(\tau_f + \tau_l + \tau_p\right)^2} + \frac{1}{2}\frac{\tau_l + \tau_p}{\left(\tau_f + \tau_l + \tau_p\right)^2}\right) \\
& -\frac{1}{c}\frac{1}{2}\exp\left(2\bar{f} + \frac{1}{(\tau_f + \tau_l + \tau_p)} + 2\left(\frac{\left(\tau_l + \tau_p\right)^2\frac{1}{\tau_f}}{\left(\tau_f + \tau_l + \tau_p\right)^2} + \frac{\tau_l + \tau_p}{\left(\tau_f + \tau_l + \tau_p\right)^2}\right)\right) \\
= \; & \frac{1}{c}\exp\left(2\bar{f} + \frac{1}{\tau_f}\frac{\tau_f + 2\tau_l + 2\tau_p}{\tau_f + \tau_l + \tau_p}\right) - \frac{1}{c}\frac{1}{2}\exp\left(2\bar{f} + \frac{1}{\tau_f}\frac{\tau_f + 2\tau_l + 2\tau_p}{\tau_f + \tau_l + \tau_p}\right) \\
= \; & \frac{1}{c}\frac{1}{2}\exp\left(2\bar{f} + \frac{1}{\tau_f}\frac{\tau_f + 2\tau_l + 2\tau_p}{\tau_f + \tau_l + \tau_p}\right).
\end{aligned}$$

Therefore the maximization problem can be viewed as maximizing the following expression in k :

$$\exp\left(\frac{\tau_f + 2\tau_l + 2\tau_p}{\tau_f + \tau_l + \tau_p}\right),$$

and this is equivalent to maximizing τ_p which is maximized at $\tau_c/\tau_s\tau_\xi$. QED.

Proof of Proposition 7:

For α large enough $H(k)+J(k)$ evaluated at $k_{OP}=\tau_c/r$ is approximately:

$$\frac{\tau_c^2}{r^3}\left(\tau_c+r\right)\left(2\tau_c r-\tau_c\tau_s+2\tau_f r-\tau_f\tau_s+2\tau_l r-2\tau_l\tau_s-r\tau_s+2r^2\right)$$

which is negative for r small. Moreover it may be decreasing in r for r small but eventually increases and becomes positive. This means that there is a cutoff $\bar{r}$ for r such that for $r<\bar{r}$ we have $k^*<k_{OP}$ and for $r>\bar{r}$ we have $k^*>k_{OP}$. QED.

Proof of Proposition 8: (a) The market clearing price is

$$P=\exp\left(\frac{\left(1+k\right)\tilde{f}+k\sigma_c\tilde{\epsilon}_c-g+\tilde{\xi}}{\alpha\sigma_s}\right),$$

and its excess volatility is defined as non-fundamental volatility which can be written as the volatility of the following:

$$z(P)-\tilde{f}=\frac{g+\alpha\sigma_s\ln(P)}{1+k}-\tilde{f}=\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{\sigma_s}{1+k}\tilde{\xi}.$$

It is straightforward to show that when $k=k_{OP}=\tau_c/\tau_s\tau_\xi$, its excess volatility is the lowest and is

$$\text{Excess Volatility}\left(\text{Asset Price}\right)=\frac{1}{\tau_c+\tau_s\tau_\xi}.$$

The rest of the statement follows immediately. QED.

(b) We know that:

$$I=\frac{1}{c}\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p\left(\tilde{f}+\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{\sigma_s}{1+k}\tilde{\xi}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right).$$

Take logs on both sides, we obtain:

$$\ln I=\ln\left(\frac{1}{c}\right)+\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p\left(\tilde{f}+\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{\sigma_s}{1+k}\tilde{\xi}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right).$$

We can define the excess volatility of the real investment as the volatility of the following:

$$\frac{\left(\tau_f+\tau_l+\tau_p\right)\left(\ln I-\ln\left(\frac{1}{c}\right)\right)-\frac{1}{2}-\tau_f\bar{f}}{\tau_l+\tau_p}-\tilde{f}=\frac{\tau_l\sigma_l\epsilon_l+\tau_p\left(\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{\sigma_s}{1+k}\tilde{\xi}\right)}{\tau_l+\tau_p}$$

It is straightforward to show that when $k=k_{OP}=\tau_c/\tau_\xi$, $\tau_p=\tau_c+\tau_s\tau_\xi$,and the excess volatility of the real investment is the lowest which is

$$\text{Excess Volatility}\left(\text{Real Investment}\right)=\frac{1}{\tau_l+\tau_c+\tau_s\tau_\xi}.$$

The rest of the statement follows immediately. QED.