Time horizon and the discount rate

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[1]We are grateful to Claude Henry, Jean-Pierre Florens, Jennifer Gann, Jean-Jacques Laffont, Pierre Lasserre, Jean-Charles Rochet and Patrick Roger for their helpful comments. Financial support from the European Commission is gratefully acknowledged.

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Abstract

We consider an economy à la Lucas [12] with a risk-averse representative agent. The exogenous growth rate of the economy follows a random walk. We characterize the set of utility functions for which it is efficient to discount more distant cash flows at a lower rate. The benchmark result is that, when the growth rate is almost surely nonnegative, the yield curve is decreasing if and only if relative risk aversion is decreasing with wealth. Relaxing the assumption on the absence of recession requires more restrictions on preferences, as increasing relative prudence.

Keywords: Discounting, uncertain growth, prudence, time horizon.

JEL Classification: D81, D91, Q25, Q28.

1 Introduction

The objective of this paper is to examine the relationship that exists between the socially efficient discount rate and the time horizon. We consider a simple economy à la Lucas [12] with a representative risk-averse agent facing an exogenous and uncertain growth of her consumption over time. This paternalistic agent wants to maximize the net present value of the flow of future expected utility. We characterize the set of utility functions for which it is optimal for the representative agent to discount more distant cash flows at a lower rate. This question has not been addressed in the literature so far, except by Cox, Ingersoll and Ross [2,3] in the special case of power utility functions, in which case the socially efficient discount rate is independent of the time horizon.

This paper is motivated by the difficulty in using the standard cost-benefit analysis with a constant discount rate for public investment projects whose costs and benefits are generated over a long period of time, as is the case for projects related to mitigating global warming, or for the management of nuclear wastes. Discounting far distant costs and benefits at the same rate as for the shorter terms is equivalent to ignoring these long term effects. Using discount rates that are decreasing with the time horizon would reduce the exponential effect of discounting.

Before examining the effect of time on the socially efficient discount rate, it is useful to recall the determinants of the level of the efficient discount rate. With a sure positive growth of the economy, we don't want to benefit overmuch future generations which will enjoy a larger GNP per capita. Under decreasing marginal utility of consumption, one more unit of consumption in the future is less valuable than one more unit of consumption today. This wealth effect is the standard argument for using a positive discount rate. But the growth of the economy is affected by random shocks which should be taken into account in the selection of the discount rate. The effect of a future risk on the willingness to invest for the future is well-known since Leland [11] and Drèze and Modigliani [4]: if people are prudent, the uncertainty about the growth of incomes should induce them to invest more for the future. This precautionary effect provides an argument to reduce the discount rate. This effect increases with the degree of prudence, an index introduced by Kimball [9] to measure the propensity to accumulate savings in the face of future risk.

We want to determine how the wealth effect interacts with the precautionary effect when the time horizon is expanded. We first examine the case of an economy facing no risk of recession. In that case, we show that the socially efficient discount rate is decreasing with the time horizon if and only if relative risk aversion is decreasing. The case of an economy with a risk of a recession is more complex. In such a situation, there are two conflicting effects of a longer time horizon on the valuation of future cash flows. When the expected growth of the economy is positive, a longer time horizon raises the expected consumption and the risk affecting this consumption. Thus, a longer time horizon yields a positive wealth effect and a negative precautionary effect on the socially efficient discount rate. If the precautionary effect dominates the wealth effect, one should recommend to select a smaller discount rate for longer horizons. In this paper, we provide conditions for this dominance to hold. Without surprise, our conditions depend upon properties of the index of prudence. In spite of this simple intuition, the reader should not expect to get simple necessary and sufficient conditions. The basic problem is to determine how temporal growth risks interact with each other. The recent literature on multiple risks, as initiated by Pratt and Zeckhauser [14], clearly shows that these interactions are complex, and that conditions on the third and fourth derivatives of the utility function are necessary to yield unambiguous comparative statics results.

This work is related to a recent paper by Weitzman [16] who also proves that the discount rate should be decreasing with the time horizon. Weitzman's conclusion is obtained in a different framework, with risk neutral agents together with a simple early revelation of future uncertain productivity of capital.

2 Description of the economy

There is a paternalistic representative consumer who maximizes the sum of future expected utility discounted at rate $\delta = \beta^{-1} - 1$. Parameter δ measures the rate of pure preference for the present. It must be constant over time to guarantee the time consistency of the decision process. The utility function u on consumption is assumed to be three times continuously differentiable, increasing and concave. Let z_t denote the level of consumption per capita at date t. The growth rate of z_t is uncertain. We assume that the growth rate follows a random walk. Let $\widetilde{x}_t$ denote the random growth rate of consumption in period t.

Let us consider an investment that costs one unit of consumption per capita at date 0. The investment yields a unique sure cash flow C_n at date n. Its gross rate of return per period equals $y_n = \sqrt[n]{C_n}$. The socially efficient discount rate corresponding to maturity n is the critical rate of return y_n that leaves the expected discounted utility of the representative agent unchanged. For a marginal investment project, it is given by

$$(y_n(z))^n \beta^n E\left[u'\left(\widetilde{z}_n\right) \mid \widetilde{z}_0 = z\right] = u'(z), \tag{1}$$

where $z = z_0$ is the consumption per capita at date 0. The pricing formula is obtained by rewriting condition (1) as

$$(y_n(z))^n = \frac{u'(z)}{\beta^n E u'\left[z \prod_{i=1}^n \widetilde{x}_i\right]}. \tag{2}$$

Notice that y_n would be the equilibrium interest rate for maturity n in competitive and frictionless markets for credit.

The yield curve is a plot of the term structure, that is, a plot of y_n against n. The most cited references on the yield curve are Vasicek [15] and Cox, Ingersoll, and Ross [2,3]. These authors show that the yield curve depends in a complex way upon the attitude towards risk and time, and upon the statistical relationships that may exist in the per period growth rates of the economy. This literature does not provide any result on whether the yield curve should be increasing or decreasing, except in the case of constant relative risk aversion. Under this condition, the yield curve is completely flat when shocks are multiplicative and i.i.d..[1] This is easily checked from condition (2). If $u'(z) = z^{-\gamma}$ for some constant relative risk aversion γ, this condition implies that

$$(y_n(z))^n = \frac{z^{-\gamma}}{\beta^n z^{-\gamma} E\left[\prod_{i=1}^n \widetilde{x}_i\right]^{-\gamma}} = \left[\frac{1}{\beta E \widetilde{x}_1^{-\gamma}}\right]^n = (y_1(z))^n.$$

However, the assumption that relative risk aversion be constant is questionable. For example, Ogaki and Zhang [13] tested whether relative risk aversion is decreasing or increasing from various consumption data in developing countries. They obtained strong evidences that relative risk aversion is decreasing. Another argument in favor of DRRA is based on the observation[2] that the share of wealth invested in risky assets is increasing with wealth in most developed countries. This is possible only under decreasing relative risk aversion. Finally, Guiso and Paiella [8] rejected the hypothesis that preferences of Italian households have constant relative risk aversion by using a large data set about the certainty equivalence for lotteries.

3 The yield curve when there is no risk of recession

Our benchmark result when relative risk aversion is not constant relies on the unrealistic assumption that there is no risk of recession, i.e., that $\widetilde{x}_t$ is larger than unity almost surely. This assumption will be relaxed later on. Let $\rho(z)$ denote the short term interest rate that will prevail at date $t = 1$ if the consumption per capita is z. We have

$$\rho(z) = \frac{u'(z)}{\beta E u'(z\widetilde{x}_2)}. \tag{3}$$

The distribution of the future interest rate is useful in determining the current long-term interest rate. Indeed, we can rewrite condition (2) as

$$(y_2(z_0))^{-2} = (y_1(z_0))^{-1} E\left[\frac{u'(z_0\widetilde{x}_1)}{Eu'(z_0\widetilde{x}_1)}(\rho(z_0\widetilde{x}_1))^{-1}\right]. \tag{4}$$

From this equation, we see that the yield curve is decreasing if and only if

$$E\left[\frac{u'(z_0\widetilde{x}_1)}{Eu'(z_0\widetilde{x}_1)}(\rho(z_0\widetilde{x}_1))^{-1}\right] \geq (y_1(z_0))^{-1}. \tag{5}$$

Of course, if one anticipates a smaller short-term interest rate in the second period, for example because of $\widetilde{x}_2$ being dominated by $\widetilde{x}_1$ in the sense of first-order dominance, then the yield curve in the first period will be decreasing. On the contrary, we want to consider the case where $\widetilde{x}_1$ and $\widetilde{x}_2$ are somewhat similar. From now on, we make the following ceteris paribus assumption:

$$\rho(z_0) = y_1(z_0) \tag{6}$$

or

$$Eu'(z_0\widetilde{x}_2) = Eu'(z_0\widetilde{x}_1).$$

This means that, given the same consumption per capita z_0, the short-term interest rate would be the same in the two periods. This is the case for example when $\widetilde{x}_1$ and $\widetilde{x}_2$ are i.i.d..

Suppose that there is no risk of recession in the short run, i.e., that $\widetilde{x}_1$ is larger than unity almost surely. Then we would be done if the future short term rate $\rho(z_0 x_1)$ were decreasing in x_1. Indeed, this would imply that $\rho(z_0\widetilde{x}_1)$ would be smaller than $\rho(z_0) = y_1(z_0)$ almost surely. Condition (5) would then be satisfied.

Under which condition can we guarantee that $\rho(z)$ is decreasing? Differentiating ρ defined by equation (3), we obtain that

$$z\rho'(z) = \rho(z)\left[E\left[\frac{u'(z\widetilde{x}_2)}{Eu'(z\widetilde{x}_2)}R(z\widetilde{x}_2)\right] - R(z)\right], \tag{7}$$

where $R(z) = -zu''(z)/u'(z)$ is the index of relative risk aversion of the representative agent. This implies that

$$\rho'(z) \leq 0 \quad \Longleftrightarrow \quad E\left[\frac{u'(z\widetilde{x}_2)}{Eu'(z\widetilde{x}_2)}R(z\widetilde{x}_2)\right] \leq R(z). \tag{8}$$

Let us assume that $\widetilde{x}_2$ is also larger than unity almost surely, i.e., that there is no risk of recession in the long term. If we combine this assumption with decreasing relative risk aversion (DRRA), we have that $R(z\widetilde{x}_2)$ is smaller than $R(z)$ almost surely, for all z. Condition (7) then implies that $\rho(z)$ would be decreasing. Thus, if $\widetilde{x}_1$ and $\widetilde{x}_2$ are larger than unity almost surely, DRRA implies that the yield curve is decreasing. We can also prove that the same property holds if $\widetilde{x}_1$ and $\widetilde{x}_2$ are smaller than unity almost surely. In that case, ρ would be increasing, implying that $\rho(z_0\widetilde{x}_1)$ is smaller than $\rho(z_0) = y_1(z_0)$ with probability one. This concludes the proof of the following result.[3]

Proposition 1 *Consider an economy satisfying the stationarity condition (6). Suppose also that the growth rate of consumption per capita never changes sign. Then, the yield curve is decreasing (resp. increasing) if relative risk aversion is decreasing (resp. increasing).*

Decreasing relative risk aversion means that the share of wealth that one is ready to pay to eliminate a multiplicative risk is decreasing with wealth. In addition to this intuitively appealing property, DRRA is compatible with the observation that wealthier agents invest a larger share of their wealth in stocks. It is also compatible with the recent findings of Ogaki and Zhang [13]. Thus, the above Proposition provides an argument for a discount rate that is decreasing with the time horizon. Several functional forms are compatible with DRRA. For example, the classical HARA utility function $u(z) = (z - k)^{1-\gamma}/(1-\gamma)$ with a positive level of subsistence k is DRRA.

The problem with Proposition 1 is that DRRA is sufficient for a decreasing yield curve only if the growth is either positive with probability 1 or negative with probability 1. If $\widetilde{x}_1$ or $\widetilde{x}_2$ has a support containing 1, DRRA is not sufficient anymore. Let us illustrate this point by the following example, using One-Switch utility functions introduced by Bell [1]. Take $u'(z) = a + z^{-b}$ with $a > 0$ and $b > 0$. It yields $-zu''(z)/u'(z) = b\left[az^{-b}+1\right]^{-1}$, which is decreasing in z. In addition, take $a = b = 1$ together with a pair $(\widetilde{x}_1, \widetilde{x}_2)$ of i.i.d. variables that are distributed as $\widetilde{x}$, with $\widetilde{x} - 1 \sim (-50\%, 1/3; +100\%, 2/3)$. In such a situation, straightforward computations generate $y_1 = y_2 = 0$: the yield curve is flat in spite of DRRA! Thus, extending the analysis to economies with a risk of recession is possible only by restricting further the set of DRRA utility functions.

4 The yield curve with a risk of recession in the future

In this section, we maintain the assumption that there is no risk of recession in the short run: $\widetilde{x}_1$ is larger than unity with probability 1. But we relax the condition that $\widetilde{x}_2$ be larger than unity almost surely. We replace this condition by the condition that $\beta\rho(z)$ is larger than 1 for all z in the support of $z_0\widetilde{x}_1$. This means that the wealth effect of growth on the interest rate dominates the precautionary effect, i.e. that the equilibrium interest rate is larger than the rate of pure preference for the present. This assumption is realistic, as the risk on growth has historically been small with respect to its mean value over the century. It is obviously much weaker than the absence of risk of recession.

Because we continue to assume that $\widetilde{x}_1 \geq_{a.s.} 1$, it is still true that the yield curve will be decreasing if the future short term interest rate is decreasing with GDP, i.e., if

$$\frac{-Ez\widetilde{x}_2 u''(z\widetilde{x}_2)}{Eu'(z\widetilde{x}_2)} \leq \frac{-zu''(z)}{u'(z)}, \tag{9}$$

for all z in the support of $z_0\widetilde{x}_1$. We want condition (9) to hold whenever $\beta\rho(z) \geq 1$. This condition can be rewritten as

$$Eu'(z\widetilde{x}_2) \leq u'(z) \quad \Longrightarrow \quad \frac{-Ez\widetilde{x}_2 u''(z\widetilde{x}_2)}{Eu'(z\widetilde{x}_2)} \leq \frac{-zu''(z)}{u'(z)}. \tag{10}$$

We can rewrite the above condition in its additive form, with $U'(Z) = u'(\exp Z)$, $Z = \ln z_0$ and $X = \ln x_2$:

$$EU'(Z+\widetilde{X}) \leq U'(Z) \Longrightarrow \quad \frac{-EU''(Z+\widetilde{X})}{EU'(Z+\widetilde{X})} \leq \frac{-U''(Z)}{U'(Z)}. \tag{11}$$

In words, this condition means that any expected-marginal-utility decreasing risk reduces the degree of absolute risk aversion towards any other small independent risk. This question is linked to the literature on the interaction of independent risks whose principal papers are Pratt and Zeckhauser [14], Kimball [10] and Gollier and Pratt [6]. The closest work to this problem is that of Kimball [10] who examines the condition on preferences that guarantees that any expected-marginal-utility increasing risk increases the demand for any independent risky asset. Kimball [10] showed that this condition holds if and only if both absolute risk aversion and absolute prudence are decreasing. We obtain a similar result here. Absolute measures of concavity are replaced by the corresponding relative measures, because we focus on multiplicative risks.

Proposition 2 *Consider an economy satisfying the stationarity condition (6). Suppose also that there is no risk of recession in the short run and that the future short term interest rate will be almost surely larger than the rate of pure preference for the present. Then, the yield curve is decreasing if and only if relative risk aversion R is decreasing and relative prudence P is increasing, with $R(z) = -zu''(z)/u'(z)$ and $P(z) = -zu'''(z)/u''(z)$.*

Proof: By definition, we have

$$\frac{-zu''(z)}{u'(z)} = \frac{-U''(Z)}{U'(Z)} \quad \text{and} \quad \frac{-zu'''(z)}{u''(z)} = \frac{-U'''(Z)}{U''(Z)} + 1.$$

Decreasing relative risk aversion of u is equivalent to decreasing absolute risk aversion of U. The increasing relative prudence of u is equivalent to the increasing absolute prudence of U. Thus, we have to prove that property (11) holds for all Z and $\widetilde{X}$ if and only if U exhibits decreasing absolute risk aversion and increasing absolute prudence. Define the precautionary equivalent $\psi(Z)$ of risk $\widetilde{X}$ at wealth Z by $EU'(Z+\widetilde{X}) = U'(Z+\psi(Z))$. Because we assume that $EU'(Z+\widetilde{X})$ is smaller than $U'(Z)$, $\psi(Z)$ is positive. As shown by Kimball [9], ψ is decreasing in Z if the absolute prudence of U is increasing. We then easily obtain that

$$\frac{-EU''(Z+\widetilde{X})}{EU'(Z+\widetilde{X})} = (1+\psi'(Z))\frac{-U''(Z+\psi)}{U'(Z+\psi)} \le \frac{-U''(Z+\psi)}{U'(Z+\psi)} \le \frac{-U''(Z)}{U'(Z)}.$$

The first inequality comes from increasing prudence, whereas the second inequality comes from decreasing risk aversion. This proves the sufficiency part of the Proposition. The necessity is proven in a similar way by contradiction. ■

Under the same conditions on $\widetilde{x}_1$ and $\widetilde{x}_2$, the yield curve would be increasing under increasing relative risk aversion and decreasing relative prudence. The proof of this result is left to the reader. The bottom line is that allowing for a risk of recession in the long run requires more restrictions on the utility function to generate the same unambiguous comparative property of a longer time horizon. To our knowledge, this is the first time that increasing absolute prudence arises as a useful condition in the economics of uncertainty. The only available reference is Kimball [9,10] who justifies the assumption of decreasing absolute prudence. But decreasing absolute prudence is compatible with either increasing or decreasing relative prudence.

5 The yield curve when there is a risk of a recession in each period

When there is a chance of a recession also in the first period, we cannot rely anymore on whether the future short term interest rate is decreasing or increasing with GDP, as we did before. Our objective in this section is to show that the set of constraints on preferences that would guarantee a decreasing yield curve when there is a risk of recession in the short run becomes quite sophisticated. To do this, let us examine the special case where the equilibrium interest rate in period 1 is just equal to the rate of pure preference for the present, i.e., where $\beta y_1(z_0)$ equals unity. The yield curve would thus be decreasing if $\beta y_2(z_0)$ is smaller than unity. This condition is summarized as follows:

$$\left.\begin{array}{rcl} Eu'(z_0\widetilde{x}_1) & = & u'(z_0) \\ Eu'(z_0\widetilde{x}_2) & = & u'(z_0) \end{array}\right\} \quad \Longrightarrow \quad Eu'(z_0\widetilde{x}_1\widetilde{x}_2) \geq u'(z_0). \qquad (12)$$

The first condition to the left corresponds to our assumption that $\beta y_1(z_0) = 1$, whereas the second condition is the stationarity assumption that we have made throughout this paper. The condition to the right means that $\beta y_2(z_0)$ is less than unity. The interpretation of condition (12) in terms of saving behavior is simple. Suppose that the agent does not want to save when the uncertain growth per period of her income is either $\widetilde{x}_1$ or $\widetilde{x}_2$. Does this imply that she would save in the presence of growth risk $\widetilde{x}_1\widetilde{x}_2$? Condition (12) states that she would want to save more in this second situation.

Define function v in such a way that $v(z) = -u'(z)$ for all z. Under risk aversion ($u'' \leq 0$) and prudence ($u''' \geq 0$), function v is increasing and concave, i.e., it looks like a utility function. Next, we define function V as $V(Z) = v(\exp Z)$. We can then rewrite property (12) as follows:

$$\left.\begin{array}{rcl} EV(Z+\widetilde{X}_1) & = & V(z_0) \\ EV(Z+\widetilde{X}_2) & = & V(z_0) \end{array}\right\} \quad \Longrightarrow \quad EV(Z+\widetilde{X}_1+\widetilde{X}_2) \leq V(Z). \qquad (13)$$

In words, condition (13) means that two lotteries on which the agent with utility function V is indifferent when taken in isolation are jointly undesirable. In some sense, this is equivalent to saying that independent risks are not complements. Pratt and Zeckhauser [14] called this condition "properness". A proper utility function V is a function which satisfies condition (13) for any Z and any pair of random variables ($\widetilde{X}_1, \widetilde{X}_2$). They showed that this condition is satisfied for all functions V that are $HARA$, a set of functions that contains all power, logarithmic and exponential functions. Now, remember that $V(Z) = -u'(\exp Z)$. This implies, for example, that One-switch utility functions satisfy condition (12). Indeed, with $u'(z) = a + z^{-\gamma}$, we have that $V(Z) = -a + b \exp Z$, which belongs to the class of HARA functions.

More interestingly, Pratt and Zeckhauser [14] also showed that a necessary condition for V to be proper is that

$$\left[\frac{-V''(Z)}{V'(Z)}\right]'' \geq \left[\frac{-V''(Z)}{V'(Z)}\right]' \left[\frac{-V''(Z)}{V'(Z)}\right]. \tag{14}$$

In fact, Pratt and Zeckhauser showed that this condition, when combined to decreasing absolute risk aversion of V, is necessary and sufficient for condition (13) to hold when $\widetilde{X}_1$ and $\widetilde{X}_2$ are small risks. With $V(Z) = -u'(\exp Z)$, we obtain that

$$\frac{-V''(Z)}{V'(Z)} = P(\exp Z) - 1 \tag{15}$$

$$\left[\frac{-V''(Z)}{V'(Z)}\right]' = (\exp Z)P'(\exp Z) \tag{16}$$

$$\left[\frac{-V''(Z)}{V'(Z)}\right]'' = (\exp Z)P'(\exp Z) + (\exp Z)^2 P''(\exp Z). \tag{17}$$

This yields the following result.

Corollary 3 *Consider an economy satisfying the stationarity condition (6). A necessary condition for the yield curve to be nonincreasing is*

$$zP''(z) \geq P'(z)(P(z) - 2) \tag{18}$$

for all z, where $P(z) = -zu'''(z)/u''(z)$ is relative prudence. This condition combined with condition $P'(z) \leq 0$ for all z is necessary and sufficient when the risk on growth is small and the short term interest rate equals the rate of pure preference for the present.

Necessary condition (18) is sophisticated, as it requires conditions on the fifth derivative of the utility function. To our knowledge, it is the first example of a condition on the fifth derivative of the utility function that is necessary to yield an unambiguous comparative statics result. There is no hope that this condition could be tested in the near future.

6 Conclusion

Economists who have already confronted the cost-benefit analysis of actions having an impact to the welfare of future generations have felt the difficulty in discounting future benefits at a constant rate of return. In this paper, we have shown that it is potentially socially efficient to discount far distant cash flows at a decreasing rate. This is the case for example when there is no risk of recession if relative risk aversion is decreasing. This assumption on preferences is plausible, as it corresponds to the well-documented observation that wealthier people invest a larger share of their wealth in risky assets. We have also shown that the problem becomes more complex when a risk of recession is introduced. In that case, more conditions on preferences must be added to guarantee that the yield curve be decreasing.

FOOTNOTES

[1] Notice that Cox, Ingersoll and Ross [3] consider a different model with mean-reversion in the short-term interest rate. Restrictive conditions on the distribution of the growth of the economy are necessary to generate such a property for the short term rate.

[2] See for example Guiso, Jappelli and Terlizzese, [8].

[3] Gollier [5] proved this result by using properties of log-supermodular functions.

REFERENCES

1. D.E. Bell, One-switch utility functions and a measure of risk, *Management Science* **34** (1988), 1416-1424.

2. J. Cox, J. Ingersoll and S. Ross, A theory of the term structure of interest rates, *Econometrica* **53** (1985a), 385-403.

3. J. Cox, J. Ingersoll and S. Ross, An intertemporal general equilibrium model of asset prices, *Econometrica* **53** (1985b), 363-384.

4. J.H. Drèze and F. Modigliani, Consumption decisions under uncertainty, *Journal of Economic Theory* **5** (1972), 308-335.

5. C. Gollier, Discounting an uncertain future, *Journal of Public Economics* (2000), forthcoming.

6. C. Gollier and J.W. Pratt, Risk vulnerability and the tempering effect of background risk,*Econometrica* **64** (1996), 1109-1123.

7. L. Guiso, T. Jappelli and D. Terlizzese, Income risk, borrowing constraints, and portfolio choice, *American Economic Review* **86** (1996), 158-172.

8. L. Guiso and M. Paiella, Risk aversion, wealth and financial market imperfections, Discussion Paper (2000), University of Sassari, Italy.

9. M.S. Kimball, Precautionary saving in the small and in the large, *Econometrica* **58** (1990), 53-73.

10. M.S. Kimball, Standard Risk Aversion, *Econometrica* **61** (1993), 589-611.

11. H.E. Leland, Saving and uncertainty: The precautionary demand for saving, *Quarterly Journal of Economics* (1968), 465-473.

12. Lucas, Asset Prices in an Exchange Economy, *Econometrica* **46** (1978), 1429-1446.

13. M. Ogaki and Q. Zhang, Decreasing Relative Risk Aversion And Tests of Risk Sharing, *Econometrica* (2000), forthcoming.

14. J.W. Pratt and R.J. Zeckhauser, Proper risk aversion, *Econometrica*, **55** (1987) 143-154.

15. O. Vasicek, An equilibrium characterization of the term structure, *Journal of Financial Economics* **5** (1977) 177-188.

16. M.L. Weitzman, Why the far-distant future should be discounted at its lowest possible rate, *Journal of Environmental Economics and Management* **36** (1998), 201-208.