The Social Cost of Heterogeneous Information*

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Preliminary

Abstract

I study the welfare effects of a lack of common knowledge in a dynamic price-setting model with incomplete nominal adjustment due to incomplete, heterogeneous information. I identify two welfare effects of informational heterogeneity: First, it affects the dynamic adjustment of prices in response to aggregate shocks, and thereby the magnitude of aggregate consumption volatility. Second, informational heterogeneity leads to price dispersion across firms, which leads to an ex-post resource mis-allocation. There is a trade-off between the two: Better public information always improves the ex post resource allocation, but by acting as a focal point of beliefs, it may increase consumption and price volatility. Better private information reduces aggregate volatility, but may increase price dispersion. The aggregate welfare implications are driven by the latter: Better public information is always welfare-improving, but better private information may lead to welfare losses. Finally, I compare equilibrium strategies to the first-best use of information, and show that the equilibrium relies too heavily on private information, relative to the social optimum. This inefficiency is the result of a forecasting externality, whereby individual decision rules do not internalize their effect on the forecasting problem other firms face in equilibrium.

Keywords: Higher-Order Uncertainty, Incomplete Nominal Adjustment, Transparency

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1 Introduction

Will better information lead to better decision-making, and hence induce higher welfare? For single-agent decision problems, access to better information leads to more accurate forecasts, and is hence welfare-improving. However, many market environments are characterized by the interaction of a large number of agents who have access to potentially heterogeneous sources of information, yet ideally would like to coordinate their decisions. The social benefits and costs of changes in information provision then depend on how well the decentralized use of this information internalizes the social tradeoffs that result from the coordination of individual decisions. To the extent that macroeconomic policy is directly concerned with the effect of information disclosures on market outcomes, understanding these tradeoffs is of direct relevance for assessing the desirability of transparent information policies by policy makers and other public information sources.

In this paper, I study the costs and benefits of information provision and transparency within the context of a simple model of monopolistic price-setting under heterogeneous information. For such an environment, Woodford (2002) has recently argued that informational heterogeneity in combination with the strategic complementarity in price-setting under monopolistic competition may lead to persistent delays of price adjustment, even when changes in underlying fundamentals are precisely, albeit privately, observed. Hellwig (2002a) extends Woodford's analysis to discuss the effects of public information disclosures on price adjustment, and shows that while the provision of public information reduces adjustment delays, the complementarity also amplifies the response to public information, and may thereby induce excess sensitivity to noise in public signals. Because of the resulting excess volatility, Morris and Shin (2002 - henceforth MS) conclude that the provision of better public information may lead to welfare losses.

While all these papers capture important stylized features of imperfect competition and incomplete nominal adjustment, they are not fully micro-founded, and hence do not lend themselves easily to a rigorous discussion of the welfare effects of information provision. The main objective of this paper is to provide a framework for such an analysis. I construct a stochastic dynamic general equilibrium model with monopolistic competition along the lines of Blanchard and Kiyotaki (1987), but with incomplete nominal adjustment due to incomplete and heterogeneous information. The framework generates the main conclusions regarding adjustment dynamics that emerge from Woodford's and Hellwig's analysis; in addition, it provides a simple and tractable characterization of social welfare. I further identify and separate the welfare effects of informational heterogeneity from the effects of monopolistic price competition that is the source of the strategic interaction. Finally, to avoid that observable prices are fully informative of the underlying state, I consider an incomplete asset market structure, in which the number of contemporaneously observable signals is smaller than the number of underlying shocks. The model also controls for the allocative effects of this market incompleteness.

Two important conclusions emerge within this framework: First, I consider how changes in information provision affect equilibrium welfare. I show that better public information always improves welfare. Better private information, on the other hand, may be welfare-decreasing. Second, I compare the equilibrium use of information to the social optimum. I show that the equilibrium uses the available information inefficiently, and in particular relies too much on private information and too little on public information, as compared to the social optimum.

To understand these results, it is useful to identify the different welfare effects of informational heterogeneity. First, the provision of public and private information determines how much prices adjust to fundamental changes, and hence the aggregate volatility of consumption and prices. But informational heterogeneity also leads to an ex post mis-allocation of resources, which lowers output: different signal realizations induce some firms to set prices below, others above the average price. With decreasing returns to scale in the production technology, it follows that some firms who set low prices will produce at a higher marginal cost than other firms who set higher prices. Since information is the only source of heterogeneity among firms, this ex post mis-allocation of resources is its direct consequence, and its magnitude depends on the degree to which individual decisions are conditioned on private information.[1]

The interaction between aggregate volatility and the mis-allocation of resources leads to a trade-off for the welfare effects of heterogeneous information: As in MS, the provision of better public information may actually increase aggregate volatility, because public information "acts as a focal point for individual beliefs", inducing excessive coordination of individual decisions, and hence excess volatility in outcomes. However, it also crowds out the private signal, and thereby unambiguously reduces resource mis-allocation. Better private information, on the other hand, always reduces aggregate volatility, but may raise price heterogeneity, thus potentially increasing the resulting deadweight loss.[2] The overall welfare implications of changes in public or private

[1] This effect is ruled out in MS due to the zero-sum nature of the strategic interaction.

[2] The adverse effects of better public information on aggregate volatility and of better private information on heterogeneity arise only for sufficiently diffuse public or private signals: When public information becomes infinitely

information provision therefore appear to be ambiguous at first sight; however, this ambiguity is resolved by the first main result of the paper, which also reverses the welfare result of MS.

To understand the non-monotonic welfare effect of private information, it is useful to characterize the decentralized information optimum, i.e. the linear decision rule that makes socially optimal use of the available information. This leads to the second conclusion, that the equilibrium makes inefficient use of the available information. In particular, while the equilibrium discounts private information relative to its value in forecasting fundamentals, its equilibrium weight is still too high, compared to the decentralized information optimum.

Comparing individual incentives to condition prices on private information to the resulting social costs reveals a negative forecasting externality as the source of the equilibrium inefficiency and the excessive reliance on private information. In equilibrium, each agent faces a signal extraction problem not only about the underlying state of the world, but also about the other agents' actions. The more these actions are conditioned on private information, the more difficult they are to forecast, and the larger is the aggregate strategic uncertainty. This strategic uncertainty at the individual level mirrors the welfare loss from mis-allocation of resources at the social level. There is a wedge between the private and the social value of private information: When conditioning actions on private information, agents do not take into account that this makes them more difficult to forecast for other agents, i.e. equilibrium decisions do not internalize the individual contribution to aggregate strategic uncertainty. Thus, in equilibrium, there is excess reliance on private information, excess price heterogeneity, and too little aggregate volatility.

While this entire discussion is based on the strategic interaction, the implicit risk aversion of firms (due to decreasing returns to scale), and the existence of an information externality as three salient features of the environment, and hence its conclusions potentially more generally applicable, one final implication of the present model comes as a direct consequence of the monopolistic competition framework: the more competitive the market is, i.e. the higher the elasticity of substitution between goods, the more complementary pricing decisions become. Hence, while increased competition reduces monopoly rents, it also increases strategic risk, which increases the welfare costs of information heterogeneity. These costs are especially important, when the welfare costs of market power are small; thus with heterogeneous information, too much competition might even have negative welfare effects.

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precise, aggregate volatility necessarily goes to zero; similarly, when the precision of private information becomes infinite, price heterogeneity vanishes.

Related Literature: This paper contributes to the recent literature on dynamic adjustment with heterogeneous information and strategic complementarities. Woodford (2002) points out within a stylized example that heterogeneity in beliefs may lead to substantial adjustment delays, even when the underlying fundamentals are precisely observed. In solving his model, Woodford faces an infinite regress problem similar to Townsend (1983), which he solves by numerical approximation. Hellwig (2002a) extends Woodford's model to incorporate public information, showing that public information reduces adjustment delays, but generate excess sensitivity of prices and output to public information noise. Furthermore, that paper provides a simple, yet general structure in which to avoid the infinite regress problem.[3] All these papers focus on adjustment dynamics in a simple model that captures the main stylized features of monopolistic competition, but they do not provide a complete modelling of the underlying microfoundations; in contrast, the present paper provides a fully specified model that captures the main insights of nominal adjustment under heterogeneous information in a simple framework.[4]

The welfare analysis of this paper makes explicit the link between the static analysis in MS and dynamic adjustment literature discussed in the preceding paragraph. As I had discussed at some length in the earlier paper, the MS results arise naturally, if one combines Woodford's stylized price-adjustment model with a reduced form welfare function as in Barro and Gordon (1983). In contrast, this paper comes to the conclusion that such a reduced form welfare analysis misses important welfare effects of public and private information provision. The present results thus highlight the critical importance of proper microfoundations for a consistent welfare analysis. In independent work, Angeletos and Pavan (2004) also find that the results in MS may be overturned under natural alternative modelling assumptions. However, they consider an investment model with technological spill-overs, in which the excess volatility induced by public information is socially beneficial, because it improves the coordination of individual investment decisions; furthermore, their analysis does not separate the welfare effects of information heterogeneity and technological spill-overs. The connections and differences between the models and results is fully discussed in Appendix 2, where I analyze in detail the private and social costs and benefits that arise in each of the papers within the context of a static linear-quadratic model, and I discuss under what conditions Morris and

[3] Amato and Shin (2003) extend Woodford's solution techniques to the public information setting, and derive similar implications for the dynamic effects of public information.

[4] In current work in progress, I extend the technical results of Hellwig (2002a) to a very general class of linear interaction models, and I further discuss the underlying micro-foundations of information heterogeneity in dynamic price-adjustment models. Notes are available on request.

Shin's results remain valid, and when they are overturned. The discussion in the appendix further emphasizes the essential role of the underlying model structures in determining the social costs and benefits of better public and private information.

Finally, I point out the connection to results that are implicitly, if not explicitly, known from the global games literature. The central result of this literature states that in a binary action coordination game with locally multiple equilibria under common knowledge, the presence of small idiosyncratic noise in the observation of payoffs leads to the selection of a unique risk-dominant equilibrium (Carlsson and van Damme 1993, Morris and Shin 1998). Following up on this result, various authors (see for example Morris and Shin 2003 and 2004, and Hellwig 2002b) have considered the effects of public information and concluded that the provision of sufficiently precise public information may lead back to multiplicity, and in the limit as both signals become infinitely precise, the selection of the payoff-dominant equilibrium becomes feasible. In a way similar to the results of this paper, these equilibrium selection results thus suggest that the provision of private information reduces the scope for coordination on a payoff-dominant equilibrium, while the provision of public information facilitates such coordination.

Section 2 presents the model, defines the equilibrium, and derives a series of preliminary results. In section 3, I characterize the equilibrium, and discuss the effects of information heterogeneity for nominal adjustment. In section 4, I present the main welfare results of this paper. All proofs are collected in a first appendix. In a second appendix, I formally discuss the connection between the results of this paper, and those in MS, as well as Angeletos and Pavan (2004).

2 The Model

Apart from the specific assumptions regarding the information structure, I consider what is otherwise a standard model of incomplete nominal adjustment with monopolistic firms, along the lines of Blanchard and Kiyotaki (1987), with nominal prices being preset, conditional on available information, before markets open. Time is discrete and infinite. There is a measure 1 continuum of different intermediate goods, indexed by $i \in [0, 1]$, each produced by one monopolistic firm, also indexed by i, using labor as the unique input into production. There is a final consumption good, which a perfectly competitive final goods sector produces from the continuum of intermediates according to a CES technology with constant returns to scale. On the consumption side, there is an infinitely-lived representative household, with preferences defined over consumption of the final good and labor supply in each period. The household faces a Cash-in-Advance constraint, and has to finance the current period's consumption out of the current period's nominal balances. Timing in each period is as follows: at the beginning of the period, a nominal shock, which takes the form of a stochastic lump sum transfer to the representative household is realized. Each input-producing firm observes the shocks with noise, with one signal being private, and one signal being public. Based on the available signals, each intermediate goods producer sets the nominal price of his product. After intermediate goods prices are set, markets open. Intermediates are traded at the posted prices, and intermediate producers hire labor to satisfy the demand for their products at the posted prices. The wage rate and the final goods price adjust to clear the labor, goods and money markets.

2.1 Household preferences

The representative household's preferences over final good consumption and labor supply $\{C_{t+\tau}, n_{t+\tau}\}_{\tau=0}^{\infty}$ are given by

$$U_t = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} \left(\log C_{t+\tau} - n_{t+\tau} \right) \right] \tag{1}$$

where $\beta < 1$ denotes the discount rate, and $\mathbb{E}_t(\cdot)$ denotes the household's expectations as of date t. The household's objective is to maximize (1) subject to its sequence of flow budget constraints, for $\tau = 0, 1, ...$

$$P_{t+\tau}C_{t+\tau} + M^d_{t+\tau} = W_{t+\tau}n_{t+\tau} + M^d_{t+\tau-1} + T_{t+\tau} + \Pi_{t+\tau} \tag{2}$$

where $M^d_{t+\tau}$ denotes the household's demand for nominal balances, $P_{t+\tau}$ the price of the final consumption good, $W_{t+\tau}$ the nominal wage rate, $T_{t+\tau}$ a stochastic monetary transfer the household receives at the beginning of each period, and $\Pi_{t+\tau}$ the aggregate profits of the corporate sector, which are rebated to the household. Wage payments and corporate profits are transferred to the household at the end of each period and the consumption good has to be purchased with cash, so that in addition, the household has to satisfy a Cash-in-Advance constraint in each period and finance consumption purchases out of its nominal balances after receiving the monetary transfer; i.e. for $\tau = 0, 1, ...$

$$P_{t+\tau}C_{t+\tau} \leq M^d_{t+\tau-1} + T_{t+\tau} \tag{3}$$

The nominal money supply is stochastic, with the government making a lump sum transfer

$T_{t+\tau} = M^s_{t+\tau} - M^s_{t+\tau-1}$ to the representative household at the beginning of each period. For simplicity, consider the case, where $\log \mu_t$ is i.i.d. over time with mean 0, so that $m_t \equiv \log M^s_t$ follows a random walk:

$$m_t = m_{t-1} + \frac{1}{\sqrt{\tau_\mu}}\mu_t$$

where $\mu_t \sim \mathcal{N}(0,1)$ is i.i.d. over time, and τ_μ is a scaling parameter representing the shock's precision. The information about money supply shocks is private to the household, and in particular is not accessible to the input-producing firms. In each period, the household chooses final good consumption C_t, labor supply n_t, and money demand M^d_t to maximize (1), subject to the constraints (2) and (3). Finally, I assume that τ_μ satisfies

$$\gamma^{-1} \equiv \beta e^{\frac{1}{2\tau_\mu}} < 1 \tag{4}$$

As I will show below, this assumption guarantees that the Cash-in-Advance constraint is binding in every state and date.

2.2 Final Good Producers

A large number of final goods producers uses the intermediate goods to produce the final output according to a constant returns to scale technology, which is given by the CES aggregator

$$C_t = \left[\int_0^1 \left(c^i_t\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}. \tag{5}$$

Final goods producers maximize profits, taking as given the market prices of intermediate and final goods. Due to constant returns to scale, their profits will be zero in equilibrium, and for a total demand C_t of the final good by the household, a final goods price P_t, and input prices p^i_t, the demand for intermediate good i by the final good sector is given by

$$c^i_t = c\left(p^i_t\right) = C_t \left(\frac{p^i_t}{P_t}\right)^{-\theta}. \tag{6}$$

The final goods price P_t is given by the Dixit-Stiglitz aggregator

$$P_t = \left[\int_0^1 \left(p^i_t\right)^{1-\theta} di\right]^{\frac{1}{1-\theta}}. \tag{7}$$

To close the model, I need to specify the objective function and price-setting behavior for intermediate producers.

2.3 Intermediate good producers

Each intermediate good is produced by a single firm using labor as the only input into production, according to a technology with decreasing returns to scale. In order to produce y units of good i, firm i needs to hire $n(y)$ units of labor, where $n(y)$ is given by

$$n(y) = \frac{1}{\delta} y^{\delta}, \tag{8}$$

with $\delta > 1$.[5] In each period, input producers set prices, conditional on available information $\Im_t^i$. Specifically, I assume that at date t, each firm has access to a private signal about m_t, denoted x_t^i:

$$x_t^i = m_t + \frac{1}{\sqrt{\tau_\xi}} \xi_t^i,$$

where $\xi_t^i \sim \mathcal{N}(0,1)$ is i.i.d. over time and across the population, and is independent of μ_t. τ_ξ is a scalars representing the precision of the private signal. For notational purposes, I let $\Phi(\cdot \mid m_t)$ denote the normal cdf of the private signal distribution, conditional on a realization m_t. In addition, all firms have access to a public signal z_t,

$$z_t = m_t + \frac{1}{\sqrt{\tau_v}} v_t,$$

where $v_t \sim \mathcal{N}(0,1)$ is i.i.d. over time, and independent of μ_t and all ξ_t^i, and τ_v represents the precision of the public signal. Finally, I assume that m_{t-1} is commonly known at the beginning of date t.[6] The information set $\Im_t^i$ is then given by $\Im_t^i = \left\{ x_{t-s}^i, z_{t-s}, m_{t-s-1} \right\}_{s=0}^{\infty}$. Firm i's nominal profits π_t^i, as a function of its price p_t^i, are given by:

$$\pi_t^i = p_t^i c\left(p_t^i\right) - W_t n\left(c\left(p_t^i\right)\right), \tag{9}$$

where $c\left(p_t^i\right)$ denotes the demand firm i faces for its product, given by (6). From the firm's perspective, this demand will be stochastic in equilibrium.

With homogeneous information and a complete asset market, the firm's objective is determined simply by evaluating profits according to state-prices. Here, such an approach would lead to the added complication that, if available to the firms, these asset prices would fully and commonly reveal the underlying state; on the other hand, when markets are taken to be incomplete, the firm's

[5] Alternatively, the production function $y(n)$ is given by $y(n) = [\delta n]^{1/\delta}$

[6] Here, this will follow naturally from the assumption that firms observe the market-clearing wage rate W_t.

objective need not be unambiguously specified. To get around this issue, I assume that Arrow-Debreu prices are not available to firms, but instead each firm sets its price p_t^i to maximize *expected shareholder value*, which I will define below as $E\left(\frac{\pi_t^i}{W_t} \mid \Im_t^i\right)$. The idea behind this objective is that each firm is instructed to set prices so as to maximize the representative household's welfare, taking as given the other firms' equilibrium pricing behavior. Under complete information, this approach is equivalent to evaluating profits according to state-prices. Here, it provides an alternative that avoids the issue of complete information relevelation through fully observable asset prices.

Let $E_t^i(\cdot) = E\left(\cdot \mid \Im_t^i\right)$ denote the expectations operator, conditional on $\Im_t^i$. Then, firm i's objective is

$$\max_{p_t^i} E_t^i \left[\frac{p_t^i c\left(p_t^i\right)}{W_t} - \frac{1}{\delta}\left[c\left(p_t^i\right)\right]^\delta \right] \tag{10}$$

To aggregate prices and profits, I assume that the realized distribution of private signals across firms (conditional on fundamental realizations m_t) is given by the distribution of x_t^i, conditional on m_t, almost surely, which implies that the population average of the private signal, $\int x_t^i di$ equals m_t, almost surely.[7] If prices p_t^i are a measurable of private signals x_t^i, it then follows that the CES price index P_t is given by $P_t = \int \left(p_t^i\right)^{\frac{\theta-1}{\theta}} d\Phi\left(x_t^i \mid m_t\right)$, almost surely, and aggregate profits are given by $\int \frac{\pi_t^i}{W_t} di \equiv \frac{\Pi_t}{W_t} = \int \frac{\pi_t^i}{W_t} d\Phi\left(x_t^i \mid m_t\right)$, almost surely.

2.4 Equilibrium Definition

The entire aggregate history of states in period t is $h_t = \{z_{t-s}, m_{t-s}\}_{s=0}^{\infty}$, while the current state in period t is $s_t \equiv \{z_t, m_t, m_{t-1}\}$. Here I focus on stationary Markov equilibria, in which (i) intermediate good prices p_t^i are functions only of the firms' contemporaneous information sets $\mathcal{I}_t^i \equiv \left\{z_t, x_t^i, m_{t-1}\right\}$, and (ii) the representative household's equilibrium demand for the final good and nominal balances and its supply of labor, as well as the final good price and the nominal wage rate, are all functions only of the current state s_t. Let $\mathcal{S} \equiv \mathbb{R}^3$ denote the set of current state realizations and $\mathcal{J}$ the set of contemporaneous information sets. A stationary Markov equilibrium is then defined as follows:

Definition 1 *A symmetric, stationary Markov equilibrium is defined as a set of functions $C : \mathcal{S} \to \mathbb{R}$, $P : \mathcal{S} \to \mathbb{R}$, $M^d : \mathcal{S} \to \mathbb{R}$, $W : \mathcal{S} \to \mathbb{R}$, $n : \mathcal{S} \to \mathbb{R}$, and $p : \mathcal{J} \to \mathbb{R}$, such that:*

(i) $\left\{C(\cdot), M^d(\cdot), n(\cdot)\right\}$ maximize (1) subject to (2) and (3).

[7] see Judd 1985 for the Law of Large Numbers with a continuum of random variables

(ii) zero profits for final good producers: $P(\cdot)$ *is given by (7), where* $p_t^i = p\left(\mathcal{I}_t^i\right)$

(iii) $p(\cdot)$ *maximizes (10), where* $c(p; s_t)$ *is given by*

$$c(p; s_t) = C(s_t)\,[P(s_t)]^{\theta}\,p^{-\theta}$$

(iv) All markets clear.

The equilibrium definition imposes symmetry across intermediate good producers, i.e. all firms use an identical pricing rule $p(\cdot)$. The money-market clearing condition is

$$\log M_t^d(s_t) = m_t \tag{11}$$

whereas the labor market clearing requires

$$n(s_t) = \frac{1}{\delta}\,[C(s_t)]^{\delta}\,[P(s_t)]^{\theta\delta} \int \left[p\left(\mathcal{I}_t^i\right)\right]^{-\theta\delta} d\Phi\left(x_t^i \mid m_t\right) \tag{12}$$

In addition, the final goods market clears from the zero profit condition, and intermediate goods markets clear, since demand is met at preset prices. By Walras Law, (11) then implies (12), in a symmetric equilibrium.

2.5 Preliminary Results

To characterize the equilibrium, I procede in two steps: I first characterize optimal household behavior and ex post market-clearing, for a given realization of prices $\left\{p_t^i\right\}_{i\in[0,1]}$. I then use these results to characterize the optimal price-setting among the intermediate firms as the solution to a fixed point problem.

2.5.1 Household Problem

Lemma 1 characterizes the household's optimal behavior. Its proof is standard and deferred to the appendix:

Lemma 1 *Under condition (4), the Cash-in-Advance constraint is always binding, and the household's optimal consumption in equilibrium is given by:*

$$C(s_t) = \frac{M_t^s}{P_t} \tag{13}$$

while the equilibrium wage rate satisfies

$$W(s_t) = \gamma M_t^s \tag{14}$$

Substituting the resulting budget constraint into the household's objective, we find:

$$\mathbb{E}_{t-1}\left(U_t\right) = \mathbb{E}_{t-1}\sum_{s=t}^{\infty}\beta^s\left[\log\left(\frac{M_{t+s}^s}{P_{t+s}}\right) - \frac{1}{\gamma}\left(1 - \frac{\Pi_{t+s}}{M_{t+s}^s}\right)\right] \tag{15}$$

As a consequence of (15), the representative household's expected utility is maximized when intermediate good producers maximize $E_t^i\left(\frac{\pi_t^i}{W_t}\right)$, or equivalently $E_t^i\left(\frac{\pi_t^i}{M_{t+s}}\right)$. This justifies the earlier definition of shareholder value for firms.

2.5.2 Input producers

After substituting (6), (13) and (14) into (10), the intermediate firms' maximization problem is given by:

$$\max_{p_t^i} E_t^i\left[\left(p_t^i\right)^{1-\theta} P_t^{\theta-1} - \frac{\gamma}{\delta}\left(p_t^i\right)^{-\theta\delta}\left(M_t^s\right)^{\delta} P_t^{\delta(\theta-1)}\right] \tag{16}$$

The corresponding first-order condition for p_t^i is

$$\left(p_t^i\right)^{1+\theta\delta-\theta} = \frac{\gamma\theta}{\theta-1}\frac{E_t^i\left[\left(M_t^s\right)^{\delta} P_t^{\delta(\theta-1)}\right]}{E_t^i\left[P_t^{\theta-1}\right]}. \tag{17}$$

Conjecture that $\log P_t$ and m_t, conditional on $\mathcal{I}_t^i$, will be jointly normally distributed in equilibrium. (17) can then be rewritten as:

$$\begin{aligned}\log p_t^i &= (1-r)\left[\frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{1}{2\delta}\left(V_1 - V_2\right)\right] \\ &\quad + (1-r)\,E_t^i\left(m_t\right) + rE_t^i\left(\log P_t\right)\end{aligned} \tag{18}$$

where

$$\begin{aligned} r &\equiv \frac{(\theta-1)(\delta-1)}{1+\theta\delta-\theta} \\ V_1 &\equiv \delta^2 V_t^i\left[\log\left(M_t^s P_t^{\theta-1}\right)\right] \\ V_2 &\equiv V_t^i\left[\log\left(P_t^{\theta-1}\right)\right]\end{aligned}$$

$r \in (0,1)$ denotes the degree of strategic complementarities in price-setting. (18) captures the strategic interaction that results from monopolistic price competition: a firm's optimal pricing rule is an increasing function of its expectation about the average price in the market. The novel aspect of the model presented here lies in the assumption that firms are *heterogeneous* in their information. This implies a need not only to form expectations about nominal spending, but also about the pricing decisions of other firms.

Let Σ_p^2 denote the conjectured normal variance of prices across the population of firms, and note that $\log P_t = \overline{\log p_t} - \frac{\theta-1}{2}\Sigma_p^2$, where $\overline{\log p_t} \equiv \int \log p_t^i di = \int \log p_t^i d\Phi\left(x_t^i \mid m_t\right)$, almost surely. We can then rewrite (18) as

$$\begin{aligned} \log p_t^i &= (1-r)\left[\frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{1}{2\delta}\left(V_1 - V_2\right)\right] - r\frac{\theta-1}{2}\Sigma_p^2 \\ &\quad + (1-r)E_t^i\left(m_t\right) + rE_t^i\left(\overline{\log p_t}\right). \end{aligned} \tag{19}$$

3 Equilibrium characterization

Equation (19) implicitly defines the equilibrium pricing rule as the solution to a fixed point problem. The resulting equilibrium characterization is provided in proposition 1.

Proposition 1 *In the unique equilibrium in linear strategies, firms set prices according to:*

$$\log p_t^i = \Gamma_0 + m_{t-1} + \frac{\tau_\xi(1-r)}{\tau_\mu + \tau_v + (1-r)\tau_\xi}\left(x_t^i - m_{t-1}\right) + \frac{\tau_v}{\tau_\mu + \tau_v + (1-r)\tau_\xi}\left(z_t - m_{t-1}\right) \tag{20}$$

and P_t and C_t satisfy:

$$\log P_t = \Gamma + m_{t-1} + \frac{\tau_v + \tau_\xi(1-r)}{\tau_\mu + \tau_v + (1-r)\tau_\xi}\frac{\mu_t}{\sqrt{\tau_\mu}} + \frac{\tau_v}{\tau_\mu + \tau_v + (1-r)\tau_\xi}\frac{v_t}{\sqrt{\tau_v}} \tag{21}$$

$$\log C_t = -\Gamma + \frac{\tau_\mu}{\tau_\mu + \tau_v + (1-r)\tau_\xi}\frac{\mu_t}{\sqrt{\tau_\mu}} - \frac{\tau_v}{\tau_\mu + \tau_v + (1-r)\tau_\xi}\frac{v_t}{\sqrt{\tau_v}} \tag{22}$$

where Γ_0 and Γ, as functions of the undetermined coefficients V_1, V_2 and Σ_p^2, are given by:

$$\begin{aligned} \Gamma &= \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{1}{2\delta}\left(V_1 - V_2\right) - \frac{\theta-1}{2(1-r)}\Sigma_p^2 \\ \Gamma_0 &= \Gamma + \frac{\theta-1}{2}\Sigma_p^2 \end{aligned} \tag{23}$$

To complete the equilibrium characterization, I solve for the equilibrium values of the coefficients V_1, V_2 and Σ_p^2, and consequently of Γ:

Corollary 1 *V_1, V_2, Σ_p^2 and Γ are given by:*

$$\begin{aligned}
V_1 &= \delta^2 \frac{\tau_\mu + \tau_v + \tau_\xi}{\left[\tau_\mu + \tau_v + (1-r)\,\tau_\xi\right]^2} \left[1 + \frac{\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi} \frac{r}{\delta - 1}\right]^2 \\
V_2 &= \delta^2 \frac{\tau_\mu + \tau_v + \tau_\xi}{\left[\tau_\mu + \tau_v + (1-r)\,\tau_\xi\right]^2} \left[\frac{\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi} \frac{r}{\delta - 1}\right]^2 \\
\Sigma_p^2 &= \frac{\tau_\xi \,(1-r)^2}{\left[\tau_\mu + \tau_v + (1-r)\,\tau_\xi\right]^2} \\
\Gamma &= \frac{1}{\delta} \log\left(\frac{\gamma\theta}{\theta - 1}\right) + \frac{\delta}{2} \frac{\tau_\mu + \tau_v + \theta\,(1-r)\,\tau_\xi}{\left[\tau_\mu + \tau_v + (1-r)\,\tau_\xi\right]^2}
\end{aligned}$$

Proposition 1 illustrates the two main sources of informational heterogeneity that were laid out in the introduction: namely, the aggregate volatility effect operating through the effect of fundamental and monetary shocks on output, and the effect of informational heterogeneity on the expected level of prices and output, as measured by Γ. This term captures the deadweight loss in output that results from the mis-allocation of resources due to price dispersion. Proposition 2 below lays out in detail the comparative statics of the informational parameters τ_ξ, τ_v, and τ_μ for the deadweight output loss Γ, the overall dispersion of prices Σ_p^2, and the volatility of consumption, σ_C^2. As a function of τ_ξ, τ_v, and τ_μ, the latter is given by:

$$\sigma_C^2 = \frac{\tau_\mu}{\left[\tau_\mu + \tau_v + (1-r)\,\tau_\xi\right]^2} + \frac{\tau_v}{\left[\tau_\mu + \tau_v + (1-r)\,\tau_\xi\right]^2}. \tag{24}$$

The first term in (24) measures the sensitivity of consumption to monetary shocks, due to incomplete nominal adjustment. The second term measures the sensitivity of consumption and prices to public information.

Proposition 2 *Informational heterogeneity has the following implications for prices and consumption:*

1. ***Strategic discount on private information:*** *The response of individual price decisions to private signals are discounted (relative to its signal precision τ_ξ). The weight on public and the prior is amplified, relative to their information value about fundamentals.*

2. ***Dynamic Adjustment:*** *Holding $\tau_\xi + \tau_v$ constant, a shift in signal precision from public to private information reduces the adjustment of prices to fundamental shocks, and amplifies the response of prices to public signals.*

3. ***Sensitivity of consumption to monetary shocks:*** $\frac{\tau_\mu}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}$ *is decreasing in* τ_ξ *and* τ_v*, but non-monotonic in* τ_μ*:. Whenever* $\tau_\mu > \tau_\xi (1-r) + \tau_v$*,* $\frac{\tau_\mu}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}$ *is decreasing in* τ_μ*, otherwise it is increasing.*

4. ***Sensitivity of consumption to public information:*** $\frac{\tau_v}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}$ *is decreasing in* τ_ξ *and* τ_μ*, but non-monotonic in* τ_v*. Whenever* $\tau_v > \tau_\xi (1-r) + \tau_\mu$*,* $\frac{\tau_v}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}$ *is decreasing in* τ_v*, otherwise it is increasing.*

5. ***Consumption volatility:*** σ_C^2 *is decreasing in* τ_ξ*, but non-monotonic in* τ_v *and* τ_μ*. Whenever* $\tau_v + \tau_\mu > \tau_\xi (1-r)$*,* σ_C^2 *is decreasing in* τ_v *and* τ_μ*, otherwise it is increasing.*

6. ***Price Dispersion:*** Σ_p^2 *is decreasing in* τ_v *and* τ_μ*, but non-monotonic in* τ_ξ*. Whenever* $\tau_\xi (1-r) > \tau_v + \tau_\mu$*,* Σ_p^2 *is decreasing in* τ_ξ*, otherwise it is increasing.*

7. ***Deadweight output loss:*** Γ *is decreasing in* τ_v *and* τ_μ*, but non-monotonic in* τ_ξ*. Whenever* $\tau_\xi (1-r) > \frac{\theta-2}{\theta} (\tau_v + \tau_\mu)$*, is decreasing in* τ_ξ*, otherwise it is increasing.*

Proposition 2 outlines the aggregate effects of information heterogeneity for the dynamics of output and prices, and establishes the tradeoff between aggregate volatility σ_C^2 and the deadweight output loss Γ that was discussed in the introduction.

The first point follows directly from comparing the equilibrium pricing rule to the relative information weights in first-order expectations. As a consequence of the shift towards the prior and the public signal, we find that prices respond less to monetary shocks than they would if all information was common, so that monetary shocks have more important real effects. Furthermore, such a shift from public to private information may amplify the response of prices to noise in public information, and increase overall consumption volatility, as stated in the second point. The present model thus replicates the main insights of Woodford (2002) and Hellwig (2002a) in a substantially simplified version.

The third, fourth and fifth points lay out the effects of information heterogeneity for the volatility of aggregate consumption, and illustrate in particular the effect highlighted by Morris and Shin that aggregate volatility may be increasing in the precision of public information. Note that some heterogeneity in information is essential for this result to hold, i.e. it is necessary that $\tau_\xi > 0$.

The last two points of the proposition lay out the effect of information heterogeneity on the deadweight output loss (or equivalently, the expected price level), and related to that, the degree of heterogeneity in prices. Equation (23) decomposes Γ into three terms. The first term measures the mark-up in prices that is due to monopolistic mark-ups, as well as the expected inflation tax. This term arises identically under complete information. The second term measures the fundamental and strategic risk that firms face in setting their prices, i.e. $\frac{1}{2\delta}(V_1 - V_2)$. The value of this strategic risk premium is endogenous to equilibrium strategies, since uncertainty about average prices depends on how strongly firms condition their prices on private information. The last term of (23) accounts for a negative substitution bias that arises out of the price dispersion. The equilibrium value of Γ reveals that the strategic uncertainty premium dominates the negative substitution bias. The comparative statics of Γ with respect to τ_ξ, τ_v and τ_μ are driven by the former, and we observe that increasing the precision of private information may increase the expected price level, because of the deadweight output loss that is due to price heterogeneity.

Proof of Propositions 1 and 2: A standard approach to solve for an equilibrium is by conjecturing and verifying a linear pricing rule for $\log p_t^i$. Here, I provide an alternative route for characterizing the equilibrium pricing rule that highlights the role of higher-order expectations about fundamentals for the equilibrium pricing rule, and provides some insights into the effects of private and public information about fundamentals for dynamic adjustment of prices. This approach is laid out in the following two lemmas. The first is equivalent to results used in Woodford (2002), and MS, the second is a special case of the dynamic equilibrium characterization in Hellwig (2002a), and appears in similar form for the static game studied by Morris and Shin.

Lemma 2 *The unique fixed point solution of (19) is characterized by:*

$$\log p_t^i = \Gamma_0 + (1-r) \sum_{s=0}^{\infty} r^s E_t^i \left[\overline{E}_t^{(s)} (m_t) \right] \tag{25}$$

where $\overline{E}_t^{(s)}(m_t)$ *is recursively defined by:* $\overline{E}_t^{(0)}(m_t) = m_t$; $\overline{E}_t(m_t) = \int E_t^i(m_t)\, d\Phi\left(x_t^i \mid m_t\right)$; *and* $\overline{E}_t^{(s)}(m_t) = \overline{E}_t\left[\overline{E}_t^{(s-1)}(m_t)\right]$.

In words, the log price is represented as a weighted average of higher-order expectations; i.e. i's expectation of the average expectation of the average expectation... (repeat s times) ... of m_t. When all information is common, the law of iterated expectations implies that the higher-order expectations all collapse to the common first-order expectation, however, the law of iterated expectations does not apply to average expectations in the presence of information heterogeneity. Lemma 2 transforms the fixed point problem of forecasting equilibrium prices into one of determining a weighted average of higher-order expectations. Since this weighted average is uniquely defined from the information structure, the fixed point solution has to be unique. The second lemma computes the weighted average of these higher-order expectations:

Lemma 3 $(1-r)\sum_{s=0}^{\infty} r^s E_t^i\left[\overline{E}_t^{(s)}(m_t)\right]$ *is given by:*

$$(1-r)\sum_{s=0}^{\infty} r^s E_t^i\left[\overline{E}_t^{(s)}(m_t)\right] = m_{t-1} + \frac{(1-r)\tau_\xi}{\tau_\mu+\tau_v+(1-r)\tau_\xi}\left(x_t^i - m_{t-1}\right) + \frac{\tau_v}{\tau_\mu+\tau_v+(1-r)\tau_\xi}\left(z_t - m_{t-1}\right)$$

Lemma 3 illustrates the effect of heterogeneous information on higher-order expectations that underlied the previous detailed discussion of the equilibrium effects of heterogeneous information: The weights of the prior and the public signal increase in the higher-order expectations, relative to the first order, while the weight of the private signal is reduced. The intuition behind the shift in weights is transparent from the comparison between the inference problem underlying the first-order expectation, and that underlying the second-order expectations about m_t. These two are given by:

$$\begin{aligned}
E_t^i(m_t) - m_{t-1} &= \frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\left(x_t^i - m_{t-1}\right) + \frac{\tau_v}{\tau_\mu+\tau_v+\tau_\xi}\left(z_t - m_{t-1}\right) \\
E_t^i\left[\overline{E}_t(m_t)\right] - m_{t-1} &= \frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\left[E_t^i(m_t) - m_{t-1}\right] + \frac{\tau_v}{\tau_\mu+\tau_v+\tau_\xi}\left(z_t - m_{t-1}\right) \\
&= \left[1 + \frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right]\frac{\tau_v}{\tau_\mu+\tau_v+\tau_\xi}\left(z_t - m_{t-1}\right) \\
&\quad + \left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^2\left(x_t^i - m_{t-1}\right)
\end{aligned}$$

The public component of the information structure is doubly useful in forming second order expectations: On the one hand, the public component of the information set is perfectly forecastable, because it is shared by all firms. On the other hand, both signals are useful for forecasting m_t, and consequently also to forecast another firm's private signal. The private signal only serves to forecast another firm's private signal, and therefore it becomes comparatively less important in forecasting higher-order rather than first-order expectations.

4 Welfare Results

In this section, I establish the two main results of this paper regarding the welfare implications of heterogeneous information. In Theorem 1, I establish the comparative statics of the representative household's expected utility with respect to the informational parameters and argue that improving private information may lead to lower welfare. In Theorem 2, I compare the equilibrium to the socially optimal use of the available information, and argue that there is excess reliance on private information. I first derive a few preliminary results to facilitate the welfare analysis.

4.1 Preliminaries

To begin, consider the representative household's expected per period utility:

$$\mathbb{U} \equiv \mathbb{E}_{t-1}(u_t) = \mathbb{E}_{t-1}(\log C_t) - \mathbb{E}_{t-1}(n_t) \tag{26}$$

The next lemma determines $\mathbb{U}$ under the household's expectation that $\log C_t$ is normally distributed with mean $\mathbb{E}_{t-1}(\log C_t)$, and that furthermore, prices are normally distributed across firms, as a function of expected consumption $\mathbb{E}(\log C_t)$, the volatility of consumption σ_C^2 and the price dispersion Σ_p^2.

Lemma 4 *Suppose that the firms' prices are distributed according to* $\log p_t^i \sim \mathcal{N}\left(\overline{\log p_t}, \Sigma_p^2\right)$, *and* $\log C_t \sim \mathcal{N}\left(\mathbb{E}(\log C_t), \sigma_C^2\right)$. *Then,* $\mathbb{U}$ *is given by:*

$$\mathbb{U} = \mathbb{E}(\log C_t) - \frac{1}{\delta}\exp\left\{\delta\mathbb{E}(\log C_t) + \frac{\delta^2}{2}\left[\sigma_C^2 + \frac{\theta}{1-r}\Sigma_p^2\right]\right\} \tag{27}$$

For any linear pricing rule of the form

$$\log p_t^i = m_{t-1} + \Lambda_0 + \Lambda_1\left(x_t^i - m_{t-1}\right) + \Lambda_2(z_t - m_{t-1}) \tag{28}$$

that is stationary over time, and fully incorporates the past money supply into the price, $\mathbb{U}$ is constant over time, and $\mathbb{U}/(1-\beta)$ equals the representative household's expected life-time utility prior to knowing the current period's monetary shock. Furthermore, any linear pricing rule of the form of (28) will satisfy the assumptions of lemma 4. We then have the following lemma which characterizes the representative household's utility as a function of the parameters $\{\Lambda_0, \Lambda_1, \Lambda_2\}$ of the linear pricing rule:

Lemma 5 *Suppose that firms set prices according to (28). Then,* $\mathbb{E}(\log C_t)$, σ_C^2,Σ_p^2 *and* $\mathbb{U}$ *are given by:*

$$\begin{aligned}
\mathbb{E}(\log C_t) &= -\Lambda \equiv -\Lambda_0 + \Lambda_1^2 \frac{\theta-1}{2}\frac{1}{\tau_\xi} \\
\Sigma_p^2 &= \Lambda_1^2 \frac{1}{\tau_\xi} \\
\sigma_C^2 &= (1-\Lambda_1-\Lambda_2)^2 \frac{1}{\tau_\mu} + \Lambda_2^2 \frac{1}{\tau_v} \\
\mathbb{U} &= -\Lambda - \frac{1}{\delta}\exp\left\{-\delta\Lambda + \frac{\delta^2}{2}\left[(1-\Lambda_1-\Lambda_2)^2 \frac{1}{\tau_\mu} + \Lambda_2^2\frac{1}{\tau_v} + \Lambda_1^2 \frac{\theta}{1-r}\frac{1}{\tau_\xi}\right]\right\} \qquad (29)
\end{aligned}$$

4.2 Equilibrium welfare

Evaluating the equilibrium welfare using (29) and the equilibrium pricing coefficients

$$\begin{aligned}
\Lambda^{eq} &\equiv \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{\delta}{2}\frac{\tau_\mu+\tau_v+\theta(1-r)\tau_\xi}{[\tau_\mu+\tau_v+(1-r)\tau_\xi]^2} \\
\Lambda_1^{eq} &\equiv \frac{(1-r)\tau_\xi}{\tau_\mu+\tau_v+(1-r)\tau_\xi} \\
\Lambda_2^{eq} &\equiv \frac{\tau_v}{\tau_\mu+\tau_v+(1-r)\tau_\xi}
\end{aligned}$$

now delivers the first main result of this paper:

Theorem 1 *Equilibrium welfare is given by*

$$\begin{aligned}
\mathbb{U}^{eq} &= -\Gamma - \frac{1}{\delta}\frac{\theta-1}{\gamma\theta} \qquad (30) \\
&= \frac{1}{\delta}\left[\log\left(\frac{\theta-1}{\gamma\theta}\right) - \frac{\theta-1}{\gamma\theta}\right] - \frac{\delta}{2}\frac{\tau_\mu+\tau_v+\theta(1-r)\tau_\xi}{[\tau_\mu+\tau_v+(1-r)\tau_\xi]^2}
\end{aligned}$$

and is strictly increasing in τ_μ *and* τ_v*, but non-monotonic in* τ_ξ*:* $\partial\mathbb{U}^{eq}/\partial\tau_\xi > 0$*, if and only if* $\tau_\xi(1-r) > \frac{\theta-2}{\theta}(\tau_v+\tau_\mu)$*, and* $\partial\mathbb{U}^{eq}/\partial\tau_\xi \leq 0$ *otherwise*

Theorem 1 establishes the first main result of the paper, stating that equilibrium welfare is strictly increasing in the precision of public information, and non-monotonic in the precision of private information: initially decreasing, but increasing for τ_ξ sufficiently large. As can be seen from (30), the welfare considerations are dominated by the strategic risk premium, which is decreasing in the precision of public information, and non-monotonic in the precision of private information.

As can be seen in (30), the characterization of equilibrium welfare separates the social cost of the inflation tax and the market power from the social cost of incomplete and heterogeneous information. The first component of the RHS of (30) measures the inflation tax and the market power, this term would is maximized when $\frac{\theta-1}{\gamma\theta} = 1$, i.e. in the limiting case when $\theta \to \infty$ (infinite elasticity of substitution, or zero mark-ups) and $\gamma \to 1$ (eliminating the inflation tax, so that the Cash-in-Advance constraint is no longer binding). This first term would arise identically in a complete information model. The second component of the RHS of (30) measures the social cost of heterogeneous information, and the resulting deadweight loss of output.

The conclusion of Theorem 1 is diametrically opposite to the results obtained by MS. The qualitative effects of information provision on aggregate volatility is identical in the two papers, the difference in the conclusions, arises from the difference in the planner's preferences: In MS, the principal's preferences are given by a quadratic loss function of the deviation of the *average action* from a stochastic target; hence they do not take into account the coordination motive that the agents face. Alternatively, the heterogeneity in actions across agents bears no social cost. Since the coordination motive is relevant for the individual agents' decisions, this leads to a tension between the agent's and the planner's preferences. Consequently, in MS, the planner will always benefit from more precise private information, but may be worse off with better public information. The agents, however, always benefit from more precise public information, but may be worse off, if private information becomes more precise. In the present model, the planner's objective is aligned with the firms' objective, and welfare always increases with improved public information, but there may be welfare losses resulting from better private information.

It is possible to replicate the results of MS in a reduced-form version of the present model that evaluates social welfare according to a quadratic loss function in inflation and output, along the lines of Barro and Gordon (1983). The consideration of micro-foundations in the present model revelas that strategic uncertainty rather than the aggregate volatility effect highlighted by MS are the determinant force behind the welfare effects of improving the precision of public and private information. To the extent that both the aggregate volatility and the strategic uncertainty effect are relevant for applications, a carefully micro-founded model might therefore yield radically different conclusions from a reduced-form approach that only takes into consideration that average deviation from the target.

4.3 Decentralized Information Optimum

How efficiently is the available information used in equilibrium? To answer this question, I next derive the *Decentralized Information Optimum*, i.e. the linear decision rule that makes socially optimal use of the locally available information.[8] To derive this rule, I maximize (29) with respect to the coefficients Λ, Λ_1 and Λ_2 to find:

Proposition 3 *The decentralized information optimum is given by the linear pricing rule*

$$\log p_t^i = m_{t-1} + \Lambda_0^* + \Lambda_1^* \left(x_t^i - m_{t-1}\right) + \Lambda_2^* \left(z_t - m_{t-1}\right) \tag{31}$$

where

$$\begin{aligned}
\Lambda^* &= \frac{\delta/2}{\tau_\mu + \tau_v + \frac{1-r}{\theta}\tau_\xi} \\
\Lambda_0^* &= \Lambda^* - (\Lambda_1^*)^2 \frac{\theta - 1}{2} \frac{1}{\tau_\xi} \\
\Lambda_1^* &= \frac{\frac{1-r}{\theta}\tau_\xi}{\tau_\mu + \tau_v + \frac{1-r}{\theta}\tau_\xi} \\
\Lambda_2^* &= \frac{\tau_\mu}{\tau_\mu + \tau_v + \frac{1-r}{\theta}\tau_\xi}
\end{aligned}$$

The decentralized information optimum achieves a welfare level:

$$\mathbb{U}^* = -\frac{1}{\delta} - \frac{\delta/2}{\tau_\mu + \tau_v + \frac{1-r}{\theta}\tau_\xi} \tag{32}$$

We note that the decentralized information optimum differs from the equilibrium in two ways: first, the decentralized optimum corrects the price level for the positive mark-up that was due to monopoly power and the inflation tax. Second, the optimal rule also shifts the weight in pricing decisions from private towards public information. We immediately have the second theorem:

Theorem 2 *The equilibrium pricing rule puts too little weight on public information, and too much weight on private information, relative to a socially optimal use of the available information, i.e.*

$$\Lambda_1^{eq} \geq \Lambda_1^* \text{ and } \Lambda_2^{eq} \leq \Lambda_2^*$$

with strict inequalities, whenever $\tau_\xi > 0$.

[8] Restricting attention to linear decision rules is obviously not without loss of generality. Nevertheless, it provides a very simple first step towards understanding the difference between private and social costs and benefits of information use.

Thus, even though the equilibrium weight on private information is smaller than its Bayesian weight in forecasting first-order expectations, the market equilibrium puts too much weight on private information.

The equilibrium inefficiency that results with heterogeneous information is the consequence of an externality in the problem of forecasting the equilibrium actions of other firms: The more the firms condition their prices on private information, the larger is the aggregate degree of uncertainty about equilibrium prices, i.e. the harder it becomes for any individual firm to forecast the average price. The uncertainty about equilibrium prices is reflected in the firm's pricing equation in the constant term Λ, which incorporates the risk premium due to the strategic uncertainty that firms face. This constant depends on the equilibrium values of Λ_1^{eq} and Λ_2^{eq}, but is taken as given in the individual decisions about conditioning prices on private and public information. Thus, the private benefit of conditioning prices on private signals exceeds the social benefit, and hence the inefficient use of information in equilibrium.

This externality can be illustrated by comparing the first-order conditions of the social planner's problem to the coefficients of the linear equilibrium rule. The social planner's first-order conditions for Λ_1 and Λ_2 are given by:

$$\begin{aligned}(1-\Lambda_1-\Lambda_2)\frac{1}{\tau_\mu} &= \Lambda_2\frac{1}{\tau_v} \qquad \text{for } \Lambda_2\\ (1-\Lambda_1-\Lambda_2)\frac{1}{\tau_\mu} &= \Lambda_1\frac{\theta}{1-r}\frac{1}{\tau_\xi} \qquad \text{for } \Lambda_1.\end{aligned}$$

The equilibrium coefficients Λ_1^{eq} and Λ_2^{eq}, on the other hand, satisfy:

$$(1-\Lambda_1-\Lambda_2)\frac{1}{\tau_\mu} = \Lambda_2\frac{1}{\tau_v} = \Lambda_1\frac{1}{1-r}\frac{1}{\tau_\xi} < \Lambda_1\frac{\theta}{1-r}\frac{1}{\tau_\xi}.$$

Thus, while the equilibrium fully internalizes the effect of Λ_2 for aggregate volatility, it does not internalize the trade-off between aggregate volatility and cross-sectional price heterogeneity in Λ_1.

It is useful to contrast these efficiency results with the case where information is homogenous. Setting $\tau_\xi = 0$ implies that the equilibrium pricing rule becomes

$$\log p_t^i = \log P_t = \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{\delta/2}{\tau_v+\tau_\mu} + m_{t-1} + \frac{\tau_v}{\tau_v+\tau_\mu}(z_t - m_{t-1}). \tag{33}$$

The decentralized information optimum is

$$\log P_t = \frac{\delta/2}{\tau_v+\tau_\mu} + m_{t-1} + \frac{\tau_v}{\tau_v+\tau_\mu}(z_t - m_{t-1}) \tag{34}$$

Proposition 4 *In the absence of informational heterogeneity,*

1. *The equilibrium pricing rule responds to the public signal exactly according to its Bayesian weight* $\frac{\tau_v}{\tau_v+\tau_\mu}$.
2. *There is no strategic uncertainty: Equilibrium prices can be perfectly forecast, and the risk premium* $\frac{\delta/2}{\tau_v+\tau_\mu}$ *only takes into account the exogenous uncertainty about fundamentals.*
3. *The equilibrium makes socially efficient use of the available information, i.e.* $\Lambda_2^{eq} = \Lambda_2^*$.

Thus, under common information, prices make efficient use of the available information, and the only inefficiency in the market results from the firms' market power, and the inflation tax.

4.4 The welfare effects of monopolistic competition

Besides the results about the welfare effects of public and private information, the present model also allows us to revisit the welfare effects of monopolistic competition. In this class of models, monopolistic competition is typically parametrized by the elasticity of substitution between intermediates θ, or the resulting equilibrium mark-up. Under homogeneous information, one immediately arrives at the conclusion that raising the degree of competition lowers mark-ups, hence lowers monopoly wedges and raises welfare. In the present model, it has a second effect, in that the degree of competition influences the degree to which pricing decisions are complementary, i.e. the parameter r, and consequently the cost of informational heterogeneity. As can be readily seen from (30) and (32), this welfare cost of monopolistic competition is increasing in θ; thus more competition leads to a higher cost of informational heterogeneity. Furthermore, the welfare loss relative to the decentralized information optimum is increasing in θ. When information is very precise, but mostly private, this welfare cost can become arbitrarily large. To see this, note the limiting properties as $\theta \to \infty$ of (30) and (32):

$$\begin{aligned}\lim_{\theta\to\infty} \mathbb{U}^{eq} &= \frac{1}{\delta}\left[\log\left(\frac{1}{\gamma}\right) - \frac{1}{\gamma}\right] - \frac{\delta}{2}\frac{1}{\tau_\mu+\tau_v}\left[1 + \frac{\delta}{\delta-1}\frac{\tau_\xi}{\tau_\mu+\tau_v}\right] \\ \lim_{\theta\to\infty} \mathbb{U}^{*} &= -\frac{1}{\delta} - \frac{\delta}{2}\frac{1}{\tau_\mu+\tau_v}\end{aligned}$$

Thus, when evaluating the welfare effects of enhanced competition, the present model suggests that one needs to trade off the gains from reduced mark-ups against the losses due to increased impact of information heterogeneity. This leads to the conclusion that too much competition may be costly, when information is heterogeneous, and private information sufficiently precise.

5 Concluding Remarks

In this paper, I have studied the welfare effects of public and private information provision in a dynamic price-setting model with incomplete nominal adjustment due to incomplete, heterogeneous information, along the lines of Woodford (2002). I have shown that increased transparency, in the form of improved public information is always beneficial, while better private information may increase the heterogeneity in prices, and thus lead to welfare losses. I have also compared the equilibrium to the socially optimal use of information, and shown that equilibrium price-setting relies too heavily on private information. The source of this inefficiency is an information externality, whereby the firms do not internalize the full social cost of price heterogeneity for the allocation of resources.

While the paper studies a model of monopolistic price-setting, its main insights are the consequence of the strategic interaction, the implicit risk aversion of firms, and the negative forecasting externality. There is a large number of other applications which share the strategic complementarity and risk aversion as salient model features, such as investment models with technological spill-overs, agglomeration and demand externalities, or models of search and thick-market externalities. However, the paper has also highlighted the crucial role of the information externality for the resulting welfare analysis. Thus, in order to assess whether similar conclusions arise in different environments, one needs to assess whether similar (or other) externalities arise from the microfoundations of such models. In the appendix 2 below, I generalize Morris and Shin's reduced form model to discuss how the welfare implications of public and private information provision differ depending on the underlying preference structure, and I analyze the different implications of different externalities. Finally, a recent paper by Angeletos and Pavan (2004) studies the effects of transparent information provision in an investment model with technological spill-overs, and arrives at welfare conclusions similar to the ones presented here. I also explore the connection to Angeletos and Pavan's model, and show that their results can be traced to an externality due to aggregate volatility, whereby individual actions do not fully internalize the social benefits of aggregate investment.

Finally, although the model remains silent about the conduct of monetary policy, its results raise new questions in that direction. The traditional literature on the costs and benefits of different monetary regimes emphasizes the role of transparent information and policy decisions for monitoring purposes within a principal-agent framework with time-inconsistency in the absence of policy commitment. The present model suggests a role for monetary transparency even with full commitment, namely by influencing the market information structure, and thereby improving the coordination of price-setting and investment decisions among market participants. Two channels for policy interventions in the market appear to be worthwhile investigating: First, starting from the premise that a policy authority has some private information about fundamentals, the transmission of this information to market participants reduces the heterogeneity of beliefs among market participants therefore influencing adjustment dynamics and price heterogeneity.[9] Second, even in the absence of private information by a policy maker, the model identifies a forecasting externality in the processing of heterogeneous information. Thus, even without having additional private information, there might be a role for active policy intervention to influence the trade-offs in the use of public and private information. I leave an analysis of these questions to future work.

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6 Appendix 1: Proofs

Proof of lemma 1. Substituting the budget constraint, the household's optimization problem can be rewritten as:

$$\begin{aligned} U_t &= \max_{\{C_{t+\tau}, M^d_{t+\tau}\}_{\tau=0}^{\infty}} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \left(\log C_{t+\tau} - \frac{P_{t+\tau} C_{t+\tau}}{W_{t+\tau}} - \frac{M^d_{t+\tau} - M^d_{t+\tau-1} - T_{t+\tau}}{W_{t+\tau}} + \frac{\Pi_{t+\tau}}{W_{t+\tau}} \right) \right] \\ \text{s.t. } P_{t+\tau} C_{t+\tau} &\leq M^d_{t+\tau-1} + T_{t+\tau} \end{aligned}$$

Let $\theta_{t+\tau}$ denote the Lagrange multiplier on the period $t+\tau$ Cash-in-Advance Constraint. The resulting first-order conditions are:

$$\begin{aligned} \frac{1}{P_{t+\tau} C_{t+\tau}} &= \frac{1}{W_{t+\tau}} + \theta_{t+\tau} \\ \frac{1}{W_{t+\tau}} &= \beta \mathbb{E}_t \left(\frac{1}{W_{t+\tau+1}} + \theta_{t+\tau+1} \right) \end{aligned}$$

while the transversality condition is

$$\lim_{\tau \to \infty} \beta^\tau \mathbb{E}_t \left(\frac{M^d_{t+\tau}}{W_{t+\tau}} \right) = 0.$$

We check that the proposed solution $C_{t+\tau} = \frac{M^s_{t+\tau}}{P_{t+\tau}}$, $W_{t+\tau} = \gamma M^s_{t+\tau}$ and $M^d_{t+\tau} = M^s_{t+\tau}$ satisfies the first-order conditions and transversality condition. For this solution, we find that $\theta_{t+\tau} = \frac{1}{P_{t+\tau} C_{t+\tau}} - \frac{1}{W_{t+\tau}} = \frac{1}{M^s_{t+\tau}} \left(1 - \frac{1}{\gamma} \right) > 0$, so that the Cash-in-Advance constraint is binding in every period and in every state (justifying that $C_{t+\tau} = \frac{M^s_{t+\tau}}{P_{t+\tau}}$). Next, we check that the second first-order condition is satisfied. To see that this is the case, note that

$$\begin{aligned} \frac{1}{W_{t+\tau}} - \beta \mathbb{E}_t \left(\frac{1}{W_{t+\tau+1}} + \theta_{t+\tau+1} \right) &= \frac{1}{\gamma M^s_{t+\tau}} - \beta \gamma \mathbb{E}_t \left(\frac{1}{\gamma M^s_{t+\tau+1}} \right) \\ &= \frac{1}{\gamma M^s_{t+\tau}} \left[1 - \beta \gamma \mathbb{E}_t \left(\frac{M^s_{t+\tau}}{M^s_{t+\tau+1}} \right) \right] \\ &= \frac{1}{\gamma M^s_{t+\tau}} \left[1 - \beta \gamma e^{\frac{1}{2\tau_\mu}} \right] = 0 \end{aligned}$$

Thus, the two first-order conditions are satisfied. It is also immediate that the transversality condition holds, and that the money market clears. ■

Proof of lemma 2. Averaging equation (19) over the population, we find:

$$\overline{\log p_t} = (1-r)\,\Gamma_0 + (1-r)\,\overline{E}_t\,(m_t) + r\overline{E}_t\left(\overline{\log p_t}\right)$$

where

$$\Gamma_0 = \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{1}{2\delta}\left(V_1 - V_2\right) - \frac{r}{1-r}\frac{\theta-1}{2}\Sigma_p^2$$

Successively substituting forward, we find:

$$\begin{aligned}
\overline{\log p_t} &= (1-r)\,\Gamma_0 + (1-r)\,\overline{E}_t\,(m_t) + r\overline{E}_t\left(\overline{\log p_t}\right) \\
&= (1-r)\,(1+r)\,\Gamma_0 + (1-r)\,\overline{E}_t\,(m_t) + (1-r)\,r\overline{E}_t^{(2)}\,(m_t) + r^2\overline{E}_t^{(2)}\left(\overline{\log p_t}\right) \\
&= (1-r)\,\Gamma_0\sum_{s=0}^{k-1} r^s + (1-r)\sum_{s=0}^{k-1} r^s\overline{E}_t^{(s+1)}\,(m_t) + r^k\overline{E}_t^{(k)}\left(\overline{\log p_t}\right)
\end{aligned}$$

Taking the limit as $k \to \infty$:

$$\begin{aligned}
\overline{\log p_t} &= \Gamma_0 + (1-r)\sum_{s=0}^{\infty} r^s\overline{E}_t^{(s+1)}\,(m_t) \\
&\quad + \lim_{k\to\infty}\left[r^{k-1}\overline{E}_t^{(k)}\,(m_t) + r^k\overline{E}_t^{(k)}\left(\overline{\log p_t}\right)\right]
\end{aligned}$$

But as is shown in Theorem 1 in Samet (1998), $\lim_{k\to\infty}\overline{E}_t^{(k)}\left(\overline{\log p_t}\right) = E_t\left[\log p_t \mid z_t, m_{t-1}\right]$ and $\lim_{k\to\infty}\overline{E}_t^{(k)}\,(m_t) = E_t\left[m_t \mid z_t, m_{t-1}\right]$, so that $\lim_{k\to\infty} r^k\overline{E}_t^{(k)}\left(\overline{\log p_t}\right) = 0$ and $\lim_{k\to\infty} r^{k-1}\overline{E}_t^{(k)}\,(m_t) = 0$. ■

Proof of lemma 3. The first-order expectation of m_t is given by:

$$E_t^i\,(m_t) = m_{t-1} + \frac{\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi}\left(x_t^i - m_{t-1}\right) + \frac{\tau_v}{\tau_\mu + \tau_v + \tau_\xi}\left(z_t - m_{t-1}\right)$$

Averaging over i, we find the first-order average expectation:

$$\overline{E}_t\,(m_t) = m_{t-1} + \frac{\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi}\left(m_t - m_{t-1}\right) + \frac{\tau_v}{\tau_\mu + \tau_v + \tau_\xi}\left(z_t - m_{t-1}\right)$$

and the second-order expectation:

$$\begin{aligned}
E_t^i\left[\overline{E}_t\,(m_t)\right] &= m_{t-1} + \frac{\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi}\left[E_t^i\,(m_t) - m_{t-1}\right] + \frac{\tau_v}{\tau_\mu + \tau_v + \tau_\xi}\left(z_t - m_{t-1}\right) \\
&= m_{t-1} + \left(\frac{\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi}\right)^2\left(x_t^i - m_{t-1}\right) \\
&\quad + \left[1 + \frac{\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi}\right]\frac{\tau_v}{\tau_\mu + \tau_v + \tau_\xi}\left(z_t - m_{t-1}\right)
\end{aligned}$$

Successively averaging and substituting forward, we find:

$$\begin{aligned} E_t^i\left[\overline{E}_t^{(s)}(m_t)\right] &= m_{t-1} + \left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^{s+1}\left(x_t^i - m_{t-1}\right) \\ &\quad + \frac{\tau_v}{\tau_\mu+\tau_v+\tau_\xi}\frac{1-\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^{s+1}}{1-\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)}\left(z_t - m_{t-1}\right) \end{aligned}$$

And summing over s:

$$\begin{aligned} & (1-r)\sum_{s=0}^{\infty} r^s E_t^i\left[\overline{E}_t^{(s)}(m_t)\right] \\ = \;& m_{t-1} + (1-r)\sum_{s=0}^{\infty} r^s\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^{s+1}\left(x_t^i - m_{t-1}\right) \\ & + (1-r)\sum_{s=0}^{\infty} r^s\frac{1-\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^{s+1}}{1-\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)}\frac{\tau_v}{\tau_\mu+\tau_v+\tau_\xi}\left(z_t - m_{t-1}\right) \\ = \;& m_{t-1} + (1-r)\frac{\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}}{1-r\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}}\left(x_t^i - m_{t-1}\right) \\ & + \frac{1}{1-\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)}\left[1-(1-r)\frac{\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}}{1-r\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}}\right]\frac{\tau_v}{\tau_\mu+\tau_v+\tau_\xi}\left(z_t - m_{t-1}\right) \\ = \;& m_{t-1} + \frac{(1-r)\tau_\xi}{\tau_\mu+\tau_v+(1-r)\tau_\xi}\left(x_t^i - m_{t-1}\right) + \frac{\tau_v}{\tau_\mu+\tau_v+(1-r)\tau_\xi}\left(z_t - m_{t-1}\right) \end{aligned}$$

■

Proof of Proposition 1 and Corollary 1. I prove this proposition in three steps: The first two steps are given by lemmas 2 and 3 above. The final step consists in deriving the remaining undetermined coefficients. For the definition of Γ, note that from the definition of P_t,

$$(1-\theta)\log P_t = \log\int(1-\theta)\log p_t^i di = (1-\theta)\overline{\log p_t} + \frac{(1-\theta)^2}{2}\Sigma_p^2$$

and hence

$$\begin{aligned} \Gamma &= \Gamma_0 - \frac{\theta-1}{2}\Sigma_p^2 \\ &= \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{1}{2\delta}\left(V_1 - V_2\right) - \frac{1}{1-r}\frac{\theta-1}{2}\Sigma_p^2. \end{aligned}$$

Since the equilibrium pricing rule takes the form given by lemmas 2 and 3, it immediately follows that $\Sigma_p^2 = \frac{(1-r)^2 \tau_\xi}{[\tau_\mu + \tau_v + (1-r)\tau_\xi]^2}$. Using the fact that $(\theta - 1)(1-r) = \frac{r\delta}{\delta - 1}$, we have

$$\begin{aligned}
V_1 &= \delta^2 V_t^i \left[\log\left(M_t^s P_t^{\theta-1}\right)\right] = \frac{\delta^2}{\tau_\mu + \tau_v + \tau_\xi}\left[1 + \frac{(\theta-1)(1-r)\tau_\xi}{\tau_\mu + \tau_v + (1-r)\tau_\xi}\right]^2 \\
&= \frac{\delta^2}{\tau_\mu + \tau_v + \tau_\xi}\left[1 + \frac{\frac{r\delta}{\delta-1}\tau_\xi}{\tau_\mu + \tau_v + (1-r)\tau_\xi}\right]^2 \\
&= \delta^2 \frac{\tau_\mu + \tau_v + \tau_\xi}{[\tau_\mu + \tau_v + (1-r)\tau_\xi]^2}\left[\frac{\tau_\mu + \tau_v + (1-r)\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi} + \frac{\frac{r\delta}{\delta-1}\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi}\right]^2 \\
&= \delta^2 \frac{\tau_\mu + \tau_v + \tau_\xi}{[\tau_\mu + \tau_v + (1-r)\tau_\xi]^2}\left[1 + \frac{\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi}\frac{r}{\delta - 1}\right]^2 \\
V_2 &= V_t^i \left[\log\left(P_t^{\theta-1}\right)\right] = \frac{1}{\tau_\mu + \tau_v + \tau_\xi}\left[\frac{(\theta-1)(1-r)\tau_\xi}{\tau_\mu + \tau_v + (1-r)\tau_\xi}\right]^2 \\
&= \delta^2 \frac{\tau_\mu + \tau_v + \tau_\xi}{[\tau_\mu + \tau_v + (1-r)\tau_\xi]^2}\left[\frac{\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi}\frac{r}{\delta - 1}\right]^2
\end{aligned}$$

$$\begin{aligned}
\Gamma &= \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta - 1}\right) + \frac{1}{2\delta}(V_1 - V_2) - \frac{1}{1-r}\frac{\theta - 1}{2}\Sigma_p^2 \\
&= \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta - 1}\right) + \frac{\delta}{2}\frac{\tau_\mu + \tau_v + \tau_\xi}{[\tau_\mu + \tau_v + (1-r)\tau_\xi]^2}\left[1 + 2\frac{\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi}\frac{r}{\delta - 1}\right] \\
&\quad - \frac{\theta - 1}{2}\frac{(1-r)\tau_\xi}{[\tau_\mu + \tau_v + (1-r)\tau_\xi]^2} \\
&= \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta - 1}\right) + \frac{\delta}{2}\frac{\tau_\mu + \tau_v + \tau_\xi}{[\tau_\mu + \tau_v + (1-r)\tau_\xi]^2}\left[1 + \frac{\tau_\xi}{\tau_\mu + \tau_v + \tau_\xi}\frac{r}{\delta - 1}\right] \\
&= \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta - 1}\right) + \frac{\delta}{2}\frac{\tau_\mu + \tau_v + \theta(1-r)\tau_\xi}{[\tau_\mu + \tau_v + (1-r)\tau_\xi]^2}.
\end{aligned}$$

■

Proof of Proposition 2. 1 and 2 are immediate consequences of proposition 1. For 3 through 7, the first part is also immediate. For the non-monotonicity results in 3 through 6, note that for generic $x_1, x_2 > 0$,

$$\frac{\partial}{\partial x_1}[\ln x_1 - 2\log(x_1 + x_2)] = \frac{1}{x_1} - \frac{2}{x_1 + x_2} = \frac{x_2 - x_1}{x_1(x_1 + x_2)}$$

which is positive if $x_1 < x_2$ and negative otherwise. For 7,

$$\begin{aligned} & \frac{\partial}{\partial \tau_\xi} \left[\ln\left(\tau_\mu + \tau_v + \theta\left(1-r\right)\tau_\xi\right) - 2\log\left(\tau_\mu + \tau_v + \left(1-r\right)\tau_\xi\right)\right] \\ = & \frac{\theta\left(1-r\right)}{\tau_\mu + \tau_v + \theta\left(1-r\right)\tau_\xi} - \frac{2\left(1-r\right)}{\tau_\mu + \tau_v + \left(1-r\right)\tau_\xi} \\ = & \frac{\left(1-r\right)\left[\theta\left(\tau_\mu + \tau_v + \left(1-r\right)\tau_\xi\right) - 2\left(\tau_\mu + \tau_v + \theta\left(1-r\right)\tau_\xi\right)\right]}{\left(\tau_\mu + \tau_v + \theta\left(1-r\right)\tau_\xi\right)\left(\tau_\mu + \tau_v + \left(1-r\right)\tau_\xi\right)} \end{aligned}$$

which is positive, if and only if $\left(\theta-2\right)\left(\tau_\mu + \tau_v\right) > \theta\left(1-r\right)\tau_\xi$, and negative otherwise. ■

Proof of lemma 4. Note that

$$\begin{aligned} n_t &= \frac{1}{\delta}\int_0^1 \left[c\left(p_t^i\right)\right]^\delta di = \frac{1}{\delta}\int_0^1 \left[C_t \frac{\left(p_t^i\right)^{-\theta}}{P_t^{-\theta}}\right]^\delta di = \frac{1}{\delta} C_t^\delta P_t^{\delta\theta} \int_0^1 \left(p_t^i\right)^{-\theta\delta} di \\ &= \frac{1}{\delta} C_t^\delta P_t^{\delta\theta} \exp\left\{-\theta\delta\overline{\log p_t} + \frac{\left(\theta\delta\right)^2}{2}\Sigma_p^2\right\} \\ &= \frac{1}{\delta} C_t^\delta \exp\left\{\frac{\left(\theta\delta\right)}{2}\left[1-\theta+\theta\delta\right]\Sigma_p^2\right\} \end{aligned}$$

using the fact that $\log P_t = \overline{\log p_t} - \frac{\theta-1}{2}\Sigma_p^2$. It follows that

$$\begin{aligned} \mathbb{E}_{t-1}\left(n_t\right) &= \frac{1}{\delta}\exp\left\{\delta\mathbb{E}\left(\log C_t\right) + \frac{\delta^2}{2}\sigma_C^2 + \frac{\theta\delta^2}{2}\frac{\left[1-\theta+\theta\delta\right]}{\delta}\Sigma_p^2\right\} \\ &= \frac{1}{\delta}\exp\left\{\delta\mathbb{E}\left(\log C_t\right) + \frac{\delta^2}{2}\sigma_C^2 + \frac{\theta\delta^2}{2\left(1-r\right)}\Sigma_p^2\right\} \end{aligned}$$

from which the result follows immediately. ■

Proof of lemma 5. For a linear pricing rule as posited, it follows that $\mathbb{E}\left(\log C_t\right) = -\Lambda_0 + \frac{\theta-1}{2}\Sigma_p^2 = \Lambda$. The other terms follow immediately by substituting from the linear pricing rule. ■

Proof of theorem 1. Given the previous results, the theorem follows immediately, once it is shown that

$$\exp\left\{\delta\mathbb{E}\left(\log C_t\right) + \frac{\delta^2}{2}\sigma_C^2 + \frac{\theta\delta^2}{2\left(1-r\right)}\Sigma_p^2\right\} = \frac{\theta-1}{\gamma\theta}$$

Since

$$\sigma_C^2 + \frac{\theta}{1-r}\Sigma_p^2 = \frac{\tau_\mu + \tau_v + \theta\left(1-r\right)\tau_\xi}{\left[\tau_\mu + \tau_v + \left(1-r\right)\tau_\xi\right]^2},$$

we find, after substituting for $\mathbb{E}(\log C_t)$,

$$\begin{aligned} & \exp\left\{\delta\mathbb{E}(\log C_t) + \frac{\delta^2}{2}\sigma_C^2 + \frac{\theta\delta^2}{2(1-r)}\Sigma_p^2\right\} \\ = & \frac{\theta-1}{\gamma\theta}\exp\left\{\frac{\delta^2}{2}\left[\sigma_C^2 + \frac{\theta}{1-r}\Sigma_p^2\right] - \frac{\tau_\mu + \tau_v + \theta(1-r)\tau_\xi}{\left[\tau_\mu + \tau_v + (1-r)\tau_\xi\right]^2}\right\} \\ = & \frac{\theta-1}{\gamma\theta}. \end{aligned}$$

■

Proof of Proposition 3. Taking First-order conditions of (29) with respect to Λ, Λ_1, and Λ_2, one finds:

$$\begin{aligned} 1 &= \exp\left\{-\delta\Lambda + \frac{\delta^2}{2}\left[(1-\Lambda_1-\Lambda_2)^2\frac{1}{\tau_\mu} + \Lambda_2^2\frac{1}{\tau_v} + \Lambda_1^2\frac{\theta}{1-r}\frac{1}{\tau_\xi}\right]\right\} \\ (1-\Lambda_1-\Lambda_2)\frac{1}{\tau_\mu} &= \Lambda_2\frac{1}{\tau_v} \\ (1-\Lambda_1-\Lambda_2)\frac{1}{\tau_\mu} &= \Lambda_1\frac{\theta}{1-r}\frac{1}{\tau_\xi} \end{aligned}$$

Solving these conditions with respect to Λ, Λ_1, and Λ_2 yields Λ^*, Λ_1^*, and Λ_2^*, and $\Lambda_0^* = \Lambda^* + \frac{\theta-1}{2}\frac{(\Lambda_1^*)^2}{\tau_\xi}$. ■

Proof of Theorem 2. Follows immediately from Propositions 1 and 3 ■

Proof of Prop. 4. Follows immediately from Propositions 1 and 3, setting $\tau_\xi = 0$. ■

7 Appendix 2: the *Social Value of Public Information* revisited

The results presented in the paper are in stark contrast with those reported in MS. This raises several issues: First, which of the modelling features drive the differences in results between MS and the present analysis? Second, under what conditions is the MS analysis appropriate for discussing the welfare effects of information provision, and when are the conclusions of the present model applicable?

In this appendix, I generalize the linear-quadratic model studied in MS to discuss the connections between this paper, MS and Angeletos and Pavan. The purpose of this is threefold: First, I show how the conclusions that were derived here also arise under a natural modification of the MS set-up; second, I discuss explicitly the role of the forecasting externality under heterogeneous information; and third, I show how a general class of linear quadratic models may lead to observationally equivalent conclusions regarding equilibrium actions, yet may lead to very diverse welfare implications. This last point suggests that an appropriate modelling of the underlying microfoundations is essential for a proper understanding of the welfare effects of private and public information.

I begin by reviewing the formal results of MS. MS consider a one-shot game, in which a measure 1 continuum of agents, indexed by $[0,1]$ takes actions $a_i \in \mathbb{R}$. There is a fundamental parameter θ, drawn from a normal distribution, $\theta \sim \mathcal{N}\left(z, \alpha^{-1}\right)$, where this prior parametrizes the publicly available information. In addition, agents have access to a private signal x_i, $x_i \sim \mathcal{N}\left(\theta, \beta^{-1}\right)$. The private signals are iid across the population. Thus the model yields an information structure equivalent to the one in the paper, for parameters $\alpha = \tau_\mu + \tau_v$ and $\beta = \tau_\xi$.

An agent's objective function is given by:

$$\begin{aligned} u_i(\mathbf{a}, \theta) &= -(1-r)(a_i - \theta)^2 - r\left(L_i - \overline{L}\right) \\ \text{where } L_i &= \int_0^1 (a_i - a_j)^2\, dj \text{ and } \overline{L} = \int_0^1 L_j dj \end{aligned}$$

The resulting first-order condition for a_i is

$$\begin{aligned} a_i &= rE_t^i(a) + (1-r)E_t^i(\theta) \\ \text{where } a &= \int_0^1 a_j dj \end{aligned}$$

Solving for the unique linear equilibrium yields:

$$a_i = \frac{\alpha z + (1-r)\beta x_i}{\alpha + (1-r)\beta}$$

The decentralized information optimum is found by maximizing

$$W(\mathbf{a}, \theta) = \int_0^1 u_i(\mathbf{a}, \theta)\, di = -(1-r)\int_0^1 (a_i - \theta)^2\, di$$

It immediately follows that the socially optimal rule is given by $a_i^* = \frac{\alpha z + \beta x_i}{\alpha + \beta}$ and that the equilibrium puts too much weight on public information, and too little on private information; as a consequence equilibrium welfare is non-monotonic in public information.

The stark modelling assumption in MS is that the coordination motive enters individual preferences through a zero-sum component that washes out in the social welfare function. As I will now illustrate, there is a general class of models that all lead to the same linear equilibrium strategies as MS, but yield radically different implications in terms of equilibrium welfare. To begin, it is useful to rewrite L_i, $\overline{L}$ and $u_i(\mathbf{a},\theta)$ as follows:

$$\begin{aligned} L_i &= \int_0^1 (a_i - a_j)^2 dj = (a_i - a)^2 + \int_0^1 (a_j - a)^2 dj \\ \overline{L} &= \int_0^1 L_j dj = 2\int_0^1 (a_j - a)^2 dj \\ u_i(\mathbf{a},\theta) &= -(1-r)(a_i-\theta)^2 - r(a_i - a)^2 + r\int_0^1 (a_j - a)^2 dj \end{aligned}$$

This last formulation captures the different components of the individual welfare function: The first component captures the loss due to "missing the target" θ. The second component captures the loss that i suffers from missing the average action a. The last term captures an externality imposed on i by the heterogeneity of the actions taken by the other agents. In other words, this specification highlights the welfare effects that were underlying the discussion in the paper, with the first term capturing the aggregate volatility effect, the second term the cost of heterogeneity that is internalized by individual decisions, and the final term capturing the welfare effect of heterogeneity that acts as a pure externality. Under the specification considered by MS, this is a positive externality, i.e. the more heterogeneous actions are, the better for any individual. We can consider a more general class of individual objectives:

$$u_i(\mathbf{a},\theta;k) = -(1-r)(a_i-\theta)^2 - r(a_i - a)^2 + k\int_0^1 (a_j - a)^2 dj$$

where the coefficient k measures the size and the sign of the welfare effects of aggregate heterogeneity. As special cases, we can consider the cases $k = r$, which corresponds to MS, $k = -r$, which arises naturally when the term $\overline{L}$ is dropped from the MS objective, and $r = 0$, which, as we will see, corresponds to the case where the cost of heterogeneity is fully internalized by each individual, and the equilibrium makes socially optimal use of the available information. For all these specifications, the linear equilibrium rule is given by $a_i = \frac{\alpha z + (1-r)\beta x_i}{\alpha + (1-r)\beta}$; however the social optimum maximizes

$$W(\mathbf{a},\theta;k) = -(1-r)\int_0^1 (a_i-\theta)^2 di + (k-r)\int_0^1 (a_j - a)^2 dj$$

Substituting a linear decision rule $a_i = \lambda x_i + (1-\lambda) z$, we find $a - \theta = (1-\lambda)(z-\theta)$ and $a_i - a = \lambda(x_i - \theta)$, so that

$$W(\lambda;k) = -\frac{\lambda^2}{\beta} - (1-r)\frac{(1-\lambda)^2}{\alpha} + k\frac{\lambda^2}{\beta}$$

The resulting first-order condition for λ is $(1-r)\frac{(1-\lambda)}{\alpha} = (1-k)\frac{\lambda}{\beta}$, or

$$\lambda^* = \frac{\beta(1-r)}{\beta(1-r)+\alpha(1-k)}$$

Therefore, whether the equilibrium makes too heavy or too little use of public information depends on the sign of the forecasting externality. When $k=0$, every individual fully internalizes the cost of action heterogeneity, and the equilibrium makes efficient use of the available information. In the case of MS, the forecasting externality has a positive welfare effect, and the equilibrium makes too heavy use of the available public information, not fully internalizing the social benefit of action heterogeneity. Finally, in the case where there is a negative externality, the equilibrium doesn't fully internalize the cost of heterogeneity, and makes too heavy use of private information.

Using the equilibrium rule, we revisit the equilibrium comparative statics with respect to public and private information:

$$\begin{aligned} W^{eq} &= -\frac{1-r}{\alpha+(1-r)\beta} + k\beta\left[\frac{1-r}{\alpha+(1-r)\beta}\right]^2 \\ \frac{\partial W^{eq}}{\partial\alpha} &= \frac{1-r}{[\alpha+(1-r)\beta]^2}\frac{\alpha+(1-2k)(1-r)\beta}{\alpha+(1-r)\beta} \\ \frac{\partial W^{eq}}{\partial\beta(1-r)} &= \frac{(1-r)^2}{[\alpha+(1-r)\beta]^2}\frac{\alpha(1+k)+(1-k)(1-r)\beta}{\alpha+(1-r)\beta} \end{aligned}$$

Therefore, W^{eq} is non-monotonic in α whenever $k > 1/2$, and is increasing in α if and only if $\alpha > (2k-1)(1-r)\beta$. If $k < 1$ (so that heterogeneity in actions is costly from a social point of view), W^{eq} is increasing in β if and only if $(1-r)\beta > -\frac{1+k}{1-k}\alpha$; this implies that W^{eq} is monotonic in β, if and only if $k \geq -1$; otherwise W^{eq} is non-monotonic.[10]

As I discussed in the main text, the present model of nominal adjustment under heterogeneous information generates an externality due to price heterogeneity. Indeed, the main results of the paper are identically replicated in this quadratic example when $k = 1-\theta < 0$. In that case, $\lambda^* = \frac{\beta\frac{1-r}{\theta}}{\beta\frac{1-r}{\theta}+\alpha}$, W^{eq} is strictly increasing in α, and increasing in β, if and only if $(1-r)\beta > \frac{\theta-2}{\theta}\alpha$, as was stated in the results leading up to theorem 2.

Up to this point, I have considered quadratic models, in which the only externality came from the heterogeneity in actions. An alternative externality in information processing might arise from

[10] If $k > 1$, W^{eq} is non-monotonic in β and is increasing iff $(1-r)\beta < -\frac{1+k}{k-1}\alpha$ (For high enough β, a further increase reduces the heterogeneity in actions, which now comes at a social cost).

the aggregate volatility effect, whereby individuals do not fully internalize their impact on aggregate volatility. To illustrate this point, suppose that the household's objective is given by an arbitrary quadratic form in $(a_i - \theta, a_i - a)'$:

$$u_i(\mathbf{a}, \theta) = -c_1 (a_i - \theta)^2 - c_2 (a_i - a)^2 - 2c_3 (a_i - \theta)(a_i - a)$$

where $c_1 + c_2 + 2c_3 = 1$. This objective can be rewritten as:

$$\begin{aligned} u_i(\mathbf{a}, \theta) &= -(c_1 + c_3)(a_i - \theta)^2 - (c_2 + c_3)(a_i - a)^2 \\ &\quad + c_3 \left[(a_i - \theta)^2 - 2(a_i - \theta)(a_i - a) + (a_i - a)^2 \right] \\ &= -(c_1 + c_3)(a_i - \theta)^2 - (c_2 + c_3)(a_i - a)^2 \\ &\quad + c_3 \left[(a_i - \theta) - (a_i - a) \right]^2 \\ &= -(c_1 + c_3)(a_i - \theta)^2 - (c_2 + c_3)(a_i - a)^2 \\ &\quad + c_3 (a - \theta)^2 \end{aligned}$$

Setting $r = c_2 + c_3$ and $c = c_3$, this objective takes the form

$$u_i(\mathbf{a}, \theta) = -(1 - r)(a_i - \theta)^2 - r(a_i - a)^2 + c(a - \theta)^2$$

Therefore, the resulting first-order condition is formally equivalent to the previous one, and in the unique equilibrium, $a_i = \frac{\alpha z + (1-r)\beta x_i}{\alpha + (1-r)\beta}$. In this case, there is a spill-over from aggregate volatility which is not internalized in the equilibrium pricing rule. After substituting for a linear decision rule $a_i = \lambda x_i + (1 - \lambda) z$ and integrating over i, we find

$$W(\lambda; c) = -\frac{\lambda^2}{\beta} - (1 - r)\frac{(1 - \lambda)^2}{\alpha} + c\frac{(1 - \lambda)^2}{\alpha}.$$

The FOC for the first-best rule is $\frac{\lambda}{\beta} = (1 - r - c)\frac{(1-\lambda)}{\alpha}$ or

$$\lambda^* = \frac{\beta(1 - r - c)}{\beta(1 - r - c) + \alpha}$$

Therefore, the equilibrium makes too heavy use of private information, when $c > 0$, and too little use of it, when $c < 0$ - thus, when there is a positive spill-over due to aggregate volatility, then more conditioning on public information (and an increase in volatility) would be socially beneficial, the opposite is true when $c < 0$. Consequently, equilibrium welfare as a function of the precisions of public and private information is:

$$\begin{aligned} W^{eq} &= -\frac{1-r}{\alpha+(1-r)\beta}+c\frac{\alpha}{[\alpha+(1-r)\beta]^2} \\ \frac{\partial W^{eq}}{\partial\alpha} &= \frac{1}{[\alpha+(1-r)\beta]^2}\left[1-r-2c\frac{\alpha}{\alpha+(1-r)\beta}+c\right] \\ \frac{\partial W^{eq}}{\partial\beta(1-r)} &= \frac{1}{[\alpha+(1-r)\beta]^2}\left[1-r-2c\frac{\alpha}{\alpha+(1-r)\beta}\right] \end{aligned}$$

Hence, provided that $c < 1-r$ (so that the noise in public information carries a social cost) we find as comparative statics that W^{eq} is increasing in β if and only if $(1-r)\beta > \left[\frac{2c}{1-r}-1\right]\alpha$, which implies that W^{eq} is monotonic in β, if and only if $c < \frac{1-r}{2}$ (otherwise it is non-monotonic). W^{eq} is increasing in α, if and only if $\alpha > -\left[\frac{1-r+c}{1-r-c}\right](1-r)\beta$. If $c > -(1-r)$, W^{eq} is everywhere increasing in α, otherwise it is non-monotonic.

One example of a game with an externality in aggregate volatility is the investment model with spill-overs considered by Angeletos and Pavan (2004). In that set-up, agents take investment decisions a_i according to an individual profit function

$$u_i(\mathbf{a},\theta) = 2Aa_i - a_i^2,$$

where the productivity parameter A is given by $A = ra + (1-r)\theta$, and $r < 1/2$. The resulting first-order condition again implies that

$$a_i = rE^i(a) + (1-r)E^i(\theta) = \frac{\alpha z + (1-r)\beta x_i}{\alpha+(1-r)\beta}.$$

Integrating u_i with respect to i, and taking expectations, Angeletos and Pavan show that the social welfare can be expressed as:

$$W = -(1-r)\mathbb{E}(a-\theta)^2 - \int_0^1 (a_i-a)^2\,di + r\mathbb{E}(a-\theta)^2$$

This formulation again separates the welfare effects into terms due to aggregate volatility, and terms due to heterogeneity in actions. The first two terms in the second line are identical to the ones that would arise in the absence of any information externality. It follows that individual decisions fully internalize the cost of action heterogeneity (second term), but there is a positive externality from aggregate volatility (owing to the last two term). Thus, we find the same structure as in the previous model, with $c = r$.The decentralized information optimum is

$$\lambda^* = \frac{\beta(1-2r)}{\beta(1-2r)+\alpha} < \frac{(1-r)\beta}{\alpha+(1-r)\beta}.$$

Thus, the equilibrium makes too heavy use of private information, and too little use of public information, since the positive spill-over that is associated with aggregate volatility is not internalized. Since $r < 1/2$, W^{eq} is everywhere increasing in α, but is decreasing in β for small β, if $r > 1/3$.

To conclude, this appendix has shown that the private and social benefits and costs of information use, and consequently the desirability of transparent public and private information provision can be linked to existing externalities that operate either through the cost of heterogeneity, or through the cost of aggregate volatility. While all these models lead to an identical strategic interaction, and consequently an identical rule regarding equilibrium play, their welfare implications, and the resulting costs and benefits of public and private information provision are quite diverse, and they depend on the existing externalities at work. The analysis has identified two potential sources of externalities: If there is a negative (positive) externality operating through action heterogeneity, then (i) actions are too heterogeneous relative to first-best, (ii) the equilibrium makes too heavy (little) use of private information, and too little (heavy) use of public information, and (iii) the equilibrium welfare is potentially non-monotonic in the precision of private (public) information. On the other hand, if there is a negative (positive) externality operating through aggregate volatility, then (i) the equilibrium exhibits too much (little) volatility, (ii) makes too heavy (little) use of public information, and too little (heavy) use of private information, and (iii) equilibrium welfare is potentially non-monotonic in the precision of public (private) information. Which of these scenarios is applicable for the welfare effects of information provision depends on the specific context, and has to be determined from the underlying model structure.