Abstract

Conventional wisdom suggests that the optimal policy response to rising income inequality is greater redistribution via higher marginal tax rates and more progressive tax schedules. In this paper we study an economy in which trade is associated with a costly entry into the foreign market, so that only the most productive agents can profitably participate in foreign trade. In this model, trade integration simultaneously leads to rising income inequality and a more sensitive efficiency margin of taxation, both driven by the extensive margin of trade. As a result, the optimal policy response may be to reduce the marginal taxes, thereby further increasing inequality. In order to reap most of the welfare gains from trade, countries may need to accept increasing income inequality.

1 Introduction

The idea that trade leads to distributional conflict is one of the most widely accepted views among economists. The 1980s and 90s saw rapid globalization and a concomitant increase in wage and income inequality in the U.S., the U.K. and many other developed countries.\footnote{See, for example, Burtless and Jencks (2003) and Machin and Van Reenen (2007).} The contribution of trade, as opposed to skill-biased technical change, to rising inequality is intensely debated. Feenstra and Hanson (1999)’s estimates suggest that outsourcing alone could account for as much as 40\% of the increase in the U.S. skill premium in the 1980s. Other studies, summarized in Krugman (2008), arrive at more conservative estimates suggesting that trade accounted for about 15-20\% of the increase in income inequality.\footnote{A recent study by Bloom, Draca, and Van Reenen (2008) finds that trade with China is an important force behind differential technology adoption across British firms which in turn leads to growing wage inequality. Another recent study by Broda and Romalis (2008) suggests, however, that the same trade with China has likely reduced inequality on the consumption side in the US by reducing the price of the consumption bundle for the poor relative to that for the rich.} Even more striking is the evidence for developing countries. Goldberg and Pavcnik (2007) summarize a body of literature studying the consequences of trade liberalization across a number of developing countries after 1970s, all finding a significant increase in inequality.

The above evidence raises the question as to how optimal national redistribution policies should respond to increasing world trade. Optimal redistribution policy has to balance equity and efficiency considerations. The conventional view is that greater inequality in a closed economy should be offset by more progressive taxation and higher marginal tax rates. This view is often extended to trade-induced inequality: when trade causes rising inequality, the optimal policy response should be to increase redistribution, rather than to limit trade.\footnote{For example, Irwin (2008, pp. 142–3) summarizes this intuition: “I suspect the policy advice would be the same regardless of whether trade was found to have been responsible for 4 percent or 40 percent of a given amount of wage inequality. That response would probably be as follows: inequality may be undesirable, but it should be addressed not by closing markets through greater protectionism, but by more progressive income taxation, a stronger social safety net, and more assistance for displaced workers.”} This conventional intuition does not hold, however, when the original cause of rising inequality also intensifies the efficiency margin in the economy. In this paper we show that in a class of models, trade-induced inequality is intricately linked with a more sensitive efficiency response of economy to taxation, both being caused by an extensive margin of trade. Increasing marginal tax rates, therefore, is not necessarily the optimal response to trade-induced inequality. In fact, under some circumstances, it is optimal to reduce marginal tax rates and taxation progressivity in response to trade liberalization, which further worsens the inequality outcome. That is, countries might need to accept increasing inequality in order to reap the most welfare gains from trade.
To address the issue of optimal redistribution in an open economy, one needs a particular modeling framework that allows for analyzing the distributional consequences of trade. Traditionally this has been the Stolper-Samuelson Theorem of the Heckscher-Ohlin (HO) model. Recently however the empirical limitations of this framework have become apparent. As already mentioned, trade liberalizations led to a sharp increase in inequality in unskilled-labor abundant developing countries, a phenomenon at odds with the prediction of the HO model.\(^4\) In addition, the contribution of the residual component of wage inequality within groups of workers with similar observable characteristics appears to be at least as important as the growing skill premium across groups, as emphasized by the HO model.\(^5\) Finally, contrary to the main mechanism of adjustment in the HO model, the reallocation within sectors appears to be more important than across sectors for both adjustment to trade and inequality dynamics.\(^6\)

In this paper we propose a simple modeling framework for thinking about the distributional effects of trade and analyzing the optimal redistribution policies in an open economy. Although this framework is highly stylized, its predictions for the relationship between trade and inequality are consistent with those arising from a more detailed model of product and labor markets, developed in Helpman, Itskhoki, and Redding (2008b), who proposed an alternative to HO framework consistent with a number of empirical regularities.\(^7\)

The key ingredients of the current model are unobservable agent heterogeneity and fixed costs of exporting, as in Melitz (2003) and Yeaple (2005), which allow only the most productive agents to participate in international trade.\(^8\) Consequently, trade disproportionately benefits the most productive agents within sectors and occupations, leading to greater income inequality in a trading equilibrium than in autarky. In addition, selection into the exporting activity generates an extensive margin of trade, which is sensitive to national redistribution policies and contributes to the overall efficiency margin of taxation.

\(^4\)A related observation is that the movements in relative prices of skilled to unskilled goods, which are at the core of the Stolper-Samuelson mechanism, tended to be small (e.g., see Lawrence and Slaughter, 1993).


\(^6\)For example, Faggio, Salvanes, and Van Reenen (2007) show that most of the increase in wage inequality in UK happened within industries, while Levinsohn (1999) shows the relative importance of within-industry reallocation in response to trade liberalization in Chile.

\(^7\)Helpman, Itskhoki, and Redding (2008a,b) construct a model of firm heterogeneity, unobservable worker heterogeneity, random search and endogenous screening which allows to account for size and exporter wage premium, residual component of wage inequality and patterns of inter- and intra-sectoral reallocation in response to trade liberalization.

\(^8\)Empirically only a small fraction of firms export even in the most tradable sectors (Bernard and Jensen, 1999) and fixed costs of trade appear to be quantitatively very important (Das, Roberts, and Tybout, 2007; Bernard and Jensen, 2004).
The unobservable agent heterogeneity points towards the Mirrlees (1971) framework for analyzing the optimal redistribution policy. In our analysis we deviate as little as possible from the baseline Mirrlees closed economy. The departure that we consider is the introduction of imperfect substitutability between individual varieties of the differentiated final good. Imperfect substitutability results in love-of-variety which is the source of trade in our model as in Krugman (1980) and Helpman and Krugman (1985). Agents in our economy are worker-entrepreneurs each producing a distinct variety of the final good. Unobservable productivity heterogeneity across agents generates income inequality. Inequality averse society designs incentive-compatible redistribution policies in order to partly offset equilibrium inequality by optimally balancing the equity-efficiency considerations. For the purposes of tractability, we restrict the analysis to a limited set of policy instruments, which however allows for differential marginal taxes on exporters and non-exporters, as well as a subsidy for entry into the foreign market.

The main result of our analysis is that selection into foreign trade activities leads to both greater inequality and greater efficiency losses from redistribution in an open economy relative to autarky. In more technical language, both inequality and efficiency margins intensify in an open economy causing the equity-efficiency trade-off to become more tight. As a result, the optimal redistribution policy response to trade liberalization is in general ambiguous. On the one hand, selection into exporting activity leads to an increase in the dispersion of relative revenues across the groups of exporters and non-exporters. This causes greater income inequality and calls for more redistribution. On the other hand, selection into exporting activity also results in an active extensive margin of trade, which is sensitive to the stance of the redistribution policy. Therefore, the efficiency losses from redistribution also increase in an open economy as they now combine both intensive and extensive margins. On net, the optimal policy response to increasing trade can be both to raise or to reduce marginal taxes and the progressivity of the tax schedule.

Societies might have to accept growing income inequality as a necessary outcome in order to capture the welfare gains from trade. If redistribution policies are determined by highly

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9 Imperfect substitutability between different types of labor in the Mirrlees model were studied by Feldstein (1973) and Stiglitz (1982) in a two-class economy.

10 Love-of-variety models are the principal explanation for intra-industry trade which currently accounts for the majority of world trade (e.g., see Helpman, 1999). Moreover, Broda and Weinstein (2006) show the empirical importance of the love-of-variety effect and estimate large welfare gains from increased variety through international trade.

11 Higher marginal taxes do not necessarily affect the extensive margin directly, however, they have an indirect effect through the response of the optimal scale of production. Fixed costs activities require a certain scale in order to be justified. Since higher marginal taxes reduce the optimal scale for all firms they also negatively affect the extensive margin.
inequality-averse agents, society will be bound to forgo a large fraction of the welfare gains from trade. It is important to note here that this argument does not rely on whether there are losers from trade or whether every agent gains from trade. It only requires that gains from trade are not equally distributed, with the most gains concentrated among highly productive agents who can take the advantage of opening up by exporting their products. In fact, in the present model, all agents gain from trade in absolute terms provided that there is no redistribution policy response to increasing trade.\textsuperscript{12}

In Section 2 we start our analysis by considering the case of the closed economy. We show that greater dispersion of revenues coming from the underlying ability distribution robustly leads to higher marginal tax rates and more progressive average taxes. In Section 3 we study the baseline model of an open economy without fixed costs of trade in which agents of all ability levels participate in trade and export. We show, quite surprisingly, that in this environment neither inequality nor efficiency margins respond to trade.\textsuperscript{13} The reason is that trade increases revenues proportionally for all agents, and hence, leads to no distributional conflict. In addition, this model does not have an active extensive margin of trade, and hence, the efficiency margin is not affected. It follows that trade in a model without selection into the export market induces no inequality response and leaves the optimal redistribution policy unaltered.

In Section 4 we study the case of the open economy with fixed costs of exporting. We start by showing that inequality of both revenues and utilities is greater in an open economy than in autarky even though trade is beneficial for all agents. Fixed costs of trade allow only the most productive agents to profitably engage in exporting. These are the agents who benefit the most from trade liberalization, and this exacerbates the distribution conflict. We then show that the presence of an extensive margin of trade also magnifies the efficiency consequences of taxation. As a result, the optimal progressivity of taxation can either increase or decrease.

We start our analysis with a single linear tax instrument and then introduce additional tax instruments such as export market entry subsidy and differential tax brackets for exporters and non-exporters. The key insight here is that a limited set of tax instruments has to balance

\textsuperscript{12}When domestic redistribution policy responds to trade some agents may lose. Therefore, in this model there are no losers from trade per se, but trade may induce a domestic policy response which harms some of the agents in absolute terms relative to autarky.

\textsuperscript{13}This is the case with symmetric countries and cooperative policy determination (defined as the Nash bargaining solution between the two countries). We also study the case of non-cooperative policy determination. In this case terms of trade partly shield the country from distortionary domestic taxation and result in inefficiently high level of taxes in both countries. A similar effect is studied by Epifani and Gancia (2008) in a different model of taxation.
the goals of redistributing income and inducing optimal entry. When some instruments are not feasible, the other instruments have to adjust in order to replicate the optimal allocation as close as possible. For example, if entry subsidy is infeasible, marginal taxes will be regressive in order to encourage the right amount of entry at the cost of providing less income redistribution. In contrast, when entry subsidy is available, the optimal taxation scheme can be strongly progressive.

Finally, Section 5 shows how the insights of our analysis can be extended to other areas such as technology adoption. There is substantial evidence that technology adoption is another activity which requires considerable fixed costs so that only the most productive agents can take advantage of new technologies. Therefore, the results of our analysis can be readily applied to studying the relationship between technology adoption and inequality, as well as the optimal redistribution policy response. Section 5 also provides our concluding remarks. The technical details of the derivations and proofs are relegated to the Appendix.

**Related literature:** The issue of optimal redistribution in an open economy has received little attention in the literature. Following Dixit and Norman (1980, 1986) the literature mainly focused on the possibility of compensating losers from trade, typically in the context of a HO model, rather than on the design of the optimal redistribution policies. A few studies in this body of literature are most closely related to this paper. Spector (2001) introduces Mirrlees incentive constraints into an otherwise standard HO model and shows that trade may lead to welfare losses by endogenously limiting the set of instruments available to the government for redistributional purposes. Ichida (2003) uses a two-sector model with unobservable agent heterogeneity to study the possibility of attaining a Pareto improvement from trade without overcompensating some of the agents. Davidson and Matusz (2006) design the lowest cost compensation policies for the losers from trade in a two-sector economy with heterogeneous agents and participation decisions, but fixed labor supply. A recent paper by Egger and Kreickemeier (2008) analyzes a model of firm heterogeneity and fair wages, and studies the possibility of ensuring both welfare gains from trade and a more equal wage distribution when only a linear profit tax is allowed.

In the closed economy literature, the closest paper is by Saez (2002), who studies optimal redistribution in a model with both intensive and extensive margins of labor supply responses.

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14There exists, however, a vast literature on optimal capital and profit taxation in an open economy surveyed in Gordon and Hines (2002). Another strand of literature, started by Cameron (1978) and Rodrik (1998), studies the optimal size of the government in an open economy. Finally, there recently has emerged an active literature on optimal dynamic redistribution in closed economy macro models, as surveyed in Golosov, Tsyvinski, and Werning (2006).
He finds that Negative Income Tax programs are no longer optimal in a model with labor force participation decisions. The presence of an extensive margin of labor supply leads to the optimality of negative marginal taxes for the agents who are most likely to drop out of labor force.

2 Closed Economy

In this section we lay out the setup of the baseline closed economy and derive a general formula for the optimal tax rate which, as we show later, extends to an open economy environment. In the closed economy, our main emphasis is on the effect of income dispersion on marginal tax rates.

2.1 Economic Environment

The economic environment departs minimally from the original Mirrlees (1971) economy in order to allow for a meaningful analysis of international trade in later sections. We consider a static one sector economy. The economy is populated by a measure $L$ of worker-entrepreneurs heterogenous in their ability $n$, which is distributed on $[n_{\text{min}}, n_{\text{max}}] \subset \mathbb{R}_+$ according to a cumulative distribution function $H(n)$.\footnote{In particular, we allow both $n_{\text{min}} = 0$ and $n_{\text{max}} \to \infty$. For convenience, we write 0 and $\infty$ as the limits of integration implying that $H(n) \equiv 0$ for $n \in [0, n_{\text{min}})$ and $H(n) \equiv 1$ for $n \in (n_{\text{max}}, \infty)$.} Each agent can produce his own variety of the final good according to a linear production technology:

$$y_n = n\ell_n,$$

where $y_n$ is output and $\ell_n$ is labor input of an agent with productivity $n$. Since all agents with productivity $n$ are symmetric, we use $n$ to index the agents. Note that the amount of product variety in the economy is determined by the measure of agents who choose to produce and service the market in equilibrium.

The final good is a Dixit and Stiglitz (1977) CES aggregator of individual varieties:

$$Q = \left[ L \int_0^\infty y_n^\beta dH(n) \right]^{1/\beta}, \quad 0 < \beta \leq 1,$$

where $1/(1 - \beta) > 1$ is the elasticity of substitution between the varieties. Most of the public finance literature following Mirrlees (1971) assumes perfect substitutability between
the produced varieties ($\beta = 1$). The baseline case of this paper is the case of imperfect substitutability between varieties ($\beta < 1$), which will be the source of international trade in the following sections.

The assumption of imperfect substitutability between varieties has a number of implications. First, it leads to well-defined boundaries of the firms as opposed to the original Mirrlees (1971) model in which the boundaries of the firms are indeterminant. With perfect substitutability among varieties, the equilibrium allocations are identical for any number of firms that hire any fraction of the total labor supply in the competitive labor market. With imperfect substitutability, each agent produces his own variety and hence is an entrepreneur operating a separate firm, while the labor market is effectively non-existent. We view this feature of the model as a useful abstraction for the purposes of this paper. Second, imperfect substitutability results in monopolistic power and monopolistic competition among the producers of individual varieties. This introduces additional distortions to the equilibrium allocation that an optimal redistribution policy will have to take into account.

Consumption aggregator in (2) leads to the following (real) revenue function for each individual variety:

$$r_n = Q^{1-\beta}y_n^{\beta}. \tag{3}$$

The revenue is increasing and concave in the agent’s output and shifts out with an increase in the economy-wide real consumption. CES preferences imply that tighter product market competition, causing higher real consumption, increases revenues for every producer in the market. The opposite prediction arises in a two-sector model (e.g., Melitz and Ottaviano, 2008; Helpman and Itskhoki, 2008), although the implications for the relative revenues of different agents, central for the analysis of optimal redistribution, are robust across these models.

Utility of every agent in the economy is given by $U(c, \ell)$, where $c$ is consumption and $\ell$ is labor effort. To make the analysis tractable, we adopt the GHH preferences (due to Greenwood, Hercowitz, and Huffman, 1988), featuring no income effects on labor supply, a

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16Important exceptions to this are Feldstein (1973) and Stiglitz (1982) who study the case of imperfect substitutability between different types of labor input in a two class economy which results in an optimal income subsidy for high productivity types. This result is driven by the attempt of the optimal income taxation to manipulate equilibrium relative prices of different varieties. Similar effects will be at play in the present paper.

17Helpman, Itskhoki, and Redding (2008a,b) develop a detailed model of product and labor market equilibrium which has similar implications for the effects of trade on inequality.

18CES aggregator implies constant elasticity demand, $y_n = Q \cdot (p_n/P)^{-1/(1-\beta)}$, where $p_n$ is the variety’s price and $P = \left[ L \int_0^\infty p_n^{-\beta/(1-\beta)} dH(n) \right]^{-1/(1-\beta)/\beta}$ is the ideal price index associated with consumption aggregator (2). Nominal revenue is then given by $p_n y_n = PQ^{1-\beta}y_n^{\beta}$ and real revenue is $r_n = p_n y_n / P$. The price level, $P$, plays no role in the main text analysis and can be normalized to 1 without loss of generality.
constant wage elasticity of labor supply and a constant relative risk aversion:\(^{19}\)

\[
U(c, \ell) = \frac{1}{1 - \rho_a} \left( c - v(\ell) \right)^{1 - \rho_a}, \quad v(\ell) = \frac{1}{\gamma} \ell^\gamma, \quad \gamma = 1 + \frac{1}{\varepsilon}.
\]  

(4)

Here \(\rho_a\) denotes constant relative risk aversion of the agents and \(\varepsilon\) is the constant labor supply elasticity.

All agents face the same tax schedule \(T(r)\), which is a function of only their income (revenues), assuming that their labor effort is unobservable. As a result, the budget constraint of agent \(n\) is given by:

\[
c_n = r_n - T(r_n), \quad r_n = Q^{1-\beta}(n\ell_n)^\beta,
\]  

(5)

where real consumption equals real after tax revenues and the production function (1) is substituted into the expression for real revenues (3). Therefore, each agent maximizes utility (4) subject to his budget constraint (5). We denote the resulting utility by \(U_n\).

Finally, the government chooses the tax schedule \(T(\cdot)\) to maximize the social welfare function

\[
W = L \int_0^\infty G(U_n) \, dH(n)
\]  

subject to the government budget constraint

\[
L \int_0^\infty T(r_n) \, dH(n) \geq 0
\]  

(7)

and behavioral responses of the agents (i.e., incentive compatibility constraints) which require that \(\{c_n, \ell_n, r_n, U_n\}\) are the outcomes of agent optimization given a tax schedule \(T(\cdot)\). Government budget constraint (7) implicitly assumes that the only purpose of taxation is redistribution. In general, \(G(\cdot)\) is a strictly increasing and weakly concave function. We restrict it to the case of constant relative inequality aversion:

\[
G(u) = \frac{1}{1 - \rho_g} u^{1-\rho_g}, \quad \rho_g \geq 0,
\]

where \(\rho_g\) is the constant relative inequality aversion parameter of the government. The case of \(\rho_g = 0\) corresponds to the utilitarian planner and the other limiting case \(\rho_g \to \infty\) corresponds to the Rawlsian planner. Note that the overall measure of inequality aversion in the economy is given by \(\rho = \rho_a + \rho_g\), which is what matters for the optimal redistribution policy rather than its decomposition into the individual and aggregate components (\(\rho_a\) and

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\(^{19}\)The key simplifying assumption here is the absence of income effects on labor supply, which is a common benchmark in the public finance literature (e.g., Diamond, 1998; Saez, 2002). Since our focus in this paper is on the effects of trade on inequality and optimal redistribution, we choose to shut down the additional effects operating through the utility function in the baseline analysis.
Therefore, without loss of generality, we assume for notational convenience that $\rho_a = 0$ and $\rho = \rho_g \geq 0$.

We start our analysis with the case of linear taxation, so that $T(r) = -\Delta + tr$, where $t$ is the marginal tax rate common for all agents and $\Delta$ is the uniform transfer. The government budget constraint in this case becomes $\Delta = tR$, where

$$R \equiv \int_0^\infty r_n dH(n)$$

is the average revenue which, from (2) and (3), can be seen to equal the per capita consumption ($R = Q/L$). In the open economy case we allow for the introduction of additional tax instruments, including differential marginal tax rates for exporters and non-exporters, as well as an entry subsidy into the exporting activity.\(^{21}\)

### 2.2 Optimal Redistribution in Closed Economy

In this section we derive a general formula for the optimal linear tax which generalizes to the case of open economy studied in later sections. We lay out in detail only the steps of the derivation that prove to be useful in developing intuition for the results that follow, relegating the formal proofs to the Appendix. We then provide the main result of this section: the level of marginal tax increases in the equilibrium dispersion of relative revenues, which is pinned down by the exogenous distribution of abilities in the closed economy.

Using the government budget constraint (7) and substituting the agent’s budget constraint (5) into the utility function (4), agent $n$’s optimization yields:

\[
\begin{align*}
U_n &= \max_y \left\{ \Delta + (1-t)Q^{1-\beta}y^\beta - \frac{1}{\gamma} \left( \frac{y}{n}\right)^\gamma \right\} \\
&= tR + (1 - \beta/\gamma)(1-t)r_n,
\end{align*}
\]

where $r_n = Q^{1-\beta}y_n^\beta$ and $y_n$ is the optimal output of agent $n$ that satisfies the first order condition

$$\beta(1-t)Q^{1-\beta}y_n^{\beta-1} = v'(\frac{y_n}{n}) \frac{1}{n} = \frac{y_n^{\gamma-1}}{n^\gamma}.$$ 

\(^{20}\)The optimal amount of redistribution depends on the cross-sectional distribution of

$$G'(U_n) \cdot \frac{\partial U(c_n, \ell_n)}{\partial c} = U_n^{-\rho_a} \cdot U_n^{-\rho_a} = \left(c_n - v(\ell_n)\right)^{(\rho_a+\rho_a)}.$$ 

Since there is no uncertainty and the model is static, the value of $\rho_a$ also does not affect the optimal individual allocations ($c_n, \ell_n$).

\(^{21}\)A numerical investigation of an unconstrained Mirrlees (1971) optimal policy is intended in the next versions of the paper.
This optimality condition also implies \( v(y_n/n) = \beta/\gamma(1-t)r_n \), which we have used in the second line of (8).

Tax rate \( t \) affects agent \( n \)'s utility both directly and indirectly. There is a positive direct effect through the amount of transfer proportional to the average revenue in the economy \( R \) and a negative direct effect from taxing away a fraction of individual revenue \( r_n \). The indirect effect of a tax rate operates through the equilibrium impact of \( t \) on aggregate consumption and average revenue \( (Q \) and \( R) \): An increase in the marginal tax rate reduces the per capita taxation base \( (R) \) and, in addition, reduces revenue of every agent by reducing \( Q \).

Therefore, the overall effect of taxes on utility is:

\[
dU_n = \left[ \frac{\partial U_n}{\partial t} + \frac{\partial U_n}{\partial Q} \frac{dQ}{dt} \right] dt = (R - r_n) dt + \frac{dQ}{Q} \left[ tR + (1-t)(1-\beta)r_n \right]. \tag{9}
\]

The first term is the redistributional component from agents with above-average revenues towards agents with below-average revenues. The second term is the efficiency component which is proportional to the effect of taxes on real consumption. Note that agents with lower revenue, and hence, lower utility always gain more (or lose less) from an increase in taxes.

Finally, the optimality condition for the marginal tax rate is given by

\[
\int_0^\infty G'(U_n) \frac{dU_n}{dt} dH(n) = 0. \tag{10}
\]

The government searches for a tax rate which equalizes to zero the cross-sectional weighted average utility gains from a marginal increase in the tax rate. The agents are weighted by their marginal contribution to the social welfare function, \( G'(U_n) \). With positive inequality aversion \( (\rho > 0) \), the government puts more weight on the agents with lower utility.

It proves useful to introduce the following notation. Denote by

\[
\bar{\epsilon} \equiv \frac{\ln Q}{\ln(1-t)} = -\frac{(1-t) dQ}{Q dt}
\]

the elasticity of output (or consumption) with respect to the marginal tax rate. This elasticity quantifies the efficiency loss from taxation and is the first key object that will shape optimal taxes throughout this paper. We refer to it as the efficiency margin of taxation. With this definition we can combine (9) and (10) to rewrite the optimality condition for the marginal tax rate as:

\[
\int_0^\infty G'(U_n) \left[ \left(1 - \frac{r_n}{R} \right) - \bar{\epsilon} \left( \frac{t}{1-t} + (1-\beta)\frac{r_n}{R} \right) \right] dH(n) = 0.
\]

\[22\]The indirect effect of taxes on utility operating through their impact on the scale of individual production \( (y_n) \) is nil by the Envelop Theorem. The effect on utility through \( Q \) is not nil since individual optimization does not internalize the CES externality or, in other words, agents exercise their monopolistic power and underproduce relative to the efficient allocation.
Already this expression suggests that what matters for the optimal tax rate is the dispersion of relative revenues.

Rearranging the expression above, we obtain the general formula which determines the optimal linear tax rate:

\[
\frac{t}{1 - t} = \frac{1}{\bar{\varepsilon}} \cdot \alpha - (1 - \beta)(1 - \alpha),
\]

where

\[
\alpha = \int_{0}^{\infty} \lambda^{-1}G'(U_n) \left(1 - \frac{r_n}{R}\right) dH(n) = -\text{cov} \left(\lambda^{-1}G'(U), \frac{r}{R}\right)
\]

and \(\lambda = \int_{0}^{\infty} G'(U_n) dH(n)\) is the average marginal utility (or, more precisely, average marginal contribution of agents to the social welfare), which is also equal to the shadow value of one real dollar (unit of consumption) in the hands of the government. Observe from (12) that \(\alpha\) is the cross-sectional covariance between two normalized variables, both with a mean of 1. It is the second key object affecting the level of optimal taxation. We refer to \(\alpha\) as the inequality margin of taxation since it is a measure of dispersion of relative utilities resulting from the dispersion of relative revenues.

We prove in the Appendix the following:

**Lemma 1** \(0 \leq \alpha \leq 1\). \(\alpha = 0\) if and only if either \(\rho = 0\) or \(r_n \equiv R\) for all agents. \(\alpha \to 1\) if and only if \(\rho \to \infty\) and \(r_{n_{\text{min}}} = 0\).

Formula (11) and Lemma 1 provide a general characterization of the optimal linear tax rate. Since \(\alpha \in [0, 1]\), \(t/(1 - t)\) equals a convex combination of \(1/\bar{\varepsilon}\) and \(-(1 - \beta)\). When \(\alpha = 0\), which from Lemma 1 occurs either when there is no inequality or when the society does not care about inequality, the optimal marginal tax rate is \(t = -(1 - \beta)/\beta\). This optimal subsidy offsets the monopolistic distortion – a constant mark-up equal to \(1/\beta\) – and implements an efficient allocation. When \(\alpha \to 1\), which happens under the Rawlsian planner and provided that the least able agent does not produce, the optimal linear tax rate \(t \to 1/(1 + \bar{\varepsilon})\), which is the revenue maximizing tax rate at the peak of the Laffer curve.\(^{23}\) More generally, when \(\alpha \in (0, 1)\), the optimal marginal tax rate trades off efficiency for equity. Finally, note that when \(\beta \in (0, 1)\), optimal taxation has to balance redistributive considerations with offsetting the monopolistic distortion. If agents were price takers, the second term would be absent from (11) and \(\beta < 1\) would have no effect on the optimal taxes.

We summarize the discussion above in:

\(^{23}\)Similar results were obtained by Sheshinski (1972) and Hellwig (1986) for a general utility function, but with perfectly substitutable labor supply of different agents.
Proposition 1 The optimal linear income tax rate satisfies

\[
\frac{t}{1-t} = \frac{1}{\bar{\varepsilon}} \cdot \alpha - (1-\beta)(1-\alpha), \quad 0 \leq \alpha \leq 1.
\]  

(11')

Therefore,

\[
\frac{1-\beta}{\beta} \leq t \leq \frac{1}{1+\bar{\varepsilon}},
\]

where \((1-\beta)/\beta\) is the efficiency maximizing subsidy which is optimal if and only if there is either no inequality or no inequality aversion; \(1/(1+\bar{\varepsilon})\) is the revenue maximizing tax which is optimal if and only if there is extreme inequality aversion (Rawlsian planner) and the least productive agent does not produce.

The optimal tax rate is determined by the interaction between the efficiency margin \(\bar{\varepsilon}\) and the inequality margin \(\alpha\): greater efficiency margin reduces the optimal tax rate, while greater inequality margin increases it. Note that (11') characterizes the optimal tax rate only implicitly since \(\alpha\) is an endogenous object which in particular depends on the tax rate \(t\). Throughout the paper we assume that \(\partial \alpha / \partial t < 0\), which intuitively implies that a higher tax rate reduces the inequality margin. This assumption is also sufficient for the concavity of the welfare function in \(t\) and hence for the uniqueness and optimality of the tax rate determined by the first order condition (11'). In the Appendix we discuss sufficient conditions for \(\partial \alpha / \partial t < 0\) (see also Lemma 3 below).

The discussion so far has been general in the sense that all previous results extend to the open economy environment that we consider in the following sections. We now characterize the closed economy values of \(\bar{\varepsilon}\) and \(\alpha\). We have the following:

Lemma 2 In the closed economy \(\bar{\varepsilon} = \varepsilon\).

That is, the elasticity of final good production with respect to the marginal tax rate (the efficiency margin) is equal to the labor supply elasticity, which is also the intensive margin of agents’ responses to an increase in the marginal tax rate. The intuition is that an increase in \(t\) leads all agents to reduce their output proportionally with elasticity \(\varepsilon\). This in turn leads to a proportional reduction in final output which is a homothetic aggregator of individual outputs.

We now turn to the value of \(\alpha\), which belongs to the interval \([0, 1]\) by Lemma 1. From (12), \(\alpha\) is a covariance between \(r/R\) and \(U^{-\rho}/E\{U^{-\rho}\}\). From (8), note that \(U_n\) is a linear transformation of \(r_n/R\). This suggests that \(\alpha\) should increase in the dispersion of relative revenues and in the value of \(\rho\) which makes marginal utility steeper. We prove in the
Appendix the following lemma based on the first order approximation of $U^{-\rho}$ around $(\mathbb{E}U)^{-\rho}$, which becomes exact as dispersion of ability $n$ decreases towards zero:\textsuperscript{24}

**Lemma 3** In the closed economy, the second order approximation to $\alpha$ around $r_n = R$ for all $n$ is given by:

$$\alpha \approx \rho \cdot \frac{1}{1 + \frac{t}{1-t} \frac{1}{1-\beta/\gamma}} \cdot \text{var} \left( \frac{r}{R} \right).$$

(13)

Consistent with the intuition, the approximate solution implies that $\alpha$ increases in the inequality aversion $\rho$ and in the variance of relative revenues which also equals the square of the coefficient of variation of revenues. Importantly, from the approximation in Lemma 3 it follows that $\partial \alpha / \partial t < 0$, i.e. the inequality margin is decreasing in the level of marginal tax. As discussed above, this is sufficient to ensure that the second order condition is satisfied and the first order condition ($11'$) identifies the unique optimal marginal tax rate.

Finally, we characterize the distribution of relative revenues in equilibrium. From (3) we have $r_n/R = L(y_n/Q)^{\beta}$ and from the optimality condition for the agent’s problem (8) it follows that

$$y_n = \left[ \beta(1-t) \right]^{\frac{1}{1-\beta}} Q^{\frac{1}{1-\beta+\frac{\gamma}{\gamma}}} n^{\frac{\gamma}{\gamma-\beta}}.$$

Therefore, relative revenues are pinned down exclusively by the distribution of underlying ability:

**Lemma 4** In the closed economy equilibrium,

$$\frac{r_n}{R} = \frac{n^{\frac{\beta\gamma}{\gamma-\beta}}}{\int_0^\infty n^{\frac{\beta\gamma}{\gamma-\beta}} dF(n)}.$$

Intuitively, relative revenues do not depend on the equilibrium level of taxes which reduce revenues of all agents proportionally. This has an immediate implication that $\text{var}(r/R)$ is determined uniquely by the distribution of underlying ability $n$ and primitive elasticities $\beta$ and $\gamma$. More dispersion in the underlying ability translates into more dispersion in the relative revenues, which in turn leads to a higher $\alpha$ and higher optimal linear tax rate.

We summarize the implications of the above analysis in the following:

**Proposition 2** Assuming the approximation in Lemma 3 to be accurate, the optimal linear tax rate in the closed economy (i) increases in the inequality aversion $\rho$ and in the dispersion of relative revenues exogenously determined by the dispersion of ability; (ii) does not depend on the size of the economy $L$; (iii) depends ambiguously on the labor supply elasticity $\varepsilon$.

\textsuperscript{24}Exact results for the comparative statics of $\alpha$ under a general skill distribution are unavailable. Therefore, we rely on this approximation in the analytical discussion and verify its implications numerically in Section 4 for a special distribution of skills.
The central result is that in the closed economy marginal tax rate increases in the dispersion of relative revenues, which from Lemma 4 is exogenously determined by the dispersion of underlying ability distribution. Greater dispersion of relative revenues increases the inequality margin, $\alpha$, which leads to a higher optimal tax rate.

Interestingly, the size of the economy does not affect the level of taxes, although the set of available varieties and the per capita consumption increase with $L$. The intuition is that greater $L$ increases the scale of production and revenues proportionally for all agents without affecting $\alpha$, and also does not affect the elasticity of labor supply and hence $\tilde{\varepsilon}$. This result is useful for thinking about the effects of trade in the following section. Finally, a change in the elasticity of labor supply $\varepsilon$ has an ambiguous effect on the optimal tax rate since it not only affects the efficiency margin ($\bar{\varepsilon}$), but also changes the equilibrium dispersion of relative revenues (see Lemma 4 and recall that $\gamma = 1 + 1/\varepsilon$).

3 Open Economy I: No Fixed Costs

In this section we open our economy to trade and study how this affects the optimal taxation. The source of trade in this economy is love-of-variety, as in Helpman and Krugman (1985) style models, generated by imperfect substitutability ($\beta < 1$) between different varieties produced at home and abroad. Love-of-variety models are the principal explanation for intra-industry trade which currently accounts for the majority of world trade (e.g., see Helpman, 1999). Moreover, Broda and Weinstein (2006) show the empirical importance of the love-of-variety effect and estimate large welfare gains from increasing variety through international trade.

In the baseline open economy, each agent can sell part of his output at home and export the other part abroad. Exporting involves no fixed cost, but is subject to an iceberg-type variable trade cost. Specifically, for one unit of good to arrive abroad, $\tau > 1$ units have to be shipped out. Costs of trade is what distinguishes international trade from trade within national borders. Without fixed cost, every agent will participate in trade since

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25This conclusion can be shown to be robust to a number of extensions, including the introduction of income effects and non-constant elasticity of labor supply. Similar insights for the marginal tax on a median agent can be obtained in a model with general non-linear taxes.

26Specifically, $\begin{align*}
d \ln(Q/L) &= (1 + \varepsilon)(1 - \beta)/\beta \cdot d \ln L.\end{align*}$

27Increasing returns, another common ingredient of Helpman-Krugman style models, are not needed in this framework as the amount of variety is restricted by the number of agents in the two countries. As a result, the free entry condition is missing as well, which in particular has implications for equilibrium response of the size of the individual firms to trade. We chose this modeling approach to depart minimally from the original Mirrlees economy. Free entry introduces additional interesting public finance considerations, which are arguably more relevant in the studies of optimal capital and profit taxation.
trade allows the agents to partly escape decreasing demand and concavity of the revenue function in the domestic market. We study how opening up to trade in this economy affects income distribution and optimal income taxation. Throughout the analysis we assume no international labor mobility.

Although in principle there can be interesting interactions between optimal taxes and tariffs, we do not address the issue in this paper. Rather, we focus our analysis on the national income taxation policies in an open economy. One can rationalize this assumption in the following way: while tariffs are bound to be low by WTO agreements, national redistribution policies are set unilaterally by sovereign states.

### 3.1 Properties of Open Economy Equilibrium

For simplicity of exposition, here we describe the case of two symmetric trading economies. The case of asymmetric countries is studied in the Appendix. We characterize equilibrium allocations in the home economy with analogous characterizations for the foreign. Foreign variables are denoted with an asterisk.

An agent producing $y$ units of his variety supplies $y_d$ to the domestic market and exports the remaining $y_x = y - y_d$. Domestic sales generate revenues $Q^{1-\beta}y_d^\beta$, as explained in footnote 18. Export revenues are given by $Q^{*1-\beta}(y_x/\tau)^\beta$ since only $y_x/\tau$ units of the exported good reach the foreign market. In the symmetric equilibrium $Q = Q^*$. The optimal allocation of output for domestic sales and exports results in the following revenues:

$$r(y) = \max_{y_d, y_x} \left\{ Q^{1-\beta}y_d^\beta + Q^{*1-\beta}(y_x/\tau)^\beta \right\} = \Upsilon_x^{1-\beta}Q^{1-\beta}y^\beta,$$

where

$$\Upsilon_x \equiv 1 + \tau^{-\beta}Q^*/Q > 1$$

is the foreign market access variable which quantifies how much the access to the foreign consumers would boost producer’s revenues. $\Upsilon_x$ decreases in the variable trade cost $\tau$ and increases in the relative size of demand $Q^*/Q$. Note that the revenue of every agent in the economy is magnified proportionally by the access to international trade.

As in the closed economy, agents choose output to maximize their utility:

$$y_n = \arg \max_y \left\{ \Delta + (1-t)r(y) - \frac{1}{\gamma} \left( \frac{y}{n} \right)^\gamma \right\} = [\beta(1-t)]^{\frac{1}{1-\beta}}\Upsilon_x^{\frac{1-\beta}{1-\beta}}Q^{1-\beta}n^{-\frac{\gamma}{1-\beta}},$$

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18

The discussion here assumes that the price levels in both countries are normalized to 1, $P = P^* = 1$. This is a valid assumption in a symmetric equilibrium or when $\tau \rightarrow 1$. In the Appendix we consider a general non-symmetric case in which we explicitly allow for $P \neq P^*$.

29

Optimal allocation across markets requires equalization of marginal revenues in the two destinations which implies $y_x/y_d = \tau^{-\beta(1-\beta)} = \Upsilon_x - 1$ or equivalently $y_d/y = 1/\Upsilon_x$ and $y_x/y = (\Upsilon_x - 1)/\Upsilon_x$. 


where $\Delta$ is the transfer from the government, which satisfies the government budget constraint $\Delta = tR$. Maximized utility of agent $n$ is again given by $U_n = tR + (1 - \beta/\gamma)(1 - t)r_n$, where his revenues equal

$$r_n = \gamma x^1 Q^{1-\beta} y^\beta.$$  

Consequently, average revenues are given by

$$R = \int_0^\infty r_n \text{d}H(n) = \gamma x^1 Q^{1-\beta} \int_0^\infty y^\beta \text{d}H(n).$$

Finally, balanced trade implies that aggregate revenues of domestic agents equal aggregate consumption, $LR = Q$. This concludes the description of symmetric open economy equilibrium.

Without further characterization we can prove an important result about inequality in an open economy:

**Proposition 3** In an open economy without fixed costs of trade, the dispersion of relative revenues is the same as in the closed economy and is determined by the dispersion of underlying exogenous ability distribution, as described in Lemma 4. Therefore, trade does not lead to an increase in income inequality, provided that all agents participate in the exporting activity.

As we show in the Appendix, this proposition does not require the countries to be symmetric and does not depend on how the marginal tax rates in the two countries respond to trade. The result follows from the fact that international trade in this economy increases revenues proportionally for all agents, and given constant labor supply elasticity, all agents respond by proportionally increasing their outputs. As a result, the distribution of relative revenues is unaffected and the amount of income inequality is unchanged. Changes in taxes have similar proportional effects and do not alter the distribution of relative revenues (cf. Lemma 4).

To summarize, the model without fixed costs predicts no inequality effects of globalization. In Section 4, we study how this prediction is altered in a model with fixed costs and selection into export activity.

**Gains from Trade**

We now discuss who gains from trade in this economy. This discussion allows for arbitrary asymmetries across countries and the formal details are found in the Appendix. Define the social welfare function by

$$W(t, t^*) = \int_0^{\infty} G(U_n) \text{d}H(n),$$  

(15)
where $U_n$ is the equilibrium utility of agent $n$ in an open economy with domestic marginal tax rate $t$ and foreign marginal tax rate $t^*$. We denote by $W^a(t)$ the social welfare function in autarky. It is immediate to prove a general gains from trade result in this economy:

**Proposition 4** Welfare is higher in an open economy than in the closed economy.

**Proof.** For any $t^*$, $W(t, t^*) \geq W^a(t)$ since $r_n$ increases proportionally in an open economy for all agents, given that taxes are unchanged. Open economy welfare is equal to $\max_{t'} W(t', t^*) \geq W(t, t^*)$ for any $t$. Therefore, open economy welfare is greater than closed economy welfare $\max_{t'} W^a(t')$. A more formal argument is developed in the Appendix. ■

This proposition implies that regardless of the taxation policy response at home and abroad, opening up to trade leads to welfare gains in both countries in the aggregate. The intuition is that trade in this model unambiguously increases the choice set of the country irrespective of the taxation policy abroad. This result stands in contrast to Spector (2001) and Epifani and Gancia (2008), who demonstrate a possibility of welfare loses in related types of economies.\(^{30}\)

Aggregate gains from trade do not guarantee, however, that every agent in the economy gains from trade. As was mentioned above, every agent gains proportionally from trade if the open economy taxes are the same as in autarky. If opening up to trade leads to a change in the tax policy, however, in principle some agents may lose from trade. Specifically, if taxes go up in the open economy, the most productive agents might end up losing from opening up to trade; by contrast, if taxes go down in the open economy, the least productive agents in the economy might end up losing.\(^{31}\) The Appendix provides formal conditions for high (low) productivity agents to lose from trade. It is important to note that in this economy agents lose not from trade per se, but from the endogenous response of the redistributional policy to trade. Therefore, a source of protectionism might not be the distributional consequences of trade per se, but rather the expectation that trade will cause unfavorable changes in the domestic policies.

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\(^{30}\)In our economy every agent produces a distinct variety. In Spector (2001), every variety produced at home is also produced abroad. As a result, after opening up to trade, the government loses the ability to manipulate prices of the varieties and hence its choice set is restricted. Epifani and Gancia (2008) consider a love-of-variety model of trade without agent heterogeneity, but with a public good provided by the government.

\(^{31}\)Note the implication that if there is tax rate convergence across trading partners, the poor may lose in the country with initially high taxes and the rich may lose in the country with initially low taxes.
3.2 Optimal Redistribution

The immediate implication of Proposition 3 is that the dispersion of relative revenues does not depend on the size of variable trade cost $\tau$. Moreover, from the expression for equilibrium utility we observe that, given that the marginal tax rate remains unchanged, the distribution of relative utilities across agents is unchanged, and every agent gains from trade proportionally.\(^{32}\) This immediately implies that $\alpha$, as defined in (12), also remains the same as long as marginal taxes are not adjusted. In other words, opening up this economy to trade does not have distributional consequences, and only the potential change in the efficiency margin can call for tax policy adjustment. This is the focus of the remainder of this section.

In the text, we focus again on the case of symmetric countries and cooperative policy determination. Asymmetric countries and non-cooperative policies are characterized in the Appendix and briefly discussed below. We define cooperative policies as the outcome of a Nash bargaining solution between the two countries, where the non-cooperative Nash equilibrium is taken as the status quo point. Cooperative solution ensures that countries do not forgo any possible Pareto gains from policy coordination.

In a symmetric world equilibrium $t^* = t$ and $W(t, t) = W^*(t, t)$. In the Appendix we show that optimal tax rates are determined by the following first order condition:

$$\frac{dW(t, t)}{dt} = \frac{\partial[W(t, t) + W^*(t, t)]}{\partial t} = \frac{\partial W(t, t)}{\partial t} + \frac{\partial W(t, t)}{\partial t^*} = 0,$$

which takes into account the effect of the domestic tax rate on welfare both at home and abroad. We further show that this condition again implies

$$\frac{t}{1 - t} = \frac{1}{\tilde{\epsilon}} \cdot \alpha - (1 - \beta)(1 - \alpha),$$

as in the closed economy. The inequality margin, $\alpha$, is still defined by (12) and

$$\bar{\epsilon} \equiv \frac{d \ln Q}{d \ln (1 - t)} \bigg|_{t = t^*} = \tilde{\epsilon}.$$

Therefore, neither efficiency margin ($\bar{\epsilon}$), nor inequality margin ($\alpha$) change in a symmetric open economy. Consequently, in a symmetric open economy, cooperatively-set marginal taxes are the same as in autarky. The intuition is straightforward: Opening up to trade in this economy does not induce redistributional effects as discussed in Section 3.1, and the output response to a coordinated change in taxes is still given by the labor supply elasticity

\(^{32}\)Since aggregate revenue increases in an open economy, the size of the transfer $\Delta = tR$ increases as well even with constant marginal tax rate $t > 0$. This ensures that even very low productivity agents gain from trade proportionally in the utility terms.
This is the case as output of each variety for domestic sales and exports is reduced proportionally in response to a global increase in the income tax rate. In other words, the efficiency margin is still determined by the intensive margin of production, which equals the labor supply elasticity.

We summarize this in:

**Proposition 5** Cooperatively-set taxes in a symmetric world economy without fixed costs of trade are the same as in the closed economy. Both the efficiency and the inequality margins of optimal taxation are the same in the open economy as in autarky.

Opening up to trade in this economy is similar to increasing the measure of agents, \( L \), in the closed economy, which according to Proposition 2 has no effect on the optimal tax rate. In the next section we show how this result contrasts with the implications of a model with fixed costs of trade and selection into exporting activity.

Consider now what happens when policies are determined non-cooperatively and countries are asymmetric. With non-cooperative policy determination, the countries are shielded by endogenous terms-of-trade from distortionary domestic policies. Specifically, a fraction of welfare losses from distortionary domestic taxation is born by the consumers in the trade partner, who now have to face higher import prices and depreciated terms of trade. As a result, both trading countries will tend to set taxes inefficiently high in the sense that a coordinated reduction in taxes would induce a Pareto improvement.\(^{33}\) Finally, when countries are asymmetric, Proposition 5 no longer holds, even cooperatively-set optimal taxes in the open economy are no longer the same as in autarky, and under certain condition the model predicts convergence in the tax rates across countries. The Appendix provides a detail discussion of these issues.

### 4 Open Economy II: Fixed Costs of Trade

We now introduce fixed costs of entering the export market into the trade model of the previous section. This follows a vast theoretical literature started by Melitz (2003). A model with fixed costs of trade leads to a selection of agents into foreign trade with only the most

\(^{33}\)The terms-of-trade externality is predicted to be stronger for smaller and more open economies, which empirically tend to have larger governments (Rodrik, 1998; Alesina and Wacziarg, 1998). The importance of terms-of-trade externality on the level of optimal taxation was recently studied by Epifani and Gancia (2008), both theoretically and empirically. Finally, Mendoza and Tesar (2005) in a quantitative model of taxation in the open economy find the gains from policy coordination to be small.
productive of them being able to profitably export. This is consistent with the empirical facts that only a small fraction of firms participate in international trade and these firms appear to be more productive (Bernard and Jensen, 1999; Eaton, Kortum, and Kramarz, 2004). Moreover, empirical estimates of the fixed costs of entering the foreign market appear to be large quantitatively (Das, Roberts, and Tybout, 2007; Bernard and Jensen, 2004).

Fixed costs of trade and selection into exporting have two important effects. First, they lead to discontinuously higher revenues for exporters relative to non-exporters, even controlling for their productivity. This is because revenues need to be discreetly higher for exporters in order for them to cover the fixed cost of trade. Second, selection into exporting creates an additional extensive margin of trade, which is sensitive to domestic redistribution policies. In this section, we study the interaction between these two effects and their implications for optimal taxation. We restrict our attention to the case of symmetric countries each with unit continuum of agents \((L = 1)\) and to cooperative policy determination. We start by considering the optimal linear tax and then add additional policy instruments.

4.1 Equilibrium Properties

We start our analysis by characterizing an open economy equilibrium holding the tax policy \(t\) constant. Given fixed costs of trade, each agent now has to decide whether to serve the domestic market only at no fixed cost or to pay the fixed cost and both serve the domestic market and export. In the former case his real revenue is \(Q^{1-\beta} y^{\beta}\) as in the closed economy, while in the latter case his revenue is \(\Upsilon_{x}^{1-\beta} Q^{1-\beta} y^{\beta}\) as in the open economy of the previous section. Since the countries are symmetric, \(\Upsilon_{x} = 1 + \tau^{-\beta/(1-\beta)} > 1\). The fixed cost equals \(f_{x}\) in the units of the final good. For an agent to profitably engage in foreign trade the increase in revenues from the access to the foreign market should exceed both the fixed cost and the variable cost of extra production for the export market. As we show below, only the most productive agents participate in the exporting activity, provided that the fixed cost is high enough.

We assume that the tax is levied on the revenues of the agents gross of fixed cost, or in other words, that the fixed cost of trade is not tax deductible. We make this assumption to preserve the tractability of the analysis. In the Appendix we show that the qualitative predictions remain unchanged if the fixed costs are tax deductible, while in the numerical investigation of Section 4.3 we show that the predictions of the specification with non-tax-

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\(^{34}\)Alternative modeling strategies can generate selection into exporting without fixed costs. For example, Bernard, Eaton, Jensen, and Kortum (2003) consider a model of Bertrand competition and Melitz and Ottaviano (2008) develop a model with non-CES demand.
deductible fixed costs are robust to making the fixed costs tax-deductible.

Under these circumstances, the problem of the agent with productivity \( n \) is given by:

\[
U_n = \max_{y, I_n \in \{0, 1\}} \left\{ \Delta + (1 - t) \left( 1 + I_n \tau^{\frac{\alpha}{1 - \alpha}} \right)^{1 - \beta} Q^{1 - \beta} y^\beta - v(y/n) - I_n f_x \right\}
\]

\[
= tR + (1 - t)(1 - \beta/\gamma) r_n - I_n f_x,
\]

where \( I_n \) is the indicator variable of the agent’s export status. The appendix derives the optimality conditions for the choice of \( y_n \) and \( I_n \): the condition for \( y_n \) is similar to the ones in the previous sections, while the entry cutoff condition is given by

\[
(1 - t)(1 - \beta/\gamma) \left[ r_{n_x^+} - r_{n_x^-} \right] = f_x,
\]

where \( n_x \) denotes the productivity cutoff, i.e. the productivity of an agent who is indifferent between serving and not serving the export market; \( n_x^+ \) (\( n_x^- \)) denotes the right (left) limit as the revenues jump discontinuously for an agent who decides to be an exporter. This condition implies that the increase in after-tax revenues net of the cost of effort should be large enough to cover the fixed cost for an agent to export profitably.\(^{35} \) The fixed cost \( f_x \) is assumed to be large enough so that \( n_x > n_{\text{min}} \). All agents with \( n > n_x \) serve the foreign market, while agents with \( n < n_x \) serve only the domestic market.

Equilibrium in the product market implies that aggregate output (which is split between private consumption and covering fixed costs of exporting) equals aggregate revenues, \( Q = R \). This equilibrium condition also implies balanced trade. Aggregate revenues are given by

\[
R = \int_0^\infty r_n dH(n) = Q^{1-\beta} \int_0^\infty \left( 1 + I_n \tau^{-\frac{\beta}{1-\beta}} \right)^{1-\beta} y_n^\beta dH(n).
\]

Together with the agent’s optimal choice of \( \{y_n, I_n\} \), this condition allows us to characterize the symmetric world equilibrium.

In the Appendix we show that the relative revenue of agent \( n \) in the economy with fixed costs is given by:

\[
\frac{r_n}{R} = \left( \frac{1 + I_n \tau^{-\frac{\beta}{1-\beta}}}{1 + I_n \tau^{-\frac{\beta}{1-\beta}}} \right)^{\frac{1-\beta}{\gamma-\beta}} \frac{n^{\beta/\gamma}}{n^{1-\beta/\gamma}} \prod_{n_{\text{min}}, n_x} \left\{ \left( 1 + \tau^{-\frac{\beta}{1-\beta}} \right)^{\frac{1-\beta}{\gamma-\beta}} n^{\beta/\gamma}, \quad n \in [n_{\text{min}}, n_x], \right. \\
\left. \left( 1 + \tau^{-\frac{\beta}{1-\beta}} \right)^{\frac{1-\beta}{\gamma-\beta}} n^{\beta/\gamma}, \quad n \in (n_x, n_{\text{max}}]. \right.
\]

Therefore, the distribution of relative revenues depends on the exporting cutoff \( n_x \) and on the foreign market access variable \( \Upsilon_x \). The former depends among other things on the fixed

\(^{35}\)If the fixed cost is tax deductible, agents will compare before-tax net revenues with the fixed cost. However, since taxes reduce the scale of production for all agents, an increase in taxes will have the same effect on the exporting cutoff \( n_x \) independently of whether the fixed cost is tax deductible or not.
cost of trade \( f_x \), while the latter depends on the variable cost of trade \( \tau \). Contrast this result with Lemma 4, which characterizes the distribution of relative revenues for the closed economy, as well as for the open economy without fixed costs. In closed economy and in open economy without selection into the export market, the distribution of relative revenues is uniquely pinned down by the underlying distribution of ability and does not respond to growing trade driven, for example, by a fall in \( \tau \).

Figure 1: Relative Revenues (left) and Utility (right) in Open Economy with Fixed Costs

Note: Although revenues and utilities increase for all agents in open economy relative to autarky, relative revenues fall for non-exporters and rise for exporters. Revenues increase discontinuously for exporters, while utility is continuous and experiences a kink at \( n_x \). Parameters for this figure are chosen as discussed in Section 4.3.2.

While revenues increase discontinuously for exporters, utility increases continuously with productivity \( n \), but experiences a kink at \( n_x \), as illustrated in Figure 1. Therefore, the access to foreign trade benefits disproportionately high-productivity agents who can easily cover the fixed cost and profitably export. This lies at the core of the distributional conflict in the open economy (cf. Helpman, Itskhoki, and Redding, 2008b). While in this model all agents gain from trade in absolute terms, trade worsens the inequality of relative revenues and utilities, as we discuss below. However, before we turn to the analysis of inequality, we summarize the comparative statics in the open economy with fixed costs:

Proposition 6 (i) A reduction in the cost of trade (variable \( \tau \) or fixed \( f_x \)) leads to an increase in real output \( Q \) and a reduction in exporting cutoff \( n_x \). (ii) All agents gain from greater trade integration, although these gains are not proportionally distributed.

We now characterize the patterns of income and utility inequality in the open economy.

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36This proposition requires a mild stability condition which is satisfied when \( \beta \) is not too small. Appendix contains a formal proof.
with fixed costs using (18) and (16). First, observe from (18) that when either no agent participates in trade \((n_x = n_{\text{max}})\) or all agents export \((n_x = n_{\text{min}})\), the distribution of relative revenues is determined by the underlying ability distribution, as described in Lemma 4. This is not surprising since the former case corresponds to the closed economy and the latter case is similar to an open economy with no (or negligibly small) fixed costs, as we studied in previous sections.

Figure 2: Inequality of Revenues in Open Economy with Fixed Costs relative to Autarky: against variable trade costs (left) and fraction of exporting agents (right)

Note: As variable cost of trade \(\tau\) falls, the export productivity cutoff \(n_x\) decreases and the fraction of exporting agents \(1 - H(n_x)\) increases. The two limiting cases \((n_x = n_{\text{max}}\) and \(n_x = n_{\text{min}}\) are attained respectively when \(\tau \to \infty\) and when \(\tau\) is just above 1. In these two limiting cases, the inequality of revenues is the same in the open economy as in autarky. Baseline parameters as in Figure 1.

Our central result, which we prove in the Appendix, is that whenever some but not all agents export, the dispersion of relative revenues is strictly greater than in autarky. Intuitively, when \(n_x \in (n_{\text{min}}, n_{\text{max}})\), the ranking of agents by revenues is the same as in autarky, but agents with high revenues \((n > n_x)\) earn relatively more, while agents with low revenues \((n < n_x)\) earn relatively less. Further, since inequality is the same in autarky and in the fully open economy, falling trade costs have a non-monotonic effect on inequality, first increasing it and later decreasing it, as illustrated in Figure 2.\(^{37}\) These results replicate the findings of Helpman, Itskhoki, and Redding (2008b) within a richer product and labor market equilibrium framework.

Finally, the implications for the inequality of utilities are largely the same as those for inequality of revenues. The only difference is that inequality of utilities is higher in an open economy with positive fixed cost than in autarky even when all agents export \((n_x = n_{\text{min}})\).

\(^{37}\)One can show that decreasing fixed costs of trade have an inverted U-shape effect on inequality.
This is because fixed costs of trade constitute a disproportionately higher burden for low productivity agents with relatively low revenues.

We summarize these findings in (see the Appendix for a formal proof):

**Proposition 7** The inequality of relative revenues and utilities is higher in the open economy than in autarky when some, but not all agents export. Falling trade costs first increase and then decrease income inequality.

Contrast this proposition with the result of Section 3.1 that opening up to trade in an economy without fixed costs has no effect on inequality. This emphasizes that what generates inequality response to trade in this type of models is not agent heterogeneity per se, by rather the selection into the exporting activity.

We now study how these distributional consequences of trade affect optimal taxation.

### 4.2 Optimal Linear Tax Rate

Through what channels do taxes affect welfare in the open economy with fixed costs? The effect of a linear tax on agents’ utilities in the open economy with fixed costs is still given by (9). The effect of taxes on exporting cutoff \( n_x \) does not influence agent \( n_x \)'s utility directly since on the margin this agent is indifferent between exporting and not exporting. The only indirect effect of taxes on utility is still coming through their effect on aggregate output \( Q \). Further, in the cooperative solution with symmetric countries, the first order condition for the optimal tax is still given by (10). Therefore, optimal tax rate is still determined by (11'):

\[
\frac{t}{1-t} = \alpha \cdot \frac{1}{\bar{\varepsilon}} - (1 - \beta)(1 - \alpha),
\]

where \( \bar{\varepsilon} \) and \( \alpha \) are defined as before. What changes from the closed economy environment is the output elasticity with respect to the tax rate, \( \bar{\varepsilon} \), and the equilibrium distribution of revenues which affects the inequality margin, \( \alpha \).

We start with the inequality margin. Proposition 7 states that dispersion of relative revenues and utilities increases in the open economy with fixed costs. This intensifies the inequality margin of the optimal taxation. In the Appendix we discuss conditions under which we can unambiguously predict that \( \alpha \) increases in the open economy. The numerical illustration provided in the left panel of Figure 3 demonstrates that \( \alpha \) tracks very closely the dispersion of relative revenues.

Consider now the efficiency margin. In the Appendix we prove the following:
Figure 3: Equity Margin (left) and Efficiency Margin (right)

Note: The left panel shows that inequality margin, $\alpha$, tracks closely the dispersion of relative revenues, $\text{var}(r/R)$. Note two differences: (i) for high values of $\tau$, $\alpha$ is much closer to $\alpha^a$ than $\text{var}(r/R)$ to $\text{var}(r^a/R^a)$. This is because with high $\tau$ and low $Y_x$, increased revenues of high-productivity exporters are almost entirely consumed by the fixed cost of trade; (ii) $\alpha > \alpha^a$ even when $\tau$ is low enough so that all agents participate in exporting. This is because fixed costs of trade constitute a disproportional burden on low-productivity agents with relatively low revenues, and as a result, the inequality of utilities (unlike the inequality of revenues) is still higher in this case than in autarky. The right panel illustrates how the efficiency margin $\tilde{\varepsilon}$ increases in the open economy over and above the labor supply elasticity $\varepsilon$ due to the extensive margin of trade; $\tilde{\varepsilon}/\varepsilon$ tracks closely $h(n_x)n_x$ until $n_x = n_{\text{min}}$ (here the productivity distribution is chosen to be Pareto). Baseline parameters as in Figure 1; in autarky, $\alpha^a = 0.204$ and $\text{var}(r^a/R^a) = 0.84$; linear tax rate held constant at the optimal autarky level ($t^a = 17.3\%$).

**Lemma 5** Assume the density $h(n)$ associated with cdf $H(n)$ exists at $n_x$. In an open economy equilibrium with fixed costs,

$$
\tilde{\varepsilon} \equiv \frac{\ln Q}{\ln(1-t)} = \varepsilon \cdot \frac{1 + \nu_x}{1 - \varepsilon(1 - \beta)\nu_x},
$$

where $\nu_x \equiv \frac{\gamma f_x}{\beta(1-t)R}h(n_x)n_x \geq 0$.

In addition, stability of the equilibrium requires $\varepsilon(1 - \beta)\nu_x < 1$, which is always satisfied for high enough $\beta$.

Note that whenever $h(n_x) > 0$, $\tilde{\varepsilon} > \varepsilon$, i.e. trade with fixed costs and selection into the export market magnifies the efficiency margin of taxation. The intuition is straightforward: Given selection into the export market, taxation negatively affects not only the intensive margin, but also the extensive margin of trade. Therefore, the overall efficiency loss is greater than in the case when the extensive margin does not respond. The behavior of the efficiency margin in an open economy with fixed costs is illustrated in the right panel of Figure 3. The conclusion here is that the same feature of the model which leads to greater inequality in an open economy, i.e. the fixed costs of trade and selection into the exporting activity, also necessarily magnifies the efficiency margin.

To summarize, efficiency and inequality margins are linked tightly together, and both are
driven by selection into the exporting activity. As a result, the optimal marginal tax rate can both go up or down in the open economy relative to autarky despite the fact that inequality increases. In other words, in order to capture the welfare gains from trade, a country has to accept an increase in income inequality, since trying to minimize income inequality resulting from trade brings about excessive efficiency losses.

![Figure 4: Optimal Linear Tax](image)

Note: This figure plots the optimal linear tax rate as a function of variable trade cost \( \tau \), for both the specifications with tax-deductible and non-tax-deductible fixed costs of trade. The behavior of the optimal tax rate reflects the balance between the efficiency and inequality margins in an open economy. For this calibration, the efficiency margin dominates, and the optimal linear tax rate is lower in an open economy in which only a fraction of agents export than in autarky. Note that when all agents export, the tax rate in an open economy can be greater than in autarky. This is because \( \alpha > \alpha^a \) in an open economy with fixed costs even when all agents export (see Figure 3). Baseline parameters as in Figure 1; the optimal autarky tax rate: \( t^a = 17.3\% \).

We illustrate these findings in Figure 4, where we plot the optimal linear tax rate as a function of variable trade cost \( \tau \), both for the case of non-tax-deductible and tax-deductible fixed cost. Qualitatively, the results are the same in both cases, although the quantitative response of the tax rate is larger when the fixed cost is not tax-deductible. Note that for our calibration, the efficiency margin dominates the inequality margin. As a result, the optimal linear tax is lower in an open economy in which only a fraction of agents export than in autarky. A reduction in the marginal tax rate further exacerbates the inequality effects of trade. In the following section we study how these conclusions hold to the introduction of additional tax instruments.

Contrast this policy response to the policy response in the closed economy to exogenously growing income inequality driven by the dispersion of underlying ability (see Proposition 2).

---

[^38]: It is also easy to develop an example in which the optimal tax is higher in an open economy equilibrium than in autarky. This simply requires choosing \( h(n_x) \approx 0 \). However, even in this case, the optimal tax rate will not be high enough to reduce the inequality margin \( \alpha \) to its autarky level.
In the closed economy, an increase in the dispersion of relative revenues leads to an increase in the inequality margin \( \alpha \), leaving the efficiency margin \( \bar{\epsilon} \) unchanged (Lemmas 2 and 3). Therefore, the optimal tax rate necessarily increases. We illustrate this in Figure 5. The left panel plots the optimal tax rates in the closed and open economies for the same equilibrium values of \( \alpha \); in an open economy \( \alpha \) is driven by a reduction in variable trade costs \( \tau \), while in the closed economy the dispersion of ability distribution is adjusted to induce the same values for \( \alpha \). As variable trade cost falls, the optimal tax rate in an open economy decreases despite an increase in the inequality margin \( \alpha \) (see left panel of Figure 3), while the same increase in \( \alpha \) would lead to a higher optimal tax rate in the closed economy. The right panel of Figure 5 plots the response of the optimal tax rates against the inequality margin \( \alpha \). Again in the open economy case, \( \alpha \) is driven by a reduction in variable trade cost, while in the closed economy it is driven by a corresponding increase in the dispersion of underlying ability distribution. The conclusion here is that the optimal policy response to growing income inequality can be very different depending on the original source of inequality increase.

### 4.3 Additional Tax Instruments

The restriction to a single tax rate appears to be particularly restrictive in the open economy case with fixed costs since there are two well-separated groups of agents – exporters and non-exporters – and an entry decision in addition to a baseline intensive margin. Therefore, we study here how our conclusions change when we introduce additional policy instruments such
as an entry subsidy and differential marginal tax rates for exporters and non-exporters.

Denote by $t_d$ and $t_x$ the marginal tax rates on gross revenues for non-exporters and exporters respectively. Denote by $s$ the export market entry subsidy. With this notation, the problem of an agent becomes:

$$U_n = \max_{y, I_x} \left\{ \Delta + \left[ 1 - t_d(1 - I_x) - t_x I_x \right] \left( 1 + I_x \tau^{\frac{\beta}{1 - \beta}} \right)^{1 - \beta} Q^{1 - \beta} y^\beta - v \left( \frac{y}{n} \right) - (f_x - s) I_x \right\}$$

and the government budget constraint implies

$$\Delta = \int_0^\infty \left\{ [t_d(1 - I_n) + t_x I_n] r_n - s I_n \right\} dH(n),$$

where $r_n = \left( 1 + I_n \tau^{\frac{1 - \beta}{1 - \beta}} \right)^{1 - \beta} Q^{1 - \beta} y_n^\beta$. The expression for aggregate output (17) remains unchanged; however, agents’ optimality conditions determining intensive and extensive margins are now different, as described in the Appendix.

Analytical characterization of the optimal policy with three instruments is intractable. We thereby rely on the numerical methods to characterize the optimal policy in this case. But before we do so, we characterize the optimal entry subsidy taking the marginal tax rates in the two brackets as given. This exercise allows us to develop intuition for interpreting the numerical results to follow.

### 4.3.1 Optimal Entry

First, consider the case of no inequality aversion ($\rho = 0$) which implies a utilitarian planner and no risk-aversion at the individual level. In this case we have the following result (see the Appendix):

**Proposition 8** With no inequality aversion ($\rho = 0$), the first best allocation requires $t_d = t_x = -(1 - \beta)/\beta$ and $s = 0$, i.e. a constant negative marginal tax rate to offset the monopolistic distortion and no entry subsidy.

This proposition implies that entry is efficient provided that taxes fully offset the monopolistic distortion.\(^{40}\) At the individual level the fixed cost is traded off for additional revenues from entry into the export market. At the aggregate level entry increases variety at the cost of allocating a greater share of output to cover the fixed costs. It turns out that when the fixed cost is not tax deductible. One can think alternatively about this setup as a two-bracket tax system.

\(^{39}\)Note that with these three instruments available, it is without loss of generality (for the set of feasible allocations) to assume that the fixed cost is not tax deductible. One can think alternatively about this setup as a two-bracket tax system.

\(^{40}\)Interestingly, in the Melitz (2003) setup the allocation is efficient without any government intervention as the monopolistic profits are fully used to cover the entry costs as in Dixit and Stiglitz (1977).
monopolistic distortions are offset, private incentives are perfectly aligned with the public welfare. Finally, note that under no inequality aversion a single linear tax is sufficient to fully restore efficiency and attain the first best allocation.

We now characterize the optimal utilitarian entry subsidy with exogenously set marginal tax rates on exporters and non-exporters (see the Appendix):

**Proposition 9** Let the marginal tax rates $t_d$ and $t_x$ be given exogenously. Then: (i) With no inequality aversion ($\rho = 0$) and given $t_d = t_x = t$, the optimal entry subsidy is

$$s^o = f_x \frac{1 - \beta(1 - t)}{1 - \frac{\beta}{\gamma}(1 - t)}.$$  

Optimal utilitarian subsidy $s^o$ is increasing in $t$. When $t_d \neq t_x$, $\partial s^o / \partial t_d < 0$ and $\partial s^o / \partial t_x > 0$. (ii) With positive inequality aversion ($\rho > 0$), the optimal subsidy is strictly smaller than $s^o$ given the same level of marginal tax rates $t_d$ and $t_x$.

This proposition is useful in developing intuition for the numerical results below. First, distortionary taxes ($t > -(1 - \beta)/\beta$) negatively affect the entry decision and need to be offset by an entry subsidy. Second, this effect is stronger, the larger is the difference between the marginal tax rates on exporters and on non-exporters (formally, the smaller is $(1 - t_x)/(1 - t_d)$). Third, setting $t_d > t_x > -(1 - \beta)/\beta$ may partly or fully offset the need to use an entry subsidy.

The overall lessons here are twofold: (1) when marginal taxes are used to reduce inequality or for other exogenous reasons, entry has to be subsidized to avoid significant efficiency losses due to adjustment on the extensive margin; (2) proper entry incentives can be generated via different combinations of policy instruments (e.g., low $t_x$ relative to $t_d$ instead of positive $s$), which can be useful when entry subsidy is unavailable.\(^{41}\)

Finally, since entry has distributional consequences, under inequality aversion optimal entry subsidy will be lower than the utilitarian entry subsidy, and as a result entry will be less than efficient (from the point of view of a society that does care about inequality). With these results in mind, we turn to our numerical exercise.

### 4.3.2 Numerical Solution

Before discussing the results of our numerical analysis, we briefly comment on the calibration of the parameters of the model. We choose the Pareto productivity distribution with a

\(^{41}\)As we discuss in the numerical section, the tax system will be adjusted in a way to both reduce inequality and provide entry incentives if some of the instruments are not available.
shape parameter of 2.2, consistent with the findings in Saez (2001) for the upper tail of the productivity distribution, where the entry decision is particularly relevant in our calibration. In the baseline case, the inequality aversion \( \rho \) is set to 2, which can be viewed as a combination of logarithmic individual preferences \( (\rho_a = 1) \) and logarithmic social welfare function \( (\rho_g = 1) \). The demand parameter, \( \beta \), is calibrated to 0.75, which implies an elasticity of substitution of 4, consistent with the estimates in Bernard, Eaton, Jensen, and Kortum (2003) and Broda and Weinstein (2006). For labor supply elasticity \( \varepsilon \), we use the value of 0.5, consistent with the empirical estimates surveyed in Tuomala (1990, Chapter 3) and used in Saez (2001, 2002).

The fixed costs of trade \( f_x \) are calibrated such that 40% of output is produced by exporting agents and exports account for 15% of consumption, also consistent with the evidence. Finally, \( \tau \) is set in the baseline case to 1.5, corresponding to a variable trade cost of 50% in line with the estimates in Anderson and van Wincoop (2004). In our analysis we solve numerically for the optimal tax policy (given different sets of available policy instruments) for different values of the variable trade cost \( \tau \in [1, 2] \).

We consider the following sets of policy instruments: (1) a single linear tax rate \( (t) \) with both deductible and non-deductible fixed costs; (2) a single linear tax rate with an entry subsidy \( (t \text{ and } s) \); (3) two separate tax rates for exporters and non-exporters \( (t_d \text{ and } t_x) \); (4) all three policy instrument \( (t_d, t_x \text{ and } s) \). In some cases we also solve for the optimal linear tax rate given a utilitarian subsidy, \( s^\circ \).

The left panel of Figure 6 plots our first results regarding the optimal amount of entry in the export market \( (n_x) \). The dotted line characterizes the first-best entry under a utilitarian planner \( (n_x^\circ) \). With positive inequality aversion and for any set of available policy instrument, the optimal entry is strictly less then under the utilitarian planner \( (n_x > n_x^\circ) \). Note that this gap is greater when only a single tax instrument is available, but it becomes very small when two or more policy instruments are available.

The right panel of Figure 6 plots the fraction of welfare gains attained with a given set of policy instruments, where 100% corresponds to the welfare gains with all three policy instruments relative to no taxation. As variable trade costs fall and the trade flows increase, having a richer set of policy instruments becomes increasingly important. Having only a single tax rate may allow to capture as little as 50% of the welfare gains when \( \tau \) is small, while having a single tax rate and an entry subsidy always guarantees at least 85% of the welfare gains. The case with the marginal tax rates but no entry subsidy falls in between.

The overall lesson from the two panels of Figure 6 is that having a single linear tax in an environment with entry into the export market is very restrictive, both for encouraging
the right amount of entry and for capturing the welfare gains. A single tax rate is not very successful in balancing the entry decision with redistribution. However, as soon as two or more policy instruments are available, nearly optimal entry can be ensured and a large fraction of the welfare gains can be captured.

Figure 7 plots the optimal entry subsidy (as a fraction of fixed cost $f_x$) as a function of variable trade costs and for different sets of policy instruments. First, note that when only a single tax rate is available (two lines in the middle), the optimal subsidy $s$ is very close to the utilitarian subsidy $s^\circ$, but $s < s^\circ$ in all cases, consistent with Proposition 9. Second, the lower line corresponds to a hypothetical utilitarian subsidy in the case when only two marginal tax rates are available. Note that this subsidy is relatively small, suggesting that the tax system with two marginal tax rates will be adjusted in order to encourage entry (i.e., $t_x < t_d$ as we show below). Finally, the upper line is the optimal subsidy in the case when
all three policy instruments are available. Note that in this case, \( s \approx f_x \), i.e. almost all fixed costs of entry are reimbursed by the government. As we show below, this is the case because the government uses the available marginal tax rates to aggressively redistribute away from the high productivity exporters (i.e., \( t_x \gg t_d \)).

Further, Figure 8 plots the marginal tax rates as a function of variable trade costs with different sets of policy instruments. The left panel compares the cases with and without an entry subsidy when only a single marginal tax rate is available. The result is quite intriguing, although very intuitive. When an entry subsidy is unavailable, the marginal tax rate has to go down in the open economy in order to encourage more entry (cf. Figure 4 in Section 4.2). As a result, equality has to be traded off to ensure moderately efficient entry, which leads to a poor welfare performance of a single linear tax rate (Figure 6). In contrast, when an entry subsidy is available, the marginal tax rate goes up. The entry subsidy ensures the right amount of entry, while a higher marginal tax rate allows to moderate the increased inequality. As Figure 6, this combination of policy instruments is very successful from the welfare point of view.

Figure 8: Marginal Taxes

The right panel of Figure 8 plots the marginal taxes on exporters (dashed lines) and non-exporters (solid lines) both when an entry subsidy is and is not available. The results here are even more intriguing. When the entry subsidy is unavailable, the tax system becomes increasingly regressive as the trade costs fall, i.e. the marginal tax rate on poor low-productivity non-exporters \( t_d \) increases, while the marginal tax rate on rich high-productivity exporters \( t_x \) falls. At first glance counter-intuitive, this tax system adjustment is necessary to encourage the right amount of entry. However, it only further exacerbates increased inequality, which is why this tax system does not perform that well on the welfare account (right panel of Figure 6). Finally, when the entry subsidy is available, the tax system becomes strongly
progressive, with a marginal tax rate on exporters almost twice as high as on non-exporters for low values of $\tau$. Here again, the entry subsidy ensures the right amount of entry, while the marginal taxes allow to aggressively redistribute away from the relative winners towards relative losers from trade.

The overall lesson from both panels of Figure 8 is that once a policy instrument (in this case an entry subsidy) is restricted, the optimal choice for other instruments may dramatically change in order to replicate the effects of the missing instrument. In particular, the tax schedule may revert from very progressive to strongly regressive.

Finally, in Figure 9 we plot the welfare and inequality outcomes of optimal policy for different sets of instruments. The right panel plots welfare as a function of trade costs – the dashed line in case with no redistribution policy and solid lines when optimal policies are in place. Note that the gains from trade are present even when $t = 0$ or $t = t^a$ is held fixed. However, having three tax instruments in place, allows to magnify the welfare gains from trade quite substantially when trade costs are small.

![Figure 9: Equilibrium Welfare and Inequality](image-url)

Lastly, the right panel of Figure 9 plots the resulting inequality of revenues the optimal policies are in place (the dashed line again corresponds to $t = 0$). Note that in all cases, even when the policy responds optimally to trade, trade results in higher inequality relative to autarky. Moreover, the optimal policy response often exacerbates inequality relative to a passive redistribution policy. For example, this is the case when three instruments are available and trade costs are high ($\tau > 1.5$), and this is the case when only a linear tax is available and costs are low ($\tau < 1.4$). This confirms our conclusion in the end of Section 4.2 that in order to capture the most gains from trade, the countries may need to accept increased income inequality.
5 Discussion

Selection into export markets has consequences for both income inequality and efficiency losses from taxation. Trade openness intensifies both sides of the equity-efficiency trade-off, making the optimal redistribution policy response to trade ambiguous. More generally, when the original source of increasing income inequality is also inherently connected with the extent of welfare losses from taxation, the optimal policy response should not necessarily be to offset rising inequality.

The results of the paper immediately extend to any activity that features fixed costs and a non-trivial entry decision so that only the most productive agents can profitably participate. Specifically, consider the case of technology adoption. If new technology adoption requires paying a fixed cost, this will lead to similar implications for inequality and optimal policy response. Note that even with one heterogenous factor of production and no built-in skill-bias of technology, fixed costs and participation decision are enough to cause rising inequality in response to skill-neutral technological change.

The prediction of the model for the tax system is the following. In economic activities with extensive margin response, it is optimal to subsidize entry by covering a fraction of fixed costs and then redistribute revenues away from high-productivity agents who take advantage of the fixed cost activity using a progressive tax schedule. When, however, certain policy instruments are unavailable, other policy instruments should adjust in order to replicate as close as possible the optimal allocation. Specifically, when an entry subsidy is infeasible, it might well become optimal to use a regressive tax schedule, which encourages entry at the cost of less redistribution.

In the next versions of this paper we plan to solve numerically for the fully unrestricted Mirrlees policy, and study how closely a two-brackets tax system replicates the unrestricted allocation. Finally, an interesting avenue for further research is to contrast the optimal taxation of activities that are and are not subject to free entry conditions. This may shed light on the differential response of profit and income (or capital and labor) taxation in an open economies.
Appendix

A Derivations and Proofs for Section 2

Consider the problem of the government:

$$W(t) \equiv L \int_0^\infty G(U_n) dH(n) \rightarrow \max_t$$

subject to

$$U_n = \max_y \left\{ tR + (1 - t)Q^{1-\beta}y^{\beta} - v(y/n) \right\},$$

where $LR = Q = \left[ L \int_0^\infty y_\beta dH(n) \right]^{1/\beta}$ and $y_n$ maximizes agent $n$'s utility, which with our function form $v(x) = x^{\gamma/\gamma}$ yields

$$y_n = \left[ \beta(1-t) \right]^{\frac{\gamma}{\gamma-\beta}} Q^{\frac{1-\beta}{\gamma-\beta}} n^{\frac{\gamma}{\gamma-\beta}}.$$

With this solution for output $y_n$, we have the following individual revenue and utility:

$$r_n = \left[ \beta(1-t) \right]^{\frac{\beta}{\gamma-\beta}} Q^{\frac{1-\beta}{\gamma-\beta}} n^{\frac{\beta}{\gamma-\beta}},$$

$$U_n = tR + (1 - t)(1 - \beta/r_n).$$

The equilibrium differential of the agent’s utility with respect to the tax rate is

$$\frac{dU_n}{dt} = (R - r_n) + t \frac{dR}{dt} + (1 - t)(1 - \beta)r_n \frac{1}{Q} \frac{dQ}{dt}, \quad r_n = Q^{1-\beta} y_n^{\beta}.$$

The term $dy_n/dt$ enters into the differential with a coefficient of zero due to the Envelop Theorem since agents choose $y_n$ optimally.\textsuperscript{42} Further, since $R = Q/L$, we have $d\ln R = d\ln Q$ and we can rewrite:

$$\frac{dU_n}{dt} = (R - r_n) - \tilde{\varepsilon} \left[ \frac{t}{1-t} R + (1 - \beta) r_n \right], \quad \tilde{\varepsilon} \equiv \frac{d\ln Q}{d\ln(1-t)} = -\frac{(1-t)dQ}{Q d\ln t}.$$

Consider now the first order condition for the government’s problem:

$$\frac{\partial W(t)}{L \partial t} = \int_0^\infty G'(U_n) \frac{dU_n}{dt} dH(n) = \int_0^\infty G'(U_n) \left\{ (R - r_n) - \tilde{\varepsilon} \left[ \frac{tR}{1-t} + (1 - \beta) r_n \right] \right\} dH(n) = 0.$$

Expressing out $t/(1 - t)$, we have:

$$\frac{t}{1 - t} = \frac{1}{\tilde{\varepsilon}} \cdot \alpha - (1 - \beta)(1 - \alpha),$$

where

$$\alpha = \int_0^\infty G'(U_n) (R - r_n) dH(n) \lambda R, \quad \lambda \equiv \int_0^\infty G'(U_n) dH(n).$$

\textsuperscript{42}If agents were price takers, $dy_n/dt$ would not be zero since agents are not fully optimizing in this case. Instead the sum $[(1 - \beta)d\ln Q + \beta d\ln y_n]/dt$ would be zero. Therefore, both terms with $dQ/dt$ and $dy_n/dt$ drop out from the expression for $dU_n/dt$. This then results in (11) becoming $t/(1 - t) = \alpha/\tilde{\varepsilon}$, just as in the case with $\beta = 1$. 

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Note that $\alpha$ is a cross-sectional covariance between two normalized variables, $\lambda^{-1}G'(U)$ and $1-r/R$, both with a mean of 1. Alternatively, $\alpha$ is minus the covariance between $\lambda^{-1}G'(U)$ and $r/R$.

With this notation we can rewrite the first order condition as:

$$\frac{\partial W(t)}{\partial t} = \lambda^{-1}R \left[ \alpha - \tilde{\varepsilon} \left( \frac{t}{1-t} + (1-\beta)(1-\alpha) \right) \right] = 0.$$ 

Then the second order condition can be written as:

$$\frac{\partial^2 W(t)}{\partial t^2} = \lambda^{-1}R \left[ \frac{\partial \alpha}{\partial t} \left( 1 + \tilde{\varepsilon}(1-\beta) \right) - \frac{\tilde{\varepsilon}}{(1-t)^2} \right] ,$$

where we used the result of Lemma 2 that $\tilde{\varepsilon} = \varepsilon$ and does not depend on $t$ in the closed economy. Note that $\partial\alpha/\partial t < 0$ is sufficient, but is not necessary for the second order condition to be satisfied. Finally, one can show that $\partial\alpha/\partial t < 0$ is also sufficient for the concavity of $W(t)$ on the relevant range (i.e., for $t < 1/(1+\varepsilon)$).

We can now prove:

**Proof of Lemma 1** Since $G(\cdot)$ is increasing and concave, we have $G'(\cdot) \geq 0$ and $G''(\cdot) \leq 0$. Therefore, $G'(U_n)$ is positive and decreasing with $r_n$, since $U_n$ increases with $r_n$. This implies that $\alpha \geq 0$ as the negative of a covariance between two variables that move in the opposite directions. Next note that we can write

$$\alpha = 1 - \int_0^\infty \lambda^{-1}G'(U_n) \frac{r_n}{R} dH(n) \leq 1,$$

since the second term is non-negative as an integral of a two non-negative function. Therefore, we conclude that $0 \leq \alpha \leq 1$.

Further, $\alpha = 0$ if and only if either $r_n/R \equiv 1$ or $G'(U_n) = \text{const}$. The former is the case when $H(n)$ is degenerate with all mass at one value of $n = n_{\text{min}} = n_{\text{max}}$ implying $r_n \equiv R$ for all agents. The latter is the case when $G'(\cdot) \equiv 1$, i.e. $\rho = 0$ and there is no inequality aversion.

Finally, $\alpha = 1$ if and only if $G'(U_n) \cdot r_n = 0$ with probability 1. This is the case if and only if the planner has all weight on agents with no revenues. This, however, requires both that the planner is Rawlsian ($\rho \to \infty$) and that the least productive agent does not produce ($r_{n_{\text{min}}} = 0$). The utility of this agent is given by $U_n = tR$ and its maximization results in $t/(1-t) = 1/\tilde{\varepsilon}$, or $t = 1/(1+\tilde{\varepsilon})$, which is at the peak of the Laffer curve. ■

**Proof of Lemma 2** Log-differentiating the definition of $Q$, (2), we have

$$\frac{d \ln Q}{d \ln (1-t)} = L \int_0^\infty \left( \frac{y_n}{Q} \right)^\beta \frac{d \ln y_n}{d \ln (1-t)} dH(n).$$

From the expression for agents’ optimal output $y_n$, we have:

$$\frac{d \ln y_n}{d \ln (1-t)} = \frac{1}{\gamma - \beta} + \frac{1-\beta}{\gamma - \beta} \frac{d \ln Q}{d \ln (1-t)}.$$ 

Combining these two expressions and observing that $L \int_0^\infty (y_n/Q)^\beta dH(n) = 1$ from the definition of $Q$, we obtain:

$$\tilde{\varepsilon} \equiv \frac{d \ln Q}{d \ln (1-t)} = \frac{1}{\gamma - 1} = \varepsilon.$$ ■
Comparative Statics for $\alpha$. Denote $x = r/R$. From Lemma 4, the distribution of $x$, $\varphi(x)$, is the normalized distribution of $n^{\beta \gamma / (\gamma - \beta)}$ with mean 1, and hence it does not depend on $t$ or $\rho$. Therefore, we can treat $\varphi(x)$ as being given exogenously. Denote $\delta = (1 - \beta / \gamma)(1 - t)/t$. Then we can write $\alpha$ as

$$\alpha = \text{cov} \left( 1 - x, \frac{(1 + \delta x)^{-\rho}}{\mathbb{E}(1 + \delta x)^{-\rho}} \right) = \frac{\int_0^\infty (1 - x)(1 + \delta x)^{-\rho} \varphi(x) dx}{\int_0^\infty (1 + \delta x)^{-\rho} \varphi(x) dx}.$$

One can show that for $\rho > 0$,

$$\text{sign} \left( \frac{\partial \alpha}{\partial \delta} \right) = -\text{sign} \left( \frac{\partial \alpha}{\partial \delta} \right) = \text{sign} \left( \text{cov} \left( 1 - x, \frac{x(1 + \delta x)^{-\rho - 1}}{\mathbb{E}(1 + \delta x)^{-\rho - 1}} \right) - \frac{(1 + \delta x)^{-\rho}}{\mathbb{E}(1 + \delta x)^{-\rho}} \right).$$

The condition for $\partial \alpha/\partial t < 0$ can be written as follows:

$$\int_0^\infty \frac{x \varphi(x) dx}{(1 + \delta x)^{1+\rho}} \int_0^\infty \frac{x \varphi(x) dx}{(1 + \delta x)^{\rho}} < \int_0^\infty \frac{x^2 \varphi(x) dx}{(1 + \delta x)^{\rho + 1}} \int_0^\infty \frac{\varphi(x) dx}{(1 + \delta x)^{\rho}}.$$

Although this inequality is very close to Cauchy-Schwarz inequality, it does not hold for a general distribution $\varphi(x)$. However, this inequality can be shown numerically to hold for Uniform and Pareto distributions.

Proof of Lemma 3 Using the expression for $U_n$, we have

$$G'(U_n) = (tR + (1-t)(1-\beta / \gamma)r_n)^{-\rho} = \left( [t + (1 - \beta / \gamma)(1 - t)] R \right)^{-\rho} \left[ 1 + \frac{1}{1 + \frac{1}{1-t} \frac{1}{1-\beta / \gamma}} \left( \frac{r_n}{R} - 1 \right) \right]^{-\rho}.$$

Taking the first order Taylor approximation to $G'(U_n)$ around $r_n/R = 1$ yields:

$$G'(U_n) = \left( [t + (1 - \beta / \gamma)(1 - t)] R \right)^{-\rho} \left[ 1 - \rho \frac{1}{1 + \frac{1}{1-t} \frac{1}{1-\beta / \gamma}} \left( \frac{r_n}{R} - 1 \right) \right] + O \left( \frac{r_n}{R} - 1 \right)^2.$$

This implies

$$\lambda = \int_0^\infty G'(U_n) dH(n) = \left( [t + (1 - \beta / \gamma)(1 - t)] R \right)^{-\rho} + O_p \left( \frac{r}{R} - 1 \right)^2$$

and, therefore,

$$\lambda^{-1} G'(U_n) = 1 - \rho \frac{1}{1 + \frac{1}{1-t} \frac{1}{1-\beta / \gamma}} \left( \frac{r_n}{R} - 1 \right) + O_p \left( \frac{r}{R} - 1 \right)^2.$$

Now using the definition of $\alpha$, we have

$$\alpha = -\text{cov} \left( \lambda^{-1} G'(U), \frac{r}{R} \right) = \rho \frac{1}{1 + \frac{1}{1-t} \frac{1}{1-\beta / \gamma}} \text{var} \left( \frac{r}{R} \right) + O_p \left( \frac{r}{R} - 1 \right)^3. \quad \blacksquare$$

Proof of Lemma 4 The solution to agent $n$’s problem yields:

$$r_n = \left[ \beta (1 - t) \right]^{\frac{\beta}{\gamma - \beta}} Q^{\frac{1 - \beta}{\gamma - \beta}} n^{\frac{\beta}{\gamma - \beta}}.$$

Since $R = \int_0^\infty r_n dH(n)$, we have

$$\frac{r_n}{R} = \frac{n^{\frac{\beta}{\gamma - \beta}}}{\int_0^\infty n^{\frac{\beta}{\gamma - \beta}} dH(n)}.$$
Therefore, the distribution of \( r_n/R \) is the same as the normalized distribution of \( n^{\beta \gamma / (\gamma - \beta)} \). An increase in the dispersion of \( n^{\beta \gamma / (\gamma - \beta)} \) holding its mean constant leads to an increase in the dispersion of relative revenues.

**Proof of Proposition 2**  Assuming that the approximation in Lemma 3 is accurate, we have

\[
\frac{t}{1 - t} = \alpha \left( \frac{1}{\varepsilon} + (1 - \beta) \right) - (1 - \beta), \quad \alpha \approx \rho \frac{1}{1 + \frac{t}{1 - t} \frac{1}{1 - \beta/\gamma}} \text{var} \left( \frac{r}{R} \right).
\]

This immediately implies \( \partial t / \partial \rho > 0 \) and \( \partial t / \partial \text{var}(r/R) > 0 \). Next, Lemma 4 implies that \( L \) does not affect the equilibrium distribution of \( r/R \), and hence, it also does not affect the optimal tax rate, \( t \). Finally, combining Lemmas 3 and 4, we have

\[
\alpha \approx \rho \frac{1}{1 + \frac{t}{1 - t} \frac{1}{1 - \beta/\gamma}} \left( \frac{\beta \gamma}{\gamma - \beta} \right)^2 \text{var}(\ln n).
\]

Choose \( \rho \text{var}(\ln n) \) such that optimal \( t = 0 \). Then, using \( \gamma = 1 + 1/\varepsilon \), we have

\[
\text{sign} \left( \frac{\partial t}{\partial \varepsilon} \right) = \text{sign} \left( 2\beta - 1 - \frac{1}{\varepsilon} \right) \gtrless 0. \quad \blacksquare
\]

**B Results, Derivations and Proofs for Section 3**

**Equilibrium with Asymmetric Countries**  As discussed in footnote 18, the nominal revenues in the domestic market are given by \( PQ^{1-\beta} y_1^{\beta} \). It is convenient to denote \( Z \equiv QP^{1/(1-\beta)} \), so that nominal revenues can be written as \( Z^{1-\beta} y_1^{\beta} \). Nominal revenues from exports are then given by \( Z^{1-\beta} (y_x/\tau)^\beta \). The agent optimally splits his production \( y \), to supply \( y_d \) in the domestic and to export the remaining \( y_x = y - y_d \) to the foreign market, in order to maximize real revenues

\[
\left[ Z^{1-\beta} y_1^{\beta} + Z^{1-\beta} (y_x/\tau)^\beta \right] / P.
\]

This results in the following real revenue function:

\[
r(y) = \Upsilon_x^{1-\beta} Q^{1-\beta} y_x, \quad \Upsilon_x \equiv 1 + \tau^{-\beta} \frac{Z^*}{Z}.
\]

In addition, the split of output between the domestic and foreign market is given by \( y_d = y / \Upsilon_x \) and \( y_x = (\Upsilon_x - 1)y / \Upsilon_x \). Note the more general relative to (14) definition of the market access variable, \( \Upsilon_x \), which now increases in the relative measure of nominal demand, \( Z^*/Z \). \( Z^*/Z \) can be high either because consumption is relatively high abroad or because the price level is relatively high abroad. Of course, in equilibrium \( Q^*/Q \) and \( P^*/P \) are closely connected.

The problem of an agent with productivity \( n \) is given by

\[
U_n = \max_y \left\{ \Delta + (1 - t) \Upsilon_x^{1-\beta} Q^{1-\beta} y_x^{\beta} - \frac{1}{\gamma} \left( \frac{y}{n} \right)^\gamma \right\}.
\]

It results in

\[
y_n = [\beta(1 - t)]^{1/\gamma} \Upsilon_x^{1-\beta} Q^{1-\beta} n^{\gamma}, \\
r_n = \Upsilon_x^{1-\beta} Q^{1-\beta} y_n, \\
U_n = tR + (1 - t)(1 - \beta/\gamma)r_n,
\]
where the last expression utilizes the government budget constraint $\Delta = tR$ and $R = \int_0^\infty r_n dH(n)$ is the average revenue.

Denote by $Y$ the per capita total domestic production of the final good: $$Y = \left[ \int_0^\infty y_n^\beta dH(n) \right]^{1/\beta}.$$ Then we have $$R = \Upsilon_x^{1-\beta} Q^{1-\beta} Y^\beta$$ and from agents’ optimization $$Y^\beta = \left[ \beta (1-t) \right]^{\beta \frac{1-\beta}{\gamma-\beta}} \Upsilon_x^{\frac{1-\beta}{\gamma-\beta}} Q^{\frac{1-\beta}{\gamma-\beta}} \Theta, \quad \text{where} \quad \Theta \equiv \int_0^\infty n^\frac{\beta \gamma}{\gamma-\beta} dH(n).$$ Domestic consumption is given by $$Q = \left[ L \left( \frac{1}{\Upsilon_x} \right)^\beta Y^\beta + \tau^{-\beta} L^* \left( \frac{Y^*_x - 1}{Y^*_x} \right)^\beta Y^*_x \right]^{1/\beta}.$$ Balanced trade implies $LR = Q$. Using the expressions for $R$ and $Q$, balanced trade condition can be rewritten as $$\frac{LY^\beta}{L^* Y^*_x} = \left( \frac{Z}{Z^*} \right)^{1+\beta} \left( \frac{\Upsilon_x}{\Upsilon^*_x} \right)^\beta.$$ Using the expression for $R$, this can be rewritten as

$$\left( \frac{Q}{Q^*} \right)^\beta = \frac{\Upsilon_x}{\Upsilon^*_x} \left( \frac{Z}{Z^*} \right)^{1+\beta}, \quad \frac{\Upsilon_x}{\Upsilon^*_x} = \frac{Z^*}{Z} \frac{Z + \tau^{-\beta} Z^*}{Z + \tau^{-\beta} Z^*}.$$ \hfill (21)

This condition constitutes an equilibrium relationship between $Q/Q^*$ and $Z/Z^*$, and by consequence with $P/P^*$. Next, combining the expression for $Y$ and $R$, we obtain the expression for domestic consumption $$Q^\beta \frac{\gamma-1}{\gamma-\beta} = \left[ \beta (1-t) \right]^{\frac{\beta}{\gamma-\beta}} L^\gamma \Theta \Upsilon_x^{\frac{1-\beta}{\gamma-\beta}}.$$ \hfill (22)

Conditions (21), (22) and its counterpart for the foreign country are sufficient to solve for equilibrium $Q$, $Z/Z^*$, $\Upsilon_x$ and so forth. All the comparative statics can be done on these two conditions. Finally, (22) and its foreign counterpart result in

$$\left( \frac{Q}{Q^*} \right)^{\beta \frac{\gamma-1}{\gamma-\beta}} \left( \frac{Z}{Z^*} \right)^{(1+\beta)\frac{\gamma-1}{\gamma}} = \left( \frac{L^\gamma \Theta}{L^* \Theta^*} \right)^{\frac{\gamma-\beta}{\gamma-\beta}} \left( \frac{1-t}{1-t^*} \right)^{\frac{\gamma}{\gamma-\beta}}.$$ \hfill (23)

**Proof of Proposition 3** Combining the expressions for $r_n$ and $y_n$, we have $$r_n = \left[ \beta (1-t) \right]^{\frac{\beta}{\gamma-\beta}} \Upsilon_x^{\frac{1-\beta}{\gamma-\beta}} Q^\beta \frac{\gamma-1}{\gamma-\beta} n^\frac{\beta \gamma}{\gamma-\beta}.$$ Therefore, relative revenue is $$\frac{r_n}{R} = \frac{\int_0^\infty r_n dH(n)}{\int_0^\infty n^\frac{\beta \gamma}{\gamma-\beta} dH(n)} = \frac{n^\frac{\beta \gamma}{\gamma-\beta}}{\int_0^\infty n^\frac{\beta \gamma}{\gamma-\beta} dH(n)},$$
as in the closed economy (Lemma 4). Note that this prove is invariant to the asymmetry of countries, the level of taxes in the two countries \( t \) and \( t^* \) and other details of the open economy equilibrium. ■

**Proof of Proposition 4** From (22), we have that open economy equilibrium aggregate consumption is given by

\[
Q = \left[ \beta (1 - t) \right]^{\varepsilon} (LE)^{\varepsilon \frac{\gamma - \beta}{\gamma}} \Upsilon_x^{\frac{1 - \beta}{\gamma}}, \quad \varepsilon = \frac{1}{\gamma - 1}.
\]

Note that in an open economy equilibrium with positive trade flows \( \Upsilon_x > 1 \) and the autarky obtains as a special case when \( \tau \rightarrow \infty \) and hence \( \Upsilon_x \rightarrow 1 \). Therefore, we have

\[
\frac{Q}{Q^a} = \left( \frac{1 - t}{1 - t^a} \right)^{\varepsilon} \Upsilon_x^{\frac{1 - \beta}{\gamma}},
\]

where superscript \( a \) stands for autarky. Since \( LR = Q \) both in open economy and in autarky, we have \( \frac{R}{R^a} = \frac{Q}{Q^a} \). Finally, since Proposition 3 implies \( r_n/R = r_n^a/R^a \), and in equilibrium \( \Upsilon_n = tR + (1 - t)(1 - \beta/\gamma)r_n \), we have

\[
\forall n \quad \frac{\Upsilon_n}{\Upsilon_n^a} = \frac{r_n}{r_n^a} = \frac{R}{R^a} = \frac{Q}{Q^a} = \left( \frac{1 - t}{1 - t^a} \right)^{\varepsilon} \Upsilon_x^{\frac{1 - \beta}{\gamma}}.
\]

Therefore, setting \( t = t^a \) in the open economy allows to increase utilities proportionally for all agents, and hence, increases welfare. This is independent of the tax rate in the trade partner and any asymmetries between countries since in \( \Upsilon_x \geq 1 \) and in any open economy equilibrium with positive trade flows the inequality is strict. Finally, by choosing \( t \) optimally, the government will only further increase welfare. In other words, trade necessarily improves the choice set of the government. A similar proof allows to demonstrate welfare gains from any marginal reduction in trade costs \( \tau \).■

**Losers from Trade** Consider agents with lowest skill \( n_0 = 0 \) and highest skill \( n_\infty = \infty \) (even if these agents are imaginary and never occur in the model economy). The utilities of these agents are \( \Upsilon_0 = tR \) and \( \Upsilon_\infty = (1 - t)(1 - \beta/\gamma)r_\infty \) as the former agent does not produce and for the latter agent the government’s transfer is negligible relative to his after-tax revenues. From the discussion above we know that \( r_n \) for all \( n \) and \( R \) increase proportionally in the open economy. Therefore, if both \( n_0 \) and \( n_\infty \) gain from trade, then all agents in the economy gain from trade. Moreover, since there are aggregate gains from trade, \( n_0 \) necessarily gains when \( t > t^a \) and \( n_\infty \) necessarily gains when \( t < t^a \). The conditions for these agents to lose from trade can be written as follows:

\[
\frac{d \ln \Upsilon_0}{d \ln Q} = -\frac{1 - t}{t}d \ln (1 - t) + d \ln Q < 0,
\]

\[
\frac{d \ln \Upsilon_\infty}{d \ln Q} = d \ln (1 - t) + d \ln Q < 0,
\]

where \( t \) adjusts in response to trade and \( Q \) changes in responds to both trade and a change in \( t \). Comparative statics below allows to rewrite these conditions in terms of primitives. As discussed above, only one condition can be satisfied at a time. When neither of the conditions is satisfied, all agents in the economy gain from trade.

---

43One additional step in this proof is to show that \( \Upsilon_x \) necessarily increases as \( \tau \) falls (see comparative statics below).
Comparative Statics with Asymmetric Countries  Take the total log-differential of (21)-(23) and the expression for \( \Upsilon_x \):

\[
\beta \left( \dot{Q} - \dot{Q}^* \right) = (1 + \beta) \left( \dot{Z} - \dot{Z}^* \right) + \left( \dot{\Upsilon}_x - \dot{\Upsilon}^*_x \right)
\]

\[
\dot{Q} = \epsilon d \ln(1 - t) + \frac{1 - \beta}{\beta} (1 + \epsilon) \dot{\Upsilon}_x
\]

\[
\epsilon (\gamma \beta - 1) \left[ \dot{Q} - \dot{Q}^* \right] + \frac{1 - \beta}{\beta} (1 + \epsilon) (1 + \beta) \left[ \dot{Z} - \dot{Z}^* \right] = \epsilon \left[ d \ln(1 - t) - d \ln(1 - t^*) \right]
\]

\[
\dot{\Upsilon}_x = \frac{1 + \beta}{\beta \kappa} \left[ - \frac{\beta}{1 - \beta} \dot{r} + (\dot{Z}^* - \dot{Z}) \right], \quad \kappa = \frac{\beta \Upsilon_x - 1}{1 + \beta} \frac{1}{\Upsilon_x} = \frac{\beta}{1 + \beta} + \frac{\tau^{-\beta}(Z^*/Z)}{1 + \tau^{-\beta}(Z^*/Z)} \in [0, 1/2),
\]

where hats denote log-differentials. This is a system of four linear equations which allows to solve for \( \dot{Q}, \dot{Z}, \dot{\Upsilon}_x \) and their foreign counterparts as functions of \( \dot{r}, d \ln(1 - t) \) and \( d \ln(1 - t^*) \).

We now provide this solution for the special case \( \gamma \beta = 1 \), which greatly simplifies the resulting expressions:

\[
\dot{Q} = \epsilon d \ln(1 - t) + \dot{\Upsilon}_x,
\]

\[
\frac{1 + \beta}{\beta} \left[ \dot{Z} - \dot{Z}^* \right] = \epsilon \left[ d \ln(1 - t) - d \ln(1 - t^*) \right],
\]

\[
\dot{\Upsilon}_x = - \frac{1 + \beta}{1 - \beta} \kappa \dot{r} - \kappa \epsilon \left[ d \ln(1 - t) - d \ln(1 - t^*) \right],
\]

which leads to

\[
\dot{Q} = - \frac{1 + \beta}{1 - \beta} \kappa \dot{r} + \epsilon (1 - \kappa) d \ln(1 - t) + \epsilon \kappa d \ln(1 - t^*).
\]

Note that real consumption increases as trade cost \( \tau \) falls. Real consumption falls both in the levels of domestic and foreign tax rates. The elasticity with respect to domestic (foreign) tax rate is smaller (larger) the greater is the value of \( \kappa \); \( \kappa \) is high when trade cost \( \tau \) is low or when the domestic country is relatively small (in terms of effective demand \( Z^*/Z \)). In the closed economy \( \kappa = 0 \). The results in the general case with \( \gamma \beta \neq 1 \) are qualitatively the same.

Finally, we evaluate the comparative statics for agents’ utilities and aggregate welfare. From agent \( n \)’s problem and using Envelop Theorem for the choice of \( y_n \), we have the following utility differential:

\[
d \mathcal{U}_n = (R - r_n) d t + \left[ \frac{t}{1 - t} + (1 - \beta) \frac{r_n}{R} \left( 1 + \frac{\dot{\Upsilon}_x}{Q} \right) \right] (1 - t) R \dot{Q}.
\]

Holding \( \tau \) constant and using the above comparative statics, we obtain:

\[
\frac{d \mathcal{U}_n}{dt} = (R - r_n) - \epsilon (1 - \kappa) \left[ \frac{t}{1 - t} + (1 - \beta) \frac{r_n}{R} \frac{1 - 2 \kappa}{1 - \kappa} \right] R
\]

and

\[
\frac{d \mathcal{U}_n}{dr_n} = - \epsilon \kappa \left[ \frac{t}{1 - t} + 2 (1 - \beta) \frac{r_n}{R} \right] R.
\]

Therefore,

\[
\frac{d \mathcal{U}_n}{dt} + \frac{d \mathcal{U}_n}{dr_n} = (R - r_n) - \epsilon \left[ \frac{t}{1 - t} + (1 - \beta) \frac{r_n}{R} \right] R,
\]

equivalent to \( d \mathcal{U}_n^a / dt \) in the closed economy.
The differential of the welfare function is given by
\[ dW(t, t^*) = \int_0^\infty G'(U_n) dU_n dH(n). \]

Therefore,
\[ \frac{\partial W(t, t^*)}{\partial t} = \lambda R \left[ \alpha - \varepsilon (1 - \kappa) \left( \frac{t}{1 - t} + \frac{1 - 2\kappa}{1 - \kappa} (1 - \beta)(1 - \alpha) \right) \right], \]
\[ \frac{\partial W(t, t^*)}{\partial t^*} = -\lambda R \left[ \varepsilon \kappa \left( \frac{t}{1 - t} + 2(1 - \beta)(1 - \alpha) \right) \right], \]
where \( \alpha \) and \( \lambda \) are as defined in Section 2. Note that
\[ \frac{\partial W(t, t^*)}{\partial t} + \frac{\partial W(t, t^*)}{\partial t^*} = \lambda R \left[ \alpha - \varepsilon \left( \frac{t}{1 - t} + (1 - \beta)(1 - \alpha) \right) \right], \]
equivalent to \( \partial W^a(t)/\partial t \) in the closed economy. This result holds even for \( t \neq t^* \).

**Non-cooperative Determination of Taxes**  Non-cooperative solution is a Nash equilibrium in which both countries unilaterally choose their tax rate taking the tax rate in the trading partner as given. Formally, \((t_0, \tau_0^*)\) is a Nash equilibrium if \(t_0 \in \arg \max_t W(t, \tau_0^*)\) and \(\tau_0^* \in \arg \max_{t^*} W^*(t^*, t_0)\). The first order condition for the choice of \(t_0\) is
\[ \frac{\partial W(t_0, \tau_0^*)}{\partial t} = \lambda R \left[ \alpha - \varepsilon (1 - \kappa) \left( \frac{t_0}{1 - t_0} + \frac{1 - 2\kappa}{1 - \kappa} (1 - \beta)(1 - \alpha) \right) \right] = 0, \]
which results in
\[ \frac{t_0}{1 - t_0} = \frac{1}{\varepsilon (1 - \kappa)} \cdot \alpha - \frac{1 - 2\kappa}{1 - \kappa} (1 - \beta)(1 - \alpha), \quad \kappa \in [0, 1/2). \]
A symmetric condition holds for the foreign country. We also denote \(W_0 = W(t_0, \tau_0^*)\) and \(W_0^* = W^*(\tau_0^*, t_0)\).

Note the difference between this condition and (11') in the closed economy. Trade openness implies \( \kappa > 0 \), and as a result the efficiency margin of taxation decreases. There are two effects. First, note that \( \varepsilon \delta d \ln Q/d \ln (1 - t) = \varepsilon (1 - \kappa) < \varepsilon \). That is, domestic consumption is less sensitive to the domestic taxes in the open economy when the foreign tax rate is held fixed. The reason is that agents can substitute away from more expensive local goods to the foreign goods for which prices do not change. In other words, terms of trade improve in response to an increase in domestic income taxes and partly shield the country from the efficiency losses. Moreover, from the discussion above (Proposition 5) it follows that world efficiency losses are still given by \( \varepsilon \), implying that the remainder of the efficiency losses are born by the trade partner through the deterioration of the terms of trade.44

Second, in the expression for \( t_0/(1 - t_0) \), the weight on the negative monopolistic distortion component \((1 - \beta)\) is reduced. This component is supposed to offset the monopolistic mark-up in the economy. There are less incentives to fully offset the monopolistic mark-up in an open economy

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44An opposite result obtains in a model with free entry (see Helpman and Itskhole, 2008): there effects on entry dominate the terms of trade movements. As a result, more distortions at home improve welfare in the trade partner while they reduce efficiency at home by more than in the closed economy. Since models with free entry are more relevant for profit taxation rather than income taxation, this may have implications for the differential response of profit versus income taxes and capital versus labor taxes in the open economy (cf. Mendoza and Tesar, 2005).
for two related reasons: (1) higher monopolistic prices induce terms of trade improvement beneficial for the country and (2) inefficiency of monopolistic pricing harms domestic consumers only partially now as the effects are spilled over onto the foreign consumers who do not enter domestic welfare.

Overall, both of the two new effects operate in the direction to reduce equilibrium taxes. Both effects are stronger the larger is $\kappa$. This is the case when the economy is smaller (in terms of its demand level $Z$) and more open (in terms of its ratio of trade to GDP which decreases in $\tau$). This is intuitive, since for more open and smaller countries trade provides a better shield from inefficient domestic policies. This may rationalize the finding in Rodrik (1998) and Alesina and Wacziarg (1998) that both smaller and more open economies tend to have larger governments.

We summarize these findings in:

**Proposition 10** In an open economy without fixed costs, non-cooperatively-set tax rate is higher than the optimal tax rate in the closed economy. The difference is larger the smaller is the country and the more open it is to trade. Non-cooperatively-set taxes are inefficient high.

Contrast this proposition with Proposition 5: there is a stark difference between cooperatively and non-cooperatively-set income taxes. Since higher tax rate at home harms the trade partner, and access to foreign trade partially shields the home economy from inefficiency, non-cooperative policy determination always results in higher taxes. This result is similar to findings of Epifani and Gancia (2008) in a different model of taxation in an open economy, who also provide empirical evidence for the importance of the terms of trade externality in affecting the size of the government in an open economy. At the same time, the quantitative calibration of Mendoza and Tesar (2005) suggests that the gains from tax policy coordination are small in the EU, which points towards small calibrated values of $\kappa$’s.

Finally, a direct implication of this discussion is that non-cooperatively set taxes are inefficient, in the sense that they leave the two countries strictly inside the Pareto frontier. Formally, there exist tax policies $\tilde{t}$ and $\tilde{t}^*$ such that $W(\tilde{t}, \tilde{t}^*) > W_0$ and $W^*(\tilde{t}^*, \tilde{t}) > W^*_0$. To see this, set $\tilde{t} = t_0 - \delta$ and $\tilde{t}^* = W_0 - \delta$ for $\delta$ small enough. In the Nash equilibrium, we have $\partial W(t_0, t_0^*)/\partial t = 0$. Moreover, from the comparative statics above, we know that $\partial W(t, t^*)/\partial t^* < 0$. In words, reducing $t$ has a second-order negative effect on $W$, while reducing $t^*$ has a first-order positive effect. Therefore, when $t$ and $t^*$ are both reduced by a small amount, $W$ necessarily increases. A similar argument works for the foreign country.

Cooperatively-set national policies make sure that these welfare gains from coordination are not forgone.

**Cooperative Determination of Taxes** is a Nash bargaining solution in the game of the two countries, where the non-cooperative Nash equilibrium is taken as a status quo. Hence cooperatively-set taxes satisfy

$$\max_{t, t^*} \left\{ (W(t, t^*) - W_0)(W^*(t^*, t) - W^*_0) \right\}.$$ 

Therefore, the optimality condition for the tax rate is

$$\frac{1}{W(t, t^*) - W_0} \frac{\partial W(t, t^*)}{\partial t} + \frac{1}{W^*(t^*, t) - W^*_0} \frac{\partial W^*(t^*, t)}{\partial t} = 0.$$ 

A similar condition holds for $t^*$. When the two countries are symmetric, this condition reduces to $\partial W(t, t^*)/\partial t + \partial W^*(t^*, t)/\partial t = 0$. See the proof of Proposition 5.

\[\text{Formally, the proof requires that } \partial \alpha/\partial t < 0 \text{ globally which we assume is the case (see Lemma 3 and the discussion that follows).}\]
We now consider the case of asymmetric countries. Using the comparative statics results, we can combine the two optimality conditions in this case to obtain:

$$
\eta \varepsilon \left[ \frac{t}{1-t} - \alpha / \varepsilon + (1 - \beta)(1 - \alpha) \right] + \eta^* \varepsilon^* \left[ \frac{t^*}{1-t^*} - \alpha^* / \varepsilon^* + (1 - \beta)(1 - \alpha^*) \right] = 0,
$$

where $\eta \equiv R / (W - W_0)$. This condition has two implications. First, condition (11') still applies, but only on average across the two economies. Generally, the tax rates in the open economy will be different from those in the closed economy, unless the two countries chose the same tax rate in autarky. Second, if one country has a higher tax rate than in autarky, the other country necessarily has a lower tax rate than in autarky.\(^{46}\) Moreover, under the assumption that the country that had a higher tax rate in autarky still has a higher tax rate in the open economy,\(^{47}\) there has to be a convergence of tax rates across countries when they start trading.

Proof of Proposition 5  As discussed above, the optimality condition for cooperatively-set taxes with symmetric countries reads as follows:

$$
\frac{\partial W(t, t^*)}{\partial t} + \frac{\partial W^*(t^*, t)}{\partial t} = 0,
$$

with a symmetric condition holding for $t^*$. We now make use of the comparative statics result to obtain:

$$
\frac{\partial W(t, t^*)}{\partial t} + \frac{\partial W^*(t^*, t)}{\partial t} = \lambda R \left[ \alpha - \varepsilon (1 - \kappa) \left( \frac{t}{1-t} + \frac{1 - 2\kappa}{1 - \kappa} (1 - \beta)(1 - \alpha) \right) \right] - \lambda^* R^* \left[ \varepsilon^* \kappa^* \left( \frac{t^*}{1-t^*} + 2(1 - \beta)(1 - \alpha^*) \right) \right] = 0,
$$

and imposing the symmetry, it is equivalent to

$$
\lambda R \left[ \alpha - \varepsilon \left( \frac{t}{1-t} + (1 - \beta)(1 - \alpha) \right) \right] = 0.
$$

This condition is the same as (11') in the closed economy. It directly implies that the efficiency margin is still the labor supply elasticity, $\varepsilon$. Finally, Proposition 3 implies that the inequality margin, $\alpha$, is also unchanged. \(\blacksquare\)

C Derivations and Proofs for Section 3

Agent’s Problem, Open Economy Equilibrium and Comparative Statics: Solution to agent $n$’s problem (16) leads to the following optimal allocation:

$$
y_n = [\beta (1 - t)]^{\gamma - \beta} \left( 1 + I_n \tau^{-\beta \gamma} Q^{-\beta} n^{-\beta} \right)^{1-\beta \gamma} Q^{\frac{1-\beta}{\gamma-\beta}} n^\frac{\gamma}{\gamma-\beta},
$$

and $I_n = \mathbb{I}\{n > n_x\}$, where $n_x$ is defined by

$$
(1 - t)(1 - \beta / \gamma) Q^{\frac{1-\beta}{\gamma-\beta}} \left( \tau_x^{\frac{1-\beta}{\gamma-\beta}} - 1 \right) n_x^{\frac{\beta \gamma}{\gamma-\beta}} = f_x,
$$

\(^{46}\)This statement requires the assumption that $\partial \alpha / \partial t < 0$ globally. Alternatively, it is a local result for small asymmetries across countries: there a relevant condition is satisfied due to the second order condition in the symmetric case.

\(^{47}\)One can show that this assumption is not automatically granted, however, it holds when $\beta$ is sufficiently close to one and the two countries are not too asymmetric.
where $\Upsilon_x = \left(1 + \tau^{-\frac{\beta}{1-\beta}}\right)$. This results in the following revenue function:

$$r_n = \left[\beta(1-t)\right]^{\frac{\beta}{1-\beta}} \left(1 + I_n \tau^{-\frac{\beta}{1-\beta}}\right) \gamma^{\frac{1-\beta}{1-\beta}} n^{\frac{\beta}{1-\beta}} Q^{\frac{1-\beta}{1-\beta}} n^{\frac{\beta}{1-\beta}}$$

and aggregate revenues given by

$$R = \left[\beta(1-t)\right]^{\frac{\beta}{1-\beta}} Q^{\frac{1-\beta}{1-\beta}} \int_0^\infty \left(1 + I_n \tau^{-\frac{\beta}{1-\beta}}\right) \gamma^{\frac{1-\beta}{1-\beta}} n^{\frac{\beta}{1-\beta}} dH(n).$$

Combining these two expressions yields (18) in the text. Finally, the maximized utility of agent $n$ is given by

$$U_n = tR + (1-t)(1 - \beta/\gamma) r_n - I_n f_x,$$

since agents’ optimality conditions imply $v(y_n/n) = \beta(1-t)/\gamma \cdot r_n$.

Balanced trade implies $Q = R$. Using the expression for $R$, we can solve for equilibrium $Q$ as a function of $n_x$:

$$Q^{\frac{\gamma-1}{\gamma-\beta}} = \left[\beta(1-t)\right]^{\frac{\beta}{1-\beta}} \int_0^\infty \left(1 + I_n \tau^{-\frac{\beta}{1-\beta}}\right) \gamma^{\frac{1-\beta}{1-\beta}} n^{\frac{\beta}{1-\beta}} dH(n). \quad (25)$$

Together with (24), this expression characterizes open economy equilibrium with fixed costs of trade.

These expressions also allow to obtain comparative statics. The log-differentials of (24) and (25) can be written as:

$$\left(1 - \beta\right) \dot{Q} + \beta \dot{n}_x = -d \ln(1-t) - (1 - \beta) \zeta \dot{\Upsilon}_x + (1 - \beta/\gamma) \dot{f}_x,$$

$$\dot{Q} + \varepsilon \nu_x \dot{n}_x = \varepsilon d \ln(1-t) + (1 + \varepsilon) \frac{1 - \beta}{\beta} \zeta \frac{f_x \int_{n_x}^\infty (n/n_x)^{\frac{\beta}{1-\beta}} dH(n)}{(1 - \beta/\gamma)(1-t)R} \dot{\Upsilon}_x, \quad (27)$$

where $\zeta \equiv \Upsilon_x^{\frac{1-\beta}{\gamma-\beta}} / \left[\Upsilon_x^{\frac{1-\beta}{\gamma-\beta}} - 1\right]$ and

$$\nu_x \equiv \frac{\gamma}{\beta(1-t)R} h(n_x)n_x.$$

This fully describes the comparative statics of $Q$ and $n_x$ in response to changes in $t$, $\tau$ and $f_x$.

Note that the equilibrium system is stable when $\varepsilon (1 - \beta) \nu_x < 1$. For every distribution $H(\cdot)$, there exists a $\beta < 1$ such that the stability condition above is satisfied for almost every $n_x$. When the stability condition is not satisfied, the equilibrium is locally unstable. If $\beta$ is too low (a very large CES externality) so that this condition is never satisfied, the model admits only two types of equilibria – when no agent exports or when every agents exports.

Finally, consider the change in agent $n$’s utility:

$$dU_n = (R - r_n) dt + \dot{Q} \left[ tR + (1-t)(1 - \beta)r_n \right] + (1 - \beta) \dot{\Upsilon}_x I_n r_n - \dot{f}_x I_n f_x. \quad (28)$$

Note that changes in $y_n$ and $n_x$ do not have an effect on $U_n$ by the Envelop Theorem. The reduction in trade costs (fixed $f_x$ and variable through $\Upsilon_x$) directly affects only the agents who already participate in exporting. All indirect effects work exclusively through $Q$.

**Proof of Proposition 6** (i) A reduction in $\tau$ leads to an increase in $\Upsilon_x$. From (26)-(27) it follows that $Q$ increases and $n_x$ decreases as $f_x$ falls or as $\Upsilon_x$ increases. Stability condition $\varepsilon (1 - \beta) \nu_x < 1$ is required.
(ii) Greater trade integration implies either a fall in $\tau$ (an increase in $\Upsilon_x$) or a fall in $f_x$. Tax rate $t$ is assumed to be held fixed. Therefore, from (28), there are three effects on utility – through $Q$, through $\Upsilon_x$ and through $f_x$. By part (i) of this proposition, greater trade integration also results in higher $Q$. Therefore, all effects on utility are positive, hence all agents gain from trade. Note, however, that only the effect of $Q$ is common across all agents, while the direct effects of $f_x$ and $\Upsilon_x$ are experienced only by the pre-trade-liberalization exporters, who gain disproportionately more from trade liberalization. ■

Proof of Proposition 7 From (18) it is apparent that the distribution of revenues is equivalent to the one in Lemma 4 when $n_x = n_{\min}$ or $n_x = n_{\max}$. This implies that inequality in these two cases is the same as in autarky.

Now compare $r_n/R$ defined by (18) with $r_n^a/R^a$ defined in Lemma (4) in the case when $n_{\min} < n_x < n_{\max}$. Note that one is a mean-preserving transformation of the other. Specifically, there exist $\alpha_1 > 1 > \alpha_0 > 0$ such that

$$r_n(R) = \begin{cases} 
\alpha_0 r_n^a/R^a, & n < n_x, \\
\alpha_1 r_n^a/R^a, & n \geq n_x,
\end{cases}$$

and $\int_0^\infty r_n/RdH(n) = \int_0^\infty r_n^a/R^a dH(n) = 1$. This immediately implies that the cumulative distribution function of $r_n/R$ is to the left (right) from that for $r_n^a/R$ for $n < n_x$ ($n > n_x$). Therefore, the distribution of $r^a/R^a$ strictly second order stochastically dominates the distribution of $r_n/R$, which implies

$$\text{var} \left( \frac{r}{R} \right) > \text{var} \left( \frac{r^a}{R^a} \right).$$

By consequence, this implies that income (revenue) inequality increases for $n_x \lesssim n_{\max}$ (high trade costs) and decreases for $n_x \gtrsim n_{\min}$ (low trade costs). In addition, one can show by direct computation that $\text{var}(r/R)$ has an inverted U-shape as $f_x$ fall from $\infty$ towards $0$.

Similar arguments can be developed for relative utilities when $n_x < n_{\max}$. The difference is that when $n = n_{\min}$, the dispersion of relative utilities is still greater than in autarky. To see this, consider

$$\frac{U_n}{\int_0^\infty U_n dH(n)} = \begin{cases} 
t + (1-t)(1-\beta/\gamma)r_n/R, & \text{when no agent exports}, \\
t + (1-t)(1-\beta/\gamma)(r_n - f_x)/R, & \text{when all agents export},
\end{cases}$$

Since in both cases $r_n/R = r_n^a/R^a$, the former distribution second order stochastically dominates the latter (strictly dominates when $f_x > 0$). ■

Inequality Margin $\alpha$: Following the same steps as in the proof of Lemma 3, we arrive at the following second order approximation, which becomes precise as the dispersion of $n$ goes towards zero:

$$\alpha \approx \rho \cdot \frac{1 - (1-t)(1-\beta/\gamma)R}{1 + \frac{1}{(1-t)(1-\beta/\gamma)R} - \frac{\pi_x f_x}{(1-t)(1-\beta/\gamma)R}},$$

where $\pi_x \equiv 1 - H(n_x)$ is the fraction of exporting agents. One can show that the variance term is strictly greater in open economy than the autarky variance of $r^a/R^a$. However, the preceding term is strictly less than 1, which leads to ambiguity in the comparative statics of $\alpha$ with respect to greater trade openness. To be completed.
**Proof of Lemma 5** From (26)-(27), it follows immediately that

$$
\hat{\varepsilon} \equiv \frac{d \ln Q}{d \ln (1 - t)} = \varepsilon \cdot \frac{1 + \nu_x}{1 - \varepsilon (1 - \beta) \nu_x},
$$

where $\nu_x$ is as defined above. The stability condition $\varepsilon (1 - \beta) \nu_x < 1$ needs to be satisfied. $\nu_x > 0$ whenever $h(n_x) > 0$, which leads to $\hat{\varepsilon} > \varepsilon$. ■

**Tax-deductible Fixed Costs of Trade** To be completed.

**Equilibrium Characterization with Additional Tax Instruments** The optimality conditions for the agent’s problem (19) are

$$
n < n_x \quad \beta (1 - t_d) Q^{1-\beta} y_\alpha^n = \gamma v(y_n / n),
$$

$$
n > n_x \quad \beta (1 - t_x) Q^{1-\beta} y_\alpha^n = \gamma v(y_n / n),
$$

where $n_x$ is defined by

$$(1 - \beta / \gamma) \beta^{\gamma / \beta} Q^{1-\beta} \left[ \gamma \frac{1-\beta}{\gamma} (1 - t_d) \frac{\gamma}{\gamma} - (1 - t_d) \frac{\gamma}{\gamma} \right] n_x^{\beta / \gamma} = f_x - s. \quad (29)$$

The equilibrium output is given by

$$Q^{\beta / \gamma - 1} = \beta^{\beta / \gamma} \left[ (1 - t_d) \frac{\beta}{\gamma} \int_{n_x} \frac{\beta}{\gamma} dH(n) + (1 - t_x) \frac{\beta}{\gamma} \int_{n_x} \frac{\beta}{\gamma} \right].$$

The last two equations allow to solve for equilibrium and obtain comparative statics.

**Optimal Entry** The utilitarian welfare is given by

$$W^u \equiv W^u(t_d, t_x, s) = \int_0^\infty U_n dH(n),$$

where $U_n$ is equilibrium utility of agent $n$ facing the tax system $(t_d, t_x, s)$. Since $U_n = c_n - v(y_n / n)$, we have

$$W^u = \int_0^\infty \left[ c_n - v(y_n / n) \right] dH(n) = Q - \pi_x f_x - \int_0^\infty v(y_n / n) dH(n),$$

where we have used the fact total consumption is equal to total output of the final good minus the fraction of output that is spent on fixed costs. Here again $\pi_x = 1 - H(n_x)$ is the fraction of agents that export, and hence, have to bear the fixed cost. From the definition of $Q$, we have

$$Q = \left[ \int_0^\infty \left( 1 + I_n \tau^{-\beta / \gamma} \right)^{1-\beta} y_\alpha^n \right]^{1/\beta}.$$ 

Note that entry subsidy $s$ does not enter directly the expression for utilitarian welfare, since the distributional effects induced by $s$ do not impact $W^u$. The only effect of $s$ on $W^u$ is indirect, through its effect on $n_x$. Thus, we can solve for optimal entry $n_x$ under the utilitarian welfare, and then recover what it implies for the level of the optimal utilitarian subsidy $s^\circ$.

Consider the optimal utilitarian entry given exogenously-set tax rates $t_d$ and $t_x$:

$$\frac{d W^u}{dn_x} = - h(n_x) \cdot \left[ \frac{1}{\beta} Q^{1-\beta} \left( \gamma x_{-x}^{1-\beta} y^{\beta}_{-x} - y^{\beta}_{x+} \right) - \left( v \left( \frac{y_{x+}}{n_x} \right) - v \left( \frac{y_{x-}}{n_x} \right) \right) - f_x \right],$$

$$- h(n_x) \cdot \left[ Q^{1-\beta} \left[ \frac{1}{\beta} - \frac{\beta (1 - t_x)}{\gamma} \right] y^{\beta}_{x+} - \left( \frac{1}{\beta} - \frac{\beta (1 - t_x)}{\gamma} \right) y^{\beta}_{x-} - f_x \right] = 0,
where we have used agents’ optimality conditions which imply $v(y_n/n) = \beta(1-t_d)/\gamma r_n$ for $n < n_x$ and $v(y_n/n) = \beta(1-t_x)/\gamma r_n$ for $n > n_x$. We now use the optimal $y_n$’s to rewrite this optimality condition as

$$\beta^{\frac{1}{\gamma - \beta}} Q^{\frac{1}{\gamma - \beta}} \left[ \left( \frac{1}{\beta} - \frac{\beta(1-t_x)}{\gamma} \right) \gamma^{\frac{1}{\gamma - \beta}} (1-t_x)^{\frac{\beta}{\gamma - \beta}} - \left( \frac{1}{\beta} - \frac{\beta(1-t_d)}{\gamma} \right) (1-t_d)^{\frac{\beta}{\gamma - \beta}} \right] \frac{n^{\gamma}}{x^{\gamma - \beta}} = f_x. \quad (30)$$

Contrast this optimality condition with the equilibrium condition for entry (29). This allows us to find the level of subsidy $s^*$ which would ensure the optimal utilitarian entry for any given the tax rates $t_d$ and $t_x$.

**Proof of Proposition 8** Compare (29) with (30) in the special case when $t_d = t_x = -(1-\beta)/\beta$:

$$\left( \frac{1}{\beta} - \frac{1}{\gamma} \right) \beta^{\frac{1}{\gamma - \beta}} Q^{\frac{1}{\gamma - \beta}} \left[ \gamma^{\frac{1}{\gamma - \beta}} (1-t_x)^{\frac{\beta}{\gamma - \beta}} - 1 \right] \frac{\gamma}{x^{\gamma - \beta}} = f_x - s,$$

$$\left( \frac{1}{\beta} - \frac{1}{\gamma} \right) \beta^{\frac{1}{\gamma - \beta}} Q^{\frac{1}{\gamma - \beta}} \left[ \gamma^{\frac{1}{\gamma - \beta}} (1-t_x)^{\frac{\beta}{\gamma - \beta}} - 1 \right] \frac{\gamma}{x^{\gamma - \beta}} = f_x.$$

These two conditions imply the same amount of entry when $s^* = 0$, i.e. with $t_d = t_x = -(1-\beta)/\beta$ the optimal utilitarian subsidy is zero.

Now consider the optimal allocation $y_n$ for an agent $n \leq n_x$ from the point of view of utilitarian welfare:

$$\frac{dW^u}{dy_n} = h(n_x)y_n^{-1} \cdot \left[ (1 + I_n)^{\frac{1}{\gamma - \beta}} Q^{\frac{1}{\gamma - \beta}} y_n^{\frac{\beta}{\gamma - \beta}} - \gamma v \left( \frac{y_n}{n} \right) \right] = 0.$$

This condition is equivalent to agents optimality if and only if $\beta(1-t_d) = \beta(1-t_x) = 1$. Therefore, the only combination of taxes and entry subsidy that implement the utilitarian optimum is $t_d = t_x = -(1-\beta)/\beta$ and $s = 0$. ■

**Proof of Proposition 9** Consider now the case when $t_d = t_x = t \neq -(1-\beta)/\beta$. Then the comparison between (29) and (30) becomes:

$$\frac{(1-\beta)}{\beta(1-t)} \frac{(1-t)}{(1-\beta)} \beta^{\frac{1}{\gamma - \beta}} Q^{\frac{1}{\gamma - \beta}} \left[ \gamma^{\frac{1}{\gamma - \beta}} (1-t_x)^{\frac{\beta}{\gamma - \beta}} - 1 \right] \frac{\gamma}{x^{\gamma - \beta}} = f_x - s,$$

$$\frac{(1/\beta - (1-t)/\gamma)}{(1/\beta - (1-t)/\gamma)} = \frac{f_x - s}{f_x} \Rightarrow \frac{s^*}{f_x} = \frac{1 - \beta(1-t)}{1 - \beta^2(1-t)/\gamma},$$

as stated in the proposition. This expression implies $\partial s^*/\partial t > 0$ since $\gamma > \beta$.

More generally, when $t_d \neq t_x$, the condition for $s^*$ is more tedious:

$$\frac{f_x - s^*}{f_x} = \frac{(1-\beta)}{\beta(1-t)} \left[ \frac{\gamma}{x^{\gamma - \beta}} (1-t_x)^{\frac{\beta}{\gamma - \beta}} - (1-t_d)^{\frac{\beta}{\gamma - \beta}} \right] \left[ \left( \frac{1}{\beta} - \frac{\beta(1-t_d)}{\gamma} \right) \gamma^{\frac{1}{\gamma - \beta}} (1-t_x)^{\frac{\beta}{\gamma - \beta}} - \left( \frac{1}{\beta} - \frac{\beta(1-t_d)}{\gamma} \right) (1-t_d)^{\frac{\beta}{\gamma - \beta}} \right]$$

One can show that the right-hand side increases in $t_d$ and decreases in $t_x$. Therefore, $\partial s^*/\partial t_d < 0$ and $\partial s^*/\partial t_x > 0$.

Finally, marginally reducing $s$ has only a second order effect on entry, while it has a first order effect on improving the income distribution since it increases the transfer $\Delta$ and reduces the effective transfer to agents with $n > n_x$. Therefore, with positive inequality aversion, optimal subsidy $s < s^*$. ■
References


